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EULER, JACOBI, AND MISSIONS TO COMETS AND ASTEROIDS.

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ABSTRACT

Whenever a freely spinning body is found in a complex rotational state, this means that either the body experienced some interaction within its relaxation-time span, or that it was recently “prepared” in a non-principal state. Both options are encountered in astronomy where a wobbling rotator is either a recent victim of an impact or a tidal interaction, or is a fragment of a disrupted progenitor. Another factor (relevant for comets) is outgassing. By now, the optical and radar observational programmes have disclosed that complex rotation is hardly a rare phenomenon among the small bodies. Due to impacts, tidal forces and outgassing, the asteroidal and cometary precession must be a generic phenomenon: while some rotators are in the state of visible tumbling, a much larger amount of objects must be performing narrow-cone precession not so easily observable from the Earth.

The internal dissipation in a freely precessing top leads to relaxation (gradual damping of the precession) and sometimes to spontaneous changes in the rotation axis. Recently developed theory of dissipative precession of a rigid body reveals that this is a highly nonlinear process: while the body is precessing at an angular rate ω , the precession-caused stresses and strains

in the body contain components oscillating at other frequencies. Dependent upon the spin state, those frequencies may be higher or, most remarkably, lower than the precession rate. In many states dissipation at the harmonics is comparable to or even exceeds that at the principal frequency.

For this and other reasons, in many spin states the damping of asteroidal and cometary wobble happens faster, by several orders, than believed previously. This makes it possible to measure the precession-damping rate. The narrowing of the precession cone through the period of about a year can be registered by the currently available spacecraft-based observational means. We propose an appropriate observational scheme that could be accomplished by comet and asteroid-aimed missions. Improved understanding of damping of excited rotation will directly enhance understanding of the current distribution of small-body spin states. It also will constrain the structure and composition of excited rotators.

However, in the near-separatrix spin states a precessing rotator can considerably slow down its relaxation. This lingering effect is similar to the one discovered in 1968 by Russian spacecraft engineers who studied free wobble of a tank with viscous fuel.

I. PROLEGOMENA

In 1730, 23-year-old lieutenant Leonhard Euler retired from the Russian navy, to become a professor of physics at the Russian Academy of Sciences. Eleven years later he interrupted his tenure, to assume the post of Director of Mathematics and Physics at the Berlin Academy, offered to him by King Friedrich the Second. There he stayed until 1765 when he was invited back to St.Petersburg by the enlightened Empress Catherine the Great. It is during that 25-year-long Berlin period of his life that Euler made his major contributions to the mechanics of a rotating body. After publishing some prefatory results in 1750 and 1758, Euler wrote down, in 1760, his now celebrated equations describing free spin of an unsupported top with arbitrary moments of inertia I_1, I_2, I_3 :

$$I_i \dot{\Omega}_i - (I_j - I_k) \Omega_j \Omega_k = \tau_i \quad . \quad (1.1)$$

$\Omega_{1,2,3}$ being the the angular-velocity components in the coordinate system defined by the three principal axis of inertia, (1, 2, 3). For a freely rotating body, the external torques $\tau_{1,2,3}$ standing in the right-hand side are nil. Without loss of generality, one may assume that $I_3 \geq I_2 \geq I_1$, and thus, always to consider axis (3) to be the major-inertia axis.

This result was published only five years later, in Chapter X of Euler's book *Theoria motus corporum solidorum seu rigidorum ex primis nostrae cognitionis principiis stabilita et ad omnes motus qui in huiusmodi corpora cadere possunt accomodata*.

In Chapter XI, Euler easily found the solution for a prolate symmetrical case¹: if $I_3 = I_2 > I_1$ (like, for example, in the case of an elongated rod), the vector of inertial angular velocity Ω describes a circular cone about the minor-inertia axis (1) of the body:

$$\Omega_1 = const \quad , \quad \Omega_2 = \Omega_{\perp} \cos \omega t \quad , \quad \Omega_3 = \Omega_{\perp} \sin \omega t \quad , \quad (1.2)$$

$\omega = (I_1/I_3 - 1)\Omega_1$ being the precession rate. It is possible to show that the angular-momentum vector \mathbf{J} behaves like Ω , i.e., precesses about axis (1) at the same angular rate. This picture

¹Later the symmetric top was also treated by Lagrange (1788) and Poisson (1813).

of Ω and \mathbf{J} precessing about axis (1) is the one seen by an observer associated with the body frame. In an inertial observer's opinion, things will look different: from his viewpoint the angular momentum \mathbf{J} of an unsupported top will conserve, while its angular velocity Ω and the minor-inertia axis (1) will be precessing about \mathbf{J} .

