

Evaporation and combustion of thin films of liquid fuels

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We consider the evaporation and subsequent burning of thin films of liquid fuels. Previous studies on liquid films, with and without evaporation, have primarily considered the gas phase to be passive. The new element in this study is the introduction of combustion and the examination of both the liquid and gas phases and their effect on the film's behaviour. By exploiting the various length scales associated with the problem and the disparity between the density and viscosity of the liquid and its vapor, the problem was systematically reduced to a single nonlinear evolution equation for the film thickness. All remaining variables, which are simply expressed in terms of the function describing the instantaneous position of the liquid-vapor interface, are subsequently determined. This equation is then analyzed to understand the dynamics of a liquid film burning in quiescent air.

The planar configuration is analyzed first. Predictions for the total evaporation time are obtained, along with the time history of the film thickness, the interfacial surface temperature, the flame standoff distance and its temperature, and the mass burning-rate. The dependence of the burning characteristics on the fuel and oxidant Lewis numbers, which measure the relative importance of thermal and molecular diffusivities, is also determined. Second, we analyze the case of a non-planar interface, where temperature variations along the film's surface causes fluid motion in the liquid that could either dampen or amplify spatial nonuniformities. We show that, while thermocapillarity has the tendency to destabilize the planar interface, combustion acts to reduce this effect. In particular, when the heat release by combustion is substantial, all disturbances are obliterated, the film remains nearly planar and the burning occurs along nearly horizontal surfaces.

1. Introduction

Thin liquid films play substantial roles in various scientific areas such as biology, physics, chemistry and geology. In engineering applications, they serve as agents for heat and mass transport and are common in lubrication systems. Our interest stems from applications involving films of liquid fuels. An accidental spillage of a flammable fluid can be a fire hazard. A leak of liquid fuel at high pressure in industrial applications may produce a flow of burning liquid over solid surfaces, a situation referred to as a "running liquid fire" in the petrochemical industries. In combustion devices where a liquid fuel jet is atomized into droplets, the development of a thin liquid films flowing along the rela-

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tively colder walls is often possible. Understanding the burning characteristics of these films is essential for the overall control of the combustion efficiency.

Over the past few decades much work has been done on the stability of thin liquid films with and without evaporation and in the presence of various effects such as surface tension, vapor recoil and molecular forces as well as with the presence of soluble and insoluble surface active agents (c.f. Oron, Davis & Bankoff 1997). A similar theoretical study which examines the combustion of thin films has not been done. This investigation provides a general formulation for such studies. Unlike most previous works where the gas-phase processes were considered passive, our formulation accounts for the interaction between the liquid and the gas and recognizes the fact that the dynamics could be profoundly influenced by interfacial effects.

The burning of liquid fuels is a complex phenomenon; it is an unsteady process, since as a result of vaporization the surface of the film recedes continually, and a process that exhibits a strong coupling between the liquid and gas phases. The fuel vapor produced as a result of evaporation can either burn as a “pool fire”, with nearly uniform horizontal surfaces, or as a flame spreading along the surface of the film. In this work we will be primarily concerned with a configuration that is relevant to pool fires, but our formulation is quite general and has far more reaching applications.

A key element in the analysis is the fact that the gas-to-liquid density ratio is small. As a result, the film recedes relatively slowly and the gas-phase processes are quasi-steady. For simplicity we have assumed in this work that the chemistry is relatively fast so that the burning occurs along a thin diffusion flame located where the fuel vapor meets the oxidant at stoichiometric proportions. Incomplete combustion including the possibility of extinction could be easily incorporated using more general jump relationships across the diffusion flame surface (Cheatham & Matalon 2000). The fluid dynamics in the liquid-phase is simplified by adopting a “lubrication approximation” which exploits the disparity between the film thickness and the typically larger transverse characteristic length. These simplifications lead to a formulation which consists of a nonlinear evolution equation that describes the local and instantaneous thickness of the film. All remaining variables, which are simply expressed in terms of the function describing the instantaneous position of the liquid-vapor interface, are subsequently determined. Similar to previous studies (Williams & Davis 1982, Burelbach, Bankoff & Davis 1988, Krishnamoorthy, Ramaswamy & Joo 1995, Oron *et al.* 1997) this equation accounts for gravity and thermocapillary effects. However, it also depends on the local surface temperature and on the mass flux leaving the surface of the film, two quantities that are determined by the details of the gas-phase processes.

2. Formulation

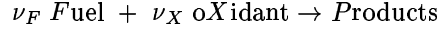
A thin liquid layer lies on a solid surface located at $y = 0$ and is maintained at a uniform temperature T_b . The film is laterally unbounded and consists of a combustible, incompressible fluid of constant material properties. The liquid is evaporating at its surface and the fuel vapor thus formed is burning in the gas-phase with the ambient oxidant. The liquid-vapor interface is described by $y = \varphi(x, t)$ where x is the lateral coordinate and t is the time variable. Thus, unit normal and tangent vectors to the interface are respectively given by

$$\mathbf{n} = \frac{(-\partial\varphi/\partial x, 1)}{\sqrt{1 + (\partial\varphi/\partial x)^2}}, \quad \mathbf{t} = \frac{(1, \partial\varphi/\partial x)}{\sqrt{1 + (\partial\varphi/\partial x)^2}},$$

and the velocity of the interface along the normal is

$$V_I = \frac{\partial \varphi / \partial t}{\sqrt{1 + (\partial \varphi / \partial x)^2}}.$$

The chemical activity is modeled by an overall one-step chemical reaction of the form



where ν_i is the stoichiometric coefficient of species i and the subscripts F , X stand for fuel and oxidant, respectively. It is assumed that the fuel vapor burns to completion within a finite “boundary layer” of thickness H . The configuration is shown in figure 1.

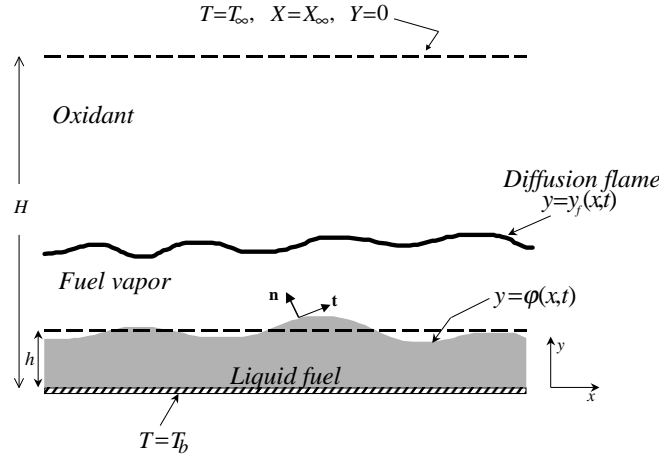


FIGURE 1. Configuration of the problem

We denote the velocity field by \mathbf{v} , the temperature and density by T and ρ and the mass fractions of fuel and oxidant by Y and X , respectively. Furthermore, the accent $\hat{}$ identifies the variables in the liquid phase. Conservation of mass, momentum and energy in each of the two phases are:

liquid phase:

$$\nabla \cdot \hat{\mathbf{v}} = 0 \quad (2.1)$$

$$\rho_\ell \frac{D\hat{\mathbf{v}}}{Dt} = \nabla \cdot \hat{\mathbf{\Sigma}} + \rho_\ell \mathbf{g} \quad (2.2)$$

$$\rho_\ell \frac{D}{Dt} \left(c_{p_\ell} \hat{T} + \frac{1}{2} \mathbf{v}^2 \right) - \lambda_\ell \nabla^2 \hat{T} = 0 \quad (2.3)$$

gas phase:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \quad (2.4)$$

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \mathbf{\Sigma} + \rho \mathbf{g} \quad (2.5)$$

$$\rho c_{p_g} \frac{DT}{Dt} - \lambda_g \nabla^2 T = Q\omega \quad (2.6)$$

$$\rho \frac{DY}{Dt} - \rho \mathcal{D}_F \nabla^2 Y = -\nu_F W_F \omega \quad (2.7)$$

$$\rho \frac{DX}{Dt} - \rho \mathcal{D}_X \nabla^2 X = -\nu_X W_X \omega. \quad (2.8)$$

In these equations D/Dt is the convective derivative, $\hat{\Sigma}$ and Σ are the stress tensors, given by $\hat{\Sigma} = -\hat{p}\mathbf{I} + \mu_\ell (\nabla \hat{\mathbf{v}} + \nabla \hat{\mathbf{v}}^T)$ in the liquid and $\Sigma = -p\mathbf{I} + \mu_g \{(\nabla \mathbf{v} + \nabla \mathbf{v}^T) - \frac{2}{3}(\nabla \cdot \mathbf{v})\mathbf{I}\}$ in the gas with the superscript T denoting the transpose, \mathbf{I} a unit tensor and \hat{p} , p the pressure in the respective phases. The gravitational force (per unit mass) is denoted by \mathbf{g} and is acting downwards, in the negative y -direction. The coefficients μ_ℓ , c_{p_ℓ} , λ_ℓ and μ_g , c_{p_g} , λ_g , which are all assumed constant, represent the viscosity, specific heat and thermal conductivity of the liquid and gaseous mixture, respectively. The liquid density ρ_ℓ is also taken as a constant, but the density of the gas ρ varies with temperature. The molecular weight of species i is denoted by W_i and the molecular diffusivity \mathcal{D}_i is assumed to vary with temperature such that $\rho \mathcal{D}_i$ is constant. The chemical reaction rate, is assumed to obey the Arrhenius law

$$\omega = \mathcal{B} \left(\frac{\rho Y}{W_F} \right) \left(\frac{\rho X}{W_X} \right) e^{-E/R^\circ T} \quad (2.9)$$

where \mathcal{B} is a pre-exponential factor, E is the overall activation energy and R° is the universal gas constant. The total chemical heat release is denoted by Q .

