A BIFURCATION THEOREM FOR CRITICAL POINTS OF VARIATIONAL PROBLEMS

BY

SHUI-NEE CHOW

AND

REINER LAUTERBACH

IMA Preprint Series # 179

September 1985

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

514 Vincent Hall

206 Church Street S.E.

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A Bifurcation Theorem for Critical Points of Variational Problems

Shui-Nee Chow*

Department of Mathematics
Michigan State University
Wells Hall
East Lansing, MI 48824
USA

and

Reiner Lauterbach**

Institut für Mathematik
Universitat Augsburg
Memminger Str. 6
D-89 Augsburg
West Germany

* Research partially supported by National Science Foundation Grant DMS 8401719

** Supported by Deutsche Forschungemeinschaft LA 525/1-1

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§1 Introduction and main theorem

In this note we consider the problem

$$\min_{x \in E} F(x,\lambda)$$

where E is a real Hilbert space, $\lambda \in \mathbb{R}$ and

(1.1) $$F: E \times \mathbb{R} \to \mathbb{R}$$

is a C^2-functional

(1.2) $$f(x,\lambda) = D_x F(x,\lambda)$$

be the gradient of F, which by the Riesz representation theorem may be identified with an element of E. An element $(x_0,\lambda_0) \in E \times \mathbb{R}$ such that

(1.3) $$f(x_0, \lambda_0) = 0$$

is called a critical point of F. We assume that a C^2-smooth curve of critical points of F is given, and that it may be parameterized

(1.4) $$x = x(\lambda).$$

Without loss of generality, we may assume that $x(\lambda) \equiv 0$, i.e. $(0,\lambda)$ is a critical point of F for all $\lambda \in \mathbb{R}$. We want to investigate the branching of further critical points from this curve, which we shall call the trivial solution (of (1.3)). Let $D_x f(0,\lambda) = A(\lambda)$ and assume

(1.5) $$0 < n = \dim \ker A(0) < \infty.$$

Moreover, assume that 0 is isolated in the spectrum $\sigma(A(0))$. Then, for small $|\lambda| \neq 0$, there exist n eigenvalues near zero, we assume that none of these is zero, i.e. there exists a number $\varepsilon_0 > 0$, such that

$$\ker A(\lambda) = \{0\} \text{ for } 0 < |\lambda| < \varepsilon_0.$$

Let $\mathrm{eig}_0(A(\lambda))$ be the set of eigenvalues of $A(\lambda)$, which approach 0 as $\lambda \to 0$, in Kato's terminology, the 0-group [3]. Let $r(A(\lambda))$ be the number of elements in

$$\mathrm{eig}_0\,(A(\lambda)) \cap \{t \in R \mid t<0\}$$

Assume

$$r^+_{A(\lambda)} = \lim_{\substack{\lambda\to 0\\ \lambda>0}} r(A(\lambda)) \tag{1.6}$$

and

$$r^-_{A(\lambda)} = \lim_{\substack{\lambda\to 0\\ \lambda<0}} r(A(\lambda) \tag{1.7}$$

exist. Then we have

<u>Theorem</u> Let $F:E \times \mathbb{R} \to \mathbb{R}$ be C^2 and $f(x,\lambda) = D_xF(x,\lambda)$. Suppose that $f(0,\lambda) = 0$ for all $\lambda \in \mathbb{R}$ and 0 is an eigenvalue of $D_xf(0,0) = A(0)$. If

$$r^+_{A(\lambda)} - r^-_{A(\lambda)} \neq 0,$$

then $(0,0)$ is a bifurcation point of the equation $f(x,\lambda) = 0$.

<u>Remark 1</u> In applications, we consider partial differential equations expressed in terms of integral equations. For example, many boundary value problems may be written as an operator equation

$$(\lambda I - L)x + N(x,\lambda) = 0$$

where I is the identity, $L : E \to E$ is linear, compact and selfadjoint, and $N = o(|x|)$ as $|x| \to 0$ uniformly for bounded λ. Thus, our theorem is applicable if $f(x,\lambda) = (\lambda I-L)x + N(x,\lambda)$.

