Essays on Sovereign Bond Markets

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Dedication

To my lovely mother, Jaehyun Kim, my father, Seongho Yang, and my sister, Jinyoung Yang, who have always been a source of encouragement, urging me to be ambitious and supporting me every step of the way. Additionally, I am deeply grateful to all my relatives in my hometown, Jeju, for their unwavering support.

Abstract

This dissertation comprises three chapters investigating the impact of information frictions and market segmentation on debt financing in a small open economy.

In the first chapter, I examine debt crises in countries relying on bond auctions to finance outstanding debt. Using a noisy rational expectation model, I show that information frictions can trigger debt crises even when fundamentals alone wouldn't. Pessimistic signals can lead to defaults with low default risk fundamentals, while optimistic signals can avert defaults with high default risk fundamentals.

The second and third chapters study sovereign credit ratings' implications for developing countries. I explore the impact of market segmentation from ratings on default risk and borrowing behavior. Using emerging market panel data, I estimate bond spread responses to downgrades to junk rating, emphasizing regulatory thresholds.

The third chapter introduces a quantitative sovereign default model incorporating credit ratings and segmented markets. I find higher spreads imply a 200-basis-point higher discount rate on junk bonds. Beyond a threshold, downgrades lead to higher interest rates, reducing default risk and raising bond prices. Segmentation benefits low-debt states, mitigating overborrowing friction and providing welfare gains. A looser rating rule diminishes these gains.

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Chapter 1

The Role of Information in Bond Auction Markets

1.1 Introduction

Consider an environment where government has temporarily limited resources to finance its outstanding debt. This situation is expected to last for a short period, and the resources are guaranteed to return to their original level in the next period. This is a common knowledge among lenders. The government issues a bond and promises to repay it in the next period as long as bond revenue is sufficient to finance the debt. If the government fails to finance the debt, it defaults. Although lenders do not know the exact amount of the government's debt, each lender has an estimate of it. With this knowledge, lenders bid in bond auction to maximize their expected profit from the bond investment.

In this auction, the value of the auctioned item, i.e., the profit of bond investment, is endogenously determined, particularly by an equilibrium bond price in the auction. Equilibrium bond price plays a significant role as it not only determines the cost of bond investment but also influences bond revenue, default outcome, and the profit of bond investment. Moreover, it is the outcome of aggregating all bids in the bond auction market. It can be a another signal of the amount of the government's debt.

In this chapter, I analyze the role of information in a setup where the government faces a rollover debt problem and the amount of debt is not publicly known. Each lender observes a private, noisy signal of the debt. The government issues a bond with a face value of $\bar{B}$[1] , promising to repay it next period as long as bond revenue is enough to finance the debt. If not, the government defaults. Lenders bid in a bond auction market to maximize their expected payoff, inferring the unknown amount of debt from their private signal and the equilibrium bond price, which is determined by the aggregation of all bids in the market. However, the market has a noise, causing the equilibrium bond price not to reveal the fundamental perfectly.

To investigate this setup, I construct a noisy rational expectation model and focus on a monotone and symmetric equilibrium, where all lenders have the same optimal bidding strategy, which monotonically decreases in the private signal. I examine how more precise information about the government's debt affects the equilibrium bond price and default outcome.

I first find that having sufficiently precise information can lead to multiple equilibrium bond prices. Specifically, it generates multiple prices which clear the bond market with a fixed face value. This result is consistent with the findings of [Hellwig et al., 2006] and [Bassetto and Galli, 2019]. The limit case of the precise information, which is the case of perfect information, also has multiple equilibria: one where the market values the bond price low, the government defaults, and this verifies the market's evaluation; and the other where the market values the bond high, the government repays, and this also verifies the market's evaluation. This result aligns with the concept of multiple equilibria in a self-fulfilling crisis, as described by [Calvo, 1988] and [Cole and Kehoe, 2000].

[1] The government is guaranteed to have resources as much as $\bar{B}$ in the next period, provided the government does not default. The bond is issued based on this fact, which lenders know.

Under a parameterization that delivers a unique equilibrium, I find the following main result: precise information makes the price schedule more sensitive to the market-clearing price, and this increased sensitivity can have an adverse (or advantageous) impact on low (or high) debt states.[2] A marginal lender with rational expectations can infer debt state and market noise from a market-clearing price, which is called a market signal. He evaluates a bond price based on this market signal. As the private signal becomes more precise, the marginal lender has a more concentrated understanding of the debt state, which causes the price schedule to react more sensitively to the market signal. The increased sensitivity implies that the bond price appreciates more with a good market signal and depreciates more with a bad market signal.

An important feature of this paper is that the marginal lender cannot fully separate fundamental (i.e., debt state) and fundamental-irrelevant noise (i.e., market noise) from the market signal, even with highly precise information. The market signal is derived from the market-clearing bond price and is used by the lender to infer the debt state. There are some states where the debt level is low, but the marginal lender perceives it as a bad market signal. Conversely, in some states, the debt level is high, but the marginal lender perceives it as a good market signal. There is a discrepancy between the actual debt state and the inferred debt state based on the market signal.

Due to the combination of this discrepancy and the sensitive price schedule coming from precise information, there are cases where the debt level is low (or high) but perceived as a bad (or good) signal, resulting in a depreciated (or appreciated) price. This effect is significant enough that it can cause a government with low debt to default even though the debt level is relatively low. The government could have repaid it without the sensitive price schedule. On the other hand, the sensitive

[2] I restrict the degree of information precision parameter to guarantee a unique market-clearing price. Then, I proceed with comparative statics for equilibrium bond price and default outcome with respect to the precision parameter.

price schedule appreciates the bond price sufficiently for high-debt states, simply because the states are perceived as good market signals. Consequently, high-debt states can repay, even though the actual debt level is relatively high, and it could have defaulted without the sensitive price schedule.

Certainly, a marginal lender is better able to distinguish true fundamentals and market noise from a market signal as the private signal becomes more precise. It is true that among a set of good signals, there are more states that are also fundamentally good. However, the effect of price depreciation to bad signals dominates the effect of the market signal in more accurately revealing good fundamentals. As a result, low-debt states are adversely affected by the high precision of information. Conversely, the opposite happens to bad fundamentals when high-debt states benefit from highly precise information. This is because the effect of price appreciation on a good signal dominates the effect of the market signal in more accurately revealing bad fundamentals.

Finally, I demonstrate that the main result is robust in an environment where there is a prior belief about the debt level. In an economy with a low prior belief, it is common knowledge that the debt level is likely to be low, so the marginal lender is more positive toward the market signal. Conversely, in an economy with a high prior belief, the marginal lender is more negative toward the market signal. This means that the same market signal can be perceived as a good signal in a low prior economy, whereas it is interpreted as a bad signal in a high prior economy. Furthermore, the price schedule is generally higher in a low prior economy compared to a high prior economy. A low prior economy has a higher average bond price and a higher probability of repayment compared to a high prior economy, regardless of the degree of information precision.

As the information becomes more precise, the probability of repayment in a low prior economy decreases compared to that of low precision, and the opposite occurs in a high prior economy. It is low-debt states that are negatively affected by high information precision (i.e., lower bond price and a higher likelihood of default), while high-debt states are positively affected by high information precision (i.e., higher bond price and a higher likelihood of repayment). Since a low prior economy is more likely to realize low debt states, it is adversely affected by high information precision, resulting in a lower average bond price and a lower probability of repayment. Conversely, a high prior economy is more likely to realize high debt states, leading to a higher average bond price and a higher probability of repayment with high information precision.

The rest of the chapter proceeds as follows: first I document literature related to this paper. In Section 1.2, I describe the model and define the equilibrium of the model. In Section 1.3, I explain how I solve the model and present a bond price characterizing equation, which is the key equation of the paper. I also show the possibility of multiple market-clearing prices. In Section 1.4, I go through a numerical exercise to proceed with comparative statics on bond price and default outcome, having information precision as a parameter of interest. I present how information precision affects price schedule and market signal, after which the main result comes. In Section 1.5, I extend the model by adding prior belief. I present how prior belief can change the price schedule and show that the main result is robust to the extended model.

1.1.1 Related literature

[Cole et al., 2018] establish a model where lenders have asymmetric information regarding a country's fundamentals and participate in a bond auction market with rational expectation. They compare the expected bond yields and yield volatility between uniform price and discriminatory price auction protocols. Their model assumes exogenous default which is solely determined by a country's fundamentals.

In contrast, this paper assumes endogenous default, specifically occurring when bond revenue is insufficient. In my model, a default does not necessarily arise from bad fundamentals but rather when the bond price is not high enough compared to debt level.

Since this paper assumes an environment of rollover risk and default due to insufficient bond revenue, it aligns with the concept of self-fulfilling crises and multiple equilibrium, as discussed in [Calvo, 1988] and [Cole and Kehoe, 2000]. Among the sizable literature on self-fulfilling crises, this paper falls into the category of studies involving lenders with heterogeneous information. [He et al., 2019], for example, examine the role of heterogeneously informed lenders in a country's rollover risk. They assumes that the country defaults due to insufficient bond revenue, and lenders decide where to invest among two countries of different sizes. This paper also assumes the same environment, where a low market valuation can trigger the country's default; however, I incorporate a more complex action for lenders. Specifically, lenders participate in a bond auction market, making price-contingent investment decisions rather than a binary choice of investing or not investing.

This paper is also closely connected to a broad body of literature that examines noisy rational expectation equilibrium. [Bassetto and Galli, 2019] and [Hellwig et al., 2006] employ noisy rational expectation models to study situations involving endogenous default risk and currency crisis, respectively. Both papers discover the possibility of multiple market-clearing price arising from endogenous default or endogenous currency regime choices, respectively. [Bassetto and Galli, 2019] further find that the price is more sensitive to market signal when lenders possess more precise information. This paper contributes to the existing literature by analyzing the default outcome in greater detail. Building on the insight regarding the sensitivity of price schedules, I examine how a sensitive price schedule can influence the default outcome and whether such sensitivity is advantageous for economies with low or high levels of debt.

1.2 The model

I consider an environment where a government faces a rollover debt problem. If the government successfully finances its debt, it is guaranteed to have resources $\bar{B}$ in the next period. Based on this, the government issues a bond with a face value of parameter $\bar{B}$, and the bond revenue is utilized to finance the debt if feasible. However, the government must default if the bond revenue falls short in comparison to the debt level θ. The precise level of debt is not perfectly observed, and each lender receives a private noisy signal s_i that provides information about it. Lenders participate in the bond auction market and infer θ not only through their respective s_i but also by considering the bond price p using rational expectation. Additionally, there exists a market noise that is unrelated to θ, which leads to limited revelation of the debt level from the bond price.

1.2.1 Players, actions, and payoffs

In the model, there are a government and an unit mass of lenders, indexed by $i \in [0, 1]$. The government faces an outstanding debt, θ. If the debt is financed successfully, the government is guaranteed to receive an endowment $\bar{B}$ in the next period. It can be understood that the government has limited resources temporarily and as long as there is no default, it gets back to the original resource level for sure.

With this guarantee, the government issues a bond with face value $\bar{B}$ and raises bond revenue through a bond auction market. The government will repay its obligations only if the bond revenue exceeds than the level of debt. Mathematically, this condition is expressed as

$$\theta \leq p\bar{B} \tag{1.1}$$

where p represents the price of the bond determined in the bond auction market. In the event that $\theta > p\bar{B}$, it defaults on all its obligations, including newly issued debt. I assume $\bar{B}$ to be a parameter, which implies that the government follows a predetermined strategy in issuing bonds. It issues a fixed amount of bonds and adheres to the repayment rule as in (1.1).

The strategic players of interest are lenders who participate in the bond auction market. Each lender submits a bid for bond investment, and if they win the auction, they commit to purchasing the specified quantity of bonds as indicated in their bid. I denote b_i as the quantity of bonds that lender i promises to purchase upon winning the auction. For tractability, I assume that each lender is limited to purchase one unit of bond auctioned by the government.[3] Consequently, $b_i \in \{0, 1\}$ for $i \in [0, 1]$. I define a bid as a price-contingent bond purchase schedule, denoted as $\{b(p)\}_p$, where $b(p)$ represents the quantity of bonds to be purchased at a given bond price p.

Each lender maximizes their expected payoff [4] by strategically submitting a bid and investing in bonds accordingly. The lender's payoff is determined by whether they win the auction, and the auction rules are detailed in subsection 1.2.2. Additionally, the lender's payoff is contingent on the government's default. Assuming a lender invests one unit of bond, if the government repays, the lender will receive one unit of numeraire in the next period. However, if the government defaults, the lender will not receive any returns on their investment. I assume zero discounting between periods, and the payoff of the outside option is normalized to 0, meaning that the payoff of not investing in a bond is 0.

1.2.2 Auction protocol and auction market

In sovereign bond auction, there are mainly two auction protocols: uniform-price auction and discriminatory auction (or multiple-price auction). Uniform-price auction is where all auctioned items are executed at a single price which is called a marginal price. Discriminatory auction is where auctioned items are executed at

[3] One unit of bond promises a claim to one unit of the numeraire conditional on government repayment.

[4] I assume lenders have linear utility.

the price each bidder bids at. [Brenner et al., 2009] categorize countries by the type of protocol they use for sovereign bond auctions. They document that Germany, France, Greece, and many other European countries use discriminatory auction protocol, and USA, South Korea, and Argentina use uniform-price auction protocol. I focus on uniform-price auction protocol in this paper.

Table 1.1: A lender's ex-post payoff under a marginal price, p

	Default	Repay
$b(p) = 1$	$-p$	$-p + 1$
$b(p) = 0$	0	0

The bond auction in the model proceeds as follows: Each lender simultaneously submits their bid $\{b_i(p)\}$. The auctioneer collects all bids and determines a marginal price p at which the bond will be executed. Lenders who bid to purchase the bond at p, denoted as $\{i|b_i(p) = 1\}$, win the auction and are obligated to purchase the bond at the price p. Those who bid not to purchase the bond at p, denoted as $\{i|b_i(p) = 0\}$, lose the auction and neither gain nor lose anything. In the next period, the lenders who purchased the bond will receive one unit of numeraire only if the government does not default. The lender's payoff under a marginal price p is summarized in Table 1.1.

To prevent the marginal price from fully revealing θ, I introduce a market noise as the form of noisy trader shock. Specifically, I assume that noisy traders bid sufficiently high and always win the auction, demanding a portion of total bonds given by $\Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)$. Here, Φ is the cumulative standard normal distribution function, $\sigma > 0$ is a parameter, and the random variable $\mu \in \mathbb{R}$ represents noisy trader shock. This shock μ is independent of θ, meaning a fundamental-irrelevant shock. It follows a normal distribution with mean zero and precision $\alpha > 0$, $\mu \sim N(0, \frac{1}{\alpha})$. The parameter σ governs the concentrated impact of the market noise μ on the market. A low σ results in more concentrated demand from noisy traders within a given range of μ, whereas a high σ leads to more dispersed demand within the same range of μ.

The marginal price p is determined to clear the bond market. Bond market clearing condition is as follows:

$$\int b_i(p)di + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B} = \overline{B} \tag{1.2}$$

1.2.3 Information structure and timing

I assume that lenders cannot observe θ. Instead, each lender privately observes a noisy signal of θ, denoted as $s_i = \theta + \epsilon_i$. Here, the random variable ϵ is drawn from a normal distribution with mean zero and precision $\beta > 0$, $\epsilon \sim N(0, \frac{1}{\beta})$. A natural interpretation of s is each lender's estimate of the fundamental (the level of debt that government need to finance). A lender with high s believes that the government owes a high level of debt, whereas a low s lender has a optimistic view on the debt situation.

The parameter β represents the precision of individual information regarding the fundamental, which is a crucial parameter in this paper. High β indicates that lenders are better informed about the fundamental. I assume that the number of lenders is sufficiently large, and consequently the law of large numbers (LLN) applies. Consequently, the proportion of lenders with a signal lower than $x \in \mathbb{R}$ corresponds to the probability that the random variable s is lower than x. The prior of θ is uniform in $\mathbb{R}$ (improper uniform distribution) for tractability. However, in section 1.5, I generalize it to a more realistic distribution.

The timing in the model is as follows: In period 1, nature draws the values for $(\theta, \mu, \{\epsilon_i\}_{i\in[0,1]})$ from a given distribution, respectively. The government then issues a bond in the bond auction market. Each lender i privately observes their signal s_i

and simultaneously submits a bid. The auction closes, and the market clearing price p is determined by (1.2). The winner of the auction is determined, and the bond is executed accordingly. In period 2, the government observes θ, and the outcome of default or repayment is determined by (1.1). The lenders' payoffs are determined based on default/repayment outcome.

1.2.4 Strategy and symmetric equilibrium

Lender i's strategy is a function that maps the signal $s_i \in \mathbb{R}$ to the bid $\{b(p)\}_p$. In this analysis, I focus on a symmetric equilibrium where all lenders adopt the same equilibrium strategy. Henceforth, I denote the strategy as $b(s,p)$, which represents the quantity of bond to be purchased given a signal s at a marginal price p.[5] The lender chooses the optimal bidding strategy $b(s,p)$ to maximize the expected payoff from bond investment for all (s,p).

$$\max_b \mathrm{E}[(-pb+b)\mathbf{1}\{p\overline{B} \geq \theta\} + (-pb)\mathbf{1}\{p\overline{B} < \theta\}|s,p] \quad \forall (s,p)$$

The first part represents the payoff conditional on repayment, while the second part represents the one conditional on default. Given (s,p), the lender forms a posterior belief in θ, which is crucial for determining the default probability. The optimization problem can be reformulated as follows:

$$\max_b \mathrm{E}[-pb + b\mathbf{1}\{p\overline{B} \geq \theta\}|s,p] \quad \forall (s,p) \tag{1.3}$$

An equilibrium of interest consists of lender's strategy $b(s,p)$ and lender's posterior belief $\hat{\theta}(s,p)$ where $b(s,p)$ maximizes the lender's expected payoff for all (p,s) and belief consistency holds. The equilibrium outcome of interest is the bond price p and resulting default outcome.

[5] It is worth noting that lenders are ex-ante identical but exhibit ex-post heterogeneity based on the signals they receive. However, since the focus is on symmetric equilibrium, the subscript i can be dropped.

Definition 1. *A **Symmetric Perfect Bayesian Equilibrium** consists of bidding strategy $b(s, p)$, bond price function $p(\theta, \mu)$ and posterior belief $\hat{\theta}(s, p)$ such that*

(i) Given $\hat{\theta}(s, p)$, bidding strategy $b(s, p)$ solves lender's maximization problem as in (1.3)

(ii) Given $b(s, p)$, bond price p clears the bond market for all (θ, μ) as in (1.2)

(iii) posterior belief $\hat{\theta}(s, p)$ satisfies Bayes' rule

1.3 Solving the model

1.3.1 Optimal bidding strategy

I define $\delta(s, p)$ as the expected repayment probability conditional on a marginal price p and the signal s. Then, the expected payoff from bidding $b(s, p)$ given (s, p) is

$$\delta(s,p)\{-b(s,p)p + b(s,p)\} + (1 - \delta(s,p))\{-b(s,p)p\} = b(s,p)\{\delta(s,p) - p\}$$

Since the payoff is linear in $b(s, p)$, the optimal bidding strategy is in the form of cutoff strategy:

$$b(s,p) = \begin{cases} 1, & \text{if} \quad \delta(s,p) > p \\ 1 \quad \text{or} \quad 0, & \text{if} \quad \delta(s,p) = p \\ 0, & \text{if} \quad \delta(s,p) < p \end{cases}$$

where p is cost for investing the bond regardless of default outcome, whereas $\delta(p, s)$ is the expected benefit from investing the bond, and it is high as the lender expects the government repayment with high probability.