In writing these equations it has been assumed that the characteristic gas-phase velocities are small compared to the speed of sound. As a result the kinetic energy $\rho \mathbf{v}^2/2$ in the energy equation (2.6) has been neglected, as was the rate of dissipation of mechanical energy due to shear viscosity. Another consequence of this assumption is that the gas-phase processes are nearly isobaric, with the pressure $P \approx P_\infty$, the pressure in the ambient gas. The equation of state reduces to

$$\rho T = \frac{\overline{W}}{R^\circ} P_\infty \quad (2.10)$$

with \overline{W} the average molecular weight of the mixture. The pressure p appearing in Σ is the small pressure variation $P - P_\infty$, of the order of the square of the representative Mach number.

The boundary conditions are

$$\hat{\mathbf{v}} = 0, \quad \hat{T} = T_b \quad \text{at } y = 0 \quad (2.11)$$

corresponding to the no-slip and isothermal conditions at the solid surface, and

$$T = T_\infty, \quad \rho = \rho_\infty, \quad X = X_\infty, \quad Y = 0 \quad \text{at } y = H \quad (2.12)$$

corresponding to the specified uniform state at the edge of the combustion layer. The velocity field at $y = H$ is also specified by the external flow.

At the interface, $y = \varphi(x, t)$, the vapor-liquid jump conditions are as follows: The overall mass balance is

$$\rho_\ell (\hat{\mathbf{v}} \cdot \mathbf{n} - V_I) = \rho (\mathbf{v} \cdot \mathbf{n} - V_I) \equiv m \quad (2.13)$$

where m denotes the net mass flux across the interface. The mass balance for the fuel and oxidant are

$$mY - \rho \mathcal{D}_F \frac{\partial Y}{\partial n} = mY_\ell \quad (2.14)$$

$$mX - \rho \mathcal{D}_X \frac{\partial X}{\partial n} = 0 \quad (2.15)$$

where Y_ℓ is the fuel mass fraction in the liquid. The balance of normal-stress is

$$m(\hat{\mathbf{v}} - \mathbf{v}) \cdot \mathbf{n} - \mathbf{n} \cdot (\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}) \cdot \mathbf{n} = \sigma \nabla \cdot \mathbf{n} \quad (2.16)$$

where $\nabla \cdot \mathbf{n}$ is twice the mean curvature and σ is the surface tension. The balance of shear stress is

$$m(\hat{\mathbf{v}} - \mathbf{v}) \cdot \mathbf{t} - \mathbf{t} \cdot (\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}) \cdot \mathbf{n} = -\mathbf{t} \cdot \nabla \sigma. \quad (2.17)$$

The balance of energy is

$$\lambda_g \frac{\partial T}{\partial n} - \lambda_\ell \frac{\partial \hat{T}}{\partial n} = mL_v - \frac{1}{2}m(\hat{\mathbf{v}} \cdot \mathbf{n})^2 \quad (2.18)$$

where L_v is the latent heat of vaporization. Finally, no slip at the interface between the two fluids

$$(\hat{\mathbf{v}} - \mathbf{v}) \cdot \mathbf{t} = 0, \quad (2.19)$$

and continuity in temperature at the interface $T = \hat{T}$ must be applied.

The Clausius-Clapeyron condition that relates the partial pressure of the fuel vapor to the temperature in the vicinity of the interface is expressed as

$$\frac{Y}{W_F/\bar{W}} = \exp \left\{ \frac{L_v}{R^\circ/W_F} \left(\frac{1}{T_B} - \frac{1}{T} \right) \right\} \quad (2.20)$$

where T_B is the boiling temperature evaluated at the ambient pressure P_∞ . The surface tension σ is assumed to depend linearly on the temperature, namely

$$\sigma = \sigma_0 - \gamma(\hat{T} - T_\infty) \quad (2.21)$$

where σ_0 is evaluated at the reference state where $T = T_\infty$. For most common liquids, the gradient $\gamma = -d\sigma/dT > 0$.

3. Scaling and nondimensionalization

A key element in scaling the equations is the realization that the gas-to-liquid density and viscosity ratios are small. Typical values for representative alcohol and hydrocarbon fuels are shown in table 1. Let ρ_g and μ_g be the characteristic density and viscosity of the gaseous mixture, determined mostly by the properties of air, we see that $\rho_g/\rho_\ell \sim 10^{-3}$ and $\mu_g/\mu_\ell \sim 10^{-2}$. Thus, we write

$$\rho_g/\rho_\ell \equiv \epsilon^2$$

with $\epsilon^2 \ll 1$, and consequently

$$\mu_g/\mu_\ell = \epsilon^2 \mu$$

with $\mu = O(1)$. The ratio of thermal conductivities,

$$\lambda_g/\lambda_\ell = \lambda$$

is treated as $O(1)$.

Apart from the chemical reaction time the problem involves two time scales: the evaporative time scale $\mathcal{D}_{\text{th}}^{-1} h^2 (\rho_\ell/\rho_g)$ where $\mathcal{D}_{\text{th}} = \lambda_g/\rho_g c_{p_g}$ is the thermal diffusivity and h the initial film thickness, and the viscous time scale $\rho_\ell h^2/\mu_\ell \sim \rho_g h^2/\mu_g$. As expected, the viscous time is much shorter than the evaporative time, their ratio being $O(\epsilon^2)$. In the following we shall use $\rho_\ell h^2/\mu_\ell \epsilon^2$ as a unit of time for nondimensionalization which apart from an $O(1)$ constant, say α , is equivalent in magnitude to the evaporative time. The coefficient $\alpha = \mu/Pr$ where $Pr = \mu_g c_{p_g}/\lambda_g$ is the gas-phase Prandtl number. We

	Methanol	Ethanol	Hexane	Octane	Air
ρ_l [kg/m ³]	791	789	660	703	—
ρ_g [kg/m ³]	—	—	—	—	1.23
μ_l [kg/m s]	0.6×10^{-3}	1.2×10^{-3}	0.3×10^{-3}	0.5×10^{-3}	—
μ_g [kg/m s]	1.0×10^{-5}	0.9×10^{-5}	0.7×10^{-5}	0.5×10^{-5}	1.8×10^{-5}

TABLE 1. Density and viscosity of representative liquid fuels and air.

note that although one could absorb the constant α in the time scale, it is left out so that the resulting evolution equation resembles in form those appearing in previous thin-film studies.

Another key assumption in scaling the equations is that the deviation of the interface from its mean position h has a relatively long wavelength. Hence the vertical dimension is scaled with respect to h but the horizontal coordinate with respect to h/ϵ . In order to conserve mass \hat{v}/\hat{u} and v/u must both be $O(\epsilon)$. We thus scale the horizontal velocity components using $\mu_l \epsilon / \rho_l h$ and $D_{th}/h\epsilon$ as units, for the liquid and gas phases respectively, and scale the vertical velocity components accordingly. The net mass flux m leaving the surface is thus made dimensionless with respect to λ_g/hc_{p_g} . The pressure in the liquid and gas phases is respectively referred to $\mu_l^2/\rho_l h^2$ and $\rho_g D_{th}^2/h^2 \epsilon^2$, the temperature is scaled with respect to the ambient temperature T_∞ and, consistent with these choices, $\rho_g = \rho_\infty$. Finally, the fuel and oxidant mass fractions are both scaled with respect to Y_ℓ .

The dimensionless governing equations and boundary conditions, after taking the limit $\epsilon \rightarrow 0$, become (the same symbols are used below for the dimensionless quantities as well):

liquid phase:

