Modeling The Economics of Differentiated Durable-Goods Markets *

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*Due to the time constraint, the paper was completed in a rush and thus some of the economic interpretation may not be completely accurate.

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Abstract

Leasing has traditionally been one of the tools that firm employs to increase market share. It is not intuitive that this strategy would actually be beneficial. It is certainly not true if we consider only perishable goods. However, there are goods in the market that does not perish the instant we consume it or durable goods. Having leasing option in durable goods creates another market for used goods as a residue when the lease term ends. In the past, we have seen companies implementing this policy exclusively even abandoning their selling option. Here we are only considering the case where firms never sells their new goods due to the high transaction cost incurred. Durable goods is interesting on its own accord since we are faced with a dynamic problem in which each period depends on the previous periods action since we might still have an option to reuse the product we purchased from the previous periods. Thus the decision process for the market participants are intrinsically dynamic. Transaction costs in selling used goods prompt consumers to think ahead in order to avoid adverse lock-ins, and hence have a strong influence on the endogenous formation of these patterns. Likewise, producers have to choose their production/pricing strategies so that the demands on new and used goods are balanced, and the percentages of sold and leased new goods are properly allocated. We consider exogenuous products with vertical differentiation. There exist a continuum of consumer types. Producers have an option to concurrently sell and lease new goods with the possibility of incurring a transaction costs. Once consumers select their consumption pattern, they will continue to follow it ad infinitum. Thus the equilibrium aggregate market shares are essentially static, product differentiation in durable-goods market becomes almost the same as that in search-goods markets with only minor modifications. The resulting pricing game practically becomes a mechanism design problem with one agent and either one or two principals in monopoly and duopoly settings respectively. Leasing in our model turns out to be a tool to facilitate price discrimination. Following previous study done by Huang, Kuzyutin (2002) on the utilization of concurrent selling and leasing for competition, we consider further in this paper what will happen if the action space of the agents are reduced and show that concurrent leasing and selling would be the optimal setting in the monopoly market structure. In addition, we will be analyzing the comparative advantage of each approach (pure-selling, pure-leasing, and concurrent selling and leasing) in each consumption regions.

1 Introduction

Durable good market is quite different from perishable goods market in several aspects. First, consumer's actions may not be static, they might change their consumption preference every period due to the intertemporal link. Thus we cannot really have a static model to describe the consumer behaviors. In addition, there are more available actions than its counterparts (perishable goods). It does not make sense to talk about leasing option when we have perishable goods. However, leasing might be profitable if we are dealing with durable goods due to the intertemporal effect. We might also choose to just consume the used goods we obtain in the previous transaction. In order to stay competitive, producer must understand the relationship between new and used goods, and which policy (concurrent selling and leasing, pure selling, or pure leasing) to adopt that is most suitable. The standard approach of static games, where consumers just choose the type of goods and not the method of obtaining them is no longer sufficient in this case.

Recently, Huang and Kuzyutin (2002) proposed a framework for pricing differentiated product for durable goods market. In their work, they view consumer's decision process in a durable-goods market as a process of choosing an appropriate consumption pattern rather than choosing a single action at each time period. Thus it is possible to view the aggregate consumer behavior as approximately time independent. The producer in turn can treat this steady aggregate level of consumption pattern as a type of search good. Their work also generalizes the previous work of Huang, Yang and Anderson (2000) who proposes a framework for modeling a finitely durable goods monopoly with an explicit used goods market and transaction costs for selling used goods by generalizing the framework where there are 2 differentiated goods in the market.

In this paper, we are interested mainly in showing that the structure proposed by Huang and Kuzyutin (2002) induces a positive welfare effect for both consumers and produces in both economic settings (monopolist and duopolist). Here we consider a 2 stage game; the first stage the producer determines the products and the second stage is a Bertrand price competition among the producers who choose to enter the market at the first stage. We focus only at the price competition stage and leave the market structure and product choice to be exogenous. We assume that the entry barrier is very high and costly development of new products. Thus, we can view the steady limit model as a generalization of models of vertical differentiation in search-good markets.

In the next section, we cite the model and solution concept introduced in Huang and Kuzyutin (2002) work in order for the paper to be self contained. We also show how Incentive Compatibility manifests itself very naturally in the setting of the game. In the following sections, we present the results of pure leasing(Chapter 3) and pure selling(Chapter 4) situations and compare it with the results of the previous paper. We consider in both settings, the monopoly and duopoly equilibrium in one case where the two brands of goods are well differentiated and in another case where the two brands of goods are close substitutes. We then do an analysis of economic implications of all the cases considered(Chapter 5). We summarize our main findings of this work and outline new directions of research in the last section.

2 The Model and Solution Concept

Time, t, is measured discretely. There are potentially two brands of goods, A and B, on the markets. All new goods of the same brand are of equal quality and have a lifetime of two periods. All lease terms last one period.

Consumers: Consumers are assumed to be price-takers. They live forever. Their heterogeneity is represented by a type parameter θ, whose value is assumed to be anonymous for each consumer. In order to obtain results that can be expressed in closed forms, we further assume a uniform distribution for $\theta \in [0, 1]$. The total consumer-population is a constant and is normalized to unity.[1]

The actions a consumer can take at each period are defined as in their natural transactions. There are totally seven choices: $\{\mathrm{L}_A, \mathrm{N}_A, \mathrm{U}_A, \mathrm{L}_B, \mathrm{N}_B, \mathrm{U}_B, \mathrm{I}\} \equiv \mathcal{A}$, where L_A (L_B) stands for leasing a new good of brand A (B) for one period and returning the used good to the lessor at the lease-end, N_A (N_B) for purchasing a new good of brand A (B) for its entire life span (as people do in real life), and U_A (U_B) for purchasing a used good of brand A (B) for its remaining life span (one period). Prices associated with these actions are defined analogously. For example, r_A is the price for action L_A, q_A is the price for action N_A and s_A is the price for action U_A. When a consumer inherits a used good from an earlier transaction, U_i is also used to denote the action of consuming a used brand "i" without paying additional price. The action "I" stands for not participating in the market or inaction for one period.

On the other hand, it is more convenient to define physical utilities as flows, i.e. on a per period basis. Normalizing the physical utility for consuming the entire life span of brand A to 1 for consumer $\theta = 1$, the following functional forms for the physical utility flows are chosen:

$$\begin{array}{lll} v_A^N(\theta) = (1-\delta)\theta, & v_A^U(\theta) = \delta\theta, & v_A^L(\theta) = v_A^N(\theta), \\ v_B^N(\theta) = \lambda(1-\delta)\theta, & v_B^U(\theta) = \lambda\delta\theta, & v_B^L(\theta) = v_B^N(\theta)\,. \end{array} \tag{1}$$

This choice implies that all consumers have the same ordering in their consumption preferences, a common characteristic in vertical differentiation models. The exogenous parameters, λ and δ, characterize the relative qualities of the new and used goods. We restrict $\lambda \in (0, 1)$ and $\delta \in (0, 1/2)$, reflecting the chosen conventions that brand A is more desirable for any given consumer than brand B of the same vintage, and that a new good is more desirable than a used one of the same brand. The full per period payoff matrix for consumer θ with state s and action a, $\Pi_\theta[a, s]$, is given in Table 1. The transaction cost α comes in only when a consumer needs to sell a used good.

Producers: Fixed costs are sunk, and hence will not have any effect on our analysis. No capacity constraints are imposed, and no used goods are scrapped. Marginal costs for brand A and B are denoted by $c_A \in (0, 1)$ and $c_B \in (0, \lambda)$, respectively. Producers are assumed to be the lessors of their own products for simplicity. They need to incur a disposal cost, β, to dispose each returned off-lease good in the used-goods market. It is natural to expect that a producer is more efficient in disposing a used good than any individual consumer. This is because producers have better market information, easier market access and larger economy of scale. In addition, they usually have better reputation, and hence suffer less from adverse selection problems. We further expect that all transaction costs are relatively small. Therefore, the following conditions should hold

$$0 < \beta < \alpha \ll 1\,. \tag{2}$$

[1]This normalization amounts to ignoring an overall constant factor that can depend on the values of quality parameters, λ and δ, to be introduced later. Since we treat the goods as exogenous this convenient normalization is appropriate.