The expected repayment probability is influenced by three factors. First, it is affected by the signal s. A low s suggests a higher probability of a low θ. The lender anticipates a higher probability of government repayment at any p, leading to their willingness to invest in the bond. Second, the bond price p determines the bond revenue, $p\bar{B}$, which in turn impacts the default outcome. Given s and the corresponding posterior belief $\hat{\theta}$, a high p indicates a high bond revenue, suggesting a higher probability of government repayment. The reason p can affect the expected repayment probability and subsequent bond demand, based on a posterior belief, is because default outcomes depend on the market's valuation of the bond. This implies the possibility of self-fulfilling default: at a low p, lenders anticipate a low repayment probability and abstain from purchasing the bond. As a result, aggregate bond demand decreases, and the bond price p is set low, confirming the lenders' initial pessimistic expectations.

Lastly, the marginal price p can be another signal for the unobserved θ. Bond market clearing condition explains it clearly.

$$\int b_i(p, s_{i|\theta})di + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B} = \overline{B} \tag{1.4}$$

Note that the equation above represents a market clearing p, and a market clearing p should satisfy the equation. It is important to recognize that the equation includes states of the world, (θ, μ). The quantity of bonds demanded by the noisy trader is determined by μ, and θ determines the distribution of s_i, impacting the bidding strategies of all lenders and the overall demand for the bond. In essence, the market clearing equation establishes a relationship between the bond price p and the state (θ, μ). For a high p to be a market clearing price, one of two conditions must hold: either the fundamental is favorable (indicating a low θ), or the aggregate bond demand is bolstered by the noisy trader, despite an unfavorable fundamental (indicating a high θ and a high μ). With this inference, rational lenders can update their belief in θ accordingly.

1.3.2 Monotone equilibrium

There will be many classes of equilibrium for this model, but I focus on a monotone equilibrium, where optimal bidding strategy $b(s, p)$ is monotonic decreasing in s. This means that in equilibrium, lenders bid to purchase the bond as they receive a low signal. The solution strategy will be first solving for the market clearing p and then verifying that the monotone strategy is indeed optimal.

In monotone equilibrium, the equilibrium bidding strategy is characterized as a signal cutoff $s^*(p)$ where a given p, $b(s, p) = 1$ for $s \leq s^*(p)$ and $b(s, p) = 0$ for $s > s^*(p)$. By definition, $s^*(p)$ has to satisfy $\delta(s^*(p), p) = p$. Also, equilibrium default outcome is characterized as a default cutoff $\theta^*(p)$ where a given p, the government defaults if and only if $\theta > \theta^*(p)$ and the government repays if and only if $\theta \leq \theta^*(p)$. By definition, $\theta^*(p)$ has to satisfy $p\bar{B} = \theta^*(p)$. From now on, I normalize $\bar{B} = 1$ so that the equation can be simplified as $p = \theta^*(p)$.

For a given cutoff $s^*(p)$, lenders' aggregate bond demand is $\int b_i(p, s_{i|\theta})di = prob\{s_i < s^*(p)|\theta\} = \Phi(\sqrt{\beta}(s^*(p) - \theta))$. For a state (θ, μ), p is a market clearing price if and only if $\Phi(\sqrt{\beta}(s^*(p) - \theta)) + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right) = 1$, or equivalently

$$s^*(p) = \theta - \frac{\mu}{\sqrt{\beta\sigma}} \equiv x \tag{1.5}$$

This equation mathematically shows how a state (θ, μ) is inferred from a market clearing price p. Given a p, lenders can infer that the unknown state (θ, μ) should satisfy (1.5) using the fact that a market clearing p satisfies equation (1.4) and the knowledge of the cutoff $s^*(p)$. I define the right-hand side of this equation as x, which represents a market signal about the state inferred from the market price p.

The market signal x is a public signal among lenders, since p is public knowledge, and every lender observes p. Another important feature is that the market signal cannot distinguish between θ and μ, even though it provides an unbiased estimate of θ. The market signal is informative in Bayesian' updating, allowing lenders to narrow down the entire state space to $\{(\theta, \mu)|\theta - \frac{\mu}{\sqrt{\beta\sigma}} = x\}$. However, it still cannot precisely determine the true (θ, μ). As mentioned, x is an unbiased estimate of θ, and it follows a normal distribution $x \sim N\left(\theta, \frac{1}{\beta\sigma\alpha}\right)$. The precision of x depends not only on β, α but also on σ. A higher value of σ concentrates the impact of a noisy trader within a given interval of μ, which has a similar effect to a higher precision of μ (represented by a higher value of α).

1.3.3 Characteristic equation for bond price

Given x and its distribution, a lender i with its signal s_i updates posterior belief as follows:

$$\theta | s_i, x \sim N\left(\frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}, \frac{1}{\beta + \sigma\alpha\beta}\right).$$

For the marginal trader with its signal $s^*(p)$, posterior belief is

$$\theta | s^*(p), x \sim N\left(x, \frac{1}{\beta + \sigma\alpha\beta}\right).$$

The marginal lender is as optimistic as the market signal. Those with $s_i > x$ expect θ to have mean greater then x, meaning less optimistic, and $b(s_i, p) = 0$, whereas those with $s_i < x$ expects θ to be less then x on average, meaning more optimistic, and $b(s_i, p) = 1$. With this update, I can express the expected repayment probability for the lender i and the marginal lender as follows:

$$\delta(s_i, p) \equiv prob\{\theta \leq \theta^*(p) | s_i, p\} = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$$

.

$$\delta(s^*(p), p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\,(\theta^*(p) - x)\right)$$

The equilibrium bond price p is characterized by the marginal lender having zero expected payoff of bond investment. Substituting $\theta^*(p)$ to p, the market clearing bond price p given x can be implicitly characterized, which is the key equation in this paper.

$$p = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\,(p - x)\right) \tag{1.6}$$

p is determined not only by θ but by x naturally comes from the fact that the market signal cannot separate θ from x. The marginal lender neither can separate θ from x and he decides market price p depending on what his signal and market signal, x, is.

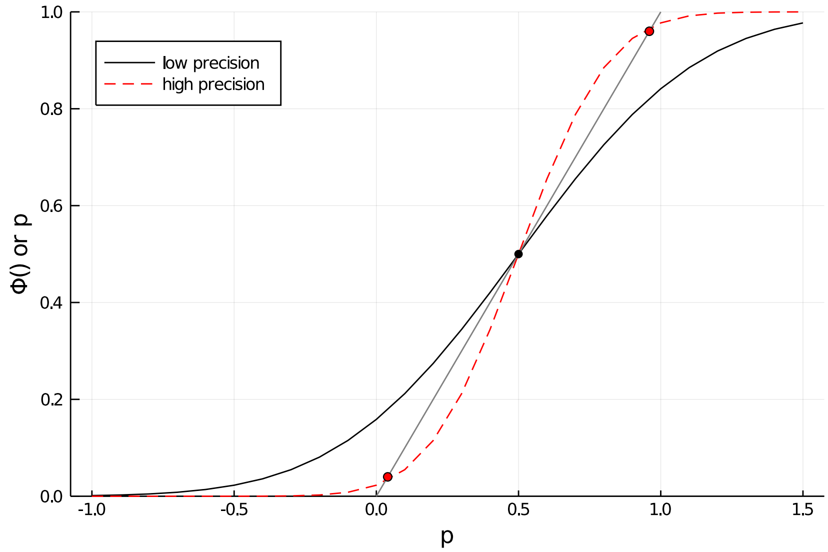
What remains is verifying that the monotone strategy is the optimal strategy. It is straightforward given the fact that the expected repayment probability is strictly decreasing in the signal. Recall $\delta(s_i, p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$. Given p, and for $s_i < s^*(p), \delta(s_i, p) > \delta(s^*(p), p) = p$ by the indifference condition of the marginal lender. Therefore, $\delta(s_i, p) > p$ and $b_i(s_i, p) = 1$. The similar logic applies to $s_i > s^*(p)$.

1.3.4 Multiple market clearing prices

There could be multiple p satisfying (1.6), implying a possibility of multiple market clearing prices. Figure 1.1 shows it graphically by plotting the right-hand side of (1.6) as a function of p with a various precision of marginal lender's posterior belief, $\beta + \beta\sigma\alpha$.

It shows that the high precision case has three fixed points, meaning three market clearing prices, and the low precision case has a unique market clearing price. Multiple bond prices with high enough precision can be related to multiple equilibria under perfect information. Consider $\theta \in [0, 1]$, and the value of θ is a common knowledge. The common knowledge version of the model has two equilibria; one equilibrium where the market values the bond low and $p = 0$, the government defaults, and it verifies the initial evaluation of $p = 0$. The other equilibrium is where the market values the bond high and $p = 1$, the government repays, and it verifies the initial evaluation of $p = 1$. These two equilibria can be seen as a limit case of equilibrium prices in high precision, one with very low p and the other with very

Figure 1.1: Equilibrium bond prices with different precision values



Note: This plot is for illustrating equilibrium bond prices with different precision parameters β. For a given p, I plot the right-hand side of (1.6) as a function of p. Parameter values are assigned as Table 1.2. The black solid line is for low β, and the red dash line is for high β. The gray line is a 45-degree line, so that the intersections are the fixed point of (1.6), or bond prices that satisfies the equilibrium condition.

high p. The relationship between the multiplicity and parameter values is explained in details in Appendix A.

1.4 Numerical exercise

1.4.1 Parameterization

Parameter values for (β, σ, α) are assigned as shown in Table 1.2. To analyze the role of information in the rollover-debt problem, I specifically focus on β, which represents the precision of the private signal. I compare high and low β economies while fixing the values of other parameters. In the high β economy, lenders' private

Table 1.2: Parameter values

Parameter	value
σ	0.7
α	0.1
Low β	1.0
High β	2.0

signals are more centered around the true θ, allowing the marginal lender to have more accurate information about the fundamental. I am interested in observing how the bond price and default outcome are affected by an increase in the precision of private signals. Further details on how I construct grid points and how to solve the equilibrium can be found in Appendix A.

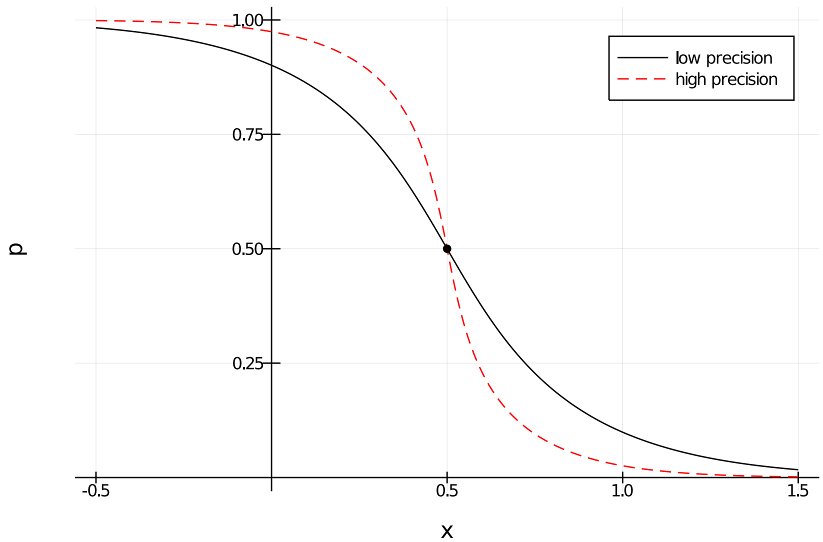
1.4.2 Price schedule and the role of information precision

Figure 1.2 plots the equilibrium bond price p as a function of market signal x. The high precision plot uses high β, and the low precision plot uses low β.

The first observation is that both price schedules are decreasing in x, implying that the bond is priced low with a high value of market signal. Even though the market signal is affected by market noise, it tells what the level of θ, on average, is. A high x means that the market predicts θ is likely to be high. The marginal lender expects a higher probability of default, and devalues the bond accordingly.

The second observation is that with high β, the price schedule responds more sensitively to x. This is graphically shown as a higher degree of concavity/convexity under high β. The market signal is from aggregating all private signals through auction bidding, and more precise private signals under high β lead to the market signal being a more precise estimate of θ. The marginal lender's posterior belief is more concentrated on the true θ, and the posterior repayment probability changes

Figure 1.2: Equilibrium bond price schedule as a function of market signal



Note: This figure depicts the equilibrium bond price as a function of market signal, x, for different values of information precision. The black solid line is for low β, and the red dash line is for high β. Parameter values are assigned as Table 1.2. For a given x, the equilibrium bond price is the fixed point of (1.6).

more dramatically by a small change in x.

Lastly, a sensitive price schedule with high β reacts depending on what the value of x is. Sensitive price function does not necessarily appreciate p for all x, but only when x is low. When x is high, on the other hand, price function with high β gives lower p than what price function with low β does. The different responses happen around the inflection point, and I define the infection point as market signal cutoff x^*. It is the market's criteria for interpreting whether x is a good signal or a bad signal. In Figure 1.2, $x^* = 0.5$, and $x < 0.5$ is interpreted as a good market signal. Given this good signal, the sensitive price schedule appreciates p more than the low β price schedule: $p(x)$ with high $\beta > p(x)$ with low β for any $x < 0.5$. The reverse happens when $x > 0.5$, which is interpreted as a bad market signal. To a bad signal, the sensitive price schedule depreciates more, and p is lower. Essentially, whether high precision is advantageous to debt rollover depends on whether the market interprets x as a good signal or not. Mathematically it depends on $x < x^*$ or $x > x^*$. I will keep this comparison as I continue the following exercises.

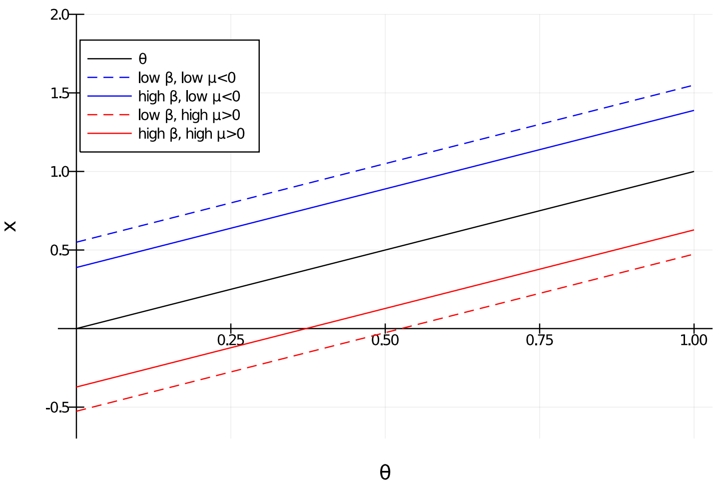
1.4.3 Market signal and the role of information precision

So far, I have shown that more accurate information makes the price schedule more sensitive to the market signal and how the schedule reacts differently to good and bad signals. In this section, I show how high precision of information affects the market signal conditional on a debt level. In short, the higher β is, the less the market signal deviates from the true θ. Mathematically, the deviation $|x - \theta| = \left|\frac{\mu}{\sqrt{\beta}\sigma}\right|$, so high β makes low deviation. Intuitively, as a private signal is more precise, x also becomes a more precise indicator for true θ because x is constructed by aggregating all private information in the bond auction market.

Figure 1.3 shows it graphically. For a fixed value of noisy trader shock, I plot the values of market signal across the realized values of the debt level. I use two values for μ: a positive value (blue lines) and a negative value (red lines). The dash lines are low β cases, and the solid lines are the high β cases. I plot the realized value of θ as a reference.

The figure shows that the solid line, X with high β, is closer to the black line, true θ, compared to dash line, X with low β, for all θ realization and in both μ cases. The difference between $\mu < 0$ case and $\mu > 0$ case is that x is in general low in $\mu > 0$ whereas x is in general high in $\mu < 0$. It means even with the same level of θ, x is interpreted as a good signal with high demand case and a bad signal with low demand case. It is due to the fact that lenders can infer true state from market price as a market signal but they cannot infer θ and μ separately. With the same value of x, it could come from low θ with low demand shock but it could also come from high θ with high demand shock. Therefore, whether high information precision

Figure 1.3: Sign effect of noisy traders shock to market signal



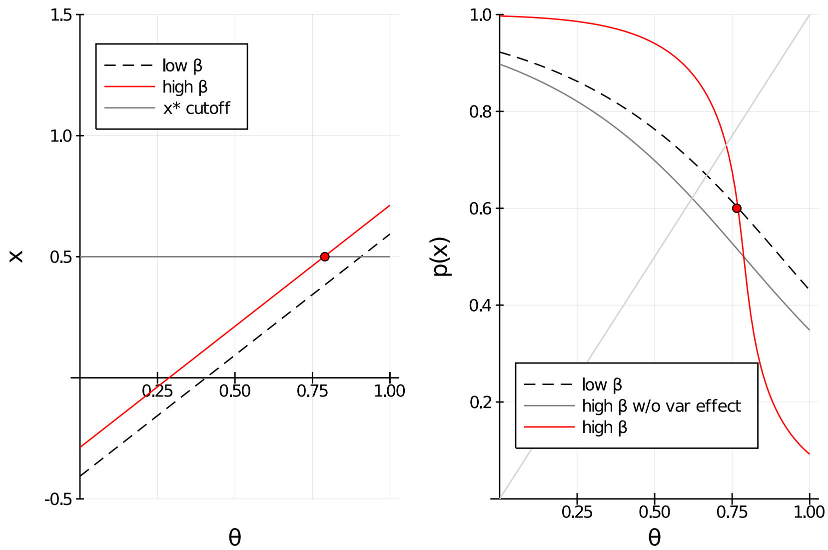
Note: This plot depicts the market signal, x, as a function of debt state, θ, for two value of μ and for two values of information precision, β. For a given μ and parameter values, for each θ level, the market signal is calculated by satisfying (1.5). The solid lines are for the high β, and the dash lines are for the low β. The blue lines are for a negative μ, and the red lines are for a positive μ. Parameter values are assigned as Table 1.2.

is good or bad also depends on whether the economy has low or high demand shock because it is the demand shock that affects x being generally a good signal or not. In Appendix A, I add how different value of μ but the same sign affect the market signal.

1.4.4 Main results

With observations from the previous sections, I present the main result of this paper. Precise information makes the government with a relatively low debt level default, whereas it makes the government with a relatively high debt level repay. With less precise information, those relatively low θ states could have repaid, and relatively high θ states could have defaulted. This might sound counter-intuitive, but it happens because there always exist a state susceptible to market noise, and the change in default outcome happens in this susceptible state combined with a sensitive price schedule.

Figure 1.4: Market signal and bond price under positive noisy trader shock



Note: this figure illustrates the market signal, x, and according equilibrium bond price, $p(x)$, for a given value of μ. Here, a positive value of μ is given, which implies high noisy trader shock. The black dash lines are for low β, and the red solid lines are for high β. Parameter values are assigned as Table 1.2. The left panel depicts the market signal as a function of θ by satisfying (1.5), and the market signal cutoff $x^* = 0.5$ is indicated as a horizontal gray line. The right panel depicts the equilibrium bond price as a function of debt state, θ. The price is obtained by using (1.6) and the corresponding market signal, x, for each debt state, θ, obtained as in the left panel. The gray line is obtained by for each debt state, θ, solving for p in (1.6) using low β value for β. For each θ, I use the same x value that I used for plotting high β (red solid) line.