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = 0 \quad (3.1)$$

$$\frac{\partial \hat{p}}{\partial y} + G = 0, \quad \frac{\partial \hat{p}}{\partial x} = \frac{\partial^2 \hat{u}}{\partial y^2} \quad (3.2)$$

$$\frac{\partial^2 \hat{T}}{\partial y^2} = 0 \quad (3.3)$$

gas phase:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (3.4)$$

$$\frac{\partial p}{\partial y} = 0, \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Pr \frac{\partial^2 u}{\partial y^2} \quad (3.5)$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} - \frac{\partial^2 T}{\partial y^2} = q\omega \quad (3.6)$$

$$\rho u \frac{\partial Y}{\partial x} + \rho v \frac{\partial Y}{\partial y} - L_F^{-1} \frac{\partial^2 Y}{\partial y^2} = -\omega \quad (3.7)$$

$$\rho u \frac{\partial X}{\partial x} + \rho v \frac{\partial X}{\partial y} - L_X^{-1} \frac{\partial^2 X}{\partial y^2} = -\nu \omega \quad (3.8)$$

$$\omega = D\rho^2 X Y e^{-\varepsilon/T}, \quad \rho T = 1$$

boundary conditions:

$$\hat{u} = \hat{v} = 0, \quad \hat{T} = T_b \quad \text{at } y = 0 \quad (3.9)$$

$$T = 1, \quad Y = 0, \quad X = X_\infty \quad \text{at } y = H. \quad (3.10)$$

liquid-gas interfacial conditions, at $y = \varphi(x, t)$:

$$m \equiv \rho(-u \frac{\partial \varphi}{\partial x} + v), \quad -\hat{u} \frac{\partial \varphi}{\partial x} + \hat{v} - \frac{\partial \varphi}{\partial t} = \alpha m \quad (3.11)$$

$$u = 0 \quad (3.12)$$

$$\hat{p} - \alpha^2 p = -3S \frac{\partial^2 \varphi}{\partial x^2} \quad (3.13)$$

$$-\frac{\partial \hat{u}}{\partial y} + \alpha^2 \left(Pr \frac{\partial u}{\partial y} + mu \right) = 2M \left(\frac{\partial \hat{T}}{\partial x} + \frac{\partial \varphi}{\partial x} \frac{\partial \hat{T}}{\partial y} \right) \quad (3.14)$$

$$\lambda^{-1} \frac{\partial \hat{T}}{\partial y} - \frac{\partial T}{\partial y} = -mL \quad (3.15)$$

$$L_F^{-1} \frac{\partial Y}{\partial y} = m(Y - 1), \quad L_X^{-1} \frac{\partial X}{\partial y} = mX \quad (3.16)$$

$$Y = K e^{-A/T}. \quad (3.17)$$

The scaling we have adopted yields in the liquid phase equations which are consistent with lubrication theory, and in the gas phase the boundary layer equations.

Several parameters appear in equations (3.1)-(3.17): The gas-phase Prandtl number Pr , which has been defined earlier, represents the ratio of viscous to thermal diffusivities. The fuel and oxidant Lewis numbers, $L_F = \mathcal{D}_{th}/\mathcal{D}_F$ and $L_X = \mathcal{D}_{th}/\mathcal{D}_X$, represent the ratio of the thermal diffusivity of the mixture to the mass diffusivities of fuel or oxidant, respectively. Although Lewis numbers are typically near one, small variations above or below one are known to have significant effects on combustion processes. Such variations could result either because of different mobility of the fuels considered or because of the existence of an abundant inert in the gas phase that could alter the average thermal diffusivity. Heavier fuels such as hydrocarbons, for example, have a larger value of L_F than alcohols and, the value of L_X could be reduced by diluting the ambient with CO_2 , say, because of its larger heat capacity. The heat release parameter q is given by $q = (Q/c_{p_g} T_\infty)(Y_\ell/\nu_F W_F)$. In the parametric study presented below the related parameter $q' = qX_\infty/\nu$, with $\nu = \nu_X W_X/\nu_F W_F$ the mass-weighted stoichiometric coefficient, is found to be more useful; it represents the ratio of the energy liberated per unit mass of available oxidant to the total energy in the ambient gas. The latent heat parameter L is given by $L = L_v/c_{p_g} T_\infty$. The surface tension parameter $S = \sigma_0 \rho_\ell h \varepsilon^2/3\mu_\ell^2$ and Marangoni number $M = \gamma h T_\infty \rho_\ell/2\mu_\ell^2$ measure the level of surface tension and its gradient, respectively; they will be further discussed below. The reciprocal of the Froude number $G = \rho_\ell^2 h^3 |\mathbf{g}|/\mu_\ell^2$ measures the importance of gravity relative to inertia effects. The Damköhler number $D = \nu_F \rho_g Y_\ell h^2 \mathcal{B}/W_X \mathcal{D}_{th}$ is the ratio of the diffusion time to the chemical reaction time and is typically large suggesting that the chemical reaction

rate is relatively fast. The activation energy parameter $\mathcal{E} = E/R^\circ T_\infty$ measures the sensitivity of the chemical reaction rate to temperature. Finally, the equilibrium coefficients $K = (W_F/Y_t \bar{W}) \exp[L_v/R^\circ T_B/\bar{W}]$ and $A = \bar{W}L_v/R^\circ T_\infty$ appear from scaling the Clausius-Clapeyron relation. With the exception of the Damköhler number D , all the parameters will be treated as $O(1)$ quantities.

4. Evolution equation

4.1. The gas phase.

Although the mathematical formulation given above allows for a general external flow, we shall be concerned here with the special case in which the ambient gas is quiescent. This assumption, together with the requirement (3.12) imply that $u \equiv 0$ and, consequently, $p = 0$. The fuel vapor emanating from the surface of the film is transported everywhere vertically with a mass flow rate $m = \rho v$ that depends on the transverse coordinate and on time.

As noted earlier, the Damköhler number D is typically large. In the limit $D \rightarrow \infty$, known as the Burke-Schumann limit (Burke & Schumann 1928), the chemical reaction proceeds very fast so that fuel and oxidant cannot co-exist. Complete combustion occurs along a reaction sheet, located at $y = y_f(x, t)$ say, where the fuel and oxidant meet at stoichiometric proportions. These conditions, which can be expressed as jump relationships across the sheet (c.f. Cheatham & Matalon 2000)

$$[T]_{y_f^-}^{y_f^+} = [Y]_{y_f^-}^{y_f^+} = [X]_{y_f^-}^{y_f^+} = 0 \quad (4.1)$$

$$\frac{1}{q} \left[\frac{\partial T}{\partial y} \right]_{y_f^-}^{y_f^+} = -\frac{1}{L_F} \left[\frac{\partial Y}{\partial y} \right]_{y_f^-}^{y_f^+} = -\frac{1}{\nu L_X} \left[\frac{\partial X}{\partial y} \right]_{y_f^-}^{y_f^+}, \quad (4.2)$$

together with the requirements

$$Y|_{y_f^+} = 0, \quad X|_{y_f^-} = 0, \quad (4.3)$$

are sufficient to determine the combustion field as well as the instantaneous location of the reaction sheet. To this end one needs to solve the transport equations

$$m \frac{\partial T}{\partial y} - \frac{\partial^2 T}{\partial y^2} = 0$$

$$m \frac{\partial Y}{\partial y} - L_F^{-1} \frac{\partial^2 Y}{\partial y^2} = 0, \quad m \frac{\partial X}{\partial y} - L_X^{-1} \frac{\partial^2 X}{\partial y^2} = 0$$

on either side of the reaction sheet subject to (4.1)-(4.3) and the boundary conditions (3.10) and (3.16). One finds

$$Y = \begin{cases} 1 - e^{-m L_F (y_f - y)} & y < y_f \\ 0 & y > y_f \end{cases}$$

$$X = \begin{cases} 0 & y < y_f \\ X_\infty \left(\frac{e^{m L_X (y - y_f)} - 1}{e^{m L_X (H - y_f)} - 1} \right) & y > y_f \end{cases}$$

$$T = \begin{cases} T_s + \{1 - T_s + q[e^{m(H-y_f)} - 1]\} \frac{e^{my} - e^{m\varphi}}{e^{mH} - e^{m\varphi}} & y < y_f \\ 1 + \{T_s - 1 + q[1 - e^{-m(y_f-H)}]\} \frac{e^{mH} - e^{my}}{e^{mH} - e^{m\varphi}} & y > y_f \end{cases}$$

where T_s denotes the (surface) temperature at the interface that remains to be determined. The location of the reaction sheet is given by

$$y_f = H - \frac{1}{m} \ln[(1 + \nu^{-1} X_\infty)^{1/Lx}] \quad (4.4)$$

and the flame temperature T_f is obtained by evaluating T at $y = y_f$.

The only coupling between the liquid and gas phases remains in the mass and energy balances (3.11) and (3.15) and the Clausius-Clapeyron equation (3.17).

4.2. The liquid phase.

The temperature in the film is easily obtained by integrating (3.3), leading to

$$T = T_b + (T_s - T_b) \frac{y}{\varphi}.$$

Integrating the momentum equations (3.2) and applying the conditions (3.9) and (3.13)-(3.14) one finds

$$\begin{aligned} \hat{p} &= -3S \frac{\partial^2 \varphi}{\partial x^2} + G(\varphi - y) \\ \hat{u} &= E_1 y^2 + E_2 y; \quad \hat{v} = - \left(\frac{\partial E_1}{\partial x} \right) \frac{y^3}{3} - \left(\frac{\partial E_2}{\partial x} \right) \frac{y^2}{2} \end{aligned}$$

where

$$\begin{aligned} E_1 &= \frac{1}{2} G \frac{\partial \varphi}{\partial x} - \frac{3}{2} S \frac{\partial^3 \varphi}{\partial x^3} \\ E_2 &= 3S \varphi \frac{\partial^3 \varphi}{\partial x^3} - 2M \frac{\partial T_s}{\partial x} - G \varphi \frac{\partial \varphi}{\partial x}. \end{aligned}$$

At this stage the velocity and pressure fields are completely determined in terms of the yet unknown function $\varphi(x, t)$.