Similarly, in the case of oblate symmetry ($I_3 > I_2 = I_1$) both Ω and \mathbf{J} will perform, in the body frame, a circular precession about the major-inertia axis (3):

$$\Omega_1 = \Omega_{\perp} \cos \omega t, \quad \Omega_2 = \Omega_{\perp} \sin \omega t, \quad \Omega_3 = \text{const} \quad (1.3)$$

where $\omega = (I_3/I_1 - 1)\Omega_3$. In an inertial frame, though, it will be Ω and axis (3) describing circular cones about \mathbf{J}

In Chapter XIII Euler tackled the general case of $I_3 > I_2 \geq I_1$ and solved it in terms of functions presently known as elliptic integrals. These were pioneered a century earlier by John Wallis and Isaac Newton, and went in the late XVIII - early XIX centuries under the name of elliptic functions². (Euler, though, used neither of these names, and did not refer to Wallis or Newton.) Nowadays the name "elliptic" belongs to functions sn , cn , dn and their kin, that were not known at the time of Euler. They were introduced by Karl Jacobi in 1829 (Jacobi 1829), studied by Legendre (1837), and later employed (Jacobi 1849, 1882) in the rotating-top studies. These functions are, in a way, generalisations of our customary trigonometric functions: while for symmetric prolate and oblate bodies the circular precession is expressed by (1.2) and (1.3) correspondingly, in the general case $I_3 \geq I_2 \geq I_1$ the solution will read:

$$\Omega_1 = \gamma \, dn(\omega t, k^2), \quad \Omega_2 = \beta \, sn(\omega t, k^2), \quad \Omega_3 = \alpha \, cn(\omega t, k^2), \quad (1.4)$$

for $\mathbf{J}^2 < 2 I_2 T_{kin}$, and

$$\Omega_1 = \tilde{\gamma} \, cn(\tilde{\omega} t, \tilde{k}^2), \quad \Omega_2 = \tilde{\beta} \, sn(\tilde{\omega} t, \tilde{k}^2), \quad \Omega_3 = \alpha \, dn(\tilde{\omega} t, \tilde{k}^2) \quad (1.5)$$

for $\mathbf{J}^2 > 2 I_2 T_{kin}$. Similarity between (1.2) and (1.4), as well as between (1.3) and (1.5), is evident. In the above expressions, the precession rate ω and the parameters

²See, for example, <http://www-groups.dcs.st-andrews.ac.uk/history/HistTopics/>

$\alpha, \beta, \tilde{\beta}, \gamma, \tilde{\gamma}, \tilde{\omega}, k$ and \tilde{k} are some sophisticated combinations of $I_{1,2,3}, T_{kin}$ and \mathbf{J}^2 . What is important, is that solution (1.4) approaches (1.2) in the limit of prolate symmetry, $(I_3 - I_2)/I_1 \rightarrow 0$, while solution (1.5) approaches (1.3) in the limit of oblate symmetry, $(I_2 - I_1)/I_1 \rightarrow 0$. To adumbrate in an illustrative manner the applicability realms of (1.4) and (1.5), let us turn to Figure 1. In the course of free spin, two quantities (integrals of motion) are conserved. One is the angular momentum

$$\mathbf{J}^2 = I_1^2 \Omega_1^2 + I_2^2 \Omega_2^2 + I_3^2 \Omega_3^2, \quad (1.6)$$

another is the kinetic energy

$$T_{kin} = \frac{1}{2} \{ I_1 \Omega_1^2 + I_2 \Omega_2^2 + I_3 \Omega_3^2 \}. \quad (1.7)$$

