Optimization of a Telecommunication Network with Financial Considerations[1]

M. M. Ali
School of Computational and Applied Mathematics
Witwatersrand University,
Private Bag-3, Wits-2050, Johannesburg,
mali@cam.wits.ac.za

Abstract

In this paper we have presented a methodology for a rural and semi-urban telecommunication network placement. In order to optimally place the network and to ensure that the network is realistic and viable, we address four key issues, namely the demographic and socio-economic issues, geographical estimation, optimization of the network placement and financial optimization. A digital representation of the map of the region where the network has to be placed is used. A continuous optimization algorithm is applied to optimally place the backbone rings, and a combinatorial optimization algorithm is applied to obtain the optimal rollout order for the network. Mathematical formulations for both the optimization problems are presented. Optimal financial indicators are obtained.

Keywords: Rural telecommunication network, geographical estimation, continuous optimization, combinatorial optimization, point of presence, backbone, ring topology, rollout order, net present value, internal rate of return.

1 Introduction

Telecommunication is becoming an increasing critical component of economic growth for an evolving global information society. The establishment of a modern reliable and rapidly expanding telecommunication infrastructure contributes considerably to the promotion of a variety of activities of economics expansion [1]. Telecommunication offers an unique opportunity for rural development. Despite such an opportunity for rural development, the vast majority of rural people in the Southern African region are not connected to it and as a result they are left behind and denied the economic and quality of life opportunities that this technology has created. In recent times, opportunities have been created to bring many rural people in contact with telecommunication networks. This has been possible due to deregulated markets and governments' initiatives.

Our current research forms part of a telecommunication project involving a Southern African rural and semi-urban region of 500 by 500 kilometers ($500km^2$), where an entire network has to be serviced. The aim is to first collect the socio-economic data for assessing the commercial viability of the project, then to create a computational environment where the telecommunication network is optimized with respect to its placement and profitability. In our solution approach we first look at a hierarchy of the problem. We then solve all individual subproblem in the hierarchy one by one, feed the results of one subproblem to the next as input data, and assess the profitability from the final results. Although, this

[1]This telecommunication network is a combined telephone and data network such as VIP (voice over Internet Protocol).

approach is directed to solving a particular problem, our methodology can be applied to solve many telecommunication problems where the financial aspects can be incorporated. The methodology addresses four core problem specific issues. The first issue we consider is the socio-economic estimation. The second issue is the geographical estimation of the entire region. Thirdly, we have proposed a modified global optimization algorithm to tackle the optimization problem involving the network placement, in particular the placement of backbone rings. And finally, we have derived a mathematical model for the profits which we have optimized to obtain a rollout order of the network. These are consecutively presented in the sections below.

In section 2, we discuss the demographic and socio-economic issues. In section 3, an analysis of the geography of the region is given. In section 4, the optimization problem involving the backbone placement of the network is presented. A combinatorial optimization problem involving the rollout order is derived in section 5. In the same section, the results of the combinatorial optimization problem are presented. Numerical results are presented in section 6 and conclusions are given in section 7.

2 Demographic and Socio-economic Data

We have investigated some demographic and socio-economic data that directly or indirectly affect the viability of the telecommunication network in our study. Most of these data have simply been collected. The demographic data measures the population density, which is the subscriber base, hence knowledge of the operators (investors) market size (teledensity). Also the study of population identifies key locations for installation of telecommunication equipments as well as the distribution and concentration of the types user, e.g., business and household users and their demand per year. We have taken into account the literacy and unemployment rate (telepropensity), as they are good indicators of the extent and nature of the telecommunication network. The study of the economic domain is also important in identifying key indicators used to determine if a telecommunication network is viable in an area. We have considered the Gross Domestic Product (GDP) per capita as this is the most useful predictor of land-line telephone penetration.

The first step in designing a telecommunication network is the estimation of demand for various services on a geographical basis. Demand will depend on the concentration and the nature of the users. Our investigation has obtained that there will be 30 thousand initial users in the proposed area where the network will be placed, with an estimated demand of 36936 gigabytes per year. We have estimated the number of users and their demands on geographical basis and these are presented later in the paper.

The interest rate and the inflation rate are taken to be 12% and 5% respectively. They are quoted by *Merrill Lynch South Africa*-an investment company. The growth rate for demand for services are found to be 6%. This was estimated by using Revenues of some telecommunication companies working in South Africa [2]. The full details of all favourable demographic and socio-economic data such as GDP, teledensity and telepropensity that indicated the commercial viability of this project can be found in [2]. We will use the inflation and interest rate, the number of users and their yearly demand, and the growth rate as inputs date to our optimization problem.

3 Geographical Estimation

To minimize the cost of the network, the network placement has to be done in an optimal way. Therefore, an analysis of the geography must be done to identify key landmarks, such as roads, mountains, rivers etc. This is the most important factor to be considered when planning a network and should be as accurate as possible. For instance, if a large mountain or river is overlooked and a vast backbone structure is placed where the river or mountain exists, it creates enormous unexpected cost when trying to compensate for the error. We consider two maps, the topographical and the digital terrain map, of the entire region to identify landmarks and the potential users (customers). A digital representation of the map in a matrix format is created by examining these maps[2]. We have given positive values to the potential users and negative values to obstacles (see Table 1), and then transformed the features of both the maps into a matrix. The maps do not contain the individual users, only clusters of houses or buildings are indicated on the topographical coloured map. The ground level is taken to be the height above the sea level where most of the potential users live. Roads, streams, small hills and trees are not considered as major obstacles. We let each 5 by 5 km area (a cell) in the maps to be associated with a matrix element in a 100 by 100 matrix. A value is then assigned to each element of the matrix according to the landmark residing in the block. The size of the matrix can be adjusted. If both a large river and potential users reside in the same cell then 5 will be assigned to the users and -2 for the river. The resulting value will be $5 - 2 = 3$. An Excel spreadsheet consisting of a 100 by 100 grid with values of the cell as the values of the matrix elements is shown in Appendix A. If no value in the grid is recorded, then this corresponds to the value zero assigned to the matching matrix element. A side linkage clustering (SLC) [3] technique has been

Table 1: Mathematical representation of landmarks

Landmark	Value
Potential Users	5
Major River	-2
Ocean	-10
Mountain	-1 for every 50m above the ground level

used to identify four different clusters (regions) where the potential users are located (see Appendix A). We represent region i by R_i ($i = 1, 2, \cdots, 4$). For more details of the side linkage clustering (SLC), in the context of telecommunication, see [3].