Remark 2: This theorem generalizes earlier results by Rabinowitz [5,8], Clark [9], Fadell & Rabinowitz [10], Böhme [1], Marino [4], Berger [11] and Takens [12]. The major difference between these earlier results and ours is that we allow an arbitrary dependence of F on λ, while the other authors required that F depends linearly on λ.

Remark 3: The major difference between our proof and the proof given by Rabinowitz [5] is that he uses the Ljapunov-Schmidt method, while we use center manifold theory. The difficulty using Ljapunov-Schmidt method arises from the fact that the bifurcation equation might not possess a variational structure, compare also Chow & Hale [2].

Remark 4: Below we shall indicate a slight generalization to those real Banach spaces E which admit a bounded linear map

$$K: H \to E \tag{1.8}$$

with range $R(K)$ dense E, where H is a real Hilbert space. Let $J: H \to H^*$ be the isomorphism given by the Riesz representation theorem, if $H \overset{K}{\to} E$ then

$$E \leftarrow H \to H^* \leftarrow E^* \text{ , i.e.} \tag{1.9}$$

$$\tilde{K} = K\, J^{-1}\, K^* \tag{1.10}$$

is a bounded linear operator, which in fact is injective, since K has dense range.

Remark 5: The construction (1.9) to (1.10) applies to all Sobolev spaces $W^{m,p}(\Omega)$ for bounded domains Ω R^N and $1 < p < \infty$.
If $1 < p < 2$ then

$$W^{m,2}(\Omega) \overset{K}{\to} W^{m,p}(\Omega) \tag{1.11}$$

and if $p > 2$ then

$$W^{m',2}_{(\Omega)} \overset{K}{\to} W^{m,p}(\Omega)$$

where m' satisfies

$$\frac{1}{p} = \frac{1}{2} - \frac{m'-m}{N} .$$

Remark 6: Problems in nonlinear elasticity lead to variational problems in $W^{1,p}(\Omega)$ for $\Omega \subset R^N$, $N = 1,2,3$. Compare Marsden and Hughes [14]. Our more general formulation given below applies to the problems in hyperelasticity which have a convex strain energy function.

II. Proof of the theorem

We consider the differential equation

$$\dot{x} = - f(x,\lambda) \tag{2.1}$$

$$\dot{\lambda} = 0 \tag{2.2}$$

where the dot refers to differentiation with respect to time. Since $F \in C^2(E \times \mathbb{R}, \mathbb{R})$ the hypotheses of the center manifold theorem (Henry [6], p. 169, see also Bates and Jones [13]) are satisfied for (2.1), (2.2) at $(x,\lambda) = (0,0)$. This implies that there exists a $(n+1)$ - dimensional manifold $M \subset E \times \mathbb{R}$ which is

(i) locally invariant under the flow generated by (2.1), (2.2) and such that

(ii) there exists a $U \subset E \times \mathbb{R}$ of $(0,0)$ such that any solution $(x(t),\lambda)$ of (2.1), (2.2) with $(x(t), \lambda) \in U$ for all $t \in \mathbb{R}$ lies on M U.

(iii) M is tangent to ker $A(0) \times R$ at $(0,0)$. We remark that each slice M_λ defined by

$M_\lambda = \{(x,\lambda) | x \in E, \lambda \in R, |\lambda|$ sufficently small$\}$ is locally invariant under the flow generated by (2.1). The center manifold M is defined by a map

$$G : (\ker A \times \mathbb{R}) \cap U \to (E \times \mathbb{R}) \cap W \tag{2.3}$$

such that

$$M = \{(v + G(v,\lambda),\lambda) \mid (v,\lambda) \in (\ker A(0) \times \mathbb{R}) \cap U\}$$

where W is a neighborhood of $(0,0) \in E \times \mathbb{R}$ defined by

$$W = \{(v,\lambda) \in \ker A(0) \times \mathbb{R} \mid \|v\|_E < a_1,\ |\lambda| < a_2\}.$$

for a pair of sufficiently small real numbers a_1, a_2. For fixed λ, the flow M_λ is given by

