Argyres-Seiberg Duality and New SCFT’s

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w/ Philip Argyres and Paul Esposito
Outline

I Argyres-Seiberg Duality

II Criteria For Duality

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IV Review of Seiberg-Witten Theory

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VII Constructing New Seiberg-Witten Curves

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I Argyres-Seiberg Duality

Philip Argyres & Nathan Seiberg 0711.0054

\[ g[d_i] \text{ w/ } r \simeq \tilde{g}[\tilde{d}_i] \text{ w/ } (\tilde{r} \oplus \text{SCFT}[d : \mathfrak{h}]) \]

- LHS: There is a gauge group $g$ with matter charged in representations $r$.

- RHS: There is a rank 1 SCFT with mass dimension of the Coulomb branch moduli $d$ and flavor symmetry $\mathfrak{h}$. Then $\tilde{g} \subset \mathfrak{h}$ is gauged with matter charged in representations $\tilde{r}$.
II Criteria for Duality

Philip Argyres & JRW 0712.2028

- the spectrum of dimensions of Coulomb branch vevs
  \[ \{d_i\} = \{\tilde{d}_i\} \cup \{d\} \]
- the flavor symmetry algebras
  \[ f = \tilde{f} \oplus H \]
- the beta function from weakly gauging the flavor symmetry
  \[ T(r) = T(\tilde{r}) + k_{\tilde{h}} \cdot I_{H \leftarrow \tilde{h}} \]
- the number of marginal couplings
  \[ 2 \cdot T(\tilde{g}) = T(\tilde{r}) + k_{\tilde{h}} \cdot I_{\tilde{g} \leftarrow \tilde{h}} \]
Criteria for Duality (cont’d)

• contribution to the $u(1)_R$ central charge (for the SCFT)
  \[ \frac{3}{2} k_R = 24 c = 4 (|g| - |\tilde{g}|) + (|r| - |\tilde{r}|) \]

• contribution to the $a$ conformal anomaly (for the SCFT)
  \[ 48 a = 10 (|g| - |\tilde{g}|) + (|r| - |\tilde{r}|) \]

• existence of a global $\mathbb{Z}_2$-obstruction to gauging the flavor symmetry
Criteria for Duality \((a\) and \(c\) anomalies\)

- in Lagrangian theories, the \(a\) and \(c\) anomalies can be computed by t’ Hooft anomaly matching
  \[
  4 \cdot (2 \cdot a - c) = |g| = \sum_i (2 \cdot d_i - 1)
  \]

- looking back at the criteria the SCFT satisfies a similar relation
  \[
  4 \cdot (2 \cdot a - c) = (2 \cdot d - 1)
  \]

- Shapere and Tachikawa have given a proof that this formula holds for a large class of \(N=2\) SCFT’s in 0804.1957
Criteria for Duality (\(\mathbb{Z}_2\)-obstruction)


\(G_2 \; \text{w/} \; 8 \cdot 7 \cong \text{su}(2) \; \text{w/} \; (2 \oplus \text{SCFT}[6 : \text{sp}(5)])\)

- LHS: the 7 of \(G_2\) is real \(\Rightarrow\) the flavor symmetry is \(\text{sp}(4)\)
  when the \(\text{sp}(4)\) is gauged there is a \(\mathbb{Z}_2\)-obstruction because the 8 is pseudoreal

- RHS: the \(\text{su}(2)\) has an anomaly related to the single 2 \(\Rightarrow\)
  \(\text{sp}(5)\) must posses a \(\mathbb{Z}_2\)-obstruction to gauging in order to cancel this since the LHS is anomaly free \(\Rightarrow\)
  this gives \(\text{sp}(4)\) a \(\mathbb{Z}_2\)-obstruction since \(I_{f \hookrightarrow h} = 1\) for \(f\) either \(\text{su}(2)\) or \(\text{sp}(4)\)
III Examples of Duality and Results (1 Marginal Operator)

