Chiral gauge dynamics and dynamical supersymmetry breaking

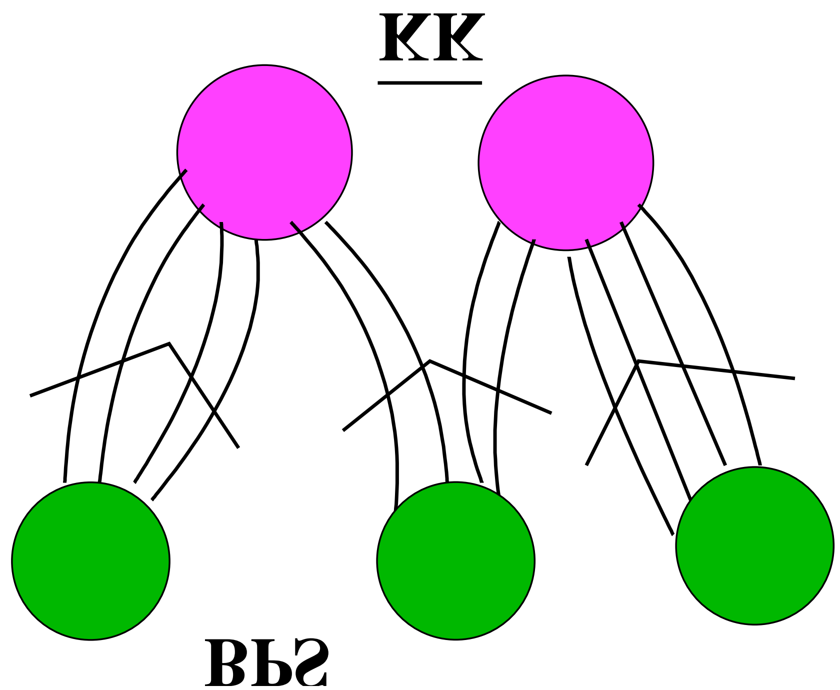
Shifmania, 14 May 2009

Mithat Ünsal, SLAC, Stanford University

arXiv:0905.0634 [hep-th],

joint work with Erich Poppitz,
based on earlier works with Misha Shifman

Happy birthday Misha!



(Non-susy) chiral gauge theories

Well-defined, free of internal anomalies, may have relevance to Nature. Pert. th. works, however,

Large N not so useful so far.

Lattice does not work.

String theory was not useful so far either.

No dynamical framework to address their dynamics until last year.

‘t Hooft’s beautiful ideas.... Dimopoulos, Raby, Susskind: (early 80’s) advocated their relevance for phenomenology, the possibility of light composites fermions. Early proposals of Dynamical Susy Breaking also involve many chiral examples.

Recent progress

- A new method to study the non-perturbative dynamics of (non-susy) non-abelian gauge theories, which we refer to as deformation theory is developed.

- Deformation theory usefully applies to chiral gauge theories. The first dynamical framework to address non-perturbative chiral dynamics.

- This talk: SU(2) susy and non-susy chiral gauge theory with I=3/2 matter, both of which circumvented all non-perturbative controllable approaches so far.

Deformation theory

to make the non-susy world as comfortable as superworld.

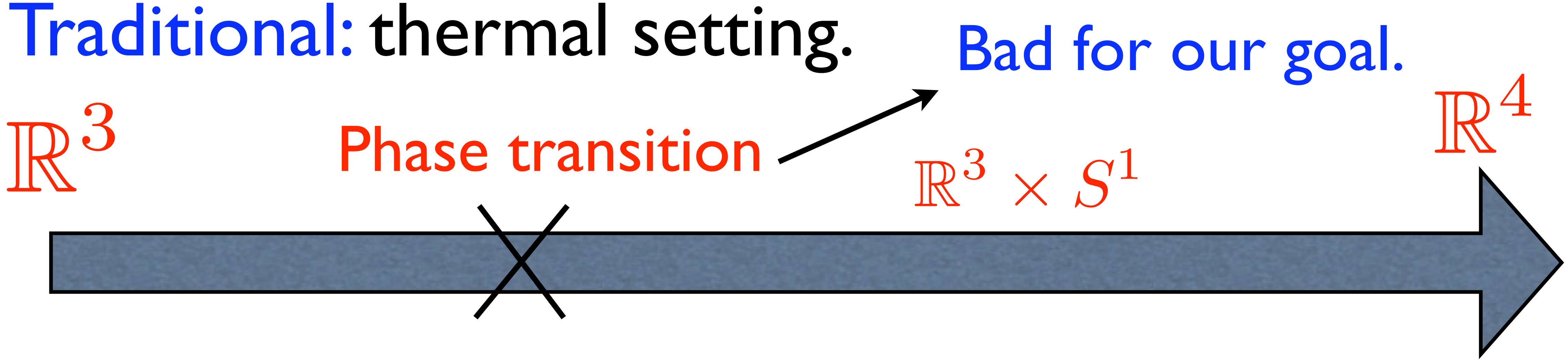
A quick overview of main idea

Theories on $\mathbb{R}^4$ (target theories , but hard)