(2.4) $$\dot{v} = -Pf(v + G(v,\lambda),\lambda)$$

where $P = \frac{1}{2\pi}\int_\Gamma R(z,A(0)\,dz$, with $R(z,A(0))$ is the resolvent of the linear operator $A(0)$ at $z \in \Gamma$ and Γ is a closed curve in C, such that 0 is the only point in the spectrum lying in the bounded region defined by Γ. Write (2.4) as

(2.5) $$v = -B(\lambda)v+g(v,\lambda)$$

with $g(v,\lambda) = O(\|v\|)$.

The main observation for using the center manifold in bifurcation theory is, that

$$r(A(\lambda)) = r(-B(\lambda))$$

and

$$r^+_{A(\lambda)} - r^-_{A(\lambda)} = r^+_{-B(\lambda)} - r^-_{-B(\lambda)}.$$

Therefore the Morse index of the trivial solution changes. For the moment we assume that our theorem is not true. Then 0 will be an isolated critical point of $F(\cdot,0)$, i.e. there exists a neighborhood V of $x = 0$ in E, such that $F(\cdot,0)$ has no other critical point in V. From the fact that $\dot{x} = -f(x,0)$ defines a gradient flow, it follows that $\{x=0\}$ is an isolated invariant set for (2.1) at $\lambda=0$. Therefore there exists an isolating neighborhood $N \subset V \times \{0\}$ for $(x,\lambda) = (0,0)$. This isolating neighborhood N continues for small $|\lambda|$, let us say $|\lambda| < \varepsilon_2$. Again from the fact that (2.1) defines a gradient flow and the assumption that our theorem is not true it follows that $\{x = 0\}$ is an isolated invariant set for $|\lambda| < \varepsilon_2$ with isolating neighborhood $N \times \{\lambda\}$. This gives a continuation connecting two maximal invariant, isolated sets (our critical point 0) having different Morse index. This contradicts Conley's continuation theorem (Conley [7]). Therefore our theorem is proved. $\square$

III. A Generalization

There are two directions in which one could try to generalize our theorem:

(a) more general spaces

(b) less smoothness assumptions on F.

For example, if one wants to prove a similar theorem which is applicable to problems in elasticity one needs a generalization in both directions. The simplest problems in hyperelasticity as for example the buckling of a rod lead to variational problems as

$$(3.1) \qquad \min_{u \quad W^{m,p}([0,1]} \int_0^1 W(u_x,\lambda)dx$$

where $W^{m,p}([0,1])$ is the Sobolev space of functions in $L^p([0,1])$ having weak derivatives in $L^p([0,1])$. We note that "real problems" are formulated over domains $\Omega \subset R^3$. We want to assume $p>1$ although in applications this is not always the case. The functional.

$$(3.2) \qquad F = \int_0^1 W(u_x,\lambda)dx$$

is not C^2 and if $p \neq 2$, $W = W^{m,p}_{(\Omega)}$ is not a Hilbert space. If $p<2$, and $\Omega \subset R^N$ is bounded, then the construction given in remark 4 above applies with $H = W^{m,2}(\Omega)$ if $p>2$, take m' satisfying

$$\frac{1}{p} = \frac{1}{2} - \frac{m' - m}{N}$$

and $H = W^{m',2}(\Omega)$. In these cases we look at the differential equation

(3.3) $$X = -Kf(x,\lambda).$$

If F is differentiable (in some sense) we get for a solution of (3.3) ($\langle .,. \rangle_{(E^*,E)}$ refers to the pairing E^*,E, while $\langle .,. \rangle_H$ denote the inner product in H)

