Theory of superconductor to metal transitions in highly conducting systems

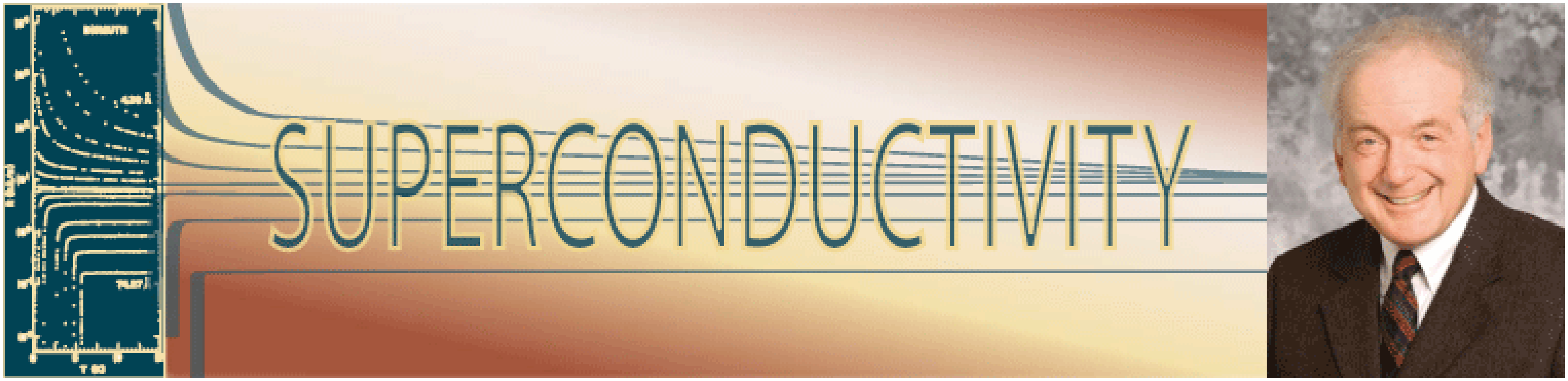
B. Spivak, P. Oreto, and SAK

PRB **77**, 214523 (2008).

Previous work with S. Chakravarty,
A. Luther, G. Ingold, B.I.Halperin, and A. Luther

Closely related work (here) by Feigelman and Larkin

U.M. - 2009



Thank you for (many) years of fascinating physics,
inspiring enthusiasm,
and being a model for how to ENJOY physics.

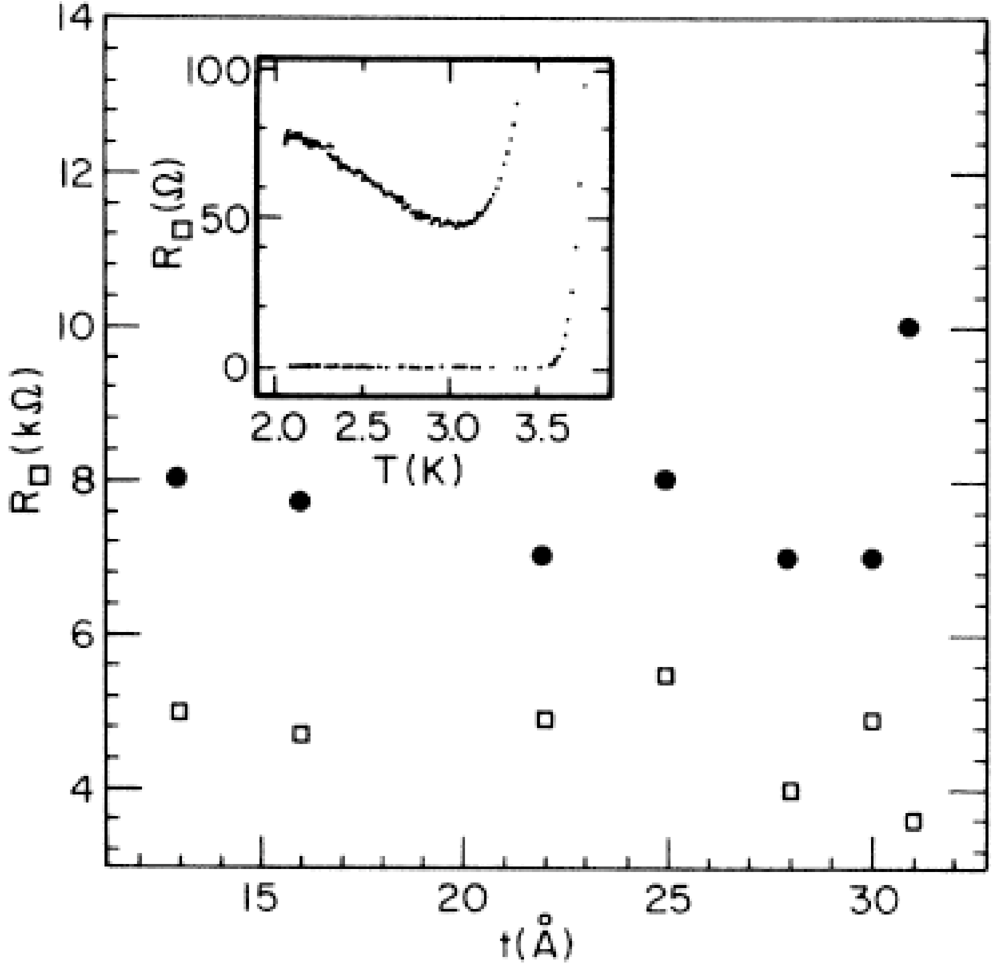


FIG. 1. Values of $R_\square(T=20\ \mathrm{K})$ vs thickness t.

B. G. Orr, H. M. Jaeger, A. M. Goldman, and C. G. Kuper, PRL **56**, 378 (1986).

Quantization of the RSJ Model

$$S = \int d\tau \left\{ \frac{C}{2} \left(\frac{h}{2e} \right)^2 \dot{\theta}^2 + V[1 - \cos(\theta)] \right\} + \left(\frac{R_Q}{R} \right) \int d\tau d\tau' \frac{|\theta(\tau) - \theta(\tau')|^2}{|\tau - \tau'|^2}$$

Transition from a gapped (superconducting) phase for $R < R_Q$

to

Non-superconducting (free) phase for $R > R_Q$

A dissipation driven transition with a critical *normal state resistance*.

S.Chakravarty *et al*, PRL 1986

S Chakravarty PRL 1982 A. Schimd PRL 1983.