Evidently, these two expressions define ellipsoids in the angular-velocity space $(\Omega_1, \Omega_2, \Omega_3)$. Intersection of these two surfaces will be the trajectory described by vector $\boldsymbol{\Omega}$ in the said space. On Figure 1, the angular-momentum ellipsoid is depicted. On its surface, we have marked the lines of its intersection with several different kinetic-energy ellipsoids appropriate to different energies. It does not take much space imagination to understand that, for a fixed angular-momentum surface (1.6), there exist an infinite family of kinetic-energy surfaces (1.7) intersecting with it. The largest surface of kinetic energy (corresponding to the maximal value of T_{kin}) is an ellipsoid that fully encloses our angular-momentum ellipsoid and only touches it in point A and its opposite. Similarly, the smallest surface of kinetic energy (corresponding to minimal T_{kin}) would be an ellipsoid fully contained inside our angular-momentum ellipsoid and only touching it from inside, at point C and its opposite. It is easy to demonstrate that, for a fixed \mathbf{J} , the maximal and minimal possible values of the kinetic energy are achieved during rotations about the minimal-inertia and maximal-inertia axes, correspondingly. It can also be shown, from (1.1), that in the case of a non-dissipative and torque-free rotation, the tip of the

FIGURES

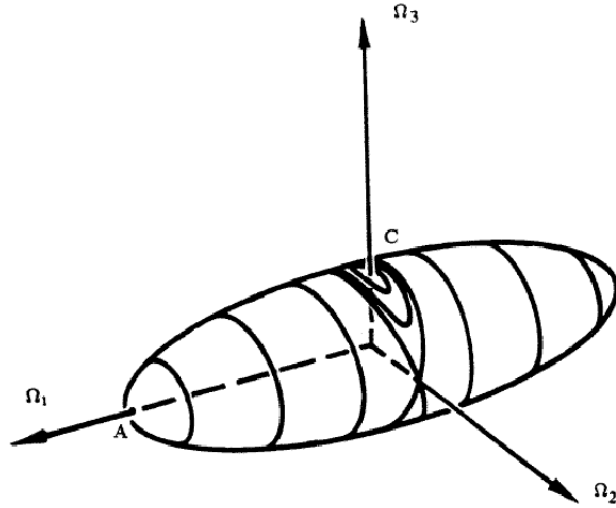


FIG. 1. The constant-angular-momentum ellipsoid, in the angular-velocity space. The lines on its surface are its intersections with the kinetic-energy ellipsoids corresponding to different values of the rotational energy. The quasi-stable pole A is the maximal-energy configuration, i.e., the state wherein the body spins about its minimal-inertia axis. The stable pole C symbolises the minimal-energy state, i.e., rotation about the maximal-inertia axis. The angular-velocity vector describes the constant-energy lines, and at the same time slowly shifts from one line to another, approaching pole C. The picture illustrates the case of an elongated body: $I_3 \gtrsim I_2 > I_1$. The trajectories are circular near A and remain (in the case of an elongated body) virtually circular almost up to the separatrix. The trajectories will regain a circular shape only in the closest proximity of C.

vector Ω will be describing, on Figure 1, a curve along which the angular-momentum and energy ellipsoids intersect (Lamy & Burns 1972). Hence, these intersections may be called trajectories. Solution (1.4) is valid for higher energies, i.e., from pole A through the separatrix; solution (1.5) works for lower energies, i.e., from the separatrix through pole C. Wherever the trajectories are almost circular, the solutions (1.4) and (1.5) may be approximated by (1.2) and (1.3), correspondingly.

The formalism developed by Euler and refined by Jacobi might be a perfect tool for description of rotation of asteroids, comets, cosmic-dust granules, spacecrafts and whatever other unsupported rigid rotators, if not for one circumstance, inner dissipation. Because of this circumstance, the Euler-Jacobi theory of precession works only for time spans short enough to neglect dissipation. The presence of inner dissipation may be guessed even on a heuristic level. The bounded range of permissible energies makes one think that a freely spinning body of a fixed angular momentum must be seeking ways of relaxation, i.e., of getting rid of the excessive energy, in order to approach the minimal-energy configuration. Thence the necessity of some dissipation mechanism.

Two such mechanisms are known. One is relevant only for mesoscopic rotators, like interstellar-dust grains, and therefore plays a certain role in the cosmic-dust alignment. This is the Barnett dissipation, a phenomenon called into being by the periodic remagnetisation of a precessing paramagnetic body (Lazarian & Draine 1997).

The second mechanism, inelastic dissipation, is, too, relevant for mesoscopic grains (Lazarian & Efroimsky 1999), and it plays the decisive role in the macroscopic bodies' relaxation. The effect results from the alternating stresses produced inside a wobbling body by the time-dependent acceleration of its parts. The stresses deform the body, and the inelastic effects cause dissipation of the rotational energy.

