NUMERICAL EVALUATION OF H-FUNCTION BY CONTINUED FRACTIONS

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ABSTRACT

In the present paper an attempt has been made to evaluate for different values of parameters m, n, p, q and the variable Z in the range 0.1 to 10.0 by the application of continued fractions

INTRODUCTION

Fox H-function is one of the most generalized function among all special functions. It covers wider range and gives deeper, more general and more useful results directly applicable in various problems arising in physical and biological sciences, engineering and statistics.

With an aim to provide assistance to those mathematicians, engineers, scientists and statisticians who discover that they need to generate numerical value of the special function in the course of solving their problems, Lozier and Olver [3] have prepared a report which consist reviews of packages, libraries and systems usable in the evaluation of classical functions but not for more generalized functions like E-function, G-function, H-function etc. Lozier [2] has developed a software test device for use in testing the accuracy or numerical precision of mathematical software for elementary special functions.

Special functions can be evaluated by continued fractions using qd scheme. Such type of work was initiated long back in 1954. Detailed account of which can be seen to the book of P. Henrici [6].

Historical survey of continued fractions and Pade approximation has been compiled in the work of Brezinski [1]. A code of evaluate modified Bessel's functions based on the continued fraction method has been investigated by Segura et. al [4] and for higher order function, the device has been developed by Ratis and Cardoba [8]. Ratios of contiguous hypergeometric functions of the type ${}_3F_2$ have been represented in the form of continued fractions by Singh [7]. A set of fast codes to calculate Bessel functions of integral and fractional orders, modified Bessel functions etc., based on the continued fractions method has been presented by Monreal et. al [5].

We are making an attempt to evaluate a more generalized function from among special functions, the H- function by the application of continued fraction for values of parameters in the ranges :-

m varying from 1 to 4

n has been fixed as '0'

p varying from 0 to 1

q varying from 1 to 4

and variable Z varying in the ranges 0.1 to 10.0

1. GENERAL FORMULATION

Application of initial conditions to the qd schemes associated with confluent hypergeoemtric function, Gauss hypergeometric function and Bessel's function have yielded results, which has been compiled in the book of Henirici [6] but does not throw any light on the evaluation of generalized function like G-function, H-function etc.

We have generated following results under qd scheme for the H-function

$$\left.\begin{aligned}
q_1^{(n)} &= \frac{(\alpha+n)}{(\beta+n)}\gamma \\
&\quad n = 0,1,2,3,------- \\
e_k^{(n)} &= -\frac{(\alpha+n)(\alpha+n+k-1)}{(\beta+n+2k-2)(\beta+n+2k-1)}\gamma \\
&\quad (k = 1,2,3\text{---------}) \\
q_k^{(n)} &= \frac{(\beta-\alpha+k-1)}{(\beta+n+2k-3)(\beta+n+2k-2)}\gamma \\
&\quad (k = 2,3,4\text{---------})
\end{aligned}\right\} ----- (1.1)$$

$$\left.\begin{aligned}
q_1^{(n)} &= \frac{(\alpha+n)(\beta+n)}{(\gamma+n)}\delta \\
&\quad n = 0,1,2,3,------- \\
q_k^{(n)} &= \frac{(\alpha+n+k-1)(\gamma-\beta+k-1)}{(\gamma+n+2k-3)(\gamma+2k+n-2)}\delta \\
&\quad (k = 2,3,4\text{---------}) \\
e_k^{(n)} &= \frac{k(\gamma-\alpha-\beta-n+k)}{(\gamma+n+2k-2)(\gamma+n+2k-1)}\delta \\
&\quad (k = 1,2,3\text{---------})
\end{aligned}\right\} ----- (1.2)$$

Corresponding contined fraction for the function can be obtained with the help of following results:-

$$a_2 = -q_1^0$$
$$a_{2k+1} = -e_k^0$$
$$a_{2k+2} = -q_{k+1}^0$$

The division algorithm for the H-function based has been produced as

$$\frac{J_\nu(Z)}{J_{\nu-1}(Z)} = \frac{Z}{(2\nu)-}\,\frac{Z^2}{(2\nu+2)-}\,\frac{Z^2}{(2\nu+4)-}\,\frac{Z^2}{(2\nu+6)-}\,\frac{Z^2}{(2\nu+8)-}----\Big\}------(1.3)$$

2. NUMERICAL EVALUATION OF THE FUNCTION

In the present section we have defined various H- functions by RITZ method and then qd - scheme has been utilized in order to tabulate the function for value of α, β and Z .