4 The Optimal Placement of Backbone Network

To solve the network placement problem as effectively as possible, it is necessary to first determine the optimal placement of the backbones. If this initial step is not sensible the whole design could fail. The problem is to place the backbone rings (ring topology is chosen) around or through the regions so that most of the users can be linked to the backbones using the minimum amount of technologies. For instance, it is more cost effective to join a user to a backbone through only one PoP (point of presence) rather than a series of PoPs. To optimally place the backbone rings we have used a modified differential evolution (DE) algorithm. We now present the background for the mathematical formulation of the optimization problem involved in the placement of the backbone structures in the entire region.

After a cluster has been formed using SLC and the associated region has been identified and its area is known. The next question will be how many backbone rings are to be placed in that area. Rings are considered as circles or ellipses. The mathematical formula for an ellipse is given by

$$r^2 = (x - a)^2 + b(y - c)^2. \tag{1}$$

The number of rings in each region depends on the area of the region and the range (maximum length of circumference allowed for a backbone ring) of the optical cable. The range of the optical cable for this project is taken to be $100km$. We approximate the number of backbone rings in a region in the following way. If the range (circumference) of the optical cable ring is R, the area covered by the (circular) ring would be $C_A = R^2/4\pi$. Let A_i be the area of the region R_i, then the number of rings in the region R_i is approximated by the nearest integer to A_i/C_A. In this way the number of rings in region R_i is found,

[2] Ideally image processing techniques could be used on the map of the area, but we mimic the result of image processing by digital representation of the map in a matrix format.

before the optimization starts. The location of the centres (a, c) for the ellipses distributed in all regions, the horizontal radius r and the parameter value b for each ellipse are to be found optimally. For a single ellipse the number of variables involved is 4. The ranges of a and c are chosen so that each centre (a, c) lies in the respective region. The range of b is taken to be $0.5 \leq b \leq 1.5$. The radius r for each ring is chosen to lie between $R/4\pi$ and $R/2\pi$. For more details of the ranges for the optimization parameter, see [3].

The objective function to be minimized is the total cable length required in all regions. This is the sum of distances from the user locations to their respective closest rings in all regions. For instance in each R_i the location of the centres (a, c) for the ellipses, the horizontal radii r and the parameter values b are to be found so as to minimize the distance from each user location to its nearest ring. We now explain how the distance from each user location to its closest ring is found. We do this by calculating the distances to every ring in the region and selecting the minimum. We calculate the distance from a user location to a ring in the following way. We consider the ring to be made up of four arcs and each arc lying in one quadrant. The quadrants are separated by $x = a$ and $y = c$. The user will lie within one of the four quadrants and the distance is taken as the distance from the user location to the arc in the same quadrant. We find the (minimum) distance to the user by considering the distance from the user to a number of points on the arc (see Figure 1). The accuracy is determined by the number of points on the arc. It is found that it is fairly optimal if the number of points is chosen to be $r/\sqrt{b}$. Since the placement of rings in each region is independent from the placement of rings in others regions, we consider four optimization subproblems, one for each region. The dimensions of these subproblems and the ranges for a and c are not the same. The differential evolution algorithm [4] is suitably adapted to minimize the sum of these distances in all four regions.

Remark 1 : The number of rings was found to be 3, 2, 2 and 4 respectively for the regions 1 to 4. The total number of rings is 11 which is the sum of the number of rings in each region. See the optimal placement of these rings in Figure 3.

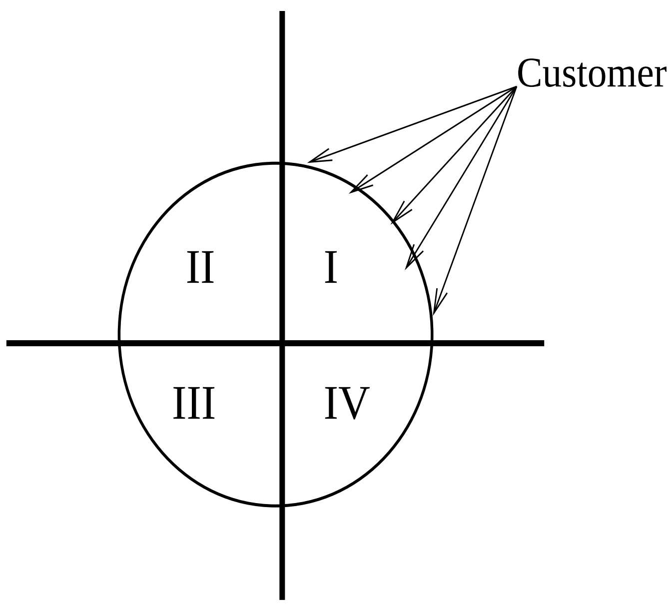

Figure 1: Distances from the user to the backbone network

4.1 Differential Evolution

Differential Evolution (DE) [4] is a population based direct search method. The overall structure of the DE algorithm resembles that of controlled random search (CRS) [5] and genetic algorithm (GA) [6]. Like other population based direct search methods, DE also attempts to guide an initial population set

$$S = \{x_1, x_2, \cdots, x_N\} \tag{2}$$

of points in the search region $\Omega \subset IR^n$ to the vicinity of the global minimum through repeated cycles of mutation, crossover (recombination) and acceptance. Here N is the population size and n is the dimension of the problem. DE attempts to replace all points in S by new better points at each generation. In each generation, N competitions are held to determine the members of S for the next generation. The i-th ($i = 1, 2, \cdots, N$) competition is held to replace x_i in S. Considering x_i as the target point a trial point y_i is found from two points (parents), the point x_i, i.e., the target point and the point $\hat{x}_i$ determined by the mutation operation. In its mutation phase DE randomly selects three distinct points x_p, x_q and x_r from the current set S. None of these points should coincide with the current target point x_i. The weighted difference of any two points is then added to the third point which can be mathematically described as :

$$\hat{x}_i = x_p + F_c \times (x_q - x_r) \,, \tag{3}$$

where $F_c \leq 1$ is a scaling factor. The trial point y_i is found from its parents x_i and $\hat{x}_i$ using the following rule :

$$y_i^j = \begin{cases} \hat{x}_i^j & \text{if } z^j \leq CR \\ x_i^j & \text{if } z^j > CR \,, \end{cases} \tag{4}$$

where the superscript j represents the j-th component of respective vectors; $z^j \in (0, 1)$, drawn randomly for each j. The entity CR is a constant (eg. 0.5). The acceptance mechanism follows the crossover. In the acceptance phase the function value at the trial point, $f(y_i)$, is compared to $f(x_i)$, the value at the target point. If $f(y_i) < f(x_i)$ then y_i replaces x_i in S, otherwise, S retains the original x_i. The process continues until all members of S are targeted. The rules (3) and (4) are respectively known as mutation and crossover. More details of DE can be found in [6]. In the next section we describe how the DE algorithm is adapted for this backbone placement problem.

