Anything but ... *Leptogenesis*

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CP Violation in $\mu - e$ Conversion

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1. Why is $\not{C}P$ in muon physics interesting?
   - in general
   - leptogenesis

2. $\not{C}P$ in triple products
   - *rappelle*
   - CPT, unitarity and all that...
   - looking for $\not{C}P$ in rare muon decays

3. $\mu - e$ conversion

4. Simple Dirac algebra (even I can do by hand)

5. Summary and things to do

Why is μ physics interesting, in the shadow of the LHC?

$Assume$ find new physics (... LHC?), precision low energy data can be useful for

1. obtaining model parameters (eg to which LHC is not sensitive)
   $BR(\mu \to e\gamma)$ angular distributions in $\mu \to e\gamma$,, ...

2. model testing (analogy: inputs to the electroweak fit of LEP data : the SM, G_F, α_{em}, m_Z)
   $(g-2)_\mu \leftrightarrow LHC$
   $\mu \to e\gamma \leftrightarrow \mu N \to eN$

...and if LHC does not find BSM, most low energy experiments still $could$ ...

(But must check which area of parameter space excluded by LHC).

Why is $\not{C}\mathrm{P}$ interesting?

- CP : particules $\longrightarrow$ anti-particles
- In Lagangian $\mathcal{L}$ implemented by taking complex conjugate of all coupling constants

$$\not{C}\mathrm{P} \; \Leftrightarrow \text{unremoveable phases}$$

- quark sector of the SM: one $\mathcal{O}(1)$ CKM phase
- *IF* new physics at the weak scale: no electric dipole moments observed $\Rightarrow$ combinations of the new physics phases $\ll 1$

Enigmatic origin of CP Violation : are all coupling constants equiped with $\mathcal{O}(1)$ phases, or is CPV a particularity of the quark sector?

Why is $\not{C}P$ in muon physics interesting?

- baryon asymmetry of the U $\Leftrightarrow$ $\not{C}P$ (but all mechanisms need more than CKM)

- IF baryogenesis via leptogenesis ($e.g.$ in the seesaw) $\Rightarrow$ $\not{C}P$ phases in the lepton sector

- !! third generation ν beam (νfact, β-beam or superbeam) sensitive to the phase δ of the lepton mixing matrix !!

- these are expensive machines ($10^9\ \$$, € for δ?)

can leptonic CP Violation can be found somewhere else?

Outline (again)

1. Why is $\not{C}P$ in muon physics interesting?
   - in general
   - leptogenesis

2. **$\not{C}P$ in triple products** (LR, FB asyms. No antiparticle process...)
   - *rappelle*
   - **CPT, unitarity and all that...**
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Discrete symmetries ...

In a local Lorentz-invariant theory, described by a unitary S-matrix (and respecting spin-stats), CPT is a symmetry. (Not quite : $CP = T^{-1}$)

$$CP : |\text{particle}(\vec{p}, \vec{s})\rangle \to |\text{antipart}(-\vec{p}, \vec{s})\rangle$$

$$T : |\text{particle}(\vec{p}, \vec{s})\rangle \to \langle\text{particle}(-\vec{p}, -\vec{s})|$$

For example, matrix element for $\mu \to e\gamma$

$$CP : \mathcal{M}\Big(\mu(\vec{p}_\mu, \vec{s}_\mu) \to e(\vec{p}_e, \vec{s}_e) + \gamma(\vec{p}_\gamma, \vec{s}_\gamma)\Big) \longrightarrow \mathcal{M}\Big(\bar{\mu}(-\vec{p}_\mu, \vec{s}_\mu)\Big) \to \bar{e}(-\vec{p}_e, \vec{s}_e) + \gamma(-\vec{p}_\gamma,$$

$$T : \mathcal{M}\Big(\mu(\vec{p}_\mu, \vec{s}_\mu) \to e(\vec{p}_e, \vec{s}_e) + \gamma(\vec{p}_\gamma, \vec{s}_\gamma)\Big) \longrightarrow \mathcal{M}\Big(e(-\vec{p}_e, -\vec{s}_e) + \gamma(-\vec{p}_\gamma, -\vec{s}_\gamma) \to \mu(-\vec{p}_\mu, -\vec{s}$$

$\not{C}\mathrm{P}$ triple products

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If "triple products" appear in $|\mathcal{M}|^2$ (can arise from Tr$\Big[\gamma_0\gamma_i\gamma_j\gamma_k\gamma_5\Big] = 4i\epsilon_{0ijk}$)

$$|\mathcal{M}|^2 \propto \ldots + [\ldots]\vec{p}_b \cdot (\vec{s}_c \times \vec{p}_d) + [\ldots]\vec{p}_b \cdot (\vec{s}_c \times \vec{s}_d) + \ldots$$

and if they are multiplied by complex coupling constants

$\Rightarrow$ could be a T (maybe CP) odd term in the differential rate !

