Argyres–Seiberg Duality and New SCFT's

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I Argyres-Seiberg Duality

Philip Argyres & Nathan Seiberg 0711.0054

$$\mathfrak{g}[d_i] \text{ w/ } \mathbf{r} \simeq \tilde{\mathfrak{g}}[\tilde{d_i}] \text{ w/ } (\tilde{\mathbf{r}} \oplus \text{SCFT}[d:\mathfrak{h}])$$

- LHS: There is a gauge group $\mathfrak{g}$ with matter charged in representations $\mathbf{r}$.

- RHS: There is a rank 1 SCFT with mass dimension of the Coulomb branch moduli d and flavor symmetry $\mathfrak{h}$. Then $\tilde{\mathfrak{g}} \subset \mathfrak{h}$ is gauged with matter charged in representations $\tilde{\mathbf{r}}$.

II Criteria for Duality

Philip Argyres & JRW 0712.2028

- the spectrum of dimensions of Coulomb branch vevs
  $\{d_i\} \;=\; \{\widetilde{d}_i\} \;\cup\; \{d\}$

- the flavor symmetry algebras
  $f \;=\; \widetilde{f} \;\oplus\; \mathsf{H}$

- the beta function from weakly gauging the flavor symmetry
  $\mathsf{T}(\mathbf{r}) \;=\; \mathsf{T}(\widetilde{\mathbf{r}}) \;+\; \mathbf{k}_{\mathfrak{h}} \cdot \mathbf{I}_{\mathsf{H}\hookrightarrow\mathfrak{h}}$

- the number of marginal couplings
  $2 \cdot \mathsf{T}(\widetilde{\mathfrak{g}}) \;=\; \mathsf{T}(\widetilde{\mathbf{r}}) \;+\; \mathbf{k}_{\mathfrak{h}} \cdot \mathbf{I}_{\widetilde{\mathfrak{g}}\hookrightarrow\mathfrak{h}}$

Criteria for Duality(cont'd)

- contribution to the $u(1)_R$ central charge (for the SCFT)
  $3/2 \cdot k_R \;=\; 24 \cdot c \;=\; 4 \cdot (|\mathfrak{g}| \;-\; |\widetilde{\mathfrak{g}}|) \;+\; (|\mathbf{r}| \;-\; |\widetilde{\mathbf{r}}|)$

- contribution to the a conformal anomaly (for the SCFT)
  $48 \cdot a \;=\; 10 \cdot (|\mathfrak{g}| \;-\; |\widetilde{\mathfrak{g}}|) \;+\; (|\mathbf{r}| \;-\; |\widetilde{\mathbf{r}}|)$

- existence of a global $\mathbb{Z}_2$-obstruction to gauging the flavor symmetry

Criteria for Duality (a and c anomalies)

- in Lagrangian theories, the a and c anomalies can be computed by t' Hooft anomaly matching
  $4 \cdot (2 \cdot a \; - \; c) \; = \; |\mathfrak{g}| \; = \; \sum_i (2 \cdot d_i \; - \; 1)$

- looking back at the criteria the SCFT satisfies a similar relation
  $4 \cdot (2 \cdot a \; - \; c) \; = \; (2 \cdot d \; - \; 1)$

- Shapere and Tachikawa have given a proof that this formula holds for a large class of $N = 2$ SCFT's in 0804.1957

Criteria for Duality ($\mathbb{Z}_2$-obstruction)

E. Witten, An su(2) Anomaly, Phys.Lett.B117:324-3278,1982

G_2 w/ $8 \cdot \mathbf{7}$ $\simeq$ su(2) w/ $(\mathbf{2} \oplus \mathrm{SCFT}[6 : \mathrm{sp}(5)])$

- LHS: the $\mathbf{7}$ of G_2 is real $\Rightarrow$ the flavor symmetry is sp(4) when the sp(4) is gauged there is a $\mathbb{Z}_2$-obstruction because the $\mathbf{8}$ is pseudoreal

- RHS:the su(2) has an anomaly related to the single $\mathbf{2}$ $\Rightarrow$ sp(5) must posses a $\mathbb{Z}_2$-obstruction to gauging in order to cancel this since the LHS is anomaly free $\Rightarrow$ this gives sp(4) a $\mathbb{Z}_2$-obstruction since $\mathrm{I}_{f \hookrightarrow \mathfrak{h}} = 1$ for f either su(2) or sp(4)

III Examples of Duality and Results (1 Marginal Operator)