Keep locally d=4 such as $\mathbb{R}^3 \times S^1$

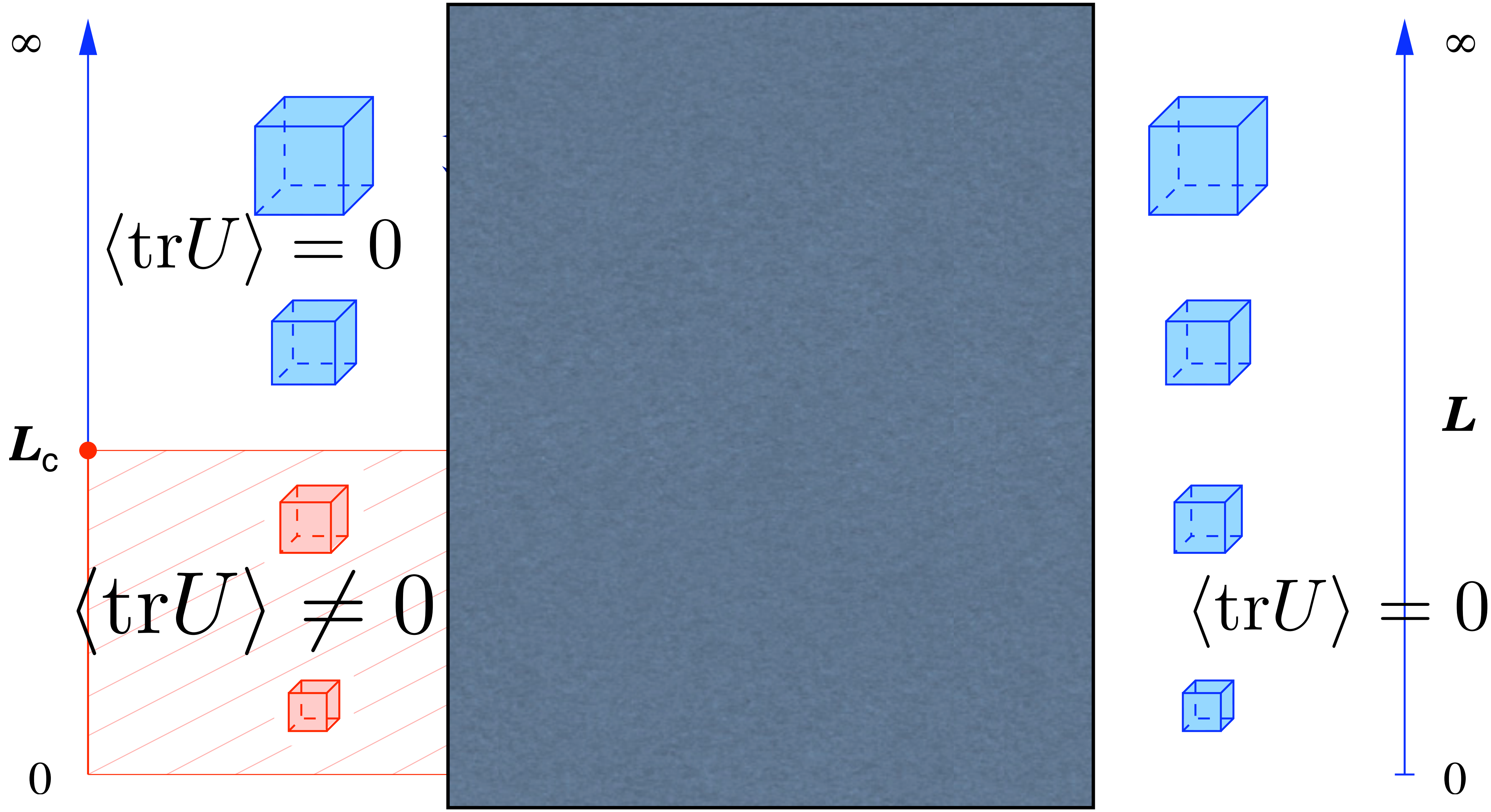
Take advantage of circle (as control parameter).
Small circle, AF and weak coupling:

Traditional: thermal setting.