Defining the function $H_{0,1}^{1,0}[Z/(\alpha,1)]$ by RITZ method

$$\left.\begin{aligned} & Z^{\alpha}\left[\frac{1}{1+}\frac{a_2 Z}{1+}\frac{a_3 Z}{1+}\frac{a_4 Z}{1+}------------\right] \\ & = \frac{Z^{\alpha}}{1+}\frac{a_2 Z}{1+}\frac{a_3 Z}{1+}\frac{a_4 Z}{1+} \end{aligned}\right\} ---------(2.1)$$

and then taking qd Schemes

$$\left.\begin{aligned} a_2 &= \frac{1}{1} \\ a_{2k+1} &= -\frac{1}{2(2k-1)} \\ a_{2k+2} &= \frac{1}{2(1+2k)} \end{aligned}\right\} ----------(2.2)$$

where k = 1,2,3,4-----------------

we can tabulate the functions by making the substitutings

Z= 0.1, 0.2, 0.3, 0.4,0.5,0.6,0.7,0.8,0.9, 1.0

α = 0.2, 0.3,0.4,0.5, 1.0

in the from of table (Appendix-I)

The function $H_{0,2}^{2,0}\left[Z\middle|(\alpha,1),(\beta,1)\right]$ can be defined with the help of RITZ[-1] method in the following form:-

$$\frac{\sqrt{2\pi}\times Z^{\frac{\alpha+\beta}{2}}}{e^{2\sqrt{Z}}}\left[\frac{Z^{-3/2}}{1\quad +}\;\frac{a_2\,\frac{1}{\sqrt{Z}}}{1\quad +}\;\frac{a_3\,\frac{1}{\sqrt{Z}}}{1\quad +}\;\frac{a_4\,\frac{1}{\sqrt{Z}}}{1\quad +}\;\frac{a_5\,\frac{1}{\sqrt{Z}}}{1\quad +}\;\underline{\qquad\qquad}\right]$$

$$=\left[\frac{\sqrt{2\pi}\times Z^{\frac{\alpha+\beta-3}{2}}}{e^{2\sqrt{Z}}\quad +}\;\frac{a_2\,e^{2\sqrt{Z}}}{\sqrt{Z}\quad +}\;\frac{a_3}{1\quad +}\;\frac{a_4}{\sqrt{Z}\quad +}\;\frac{a_5}{1\quad +}\;\frac{a_6}{\sqrt{Z}\quad +}\;\underline{\qquad\qquad}\right] \tag{2.3}$$

$$\left.\begin{aligned}
a_2 &= \frac{1}{4}\left(\alpha-\beta+\tfrac{1}{2}\right)\left(\beta-\alpha+\tfrac{1}{2}\right)\\
a_{2k+1} &= \frac{k}{8(2k-1)}\\
a_{2k+2} &= \frac{\left(\tfrac{1}{2}+\alpha-\beta+k\right)^2}{2k(1+2k)}\\
& \qquad\qquad\qquad k = 1,2.3\text{-----},
\end{aligned}\right\}\text{-------}(2.4)$$

we can tabulated the function by making the substitutings

Z = 0.1,0.2,0.3,.0.4,0.5,0.6,0.7,0.8,0.9,1.0,2.0,3.0, 4.0,5.0,

α = 1,2,3,4,

β = 1

in the form of table (Appendix-II)

The function $H_{0,4}^{4,0}\left[Z(\alpha,1),(\alpha+\frac{1}{2},1),(\beta,1)(\beta+\frac{1}{2},1)\right]$ can be defined with the help of RITZ^{-1} method in the following form

$$\frac{(2\pi)^{\frac{3}{2}}Z^{\frac{\alpha+\beta}{2}}}{e^{4Z^{\frac{1}{2}}}}\left[\frac{Z^{-\frac{5}{4}}}{1+}\frac{a_2\frac{1}{Z^{\frac{1}{4}}}}{1+}\frac{a_3\frac{1}{Z^{\frac{1}{4}}}}{1+}\frac{a_4\frac{1}{Z^{\frac{1}{4}}}}{1+}\frac{a_5\frac{1}{Z^{\frac{1}{4}}}}{1+}-----\right]$$