- need to measure spin, $\vec{p}$ ...
- not need to measure process involving antiparticles
- *not* need loop/strong and weak phases

Unitarity, loops and all that...

Recall (from leptogenesis), that a CP asym in an integrated partial decay rate requires loop(s), whose on-shell intermediate state particles give "strong" phase, that multiplies phase of coupling constants. This follows from

1. unitarity of $S = I + i(2\pi)^4\delta^4(P_i - P_f)\mathcal{M}(i \to f)$: ($SS^\dagger = [1 + iT][1 - iT^\dagger]$)

$$\begin{aligned} \mathcal{M}(\mu \to e\gamma) - \mathcal{M}(e\gamma \to \mu) &= \mathcal{M}^\dagger\mathcal{M}(\mu \to e\gamma) \\ \Rightarrow |\mathcal{M}(\mu \to e\gamma)|^2 - |\mathcal{M}(e\gamma \to \mu)|^2 &= \mathcal{O}(\mathcal{M}\mathcal{M}^\dagger\mathcal{M})(\mu \to e\gamma) + ... \end{aligned}$$

2. CPT:

$$\left|\mathcal{M}\Big(e(P_e, s_e) + \gamma(P_\gamma, s_\gamma) \to \mu(P_\mu, s_\mu)\Big)\right|^2 = \left|\mathcal{M}\Big(\bar{\mu}(P_\mu, -s_\mu) \to \bar{e}(P_e, -s_e) + \gamma(P_\gamma, -s_\gamma\right.$$

... unitarity + CPT say that a CP asymmetry *where spins, momenta are summed over*, is higher order in coupling constant/loop expansion

$\Rightarrow$ obtain phases of coupling constants at tree level by measuring asymmetries associated with triple products (FB asyms, spin asyms, etc)

Triple products in rare muon decays

$\mu(P_\mu, s_\mu) \rightarrow e\bar{e}e$

triple products galore...

Okada Okumura Shimizu

$$BR(\mu \rightarrow e\bar{e}e) < 1.0 \times 10^{-12}$$

no new experiments planned...?PSI thinking?

difficulty: multiple final state particles, accidental coincidence backgrounds

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$\mu(P_\mu, s_\mu) \to e(p_e, s_e) + \gamma(q)$

$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11} \qquad MEG@PSI \to \sim 10^{-13} \text{ soon}$$

only possible triple product: $\vec{s}_\mu \cdot (\vec{s}_e \times \vec{p}_e)$, ... *but* multiplied by $q^2 = 0$

if measure also photon polarisation, there is sensitivity to $\not{C}\not{P}$ phases Farzan

backgrds: ...+ accidental coincidences, increase as (rate)^2

$\Rightarrow \mu - e$ conversion on nuclei: one outgoing e, $q^2 \neq 0$.

$\mu - e$ conversion on nuclei

μ^- captured by nucleus, cascades down to $1s$ state, then (in SM) beta decay of nucleus by muon capture (emit ν).

Kuno Okada, ...

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Or: Beyond the SM, could have Lepton Flavour Violation. Parametrise BSM in off-shell effective field theory, μe conversion mediated by

$$-\frac{4G_F m_\mu}{\sqrt{2}}\overline{\mu}\sigma_{\alpha\beta}(A_L P_R + A_R P_L)eF^{\alpha\beta} + h.c + \text{four fermion}(\bar{e}\mu\bar{q}q)$$

$$\frac{\Gamma(\mu\text{Ti} \to e\text{Ti})}{\Gamma(\mu\text{Ti} \to capture)} < 4.3 \times 10^{-12} \quad , \quad \frac{\Gamma(\mu\text{Au} \to e\text{Au})}{\Gamma(\mu\text{Au} \to capture)} < 7 \times 10^{-13} \quad .$$

$\mu 2e$(FNAL), COMET/PRISM (J-PARC) $\to 10^{-16}, 10^{-18}$

backgrds: muon decay in orbit, in flight... Improve beam quality with intensity, can improve sensitivity...

disciminate operators via rate diff on various nuclei?

Kuno Okada, ...

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triple product $\vec{s}_\mu \cdot (\vec{s}_e \times \vec{p}_e)$ in μe conversion? Need μ polarisation...$\sim$ 16% after cascade?
Worse for target with spin, better for polarised target.