<table>
<thead>
<tr>
<th>g</th>
<th>w/ r</th>
<th>=</th>
<th>˜g</th>
<th>w/ ˜r ⊕ SCFT [d : h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>sp(3)</td>
<td>14 ⊕ 11 · 6</td>
<td>=</td>
<td>sp(2)</td>
<td></td>
</tr>
<tr>
<td>su(6)</td>
<td>20 ⊕ 15 ⊕ 15 ⊕ 5 · 6 ⊕ 5 · 6</td>
<td>=</td>
<td>su(5)</td>
<td>5 ⊕ 5 ⊕ 10 ⊕ 10</td>
</tr>
<tr>
<td>so(12)</td>
<td>3 · 32 ⊕ 32' ⊕ 4 · 12</td>
<td>=</td>
<td>so(11)</td>
<td>3 · 32</td>
</tr>
<tr>
<td>G_2</td>
<td>8 · 7</td>
<td>=</td>
<td>su(2)</td>
<td>2</td>
</tr>
<tr>
<td>so(7)</td>
<td>4 · 8 ⊕ 6 · 7</td>
<td>=</td>
<td>sp(2)</td>
<td>5 · 4</td>
</tr>
<tr>
<td>su(6)</td>
<td>21 ⊕ 21 ⊕ 20 ⊕ 6 ⊕ 6</td>
<td>=</td>
<td>su(5)</td>
<td>10 ⊕ 10</td>
</tr>
<tr>
<td>sp(2)</td>
<td>12 · 4</td>
<td>=</td>
<td>su(2)</td>
<td></td>
</tr>
<tr>
<td>su(4)</td>
<td>2 · 6 ⊕ 6 · 4 ⊕ 6 · 4</td>
<td>=</td>
<td>su(3)</td>
<td>2 · 3 ⊕ 2 · 3</td>
</tr>
<tr>
<td>so(7)</td>
<td>6 · 8 ⊕ 4 · 7</td>
<td>=</td>
<td>G_2</td>
<td>4 · 7</td>
</tr>
<tr>
<td>so(8)</td>
<td>6 · 8 ⊕ 4 · 8' ⊕ 2 · 8''</td>
<td>=</td>
<td>so(7)</td>
<td>6 · 8</td>
</tr>
<tr>
<td>so(8)</td>
<td>6 · 8 ⊕ 6 · 8'</td>
<td>=</td>
<td>G_2</td>
<td></td>
</tr>
<tr>
<td>sp(2)</td>
<td>6 · 5</td>
<td>=</td>
<td>su(2)</td>
<td>3 · 2</td>
</tr>
<tr>
<td>sp(2)</td>
<td>4 · 4 ⊕ 4 · 5</td>
<td>=</td>
<td>su(2)</td>
<td></td>
</tr>
<tr>
<td>su(4)</td>
<td>10 ⊕ 10 ⊕ 2 · 4 ⊕ 2 · 4</td>
<td>=</td>
<td>su(3)</td>
<td>3 ⊕ 3</td>
</tr>
<tr>
<td>su(3)</td>
<td>6 · 3 ⊕ 6 · 3</td>
<td>=</td>
<td>su(2)</td>
<td>2 · 2</td>
</tr>
<tr>
<td>su(4)</td>
<td>6 · 6 ⊕ 6 · 6</td>
<td>=</td>
<td>sp(2)</td>
<td>6 · 4</td>
</tr>
<tr>
<td>su(3)</td>
<td>3 ⊕ 3 ⊕ 6 ⊕ 6</td>
<td>=</td>
<td>su(2)</td>
<td>n · 2</td>
</tr>
</tbody>
</table>

• predicted dualities with one marginal operator
### Examples of Duality and Results (2 Marginal Operators)