The dissipation entails relaxation of the precession: the major-inertia axis of the body and its angular-velocity vector Ω tend to align along the angular momentum \mathbf{J} . In other words, the precession cone described by Ω about \mathbf{J} will be narrowing until Ω aligns along \mathbf{J}

completely. A simple calculation (Efroimsky 2001, Efroimsky 2000, Efroimsky & Lazarian 2000, Lazarian & Efroimsky 1999) shows that in this case the major-inertia axis of the body will align in the same direction, so that, from the body-frame viewpoint, Ω will eventually be pointing along this axis. This configuration will correspond to the minimal kinetic energy, the angular momentum being fixed.

An inertial observer will thus see the unsupported body miraculously changing its rotation axis. This is exactly what happened in 1958 when, to mission experts' surprise, satellite Explorer I changed its rotation axis. The spacecraft was a very elongated body. It had been supposed to spin about its least-inertia axis (i.e., about its longest dimension), but refused to do so, and instead started precessing (Thomson 1961).

This was probably the first example of a practical need for a further development of the Eulerian theory of a free top, a development that would address an unsupported top with dissipation. Another motivation for this work was put forward in the same year by Prendergast (1958) who studied the asteroid population of the Solar System and enquired as to how many asteroids could be in non-principal (i.e., precessing) spin states, and how this could evidence of the impact frequency in the main belt. (Prendergast implied that it is collisions that might drive asteroids out of the principal state and make them wobble.) An important point made by Prendergast was the generation of the second harmonic: if a body is precessing at an angular rate ω , then the dissipation is taking place not only at this frequency but also at double thereof. Prendergast failed to notice the emergence of the higher harmonics, but even his noticing of the second harmonic was an important observation. In several other aspects the mathematical treatment of the problem, offered by Prendergast, was erroneous and gave him no chance to come to a reasonable solution. Moreover, at that time the observational astronomy lacked any reliable data on wobbling asteroids. So, Prendergast's paper was forgotten (even though once in a while it appeared in the references), and his successors had to start up from scratch.

The interest in the asteroidal precession re-emerged in 70-s, after the publication of the important work (Burns & Safronov 1973) that suggested estimates for the relaxation time, based

on the decomposition of the deformation pattern into bulge flexing and bending, and also on the conjecture that “the centrifugal bulge and its associated strains wobble back and forth relative to the body as the rotation axis ω moves through the body during a wobble period.” As turned out later, the latter conjecture does not work, because the inelastic dissipation, for the most part of it, is taking place not near the surface but in the depth of the body, i.e., not right under the bulge but deep beneath it. Thus, the bulge is much like an iceberg tip. This became clear when the distribution of precession-caused stresses was calculated, with improved boundary conditions (Efroimsky & Lazarian 2000), (Lazarian & Efroimsky 1999). Another, main, problem of Burns & Safronov’s treatment was their neglect of the nonlinearity, i.e., of the second and higher harmonics. The nonlinearity, in fact, is essential. Neglect thereof leads to a large underestimation of the damping rate, because the leading effect comes often from the second and higher harmonics (Efroimsky & Lazarian 2000), (Efroimsky 2000). All in all, the neglect of nonlinearity and mishandling of the boundary conditions leads to a several-order underestimate of the precession-damping rate.

In the same year, Peale published an article dealing with inelastic relaxation of nearly spherical bodies (Peale 1973), and there he did take the second harmonic into account.

In 1979 Purcell addressed a similar problem of interstellar-grain precession damping. He ignored the harmonics and mishandled the boundary conditions upon stresses: in (Purcell 1979) the normal stresses had their maximal values on the free surfaces and vanished in the centre of the body (instead of being maximal in the centre and vanishing on the surfaces). These oversights lead to a several-order underevaluation of the dissipation effectiveness and, thereby, of the relaxation rate.