$$=\left[\frac{(2\pi)^{\frac{3}{2}}\times Z^{\frac{2\alpha+2\beta-5}{4}}}{\left(e^{4Z^{\frac{1}{4}}}\right)+}\frac{a_2e^{4Z^{\frac{1}{4}}}}{Z^{\frac{1}{4}}+}\frac{a_3}{1+}\frac{a_4}{Z^{\frac{1}{4}}}\frac{a_5}{1+}\frac{a_6}{Z^{\frac{1}{4}}}-----\right]------(2.5)$$

and then taking qd schemes

$$\left.\begin{aligned}
a_2 &= \frac{1}{8}\left(2\alpha-2\beta+\tfrac{1}{2}\right)\left(2\beta-2\alpha+\tfrac{1}{2}\right)\\
a_{2k+1} &= \frac{1}{16}\frac{k}{(2k-1)}\\
a_{2k+2} &= \frac{1}{16}\frac{\left(\frac{1}{2}+2\alpha-2\beta+k\right)^2}{k(1+2k)}
\end{aligned}\right\}-----(2.6)$$

$$where\ k=1,2,3,-----$$

we can tabulate the functions by making the substituting

Z = 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0
$\alpha = 1,2$
$\beta = 1,2$

In the form of table (Appendix-III)

The function $H_{1,2}^{1,0}\left[Z\middle|_{\beta,1}^{\alpha,1}\right]$ can be defined with the help of RITZ method in the following form

$$\left.\begin{aligned}&\frac{1}{\Gamma(\alpha-\beta)}Z^{\beta}\left[\frac{1}{1+}\,\frac{a_2Z}{1+}\,\frac{a_3Z}{1+}\,\frac{a_4Z}{1+}\,\frac{a_5Z}{1+}\,\frac{a_6Z}{1+}\,\frac{a_7Z}{1+}----\right]\\&=\frac{Z^{\beta}}{\Gamma(\alpha-\beta)+}\,\frac{a_2\Gamma(\alpha-\beta)Z}{1+}\,\frac{a_4Z}{1+}\,\frac{a_5Z}{1+}\,\frac{a_6Z}{1+}----\end{aligned}\right\}------(2.7)$$

and then taking qd schemes as

$$\left.\begin{aligned}&a_2=-(\beta-\alpha+1)\\&a_{2k+1}=\frac{(k-\beta+\alpha-1)}{2(2k-1)}\\&a_{2k+2}=\frac{-(k+\beta-\alpha+1)}{2(2k+1)}\\&\qquad k=1,2,3----\end{aligned}\right\}-------(2.8)$$

we can tabulate the function by making the substitutings according condition |z|<1

Z= 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9

α = 1,3/2, 2

β = 1/2, 1

in the form of table (Appendix-IV)

The function $H_{1,2}^{1,0}\left[Z\left|{}^{(\frac{1}{2},1)}_{(\alpha,1)(-\alpha,1)}\right.\right]$ can be defined with the help of RITZ method in the following form

$$\left.\begin{aligned}&\frac{1}{\sqrt{\pi}}\frac{\cos(\alpha\pi)(\frac{Z}{4})^{\alpha}}{\Gamma(1+\alpha)}\left[\frac{1}{1+}\frac{a_2Z}{1+}\frac{a_3Z}{1+}\frac{a_4Z}{1+}\frac{a_5Z}{1+}\frac{a_6Z}{1+}----\right]\\&=\frac{\cos(\alpha\pi)Z^{\alpha}}{\left\{\sqrt{\pi}\Gamma(1+\alpha)4^{\alpha}\right\}+}\frac{a_2\sqrt{\pi}\Gamma(1+\alpha)4^{\alpha}Z}{1+}\frac{a_3Z}{1+}\frac{a_4Z}{1+}\frac{a_5Z}{1+}----\end{aligned}\right\}-----(2.9)$$

and then taking qd schemes as

$$\left.\begin{aligned}&a_2=-\frac{1}{2}\\&a_{2k+1}=\frac{(2\alpha+1)}{4(\alpha+k)}\\&a_{2k+2}=-\frac{1}{4(\alpha+k)}\\&\qquad k=1,2,3----\end{aligned}\right\}------(2.10)$$

we can tabulate the function by making the substituting

Z= 0.2,0.4,0.6,0.8,1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0

$\alpha = 0,1,2$

in the form of table (Appendix-V)