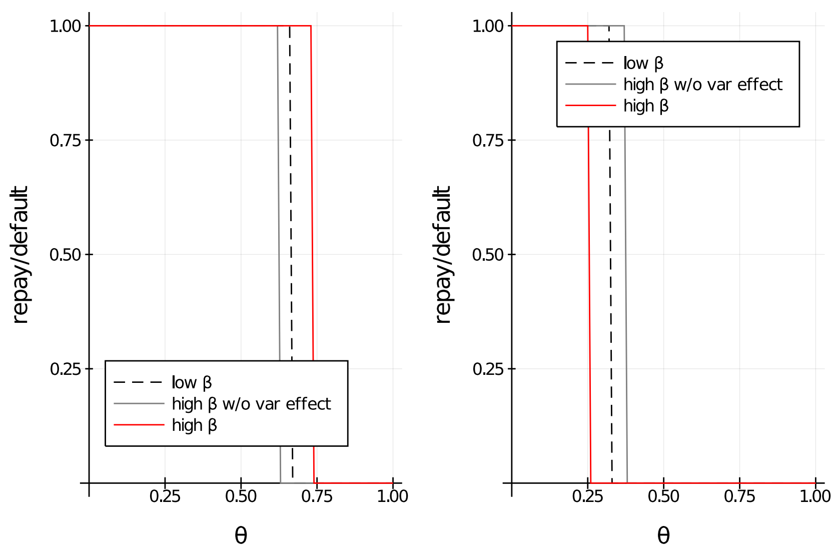
Figure 1.4 shows how the market signal and the accordingly price schedule are formed across the realized debt level for a fixed noisy trader shock, particularly $\mu > 0$. A positive noisy trader shock implies that noisy traders demand bonds relatively high, on average, makes the market signal perceived as a good signal by the marginal lender, hence $x < \theta$ for all θ. However, only those market signals that are lower than the market signal cutoff, $x < x^* = 0.5$, are interpreted as a optimistic signal by the market, and a corresponding $\theta < 0.765$ benefits from the sensitive price schedule which appreciates more in response to those optimistic signals. The right panel of Figure 1.4 shows that θ in a optimistic signal region has a higher bond price under high precision. To see how appreciated p affects default outcome, compare p and θ using a 45-degree line in the right panel of Figure 1.4. The debt region $\theta \in [0.625, 0.735]$ is where under low information precision the marginal lender values the bond low, and the government would default; however, the bond price under high information precision is high so that the government would repay.

This is also illustrated in the left panel of Figure 1.5. It is not necessarily because the debt level itself is low and the government bond has less default risk. Rather it is because the marginal lender infers the debt level as low under the favor of a positive noisy trader shock. Information precision makes the marginal lender respond favorably to those market signals that are interpreted as optimistic. Hence, even though the government as a fairly high amount of debt, as long as the market perceives the debt status as optimistic, the bonds are high-valued. The bond revenue is high enough for the government to successfully finance its outstanding debt, staving off the government default.

It is important to note that high information precision has a trade-off; although high information precision makes the price schedule more sensitive, leading to bond price appreciation in some states of high level of debt, it makes the market signal more accurately revealing what the true debt state is. However, the effect from a higher sensitivity of the price schedule dominates the other effect, and the bond revenue under the states of fairly high-level of debt is higher than the debt level, resulting in the government not defaulting.

As shown the left panel of Figure 1.4, high precision moderates a positive market noise, and across all debt levels, the market signal is perceived as less optimistic under high precision than how it would be perceived under low precision. It indeed disadvantageously affects the equilibrium bond price. If the price schedule were as sensitive as it is in low precision case, the equilibrium would have had a low bond price for all debt states, as shown in the right panel of Figure 1.4. Accordingly, there would have had more default as shown in the left panel of Figure 1.5. However, the sensitive price schedule dominates the effect of less optimistic market signal. Even though the market signal is not as optimistic as that under the low precision economy, there are debt states where the government benefits from high precision.

Figure 1.5: Default outcome in positive (left) and negative (right) noisy trader shock



Note: This figure illustrates the government's default outcome in equilibrium for each debt state, θ, for two values of noisy trade shock, μ. The left panel is for a given $\mu < 0$, which implies low noisy trader shock, and the right one is for a given $\mu > 0$, which implies high noisy trader shock. The value 1.0 represents the government repayment, and the value 0.0 represents the government default. The default outcome satisfies (1.1) with $\bar{B} = 1$. The black dash lines are for low β, and the red solid lines are for high β. Parameter values are assigned as Table 1.2. The gray lines is obtained similarly as in Figure 1.4.

Similarly, under a negative noisy trader shock, $\mu < 0$, high information precision can result the government defaulting there exist debt states where the government fails to rollover its debt, and the government defaults because the market perceives the debt status in a pessimistic way. The reason why it happens is because market cannot separate repay relevant fundamental θ from market signal x and there exists some θ that itself is a good fundamental but are seen as a bad signal by the market with sufficient market noise. In Appendix A, I illustrate what happens under a negative noisy demand shock.

1.5 Extension: a normal distribution for prior belief

The model assumes that θ is drawn from an improper uniform distribution over the real line, and it has no notion of the mean of θ. It is a convenient assumption for tractability but a bit far from being realistic. Some governments maintain a low debt level, whereas other countries struggle from high indebtedness. The current specification of the prior distribution implies that the equilibrium bond price solely depends on what value the market signal turns out to be. From now on, I extend the common prior distribution to a normal distribution, so that those parameters governing this distribution jointly affect the equilibrium bond price, along with the value of market signal. In conclusion, I show that the main result is robust to this richer prior specification.

1.5.1 The model with prior belief

I assume that θ is now drawn from a normal distribution as follows: $\theta \sim N(\theta_0, \frac{1}{\gamma})$, where γ is the precision of θ distribution and θ_0 is a prior mean, which is common knowledge among all lenders. Almost all parts of the analysis hold as the same except for posterior belief. For lender i, their posterior belief is now

$$\theta|_{s_i,x,\theta_0} \sim N\left(\frac{\gamma\theta_0 + \beta s_i + \beta\sigma\alpha x}{\gamma + \beta + \beta\sigma\alpha}, \quad \frac{1}{\gamma + \beta + \beta\sigma\alpha}\right).$$

For the marginal lender, their posterior belief becomes

$$\theta|_{s^*(p),x,\theta_0} \sim N\left((1-\tau)\theta_0 + \tau x, \quad \frac{1}{\gamma + \beta + \beta\sigma\alpha}\right)$$

where $\tau = \frac{\beta+\beta\sigma\alpha}{\gamma+\beta+\beta\sigma\alpha}$. Adding prior gives additional information of the debt state, and lenders update posterior belief based on θ_0, the average value the debt level tends to be, as well as based on based on the market signal and the private signal. The mean of the posterior is determined as an average of the prior mean θ_0, the private signal s_i, and the market signal x, weighted by their precision respectively.

As before, in equilibrium, the private signal that the marginal lender receives has the same value as the market signal. This means that the marginal lender anticipates the debt level as same as the level that the market signal implies. Those lender whose private signal is higher than the market signal, $s_i > x$, are less optimistic than how the marginal lender is. Those whose signal is lower than the market signal, $s_i < x$, are more optimistic, and they are the ones who bid to purchase bonds.

The following equation is a characteristic equation for the equilibrium bond price:

$$p = \Phi\left(\sqrt{\gamma + \beta + \beta\sigma\alpha}\,(p - (1-\tau)\theta_0 - \tau x)\right) \tag{1.7}$$

Again, the equilibrium bond price is a fixed point of (1.7). Sufficiently high precision inducing multiple prices still holds here, and I adjust parameter values accordingly to guarantee a unique price. The specified values are listed in Table 1.3. I set a low prior case $\theta_0 = 0.1$ representing the low-debt country, and a high prior case with $\theta_0 = 0.9$ representing the high-debt country.

1.5.2 Main results

Price schedule and the role of prior mean Similar to section 1.4, the equilibrium bond price is a function of the market signal. In Figure 1.6, various cases of price schedules are plotted under different precision levels and prior means. As in the benchmark case, the price schedule is a decreasing function of market signal.

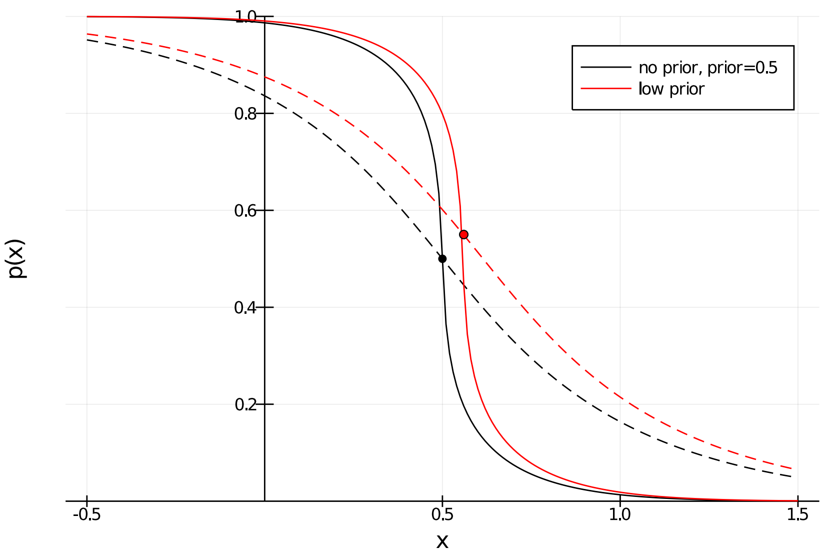
Table 1.3: Parameter values in the model with prior belief

Parameter	value
σ	0.5
α	0.1
γ	0.3
Low β	1.0
High β	2.0
Low θ_0	0.1
High θ_0	0.9

It also has an inflection point where a sensitive price schedule with high precision responds differently to the market signal: around which the bond price is higher or lower compared to a less sensitive case with low precision.

The prior mean θ_0 affects the price schedule by influencing the market signal cutoff. When $\theta_0 = 0.5$, the market signal cutoff is $x^* = 0.5$, and the price schedule is the same as in the benchmark case. Additionally, the market signal cutoff in the benchmark model is the same as in the $\theta_0 = 0.5$ case. This suggests that the benchmark model can be interpreted as an extension of the model with $\theta_0 = 0.5$. The schedule with the low θ_0 shows that its market signal cutoff is higher than 0.5, or $x^* > 0.5$. Incorporating Figure A.3 from Appendix A, where I compare the price schedules with low and high θ_0, I can generalize the finding. The market signal cutoff with the high θ_0 is lower than that with the low θ_0. In other words, as the prior mean θ_0 decreases, the market signal cutoff x^* increases. This finding implies that, for a given x, the market tends to infer a more favorable fundamental in an economy with low prior mean compared to an economy with high prior mean.

Figure 1.6: Price schedule of the extended model with low θ_0



Note: I plot the price schedule as a function of the market signal, x. Dash lines are under low precision of private signal (low β), and solid lines are under high precision of private signal (high β). Black lines correspond to the benchmark economy with improper uniform distribution, and red lines correspond to the economy with normal distribution prior with low mean (low θ_0). Parameter values are assigned as in Table 1.3.

The main result with prior With the observation, I do the same exercise. I compare how price and default outcome changes as β gets high between the low θ_0 and high θ_0 case. The result is robust in the extended version of the model, and it can be extended to the conclusion that the economy with the low prior mean has a higher default probability with precise information. In this economy, the government tends to have states where its debt level is low, but due to noisy trader shock, the market infers the state as high default risk fundamental. A sensitive price schedule with precise information is disadvantageous to those states.

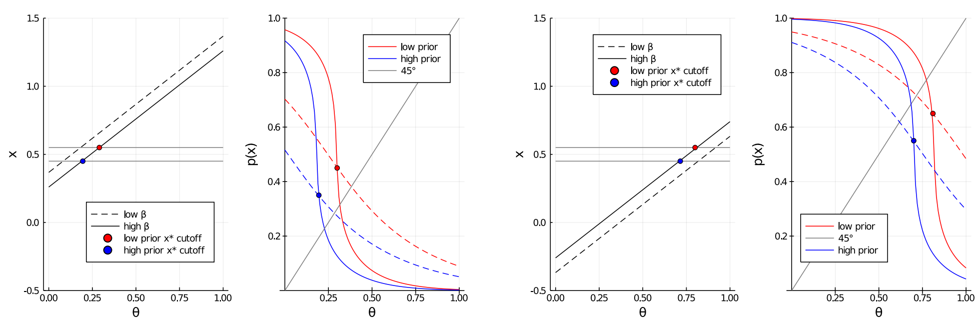
Note that for any states (θ, μ), the value of the market signal is the same in both low and high θ_0 economies. It is the realized value of θ and the distribution of s_i

that determine what x is; this is unrelated to the prior distribution of θ. In both economies, x is determined by the measure of lenders who receive $s_i|\theta < s^*(p)$, and the conditional distribution of s is independent of the value of θ_0.

Both the low θ_0 and the high θ_0 economies, therefore, have the same value of x for a given state (θ, μ). What distinguishes the two economies is how the market interprets the same x. The low θ_0 economy has a higher x^* and a wider range of interpreting x as a favorable signal. On the other hand, the high θ_0 economy has a narrower range of favorable signals. This distinction arises from the fact that the low θ_0 economy has a lower prior, and it occurs regardless of the realization of μ.

Figure 1.7: Market signal and price schedule for a given noisy trader shock

(a) low noisy trader ($\mu < 0$) (b) high noisy trader ($\mu > 0$)



Note: This plot depicts the market signal and the according equilibrium price for each debt state, θ, for two different values for noisy trader shock, μ. The panel (a) represents the case of low noisy trader, $\mu < 0$, and the panel (b) represents the case of high noisy trader, $\mu > 0$. For each panel, the left plot is the market signal as a function of debt state, and the right plot is equilibrium price as a function of debt state. In the left plot, market signal cutoffs, x^*, for each low and high prior mean economy are represented as horizontal gray lines. The red lines represent the low prior mean economy with low θ_0, and the blue lines represent the high prior one. The solid lines represent high precision economy, and the dash lines represent low precision economy. Parameter values are assigned as Table 1.3.

Given this different interpretation, equilibrium bond price reacts accordingly, as shown in Figure 1.7. Note that the low prior mean economy has a higher p than the high prior mean economy for all θ and regardless of μ being positive or negative. This is a combined result of the price schedule of the low prior being high in general and low prior having a larger range of good signal. Not only that low prior economy benefits from sensitive price schedule in a larger θ range since it has a larger range of good signal.

The low prior economy still has a θ range where it is disadvantages from a sensitive price schedule and has lower p. The sensitive price schedule devalues unfavorable signals sufficiently so that the government with $\theta \in [0.315, 0.380]$ ends up default with high precision where it would not have defaulted with low precision. In the high prior economy, the government type of $\theta \in [0.230, 0.285]$ would default with higher information precision. Both happen because, as before, θ is not low enough to be interpreted as a favorable signal with negative market noise and p depreciates sufficiently. On the other hand, in the positive market noise case as shown in (b) of Figure 1.7, the opposite happens. $\theta \in [0.715, 0.775]$ changes to repay in low prior economy, and it is $\theta \in [0.620, 0.685]$ for the high prior economy.

Default outcome Figure 1.8 shows the repayment/default outcome for a given $\mu < 0$ (left panel) or $\mu > 0$ (right panel) case. Similar to section 1.4, it is low θ states where high precision results the government default. Conversely, it is high θ states where high precision allows the government to rollover its debt. It happens in both the low and the high θ_0 economy. The difference is that since the low prior economy is more generous to favorable signals, the market generally values the bonds higher for the low prior economy than the high prior economy. As a result, the low prior economy has larger a θ range of repay compared to what the high prior economy has.

I calculate repayment probability, defined as the average of repayment outcome in all states (θ, μ) weighted by the join probability, and the result is in the left panel of Table 1.4. Note that the repayment probability with a neutral prior ($\theta_0 = 0.5$) is 0.5 which is the reference number for determining whether the government repayment is likely or not. The table shows that the low prior economy has a lower repayment probability under high information precision, whereas the high β leads the high prior economy to have a lower default probability. The reason is as follows. It is those states (θ, μ) with relatively low θ that are associated with depreciated p by a sensitive price schedule and change to default from repay. Reversely, it is those states (θ, μ) with relatively high θ that that are associated with appreciated p by a sensitive price schedule and change to repay from default. In the neutral prior or no prior case, both disadvantageous and advantageous effects from high precision are equally likely to happen, cancel each other out, and have no change in repayment probability with high precision.[6]

The low prior economy is more likely to have relatively low θ and, although those θ are good fundamentals, it is more vulnerable to being interpreted as a bad signal with a negative market noise and get changed to default with high precision. On the other hand, the high prior economy is more likely to have relatively high θ. This change can be interpreted as favorable market signals with positive market noise, and when this happens, it can get benefit from precise information since it appreciates p sufficiently so that it has higher p than θ even though θ itself is relatively high. However, note that the low prior economy has higher repayment probability than the high prior economy in both low and high information precision systems. The market has more generous good signal criteria toward the low prior economy due to the fact that the economy in general has a low level of debt. However, the market cannot distinguish the relevant fundamental and market noise so that price response and according default outcome can happen adversely to the states with low θ.

Next, I calculate the average bond price, defined as the average of p in all states (θ, μ) weighted by the joint probability, and the right panel of Table 1.4 shows the result. The low prior economy has a higher average bond price than the high prior economy in both low precision and high precision case. This means that lenders

[6] I set the μ grid to be symmetric so that positive market noise can realize as equally likely as negative market noise.

Table 1.4: Default risk and average bond price

Repay probability

	low β	High β
Low prior	0.5500	0.5438
High prior	0.4499	0.4561
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

Average bond price

	low β	High β
Low prior	0.5585	0.5501
High prior	0.4414	0.4498
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

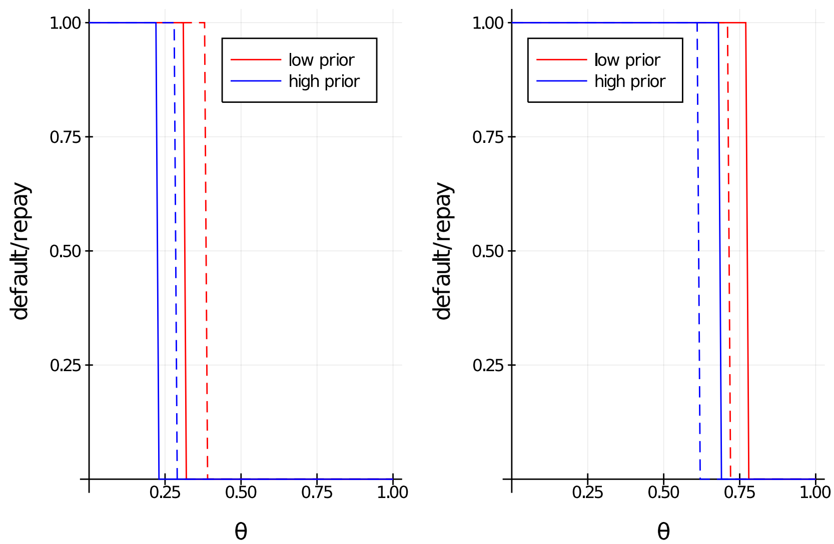
Note: This table shows the repayment probability (left) and the average equilibrium bond price (right) for each specified economy. No prior economy represents the benchmark economy as in section 1.2. Other economies represents the economy with prior as in section 1.5 with a prior mean, θ_0 as specified. Low prior represents an economy with $\theta_0 = 0.1$, and high prior represents an economy with $\theta_0 = 0.9$. Parameter values are assigned as Table 1.2 and Table 1.3.

understand the fact that θ is likely to be low in general. However, as shown in Table 1.4, the low prior economy has a lower average price in high precision case. Low θ state is more likely to be adversely affected by precise information, resulting in depreciation of bond price by sensitive price schedule. It is the low prior economy that is more likely to have those states realized.