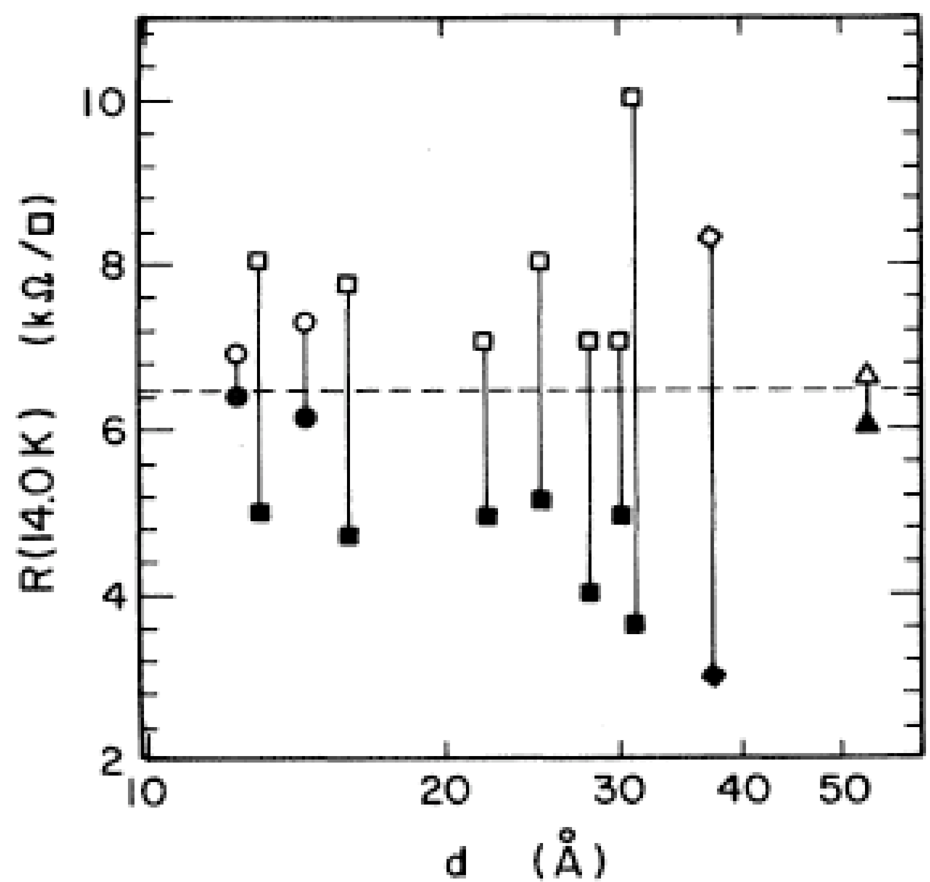


FIG. 6. The pairs of connected points represent the normal-state resistance of successive depositions in sequences of Ga (○), Sn (□), Pb (◇), and Al (△) films. The nominal thickness

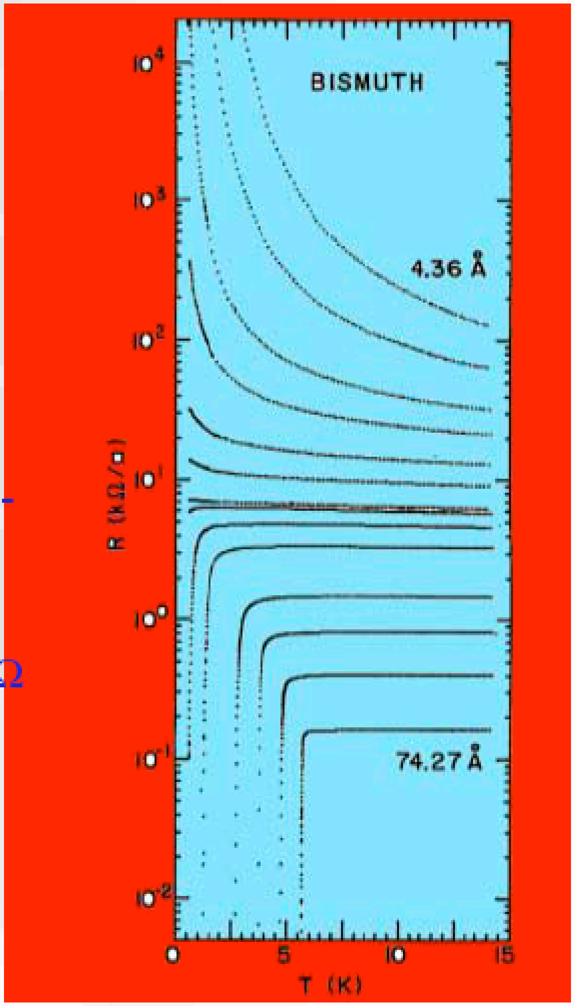
H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman
Phys. Rev. B 40, 182 (1989).

Amorphous Bi grown on amorphous Ge at liquid helium temperatures

Cyclic evaporation leads to evolution of superconductivity with thickness.

Apparent separation between superconducting and insulating behavior.

Critical resistance close to $h/4e^2 = 6450\ \Omega$



The normal state resistance is also close to $h/4e^2$

The metallic state with $R \sim R_Q$ is still not understood - no “small parameter.”

$$\boxed{\text{Progress on the superconductor to metal transition in highly conducting systems - } R << R_Q}$$

Motivated by the anomalous metallic phase found by Kapitulnik, Mason, …

Under what circumstances is
there a superconductor to a metal transition?
Why does everyone talk about the metal-insulator transition, instead?

1) Anderson's theorem for ordinary s-wave SC:

To destroy SC on mean-field level requires $k_F\, l \sim 1$.

There are many situations in which Anderson's theorem does not apply, so SC destroyed when $k_F l >> 1$.

Under what circumstances is
there a superconductor to a metal transition?
Why does everyone talk about the metal-insulator transition, instead?

1b) In 2D, all single particle states are localized.

Some cases we will consider will be 3D.

Even in 2D, if $k_F\, l \gg 1$,

$$k_F\, \xi_{loc} \sim \exp[\, + (\pi/2)\, k_F l\,] \gg k_F\, l$$

Also, deep question about whether fluctuational effects change even true asymptotics.

Under what circumstances is
there a superconductor to a metal transition?
Why does everyone talk about the metal-insulator transition, instead?

2) In "dirty boson" models, the usual physics gives rise to a superconductor to insulator transition.
A metallic state is an exotic possibility which has widely been believed to be impossible to achieve*.

* However, see PP and D and D and MPAF and …

Under what circumstances is
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2) In “dirty boson” models, the usual physics gives rise to a superconductor to insulator transition.
A metallic state is an exotic possibility which has widely been believed to be impossible to achieve.

We will consider only “weak coupling” situations in which $\Delta_o << E_F$ and $k_F\xi_0 >> 1$.

There are generally gapless quasi-particles near criticality, so dirty boson asymptotics unclear.

Under what circumstances is
there a superconductor to a metal transition?
Why does everyone talk about the metal-insulator transition, instead?

3) Quantum fluctuations are strongly suppressed if
$G = \sigma/\sigma_Q = R_Q/R > 1$.