<table>
<thead>
<tr>
<th>g</th>
<th>w/</th>
<th>r</th>
<th>(=)</th>
<th>(\tilde{g})</th>
<th>w/</th>
<th>(\tilde{r})</th>
<th>SCFT ([d : h])</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.</td>
<td>su(2) ⊕ su(3)</td>
<td>2 · (2, 1) ⊕ (2, 3 ⊕ 3) ⊕ 4 · (1, 3 ⊕ 3)</td>
<td>=</td>
<td>su(2) ⊕ su(2)</td>
<td>2 · (2, 1) ⊕ 2 · (1, 2)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>su(2) ⊕ sp(2)</td>
<td>2 · (2, 4) ⊕ 8 · (1, 4)</td>
<td>=</td>
<td>su(2) ⊕ su(2)</td>
<td>3 · (2, 1)</td>
<td>4 : E₇</td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>su(2) ⊕ sp(2)</td>
<td>3 · (2, 1) ⊕ (2, 5) ⊕ 4 · (1, 5)</td>
<td>=</td>
<td>su(2) ⊕ su(2)</td>
<td>2 · (2, 1)</td>
<td>[4 : sp(3) ⊕ su(2)]</td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td>su(2) ⊕ G₂</td>
<td>(2, 1) ⊕ (2, 7) ⊕ 6 · (1, 7)</td>
<td>=</td>
<td>su(2) ⊕ su(2)</td>
<td>(2, 1) ⊕ (1, 2)</td>
<td>[6 : sp(5)]</td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td>su(3) ⊕ su(3)</td>
<td>2 · (3, 3) ⊕ 2 · (3, 3)</td>
<td>=</td>
<td>su(2) ⊕ su(3)</td>
<td>2 · (2, 1)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td>su(3) ⊕ su(3)</td>
<td>(3 ⊕ 3, 3 ⊕ 3)</td>
<td>=</td>
<td>su(2) ⊕ su(3)</td>
<td>2 · (2, 1)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td>su(3) ⊕ su(3)</td>
<td>3 · (3 ⊕ 3, 1) ⊕ (3, 3) ⊕ 3 · (1, 3 ⊕ 3)</td>
<td>=</td>
<td>su(2) ⊕ su(3)</td>
<td>2 · (2, 1)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td>su(3) ⊕ sp(2)</td>
<td>(3 ⊕ 3, 1) ⊕ (3 ⊕ 3, 5)</td>
<td>=</td>
<td>su(2) ⊕ sp(2)</td>
<td>2 · (2, 1)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>su(3) ⊕ sp(2)</td>
<td>2 · (3 ⊕ 3, 1) ⊕ (3 ⊕ 3, 4) ⊕ 6 · (1, 4)</td>
<td>=</td>
<td>su(2) ⊕ sp(2)</td>
<td>2 · (2, 1) ⊕ 6 · (1, 4)</td>
<td>3 : E₆</td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td>sp(2) ⊕ sp(2)</td>
<td>2 · (5, 1) ⊕ (5, 4) ⊕ 7 · (1, 4)</td>
<td>=</td>
<td>su(2) ⊕ sp(2)</td>
<td>2 · (3 ⊕ 3, 1)</td>
<td>4 : E₇</td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td>sp(2) ⊕ sp(2)</td>
<td>4 · (4, 1) ⊕ 2 · (4, 4) ⊕ 4 · (1, 4)</td>
<td>=</td>
<td>su(2) ⊕ sp(2)</td>
<td>7 · (1, 4)</td>
<td>4 : sp(3) ⊕ su(2)</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>sp(2) ⊕ G₂</td>
<td>5 · (4, 1) ⊕ (4, 7) ⊕ 4 · (1, 7)</td>
<td>=</td>
<td>su(2) ⊕ G₂</td>
<td>4 · (1, 7)</td>
<td>4 : E₇</td>
<td></td>
</tr>
</tbody>
</table>

- Predicted dualities with two marginal operators
Examples of Duality and Results (New SCFT’s)

