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ABSTRACT

The present study attempts to develop a mathematical model for description of the village ecosystem within a larger perspective. However, on exemplary basis the application of the technique has been demonstrated for a sample valley village of Almora district with the hope that it should hold good for all other villages under almost similar natural and geographical conditions.

The ecosystem has been defined with the help of distinct subsystems comprising spatial variables. Human resource, Household items, Land inventory, Farm machinery & implements, Energy consumption, Settlement and Livestock. Each subsystem has been further subdivided into distinct subsystems. Association or link between two or more entities has been defined with the help of entity-relationship of different types (one to one, one to many and many to one).

The relationship of sustainable productivity with different state effective variables (inferences derived on the basis of different elementary quantification tools) have generated different set of equations during Rabi and Kharif seasons but with varying degree of efficiency. It has prompted us to apply assignment problem of linear programming in order to have the best assignment to attain the objectives of live together and guard together. Optimal solutions have been transmogrified into easily understandable terms.

State of the system has been defined in terms of theoretical general state vector linear model and the empirically observed values have been compared with the theoretical values. The relative error values for different values of ‘t’ have made us to change the modeling parameter function by another polynomial function. This function has been also tested on computer by simulation process.

KEYWORDS

State of the system, subsystem, sustainable productivity, regression equations, assignment problem, linear programming, effect ness matrix, simulation process.

INTRODUCTION

The basic warp and woofs of policies for sustainable development are based on the classic Brundtland definition that is, "to meet the needs of the present without compromising the ability of future generations to meet their own needs". It takes care of the present and leads to future welfare. The general acceptance of this concept does not coincide with the true technical meaning in vogue, and the commonness of its understanding is somewhat different from its concrete meaning. It neither spells the sheer replacement of 'development' nor negates meaning of environmental conservation and protection. It is a gestaltic and holistic concept which makes present pleasant and future secured as presented by Raffensperger et al (1999) in their book.

There exist two completely different concepts of securing a sustainable base for future development, the one described as "strong sustainability" and the other "weak sustainability". The first concept calls for keeping natural as well as other resources related with human activities while the second suggests maintaining an unchanged total amount of resources. Man's intrusion inevitably disturbs the delicate balance of organisms, substances and their activity that nature has evolved. Ecosystems can adopt to man's reckless activities but only up to a certain extent. Sustainability can not exist without guaranteeing the protection of elementary environment from the reckless attacks of the activities of mankind.

Concept of sustainability argues in favor of a relevant policy and not in favor of pseudo and irrelevant ones. The relevancy of the sustainable policy forces us to proceed on the lines of logical positivism in order to develop abstract as well as concrete models guiding what is adequate and compatible for the present as well as for the future.

Undoubtedly the nature valuation methodology has improved considerably in the academic world during the last few decades, allowing more and more precise estimations of the values social and ecological, economic etc, of various resources. The results of such analyses are of utmost importance in planning the environmental protection strategies as they explain the nature of the benefits that may be expected after keeping or restoring particular elements of natural environment.

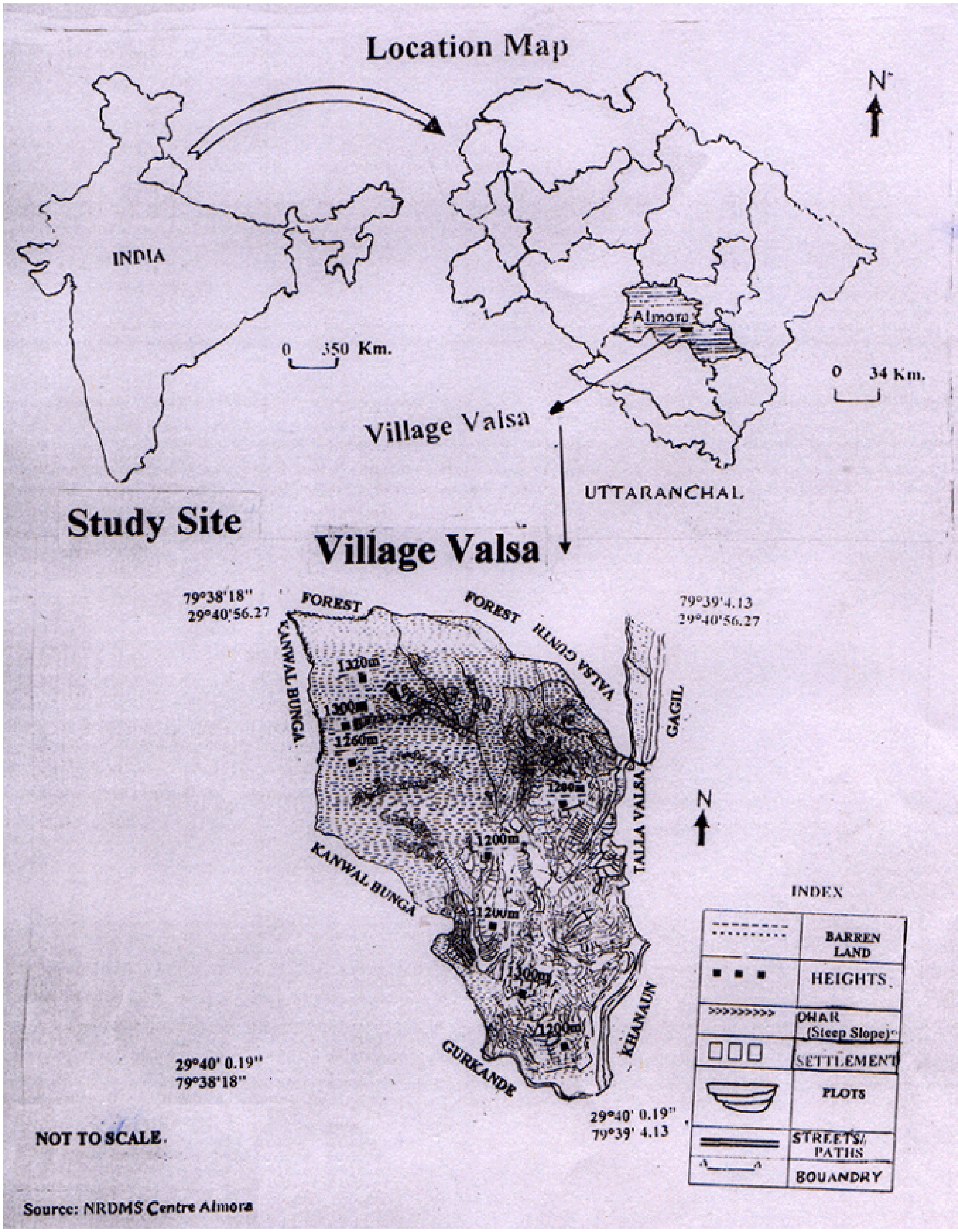
There are many scientific programmes whose aim is to help implement sustainable development. These programs generally focus on assessing and forecasting changes in biodiversity, structure, function and dynamics of ecosystems and their services; relationships between society, economy, biodiversity and habitats; integrated assessments of drivers affecting biodiversity and mitigation of biodiversity loss; risk assessment, management, conservation and rehabilitation options.

The traditional reaction of man to the apparent complexity of the world around him has been to make a simplified and intelligible picture of the world. The mind decomposes the real world into a series of simplified systems and thus achieves in one act ‘an overview of the essential characteristics of a domain’. The simplified statements about these structural interdependences have been termed ‘models’. The impact on ecosystem can only be described quantitatively by the application of ecological models and systems ecology, and their reaction to perturbations can only be understood by the use of ecosystem theory. A system may be defined as a set of elements interacting and functioning to represent the whole. It is generally agreed that a “System” in a model of general nature is, a conceptual analog of certain rather than universal traits of observed entities. A model is thus a simplified structuring of a reality which presents supposedly significant features or relationship in a generalized form.

