A REMARK ON A PRIORI BOUNDS
FOR
GLOBAL SOLUTIONS OF SEMILINEAR EQUATIONS

BY

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#	Author(s) Title
1	**Workshop** Summaries from the September 1982 Workshop on Statistical Mechanics, Dynamical Systems and Turbulence
2	**Raphael De laLlave**, A Simple Proof of C. Siegel's Center Theorem
3	**H. Simpson, S. Spector**, On Copositive Matrices and Strong Ellipticity for Isotropic Elastic Materials
4	**George R. Sell**, Vector Fields in the Vicinity of a Compact Invariant Manifold
5	**Milan Miklavcic**, Non-linear Stability of Asymptotic Suction
6	**Hans Weinberger**, A Simple System with a Continuum of Stable Inhomogeneous Steady States
7	**Bau-Sen Du**, Period 3 Bifurcation for the Logistic Mapping
8	**Hans Weinberger**, Optimal Numerical Approximation of a Linear Operator
9	**L.R. Angel, D.F. Evans, B. Ninham**, Three Component Ionic Microemulsions
10	**D.F. Evans, D. Mitchell, S. Mukherjee, B. Ninham**, Surfactant Diffusion; New Results and Interpretations
11	**Leif Arkeryd**, A Remark about the Final Aperiodic Regime for Maps on the Interval
12	**Luis Magalhaes**, Manifolds of Global Solutions of Functional Differential Equations
13	**Kenneth Meyer**, Tori in Resonance
14	**C. Eugene Wayne**, Surface Models with Nonlocal Potentials: Upper Bounds
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16	**George R. Sell**, Smooth Linearization Near a Fixed Point
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18	**Pierre Collet**, Local C^{∞} Conjugacy on the Julia Set for some Holomorphic Perturbations of $z \to z^2$
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20	**George R. Sell**, Linearization and Global Dynamics
21	**P. Constantin, C. Foias**, Global Lyapunov Exponents, Kaplan-Yorke Formulas and the Dimension of the Attractors for 2D Navier-Stokes Equations
22	**Milan Miklavcic**, Stability for Semilinear Parabolic Equations with Noninvertible Linear Operator
23	**P. Collet, H. Epstein, G. Gallavotti**, Perturbations of Geodesic Flows on Surfaces of Constant Negative Curvature and their Mixing Properties
24	**J.E. Dunn, J. Serrin**, On the Thermodynamics of Interstitial Working
25	**Scott J. Spector**, On the Absence of Bifurcation for Elastic Bars in Uniaxial Tension
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28	**Luis Magalhaes**, The Asymptotics of Solutions of Singularly Perturbed Functional Differential Equations: and Concentrated Delays are Different
29	**Charles Tresser**, Homoclinic Orbits for Flow in R^3
30	**Charles Tresser**, About Some Theorems by L.P. Sil'nikov
31	**Michael Aizenmann**, On the Renormalized Coupling Constant and the Susceptibility in ϕ_4^4 Field Theory and the Ising Model in Four Dimensions
32	**C. Eugene Wayne**, The KAM Theory of Systems with Short Range Interactions I
33	**M. Slemrod, J. E. Marsden**, Spatial Chaos in a Van der Waals Fluid Due to Periodic Thermal Fluctuations
34	**J. Kirkwood, C.E. Wayne**, Percolation in Continuous Systems
35	**Luis Magalhaes**, Invariant Manifolds for Functional Differential Equations Close to Ordinary Differential Equations
36	**C. Eugene Wayne**, The KAM Theory of Systems with Short Range Interactions II
37	**Jean De Canniere**, Passive Quasi-Free States of the Noninteracting Fermi Gas
38	**Elias C. Aifantis**, Maxwell and van der Waals Revisited
39	**Elias C. Aifantis**, On the Mechanics of Modulated Structures
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47	**E. Fabes, D. Stroock**, The L^P-intergrability of Green's Functions and Fundamental Solutions for Elliptic and Parabolic Equations
48	**H. Brezis**, Semilinear Equations in R^N without Conditions at Infinity
49	**M. Slemrod**, Lax-Friedrichs and the Viscosity-Capillarity Criterion
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51	**Andrew Postlewaite, David Schmeidler**, Revelation and Implementation under Differential Information
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59	**Abstracts** for the Workshop on Price Adjustment, Quantity Adjustment, and Business Cycles
60	**Rafael Rob**, The Coase Theorem an Informational Perspective
61	**Joseph Jerome**, Approximate Newton Methods and Homotopy for Stationary Operator Equations
62	**Rafael Rob**, A Note on Competitive Bidding with Asymmetric Information
63	**Rafael Rob**, Equilibrium Price Distributions
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69	**Steve Williams**, Realization and Nash Implementation: Two Aspects of Mechanism Design
70	**Steve Williams**, Sufficient Conditions for Nash Implementation
71	**Nicholas Yannelis, William R. Zame**, Equilibria in Banach Lattices Without Ordered Preferences
72	**W. Harris, Y. Sibuya**, The Reciprocals of Solutions of Linear Ordinary Differential Equations
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75	**J.S. Jordan**, Information Flows Intrinsic to the Stability Economic Equilibrium
76	**J. Jerome**, An Adaptive Newton Algorithm Based on Numerical Inversion: Regularization Post Condition
77	**David Schmeidler**, Integral Representation Without Additivity