$$\frac{d}{dt}F(x(t),\lambda) = - \langle f(x,\lambda),\ x)_{(E^*,E)}$$

$$= - \langle f(x,\lambda)Kf(x_1\lambda)\rangle_{(E^*,E)}$$

$$= - \langle f(x,\lambda),K^*J^{-1}Kf(x,\lambda)\rangle_{(E^*,E)}$$

$$= - \langle Kf(x,\lambda),\ J^{-1}\ Kf(x,\lambda)\rangle_H < 0\ .$$

By the injectivity of K we conclude that any point x_0 in the (strong) ω-limit set of $x(t)$ is an equilibrium for (3.2) and a critical point of F. The main ingredient in our proof is the center manifold theorem, which holds under more general hypotheses, then we needed above (Henry [6]). In elasticity one looks at strain energy function $W(u_x,\lambda)$ where u is the deformation, u_x the deformation gradient and the problem is to minimize the function

(3.3) $$F = \int_0^1 W(u_x,\lambda') + b(u,x)dx$$

over $W^{m,p}([0,1])$, where b incorporates external forces.

We assume

(i) $W \in C^2(R,R)$

(ii) $\frac{\partial^2}{\partial z^2}W(z,\lambda) > 0$, i.e. $W(\cdot,\lambda)$ is convex for each λ.

The Euler Lagrange equation of (3.3) is quasilinear, namely

(3.4) $$W''(u_x,\lambda)u_{xx} + b(u,x) = 0$$

We look at

(3.5) $$\frac{\partial u}{\partial t} = + (W''(u_x,\lambda)\, u_{xx} + b(u,x))$$

Changing the time variable as in Henry [G], p. 59, we obtain the semilinear parabolic equation

$$\frac{\partial u}{\partial t} = + u_{xx} + \frac{1}{W''(u_x,\lambda)} b(u,x)$$

which may be treated by our theory.

Acknowledgement: The second author wishes to thank John Maddocks for mentioning this problem.

References:

[1] Böhme, R: Die Lösung de Verzweigungsgleicheng für nichtlinare Eigenwertprobleme, MZ 127(1972) 105-126.

[2] Chow, S.N. and Hale, J.K., Methods in Bifurcation Theory, Grundl. d. Math. Wiss. 250, Springer Verlag, Berlin-New York, (1983)

[3] Kato, T: Perturbation Theory for Linear Operators, Grund. der Math Wiss. 132, Springer Verlag, Berlin-New York (1966)

[4] Marino, A. : La biforcarione nel caso variationale, Conf. Sem. Mat. Bari (1973).

[5] Rabinowitz, P.H.: A bifurcation theorem for potential operators, J. Funct. Anal. 25 (1977) 412-424.

[6] Henry, D.: Geometric Theory of Semilinear Parabolic Equations, Springer Lect. Notes in Mat. 840, Springer Verlag (1982)

[7] Conley, C: Isolated Invariant Sets & the Morse Index, Conf. Board of the Math. Sc., Reg. Conf. Ser. in Math, 38 (1976).

[8] Rabinowitz, P.H.: Variational Methods for Non-linear Eigenvalue Problems, C.I.M.E, Varenna (1974).

[9] Clark, D.C.: Eigenvalue Perturbation for odd gradient operators, Rocky Mt. J. Math 5, (1975), 317-336.

[10] Fadell, E.R. and Rabinowitz, P.H: Bifurcation for odd potential operators and an alternative index, J. Funct. Anal. 26 (1977) 48-67.

[11] Berger, M.S.: Bifurcation Theory and the type numbers of Marston Morse, Proc. Nat. Acad. Sc. 69 (1972), 1737-1738.

[12] Takens, F.: Some Remarks on the Böhme Berger Bifurcation Theorem, MZ 129 (1972), 359-364.

[13] Bates, P.W. and Jones, C.K.R.T: The Center Manifold Theorem with Applications, Preprint, 1985.

[14] Marsden, J.C. and Hughes, T.J.R: Mathematical Foundations of Elasticity, Prentice-Hall, Englewood, 1983.

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