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\mathfrak{h}$</th>
<th>$k_{\mathfrak{h}}$</th>
<th>$3/2 \cdot k_R$</th>
<th>$48 \cdot a$</th>
<th>$\mathbb{Z}_2$ anomaly?</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$E_8$</td>
<td>12</td>
<td>124</td>
<td>190</td>
<td>no</td>
</tr>
<tr>
<td>6</td>
<td>sp(5)</td>
<td>7</td>
<td>98</td>
<td>164</td>
<td>yes</td>
</tr>
<tr>
<td>4</td>
<td>$E_7$</td>
<td>8</td>
<td>76</td>
<td>118</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>sp(3) $\oplus$ su(2)</td>
<td>5 $\oplus$ 8</td>
<td>58</td>
<td>100</td>
<td>yes $\oplus$ no</td>
</tr>
<tr>
<td>3</td>
<td>$E_6$</td>
<td>6</td>
<td>52</td>
<td>82</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>$2 \leq \text{rank}(\mathfrak{h}) \leq 6$</td>
<td>$(8 - n)/\text{I}_{\text{su}(2) \hookrightarrow \mathfrak{h}}$</td>
<td>38-2$n$</td>
<td>68-2$n$</td>
<td></td>
</tr>
</tbody>
</table>

- The flavor central charges, $k_{\mathfrak{h}}$, were confirmed for $E_6$, $E_7$, and $E_8$ through an F-theory calculation by Aharony and Tachikawa in 0711.4532.
the physics is encoded by:

- the Seiberg-Witten curve:
  \[ y^2 = x^3 + f(u, m_i)x + g(u, m_i) \]

- and the Seiberg-Witten 1-form: \( \lambda_{SW} \)
  \[ \partial_u \lambda_{SW} = \frac{dx}{y} + \partial_x(\star)dx \]

charged states are encoded by:

- \( u(1) \) charges of a physical state are given by the homology class of a cycle, \( \gamma = n_e[\alpha] + n_m[\beta] \), when \( m_i = 0 \)

- these states have central charge, \( Z = \oint_\gamma \lambda_{SW} \)
Seiberg-Witten Theory (Singlarities)

the singularities of the Seiberg-Witten curve:

- are located at $\Delta = 4 \cdot f^3 - 27 \cdot g^2 = 0$

- physically they correspond to a breakdown of the low-energy description  
  charged states becoming massless
Central Charges and Curves

Shapere and Tachikawa 0804.1957

the twisted version of Seiberg-Witten theory relates the anomalies and central charges to:

- the mass dimension of the vev on moduli space
- \# of neutral hypermultiplets on moduli space
- \# of singular points of the Seiberg-Witten curve
Central Charges and Curves (twisted PI)

\[ \int [du][dq] A^\chi B^\sigma C^n e^{-S_{low-energy}} \]

- \( \chi \) is the Euler characteristic
- \( \sigma \) is the signature
- \( n \) is an instanton number
- \( A^2 = \det \left[ \frac{\partial w_i}{\partial a_j} \right] \)
- \( B^8 = \text{Radical}[\Delta] \)
Central Charges and Curves (master equations)

- the scaling behavior of the measure determines the R-charge of the states becoming massless at a singularity

- the normalization is: $R(\star) = 2 \cdot D(\star)$

- $48 \cdot a = 12 \cdot R(A) + 8 \cdot R(B) + 10 \cdot r + 2 \cdot h$

- $24 \cdot c = 8 \cdot R(B) + 4 \cdot r + 2 \cdot h$

- $4(2 \cdot a - c) = 2 \cdot R(A) + r = \sum_{i=1}^{r} 2 \cdot (d_i - 1) + r = \sum_{i=1}^{r} (2 \cdot d_i - 1)$

$r \equiv$ the complex dimension of moduli space

$h \equiv$ the # of massless neutral hypermultiplets
Central Charges and Curves ($r = 1$)

- $R(A) = d - 1$

- $R(B) = \frac{1}{4} \cdot Z \cdot d$

  $Z = \# \text{ of singular points of the Seiberg-Witten curve}$

- $24 \cdot c = 2 \cdot Z \cdot d + 4 + 2 \cdot h$

- $k_{\hbar} = 2 \cdot d - h$
### Central Charges and Curves (Results)