The real world problems often require description, explanation, prediction or justification or within a larger perspective it may be said that there is a necessity of technological process conducive to decision ranking planning and such like. All these desired goals can be achieved efficiently by the use of mathematical tools which in turn pave the way for the emergence of mathematical modeling.

Mathematical modeling essentially consists of translating real world problem into mathematical problems, solving the mathematical problems and interpreting these solutions in the language of real world. Developing a mathematical model can be seen as a search for the description or even as a ‘cause’ for an observed fact constructing a design for something which does not preexist, or for developing an action plan.

Ecosystems, as examples of large-scale multifaceted systems, require that a multiplicity of models be developed since a single all-encompassing model, however desirable as a conceptual goal, is not a practical object. By decomposing questions and modeling objectives into an ordered structure of elements called experimental frames (Zeigler, 1976), useful partial models may be constructed, validated, and employed, each one attuned to a particular experimental frame. Concomitant with the pluralism of such partial models is the recognition that models are expressible in different formalisms, each offering, conceptual and computational advantages within its domain of application (Zeigler and Barto, 1977).


Location Map
INDIA
0 350 Km.
N
Almora
0 34 Km.
Village Valsa
UTTARANCHAL
Study Site
Village Valsa
79°38'18"
29°40'56.27
79°39'4.13
29°40'56.27
FOREST
FOREST
VALSA GUNTH
GAGIL
KANWAL BUNGA
TALLA VALSA
KANWAL BUNGA
KHANAUN
GURKANDE
29°40' 0.19"
79°38'18"
29°40' 0.19"
79°39' 4.13
NOT TO SCALE.
INDEX
BARREN LAND
HEIGHTS
DHAR (Steep Slope)
SETTLEMENT
PLOTS
STREETS/ PATHS
BOUANDRY
Source: NRDMS Centre Almora

PRELIMINARY INTERPRETATION OF THE COMPILED DATA

The functioning of any village ecosystem (agricultural and allied activities) is governed by the working age group which in the present case formed about 66.14 percent of the Total population. The female workforce contributes substantially more in agriculture and related activities 31.12 percent female's vs11.11 males of the total working population. 53.37 percent total Population of village ecosystem is of males, while 46.63 percent belongs to females.

General caste population is 76.28 percent while 24.58 percent is of Schedule caste Migration is a common phenomenon in the villages. Presently 27.39 percent villagers have migrated from the village. Education is one of the factors of any village ecosystem. There is 82.65 percent literacy in the village.

Live Stock

Agro ecosystems live stock is the backbone of central Himalayan region as only they provide are the source of draught power and farmyard manure. A number of live stock are maintained by each household for specific purpose, i.e. cows and buffaloes for milk, bullocks for traction power and goats, sheep, horses and mules for other economic activities including transport. It has been replicated in our study as live stock. Milk yielding animals accounted for about 43.05 percent of the total live stock.

Some other prominent features of the village ecosystem:

Village Land use and classification

Total Land(ha)

Cultivated land (%)	36.32
Irrigated Land (%)	7.47

Abandoned land	(%)	0.89
Barren land	(%)	2.86
Forest land	(%)	50.28
Reserve land	(%)	2.19

Fertilizers used by valsa villagers 100% Inorganic

Energy Consumption in Summer Season 37.81%

Energy Consumption in Winter Season 57.82%

Farm Machinery Implements (Mai and Plough) used by farmers 25% and 75% (Darati, Kutla, etc.,) used by Farmers.

Seeds used by Farmers are given below (in %):

Wheat 6.37%, Barley 0.86%, Peas 0.032%, Massor 0.64%, Mustard 0.17%, Spinich 0.05%, Paddy 9.65%, Capsicum 0.029%, Chilly 0.42%, Tomato 0.034%, Bean 0.04%, Madua 2.058%, Soyabean 0.65%, Urd 0.48%, Gahat 0.58% Potato 0.71%.

Quantification

a. Food grain Production is estimated (from enquiring in 44 households) to be 32800 kg or 328 qt.

b. Fodder Production from cultivated land

$$= (32800 \times 3)/25 = 3936 \text{ head loads/year}$$

Fodder Production from support area:

Individual land (closed to grazing):

$$589 \text{ nails} \times 3 \text{ head loads /nail/ year} = 1767 \text{ head loads/year}$$

Common land (Closed to grazing):

100 nalis ×2 head loads/nail/year = 200 head loads/year

Common land (open to grazing):

35 nalis × 1 head loads/ nail/year = 35 head loads/year

Total = 1767 + 200 + 35 = 2002 head loads/year

From the total, 100 loads of maize and lahi stalks are are deducted. They are used as fuel.

Actual fodder production = 2002-100=1902, of this 120 loads were sold = 1902 – 120 = 1782

Actual fodder consumed = 1782

The fodder consumption per animal unit becomes

(1782/156) = 12 loads/ year

c. Milk Production being-

12 Cows × 90 kg./ Cow/ year = 1080 kg/year

14 buffaloes × 260 kg./ buffalo/ year = 3640 kg/year

Total = 1080 + 3640 = 4720 kg/year

d. Wood consumption is estimated 25 head loads persons 1 year

Total village consumption is thus

206 × 25 = 5150 loads /years.

Production estimated as:

From terrace risers 200 nali × 25 = 5150 loads / year.

From Support area (individual)

589 × (1/5)× 6 = 707 head loads/ year

From homesteads: 80 head loads /year

Crop residues: 100 head loads/ year

From reserve forest: 328 head loads/ year

Total : 1815 head loads/year

e. Spring number 1 had a measured outflow in October of 30 liters /min. This is estimated to fall to 20 liters 1 min. in May. Assuming a linear decline, outflow in march would be about 20 liters/ min. and the average out flow for the period.

Evaluation

1. Adequacy of Subsistence materials

Average daily consumption per person is

$$(4720 \times 1000) / (206 \times 365) = 63 \text{ ml.}$$

We assume that a minimum level of milk consumption should be 250 ml./person/day, or 91 kg./ person/ year.

Concerning wood consumption we assume that for good health and comfort, 26 head loads/ person/ year are needed.

The percent consumption of 22 loads is considered inadequate, and is due to a Scarcity of trees in the support area.

2. Current carrying capacity

Carrying capacity of village ecosystem in terms of milk is carrying capacity (milk) =

$$(4720)/91 = 52 \text{ people}$$

In terms of wood, it is:

Carrying capacity (wood) = (1815)/26 = 70 people

Taking the smaller of those two figures, we will say that the carrying capacity of the village ecosystem is 52.

3. Current index of Sustainability

Since there are at present 206 residents in the village,

Sustainability index = $\frac{52}{206}$ = 0.26

This low value confirms our general conclusion, based upon our observation of the denuded support area, that the villager ecosystem is being used very unsustainably.

