GENERALIZED HORN'S FUNCTIONS OF MATRIX ARGUMENTS

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ABSTRACT

Four results have been established here- three for the function ${}^{(k)}H_3^{(n)}$ and one for the function ${}^{(k)}H_4^{(n)}$ with matrix arguments along with two special cases.

INTRODUCTION

The generalized Horn's functions ${}^{(k)}H_3^{(n)}$ and ${}^{(k)}H_4^{(n)}$ follow as the generalizations of the Horn's functions H_3 and H_4 of two variables. We have already defined the functions ${}^{(k)}H_3^{(n)}$ and ${}^{(k)}H_4^{(n)}$ with matrix arguments in our previous papers [5,6]. Here, we have given further results for these functions. All the matrices appearing in this paper are (pxp) real symmetric positive definite matrices and the meanings of all the other symbols used are the same as in the works of Mathai [2, 3].

1. Preliminary Definition

DEFINITION 1.1: The $\Phi_3^{(n)}$ - function of matrix arguments

$$\Phi_3^{(n)} = \Phi_3^{(n)}(a_2, \cdots, a_n; c; -X_1, \cdots, -X_n)$$

is defined as that class of functions for which the matrix transform (M-transform) is the following:

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$$M(\Phi_3^{(n)}) = [\int_{X_1>0} \cdots \int_{X_n>0} |X_1|^{\rho_1-(p+1)/2} \cdots |X_n|^{\rho_n-(p+1)/2} \times$$

$$\Phi_3^{(n)}(a_2,\cdots,a_n;c;-X_1,\cdots,-X_n)dX_1\cdots dX_n]$$

$$= \frac{\Gamma_p(a_2-\rho_2)}{\Gamma_p(a_2)} \cdots \frac{\Gamma_p(a_n-\rho_n)}{\Gamma_p(a_n)} \frac{\Gamma_p(c)\Gamma_p(\rho_1)\cdots\Gamma_p(\rho_n)}{\Gamma_p(c-\rho_1-\cdots-\rho_n)} \qquad \cdots\cdots(1.1)$$

for $\mathrm{Re}(a_2-\rho_2,\cdots,a_n-\rho_n,c-\rho_1-\cdots-\rho_n,\rho_1,\cdots,\rho_n) > (p-1)/2.$

2. The ${}^{(k)}H_3^{(n)}$ Function of Matrix Arguments

THEOREM 2.1:

$${}^{(k)}H_3^{(n)}(a, b_{k+1},\cdots,b_n;c;-X_1,\cdots,-X_n)$$

$$= \frac{1}{\Gamma_p(a)} \int_{S>0} e^{-\mathrm{tr}(S)} |S|^{a-(p+1)/2} \Phi_3^{(n-k+1)}(b_{k+1},\cdots,b_n;c;$$

$$-S(X_1+\cdots+X_k)S', -S^{1/2}X_{k+1}S^{1/2},\cdots,-S^{1/2}X_nS^{1/2})dS \qquad \cdots\cdots(2.1)$$

for $S = S' > 0$ and $\mathrm{Re}(a) > (p-1)/2.$

PROOF: We take the M-transform of the right side of eq.(2.1) with respect to the variables $X_1,\cdots,X_n$ and the parameters $\rho_1,\cdots,\rho_n$ respectively to obtain,

$$\int_{X_1>0} \cdots \int_{X_n>0} |X_1|^{\rho_1-(p+1)/2} \cdots |X_k|^{\rho_k-(p+1)/2} |X_{k+1}|^{\rho_{k+1}-(p+1)/2} \times$$

$$\cdots |X_n|^{\rho_n-(p+1)/2} \Phi_3^{(n-k+1)}(b_{k+1},\cdots,b_n;c;-S(X_1+\cdots+X_k)S',$$

$$-S^{1/2}X_{k+1}S^{1/2},\cdots,-S^{1/2}X_nS^{1/2})dX_1\cdots dX_k dX_{k+1}\cdots dX_n \qquad \cdots\cdots(2.2)$$

Applying the transformations,

$Y_i = SX_iS'$, and $Y_j = S^{1/2}X_jS^{1/2}$ with $dY_i = |S|^{p+1}dX_i; dY_j = |S|^{(p+1)/2}dX_j$ and