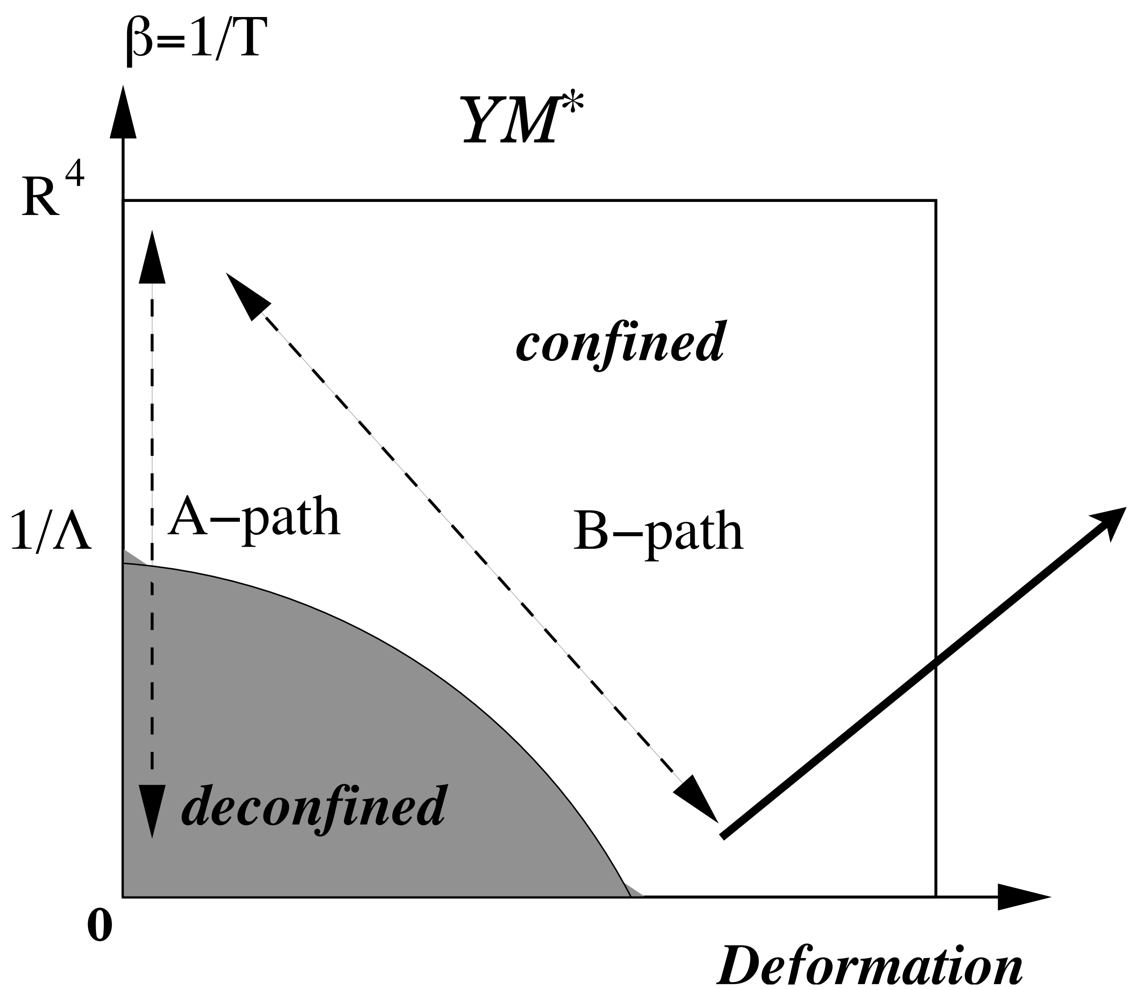


Order Parameter/center symmetry

ordinary Yang–Mills deformed Yang–Mills



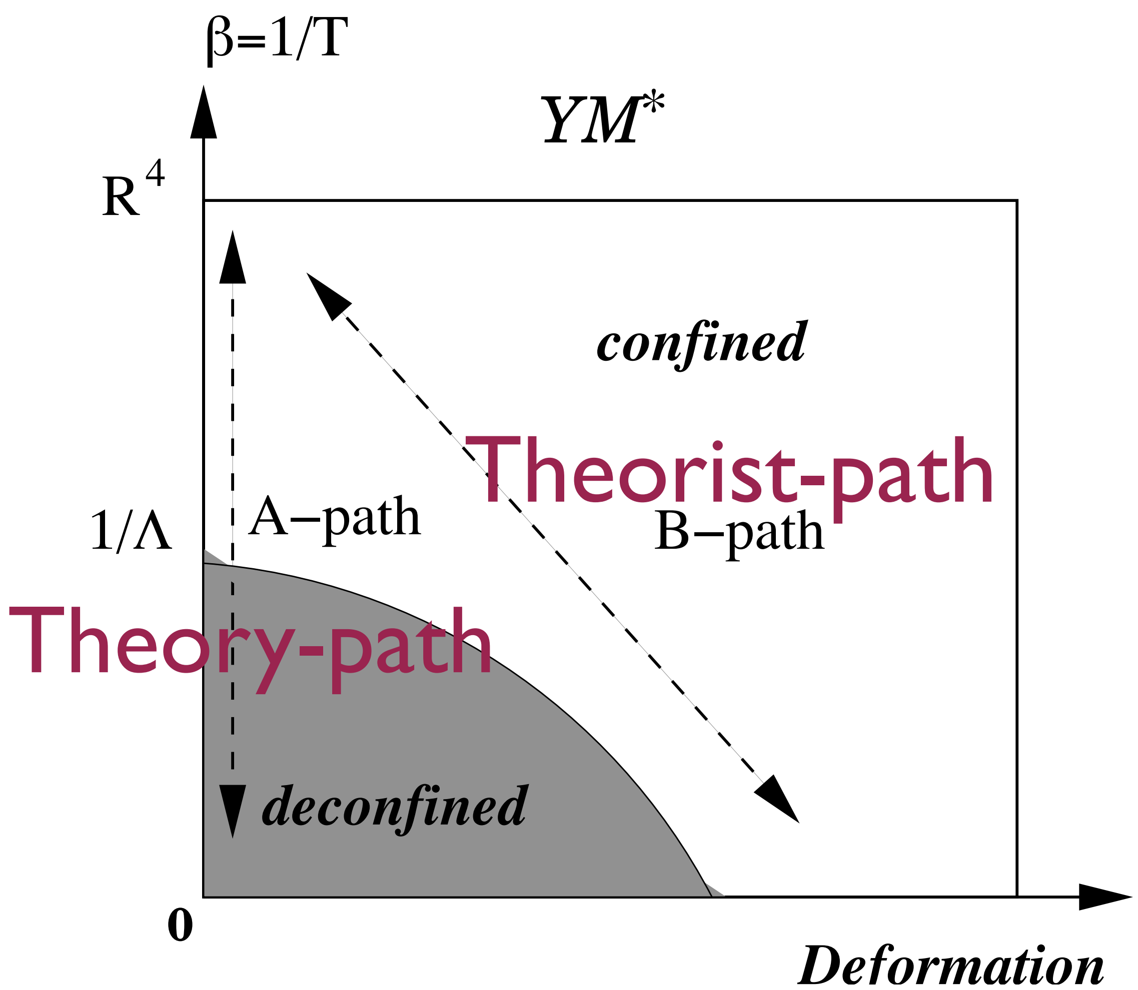
Deformed YM theory at finite N



Small volume theory becomes solvable in the same sense as Polyakov model or Seiberg-Witten theory by abelian duality.

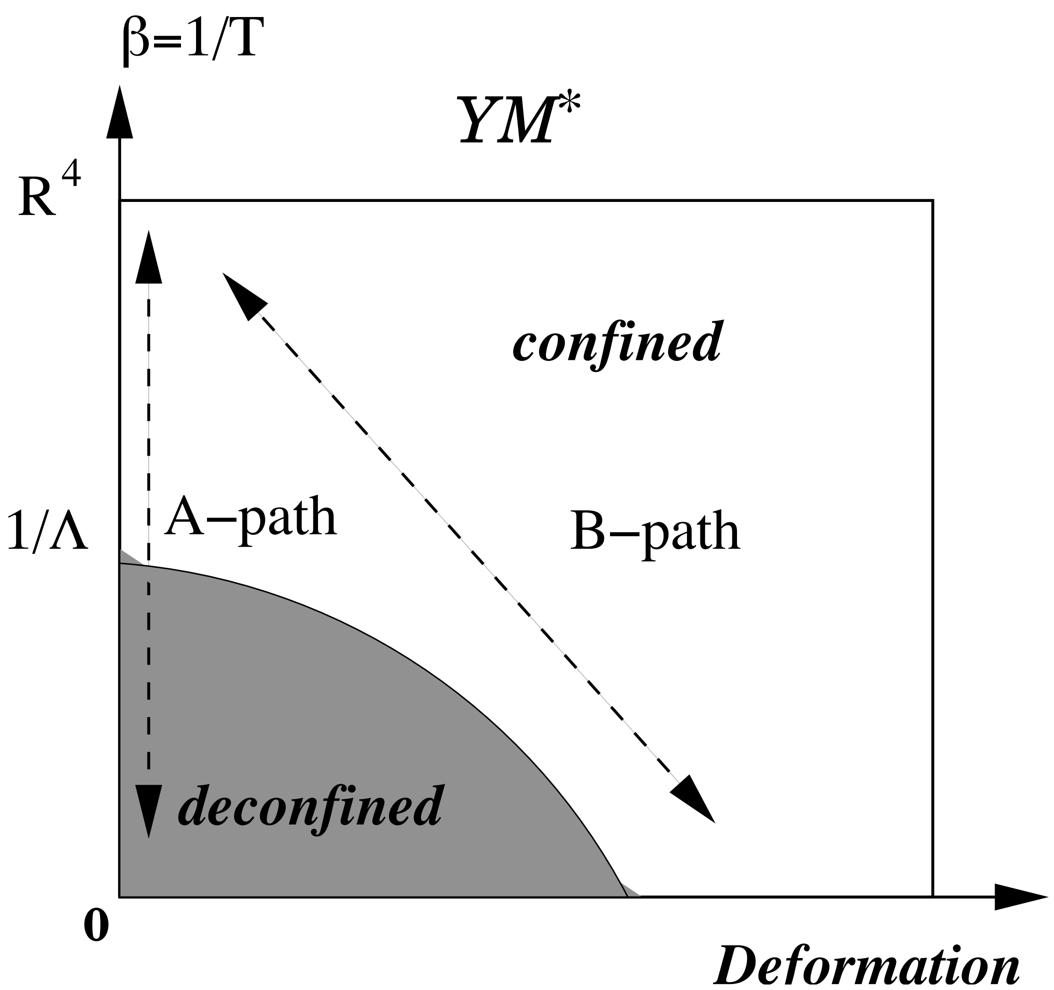
One can show the mass gap and linear confinement. Although the region of validity does not extend to large circle, it is continuously connected to it with no gauge invariant order parameter distinguishing the two regimes.