The function $H_{1,2}^{2,0}\left[Z\Big|_{(\beta,1)(-\beta,1)}^{(\frac{1}{2},1)}\right]$ can be defined with the help of RITZ^{-1} method in the following form

$$\left.\begin{aligned}&\frac{1}{e^{Z}\sqrt{2}}\left[\frac{1}{Z^{2}+}\frac{a_{2}Z}{1+}\frac{a_{3}\frac{1}{Z}}{1+}\frac{a_{4}\frac{1}{Z}}{1+}\frac{a_{5}\frac{1}{Z}}{1+}\frac{a_{6}\frac{1}{Z}}{1+}----\right]\\&=\frac{1}{Z^{2}\sqrt{2}e^{Z}+}\frac{Za_{2}e^{Z}\sqrt{2}}{1+}\frac{a_{3}}{Z+}\frac{a_{4}}{1+}\frac{a_{5}}{Z+}\frac{a_{6}}{1+}----\end{aligned}\right\}-----(2.11)$$

and then taking qd schemes as

$$\left.\begin{aligned}&a_{2}=\left(\frac{1}{2}+\beta\right)\left(\frac{1}{2}-\beta\right)\\&a_{2k+1}=\frac{k}{2(2k-1)}\\&a_{2k+2}=+\frac{(\beta+k+\frac{1}{2})^{2}}{2k(1+2k)}\\&\qquad k=1,2,3----\end{aligned}\right\}------(2.12)$$

we can tabulate the function by making the substituting

Z= 0.2,0.4,0.6,0.8,1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0

β = 0,1,2

in the form of table (Appendix-VI)

Application of division algorithm to the expansion of $H_{0,2}^{1,0}\left[Z\middle|(\alpha,1),(\beta,1)\right]$ by RITZ method yields.

$$Z^{\frac{\alpha+\beta}{2}} J_{\alpha-\beta-1}(2\sqrt{Z})\left[\frac{2\sqrt{Z}}{(2(\alpha-\beta))\ -}\ \frac{4Z}{(2(\alpha-\beta)+2)\ -}\ \frac{4Z}{(2(\alpha-\beta)+4)\ -}\ \frac{4Z}{(2(\alpha-\beta)+6)\ -}\ \text{———}\right]$$

$$=\left[\frac{2Z^{\frac{\alpha+\beta+1}{2}} J_{\alpha-\beta-1}(2\sqrt{Z})}{(2(\alpha-\beta))\ -}\ \frac{4Z}{(2(\alpha-\beta)+2)\ -}\ \frac{4Z}{(2(\alpha-\beta)+4)\ -}\ \frac{4Z}{(2(\alpha-\beta)+6)\ -}\ \text{———}\right]$$

$$=\left[\frac{Z^{\frac{\alpha+\beta+1}{2}} J_{\alpha-\beta-1}(2\sqrt{Z})}{((\alpha-\beta))\ -}\ \frac{Z}{((\alpha-\beta)+1)\ -}\ \frac{Z}{((\alpha-\beta)+2)\ -}\ \frac{Z}{((\alpha-\beta)+3)\ -}\ \text{———}\right] \text{----(2.13)}$$

The table can be constructed by making the substitutings

Z= 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,2.0,3.0,4.0,5.0

α = 1,2,3

β = 1,2

as compiled in (appendix - VII)

Above defined proceedure, when applied to the

$H_{0,4}^{2,0}\left[Z(\alpha,1),(\alpha+\frac{1}{2},1),(\beta,1)(\beta+\frac{1}{2},1)\right]$ produces following expression :-

$$Z^{\frac{\alpha+\beta}{2}} J_{2(\alpha-\beta)-1}(4Z^{\frac{1}{4}})\left[\frac{4Z^{\frac{1}{2}}}{\{4(\alpha-\beta)\}-}\frac{4^2 Z^{\frac{1}{2}}}{\{4(\alpha-\beta)+2\}-}\frac{4^2 Z^{\frac{1}{2}}}{\{4(\alpha-\beta)+4\}-}\frac{4^2 Z^{\frac{1}{2}}}{\{4(\alpha-\beta)+6\}-}---\right]$$