4.3. Interfacial conditions.

When the kinematic condition (3.11) is applied at the liquid-gas interface and use is made of the solutions in the gas and liquid phases, one finds

$$\frac{\partial \varphi}{\partial t} = -S \frac{\partial}{\partial x} \left(\varphi^3 \frac{\partial^3 \varphi}{\partial x^3} \right) + M \frac{\partial}{\partial x} \left(\frac{\partial T_s}{\partial x} \varphi^2 \right) + \frac{1}{3} G \frac{\partial}{\partial x} \left(\varphi^3 \frac{\partial \varphi}{\partial x} \right) - \alpha m \quad (4.5)$$

with m and T_s obtained from

$$m = \frac{L_F^{-1}}{H - \varphi} \ln \left\{ \frac{(1 + \nu^{-1} X_\infty)^{L_F/Lx}}{1 - K e^{-A/T_s}} \right\}, \quad (4.6)$$

$$\frac{1 - T_s + q \left[(1 + \nu^{-1} X_\infty)^{1/Lx} - 1 \right]}{e^{m(H-\varphi)} - 1} - \lambda^{-1} \frac{T_s - T_b}{m\varphi} = L, \quad (4.7)$$

which follow from applying conditions (3.15) and (3.17). The problem thus reduces to a single nonlinear partial differential equation (4.5) for the unknown $\varphi(x, t)$. It should be noted that equation (4.6) implies that T_s must satisfy the inequality $K \exp(-A/T_s) < 1$

which, in dimensional form, simply states that the surface temperature must be below the boiling temperature T_B .

5. Planar interface

The planar interface, which we shall denote here by $y = \eta(t)$ for convenience, recedes at a rate

$$\frac{d\eta}{dt} = -\alpha m \quad (5.1)$$

starting from the initial value $\eta(0) = 1$ to the disappearance of the film at $t = t_E$. Of primary interest is to determine the total evaporation time t_E , the surface temperature T_s , the evaporation or burning rate m and the flame standoff distance and temperature y_f and T_f , respectively.

5.1. Isothermal film

It is instructive to consider first the special case of an isothermal film. The temperature in the liquid is constant and remains equal to T_b at all times, so that $T_s = T_b$. This condition replaces the surface equilibrium condition (4.6) which must therefore be abandoned. Substituting in (4.7) and solving for m one obtains

$$m = \frac{\ln(1+B)}{H-\eta}, \quad (5.2)$$

where

$$B = \frac{1}{L} \left\{ 1 - T_b + q \left[(1 + \nu^{-1} X_\infty)^{1/L_x} - 1 \right] \right\} \quad (5.3)$$

is the conventional transfer number (Spalding 1952). The numerator of (5.3) consists of the difference in enthalpy between the ambient and the interface added to the heat released by the chemical reaction. Thus B is the ratio of the added energy which enhances the transfer to the latent heat of vaporization resisting the transfer. In the absence of combustion $B = (1 - T_b)/L$, which is the transfer number for the pure vaporization case. Equation (5.1), with m given by (5.2), can now be integrated to give

$$\eta = H - \sqrt{(H-1)^2 + 2\alpha \ln(1+B) t}$$

from which the total time for film evaporation can be deduced by setting $\eta = 0$, giving

$$t_E = \frac{2H-1}{2\alpha \ln(1+B)}. \quad (5.4)$$

Clearly $t_E > 0$ since $H > 1$. Furthermore, t_E increases with increasing L and decreasing q . By rewriting (5.4) in dimensional form it is readily seen that the characteristic evaporation time is indeed $(h^2 \rho_\ell / \mathcal{D}_{\text{th}}) \rho_g$, which is the unit of time we have chosen in scaling the equations divided by α . As noted earlier the factor α was introduced for convenience and has no particular physical meaning; it is used below as a scaling factor to enhance graph visualization. With m and B known, one obtains for the flame position and temperature

$$y_f = H - (H - \eta) \frac{\ln[(1 + \nu^{-1} X_\infty)^{1/L_x}]}{\ln[1+B]}. \quad (5.5)$$

and

$$T_f = T_b + L \left[\frac{1+B}{(1 + \nu^{-1} X_\infty)^{1/L_x}} - 1 \right]. \quad (5.6)$$

The flame temperature remains constant at all times. The flame position varies in time in a similar way as η ; its initial position depends on the oxidant concentration or mixture strength.

The results presented above are independent of the fuel Lewis number L_F which, as we shall see, is a consequence of the isothermal assumption. There is, however, an explicit dependence on the oxidant Lewis number L_X ; increasing L_X , for example by appropriately diluting the ambient gas, causes a decrease in B and therefore in the burning rate, thus extending the evaporation time t_E .

Since for an isothermal film, all the heat available for evaporation is provided from the gas phase, an increase in the ambient temperature results in faster evaporation. Thus a decrease in T_b , the ratio of the film to the ambient temperature, causes an increase in B which produces a larger value of m and a shorter evaporation time t_E . Consistent with that we find that the flame temperature T_f decreases and the flame moves away from the film surface.

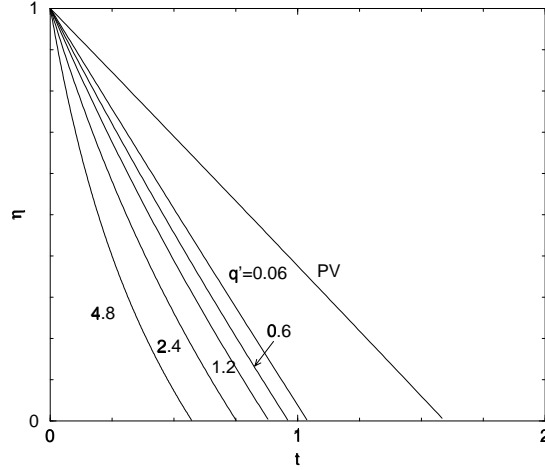


FIGURE 2. The position of a planar interface as a function of time for various values of the heat release parameter q' . Here PV corresponds to the case of pure vaporization.

5.2. Variable temperature film

When there is a variation in temperature throughout the film, the surface temperature T_s is an unknown that must be determined along with m and η from equations (4.6)-(4.7) and (5.1). We can write for the mass burning rate, similar to (5.2), an expression of the form

$$m = \frac{\ln(1 + B_{\text{eff}})}{H - \eta}$$

except that now it depends on an “effective transfer number” given by

$$B_{\text{eff}} = \frac{(1 + \nu^{-1} X_{\infty})^{1/L_X}}{(1 - K e^{-A/T_s})^{1/L_F}} - 1.$$

The surface temperature is determined from

$$\frac{1 - T_s + q[(1 + \nu^{-1} X_{\infty})^{1/L_X} - 1]}{B_{\text{eff}}} - \lambda^{-1} \frac{T_s - T_b}{m \eta} = L.$$

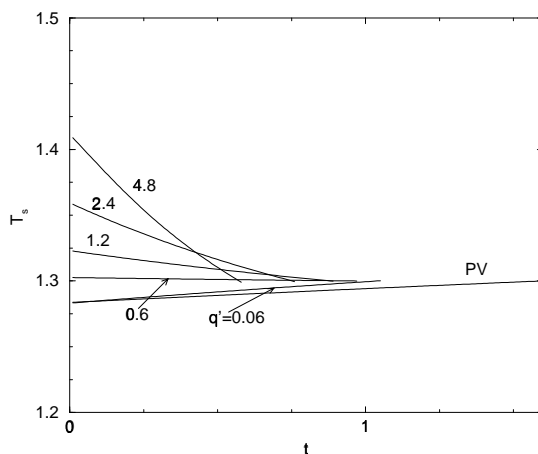


FIGURE 3. The surface temperature of a planar interface as a function of time for various values of the heat release parameter q' . Here PV corresponds to the case of pure vaporization.

As before B_{eff} represents the empetous to resistance of heat transfer, but because the resistance to the transfer depends on the local equilibrium, it is a function of time. For an isothermal film, obtained formally by taking the limit $\lambda \rightarrow \infty$, we see that B_{eff} reduces to the conventional transfer number B . Because of the transcendental nature of these relations, solutions are sought numerically. The results are shown in figure 2 and 3 where the dependence of η and T_s are plotted against time for different values of q' which, as noted earlier, is the more relevant parameter. The calculations reported are based on the following parameter values which are representative of alcohol and hydrocarbon fuels burning in air:

$$\begin{aligned} \lambda = 0.2, \quad \mu = 2, \quad \nu^{-1}X_\infty = 0.067, \quad H = 6, \\ L = 1.3, \quad A = 20, \quad K = 6 \times 10^5, \quad T_b = 1.3, \\ L_F = 1, \quad L_X = 1, \quad Pr = 0.7. \end{aligned} \quad (5.7)$$

Based on the value of μ and Pr , the factor $\alpha = 28.5$. The pure vaporization case is denoted by PV and has the longest evaporation time. The dependence of η on t which is close to linear for the pure vaporization case, becomes significantly nonlinear as q' increases.

For the pure vaporization case the surface of the film receives heat only from the hotter liquid below it. As the surface recedes, its heats up reaching T_b as $t \rightarrow t_E$. The surface temperature thus increases in time during the whole process. With sufficient combustion, however, this trend is reversed and the slope of T_s changes from positive to negative. The surface is now being heated mostly from above and cools off as it recedes, approaching T_b as $t \rightarrow t_E$. Since T_s can be significantly higher in this case, the evaporation time t_E is much shorter. The dependence of t_E on q' is shown in figure 4 along with the equivalent result for the case of an isothermal film. The evaporation times for the constant temperature film are much smaller, as expected, because by replacing the Clausius-Clapeyron equilibrium condition with a constant T_s , it is implicitly implied that the surface temperature is at or near the boiling temperature.