Deformed YM theory at finite N



Deformed YM theory at finite N

Shifman-MU, Yaffe-MU



$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

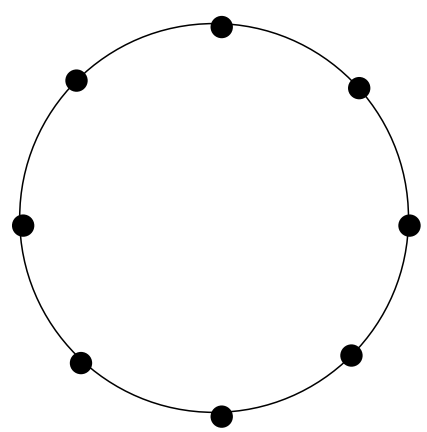
$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left| \mathrm{tr}\left(U^n\right)\right|^2$$

Lattice studies by Ogilvie, Myers, Meisinger backs-up the smoothness conjecture.

Ogilvie, Myers also independently proposed the above deformations to study phases of partially broken center.

Deformation and Polyakov loop

With deformation, eigenvalues repel. Minimum at



$$U = \mathrm{Diag}(1, e^{i2\pi/N}, \ldots, e^{i2\pi(N-1)/N})$$

$$\langle \mathrm{tr} U \rangle = 0$$

At weak coupling, the fluctuations are small, a "Higgs regime"

$$SU(N) \to [U(1)]^{N-1}$$

Georgi-Glashow model with compact adjoint Higgs field.

Compactness implies N types of monopoles, rather than N-1.

Long-distance 3d dual theory

$$S^{\text{dual}} = \int_{\mathbb{R}^3} \left[\frac{1}{2L} \left(\frac{g}{2\pi} \right)^2 (\nabla\sigma)^2 - \zeta \sum_{i=1}^{N} \cos(\alpha_i \cdot \sigma) \right].$$

Maxwell term

Monopole Operator

Abelian duality

$$F_{\mu\nu}^{(j)} = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\rho} \partial_\rho \sigma^j$$

monopole due to
compactness of Higgs scalar

Lee, Yi, Kraan, vanBaal

Monopole charges $\Delta^0_{\text{aff}} \equiv \{\underbrace{\alpha_1, \alpha_2, \ldots, \alpha_{N-1}}_{\text{usual N-1 monopoles}}, \overline{\alpha_N}\}.$

Use deformation theory to study chiral dynamics.

What is the mechanism of confinement, if it confines? Is it different from vector-like theories or YM theory?

What is the realization of chiral symmetry?

Will discuss these in simplest chiral gauge theory in terms of rank-matter content.

Chiral SU(2) with J=3/2

Well-defined, gauge and global (Witten) anomaly free.
No framework to address its dynamics until recently.
SUSY version: Simplest dynamical ~~SUSY~~?
(Controversial: Intriligator, Seiberg, Shenker, 94, Vainshtein, Shifman 99, (b0=1),
Intriligator (a-max.) conformal window, Erich Poppitz, MU 09, (circle deformation)

Instantons:

$$I(x) = e^{-S_{\rm inst}}\psi^{10}$$

Symmetry:

$$\mathbb{Z}_{10}: \psi \to e^{i\frac{2\pi k}{10}}\psi,$$

Shifman, M.U. 08 for new techniques applied to chiral quiver gauge theories,
Poppitz, MU, relatively simpler applications.

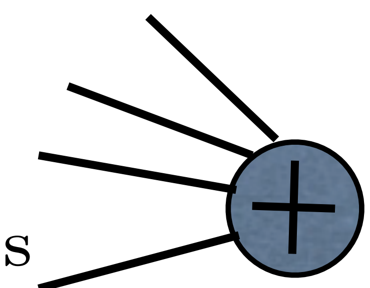
In theories with massless fermions,

such as

SU(2) chiral theory

monopoles operators has fermionic zero modes.

$$e^{-S_0} e^{i\sigma} \qquad \underbrace{\psi \ldots \psi}_{\text{fermion zero modes}}$$



Hence, cannot generate confinement and mass gap.

(Unlike monopole-instantons of Polyakov mechanism)

Is a new mechanism at work? What is the index?

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

Appendix A. Adiabatic limits of η-invariants

$$\begin{aligned} \operatorname{ind}\,(D_{\mathbb{A}}^{+}) &= \int_X \operatorname{ch}(\mathbb{E}) + \frac{1}{\mu_0} \sum_{\mu} \epsilon_\mu c_1(E_\mu)[S^2_\infty] \\ &= \int_X \operatorname{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\lim} \end{aligned} \tag{22}$$

Last formula in the paper.

Indexology

- Thankfully, in collaboration with Erich Poppitz, we were able to re-derive the Nye-Singer formula by using quantum field theory methods and in full generality such that it will be useful for concrete gauge theory applications. We were also able to get useful numbers.

- Some relevant index theorems...

- $\mathbb{R}^4$: Atiyah- M. I. Singer 75

- $\mathbb{R}^3$ Callias 78 E. Weinberg 80 (physics derivation)

- $\mathbb{R}^3 \times S^1$ Nye- A. M. Singer 00, Poppitz-MU 08

- Our strategy was similar to E.Weinberg. Results interpolates nicely.

Indexology

- Thankfully, in collaboration with Erich Poppitz, we were able to re-derive the Nye-Singer formula by using quantum field theory methods and in full generality such that it will be useful for concrete gauge theory applications.