4.2 Implementation of DE

We have suitably adapted the DE algorithm for our optimization problem. We now present how DE has been implemented to optimally place the backbone rings in four different regions. The subproblem in each region is different in that each has a different search region–R_i has a search region Ω_i for $i = 1, \cdots, 4$. The dimensions for each Ω_i are also different (the number of rings in each region is not the same). For instance, $\Omega_1 \subset IR^{12}, \Omega_2, \Omega_3 \subset IR^8$ and $\Omega_4 \subset IR^{16}$. Therefore, each x_j in the population set S in (2) can be represented as

$$x_j = (x_{j1}, x_{j2}, x_{j3}, x_{j4}) \,, \tag{5}$$

where $x_{ji} \in \Omega_i$. Since the subproblems are independent, we consider f_i to be the objective function associated with the subproblem i. The DE algorithm minimizes the objective function

$$f = \sum_{i=1}^{4} f_i, \tag{6}$$

which in turn minimizes f_i. Clearly, the objective function f_i for the subproblem i is defined on Ω_i and the dimension, n, of f is 44. At each iteration DE uses mutation and crossover to generate trial points

$$y_j = (y_{j1}, y_{j2}, y_{j3}, y_{j4}) \tag{7}$$

to replace target points x_j defined by (5). However, unlike regular DE, the adapted DE algorithm performs separate mutation and crossover for each member of y_j in (7). In particular, the k-th component y_{ji}^k of y_{ji} is found from

$$y_{ji}^k = \begin{cases} \hat{x}_{ji}^k & \text{if } z^k \le CR \\ x_{ji}^k & \text{otherwise} , \end{cases} \tag{8}$$

where $\hat{x}_{ji}^k$ is the k-th component of the mutation vector

$$\hat{x}_{ji} = x_{pi} + F_c \times (x_{qi} - x_{ri}) ,$$

where x_{pi}, x_{qi} and x_{ri} $(i = 1, 2, \cdots, 4)$ are defined like x_{ji} in (5) and z^k and CR are as in (4). If the function value f' at y_j is less than f at x_j, i.e., $f' < f$ then y_j will replace x_j in the population set S. Otherwise S retains x_j. The above adaptation of DE is well suited to our minimization problem. As

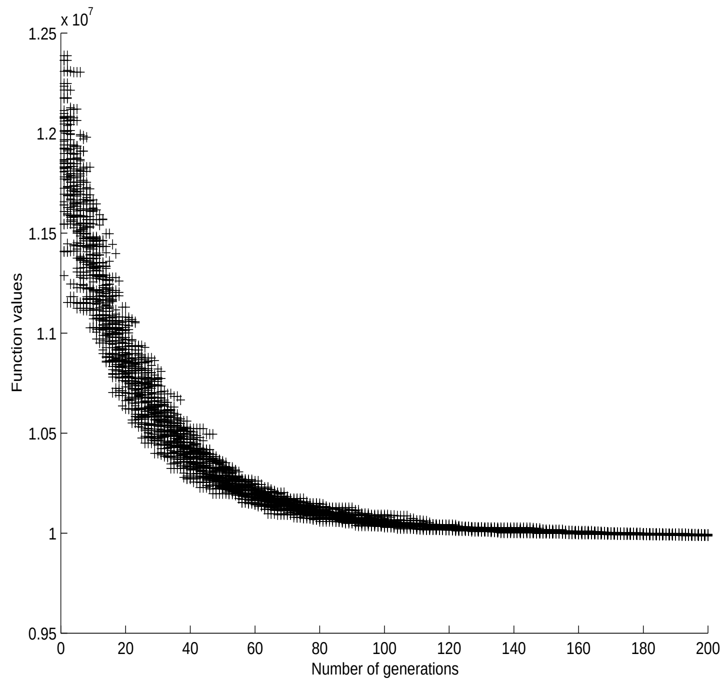

Figure 2: Number of generation vs function values for $F_c = 0.2$ and $N = 50$

stated earlier, the objective function f we considered is not the cost of the network but a total distance to be minimized. This minimization guides the DE algorithm to place backbone rings optimally in the potential regions where the users are located. In Figure 2 we present the function values in the population set S against the number of generations of modified DE. From this figure it is also clear that DE converges within 200 generations. The optimal solutions obtained by DE using $N = 50$ and $F_c = 0.2$ are plotted in Figure 3.

Remark 2 :We have used the optimal values of the centre, the radius and the circumference for all the rings in determining the number of PoPs per ring and their locations.

5 Financial Optimization

In this section the rollout order of the entire network has been modelled as a combinatorial optimization problem. The investors want to make their investment decisions given what their investment will be

