The Design of a Microactuator

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Abstract

Our task is to devise a mathematical model for a simple microactuator and analyze it from both analytical and numerical points of view. Our goal is to use the model to design an efficient actuator, that is, one that moves as quickly as possible over three microseconds.

1 Introduction

With advances in microelectronic device fabrication technology, it has become possible to make small mechanical devices on computer chips. These devices can be used widely in several electronical machines such as ink-jet printers and micro-medicine-pumps implanted under the patient's skin. The basic idea in the design of these pumps is that we have a beam with two layers made of two different materials with two different thermal expansion coefficients. The two layers are strongly joined together. When the layer with larger thermal expansion coefficient is being heated wants to expands more than the joint layer, but it does not allow the expansion. This reaction makes the the beam to be bent. If we choose the material and the heating carefully this bending can be made fast. By placing this device into a fluid tank with a pipe it is able to pump out the fluid from the tank. The size of these micro-pumps is the order of a hundred microns. Our goal is to design a model for a simple microacturator that simulate the work of the real device.

2 Formulation of the Problem

In order to deal with the problem we had to divide the problem into two parts. The first part is the thermal aspect of the problem and the second it the mechanical part. By combining these parts we could built a fairly accurate thermodynamical model. But, this model is to be used only in vacuum or in air. If we want to use our model in fluid we must count with the resistance

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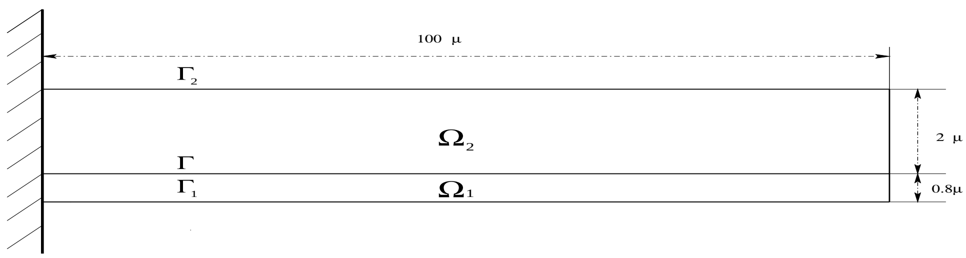
of the fluid. We built in this fluid resistance into our final model. We were also concerned about the influence of gravity, but we found that the force of gravity is very small compared to the other forces acting on the beam so, we did not include its influence in our model.

2.1 The Thermal Aspect



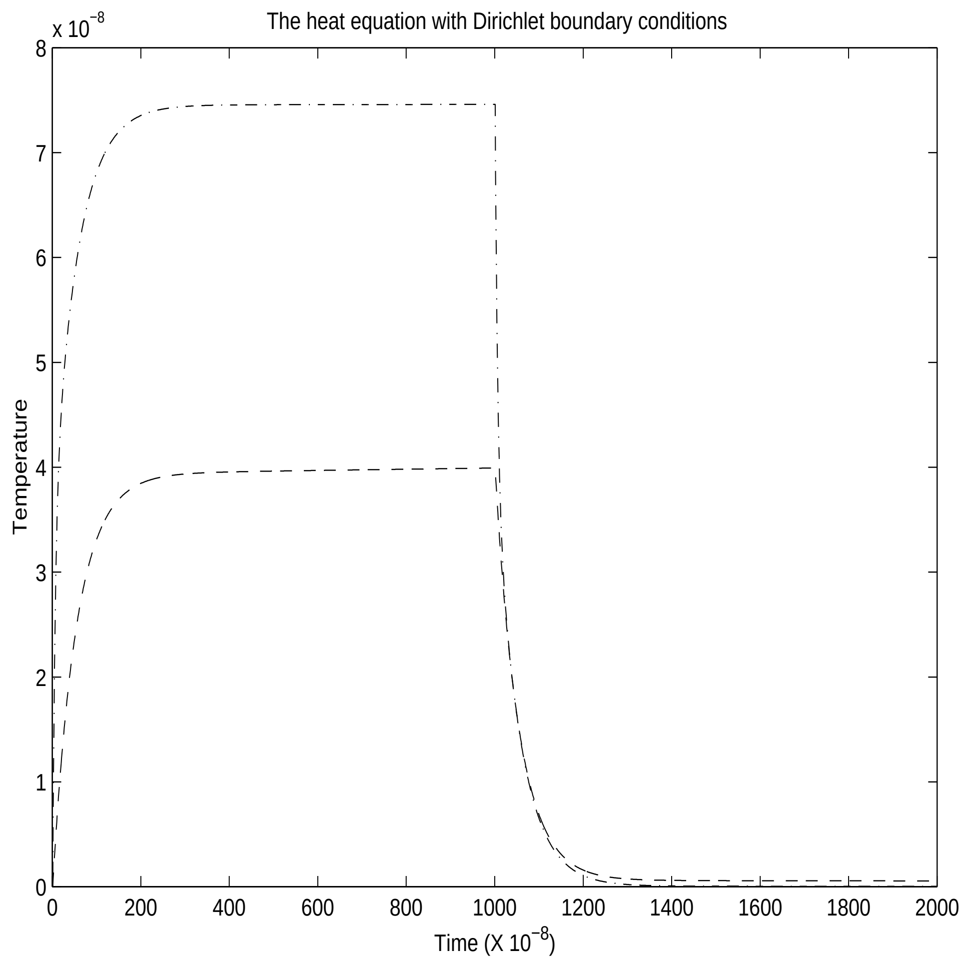
The above figure represents a longitudinal cross-section of the device. The indicated dimensions are possible dimensions for an actual actuator.