II. PRECESSION DAMPING

The dynamics of precession relaxation is described by the angular rate of alignment of the maximal-inertia axis (3) along the angular momentum \mathbf{J} , i.e., by the decrease in angle θ between these. In the case of oblate symmetry (when $I_3 > I_2 = I_1$), this angle remains

adiabatically unchanged over the precession period, which makes $d\theta/dt$ a perfect measure of the damping rate (Efroimsky & Lazarian 2000). However, in the general case of a triaxial body angle θ evolves periodically through the precession cycle. To be more exact, it evolves *almost* periodically, and its value at the end of the cycle is only slightly different from that in the beginning of the cycle. The relaxation is taking place through accumulation of these slight variations over many periods. This is called adiabatic regime, i.e., regime with two different time scales: we have a “fast” process (precession) and a “slow” process (relaxation). Under the adiabaticity assumption, one may average θ , or some function thereof, over the precession cycle. Then the damping rate will be described by the evolution of this average. Technically, it is convenient to use the average of its squared sine (Efroimsky 2000). One can write for a triaxial rotator:

$$\frac{d \langle \sin^2 \theta \rangle}{dt} = \frac{d \langle \sin^2 \theta \rangle}{dT_{kin}} \frac{dT_{kin}}{dt} , \quad (2.1)$$

while for an oblate one the expression will look simpler:

$$\left(\frac{d\theta}{dt} \right)_{(oblate)} = \left(\frac{d\theta}{dT_{kin}} \right)_{(oblate)} \frac{dT_{kin}}{dt} . \quad (2.2)$$

The derivatives $d \langle \sin^2 \theta \rangle / dT_{kin}$ and $(d\theta/dT_{kin})_{(oblate)}$ appearing in (2.1) and (2.2) indicate how the rotational-energy dissipation affects the value of $\langle \sin^2 \theta \rangle$ (or simply of θ , in the oblate case). These derivatives can be calculated from the equations of motion (see Efroimsky & Lazarian (2000) and Efroimsky (2000)). The kinetic-energy decrease, dT_{kin}/dt , is caused by the inelastic dissipation:

$$dT_{kin}/dt = \langle dW/dt \rangle , \quad (2.3)$$

W being the energy of the alternating stresses, and $\langle \dots \rangle$ denoting an average over a precession cycle. (This averaging is justified within our adiabatic approach.) Finally, in the general case of a triaxial top, the alignment rate will read:

$$\frac{d \langle \sin^2 \theta \rangle}{dt} = \frac{d \langle \sin^2 \theta \rangle}{dT_{kin}} \frac{d \langle W \rangle}{dt} , \quad (2.4)$$

and for a symmetrical oblate top:

$$\left(\frac{d\theta}{dt}\right)_{(oblate)} = \left(\frac{d\theta}{dT_{kin}}\right)_{(oblate)} \frac{d \langle W \rangle}{dt} . \quad (2.5)$$

Vibrations in the material, at frequency ω , are accompanied by dissipation

$$\frac{dW(\omega)}{dt} = \frac{2 \omega \langle W(\omega) \rangle}{Q(\omega)} \quad (2.6)$$

$\langle W(\omega) \rangle$ being the averaged-over-cycle elastic energy of deformation at frequency ω , and $Q(\omega)$ being the so-called quality factor of the body material. To calculate $\langle W(\omega) \rangle$, one has first to find the time-averaged elastic-energy density, $d \langle W(\omega) \rangle / dV$ (that can be calculated from the known picture of stresses and strains) and then to integrate it over the volume of the body. This yields:

$$\frac{dW(\omega)}{dt} = \frac{2 \omega}{Q(\omega)} \int \left(\frac{d \langle W(\omega) \rangle}{dV} \right) dV . \quad (2.7)$$

In a more refined approach, though, one should take into account the generation of harmonics and the inhomogeneity of the wobbling top:

$$\frac{dW}{dt} = \sum_n 2 \omega_n \int \frac{1}{Q(\omega_n)} \left(\frac{d \langle W(\omega_n) \rangle}{dV} \right) dV \quad (2.8)$$

where it is implied that Q depends not only upon the frequency but also upon the coordinates inside the body. The Q -factor is empirically introduced in seismology and acoustics in order to make up for our inability to describe the total effect of several attenuation mechanisms (Nowick and Berry 1972), (Burns 1986), (Burns 1977), (Knopoff 1963). These bear a complicated dependence upon frequency, but the overall frequency dependence of Q is usually very slow and smooth (except some narrow resonances revealing a local domination of one or another particular mechanism). The temperature dependence of Q is inseparably connected with the frequency dependence (see (Efroimsky & Lazarian 2000) and references therein). The dependence of Q upon the humidity (and upon the presence of some other saturants) still poses a challenge to material scientists. In many minerals, for example in silicate rocks, several monolayers of water may decrease Q by a factor of about 55 as described, for example, by Tittman, Ahlberg,

and Curnow (1976) who studied samples of the lunar rock³. Presumably, humidity and other saturants affect the inter-grain interactions in rocks.