The input and output state effecting parameters and their averages have been tabulated as under-

Sl.	**INPUT**	**MEAN**	**OUTPUT**	**MEAN**
1.	Age (Male & Female)	51.04	Cleaning of food grains	33.3
2.	Education (Male& Female)	2.514	Crop residues	2.125
3.	Marital Status (Male& Female)	4.07	Dung Cake	0
4.	Occupation (Male& Female)	2.198	Other use of Goth	1.0
5.	Health and Nutrition (Male& Female)	5.89	Total Dung Product	0.114
6.	Kitchen ware	1.667	Present Value	0.0094
7.	Household items	8.4	Milk and Milk Products	
8.	Ploughing	0.923	Harvesting	15.13
9.	Sowing	1.105	Weeding	2.601
10.	Irrigation	10.406	Breeding	78
11.	Owned land	0.856	Initial Investment	87.820

12.	Leased in	39.96	Source of Finance	39
13.	Leased out	39.96	Fodder requirement	0.039
14.	Cultivated	1.304	Wooden plough	8
15.	UnIrrigated	1.147	Mud setter	9.6
16.	Abandoned	44.4	Spray Pump	0.1
17.	Fallow Land	0	Storage bins	15.4
18.	Protected Pasture for Grazing	0	Density	0
19.	Barren	20.188	Altitudinal Variation	0
20.	Civil	7.568	Whether extra land or any part of abandoned land is converted for terraces	13.32
21.	Soyam	13.32		
22.	Forest	0.783		
23.	Reserve forest	7.654		
24.	Fuel Wood	17.6		
25.	Electricity Unit	1.517		
26.	Kerosene	12.022		
27.	L.P.G. Cylinder	0.88		
28.	Used wood species for house construction	14.66		
29.	Milking No.	6		
30	Non Milking No.	4.457		

In order to demonstrate inter-relationship between these entities or state affecting variables, we have generated a model known as Entity-Relationship model between groups of date elements taken from subsystems and their distinct subsystems as elucidated in fig-I.

The relationships have been marked as M: 1 (Many to one) which symbolizes that to each subsystem in the left there is one subsystem in the right. For example to each male there is a house but the house may be giving shelter to many males; 1:1 (each subsystem on the left corresponds to a subsystem on the right and vice-versa) Example may be of the fact that to each human resource there corresponds male/female and each male/female corresponds to a human resource; M:M (Many to Many), also known as complex relationship glorifies the right that of subsystem on the left correspond to a number of elements of the subsystem on the right and vice-versa. For example the relationship between human resource and settlement is M; M. The study of energy consumption pattern for various activities in different seasons under the head agricultural sector, lead is to observe that activities like land preparation, Weeding, Irrigation, Harvesting, and Threshing, require difference with from human beings as well as animals. It suggests the within- entities relationship as depicted is the figure -I between elements of these subsystems. This type of relationship model finds its origin in set theory where different elements of a set are in one or another way related with elements of other set and thus establishes mappings like injective, surjective etc, or functional relationships of different kinds.

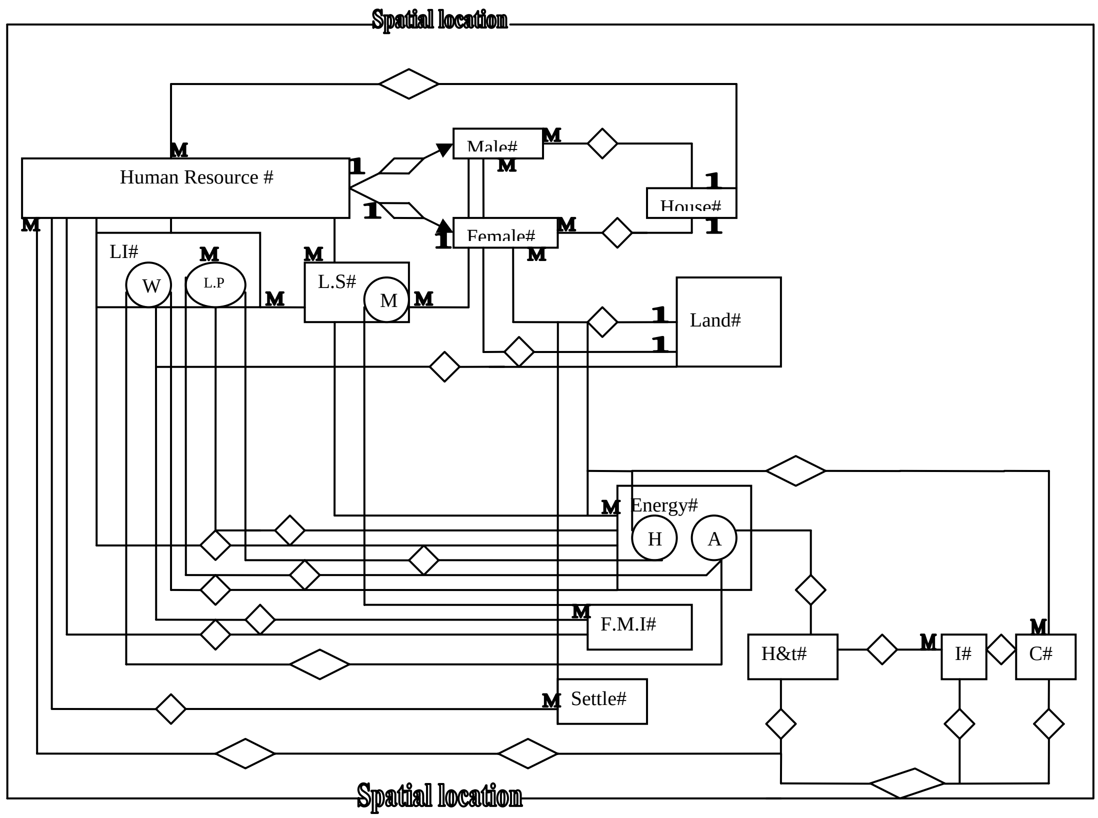


Figure 1

Spatial location (Space, longitude, latitude, altitudinal variation, area, slope, density, Succession)

Human Resource# (Human Resource Inventory).

Male# (Male) age, education, marital status, migration, occupation, Health and nutrition.

Female# (Female) age, education, marital status, migration, occupation, Health and nutrition fertility behavior.

House# (Household Items) Kitchen ware, Entertainment items.

Land# (Land Holding) Owned, leased in, leased out, cultivated, Unirrigated, Irrigation, abandoned, fallow, protected, for pasture, pasture for grazing, barren, civil, soyam, forest, reserve forest, whether extra land any part of abandoned land is converted for terraces.

Energy# (Energy consumption) fuel wood, crop residues, electricity unit, kerosene, dung cake, L.P.G. cylinder, Human, animal.

(i) H&t# (Harvesting and threshing).

(ii) I# (Irrigation)

(iii) C# (Cleaning of food grains).

F.M.I. # (Farm Machinery Implements Wooden plough, mud settler (Mai) spray pump, storage bins).

Settle# (Settlement)

(i) location site, caste association, dimension of house, type of house, no. of stories, Rooms, angan, year of construction, used wood species for house construction, cooking, fuel wood,

(ii) Association bakhali system, associated beliefs, internal parts of the room roof construction material of wall, windows, other uses of goth, social belief associated with the internal organization of the house.

LI.# (Land Inventory) LP (Land Preparation).

W (Weeding).

L.S.# (Live Stock) milking no., non-milking no., breading, Initial investment, source of finance, fodder requirement, total; dung product, present value, Milk and milk products.

Above systems and their subsystems shall be referred as **set of systems** in ensuing pages.

1. GENERAL FORMULATION

In this section we shall develop some formulae which are the prerequisites for the development of a model.

Let us denote an ecosystem composed of p distinct subsystems

$S_i, i = 1,2,....., p$,   that is,

$$S = \bigcup_{i=1}^{p} S_i \qquad (1.1)$$

A set **S** = {S} of ecosystems will denote a collection of ecosystems each member S of the collection will necessary contain the same distinct subsystems in order to be in **S**.

For the real world problem being discussed in our this work, the subsystems shall consist of:-

(1) Spatial Location

(2) Male

(3) Female

(4) Household Items

(5) Land Holding

(6) Land preparation

(7) Energy Consumption)

(8) Farm Machinery and equipments

(9) Settlement

(10) Live Stock

as subsystems. It is obvious that here p = 10.