Table 1: *The payoff matrix for consumer θ at period t:* $\Pi_\theta[a,s]$

$a\backslash s$	N_A^{t-1}	N_B^{t-1}	L_A^{t-1}, U_A^{t-1}, L_B^{t-1}, U_B^{t-1}, I^{t-1}
N_A^t	$v_A^N(\theta)-q_A^t+s_A^t-\alpha$	$v_A^N(\theta)-q_A^t+s_B^t-\alpha$	$v_A^N(\theta)-q_A^t$
U_A^t	$v_A^U(\theta)$	$v_A^U(\theta)-s_A^t+s_B^t-\alpha$	$v_A^U(\theta)-s_A^t$
L_A^t	$v_A^L(\theta)-r_A^t+s_A^t-\alpha$	$v_A^L(\theta)-r_A^t+s_B^t-\alpha$	$v_A^L(\theta)-r_A^t$
N_B^t	$v_B^N(\theta)-q_B^t+s_A^t-\alpha$	$v_B^N(\theta)-q_B^t+s_B^t-\alpha$	$v_B^N(\theta)-q_B^t$
U_B^t	$v_B^U(\theta)-s_B^t+s_A^t-\alpha$	$v_B^U(\theta)$	$v_B^U(\theta)-s_B^t$
L_B^t	$v_B^L(\theta)-r_B^t+s_A^t-\alpha$	$v_B^L(\theta)-r_B^t+s_B^t-\alpha$	$v_B^L(\theta)-r_B^t$
I^t	$s_A^t-\alpha$	$s_B^t-\alpha$	0

There are three parts in the revenue stream associated with each brand: sale of new goods, lease of new goods, and sale of returned off-lease goods.[2] These three parts in turn depend on the aggregate consumer behaviors in the current and previous periods. Let $P_A^L(t)$ ($P_B^L(t)$) and $P_A^N(t)$ ($P_B^N(t)$) denote the market shares of consumers who lease and purchase new goods of brand A (B) at time t. Then the profit functions associated with brand A and B at time t are given by

$$\Pi_A(t)=(q_A^t-c_A)P_A^N(t)+(r_A^t-c_A)P_A^L(t)+(s_A^t-\beta)\,P_A^L(t-1)\,, \tag{3}$$

$$\Pi_B(t)=(q_B^t-c_B)P_B^N(t)+(r_B^t-c_B)P_B^L(t)+(s_B^t-\beta)\,P_B^L(t-1)\,. \tag{4}$$

Rules of the Game: As in any standard game, all the players in our model are assumed to be rational and to maximize their own net present values (NPV) with a discount factor $0\le\rho\le1$. Except the anonymity of θ, all other information is common knowledge. We limit the strategy space for all consumers to be pure Markov strategies.[3]

In a durable-goods market it is natural to assume that prices can be adjusted much more easily than product attributes. Therefore, the control variables for producers at each period are the prices for new goods, q_A^t, r_A^t, q_B^t and r_B^t. We assume that the used-goods markets are competitive, and thus the prices for used goods, s_A^t and s_B^t, will be determined by the clearance conditions that equalize the demand and supply for each brand of used goods at all times.

The timing of the game in the monopoly setting is such that the monopolist picks new sales and lease prices for both brands at the beginning of each period. These prices are immediately announced to consumers. The timing of the game in the duopoly setting is similar, except that the two producers play a Bertrand pricing game at the beginning of each period, contingent on all the actions in the preceding period. The outcome of this Bertrand pricing game is then announced immediately. Consumers, who are price takers, play the game strategically against the producer(s), but not against each other. Each

[2]It will be shown in the next section that no consumer sells any used good in the steady limit.

[3]Behavior strategies, or mixed strategies, are not expected to play a significant role in the model, because the number of consumers who happen to be indifferent has measure zero. Therefore, we do not consider these strategies.

consumer's action at time t depends only on his own action in the previous period and the announced q_A^t, r_A^t, q_B^t and r_B^t. The aggregate consumer actions and the used-good prices need to be consistent with the clearance conditions of the used-goods markets.

The above process is repeated forever. An infinite time horizon will allow us to analyze the long run behaviors of the producer(s) and consumers, and to alleviate the potential adverse effects of otherwise artificially specified terminal conditions.

Solution Concept: The solution concept in the monopoly setting is identical to the one developed in Huang, Yang and Anderson (2000). It involves a Markov perfect equilibrium, within which a general equilibrium is embedded at every period. In a duopoly setting this concept needs to be generalized, since the game no longer belongs to the class of sequential games with perfect information. Fortunately, this generalization merely requires us to replace the backward induction used in Huang, Yang and Anderson (2000) by the generalized backward induction.[4] The generalized backward induction can handle dynamic games with stage games involving imperfect information, such as simultaneous moves by some players in the game. The end result of this generalization is a pair of Bellman equations for producers, instead of a single Bellman equation in the monopoly case, whose solutions characterize the best responses of the two producers to each other, as well as the best reaction functions of the producers to consumers' actions in the preceding period.

Because our interest in this paper is at the long run limit we will not pursue along the line of Huang, Yang and Anderson (2000) to derive the full Bellman equations. Instead, we will simply write down the Bellman equation for consumer θ at the steady limit. Given all the prices, the reaction function for consumer θ is solved from

$$R_\theta[s] = \arg\max_{a\in\mathcal{A}} \{\Pi_\theta[a,s] + \rho\, V_\theta[a]\}\ , \qquad \forall s \in \mathcal{A}\,, \tag{5}$$

where $V_\theta[s]$ is the NPV function for consumer θ at state s. This is a finite dimensional dynamic programming problem and is known to possess a unique solution.[5]

Following a similar line of argument in Huang, Yang and Anderson (2000) one can show that the optimization problems for producers at the steady limit become static, after consumers' time-consistency in choosing their consumption patterns has been properly taken into account. So, we can use a procedure to solve the equilibrium at the steady limit that is essentially the same as in a one-shot game.

Closed-form expressions for all interesting quantities, such as the equilibrium prices, market shares, profits and equations that characterize appropriate validity regions, can be derived straightforwardly, at least using some software packages that can perform symbolic manipulation such as Mathematica.[6] Unfortunately, except for some special cases, these expressions are too lengthy and are not particularly revealing. So, we will present explicit results only when they are sufficiently concise. Otherwise, we will merely

[4] A good discussion of the generalized backward induction can be found in Chapter 9.B of Mas-Colell, Whinston and Green (1995).

[5] See, for example, Bertsekas (1995).

[6] We will be glad to provide the Mathematica codes upon request through email.

provide definitions starting from which final results can be obtained by following standard procedures; and rely on a combination of graphic and verbal descriptions to convey the main points.

3 Pure Selling Case

In this section, we consider the case where firms only offer new goods for sale. Thus the actions a consumer can take at each period is restricted to 3 choices $\{\mathrm{N}_A, \mathrm{U}_A, \mathrm{N}_B, \mathrm{U}_B, \mathrm{I}\} \equiv \mathcal{A}$, where they are all defined as previously stated. We can view this case almost as perishable good case since we assume that the individual transaction cost is so high that sale of used new goods are not possible and thus there is no market for used goods being created. This assumption will in turn create a model very similar to perishable goods. This case will serve as a basis for our comparison in the economic implications since the model reduces to previous well known perishable goods model.

3.1 Consumer Incentive Compatibility

Consumers' individual rationality is guaranteed since we have an inaction with zero utility flow in the action set.

Lemma 1 *At a steady limit, the undominated consumption patterns are one of those composed by infinitely repeating one of the following three basic patterns.*

$$\{N_A U_A,\ N_B U_B,\ II\} \tag{6}$$

for an arbitrary consumer θ.

A consumer θ would be better off to play one of these three patterns than any other. This lemma further implies that no used goods are sold by any individual consumer. The proof of this lemma follows automatically from Lemma 1 in Huang and Kuzyutin (2002) paper.

It can be recognized that the requirement of $(1-\rho)\,\frac{\partial V_\theta[\cdot]}{\partial \theta} \geq 0$ is nothing but an endogenized sorting condition with a continuum type-variable and a discrete action-variable. Therefore it is also valid for general $\lambda \in (0,1)$, $\delta \in (0, 1/2)$ and $\rho \in [0,1]$, which implies that the task of pricing in durable-goods markets with heterogeneous consumers is practically a mechanism design problem. Look at Table 2 for the ordering of physical utility.

Table 2: *Ordering of average physical utility flows in terms of consumption patterns.*

Region	Ordering of average physical utility flows	Relationship between δ and λ
(A $\succ\!\!\succ$ B)	$\mathrm{N}_A\mathrm{U}_A > \mathrm{N}_B\mathrm{U}_B > \mathrm{II}$	All λ, δ

3.2 Equilibrium Solution in Region (A $\succ\!\succ$ B)

3.2.1 Consumer Behavior

Consumer average surplus function for the 3 consumption classes can be constructed as

$$\begin{cases} \tilde{V}_\theta[\mathrm{N}_A\mathrm{U}_A] = (\theta - q_A)/2\,, & \text{when} \quad \theta \in (\theta_1, 1) \quad \text{or} \quad \theta \in \mathrm{N}_A\mathrm{U}_A \\ \tilde{V}_\theta[\mathrm{N}_B\mathrm{U}_B] = (\lambda\theta - q_B)/2\,, & \text{when} \quad \theta \in (\theta_2, \theta_1) \quad \text{or} \quad \theta \in \mathrm{N}_B\mathrm{U}_B \\ \tilde{V}_\theta[\mathrm{II}] = 0\,, & \text{when} \quad \theta \in (0, \theta_2) \quad \text{or} \quad \theta \in \mathrm{II} \end{cases} \tag{7}$$

The class-division points are given by

$$\begin{cases} \theta_1 = (q_A - q_A)/(1-\lambda), \\ \theta_2 = q_B/\lambda, \end{cases} \tag{8}$$

as long as q_A, q_B are such that the ordering

$$0 \leq \theta_2 \leq \theta_1 \leq 1 \tag{9}$$

is maintained. These points are found by setting the surplus function of adjacent regions to be equal.

