# *Superconductivity and Ferromagnetism in Two Dimensions*

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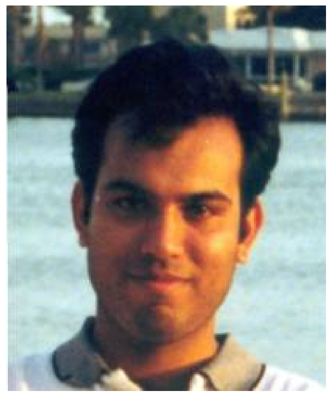

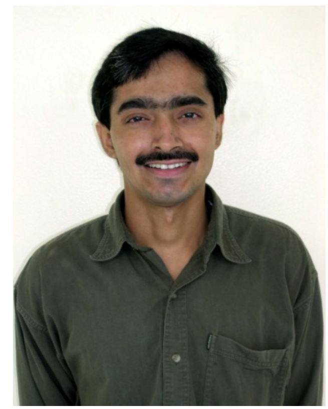

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Research supported by NSF

# Linking Questions ?

1. **Band ferromagnetism relies on itinerant electrons. When itinerancy is compromised by disorder, what happens?**

2. **Any signatures at $\hbar/e^2 = 4100\ \Omega$ ?**

3. **Is there a ferromagnetic-insulator transition?**

4. **Ferromagnetic behavior and film morphology?**

## Presentation (answers?) in four Acts !

# Prelude (some background)

At low temperatures,

$$\sigma_{2D} = \sigma_{Drude} + \Delta\sigma_{WL} + \Delta\sigma_{e-e}$$

**Disorder:-**

Lattice imperfections, grain boundaries, etc

***Which of the processes (WL or e-e interaction) is dominant at different disorder strength?***

***Quantum corrections are affected by spin alignment (ferromagnetic order).***

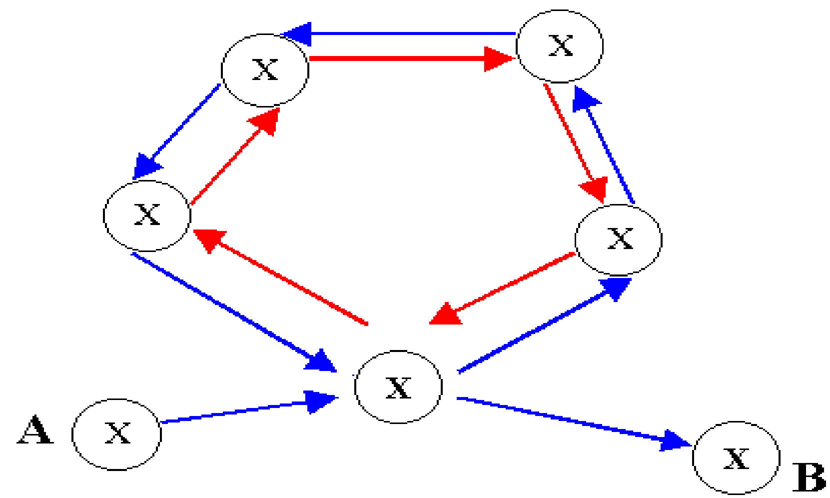
# Weak localization

## Magnetic Field

$$\vec{p} \rightarrow \vec{p} - \frac{e}{c}\vec{A}$$

$$\Delta\varphi_H = \frac{2e}{c\hbar}\oint \vec{A}\cdot d\vec{l} = 2\pi\frac{\Phi}{\Phi_0}$$

$$\sigma(H) - \sigma(0) \sim \frac{e^2}{\hbar}\ln\left(\frac{eHD\tau_\varphi}{\hbar c}\right)$$




**Negative magnetoresistance**

**Magnetic field suppresses coherent backscattering**

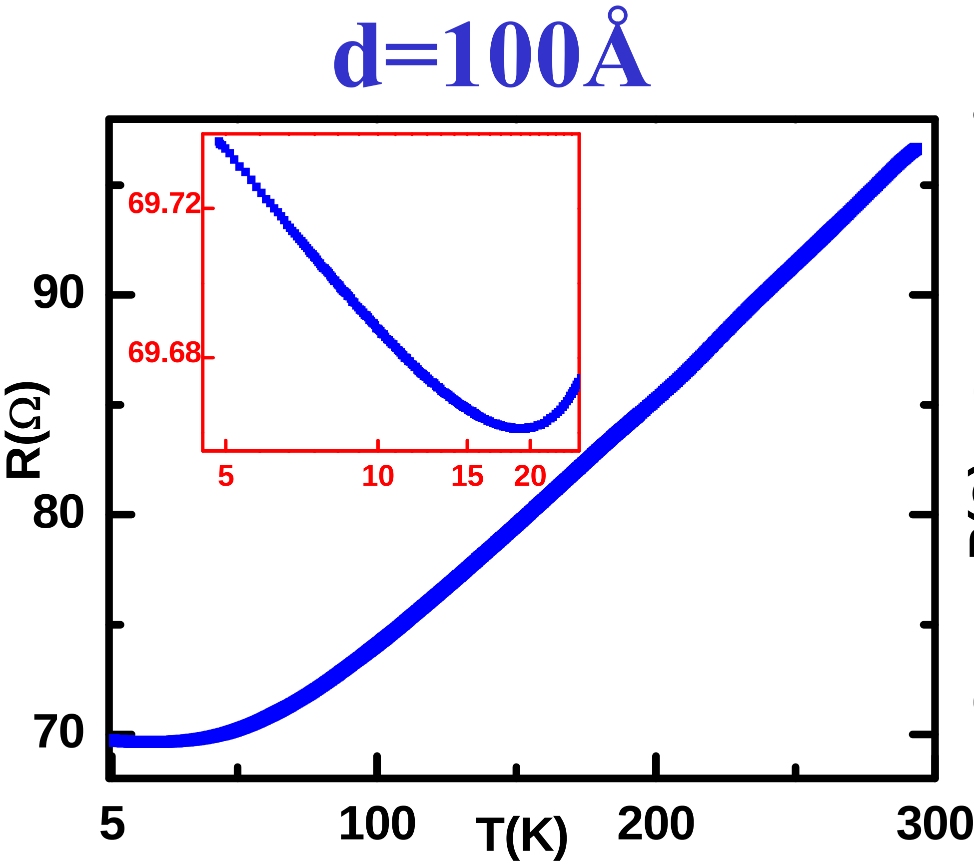
**A necessary condition for localization,**

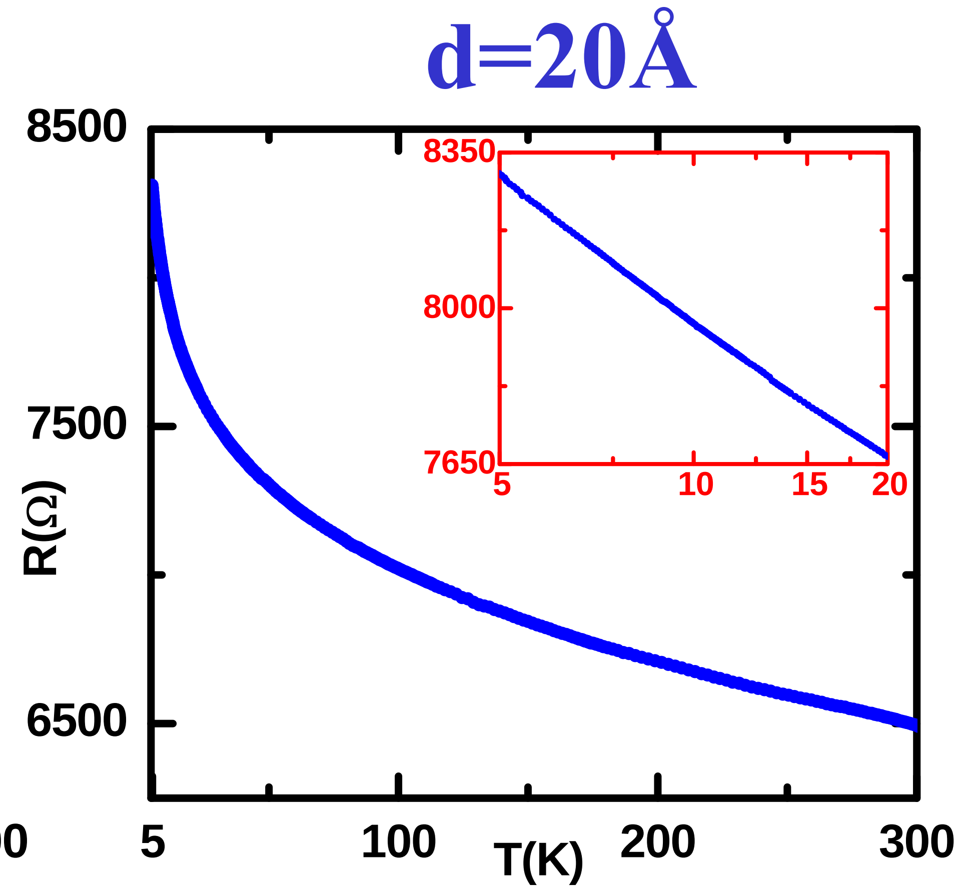
$$\max\{\tau_s^{-1}, \tau_{so}^{-1}, \omega_c\} << \tau_\varphi^{-1} << \tau_{tr}^{-1}$$