A REMARK ON A PRIORI BOUNDS FOR GLOBAL SOLUTIONS OF SEMILINEAR HEAT EQUATIONS

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1. Introduction

Consider the initial boundary value problem on Ω for a semilinear heat equation

$$u_t - \Delta u - u^p = 0 \tag{1}$$

with the Dirichlet boundary condition

$$u(x,t) = 0 \quad \text{on} \quad \partial\Omega \tag{2}$$

and initial data

$$u(x,0) = u_0(x) \geq 0 \quad \text{in} \quad \Omega \ , \tag{3}$$

where Ω is a smoothly bounded domain in $\mathbb{R}^n$ and $p>1$. Because of the maximal principle and (3) the solution $u(\cdot,t)$ must be positive in Ω for $t>0$ unless $u_0(x) \equiv 0$ so the term u^p is well-defined for every p. The asymptotic behavior of global solution $u(\cdot,t)$ as $t \to \infty$ has been studied by many authors; see for example [1,4,5]. To classify initial data by asymptotic behavior of solutions, it is important to study whether global solution has <u>a priori</u> bounds for all time. Cazenave and Lions [1] prove that every global solution is bounded in $\Omega \times (t_0,\infty)$ for every $t_0>0$ provided that $n/2 < (p+1)/(p-1)$ (<u>i.e.</u> $p <(n+2)/(n-2)$ or $n \leq 2$). For $p < 1 + 12/(3n-4)$ or $n = 1$, their bounds depend only on some norm of initial data. However, if $n > 1$ and $p \geq 1+12/(3n-4)$, the dependence of their bounds on initial data is unclear.

Our main goal in this paper is to remove this technical restriction of p. We shall show that every global solution has <u>a priori</u> bounds depending only on some norm of initial data provided that $n/2 < (p+1)/(p-1)$. This condition of p seems optimal because otherwise there is a global unbounded (unfortunately) weak solution (cf. [5]). To avoid later confusion, by a 'solution' of (1-3) in $\Omega \times [0,T)$ we shall always mean a continuous function in $\overline{\Omega} \times [0,T)$ which is smooth enough for each term appearing (1) to be continuous in $\Omega \times (0,T)$.

Main Theorem. *Let* $n/2 < (p+1)/(p-1)$. *Suppose that* $u(x,t)$ *is the solution of* (1-3) *in* $\Omega \times [0,\infty)$ *with* $u_0 \in C(\overline{\Omega})$. *Then there is a constant* M *depending only on* $\sup \{u_0(x);\ x \in \Omega\}$ *such that*

$$u(x,t) < M \quad \text{for} \quad x \in \Omega,\ t > 0.$$

As a simple application we obtain a closedness of the set of initial data which gives the global existence. To be precise, we denote

$$C = \{\ u_0 \in C(\overline{\Omega});\quad u_0 > 0\ \}$$

$$K = \{u_0 \in C\ ;\ \text{global solution of (1-3) exists with}\ \ u(x,0) = u_0(x)\}.$$

We should note that $C \setminus K$ is not empty; see for example [1,4,5,6].

Corollary. *Let* $n/2 < (p+1)/(p-1)$. *The set* K *is closed in* C.

To prove main theorem we combine energy identities [1] and a scaling argument which is previously used by Gidas and Spruck [2] for elliptic equations. Our method works for all p, $n/2 < (p+1)/(p-1)$. Although we discuss only a special nonlinear term, our argument is valid for a more general class of nonlinearity which will be briefly explained in Sect. 3.