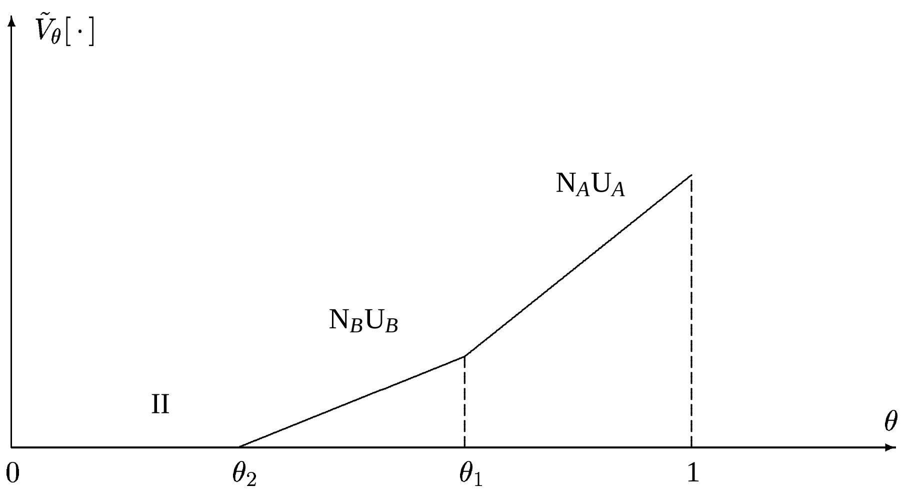

Figure 1: *Consumers' surplus functions as functions of θ in region (A $\succ\!\succ$ B).*

Depending on the value of the prices, some constraints may become binding leading to zero market shares for those particular consumption pattern classes. However, we will still have the ordering in the class-division points as dictated by the endogenized sorting condition. The price associated with such class cannot exist in a market in the sense of Kuhn-Tucker condition when solving for the equilibrium.

3.2.2 Monopoly Pricing

In this section, we assume both brands of goods are produced by the same firm. So we end up with a mechanism design problem with one principal one agent problem. The total profit of the monopolist is given by the sum of Eq.(3) and Eq.(4) under the special constraint of the no new goods being offered. In order to ensure Incentive Compatibility, monopolist maximize his profit based on the ordering on Eq. (9).

Table 3: *Sets of consumption-patterns that can be induced potentially.*

Label	Set of consumption-patterns	Corresponding multipliers
S4	$\mathrm{N}_A\mathrm{U}_A, \mathrm{N}_B\mathrm{U}_B, \mathrm{II}$	$\eta_A^N = 0, \eta_B^N = 0$
S6	$\mathrm{N}_A\mathrm{U}_A, \mathrm{II}$	$\eta_A^N > 0, \eta_B^N = 0$
S8	$\mathrm{N}_B\mathrm{U}_B, \mathrm{II}$	$\eta_A^N = 0, \eta_B^N > 0$
S16	II	$\eta_A^N > 0, \eta_B^N > 0$

Now we end up with profit optimization problem with a quadratic objective function and linear constraints since we have a uniform distribution of θ. We apply the Kuhn-Tucker method to solve this. The Lagrangian of the problem is as follows:

$$\mathcal{L}_M = \Pi_A + \Pi_B + \eta_A^N(1-\theta_1) + \eta_B^N(\theta_1 - \theta_2)\,, \tag{10}$$

with all η's non-negative. The following proposition summarizes the result of explicit maximization.

Proposition 1 *There are 4 feasible sets of consumption patterns that are actually induced by the monopolist in region (A $\succ\!\succ$ B). These sets are separated by straight lines in the (c_A, c_B)-plane. When prevailing on the market the two optimal selling prices are given by the corresponding monopoly prices: $q_A^* = (1+c_A)/2$ and $q_B^* = (c_B+\lambda)/2$. The optimal market shares are given by $P_A^N = (-1+c_A-c_B+\lambda)/(2(1+\lambda))$ and $P_B^N = (c_B - c_A\lambda)/(-2\lambda+2\lambda^2)$ These holds when monopolist offers to sell brand A and B concurrently. All prices and market shares are continuous and piecewise linear in c_A and c_B.*

To see how the induced consumption-pattern sets and their boundaries are derived, readers are encouraged to consult Huang and Kuzyutin (2002) work.

For an intuitive feel of how the consumption patterns are arranged on the (c_A, c_B)-plane, we graphically show the boundaries that separate various consumtion sets at $\lambda = 0.5$, $\delta = 0.4$ and $\beta = 0.02$. The transitions between each consumption sets can be understood by closer examination of how the prices vary as a function of one cost with another cost fixed. In Fig.2 with $c_B = 0.15$, brand B is not sufficiently competetitive relative to A when c_A is low in region S6. So it is optimal for monopolist not to consequently sell new goods B in this region. As c_A increases, we get to region S4 where it is profitable for monopolist to sell both goods A and B. However, as c_A increases, since we are keeping c_B constant, we will get similar effect as S6 in S8 where monopolist will stop offering goods A. S16 (not visible in upper right hand corner) is described as the set where both good cost are sufficiently high that it is not profitable for the monopolist to offer any of them. As c_A increases, q_A^* have to be increased. When c_A enters S4, selling brand B becomes profitable, and the monopolist makes profits from both brands. Beyond S8, q_A^* becomes too high and selling brand A becomes unprofitable in S8.

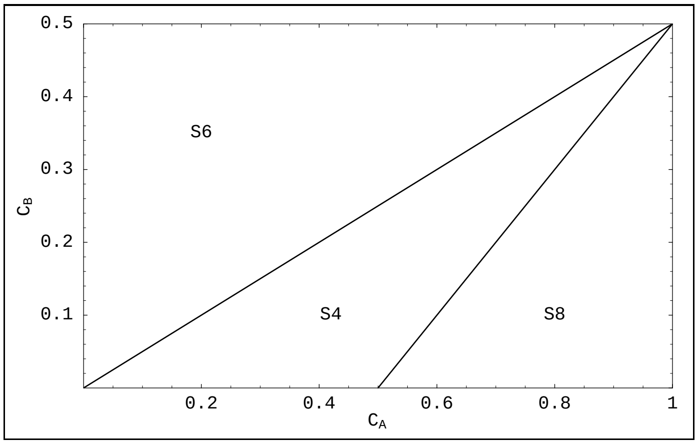

Figure 2: *Sets of consumption patterns induced by the monopolist in the* (c_A, c_B)*-plane. Quality parameters are in region (A* $\succ\!\succ$ *B):* $\lambda = 0.5$, $\delta = 0.4$*, and the disposal cost is* $\beta = 0.02$.

The previous figure also indicates that the cost of one brand has strong impact to its own prices but has little influence on the prices of another brand. This persists across all consumption sets. One can show that similar conclusion can be drawn if we fix c_A instead. This shows that in this setting, monopolist can price the two brands independently as if the other brands does not exits. Thus as long as the brands are well separated, we can extend the intuition in search good markets to durable goods market. Reader can consult Fig.3 for profit and price on different consumption region.