**$\tau_\varphi^{-1} \sim T$ in FM films due to spin conserving inelastic scattering off of spin wave excitations!**

# Weak disorder: Fe on glass substrate

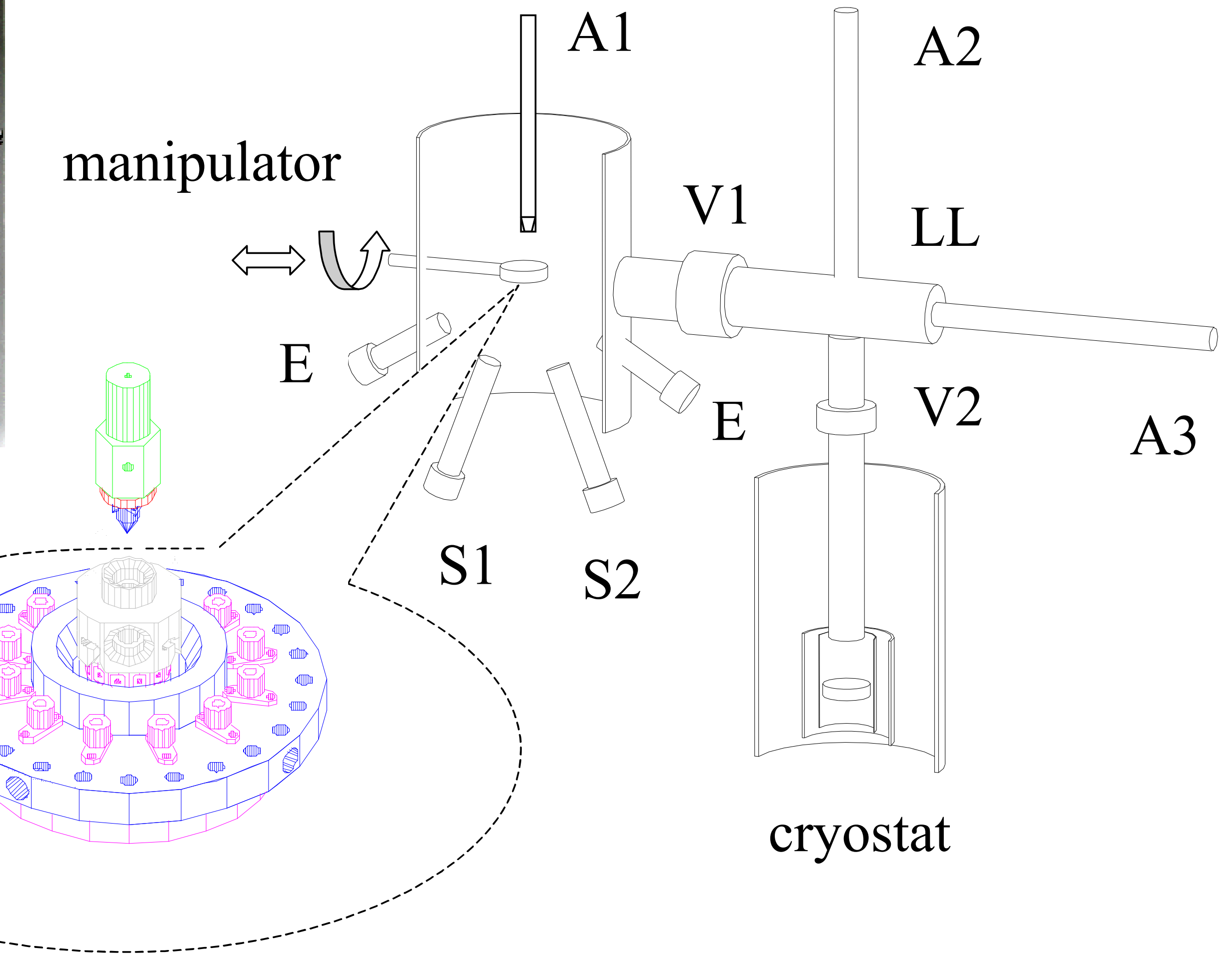
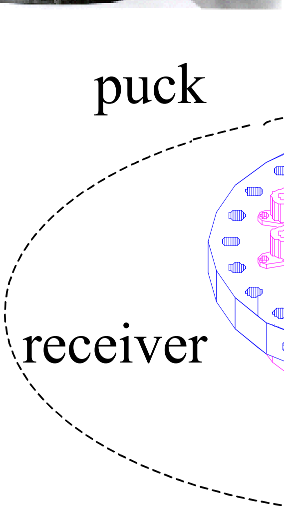
**Magnetron sputtering at room T**



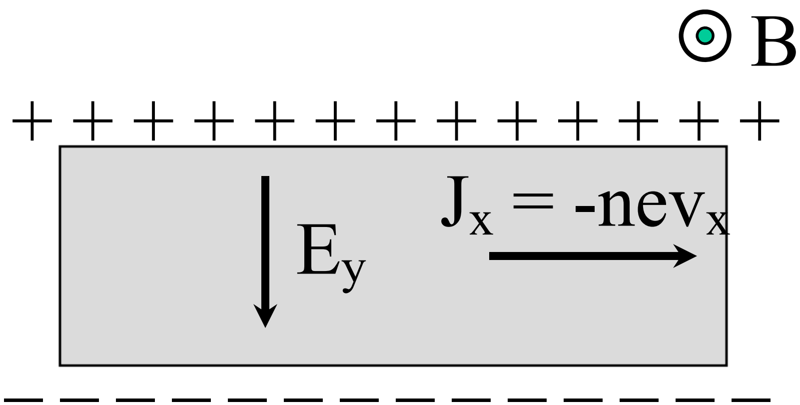

**Inset: Log(T) dependence of R at low temperature**

# SHIVA- Sample Handling In VAcuum

# Hall effect(s)

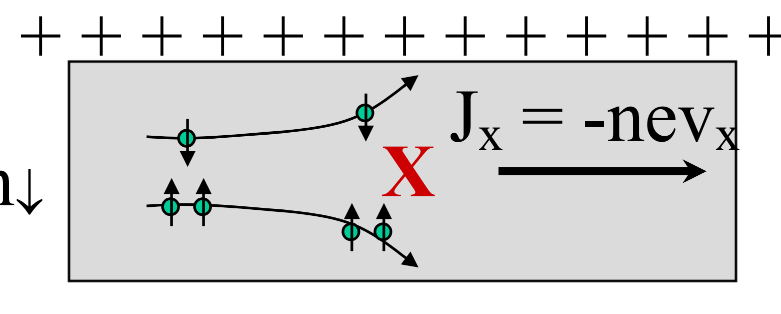
## Normal Hall Effect




$$R_{xy} = V_y/I_x = B/Ne$$

## Itinerant ferromagnetism and the Anomalous Hall Effect (AHE)

$n\uparrow > n\downarrow$




**X ≡ spin-orbit scattering**

$$R_{xy} = V_y/I_x = \mu_0 R_s M$$

# Relative resistance/conductance changes

Experimentally, $R_{xx}^{AH} >> R_{xy}^{AH}$

$$\sigma_{xy}^{AH} \approx \frac{R_{xy}^{AH}}{(R_{xx}^{AH})^2} \qquad \Rightarrow \qquad \frac{\delta\sigma_{xy}^{AH}}{\sigma_{xy}^{AH}} = \frac{\delta R_{xy}^{AH}}{R_{xy}^{AH}} - 2\frac{\delta R_{xx}^{AH}}{R_{xx}^{AH}}$$

# Quantum corrections to normal Hall conductivity

## Weak localization

**Fukuyama** showed for Hall resistance $R^n_{xy} = R_n B$

$$\delta R^n_{xy} / R^n_{xy} = 0$$

$$\frac{\delta\sigma^n_{xy}}{\sigma^n_{xy}} = -2\frac{\delta R_{xx}}{R_{xx}} = 2\frac{\delta\sigma_{xx}}{\sigma_{xx}}$$

J. Phys. Soc. Jpn. 49, 1980

## Electron interaction

**Altshuler** *et. al*. showed that

$$\frac{\delta\sigma^n_{xy}}{\sigma^n_{xy}} = 0$$

$$\frac{\delta R^n_{xy}}{R^n_{xy}} = 2\frac{\delta R_{xx}}{R_{xx}}$$

Phys. Rev B 22, 1980

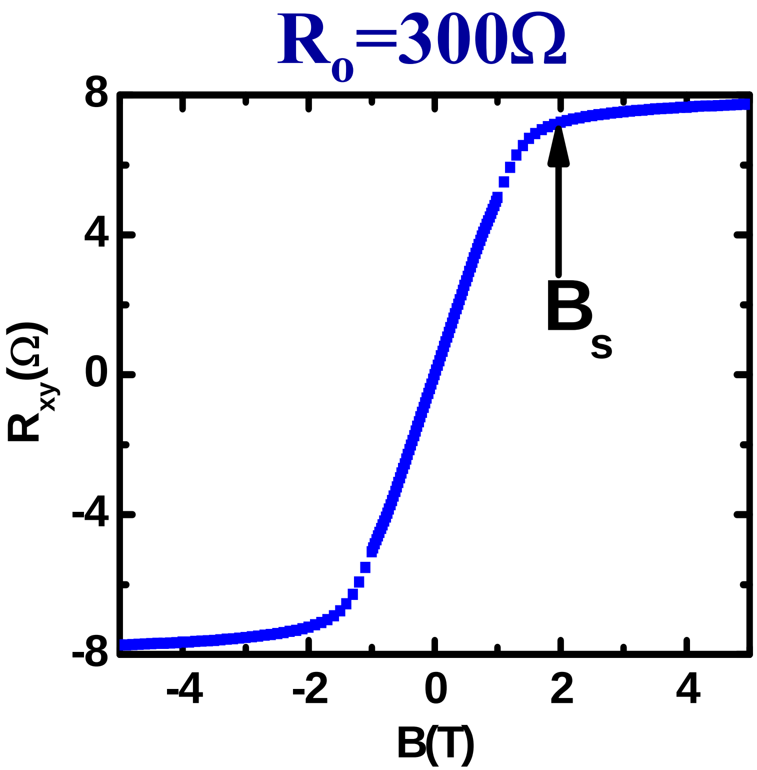
# ACT (I)

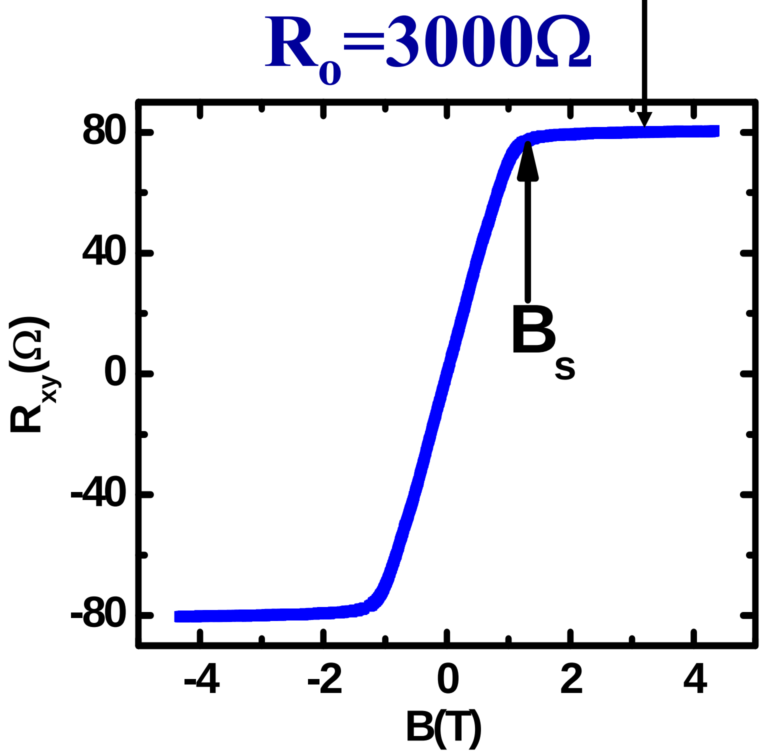
***Evidence for a disorder-dependent localization correction to the anomalous Hall (AH) conductance of Fe thin films***

# Anomalous Hall effect in iron

$$\boxed{R_{xy} = \mu_o R_s M + R_o B}$$

Remember this value




**Note: decrease in $B_s$ and increase in $R_{xy}$ with $R_o$**

# Quantum corrections to AH conductivity- Previous work

*Effect of electron interactions to* $\sigma_{xy}^{AH}$ *( skew scattering regime)*

