

Supersymmetric Gauge Theories as Curved Spacetimes

Igor Klebanov

Department of Physics and PCTS



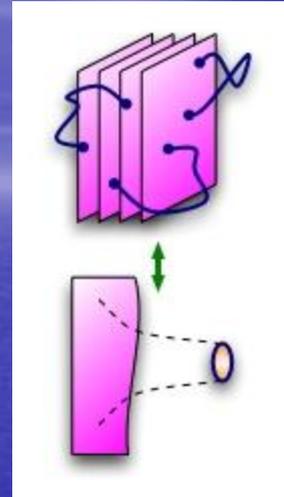
Princeton University

Talk at Shifmania
May 15, 2009

From D-branes to AdS/CFT

- A stack of N Dirichlet 3-branes realizes $\mathcal{N}=4$ supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(- (dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2)$$



which for small r approaches $AdS_5 \times S^5$

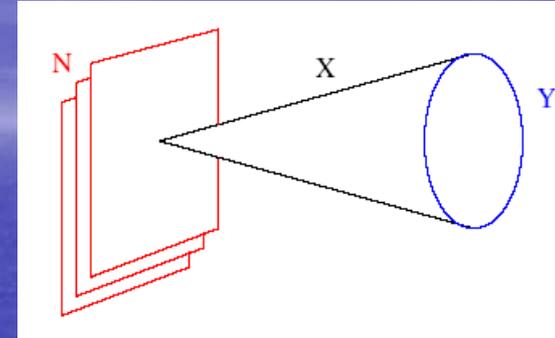
- For an introduction, see the Physics Today January 2009 article 'Solving Strongly Coupled Field theories via Curved Spacetimes' I.K., J. Maldacena.



Conebrane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y :

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



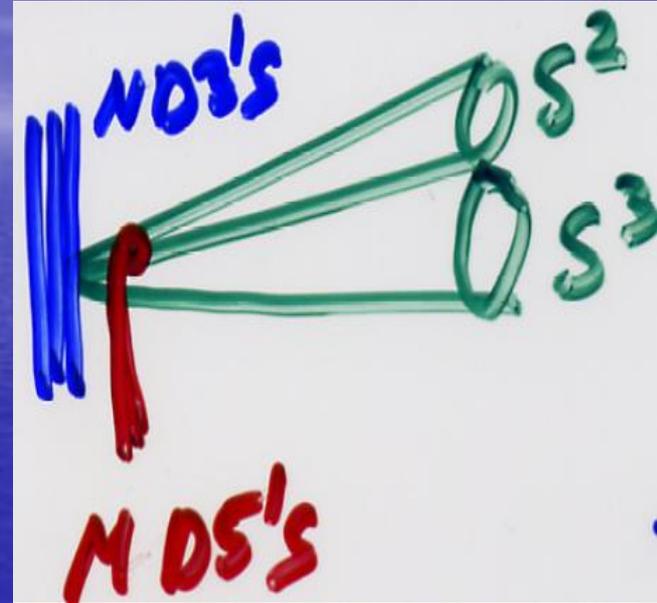
- Taking the near-horizon limit of the background created by the N D3-branes, we find the space $AdS_5 \times Y$, with N units of RR 5-form flux, whose radius is given by
- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X .

$$L^4 = \frac{\sqrt{\pi} \kappa N}{2 \text{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\text{Vol}(Y)}$$

Kachru, Silverstein; Lawrence, Nekrasov, Vafa; ...

Anatomy of a Warped Throat

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the **warped deformed conifold** (IK, Strassler)



$$ds_{10}^2 = h^{-1/2}(t) \left(- (dx^0)^2 + (dx^i)^2 \right) + h^{1/2}(t) ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^4 z_i^2 = \epsilon^2$$

- The warp factor is finite at the 'tip of the cigar' $t=0$, as required for the confinement: $h(t) = \frac{2^{-8/3}}{\gamma} I(t)$

$$I(t) = \int_t^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{1/3}, \quad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3}$$