3.3 Duopoly Pricing

In this subsection, assume that brand A is produced by firm A and brand B is produced by firm B. Thus we have a Bertrand price game in the steady limit. However, the demand curve is endogenuously defined and satisfy Incentive Compatibility condition. Thus we have a mechanism design problem with two principal one agent problem. We need to formulate the response function and equate the response function of all the producers to obtain the Nash equilibrium

$$\{Q_A(q_B)\} \in \arg\max_{q_A} \{\Pi_A[q_A, q_B] + \eta_A^N(1-\theta_1) + \eta_B^N(\theta_1 - \theta_2)\} \ , \tag{11}$$

$$\{Q_B(q_A)\} \in \arg\max_{q_B} \{\Pi_B[q_A, q_B] + \psi_A^N(1-\theta_1) + \psi_B^N(\theta_1 - \theta_2)\} \ . \tag{12}$$

The best response for each region are defined by

$$Q_A(q_B) = \begin{cases} (1 + c_A - \lambda + q_B)/2\,, & \text{if } \eta_A^N = 0,\ \eta_B^N = 0, \\ q_B/\lambda, & \text{if } \eta_A^N = 0,\ \eta_B^N > 0, \end{cases} \tag{13}$$

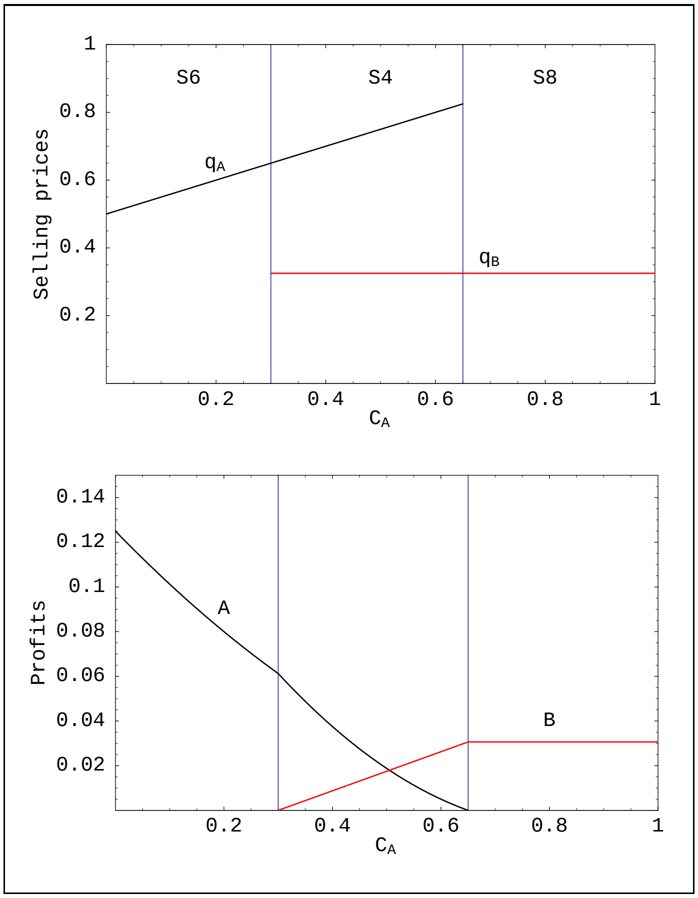

Figure 3: *Optimal monopoly prices and profits as functions of* c_A *at fixed* $c_B = 0.15$*. Quality parameters are in region (A* $\succ\!\succ$ *B):* $\lambda = 0.5$, $\delta = 0.4$, *and the disposal cost is* $\beta = 0.02$.

$$Q_B(q_A) = \begin{cases} (c_B + \lambda q_A)/2\,, & \text{if } \psi_A^N = 0,\ \psi_B^N = 0, \\ -1 + \lambda + q_A\,, & \text{if } \psi_A^N > 0,\ \psi_B^N = 0, \end{cases} \tag{14}$$

One can show that these response function are continuous when crossing boundaries between feasible regions. Given that those respond function represent the best action you can take as a function as the other party action, the intersection of those 2 respond function would define a Nash equilibrium. In addition, depending on what the marginal costs of production, the producers might sometime find it advantageous to phase out certain consumption patterns.

By comparing the profit and price of the monopoly and duopoly cases, we can see that both are lower in the duopoly market structure than in the monopoly since the surviving products or firms need to resist the entry pressure from the other brand by setting the price lower than the monopoly values and hence forcing the other brand out of the market. However, this does not hold true in the region S6 and S8 as shown in Fig.4 because we actually have pure monopoly here since the price that the other brand/firm will charge if they enter at that region will be below their marginal cost and thus will not ever happen. Thus this reduces to the monopoly case.

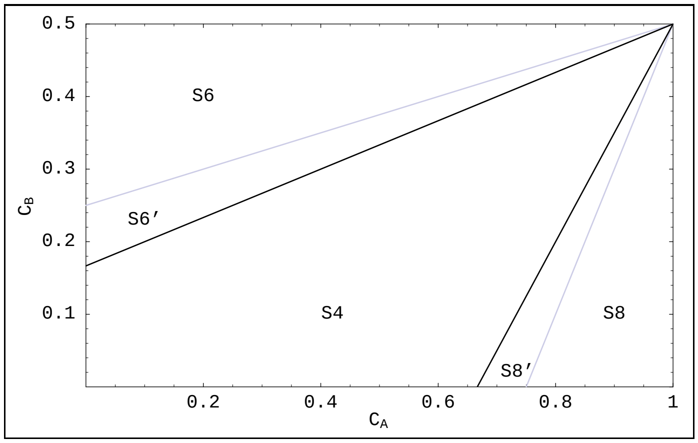

Figure 4: *Sets of consumption patterns induced by the duopolist in the* (c_A, c_B)*-plane. Quality parameters are in region (A* $\succ\!\succ$ *B):* $\lambda = 0.5$*,* $\delta = 0.4$*, and the disposal cost is* $\beta = 0.02$*.*

As can be observed from the figure, when the brands are well differentiated, the cost of one brand in duopoly setting has some impact on the price of the other brand, though not as strong as on its own price. The strength of the effect depends on how close the brand is being pushed out of the market either in the pure monopoly sense or in the monopoly where the surviving brand needs to enforce entry pressure.

4 Pure Leasing Case

In this section, we consider the situation where new goods are not offered by the producers. Instead, they only offer the choice to lease their goods and they will sell the goods as used when the lease ends at the succesive periods. Thus the actions a consumer can take at each period in restricted to 5 choices $\{\mathrm{L}_A, \mathrm{U}_A, \mathrm{L}_B, \mathrm{U}_B, \mathrm{I}\} \equiv \mathcal{A}$, where they are all defined as previously stated.

4.1 Consumer Incentive Compatibility

Consumer rationality is obtained automatically since we have a zero utility flow and zero price action in the action set.

Lemma 2 *At a steady limit, the undominated consumption patterns are one of those composed by infinitely repeating one of the following seven basic patterns.*

$$\{L_AL_A,\ U_AU_A,\ L_BL_B,\ U_BU_B,\ II\} \tag{15}$$

for an arbitrary consumer θ*.*

A consumer θ would be better off to play one of these five patterns than any other. This lemma further implies that no used goods are sold by any individual consumer. The proof of this lemma follows automatically from Lemma 1 in Huang and Kuzyutin (2002) paper. We notice that the payoff table is

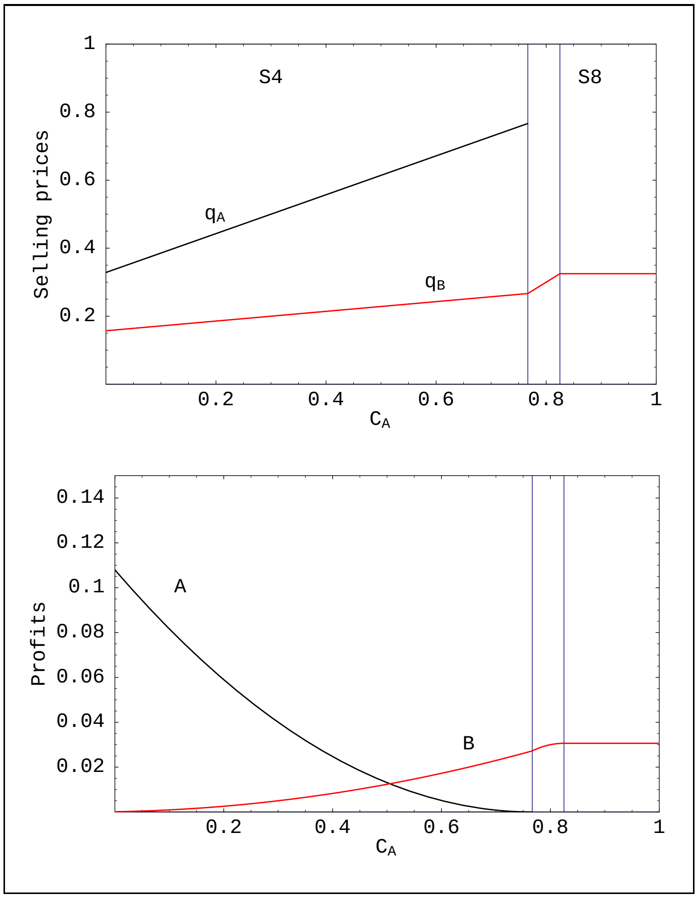

Figure 5: *Equilibrium duopoly prices and profits as functions of c_A at fixed $c_B = 0.15$. Quality parameters are in region (A $\succ\!\!\succ$ B): $\lambda = 0.5$, $\delta = 0.4$, and the disposal cost is $\beta = 0.02$.*

simplified since the process becomes independent, i.e. the action we took at each state is independent of the previous state. We end up having only the last column of the payoff table. Thus the previous lemma follows automatically.