1.6 Conclusion

This paper explores the impact of private information precision on default outcomes through bond prices. The key finding is that precise information increases the sensitivity of the price schedule to the market signal. However, this heightened sensitivity can have adverse consequences for states with low debt levels. The reason behind this is that the market struggles to distinguish between the fundamental and irrelevant market noise within the market signal. As a result, states that are fundamentally good with low debt levels are more susceptible to negative market noise compared to states with bad fundamentals. Consequently, fundamentally good states are more likely to be perceived as a pessimistic market signal, prompting the sensitive price schedule to devalue the bond price. This devaluation effect is substantial enough to push a government with low debts into default, despite the state itself being fundamentally sound and capable of repayment without the sensitive response.

Figure 1.8: Default outcome in prior economy: positive (left) and negative (right) noisy trader



Note: This figure illustrates the government's default outcome in equilibrium for each debt state, θ, for two values of noisy trade shock, μ, and two different prior mean, θ_0. The left panel is for a given $\mu < 0$, which implies low noisy trader shock, and the right one is for a given $\mu > 0$, which implies high noisy trader shock. The value 1.0 represents the government repayment, and the value 0.0 represents the government default. The default outcome satisfies (1.1) with $\bar{B} = 1$. The red lines represent the low prior mean economy with low θ_0, and the blue lines represent the high prior one. The solid lines represent high precision economy, and the dash lines represent low precision economy. Parameter values are assigned as Table 1.3.

Chapter 2

Learning about Sovereign Credit Ratings

2.1 Introduction

Along with Chapter 3, this chapter examines the regulatory usage of sovereign credit ratings, particularly the constraints that institutional investors face in holding sovereign bonds with low ratings. The study investigates the consequences of these constraints on capital market segmentation and how a country's sovereign ratings affect the external credit supply it receives in an open economy. When a country is downgraded to junk status, it loses a group of investors who are subject to prudential regulations. As a result, a junk-rated country must rely on a different set of investors to raise funds compared to an investment-grade country.

This market segmentation by sovereign credit ratings has a greater impact on developing countries than on developed countries. Developing countries are more likely to experience downgrades to junk during recessions or political instability, whereas advanced economies are seldom rated as junk.[1] Focusing on the consequence on capital market segmentation, this paper aims to understand the role of sovereign ratings in emerging markets.

In this chapter, I document the regulatory usage of sovereign credit ratings and examine the market response upon the change in credit ratings. From an emerging market panel data, I estimate informative empirical statistic, which will be used as a targeted moment in Chapter 3. I use sovereign bond spreads and sovereign ratings data to capture the spread change when a country's rating changes from investment grade to junk. I find that countries experience an increase of a 30-basis-point in their yields when they are downgraded to junk. This increase is compared to those countries with similar economic fundamentals but not experiencing a downgrade to junk.

This higher spread is robust to other several specifications. I find that the increase in bond spread is higher for those countries that are downgraded to junk than those that are downgraded but not cross the regulatory threshold (downgraded within investment grade or junk grade). Those empirical findings are consistent with the regulatory usage of sovereign ratings.

The rest of the chapter is organized as follows: first, I explore the related literature. In Section 2.2, I provide examples of regulations that use credit ratings for classifying risky asset and prohibiting institutional investors from investing low-rating sovereign bonds. In Section 2.3, I estimate the spread movement upon downgrades. I find that the spread moves differently depending on downgrade across the

[1] According to S&P sovereign rating, Sweden, the United Kingdom, Netherlands, and the United States have downgraded once during the entire rating history and it is from AAA (the highest rating level) to AA+ (the second highest rating level). Germany and Switzerland have been keeping their rating as AAA throughout their rating history. Singapore and Hong Kong constantly improved sovereign ratings and have kept their investment-grade status throughout their history. During the European debt crisis, only Portugal and Greece experienced downgrading to junk. Spain, Italy, and Ireland experienced multi-notch downgrades, but they never downgraded to junk.

regulatory threshold or not.

2.1.1 Related literature

This paper contributes to two related strands of literature. First, this paper is related to the quantitative sovereign default literature. Second, the paper is closely related to a broad set of empirical papers that analyze the impact of credit ratings on sovereigns, especially developing countries.

The sovereign default literature bases its theoretical framework developed in [Eaton and Gersovitz, 1981] and extended by [Arellano, 2008], [Hatchondo and Martinez, 2009], and [Chatterjee and Eyigungor, 2012]. The survey papers, [Aguiar and Amador, 2014] and [Aguiar and Amador, 2021], documents the sovereign default literature in detail. The Eaton-Gersovitz model is about a government that faces income risk and borrows, using a defaultable but otherwise noncontingent bond. The risk-averse government borrows to smooth consumption, but its impatient preference generates front-loading consumption. The key feature of the model is that the government cannot commit to repay its debts. This class of model is able to replicate the countercyclical movement of interest rates in emerging markets, which is documented in [Aguiar and Gopinath, 2007] and [Neumeyer and Perri, 2005].

Including long-term bond maturity allows the model to match higher debt levels and higher volatility of interest rates. Long-term bond maturity models have a computational challenge as discussed in [Aguiar et al., 2020]. [Dvorkin et al., 2021] employs discrete choice methods to overcome the computational challenge, making the quantitative sovereign default model under long-term bonds tractable. I follow their methods and include extreme value shocks to handle the computational challenge.

There are ample empirical papers about sovereign credit ratings and their impact on sovereign bond spreads. [Cantor and Packer, 1996] estimate the weights of each sovereign's macroeconomic fundamentals in the determinants of sovereign credit ratings. Using cross-section data, they find that per-capital income, external debt, and inflation are crucial determinants of sovereign credit ratings. The paper shows that sovereign credit ratings summarize the country's macroeconomic fundamentals well and are highly correlated with sovereign bond spreads. Using event studies, the paper highlights that credit ratings can independently affect spreads and shows that spread movement follows the announcement of the change in the country's credit ratings in the expected direction. The paper points out that the announcement for non-investment grade, or junk grade, affects spread to a greater degree than that for investment grade bonds. Following the findings, I incorporate regulations on investing junk bonds to understand the distinction between investment grade and junk.

[Hanusch et al., 2016] and [Drago and Gallo, 2016] are the closest papers to the empirical evidence of this paper. Both papers focus on the regulatory usage of sovereign ratings. [Hanusch et al., 2016] use panel regression to estimate the impact of downgrades from investment grade to junk. They estimate the impact on short-term borrowing using Treasury Bills, whereas this paper finds evidence on long-term borrowing which is consistent with the quantitative model. [Drago and Gallo, 2016] use an event study approach to provide evidence of a regulatory threshold. They compare the spread response to downgrades crossing investment grade/junk classification and to downgrades without crossing it. They find that the CDS spread reacts intensely to downgrades crossing the threshold. This paper takes a similar approach, and its findings are consistent with the literature. The contribution of this paper is building a quantitative model that can generate empirical findings, which enables us to estimate the degree of investment grade/junk segmentation and to do welfare exercises.

2.2 Ratings and regulations on institutional lenders

Sovereign credit ratings are the assessment by credit rating agencies, for example, Moody's and Standard and Poors', of the likelihood of a government defaulting on its debt obligations. In recent decades, the demand for sovereign ratings has risen rapidly. In the early phase of the 20^{th} century, Moody's rated roughly 50 bonds of central governments. These were mainly issued by developed countries, including the United States. The number has risen to more than 130 countries as of today. This growth is because sovereign ratings facilitate countries to access international capital markets: International investors, particularly United States investors, prefer rated government securities over unrated securities of similar default risk ([Cantor and Packer, 1995], [Luitel and Vanpée, 2018]). In recent decades, developing and low-income countries, for example, African countries, have newly had their sovereign bonds assessed for credit ratings. Developing countries pay considerable attention to sovereign ratings as a means of raising external funds from international capital markets ([Cantor and Packer, 1995]).

Another reason developing economies care about their credit ratings is because their rating change could trigger regulatory constraints on investing their bonds. Rating-based regulations on institutional investors are intended to prevent speculative investing under prudential measures. Those regulations originated from the Volcker Rule and the Dodd-Frank law in the United States [2] (see [Duffie, 2012], [Bernstein, 2019]), and the Basel II framework spread rating-based rules widely in advanced economies as well. The restrictions are often in the form of imposing the minimum level of rating for securities investment and holdings. A well-known notion for the minimum level is the speculative grade or junk: equal to or below BB+ under S&P and Fitch or Ba1 under Moody's. Therefore, when a developing country has its sovereign rating downgraded to junk, its sovereign bonds are subject

[2] The Volcker Rule exempts the Unites States treasuries and federal agency bonds, but it does not exempt securities issued by foreign countries.

to prudential regulations, and potential investors are limited to holding junk bonds due to regulatory concerns.

South Africa in 2020 provides anecdotal evidence of developing countries' concern about being junk-rated. In November 2022, Moody's and Fitch downgraded South Africa deeper into junk territory. South Africa's Minister of Finance said, "The decision by Fitch and Moody's ... is a painful one. ... Continuous rating downgrades will translate to unaffordable debt costs, deteriorating asset values."[3] According to JP Morgan estimates, the downgrade of South Africa could trigger the forced selling of up to $2.4 billion worth of South Africa's dollar-denominated Eurobonds.[4] Another piece of anecdotal evidence is Greece in 2010. As its debt crisis evolved, Greece was downgraded to junk by S&P and Moody's. According to Barclays Capital, a British universal bank, this downgrade is estimated to result in Greek government bonds worth $252 billion being excluded from its bond indexes.[5] [6]

As documented in [Kiff et al., 2012], [BIS, 2009a], credit ratings are often used by authorities to classify securities in legislation, regulations, and supervisory policies. Credit ratings affect the bond demand of institutional lenders and serve as triggers in investment decisions under regulations. These regulations are mostly motivated by prudential measures and have the goal of sheltering institutional investors' portfolios from high risks. There are nationally recognized statistical rating organizations (NRSROs) under the United States Securities and Exchange Commission (the U.S. SEC) and these regulations adopt credit ratings released by NRSROs.

Basel II is one of the prominent examples of the usage of credit ratings on

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[3] [Mukherjee, 2020]

[4] [Strohecker, 2017]

[5] [Wilson, 2010]

[6] Investment decision by credit ratings is also present in internal rules of index funds. Investment-grade (rating equal to or above BBB- under S&P and Fitch or Baa3 under Moody's) bond indexes are forced to sell their junk bonds following internal guidelines. Dropping out major bond indices could trigger additional forced-sell by other investors.

regulations. Risk-based capital requirements under a standardized approach use credit ratings to map credit risks to risk weights or capital charges. Many advanced economies, for example, the European Union, Australia, Canada, Japan, and the United States, incorporate the Basel II framework into assessing the credit quality of securities in banking sectors. Also, the central banks set a limitation for acceptable collateral and margin requirements. For example, the European Central Bank (ECB) specifies the minimum rating level for the eligible collateral of commercial banks ([ECB, 2013]).

Not only banking sector is affected, but insurance sector is also affected. Mostly, insurance companies or pension funds are regulated to hold bonds above the minimum rating. In Japan, for example, estimating credit risks for insurance companies is done by calculating the solvency margin ratios, and credit ratings are used for the calculation. In the case of the United Kingdom, the Insurance Prudential Sourcebook relies on credit ratings for insurance capital requirements. In Canada, the Office of the Superintendent of Financial Institutions (OSFI) states, in its life insurer capital guideline, that "A company must consistently follow the latest ratings from a recognized, widely followed credit rating agency. Only where that rating agency does not rate a particular instrument, the rating of another recognized, widely followed credit rating agency may be used." These restrictions are present also in state legislation. For example, New York state insurance law mentions that an insurer may insure municipal obligation bonds that are not investment grade as long as at least 95 % of the insurer's aggregate liability is investment grade (see [BIS, 2009b]).

Non-investment grade is often subject to these regulation, and investment/non-investment is a commonly-used regulatory threshold. Investment grade is above BBB- under the Standard and Poor's and Fitch rating system or Baa3 under Moody's system. Non-investment grade which is often called a junk grade is below BB+ under the Standard and Poor's and Fitch system or Ba1 under Moody's system. Regulatory restrictions imply that it is costly to hold non-investment grade (or junk grade) securities or that the potential holders of non-investment grade sovereign debt are limited. As documented in [Rigobon, 2002], upgrading to investment grade means that broader types of investors, for example, pension funds, and insurance companies, can hold the sovereign bonds.

2.3 Empirical evidence

In this section, I present the empirical relationship between countries' credit ratings and their spreads. I investigate spreads response as a country's credit rating changes across regulatory threshold. In what follows, I use investment grade and junk grade as the regulatory threshold.

The country's default risk affects both the bond spread and the credit rating. In regression, I control for the country's default risk using its macroeconomic fundamentals. I show that the country experiencing downgrades across the regulatory threshold is associated with a statistically higher bond spread compared to the country with similar macroeconomic fundamentals that does not experience a downgrade. I use the coefficient estimate of downgrade dummy variable to calibrate the market segmentation parameter later in the calibration section. As a robust check, I compare the spread response to downgrades across the threshold to the spread response to the same degree of the downgrades but not crossing the threshold. I find a higher spread with downgrades across the threshold than downgrades not crossing the threshold.

2.3.1 Data description

The observation of interest is how the bond market responds to a regulatory constraints triggered by a change in the credit rating of a bond. I use bond spreads as the measure of the bond market response. In particular, I use the JP Morgan's Emerging Market Bond Index (EMBI) spread in the Global Economic Monitor (GEM) in the World Bank. These spreads are defined as the weighted averages of the bond yield spreads of US dollar-denominated external debts issued by sovereigns and sovereign entities over corresponding the United States government debt securities (see [Comelli, 2012]). For a country's credit rating, I use the Standard and Poor's sovereign credit ratings. [7] Under the Standard and Poor's (S&P) criteria, the regulatory threshold of interest lies between BBB- and BB+. In other words, a country with S&P ratings above BBB- is investment-graded and the one with S&P ratings below BB+ is junk-graded.

The sample period is from January 1998 to December 2019. I choose a set of countries that have experienced being both investment-graded ratings and junk-graded ratings throughout the sample period, along with data availability. Russia has a default experience during the sample period (year 1999), and I exclude the country as an outlier. The 12 countries in the sample are Azerbaijan, Brazil, Bulgaria, Colombia, Croatia, Hungary, Mexico, Namibia, Panama, Peru, the Philippines, and South Africa. Bond spreads and ratings are calculated monthly. For each country's economic fundamentals, I use real GDP growth (in percentages) and gross debt to GDP ratio (in percentages). The data is from World Economic Outlook (WEO) in the IMF, and it is at a yearly frequency.

Table 2.1 shows the summary statistics of bond spreads by ratings. These observations are at monthly frequency. The sample is centered around the regulatory threshold, between BBB- and BB+. A low level of ratings indicates the low credibility of creditors, and as expected, a lower rating is associated with a higher bond spreads. Moreover, bond spreads become more volatile for lower ratings.

Table 2.2 shows the monthly changes in ratings. A downgrade is defined as

[7] The Standard and Poor's is one of the major credit rating agencies authorized as an NRSRO. It is fairly documented by the literature that the ratings of different credit rating agencies rarely differ much (see [Ferri et al., 1999]).[El-Shagi and von Schweinitz, 2018] documented empirically that the mean absolute difference to the average rating level is 0.5 notches. It means ratings across agencies are far less than one notch apart on average. Therefore, the empirical findings would not change much by using credit ratings of different agencies, for example, Moody's and Fitch.

Table 2.1: Summary statistics of bond spreads by ratings (bps)

	rating	mean	median	s.d.	min	max	freq.	obs.
Investment grade	A-	50	53	0.24	13	124	0.02	-
	BBB+	187	168	0.95	55	668	0.13	-
	BBB	183	172	0.83	43	696	0.20	-
	BBB-	263	226	1.14	45	924	0.20	-
Junk	BB+	331	312	1.46	103	937	0.16	-
	BB	357	315	1.68	97	985	0.16	-
	BB-	444	393	2.48	137	1,370	0.07	-
	B+	884	751	3.64	430	2,057	0.02	-
	B	782	803	1.94	506	1,365	0.01	-
	total	294	234	2.08	13	2,057	1.00	2,610

Note: this table provides summary statistics of the data set used in section 2.3. Both sovereign credit rating and spread is country-time specific. The data is in monthly frequency. I use S&P sovereign credit rating and JP Morgan's Emerging Market Bond Index (EMBI) spread in the Global Economic Monitor (GEM) in the World Bank to measure rating and spread, respectively. Under S&P rating classification, Investment grade is defined as ratings equal or higher than BBB-, and Junk is defined as ratings equal or lower than BB+.

an event where a country's rating changes to a lower level than the rating level of the previous month. An upgrade is defined as an event where a country's rating changes to a higher level than the rating level of the previous month. In my sample, every rating changes by one notch. As shown in the table, countries' ratings are persistent, and a country's credit rating has a 97% likelihood of not changing in the next month. The literature has documented that countries' ratings tend to be persistent. (See [El-Shagi and von Schweinitz, 2018]).

Table 2.2: Monthly rating changes

	freq.	obs.
No change	0.97	2,776
Downgrade	0.01	31
Upgrade	0.02	49

Note: this table provides the frequency of rating change in the data set of section 2.3. Downgrade represents an event where a country's credit rating changes to a lower rating than the one in the previous month. Upgrade represents an event where a country's credit rating changes to a higher rating than the one in the previous month. No change represents an event where a country's credit rating remains the same as the one in the previous month. I use S&P sovereign credit rating to measure rating.

2.3.2 The empirical results

After constructing the sample, I investigate how yields respond to a change in ratings. I focus on downgrades where regulatory restrictions are triggered (that is, when a country's rating changes from investment grade to junk).

I conduct the following panel fixed effect regression:

$$spread_{it} = \beta_0 + \beta_1 DowntoJunk_{it} + \Gamma X_{it} + \alpha_i + \delta_t + \epsilon_{it} \tag{2.1}$$

The variable $spread_{it}$ is country i's bond spread at time t, $DowntoJunk_{it}$ is a dummy variable which equals to one if the country i's rating is investment grade at time $t-1$ and junk at time t and equals to zero otherwise.

X_{it} is a set of control variables including the country's economic fundamentals and lagged spread. I include GDP growth and gross debt-to-GDP ratio as control variables to control a country's underlying economic fundamentals that affect both spreads and sovereign ratings. Unlike ratings and spreads in monthly frequency, economic fundamental variables are yearly. To capture the movement of the fundamentals within a year, I include the country's lagged spread (the spread of the previous month) as another control variable. I include country-fixed effect α_i and time-fixed effect δ_t in monthly frequency. The coefficient of interest is the downgrade dummy coefficient β_1.

The regression result is summarized in Table 2.3. The coefficient estimate of $DowntoJunk$ is positive and statistically significant, implying that a country's downgrade across the regulatory threshold is associated with an increase in bond spreads. The bond spreads, on average, increases by 30 bps when the country downgrades from investment grade to junk compared to the spread of a country with similar fundamentals but that has not experienced the downgrade to junk. This finding is consistent with the literature, for example, [Hanusch et al., 2016]. They also find a positive and significant spread response to a country's downgrades. Unlike them, who use a short-term bond (60-day Treasury bill) spreads, I use a long-term bond spreads, which is consistent with the specification of the quantitative model in Chapter 3.

