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By

H.S. Dhami

A.K. Singh

G.S. Negi

and

Bhupendra Singh

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INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455–0436

Phone: 612/624-6066 Fax: 612/626-7370
URL: http://www.ima.umn.edu

ESTIMATION OF BIAS AND RELATIVE ERROR FROM THE AGGREGATION MODEL DEVELOPED FOR A SAMPLE VALLEY VILLAGE OF ALMORA DISTRICT*

H.S.Dhami**,A.K.Singh,G.S.Negi and Bhupendra Singh
University of Kumaun,
Shoban Singh Jeena Campus,
lmora (Uttaranchal) India 263601

ABSTRACT

The present study attempts in the direction of examining errors and their analysis with the intent that the developed aggregate model would simulate behaviour of the real system at the highest level of accuracy possible under the employed modelling strategy.

It has been demonstrated that the aggregate models are recursive in nature and the minimum value of bias corresponding to value of ‘t’ has been obtained. It has also been depicted that the time of minimum bias does not correspond to the time of minimum relative error.

KEY WORDS

relative error, bias, aggregate model, recursive, modelling parametric function, polynomial function

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- ** To whom all correspondence be addressed
- Professor H.S.Dhami, Dept. of Mathematics, University of Kumaun, SSJ Campus Almora, Principal Investigator, IERP Project
- Dr.A.K.Singh, Reader, Dept. of Geography, SSJ Campus Almora, Co-Investigator, IERP Project
- Dr.G.S.Negi, Junior Project Fellow, IERP Project
- Bhupendra Singh, Field Surveyor, IERP Project

INTRODUCTION

The global factual reality is that everything is related to everything else- or that everything depends on some thing else for its survival or in different tone it can be said that we have interactive system of plants, animals, humans and their surrounding physical environment. This explanation tells the tail of facts about ecosystem which contains living and non-living organisms that each provide or contribute to a unique service or function upon which their organisms depend on. The notion of ecosystem recognizes the many ways an organism interacts with and is dependent for its own survival on various parts of its environment. All ecosystems are nested within a system of larger ecosystems and can be defined by different scales or sizes from as small as a puddle or a rotting log to a forest or the planet.

Ecosystem can also be defined as a dynamic entity composed of a biological community and its associated abiotic environment. Often the dynamic interactions that occur within an ecosystem are numerous and complex. The process of alterations in biotic and abiotic components of ecosystem is continuous. Some of these alterations begin first with a change in the state of one component of the ecosystem which then cascades and sometimes amplifies in the other components because of the relationship.

In recent years the impact of humans has caused a number of catastrophic changes to a variety of ecosystems found on the earth. Humans use and modify natural ecosystems through agriculture, forestry, recreation, urbanization, industrialisation and through various anthropogenic activities. Man has been considered as among the strongest powers of organic nature that has caused ecosystem anarchism. The continued ecological tensions caused and created by humans can be seen anywhere.

The complexity of environmental problems we face now and in the future is ever increasing. The traditional reaction of a man to the apparent complexity of the world around him has been to make for him a simplified and intelligible picture of the world. The mind decomposes the real world into a series of simplified systems and thus achieves in one act 'an overview of the essential characteristics of a domain'. The simplified statements of the structural independence of the system have been termed 'models'. Proceedings of the Cross Discipline Ecosystem Modelling and Analysis Workshop, held during 15-17, 2000 under the US Environmental Protection agency deals with Multimedia Ecosystem models deals with multimedia ecosystem models of varied nature.

Prominent among them being- Land Surface Hydrology, Mechanistic based Ecosystem Exposure modelling on a watershed scale, Data visualization and analysis tools, Land scape and subsurface characterization etc.

Ecosystem components and their interrelationship can always be quantified and thus lead to the formulation of quantitative models which are crucial to almost every area of ecosystem science. They provide a logical structure and framework that guides as well as informs empirical observations of ecosystem processes. They play a practically pivotal role in synthesizing and integrating our understanding of the immense diversity of ecosystem structure and function. The confidence in regional models depend n how well the field data used to develop the model represent the region of interest, how well the environmental model driving variables are represented in the model and how well regional model predictions agree with observed data for the region.