Each subsystems S_i is also composed of set of distinct subsystems elaborated under **set of systems** mentioned in the previous page.

Symbolically we may represent them as

$$S_{ij}, where \quad j = 1,2,.......19. \quad \text{That is,}$$

$$S_i = \bigcup_{j=1}^{m} S_{ij}, \quad i = 1.2,............10$$

For the sake of illustration it may be cited that

S_{11} Shall correspond to Space

S_{12} Shall Correspond to Longitude

S_{22} Shall Correspond to Female

S_{34} Shall Correspond to Migration

S_{84} represents cultivated land holding

$S_{(11)9}$ has been used for wood species for house construction under S_{11} (Settlement).

and like wise for others.

The genesis of subsystems and their distinct subsystems can be understood on *exampli gratia* basis by following set theoretic properties

$S_2 = S_{21} \cup S_{22}$

Human resource Inventory = Male U Female

Similarly $S_9 = S_{91} \cup S_{92} \cup S_{93} \cup S_{94} \cup S_{95} \cup S_{96} \cup S_{97} \cup S_{98}$

where the symbols have their usual meanings. The elements involved in the process of union shall vary according to the member of distinct subsystems of concerned subsystems.

Now let S′ denote the Modeled system, then from (1.1)

$$S' = \bigcup_{i=1}^{p'} S_i{}' \qquad (1.2)$$

where p′ denotes the number of subsystems $S_i{}'$ which makes up S′ it is obvious that p may not be equal to p′ in the model, nor will the selection of subsystems be identical. But since

$$S' = \left[\bigcup_{i=1}^{p} S_i\right] \cup \left[\bigcup_{S_i \not\subset s} S_i{}'\right] - \left[\bigcup_{S_i \not\subset s} S_i\right] \quad — (1.3)$$

and $\bigcup_{S_i \not\subset s} S_i{}'$ & $\bigcup_{S_i \not\subset s'} S_i$ — (1.4) are small and /or relatively unimportant in the analysis where S is used instead of S′, where $\bigcup_{S_i \not\subset s} S_i{}'$ and $\bigcup_{S_i \not\subset s'} S_i$ denote respectively those subsystems which were there but were not modeled and those subsystems which were modeled but were not there.

2.State of the System

The purpose of modeling an ecosystem under present predicaments shall be the prediction of the state of the system at any time, t, with respect to sustainable productivity, say, P, which varies continuously with time.

Here $x(t)$ shall be the state of system S as it satisfies following properties.

(a) $x(t) \geq 0$ for all t, $0 \leq t < \infty$

(b) $x(t)$ is continuous, $0 \leq t < \infty$ and } (2.1)

(c) $x(t)$ has meaning in ecological sense and is a meaningful measure of P.

But for the state vector, real elements are required that affect the state of the system and is turn the sustainable productivity. Our purpose in (a) is to approximate $x(t)$, the true state for a single ecosystem S′. This system S′ shall be valid for ecosystem S but not for any other unless we assume that all distinct subsystems in the collection of ecosystems are not significantly different from the one we have modeled.

In order to carry out (b) and or (c) well, it is mandatory to carryout (a) in a careful manner,

Let $x^*(t)$ be a model for $x(t)$, then $x^*(t)$ approximates $x(t)$ on the interval $0 \le t < \propto$ with accuracy $\in > 0$, if

$$\max_{0 \le t < \infty} \left| x^*(t) - x(t) \right| \le \in \qquad \text{……(2.2)}$$

More mathematical approaches may be as mentioned below-

$$\int_0^\infty \left[x^*(t) - x(t) \right]^2 dt \le \in \qquad \text{…………….. (2.3)}$$

and
$$\int_0^\infty x^*(t) - x(t) dt \quad \le \in \qquad \text{……………….. (2.4)}$$

It has been observed that sustainable productivity depends substantially on land holdings, live stocks, farm machinery implements, household items, cultivated area, human resource inventory, energy consumption, education of the inhabitants, seeds, fertilizers, irrigation and labor input, In order to approximate the unknown function $x(t)$ with a known function $x^*(t)$ we shall restrict ourselves to the selection of family of functions defined by the set of parameters given as under:-

1. Size of land holding, quality of live stock, farm machinery implements in relation with sustainable productivity

2. Perceiving the relationship among size of land holding, quality of live stock, household items concerned with sustainable productivity
3. Relation of land preparation, cultivated area, farm machinery implements with sustainable productivity
4. Relationship exhibiting Involvement of male and female population in agricultural sector, energy consumption in the farm of use of wood and kerosene during different seasons, education of inhabitants with sustainable productivity
5. Size of land holding, quality of seed, type of fertilizers alliance with sustainable productivity
6. Connection of land holding, irrigation and seed with sustainable productivity.
7. Size of land holding, labor input and bearing of live stock with sustainable productivity

Regression equations were obtained by the methods suggested by MontGomery separately for different set of parameters (discussed above) by representing them as $x_1, x_2, x_3,$ and x_4 respectively.

Parameters mentioned at serial no 1 generated following set of equations for yields during Rabi and Kharif seasons

$$\left.\begin{array}{l} x_1 = 2.266 + 0.482x_2 - 0.379x_3 + 1.114x_4 \\ x_1 = -11.09 + 1.13x_2 + 0.129x_3 + 2.425x_4 \end{array}\right] \qquad (2.5) \ ,$$

while the set of parameters mentioned at serial no 2 generated following two relations for the two yields

$$\left.\begin{array}{l} x_1 = 0.632 + 0.459x_2 + 1.121x_3 + 0.074x_4 \\ x_1 = -18.464 + 1.243x_2 + 1.444x_3 + 4.771x_4 \end{array}\right] \qquad (2.6)$$

The inhabitants of the surveyed village informed that they generally cultivate only two crops (Rabi a khrif) as the farming mainly depends on rains and growing of these two crops also takes time more than required. Since the data collected for the jayad crop was meager so the subsequent Modelling factors suggested ignoring it.

Equations were obtained for parameters mentioned at serial number 3 for inhabitants of general castes and schedule castes separately and it resulted in the form of following four equations for the two yields

$$\left.\begin{array}{l} x_1 = 11.958 - 0.056x_2 + 4.129x_3 + 1.595x_4 \\ x_1 = 1.199 + 1.144x_2 + 3.106x_3 - 1.065x_4 \\ x_1 = -547.503 - 8.148x_2 + 3.879x_3 + 95.162x_4 \\ x_1 = 0.443 + 0.22x_2 + 1.611x_3 + 0.118x_4 \end{array}\right] \qquad (2.7)$$

Parameters mentioned at serial number 4 were considered separately for energy consumption from the two prominent sources of energy prevalent in hills (wood and kerosene) and this exercise lead to the formation of sixteen equations (four is each category) during summer and winter seasons. First set deals with energy consumption from wood during summer while the second one is concerned with consumption of kerosene energy during summers. Needless to say that third and fourth sets deal with consumption from these two sources during winters.