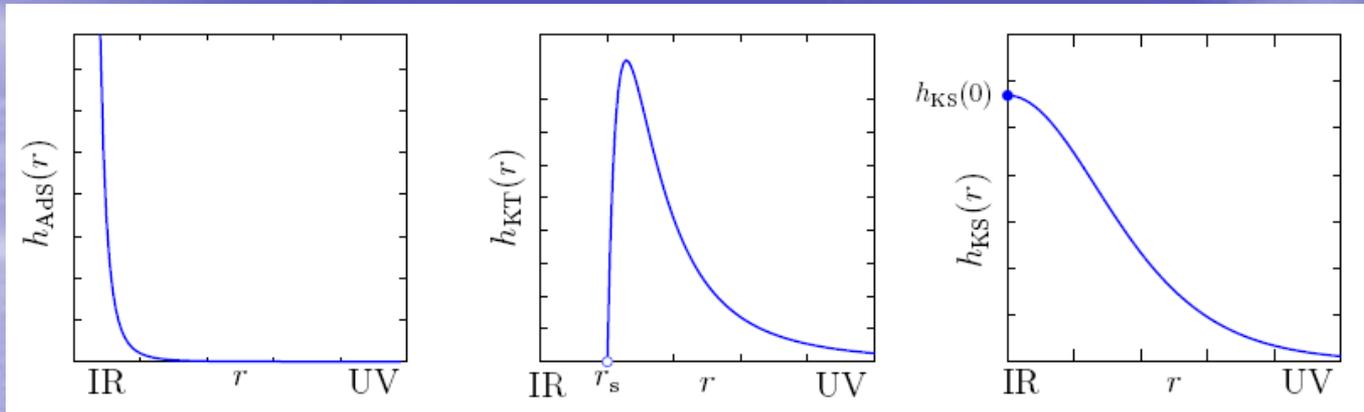
- The string tension, is proportional to $h(t)^{-1/2}$ and is minimized at $t=0$. It blows up at large t (near the boundary) where space is 'near-AdS.'
- **Dimensional transmutation** in the IR. The dynamically generated confinement scale is $\sim \varepsilon^{2/3}$
- The pattern of **R-symmetry breaking** is the same as in the $SU(M)$ SYM theory: $Z_{2M} \rightarrow Z_2$

Glauino Condensation

- Subleading terms in the 2-form field strength indicate that a dimension 3 flavor singlet fermion bilinear gets a VEV Loewy, Sonnenschein

$$\delta \left(C_2 - \frac{i}{g_s} B_2 \right) \sim \frac{M\alpha'}{4} \tau e^{-\tau} [g_1 \wedge g_3 + g_2 \wedge g_4 - i(g_1 \wedge g_2 - g_3 \wedge g_4)]$$
$$\sim \frac{M\alpha' \varepsilon^2}{r^3} \ln(r/\varepsilon^{2/3}) e^{i\psi} (d\theta_1 - i \sin \theta_1 d\phi_1) \wedge (d\theta_2 - i \sin \theta_2 d\phi_2) .$$

- The theory thus exhibits the SUGRA dual of the famous gluino condensation Novikov, Shifman, Vainshtein, Zakharov; Veneziano, Yankielowicz



- Comparison of warp factors in the AdS, warped conifold, and warped deformed conifold cases. The warped conifold solution with $\epsilon = 0$ has an unacceptable naked singularity where $h=0$.
- This is how string theory tells us that the chiral symmetry breaking and dynamical scale generation must take place through turning on the deformation ϵ . The finiteness of the warp factor at $r=0$ translates into confinement.

- The radius-squared of the S^3 at $t=0$ is $g_s M$ in string units.
- When $g_s M$ is large, the curvatures are small everywhere, and the SUGRA solution is reliable in 'solving' this confining gauge theory.
- Even when $g_s M$ is small, the curvature gets small at large t (in the UV).
- In the dual gauge theory the coupling stays strong in the UV. It is not asymptotically free, but rather undergoes a 'cascading' logarithmic RG flow.

- All of this provides us with an **exact solution** of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for studying strongly coupled gauge theory: a **'hyperbolic cow'** approximation to $\mathcal{N}=1$ supersymmetric gluodynamics.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck; Benna, Dymarsky, IK, Soloviev
- Possible applications of these models to new physics include RS warped extra dimension models, KKLT moduli stabilization in flux compactifications, as well as warped throat D-brane cosmology (KKLMMT).

What is the M2 Brane Theory?

- It is the Infrared limit of the D2-brane theory, the $\mathcal{N}=8$ supersymmetric Yang-Mills theory in 2+1 dimensions, i.e. it describes the degrees of freedom at energy much lower than $(g_{\text{YM}})^2$
- The number of such degrees of freedom $\sim N^{3/2}$ is much lower than the number of UV degrees of freedom $\sim N^2$. IK, Tseytlin
- Is there a more direct way to characterize the Infrared Scale-Invariant Theory?