$$=\frac{2Z^{\frac{2\alpha+2\beta+1}{4}} J_{2(\alpha-\beta)-1}(4Z^{\frac{1}{4}})}{\{2(\alpha-\beta)\}-}\frac{4Z^{\frac{1}{2}}}{\{2(\alpha-\beta)+1\}-}\frac{4Z^{\frac{1}{2}}}{\{2(\alpha-\beta)+2\}-}\frac{4Z^{\frac{1}{2}}}{\{2(\alpha-\beta)+3\}-}------- \quad --(2.14)$$

we can tabulated the function by making the substitutings

Z = 0.1,0.2,0.3,.0.4,0.5,0.6,0.7,0.8,0.9,1.0

α = 1,2

β = 1,2

in the form of table (Appendix-VIII)

Appedix-I

Z \ α	0.2	0.3	0.4	0.5	1.0
0.1	0.5709	0.4199	0.3602	0.2861	0.0904
0.2	0.5933	0.4787	0.4300	0.3661	0.1637
0.3	0.5822	0.4959	0.4576	0.4057	0.2222
0.4	0.5580	0.4938	0.4646	0.4239	0.2681
0.5	0.5280	0.4814	0.4596	0.4288	0.3032
0.6	0.4955	0.4628	0.4473	0.4251	0.3292
0.7	0.4623	0.4409	0.3970	0.4154	0.3476
0.8	0.4297	0.4171	0.4109	0.4018	0.3594
0.9	0.3980	0.3925	0.3897	0.3857	0.3659
1.0	0.3678	0.3678	0.3678	0.3678	0.3678

Appedix-II

α Z	β= 1 1	 2	 3	 4
0.1	0.1555	0.0823	0.1024	0.2241
0.2	0.2261	0.1541	0.2174	0.5173
0.3	0.2526	0.1978	0.2924	0.7001
0.4	0.2548	0.2198	0.3337	0.7914
0.5	0.2436	0.2021	0.3509	0.8224
0.6	0.2565	0.2578	0.4202	0.9014
0.7	0.2631	0.2818	0.4789	1.1991
0.8	0.2647	0.2996	0.5271	1.2119
0.9	0.2626	0.3118	0.5651	1.4720
1.0	0.2277	0.2797	0.5075	1.2947
2.0	0.1752	0.2890	0.6426	1.8762
3.0	0.1317	0.2597	0.6599	2.1240
4.0	0.0892	0.1997	0.5568	1.9126
5.0	0.0714	0.1771	0.5375	1.9784

Appedix-III

α,β / Z	$\alpha = 1, \beta = 1$	$\alpha = 2, \beta = 1$	$\alpha = 2, \beta = 2$
0.1	(0) 0.1122	(-1) 0.7448	(-1) +0.1122
0.2	(0) 0.1392	(0) 0.1187	(-1) +0.2784
0.3	(0) 0.1471	(0) 0.1450	(-1) +0.4413
0.4	(0) 0.1556	(0) 0.1715	(-1) +0.6224
0.5	(0) 0.1546	(0) 0.1851	(-1) +0.7730
0.6	(0) 0.1478	(0) 0.1887	(-1) +0.8868
0.7	(0) 0.1538	(0) 0.2098	(0) 0.1076
0.8	(0) 0.1571	(0) 0.2263	(0) 0.1256
0.9	(0) 0.1580	(0) 0.2386	(0) 0.1422
1.0	(0)0.1402	(0) 0.2187	(0) 0.1402

Appedix-IV

α,β Z	$\alpha = 1, \beta = 1/2$	$\alpha = 3/2, \beta = 1/2$	$\alpha = 3/2, \beta = 1$	$\alpha = 2, \beta = 1$
0.1	0.1880	0.3162	0.0594	0.1000
0.2	0.2820	0.4472	0.1261	0.2000
0.3	0.3693	0.5477	0.2023	0.3000
0.4	0.4606	0.6324	0.2913	0.4000
0.5	0.5641	0.7071	0.3989	0.5000
0.6	0.6909	0.7745	0.5352	0.6000
0.7	0.8618	0.8366	0.9708	0.7000
0.8	1.1283	0.8944	1.0092	0.8000
0.9	1.6925	0.9486	1.6057	0.9000