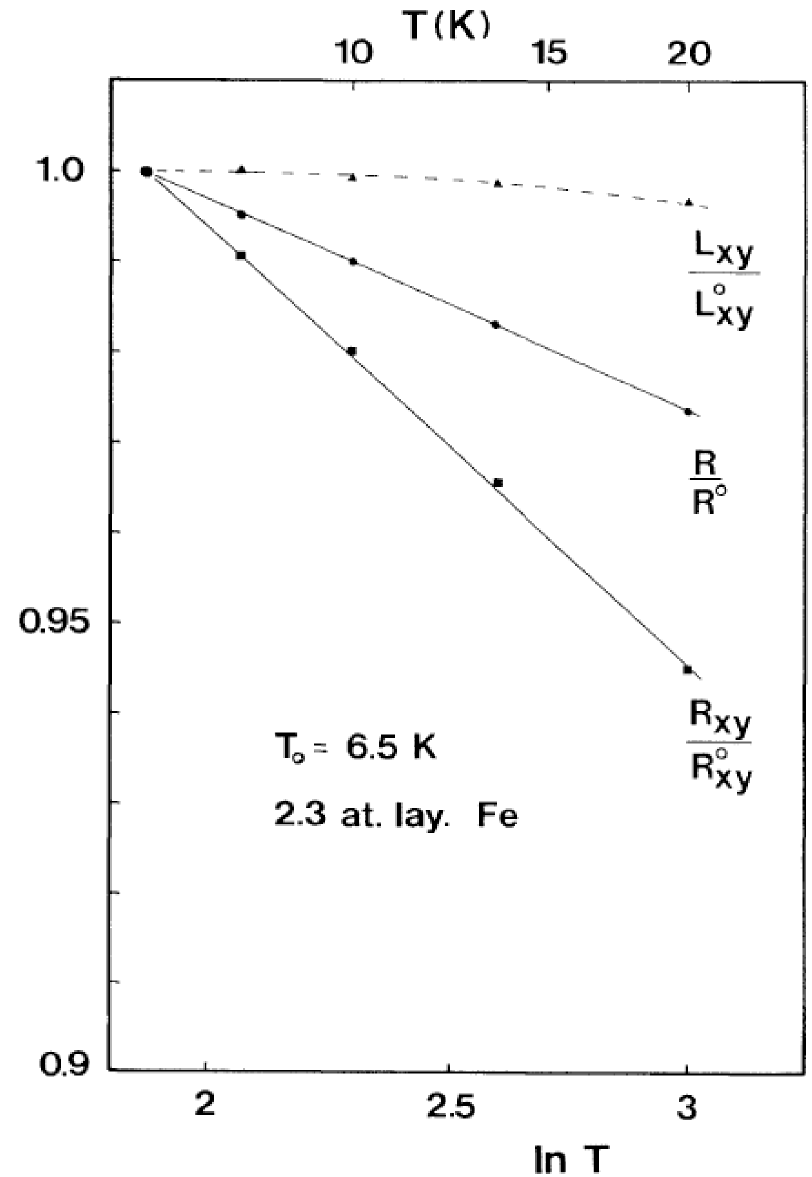
$$\frac{\delta\sigma_{xy}^{AH}}{\sigma_{xy}^{AH}} = 0 \Rightarrow \frac{\delta R_{xy}^{AH}}{R_{xy}^{AH}} = 2\frac{\delta R_{xx}}{R_{xx}}$$

*Langenfeld et. al. PRL(67)739,1991*

**Amorphous Iron films on Sb grown at T< 20K**

*Bergmann et.al. PRL(67)735,1991*

***Bergmann Scaling***

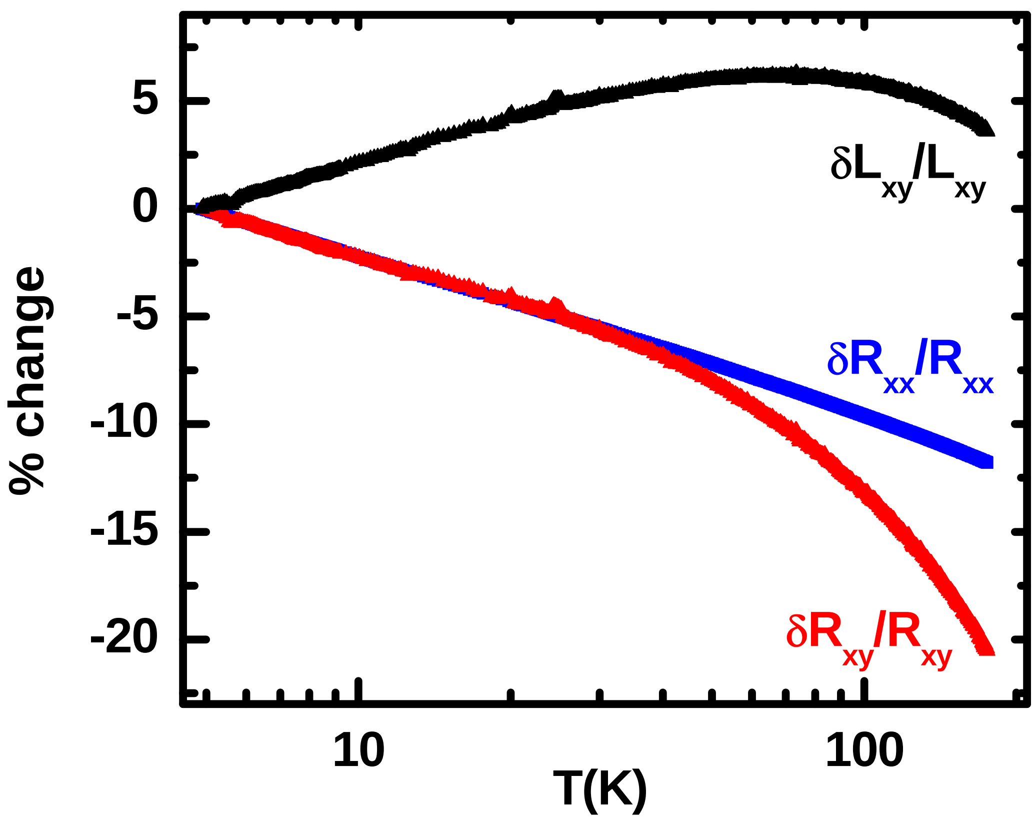



**Weak localization corrections to AH conductivity due to side jump mechanism are negligible in a 2D ferromagnet.**

*Dugaev et. al. PRB 104411 (2001)*

# Ultra thin Fe on glass: $\delta L_{xy} \neq 0$!

**$R^o = 2730\Omega$ $T^o = 5K$**




$$\frac{\delta\sigma_{xy}^{AH}}{\sigma_{xy}^{AH}} = \frac{\delta R_{xy}^{AH}}{R_{xy}^{AH}} - 2\frac{\delta R_{xx}^{AH}}{R_{xx}^{AH}}$$

**Our Result(T<20K)**

$$\boxed{\frac{\delta R_{xx}}{R_{xx}} = \frac{\delta R_{xy}}{R_{xy}} = -\frac{\delta L_{xy}}{L_{xy}}}$$

**Note the deviation at high T**

# Relative resistance(RR) scaling !

# Dimensionless Normalized Relative Change

- **Normalized Relative Change**

$$\Delta^N(Q_{ij}) = \frac{1}{L_{00}R_0}\frac{Q_{ij}(T) - Q_{ij}(T_0)}{Q_{ij}(T_0)}, \qquad \text{where } L_{00} = \frac{e^2}{2\pi^2\hbar} = (81k\Omega)^{-1}$$

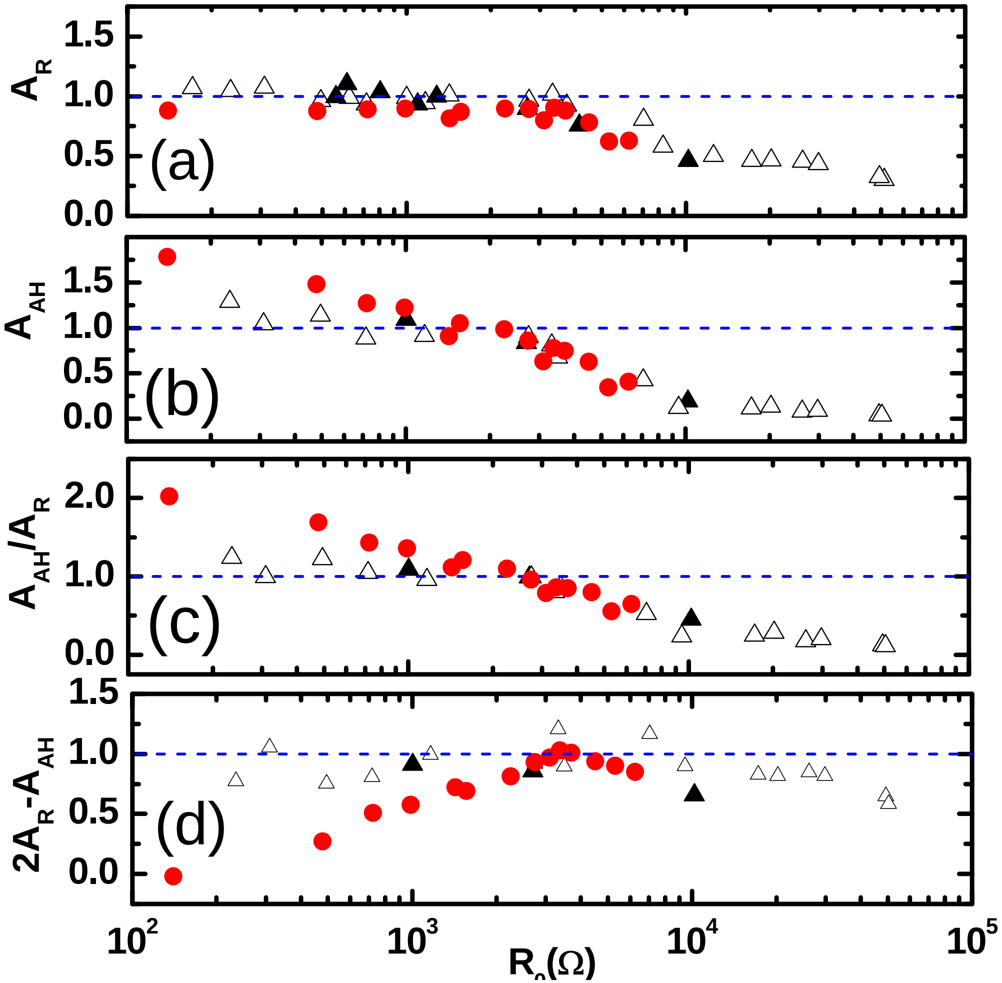
- **ln(T) dependence of $R_{xx}$, $R_{xy}$ for all $R_0$ at T<20K**

$$\boxed{\Delta^N(R_{xx}) = -A_R \ln\frac{T}{T_0}; \qquad \Delta^N(R_{xy}) = -A_{AH} \ln\frac{T}{T_0}}$$

$$\sigma_{xy}^{AH} \approx \frac{R_{xy}}{R_{xx}^2} \quad \Rightarrow \quad \frac{\delta\sigma_{xy}^{AH}}{\sigma_{xy}^{AH}} = \frac{\delta R_{xy}^{AH}}{R_{xy}^{AH}} - 2\frac{\delta R_{xx}^{AH}}{R_{xx}^{AH}}$$

$$\boxed{\Delta^N(\sigma_{xx}) = A_R \ln\frac{T}{T_0}; \qquad \Delta^N(\sigma_{xy}) = (2A_R - A_{AH})\ln\frac{T}{T_0};}$$

# $A_R$ and $A_{AH}$ for Fe films on glass (triangles) and sapphire (circles) substrates




$$\Delta^N \sigma_{xx}^{WL} = \ln(T/T_0)$$

$$\Delta^N \sigma_{xy}^{WL} = \frac{\sigma_{xy}^{SSM} \ln(T/T_0)}{(\sigma_{xy}^{SSM} + \sigma_{xy}^{SJM})}$$

**A weak localization correction within the skew scattering model is present in Fe films !**

# Theoretical calculations by Muttalib and Wölfle (I)

- $\sigma_{xy}^{SSM} = \sigma_{xx}^{0}\overline{\eta} M g_{so}\tau_{tr}/\tau; \qquad \sigma_{xy}^{SJM} = e^{2} M g_{so}\tau_{tr}/\tau,$

  where $\sigma_{xx}^{0} = e^{2}(n/m)\tau_{tr},$

  $g_{so}$ : spin orbit coupling, $M = n_{\uparrow} - n_{\downarrow}$ : net spin density,

  $\tau$ : single particle relaxation time,

  $\overline{\eta} \approx N_0 V,$ where $N_0$ is DOS and V $\rightarrow$ impurity potential

- *With* $R_0 \uparrow, \quad \dfrac{\sigma_{xy}^{SJM}}{\sigma_{xy}^{SSM}} \downarrow,$ if $\overline{\eta}$ increases sufficiently rapidly with disorder.