Employing the standard techniques of modeling heat equations, we obtained the following characterization of the temperature in a longitudinal cross-section of the beam

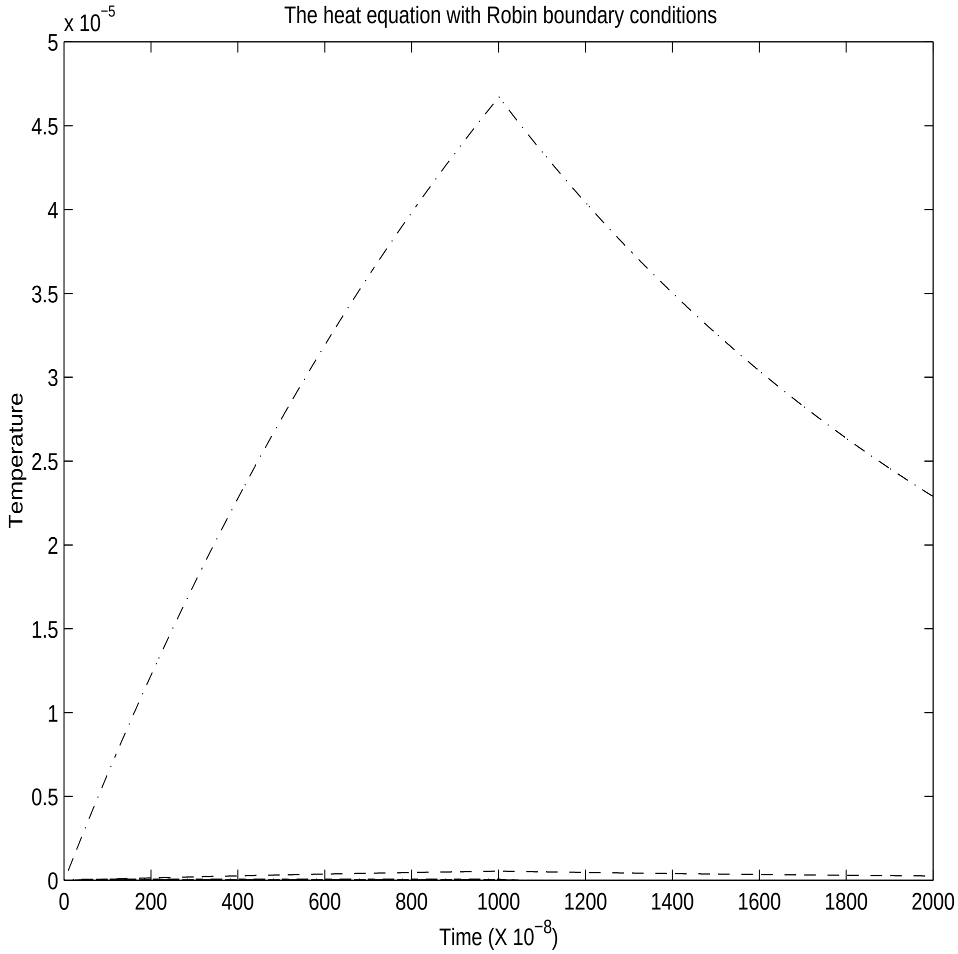
$$(H) \qquad \begin{cases} \rho_i c_i \frac{\partial u}{\partial t} = \operatorname{div}(k_i \nabla u_i) + r_i & \text{in } \Omega_i \\ u_i(t,x) = u_0 & x \in \Gamma_i \\ u_i(0,x) = u_0 & x \in \Omega_i \\ & \\ \begin{cases} u_1 = u_2 \\ k_2 \frac{\partial u_2}{\partial y} = k_1 \frac{\partial u_1}{\partial y} \end{cases} & \text{on } \Gamma \end{cases}$$