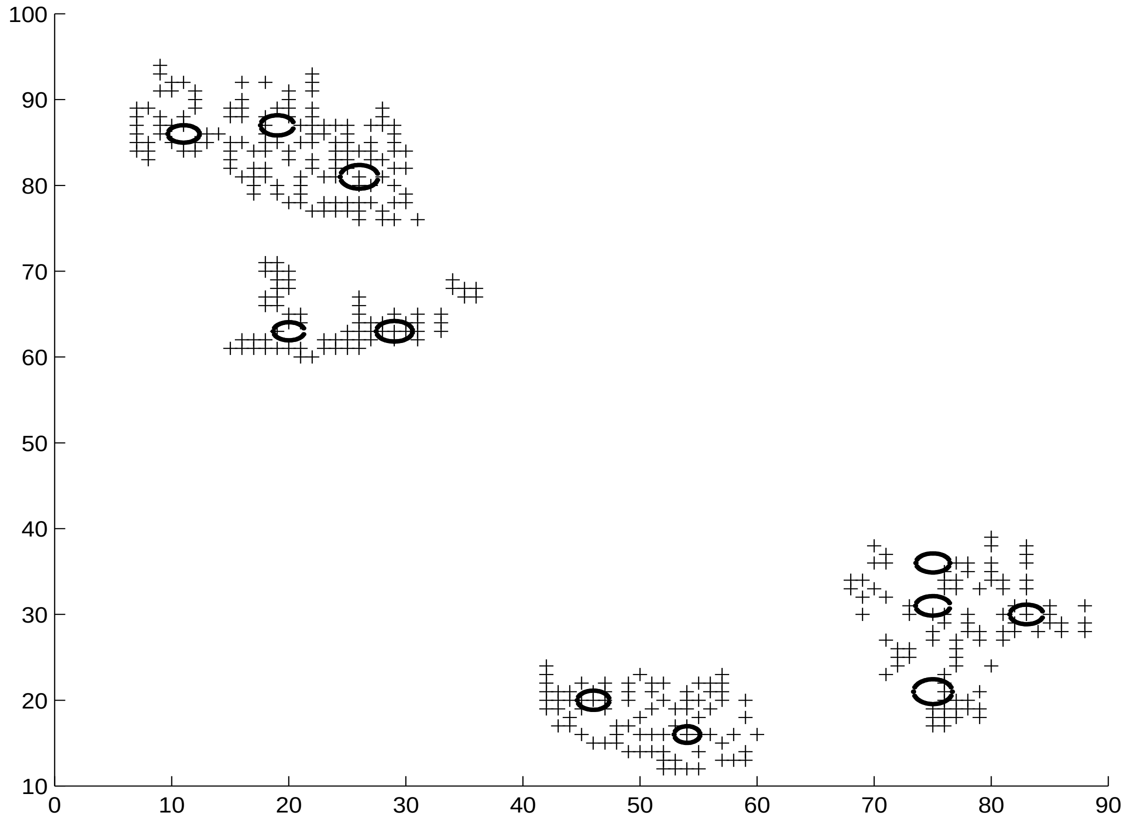

Figure 3: Optimally placed backbone rings

worth to them now. Therefore, before the implementation of the network, the investors decide a specific time horizon in which they would like to make profit. We refer to this time horizon as the ‘payback period’. The optimization problem is based on the present valuations of costs and revenues and the payback period is a parameter in the optimization function. We assume that the entire network will be installed within the payback period d and the installations are done at the beginning of each year. We also assume that the installation time as negligible, i.e., if the installation of a PoP starts at the beginning of a year it will be completed within a short period of time so as to fulfill the demand of the PoP within the same year. This implies that the revenues will be generated for d years for the PoPs installed at the first year, and for one year for the PoPs installed at d-th year. The profitability of the network has been calculated within the payback period, although there is no reason why the investors cannot make profits beyond this period. We have used the Net Present Value (NPV) and the Internal Rate of Return (IRR) [7, 8, 9] as measures of profitability. We have calculated both NPV and IRR as they are complementary measures of Discounted Cash Flow (DCF) [9]. Also, NPV and IRR together give better analysis than either alone. The interest rate and the inflation rate are taken to be constant over the payback period. In a stable economy, these rates do not vary drastically over a shorter payback period. We have used a simulated annealing algorithm to maximize the NPV of the profits from the entire network. We have then calculated the IRR from the cashflows resulted from the optimization of NPV. The optimization has been carried out with respect PoP installation, i.e., which PoP has to be installed at which point in time over the payback period so as to maximize the NPV. Therefore, all financial calculations have been carried out at the PoP level. The profit will depend on the size of the network, the users and the growth rate. Costs and revenues have been calculated at equally spaced, yearly intervals.

Table 2: PoP distribution per backbone and region

B_i	$P_i \rightarrow$	
B_1	$P_1,\ P_2,\ P_3$	R_1
B_2	$P_4,\ P_5,\ P_6$	
B_3	$P_7,\ P_8,\ P_9,\ P_{10}$	
B_4	$P_{11},\ P_{12},\ P_{13}$	R_2
B_5	$P_{14},\ P_{15}$	
B_6	$P_{16},\ P_{17},\ P_{18},\ P_{19},\ P_{20}$	R_3
B_7	$P_{21},\ P_{22},\ P_{23},\ P_{24}$	
B_8	$P_{25},\ P_{26},\ P_{27},\ P_{28}$	R_4
B_9	$P_{29},\ P_{30},\ P_{31}$	
B_{10}	$P_{32},\ P_{33},\ P_{34}$	
B_{11}	$P_{35},\ P_{36}$	

5.1 The Ingredients of Network used in Financial Calculation

We have considered the network up to the level of PoP, and used PoPs as the focal point of our financial calculation. We do not consider the lower level network architecture(s). In the lower level architecture copper wires are normally used. Therefore, for the entire network we have considered the fibre optic cables. For the entire network, as shown in Figure 3, there are 11 backbone rings distributed over four different regions. The backbones in top left region (R_1) are numbered as B_1, B_2 and B_3, in the middle region (R_2) as B_4 and B_5, in the bottom left region (R_3) as B_6 and B_7, and the backbones in the bottom right region (R_4) as B_8, B_9, B_{10} and B_{11} respectively. With careful analysis of the distribution of the users we have considered a total of 36 PoPs for the entire network. We also consider 4 main PoPs, one in each region, that carry inter region traffic. These are P_{10}, P_{12}, P_{16} and P_{25} respectively for regions R_1, R_2, R_3 and R_4. Distributions for PoPs per region as well as per backbone are summarised in Table 2. There are 10 PoPs in R_1, 5 in R_2, 9 in R_3 and 12 in R_4. All PoPs have a bandwidth capacity of 100 Megabits per second (Mbps), both ways. For these PoPs to constitute a network, connections have to be made between PoPs within each region. We refer a connection between two PoP as an edge. An edge e has a cost C_e per unit bandwidth. Therefore, allocating x_e units of bandwidth on edge e will cost $C_e x_e$. We do not consider the optimal allocation of bandwidth on edges. We assume that initial assignment of bandwidth on the edges is sufficient enough to carry the traffic for the entire payback period.

We assume that the users are concentrated at PoPs. Therefore, for a PoP P_p ($p = 1, 2, \cdots, 36$) there are revenues and costs associated with it, including the cable cost. We have correlated the results of optimization in section 4 with that of the field level survey in determining the edges. Therefore, the length of the fibre optic cable for each PoP is a estimate. For example, if two PoPs on two different rings are connected by an edge, the length of the edge is distributed between this two PoPs. Similarly, if three PoPs are place on the circumference of a ring then the (cable) length of the circumference is distributed amongst three PoPs proportionately, depending upon their positions. The financial calculation in this section is based on a concept that there should not be too few details of the financial problem that no meaningful conclusion can be drawn, yet the problem should not contain the overwhelming details of the data that the focus of this study is lost. Therefore, we refer to the reference [2] for the full details of how the cable length per PoP has been calculated. However, the data (cable length per PoP) that will be needed for our calculation is given in Table 3. The data under P_p in this table represents the cable lengths of PoP P_p in the respective regions. The four main PoPs in four regions are shown in bracket in the table.

