RECOGNIZING SINGLE-PEAKED PREFERENCES ON A TREE

By

Michael A. Trick

IMA Preprint Series # 376

January 1988

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

514 Vincent Hall

206 Church Street S.E.

Minneapolis, Minnesota 55455

RECOGNIZING SINGLE–PEAKED PREFERENCES ON A TREE

MICHAEL A. TRICK†

Abstract. The Condorcet winner is a natural choice as the winner of an election, when it exists. When preference orders are restricted to be single–peaked on a tree, it is known that there is a Condorcet winner. We give an algorithm for determining whether or not a set of preferences is single–peaked on some tree. Knowing this tree is useful for two reasons: its form may reflect the perceptions of the candidates by the electorate, and the existance of a tree guarantees stronger, more difficult to check, properties.

Key words. voting theory, single–peaked preferences

1. Introduction. When a group of electors chooses from a set of candidates, a natural choice is that candidate who is prefered over each other candidate by a majority of the electors. Such a candidate is termed the *Condorcet winner.* Unfortunately, due to the *Condorcet Paradox* (or the *Paradox of Cyclic Majorities*) it is well known that not every election has a Condorcet winner. A large body of literature in social choice theory is concerned with determining conditions under which a Condorcet winner is guaranteed to exist. One method to guarantee a Condorcet winner is to restrict the allowable preferences of the electors. The most studied restriction is that of single–peakedness, first studied by Black (1958). Single–peaked preferences are preferences derived from a linear ordering of the candidates, say along the liberal—conservative spectrum. Single–peakedness provides a number of nice properties, including non–manipulability (Moulin, 1980) and the transitivity of the majority rule (Inada, 1969) in addition to guaranteeing a Condorcet winner.

Demange (1982) generalizes the concept of single–peakedness by generalizing the underlying graph. Single–peaked preferences must be single peaked on a path; Demange allows preferences to be single–peaked on a tree. He shows that such a restriction is sufficient to guarantee a Condorcet winner (although other nice properties such as the transitivity of the majority rule do not hold). Demange also shows that any simple game has a non–empty core when the preferences are so restricted.

Given just a set of preferences, it is not at all obvious whether they satisfy a given preference restriction. Generally, an analyst would not know the underlying ordering or tree being used by the electorate. Rather, the analyst would like to find the underlying structure from the preferences to show how the candidates are perceived by the voters. Furthermore, the preferences can be single–peaked without the underlying structure corresponding to the perceptions of any single voter. In this case, the stronger properties (non–manipulability,

†Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN, 55455. This work was supported by a Postdoctoral Fellowship from the IMA.

non-emptyness of the core) still hold. Finding the underlying structure is one way of proving that these properties apply.

The recognition problem for single-peaked preferences is solved in Bartholdi and Trick (1986). They reduce the problem to the graph theoretic question of determining if a set of edges form subpaths of a single path. This is then solved by an algorithm of Booth and Lueker (1976).

In this paper we show how to determine if there is a tree for which the preferences are single-peaked. We show that this problem is a special case of the graph theoretic problem of determining whether a given collection of sets of nodes induces subtrees of a single tree. For this special case, we are able to provide a very efficient algorithm.

We begin by reviewing some basic concepts in graph theory and voting theory. We then give the algorithm and prove its correctness. We also provide some specializations when the preferences are further restricted.

2. Single–Peaked Preferences on a Tree. Consider a finite set C of candidates and a finite set E of electors. Each elector $i \in E$ has a strict, transitive preference order $\succ_i$ on C. A candidate $c \in C$ is a *Condorcet winner* if $|\{i : i \in E; c \succ_i c'\}| > |\{i : i \in E; c' \succ_i c\}|$ for all $c' \in C, c' \neq c$.

A graph G on C is a set of unordered pairs of distinct elements of C, called *edges*. A *path* of G is a sequence $c_1, c_2, c_3, \ldots, c_k$ where $c_i \in C, i \in \{1, 2, \ldots, k\}$, $c_i \neq c_j$ for $i \neq j$, and (c_i, c_{i+1}) is an edge for $i \in \{1, 2, \ldots, k-1\}$. A graph is a *tree* if any two points are linked by a unique path.

An ordering $\succ$ is *single–peaked on a path P* if, for any pair c_i, c_j, if c_k is on the path from c_i to c_j then c_k is not the least preferred of the three under $\succ$.