The BLG Theory

- In a remarkable development, Bagger and Lambert, and Gustavsson formulated an SO(4) Chern-Simons Gauge Theory with manifest $\mathcal{N}=8$ superconformal gauge theory. In Van Raamsdonk's SU(2)xSU(2) formulation,

$$\mathcal{S} = \int d^3x \operatorname{tr} \left[-(\mathcal{D}^\mu X^I)^\dagger \mathcal{D}_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu \mathcal{D}_\mu \Psi \right. \\ \left. - \frac{2if}{3} \bar{\Psi}^\dagger \Gamma^{IJ} (X^I X^J \Psi + X^J \Psi^\dagger X^I + \Psi X^I X^J) - \frac{8f^2}{3} \operatorname{tr} X^{[I} X^{J} X^{K]} X^{\dagger[K} X^J X^{\dagger I]} \right. \\ \left. + \frac{1}{2f} \epsilon^{\mu\nu\lambda} (A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda) - \frac{1}{2f} \epsilon^{\mu\nu\lambda} (\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda) \right]$$

$$X^* = -\epsilon X \epsilon$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- X^I are the 8 fields transforming in (2,2), which is the 4 of SO(4)

$$X^I = \frac{1}{2} (x_4^I \mathbb{1} + i x_i^I \sigma^i)$$

$\mathcal{N}=2$ Superspace Formulation

- Define bi-fundamental superfields rotated by $SU(4)_{\text{flavor}}$ symmetry

$$\begin{aligned} Z &= Z(x_L) + \sqrt{2}\theta\zeta(x_L) + \theta^2 F(x_L), \\ \bar{Z} &= Z^\dagger(x_R) - \sqrt{2}\bar{\theta}\zeta^\dagger(x_R) - \bar{\theta}^2 F^\dagger(x_R) \end{aligned}$$

$$Z^{\dagger A} := -\varepsilon(Z^A)^T \varepsilon = X^{\dagger A} + iX^{\dagger A+4}$$

- The superpotential is Benna, IK, Klose, Smedback,

$$W = \frac{1}{4!} \epsilon_{ABCD} \text{tr} Z^A Z^{\dagger B} Z^C Z^{\dagger D}$$

- Using $SO(4)$ gauge group notation,

$$W = -\frac{1}{8 \cdot 4!} \epsilon_{ABCD} \epsilon^{abcd} Z_a^A Z_b^B Z_c^C Z_d^D$$

The ABJM Theory

- Aharony, Bergman, Jafferis and Maldacena argued that the correct description of a pair of M2-branes is slightly different. It involves $U(2) \times U(2)$ gauge theory.
- The $SU(4)$ flavor symmetry is not manifest because of the choice of complex combinations

$$Z^1 = X^1 + iX^5,$$

$$Z^2 = X^2 + iX^6,$$

$$W_1 = X^{3\dagger} + iX^{7\dagger}$$

$$W_2 = X^{4\dagger} + iX^{8\dagger}$$

- The manifest flavor symmetry is $SU(2) \times SU(2)$

$$W = \frac{1}{4} \epsilon_{AC} \epsilon^{BD} \text{tr} Z^A W_B Z^C W_D$$

- For N M2-branes ABJM theory easily generalizes to $U(N) \times U(N)$. The theory with Chern-Simons coefficient k is then conjectured to be dual to $AdS_4 \times S^7/Z_k$ supported by N units of flux.
- For $k > 2$ this theory has $\mathcal{N}=6$ supersymmetry, in agreement with this conjecture. In particular, the theory has manifest $SU(4)$ R-symmetry.