Table 3: Cable length per PoP in kilometers

R_1	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	$\boxed{P_{10}}$		
	20	42.5	30	52.5	57.5	37.5	40	50	23	65		
R_2	P_{11}	$\boxed{P_{12}}$	P_{13}	P_{14}	P_{15}							
	68	74.5	18	40	45							
R_3	$\boxed{P_{16}}$	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}			
	72.5	42.5	60	35	27.5	25	60	32.5	55			
R_4	$\boxed{P_{25}}$	P_{26}	P_{27}	P_{28}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}
	69	27.5	27	57	55	32	50	33	32.5	25	23.5	30

5.2 The Predicted Cost

In this section we define the cost involved in the network and derive a mathematical formulation of the cost per PoP. The cost is the currency value of any asset used in order to lay out the telecommunication network. An asset is any physical component, knowledge or labour. The South African currency, Rand, has been used in our calculation. The fibre optic technology that will be installed at PoPs will have costs that can be divided up into four components costs, namely (a) Capital cost, (b) Installation cost, (c) Operating cost and (d) Miscellaneous cost. The capital cost includes the capital expenditure (offices and tents and cost for the instruments), the Installation cost is the cost for installation of instruments at PoP and the Operating cost is the cost for maintenance and operation, eg., the human capital (salary). The Miscellaneous cost is the incidental cost. We summarise these costs per PoP per year in Table 4. We take

Table 4: Costs per 100 Mbps bandwidth per PoP

Cost type	Amount in Rand (R)
Capital cost (K_c)	R1200000
Installation cost(I_c)	R350000
Miscellaneous cost (M_c)	R50000
Operating cost (O_c) per year	R2250000

into account the inflationary effect. Therefore, the costs are not fixed throughout the payback period. We have taken the cost per $100km$ of cable as the unit cable cost. The cost per unit is calculated as

$$\text{unit cost} = (\text{capital cost} + \text{installation cost} + \text{miscellaneous cost})\,/100. \tag{9}$$

Therefore, the unit cable cost is R16000. The PoP P_p has a associated cable cost c^p and this is found using data in Table 3. The Operating cost is the yearly running cost while the other costs are once-off. For instance, when a PoP has been installed the money spend on capital, installation and the miscellaneous reasons are spent only once. However, these costs vary with the inflation. The operating cost increases in direct proportion to the inflation, since the cost of purchasing new materials or hiring people will approximately follow the inflation rate. The operating cost can only be determined at the end of each year after a PoP has been installed. We assume that an estimate O_c of the operating cost per 100 megabits bandwidth is known at the end of first year. Let k_p denote the installation index, $k_p = 1, 2, \cdots, d$. If a PoP is installed at k_p-th year of the total payback period d, its operating cost $O_c(1 + z_1)^{(k_p-1)}$ will be known at $(k_p + 1)$-th year (inflation rate z_1). If we install a PoP at the beginning of first year of the payback period, the money spent is $K_c + I_c + M_c + c^p$. This cost at the k_p-th year will be

$$(K_c + I_c + M_c + c^p)(1 + z_1)^{k_p-1},\ k_p = 1, 2, \cdots, d, \tag{10}$$

where z_1 again is the inflation rate. If we denote k as the the cashflow index then $k = 1, 2, \cdots, d + 1$. We consider the costs as the negative revenues and cashflows as the accumulated revenues at index k.

Table 5: Number of users, and usage (gb), per PoP

R_1			R_2			R_3			R_4		
P_p	n_p	u_p	P_p	n_p	u_p	P_p	n_p	u_p	P_p	n_p	u_p
P_1	900	738.72	P_{11}	810	1071.144	P_{16}	2400	369.6	P_{25}	2100	2585.52
P_2	390	369.36	P_{12}	1860	4062.96	P_{17}	600	406.296	P_{26}	570	701.784
P_3	690	738.72	P_{13}	960	775.656	P_{18}	900	1108.08	P_{27}	630	775.656
P_4	600	738.7	P_{14}	600	369.36	P_{19}	510	590.976	P_{28}	690	517.104
P_5	600	1108.08	P_{15}	600	923.4	P_{20}	690	886.464	P_{29}	780	775.656
P_6	900	738.72				P_{21}	570	701.784	P_{30}	810	960.336
P_7	300	554.04				P_{22}	630	775.656	P_{31}	840	849.528
P_8	450	554.04				P_{23}	420	517.104	P_{32}	600	664.848
P_9	750	886.464				P_{24}	720	849.528	P_{33}	840	627.912
P_{10}	2130	2954.88							P_{34}	450	590.976
									P_{35}	900	812.592
									P_{36}	750	517.104

Notice that the revenue at $k = 1$ is always negative. Therefore, if the PoP P_p is installed at the beginning of k_p-th year ($k_p = 1, 2, \cdots, d$) of the payback period then the net present value, C_p, of the entire cost for this PoP is given by

$$C_p = (K_c + M_c + I_c + c^p)(1+z_1)^{(k_p-1)}(1+i)^{-(k_p-1)} + \sum_{j=k_p}^{d} O_c(1+z_1)^{(j-1)}(1+i)^{-j},\ k_p = 1, 2, \cdots, d, \tag{11}$$

where the sum is the accumulated sum of operating costs from k_p-th to the d-th year (known from $(k_p + 1)$-th to $(d + 1)$-th year). We have taken the interest rate i as the discount rate. Which PoP will be installed at the beginning of which year (rollout order) will depends upon both the quantity and types of demands, costs and revenues they generate, which in tern depend on the growth and inflation rates.

5.3 The Predicted Revenue

We assume that we have an amount of traffic using the network per year (expected usage 36936 gigabytes), and that tariffs are the main source of revenue for the investors. There are two types of tariffs : a fixed tariff, f_t, per year, per line (user) and a variable tariff, v_t, per megabytes. We have used f_t=30 and v_t=3. At each PoP P_p we have a number of user n_p and usage u_p in gigabytes (gb). We summarize the number of user and their yearly demand in Table 5, per PoP. Tariffs are not fixed throughout the payback period. They increase at a rate of z_2 per year. The revenue generated at the end of first year (year two), for a PoP P_p installed in first year (year one), is given by $n_p f_t + u_p v_t$. The fixed tariff f_t and the variable tariff v_t increase at a yearly rate of $z_2 = 3$. Notice that this rate is less than the inflation rate. This is compensated for by the growth in demand. The number of user n_p and usage u_p have a growth rate g. The values for n_p, u_p, f_t and v_t are all known at the beginning in year one and their incremental values are effective in year two. Therefore, the present value of the revenues (available at (k_p+1) year) generated at PoP P_p, installed at k_p-th year, is given by

$$R_p = \sum_{j=k_p}^{d} \left(n_p(1+g)^{(j-1)} f_t(1+z_2)^{(j-1)} + u_p(1+g)^{(j-1)} v_t(1+z_2)^{(j-1)} \right) (1+i)^{-j},\ k_p = 1, 2, \cdots, d. \tag{12}$$

This can be rewritten as follows:

$$R_p = \sum_{j=k_p}^{d} (n_p f_t + u_p v_t)(1+z_2)^{j-1}(1+g)^{(j-1)}(1+i)^{-j},\ k_p = 1, 2, \cdots, d. \tag{13}$$

For instance, the present value of the revenue from the PoP P_p install at the beginning of 5-th year for a payback period of 5 years will be given by

$$\left[(n_p \times f_t + u_p \times v_t)\,(1+g)^4(1+z_2)^4\right](1+i)^{-5}$$

5.4 The Predicted Profit

We present the net present value (NPV) of the profit first at the PoP level, then at the backbone level and finally at the network level. The NPV of profit for PoP P_p is given by

$$P_p^v = R_p - C_p, \tag{14}$$

where P_p^v is the net present value of profit at the PoP P_p ($p = 1, 2, \cdots, 36$). Similarly, the NPV for the backbone B_b ($b = 1, 2, \cdots, 11$) is given by

$$B_b^v = \sum_{i=1}^{N_b} R_{b_i} - C_{b_i}, \tag{15}$$

where B_b^v is the net present value of the profit for the backbone B_b, N_b is the number of PoP at the backbone B_b, and b_i is the associated PoP index for the backbone B_b. Finally, the net present value of the entire network is given by

$$NPV_n^v = \sum_{b=1}^{11} B_b = \sum_{p=1}^{36} R_p - C_p, \tag{16}$$

where NPV_n^v is the net present value of the total profit from the entire network.