	$\mathfrak{g}$ w/ $\mathbf{r}$		=	$\widetilde{\mathfrak{g}}$ w/ $\widetilde{\mathbf{r}}$		$\oplus$ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14} \oplus 11 \cdot \mathbf{6}$	=	sp(2)		$[6 : E_8]$
2.	su(6)	$\mathbf{20} \oplus \mathbf{15} \oplus \overline{\mathbf{15}} \oplus 5 \cdot \mathbf{6} \oplus 5 \cdot \overline{\mathbf{6}}$	=	su(5)	$\mathbf{5} \oplus \overline{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$	$[6 : E_8]$
3.	so(12)	$3 \cdot \mathbf{32} \oplus \mathbf{32}' \oplus 4 \cdot \mathbf{12}$	=	so(11)	$3 \cdot \mathbf{32}$	$[6 : E_8]$
4.	G_2	$8 \cdot \mathbf{7}$	=	su(2)	$\mathbf{2}$	$[6 : \mathrm{sp}(5)]$
5.	so(7)	$4 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{7}$	=	sp(2)	$5 \cdot \mathbf{4}$	$[6 : \mathrm{sp}(5)]$
6.	su(6)	$\mathbf{21} \oplus \overline{\mathbf{21}} \oplus \mathbf{20} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	su(5)	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$[6 : \mathrm{sp}(5)]$
7.	sp(2)	$12 \cdot \mathbf{4}$	=	su(2)		$[4 : E_7]$
8.	su(4)	$2 \cdot \mathbf{6} \oplus 6 \cdot \mathbf{4} \oplus 6 \cdot \overline{\mathbf{4}}$	=	su(3)	$2 \cdot \mathbf{3} \oplus 2 \cdot \overline{\mathbf{3}}$	$[4 : E_7]$
9.	so(7)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{7}$	=	G_2	$4 \cdot \mathbf{7}$	$[4 : E_7]$
10.	so(8)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{8}' \oplus 2 \cdot \mathbf{8}''$	=	so(7)	$6 \cdot \mathbf{8}$	$[4 : E_7]$
11.	so(8)	$6 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{8}'$	=	G_2		$[4 : E_7] \oplus [4 : E_7]$
12.	sp(2)	$6 \cdot \mathbf{5}$	=	su(2)		$[4 : \mathrm{sp}(3) \oplus \mathrm{su}(2)]$
13.	sp(2)	$4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{5}$	=	su(2)	$3 \cdot \mathbf{2}$	$[4 : \mathrm{sp}(3) \oplus \mathrm{su}(2)]$
14.	su(4)	$\mathbf{10} \oplus \overline{\mathbf{10}} \oplus 2 \cdot \mathbf{4} \oplus 2 \cdot \overline{\mathbf{4}}$	=	su(3)	$\mathbf{3} \oplus \overline{\mathbf{3}}$	$[4 : \mathrm{sp}(3) \oplus \mathrm{su}(2)]$
15.	su(3)	$6 \cdot \mathbf{3} \oplus 6 \cdot \overline{\mathbf{3}}$	=	su(2)	$2 \cdot \mathbf{2}$	$[3 : E_6]$
16.	su(4)	$4 \cdot \mathbf{6} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \overline{\mathbf{4}}$	=	sp(2)	$6 \cdot \mathbf{4}$	$[3 : E_6]$
17.	su(3)	$\mathbf{3} \oplus \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	su(2)	$n \cdot \mathbf{2}$	$[3 : \mathfrak{h}]$

- predicted dualities with one marginal operator

Examples of Duality and Results (2 Marginal Operators)

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\widetilde{\mathfrak{g}}$	w/ $\widetilde{\mathbf{r}}$	$\oplus$ SCFT$[d:\mathfrak{h}]$
18.	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{3}\oplus\overline{\mathbf{3}}) \oplus 4\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$	=	su(2) $\oplus$ su(2)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus 2\cdot(\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{4}) \oplus 8\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ su(2)		$[4:E_7]$
20.	su(2) $\oplus$ sp(2)	$3\cdot(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{5}) \oplus 4\cdot(\mathbf{1},\mathbf{5})$	=	su(2) $\oplus$ su(2)	$3\cdot(\mathbf{2},\mathbf{1})$	[4 : sp(3)$\oplus$su(2)]
21.	su(2) $\oplus$ G_2	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}) \oplus 6\cdot(\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ su(2)	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	[6 : sp(5)]
22.	su(3) $\oplus$ su(3)	$2\cdot(\mathbf{3},\overline{\mathbf{3}}) \oplus 2\cdot(\overline{\mathbf{3}},\mathbf{3})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) $\oplus$ su(3)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{3}\oplus\overline{\mathbf{3}})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) $\oplus$ su(3)	$3\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3},\overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}},\mathbf{3})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
		$\oplus\, 3\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$			$\oplus\, 3\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$	
25.	su(3) $\oplus$ sp(2)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{5})$	=	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})$	[4 : sp(3)$\oplus$su(2)]
26.	su(3) $\oplus$ sp(2)	$2\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{4}) \oplus 6\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus 6\cdot(\mathbf{1},\mathbf{4})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$2\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) $\oplus$ sp(2)	$2\cdot(\mathbf{5},\mathbf{1}) \oplus (\mathbf{5},\mathbf{4}) \oplus 7\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$7\cdot(\mathbf{1},\mathbf{4})$	[4 : sp(3)$\oplus$su(2)]
			=	sp(2) $\oplus$ su(2)	$2\cdot(\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	sp(2) $\oplus$ sp(2)	$4\cdot(\mathbf{4},\mathbf{1}) \oplus 2\cdot(\mathbf{4},\mathbf{4}) \oplus 4\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$4\cdot(\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) $\oplus$ G_2	$5\cdot(\mathbf{4},\mathbf{1}) \oplus (\mathbf{4},\mathbf{7}) \oplus 4\cdot(\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ G_2	$4\cdot(\mathbf{1},\mathbf{7})$	$[4:E_7]$
			=	sp(2) $\oplus$ su(2)	$5\cdot(\mathbf{4},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	[6 : sp(5)]