- There is no interaction induced ln(*T)* correction to $\sigma_{xy}$ (holds for both exchange & Hartree term, for both the skew scattering & side jump).

# Theoretical calculations by Muttalib and Wölfle (II)

- The weak localization condition is satisfied in the temperature range under study and the *most dominant contribution to the dephasing rate comes from the electron scattering off spin waves*.

- Calculations show that

$$\delta\sigma_{xx}^{WL} = -L_{00}\ln(\tau_\varphi/\tau); \qquad \delta\sigma_{\mathrm{xy}}^{\mathrm{WL}} = \delta\sigma_{xx}^{WL}\overline{\eta}Mg_{so}\tau_\varphi/\tau.$$

**Theoretical calculations by Dugaev et. al. shows that the WL contribution to AH conductivity from the side jump mechanism is negligible.**

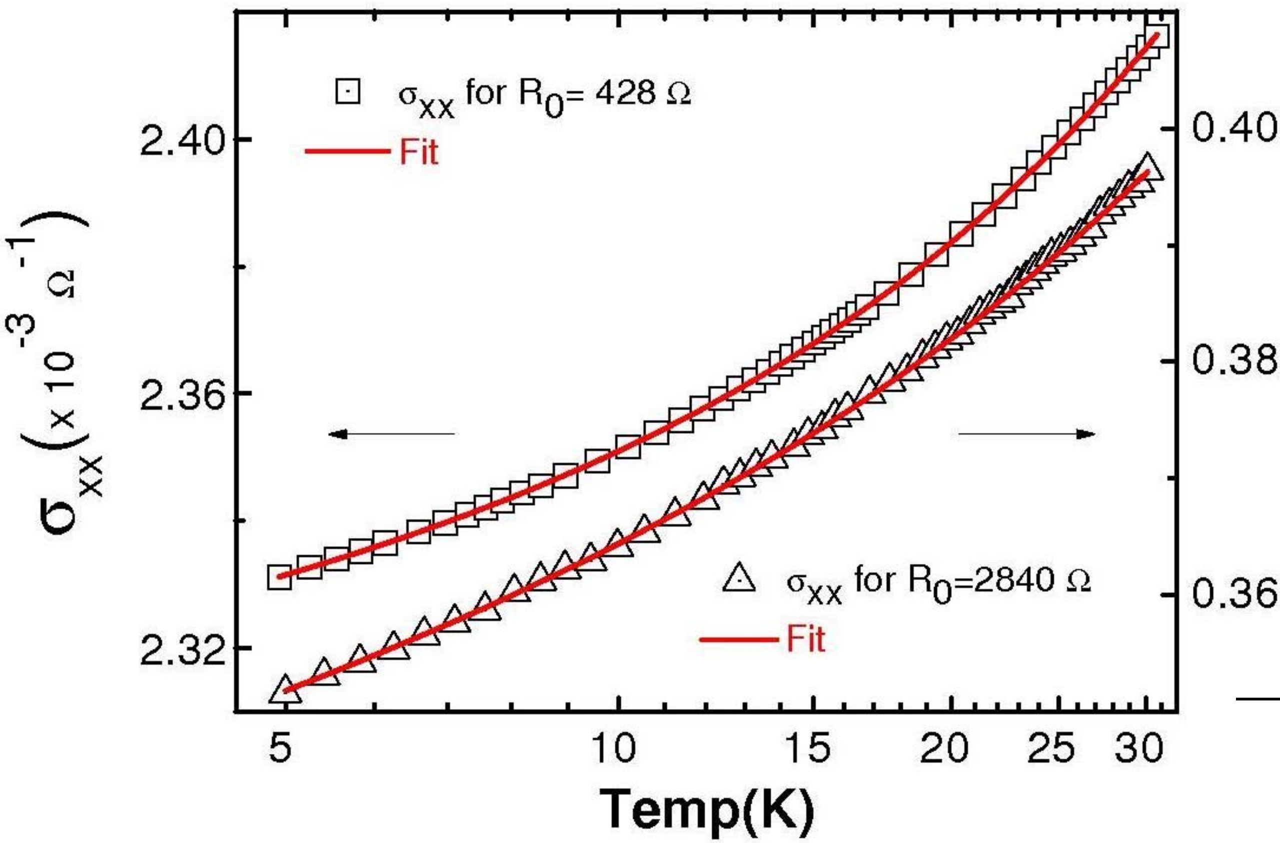
# ACT (II)

## *Spin-wave mediated quantum corrections to the conductivity in thin ferromagnetic gadolinium films*

***Given the importance of spin waves in Fe films, one might expect to observe even larger effects in FM films such as Gd (a local moment system)which has larger and more strongly coupled magnetic moments.***

# Quantum corrections to Conductivity

**Misra et al., Phys. Rev. B79, 140408(R) 2009**

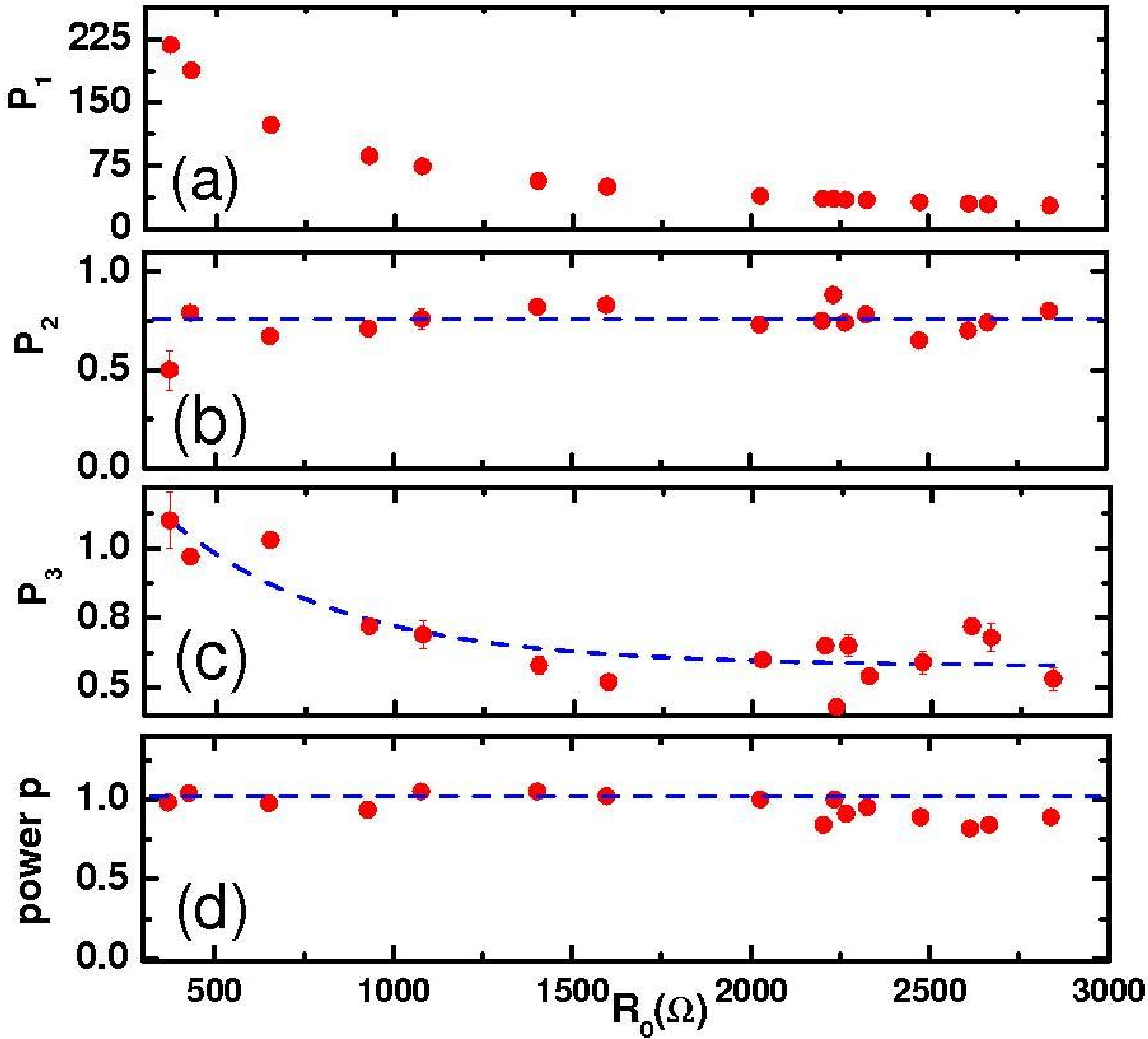



**Note logarithmic and Linear Corrections to the conductivity!**

Fitting function:

$$\frac{\sigma_{xx}}{L_{00}} = P_1 + P_2 \ln\left(\frac{T}{T_0}\right) + P_3\left(\frac{T}{T_0}\right)^P$$

# Dependence of fitting parameters on disorder parameter $R_0$ of Gd films




16 films

Fitting Equation :

$$\frac{\sigma_{xx}}{L_{00}} =$$

$$P_1 + P_2 \ln\left(\frac{T}{T_0}\right) + P_3\left(\frac{T}{T_0}\right)^P$$

# Spin wave mediated Altshuler-Aronov corrections to conductivity-

## Effective spin wave mediated interaction:

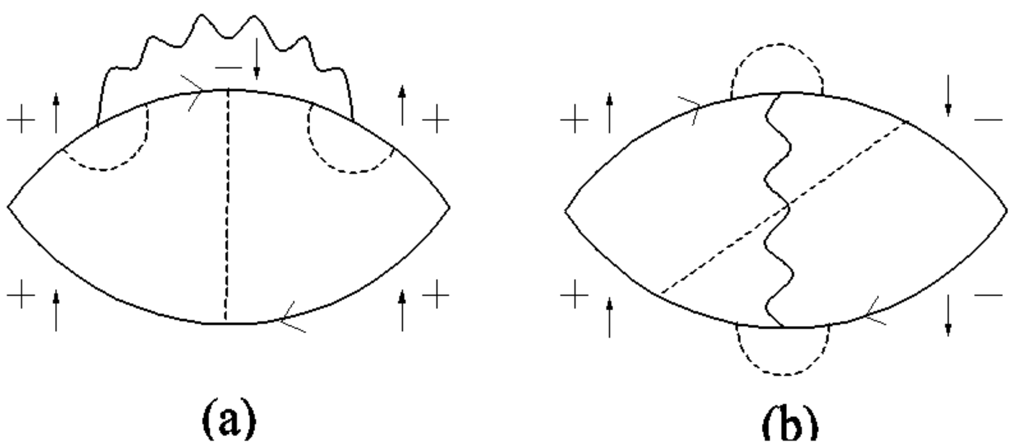
$$V_{SW}(q,\omega) = nJ^2 2\omega_q \big/ (\omega^2 + \gamma\omega\omega_q + \omega_q^2); \qquad \omega_q = \Delta + Aq^2$$

$n$ : 2D density of states J : spin exchange interaction

$\Delta$ : spin wave gap A : spin stiffness

$\gamma$ : spin wave damping B : Exchange splitting

## Spin wave mediated Altshuler-Aronov correction to conductivity:




$$\delta\sigma_{xx} \propto T$$

$$\text{for} \quad T << B$$

**wavy line: sw interaction** **dashed line: diffusons** ***K. A. Muttalib & P. Wölfle***

# Spin wave mediated Altshuler-Aronov corrections to conductivity

## The total spin wave contribution

$$\frac{\delta\sigma_{xx}}{L_{00}} \approx \left(\frac{Jk_F^2}{2\pi B}\right)^2 (\varepsilon_{\mathrm{F}}\tau)\frac{T}{Ak_F^2}$$

The disorder dependence of the linear T contribution is given by $P_3 \propto \varepsilon_F \tau$ , which decreases with increasing disorder.