The estimate is robust to other specifications. I include other downgrade dummies in the regression and compare the coefficients across the dummies. $withinJunk$ and $withinInvst$ variables represent similar downgrades. The variable $withinJunk$ captures downgrades within junk: downgrade events either from BB+ to BB or from BB to BB-. Similarly, the variable $withinInvst$ denotes downgrades within investment grade: downgrade events either from BBB+ to BBB or from BBB to BBB-. Both dummies are still near investment-grade and junk threshold (between BBB- and BB+).

Formally, the regression with different downgrade dummies is as follows:

$$\begin{aligned} spread_{it} = \beta_0 + \beta_1 DowntoJunk_{it} \\ + \beta_2 withinJunk_{it} + \beta_3 withinInvst_{it} + \Gamma X_{it} + \alpha_i + \delta_t + \epsilon_{it} \end{aligned} \quad (2.2)$$

where $withinJunk_{it} = 1$ when a country i's rating is either BBB+ at time $t-1$ and BBB at time t or BBB at time $t-1$ and BBB- at time t. $withinInvst_{it} = 1$ when a country i's rating is either BB+ at time $t-1$ and BB at time t or BB at time $t-1$ and BB- at time t. The variables of interest are β_1, β_2 and β_3.

Table 2.4 depicts the regression results. Downgrades crossing the threshold, $DowntoJunk$, are associated with higher spreads than downgrades not crossing the threshold, $withinJunk$ and $withinInvst$. This is consistent with the interpretation that a downgrade from investment-grade to junk is particularly important and could be related to the regulatory concerns. In general, downgrade events are negative news about the economy, and a positive sign for the downgrade coefficient is expected. However, the magnitude of the coefficient estimates is different whether the country's rating crosses the regulatory threshold or not. Furthermore, the downgrades crossing the threshold are statistically more significant than downgrades not crossing the threshold.

This result with different downgrade dummies is consistent with the literature. Using the CDS spreads of European countries, [Drago and Gallo, 2016] find that crossover downgrades have a significantly greater impact than non-crossover downgrades. They conclude that downgrades leading to a country's rating category change imply intense reactions from investors due to regulatory constraints. The contribution of the part of the paper is to use a broader set of countries than just European countries to show the evidence of the regulatory aspect of sovereign ratings. Next, I use this empirical evidence to discipline a quantitative sovereign default model.

In the Appendix B, I show additional tables related to the regressions and the regression result of a different specification.

2.4 Conclusion

This chapter is motivated by macroprudential regulations such as the Dodd-Frank Act and Basel II, which utilize credit ratings to classify risky assets. Institutional lenders are constrained to invest in low-rated sovereign bonds, often referred to as junk bonds. These regulations imply that a country with a junk rating faces a different capital market compared to a non-junk-rated country. The objective of this chapter, as well as Chapter 3, is to understand this credit market segmentation by credit rating, particularly in relation to its impact on a country's debt issuance behavior.

After constructing a panel data of emerging markets, I estimate how the market reacts to a change in a country's rating, measured in the bond spread. I find that countries experience an increase in their spread when they are downgraded to junk, compared to comparable countries with similar economic fundamentals but not experiencing a junk downgrade. The estimated increase is 30 basis points. This estimate will be a crucial targeted moment in Chapter 3, where a quantitative model is developed to examine the impact of capital market segmentation by ratings on emerging markets.

Additionally, countries' bond spreads respond differently to changes in their credit rating depending on whether the change occurs across the regulatory threshold or not. Specifically, I observed that when a downgrade pushes a country across the boundary between investment-grade and junk classification, the increase in bond spread is of a higher magnitude compared to a downgrade that occurs within either the investment-grade or junk classification. This finding highlights the importance of the regulatory threshold in credit ratings and provide empirical statistics for understanding how the segmentation of capital markets based on credit ratings can significantly influence emerging market economies.

Table 2.3: Regression with downgrade to junk dummy

	(1)
	spread
DowntoJunk	29.64***
	(11.47)
lag_spread	0.969***
	(0.005)
gdp	-0.312
	(0.308)
grossdebt	0.146**
	(0.057)
Observations	2528
R^2	0.981
Country FE	Y
Time FE	Y

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table reports the regression result of (2.1). DowntoJunk variable represents a dummy variable which captures an event where a country's rating is junk-rated whereas it was investment-grade in the previous month. Lag-spread represents a country's spread in the previous month. Time fixed effect is in monthly frequency.

Table 2.4: Regression with different downgrade dummies

	(1)
	spread
withinInvst	17.10*
	(9.572)
DowntoJunk	30.14***
	(11.46)
withinJunk	18.63*
	(10.09)
lag_spread	0.969***
	(0.005)
gdp	-0.249
	(0.309)
grossdebt	0.141**
	(0.057)
Observations	2528
R^2	0.981
Country FE	Y
Time FE	Y

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table reports the regression result of (2.2). DowntoJunk variable represents a dummy variable which captures an event where a country's rating is junk-rated whereas it was investment-grade in the previous month. withinInvst variable represents a dummy variable which captures an event where a country experiences a downgrade and the country was investment-grade in both t and $t-1$. withinJunk variable represents a dummy variable which captures an event where a country experiences a downgrade and the country was junk-rated in both t and $t-1$. Lag-spread represents a country's spread in the previous month. Time fixed effect is in monthly frequency.

Chapter 3

Market Segmentation by Sovereign Credit Ratings

3.1 Introduction

Along with Chapter 2, this chapter aims to understand the role of sovereign ratings in emerging markets and the consequence on capital market segmentation by ratings. While sovereign ratings have significant implications for credit access, few studies have analyzed their impacts on emerging markets and those that do often neglect the endogenous behavior of the sovereign.[1] For example, Moody's upgraded Vietnam's rating as Vietnam's National Assembly conducted prudent debt management policies (including lowering the public debt ceiling).[2] This chapter addresses this gap by providing the first quantitative analysis of how developing countries respond to regulatory constraints imposed by sovereign credit ratings in segmented market structures.

Unlike the previous literature, this paper highlights the disciplinary role of

[1] For example, [Ferri et al., 1999] is motivated by Asian financial crisis episodes and conclude that sovereign ratings can represent another systemic risk to emerging markets.

[2] https://www.moodys.com/research/Moodys-upgrades-Vietnams-rating-to-Ba2-outlook-changed-to-stable--PR_468174

sovereign ratings on countries' borrowing. Specifically, accumulating debt beyond a certain level may cause a downgrade to junk, leading to higher interest rates on the country's bonds. The country takes these consequences into account and may choose to reduce its debt level to avoid downgrades or to exit the junk territory. This disciplinary role ultimately lowers default risk and may enable the country to issue bonds at a more favorable price. The segmentation by credit ratings encourages the country to keep the default risk relatively low, and the country is less vulnerable to sudden output drop. Also, this effect can reduce overborrowing inefficiency stemming from bond maturity structure and improve the country's welfare. Incorporating the endogenous response of the sovereign is critical for this result to arise.

To this goal, I build a quantitative sovereign default model with credit ratings and a segmented market structure. In my model, the sovereign borrows in long-term bonds from the international credit market to offset income shocks and to front-load its expenditure. It also chooses to default on its outstanding debt or not. A credit rating represents the country's default risk. But the rating also determines which set of creditors the government borrows from. When the country is high-rated (or investment-grade), it issues bond in a high-rating bond market and vice versa for the low-rated (or junk) bond market. Therefore, the segmented market structure generates different bond price schedules depending on the rating. I assume that those price schedules are different by the discount rate of the creditors. Those discount rate parameters controls the degree of segmentation, and are calibrated by matching empirical statistics in Chapter 2.

By replicating this statistic, calibrated the segmentation parameter represents a higher discount rate of the low-rated bond market than that of the high-rated one. This implies that a low-rated bond is charged a higher interest rate than a bond with a high rating. This discourages the government from accumulating its debt beyond the downgrade threshold. Although it prefers front-loaded expenditure, the government finds it optimal to reduce its debt level. Debt reduction as an optimal policy is apparent particularly when the economy is near a downgrade threshold. The debt issuance in a controlled manner makes the country resilient to negative output shocks, and therefore default happens less often. This lower level of default is embedded in the pricing schedule as a lower default risk compensation. This is shown by comparing the calibrated benchmark economy to the counterfactual economy without ratings. By targeting the economy to match the average of the panel data, the model simulation shows that the frequency of default, default risk, and average spread are lower in the benchmark than in the counterfactual.

Next, I study whether countries can be better off with ratings and the resulting segmentation. I find that, under the calibration, the country is worse off with ratings when at the steady-state level of debt; however, the country can be better off at low levels of debt. There are opposite forces for this welfare implication. The welfare benefit arises from the disciplinary effect: the country can borrow at a favorable price. On the other hand, borrowing less and abstaining from consumption reduces welfare. Another welfare cost arises as the country's rating is low; higher yield from impatient lenders. The calibrated sovereign is so impatient that the welfare cost quantitatively dominates the welfare benefit. However, I find that the segmentation can make the country better off in low-debt states.

Now, I investigate how the maturity of bonds affects the welfare gains associated with sovereign ratings. I compare the benchmark economy with a counterfactual one where only one-period debt is issued. My findings show that, for all levels of debt, the use of sovereign ratings results in a decrease in welfare for the country. However, ratings do have a positive effect as they act as a commitment device for the country's future borrowing behavior. This partially resolves the debt-dilution problem under long-term debts. The issue with long-term debt is that, when a new debt is issued, the country cannot commit to its future borrowing behavior, making long-term debt more expensive than one-period debt. Ratings give lenders a rational basis to anticipate that the country will not choose to borrow more in the future, especially above the junk cutoff. This anticipatory behavior of lenders leads to pay a higher price, resulting in welfare gains under long-term debts.

Conducting a counterfactual exercise using alternative segmentation rules suggests that loose rules generate negative welfare implications because they weaken the disciplinary effect and lower the welfare benefits of segmentation.

This chapter is organized as follows: Section 3.2 describes the model, and Section 3.3 discuss how I calibrate the model. Section 3.4 shows the main quantitative results: the disciplinary effect of sovereign ratings, welfare implications, and the role of bond maturity. Lastly, Section 3.5 concludes the chapter.

3.2 The model

Motivated by the credit market segmentation that depends on credit ratings, I develop a dynamic small open-economy model. The model incorporates credit ratings and a global credit market segmented by these ratings. In addition, it includes a sovereign that borrows in long-term debts and strategically defaults under the timing framework proposed by [Eaton and Gersovitz, 1981]. Including long-term debt is important as it allows the model to quantitatively match the high levels of debt observed in countries experiencing debt crises, a challenge that one-period debt models struggle with. (see, for example, [Hatchondo and Martinez, 2009], [Chatterjee and Eyigungor, 2012]) Furthermore, bond maturity plays a vital role in determining the welfare implication of market segmentation. As the literature suggests (see, for example, [Chatterjee and Eyigungor, 2012], [Hatchondo et al., 2016]), debt dilution problem is present in long-term debts, which constitutes the main friction in the model.

3.2.1 Model Environment

Time is discrete and infinite $t \in \{0, 1, ..., \infty\}$. The economy consists of a country, a global credit market, and the country's credit rating agency. The sovereign maximizes its expected and discounted lifetime utility. Its utility comes from expenditure each period under budget constraints. The sovereign faces a stochastic output process and has an outstanding debt. The sovereign can choose to default on its outstanding debt. If so, the country is exempted from paying all debts, but faces a cost. Under no default, the country pays the debt service and rolls over its debt by issuing new bond. The credit market is segmented by a credit rating. From which of these the sovereign raises their bond revenue depends on the credit rating the sovereign is assigned.

I model the credit rating as follows: it indicates the country's credibility and is assigned every period based on the country's default risk for next period. Not only does it indicate how likely the country is to default next period, but it determines the market where the sovereign issue new bonds. The model has multiple credit markets with a representative lender for each. I assume that the representative lender in each market is different, in particular different by discount factor. This discount factor is used for pricing the bonds. I focus on Markov equilibrium.

3.2.2 Timing

Following [Eaton and Gersovitz, 1981], the timing of the model within a period is as follows:

1. Output y realizes with inherited debt b.
2. The country chooses to default on the debt or repay.
3. Conditional on repayment, the country chooses a new debt level b'.
4. Credit rating r is assigned conditional on b'.

5. Bonds are traded in the appropriate credit market conditional on the rating. The sovereign raises bond revenue and chooses its expenditure.

3.2.3 The sovereign's problem

The state of the economy is output, y, and the outstanding debt level, b. Output is stochastic and exogenous. I assume it follows an AR(1) process with persistence ρ and volatility η. I estimate the process from the emerging market panel data in the empirical section.

The debt is in a long-term, and defaultable but otherwise non-contingent bond. Following [Hatchondo and Martinez, 2009] and [Chatterjee and Eyigungor, 2012], I consider random maturity bonds. I assume that every bond randomly matures at the rate of λ in each period. Each bond pays coupon until it matures, and when maturing, it promises to pay a principal of 1. The baseline coupon payment κ is normalized so that the price of risk-free bond equals to 1 (see [Aguiar and Amador, 2021]). With this specification, a unit of bond issued today is a contract that promises to pay debt service tomorrow of $\lambda + (1-\lambda)\kappa$ and of the remaining portion $(1-\lambda)$ only if the country does not default. With long-term bonds with random decay at a constant rate, the relevant variable to keep track of is the stock of the debt that the country is owed.

At the beginning of each period, the sovereign chooses to default on its debt or repay the maturing principal and coupon. If default is chosen, the sovereign is exempted from its debt obligations, but default generates two costs: 1) it generates a direct output cost, which is captured by $def(y)$, and 2) the sovereign is excluded from the global credit markets and cannot borrow for a period of time. If repayment is chosen, the sovereign pays the total debt service which is proportional to the stock of outstanding debt and raises a new revenue by issuing new bonds. Consumption takes place using the exogenous output and new bond revenue after serving the outstanding debt. The sovereign is risk-averse. The sovereign's problem is described as follows.

The sovereign, at the beginning of each period, chooses to repay or not. The value $V(y, b)$ is the value at state (y, b), $V^R(y, b)$ is the value under repayment and $V^D(y)$ is the value under default. That is:

$$V(y, b) := \max\left\{V^R(y, b), V^D(y)\right\}$$

The function $D(y, b)$ is the default policy function where it equals to 1 when $V^R(y, b) < V^D(y)$.

Under default, the sovereign is free from its debt obligations, but suffers a direct output cost and is excluded from the credit market. It cannot borrow, but in the next period with some probability, it re-enters the credit market starting from zero debt. The parameter θ is the probability that the country re-enters the credit market under default status.

The value of default is:

$$V^D(y) = u(def(y)) + \beta\mathbb{E}_{y'|y}\left\{\theta V(y', 0) + (1 - \theta)V^D(y')\right\}$$

Under repayment, the sovereign pays the debt service, coupon payments, and the principles of matured debt, and chooses the stock of debt tomorrow, b'. By doing so, the sovereign issues new bonds, $b' - (1 - \lambda)b$, at a price of q. As shown below, the bond price is determined by the future probability of repayment and it will depend on tomorrow's stock of debt b'. An innovation of this paper is that bond price also depends on the sovereign's credit rating. The credit rating matters because it determines in which credit market the sovereign sells its bond. When the sovereign's credit rating is high, it sells its bonds in a high-rating bond market, whereas when it has a low rating, it sells in a low-rating bond market. How the two credit markets are different is explained in the market segmentation section.

The function $u(.)$ is the sovereign's utility function. β is the sovereign's discount factor. The sovereign chooses its optimal debt issuance to smooth its consumption under the budget constraint (3.1).

$$V^R(y,b) = \max_{c,b'} u(c) + \beta \mathbb{E}_{y'|y}\{V(y',b')\}$$

$$s.t. \quad c + \big(\lambda + (1-\lambda)\kappa\big)b = y + q(b', R(b',y))\big(b' - (1-\lambda)b\big) \tag{3.1}$$

The debt issuance policy function $B(y,b)$ is determined from this optimization problem.

3.2.4 Credit rating

In each period, the sovereign's credit rating, r, is assigned. The credit rating takes two values: $r \in \{h, l\}$. [3] Investment-grade bonds are high rating bonds ($r = h$), and junk bonds are low rating bonds ($r = l$). I assume that there is no incomplete or imperfect information in the model. [4]

As in the real world where credit ratings reflect the credibility of the country's debt obligations, in the model, the credit rating reflects the sovereign's default probability. If the sovereign is likely to default in the next period, a low rating is assigned to the bonds, and vice versa. I assume that the assignment of a rating follows an exogenous rule, which is characterized by a rating rule parameter, $\bar{p}$. This parameter serves as a threshold between high and low ratings: if the sovereign's default probability next period is higher than $\bar{p}$, the country's bonds are assigned low rating and vice versa. It is important to remember that a credit rating is

[3] Credit rating agencies, like Moody's and S&P, issue sovereign credit ratings with the highest AAA+ and the lowest D. There are 20 different levels of credit ratings.

[4] There are literature studies on sovereign credit ratings under the global game framework. See [Carlson and Hale, 2006], [Holden et al., 2012], [Holden et al., 2018]

assigned after the sovereign's debt issuance choice. Therefore, the sovereign takes into account that its debt choice affects how its bonds are rated.

The credit rating is assigned as follows. For each state s, given sovereign's debt choice b',

$$R(b'(s), s) = l \quad if \quad \mathbb{E}_{s'|s}\big(D(s', b'(s))\big) > \bar{p} \tag{3.2}$$

$$R(b'(s), s) = h \quad if \quad \mathbb{E}_{s'|s}\big(D(s', b'(s))\big) \leq \bar{p} \tag{3.3}$$

The function $R(b'(s), s)$ is the rating policy function under the rule $\bar{p}$.

3.2.5 Global credit markets

There are two credit markets; one for high rating bonds and one for low rating bonds. Each market is perfectly competitive and is composed of a continuum of lenders. Lenders in each market are homogeneous, and therefore, each market has a representative lender. Both representative lenders are risk-neutral and have deep pockets. I assume that the credit markets are segmented: the representative lender of the high rating bonds markets purchases and holds the high rating bonds and they cannot purchase and hold the low rating bonds, and vice versa.[5]

Each lender maximizes the expected profit from holding the sovereign's risky bonds. In each period, the representative lender purchases bonds at the market price. The payoff of the bonds is conditional on the country's repayment next period and consists of debt services and the market value of the remaining bonds next period.[6] In equilibrium, the bond price schedules of high and low rating bonds

[5] I do not allow lenders to choose which rated bonds to hold but take it as exogenous. This enables the model to be solved in a tractable way, especially the pricing schedules.

[6] When the country is either upgraded or downgraded next period, the current bondholder can no longer hold the bonds and has to sell the holding bonds to the secondary market. When the country keeps its rating, the current bondholder can sell the bonds or resell them to the secondary market. I assume both primary and secondary markets are perfectly competitive, and the secondary market has no liquidity friction. Therefore, primary and secondary markets share the same market price schedule, and I can recursively express bond prices with a single price schedule.

satisfy a zero expected profit for each lender.

The main assumption is that the two lenders are different in terms of their discount factors: $\frac{1}{R_h}$ for the high rating bond market lender and $\frac{1}{R_l}$ for the low rating bond market lender. I normalize R_h to a risk-free rate.