$$\left.\begin{aligned} x_1 &= -0.203 + 0.41x_2 - 0.029x_3 + 0.1x_4 \\ x_1 &= 0.954 + 0.611x_2 - 2.219x_3 + 0.278x_4 \\ x_1 &= -1.05 + 0.528x_2 - 0.063x_3 + 0.066x_4 \\ x_1 &= -1.79 + 0.29x_2 + 1.383x_3 + 0.148x_4 \end{aligned}\right] \qquad (2.8)$$

$$\left.\begin{aligned} x_1 &= 0.359 + 0.014x_2 + 0.639x_3 + 0.212x_4 \\ x_1 &= 6.235 - 0.179x_2 + 0.383x_3 + 0.179x_4 \\ x_1 &= -0.733 + 0.093x_2 + 0.844x_3 + 0.108x_4 \\ x_1 &= 1.467 + 0.058x_2 + 0.427x_3 + 0.135x_4 \end{aligned}\right] \qquad (2.9)$$

$$\left.\begin{aligned} x_1 &= 4.64 - 0.338x_2 - 0.013x_3 + 0.214x_4 \\ x_1 &= 2.256 + 0.37x_2 + 0.935x_3 - 0.059x_4 \\ x_1 &= 3.115 + 0.028x_2 - 0.101x_3 + 0.202x_4 \\ x_1 &= 4.618 + 0.216x_2 + 0.277x_3 - 0.063x_4 \end{aligned}\right] \qquad (2.10)$$

and

$$\left.\begin{aligned} x_1 &= 2.953 - 0.127x_2 + 0.347x_3 + 0.179x_4 \\ x_1 &= -0.084 + 0.43x_2 + 0.4x_3 + 0.115x_4 \\ x_1 &= -6.595 + 0.94x_2 + 0.843x_3 + 0.174x_4 \\ x_1 &= -1.6 + 0.773x_2 + 0.658x_3 - 0.064x_4 \end{aligned}\right] \qquad (2.11)$$

Parameters mentioned at serial no. 5 constitute the formation of following regression equation for inhabitants of general caste and schedule caste villagers during the two seasons

$$\left.\begin{aligned} x_1 &= 46.699 - 139.765x_2 + 187.997x_3 + 0.27x_4 \\ x_1 &= -4.488 + 1.492x_2 - 0.46x_3 + 1.154x_4 \\ x_1 &= -26.446 - 152.852x_2 + 206.3x_3 + 22.006x_4 \\ x_1 &= -5.262 + 2.016x_2 + 3.04x_3 + 1.005x_4 \end{aligned}\right] \qquad (2.12)$$

Following linear regression equation are generated for the parameters mentioned at serial no. 6 for the same set of individuals and conditions as mentioned for construction of equations

$$\left.\begin{aligned}
x_1 &= -1.334 + 1.667x_2 + 0.758x_3 + 0.129x_4 \\
x_1 &= -4.645 + 3.657x_2 - 0.103x_3 - 0.167x_4 \\
x_1 &= -6.267 - 7.792x_2 + 2.011x_3 + 9.864x_4 \\
x_1 &= -19.39 - 5.085x_2 + 3.61x_3 + 16.017x_4
\end{aligned}\right] \qquad (2.13)$$

The last set shall be able to procreate following four equations under the same set of individuals and conditions as mentioned for the generation of equation (2.12) and (2.13)

$$\left.\begin{aligned}
x_1 &= -0.53 + 0.069x_2 + 0.449x_3 + 1.117x_4 \\
x_1 &= -1.198 - 6.628x_2 + 0.869x_3 + 5.33x_4 \\
x_1 &= -6.361 - 8.845x_2 + 1.108x_3 + 5.608x_4 \\
x_1 &= -4.229 - 0.011x_2 + 0.305x_3 + 3.051x_4
\end{aligned}\right] \qquad (2.14)$$

3. <u>Generation of General State Vector Linear Model</u>

In Mathematical modeling it is vitally important to compare model results with observations and to analyze remaining errors. The optimization of parameter values helps to improve the model's performance and facilitates the investigation of remaining model deficiencies for the development and evaluation of new mathematical models. We seek for optimal parameters which remain constant in time, as it is usually assumed for large system models. The model is forced to its best possible solution when compared with observations and is then analyzed, focusing on its reliability. Optimization of parameters shall have its role in such type of estimation problems. A number of Biological models appear in the journals. Out of these, some models of our interest were those suggested by Anderson et al (2001) and Cox et al (2000).

We know that each parameter mentioned in the set of equations (2.5 -2.14) has its role in achieving sustainable productivity but with varying degree of efficiency, so the assignment problem of linear programming can be applied in order to have the best assignment in the best interest of the inhabitants of the village.

The equations (2.5) and (2.6) can be assimilated together to form following effectiveness matrix

$$\begin{bmatrix} 10 & 4.8 & -3.8 & 11.1 \\ 10 & 11.3 & 1.3 & 24.2 \\ 10 & 4.6 & 11.2 & 0.7 \\ 10 & 12.4 & 14.4 & 47.7 \end{bmatrix} \qquad (3.1)$$

Assignment algorithm leads to the solution of above problem with the conclusion that productivity can be optimized during Rabi season from amongst the parameters mentioned at serial no. 1 size if land holding is increased and during Kharif season we can get the best production by maximum utilization of the live stock in the same size of land holding parameters mentioned at serial no. 2 can generate optimized productivity during Rabi season if better quality of farm machinery implements is utilized in the exercise of agricultural work, The maximum production under above recommendations shall amount 80.20 quintals.

Second effectiveness matrix generated for the set of equations (2.7) is

	I	II	III	IV
A	10	-0.6	41.3	15.9
B	10	11.5	31.1	-10.5
C	10	-81.5	38.8	951.7
D	10	2.2	16.2	1.2

(3.2)

Optimal solution of this problem is as follows

A→ IV

B→ II

C→ I

D→ III

which means that the productivity can be maximized in sustainable manner if schedule caste residents of the village enhance their cultivated area for Rabi season, general caste villagers give proper attention to land preparation activities during Kharif seasons and schedule caste villagers use improved quality of farm machinery implements. Total production on account of this optimization shall reach up to 53.6 quintals.

Parameters mentioned at serial no. 4 yielded equation (2.8) which produced following effectiveness matrix for energy consumption from wood during summer season

10	4.1	-0.3	1.0
10	6.2	-22.2	2.8
10	5.3	-0.6	0.7
10	2.9	13.9	14.8

(3.3)

where optimal situation has lead to the inferences that productivity can be maximized in sustainable manner by better and active involvement of male members of general caste subject to the condition that they fully utilize their acquired knowledge in agricultural sector while schedule caste villagers can improve their production if they properly use the energy produced by the burning of wood. The total optimum output on account of this exercise shall be 32 quintal.

Optimal solution for set of equations given is (2.9) exhibits the fact that the necessary requirements to optimize the productivity should be that schedule caste villagers should not misuse kerosene oil rather they should take necessary steps to utilize it in a systematic manner in both seasons (Rabi & Kharif), general caste villagers should employ their educated children in the agricultural sector during Rabi season so that their gained knowledge could be of use for their parents. The production can be optimized up to the level of 9.3 quintals on the basis of above mentioned recommendations.

Set of equations mentioned in (2.10) suggests that productivity can be optimized during winter season if general caste villagers take proper care in utilizing energy produced by the burning of wood during Rabi season, schedule caste villagers give attention towards the education of their children and utilize their gained knowledge in their profession and general caste villagers insist upon more active involvement of their children in the agricultural sector and their other professions. The optimized productivity on account of their exercise shall amount up to 21.9 quintals

Assignment algorithm for linear regression equations given in (2.11) insinuate the fact that general caste villagers should not misuse the kerosene oil and schedule caste residents should involve more male members of their families in agricultural works. The product on the basis of these recommendations shall rise up to 14.5 quintals. The role of quality of seeds and type of fertilizers in optimizing the production can be estimated from set of equations given in (2.12). The assignment algorithm suggests that residents of general caste should give emphasis to the sowing of better and new seeds, but the area of fallow land should not exceed beyond a certain level during Kharif while schedule caste villagers are advised to use organic manures as from the available data it appears that they do not possess inorganic manures up to the desired level. The production then can be optimized up to 1345.7 quintals.