Here
Ω_1 and Ω_2 represent longitudinal cross-sections of the two beams;
Γ is the segment line corresponding to the two beams interface;
Γ_i, represents the boundary of Ω_i, excepting the segment Γ;
$r_1(t) =$ constant and $r_2 = 0$ are the rates of heating corresponding to Ω_1 and, respectively, Ω_2;
u_0 is the initial temperature of the device;
ρ_i is the density of Ω_i, $i = 1, 2$;
c_i is the thermal capacity of Ω_i;
k_i is the conductivity coefficient of Ω_i.

Using the forward-diference method for this problem we have obtained the following dependence of temperature with time



The data from direct experiments tells us that more accurate results could be obtained using Robin boundary conditions. Unfortunately our method is unstable for very small steps in both space and time. Therefore we had to increase the thickness of our device in order to obtain consistent numerical results. The next diagram shows the heat behavior for a 10 times thicker beam.



For both theoretical and numerical reasons is worth while to consider also the weak formulation of the above problem

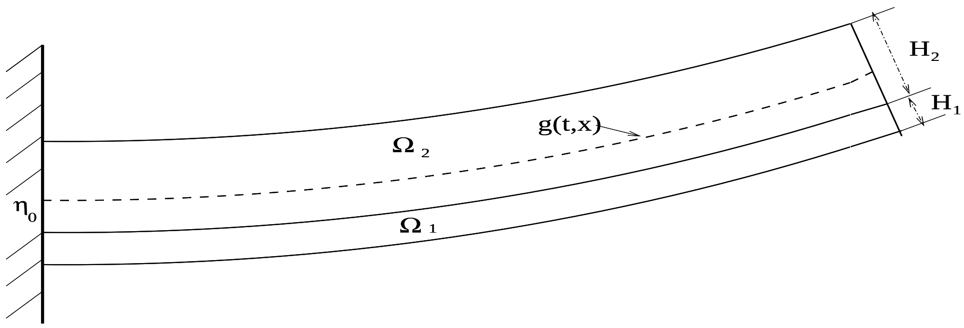
$$\int_\Omega \rho c \frac{\partial u}{\partial t} v \mathrm{d}x + \int_\Omega k \nabla u \nabla v \mathrm{d}x - \int_\Omega r v \mathrm{d}x = 0, \ \forall v \in H_0^1(\Omega).$$

(We can take $u_0 = 0$ without losing generality.)

$\rho,\ c,\ k$ and r are the functions on $\Omega = \Omega_1 \bigcup \Omega_2$ whose restrictions on Ω_i are $\rho_i,\ c_i,\ k_i$ and, respectively, r_i.

The weak formulation of the problem could be useful in numerical approaches using the finite element method, but also for theoretical purposes like existence, uniqueness, regularity etc.

2.2 The Mechanical Part



Using the Hooke's law and some infinitesimal approximations, we get the stress at a point having x_0 as x-coordinate as being

$$\frac{dF}{dA} = E(\eta - \eta_0)g''(x_0),$$

where E is a function of η reprezenting the Young's modulus.

η_0 is the y-coordinate of the neutral segment line (the segment for which the distances between points do not change when the beam bents) and $g(x,t)$ gives the shape of the neutral line with respect to time. We can find η_0 from the following simple condition

$$F = \int_{H/2}^{H/2} E(\eta)(\eta - \eta_0)g''(x_0)d\eta = 0,$$

where $H = H_1 + H_2$ is the height of the entire beam.