Kuno, Nagamine, Yamazaki

$\not{C}\mathrm{P}$ **in** $\mu - e$ **conversion —to calculate**

1)From operators : $\quad -\dfrac{4G_F m_\mu}{\sqrt{2}}\overline{\mu}\sigma_{\mu\nu}(A_L P_R + A_R P_L)eF^{\mu\nu} + h.c$

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_L + \mathcal{M}_R \\ &= -\frac{4G_F m_\mu}{\sqrt{2}}\int d^3x e^{-i\vec{k}\cdot\vec{x}}\left(A_L^*\overline{u}(k)2\sigma^{0i}E_i P_L \psi_{1s}^{(\mu)} + A_R^*\overline{u}(k)2\sigma^{0i}E_i P_R \psi_{1s}^{(\mu)}\right)\end{aligned}$$

2)Define average over muon wavefunction : $\displaystyle\int d^3x\psi_{1s}^{(\mu)}E^i \equiv \langle E^i\rangle \quad ,$

and recall spin proj. operator : $\quad \dfrac{1}{2}(I + \gamma_5 \not{s}\ )$

$$\begin{aligned}|\mathcal{M}|^2 &= ...+ \quad -2G_F^2 m_\mu^2 A_L A_R^*\Big\{(I+\gamma_5\not{s}_e)\not{k}\ \ \gamma^0\langle \not{E}\ \ P_R(I+\gamma_5\not{s}_\mu)\rangle\gamma^0 P_R\ \gamma^0\langle\not{E}\ \rangle^*\Big\} \\ &\qquad\qquad -2G_F^2 m_\mu^2 A_L^* A_R\Big\{(I+\gamma_5\not{s}_e)\not{k}\ \ \gamma^0\langle \not{E}\ \ P_L(I+\gamma_5\not{s}_\mu)\rangle\gamma^0\ P_L\gamma^0\langle\not{E}\ \rangle^*\Big\} \\ &= \quad ... + 8G_F^2 m_\mu^2 \mathrm{Im}\{A_L A_R^*\}|\langle\vec{E}\rangle|^2\vec{s}_\mu\cdot(\vec{s}_e\times\vec{k})\end{aligned}$$

The asymmetry

For 100% polarised muons, the asymmetry (normalised to conversion rate) between electrons polarised $\pm$ in direction perpendicular to momentum:

$$\frac{\mathrm{Im}[A_L^* A_R]}{8(|A_L|^2 + |A_R|^2)} \qquad (recall: -\frac{4G_F m_\mu}{\sqrt{2}} \overline{\mu} \sigma_{\mu\nu} (A_L P_R + A_R P_L) e F^{\mu\nu} + h.c)$$

- maximised for extensions of the SM that give similar, and complex, contributions to A_L and A_R
  (relative magnitude of A_L and A_R can be determined by the angular distribution of the electron, in $\mu \to e\gamma$, or $\mu - e$ conversion providing the muon is polarised)

- For 16 % μ polarisation, 10 % efficiency for detecting e polarisation, need $> 10^3$ events?
  ... BR($\mu - e$ conversion) $\gtrsim 10^{-15}$ for exptal sensitivity $\gtrsim 10^{-18}$.

$$\mathrm{But} \quad \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu e \mathrm{conversion})} \sim \frac{1}{\alpha} \Rightarrow \quad \mathrm{need} \quad BR(\mu \to e\gamma) \sim 10^{-13} \quad \ldots \quad \mathrm{PSI??}$$

Summary, Prospects...

If there is new flavoured physics at the TeV, it could mediate LFV such as $\mu \to e\gamma$, $\mu \to 3e$, $\mu - e$ conversion.

The greatest planned sensitivity is BR ($\mu - e$ conversion) $\sim 10^{-18}$.

Asymmetries in these processes, including $\mu - e$ conversion, are sensitive to $\not{C}\not{P}$ phases (of effective operators)

To do:

- what is relation of the effective operator phases to phases (flavour-diagonal, flavoured) in BSM Lagrangians
  - in SUSY seesaw — can I relate this phase to leptogenesis?
- what are asymmetries generated by all those four fermion $\bar{e}\mu\bar{q}q$ operators, which contribute to $\mu - e$ conversion(but not to $\mu \to e\gamma$, $\mu \to 3e$)?
  - Arise (in off-shell effective field theory) from box diagrams. Small in many SUSY models. Not small in, *eg* Little Higgs.
- final state interactions?
- better nuclear physics ...