It is reasonable to assume that $R_l > R_h$, which means the lender in the low-rated bond market is more impatient than the high-rated bond market lender. Because hedge funds usually seek high-return investment through short-selling and speculative investment practices, their outside investment options are higher than those of commercial banks and pension funds. Therefore, sovereign bonds that are junk-rated need to compensate hedge funds to find investing in junk bonds is as profitable as investing in high-yield outside options. In addition, investing in junk bonds involves regulatory costs that financial institutions have to bear, for example, higher capital requirements and not being counted as eligible collateral by central banks. Pricing of junk bonds incorporates the compensation for those costs. It also captures the different risk tolerance between traditional financial institutions and hedge funds.

The pricing equations for each market are as follows. For each state, and given the country's choice of b', the pricing schedule for high rating bonds is

$$
\begin{aligned}
q(b', s, &R(b', s) = h) \\
&= \frac{1}{R_h}\mathbb{E}_{s'|s}\Big[(1 - D(b', s'))\big(\lambda + (1-\lambda)\kappa + (1-\lambda)q(B(b', s'), s', R(B(b', s'), s'))\big)\Big]
\end{aligned}
$$

The pricing schedule for low rating bonds is

$$
\begin{aligned}
q(b', s, &R(b', s) = l) \\
&= \frac{1}{R_l}\mathbb{E}_{s'|s}\Big[(1 - D(b', s'))\big(\lambda + (1-\lambda)\kappa + (1-\lambda)q(B(b', s'), s', R(B(b', s'), s'))\big)\Big]
\end{aligned}
$$

It is important to notice that both pricing schedules are the same except for the discount factor. Regarding the pricing of the bonds, not only does its expected payoff next period matter, but also the bonds' rating, thus to whom the bonds are sold. Furthermore, the bond price incorporates the expected value of the remaining bonds next period, which depends on the credit rating next period. Assuming that the discount factor of the representative lender of low rating bonds market is low, the more likely it is that the sovereign's bonds will be low rating tomorrow, the lower the value of the remaining bonds next period. This expectation is embedded in the pricing schedules. The bonds could be priced relatively low even though it is high rating, if there is a high chance of downgrade next period.

Another important feature of the model is the endogenous response of the sovereign. This response is embedded in the pricing equations above. The equilibrium pricing schedules depend on the sovereign's policy functions. The sovereign internalizes how its borrowing decision affects its credit rating and therefore the pricing schedule that it faces. The impatient sovereign has a temptation to front-load consumption, which is a main driver of high borrowing. Long bond maturity is another factor of high borrowing. The opposite tensions of high borrowing are present in the model. It is not only the fact that the bond is devalued by the default risk but also the fact that borrowing over a certain threshold (so that the sovereign downgrades to low rating) triggers another bond devaluation by the low discount factor of the representative lender of low rating bonds market. The sovereign internalizes this devaluation.

3.2.6 Markov equilibrium

I define a Markov equilibrium of the economy as follows.

Definition 2. *Given the exogenous rating rule $\bar{p}$, a Markov equilibrium consists of the sovereign's value functions $V(b,s)$, $V^D(y)$, $V^R(b,s)$, the sovereign's policy functions $B(b,s), D(b,s)$, rating policy function $R(b',s)$ and bond pricing schedule*

$q(b', s)$ *such that*

(i) Given the rating policy function and the bond pricing schedule, the sovereign's value functions and policy functions satisfy the sovereign's bellman equations and maximization problems.

(ii) Given the sovereign's policy functions, the rating policy function is consistent with the rating rule $\bar{p}$.

(iii) Given rating policy functions and the sovereign's policy functions, the bond pricing schedule satisfies zero expected discounted profit conditions.

3.2.7 Bond maturity and efficiency

The literature on quantitative sovereign defaults suggests that the maturity of bonds plays a crucial role in the sovereign's borrowing behavior (see [Hatchondo and Martinez, 2009, Chatterjee and Eyigungor, 2012, Hatchondo et al., 2016]). The literature finds that debt dilution happens under long-term debt, and it creates overborrowing inefficiency. Debt dilution means the reduction in the value of outstanding debt caused by new debt issuance. This happens because sovereigns cannot commit, at the time of borrowing, the borrowing behavior of future sovereigns. Long-term bond allows the sovereign to postpone the costs of current borrowing to the future sovereign. Given today's borrowing, the future sovereign has incentives to borrow high and the current sovereign is limited to constrain it. Rational lenders anticipate that additional borrowing by the future sovereign will increase default risk and reduce the market value of bonds issued by current sovereign. The bonds issued in current period is low-priced, and the sovereign has to issue more debts in order to raise a certain level revenue.

[Chatterjee and Eyigungor, 2012] quantifies debt dilution inefficiency by comparing equilibrium with long-term and with one-period debt. They find that sovereign's welfare is highest for one-period debt and declines monotonically as maturity parameter increases. [Hatchondo et al., 2016] compare long-term bond equilibrium with dilution effect and without dilution effect, and find that the one with dilution has higher debt level and more frequent defaults. These findings suggest that long-duration maturity induces the sovereign to borrow more due to debt dilution problem, and it is welfare-reducing.

[Aguiar and Amador, 2019] find that borrowing is constraint efficient in one-period-bond [Eaton and Gersovitz, 1981] model by using dual-contracting approach. Constraint efficiency here means Pareto-optimal allocation between lenders and sovereign in the presence of market incompleteness and lack of commitment on default decision next period. Bond maturity plays a vital role in determining the welfare implications of market segmentation in this paper. I will explain the details in Section 3.4.

3.3 Calibration

The model is calibrated to a yearly frequency. The parameters of the economy are chosen either from outside of the model or by matching moments. The model targets the average across countries in the panel data. The details of calibration strategy are in the Appendix C.

The flow utility function is assumed to be a the constant relative risk aversion (CRRA) utility on consumption:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

The default cost is in units of output. I follow [Chatterjee and Eyigungor, 2012] and [Arellano, 2008], and it is non-linear in output.

$$def(y) = max\{0, d_0 + d_1 y\}$$

Table 3.1: Parameters calibrated outside of the model

parameters	description	value	source
σ	risk aversion	2	literature
ρ	output persistence	0.844	the panel data
η	output volatility	0.034	the panel data
R_h	high rating market	1.027	risk-free rate
λ	bond structure	0.1	10-years maturity
$\bar{p}$	rating rule	0.0064	Moody's rating rule

Note: This table shows

Table 3.1 shows the list of parameters that I have calibrated outside of the model. The country's risk aversion σ is set to the standard value in the literature. The stochastic output is assumed to follow an AR(1) process and is estimated from the emerging market panel data. For each country, I linearly detrend real GDP growth and estimate an AR(1) process from the cyclical components. I take the sample mean on estimated output process weighted by data observations.[7] The estimated value for persistence and volatility on output are $\rho = 0.84$ and $\eta = 0.03$ for each. Consistent with the previous literature, as in [Neumeyer and Perri, 2005] and [Aguiar and Gopinath, 2007], the estimated parameter features the emerging market's volatile output.

I normalize the discount rate of the high-rated bond market to be the international risk-free rate. I take a long-term government bond for Germany as the benchmark safe asset. That is, the risk-free rate is estimated as the average of 10-year German bond yield from FRED in 1998-2019 (the same time period of the panel data). The bond maturity parameter λ is set to resemble the 10-year maturity bond.

[7] The dataset is imbalanced. Some counties, for example Azerbaijan and Namibia, have fewer observations.

The rating rule parameter $\bar{p}$ is taken from *idealized cumulative expected default rates* [8] released by [Moody's, 2018], one of the major credit rating agencies. These rates suggest the benchmark expected default rates where a rated-counterparty will fail to perform its debt obligation ([Moody's, 2022]). The rates are available for each rating level. Since the model is in a yearly frequency, the rates in the 1-year horizon are used for the estimation. Since $\bar{p}$ captures the threshold of credit ratings under regulatory purposes, I take the mean of expected default rates of the Baa3 rating and the Ba1 rating. The estimated value of $\bar{p}$ is 0.64%. It means that in the model country is low-rated if it is expected to default at a higher rate than 0.64% next period.

3.3.1 Calibrated parameters

The rest of the parameter values are jointly calibrated by matching moments. A key parameter is the discount rate of the low rating market R_l which governs the degree of market segmentation. I calibrate the segmentation parameter to target the downgrade coefficient in the analysis of Section 4. To be specific, I run the same regression using the model simulation as in Section 4. In the model simulations, the downgrade dummy variable captures downgrades from $r = h$ to $r = l$. I use the same controls for this regression as in the data: output level y, debt-to-output ratio b/y, and the last period spread. I do not control for country-fixed effect and time-fixed effect in the model simulation because it is a single-country model and because there is no exogenous time-varying risk-free rates. [9]

[8] The word idealized may imply a potential discrepancy between the expected default rate by the time the credit rating agency assigns a rating to the entity and the default rate afterwards. The default risk estimate using historical ratings path could be different from the initial design of each rating category.

[9] Including time-fixed effect in the data is to control time-varying risk-free rates and risk premium. Another purpose is to improve on the frequency issue of macro-fundamental variables. The spread movement within a year could potentially result from the fundamental change within a year. This is hard to capture using yearly data. Month-year fixed effect can control this potential fundamental change. In the model, the current spread perfectly captures the fundamental change between last period and current period.

Following the literature, the country's discount factor β and default cost parameters d_0, d_1 are mapped to the mean debt-to-output ratio, mean spread level, and the standard deviation of the spread. The data moments are calculated as sample averages across countries in the panel data. In the end, I calibrate the segmentation parameter R_l, country's impatience β, and default cost d_0, d_1 jointly to match the four moments: the downgrade coefficient, the mean debt-to-output ratio, mean spread level, and the standard deviation of the spread.

Table 3.2: Parameters calibrated inside of the model

parameter	description	value	moments	target	model
R_l	low rating market	1.047	downgrade coefficient	29.64	29.98
β	country's impatience	0.918	mean b/y	47.1 %	25.5 %
$d0$	default cost	-0.217	mean spread	294 bps	293 bps
$d1$	default cost	0.254	s.d. spread	1.80	1.37

Table 3.2 show the calibrated values for four parameters and the targeted moments. The model has a limitation in matching the mean debt-to-output ratio. It is because high enough R_l relative to β is necessary for generating the downgrade coefficient, but that R_l discourages the sovereign from accumulating the debt in the model simulation. On the other hand, low enough β relative to R_l weakens the segmentation effect, and the downgrade coefficient is not generated under the model simulation. Including the variance of taste shock in the calibration can potentially improve on debt-to-output ratio matching. [10]

[10] Following [Dvorkin et al., 2021], I employ extreme value shock to tackle computational issues solving a sovereign default model with long-term bond. Taste shock-related parameters are directly taken from the literature for now, but potentially I can include those parameters in the joint calibration. For the literature to include taste shocks in the calibration, see [Arce, 2021].

3.4 Quantitative results

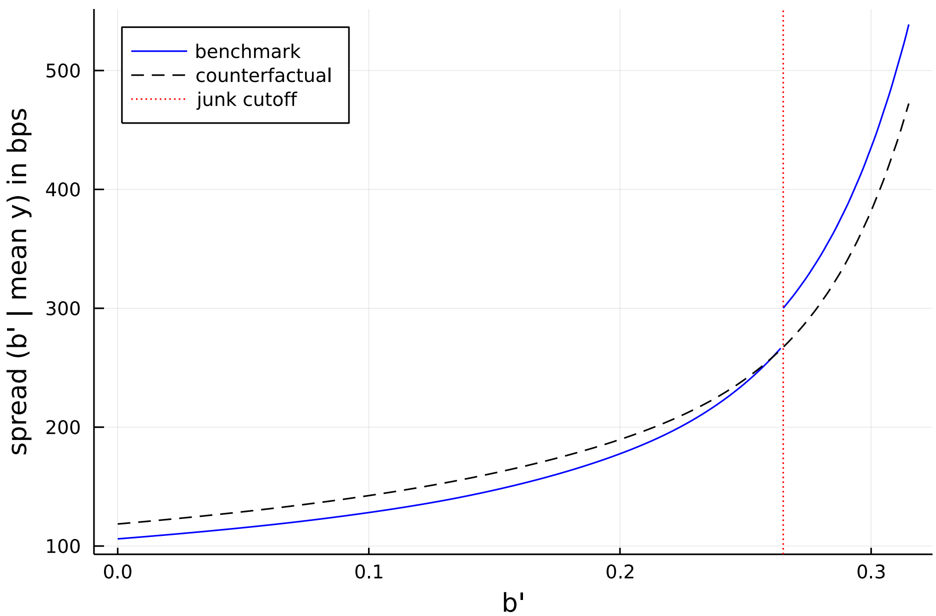
I now describe some features of the Markov equilibrium in the calibrated model. To understand the role of sovereign ratings, I compare the benchmark model to a counterfactual economy with no ratings and no market segmentation. I demonstrate the disciplinary role of ratings.

3.4.1 The equilibrium bond pricing schedule

Figure 3.1 depicts the equilibrium bond spread schedule as a function of the debt choice b'. The schedule is evaluated at the mean y level, and the b' level in the horizontal axis is relative to the mean y level. The junk cutoff indicates that the country's rating is high with a lower b' level than the cutoff, and a higher b' level than the cutoff produces the low rating for the country. As usual, the spread schedule is upward-sloping: the higher b' level increases the equilibrium spread. A high level of debt choice today means a high level of outstanding debt tomorrow, raising the sovereign's incentive to default tomorrow. That higher default risk requires a higher interest rate (or a lower bond price). The upward-sloping pricing schedule is present in both the benchmark and the counterfactual.

Unlike the counterfactual, the first distinctive feature of the benchmark is a discontinuity. This discontinuity arises from the segmentation by credit ratings. As the debt choice b' increases above the cutoff, the country's default probability increases above the rating rule threshold, $\bar{p}$, and the country's rating changes from high to low. As the country's rating changes from high to low, the lender who prices the bond switches from the patient lenders (the lenders in the high-rated bond market) to the impatient lenders (the lenders in the low-rated bond market). This switch means a discrete change in the discount rate of the equilibrium pricing equation. This discrete increase in interest rate is the driving force of the disciplinary effect on sovereign's borrowing. As shown in the counterfactual, the default risk compensation grows smoothly as the default risk increases around the threshold.

Figure 3.1: The equilibrium bond pricing schedule



Note: This figure plots the equilibrium bond spread calculated from the equilibrium bond price schedule, q, in two economies. The spread is annualized and over risk-free rate, R_h. Conditional on the mean output level, the calculated spread is plotted as a function of the debt level, b'. The x-axe represents the debt level, and the y-axe is expressed as basis points (bps). The blue line represents the benchmark economy, while the black dash line represents the counterfactual economy without the segmentation. The "junk cutoff" indicates the maximum level of debt where the country's credit rating is considered high.

Therefore, the discrete increase in the spread is not from default risk compensation but from the switch to a different credit market.

Another feature of the pricing schedule is the presence on anticipation effect. Note that the bond in the model is a long-duration bond. Thus, the bond price reflects all the future credit ratings and default probabilities. If the country downgrades to a low rating tomorrow, the market value of its bonds tomorrow will be low. Even though the sovereign's rating is high today and patient lenders price the bond with a low yield, the bond price could be noticeably low if there is a high probability of a downgrade tomorrow. That is, the expectation of future downgrades forces the devaluation of bonds in the current period. This devaluation is visualized when b is lower than the threshold but substantially high enough: the spread increases faster as b' rises closer to the threshold. This more rapid growth in the spread (or faster devaluation of the bonds) is not because the default risk evolves faster under the benchmark [11] but because downgrades in the next period are more likely to be anticipated.

Combining the first and the second features tells us how the discontinuity is determined in the model. The discontinuity is higher as the R_l parameter increases. On the other hand, the greater the anticipation effect is, the lower the discontinuity is. The country's borrowing incentives govern the anticipation effect. In particular, they govern how likely it is that a downgrade will happen next period and how likely it is for the downgrade to revert if that happened. The country's impatience and default cost parameters are relevant for the borrowing incentives. The downgrade coefficient captures this discontinuity under the simulation. To identify R_l in the model, I need to consider the interaction of this discontinuity with other parameters, which provides a rationale for the joint calibration.

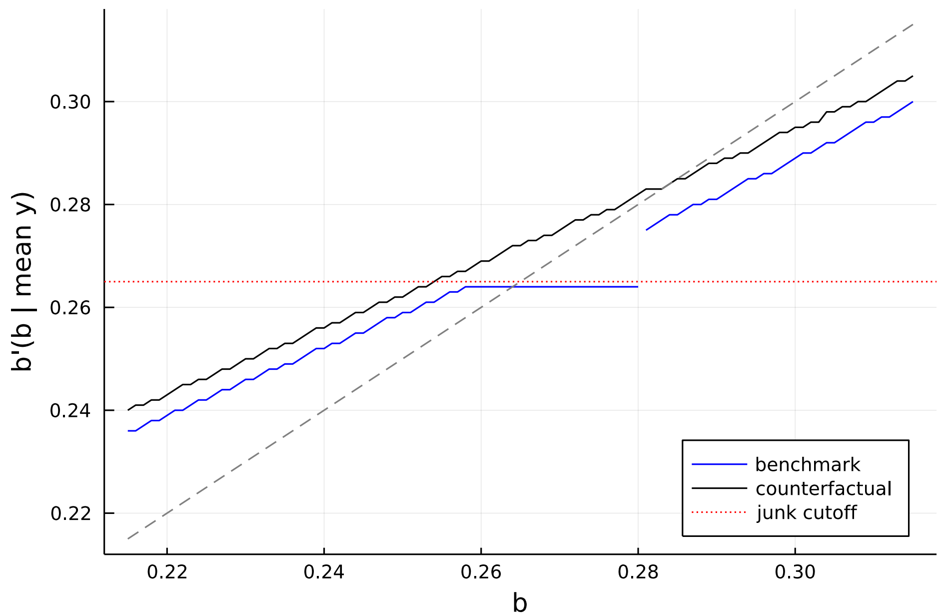
The last feature to highlight is the lower spread in the benchmark under a low enough level of b'. As long as b' is low and downgrades are less likely to happen tomorrow, the sovereign can borrow cheaply. This cheap borrowing arises from the disciplined borrowing and lower probability of default, a result that is explained in detail in the following subsections.

3.4.2 The optimal policy of the sovereign

Figure 3.2 shows the optimal policy of the sovereign in the benchmark calibration. The borrowing policy is shown as a function of the outstanding debt level b evaluated

[11] The growth rate of default probability with b' around the cutoff under the benchmark is almost equivalent to the one under the counterfactual.

Figure 3.2: Sovereign's borrowing policy



Note: This figure plots the equilibrium debt issuance policy function, $b'(b)$, in two economies. Conditional on the mean output level, the optimal debt choice is plotted as a function of the debt level, b. Both x-axes and y-axes represent the debt level relative to the mean output level. The blue line represents the benchmark economy, while the black dash line represents the counterfactual economy without the segmentation. The "junk cutoff" indicates the maximum level of debt where the country's credit rating is considered high.

at the mean y level. The level of the horizontal axis is normalized by the mean y level. The function is weakly increasing in b, which means the sovereign finds it optimal to choose a high b' when it inherits a high level of b. It is because the sovereign rolls over the inherited debt and, conditional on repayment, consumption which the sovereign gains flow utility from reduces with a higher outstanding debt. The junk cutoff denotes the level of debt such that at the given y if the government chooses b' above it, its rating is low. Therefore, the government faces today the impatient lenders of the low-rated bond market.