5.5 The Optimization Problem

The optimization problem concerned here is the maximization of profit or the maximization of the net present value of the total cashflow (16) for the entire network. The variables of the optimization problem are the integer variables k_p taking values from the set $\{1, 2, \cdots, d\}$ for $p = 1, 2, \cdots, 36$. The optimization function is given by

$$NPV_n^v = f_p(k_1, k_2, \cdots, k_{36}) = \sum_{p=1}^{36} (R_p - C_p)\ . \tag{17}$$

More precisely, the optimization problem can be stated as follows:

$$\begin{aligned}\max_{\forall k_i \in \{1,2,\cdots,d\}} f_p(k_1, k_2, \cdots, k_{36}) = \sum_{p=1}^{36} \Bigg(\sum_{j=k_p}^{d} \Big[(n_p f_t + u_p v_t)(1+z_2)^{(j-1)}(1+g)^{(j-1)}(1+i)^{-j} \\ -O_c(1+z_1)^{(j-1)}(1+i)^{-j}\Big] - (K_c + M_c + I_c + c^p)\,(1+z_1)^{(k_p-1)}(1+i)^{-(k_p-1)}\Bigg). \end{aligned} \tag{18}$$

Let $x = (k_1, k_2, \cdots, k_{36})$ be the discrete random variable and its elements taking values from $D = \{1, 2, \cdots, d\}$, $d < 36$. $k_1, k_2, \cdots, k_{36}$ is the exact sequence in which the elements from D occur for a solution to (18). Clearly, the values of $k_p, p = 1, 2, \cdots, 36$ are not distinct. An aspiration based simulated annealing algorithm (ASA) [12], presented below, has been used for the maximization of (18).

5.6 Internal Rate of Return

The Internal Rate of Return (IRR) is a discount rate at which the present value a series of cashflow (initial outlay and profits at each year from a series of cashflows) is equal to zero. From the results of the optimization problem described above we will have a optimal selection of k_p, $p = 1, 2, \cdots, 36$. At each year there will be cashflows from various PoPs. For example, there will be cashflows at years $k_p, k_p + 1, \cdots, d + 1$, due to the PoP P_p installed at the k_p-th year. If we denote our total cashflows at the beginning of k-th year by CF_k, $k = 1, 2, \cdots, (d + 1)$, then the NPV can be written as a function of IRR as follows:

$$F(IRR) = \sum_{k=1}^{d+1} CF_k(1 + IRR)^{-(k-1)}, \tag{19}$$

where IRR is the value per cent. IRR is the root of the above equation, i.e., the discount rate for which the above NPV is zero. We have used the Newton-Rapson method [10] to obtain the root of F in (19).

5.7 Simulated Annealing (SA)

The Simulated Annealing algorithms [11, 12] are characterized by a point jumping around in the search region. Trial points are generated according to some distribution over the search region and a jump to a better point is always accepted. In order for the algorithm to be able to leave local minima also a jump to a worse point is accepted but with a decreasing probability. The technique for decreasing this probability, called the cooling schedule, is theoretically based using Markov chains and secures the convergence with probability 1 to the global minimum.

In any implementation of simulated annealing, a cooling schedule must be specified. The initial temperature $Temp^{(0)}$ is usually large, so that most trials are accepted and there is little chance of the algorithm zooming in on a local minimum in early stage. A scheme is then required for reducing temperature. Finally a stopping criterion is required to stop the algorithm. The choice of a cooling schedule clearly has an important bearing on the performance of a SA algorithm. Here, we use the cooling schedule suggested by Dekkers and Aarts in [12] and modified by Ali and Storey [12].

The original simulated annealing algorithms are memoryless in the sense that no previous locations of the point are recorded (not even a possible visit at the global minimum). Therefore, in the SA process, it is possible that during a particular stage, the procedure will visit the optimal solution, but due to the acceptance-rejection mechanism, it will move on from this solution and finish at a less than optimal solution. To overcome this difficulty, a memory is incorporated in the ASA algorithm.

If $x = (k_1, k_2, \cdots, k_{36})$ is the current accepted point in the Markov chain then the next point $\hat{x}$ of the inner loop (see the SA algorithm next for the inner and outer loops) is generated with some distribution scheme. We accept $\hat{x}$ if the following condition holds

$$\exp\left(\frac{f_p(x) - f_p(\hat{x})}{Temp^{(k)}}\right) \geq Ran, \tag{20}$$

where *Ran* is a random number generated from the interval $[0, 1]$, $Temp^{(k)}$ is the temperature of the kth iteration of the outer loop. Therefore, at each stage corresponding to a fixed temperature, a Markov Chain is created for which some points will be accepted and others will be rejected. If a point generates a smaller function value than the current estimate f_p^* of the optimum than the memory and and the temperature are updated. This is carried out at each Markov chain. The ASA algorithm allows the temperature to go up from time to time. The length of the Markov is dependent on current temperature and therefore it is not constant. For more details of the ASA algorithm we refer [12]. At the end, the last x^* is taken as the final solution to (18). We now defined the simulated annealing algorithm with memory.

Algorithm : The ASA Algorithm

Step 0 **Initialization**. Calculate the initial temperature $Temp^{(0)}$ according to [12], choose a random starting point $x^0 = (k_1, k_2, \cdots, k_{36})$ and calculate $f_p(x^0)$. Initialize f_p^* with $f_p(x^0)$. Set $l = k = 0$.

Step 1 **Metropolis simulation in inner loop**. Generate the L points in the k-th Markov chain according to the following rule:

(1a) Generate the new point x^{l+1}.
If

$$\exp\left(\frac{f_p(x^l) - f_p(x^{l+1})}{Temp^{(k)}}\right) \geq Ran$$

then accept the new point and update f_p^* and L, if necessary. Set $x^l = x^{l+1}$. Set $l = l + 1$ and if $l \leq L$ go to (1a); otherwise go to Step 2.

Step 2 **Convergence**. If the stopping condition is satisfied then stop.

Step 3 **Outer loop**. Set $k = k + 1$ and $l = 0$. Reduce the temperature to $Temp^{(k+1)}$ and goto Step 1.