$|Y_i| = |S|^2|X_i|, |Y_j| = |S||X_j|$ for $i = 1,\cdots,k$ and $j = k+1,\cdots,n$

the expression (2.2) yields,

$$|S|^{-2(\rho_1+\cdots+\rho_k)-\rho_{k+1}-\cdots-\rho_n}\int_{Y_1>0}\cdots\int_{Y_n>0}|Y_1|^{\rho_1-(p+1)/2}\cdots\times$$

$$|Y_k|^{\rho_k-(p+1)/2}|Y_{k+1}|^{\rho_{k+1}-(p+1)/2}\cdots|Y_n|^{\rho_n-(p+1)/2}\times\Phi_3^{(n-k+1)}(b_{k+1},\cdots,$$

$$b_n;c;-(Y_1+\cdots+Y_k),-Y_{k+1},\cdots,-Y_n)dY_1\cdots dY_k dY_{k+1}\cdots dY_n \qquad \cdots\cdots(2.3)$$

Now, applying the transformations,

$Z_1=Y_1, Z_2=Y_1+Y_2,\cdots,Z_k=Y_1+\cdots+Y_k; Z_j=Y_j$, for $j=k+1,\cdots,n$

then from eq.(6.7) page 95 of Mathai [2] we have

$dY_1\cdots dY_k=dZ_1\cdots dZ_k$, also, $dY_j=dZ_j$ for $j=k+1,\cdots,n$ and $0<Z_1<Z_2<\cdots<Z_k, Z_j>0\ (j=k+1,\cdots,n)$

which render the expression (2.3) as below,

$$|S|^{-2(\rho_1+\cdots+\rho_k)-\rho_{k+1}-\cdots-\rho_n}\int\cdots(n)\cdots\int|Z_1|^{\rho_1-(p+1)/2}\times$$

$$|Z_2-Z_1|^{\rho_2-(p+1)/2}\cdots|Z_k-Z_{k-1}|^{\rho_k-(p+1)/2}|Z_{k+1}|^{\rho_{k+1}-(p+1)/2}\cdots\times$$

$$|Z_n|^{\rho_n-(p+1)/2}\Phi_3^{(n-k+1)}(b_{k+1},\cdots,b_n;c;-Z_k,-Z_{k+1},\cdots,-Z_n)dZ_1\cdots dZ_k\times$$

$$dZ_{k+1}\cdots dZ_n \qquad \cdots\cdots(2.4)$$

Integrating out the variables $Z_1,\cdots,Z_{k-1}$ one-by-one and in order by using a type-1 Beta integral and then using the definition (1.1), the expression (2.4) yields,

$$|S|^{-2(\rho_1+\cdots+\rho_k)-\rho_{k+1}-\cdots-\rho_n}\frac{\Gamma_p(c)\Gamma_p(b_{k+1}-\rho_{k+1})\cdots\Gamma_p(b_n-\rho_n)}{\Gamma_p(b_{k+1})\cdots\Gamma_p(b_n)}\times$$

$$\frac{\Gamma_p(\rho_1)\cdots\Gamma_p(\rho_n)}{\Gamma_p(c-\rho_1-\cdots-\rho_n)} \qquad \cdots\cdots(2.5)$$

Substituting this expression on the right side of eq. (2.1) and then integrating out S in the resulting expression by using a Gamma integral produces $M[{}^{(k)}H_3^{(n)}]$ as given by eq. (5.1) of the authors' paper [5].

THEOREM 2.2: For p=2,

$$ {}^{(k)}H_3^{(n)}(a,b_{k+1},\cdots,b_n;c;-X_1,\cdots,-X_n) $$

$$ =\frac{\Gamma_p(c)}{\Gamma_p(b_{k+1})\cdots\Gamma_p(b_n)\Gamma_p(c-b_{k+1}-\cdots-b_n)}\int\cdots(n-k)\cdots\int\left|U_{k+1}\right|^{b_{k+1}-(p+1)/2} $$

$$ \times\cdots\left|U_n\right|^{b_n-(p+1)/2}\left|I-U_{k+1}-\cdots-U_n\right|^{c-b_{k+1}-\cdots-b_n-(p+1)/2}\times $$