Experimentally, $P_3$ does indeed decrease with disorder up to $R_0 \approx 2000\Omega$ and then saturates.

*** Misra et al. PRB Rapid Communication 79, 2009**

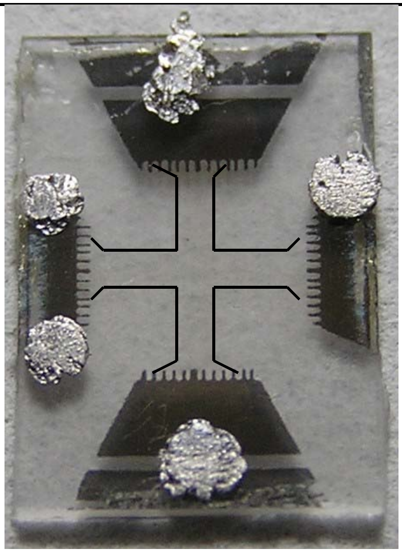
# ACT (III)

## *Finite temperature critical behavior near the Anderson quantum phase transition*

**An oxymoron???**

**Beyond the region of quantum corrections**

# Experimental details



•*Sapphire substrate*

•*Grown by r.f. magnetron sputtering at 130 K.*

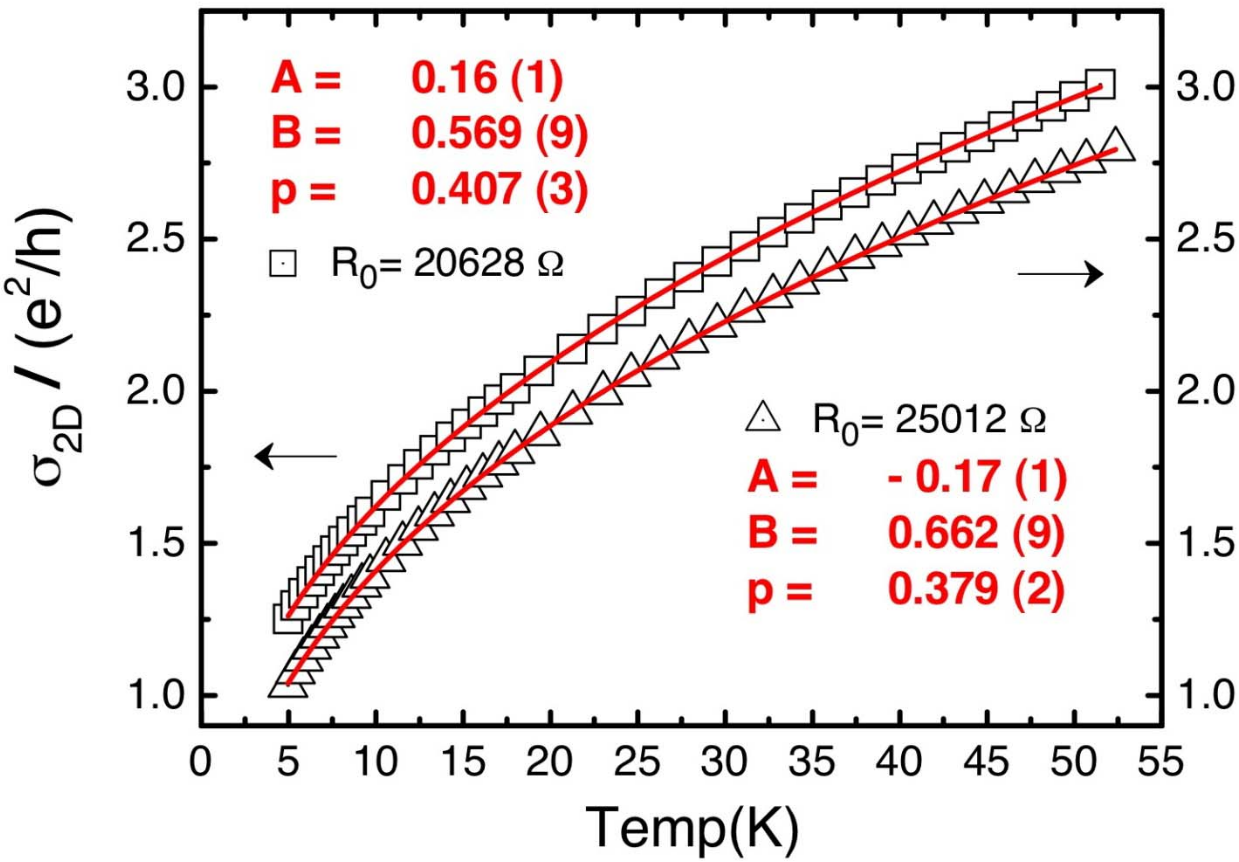
• *Hall cross geometry*.

• *Disorder* $R_0 \equiv R_{xx}(T=5K)$, $R_0$ *varying over the range* **4011 Ω (35Å) to 72KΩ (<20 Å).**

• *Thermal annealing is used to increase the amount of disorder in the film*.

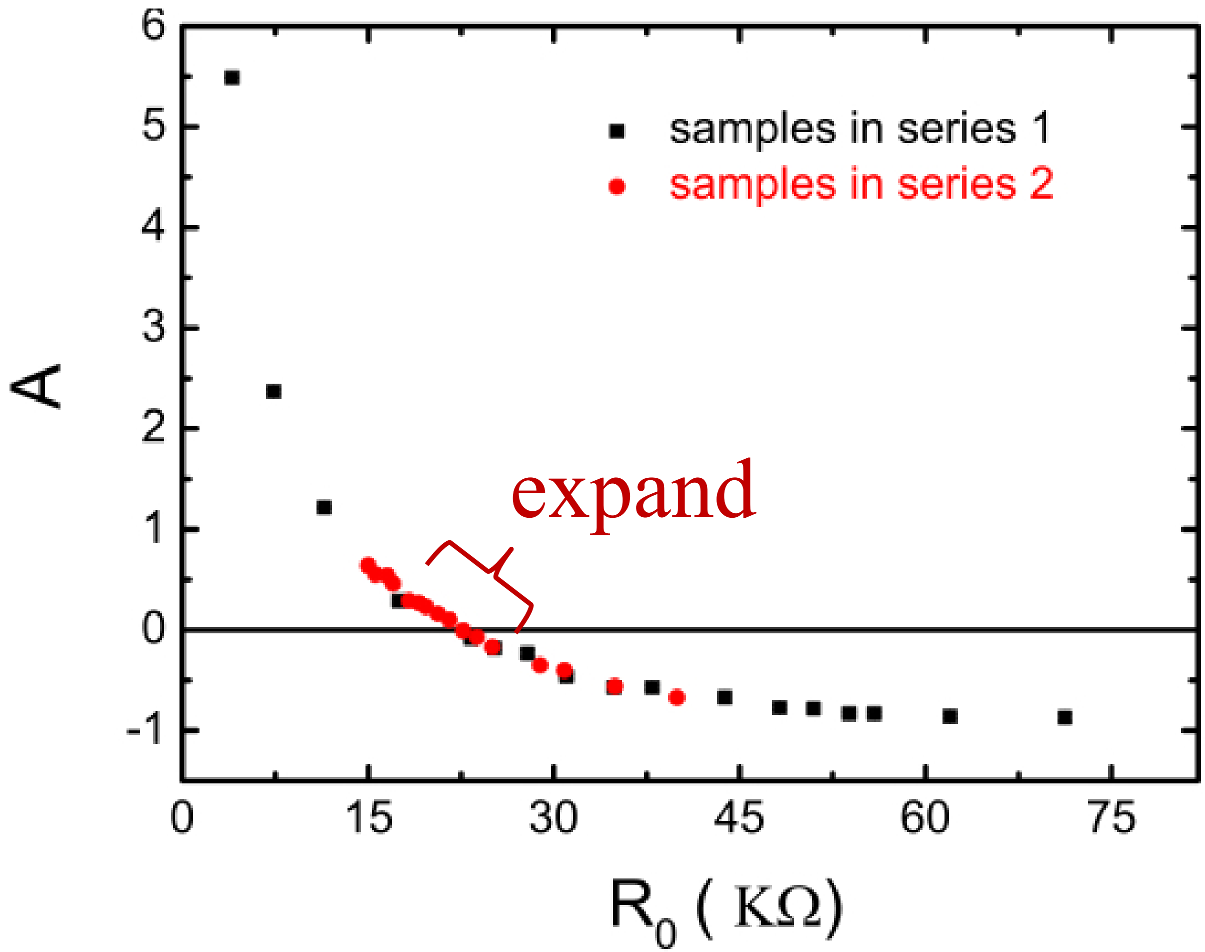
•*In-situ magneto transport (use RMFR!)* .

# Typical plot for conductivity



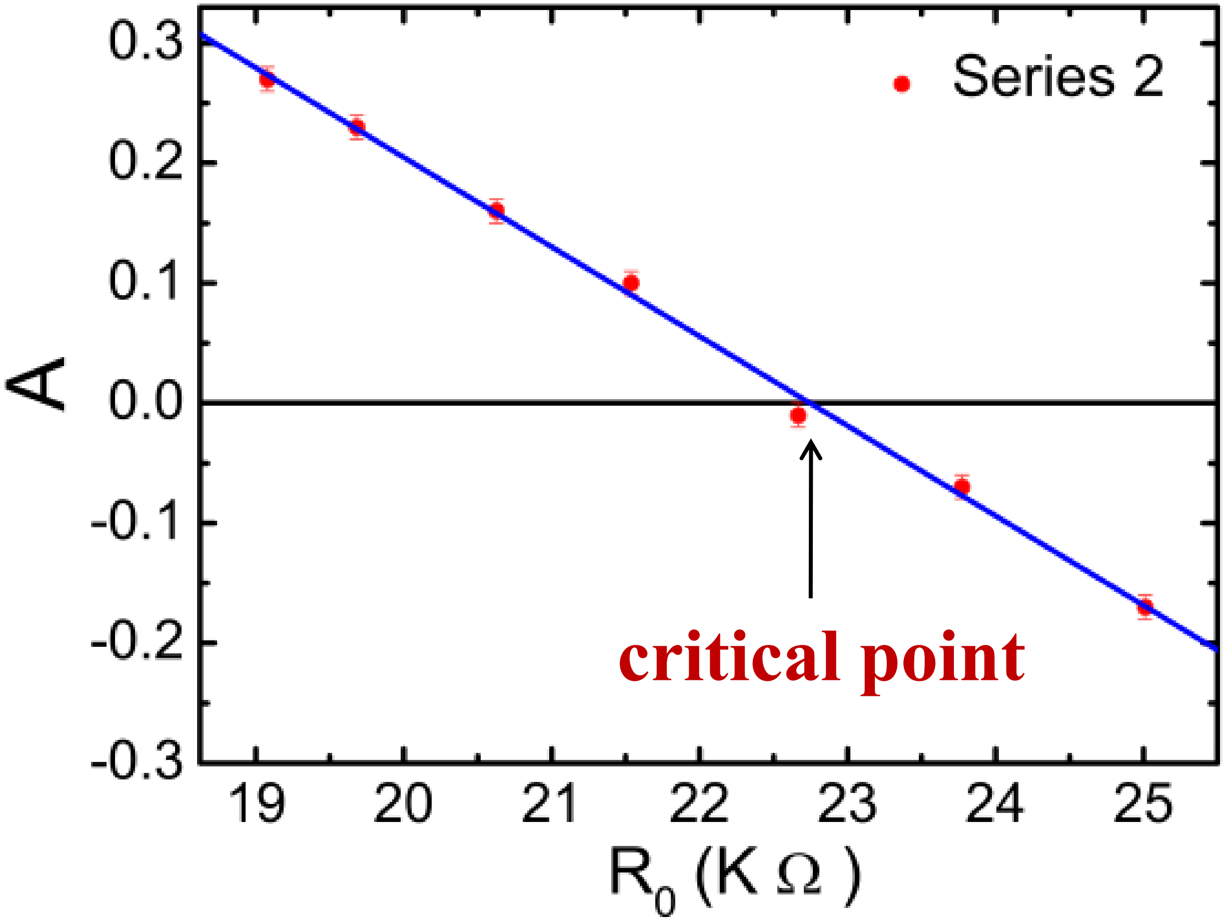

**Conductivity can be modeled as** $$\frac{\sigma_{2D}}{(e^2/h)} = A + BT^p$$

# Parameter *A* as function of disorder




**Series 1: 5 separate depositions with 2 samples undergoing 12 successive anneals.**
**Series 2: 1 sample undergoing 15 successive anneals.**

