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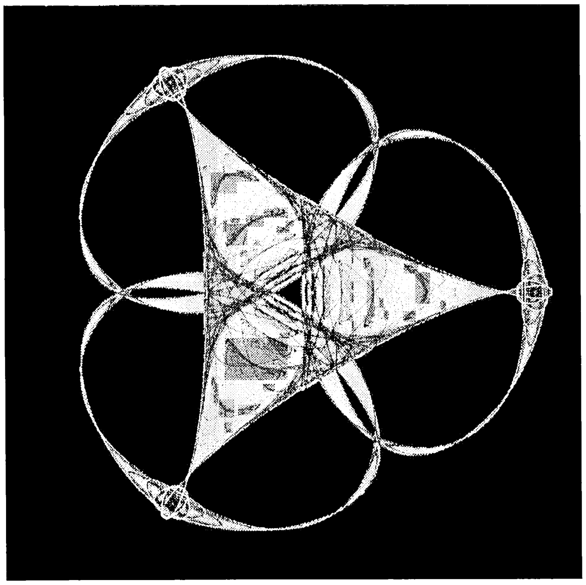

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GENESIS OF GROUPS AND ITS RAMIFICATION FROM n- LATERAL MULTIPLE HYPERGEOMETRIC FUNCTION

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In an attempt to narrow down the gap between theory and applications, the two advanced branches of mathematics, special function and group theory have been brought closer by formation of finite dimensional group for n- lateral multiple hypergeometric functions in our earlier study. Here efforts are being made to generate subgroups, ring homomorphism and isomorphism have been also established for B- function involving two variables.

Key words :- Limit Point / Lauricella function / Homomorphism / identity element.

1. INTRODUCTION

The works of Vilinkin[3] and Talman[2] had provided with a basement for derivation of different special functions of one variable and their properties from the theory of group representations. We, in our earlier work[1] have constructed finite multiplicative group for n- lateral multiple hypergeometric functions.

Here an attempt is being made to go ahead in the direction of establishing further relationship between special functions and group theory by generation of subgroups, cosets, normal subgroup, homomorphism, isomorphism, ring and topological space for our generalized B- function involving two variables.

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2. GENERAL FORMULATION

From the group (S,.) generated by result (3.7) of our earlier study, we can select subgruops of following nature,

(a). Where parameters of later half are zero.

As an example we may cite,

$$S_1 = \{B^{10}, B^{20}, B^{30}\} \quad \text{........(2.1)}$$

Whose first and third elements are inverse of each other and second element is identity .

(b). Where parameters of first half are zero.

As an example, the following subgroup may be mentioned

$$S_2 = \{ B^{01}, B^{02}, B^{03} \} \quad \text{........(2.2)}$$

Here again the first and third elements are inverse of each other and second element is identity.

(c). Where parameters of both half are considered, the subgroup of order 3 in this case shall be

$$S_3 = \{B^{11}, B^{22}, B^{33} \} \quad \text{.........(2.3)}$$

While the subgroup of order 5 shall be

$$S_4 = \{ B^{12}, B^{21}, B^{23}, B^{32},B^{22}\} \quad \text{..........(2.4)}$$

the last element of this subgroup shall be the identity element while first & fourth and second & third elements (taken in pairs) shall be inverse of each other.

Here it is evident that subgroups formed in present case do not follow the general rule (identites of group and its subgroups are same), due to the reason that parameters of one of the two halfs have been omitted.So in this case the iden tity of the group shall be equal to the product of the identities of subgroups.

3. COMPOSITION OF COSETS

Considering the subgroup S_3 (given by equation (2.3)) we can form left and right cosets as,

$$S_3 * B^{01} = \{ B^{11} * B^{01}, B^{22} * B^{01}, B^{33} * B^{01} \}$$

$$= \{ B^{11}, B^{22}, B^{32} \} \qquad \ldots\ldots\ldots\ldots\ldots(3.1)$$

and

$$B^{01} * S_3 = \{ B^{01} * B^{11}, B^{01} * B^{22}, B^{01} * B^{33} \}$$

$$= \{ B^{11}, B^{22}, B^{32} \} \qquad \ldots\ldots\ldots\ldots\ldots(3.2)$$

Which implies the condition required for our subgroup to be a normal subgroup.

4. ISOMORPHISM OF SUBGROUPS

Let $f : S \rightarrow S$ be defined as

$$f(B^{pq}) = B^{qp} \qquad \ldots\ldots\ldots\ldots\ldots(4.1)$$

then it will be proved that f is an isomorphism.

The mapping is one -one onto from mapping defined by relationship (4.1) and homomorphism can be depicted from the following exemplery case,

$$f(B^{10} * B^{03}) = f(B^{13}) = B^{31}$$

$$= B^{01} * B^{30} = f(B^{10}) * f(B^{03})$$

5. FORMATION OF RING

Let us incorporate the element B^{00} (normal B-function) in the Set S of multiplicative group (S,*) obtained in section 3 (result (3.7)) of our earlier work and to binary operators $\oplus$ and $\odot$ be defined as

$B^{ij} \oplus B^{pq}$ = products of non repeated terms of B^{ij} and B^{pq}

$B^{ij} \odot B^{pq}$ = products of repeated terms of B^{ij} and B^{pq}

then (S, $\oplus$) shall be an abelian group and (S, $\odot$) will be a semi-group.

Distributive Law shall hold good in this case as evident from the following example

$$B^{12} \odot [B^{23} \oplus B^{13}] = (B^{12} \odot B^{23}) \oplus (B^{12} \odot B^{13})$$

Here

$$B^{12} \odot [B^{23} \oplus B^{13}] = B^{12} \odot B^{30} = B^{00} \quad \text{............(5.2)}$$

and

$$(B^{12} \odot B^{23}) \oplus (B^{12} \odot B^{13}) = B^{13} \oplus B^{13} = B^{00} \quad \text{............(5.3)}$$

Which demostrates that ring shall be a ring with zero divisor. B^{00} shall be the zero of the ring and B^{22} shall be its unity element.

6. TOPOLOGICAL SPACE

Let us defined the set

$$X = \{ B^{10}, B^{20}, B^{30} \} \quad \text{...............(6.1)}$$

then the different topologies for X can be formed.

The smallest topology for X shall be

$$\Im_1 = \{ B^{00}, X \} \quad \text{...............(6.2)}$$

B^{00} shall be taken as null set.
The largest topology on X shall be

$$\Im_{14} = \{B^{00}, \{B^{10}\}, \{B^{20}\}, \{B^{30}\}, \{B^{10},B^{20}\}, \{B^{20},B^{30}\}, \{B^{10},B^{30}\}, X \} \quad \text{...............(6.3)}$$

Neighbourhood system for every element can be seen as it is evident from the following example

$N(B^{10})$ in $\Im_{14}$ shall be

$$\{ \{B^{10}\}, \{B^{10},B^{20}\}, \{B^{10},B^{30}\}, X \}$$

In the smallest topological space $(X,\Im_1)$

If we consider set $A = \{B^{10},B^{20}\}$

then each point of X shall be a limit point for A and the derived set in this case shall be X itself.

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