$$ \left|I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2}\right|^{-a}{}_2F_1[(a+1)/2,(2a+1)/4; $$

$$ c-b_{k+1}-\cdots-b_n;-4(I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2})^{-1}\times $$

$$ (I-U_{k+1}-\cdots-U_n)^{1/2}(X_1+\cdots+X_k)(I-U_{k+1}-\cdots-U_n)^{1/2}\times $$

$$ (I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2})^{-1}]dU_{k+1}\cdots dU_n \qquad \cdots\cdots(2.6) $$

where $\mathrm{Re}\,(b_{k+1},\cdots,b_n,c-b_{k+1}-\cdots-b_n)>(p-1)/2,\quad U'_{k+1}=U_{k+1}>0,$ $\cdots,U'_n=U_n>0$ and $0<U_{k+1}+\cdots+U_n<I.$

PROOF: Taking the M-transform of the right side of eq. (2.6) with respect to the variables $X_1,\cdots,X_n$ and the parameters $\rho_1,\cdots,\rho_n$ respectively, we have,

$$ \int_{X_1>0}\cdots\int_{X_n>0}\left|X_1\right|^{\rho_1-(p+1)/2}\cdots\left|X_k\right|^{\rho_k-(p+1)/2}\left|X_{k+1}\right|^{\rho_{k+1}-(p+1)/2}\times $$

$$ \cdots\left|X_n\right|^{\rho_n-(p+1)/2}\left|I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2}\right|^{-a}{}_2F_1[(a+1)/2, $$

$$ (2a+1)/4;c-b_{k+1}-\cdots-b_n;-4(I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2})^{-1}\times $$

$$ (I-U_{k+1}-\cdots-U_n)^{1/2}(X_1+\cdots+X_k)(I-U_{k+1}-\cdots-U_n)^{1/2}\times $$

$$ (I+U_{k+1}^{1/2}X_{k+1}U_{k+1}^{1/2}+\cdots+U_n^{1/2}X_nU_n^{1/2})^{-1}]dX_1\cdots dX_kdX_{k+1}\cdots dX_n \qquad \cdots(2.7) $$

Applying the transformations,

$$Y_1 = X_1, Y_2 = X_1 + X_2, \cdots, Y_k = X_1 + \cdots + X_k; Y_j = U_j^{1/2} X_j U_j^{1/2}, \text{ for } j = k+1, \cdots, n;$$

then from eq.(6.7) page 95 of Mathai [2],we have,

$dY_1 \cdots dY_k = dX_1 \cdots dX_k$, also, $dY_j = |U_j|^{(p+1)/2} dX_j$ for $j = k+1, \cdots, n$ and $0 < Y_1 < Y_2 < \cdots < Y_k$ and $Y_j > 0$ $(j = k+1, \cdots, n)$.

Using these transformations in the expression (2.7) and then integrating out the variables $Y_1, \cdots, Y_{k-1}$ one-by-one and in order by using a type-1 Beta integral as in the previous theorem, we are led to,

$$|U_{k+1}|^{-\rho_{k+1}} \ldots |U_n|^{-\rho_n} \frac{\Gamma_p(\rho_1) \cdots \Gamma_p(\rho_k)}{\Gamma_p(\rho_1 + \cdots + \rho_k)} \int_{Y_k > 0} \cdots (n-k+1) \cdots \int_{Y_n > 0} \times$$

$$|Y_k|^{\rho_1 + \cdots + \rho_k - (p+1)/2} |Y_{k+1}|^{\rho_{k+1} - (p+1)/2} \ldots |Y_n|^{\rho_n - (p+1)/2} \times$$

$$|I + Y_{k+1} + \cdots + Y_n|^{-a} {}_2F_1[(a+1)/2, (2a+1)/4; c - b_{k+1} - \cdots - b_n;$$

$$-4(I + Y_{k+1} + \cdots + Y_n)^{-1} (I - U_{k+1} - \cdots - U_n)^{1/2} Y_k (I - U_{k+1} - \cdots - U_n)^{1/2} \times$$

$$(I + Y_{k+1} + \cdots + Y_n)^{-1}] dY_k dY_{k+1} \cdots dY_n \qquad \cdots (2.8)$$