Unlike the constant temperature case, the results here depend on both Lewis numbers L_F and L_X . It is found that as L_F increases, the flame moves closer to the film thereby increasing its surface temperature. The film, however, evaporate slower because of the decrease in the fuel-vapor mobility. Increasing L_X also causes a slower evaporation of the

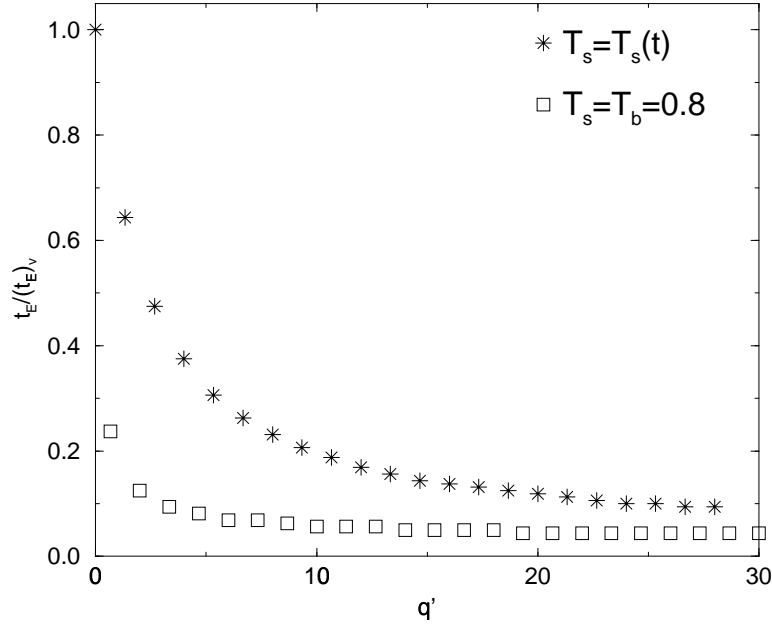


FIGURE 4. Evaporation time of a planar interface as a function of the heat release parameter q' . Time in the ordinate has been scaled with respect to the evaporation time for pure vaporization.

film, as we found earlier. As a result of the reduced mobility of the oxidant, the flame is located further away from the surface. This causes a reduction in heat flux to the surface and a slower evaporation process.

6. Spatially varying film

When the interface is no longer planar, gravity and surface tension play a significant role on the dynamics, as is evident from the evolution equation (4.5). In the following we shall consider in turn the response of the system to small disturbances, which permits linearizing the equation about the mean planar interface, and to finite disturbances where nonlinear effects become important.

6.1. Linear stability

We shall refer to the planar interface $y = \eta(t)$ discussed in the previous section as the basic state. At time $t = 0$ the surface of the film is slightly disturbed such that $\varphi(x, 0) = 1 + \psi_0 e^{ikx}$ with k the transverse wave-number of the disturbance and ψ_0 its amplitude. The disturbance is first assumed small so that $|\psi_0| \ll 1$. We thus seek solutions of the form $\varphi(x, t) = \eta(t) + \psi(t) e^{ikx}$, substitute into equations (4.5)-(4.7) and linearize about the mean position, to obtain

$$\frac{1}{\psi} \frac{d\psi}{dt} = - [b_1(t) k^4 + b_2(t) k^2 + b_3(t)] \quad (6.1)$$

where

$$b_1 = S \eta^3$$

$$b_2 = M \frac{H(\bar{T}_s - T_b)}{C_1 + C_2} \eta^2 + \frac{1}{3} G \eta^3$$

$$b_3 = \alpha \left\{ \frac{\ln(1 + B_{\text{eff}})}{(H - \eta)^2} + \delta \frac{H(\bar{T}_s - T_b)}{C_1 + C_2} \right\}.$$

Here \bar{T}_s denotes the surface temperature of the basic state, and

$$\delta = \frac{L_F^{-1} A K e^{-A/\bar{T}_s}}{\bar{T}_s^2 (1 - K e^{-A/\bar{T}_s})}$$

$$C_1 = \frac{\lambda \eta^2 \ln(1 + B_{\text{eff}})}{B_{\text{eff}}} \left\{ 1 + \frac{\delta B_{\text{eff}}}{1 + B_{\text{eff}}} \left[1 + q(1 + \nu^{-1} X_\infty)^{1/L_x} - q - \bar{T}_s \right] \right\}$$

$$C_2 = \eta(H - \eta) \left\{ 1 - \delta \frac{\bar{T}_s - T_b}{\ln(1 + B_{\text{eff}})} \right\}.$$

The basic state is considered stable, for a given k , if $\psi(t)$ is a monotonic decreasing function; it is considered unstable if $\psi(t)$ is a monotonic increasing function. In some instances, however, the amplitude of a disturbance starts decreasing, reaches a minimum value and then increases until the film's disappearance at $t = t_E$. In such a case we shall identify the state corresponding to $d\psi/dt = 0$ as the marginal state and the corresponding time, $t = t_*$, as the time of the onset of instability. Although this definition is somewhat arbitrary, because the disturbance may not grow in the remaining time to a sufficient size, it nevertheless represent the time when a significant change may occur in the film's evolution.

For an isothermal film, $\bar{T}_s = T_b$ and B_{eff} is replaced by the conventional transfer number B given by (5.3). The coefficients b_1, b_2, b_3 simplify significantly and equation (6.1) can be integrated exactly to yield

$$\psi(\eta) = \psi_0 \frac{H - 1}{H - \eta} \exp \left\{ -\alpha \frac{S k^4 + \frac{1}{3} G k^2}{\ln(1 + B)} \left[\frac{H}{4} (1 - \eta^4) - \frac{1}{5} (1 - \eta^5) \right] \right\}. \quad (6.2)$$

Since at all times, $\eta < 1$, we conclude that *all* disturbances decay in time. Hence, surface tension and gravity have a stabilizing influence on the evaporating film and any small disturbance of the surface decays in time. The absence of the Marangoni term is clearly a consequence of the constant surface temperature assumption.

For the general case, there are temperature variations along the film's interface and equation (6.1) cannot be simply integrated. Furthermore, since the coefficients b_i are time dependent, the range of the growing (unstable) modes change in time. In order to determine the regions of instability we make use of the fact that the right hand side of (6.1) is quadratic in k^2 and that the marginal states are the roots of this quadratic. Since $b_1 > 0$, the parabola is always concave down indicating that the unstable modes, at any given time, are limited to a finite range of wavenumbers. Unstable modes exist only when $b_2^2/4b_1 - b_3 > 0$. The range of wavenumbers is then limited to $k_-^2(t) < k^2 < k_+^2(t)$, where k_-^2 and k_+^2 are the two positive roots of the quadratic; when the smallest root is negative $k_-^2 = 0$. For a given set of parameters the largest and smallest values of k_\pm^2 , produced throughout the entire time interval $0 < t < t_E$, will be used to mark the boundary between stable and unstable states. The region of instability, therefore, represent the wavenumbers that are likely to be unstable at some time during the process. The wave number corresponding to the most rapidly growing mode is

$$k_{max} = \begin{cases} [-b_2/2b_1]^{1/2} & b_2 < 0 \\ 0 & b_2 \geq 0. \end{cases} \quad (6.3)$$

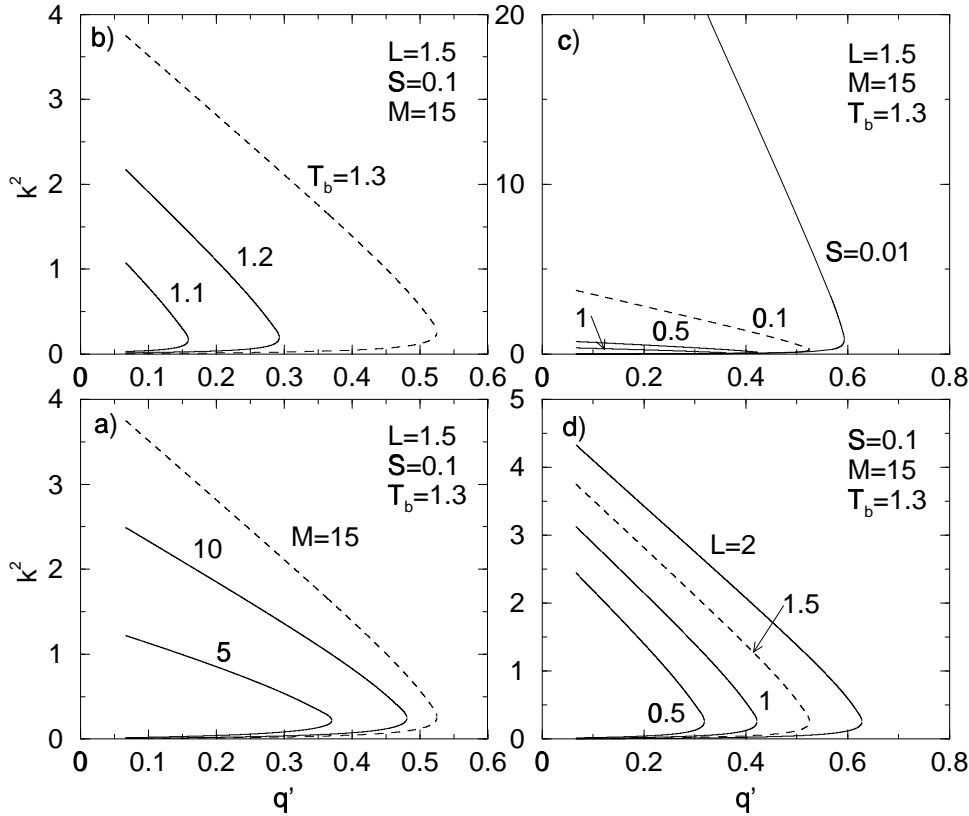


FIGURE 5. Neutral stability curves showing the dependence of the wavenumber k^2 on the heat release parameter q' , for various combinations of the parameters, in the absence of gravity ($G = 0$). The dashed curves in each of the four graphs correspond to the same parameter values. We show the effect of varying: a) the Marangoni number M , b) the bottom temperature T_b , c) the surface tension level S , and d) the latent heat L .