In Sect. 2 we give a crucial lemma of our method and in Sect. 3 we prove our main results. As a by-product of the lemma we shall prove in Sect 4 that the energy blows up if the solution blows up in finite time provided that $n/2 < (p+1)/(p-1)$.

2. A Priori Estimates

We shall derive *a priori* bounds for solutions assuming boundedness of another norm. The following lemma, which seems of itself interesting, is a crucial step of our argument.

Lemma. *Let* u *be a solution of* (1-3) *in* $Q = \Omega \times [0,T)$. *Suppose that* u *satisfies the estimate*

$$\int_0^T\int_\Omega |u_t|^2 \, dxdt < N < \infty \tag{4}$$

and that for given $t_o > 0$

$$\sup_Q u \text{ is attained in } \Omega\times(t_o,T). \tag{5}$$

Then there is a constant A _independent of_ u, u_o _and_ T (_depending on_ N _and_ t_o) _such that_

$$u(x,t) < A \text{ in } Q.$$

Our proof appeals to a scaling argument and contradiction which are used by Gidas and Spruck [2] for elliptic equations. Most of the argument is parallel to [2] except that we use parabolic theory and (4). However since the proof is by contradiction, we give not only the major part (the use of (4)) but also the outline of the whole proof.

Although we consider only a special nonlinear term the following proof works even if we replace u^p by f(u) which has the property

$$\lim_{z\to+\infty} f(z)/z^p > 0 \tag{6}$$

for some p such that $n/2 < (p+1)/(p-1)$.

Proof. We first introduce a parabolic scaled function. Our proof is by contradiction.

Suppose Lemma is false. Then there exist a sequence of function $u_k(x,t)$ satisfying (1-4) with $T = T_k > 0$ and a sequence of points (x_k,t_k), $t_k > t_o$ such that

$$M_k = \sup \{u_k(x,t);\ x \in \Omega\ ,\ t \in (0,T_k)\}$$

$$= u_k(x_k,t_k) \to \infty \quad \text{as} \quad k \to \infty\ .$$

Since $\overline{\Omega}$ is compact, we may assume $x_k \to x_\infty \in \overline{\Omega}$ as $k \to \infty$ by choosing subsequences. Let λ_k be a sequence of positive numbers such that

$$\lambda_k^{2/(p-1)} M_k = 1.$$

Since $M_k \to \infty$, evidently $\lambda_k \to 0$. We now define the scaled function

$$v_k(y,s) = \lambda_k^{2/(p-1)} u_k(x_k + \lambda_k y, t_k + \lambda_k^2 s) .$$

Clearly,

$$v_k(0,0) = 1, \tag{7}$$

and

$$v_k(y,s) \leq 1 \quad \text{where } v_k \text{ is defined.} \tag{8}$$

Following [2] we divide situation into two cases depending on whether $x_\infty \in \Omega$ or $x_\infty \in \partial\Omega$.

Case 1. $x_\infty \in \Omega$. (i) (Domain of definition of v^k). Let $Q(r)$ be a parabolic cylinder with radius r in $\mathbb{R}^{n+1}$ defined by

$$Q(r) = \{(y,s) \in \mathbb{R}^{n+1} , |y| < r, - r^2 < s < 0\} .$$

Let $2d$ denote the minimum of $2\sqrt{t_0}$ and the distance of x_∞ and $\partial\Omega$. Since $x_k \to x_\infty$, we see $v_k(y,s)$ is defined in $Q(d/\lambda_k)$ for sufficiently large k.

(ii) (The equation for v_k). Because (1) is invariant under our scaling, v_k solves

$$v_{ks} - \Delta_y v_k - v_k^p = 0 \quad \text{in} \quad Q(d/\lambda_k).$$

(iii) (Convergence of v_k). We apply parabolic L^p regularity theory [3] to get uniform bounds in some Sobolev space from (8). More precisely for given any R such that $Q(R) \subset Q(d/\lambda_k)$, $\{v_k\}$ is bounded in $W_q^{2,1}(Q(R))$ for any $q > n$.

As is usual, we find a subsequence (still denoted v_k) which converges uniformly to some function $v \geq 0$ in any $Q(R)$ and v is defined in $\mathbb{R}^n \times (-\infty, 0)$. We may assume, taking yet another subsequence if necessary that ∇v_k converges uniformly to ∇v in $Q(R)$ and that v_{ks} converges weakly to v_s in $L^2(Q(R))$.