  We were also able to get useful numbers.

$$\mathcal{I}_1 = 4, \qquad \mathcal{I}_2 = 6, \qquad \mathcal{I}_{\rm inst} = \mathcal{I}_1 + \mathcal{I}_2 = 10.$$

Erich Poppitz, MU: **Index theorem for topological excitations on R*3 * S*1 and Chern-Simons theory.**

arXiv:0812.2085 [hep-th]

Chiral SU(2) with J=3/2

relevant index theorem, Nye-Singer, 00
Poppitz, MU 08

Monopole operators

$$\mathcal{M}_1 = e^{-S_0} e^{i\sigma} \psi^4, \qquad \overline{\mathcal{M}}_1 = e^{-S_0} e^{-i\sigma} \bar{\psi}^4,$$

$$\mathcal{M}_2 = e^{-S_0} e^{-i\sigma} \psi^6, \qquad \overline{\mathcal{M}}_2 = e^{-S_0} e^{i\sigma} \bar{\psi}^6,$$

Topological symmetry

$$\mathbb{Z}_5 : \qquad \psi^4 \to e^{i\frac{2\pi}{5}} \psi^4, \qquad \sigma \to \sigma - \frac{2\pi}{5}$$

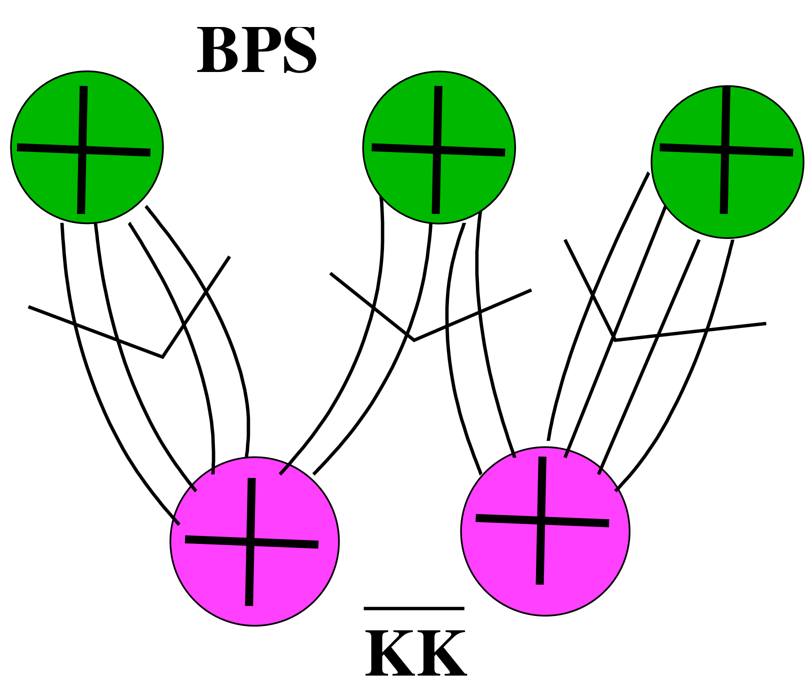
$$(\mathbb{Z}_5)_*$$

Mass gap magnetic quintet op. $\quad e^{-5S_0} \cos 5\sigma$

In the absence of fermion zero modes, the constituents of the magnetic quintet interact repulsively.

$$[\mathcal{M}_1]^3[\overline{\mathcal{M}_2}]^2 \equiv [\mathrm{BPS}]^3[\overline{\mathrm{KK}}]^2$$

$$\left(\int_{S^2_\infty} B, \ \int F\widetilde{F}\right) = \left(\pm 5, \ \pm\frac{1}{2}\right)$$



"The magnetic quintet" leading topological excitation that leads to confinement in non-susy chiral theory. (Testable on lattice)

Supersymmetric chiral SU(2)with I = 3/ 2 matter

ISS(henker) SU(2) susy-breaking proposal

Instanton operator: $I(x) = e^{-S_{\rm inst}} \psi^{10} \lambda^4, \quad U(1)_R$

$$[\lambda] = +1,$$

$$[Q] = \frac{3}{5}, \qquad [\psi] = -\frac{2}{5},$$

$$u = Q^4$$

$$[u] = \frac{12}{5}, \qquad [\psi_u] = [q^3 \psi] = \frac{7}{5}\ .$$

$W = c u^{5/6} \Lambda^{-1/3}$ allowed by symmetries but bad weak-coupling, so c=0.

If theory confines, with u - the single massless composite saturating 't Hooft (as is easily checked), adding W = u gives "simplest" susy breaking theory.

Does it? Hard to be sure. None of the usual SUSY deformations works!

Does circle deformation-the only available tool-say anything?

Index theorem and monopole operators

$$\mathcal{I}_1 = (4\psi, 2\lambda), \;\; \mathcal{I}_2 = (6\psi, 2\lambda), \;\; \mathcal{I}_{\rm inst} = (10\psi, 4\lambda) \; .$$

$$\mathcal{M}_1 = e^{-S_0} e^{-\phi + i\sigma} \psi^4 \lambda^2, \;\; \overline{\mathcal{M}}_1 = e^{-S_0} e^{-\phi - i\sigma} \bar{\psi}^4 \bar{\lambda}^2,$$

$$\mathcal{M}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^6 \lambda^2, \;\; \overline{\mathcal{M}}_2 = e^{-S_0} e^{+\phi + i\sigma} \bar{\psi}^6 \bar{\lambda}^2 \; ,$$

Compare with monopole operators in non-susy theory. One major difference, under U(1)_R:

$$\psi^4 \lambda^2 \longrightarrow e^{i\frac{2\alpha}{5}} \psi^4 \lambda^2, \qquad \psi^6 \lambda^2 \longrightarrow e^{-i\frac{2\alpha}{5}} \psi^6 \lambda^2 \; .$$

The invariance of monopole operator demands that the U(1)_R to intertwine with the topological continuous shift symmetry of the dual photon.)

$$\sigma \longrightarrow \sigma - \frac{2}{5}\alpha, \qquad [Y] = -\frac{2}{5}$$

An explicit mass term (such as magnetic quintet operators) for dual photon is forbidden.

More systematically, let us start in 3d, work our way "up" to 4d.

similar symmetry arguments in Aharony, Intriligator, Hanany, Seiberg, Strassler 97

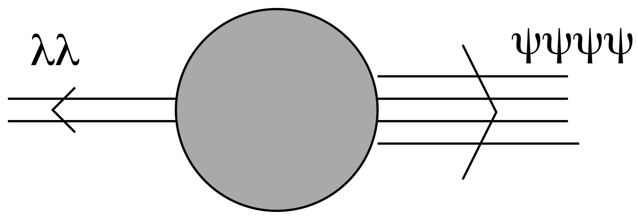
	$[U(1)_{R'}]_*$	$[U(1)_A]_*$
λ	1	0
ψ	-1	1
Q	0	1
Y	2	-4

$$Y \sim e^{-\phi + i\sigma}, \qquad u = Q^4$$

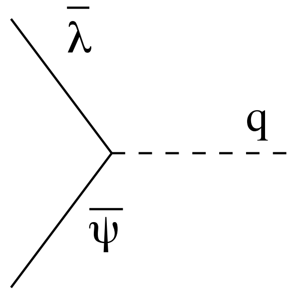
$$W[Y, u] = b\, Y u\ .$$ Symmetry allows. Is it there?