S. Chakravarty *et al* showed that for a resistively shunted Josephson junction array at T=0, the system remains superconducting even as J tends to zero for G > 1.
(PRL - 1986; PRB 1988)

Amplification of inhomogeneity near a quantum critical point (although not as extreme as quantum Griffith phases).

Existence of a broad regime of quantum fluctuations but not universal quantum critical fluctuations near criticality.

Extreme broadening of the mean-field transition
in a disordered superconductor:

$T_c \sim \exp[- 1/\lambda]$

Some distribution of local λ, even if the probability is very small, leads to extreme broadening of T_c.

Leads to an enormous violation of Anderson's theorem

$T_c(\text{disorder}) >> T_c^0$

However, usually we accept that very rare events are
unimportant due to fluctuation effects.

At T=0 in highly conducting metals, even isolated “puddles”
have an enormous superconducting susceptibility
and interactions between puddles are long-ranged.

The quantum fluctuations of an isolated superconducting grain embedded in a good metal.

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\Delta(\tau) - \Delta(\tau')|^2}{[\tau - \tau']^2}$$

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\exp[i\phi(\tau)] - \exp[i\phi(\tau')]|^2}{[\tau - \tau']^2}$$

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\phi(\tau) - \phi(\tau')|^2}{[\tau - \tau']^2}$$

$$\Delta = |\Delta| \exp[i\phi]$$

The Leggett-Caldera model

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\ \phi(\tau) - \phi(\tau')\ |^2}{[\tau - \tau']^2}$$

$$\Delta = |\Delta|\ \exp[i\phi]$$

Phenomenological derivation - quantization of the RSJ model

"Microscopic" derivation from superconducting transmission line coupled to Josephson junction
Chakravarty and Schmidt -

Susceptibility diverges as T tends to 0 for $G^{eff} > 1$.

Time-dependent GL equation:

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\Delta(\tau) - \Delta(\tau')|^2}{[\tau - \tau']^2}$$

Derivable directly from perturbation theory to lowest order in Δ.

Valid so long as Δ "small" i.e. Andreev length, $v_F/\Delta = \xi >> R$.

It is the physics of the Cooper instability so G^{eff} has nothing to do with G

$G^{eff} \sim \nu R^D / \Delta_0$

Time-dependent GL equation:

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\Delta(\tau) - \Delta(\tau')|^2}{[\tau - \tau']^2}$$

Dynamics of single grain equivalent to classical 1D inverse square XY model.

$$\boxed{\chi \sim \exp[\, 2\pi^2 G^{eff} \,]}$$

Time-dependent GL equation:

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\Delta(\tau) - \Delta(\tau')|^2}{[\tau - \tau']^2}$$

Does this lead to a quantum Griffith's phase?

$$\chi \sim \exp[\ 2\pi^2\ G^{eff}\ ]$$

$P(R) \sim \exp[\ -\alpha R^D\ ]$ so if $G^{eff} \sim g\ R^D$

contributions to $< \chi >$ from rare large grains diverges if $\alpha < 2\pi^2\ g$.

Time-dependent GL equation:

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\Delta(\tau) - \Delta(\tau')|^2}{[\tau - \tau']^2}$$

However, for large R > ξ, the perturbative conditions necessary to derive the effective action are violated. Instead …

Effective action for a "large" superconducting grain embedded in a metal

$$L = \int d\tau V(|\Delta|) + G^{eff} \int d\tau d\tau' \frac{|\exp[i\phi(\tau)] - \exp[i\phi(\tau')]|^2}{[\tau - \tau']^2}$$

$$\chi \sim \exp[\, 2\pi^2 G^{eff} \,]$$

In 3D $\quad G^{eff} \sim \sigma R/(e^2/h)$

In 2D $\quad G^{eff} \sim [\ \sigma/(e^2/h)\ ]^{1/2}$ (Feigelman and Larkin)

No quantum Griffith phase from this mechanism.
(However, see recent work of Sachdev *et al* and Vojta *et al*.)

In clean metal, the susceptibility of an isolated superconducting grain embedded in a metal can be exponentially large, although not infinite:

$$\chi \sim \exp[\ 2\pi^2\, G^{eff}\ ] \quad \text{at } T=0$$

Now, what about the Josephson coupling between two far separate grains, J(r) ?

$$J(\vec{r}_i,\vec{r}_j) \sim \frac{1}{|\vec{r}_i,\vec{r}_j|^D} F(\vec{r}_i,\vec{r}_j)$$

at T=0

$$|F(\vec{r}_i,\vec{r}_j)| \sim 1$$

A condition for global phase coherence is that between a typical pair of neighboring grains $\chi J > 1$.

This means that at T=0, and for G >> 1, the SC -M transition occurs at a critical concentration

$$N_c \sim \exp[-2\pi^2 G^{eff}]$$

In a good metal, near the quantum critical point the small inhomogeneities will be amplified: there is an emergent, but non-critical long length scale.

When “most” of the sample is “normal,” there still exist rare regions which support mean-field superconducting order.