Solution generation rule in ASA: Central to the SA algorithm is its cooling schedule and we have used the cooling schedule suggested in [12]. At each temperature SA generates a Markov chain. The length of a Markov chain is the number of trial solutions generated in the Markov chain. A solution here means the solution x. To discuss solution generation in ASA in more detail, we need first to define a neighbour structure for the discrete variable x. Let $x^l = (k_1, k_2, \ldots k_{36})$ with $k_i = d_{k_i} \in D, i = 1, \ldots 36$. The neighbourhood is defined by

$$\begin{array}{c} N_x = (\hat{k}_1, k_2, \ldots k_{36}) \cup (k_1, \hat{k}_2, \ldots k_{36}) \cup \ldots \cup (k_1, k_2, \ldots \hat{k}_{36}), \\ \text{where} \qquad \hat{k}_i = d_{k_{i+1}} \quad \text{or} \quad \hat{k}_i = d_{k_{i-1}}, \quad i = 1, \ldots, 36. \end{array} \tag{21}$$

Points in the Markov chain are generated both locally and globally. By locally we mean a new point x^{l+1} from the current accepted point x^l will be generated using (21) within the neighborhood of x^l. And by globally we mean x^{l+1} will be generated randomly, i.e., the elements of x^{l+1} will be generated uniformly from D. In the global generation, we randomly select 20% elements of x^l and replace them with the random elements from D in obtaining the coordinates of x^{l+1}. We equally generate new solution using both the local technique and the global technique. A random number $r \in (0, 1)$ is generated. If $r > 0.5$ then x^{l+1} is generated locally from the currently visited point (solution); otherwise x^{l+1} is generated randomly (globally), for each Markov chain.

Implementation of the cooling schedule requires setting of some parameter values. We set these parameter values according to the suggestions in [12, 11]. We use $\chi^{(0)} = 0.9, \delta = 0.1 \quad \epsilon = 10^{-3}$ in the cooling schedule. Moreover, we use 360 initial solutions ($n_1 + n_2 = 360$) to generate the initial temperature $Temp^{(0)}$. The lengths for all Markov chain are initially set to 300. For the description of these parameters of the cooling schedule, see Dekkers and Aarts [11], and Ali and Storey [12].

6 Numerical results

In this section we present the numerical results for the discrete optimization problem presented in the previous section. Numerical calculations have been carried out using a Pentium III of 930 MHz Processor. We run ASA algorithm with the parameter settings described in the previous section. We use the interest rate i=12%, inflation rate z_1=5%, the growth rate for demand g=6%, the yearly increase rate for tariff z_2=3%. Results for two runs using the payback periods $d = 5, 6$ are summarised in Tables 6 & 7, where P_p^v (in Table 6) is contribution of PoP P_p to NPV for the entire network. The optimal rollout order for the

Table 6: Optimal rollout order and NPV in millions, per PoP

	$d=5$		$d=6$	
p	k_p	P_p^v	k_p	P_p^v
1	3	-0.8147	4	-0.5560
2	5	-1.8902	6	-1.7383
3	1	-0.9742	1	-0.6710
4	3	1.9084	4	2.0987
5	5	-0.9935	5	-0.6924
6	1	3.7551	1	4.9342
7	5	-1.4453	6	-1.3047
8	5	-1.4429	6	-1.3023
9	1	0.9160	2	1.2836
10	1	27.4267	1	32.9894
11	3	1.6468	3	2.7367
12	1	41.5024	1	49.6715
13	1	-0.4676	2	-0.0651
14	4	-2.6914	5	-2.4546
15	3	0.5344	3	1.2710
16	1	36.8676	1	44.1785
17	5	-1.7957	6	-1.6463
18	2	2.8688	2	4.0479
19	4	-1.6092	5	-1.3996
20	2	0.6239	2	1.2755
21	5	-1.0796	6	-0.9482
22	2	-0.5013	2	-0.1124
23	4	-1.9753	4	-2.2010
24	3	-0.0080	3	0.5572
25	1	22.7183	1	27.4094
26	5	-1.0799	6	-0.9485
27	3	-0.5608	4	-0.3084
28	5	-1.5273	6	-1.3846
29	2	-0.4855	2	-0.0930
30	2	1.3782	2	2.2068
31	2	0.2641	2	0.8322
32	2	-1.6146	2	-1.4877
33	3	-1.6459	3	-1.6004
34	4	-1.6109	4	-1.6616
35	3	-0.2652	3	0.2171
36	5	-1.5225	5	-1.7404
Total		114.4087		151.3928

network and the contribution of each PoP to the NPV are presented in Table 6. Notice that in the optimal rollout order(s) in Table 6, PoPs belong to the same backbone have their installation years far apart. For instance, for $d = 5$ the optimal installation year for $P_3 \in B_1$ is year one but the installation year for P_1 of the same backbone is year five. Therefore, the backbone B_1 has to be installed in year one. This is not a major drawback from the optimization point of view since the cable cost for an entire backbone is much less than the other costs associated with a single PoP in that backbone. The optimal rollout order for both the payback periods in Table 6 shows that all four main PoPs (located in four regions and where majority of the user are concentrated) have installation year one, as expected from the practical point of view. It is also clear from Table 6 that many PoPs have negative contribution to the NPV, although the total NPV are 114.4087 and 151.3928 million Rand respectively for $d = 5$ and $d = 6$. For instance, there are 22 PoPs for $d = 5$ and 20 for $d = 6$ that have negative contributions to NPV. Therefore, the number of PoPs that contribute negatively decreases with d. We further summarise the results of Table 6 in terms of total yearly cashflows (accumulated from all PoPs) in Table 7. We present costs, revenues, cashflows per year and their present values. In Table 7, we denote the number of PoP to be installed at year k_p by p_{k_p}, the present value for revenue by NPV_{R_k}, the present value for cost by NPV_{C_k} and the cashflow per year by CF_k, $k = 1, 2, \cdots, d+1$. Therefore, in this table, columns 5 & 10 are the differences of their respective previous two columns. Similarly, the present valuations of data in columns 6 & 11 are respectively given in column 5 & 10. The last row in Table 7 is the added total for each column. We have also calculated IRR from the cashflows under CF_k, i.e., from the cashflows in 6th and 11th columns. The IRR=189.8790 and 230.5212 for these series of cashflows, respectively for the payback period 5 and 6. The optimization results in Tables 6 & 7 show a number of indicators such as NPV, IRR and present