DEFINITION. Let T be a tree on C. An ordering $\succ$ is *single–peaked on the tree T* iff it is single–peaked on every path of T.

We say that a set of orderings $\{\succ_i\}$ is single–peaked on the tree T iff each $\succ_i$ is single–peaked on T. Demange shows that if a set of orderings are single–peaked on a tree then a Condorcet winner must exist. The single–peakedness of Black (1958) and others is simply single–peakedness on a tree T, where T is itself a path.

3. Recognizing Single–Peaked Preferences on a Tree. Given a set of candidates C, a set of electors E and their preferences on C, $\{\succ_i\}$, does there exist a tree for which the preferences are single–peaked? It is conceivable that it is difficult to establish single–peakedness on a tree for there are $|C|^{|C|-2}$ distinct trees on C. For even a moderate number of candidates this is far too many trees to check one–by–one.

We show that single–peakedness on trees is recognizable in polynomial time, so that even for a large number of candidates the preferences can be tested quickly.

For a preference ordering $\succ_i$ let ${c_i}^k$ be the k^{th} most prefered element of C for $k \in \{1, 2, \ldots, |C|\}$.

DEFINITION. A subset S of the nodes of a tree T *induces a subtree* of T if and only if for every two nodes $c, c' \in S$, the path in T between c and c' is contained in S.

THEOREM 1. *The ordering $\succ_i$ is single peaked on the tree T if and only if the sets $\{{c_i}^1, {c_i}^2\}, \{{c_i}^1, {c_i}^2, {c_i}^3\}, \ldots, \{{c_i}^1, {c_i}^2, \ldots, {c_i}^{|C|-1}\}$ all induce subtrees of T.*

Proof. ($\Rightarrow$) If not, take the smallest set, say $\{{c_i}^1, {c_i}^2, \ldots, {c_i}^k\}$ that does not induce a subtree. The path from ${c_i}^1$ to ${c_i}^k$ contains ${c_i}^j$ for $j > k$. But then the preference is not single–peaked on that path. Contradiction.

($\Leftarrow$) If the ordering is not single–peaked on T then there are three nodes ${c_i}^j, {c_i}^k, {c_i}^l$ such that ${c_i}^l$ lies on the path from ${c_i}^j$ to ${c_i}^k$ but $j < k < l$. But then the set $\{{c_i}^1, {c_i}^2, \ldots, {c_i}^k\}$ does not induce a subtree. Contradiction. ▯

So the problem of determining a tree for which a set of preferences are single–peaked has been reduced to finding a tree for which a given collection of sets of nodes induce subtrees. Trick (1987) provides an algorithm for solving this general problem. The resulting time complexity is $O(|C|^3|E|)$. We can improve on this time bound by taking advantage of the fact that the sets of nodes come from preference orders.

DEFINITION. A *leaf* of a tree T is a node of T with exactly one edge incident to it.

Our first lemma gives a method for quickly identifying a candidate that must be a leaf.

LEMMA 1. *Any candidate appearing last in a preference order must be a leaf of any tree for which the preferences are single–peaked.*

Proof. Let c appear last in a preference order of elector i and let T be a tree for which the preferences are single–peaked. If c is not a leaf then it has edges to two other nodes, say c^1 and c^2. The sequence c^1, c, c^2 is a path in T but $c^1 \succ_i c$ and $c^2 \succ_i c$, contradicting the definition of single–peaked on a tree. ▯

Our next two lemmas show exactly how the leaf must interact with the rest of the candidates.

LEMMA 2. *For a leaf c, let $B_i(c)$ be the set of candidates preferred to c by i. If c is most preferred by i let $B_i(c) = \{{c_i}^2\}$ (the second most preferred candidate of i). Then, for any tree T for which the preferences are single–peaked, c is has its edge to some member of $\cap_{i \in E} B_i(c)$. If $\cap_{i \in E} B_i(c) = \emptyset$ then there is no such T.*

Proof. Suppose there is a tree T and c is adjacent to $c' \notin B_i(c)$. If c is most preferred by i then $c' \neq {c_i}^2$ so $\{{c_i}^1, {c_i}^2\}$ does not induce a subtree. Otherwise $B_i(c) \cup c$ does not induce a subtree. Both cases contradict Theorem 1. If the set is empty then for some i, c is adjacent to no member of $B_i(c)$, so the set $B_i(c) \cup c$ does not induce a subgraph. ▯

LEMMA 3. *If* $|\cap_{i \in E} B_i(c)| > 1$ *then* c *can be placed adjacent to any member of that set.*

Proof. For any i examine any set $\{c_i{}^1, c_i{}^2, \ldots, c_i{}^k\}$ containing c. Since c is in the set, so is all of $\cap_{i \in E} B_i(c)$, so being adjacent to any element of the set will still induce a subtree. □

Finally, our fourth lemma allows us to delete a leaf and solve the problem on a smaller number of candidates.