Now using the following transformation in the expression (2.8),

$$Z_k = 4(I + Y_{k+1} + \cdots + Y_n)^{-1} (I - U_{k+1} - \cdots - U_n)^{1/2} Y_k (I - U_{k+1} - \cdots - U_n)^{1/2} \times$$

$(I + Y_{k+1} + \cdots + Y_n)^{-1}$ with $dZ_k = 4^{p(p+1)/2} |I + Y_{k+1} + \cdots + Y_n|^{-(p+1)} \times$

$|I - U_{k+1} - \cdots - U_n|^{(p+1)/2} dY_k$ and $|Z_k| = 4^p |I + Y_{k+1} + \cdots + Y_n|^{-2} \times$

$|I - U_{k+1} - \cdots - U_n||Y_k|$

and then writing the M-transform of the ${}_2F_1$-function and integrating out the variables $Y_{k+1}, \cdots, Y_n$ by using a type-2 Dirichlet integral, we have,

$$4^{-p(\rho_1 + \cdots + \rho_k)} |U_{k+1}|^{-\rho_{k+1}} \ldots |U_n|^{-\rho_n} \Gamma_p(\rho_1) \cdots \Gamma_p(\rho_n) \times$$

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$$\left|I-U_{k+1}-\cdots-U_n\right|^{-(\rho_1+\cdots+\rho_k)}\frac{\Gamma_p(a-2\rho_1-\cdots-2\rho_k-\rho_{k+1}-\cdots-\rho_n)}{\Gamma_p(a-2\rho_1-\cdots-2\rho_k)}\times$$

$$\frac{\Gamma_p[(a+1)/2-\rho_1-\cdots-\rho_k]\Gamma_p[(2a+1)/4-\rho_1-\cdots-\rho_k)}{\Gamma_p[(a+1)/2]\Gamma_p[(2a+1)/4]}\times$$

$$\frac{\Gamma_p(c-b_{k+1}-\cdots-b_n)}{\Gamma_p(c-b_{k+1}-\cdots-b_n-\rho_1-\cdots-\rho_k)} \qquad \cdots\cdots(2.9)$$

Substituting this expression on the right side of eq.(2.6) and integrating out the variables $U_{k+1},\cdots,U_n$ in the resulting expression by using a type-1 Dirichlet integral and observing that,

$$4^{-p(\rho_1+\cdots+\rho_k)}\frac{\Gamma_p[(a+1)/2-\rho_1-\cdots-\rho_k]\Gamma_p[(2a+1)/4-\rho_1-\cdots-\rho_k)}{\Gamma_p(a-2\rho_1-\cdots-2\rho_k)\Gamma_p[(a+1)/2]\Gamma_p[(2a+1)/4]}$$

$$=\frac{1}{\Gamma_p(a)},\quad \text{for } p=2, \qquad \cdots\cdots(2.10)$$

from eq.(6.13) page 84 of Mathai [3], we finally have $M[{}^{(k)}H_3^{(n)}]$ as given by eq. (5.1) of the authors' paper [5]. This result is different from the corresponding result in the scalar case.

THEOREM 2.3:

$${}^{(k)}H_3^{(n)}(a,b_{k+1},\cdots,b_n;c;-X_1,\cdots,-X_n)$$

$$=\frac{1}{\Gamma_p(a)\Gamma_p(b_{k+1})\cdots\Gamma_p(b_n)}\int_{S>0}\cdots(n-k+1)\cdots\int_{T_n>0}e^{-tr(S+T_{k+1}+\cdots+T_n)}\times$$

$$|S|^{a-(p+1)/2}\left|T_{k+1}\right|^{b_{k+1}-(p+1)/2}\cdots\left|T_n\right|^{b_n-(p+1)/2}{}_0F_1[\ ;c;-S(X_1+\cdots+X_k)S'$$

$$-S^{1/2}(T_{k+1}^{1/2}X_{k+1}T_{k+1}^{1/2}+\cdots+T_n^{1/2}X_nT_n^{1/2})S^{1/2}]dSdT_{k+1}\cdots dT_n \qquad \cdots\cdots(2.11)$$

for $\mathrm{Re}(a,b_{k+1},\cdots,b_n)>(p-1)/2$.