Appedix-V

α Z	0	1	2
0.2	(0) 0.6249	(-1) -0.1298	(-3) +0.7487
0.4	(0) 0.6958	(0) -0.2895	(-2) +0.3318
0.6	(0) 0.7786	(-1) -0.4803	(-2) +0.8286
0.8	(0) 0.8754	(-1) -0.7142	(-1) +0.1637
1.0	(0) 0.9890	(-1) -0.9978	(-1) +0.2849
2.0	(0) 1.9412	(0) -0.3603	(0) 0.1998
3.0	(0) 4.1628	(0) -1.0326	(0) 0.8200
4.0	(0) 9.5011	(0) -2.7586	(0) 2.7571
5.0	(2) 0.2045	(0) -6.6068	(0) 8.4222
6.0	(2) 0.5529	(1) 0.1863	(2) 0.2442
7.0	(3) 0.1378	(2) 0.4821	(2) 0.6872
8.0	(3) 0.3480	(3) 0.1250	(3) 0.1899
9.0	(3) 0.8876	(3) 0.3249	(3) 0.5188
10.0	(4) 0.2280	(3) 0.8472	(4) 0.1407

Appedix-VI

β Z	0	1	2
0.2	(0) 1.2387	(0) 5.0293	(+3) 0.1018
0.4	(0) 0.8094	(0) 2.2056	(2) +0.2286
0.6	(0) 0.5735	(0) 1.2770	(0) 9.0870
0.8	(0) 0.4214	(0) 0.8259	(0) 4.5510
1.0	(0) 0.3162	(0) 0.5667	(0) 2.5831
2.0	(-1) +0.8736	(0) 0.1249	(0) 0.3371
3.0	(-1) +0.2691	(-1) +0.3491	(-1)+0.7346
4.0	(-2) +0.8694	(-1) +0.1067	(-1)+0.1937
5.0	(-2) +0.2886	(-2) +0.3421	(-2)+0.5623
6.0	(-3) +0.9756	(-2) +0.1127	(-2)+0.1727
7.0	(-3) +0.3338	(-3) +0.3788	(-3)+0.5503
8.0	(-3) +0.1152	(-3) +0.1289	(-3)+0.1797
9.0	(-4) +0.3953	(-3) +0.4434	(-4)+0.5980
10.0	(-4) +0.1402	(-4)+0.1537	(-4)+0.2057

Appedix-VII

Z	$\alpha = 1, \beta = 1$	$\alpha = 2, \beta = 1$	$\alpha =2, \beta = 2$	$\alpha =3, \beta = 1$
0.1	0.0912	0.0090	0.0093	0.0004
0.2	0.1692	0.0329	0.0338	0.0030
0.3	0.2295	0.0723	0.0688	0.0103
0.4	0.2684	0.1260	0.1073	0.0254
0.5	0.2834	0.1916	0.1417	0.0518
0.6	0.3070	0.2593	0.1842	0.0835
0.7	0.3187	0.3267	0.2231	0.1259
0.8	0.3183	0.4134	0.2547	0.1803
0.9	0.3059	0.4965	0..2753	0.2479
1.0	0.2238	0.5767	0.2238	0.3528
2.0	-0.3700	1.1588	-0.7401	1.9107
3.0	-1.0928	0.9312	-3.2786	4.2275
4.0	-1.5885	-0.5283	-6.3543	5.8260
5.0	-1.7112	-2.2670	-8.5564	6.2521

Appedix-VIII

α,β Z	α = 1, β = 1	α = 2, β = 1	α = 2, β = 2
0.1	(-1) +0.1103	(-1) +0.1757	(-2) +0.1103
0.2	(-1) -0.1936	(-1) + 0.4211	(-2) -0.3872
0.3	(-1) -0.6729	(-1) + 0.6168	(-1) - 0.2018
0.4	(0) - 0.1168	(-1) + 0.7612	(-1) -0.4673
0.5	(0) - 0.1721	(-1) +0.7801	(-1) -0.8607
0.6	(0) -0.2280	(-1) + 0.6384	(0) -0.1368
0.7	(0) - 0.2742	(-1) + 0.5591	(0) -0.1919
0.8	(0) -0.3193	(-1) + 0.3852	(0) -0.2555
0.9	(0) -0.3623	(-1) +0.1094	(0) -0.3260
1.0	(0) -0.3971	(-1) -0.6604	(0) -0.3971

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