Charles D.Canham et al [2] have presented different models in their book entitled “Models in Ecosystem Science”. Alldredge & Jackson [1] have defined aggregation in marine systems as one major process that significantly alters the sizes, characteristics and abundances of suspended particles and have discussed its mechanism. Jackson & Burd [4],have discussed aggregation in marine environment also.

Here we are making an attempt in the direction of developing the aggregation model and estimate the value of ‘t’ for minimum values of bias and relative error for village ecosystem with special reference to a sample valley village of Almora district.

GENERAL FORMULATION

The state of system for different values of $i = 1,2,........p$ is defined as

$$x(t) = \sum_{i=1}^{p} x_i(t) \quad ....(1.1)$$

The value of $x_i(t)$, as described in the book of Efraim Halfon [3] has been given as

$$x_i(t) = \frac{u_i}{l_i} + \left(x_i(0) - \frac{u_i}{l_i} \right) e^{-l_i t} \quad ....(1.2)$$
$$for \quad i = 1,2,.......p$$

The inferences derived on the basis of our earlier study [5], has suggested that the modelling parametric exponential function in (1.2) should be replaced by a polynomial function, which has been justified by us by adopting simulation technique. Summation of a finite number of terms is always preferable as it is most likely that summation of infinite number of terms of the exponential function may lead to wrong results. We thus have the state of system as

$$x(t) = \sum_{i=1}^{p} \left\{ x_i(\infty) + \left(x_i(0) - x_i(\infty)\right)\left(a + \frac{b}{l_i + 2t} + cl_i\right) \right\} \quad \ldots\ldots\ldots\ldots\ldots(1.3)$$

THE AGGREGATION MODEL

In this section we shall establish that aggregate models are recursive in a more complicated form.

Let $x_A(t)$ be an approximation model for $x(t)$, where

$$x_A(t) = \sum_{i=1}^{p} \left\{ x_i(\infty) + \left(\sum_{i=1}^{p} x_i(0) - \sum_{i=1}^{p} x_i(\infty) \right)\left(a + \frac{b}{l_A + 2t} + cl_A \right) \right\} \quad \ldots\ldots\ldots..(1.4)$$

The relative error in using $x_A(t)$ to approximate $x(t)$ shall be defined by q(t), where

$$q(t) = \frac{x(t) - x_A(t)}{x(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1.5)$$

and the bias given by the numeration, shall be

$$b(t) = \left[x(t) - x_A(t)\right] = \sum_{i=1}^{p} \left[\left\{x_i(0) - x_i(\infty)\right\} \left\{ \frac{(l_1 - l_A)\{c(l_i + 2t)(l_A + 2t) - b\}}{(l_1 + 2t)(l_A + 2t)} \right\} \right] \quad \ldots..(1.6)$$

Hence if $l_1 = l_2 = \ldots\ldots = l_p$,then $b(t)$ shall be zero for all t. The values of $b(t)$ and $q(t)$ shall also be zero if $x_i(0) = x_i(\infty)$ for every value of $i = 1,2,\ldots.p.$

At $t = 0$, we shall have

$$x_A(t) = \sum_{i=1}^{p} x_i(0) = x(0) \quad \ldots\ldots\ldots\ldots\ldots..(1.7)$$

$$and$$

$$q(0) = 0 \quad \ldots\ldots\ldots\ldots..(1.8)$$

and as $t \to \infty$

$$x_A(\infty) = \lim_{x \to \infty} x_A(t) = \sum_{i=1}^{p} \lim_{t \to \infty} \quad x_i(t) = \sum_{i=1}^{p} x_i(\infty) = x(\infty) \quad \ldots\ldots\ldots\ldots.(1.9)$$

so that

$$q(\infty) = 0$$

Also

$$\dot{x}(t) = \sum_{i=1}^{p}\left[\left\{\left(x_i(0) - x_i(\infty)\right) \quad \times - \left(\frac{2b}{(l_1 + 2t)^2}\right)\right\}\right] \qquad \text{.................(1.10)}$$

$$\dot{x}_A(t) = \sum_{i=1}^{p}\left[\left\{\left(x_i(0) - x_i(\infty)\right) \quad \times - \left(\frac{2b}{(l_A + 2t)^2}\right)\right\}\right] \qquad \text{..............(1.11)}$$