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\mathfrak{h}$</th>
<th>$Z$</th>
<th>$2 \cdot h$</th>
<th>rep.’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$E_8$</td>
<td>10</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>$\text{sp}(5)$</td>
<td>7</td>
<td>10</td>
<td>10(s)</td>
</tr>
<tr>
<td>4</td>
<td>$E_7$</td>
<td>9</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>$\text{sp}(3) \oplus \text{su}(2)$</td>
<td>6</td>
<td>(6, 0)</td>
<td>$6 \oplus 1(s)$</td>
</tr>
<tr>
<td>3</td>
<td>$E_6$</td>
<td>8</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>$\text{su}(3)$</td>
<td>4</td>
<td>6</td>
<td>$3 \oplus \bar{3}(c)$</td>
</tr>
</tbody>
</table>

- no neutral hypermultiplets on the LHS of the equivalence
- neutral hypermultiplets on the RHS must be charged under the flavor symmetry
VI $\mathbb{Z}_2$-obstruction Revisited

- there was a $\mathbb{Z}_2$-obstruction for the $\text{sp}(5)$ and $\text{sp}(3)$ factors

- these obstructions come from the single pseudoreal charged under them

- consider the previous example:

$$G_2 \text{ w/ } 8 \cdot 7 \simeq \text{su}(2) \text{ w/ } (2 \oplus \text{SCFT}[6 : \text{sp}(5)])$$

$$\text{su}(2) \oplus \text{sp}(4) \subset \text{sp}(5)$$

$$(2,1) \oplus (1,8) = 10$$
VII Constructing New Seiberg-Witten Curves

- the work of Shapere and Tachikawa specifies possible forms of the discriminant of the Seiberg-Witten curve

- the discriminants are determined by partitioning the total order of the singularity at $m_i = 0$ into a $\mathbb{Z}$-tuple of integers.
Constructing New Seiberg-Witten Curves (su(3))

4 singular points and 8 singularities

- \( \Delta \sim (u + ...)^5(u^3 + ...) \)
- \( \Delta \sim (u + ...)^4(u + ...)^2(u^2 + ...) \)
- \( \Delta \sim (u + ...)^3(u + ...)(u^2 + ...)^2 \)
- \( \Delta \sim (u^2 + ...)^3(u^2 + ...) \)
- \( \Delta \sim (u^4 + ...)^2 \)

A systematic search reveals 2 solutions
Constructing New Seiberg-Witten Curves \((\text{sp}(3) \oplus \text{su}(2))\)

6 singular points and 9 singularities

- \(\Delta \sim (u + ...)^4(u^5 + ...)\)
- \(\Delta \sim (u + ...)^3(u + ...)^2(u^4 + ...)\)
- \(\Delta \sim (u^3 + ...)^2(u^3 + ...)\)

A systematic search reduces the problem to solving on the order of 800 polynomial relationships amongst 160 unknowns
Constructing New Seiberg-Witten Curves (sp(5))

7 singular points and 10 singularities

• $\Delta \sim (u + \ldots)^4(u^6 + \ldots)$

• $\Delta \sim (u + \ldots)^3(u + \ldots)^2(u^5 + \ldots)$

• $\Delta \sim (u^3 + \ldots)^2(u^4 + \ldots)$

a systematic search was not attempted for this case because of the outcome found on the previous slide
Future Work

• construct Seiberg-Witten curves for the $\text{sp}(3) \oplus \text{su}(2)$ and $\text{sp}(5)$ flavor symmetries

  systematic searches are plagued with technical difficulties

• attempt matching the discriminant by adjoint breakings of the maximal flavor symmetry group ($E_7$ and $E_8$)

• every discriminant form yields at least one solution by adjoint breaking

• construct $\lambda_{SW}$ by the analysis developed by Minahan and Nemeschanskyy in:
  Nuclear Physics B 482 (1996) 142-152 and
  Nuclear Physics B 489 (1997) 24-46