LEMMA 4. *For a leaf* c*, if the preferences* $\{\succ_i\}$ *restricted to* $C - c$ *are not single–peaked on any tree then neither are* $\{\succ_i\}$ *for* C.

Proof. Suppose there is a tree T for the preferences on C. By assumption c is a leaf of T so $T - c$ is a tree. The preferences restricted to $C - c$ still give rise to induced subtrees so the preferences are single–peaked on $T - c$. Contradiction. □

We are now ready to present the algorithm for finding a suitable tree or proving none exist. This is given by Algorithm Make_Tree.

ALGORITHM MAKE_TREE.

make_tree(C*,*$\{\succ_i\}$*,*T*)*

Input: candidates C, preferences$\{\succ_i\}$

Output: tree realizing single–peakedness, T, or *failure*

if $|C| = 0$ *then* $T = \emptyset$*; return*

if $|C| = 1$ *then* $T =$ single node marked with the number of the candidate*; return*

if $|C| = 2$ *then* $T =$ two–node tree marked with the candidate numbers

    with an edge connecting nodes*; return*

let L *be set of all candidates last in a preference*

if for any $c \in L$, $\cap_{i \in E} B_i(c) = \emptyset$ *return failure*

$C' = C - L$*;* $\succ'_i = \succ_i$ *restricted to* C'

make_tree(C', $\{\succ'_i\}$, T*)*

for each $c \in L$ *attach node marked* c *to any node in* $\cap_{i \in E} B_i(c)$ *in* T

return

end (make_tree)

Algorithm Make_Tree

THEOREM 2. *Algorithm Make_tree either finds a suitable tree or shows that no tree exists.*

Proof. If the algorithm returns *failure* then a restriction found only be deleting leaves has no tree (by Lemma 2). Therefore the original problem has no solution (by Lemma 4, repeatedly).

Otherwise the algorithm must return a tree. We show that this tree is correct by induction. Clearly, if $|C| \leq 2$ then the tree is suitable. Assume the algorithm returns a suitable tree for $|C| \leq k$. At the stage where we are attempting to add the $(k+1)^{\text{st}}$ node, say, c, we know we have a suitable tree. Since $\cap_{i \in E} B_i(c) \neq \emptyset$ we have a place to add the node. By Lemmas 2 and 3, any node in that set will do, so we can add the next node. The theorem follows by induction. □

The bottleneck of this algorithm is the calculation of $B_i(c)$. A straightforward analysis gives the time complexity as $O(|C|^2|E|)$. It may be possible to preprocess the preferences to streamline the calculation of $B_i(c)$. The difficulty arises that the sets are defined with respect to restricted preferences and both candidates above c and just below c (if c is most prefered) are included. It is not obvious how to improve on the straightforward algorithm.

An interesting special case is when each candidate is most prefered by some elector. This occurs in facility location, where the feasible locations are the current customer locations and each customer most prefers its location (see Hanson and Thisse, 1981). Bartholdi and Trick (1986) coined the term *narcissistic* for this restriction.

THEOREM 3. *For narcissistic preferences, if the preferences are single–peaked on a tree then*

a) *the tree is unique, and*

b) *the tree can be found in $O(|E||C|)$ time.*

Proof. For part (a), if there were two trees T and T', then T would have an edge i, j not in T'. The path between i and j in T' must include some node k. Examine the placement of k in T. Either the path from i to k contains j or the path from j to k contains i. In the first case, since some elector most prefers i, the preferences in T show $i \succ j \succ k$ by that elector. But this is not single–peaked for T'. The other case is similar. Contradiction.