Microscopic origin of superpotential and modified monopole operators

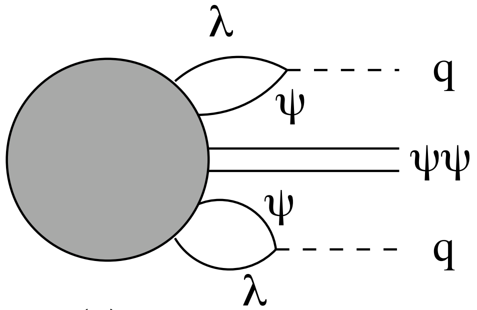
	$[U(1)_{R'}]_*$	$[U(1)_A]_*$
λ	1	0
ψ	-1	1
Q	0	1
Y	2	-4



(a)



(b)



(c)

$$Y \sim e^{-\phi+i\sigma}, \qquad u = Q^4$$

$$W[Y,u] = b\, Y u\ .$$

Yukawa lifting

$$e^{-S_0} e^{-\phi+i\sigma} \psi^4 \lambda^2(x) \left(\int d^3y\, q \bar{\lambda} \bar{\psi}(y) \right)^2 \longrightarrow \widetilde{\mathcal{M}}_1 \equiv e^{-S_0} e^{-\phi+i\sigma} q^2 \psi^2\ .$$

$$W[Y,Q] \sim YQ^4, \qquad \widetilde{\mathcal{M}}_1 = \frac{\partial^2 W}{\partial q^2} \psi\psi\ , \quad V_F(\phi,q) \sim e^{-2S_0} e^{-2\phi} q^6 (1 + \mathcal{O}(q^2))$$

Coulomb branch not lifted

No region in moduli space where both Y and u are both light.
Higgs branch: gauge multiplet is heavy,   Coulomb branch: U is heavy. How about the origin?

Micro/macro discrete parity anomalies mismatch. (b=0)

$$k_{R'R'} = \frac{1}{2}\left[3(1)^2 + 4(-1)^2\right] = \frac{7}{2} \in Z + \frac{1}{2}$$
$$k_{R'R'} = \frac{1}{2}\left[1(1)^2 + 1(-1)^2\right] = 1 \in Z$$

At the origin, need new degrees of freedom.

Most likely a CFT of strongly coupled quarks and gluons on $\mathbb{R}^3$

Chiral theory on $\mathbb{R}^3 \times S^1$

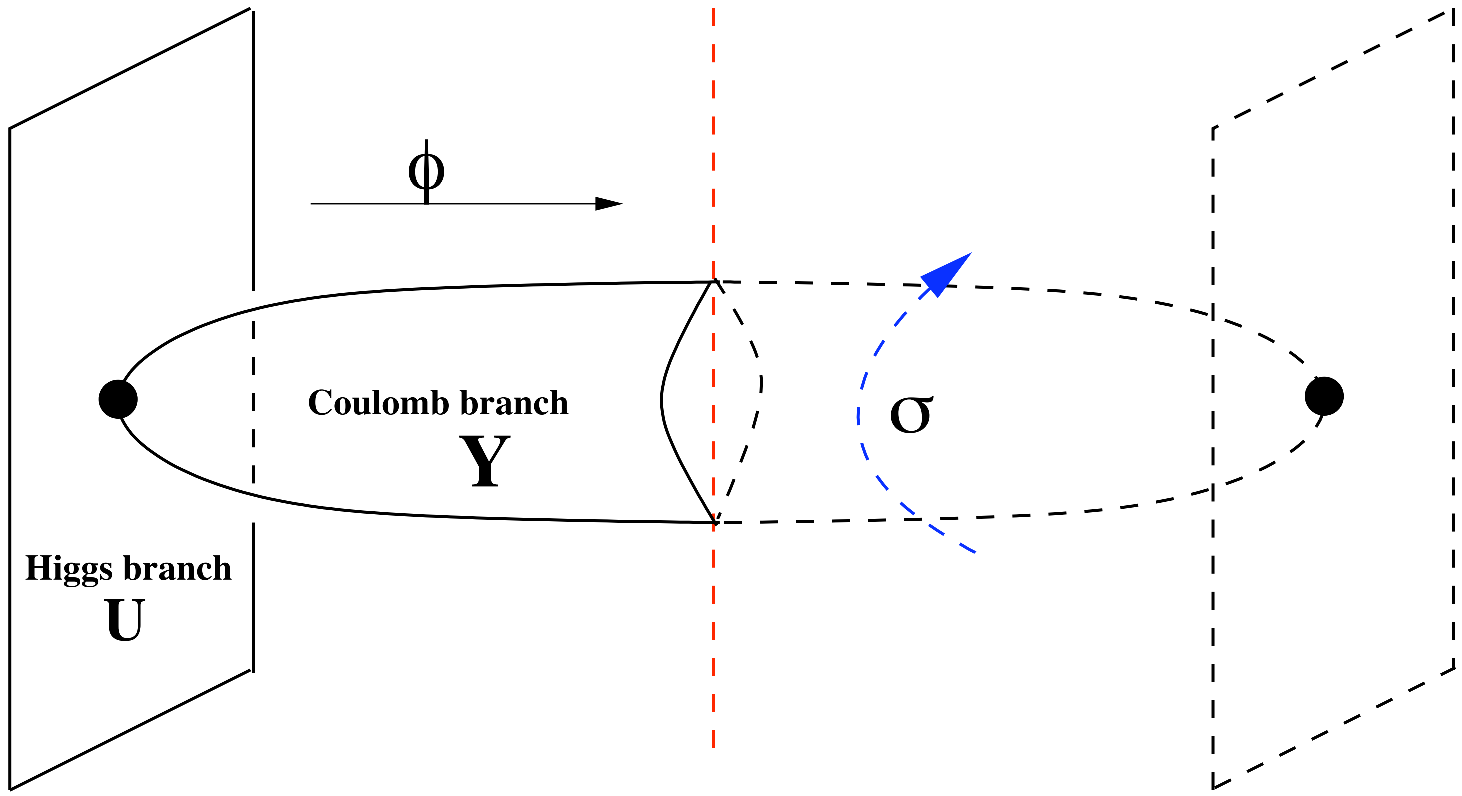
Yukawa lifting

$$\mathcal{M}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^6 \lambda^2 \longrightarrow \widetilde{\mathcal{M}_2} = e^{-S_0} e^{+\phi - i\sigma} \psi^4 q^2,$$

too many zero modes to contribute to the superpotential.

$$W_{\mathbb{R}^3 \times S^1}[u] = W_{\mathbb{R}^4}[u] = 0$$

Moduli space on $\mathbb{R}^3 \times S^1$



Decompactification and ~~SUSY~~ ?