In figure 5 we have plotted curves of marginal stability showing the range of unstable modes (those to the left of the curve) as a function of q' while varying one of the four other parameters: the surface tension S , the Marangoni number M , the ratio of the bottom of the film to the ambient temperatures T_b , and the latent heat L . This figure corresponds to the case $G = 0$, namely in the absence of gravity. Note that the dashed curves in each of the four sub-figures represent the same curve. The results clearly indicate that the range of unstable modes is wider when M , L or T_b are increased, but is reduced when the level of surface tension S is increased. Heat release by combustion acts always as a stabilizing factor as is evident from all subfigures when q' is sufficiently large. The main mechanism for instability is clearly related to thermocapillary or Marangoni effects. Its destabilizing nature can be explained by examining the response of a small disturbance to an otherwise planar interface. Consider first the case of pure vaporization, namely $q' = 0$. A depression in the liquid-gas interface lies in a region that is hotter than its neighbors since $T_b > 1$. Since surface tension is a decreasing function of temperature (see equation 2.21) interfacial stresses drive liquid along the surface away from the depression and towards the crests. As the liquid near the surface is dragged along by viscous forces, the depression deepens further. Gravity and capillary forces cannot prevent this deepening and the film's thickness continues to decrease locally until it ruptures. In the presence

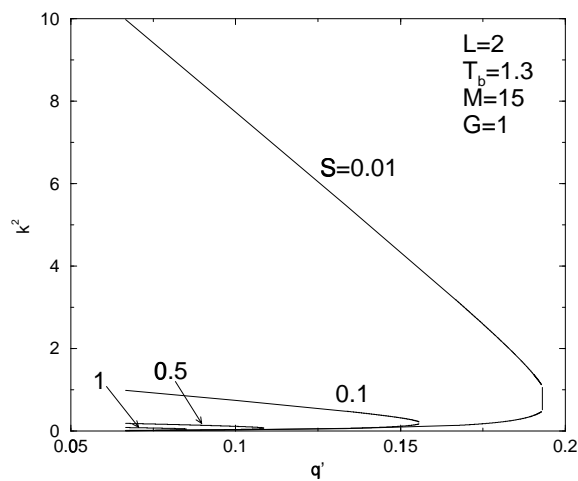


FIGURE 6. Neutral stability curves showing the dependence of the wavenumber k^2 on the heat release parameter q' for a selected set of parameter values, in the presence of gravity ($G = 1$).

of combustion the heat released in the gas phase is partially transported towards the interface. If the net heat flux at the interface is directed towards it, which would be the case when q' is sufficiently large, the intruding segments of the disturbed interface would lie in regions that are hotter than their neighbors. The flow of liquid near the interface is now directed towards the depressions, thus stabilizing the film. While heat release and thermocapillary effects are of primary importance in stabilizing/de-stabilizing the film, the role of the remaining parameters is to amplify or diminish these effects. Thus, for example, when T_b increases the temperature difference between the depressed segments of the interface and their neighbors is also increased, thus enhancing the destabilizing mechanism. Similarly an increase in the latent heat L , which increases the resistance to heat transfer from the gas phase, reduces the stabilizing mechanism. Capillary forces, however, always act to stabilize the film.

Figure 6 is similar to the sub-figure 5d, except that now $G = 1$. Gravity here, is the dominant effect and is clearly a stabilizing mechanism. Note that the critical q' for stability is reduced significantly compared to the case when $G = 0$.

6.2. Nonlinear dynamics

To examine the evolution of the interface resulting from a finite amplitude disturbance, the nonlinear partial differential equation (4.5) has been solved numerically on a closed interval with periodic boundary conditions. The integration proceeds from an initial state, corresponding to the one that is likely to grow the fastest based on the linear stability results, until rupture. Rupture is defined when one point of the interface touches the bottom $y = 0$, and the “rupture time” is the time it takes to reach that state. Fourth order central differencing was used to compute the spatial derivatives. The time marching was accomplished by using the Netlib routine LSODA (Petzold & Hindmarsh 1987). This set of routines is a robust package which automatically switches between stiff and nonstiff methods and also utilizes adaptive time steps to move forward or backward in time. A spatial grid refinement was used to determine the optimum number of grid points needed to obtain sufficiently smooth solutions. This was accomplished by increasing the number of grid points until the time to rupture no longer changed. In some cases an accurate rupture time could be found with a grid as low as $n = 40$, but it produced solutions

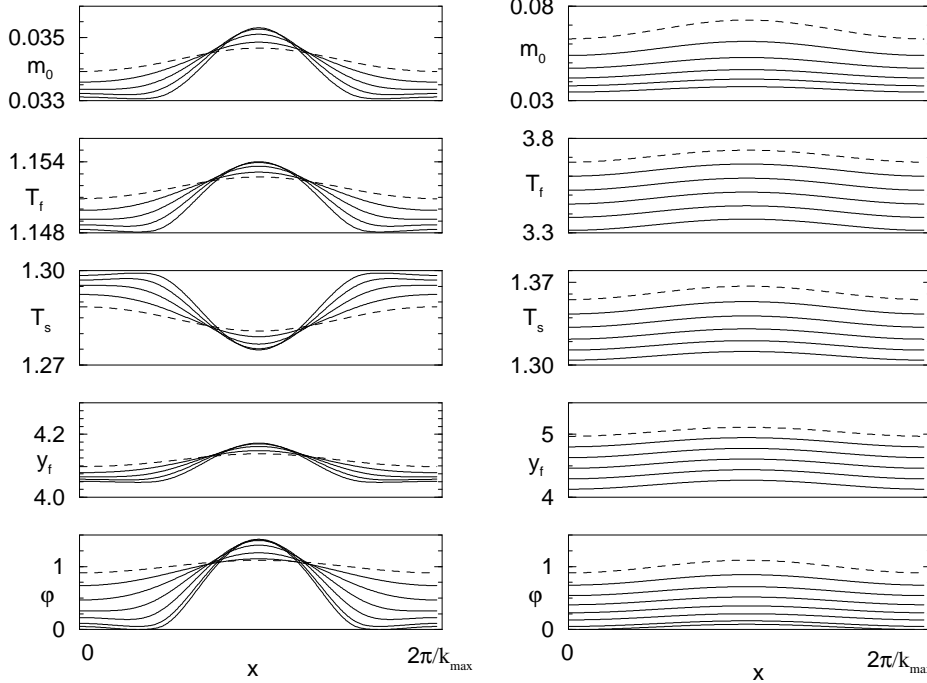


FIGURE 7. Profiles (from bottom to top) of film thickness, flame-sheet position, surface temperature, flame temperature and burning rate showing the spatial variations for consecutive times. The dashed curves in each case represents the initial profile. The graphs on the left correspond to an unstable case with $q' = 0.06$; the graphs on the right correspond to a stable case with $q' = 3$.

which did not appear sufficiently smooth. With $n = 100$ the coarseness was significantly eliminated and the solutions were acceptable. However, due to the changing nature of k_{max} , the wavenumber associated with the fastest growing mode, a grid of $n = 100$ was not always sufficient to produce smooth curves. For example, coarse results were obtained at some instances when k_{max} was small. Because of this, the solutions depicted below utilize $n = 400$ ensuring smooth solutions with reliable rupture times for all values of k_{max} considered here. Time step refinement was not necessary since LSODA selects the most appropriate time step at each iteration.

The computations reported below use, in the most part, the parameter values listed in (5.7). Few exceptions were made in an attempt to demonstrate the different physical effects and to examine the interaction between various terms in the equation. We have fixed the domain of integration to ℓ so that $0 < x < \ell$, and considered periodic boundary conditions:

$$\varphi(0, t) = \varphi(\ell, t), \quad \frac{\partial \varphi}{\partial x}(0, t) = \frac{\partial \varphi}{\partial x}(\ell, t).$$

In order to examine the solution over one wavelength, regardless of the value of the wavenumber k , we rescale the domain of integration by writing $x' = (2\pi/k)(x/\ell)$. Then $0 < x' < 2\pi/k$ and the parameters appearing in the evolution equation are modified to

$$S' = \frac{16\pi^4}{\ell^4} \frac{\sigma_0 \rho_\ell h}{3\mu_\ell^2} \epsilon^2, \quad M' = \frac{4\pi^2}{\ell^2} \frac{\gamma T_\infty h \rho_\ell}{2\mu_\ell^2}, \quad G' = \frac{4\pi^2}{\ell^2} \frac{gh}{\mu_\ell^2 / \rho_\ell^2 h^2}.$$

In the graphs reported below we have chosen $S' = 0.01$, $M' = 10$ and $G' = 0, 1$ or 5 . The

initial condition

$$\varphi(x, 0) = 1 - 0.1 \cos(k_{max}^o x) \quad (6.4)$$

was used, with k_{max}^o the maximizing wavenumber from linear theory at $t = 0$ (see equation 6.3).