SU(4)_R Symmetry

- The global symmetry rotating the 6 supercharges is SO(6)~SU(4). The classical action of this theory indeed has this symmetry. Benna, IK, Klose, Smedback

$$V^{\text{bos}} = -\frac{L^2}{48} \text{tr} \left[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C \right. \\ \left. + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right]$$

$$V^{\text{ferm}} = \frac{iL}{4} \text{tr} \left[Y_A^\dagger Y^A \psi^{B\dagger} \psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2Y_A^\dagger Y^B \psi^{A\dagger} \psi_B \right. \\ \left. - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger} \right].$$

Y^A , $A=1,\dots,4$, are complex $N \times N$ matrices.

$$Y^A = \{Z^1, Z^2, W^{1\dagger}, W^{2\dagger}\}$$

Enhanced Symmetry

- For $k=1$ or 2 the global symmetry should enhance to $SO(8)$ according to the ABJM conjecture. This is not seen in the classical lagrangian but should appear in the quantum theory.
- The key to it are probably the 'monopole' operators that create singular monopole field configurations at a point. They create magnetic flux in a diagonal $U(1)$ subgroup and are charged under the remaining gauge groups.
- For $k=1$ the singly-charged operator is $(e^\tau)_{\hat{a}}^a$ and the doubly-charged one $(e^{2\tau})_{\hat{a}\hat{b}}^{ab} = (e^{2\tau})_{\hat{b}\hat{a}}^{ba}$

Relevant Deformations

- The M2-brane theory may be perturbed by relevant operators that cause it to flow to new fixed points with reduced supersymmetry. Benna, IK, Klose, Smedback; IK, Klose, Murugan; Ahn
- For example, a quadratic superpotential deformation, allowed for $k=1, 2$, may preserve $SU(3)$ flavor symmetry

$$\Delta W = m(\mathcal{Z}^4)^a_{\hat{a}}(\mathcal{Z}^4)^b_{\hat{b}}(e^{-2\tau})^{\hat{a}\hat{b}}_{ab}$$

Squashed, stretched and warped

- The dual AdS_4 background of M-theory should also preserve $\mathcal{N}=2$ SUSY and $\text{SU}(3)$ flavor symmetry. Such an extremum of gauge SUGRA was found 25 years ago by Warner. Upon uplifting to 11-d Corrado, Pilch and Warner found a warped product of AdS_4 and of a 'stretched and squashed' 7-sphere:

$$ds_{11}^2 = \Delta^{-1} ds_4^2 + 3^{3/2} L^2 \Delta^{1/2} ds_7^2(\rho, \chi), \quad \Delta \equiv (\xi \cosh \chi)^{-\frac{4}{3}}$$

- The squashing parameter is ρ ; the stretching is χ

$$ds_8^2(\rho, \chi) = g_{IJ} dx^I dx^J = dx^I Q_{IJ}^{-1} dx^J + \frac{\sinh \chi^2}{\xi^2} (x^I J_{IJ} dx^J)^2$$

$$Q = \text{diag} \{ \rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^6, \rho^6 \}$$

$$\xi^2 \equiv x^I Q_{IJ} x^J$$

- The four complex coordinates

$$z^1 = x^1 + ix^2, \quad z^2 = x^3 + ix^4, \quad z^3 = x^5 + ix^6, \quad w = x^7 - ix^8$$

$$|z^1|^2 + |z^2|^2 + |z^3|^2 + |w|^2 = 1$$

may be expressed in terms of the 7 angles.

- The equations of motion are satisfied with

$$\rho = 3^{\frac{1}{8}}, \quad \chi = \frac{1}{2} \operatorname{arccosh} 2$$

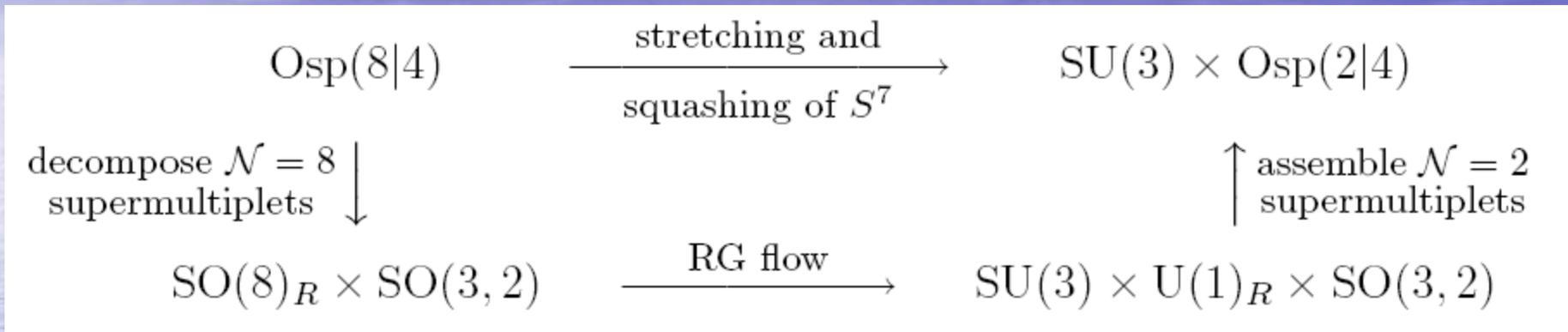
$$A_{(3)} = \frac{3^{3/4}}{4} e^{3r/L} dx^0 \wedge dx^1 \wedge dx^2 + C_{(3)} + C_{(3)}^*$$