(iv) (<u>The equation for</u> v). Combining (ii) and (iii) shows that v solves

$$v_s - \Delta_y v - v^p = 0 \text{ in } \mathbb{R}^n \times(-\infty, 0).$$

From (7) we see also

$$v(0,0) = 1.$$

(v) ($v_s = 0$). This is the only step we use (4). A simple manipulation shows that

$$\iint_{Q(d/\lambda_k)} |v_{ks}|^2 \, dyds = \lambda_k^\sigma \int_{t_k-d^2}^{t_k} \int_{|x-x_k| < d} |u_t|^2 \, dxdt$$

with $\sigma = -n + 2 + 4/(p-1)$. Applying (4), we get

$$\int\int_{Q(R)} |v_{ks}|^2 \, dyds \leq \lambda_k^\sigma N \to 0 \text{ as } k \to \infty ,$$

since $n/2 < (p+1)/(p-1)$ is equivalent to $\sigma>0$. This yields $v_s = 0$ in $Q(R)$ because v_{ks} converges weakly in $L^2(Q(R))$ to v_s and norm is lower semicontinuous under weak convergence. Here R is arbitrary so v_s identically vanishes in $\mathbb{R}^n \times (-\infty,0)$.

(vi) (<u>Application of a Liouville theorem</u>). Since $v_s = 0$, $v \geq 0$ solves

$$\Delta v + v^p = 0 \text{ in } \mathbb{R}^n .$$

A Liouville theorem [2] prevents the existence of nontrivial solution provided that $n/2 < (p+1)/(p-1)$, so $v \equiv 0$. This contradicts $v(0,0) = 1$.

Case 2. $x_\infty \in \partial\Omega$. In what follows the numbers of steps correspond to the numbers in Case 1. (0) (Coordinate change). We may assume the boundary $\partial\Omega$ is contained in the hyperplane $\{x^n = 0\}$ by taking a local coordinate $x = (x^1,..,x^n)$ around x_∞ . Unfortunately, because of the coordiante change the equation we should now discuss is a uniformly parabolic equation with smooth variable coefficients

$$u_t - \sum_{ij} a^{ij}(x) \frac{\partial^2}{\partial x^i \partial x^j} u - \sum_i b^i(x) \frac{\partial u}{\partial x^i} - u^p = 0 .$$

We may assume that $a^{ij}(0) = \delta^{ij}$.

(i) Let d_k be the distance from x_k to $\partial\Omega$. We see at least for large k , v_k is defined in $Q(\delta/\lambda_k) \cap \{y^n > - d_k/\lambda_k\}$ for some $\delta>0$.

(ii) The equation for v_k is slightly complicated because of (0).

$$v_{ks} - \sum_{ij} a^{ij}(\lambda_k y + x_k) \frac{\partial^2}{\partial x^i \partial x^j} v_k - \sum_i b^i(\lambda_k y + x_k) \frac{\partial}{\partial x^i} v_k - v_k^p = 0 .$$

(iii) This time we should use parabolic L^p theory up to the boundary [3] to the above equation in (ii). In particular, we get $|\nabla v_k|$ is uniformly bounded in $Q(R) \cap \{y^n > - d_k/\lambda_k\}$. Since $v_k(0,0) = 1$ by (7) and $v_k = 0$ on $\{y^n = 0\}$ we get $d_k/\lambda_k \geq B>0$. If $\limsup_k d_k/\lambda_k = \infty$ after a choice of subsequence the situation is reduced to case 1 with modification for variable coefficient equations (cf. [2]). We may assume $d_k/\lambda_k \to c>0$. Applying parabolic L^p theory up to the boundary yields that a subsequence (still denoted v_k) converges to v in the same sense of (iii) of case 1 except that we should replace $Q(R)$ by $Q(R) \cap \{y^n > -c + \varepsilon\}$ for arbitrary $\varepsilon>0$ and $R>0$. Note that $v=0$ on $\{y^n = - c\}$ and $v(0,0) = 1$.

(iv) The equation of v is the same as (iv) of case 1, although step (ii) is different (cf.[2]). Using $a^{ij}(0) = \delta^{ij}(0)$ and (iii), we get

$$v_t - \Delta v - v^p = 0 \quad \text{in} \quad \{y \in \mathbb{R}^n;\ y^n > -c\} \times (-\infty,0).$$

(v) This step is the same as case 1 with necessary change of domain of integration.