PROOF: Taking the M-transform of the right side of eq. (2.11) with respect to the variables $X_1,\cdots,X_n$ and the parameters $\rho_1,\cdots,\rho_n$ respectively, we get,

$$\int_{X_1>0}\cdots\int_{X_n>0}|X_1|^{\rho_1-(p+1)/2}\cdots|X_k|^{\rho_k-(p+1)/2}|X_{k+1}|^{\rho_{k+1}-(p+1)/2}\times$$

$$\cdots|X_n|^{\rho_n-(p+1)/2}\ {}_0F_1[\ ;c;-S(X_1+\cdots+X_k)S'-S^{1/2}(T_{k+1}^{1/2}X_{k+1}T_{k+1}^{1/2}+$$

$$\cdots+T_n^{1/2}X_nT_n^{1/2})S^{1/2}]dX_1\cdots dX_k dX_{k+1}\cdots dX_n \quad \cdots\cdots(2.12)$$

Applying the transformations,

$$Y_1=X_1, Y_2=X_1+X_2,\cdots,Y_k=X_1+\cdots+X_k; Y_j=T_j^{1/2}X_jT_j^{1/2}\text{, for } j=k+1,\cdots,n;$$

with $dY_1\cdots dY_k=dX_1\cdots dX_k;\ dY_j=|T_j|^{(p+1)/2}dX_j$ for $j=k+1,\cdots,n$ and $0<Y_1<Y_2<\cdots<Y_k$ and $Y_j>0\ (j=k+1,\cdots,n)$,

to the expression (2.12) and integrating out $Y_1,\cdots,Y_{k-1}$ as in the theorem (2.1) by using a type-1 Beta integral we obtain,

$$\frac{\Gamma_p(\rho_1)\cdots\Gamma_p(\rho_k)}{\Gamma_p(\rho_1+\cdots+\rho_k)}|T_{k+1}|^{-\rho_{k+1}}\cdots|T_n|^{-\rho_n}\int_{Y_k>0}\cdots(n-k+1)\cdots\int_{Y_n>0}\times$$

$$|Y_k|^{\rho_1+\cdots+\rho_k-(p+1)/2}|Y_{k+1}|^{\rho_{k+1}-(p+1)/2}\cdots|Y_n|^{\rho_n-(p+1)/2}\ {}_0F_1[\ ;c;-SY_kS'$$

$$-S^{1/2}(Y_{k+1}+\cdots+Y_n)S^{1/2}]dY_kdY_{k+1}\cdots dY_n \quad \cdots\cdots(2.13)$$

Now applying another set of transformations,

$$Z_k=SY_kS'; Z_j=S^{1/2}Y_jS^{1/2};\text{ with } dZ_k=|S|^{p+1}dY_k; dZ_j=|S|^{(p+1)/2}dY_j;$$

and $|Z_k|=|S|^2|Y_k|;|Z_j|=|S||Y_j|$; for $j=k+1,\cdots,n$

to the expression (2.13) and using the theorem 3.3 page 55 of Mathai [3], we get,

$$\Gamma_p(\rho_1)\cdots\Gamma_p(\rho_n)|T_{k+1}|^{-\rho_{k+1}}\cdots|T_n|^{-\rho_n}|S|^{-2\rho_1-\cdots-2\rho_k-\rho_{k+1}-\cdots-\rho_n}\times$$

$$\frac{\Gamma_p(c)}{\Gamma_p(c-\rho_1-\cdots-\rho_n)} \quad \cdots\cdots(2.14)$$

Substituting this expression on the right side of eq.(2.11) and then integrating out the variables S, $T_{k+1},\cdots,T_n$ in the resulting expression by using a Gamma integral produces $M[{}^{(k)}H_3^{(n)}]$ as given by eq. (5.1) of the authors' paper [5].

3. The ${}^{(k)}H_4^{(n)}$ Function of Matrix Arguments

THEOREM 3.1:

$$ {}^{(k)}H_4^{(n)}(a, b_{k+1}, \cdots, b_n; c_1, \cdots, c_n; -X_1, \cdots, -X_n) $$

$$ = \frac{1}{\Gamma_p(a)} \int_{S>0} e^{-tr(S)} |S|^{a-(p+1)/2} {}_0F_1(\ ; c_1; -SX_1S') \cdots {}_0F_1(\ ; c_k; -SX_kS') \times $$

$$ {}_1F_1(b_{k+1}; c_{k+1}; -S^{1/2} X_{k+1} S^{1/2}) \cdots {}_1F_1(b_n; c_n; -S^{1/2} X_n S^{1/2}) dS \qquad \cdots\cdots (3.1) $$

for $\mathrm{Re}(a) > (p-1)/2$.