$$C_{(3)} = \frac{3^{11/4} L^3}{4 (z^i \bar{z}_i + 3w\bar{w})} [z^{[1} dz^2 \wedge dz^3] \wedge d\bar{w} - \bar{w} dz^1 \wedge dz^2 \wedge dz^3]$$

- The internal components break parity (Englert). They preserve a flavor SU(3), and a U(1) R-symmetry

$$\frac{1}{3} (z^i \partial_{z^i} - \bar{z}_i \partial_{\bar{z}_i}) + w \partial_w - \bar{w} \partial_{\bar{w}}$$

The Spectrum via Group Theory



- There are only two ways of breaking the $\text{SO}(8)$ R-symmetry consistent with the $\text{Osp}(2|4)$ symmetry in the IR:

$$[a, b, c, d] \rightarrow \begin{cases} [a, b]_{\left(\frac{a}{3} + \frac{2b}{3} + d\right)\varepsilon} & \text{Scenario I,} \\ [a, b]_{-\left(\frac{2a}{3} + \frac{4b}{3} + c + d\right)\varepsilon} & \text{Scenario II} \end{cases}$$

	Scenario I	Scenario II
Hyper	$[n + 2, 0]_{\frac{n+2}{3}}, [0, n + 2]_{-\frac{n+2}{3}}$	$[n + 2, 0]_{-\frac{2n+4}{3}}, [0, n + 2]_{\frac{2n+4}{3}}$
Vector	$[n + 1, 1]_{\frac{n}{3}}, [1, n + 1]_{-\frac{n}{3}}$	$[n + 1, 1]_{-\frac{2n}{3}}, [1, n + 1]_{\frac{2n}{3}}$
Gravitino	$[n + 1, 0]_{\frac{n+1}{3}}, [0, n + 1]_{-\frac{n+1}{3}}$	$[n + 1, 0]_{-\frac{2n-1}{3}}, [0, n + 1]_{\frac{2n-1}{3}}$
Graviton	$[0, 0]_n, [0, 0]_{-n}$	$[0, 0]_0, [0, 0]_0$

- We find that Scenario I gives $SU(3) \times U(1)_R$ quantum numbers in agreement with the proposed gauge theory, where they are schematically given by

	Z^A	ζ^A	Z_A^\dagger	ζ_A^\dagger	Z^4	ζ^4	Z_4^\dagger	ζ_4^\dagger	x	θ	$\bar{\theta}$
SU(3)	3	3	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1	1	1	1	1	1	1
Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
R-charge	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	+1	0	-1	0	0	+1	-1

Spin-2 Perturbations

- Consider graviton perturbations in AdS

with

$$h^i{}_i = 0, \quad \partial^i h_{ij} = 0$$

$$\phi = h^i{}_j$$

satisfy the minimal scalar equation

$$\square\phi = 0$$

$$\phi = \Phi(x^i, r)Y(y^\alpha)$$

$$\square_4\Phi(r, x^i) - m^2\Phi(r, x^i) = 0$$

For the (p, q) irrep of $SU(3)$, we find the angular dependence IK, Pufu, Rocha

$$Y(y^\alpha) = a_{i_1 i_2 \dots i_p}^{j_1 j_2 \dots j_q} \left(\prod_{k=1}^p z^{i_k} \right) \left(\prod_{l=1}^q \bar{z}_{j_l} \right) w^{n_r} \\ \times \begin{cases} {}_2F_1(-j, 3 + p + q + j + n_r; 3 + p + q; 1 - w\bar{w}) & \text{if } n_r \geq 0 \\ {}_2F_1(-j + n_r, 3 + p + q + j; 3 + p + q; 1 - w\bar{w}) & \text{if } n_r < 0. \end{cases}$$