III. THE ORIGIN OF THE NONLINEARITY

What is important about formulae (2.4), (2.5), is that, after (2.8) is plugged in, one can explicitly see the contributions to the entire effect, coming from the principal frequency ω_1 and from the harmonics $\omega_n \equiv n\omega_1$. When vector Ω describes approximately circular trajectories on Figure 1, the principal frequency ω_1 virtually coincides with the precession rate ω . This doesn't hold, though, when Ω get closer to the separatrix: there ω_1 becomes *lower* than the precession rate. The analysis of the stress and strain distributions, and the resulting expressions for $d \langle W \rangle / dt$ written down in (Efroimsky & Lazarian 2000) and (Efroimsky 2000) shows that the nonlinearity is essential, in that the generation of harmonics is not a high-order effect but a phenomenon playing a key role in the relaxation process. In other words, dissipation associated with the harmonics is often of the same order as that at the principal frequency. Near the separatrix it may be even higher.

The nonlinearity emerges due to the simple fact that the acceleration of a point within a wobbling object contains centrifugal terms that are quadratic in the angular velocity Ω . In neglect of small terms caused by the body deformation, the acceleration will read:

$$\mathbf{a} = \dot{\Omega} \times \mathbf{r} + \Omega \times (\Omega \times \mathbf{r}) \quad . \quad (3.1)$$

\mathbf{a} being the acceleration in the inertial frame, and \mathbf{r} being the position of a point. In the simplest case of oblate symmetry, the body-frame-related components of the angular velocity are expressed by (1.3) plugging whereof into (3.1) produces terms containing $\sin \omega t$ and $\cos \omega t$,

³The absence of moisture in the lunar rock (except, perhaps, in some local spots) is the reason that the moonquake echo propagates for long with almost no attenuation. It would be dissipated much faster, should the lunar rocks contain just a tiny amount of water.

as well as those containing $\sin 2\omega t$ and $\cos 2\omega t$. The alternating stresses and strains caused by this acceleration are linear functions of \mathbf{a} and, thus, will also contain the second harmonic, along with the principal frequency. Calculation of the stresses, strains, and of the appropriate elastic energy W is then only a matter of some elaborate technique. This technique (presented in (Efroimsky & Lazarian 2000) and (Efroimsky 2001)) leads to an expression for W , with contributions from ω and 2ω explicitly separated. The nonlinearity is essential: in many rotation states the 2ω input in (2.8) and (2.4) is of order and even exceeds that coming from the principal frequency ω . To explain in brief the reason why the nonlinearity is strong, we would mention that while the acceleration and the stresses and strains are quadratic in the (precessing) angular velocity Ω , the elastic energy is proportional to the product of stress and strain tensors. Hence the elastic energy is proportional to the fourth power of Ω .

IV. THE NEAR-SEPARATRIX SLOWING-DOWN OF THE PRECESSION (LINGERING EFFECT)

In the general case of a triaxial rotator, precession is described by (1.4) or (1.5). The acceleration of a point inside the body (and, therefore, the stresses and strains in the material) will, according to (3.1), contain terms quadratic in the Jacobi functions. These functions can be decomposed in converging series (the so-called nome expansions) over sin's and cos'ines of $n\nu$, n being odd integers for $sn(\omega t, k^2)$ and $cn(\omega t, k^2)$ and even integers for $dn(\omega t, k^2)$. Here ν is a frequency *lower* than the precession rate ω :

$$\nu = \omega \frac{2\pi}{4K(k^2)} \quad , \quad (4.1)$$

$4K(k^2)$ being the mutual period of sn and cn . Near the poles $\nu \rightarrow \omega$, while on approach to the separatrix $\nu \rightarrow 0$. When two such expansions get multiplied by one another, they produce a series containing all sorts of products like $(\sin m\nu t \sin n\nu t)$, $(\cos m\nu t \cos n\nu t)$, and cross terms. Hence the acceleration, stress and strain contain the entire multitude of overtones. Even though the further averaging of W over the precession cycle weeds out much of these terms, we

are eventually left with all the harmonics on our hands.