The distinguishing feature under the benchmark is that the sovereign stops borrowing near the cutoff. To be more precise, the country reduces the debt stock to a level right below the cutoff. That is, the sovereign internalizes the consequence of borrowing over the cutoff. In the simulation, the sovereign finds it optimal to keep its high rating and avoids the high yield from the impatient lenders. However, reducing borrowing means less consumption and less flow utility today. With a high enough outstanding debt level, the country has no choice but to cross the threshold and as a result, its bonds are low rated. But still, the sovereign chooses less debt than it would have chosen in the counterfactual; the country exits the low-rating territory by gradually reducing debt until above, but nearby the threshold.

Not only when the state is near the cutoff, but less borrowing is also present in the entire debt state b, including when the country's rating is high and is far from the cutoff. [12] This less borrowing can be interpreted as a precautionary behavior of risk-averse agents in the incomplete market ([Aiyagari, 1994]). A sufficient negative output shock could lead to downgrades, and the country pays the high yield as a consequence. Due to an incomplete market structure, the government cannot hedge from this output shock and the following rating downgrades. Instead, the risk-averse government chooses to borrow less out of the precautionary motive, even when the state is far away from the cutoff. By doing so, the probability of downgrade, which comes from the sufficient negative output shock, is low. This motive becomes more apparent as the probability of downgrades rises: as the state is closer to the cutoff, the government chooses to borrow much less. Less borrowing in the entire debt region lowers the default probability, and the pricing schedule embraces it. Therefore, the sovereign can borrow at a lower spread under a high rating than it would have had in the counterfactual, as shown in Figure 3.1.

[12] The entire debt state refers to a set of debt state $[0, \bar{b}]$ where the default probability reaches to 1 under $\bar{b}$ given y as the mean level. In this region, the optimal borrowing in the benchmark is strictly lower than that in the counterfactual.

3.4.3 Simulation

Table 3.3 compares the model simulation result to that of the counterfactual economy. To make this comparison, I feed the same time series of stochastic variables, the output series and the re-entry shock upon default. After discarding the first 100 periods of each simulation, I imposed the same initial debt level of the benchmark simulation on the counterfactual simulation. Each simulation is over 1,000 periods, and the listed moments are calculated as the sample average over 100 simulations. Percentage change denotes the percentage change of the moment under the benchmark relative to the counterfactual. The periods when the country is in default are not included in calculating the moments.

Table 3.3: Model simulation results

	benchmark	counterfactual	% change
mean default risk	1.83 %	2.60 %	- 30 %
junk freq.	37 %	51 %	- 27 %
annual default freq.	1.7 %	2.5 %	- 32 %
mean debt/y	25.5 %	26.9 %	- 5 %
mean spread	293 bps	313 bps	- 6 %
s.d. spread	1.37	1.45	- 6 %

Note: This table reports simulated moments in two economies. Each simulation is over 1,000 periods, and the listed moments are calculated as the sample average over 100 simulations. The first 100 periods for each simulation is deleted as well as periods in financial autarky and 50 periods upon re-entering to financial market.

As documented before, a lower mean debt level relative to output in the benchmark is expected. Regarding the average spread, disciplined borrowing lowers the incentive to default at a given output level and could result in a lower spread, especially when the country's rating is high. On the other hand, the presence of impatient lenders could contribute to a higher spread on average when low ratings happen frequently. Under the calibration simulation, the country's rating is high as often as 63%, and the average spread of the benchmark economy is lower than without ratings.

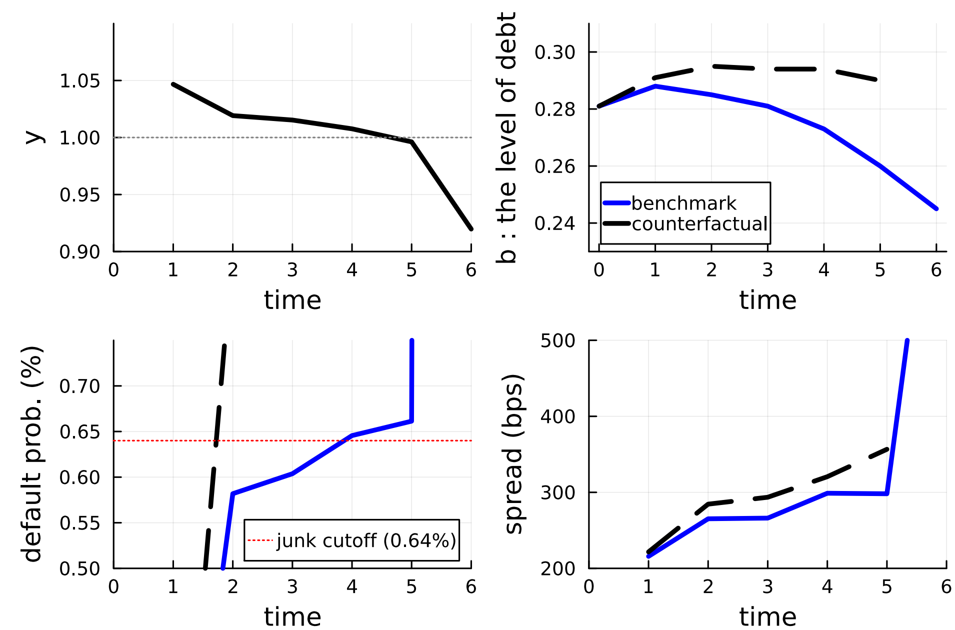
Another finding is that the country's default risk is lower with the segmentation by ratings, and fewer defaults happen on the equilibrium path. This result is consistent with a lower spread volatility and mean spread. Spread surges are observed with high default risk, which contributes to high spread volatility. Frequent downgrades could also contribute to higher spread volatility, as the spread surges discretely with the switching to impatient lenders. Despite the small magnitude of the reduction in spread volatility compared to the significant reduction in default risk, the first force quantitatively dominates the second force, resulting in lower spread volatility under the benchmark.

The fact that the reduction in default risk is relatively significant compared to the decrease in mean debt level highlights the crucial role played by the segmentation by ratings. This segmentation restricts the country from remaining in a positive default risk state, thereby preventing the default risk from evolving further. This is supported by the optimal borrowing policy, which reveals that the debt reduction is most pronounced near the cutoff or low-rating territory, where the default risk is strictly positive, and the economy is vulnerable to adverse output shocks.

3.4.4 Debt dynamics in recession

Figure 3.3 illustrates that the segmentation keeps the debt stock low, which prevents default from happening. To assess the impact of the segmentation, I simulate an output process that reflects a recession in both the benchmark and counterfactual economies. The process begins with a high level of output, gradually declines, and experience a significant drop at time $t = 6$. The two economies start with the same initial debt level. Given this output process and initial debt level, I plot the equilibrium debt level, default probability, and spreads over time. The unit of time

Figure 3.3: Debt, default probability, and the spreads dynamics during recession



Note: This figure illustrates the evolution of the debt level, default probability, and equilibrium bond spread in two economies, given a specific output process (top left). Both economies start with the same debt level, equivalent to 28% of the mean output level. The "junk cutoff" denotes the threshold level beyond which the country is rated as having a low credit rating. All x-axes represent time.

is a year.

In the counterfactual economy, the sovereign defaults at $t = 6$, whereas in the benchmark economy, it manages to survive the recession without defaulting. The likelihood of default increases as the sovereign holds a higher level of debt and as the output level drops. During a recession, defaults could happen in equilibrium when the sovereign is sufficiently indebted.At $t = 6$, when the output level drops significantly, the counterfactual economy had a debt-to-GDP ratio of up to 29%, making repayment costly for the government. In contrast, the benchmark economy had a debt-to-GDP ratio of only 26%, which is attainable because the government starts reducing the debt stock from $t = 2$. As the output level falls from $t = 2$, the default probability increases, bringing the economy closer to the junk cutoff. By $t = 4$, the country is rated as junk in the benchmark. The segmentation encourages the government to reduce its debt stock, keeping the default risk relatively low and the economy less vulnerable to sudden output drops. At $t = 5$, the default probability under the benchmark is approximately 0.66%, while under the counterfactual, it is as high as 3%.

The right bottom panel of Figure 3.3 displays the bond spreads of the two economies. In general, the spreads in the counterfactual economy are higher than those in the benchmark economy. Even though the discount rate in the counterfactual economy is equal to or lower than that in the benchmark economy, the sufficiently high default probability in the counterfactual offsets the high discount rate of the junk-rated bond market in the benchmark. One interesting observation is that due to the anticipation effect, the movement of spreads upon downgrade is relatively smooth. From $t = 3$ to $t = 4$, the government is downgraded to junk. Although the bond at $t = 4$ is discounted more by 200 bps, the spreads increase by approximately 30 bps. This is because the spreads at $t = 3$ already include the possibility of a downgrade next period and incorporate the difference in the discount rate before the downgrade.

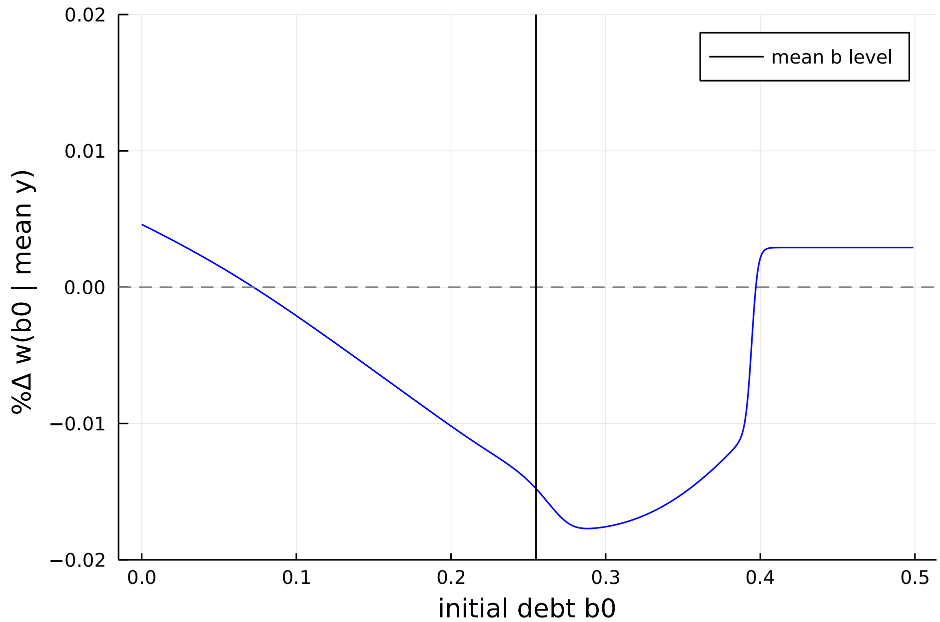
3.4.5 The welfare implications of ratings

In this section, I discuss how the ratings and resulting segmentation affects the sovereign's welfare. Figure 3.4 shows the percentage change in welfare under the benchmark compared to the counterfactual, the economy without segmentation. The welfare is in consumption equivalent units. I can define welfare in a following way:

$$1 + \Lambda(b, s) = \frac{V_{NS}(b, s)}{V_S(b, s)}^{\frac{1}{1-\sigma}} \tag{3.4}$$

where $V_{NS}(b, s)$ is the value of the sovereign at state (b, s) in benchmark economy with segmentation, and $V_S(b, s)$ is the value of sovereign at state (b, s) in counterfactual economy without segmentation. A positive number means the segmentation generates welfare gain for the sovereign. I plot the welfare for different initial debt levels b_0 and for a given mean y level.

Figure 3.4: The welfare under the segmentation



Note: This figure plots the country's welfare under segmentation as a function of initial debt level. The welfare is calculated as in (3.4), capturing the difference in the country's value in the benchmark and counterfactual economies. The positive welfare means the country has a higher level of welfare in benchmark than in counterfactual. The y-axes is in terms of percentage points.

The difference in welfare is a decreasing function of debt for low levels of the initial debt. It is because the welfare benefit from the segmentation decreases with the high debt level, whereas the welfare cost rises with the high debt level. Those opposite forces help to understand the welfare implications of segmentation. The disciplining behavior of ratings lowers the country's default risk and allows the government to borrow at a better price, which is a potential source of welfare benefit. This benefit is maximized when the debt state is far from the junk cutoff at a given y level. As the debt state is closer to the cutoff, the anticipated bond devaluation counteracts the disciplinary effect, and the welfare benefit is reduced. Although the sovereign currently maintains a high rating, there is a decent chance that the country may be downgraded to junk next period when the debt state is close enough to the cutoff.

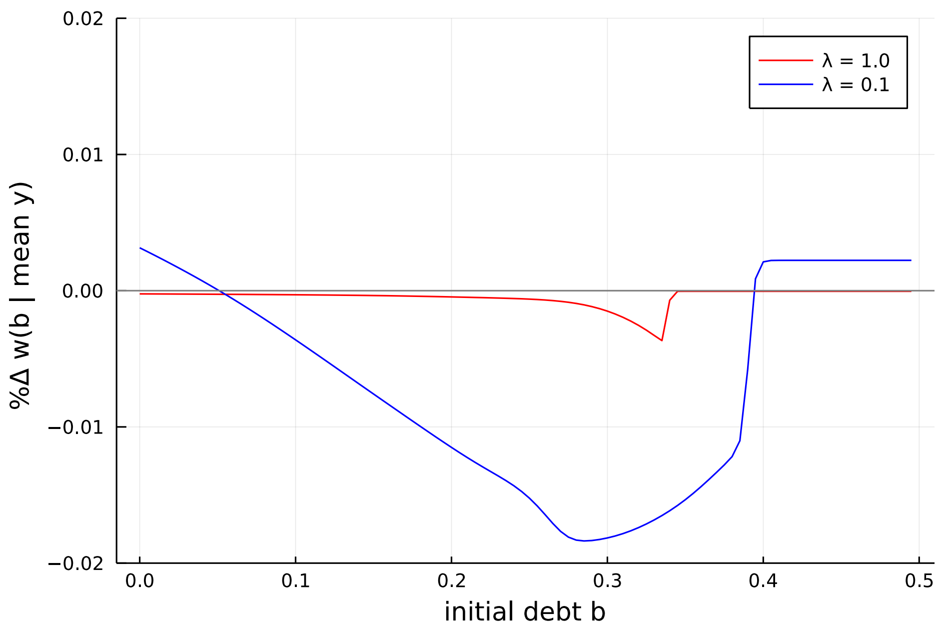
The primary source of welfare cost is low consumption from reducing debt issuance. In this model, the country's welfare is derived from the flow utility from consumption. As the debt state is closer to the cutoff at a given y level, the government actively shrinks the bond issuance, and the welfare cost from low consumption amplifies. Not only is reducing debt stock painful to the country, but the high yield from the impatient lenders is also a crucial source of welfare cost. Adverse output shocks trigger downgrades to low ratings, and the sovereign pays high yields to impatient lenders in addition to a high default risk compensation. This is not ideal from the perspective of risk sharing: the sovereign may want to hedge from low-income shock.

Figure 3.4 shows how each opposite force aggregates quantitatively. First, the segmentation delivers a welfare loss to the sovereign in the stationary debt level. The percentage change in welfare under the mean debt level is negative under the calibration. The calibrated sovereign is so impatient that it accumulates debt sufficiently close to the cutoff. The anticipated devaluation counterbalances the welfare benefit from the segmentation, at the same time the country suffers from controlled borrowing as it is near the cutoff. Moreover, the government is exposed to a decent chance of downgrades next period (coming from low-income shocks). In the end, the welfare cost quantitatively dominates the welfare benefit, and the welfare loss, in the long run, is as much as around -0.015% under the calibration.

However, the country gains welfare from the segmentation at low debt level. As long as the country is far from the cutoff, downgrades rarely occur in the near term, and the government does not need to reduce the debt issuance aggressively. The benefit of borrowing at a better price is maximized without anticipated depreciation. Quantitatively, the country's welfare is enhanced by the segmentation by roughly 0.005%. Also, the welfare gain happens under the default region. The debt region over the level of 0.4 visually shows it. In this model, the country in default status returns to the credit market with zero debt, where the welfare benefit is maximized.

3.4.6 The role of bond maturity

Figure 3.5: The welfare under the segmentation with different bond maturity



Note: This figure illustrates the country's welfare under the segmentation as a function of initial debt level in two economies: the red line represents an economy with one-period debt, and the blue line represents the benchmark economy. The welfare is calculated similarly as in Figure 3.4.

Here, I study how the welfare results change with different bond maturity. Figure 3.5 outlines the sovereign's welfare as in Figure 3.4, but I vary bond maturity parameter, λ. The blue line is the benchmark where it captures 10-years maturity whereas the red line is with $\lambda = 1.0$, which is 1-year maturity. Under one-period bond, the debt service for debt stock b is $(\lambda + (1-\lambda)\kappa)b = b$. It means that the total principle of outstanding debt should be paid in order to raise new bond revenue. This is different from the benchmark where the sovereign can raise new revenue as long as at least 10% of outstanding debt is repaid. I keep all other parameters, other than κ, the same in the counterfactual. [13]

With $\lambda = 1.0$, the sovereign's welfare under the segmentation is negative across all debt levels, suggesting that the sovereign is always worse off with the segmentation under this calibration. The main friction under one-period debt is that sovereigns cannot commit, at the time the debt is issued, on their next period default decision. Adding the segmentation cannot resolve commitment issue on not defaulting next period under [Eaton and Gersovitz, 1981] timing. Given the current debt level, the probability of default in the next period is contingent on the exogenously drawn output level, which follows a stochastic process. As the debt level increases, the sovereign's credit rating approaches the junk cutoff, and the associated welfare cost rises. At sufficiently high levels of debt, the sovereign chooses to default. In this case, the sovereign's value is independent of the debt level, and this is represented as a horizontal line for debt-to-GDP ratios higher than around 32%.

The finding that the sovereign can benefit from segmentation under long-term debts suggests that segmentation can partially address the issue of lack of commitment regarding future borrowing. The segmentation works as a commitment device by influencing market prices for future borrowing. When new debt is issued, lenders anticipate that the future sovereign is limited to borrow more, especially borrow beyond the junk cutoff. The future value of outstanding debt will not be diluted as much, and the lenders are willing to pay a higher price for current

[13] In the benchmark, coupon parameter κ is normalized so that the bond price with zero risk is equal to 1. Under one-period bond case, every bond matures next period, and there is no notion for coupon. Under one-period bond case, I set $\kappa = 0$

debt. This partially resolves the welfare-reducing debt-dilution problem, and the sovereign can benefit under sufficiently low levels of debt. This result is consistent with the existing literature. As an alternative solution to the debt-dilution problem, [Chatterjee and Eyigungor, 2012] suggests fixing the future value of outstanding debt at its value at issue. The segmentation can be viewed as a lower bound of the future value of outstanding debt.