It can be recognized that the requirement of $(1-\rho)\,\frac{\partial V_\theta[\cdot]}{\partial\theta} \geq 0$ is nothing but an endogenized sorting condition with a continuum type-variable and a discrete action-variable. Therefore it is also valid for general $\lambda \in (0,1)$, $\delta \in (0,1/2)$ and $\rho \in [0,1]$, which implies that the task of pricing in durable-goods markets with heterogeneous consumers is practically a mechanism design problem.

Table 4: *Ordering of physical utility flows in terms of consumption patterns.*

Region	Ordering of physical utility flows	Relationship between δ and λ
(A $\succ\!\!\succ$ B)	$\mathrm{L}_A\mathrm{L}_A > \mathrm{U}_A\mathrm{U}_A > \mathrm{L}_B\mathrm{L}_B > \mathrm{U}_B\mathrm{U}_B > \mathrm{II}$	$\delta > \lambda/(1+\lambda)$
(A $\sim$ B)	$\mathrm{L}_A\mathrm{L}_A > \mathrm{L}_B\mathrm{L}_B > \mathrm{U}_A\mathrm{U}_A > \mathrm{U}_B\mathrm{U}_B > \mathrm{II}$	$0 < \delta < \lambda/(1+\lambda)$

4.2 Equilibrium Solution in Region (A $\succ\!\succ$ B)

4.2.1 Consumer Behavior

Consumer surplus function for the 5 consumption classes can be constructed as

$$\begin{cases} \tilde{V}_\theta[\mathrm{L}_A\mathrm{L}_A] = (1-\delta)\theta - r_{\scriptscriptstyle A}\,, & \text{when} \quad \theta \in (\theta_1, 1) \quad \text{or} \quad \theta \in \mathrm{L}_A\mathrm{L}_A \\ \tilde{V}_\theta[\mathrm{U}_A\mathrm{U}_A] = \delta\theta - s_{\scriptscriptstyle A}\,, & \text{when} \quad \theta \in (\theta_2, \theta_1) \quad \text{or} \quad \theta \in \mathrm{U}_A\mathrm{U}_A \\ \tilde{V}_\theta[\mathrm{L}_B\mathrm{L}_B] = \lambda(1-\delta)\theta - r_{\scriptscriptstyle B}\,, & \text{when} \quad \theta \in (\theta_3, \theta_2) \quad \text{or} \quad \theta \in \mathrm{L}_B\mathrm{L}_B \\ \tilde{V}_\theta[\mathrm{U}_B\mathrm{U}_B] = \lambda\delta\theta - s_{\scriptscriptstyle B}\,, & \text{when} \quad \theta \in (\theta_4, \theta_3) \quad \text{or} \quad \theta \in \mathrm{U}_B\mathrm{U}_B \\ \tilde{V}_\theta[\mathrm{II}] = 0\,, & \text{when} \quad \theta \in (0, \theta_4) \quad \text{or} \quad \theta \in \mathrm{II} \end{cases} \tag{16}$$

The class-division points are given by

$$\begin{cases} \theta_1 = (r_{\scriptscriptstyle A} - s_{\scriptscriptstyle A})/[1-2\delta], \\ \theta_2 = (s_{\scriptscriptstyle A} - r_{\scriptscriptstyle B})/[\delta - \lambda(1-\delta)], \\ \theta_3 = (r_{\scriptscriptstyle B} - s_{\scriptscriptstyle B})/[\lambda(1-2\delta)], \\ \theta_4 = (s_{\scriptscriptstyle B})/[\lambda\delta], \end{cases} \tag{17}$$

as long as $r_{\scriptscriptstyle A}$, $s_{\scriptscriptstyle A}$, $r_{\scriptscriptstyle B}$ and $s_{\scriptscriptstyle B}$ are such that the ordering

$$0 \le \theta_4 \le \theta_3 \le \theta_2 \le \theta_1 \le 1 \tag{18}$$

is maintained. These points are found by setting the surplus function of adjacent regions to be equal.

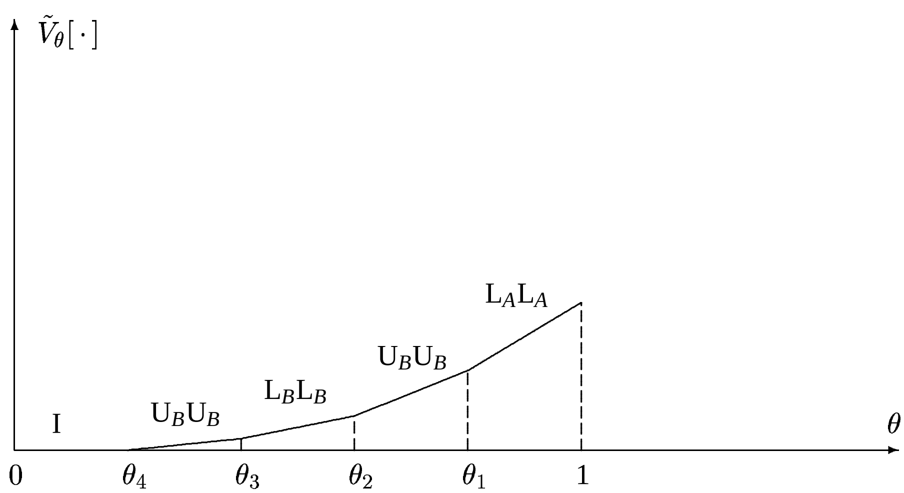

Figure 6: *Consumers' surplus functions as functions of θ in region (A $\succ\!\succ$ B).*

Depending on the value of the prices, some constraints may become binding leading to zero market shares for those particular consumption pattern classes. However, we will still have the ordering in the class-division points as dictated by the endogenized sorting condition. The price associated with such class cannot exist in a market in the sense of Kuhn-Tucker condition when solving for the equilibrium.

4.2.2 Used-goods Market

Given the preference ordering one can obtain the market shares of various consumption patterns,

$$P_A^L = 1 - \theta_1\,, \quad P_A^U = \theta_1 - \theta_2\,, \quad P_B^L = \theta_2 - \theta_3\,, \quad P_B^U = \theta_3 - \theta_4\,. \tag{19}$$

We know that the population of used goods will entirely come from the population of leased goods. Thus from the Market Clearing condition we get

$$1 - \theta_1 = \theta_1 - \theta_2\,, \qquad \theta_2 - \theta_3 = \theta_3 - \theta_4\,. \tag{20}$$

One can now solve for for the used-goods prices from these 2 conditions, yielding

$$s_A^*(r_A, r_B) = \frac{r_B - 2r_B\delta + (-1 + 2r_A + 2\delta)(\delta - \lambda + \delta\lambda)}{1 + 2(-1+\delta)\lambda}\,, \tag{21}$$

$$s_B^*(r_A, r_B) = \frac{\delta(\lambda(1 + 4\delta^2 + 4\delta(-1 + r_A) - 2r_A - 2r_B) + 2r_B)}{1 + 2(-1+\delta)\lambda}\,, \tag{22}$$

4.2.3 Monopoly Pricing

In this section, we assume both brands of goods are produced by the same firm. So we end up with a mechanism design problem with one principal one agent problem. The total profit of the monopolist is given by the sum of Eq.(4) and Eq.(5) under the special constraint of the no new goods being offered. The market shares are given by Eq.(10). The two used good prices can now be eliminated by Eq.(12) and Eq.(13) so that the total profit is an explicit function of the lease good price. In order to ensure Incentive Compatibility, monopolist maximize his profit based on the ordering on Eq. (9).

Table 5: *Sets of consumption-patterns that can be induced potentially.*

Label	Set of consumption-patterns	Corresponding multipliers
S11	$\mathrm{L}_A\mathrm{L}_A, \mathrm{U}_A\mathrm{U}_A, \mathrm{L}_B\mathrm{L}_B, \mathrm{U}_B\mathrm{U}_B$, II	$\eta_A^L = 0, \eta_B^L = 0$
S14	$\mathrm{L}_A\mathrm{L}_A, \mathrm{U}_A\mathrm{U}_A$, II	$\eta_A^L > 0, \eta_B^L = 0$
S15	$\mathrm{L}_B\mathrm{L}_B, \mathrm{U}_B\mathrm{U}_B$, II	$\eta_A^L = 0, \eta_B^L > 0$
S16	II	$\eta_A^L > 0, \eta_B^L > 0$

Now we end up with profit optimization problem with a quadratic objective function and linear constraints since we have a uniform distribution of θ. We apply the Kuhn-Tucker method to solve this. The Lagrangian of the problem is as follows:

$$\mathcal{L}_M = \Pi_A + \Pi_B + \eta_A^L(1 - \theta_1) + \eta_B^L(\theta_1 - \theta_2)\,, \tag{23}$$

with all η's non-negative. Since we impose Market Clearing condition, we automatically get the market shares of used goods to be non-negative. The following proposition summarizes the result of explicit maximization.