- The R-charge is

$$R = \frac{1}{3}(p - q) + n_r$$

- For the j-th KK mode the mass-squared is

$$m^2 = \frac{1}{L^2} \left[2j^2 + 2j|n_r| + n_r^2 + 2j(p + q + 3) + \frac{1}{3}n_r(p - q) + |n_r|(3 + p + q) + \frac{1}{9}(p^2 + q^2 + 4pq + 15p + 15q) \right].$$

- The operator dimension is determined by

$$\Delta(\Delta - 3) = m^2 L^2$$

- For operators in the MGRAV and SGRAV multiplets

$$\Delta = |R| + 3$$



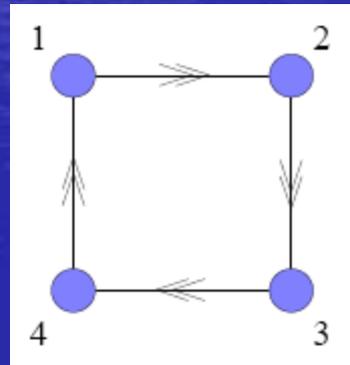
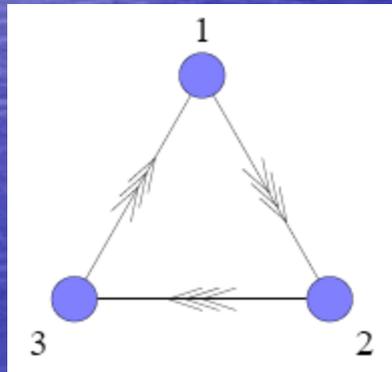
- Here are the low lying operators

$$T_{\alpha\beta}^{(0)} = \bar{D}_{(\alpha} \bar{Z}_A D_{\beta)} Z^A + i \bar{Z}_A \overleftrightarrow{\partial}_{\alpha\beta} Z^A$$