As explained in (Efroimsky 2000), higher-than-second harmonics will bring only high-order contributions to the precession-relaxation process when the rotation state is described by a point close to poles A or C. Put differently, it is sufficient to take into account only the frequencies $\nu \approx \omega$ and $2\nu \approx 2\omega$, insofar as the trajectories on Figure 1 are approximately circular (i.e., when (1.4) and (1.5) are well approximated by (1.2) and (1.3)). Near the separatrix the situation is drastically different, in that all the harmonics become important. We thus transit from the domain of essential nonlinearity into the regime of extreme nonlinearity, regime where the higher harmonics bring more in the process than ν or 2ν . We are reminded, however, that en route to the separatrix we not just get all the multiples of the principal frequency, but we face a change of the principal frequency itself: according to (4.1), the principal frequency ν will be lower than the precession rate ω ! This regime may be called “exotic nonlinearity”.

Without getting bogged down in rigorous mathematics (to be attended to in a separate paper), let me just mention here that in the limit of Ω approaching the separatrix the dissipation rate will vanish, in the adiabatic approximation. This may be guessed even from the fact that in the said limit $\nu \rightarrow 0$. We thus come to an important conclusion that the relaxation rate, being very high at a distance from the separatrix, decreases in its closest vicinity. Can we, though, trust that the relaxation rate completely vanishes on the separatrix? No, because in the limit of Ω approaching the separatrix the adiabatic approximation will fail. In other words, it will not be legitimate to average the energy dissipation over the precession cycle, because near the separatrix the precession rate will not necessarily be faster than the relaxation rate. A direct calculation shows that even on the separatrix itself the acceleration of a point within the body will remain finite (but will, of course, vanish at the unstable middle-inertia pole). The same can be said about stress, strain and the relaxation rate. So what we eventually get is not a near-separatrix trap but just lingering: one should expect relaxing tops to considerably linger near the separatrix. As for Explorer, it is now understandable why it easily went wobbling but did not rush to the minimal-energy spin state: it couldn't cross the separatrix so quickly. We would call

it "lingering effect". There is nothing mysterious in it. The capability of near-intermediate-axis spin states to mimic simple rotation was pointed out by Samarasinha, Mueller & Belton (1999) with regard to comet Hale-Bopp. A similar setting was considered by Chernous'ko (1968) who studied free precession of a tank filled with viscous liquid and proved that in that case the separatrix is crossed within a finite time interval⁴.

V. CONFIRMATION FROM NEAR'S OBSERVATIONS OF 433 EROS

A calculation (Efroimsky 2001) shows that the relaxation rate exponentially slows down at small residual angles: the narrower the precession cone the slower the relaxation rate. This means that many small bodies in the Solar System should have retained some residual narrow-cone precession. Their precession may be too narrow to be seen from the Earth, even by a radar. Still, a close monitoring from a spacecraft should reveal at least a weak residual wobble. This is especially the case for Eros: first, because it is a monolith rock and, second, because it is at the stage of leaving the main belt and is a Mars-crosser (so that its rotation was, most probably, sometimes disrupted by tidal forces or impacts through the past several millions of years).

Another good reason for Eros to retain some wobble is its almost perfect dynamic prolateness ($I_3 \approx I_2 \gg I_1$), prolateness that makes pole C on Figure 1 so tightly embraced by the separatrices. Even a weak interaction will push the tip of vector Ω across the separatrix towards pole A (Black et al 1999), while its relaxation-caused return to C will be slowed down by the aforementioned lingering effect.

Despite all these circumstances, Eros was found in a relaxed or almost⁵ relaxed spin state

⁴Such problems have been long known also to mathematicians studying the motion with a non-Hamiltonian perturbation: the perturbation wants the system to cross the separatrix, but is not guaranteed to succeed in it, because some trajectories converge towards the unstable pole (Neishtadt 1980)

⁵The word "almost" means that the half-angle of the precession cone does not exceed 0.1 angular degree. (Andrew Cheng, private communication.)

(Yeomans 2000). This means that the inelastic relaxation is indeed a very effective process, as predicted by our study. It is far more effective than believed previously⁶. In fact, judging by the absence of a visible residual wobble of Eros, there may be a possibility that the inelastic relaxation is even faster a process than our study has shown. Some probable physical reasons for this are discussed in (Efroimsky 2001).