Proposition 2 *There are 4 feasible sets of consumption patterns that are actually induced by the monopolist in region (A $\succ\!\!\succ$ B). These sets are separated by straight lines in the (c_A, c_B)-plane. The optimal selling prices are solved similarly in each region. All prices and market shares are continuous and piecewise linear in c_A and c_B.*

To see how the induced consumption-pattern sets and their boundaries are derived, readers are encouraged to consult Huang and Kuzyutin (2002) work.

For an intuitive feel of how the consumption patterns are arranged on the (c_A, c_B)-plane, we graphically show the boundaries that separate various consumtion sets at $\lambda = 0.5$, $\delta = 0.4$ and $\beta = 0.02$. The transitions between each consumption sets can be understood by closer examination of how the prices vary as a function of one cost with another cost fixed. In Fig.7 with $c_B = 0.15$, brand B is not sufficiently competetitive relative to A when c_A is low in region S14. So it is optimal for monopolist not to lease and consequently sell used goods B in this region. As c_A increases, we get to region S11 where it is profitable for monopolist to offer lease (and sell) of both goods A and B. However, as c_A increases, since we are keeping c_B constant, we will get similar effect as S14 in S15 where monopolist will stop offering goods A. S16 (not visible in upper right hand corner) is described as the set where both good cost are sufficiently high that it is not profitable for the monopolist to offer any of them. As c_A increases, r_A^* have to be increased. When c_A enters S11, selling brand B becomes profitable, and the monopolist makes profits from both brands. Beyond S15, r_A^* becomes too high and leasing brand A becomes unprofitable in S15.

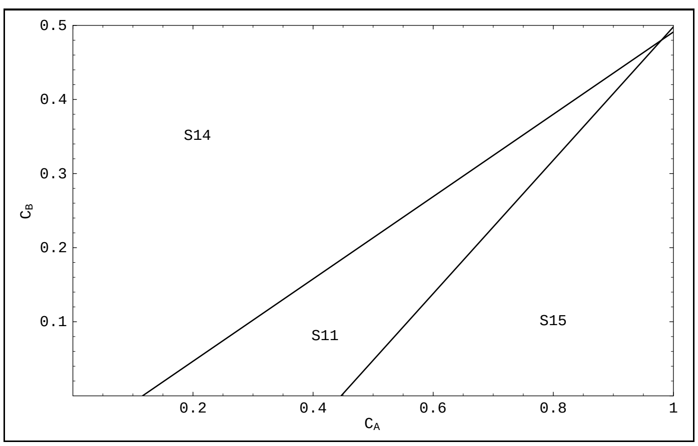

Figure 7: *Sets of consumption patterns induced by the monopolist in the (c_A, c_B)-plane. Quality parameters are in region (A $\succ\!\!\succ$ B): $\lambda = 0.5$, $\delta = 0.4$, and the disposal cost is $\beta = 0.02$.*

The previous figure also indicates that the cost of one brand has strong impact to its own prices but has little influence on the prices of another brand. This persists across all consumption sets. One can show that similar conclusion can be drawn if we fix c_A instead. This shows that in this setting, monopolist can price the two brands independently as if the other brands does not exits. Thus as long as the brands are well separated, we can extend the intuition in search good markets to durable goods market.

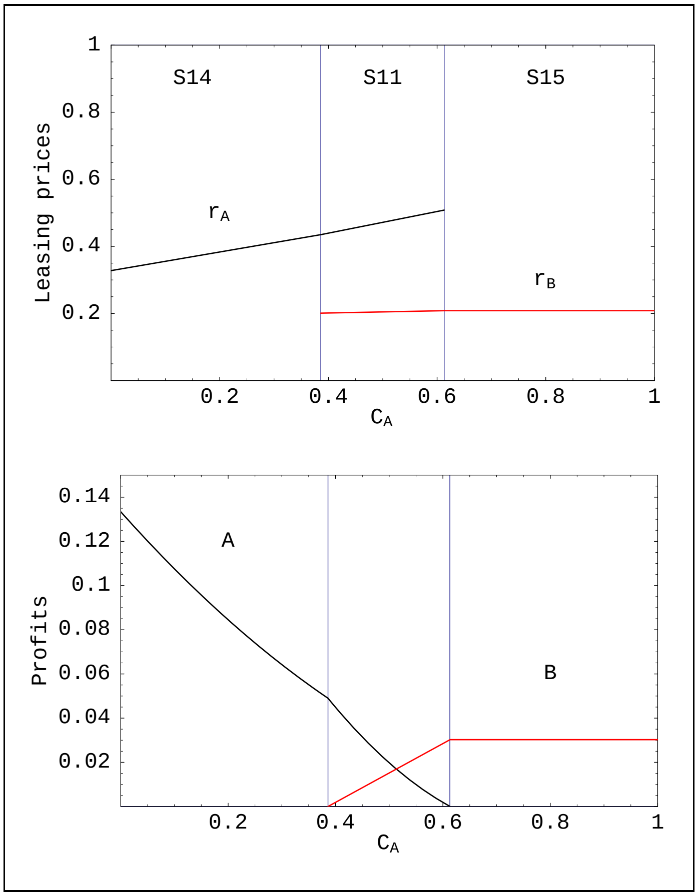

Figure 8: *Optimal monopoly prices and profits as functions of* c_A *at fixed* $c_B = 0.15$. *Quality parameters are in region (A* $\succ\!\!\succ$ *B):* $\lambda = 0.5$, $\delta = 0.4$, *and the disposal cost is* $\beta = 0.02$.

4.3 Duopoly Pricing

In this subsection, assume again that brand A is produced by firm A and brand B is produced by firm B. Thus we also have a Bertrand price game in the steady limit. However, the demand curve is endogenuously defined and satisfy Incentive Compatibility condition. Thus we have a mechanism design problem with two principal one agent problem. We need to formulate the response function and equate the response function of all the producers to obtain the Nash equilibrium

$$\{Q_A(r_B)\} \in \arg\max_{r_A} \{\Pi_A[r_A, r_B] + \eta^L_A(1-\theta_1) + \eta^L_B(\theta_1 - \theta_2)\} \ , \tag{24}$$

$$\{Q_B(r_A)\} \in \arg\max_{r_B} \{\Pi_B[r_A, r_B] + \psi^L_A(1-\theta_1) + \psi^L_B(\theta_1 - \theta_2)\} \ . \tag{25}$$

You can also calculate the response function for each region similarly. In addition, we can also show that the response function is continuous even when crossing consumption regions.

Given that those respond function represent the best action you can take as a function as the other party action, the intersection of those 2 respond function would define a Nash equilibrium. In addition,

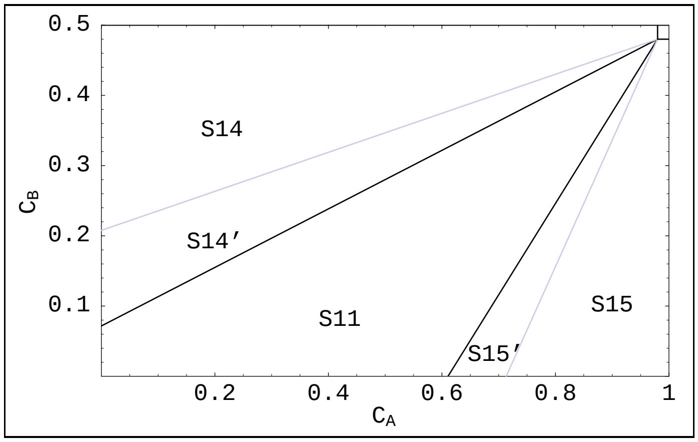

Figure 9: *Sets of consumption patterns induced by the duopolist in the* (c_A, c_B)*-plane. Quality parameters are in region (A* $\succ\!\succ$ *B):* $\lambda = 0.5$*,* $\delta = 0.4$*, and the disposal cost is* $\beta = 0.02$*.*

depending on what the marginal costs of production, the producers might sometime find it advantageous to phase out certain consumption patterns.

By comparing the profit and price of the monopoly and duopoly cases, we can see that both are lower in the duopoly market structure than in the monopoly since the surviving products or firms need to resist the entry pressure from the other brand from the other brand by setting the price lower than the monopoly values and hence forcing the other brand out of the market. However, this does not hold true in the region S14 and S15 as shown in Fig.9 because we actually have pure monopoly here since the price that the other brand/firm will charge if they enter at that region will be below their marginal cost and thus will not ever happen. Thus this reduces to the monopoly case.

As can be observed from the figure, when the brands are well differentiated, the cost of one brand in duopoly setting has some impact on the price of the other brand, though not as strong as on its own price. The strength of the effect depends on how close the brand is being pushed out of the market either in the pure monopoly sense or in the monopoly where the surviving brand needs to enforce entry pressure.