The time complexity follows easily from the observation that $|\cap_{i \in E} B_i(c)| = 1$ for all c and all restrictions of the preferences. Therefore, preprocessing the preferences so as to be able to determine if $c \succ_i c'$ (which is $O(|C||E|)$) is sufficient to calculate $\cap_{i \in E} B_i(c)$ in $O(|E|)$ time, which gives the desired time bound. □

Note that due to the uniqueness of the tree, it is unnecessary to independently check for single–peakedness on a path. If narcissistic preferences are single–peaked on a path, then that path is the unique tree realizing single–peakedness.

REFERENCES

J.J. Bartholdi and M.A. Trick, *Stable matching with preferences derived from a psychological model*, Operations Research Letters, 5 (1986), pp. 165–169.

D. Black, *The Theory of Committees and Elections*, Cambridge University Press, Cambridge, MA, 1958.

K.S. Booth and G.S. Lueker, *Testing for the consecutive 1's property, interval graphs, and graph planarity using PQ–tree algorithms*, Journal of Computers and Systems Science, 13 (1976), pp. 335–379.

G. Demange, *Single–peaked orders on a tree*, Mathematical Social Sciences, 3 (1982), pp. 389–396.

P. Hansen and J.F. Thisse, *Outcomes of voting and planning: Condorcet, Weber and Rawls locations*, Journal of Public Economics, 16 (1981), pp. 1–15.

K. Inada, *The simple majority rule*, Econometrica, 37 (1969), pp. 490–506.

H. Moulin, *On strategy–proofness and single peakedness*, Public Choice, 35 (1980), pp. 437–455.

M.A. Trick, *Induced subtrees of a tree and the set packing problem*, in preparation.