VI. MISSIONS TO COMETS

The cometary wobble is caused by tidal forces or by outgassing (Samarasinha & Belton 1995). Preliminary estimates (Efroimsky 2001) show that precession damping of a comet may be registered within a one-year time span or so, provided the best available spacecraft-based devices are used and the comet's spin state is not too close to the separatrix. Several missions to comets, including wobbling comets, are currently being planned. CONTOUR spacecraft, to be launched in 2002, will fly by Encke, Schwassmann-Wachmann 3, and d'Arrest, but the encounters will be brief. The mission will observe the spin states, but not their evolution. Another mission, Stardust, that is to visit comet P/Wild 2, will also be a fly-by one. Spacecraft Deep Impact will approach comet 9P Tempel 1 on the 3 of July 2005, and will shoot a 500-kg impactor at 10 km/s speed, to blast a crater into the nucleus, to reveal its interior. Sadly, though, this encounter too will be short.

Rosetta orbiter is designed to approach 46/P Wirtanen in 2001 and to escort it for about 2.5 years (Hubert & Schwehm 1991). Wirtanen is a wobbling comet (Samarasinha, Mueller & Belton 1996), (Rickman & Jorda 1998), and one of the planned experiments is observation of its rotation state, to be carried out by the OSIRIS camera (Thomas et al 1998). The mission will start about 1.5 year before the perihelion, but the spin state will be observed only once,

⁶To reconcile the observations with the old theory (Burns & Safronov 1973), one would have to hypothesise that 433 Eros has experienced no impact or tidal disruptions for many (dozens or hundreds) millions of years, which is very unlikely.

about a year before the perihelion, at a heliocentric distance of 3.5 AU. Three months later the comet should come within 3 AU which is to be a crucial threshold for its jetting activity: at this distance outgassing of water will begin. The strongest non-gravitational torques emerge while the comet is within this distance from the Sun. It is predominantly during this period that wobble is instigated. Hence, it may be good to expand the schedule of the spin-state observations: along with the measurement currently planned for 3.5 AU before perihelion, another measurement, at a similar heliocentric distance after the perihelion, would be useful. The comparison of these observations will provide information of the precession increase during the time spent by the comet within the close proximity to the Sun. Unfortunately, the Rosetta programme will be over soon afterwards, and a third observation at a larger distance will be impossible. The third observation performed well outside the 3 AU region might reveal the wobble-damping rate, and thereby provide valuable information about the composition and inner structure of the comet nucleus. It would be most desirable to perform such a three-step observation by the future escort missions to comets. What are the chances of success of such an experiment? On the one hand, the torques will not be fully eliminated after the comet leaves the 3 AU proximity of the Sun; though the outgassing of water will cease, some faint sublimation of more volatile species (like CO, CO₂, CH₃OH) will persist for long. On the other hand, the damping rate is high enough (Efroimsky 2001), and a comparison of two observations separated by about a year will have a very good chance of registering relaxation (provided the spin state is not too close to the separatrix).

VII. MISSIONS TO ASTEROIDS

Asteroids wobble either after tidal interactions (if they are planet-crossers), or after impacts, or if they are fragments of disrupted progenitors (Asphaug & Scheeres 1999). To register relaxation of a solid-rock monolith may take thousands of years (Efroimsky 2001). However, if the body is loosely connected (Harris 1998, Asphaug et al 1998) the inelastic dissipation in it will be several orders faster and, appropriately, its relaxation rate will be several orders higher.

Relaxation of rubble-pile planet-crossers may be observed through the same three-step scheme: observe the spin state shortly before the tidal interaction, then shortly afterwards, and then after a longer interval.

In the long run, our efforts in observation of excited spin states should target the mainbelt. As well known, it is the perturbations in the belt that drive some asteroids to planet-crossing orbits. Hence the necessity of knowing the impact frequency. To know it, one has to collect the statistics of precessing rotators, on the one hand, and to have a reliable estimate of their relaxation rate, on the other hand.

VIII. CONCLUSIONS

We may be very close to observation of the relaxational dynamics of wobbling small Solar System bodies, dynamics that may say a lot about their structure and composition and also about their recent histories of impacts and tidal interactions. Monitoring of a wobbling comet during about a year after it leaves the 3 AU zone will, most probably, enable us to register its precession relaxation.

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