3.4.7 Different segmentation rules and welfare implications

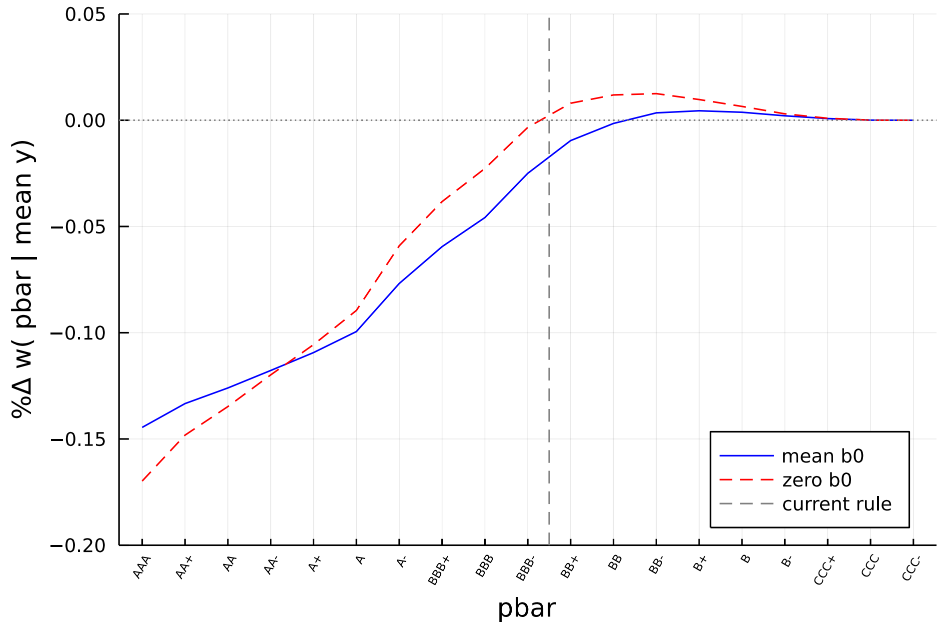
In this exercise, I vary the rating rule parameter, $\bar{p}$, and analyze how it affects the sovereign's welfare. The benchmark rule segments the credit market between BBB-/Baa3 and BB+/Ba1, which distinguishes between investment-grade and junk. Figure 3.6 shows the percentage change in welfare with different segmentation rules normalized by welfare under the benchmark. I map the $\bar{p}$ values to the rating level according to Moody's rating rule, which I use in calibration. The assumption is that each segmented market has the same discount rate as in the benchmark. The welfare change is evaluated under the mean output level y with two different initial debt levels: starting the economy with zero or the mean debt level in the benchmark.

The result suggests that the current rule is reasonable, whereas the optimal segmentation rule is a bit looser than that. Under the calibration, between BB-/Ba3 and B+/B1 is the optimal rule, and the country is better off even at the mean debt level. The sovereign is significantly impatient in the calibration, and looser rule can reduce the welfare cost. However, if the rule is loosened too much, it reduces welfare because it weakens disciplinary motives. On the other hand, tight segmentation rules aggravate the country's welfare.

3.5 Conclusion

From the regulatory usage of sovereign credit ratings, this chapter highlights the disciplinary role of ratings on governments' overborrowing. I build a model of a

Figure 3.6: Change in welfare under the counterfactual segmentation



Note: This figure illustrates the welfare under the segmentation with different rating rules. The x-axes represents a counterfactual junk cutoff below which the country is rated as high. The welfare is calculated for each counterfactual rating rule and in a similar fashion as in Figure 3.4.

country's borrowing, default, credit rating, and the consequent credit market segmentation. I incorporate the endogenous response of the government. This delivers a different implication of sovereign ratings in the international credit market, which the previous literature has neglected. I calibrate the segmentation parameter using the spread response to countries' downgrade to junk observed in the data. The consequence of downgrades to junk gives sovereigns incentives to manage their credit ratings and discourages them from borrowing over the threshold. Under the calibration, downgrades to junk imply that the impatient lenders of junk bond markets charge a high yield to the sovereign. This consequence of downgrades and the country's ability to manage its ratings are the driving forces of disciplined borrowing behavior.

This disciplined behavior lowers the country's default risk and allows it to borrow at a better price, which is a potential source of welfare benefit. The welfare cost of ratings and implied segmentation is from the anticipated devaluation of bonds before downgrades and controlled borrowing when the country is near the threshold. Under the calibration, the cost dominates the benefit, and the segmentation by sovereign ratings results in welfare loss to the impatient country in the long run. However, the government gains from the segmentation during the transition, especially when the debt stock is low.

This welfare analysis suggests the importance of moderate segmentation in the international capital market. The finding suggests a rationale for the current segmentation rule. The chapter proposes that loose segmentation policies weaken disciplinary motives and deliver negative welfare implications to developing countries. Imposing an adequate punishment as a form of market segmentation could alleviate commitment issues and give developing countries better credit access.

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Appendix A

Appendix for Chapter 1

A.1 Multiple Equilibrium in Computation

In this section, I provide more details about how I deal with equilibrium multiplicity.

This paper is focused on comparative statics in unique price equilibrium. To ensure the uniqueness of the equilibrium, I restrict parameter sets so that the precision is low enough. Numerically, I define that a unique price is attained at a given parameter β, σ, and α if fixed points with different initial guesses of p are equivalent within a certain tolerance level for all $x \in R$. Specifically, for a given value β, σ, and α, I set $0.5, 0.0$, and 1.0 as different initial guesses and compute a fixed point for each initial guess. As long as all fixed points are equivalent within tolerance level $1e - 6$ for x grid $\in [0.0, 1.0]$, I denote that β, σ, and α guarantees a unique price.

In this model, it turns out that β, σ, and α guarantees a unique price as long as $\beta + \beta\sigma\alpha < 2.439$. I continue numerical exercise with parameter sets that satisfy the condition for guaranteeing a unique price.

A.2 Computational Algorithm

For the numerical exercise, I set a grid for θ in the interval $[0.0, 1.0]$, as these values are relevant for the dependency of default outcome on $p \in [0.0, 1.0]$. For the μ grid, I symmetrically set it around 0. Given that the mean of μ is 0, $\mu > 0$ is interpreted as a high noisy demand shock relative to the average, and $\mu < 0$ as a low noisy demand shock relative to the average. In essence, $\mu > 0$ corresponds to a favorable market noise, making the market signal x more likely to be interpreted as favorable for a given θ. The interpretation of x as favorable or unfavorable critically determines whether high precision has an adverse impact on the default outcome.

The distribution of (θ, μ) is constructed conditionally within the truncated intervals.

A.3 Additional Figures

In this section, I present additional figures that provide further insights into the equilibrium dynamics.

A.3.1 The magnitude of noisy trader shocks

Along with examining the effect of different sign of noisy trader shock, μ, I analyze how different magnitudes of μ impact market signal, x. Intuitively, for a given debt level, θ, when there is higher demand from noisy trader, the market signal is likely to underestimate θ since the bond price is largely driven by μ shock relative to the fundamental.

Figure A.1 visualizes the market signal, x, under different magnitudes of μ, assuming all positive values. x is plotted as a function of the debt state, θ. The blue lines correspond to larger μ values compared to the yellow lines, and both μ values are positive shocks. In short, $\mu_{blue} > \mu_{yellow} > 0$. The dashed lines represent the market signal in a low information precision economy (low β case), while the

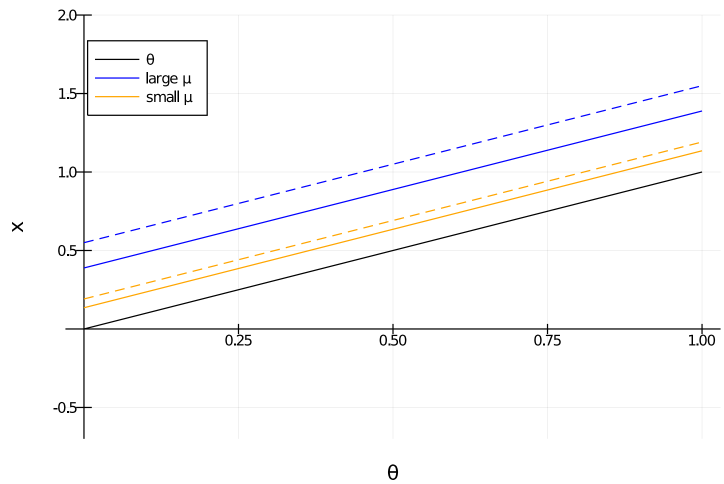

Figure A.1: Magnitude effect of μ on x

solid lines represent the market signal in a high β economy. In both cases of large and small μ, the market signal reveals the fundamental debt state more precisely as the private signals become more precise. This is shown by the solid lines, for both large and small μ, being closer to the black line representing θ compared to the dashed line, indicating a smaller deviation of x from θ. As the magnitude of the μ shock increases, the deviation of x from θ becomes larger, and the effect of high β becomes more powerful in reducing this deviation. The figure illustrates that the reduction between the solid line and the dashed line is larger in the case of a large μ compared to the case of a small μ.

A.3.2 Negative noisy trader shocks

I will now provide an explanation of what happens under negative trader shocks, in contrast to the previous explanation of positive noisy trader shocks as described in subsection 1.4.4.

Here, negative noisy trader shocks, $\mu < 0$, imply relatively low residual demands

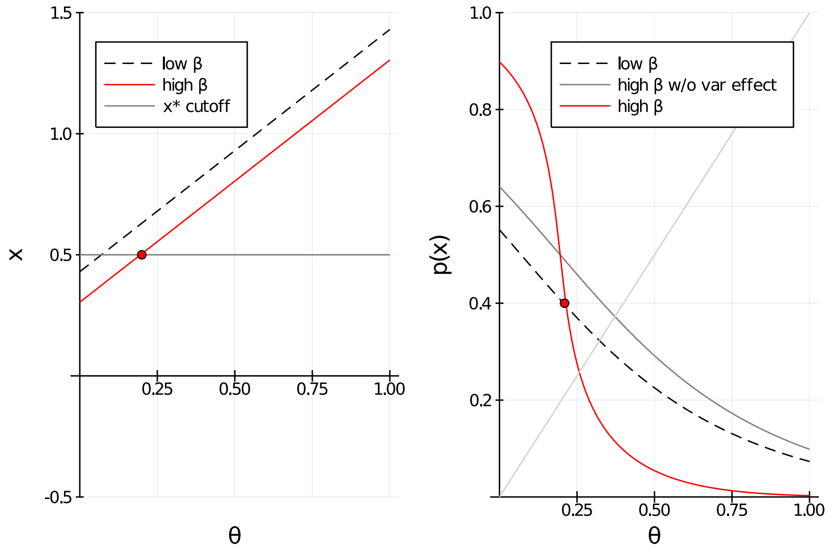

Figure A.2: x and $p(x)$ in negative μ case

for bonds, and these shocks are irrelevant to the fundamental. A low residual demand results in a low price for a given debt state, θ, which makes the market signal appear unfavorable to the marginal lender. In other words, $x > \theta$ for all θ, as illustrated in the left panel for both the low and high β cases.

Whether precise information has a negative impact on debt financing depends on the market signal relative to the market signal cutoff, x^*. The marginal lender with precise information generally responds sensitively to market signals and devalues the bond prices under unfavorable market signals relative to x^*, or when $x > x^*$. In the parameterized economy described in subsection 1.4.4, $x^* = 0.5$. The sensitive pricing induced from information precision has negative consequences for states with $\theta > 0.235$. These are the states where, for a given $\mu < 0$, $x > 0.5$ and the marginal lender interprets the fundamental as unfavorable. As a result, the government in those states sells bonds at lower prices and collects lower revenue compared to what would have been achieved with low β.

It is interesting to note that without this sensitive response, higher precision of private signals could have been beneficial because the market signal would be a more accurate estimate of the fundamental, offsetting the underestimating effect of negative noisy trader shocks. This is shown by the gray line in the right panel of Figure A.2. The gray line represents a counterfactual bond price where higher precision results in a more accurate estimate of θ through market clearing condition. However, the marginal lender does not respond as sensitively in this counterfactual economy. In this scenario, the bond price, p, is always higher with precise information. It is the marginal lender's sensitive response that generates the adverse impact, especially in those θ states where the corresponding x is higher than x^* and is inferred as unfavorable.

Default outcome The devaluation coming from the sensitive price schedule affect bond revenue and give a consequence to default outcome. When the residual demand from noisy trader is relatively low, the debt state should be sufficiently low in order for the marginal lender to infer it as favorable. In those debt states that are not as low sufficiently, a fundamental-irrelevant shock can influence default outcome. The marginal lender more confidently believes that the government's debt level is unfavorable and devalues the bond price sufficiently. This result in the government failing to raise enough bond revenue, ending up with default.

The right panel of Figure 1.5 illustrates that $\theta \in [0.265, 0.375]$ is the state where the government defaults in high β economy, whereas the default would not have occurred in the case of low β. The reason for this is that the market cannot fully separate the default-relevant fundamental, θ, from x, and there exists a θ where the marginal lender interprets it as an unfavorable signal with sufficient market noise, even though the debt level is relatively low. In those states, insolvency risk is relatively low, but rollover risk is high. Default occurs due to the failure of rollover.

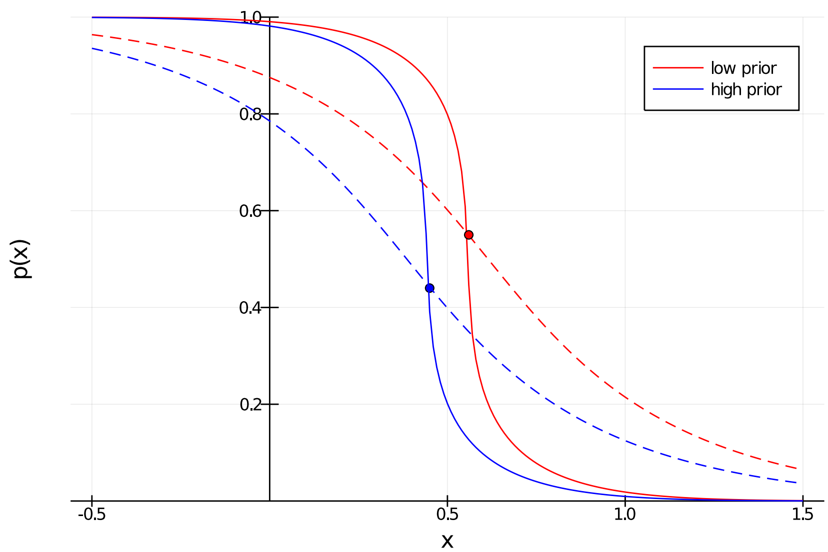

Figure A.3: Price schedule with low and high θ_0

A.3.3 Price schedule in low prior economy

Figure A.3 plots the price schedule of the low and the high prior economy, respectively. The red plots represent the low prior economy, while the blue plots represents the high prior economy. The dash lines correspond to the low β economy, and the solid lines corresponds to the high β economy. In the low prior economy, θ tends to realize as low on average, and the equilibrium bond price is always higher compared to the case of the high prior economy, regardless of the precision level.

The influence of different priors, θ_0, on the bond price schedule is manifested through the market signal cutoff, x^*. The plot shows that the high prior economy has a lower cutoff, indicating that the market has a more stringent criteria for interpreting the market signal, x. The same value of x can be interpreted as an unfavorable signal in the high prior economy, whereas it is considered a favorable signal in the low prior economy.

Appendix B

Appendix for Chapter 2

B.1 Additional tables for empirical evidence

Table B.1: Additional table for the regression (1)

	(1) spread	(2) spread	(3) spread	(4) spread	(5) spread
DowntoJunk	141.4*	33.58*	37.72**	37.42**	29.64***
	(78.73)	(18.02)	(16.85)	(16.90)	(11.47)
lag_spread		0.973***	0.976***	0.973***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.879***	1.132***	-0.312
			(0.325)	(0.379)	(0.308)
grossdebt			0.0627	0.124	0.146**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.001	0.948	0.949	0.949	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is monthly frequency across 12 countries.

Table B.2: Additional table for the regression (2)

	(1)	(2)	(3)	(4)	(5)
	spread	spread	spread	spread	spread
withininvst	174.7***	18.71	19.57	20.00	17.10*
	(65.84)	(15.07)	(14.06)	(14.10)	(9.572)
DowntoJunk	142.2*	33.84*	38.46**	38.12**	30.14***
	(78.65)	(17.99)	(16.82)	(16.86)	(11.46)
withinjunk	26.52	47.11***	50.75***	51.30***	18.63*
	(69.39)	(15.87)	(14.82)	(14.85)	(10.09)
lag_spread		0.973***	0.976***	0.974***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.977***	1.247***	-0.249
			(0.325)	(0.379)	(0.309)
grossdebt			0.0583	0.120	0.141**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.004	0.948	0.950	0.950	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note:* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is montly frequency across 12 countries.

Table B.3: Regression result with a different specification

	(1)	(2)	(3)	(4)	(5)
	spread	spread	spread	spread	spread
DowntoBBB	29.91	74.07***	77.26***	78.10***	17.74
	(104.0)	(23.78)	(22.15)	(22.20)	(15.13)
DowntoBBB-	23.81	25.55	29.52	29.85	19.38
	(93.06)	(21.27)	(19.85)	(19.89)	(13.52)
DowntoJunk	142.2*	33.83*	38.43**	38.07**	30.26***
	(78.68)	(17.99)	(16.81)	(16.85)	(11.47)
DowntoBB	174.9**	24.40	25.86	26.47	24.20**
	(78.68)	(18.00)	(16.78)	(16.82)	(11.43)
DowntoBB-	174.1	5.412	4.734	4.642	0.513
	(120.1)	(27.46)	(25.57)	(25.64)	(17.45)
lag_spread		0.973***	0.976***	0.974***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.974***	1.240***	-0.245
			(0.325)	(0.379)	(0.309)
grossdebt			0.0605	0.123	0.142**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.004	0.948	0.950	0.950	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note:* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. DowntoBBB is a dummy variable which equals to 1 when a country's rating is BBB+ at period $t-1$ and BBB at period t. Other dummy variables, DowntoBBB-, DowntoBB, and DowntoBB-, are constructed similarly (BBB in period $t-1$ and BBB- in period t, BB+ in period $t-1$ and BB in period t, and BB in period $t-1$ and BB- in period t, respectively). Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is montly frequency across 12 countries.

Appendix C

Appendix for Chapter 3

C.1 Numerical Algorithm

The algorithm iterates on value functions, $V(b,s), V^D(b,s)$ and price function, $q(b,s)$, until convergence. Using Tauchen's method, I discretize stochastic output process, y, as a Markov chain. I also discretize endogenous state variable, b into finite grid. I use 151-grid points for y, and 301-grid points for b.

The challenge of computing long-term debts has been documented in the literature. Following [Dvorkin et al., 2021], I use Extreme Value shock to resolve this issue.[1] I assume there is taste shock, additive utility shocks, associated with each possible debt level choice and default choice. For parameters governing the distribution of extreme value shock, I take the value from [Dvorkin et al., 2021].

First, I set initial guesses for $V(b,s), V^D(b,s)$, and $q(b,s)$. I update $V^D(b,s)$ using initial guess $V(b,s)$ and $V^D(b,s)$. Then, for each (b,s), I use discrete-search method to calculate the optimal debt. That is, given initial guess $V(b,s)$ and $q(b,s)$, I look for a b that gives the highest value under repayment than any other b in the grid can give. By doing so, I can define debt policy function, $B(b,s)$. Given initial $V(b,s), q(b,s)$ and defined $B(b,s)$, I can define value under repayment, $V^R(b,s)$.

[1] I am deeply grateful to Hayagreev Ramesh for providing the technical computational details.

With updated $V^D(b,s)$ and $V^R(b,s)$, I can update $V(b,s)$ and define default policy function, $D(b,s)$. Given $D(b,s)$, I can calculate next period default probability for each state (b,y). Given a rating parameter, I can define rating function, $R(b,s)$ following the rating rule as specified. Given $R(b,s)$, $D(b,s)$, and the initial $q(b,s)$, I can update price function. Rating function determines the discount rate, and initial price function is used for calculating the future value of bonds.

I check if the updated $V(b,s), V^D(b,s), q(b,s)$ are close enough to the initial $V(b,s), V^D(b,s)$, $q(b,s)$. If it close enough within a tolerance level, I stop the iteration. Otherwise, I use the updated $V(b,s), V^D(b,s), q(b,s)$ as initial guesses and re-do the iteration.