PROOF: Taking the M-transform of the right side of eq. (3.1) with respect to the variables $X_1, \cdots, X_n$ and the parameters $\rho_1, \cdots, \rho_n$ respectively, we have,

$$ \int_{X_1>0} \cdots \int_{X_n>0} |X_1|^{\rho_1-(p+1)/2} \cdots |X_k|^{\rho_k-(p+1)/2} |X_{k+1}|^{\rho_{k+1}-(p+1)/2} \times $$

$$ \cdots |X_n|^{\rho_n-(p+1)/2} {}_0F_1(\ ; c_1; -SX_1S') \cdots {}_0F_1(\ ; c_k; -SX_kS') \times {}_1F_1(b_{k+1}; c_{k+1}; $$

$$ -S^{1/2} X_{k+1} S^{1/2}) \cdots {}_1F_1(b_n; c_n; -S^{1/2} X_n S^{1/2}) dX_1 \cdots dX_k dX_{k+1} \cdots dX_n \qquad \cdots\cdots (3.2) $$

Applying the following transformations to the above expression,

$$ Y_i = SX_iS'; Y_j = S^{1/2} X_j S^{1/2}, \text{ with } dY_i = |S|^{p+1} dX_i; dY_j = |S|^{(p+1)/2} dX_j; \text{ and} $$

$$ |Y_i| = |S|^2 |X_i|; |Y_j| = |S| |X_j|; \text{ for } i = 1, \cdots, k; \ j = k+1, \cdots, n $$

and then writing the M-transforms of the ${}_0F_1$ and ${}_1F_1$ functions we obtain,

$$ |S|^{-2\rho_1-\cdots-2\rho_k-\rho_{k+1}-\cdots-\rho_n} \frac{\Gamma_p(c_1)}{\Gamma_p(c_1-\rho_1)} \cdots \frac{\Gamma_p(c_n)}{\Gamma_p(c_n-\rho_n)} \frac{\Gamma_p(b_{k+1}-\rho_{k+1})}{\Gamma_p(b_{k+1})} \times $$

$$ \cdots \frac{\Gamma_p(b_n-\rho_n)}{\Gamma_p(b_n)} \Gamma_p(\rho_1) \cdots \Gamma_p(\rho_n) \qquad \cdots\cdots (3.3) $$

On substituting this expression on the right side of eq.(3.1) and integrating out the variable S in the resulting expression by using a Gamma integral, the outcome is $M[{}^{(k)}H_4^{(n)}]$ as given by eq.(4.1) of the authors' paper [6].

THEOREM 3.2: Special Cases-

(i) $^{(0)}H_3^{(n)}(a, b_1, \cdots, b_n; c; -X_1, \cdots, -X_n) = F_D^{(n)}(a, b_1, \cdots, b_n; c; -X_1, \cdots, -X_n) \cdots\cdots (3.4)$

(ii) $^{(0)}H_4^{(n)}(a, b_1, \cdots, b_n; c_1, \cdots, c_n; -X_1, \cdots, -X_n)$

$= F_A^{(n)}(a, b_1, \cdots, b_n; c_1, \cdots, c_n; -X_1, \cdots, -X_n) \qquad \cdots\cdots (3.5)$

PROOF: (i) This result is obtained by putting k=0 in eq.(5.1) of the authors' paper [5] and then comparing the outcome with eq.(1.4) of the authors' paper [7].
(ii) Similarly, this result can be had by putting k=0 in eq.(4.1) of the authors' paper [6] and then comparing the result so obtained with eq.(1.2) of the authors' paper [7].

Corrigendum: The following more should be added at the end of the statement of the theorem (5.1) of the authors' paper [5] in order to make the statement more clear and meaningful:

"and $U'_{k+1} = U_{k+1} > 0; \cdots; U'_n = U_n > 0$ and $0 < U_{k+1} + \cdots + U_n < I$."

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