Table 7: Costs, Revenues and Cashflows in millions, per year

	$d = 5$					$d = 6$				
k	p_{k_p}	NPV_{R_k}	NPV_{C_k}	NPV_{CF_k}	CF_k	p_{k_p}	NPV_{R_k}	NPV_{C_k}	NPV_{CF_k}	CF_k
1	8	0.0000	12.8623	-12.8623	-12.8623	6	0.0000	9.6557	-9.6557	-9.6557
2	7	45.3315	26.6190	18.7125	20.9580	9	40.8336	25.6073	15.2262	17.0534
3	8	60.0477	39.5460	20.5017	25.7173	5	60.0477	35.3133	24.7344	31.0268
4	4	76.2695	45.9010	30.3685	42.6655	5	69.5402	41.9256	27.6145	38.7964
5	9	79.5307	55.8629	23.6678	37.2417	4	77.1202	46.3465	30.7737	48.4230
6		89.8870	55.8665	34.0204	59.9556	7	80.5985	53.1476	27.4509	48.3778
7							87.6238	52.3749	35.2488	69.5750
	36	351.0664	236.6577	114.4087	173.6758	36	415.7640	264.3709	151.3928	243.5967

values per PoP. Table 6 shows which PoPs are less profitable than the others. It also shows that the investors can make more profits from the four main PoPs. For instance, NPV for these PoPs are 189.8790 and 153.6334 millions respectively for $d = 5$ and 6. These results are higher than the total results (NPV) in the last row of Table 7 (column under NPV_{CF_i}). This is because of the negative contribution due to many PoPs. We have also calculated the NPV for both payback periods by assuming that all PoPs are to be installed in year 1. The NPVs for these are 96.2690 and 131.5186 millions respectively for $d = 5$ and 6. These NPVs are much lower than the optimized NPVs for both payback periods. The corresponding IRR are 59.6831 and 63.9815 respectively for d=5 and 6. This implies more negative contributions due to PoPs other than the main four PoPs. Positive NPV does not always mean a profitability indicator. For instance, if a series of cashflow with positive NPV gives an IRR which is less than 20%, the project will not considered to be profitable in South Africa. Therefore, the calculation of IRR is important. Often the investors would like to know how soon and what profit they can make from the project. Next we investigate this using a series of payback periods and summarise the results in Table 8. We keep the growth, interest and inflation rates as before. Clearly, shorter the payback period is lesser the NPV and IRR are. We have also investigated the effect of inflation rate by conducting a series of runs and the

Table 8: IRR and NPV (millions) using different d

d	1	2	3	4	5	6
NPV	-30.6272	7.5026	41.8838	77.53494	114.4087	151.3928
IRR	-47.3000	27.8730	90.2483	192.1339	189.8790	230.5212

IRR for these runs are presented in Table 9. It can be seen that the IRR drops with the increase of the inflation rate. A similar study can be carried out using interest rate and a similar trend will be expected. The investors target a time frame by which they would like to complete the network. If this time frame

Table 9: Effect of inflation rate on NPV (in million) and IRR

z_1	5	7	9	11
NPV	114.4087	103.5279	92.2956	80.6002
IRR	189.8790	161.4449	156.2774	150.6671

and the payback period are equal then they will complete the installation of all PoPs including the ones with negative NPV. If the time frame is less than the payback period then the mathematical formulation in section 5 can be extended to account for this feature. If, however, it is greater than the payback period the the investors can decide on the rollout order for the PoPs with negative NPV.

7 Conclusion

The network design methodology suggested here takes first into account geographical location of demand and optimally place the backbone rings using a continuous global optimization technique. It models financial optimization problem and optimally determine the PoP rollout order of the network using a combinatorial optimization algorithm. This determines if the investor or the operators can make the profitability targets. Moreover, when more than one architecture can be used in certain area, it will not be clear up front which architecture is the best. The methodology presented here can easily handle this by reconfiguring the network. Therefore, the combined results of these two optimization can assist in the sense that the methodology can be used to configure a network from which test data can be generated. The test data will then be used to validate the network and therefore the design methodology. While most of the other approaches appear to target urban areas where the traffic is naturally aggregated, our approach is applicable in rural or urban areas or a combination of both. The optimization results such as IRR can help the investors in adjusting the tariff to have a greater access in the competitive market. Therefore, our methodology can guide the investors. Furthermore, we have taken a combined telephone and data network for this purpose, although our methodology can be applied separately. Separate tariffs can be used for the household and the business users and this will be considered in our future research.

Th methodology can easily be extended to a decision engine which will be capable of suggesting optimum network configurations to the network designer based on a range of hard and soft inputs, such as cost of the network, ability to expand the network and ability to meet the financial target etc. Our future research is underway to include a design methodology and software program that can be used to optimize a network.

Acknowledgements :

The author thanks people at the Network Solution Group, Alcatel Altech Telecoms, South Africa for providing me with data. He also thanks Professor David Du of University of Minnesota, and Claire

Oppermann and Bryan Thomas of School of Computational and Applied Mathematics, Witwatersrand University for their assistance. This paper was written when the author participated the IMA Program on Optimization for a year. Financial support from IMA is acknowledged.

References

[1] World bank report, *World bank telecommunication sector reports*, 1999.

[2] Ali, M. M., Thomas B. and Oppermann, C. (2001), 'Optimization of Telecommunication Network : An Holistic Approach', *Research Report Number* 10, School of Computational and Applied Mathematics, Witwatersrand University, Johannesburg, South Africa.

[3] Ali, M. M., Opermann, C., Thomas, B. and Walmarans, L. (2003), 'Optimal Placement of Backbone Structures in a Rural Telecommunication Network', in *Optimization and Industry : New Frontiers*, Panos Pardalos and Victor Korotkich (Editors), Kluwer Academic Publishers, pp.241-257.

[4] Storn, R. and Price, K. (1997), 'Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces', *Journal of Global Optimization*, Vol. 11, pp.341-359.

[5] Ali, M. M. and Storey, C. (1995), 'Modified Controlled Random Search Algorithms', *International Journal of Computer Mathematics*, Vol. 54, pp.229-235.

[6] Ali, M. M. and Törn, A. (2003), 'Population Set Based Global Optimization Algorithms: Some Modifications and Numerical Studies', in Press, *Computers and Operations Research*.

[7] Dowling, E. T. (1992), *Introduction to Mathematical Economics*, McGraw-Hall.

[8] Sydsaeter, K. and Hammond, P. J. (1995), *Mathematics for Economic Analysis*, Prentice-Hall.

[9] Weston J. F. (2001), *Finance and Accounting for Non-financial Managers*, McGraw-Hill.

[10] Nocedal, J. and Wright, S. J., (1999), *Numerical Optimization*, Springer Verlag.

[11] Dekkers, A. and Aarts, E.H.L. (1991), 'Global Optimization Simulated Annealing', *Mathematical Programming*, Vol.50, pp.367-393.

[12] Ali, M. M. and Storey, C. (1997), 'Aspiration based simulated annealing algorithm', *Journal of Global Optimization*, Vol. 11, pp.181-191.

Appendix A : Numeric format of the map