- predicted dualities with two marginal operators

Examples of Duality and Results (New SCFT's)

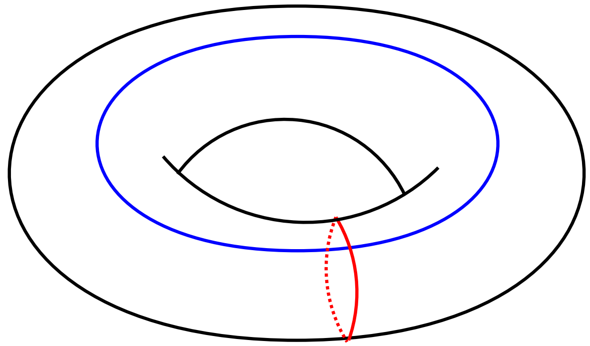
d	$\mathfrak{h}$	$k_{\mathfrak{h}}$	$3/2 \cdot k_R$	$48 \cdot a$	$\mathbb{Z}_2$ anomaly?
6	E_8	12	124	190	no
6	sp(5)	7	98	164	yes
4	E_7	8	76	118	no
4	sp(3) $\oplus$ su(2)	5 $\oplus$ 8	58	100	yes $\oplus$ no
3	E_6	6	52	82	no
3	$2 \leq \text{rank}(\mathfrak{h}) \leq 6$	$(8-n)/\mathrm{I}_{\text{su}(2)\hookrightarrow\mathfrak{h}}$	38-2n	68-2n	?

- The flavor central charges, $k_{\mathfrak{h}}$, were confirmed for E_6, E_7, and E_8 through an F-theory calculation by Aharony and Tachikawa in 0711.4532.

IV Seiberg-Witten Theory

the physics is encoded by:

- the Seiberg-Witten curve:
  $y^2 = x^3 + f(u, m_i)x + g(u, m_i)$



- and the Seiberg-Witten 1-form: λ_{SW}
  $\partial_u \lambda_{SW} = \frac{\mathrm{d}x}{y} + \partial_x(\star)\mathrm{d}x$

charged states are encoded by:

- u(1) charges of a physical state are given by the homology class of a cycle, $\gamma = n_e[\alpha] + n_m[\beta]$, when $m_i = 0$

- these states have central charge, $Z = \oint_\gamma \lambda_{SW}$

Seiberg-Witten Theory(Singularities)

the singularities of the Seiberg-Witten curve:

- are located at $\Delta \;=\; 4 \cdot f^3 \;-\; 27 \cdot g^2 \;=\; 0$

- physically they correspond to a breakdown of the low-energy description $\Rightarrow$ charged states becoming massless

V Central Charges and Curves

Shapere and Tachikawa 0804.1957

the twisted version of Seiberg-Witten theory relates the anomalies and central charges to:

- the mass dimension of the vev on moduli space

- # of neutral hypermultiplets on moduli space

- # of singular points of the Seiberg-Witten curve

Central Charges and Curves (twisted PI)

$\int[\mathrm{d}u][\mathrm{d}q]A^{\chi}B^{\sigma}C^{n}\mathrm{e}^{-S_{low-energy}}$

- χ is the Euler characteristic

- σ is the signature

- n is an instanton number

- $A^2 = det[\frac{\partial u_i}{\partial a_j}]$

- $B^8 = Radical[\Delta]$

Central Charges and Curves (master equations)