5 Economic Implication

5.1 Monopoly Market Structure

5.1.1 Relationship between Producer Surplus and Consumer Surplus in Monopoly Market Structure

Proposition 3 *The producer surplus is exactly twice of the consumer surplus in monopolist market structure.*

The consumer surplus function is essentially a piecewise linear function and thus can be computed exactly by trapezoidal rule on the region below the curve. We also know the explicit formula for the profit function of each region. We present the case for pure leasing here. The pair of producer and consumer surplus is as

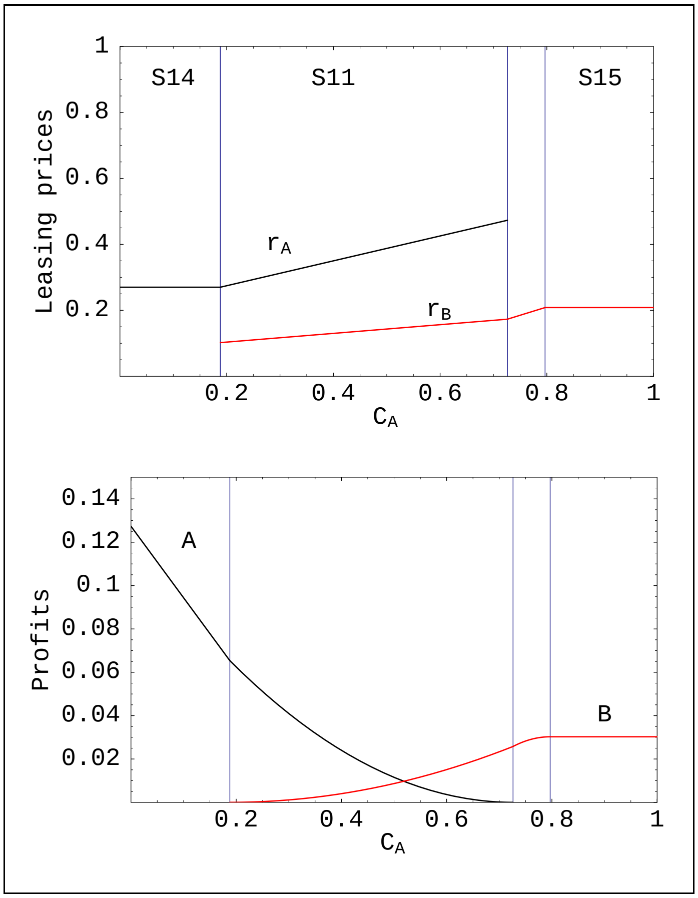

Figure 10: *Optimal duopoly prices and profits as functions of c_A at fixed $c_B = 0.15$. Quality parameters are in region (A $\succ\!\!\succ$ B): $\lambda = 0.5$, $\delta = 0.4$, and the disposal cost is $\beta = 0.02$.*

follows $((-1+\beta_A+c_A)^2/(4+8\delta),(-1+\beta_A+c_A)^2/(8+16\delta))$ for S2, $((\beta_B+c_B-\lambda)^2/(4(\lambda+2\delta\lambda)),(\beta_B+c_B-\lambda)^2/(2(\lambda+2\delta\lambda))$ for S3. As we can see, there is a factor 2 in it. One can show that similar conclusion can be made for other cases and all the regions.

5.1.2 Profits under Monopoly Market Structure

As observed in Fig.11 , we have a single crossing point in the profit function of the pure leasing and pure selling case. When we hold the producer transaction cost and one of the goods price to be fixed, as the other goods price increased, depending on what the value of the transaction cost β is, we might encounter the case where pure selling will dominate the pure leasing strategy. The intuition is beacuse when the transaction cost is fixed and one of the good cost goes sufficiently high, the advantage to segment the consumer consumption pattern in pure leasing scenario is lost. The transaction cost affects the profitability of offering used cars from the leasing by the producers. However, in certain interval of the transaction cost, the producer will end up being more profitable by just offering new cars. We can derive the condition

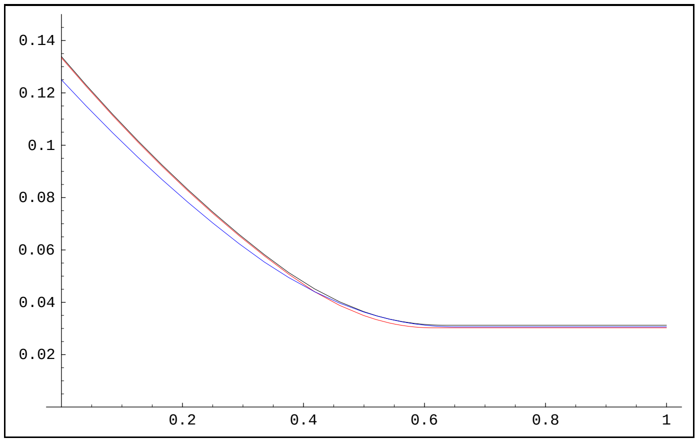

Figure 11: *Monopoly producer profit in Pure Leasing(Blue), Pure Selling(Red) and Concurrent Leasing and Selling(Black)as functions of* c_A *at fixed* $c_B = 0.15$. *Quality parameters are in region (A* $\succ\!\succ$ *B):* $\lambda = 0.5$, $\delta = 0.4$, *and the disposal cost is* $\beta = 0.02$.

for such cases by considering the extrema points of the profit functions. When brand A is not offered, the profit function for the pure selling case is $(\lambda - c_B)^2/(8\lambda)$ and that of the pure leasing case is $(\lambda - c_B - \beta_B)^2/(4\lambda(1+2\delta))$. When brand B is not offered, the profit function for the pure selling case is $(1-c_A)^2/8$ and that of the pure leasing case is $(1 - c_A - \beta_A)^2/(4(1+2\delta))$. Thus the condition on the the β in the region where only one of the brands exist in order for leasing to be better than selling are

$$\beta < (\lambda - c_B)(1 - \sqrt{(1+2\delta)/2} = \bar{\beta}_B \tag{26}$$

$$\beta < (1 - c_A)(1 - \sqrt{(1+2\delta)/2} = \bar{\beta}_A \tag{27}$$

If such condition are violated, we will get single crossing of profit function between pure leasing and pure selling strategy since we will not have pure leasing to be strictly better than pure selling.

By applying the previous lemma, we can conclude similarly that we will have single crossing for the consumer surplus function since it possess the same functional form but with a constant factor multipluer with producer surplus.

5.1.3 Comparison between Pure Selling, Pure Leasing and Selling,Leasing Strategy in Monopoly Market Structure

Looking at Fig.16, we can conclude several things. First of all, leasing is definitely a way to better price discriminate or to segregate consumer consumption patter. However, to achieve that better price discrimination, we need to incur additional disposal or transaction cost (β in this case).Depending on the relative magnitude of the margin, we can get higher or lower price discrimination power from offfering the lease option.

If we look at the blue line, the left hand corner indicates the region where leasing has the most price discrimination and thus it is higher than 1, i.e. it is better than just selling. As marginal costs for brand

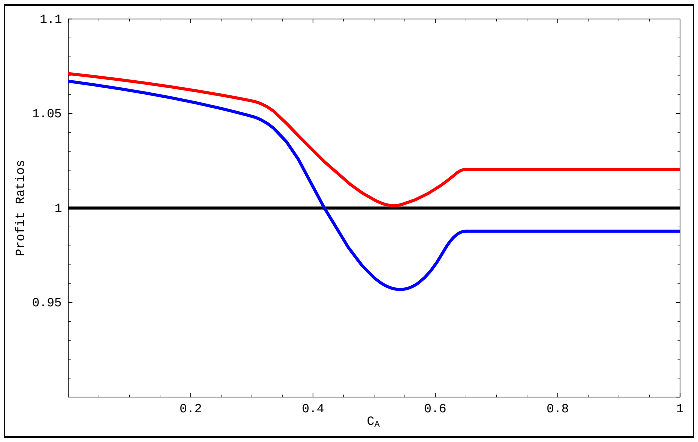

Figure 12: $Monopoly : Red = ConcurrentLeasingSellingProfit/PureSellingProfit; Blue = PureLeasingProfit/PureSellingProfit; Black = PureSellingProfit/PureSellingProfit$

A increases (we fix marginal cost of B), the power decreases until it reaches below 1, i.e. worse than just selling. This is due to the relationship of disposal cost and marginal cost of B derived earlier. Since it is more than the critical value, we expect it to the pure leasing to be taken over by pure selling on some regions where both brands A and B coexist. In this graph, this will be the middle region. The boundary of the middle region vary depending on which consumption region you are on and thus not drawn. We can think of the middle region as the region where the 2 lines start to move quite differently. We can think of the middle local minimum as the point where consumers are indifferent of brand A and B in the sense of net utility. Although consuming brand A is quite high in utility, the cost of producing A is also more costly so we can think of it as a point of indifference between those 2 brands. The right hand side is the region where brands B survives in monopoly and since we fix the cost of B, when brand A died, we expect a straight line there.