The effectiveness matrix corresponding to the set of linear regression equations given by (2.13) when solved with the help of assignment algorithm suggest that residents of general caste should increase their total land holding (area of land under abandoned and grass cultivation category should be kept at low level) and should also insist on better quality of seed for kharif crops while schedule caste villagers are required to have more live stock to their credit. Optimized production in this case can be raised up to 227 quintals.

The last set of equations pertaining to the relationship of land holding size, labor input and bearing of live stock with sustainable productivity, when utilized to form the effectiveness matrix and grilled by optimization algorithm lead to the suggestion that residents of schedule caste should thrust upon labor input for both crops while general caste villages are required to have more bearing of live stock to enhance the production , which on the basis of above recommendation can be optimized up to 44.2 quintals

These optimized values generate following matrix -

$$M(t) := \begin{bmatrix} 870.08 & 308.52 & 172.65 & 1074.16 & 516.38 & 206.48 & 89.36 & 14.8 & 43.43 \\ 271.22 & 310.3 & 140.73 & 1097.46 & 447.4 & 96.83 & 58.65 & 7.1 & 20.72 \\ 19.37 & 91.13 & 37.94 & 343.86 & 142.49 & 34.49 & 0.25 & 0 & 0 \\ 969.3 & 524.29 & 510.82 & 782.63 & 201.42 & 1083.2 & 727.95 & 104.1 & 305.11 \\ 2226.86 & 1618.79 & 1905.73 & 3166.46 & 1085.79 & 479.77 & 630.03 & 38.14 & 301.43 \\ 177.75 & 481.98 & 854.31 & 1271.26 & 1050.09 & 481.62 & 8.42 & 0 & 0.34 \\ 48.04 & 367.46 & 112.26 & 827.1 & 985.98 & 478.1 & 8.16 & 0 & 0 \\ 277.69 & 322.19 & 222.19 & 755.85 & 425.35 & 537.06 & 24.77 & 4.5 & 0.42 \\ 1473.22 & 3560.68 & 2324.74 & 2270.77 & 4331.4 & 4962.48 & 1786.64 & 390 & 1142.7 \end{bmatrix} \qquad (3.4)$$

State of the system

$$x(t) = \sum_{i=1}^{n} \sum_{j=1}^{m} m_{ij}(t)$$

and the $\sum_{i=1}^{k} m_i \times 1 \quad column\,vector$

$$x(t) = [x_{11}(t), ---, x_{j1}(t); \quad x_{12}(t), ---, x_{j2}(t); ---; x_{1n}(t), ---, x_{jkk}(t)\ ]^T$$

shall have following value in the present case

$$\begin{bmatrix} 46.96 & 193.58 & 11.66 & 24.08 & 14.8 & 64.33 & 3.26 & 14.8 & 23.54 \\ 39.22 & 194.23 & 9.97 & 20 & 0 & 39.31 & 1.57 & 7.1 & 11.2 \\ 12.1 & 67.59 & 6.42 & 0 & 0 & 13.02 & 0 & 0 & 0 \\ 104.1 & 13.45 & 0 & 97.86 & 0 & 381.01 & 22.91 & 104.1 & 165.52 \\ 1321.91 & 638.6 & 216.44 & 1125.37 & 57.58 & 83.73 & 2.53 & 11.5 & 165.42 \\ 45.66 & 334.36 & 0 & 28.76 & 439.93 & 3.52 & 0 & 0 & 0.34 \\ 34.88 & 298.28 & 0.52 & 32.34 & 563.64 & 0 & 0 & 0 & 0 \\ 38.31 & 55.31 & 6.4 & 8.5 & 112.20 & 0 & 6.102 & 4.5 & 0 \\ 1363.37 & 1805.53 & 413.4 & 551.18 & 3214.2 & 1427.56 & 85.8 & 390 & 620.1 \end{bmatrix} \qquad (3.5)$$

The general state vector linear model, as proposed theoretically is the book of Efrain Halfon (1979) shall be defined by the following vector differential equation $\dot{x}(t) = M(t) \quad x(t) + \quad u(t),$

where $M(t)$ *and* $x(t)$ have been represented by results (3.4) and (3.5), while

$u(t) = \sum_{i=1}^{k} m_i \times 1$ is a vector of input values and has been obtain as

$$\begin{bmatrix} 823.12 & 114.94 & 1.60.99 & 1059.36 & 452.05 & 142.15 & 86.1 & 0 & 19.89 \\ 116 & 116.07 & 130.76 & 1077.46 & 447.4 & 97.52 & 57.08 & 0 & 9.52 \\ 7.27 & 23.54 & 31.52 & 343.86 & 142.49 & 21.47 & 0.25 & 0 & 0 \\ 865.2 & 510.82 & 510.82 & 684.77 & 201.42 & 702.19 & 705.04 & 0 & 139.59 \\ 904.95 & 980.19 & 1689.29 & 2041.09 & 1028.21 & 396.04 & 627.5 & 26.64 & 136.01 \\ 132.59 & 147.62 & 854.31 & 1242.5 & 610.16 & 478.1 & 8.42 & 0 & 0 \\ 13.16 & 69.18 & 111.73 & 794.76 & 422.34 & 576.76 & 8.16 & 0 & 0 \\ 239.38 & 266.88 & 215.79 & 747.35 & 313.15 & 537.06 & 18.66 & 0 & 0.42 \\ 109.85 & 1755.15 & 1911.34 & 1719.59 & 1117.2 & 3534.92 & 1700.84 & 0 & 522.6 \end{bmatrix} \qquad (3.6)$$

matrix $M(t)$ can be diagonalized in the form

$$M := \begin{bmatrix} 0.14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.26 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.19 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 \end{bmatrix} \qquad (3.7)$$

by the equilibrium turnover rate formula

$$l_i = \frac{Ui}{x_i^{e}(o)}$$

where $x_i^{(e)}(o) = x_i(\infty)$;

in order to produce actual values in matrix form (given in the next page):

$$\begin{bmatrix} 703.73 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 842.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 697.93 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1287.82 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1020.65 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 925.65 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 370.47 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 558.64 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 201.66 \end{bmatrix} \quad (3.8)$$

Above diagonalized matrix has originated under the assumption that the linear model has constant coefficients,

The state vector for different values of i = 1,2,…..p, can be obtained with the help of following formula

$$x_i(t) = \frac{u_i}{l_i} + \left(x_i(0) - \frac{u_i}{l_i}\right)e^{-l_i t} \qquad (3.9)$$

We have deduced following results from our collected data during field study

$$x_1(t) = 5026.64 + (-5007.28)e^{-.04(t)}$$
$$x_2(t) = 4958.36 + (-4867.23)e^{-0.17(t)}$$
$$x_3(t) = 4652.87 + (-4614.93)e^{-0.15(t)}$$
$$x_4(t) = 4953.16 + (-4609.3)e^{-0.26(t)}$$
$$x_5(t) = 4860.48 + (-4717.99)e^{-0.21(t)}$$
$$x_6(t) = 4871.85 + (-4837.36)e^{-0.19(t)}$$
$$x_7(t) = 4630.88 + (-4630.63)e^{0.08(t)}$$
$$x_8(t) = 5078.55 + (-5078.55)e^{-0.11(t)}$$
$$x_9(t) = 5041.5 + (-5041.5)e^{-0.04(t)}$$