In supersymmetric theories there is some lore (no theorem, though) about the absence of phase transitions, based on holomorphy and the ensuing fact that singularities of the superpotential and the holomorphic gauge coupling are of codimension two and therefore one can always ``go around" them. Seiberg, Witten:94, Intriligator, Seiberg:94

Currently, there is no known example of susy gauge theories with periodic spin connections undergoing a phase transition as a function of compactification radius. Thus, we believe, we have strong evidence which indicates that the theory on decompactification limit is as well a CFT.

The theory at the origin of moduli space does not confine. Hence, W=u is quite irrelevant and its addition does not alter the long distance dynamics. Hence, no SUSY breaking in this model.

$N_f = 2 \;\; SU(2) \;\; \mathrm{SQCD} \qquad \mathbb{R}^3 \times S^1$

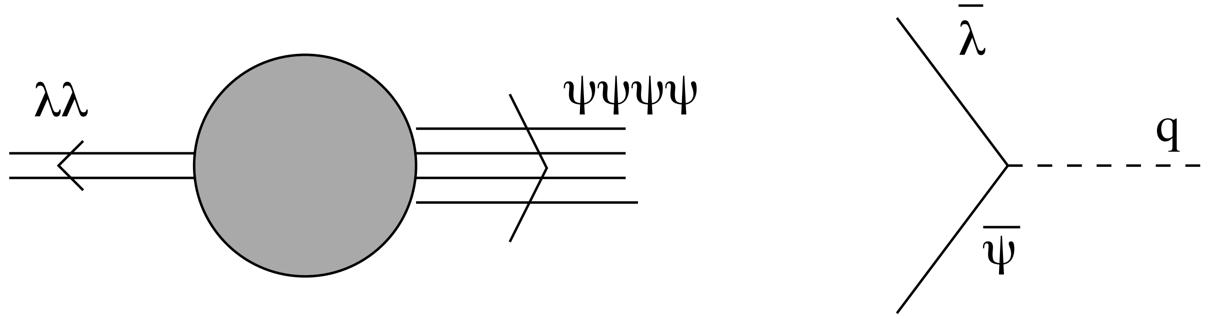
a microscopic derivation of   Aharony, Intriligator, Hanany, Seiberg, Strassler 97, 4-doublet model

$$\widetilde{\mathcal{M}}_1 + \mathcal{M}_2 = e^{-S_0} e^{-\phi + i\sigma} (q_1 q_2 \psi_3 \psi_4 + \ldots) + e^{+\phi - i\sigma} \lambda^2 \; ,$$

$\mathbb{R}^3$

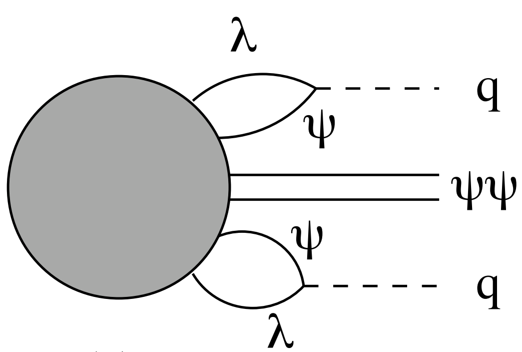
$$W = -Y \; \mathrm{Pf}(M) + \eta Y \; .$$

$\mathbb{R}^3$



(a) (b)

$$M_{ab} = Q_a \cdot Q_b$$



(c)

$\mathbb{R}^3$ ‘t Hooft parity anomalies match to Y and M
cubic W,   CFT of Y and M

$\mathbb{R}^3 \times S^1$ KK-monopole lift the Coulomb branch

Integrate out Y gives quantum modified constraint.

4d physics is reproduced on circle compactification.

Intriligator-Thomas-Izawa-Yanagida model

$$\delta W = \lambda S_{ij} M^{ij} \ .$$

$$W = (-\,\mathrm{Pf}(M) + \eta) Y + \lambda S_{ij} M^{ij} \ . \qquad \mathbb{R}^3 \times S^1$$

Susy is dynamically broken.

4d physics of DSB is reproduced on circle compactification.

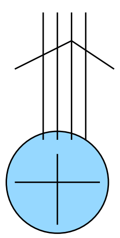
Conclusions

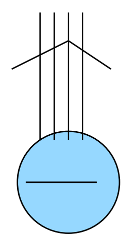
- Center stabilized deformations (in non-susy) theories gives a regime where the IR physics, including non-perturbative effects are under quantitative control.
- In some cases, the physics is conjectured (by Misha and I) to be smooth as a function of radius. Lattice data supports this.
- Confinement, when it occurs, is due to objects with non-zero magnetic charge, but vanishing index--morally similar to Polyakov--but quite exotic excitations pertinent to locally 4d.
- monopole instantons, bions, quintets. Bions most generic.
- Circle compactification rather useful in SUSY gauge theories, put forward by Seiberg, Witten, but an unpursued program.
- Primary goal: Bring the status of non-susy theories to the level of susy and I think we are getting there.

Supporting material

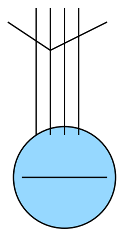
QCD(adj) $\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$

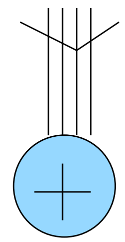
Magnetic Monopoles


BPS
(1, 1/2)


KK
(−1, 1/2)

$$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$$

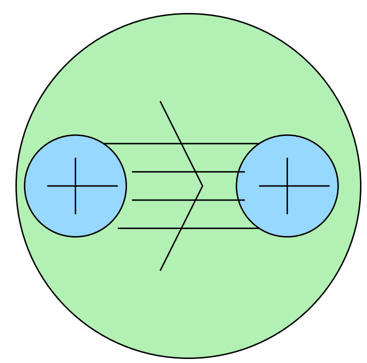

$\overline{\text{BPS}}$
(−1, −1/2)

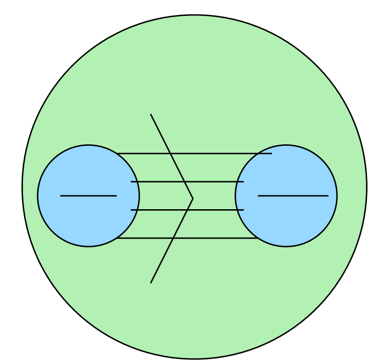

$\overline{\text{KK}}$
(1, −1/2)

$$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$$

Magnetic Bions

BPS$\overline{\text{KK}}$-composite


(2,0)


(−2, 0)