If we look at the red line, the left hand corner indicates the region where mostly everyone is leasing since leasing has the most power there so having an additional option to sell does not improve your profit by much. It will continue to decrease as we move the right since the marginal cost of A will increase and thus reduces profit. At the indifference point, the leasing power will be at the minimum point by the previous argument and thus having additional option of leasing will not bring your profit much higher than just pure selling case. If we continue to move to the right, we expect to see straight line by the same argument as before.

5.2 Duopoly Market Structure

5.2.1 Analysis of Consumer Surplus versus Producer Profit in Pure Leasing Scenario

As we can see from Fig.13,consumer surplus is higher that aggregate profits due to intensified competition between A and B. When both products/producers coexist in the market both the producer profit and consumer surplus goes down. We can view the transition from left to right as the decrease of the product

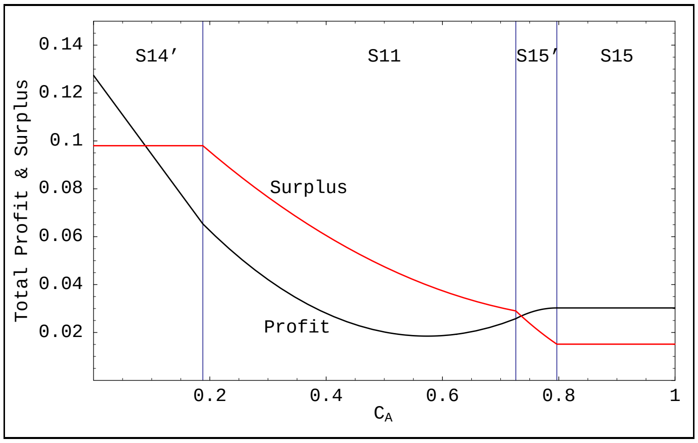

Figure 13: *Optimal Duopoly consumer surplus and producer profits as functions of c_A at fixed $c_B = 0.15$. Quality parameters are in region (A $\succ\!\succ$ B): $\lambda = 0.5$, $\delta = 0.4$, and the disposal cost is $\beta = 0.02$.*

quality (in the sense of marginal cost of production). However the profit goes down even faster than consumer surplus due to competition in this area. In the tails we have profits higher than consumer surplus due to monopoly position of either A or B.

We do not have the region S14 in this case due to the numerical parameter we choose to graph the figure. If we do have both regions of S14, what happens there is producer A forces B out of the market thus the increase of brand A marginal cost does not affect the surplus, however it affects only the profit.

In region S15', we have producer B forcing A out and thus we expect to see a crossover in this area between the surplus and profit function as it is approaching a pure monopolistic region of B (in S15) where again we expect straight line in both surplus and profit since if product A is out of the market, further increase in marginal cost of A will not have any effect.

5.2.2 Analysis of Consumer Surplus in Duopoly Market Structure

Consumer surplus functions have expected non-increasing shape. However, flat portions of surplus functions in case of pure leasing and concurrent leasing and selling comes from the fact that firm A who tries to keep dominant position in the market involuntarily decreases the price-marginal cost ratio so consumers stay unaffected. Sudden drop of the surpluses in the right portion of the graph is explained by firm B's domination of firm A and thus offering the monopoly prices to consumers that in turn decreases their surplus considerably. After B gains the dominant position in the market and A is not participating any more, increase in cost A does not affect the price of B's product thus consumer surplusis unaffected. Flat portion of the curve at the left side is longer for concurrent leasing and selling meaning that firm A is able to keep its dominant position longer.

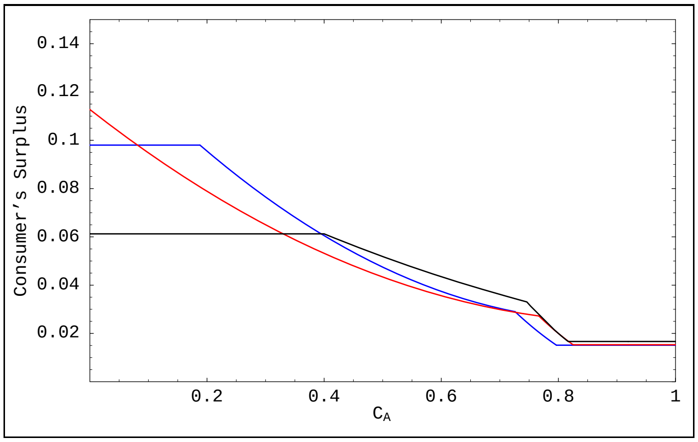

Figure 14: *Duopoly consumer surplus in Pure Leasing(Blue), Pure Selling(Red) and Concurrent Leasing and Selling(Black) as functions of* c_A *at fixed* $c_B = 0.15$. *Quality parameters are in region (A* $\ggg$ *B):* $\lambda = 0.5$, $\delta = 0.4$, *and the disposal cost is* $\beta = 0.02$.

5.2.3 Analysis if Producer Profit in Duopoly Market Structure

Looking at Fig.15 one can see that total industry profits decrease as cost of producing A is increasing since the firm A's share in bigger that firm B's and after reaching certain minimum point total industry profits start to raise again since firm A's dominant position is weakened and B's profits start to dominate. Eventually total industry profits turn to be flat since A not being in the market B's behavior is not changed by cost of A and consumer surplus stays unchanged. Dip in the middle of the graph is due to intensified competition between firm A and B.

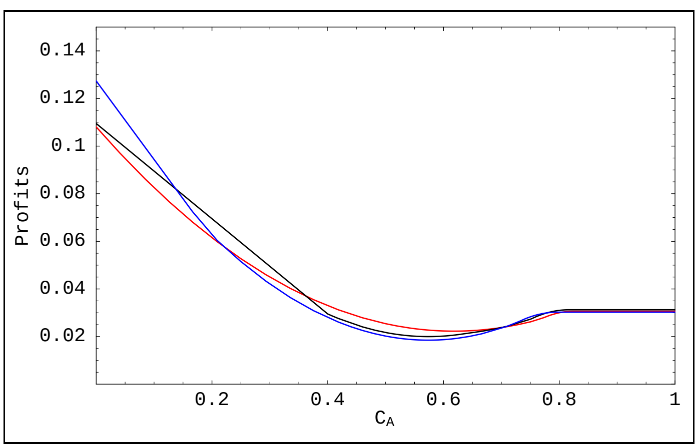

Figure 15: *Duopoly producer profit in Pure Leasing(Blue), Pure Selling(Red) and Concurrent Leasing and Selling(Black) as functions of* c_A *at fixed* $c_B = 0.15$. *Quality parameters are in region (A* $\ggg$ *B):* $\lambda = 0.5$, $\delta = 0.4$, *and the disposal cost is* $\beta = 0.02$.

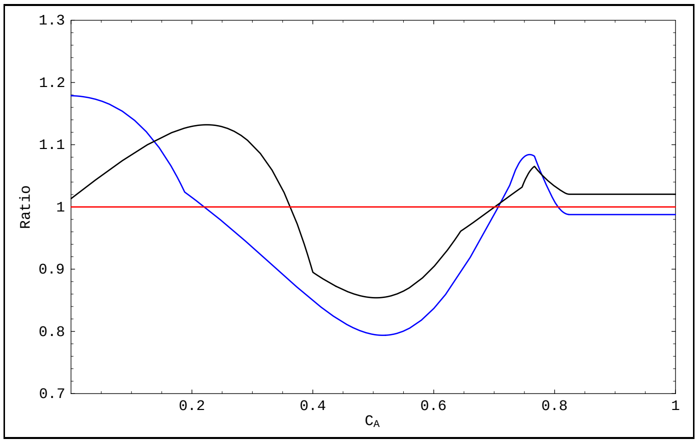

Figure 16: $Duopoly : Red = ConcurrentLeasingSellingProfit/PureSellingProfit; Blue = PureLeasingProfit/PureSellingProfit; Black = PureSellingProfit/PureSellingProfit$

6 Conclusion and Future Direction

In this paper we utilized the framework used Huang and Kuzyutin (2002) for determining the equilibrium prices for duopoly and monopoly case when firms have an option of just leasing or just selling. Then we derived the corresponding profit and consumer surplus functions. Making comparisons revealed some interesting features of interaction between different agents of durable goods markets. We have only considered for discussion the representative cases throughout the paper. Full understanding of complex features requires more analysis which is beyond the scope of this paper. We will be glad to provide the full Mathematica codes to interested reader who wants to have more general idea.

Several interesting directions for future work has emerged while working on this paper. One can add another stage to the game we have considered in this paper where before playing the price competition stage, producers choose the level of investment that affect their marginal cost in the next stage. Or choose the level of investment that affects the quality and hence the demand for their products. Or choosing the capacity level in the first stage which becomes a constraint in the subsequent periods.

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