From here

$$\eta_0 = \frac{(E_2 - E_1)H_1 H_2}{2(E_1 H_1 + E_2 H_2)}.$$

Computing the moment at x_0 and $x_0 + \Delta x$ and equating the difference with the moment due to the *shear* force, we get

$$\int_{-H/2}^{H/2} E(\eta)(\eta - \eta_0)^2 (g''(x_0 + \Delta x) - g''(x_0))d\eta = \int_{-H/2}^{H/2} \phi(x_0 + \Delta x, \eta d\eta)\Delta x.$$

Dividing by Δx and letting $\Delta x \to 0$ we have

$$\int_{-H/2}^{H/2} E(\eta)(\eta - \eta_0)^2 g'''(x_0)d\eta = \int_{-H/2}^{H/2} \phi(x_0, \eta)d\eta.$$

Finaly, if we do the same trick with this last equation (take the difference, divide by Δx and pass to the limit), we end up with

$$-\frac{\partial^2 g}{\partial t^2}(\rho_1 H_1 + \rho_2 H_2) = \int_{-H/2}^{H/2} E(\eta)(\eta - \eta_0)^2 \frac{\partial^4 g}{\partial x^4}(x_0)d\eta.$$

From this last equation and taking into consideration the initial conditions and boundary conditions we obtain the following initial boundary condition problem

$$\begin{cases} \tilde{\rho}\,\frac{\partial^2 g}{\partial t^2} + \tilde{E}\,\frac{H^3}{12}\,\frac{\partial^4 g}{\partial x^4} = -K\,\frac{\partial g}{\partial t} \\ g(0,x) = \eta_0 \\ \frac{\partial g}{\partial t}(0,x) = 0 \\ \frac{\partial^3 g}{\partial x^3}(t,L) = 0 \\ \frac{\partial^2 g}{\partial x^2}(t,L) = c(t) \\ \frac{\partial g}{\partial x}(t,0) = 0 \\ g(t,0) = \eta_0 \end{cases}$$

where

$$\tilde{\rho} = \frac{\int_{-H/2}^{H/2} \rho d\eta}{\int_{-H/2}^{H/2} d\eta}, \;\; \rho = \begin{cases} \rho_1 & \text{if } \eta \in \Omega_1 \\ \rho_2 & \text{if } \eta \in \Omega_2 \end{cases},$$

$$\tilde{E} = \frac{\int_{-H/2}^{H/2} E(\eta)(\eta - \eta_0) d\eta}{\int_{-H/2}^{H/2} (\eta - \eta_0)^2 d\eta},$$

$$E(\eta) = \begin{cases} E_1 & \text{if } \eta \in [-H/2,\, -H/2 + H_1] \\ E_2 & \text{if } \eta \in [-H/2 + H_1,\, H/2] \end{cases}$$

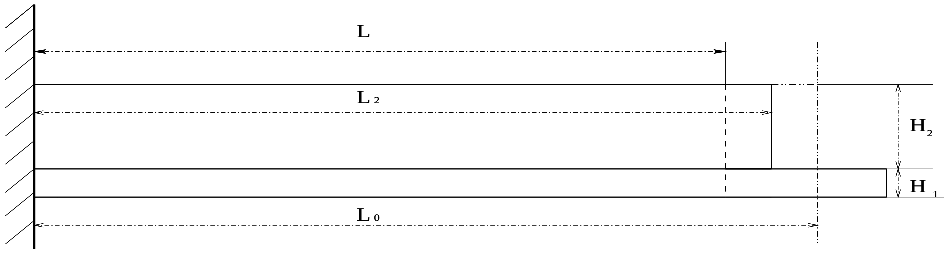
and

$$c(t) = \frac{12 \int_{-H/2}^{H/2} h(\eta)(\eta - \eta_0) d\eta}{\tilde{E} H^3},$$

with

$$h(\eta) = \begin{cases} E_1 \frac{L_0 - L_1}{L_1} & \text{if } \eta \in [-H/2,\, -H/2 + H_1] \\ E_2 \frac{L_0 - L_2}{L_2} & \text{if } \eta \in [-H/2 + H_1,\, H/2] \end{cases}.$$

Here L_0 is the lenght of the beam if it is kept straight when heated (as in the figure).



We get the expression of L_0 by equating the Hooke's law for the two layers

$$E_1 H_1 \frac{L_0 - L_1}{L_1} = -E_2 H_2 \frac{L_0 - L_2}{L_2} \quad \longrightarrow$$

$$L_0 = \frac{L_1 L_2 (E_1 H_1 + E_2 H_2)}{E_1 H_1 L_2 + E_2 H_2 L_1},$$

where $L_i = (1 + \alpha_i u_i)L$, $i = 1, 2$ is the lenght of Ω_i, α_i being the thermoextension coefficient associated to Ω_i.

The influence of the friction force due to the surronding fluid is displayed in the right-hand side of the differential equation, K being a coefficient depending on the fluid's properties.

Comment We did not include the influence of gravity into our model because of its very small influence.

References

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