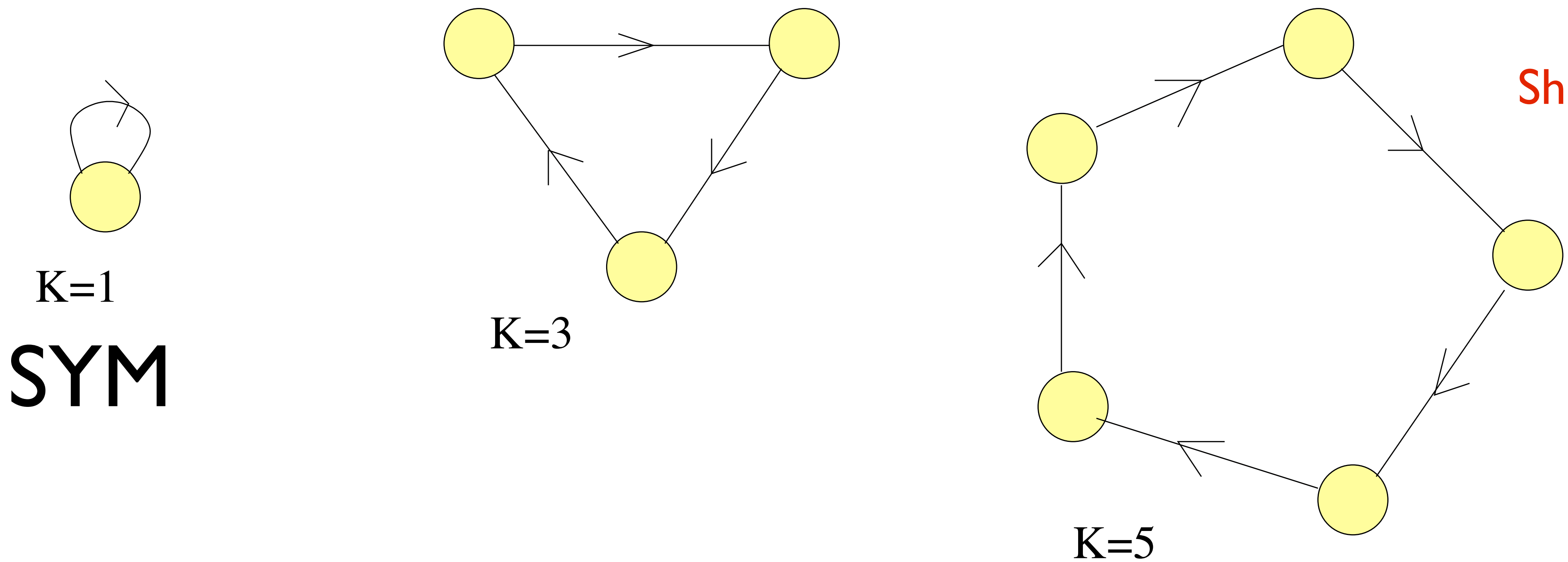
$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

Discrete shift symmetry : $\sigma \longrightarrow \sigma + \pi \qquad \psi^I \longrightarrow e^{i\frac{2\pi}{8}} \psi^I$

Chiral orbifold gauge theories: a few words

Type II: SU(5) GUT with 5 and a 10-bar. (both left handed Weyl fermions)

Type I: Related to vector-like theories via orbifold projections. (Below)



Shifman,MU 08

$$\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \ldots \times \mathrm{SU}(N)_K$$

$$\psi_J \sim (1, \ldots, N_J, \overline{N}_{J+1}, \ldots 1), \qquad J = 1, \ldots K, \;\; K+1 \equiv 1\,.$$

$$S = \sum_{J=1}^{K} \int_{R_3 \times S_1} \frac{1}{g^2} \mathrm{tr}\left[\frac{1}{2} F^2_{J,MN}(x) + i\bar{\psi}_J \bar{\sigma}_M D_M \psi_J\right],$$

$$D_M \psi_J = \partial_M \psi_J + i A_{J,M} \psi_J - i \psi_J A_{J+1,M}$$

Classical Symmetries	$\xrightarrow{\text{Instantons}}$	Quantum Symmetries

Center	Z_{N_c}		Center	Z_{N_c}
Chiral	$[U(1)]^N$		Chiral	Z_{2N_c}

also Z_K

Topological excitations

$$\text{monopoles}: \qquad e^{-S_0} e^{+i\boldsymbol{\alpha}_i \boldsymbol{\sigma}_J} \; \mathcal{O}_1(\psi)\,,$$

$$\text{bions}: \qquad e^{-2S_0} e^{+i(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_{i\pm 1})\boldsymbol{\sigma}_J}\,,$$

$$\text{BPST-instantons}: \qquad e^{-NS_0} \mathcal{O}_2(\psi)\,,$$

$$\text{flux (monopole) rings}: \qquad e^{-KS_0} e^{+i\boldsymbol{\alpha}_i \sum_J \boldsymbol{\sigma}_J} \mathcal{O}_3(\psi)\,,$$

$\mathcal{O}_3(\psi) = R^{\rm odd}$ (shown in the next page)

Non-perturbative importance: Order in topological expansion, sometimes lower order may be more important.

Dynamics is very surprising and exhibits new phenomena.

- Monopoles drops out of the chiral dynamics due to averaging over zero modes. (This never takes place in vector-like theories. It is a big surprise and is inherent to most chiral theories.)
- Mass gap for gauge fluctuations, hence confinement due to magnetic bions.
- Monopole ring operators lead to dynamical chiral symmetry breaking.
- Confinement with and without chiral symmetry breaking.
- Chiral order parameter:

$$R^{\mathrm{odd}}(x) \equiv \mathrm{tr}(\psi_1 \ldots \psi_K \psi_1 \ldots \psi_K), \qquad\qquad K \text{ odd}.$$

$$[Z_{2N}]: \quad R^{\rm odd}(x) \to e^{i\frac{2\pi K}{N}} R^{\rm odd}(x) \, .$$

$$\langle \Omega_q | \mathrm{tr}(\psi_1 \ldots \psi_K \psi_1 \ldots \psi_K) | \Omega_q \rangle = N \Lambda^{3K} e^{i\frac{2\pi q}{\widetilde{N}}} \, , \qquad q = 1, \ldots, \widetilde{N} \, .$$

$$\widetilde{K} = K \bmod N \, ,$$

$$[Z_{2N}] \to \left[Z_{2\,\mathrm{gcd}(N,\widetilde{K})} \right]$$

Disagrees with earlier guesses $[Z_{2N}] \to [Z_2]$

$$\widetilde{N} = \frac{N}{\mathrm{gcd}(N, \widetilde{K})} \, ,$$ Isolated vacua, not necessarily N

Chiral condensate may be exact. (will be happy to talk separately.)