The approximation model $x_A(t)$ for $x(t)$ shall be

$$x_A(t)=\frac{u_A}{l_A}+\left(x_A(o)-\frac{u_A}{l_A}\right)e^{-l_A t} \qquad (3.10)$$

$$x_A(t)=-64533.74e^{-2.488\ \ t}$$

The relative error in using $x_A(t)$ to approximate $x(t)$ shall be calculated with the help of following formula

$$q(t)=\frac{[x(t)-x_A(t)]}{x(t)} \qquad (3.11)$$

whose value for our observed data is

$$q(t)=\frac{\left\{\begin{matrix}44074.29-5007.28e^{-0.04(t)}-4867.23e^{-0.17(t)}-4614.95e^{-0.15(t)}-4609.3e^{-0.26(t)}-4717.99e^{-0.21(t)}\\-4837.36e^{.0.19(t)}-463.63e^{-0.08(t)}-5078.55e^{-0.11(t)}-5041.5e^{-0.04(t)}+64533.74e^{-2.488\ \ t}\end{matrix}\right\}}{\left\{\begin{matrix}44074.29-5007.28e^{-0.04(t)}-4867.23e^{-0.17(t)}-4614.95e^{-0.15(t)}-4609.3e^{-0.26(t)}-4717.99e^{-0.21(t)}\\-4837.36e^{.0.19(t)}-463.63e^{-0.08(t)}-5078.55e^{-0.11(t)}-5041.5e^{-0.04(t)}\end{matrix}\right\}}$$

The bias is given by the numeration, that is,

$$\begin{aligned}b(t)&=x(t)-x_A(t)\\&=44074.29-5007.28e^{-0.04(t)}-4867.23e^{-0.17(t)}-4614.95e^{-0.15(t)}-4609.3e^{-0.26(t)}-4717.99e^{-0.21(t)}\\&-4837.36e^{.0.19(t)}-463.63e^{-0.08(t)}-5078.55e^{-0.11(t)}-5041.5e^{-0.04(t)}+64533.74e^{-2.488\ \ t}\end{aligned}$$

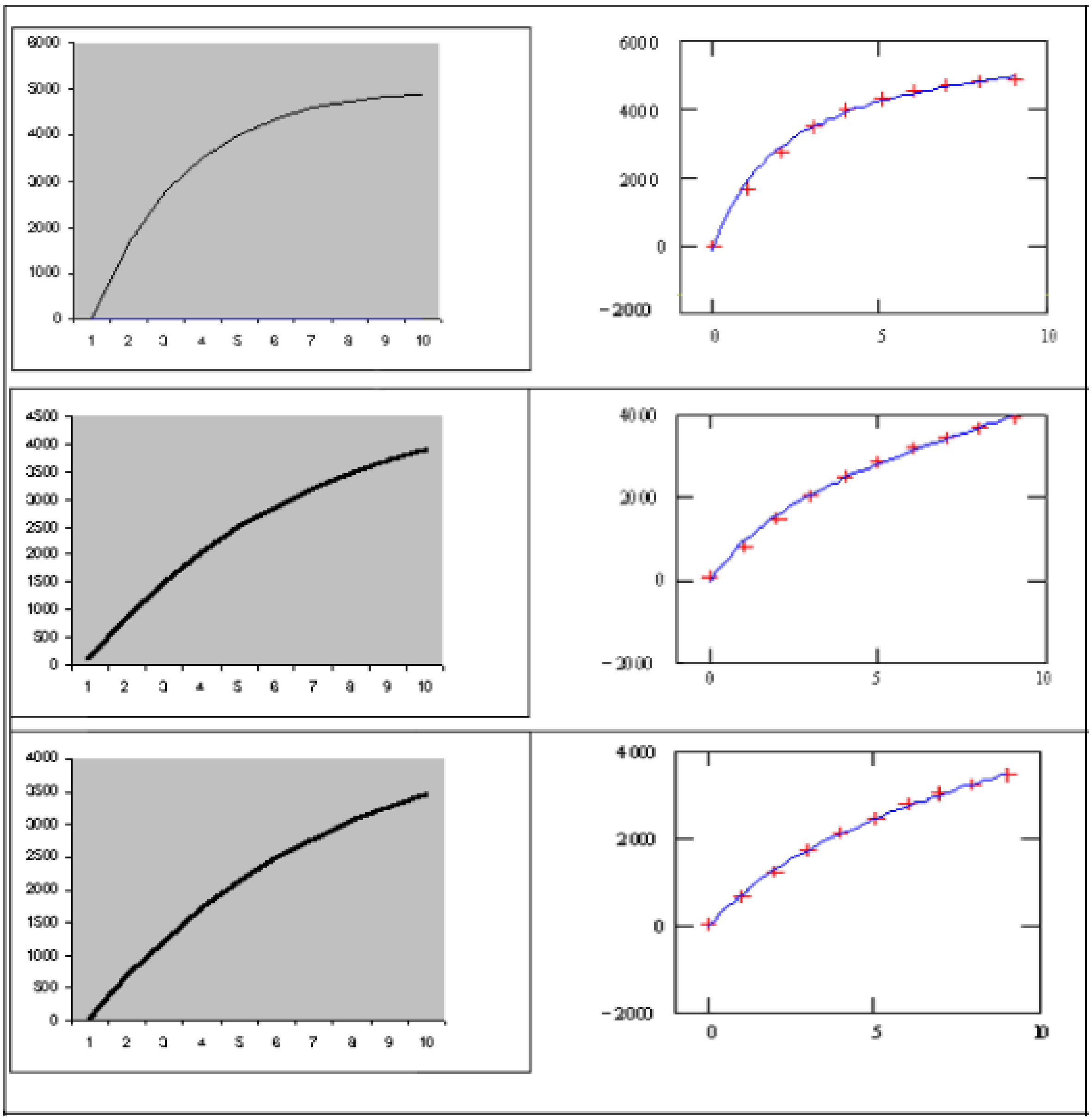
The state vector $x_i(t)$ given by equation (3.9) yields following values for the values of t varying from 0 to 9-