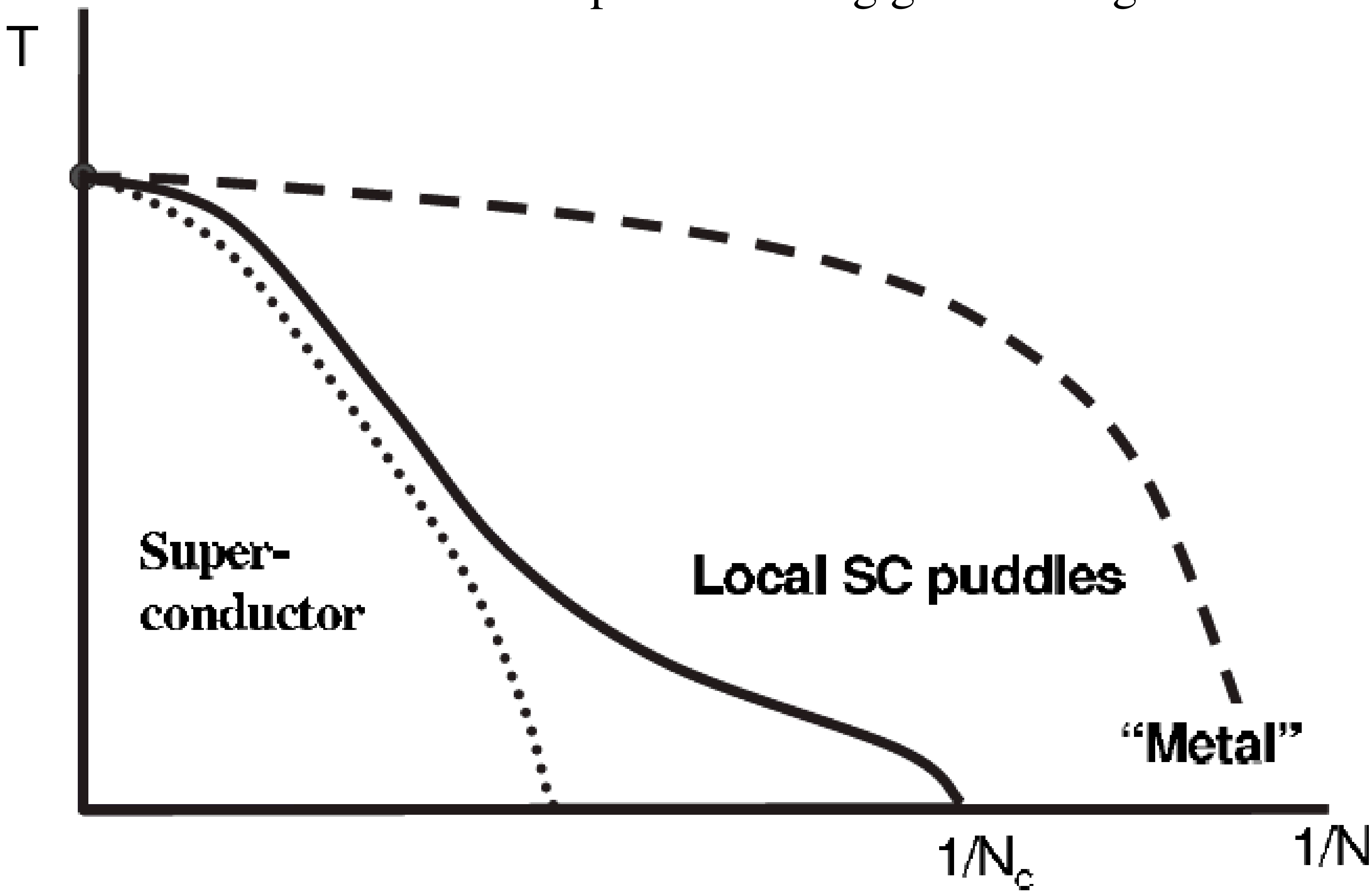
Because each such region produces a large susceptibility, and because the Josephson coupling is long-ranged, these rare regions can undergo a (quantum percolation) transition to global phase coherence, even when they are very dilute.

In a good metal, near the quantum critical point
the small inhomogeneities will be amplified:
there is an emergent, but non-critical long
length scale.

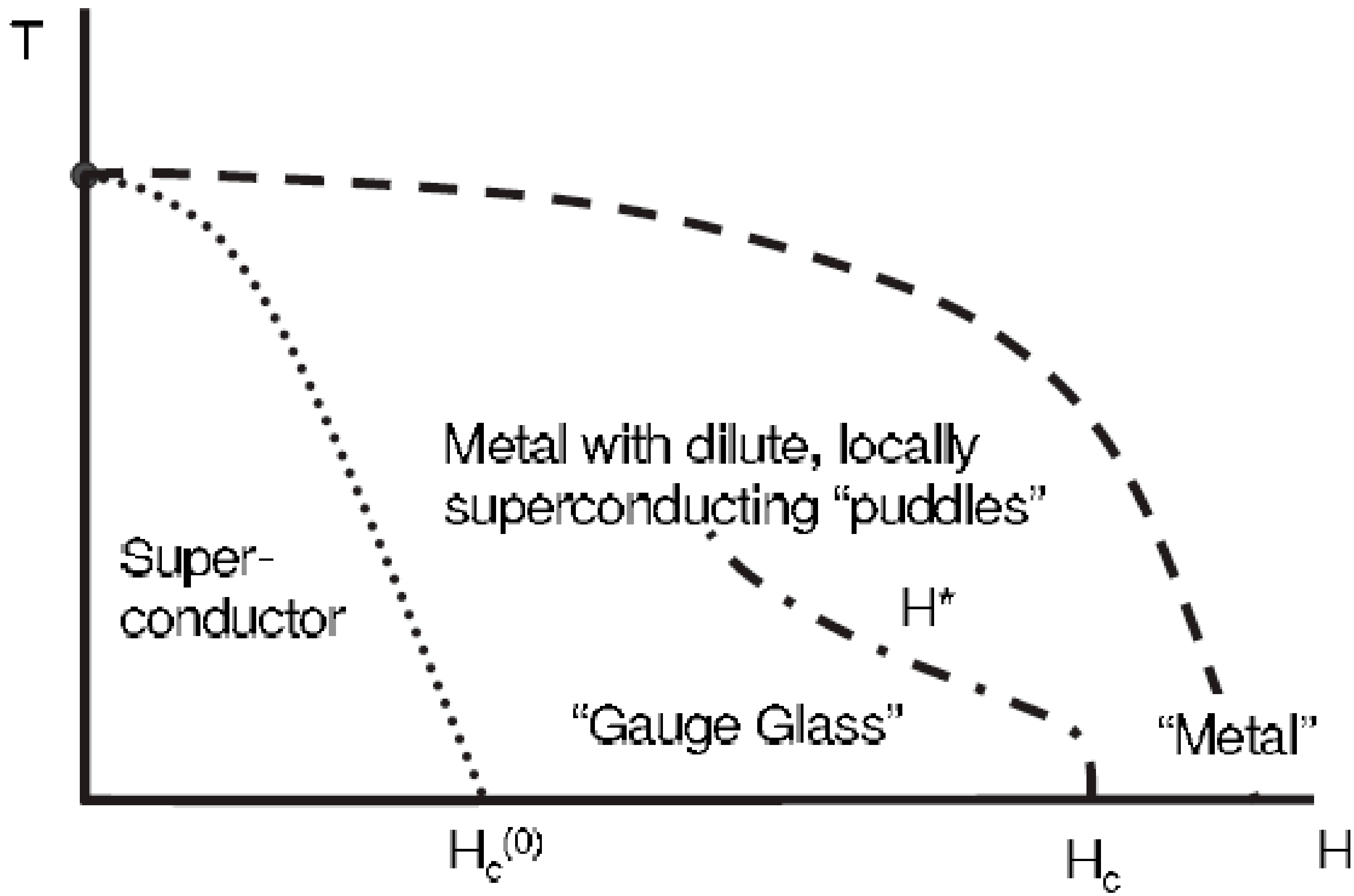
Probably cannot be approached by replica or other
effective field theory methods.

This is probably a generic feature of quantum phase
transitions in “good” metals.