(vi) Since $v_s = 0$, $v > 0$ solves

$$\Delta v + v^p = 0 \quad \text{in} \quad \{y^n > -c\}$$

with $v=0$ on $\{y^n = -c\}$. Applying a Liouville theorem in half space [2] yields $v \equiv 0$ which contradicts $v(0,0) = 1$.

We thus have contradiction in both cases which completes the proof.

3. Proof of main results

Combining energy estimates (e.g. [1]), and Lemma, we easily get our main thoerem. Although the argument is known [1], we present the outline for the reader's convenience.

Proof of Main Theorem: We may assume u_0 is not identically zero. We first note that there are constants B, $t'>0$ depending only on $\sup_\Omega u_0$ such that

$$\sup_{0<\tau<2t'} \sup_\Omega u(x,\tau) < B \ , \qquad \int_\Omega |\nabla u|^2(x,t')dx < B. \tag{9}$$

The first inequality is easy to prove. The second one follows from the regularizing property of parabolicity which is well known (c.f. [7]).

We next recall energy identities. Multiplying (1) with u and u_t and integrating over Ω yields

$$\frac{d}{dt} \int_\Omega |u|^2\, dx = -\ 4E[u](t) + a \int_\Omega |u|^{p+1} dx \tag{10}$$

and

$$\int_\Omega |u_t|^2 dx = -\frac{d}{dt} E[u] \tag{11}$$

where a is a positive computable constant and E is the energy defined by

$$E[u] = \frac{1}{2} \int_\Omega |\nabla u|^2 \, dx - \frac{1}{p+1} \int_\Omega u^{p+1} dx \ . \qquad (12)$$

The identity (11) says that the energy should decrease. This together with (10) shows that $E[u](t) \geq 0$ for $t>0$ otherwise the solution blows up in finite time. Integrating (11) over (t',T) gives

$$\int_{t'}^{T} \int_\Omega |u_t|^2 \, dxdt \leq E[u](t') \leq B/2$$

by (9) Applying Lemma yields that all solutions in $\Omega\times[t',T)$ which attain maximum outside $\Omega\times[t',2t')$ are bounded from above by a constant depending only on B and t' (independent of T). By (9) solutions which take maximum inside $\Omega\times[t',2t')$ are dominated by B. In any case

$$u(x,t) \leq M \quad \text{in} \quad \bar{\Omega} \times [t_1,\infty) \quad ,$$

where $M>B$ depends only on $\sup_\Omega u_0$. Of course $u \leq B$ in $\bar{\Omega} \times[0,t_1)$ by (9) which completes the proof of Main Theorem.

<u>Remark</u>. The nonlinear term u^p in (1) may be replaced more general one $f(u)$ satisfying (6) and following conditions which are given in [1].

(a) $f(z) = - mz + g(z)$ and $g(z)$ is a nonnegative C^1 real-valued function on $z \geq 0$.

(b) $m> - \lambda_1$ where λ_1 is the first eigenvalue of $-\Delta$ in $H_0^1(\Omega)$.

(c) There is a constant $\delta>0$ such that

$$zg(z) \geq (2+\delta) \int_0^z g(s)ds.$$

<u>Remark.</u> In [1] Cazenave and Lions obtain global <u>a priori</u> bounds provided that $1 < p < 1 + 12/(3n-4)$ or $n=1$. To get <u>a priori</u> bounds they appeal interpolation between estimates $\int\int |u_t|^2 dxdt$ and $\int(\int |\nabla u|^2 dx)^2 dt$ and get $\sup_t \int |u|^q dx$ for

some $q>1$. Unfortunately, if $p > 1 + 12/(3n-4)$ and $n > 2$, this estimate is not enough to get *a priori* bounds. Instead, for larger p they prove that all global solution is bounded by using

$$\int_T^\infty \int_\Omega |u_t|^2 \, dxdt$$

is small if T is large. It seems that such a type of argument does not work to get *a priori* bounds depending only on some norm of initial data.