# Expanded view of the critical region



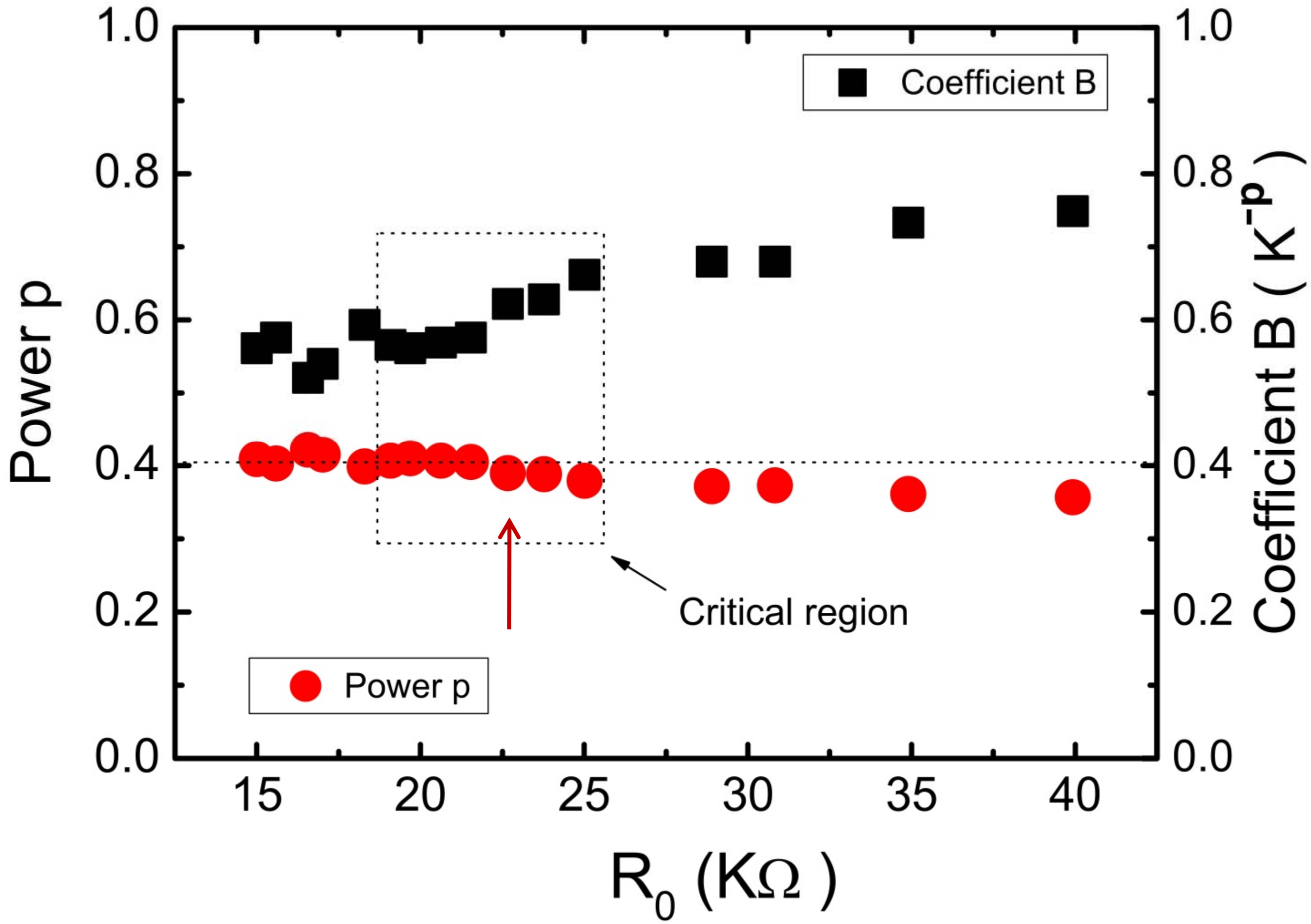

$$\frac{\sigma_{2D}}{(e^2/h)} = A + BT^p$$

$$A(R_0) \propto (R_c - R_0)^\gamma$$

$$\gamma \approx 0.98 \pm 0.07$$

At critical point where A = 0, $1/\tau_\varphi \sim T^{1.19}$

# Dependence of *p* & *B* on disorder




**With $R_0$↑, $p$↓ and $B$↑**

$$\frac{\sigma_{2D}}{(e^2/h)} = A + BT^p$$

# Primer on exponents

$\sigma \sim (1-\lambda/\lambda_c)^s$ dc conductivity, $s$ = conductivity exponent

$\sigma(\omega;\lambda_c) \sim \omega^{1/z}$ dynamical conductivity,
$z$ = dynamical exponent

$\xi \sim |1-\lambda/\lambda_c|^{-\nu}$ Correlation (localization ) length

In 3D, $\nu = s = 1.6$ (numerical calculations*) and $z = 3$

* K. Slevin and T. Ohtsuki, PRL 82, 382 (1999)

Replace frequency by the phase relaxation rate $1/\tau_\varphi$

# Scaling description of the transition

The conductivity near the transition is given by the scaling form

$$\sigma(\omega;\lambda) = \xi^{-1} G(\xi\omega^{1/3}) = B\omega^{1/3} + A(\lambda)\omega^{\delta/3} + \ldots,$$

$$\sigma(\omega;\lambda) = B\left(\frac{1}{\tau_\varphi}\right)^{1/3} + A(\lambda)\left(\frac{1}{\tau_\varphi}\right)^{\delta/3}.$$

where $A(\lambda)=0$ at $\lambda = \lambda_c$, $A(\lambda) = a_1(\lambda_c - \lambda)^{s(1-\delta)}$ for $\lambda < \lambda_c$

& $A(\lambda) = a_1'(\lambda - \lambda_c)^{s(1-\delta)}$ for $\lambda > \lambda_c$.

At critical point $1/\tau_\varphi \sim T^{1.19}$ (since we observed p = 0.39)

***Consistent with e scattering off spin wave excitations plus subleading corrections***

Also, $s(1-\delta) = 1$ with $s=1.6$ implies $\delta = 0.375$

# Our experiment

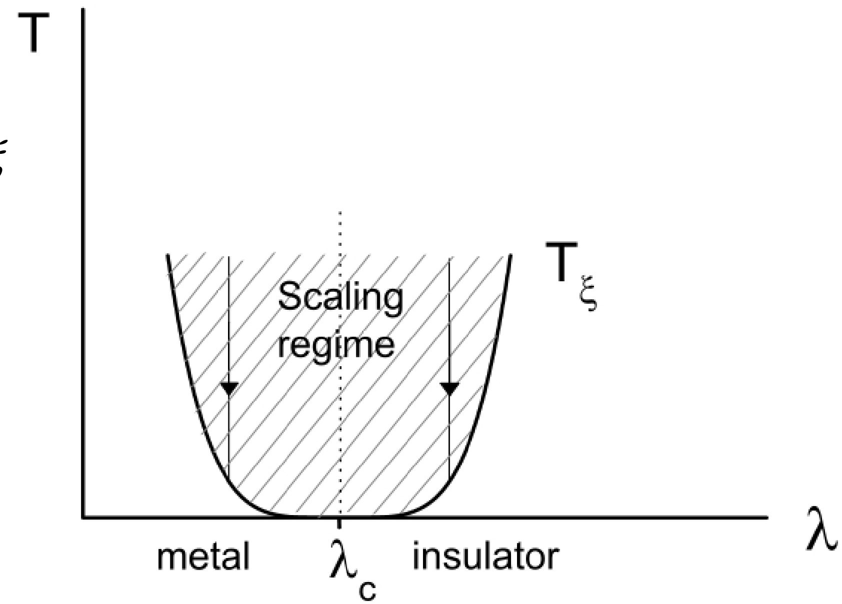
At finite T in the scaling regime,

$$\omega > \omega_\xi = \frac{1}{\tau}(\xi / \ell)^{-3} \qquad \text{or} \qquad T > T_\xi$$

the d.c. conductivity is obtained from the dynamical conductivity, with leading dynamical scaling behavior

$$\sigma(\omega) \propto \omega^{1/3}$$

by replacing frequency by the phase relaxation rate $1/\tau_\varphi$




The effective dimension is THREE, provided the temperature dependent correlation length

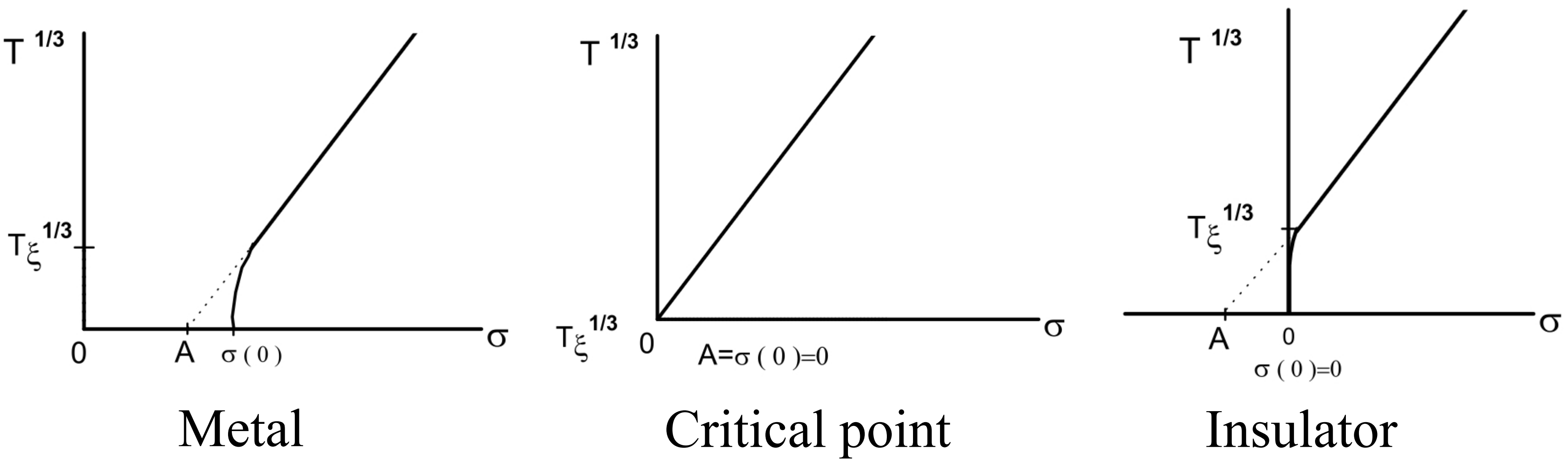
$L_\varphi < b$, (b is the thickness of the film)