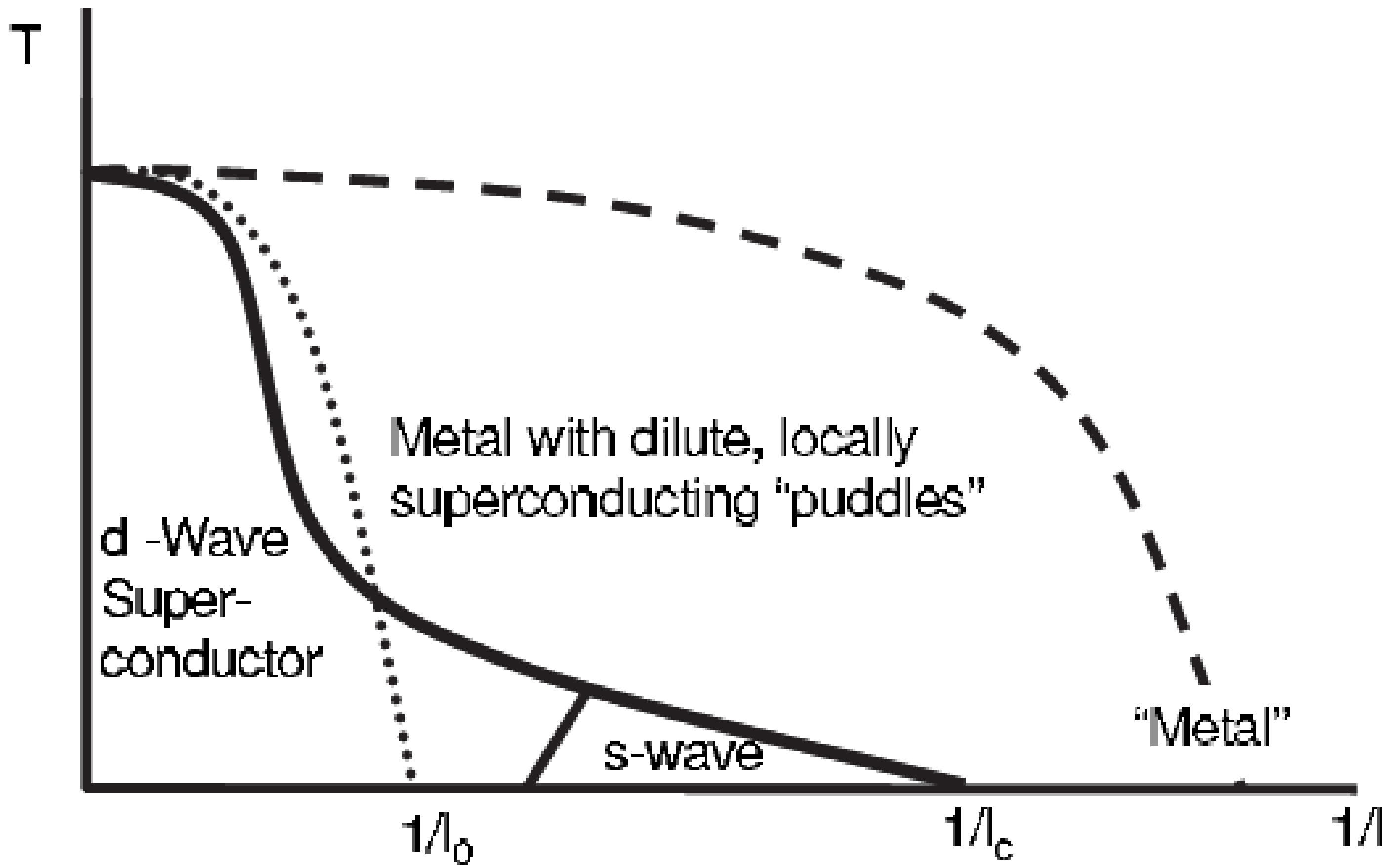
Concentration of superconducting grains in a good metal



S-wave superconductor in a transverse magnetic field.



BCS d-wave superconductor in the presence of disorder.

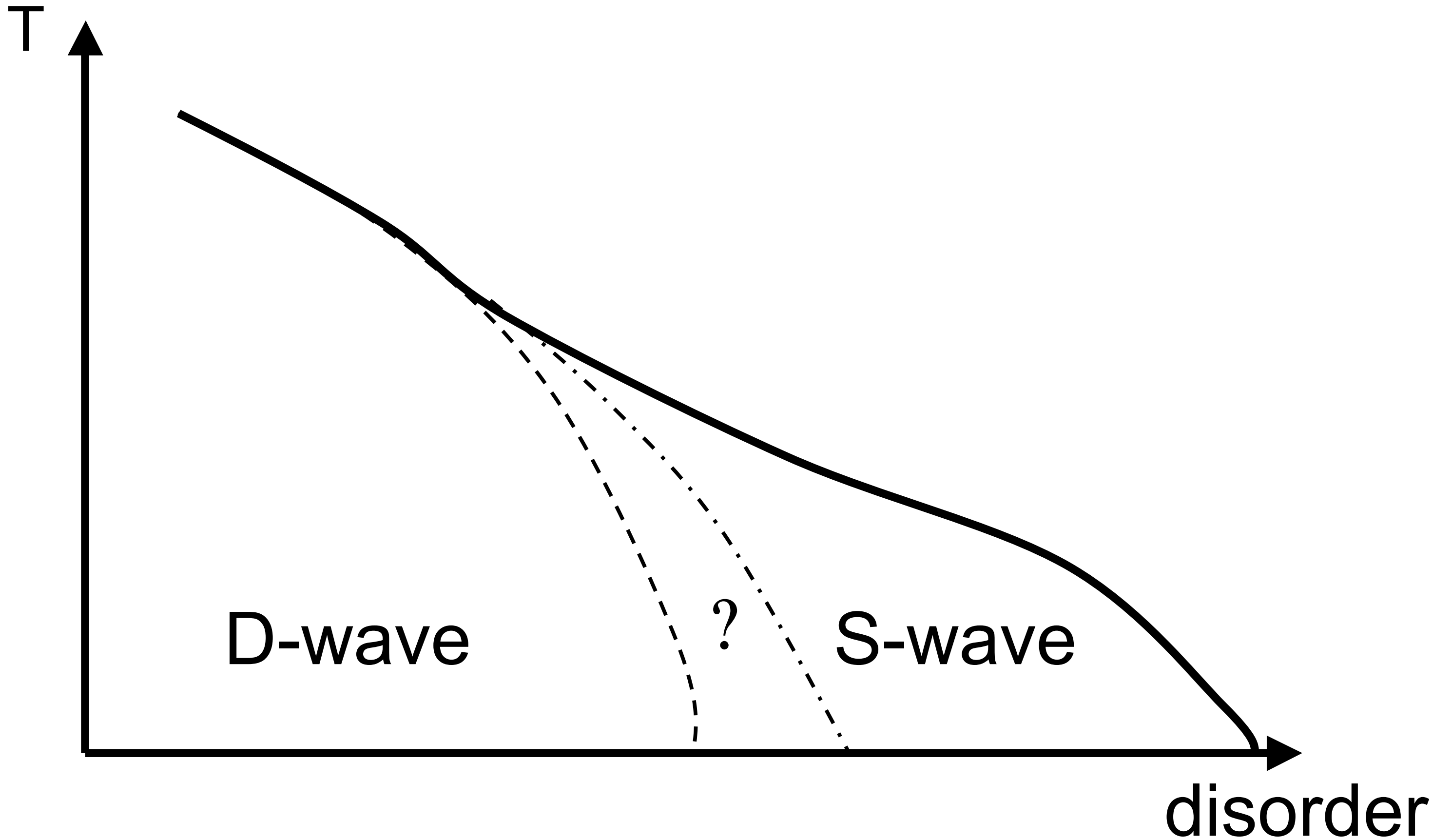


As an example, let us consider the quantum transition
from a weak coupling (BCS) d-wave superconductor
to a "good" metal.