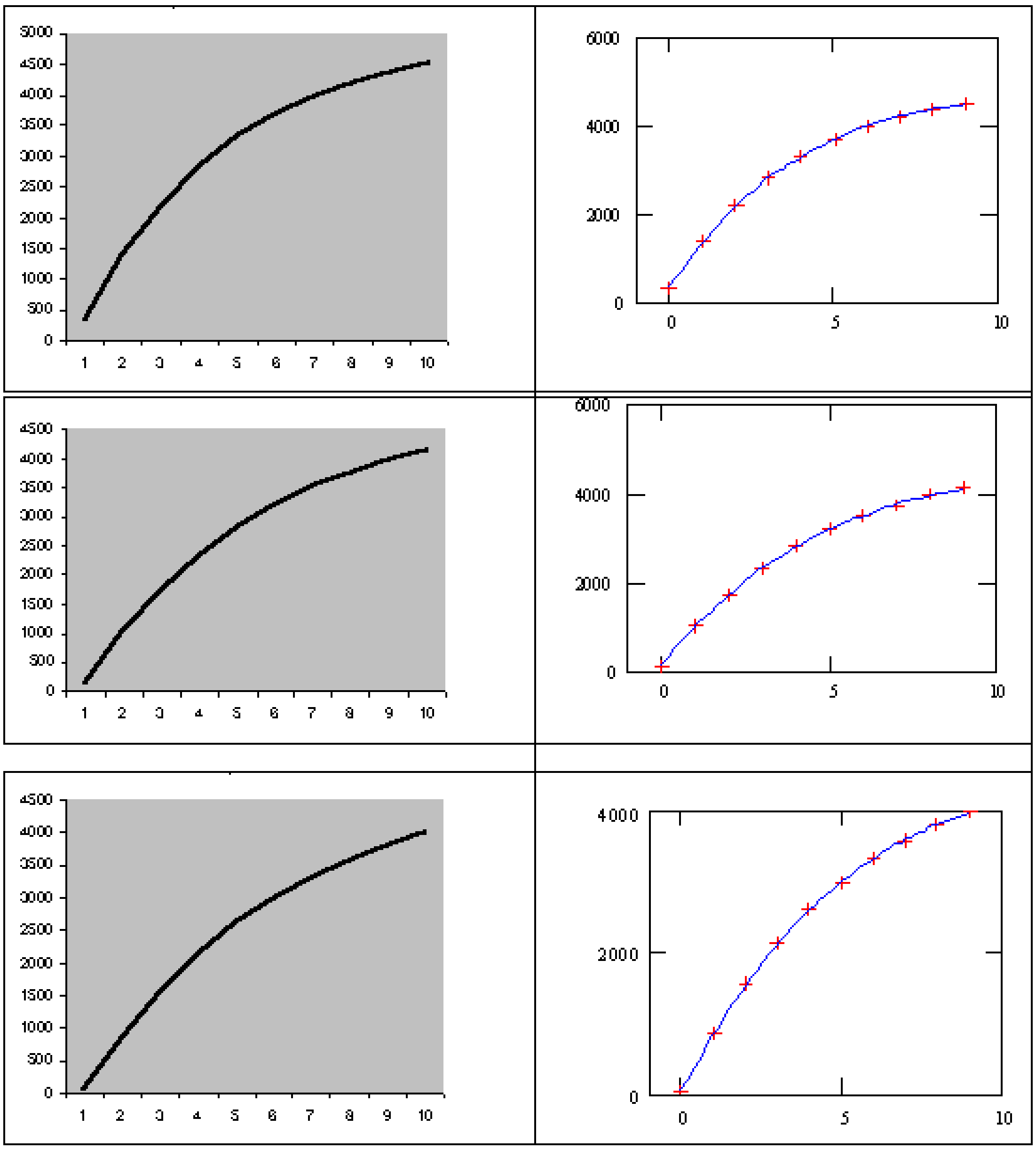
t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
0	19.36	91.13	37.94	343.86	142.49	34.49	0.25	0	0
1	1670.16	852.05	680.76	1399.15	1036.15	871.55	356.27	529.12	197.68
2	2776.72	1494.01	1234.05	2212.84	1760.54	1563.77	684.92	1002.92	387.61
3	3518.48	2035.61	1710.26	2840.23	2347.72	2136.2	988.3	1427.46	570.09
4	4015.69	2492.54	2120.14	3323.98	2823.67	2609.58	1268.35	1807.78	745.42
5	4348.98	2878.03	2472.93	3696.98	3209.48	3001.04	1526.88	2148.48	913.87
6	4572.39	3203.26	2776.58	3984.58	3522.2	3324.77	1765.52	2453.69	1075.72
7	4722.15	3477.65	3037.93	4206.33	3775.69	3592.48	1985.82	2727.11	1231.22
8	4822.53	3709.13	3262.88	4377.32	3981.17	3813.86	2189.18	2972.05	1380.62
9	4889.82	3904.43	3456.59	4509.16	4147.72	3996.94	2376.91	3191.48	1524.16

which suggests us that we should change the modeling parametric function by another polynomial function and for this purpose we are using simulation process.

GRAPHICAL REPRESENTATION BY SIMULATION PROCESS

We have observed that the state vector representing the state of the system depends mainly on $e^{-l_i t}$ but the tabulated values of $x_i(t)$ (given in the previous page) are not in agreement with this modeled parametric function. We have therefore graphically represented these values by two methods- one on Microsoft Excel and another by simulating a function with the help of Math Cad software and deductive reasoning. The outcome is in the form of following graphical results-

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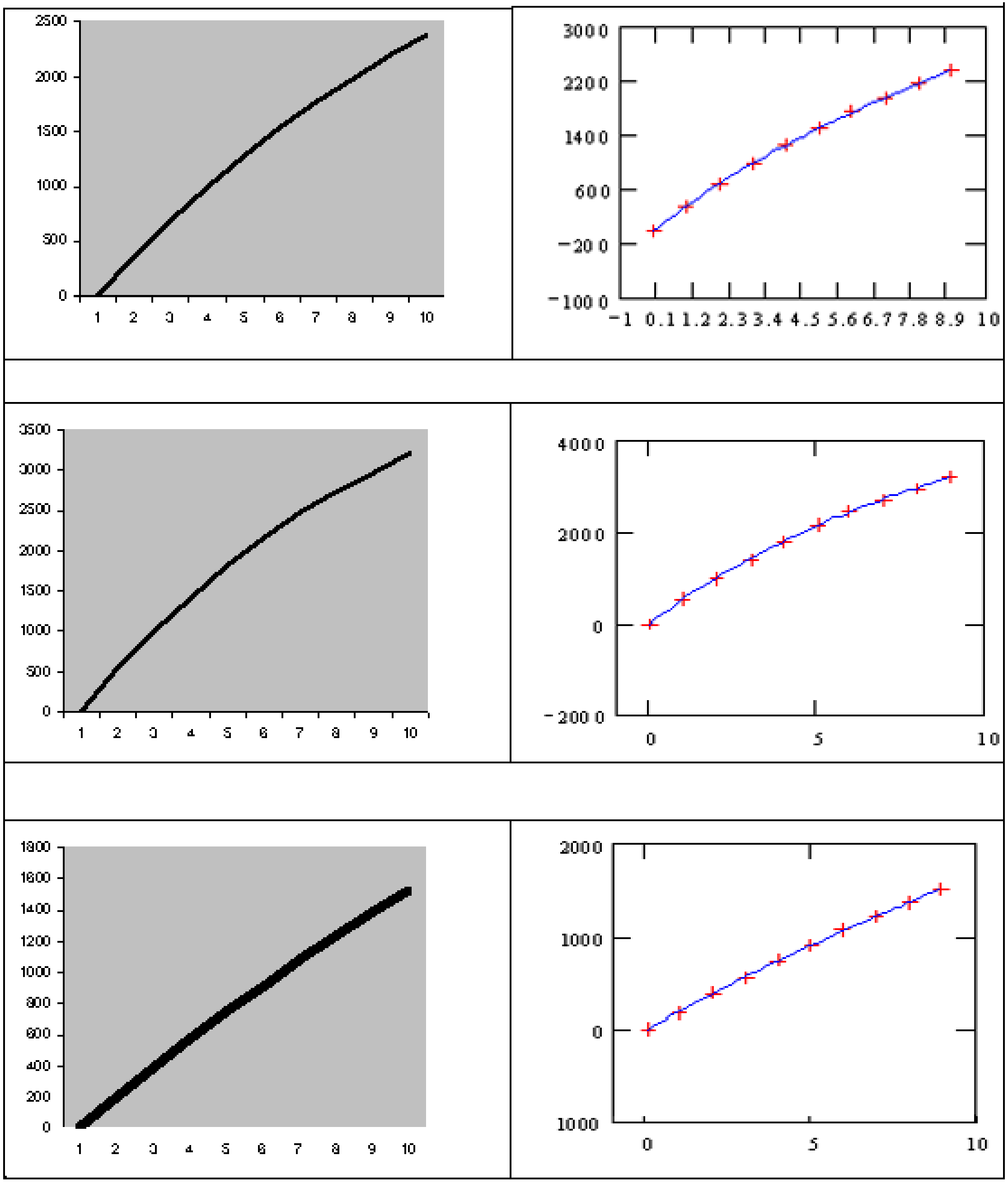

These graphs justify our choice of polynomial function in place of original modeling parametric function of the form

$$x_i(t) = \left\{ a + \frac{b}{x_i + 2t} + c x_i^{\,2} \right\}$$

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