Recent IMA Preprints

#	Author/s	Title
298	**M. Chipot, F. B. Weissler**	Some Blow-Up Results for a Nonlinear Parabolic Equation with a Gradient Term
299	**L. Kaitai**	Perturbation Solutions of Simple and Double Bifurcation Problems for Navier-Stokes Equations
300	**C. Zhangxin, L. Kaitai**	The Convergence on the Multigrid Algorithm for Navier-Stokes Equations
301	**A. Gerardi, G. Nappo**	Martingale Approach for Modeling DNA Synthesis
302	**D. N. Arnold, L. Ridgway, M. Vogelius**	Regular Inversion of the Divergence Operator with Dirichlet Boundary Conditions on a Polygon Divergence Operator with Dirichlet Boundary Conditions on a Polygon
303	**R. G. Duran**	Error Analysis in $L^p, 1 \leq p \leq \infty$, for Mixed Definite Element Methods for Linear and Quasi-Linear Elliptic Problems
304	**R. Nochetto, C. Verdi**	An Efficient Linear Scheme to Approximate Parabolic Free Free Boundary Problems: Error Estimates and Implementation
305	**K. A. Pericak-Spector, S. J. Spector**	Nonuniqueness for a Hyperbolic System: Cavitation in Nonlinear Elastodynamics
306	**E. G. Kalnins, W. Miller, Jr.**	q-Series and Orthogonal Polynomials Associated with Barnes' First Lemma
307	**D. N. Arnold, R. S. Falk**	A Uniformly Accurate Finite Element Method for Mindlin-Reissner Plate
308	**Chi-Wang Shu**	TVD Properties of a Class of Modified Eno Schemes for Scalar Conservation Laws
309	**E. Dikow, U. Hornung**	A Random Boundary Value Problem Modeling Spatial Variability in Porous Media Flow
310	**J. K. Hale**	Compact Attractors and Singular Perturbations
311	**A. Bourgeat, B. Cockburn**	The TVD-Projection Method for Solving Implicit Numeric Schemes for Scalar Conservation Laws: A Numerical Study of a Simple Case
312	**B. Muller, A. Rizzi**	Navier-Stokes Computation of Transonic Vortices over a Round Leading Edge Delta Wing
313	**J. Thomas Beale**	On the Accuracy of Vortex Methods at Large Times
314	**P. Le Talle, A. Lotfi**	Decomposition Methods for Adherence Problems in Finite Elasticity
315	**J. Douglas, Jr., J. E. Santos**	Approximation of Waves in Composite Media
316	**T. Arbogast**	The Double Porosity Model for Single Phase Flow in Naturally Fractured Reservoirs
317	**T. Arbogast, J. Douglas, Jr., J. E. Santos**	Two-Phase Immiscible Flow in Naturally Fractured Reservoirs
318	**J. Douglas, Jr., Y. Yirang**	Numerical Simulation of Immiscible Flow in Porous Media Based on Combining the Method of Characteristics with Finite Element Procedures
319	**R. Duran, R. H. Nochetto, J. Wang**	Sharp Maximum Norm Error Estimates for Finite Element Approximations of the Stokes Problem in 2-D
320	**A. Greven**	A Phase Transition for a System of Branching Random Walks in a Random Environment
321	**J. M. Harrison, R. J. Williams**	Brownian Models of Open Queueing Networks with Homogeneous Customer Populations
322	**Ana Bela Cruzeiro**	Solutions ET mesures invariantes pour des equations d'evolution Stochastiques du type Navier-Stokes
323	**Salah-Eldin A. Mohammed**	The Lyapunov Spectrum and Stable Manifolds for Stochastic Linear Delay Equations
324	**Bao Gia Nguyen**	Typical Cluster Size for 2-DIM Percolation Processes (Revised)
325	**R. Hardt, D. Kinderlehrer, F.-H. Lin**	Stable Defects of Minimizers of Constrained Variational Principles
326	**M. Chipot, D. Kinderlehrer**	Equilibrium Configurations of Crystals
327	**Kiyosi Itô**	Malliavin's C^∞ functionals of a centered Gaussian system
328	**T. Funaki**	Derivation of the hydrodynamical equation for one-dimensional Ginzburg-Landau model
329	**Y. Masaya**	Schauder Expansion by some Quadratic Base Function
330	**F. Brezzi, J. Douglas, Jr.**	Stabilized Mixed Methods for the Stokes Problem
331	**J. Mallet-Paret, G. R. Sell**	Inertial Manifolds for Reaction Diffusion Equations in Higher Space Dimensions
332	**San-Yih Lin, M. Luskin**	Relaxation Methods for Liquid Crystal Problems
333	**H. F. Weinberger**	Some Remarks on Invariant Sets for Systems
334	**E. Miersemann, H. D. Mittelmann**	On the Continuation for Variational Inequalities Depending on an Eigenvalue Parameter
335	**J. Hulshof, N. Wolanski**	Monotone Flows in N-Dimensional Partially Saturated Porous Media: Lipschitz Continuity of the Interface
336	**B. J. Lucier**	Regularity Through Approximation for Scalar Conservation Laws
337	**B. Sturmfels**	Totally Positive Matrices and Cyclic Polytopes
338	**R. G. Duran, R. H. Nochetto**	Pointwise Accuracy of a Stable Petrov-Galerkin Approximation to Stokes Problem
339	**L. Gastaldi**	Sharp Maximum Norm Error Estimates for General Mixed Finite Element Approximations to to Second Order Elliptic Equations
340	**L. Hurwicz, H. F. Weinberger**	A Necessary Condition for Decentralizability and an Application to Intemporal Allocation
341	**G. Chavent, B. Cockburn**	The Local Projection P^0P^1-Discontinuous-Galerkin-Finite Element Method for Scalar Conservation Laws
342	**I. Capuzzo-Dolcetta, P.-L. Lions**	Hamilton-Jacobi Equations and State-Constraints Problems

Recent IMA Preprints (Continued)