We all know that
d-wave superconductivity is destroyed by disorder when
$l < 1.78\ \xi_o$.

However, even when superconductivity is quenched in
most of the sample there will always exist
rare regions in which $l > 1.78\ \xi_o$

The phase diagram of a BCS d-wave superconductor



1) Near the T=0 transition, the system effectively looks granular.
2) There is inevitably an intermediate globally s-wave phase.
3) There is probably an intermediate glassy phase.

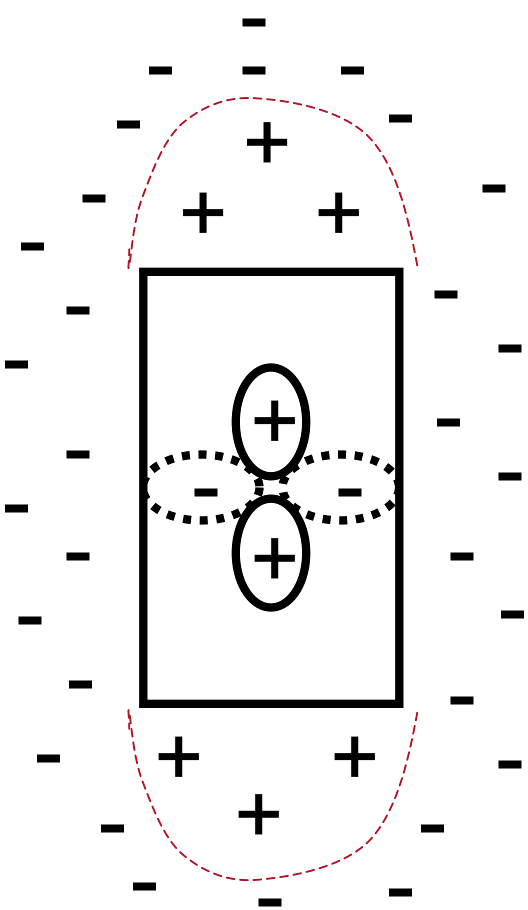
There is a "trivial" example, which can help motivate this phase diagram (Vojta *et al*, PRB 2005)

Suppose there is an attraction $1 \gg \lambda_d > \lambda_s > 0$ in both the d-wave and s-wave channels.

For zero disorder - d-wave dominates.
But d-wave killed by disorder, while s-wave is unaffected, so when $T_{cd}(l) < T_{cs}(l=\infty)$ the s-wave can dominate.

We will consider case in which there is attraction only in d-wave channel, and even repulsion is s-wave.

A warm-up model: a pure d-wave superconducting puddle is embedded into a disordered normal metal.
Outside the puddle, an s-wave component of the order parameter is generated. Only this component survives on distances larger than elastic mean free path l



Superconducting puddle with d-wave order parameter $\Delta(\mathbf{k})$

+ and – indicate sign of the s-component of the anomalous Green function of the order parameter $F(\mathbf{r},\mathbf{r})$

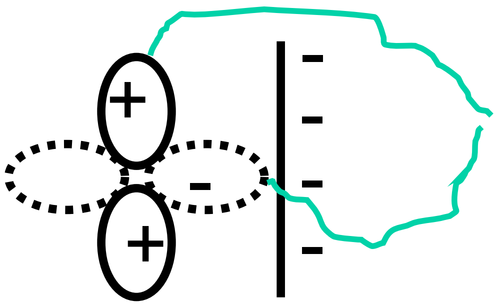
Usadel equation for s-component of the anomalous Green function F(r,r) in diffusive metal

$$D\frac{d^2\theta\left(\varepsilon,\vec{r}\right)}{d^2\vec{r}}+i\varepsilon\sin\theta\left(\varepsilon,\vec{r}\right)=0$$

$$F\left(\vec{r},\vec{r},\varepsilon\right)=-i\sin\theta\left(\varepsilon,\vec{r}\right)$$

D is the electron diffusion coefficient in the normal metal

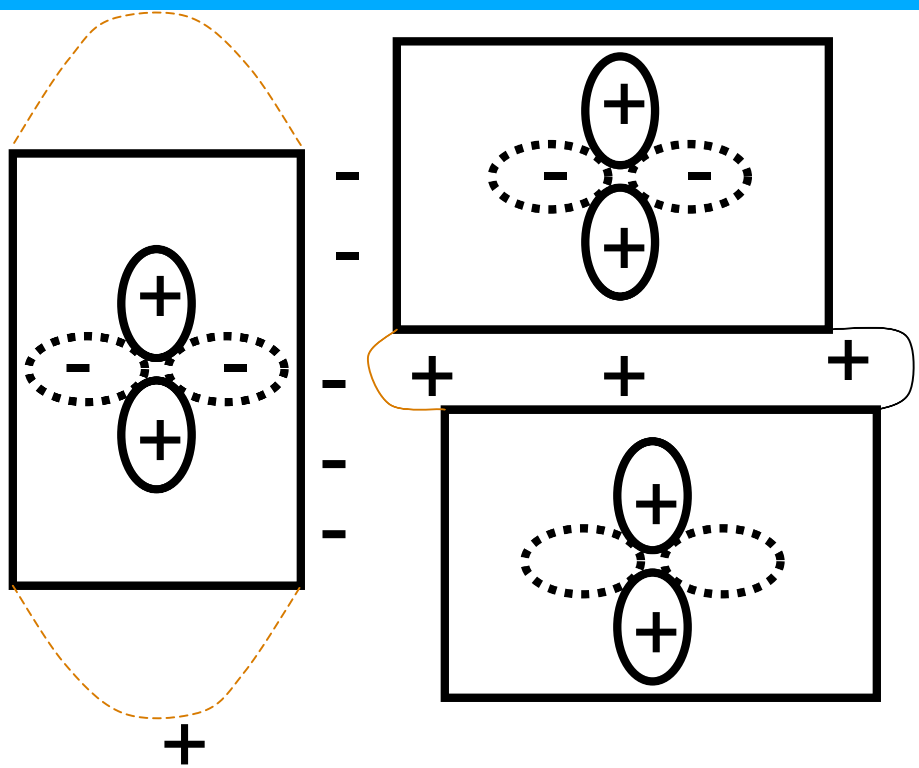
Boundary condition at d-n surface:



On large distance from the puddle decays in a long range way!