# Understanding the meaning of parameter *A*

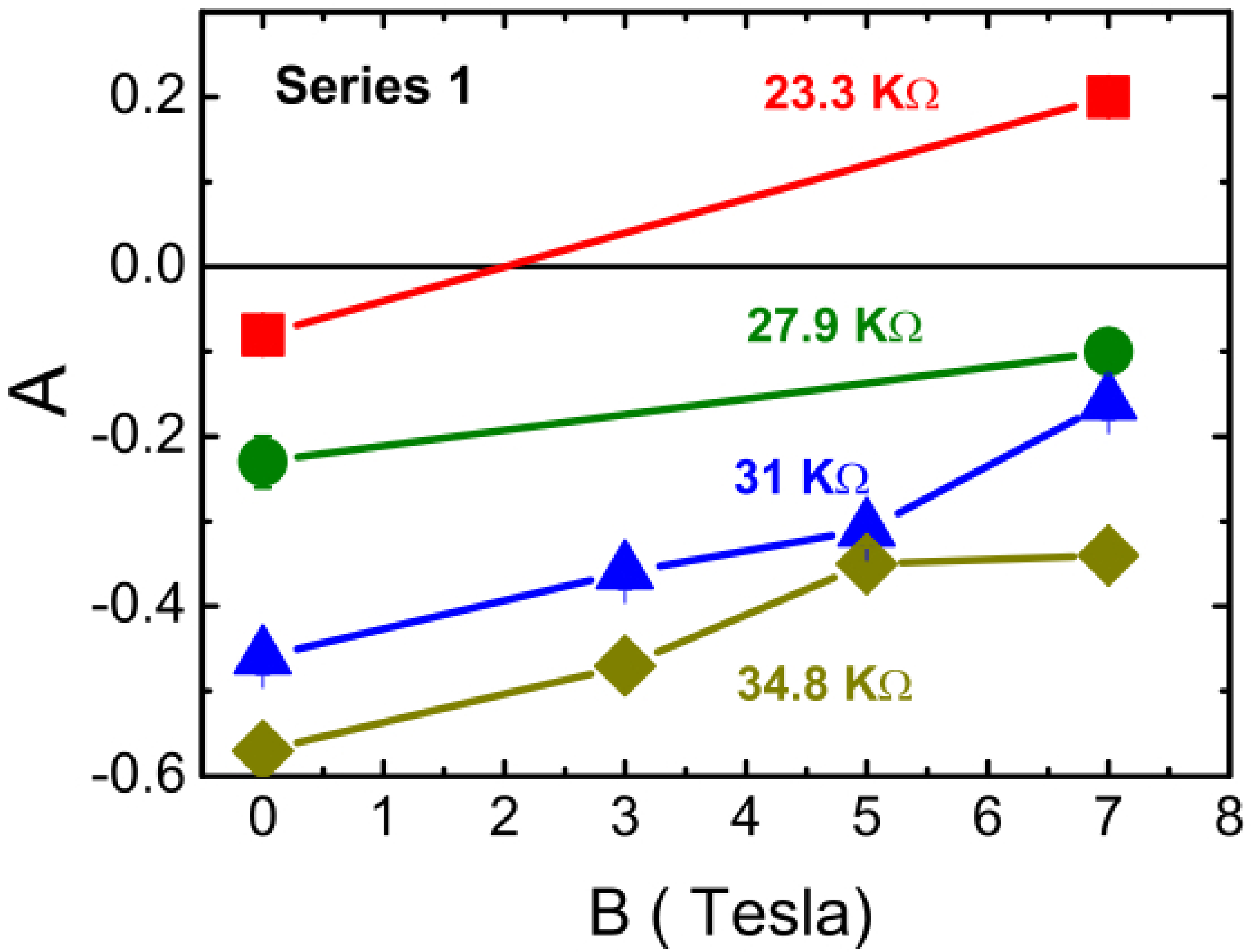
The inequality $L_\varphi < b$ is satisfied for temperatures

$$T > T_x = \left[ B(bk_F)^{-3}(\varepsilon_F \tau_\varphi)/\hbar \right]^{\frac{1}{1-p}}. \qquad T_x \approx 1.2\ K$$

Frequency is cut off by phase relaxation rate: $\omega \to \frac{1}{\tau_\varphi} \propto T$



# Field tuned insulator-to-metal transition



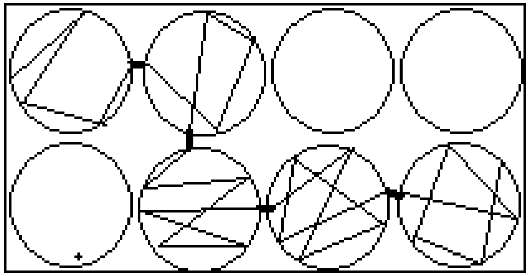

$$\frac{\sigma_{2D}}{(e^2/h)} = A + BT^p$$

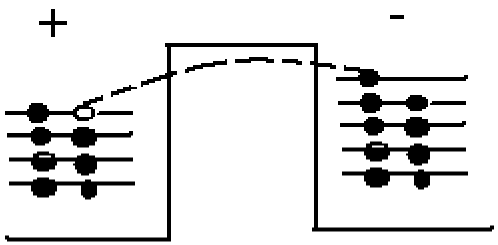
# *The Anomalous Hall Insulator* (AHI)

# Transport in granular metals

**Model:**

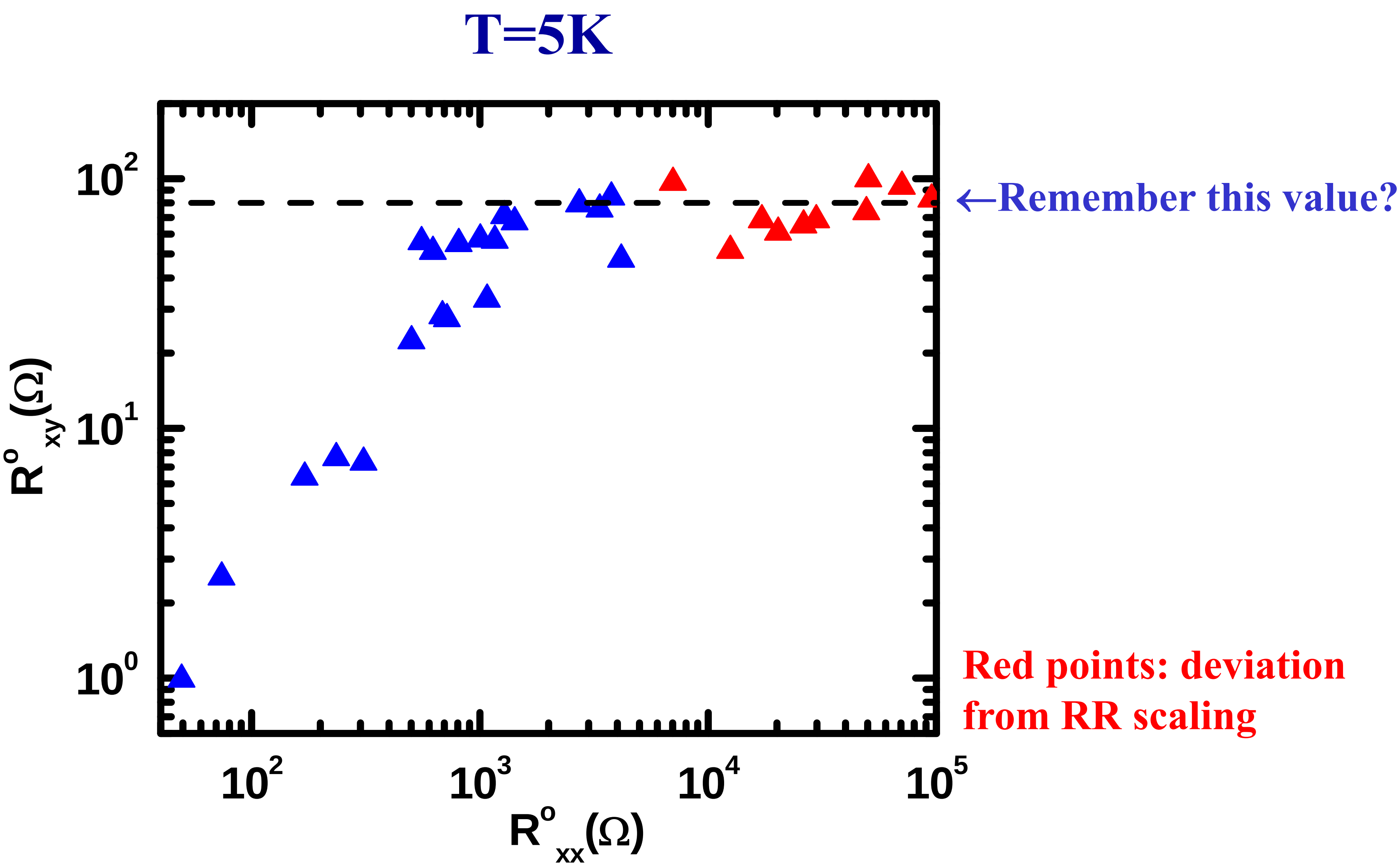
- Array of metallic grains connected by tunnel junction
- $\sigma_g >> \sigma_T$
- Diffusive motion inside grain due to GB scattering
- Electron interaction due to charging of grain
- Tunneling is 'slow' and coherent process