$[p, q]_R$	j	n_r	Δ	$m^2 L^2$	Operator
* $[0, 0]_0$	0	0	3	0	$T_{\alpha\beta}^{(0)}$
* $[0, 0]_{\pm 1}$	0	± 1	4	4	$T_{\alpha\beta}^{(0)} Z^A, T_{\alpha\beta}^{(0)} \bar{Z}_4$
$[0, 1]_{-\frac{1}{3}}, [1, 0]_{\frac{1}{3}}$	0	0	$\frac{1}{6}(9 + \sqrt{145})$	$\frac{16}{9}$	$T_{\alpha\beta}^{(0)} \bar{Z}_A, T_{\alpha\beta}^{(0)} Z^A$
* $[0, 0]_{\pm 2}$	0	± 2	5	10	$T_{\alpha\beta}^{(0)} (Z^4)^2, T_{\alpha\beta}^{(0)} (\bar{Z}_4)^2$
$[0, 0]_0$	1	0	$\frac{1}{2}(3 + \sqrt{41})$	8	$T_{\alpha\beta}^{(0)} (1 - 4a^2 Z^4 \bar{Z}_4)$
$[0, 1]_{-\frac{4}{3}}, [1, 0]_{\frac{4}{3}}$	0	-1, 1	$\frac{1}{6}(9 + \sqrt{337})$	$\frac{64}{9}$	$T_{\alpha\beta}^{(0)} \bar{Z}_A \bar{Z}_4, T_{\alpha\beta}^{(0)} Z_A Z^4$
$[0, 1]_{\frac{2}{3}}, [1, 0]_{-\frac{2}{3}}$	0	-1, 1	$\frac{1}{6}(9 + \sqrt{313})$	$\frac{58}{9}$	$T_{\alpha\beta}^{(0)} \bar{Z}_A Z^4, T_{\alpha\beta}^{(0)} Z_A \bar{Z}_4$
$[0, 2]_{-\frac{2}{3}}, [2, 0]_{\frac{2}{3}}$	0	0	$\frac{1}{6}(9 + \sqrt{217})$	$\frac{34}{9}$	$T_{\alpha\beta}^{(0)} \bar{Z}_{(A} \bar{Z}_{B)}, T_{\alpha\beta}^{(0)} Z^{(A} Z^{B)}$
$[1, 1]_0$	0	0	4	4	$T_{\alpha\beta}^{(0)} (Z^A \bar{Z}_B - \frac{1}{3} \delta_B^A Z^C \bar{Z}_C)$
$[0, 0]_{\pm 1}$	1	± 1	$\frac{1}{2}(3 + \sqrt{65})$	14	$T_{\alpha\beta}^{(0)} (2 - 5a^2 Z^4 \bar{Z}_4) Z^4, \text{c.c.}$
* $[0, 0]_{\pm 3}$	0	± 3	6	18	$T_{\alpha\beta}^{(0)} (Z^4)^3, T_{\alpha\beta}^{(0)} (\bar{Z}_4)^3$
$[1, 0]_{-\frac{5}{3}}, [0, 1]_{\frac{5}{3}}$	0	-2, +2	$\frac{1}{6}(9 + \sqrt{553})$	$\frac{118}{9}$	$T_{\alpha\beta}^{(0)} Z^A (\bar{Z}_4)^2, T_{\alpha\beta}^{(0)} \bar{Z}_A (Z^4)^2$
$[1, 0]_{\frac{1}{3}}, [0, 1]_{-\frac{1}{3}}$	1	0	$\frac{1}{6}(9 + \sqrt{505})$	$\frac{106}{9}$	$T_{\alpha\beta}^{(0)} Z^A (1 - 5a^2 \bar{Z}_4 Z^4), \text{c.c.}$
$[1, 0]_{\frac{7}{3}}, [0, 1]_{-\frac{7}{3}}$	0	2, -2	$\frac{1}{6}(9 + \sqrt{601})$	$\frac{130}{9}$	$T_{\alpha\beta}^{(0)} Z^A (Z^4)^2, T_{\alpha\beta}^{(0)} \bar{Z}_A (\bar{Z}_4)^2$
$[1, 1]_{\pm 1}$	0	± 1	5	10	$T_{\alpha\beta}^{(0)} (Z^A \bar{Z}_B - \frac{1}{3} \delta_B^A Z^C \bar{Z}_C) Z^4, \text{c.c.}$
$[2, 0]_{-\frac{1}{3}}, [0, 2]_{\frac{1}{3}}$	0	-1, 1	$\frac{1}{6}(9 + \sqrt{409})$	$\frac{82}{9}$	$T_{\alpha\beta}^{(0)} Z^{(A} Z^{B)} \bar{Z}_4, T_{\alpha\beta}^{(0)} \bar{Z}_{(A} \bar{Z}_{B)} Z^4$
$[2, 0]_{\frac{5}{3}}, [0, 2]_{-\frac{5}{3}}$	0	1, -1	$\frac{1}{6}(9 + \sqrt{457})$	$\frac{94}{9}$	$T_{\alpha\beta}^{(0)} Z^{(A} Z^{B)} Z^4, T_{\alpha\beta}^{(0)} \bar{Z}_{(A} \bar{Z}_{B)} \bar{Z}_4$
$[2, 1]_{\frac{1}{3}}, [1, 2]_{-\frac{1}{3}}$	0	0	$\frac{1}{6}(9 + \sqrt{313})$	$\frac{58}{9}$	$T_{\alpha\beta}^{(0)} (Z^{(A} Z^{B)} \bar{Z}_C - \frac{1}{3} \delta_C^{(A} Z^{B)} Z^D \bar{Z}_D), \text{c.c.}$
$[3, 0]_1, [0, 3]_{-1}$	0	0	$\frac{1}{2}(3 + \sqrt{33})$	6	$T_{\alpha\beta}^{(0)} Z^{(A} Z^{B} Z^{C)}, T_{\alpha\beta}^{(0)} \bar{Z}_{(A} \bar{Z}_{B} \bar{Z}_{C)}$

Further Directions

- Other examples of $\text{AdS}_4/\text{CFT}_3$ dualities with $\mathcal{N}=1,2,3,\dots$ supersymmetry are being studied by many groups.
- Various famous quivers assume new identities: M^{111} , Q^{222} , etc.



- Ultimate Hope: to find a 'simple' dual of a 3-d fixed point realized in Nature.

Conclusions

- Placing D3-branes at the tip of a CY cone, such as the conifold, leads to AdS/CFT dualities with $\mathcal{N}=1$ SUSY.
- Adding wrapped D5-branes at the apex produces a cascading confining gauge theory whose duals are warped deformed conifolds.
- This example of gauge/string duality gives a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space.

- Theories arising on M2-branes placed at the tip of a cone are now understood as supersymmetric Chern-Simons Gauge Theories.
- A rich spectrum of AdS_4/CFT_3 dualities is being explored.

Happy Birthday, Misha!
And Many Happy Returns...