$F \sim 1/r^D$

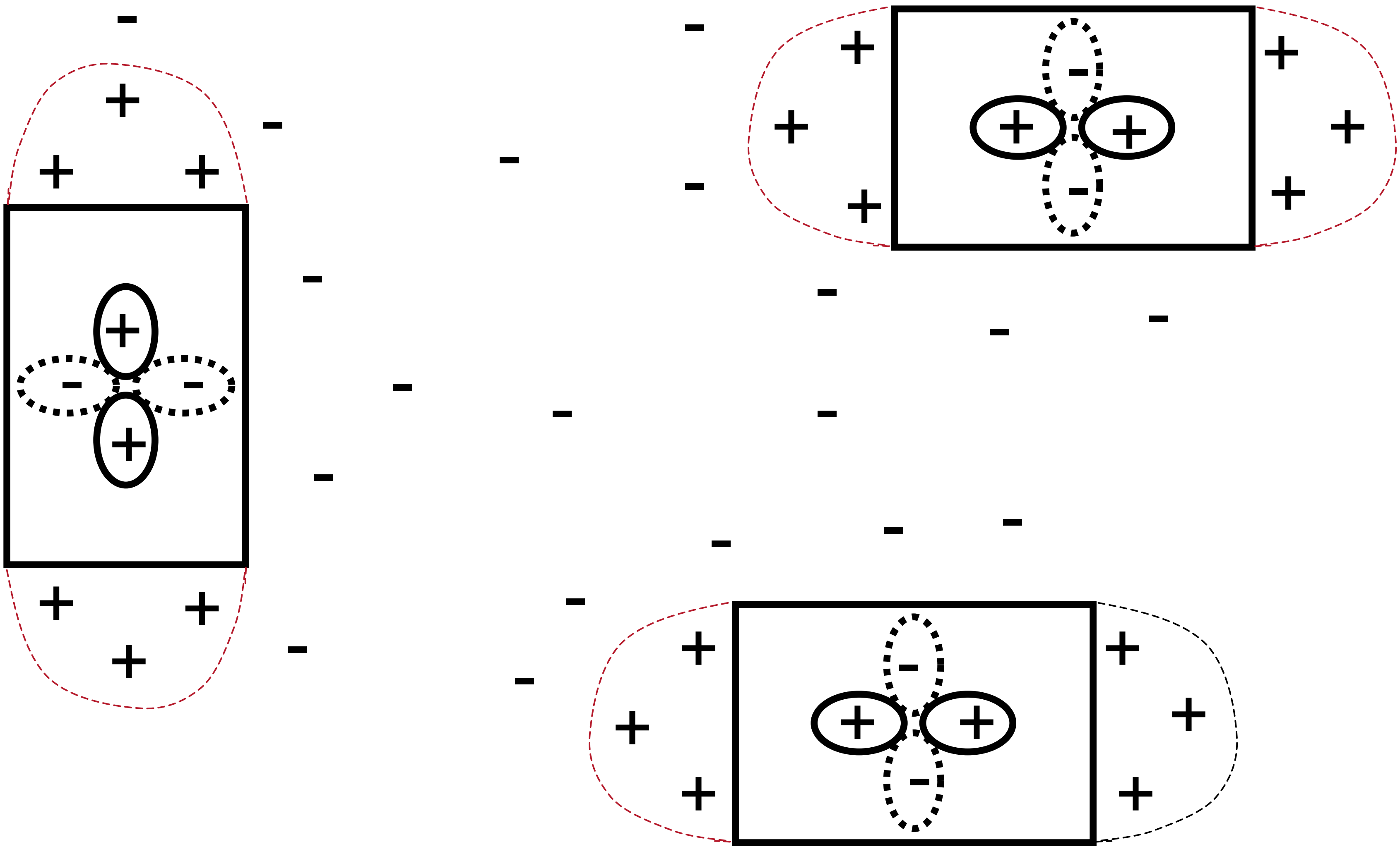
At small puddle concentrations, the order parameter has global d-wave symmetry, while s-component of the order parameter has random sample specific sign



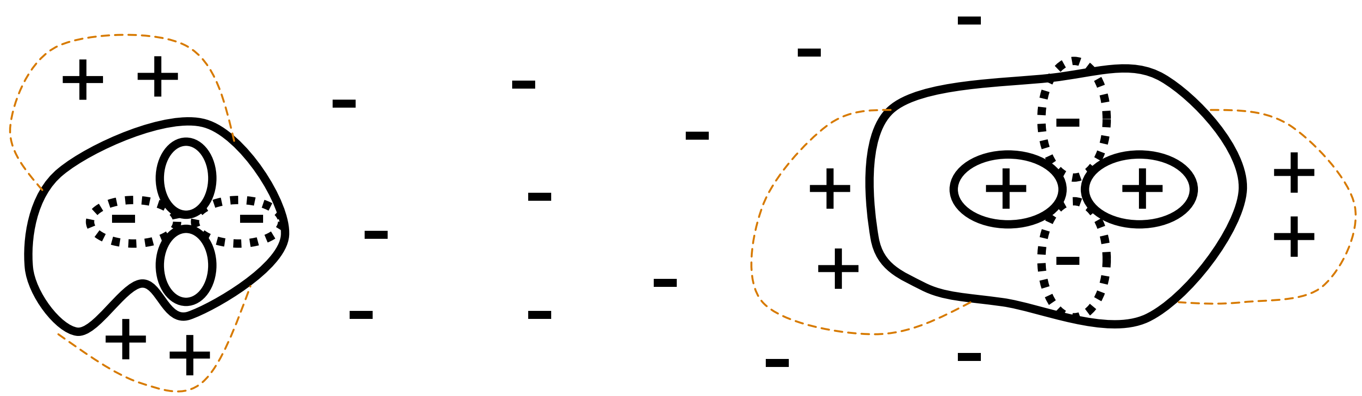
$$E = \sum_{ij} j_{ij}^{(d)} e^{i(\varphi_i - \varphi_j)} + c.c.$$

$$j_{ij} > 0 \;\; are\ Joshepson\ energies$$

If the concentration of superconducting puddles is small the order parameter has s-wave global symmetry, while d-wave component has randomly varying sign



More realistic picture of distribution of sign of s-component of anomalous Green function in a random system