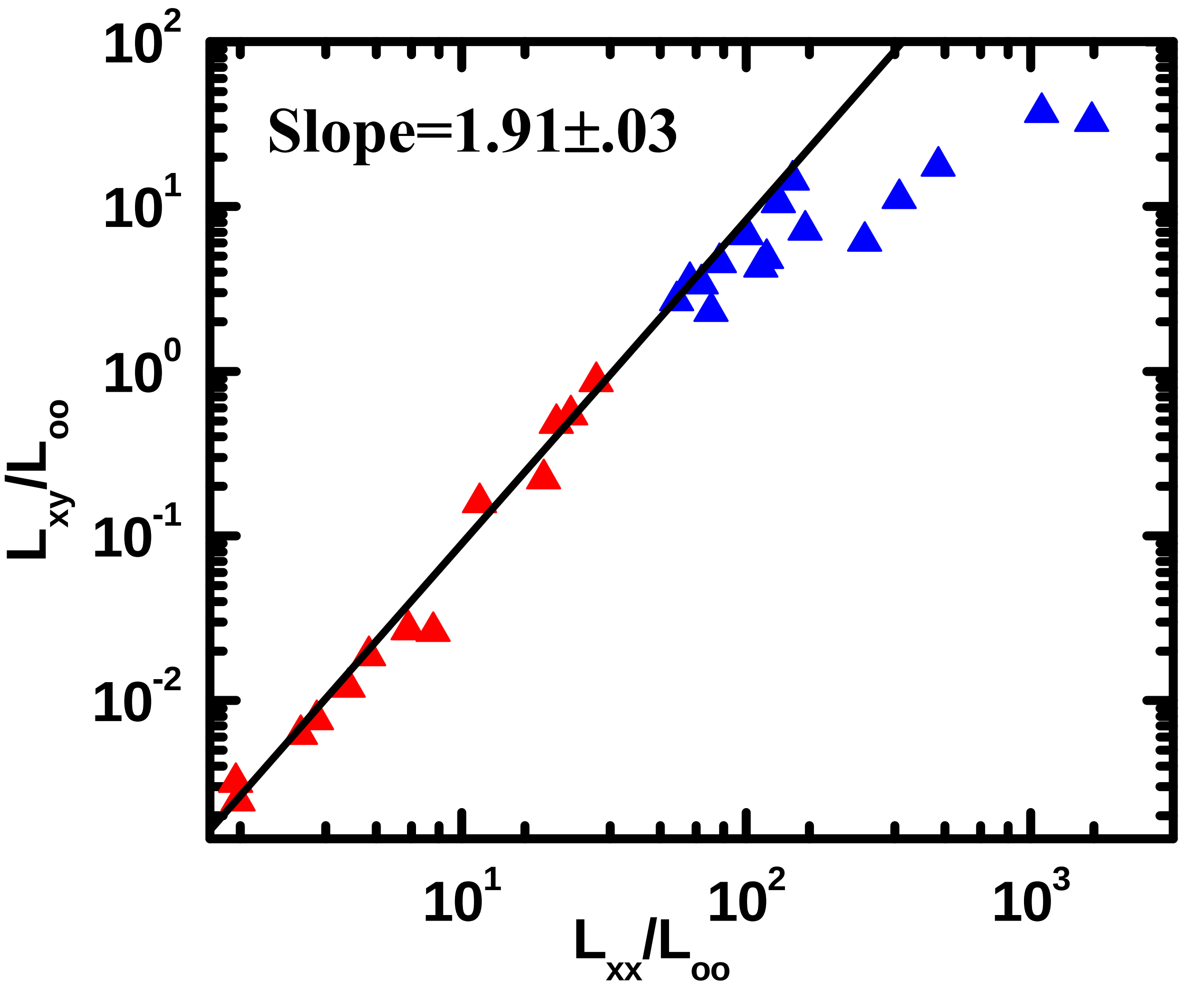



Beloborodov *et.al.* PRL '03

# Dependence of $R_{xy}^{AH}$ on $R_o$



# Anomalous Hall Insulator?




$$L_{xy} = \frac{R_{xy}}{R_{xx}^2}$$

$$L_{xx} = \frac{1}{R_{xx}}$$

$$L_{xx} \to 0, L_{xy} \to 0$$

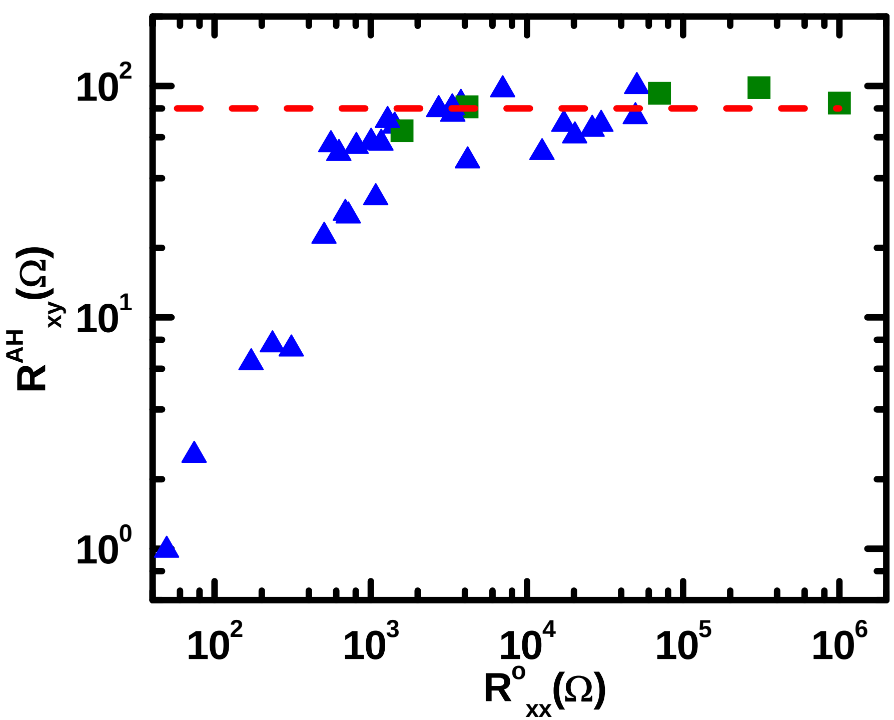
$$L_{xy} \propto L_{xx}^2$$

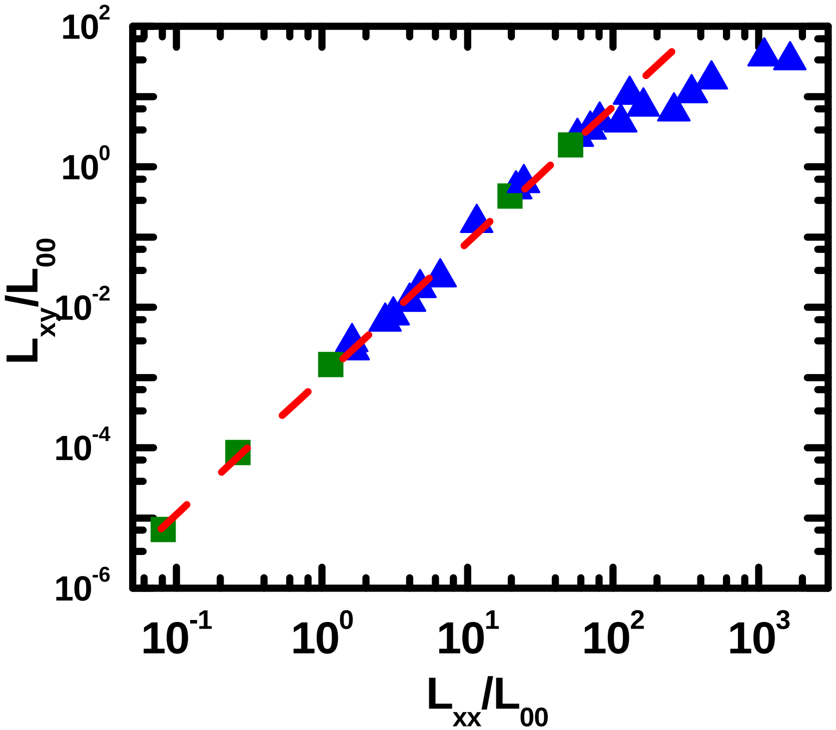
$$R_{xy} = \frac{L_{xy}}{L_{xx}^2} \approx 70\Omega$$

← More Insulating

Red points: deviation from RR scaling

# Anomalous Hall Insulator




Note: underlying $C_{60}$ layer does not affect AH resistance

**Blue points iron**

**Green points Fe/$C_{60}$ samples**

# AHE/AHI of epitaxial/granular Fe films

Scaling*: $\rho_{xy} \approx \rho_{xx}^{\gamma}$

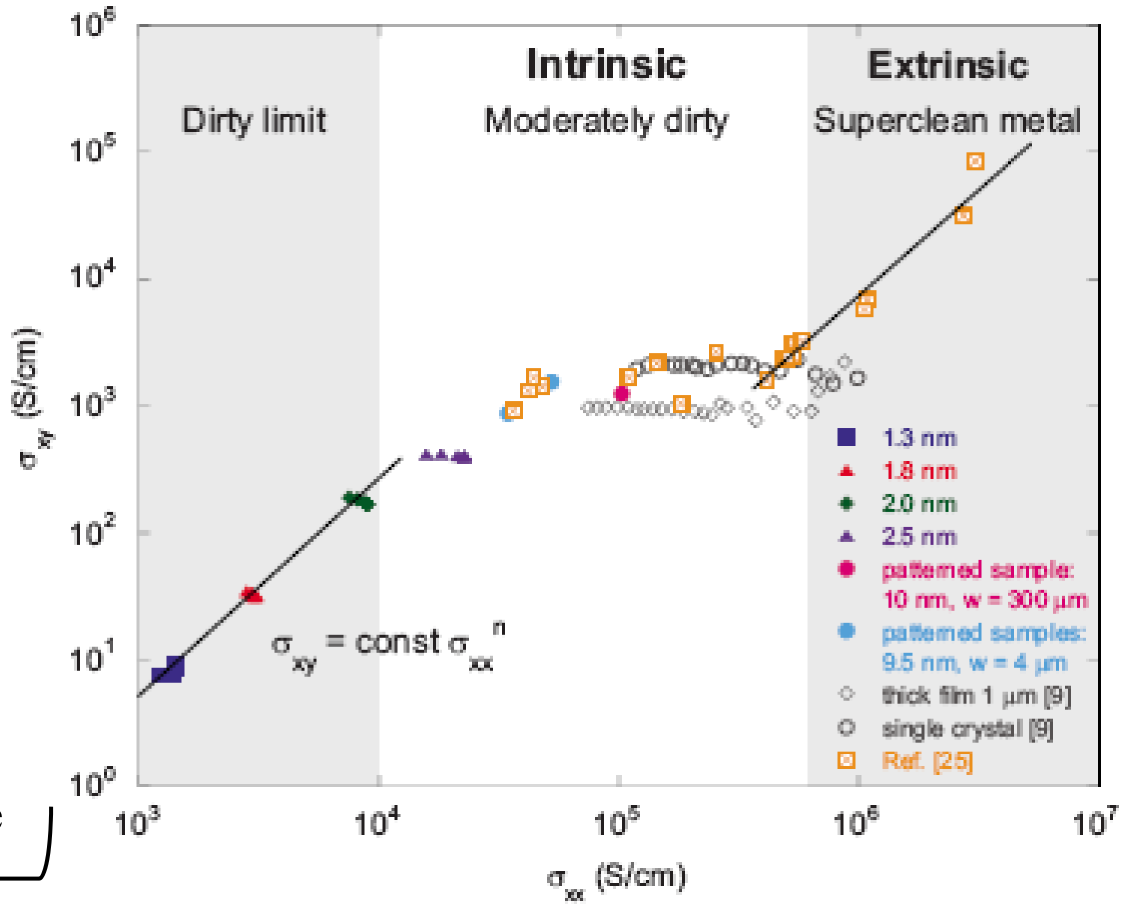
$\gamma = 2 \Rightarrow$ SJM

$\gamma = 1 \Rightarrow$ SSM

$\gamma = 0 \Rightarrow$ AHI

$\Rightarrow$ **$\rho_{xy}$ is a constant!**

***H. Meier et al. arXiv:0812.3085**




Granular (AHI) regime

$$R_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2} = R_{xx}\frac{\sigma_{xy}}{\sigma_{xx}}$$

**S. Sangiao et al. Phys. Rev. B 79, 014431 2009**

# Summary of Acts I-IV

1. WL is alive and well in itinerant ferromagnets ($\hbar/e^2$ crossover & sensitivity to morphology)
2. Spin-wave-mediated quantum corrections to conductivity of Gd films; observe a localizing linear-in-T contribution to σ. ($\hbar/e^2$ crossover)
3. Power law behavior of σ in strongly disordered films: metal-insulator transition. (compare superconductor-insulator transition?)
4. Anomalous Hall Insulator (AHI) behavior in granular films. Compare local/global behavior of superconductors and ferromagnets in extreme disorder limit.