- the scaling behavior of the measure determines the R-charge of the states becoming massless at a singularity

- the normalization is: $R(\star) \; = \; 2 \cdot D(\star)$

- $48 \cdot a \; = \; 12 \cdot R(A) \; + \; 8 \cdot R(B) \; + \; 10 \cdot r \; + \; 2 \cdot \mathsf{h}$

- $24 \cdot c \; = \; 8 \cdot R(B) \; + \; 4 \cdot r \; + \; 2 \cdot \mathsf{h}$

- $4(2 \cdot a - c) = 2 \cdot R(A) + r = \sum_{i=1}^{r} 2 \cdot (d_i - 1) + r = \sum_{i=1}^{r} (2 \cdot d_i - 1)$

$r \equiv$ the complex dimension of moduli space

$\mathsf{h} \equiv$ the # of massless neutral hypermultiplets

Central Charges and Curves ($r = 1$)

- $R(A) = d - 1$

- $R(B) = \frac{1}{4} \cdot Z \cdot d$

  $Z = \#$ of singular points of the Seiberg-Witten curve

- $24 \cdot c = 2 \cdot Z \cdot d + 4 + 2 \cdot \mathsf{h}$

- $k_{\mathfrak{h}} = 2 \cdot d - \mathsf{h}$

Central Charges and Curves (Results)

d	$\mathfrak{h}$	Z	$2 \cdot \mathrm{h}$	rep.'s
6	E_8	10	0	-
6	sp(5)	7	10	$\mathbf{10}$(s)
4	E_7	9	0	-
4	sp(3) $\oplus$ su(2)	6	$(6, 0)$	$\mathbf{6} \oplus \mathbf{1}$(s)
3	E_6	8	0	-
3	su(3)	4	6	$\mathbf{3} \oplus \overline{\mathbf{3}}$(c)

- no neutral hypermultiplets on the LHS of the equivalence

- neutral hypermultiplets on the RHS must be charged under the flavor symmetry

VI $\mathbb{Z}_2$-obstruction Revisited

- there was a Z_2-obstruction for the sp(5) and sp(3) factors

- these obstructions come from the single pseudoreal charged under them

- consider the previous example:

  G_2 w/ $8 \cdot \mathbf{7}$ $\simeq$ su(2) w/ $(\mathbf{2} \oplus \text{SCFT}[6 : \text{sp}(5)])$

  $\text{su}(2) \oplus \text{sp}(4) \subset \text{sp}(5)$

  $(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \;=\; \mathbf{10}$

VII Constructing New Seiberg-Witten Curves

- the work of Shapere and Tachikawa specifies possible forms of the discriminant of the Seiberg-Witten curve

- the discriminants are determined by partitioning the total order of the singularity at $m_i = 0$ into a Z-tuple of integers.

Constructing New Seiberg-Witten Curves (su(3))

4 singular points and 8 singularities

- $\Delta \sim (u + ...)^5 (u^3 + ...)$
- $\Delta \sim (u + ...)^4 (u + ...)^2 (u^2 + ...)$
- $\Delta \sim (u + ...)^3 (u + ...) (u^2 + ...)^2$
- $\Delta \sim (u^2 + ...)^3 (u^2 + ...)$
- $\Delta \sim (u^4 + ...)^2$

a systematic search reveals 2 solutions

Constructing New Seiberg-Witten Curves (sp(3) $\bigoplus$ su(2))

6 singular points and 9 singularities

- $\Delta \sim (u + ...)^4(u^5 + ...)$

- $\Delta \sim (u + ...)^3(u + ...)^2(u^4 + ...)$

- $\Delta \sim (u^3 + ...)^2(u^3 + ...)$

a systematic search reduces the problem to solving on the order of 800 polynomial relationships amongst 160 unknowns

Constructing New Seiberg-Witten Curves (sp(5))

7 singular points and 10 singularities

- $\Delta \sim (u + ...)^4(u^6 + ...)$

- $\Delta \sim (u + ...)^3(u + ...)^2(u^5 + ...)$

- $\Delta \sim (u^3 + ...)^2(u^4 + ...)$

a systematic search was not attempted for this case because of the outcome found on the previous slide

VIII Future Work

- construct Seiberg-Witten curves for the sp(3) $\oplus$ su(2) and sp(5) flavor symmetries

  systematic searches are plagued with technical difficulties

- attempt matching the discriminant by adjoint breakings of the maximal flavor symmetry group (E_7 and E_8)

- every discriminant form yields at least one solution by adjoint breaking

- construct λ_{SW} by the analysis developed by Minahan and Nemeschansky in:
  Nuclear Physics B 482 (1996) 142-152 and
  Nuclear Physics B 489 (1997) 24-46