Proof of Corollary. Let u_{mo} be a sequence in K such that $u_{mo} \to u_o$ in C. Main theorem implies that solution u_m of (1-3) with $u_m(x,0) = u_{mo}(x)$ satisfies the estimate $u_m < M$ in Q independent of m. This gives the uniform bounds for u of (1-3) with $u(x,0) = u_o$ which prevents the blow up in finite time. Therefore $u_o \in K$ which completes the proof.

4. A remark on blow up

It is well known that there is a initial data $u_o \in K$, *i.e.*, solution of (1-3) blows up in finite time. More precisely, there is a time $T_* < \infty$ such that u is the solution of (1-3) in $\Omega \times [0,T_*)$ and that $\sup_\Omega u \to \infty$ as $t \to T_*$. It is interesting to study what kind of quantity blows up at T_* (cf. [6]). As an application of Lemma we claim the energy E defined by (12) blows up at T_*.

Corollary to Lemma . *If the solution* u *of* (1-3) *blows up at time* $T_* < \infty$, *then the energy* $E[u](t) \to -\infty$ *as* $t \to T_*$ *provided that* $n/2 < (p+1)/(p-1)$.

Proof. If not, (11) yields that for fixed t', $0 < t' < T < T_*$,

$$\int_{t'}^T \int_\Omega |u_t|^2 \, dx\, dt < C < \infty ,$$

where C is independent of T. If $\sup_\Omega u(x,t) \to \infty$ as $t \to T_*$ it evidently contradicts to Lemma. Thus we have $E[u](t) \to -\infty$ as $t \to T_*$.

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99	**Millard Beatty**, A Lecture on Some Topics in Nonlinear Elasticity and Elastic Stability
100	**Filomena Pacella**, Central Configurations of the N-Body Problem via the Equivariant Morse Theory
101	**D. Carlson and A. Hoger**, The Derivative of a Tensor-valued Function of a Tensor
102	**Kenneth Mount**, Privacy Preserving Correspondence
103	**Millard Beatty**, Finite Amplitude Vibrations of a Neo-hookean Oscillator
104	**D. Emmons and N. Yannelis**, On Perfectly Competitive Economies: Loeb Economies
105	**E. Mascolo and R. Schianchi**, Existence Theorems in the Calculus of Variations
106	**D. Kinderlehrer**, Twinning of Crystals (II)
107	**R. Chen**, Solutions of Minimax Problems Using Equivalent Differentiable Equations
108	**D. Abreu, D. Pearce, and E. Stacchetti**, Optimal Cartel Equilibria with Imperfect Monitoring
109	**R. Lauterbach**, Hopf Bifurcation from a Turning Point
110	**C. Kahn**, An Equilibrium Model of Quits under Optimal Contracting
111	**M. Kaneko and M. Wooders**, The Core of a Game with a Continuum of Players and Finite Coalitions: The Model and Some Results
112	**Haim Brezis**, Remarks on Sublinear Equations
113	**D. Carlson and A. Hoger**, On the Derivatives of the Principal Invariants of a Second-order Tensor
114	**Raymond Deneckere and Steve Pelican**, Competitive Chaos
115	**Abstacts** for the Workshop on Homogenization and Effective Moduli of Materials and Media
116	**Abstracts** for the Workshop on the Classifying Spaces of Groups
117	**Umberto Mosco**, Pointwise Potential Estimates for Elliptic Obstacle Problems
118	**J. Rodrigues**, An Evolutionary Continuous Casting Problem of Stefan Type
119	**C. Mueller and F. Weissler**, Single Point Blow-up for a General Semilinear Heat Equation
120	**D.R.J. Chillingworth**, Three Introductory Lectures on Differential Topology and its Applications
121	**Giorgio Vergara Caffarelli**, Green's Formulas for Linearized Problems with Live Loads
122	**F. Chiarenza and N. Garofalo**, Unique Continuation for Nonnegative Solutions of Schrodinger Operators
123	**J.L. Ericksen**, Constitutive Theory for some Constrained Elastic Crystals
124	**Minoru Murata**, Positive solutions of Shrodinger Equations
125	**John Maddocks and Gareth P. Parry**, A Model for Twinning
126	**M. Kaneko and M. Wooders**, The Core of a Game with a Continuum of Players and Finite Coalitions: Nonemptiness with Bounded Sizes of Coalitions
127	**William Zame**, Equilibria in Production Economies with an Infinite Dimensional Commodity Space
128	**Myrna Holtz Wooders**, A Tiebout Theorem
129	**Abstracts** for the Workshop on Theory and Applications of Liquid Crystals