These two values of first differential coefficients are again functions of ‘t’, so we can yield second differential coefficients. It helps in finding the condition for minimum value of bias corresponding to following value of ‘t’

$$t = -\frac{1}{4}\left(l_i + l_A\right) \qquad \text{................(1.12)}$$

Corresponding to value of ‘t’ given by (1.12), we shall have the expression for minimum bias as

$$\left[b(t)\right]_{\min} = \sum_{i=0}^{p}\left[\left\{x_i(0) - x_i(\infty)\right\}(2bc)\right] \qquad \text{...................(1.13)}$$

The time of minimum bias does not correspond to the time of minimum relative error, as ‘t’ in this case is a solution of the quadratic equation

$$bct^2 + t\left\{ab + bc\left(l_i + l_A\right)\right\} + \frac{1}{4}\left\{bc\left(l_i^2 + l_i l_A + l_A^2\right) + ab\left(l_i + l_A\right) + b^2\right\} = 0 \qquad \text{.............(1.14)}$$

Additive model can be generated as under

$$x^p(t) = \sum_{i=1}^{p}\left[\left\{x_i(\infty) + \left(x_i(0) - x_i(\infty)\right)\right\}\left\{a + \frac{b}{l_i + 2t} + cl_i\right\}^p\right] \qquad \text{.................(1.15)}$$

which is of recursive nature as above expression can be expressed in the following form

$$x^{p-1}(t) + x_p(t) \qquad \text{....................(1.16)}$$

where $x^{p-1}(t)$ corresponds to

$$\sum_{i=1}^{p}\left[\left\{x_i(\infty) + \left(x_i(0) - x_i(\infty)\right)\right\} p_{C_1}\left(a + cl_i\right)^{p-1} . \left(\frac{b}{l_i + 2t}\right)\right] \qquad \text{..............(1.17)}$$

and the remaining terms shall be covered in $x_p(t)$.

The aggregate model is also recursive as is evident from following expression

$$x_A{}^p(t) = \sum_{i=1}^{p} x_i(\infty) + \left[\left\{\sum_{i=1}^{p} x_i(0) - \sum_{i=1}^{p} x_i(\infty)\right\}\left\{a + \frac{b}{l_A + 2t} + cl_A{}^2\right\}^p\right] \qquad \text{.............(1.18)}$$

which by the explanation given by the expression (1.16), shall assume the form

$$x_A{}^p(t) = \sum_{i=1}^{p}\left[x_i(\infty) + x_p(\infty)\right] + \left[\sum_{i=1}^{p-1} x_i(0) - \sum_{i=1}^{p-1} x_i(\infty)\right]\left(a + \frac{b}{l_A + 2t} + cl_A{}^2\right)^p +$$

$$+ \qquad \left[x_p(0) - x_p(\infty)\right]\left(a + \frac{b}{l_A + 2t} + cl_A{}^2\right)^p \quad \text{................(1.19)}$$

Above equation can be written in recursive form as

$$x_A{}^p(t) = x_A{}^{p-1}(t) + \left[\sum_{i=1}^{p-1} x_i(0) - \sum_{i=1}^{p-1} x_i(\infty)\right]\left(a + \frac{b}{l_A + 2t} + cl_A{}^2\right)^p +$$

$$\left[x_p(0) - x_p(\infty)\right]\left(a + \frac{b}{l_A + 2t} + cl_A{}^2\right)^p + x_p(\infty) \qquad \text{.........(1.20)}$$

The intent of finding bias and its minimum value subject to value of 't' given by equations (1.12) and (1.14) is to ultimately errors from the general state vector linear model so that the output from models may be more properly interpreted.

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