The effective energy of the system is equivalent to Mattis model in the theory of spin glasses:

$$E = -\sum_{ij} j_{ij}^{(s)} \eta_j \eta_i e^{i(\varphi_i - \varphi_j)} + c.c.$$

$$\eta_i = \pm 1 \; are \; random$$

$$in \; the \; ground \; state \;\; e^{i\varphi_i} = \eta_i$$

An effective energy at intermediate concentration of superconduting puddles

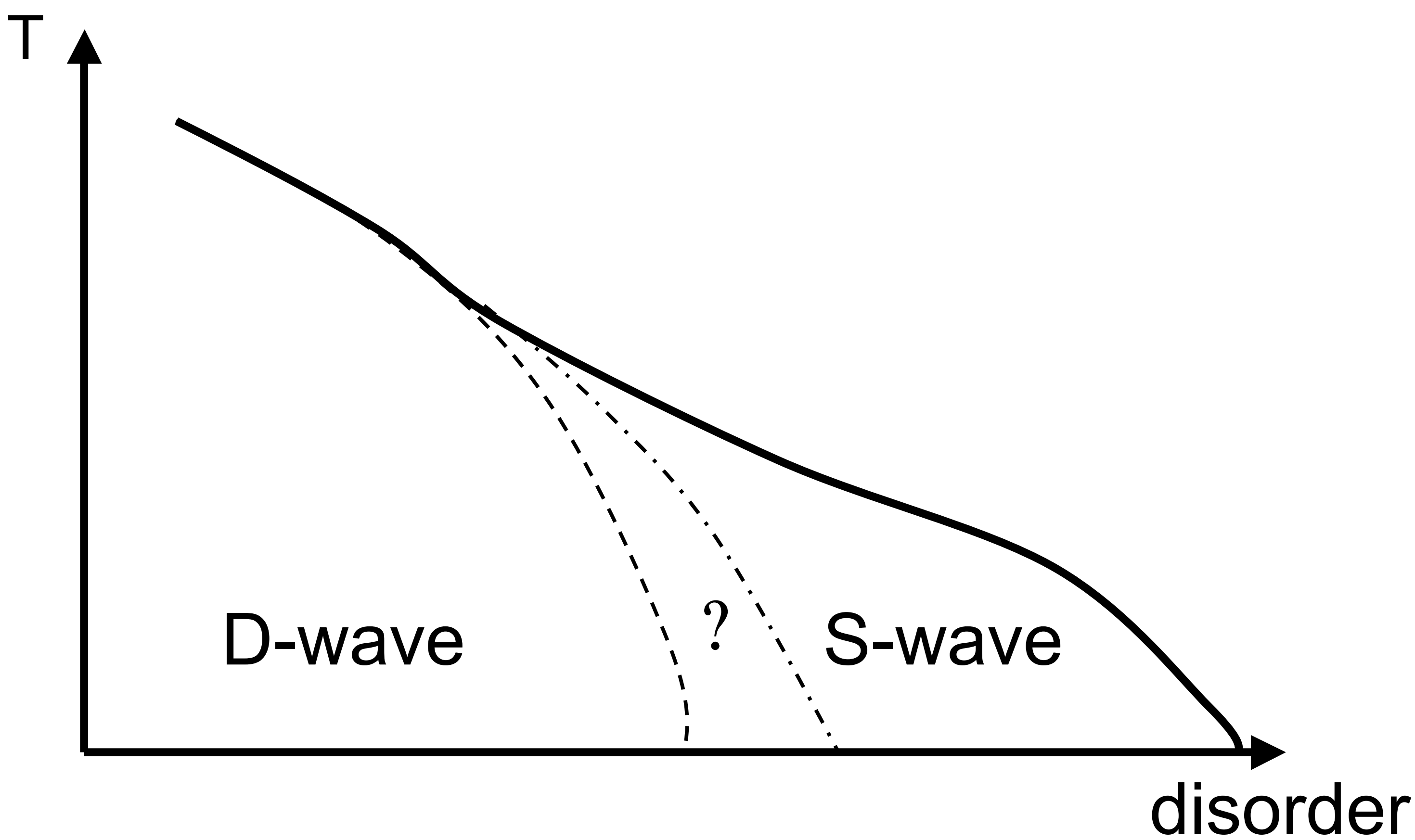
$$E = \sum_{ij}\left[j_{ij}^{(s)}\eta_j\eta_i + j_{ij}^{(d)}\right]e^{i(\varphi_i-\varphi_j)} + c.c.$$

$\eta_i = \pm 1$ *are random*

$j_{ij}^{(s)},\ j_{ij}^{(s)} > 0$ *are Joshepson coupling energies between s and d components of the order parameter respectively*

Is there a superconducting glass phase when $J^{(s)} \sim J^{(d)}$?

The phase diagram of a BCS d-wave superconductor



1) Near the T=0 transition, the system effectively looks granular.
2) There is inevitably an intermediate globally s-wave phase.
3) There is probably an intermediate glassy phase.

Non-local character of the response functions of
a metal in space and time leads to
entirely new physics near a quantum critical pt.

Amplification of the effective inhomogeneity.

Broad regime of substantial quantum fluctuations.

Suppression of the universal quantum critical regime
to a very narrow regime near criticality.

$$S=\sum_j G_i^{(\mathrm{eff})}\int d\tau d\tau'\,\frac{|e^{i\phi_j(\tau)}-e^{i\phi_j(\tau')}|^2}{(\tau-\tau')^2}+\int d\tau H_J[\{\phi\}],$$

$$S=\sum_j\left\{\alpha_j\int d\tau\left[-\frac{(\gamma-\gamma_{jc})}{2}|\Delta_j|^2+\frac{1}{4}\frac{|\Delta_j|^4}{\Delta_0^2}\right]\right.$$
$$\left.+\frac{\beta_j}{2}\int d\tau d\tau'\,\frac{|\Delta_j(\tau)-\Delta_j(\tau')|^2}{(\tau-\tau')^2}\right\}+\int d\tau H_J[\{\Delta\}]+\cdots,$$

$$H_J[\{\Delta\}]=-(1/2)\sum_{i\neq j}[J_{ij}\Delta_i^*\Delta_j+\mathrm{c.c.}], \qquad (1)$$

$$H_J[\{\phi\}] = -(1/2)\sum_{i\neq i} \widetilde{J}_{ij}\cos(\phi_i - \phi_j).$$

$$\widetilde{J}_{ij} \propto \widetilde{C}_{ij}\frac{D_{\mathrm{tr}}}{R^2}\frac{R^D}{|\mathbf{r}_i - \mathbf{r}_j|^D}\exp\left[-\frac{|\mathbf{r}_i - \mathbf{r}_j|}{L_T}\right]$$

$$J_{ij} \equiv J(\mathbf{r}_i, \mathbf{r}_j) \propto C_{ij} \frac{\nu V_i V_j}{|\mathbf{r}_i - \mathbf{r}_j|^D} \exp\left[- \frac{|\mathbf{r}_i - \mathbf{r}_j|}{L_T} \right],$$