#	Author/s	Title
343	**B. Sturmfels, N. White**,	Gröbner Bases and Invariant Theory
344	**J. L. Vazquez**,	C^{∞}-Regularity of Solutions and Interfaces of the Porous Medium Equation
345	**C. Beattie, W. M. Greenlee**,	Improved Convergence Rates for Intermediate Problems
346	**H. D. Mittelmann**,	Continuation Methods for Parameter-Dependent Boundary Value Problems
347	**M. Chipot, G. Michaille**,	Uniqueness Results and Monotonicity Properties for Strongly Nonlinear Elliptic Variational Inequalities
348	**Avner Friedman, Bei Hu**	The Stefan Problem for a Hyperbolic Heat Equation
349	**Michel Chipot, Mitchell Luskin**	Existence of Solutions to the Elastohydrodynamical Equations for Magnetic Recording Systems
350	**R.H. Nochetto, C. Verdi**,	The Combined Use of a Nonlinear Chernoff Formula with a Regularization Procedure for Two-Phase Stefan Problems
351	**Gonzalo R. Mendieta**	Two Hyperfinite Constructions of the Brownian Bridge
352	**Victor Klee, Peter Kleinschmidt**	Geometry of the Gass-Saaty Parametric Cost LP Algorithm
353	**Joseph O'Rourke**	Finding A Shortest Ladder Path: A Special Case
354	**J. Gretenkort, P. Kleinschmidt, Bernd Sturmfels**,	On the Existence of Certain Smooth Toric Varieties
355	**You-lan Zhu**	On Stability & Convergence of Difference Schemes for Quasilinear Hyperbolic Initial-Boundary-Value Problems
356	**Hamid Bellout, Avner Friedman**	Blow-Up Estimates for Nonlinear Hyperbolic Heat Equation
357	**P. Gritzman, M. Lassak**	Helly-Test for the Minimal Width of Convex Bodies
358	**K.R. Meyer, G.R. Sell**	Melnikov Transforms, Bernoulli Bundles, and Almost Periodic Perturbations
359	**J.-P. Puel, A. Raoult**	Buckling for an Elastoplastic Plate with An Increment Constitutive Relation
360	**F.G. Garvan**	A Beta Integral Associated with the Root System G_2
361	**L. Chihara, D. Stanton**	Zeros of Generalized Krawtchouk Polynomials
362	**Hisashi Okamoto**	O(2)–Equivariant Bifurcation Equations with Two Modes Interaction
363	**Joseph O'Rourke, Catherine Schevon**	On the Development of Closed Convex Curves on 3-Polytopes
364	**Weinan E**	Analysis of Spectral Methods for Burgers' Equation
365	**Weinan E**	Analysis of Fourier Methods for Navier-Stokes Equation
366	**Paul Lemke**	A Counterexample to a Conjecture of Abbott
367	**Peter Gritzmann**	A Characterization of all Loglinear Inequalities for Three Quermassintegrals of Convex Bodies
368	**David Kinderlehrer**	Phase transitions in crystals: towards the analysis of microstructure
369	**David Kraines, Vivian Kraines**	Pavlov and the Prisoner's Dilemma
370	**F.G. Garvan**	A Proof of the MacDonald-Morris Root System Conjecture for F_4
371	**Neil L. White, Tim McMillan**	Cayley Factorization
372	**Bernd Sturmfels**	Applications of Final Polynomials and Final Syzygies
373	**Avner Friedman, Michael Vogelius**	Identification of Small Inhomogeneities of Extreme Conductivity by Boundary Measurements: A Continuous Dependence Result
374	**Jan Kratochvíl, Mirko Křivánek**	On the Computational Complexity of Codes in Graphs
375	**Thomas I. Seidman**	The Transient Semiconductor Problem with Generation Terms, II
376	**Michael A. Trick**	Recognizing Single-Peaked Preferences On A Tree
377	**Michael A. Trick**	Induced Subtrees of a Tree and the Set Packing Problem
378	**Charles J. Colbourn, Ebadollah S. Mahmoodian**	The Spectrum of Support Sizes for Threefold Triple Systems
379	**Bradley J. Lucier**	Performance Evaluation for Multiprocessors Programmed Using Monitors
380	**Tomasz Luczak**	On the Size and Connectivity of the k-Core of a Random Graph
381	**B. Nicolaenko, B. Scheurer and R. Temam**	Some Global Dynamical Properties of a Class of Pattern Formation Equations
382	**Mirko Křivńek**	A Note on the Computational Complexity of Bracketing and Related Problems
383	**Eduard Harabetian**	Rarefactions and Large Time Behavior for Parabolic Equations and Monotone Schemes
384	**Victor Klee and Peter Kleinschmidt**	Polytopal Complexes and Their Relatives
385	**Joel D. Avrin**	Viscosity Solutions with Singular Initial Data for a Model of Electrophoretic Separation
386	**David W. Matula and Rakesh V. Vohra**	Calculating the Connectivity of a Directed Graph
387	**Hubert de Fraysseix, János Pach and Richard Pollack**	Small Sets Supporting Fáry Embeddings of Planar Graphs
388	**Bernardo Cockburn and Chi-Wang Shu**	The Runge–Kutta Local Projection P^1- Discontinuous–Galerkin Finite Element Method for Scalar Conservation Laws
389	**Henry Crapo, Timothy F. Havel, Bernd Sturmfels, Walter Whiteley and Neil L. White**	Symbolic Computations in Geometry