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On Taylor Series Expansions and Conditional Expectations for (Ordinary) Stratonovich Stochastic Differential Equations with Complete V–Commutativity

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Abstract. It may happen that there is not a finite maximum order bound for numerical methods approximating Stratonovich stochastic differential equations (SDEs) with some commutative structure along an appropriate functional $V(t, X_t)$. This little note does not contradict to the well–known Clark–Cameron (1980) result and aims at rigorously proving this statement with respect to the concept of mean square convergence under the assumption of "infinite smoothness" of the corresponding diffusion coefficients $\sigma^j(t, x)$ and with finite initial second moments. As a result, we obtain an infinite series expansion of the conditional expectation $\mathbb{E}[V(t, X_t)|\mathcal{F}_t^N]$ which can be used to approximate the original solution process $(X_t)_{0 \le t \le T}$ satisfying a Stratonovich diffusion by means of finite truncation of the obtained series representations on any fixed finite time interval $[0, T]$, provided that the information collected by $\mathcal{F}_T^N = \sigma\{W_{t_0}, W_{t_1}, ..., W_{t_{N-1}}, W_T\}$ is given at $N + 1$ different time instants $t_i \in [0, T]$ with $t_0 \le t_1 \le ... \le t_{N-1} \le t_N = T$. Some simple examples illustrate the suggested approach of using Taylor series expansions to approximate conditional expectations $\mathbb{E}[V(t, X_t)|\mathcal{F}_s]$ with $0 \le s < t \le T$.