The results are summarized in figures 7-11 where we have plotted over one period, from bottom to top, the shape of the interface φ , the flame position y_f , the surface temperature T_s , the flame temperature T_f and the mass burning rate m . The profiles are shown for consecutive times, for fixed intervals spanning the domain $0 \leq t < t_E$. The initial condition is denoted in all the figures by dashed curves.

Figure 7 corresponds to $q' = 0.06$ and illustrates the nonlinear development of an unstable basic state. In this case the film is heated from below ($T_b = 1.3$) and the ambient gas is relatively cold ($T_f \approx 1.15$). Although combustion is complete as a result of the assumed high Damkohler number, the diffusion flame is sitting sufficiently far from the surface. It is located at $y_f \approx 4$ where the edge of the combustion layer is at $H = 6$. Thus, the surface of the film gains heat primarily from the hot liquid beneath it so that the surface temperature T_s is higher where the film is thinner and lower at the protruding sections. As discussed above, thermocapillary forces drive the liquid near the interface away from the depressed regions and towards the crests, thus feeding the growth of the initial perturbation. In contrast, a stable situation is illustrated in figure 7 corresponding to a moderate $q' = 3$. Here the flame temperature ($T_f \approx 3 - 4$) is much higher than the surface temperature ($T_s \approx 1.3$) so that the film gains heat primarily from the gas above it. The surface temperature is thus higher at the crests and thermocapillary forces, which now drive the liquid near the interface towards the depressed regions, act to stabilize the film. The amplitude of the initial perturbation decays in time and the film becomes nearly planar as it continues to recede until $t = t_E$.

In figure 8 we show the time history of the amplitude of the film's interface for selected values of q' including the two values corresponding to the figures just discussed. The amplitude at time t is defined as half the difference between the highest and lowest points of $\varphi(x, t)$. For $q' = 0.06$ the amplitude grows in time during the whole lifetime of the film, a situation which is clearly unstable. By increasing q' the rate of growth diminishes and for $q' = 0.6$ the amplitude decays for all time; a clearly stable configuration. We have also

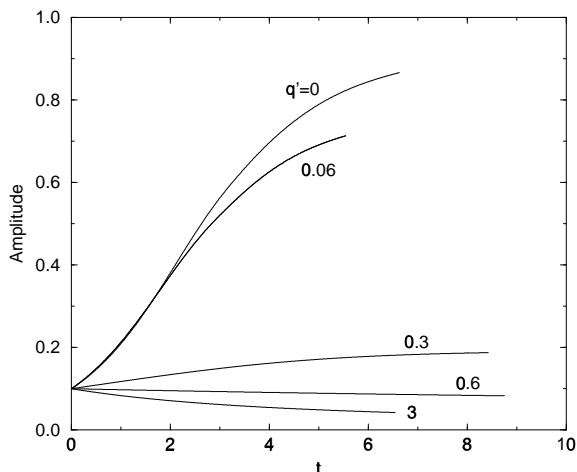


FIGURE 8. The dependence of the amplitude of the disturbed interface on time for various values of the heat release parameter q' .

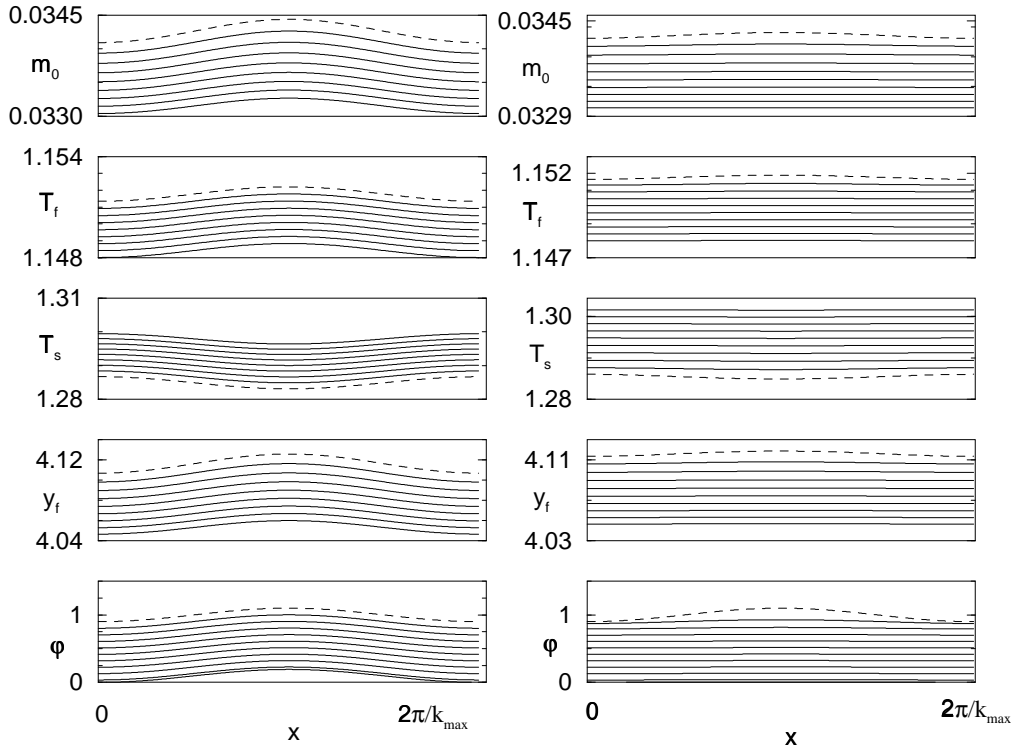


FIGURE 9. Similar to figure 7 with $q' = 0.06$, but in the presence of gravity. The left figure corresponds to $G' = 1$ and the right figure to $G' = 5$.

added in the figure a curve corresponding to the pure vaporization case ($q' = 0$) which is the most unstable scenario. Note that for a stable configuration, the ending time is approximately equal to the total evaporation time t_E of the basic state; it decreases with increasing q' as seen previously in figure 4. For an unstable configuration, however, the ending time appears to decrease with decreasing q' . This behaviour is attributed to the fact that, as the initial perturbation grows, the film ruptures first at one point and this occurs at a time $t < t_E$. The more unstable the interface is the shorter the time to rupture.

Figure 9 shows results corresponding to the same conditions as for the unstable situation depicted in figure 7 but in the presence of gravity. It is interesting to note that, although the surface temperature is higher at the depressed segments of the film and lowest at the crests, thermocapillary effects are not sufficient to promote instability; they are suppressed by gravitational forces. Although in both cases gravity is a dominant effect, the film never reaches a planar shape during the finite time interval $0 < t < t_E$ when $G' = 1$, unlike when $G' = 5$.

In figures 10-11 we show the effect of varying the Lewis numbers on the film dynamics, for unstable situations. Comparing the profiles in figure 10 we see that when $L_F < 1$, corresponding to a higher mass diffusivity of fuel, there is an increase in the mass loss from the film (increasing m) and consequently the flame stands further away from the film. Being closer to the ambient the flame temperature is lower, which explains the more unstable behaviour of the film in this case, compared to the case when $L_F > 1$ (see also figure 12). Comparing the profiles in figure 11 we see that, as a result of an increase in the oxidant diffusivity, the flame stands closer to the film when $L_X < 1$ as opposed to when

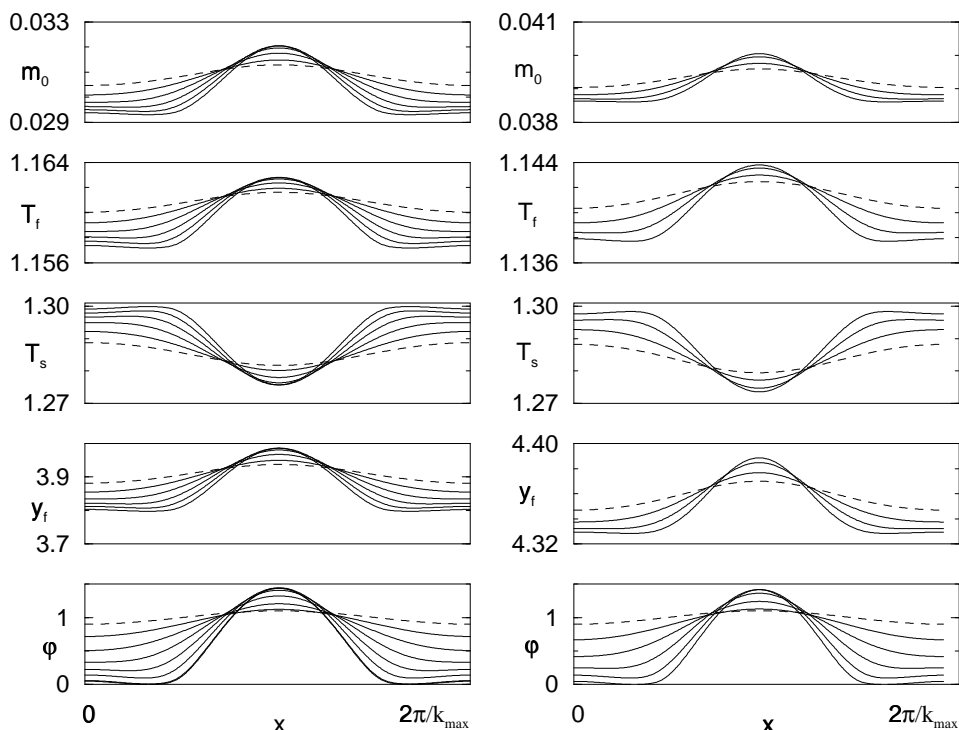


FIGURE 10. Similar to figure 7 with $q' = 0.06$, but with non-unity fuel Lewis numbers. The left figure corresponds to $L_F = 1.2$ and the right figure to $L_F = 0.8$.

$L_X > 1$. Consequently the flame temperature is higher and, since the surface of the film gains more heat from the gas-phase, it is more stable (see also figure 12). Finally, one observes from figure 12 that despite the opposing trends regarding stability, increasing either L_F or L_X lead to an increase in the total evaporation time t_E .

7. Conclusions

In this work we were concerned with the evaporation and combustion of a thin film of liquid fuel. A long-wave approximation has been used to simplify the governing equations to what amount to, boundary layer equations in the gas phase and the lubrication approximation in the liquid phase. When the burning occurs in a quiescent ambient, the problem further reduces to an evolution equation for the film's thickness. What distinguishes this equation from previous studies of thin films is two parameters that couple the dynamics of the film to the gas-phase processes. One of these parameters is the mass of fuel vapor leaving the surface of the film, closely related to the mass of fuel consumed at the diffusion flame, which depends on the total heat conducted to this surface from both phases. The second parameter is the temperature at the surface of the film which depends on the local equilibrium conditions at the liquid-vapor interface.

When the surface of the film remains planar the evolution equation describes the time rate of change of the film's thickness from its initial thickness until its disappearance. The liquid in this case remains stationary and there is only heat conduction across the liquid layer. Clearly combustion enhances the film's evaporation and disappearance. The results that include variables such a flame position and temperature, evaporation or burning rate, film thickness history and total evaporation time, exhibit an explicit dependence

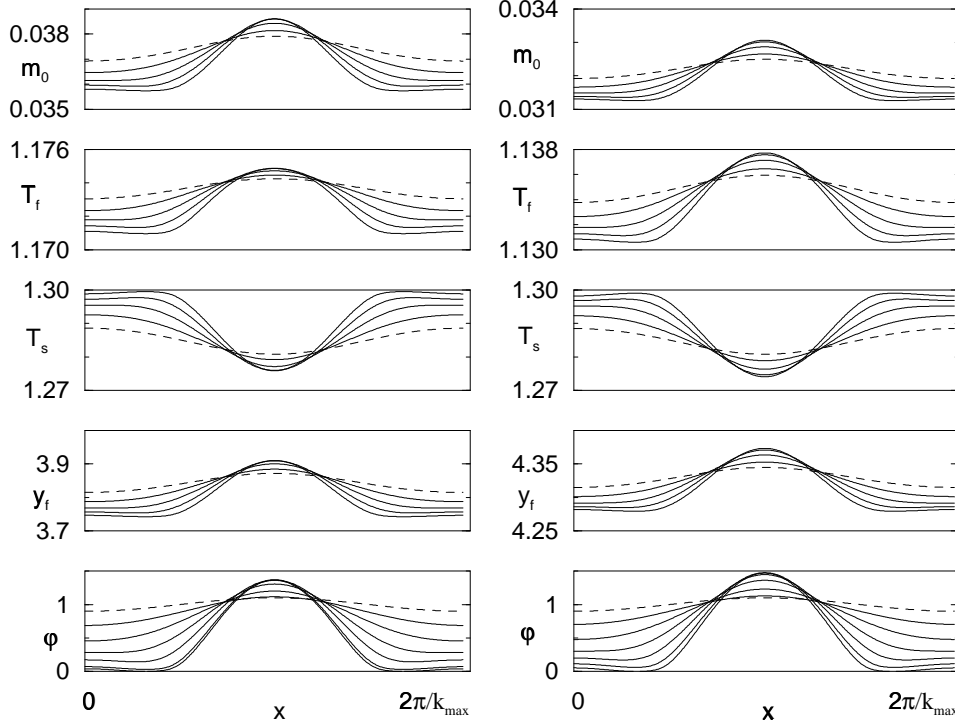


FIGURE 11. Similar to figure 7 with $q' = 0.06$, but with non-unity oxidant Lewis numbers. The left figure corresponds to $L_X = 0.8$ and the right figure to $L_X = 1.2$.

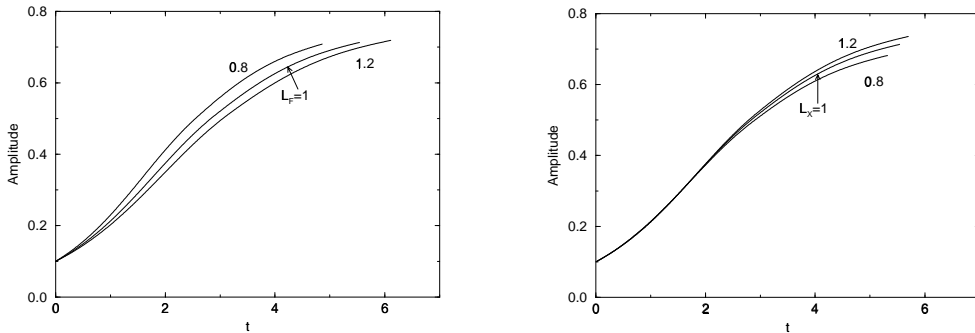


FIGURE 12. The growth of the amplitude: the graph on the left shows variations of L_X for $L_F = 1$; the graph on the right shows variations of L_X for $L_F = 1$.

on the various physico-chemical parameters. Spatial non-uniformities in the initial conditions may grow or decay at a rate different than the rate that the planar film recedes. We have thus examined the response to small disturbances, via a linear theory, and to finite amplitude disturbances by numerically solving the nonlinear evolution equation. The main mechanism that drives the film toward or away from a planar state is thermocapillary effects. In the absence of combustion, or when the heat released is small, the surface of the film gains heat primarily from the liquid beneath it. Thermocapillary effects in this case drive the liquid near the surface in such a way as to cause the depression to deepen further and further. Gravity and capillary forces cannot prevent this

deepening and the film's thickness continues to decrease locally until it ruptures. When the heat release by combustion is sufficiently high, the surface of the film gains heat primarily from the gas-phase and thermocapillary forces act to stabilize the film. The role of all other parameters, such as the temperatures of the ambient and of the bottom of the film, the latent heat of vaporization, the oxidant concentration in the ambient, the Lewis numbers, the level of surface tension and gravity, is to amplify or diminish the thermocapillary forces.

Our results have direct implications on microgravity combustion studies (King & Ross 1998), where the stabilizing effect of gravity is absent and surface tension effects become dominant. One such problem is the ignition and flame spread over liquid fuel beds, see for example Ross (1994). It is known that thermocapillary or Marangoni effect is the driving mechanism for flame spread over fuels that are below the flash temperature. For simplicity, the complex calculations of flame spread over liquid beds assume that the liquid-vapor interface is flat. Our stability results suggest that such an assumption can be justified in the downstream region where the heat conducted back from the diffusion flame stabilizes any perturbation of the surface. However, near the leading edge, since the surface is primarily heated from below, surface deformation can be amplified and therefore must be accounted for.

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REFERENCES

- BURELBACH, J.P., BANKOFF, B.G. & DAVIS, S.H. 1988 Nonlinear stability of evaporating/condensing liquid films. *J. Fluid Mech.*, **195**, 463-494.
- BURKE, S.P. & SCHUMANN, T.E.W. 1928 Diffusion flames. *Industrial Engineering Chemical*, **20** no. 10, 998-1004.
- CHEATHAM, S. & MATALON, M. A general asymptotic theory of diffusion flames with application to cellular instability. *J. Fluid Mech.*, to appear.
- KING M.K. & ROSS, H.D. *AIAA J.* **36**(8), 101-118.
- KRISHNAMOORTHY, S., RAMASWAMY, B. & JOO, S.W. 1995 Spontaneous Rupture of Thin Liquid Films Due to Thermocapillarity: A Full-scale Direct Numerical Simulation *Phys. Fluids* **7**, 2291-2293.
- ORON, A., DAVIS, S.H. & BANKOFF, S.G. 1997 Long-scale evolution of thin liquid films. *Rev. Mod. Phys.*, **69**, No. 3, 931-980.
- PETZOLD, L.R. & HINDMARSH, A.C. 1987 LSODA Computing and Mathematics Research Division, Lawrence Livermore National Laboratory, Livermore, CA.
- ROSS, H.D. 1994 Ignition of and flame spread over laboratory-scale pools of pure liquid fuels. *Progress in Energy and Combustion Science*, **20**(1), 17-64.
- SPALDING, D.B. 1952 The combustion of liquid fuels. *Fourth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, PA, 847-864.
- WILLIAMS, M.B. & DAVIS, S.H. 1982 Nonlinear theory of film rupture *J. Coll. and Int. Sci.*, **90**, No. 1, 220-228.