A UNIQUENESS THEOREM FOR AN INVERSE PROBLEM IN PERIODIC DIFFRACTIVE OPTICS

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Abstract. Consider a time-harmonic electromagnetic plane wave incident on a periodic structure in $I\!R^2$. The periodic structure separates two regions. In one region, the dielectric coefficient is assumed to be a fixed constant with nonzero imaginary part corresponding to the energy absorption. The other region contains perfectly reflecting material. The inverse problem is to determine the periodic structure or the shape of the interface from the scattered field. In this paper, a uniqueness theorem is proved by an application of Holmgren's uniqueness theorem.

Short title: *Uniqueness for an inverse diffractive optics problem*

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1. Introduction. Consider a time-harmonic electromagnetic plane wave incident from the top on a periodic structure in $I\!R^2$. The periodic structure separates two regions. In one region, above the periodic structure, the dielectric coefficient is assumed to be a fixed constant with nonzero imaginary part corresponding to the energy absorption [1]. The other region contains perfectly reflecting material. The inverse problem is to determine the periodic structure or the shape of the interface from the scattered field. In this paper, a uniqueness theorem is proved by an application of Holmgren's uniqueness theorem. Our theorem indicates that any two surface profiles are identical if they generate the same scattering fields (or patterns). This work is motivated by the study of a class of optimal design problems where one wishes to design a periodic structure that generates some specified scattering field.

Recently, the scattering of electromagnetic waves in a periodic structure, or the direct scattering problem, has been studied extensively by either integral equation methods or variational approaches. We refer to [3], [12], [5], and [9] for some interesting existence and uniqueness results.

The inverse scattering problem in periodic structure was studied by Dobson [5] and [6], Kirsch [10] where in one region (say, above the periodic structure) the dielectric coefficient was assumed to be real (lossless), and a perfect reflecting material was placed in the other region. On the contrary, our objective is to determine the periodic structure that separates an absorbent material such as a lossy dielectric or metal and a perfect reflecting material. In addition to this physical difference, our results and techniques are different from theirs. In [10], Kirsch proved a uniqueness theorem by a similar approach as for the general inverse scattering problem in [11]. The main idea was to prove the denseness of a set of special solutions. For the optical applications we are interested in, one is only allowed to use single or a small number of incident plane waves. For one incident plane wave, Dobson [6] proved a local uniqueness theorem by assuming the two profiles are sufficiently close. In [5], Dobson solved a closely related optimal design problem by a relaxation method. Our approach is similar to Borovikov [2] where a uniqueness theorem was proved in a nonperiodic structure with a normal incident wave. However, the results and many technical details differ.

The scattering theory in periodic structures has many applications in micro-optics, where periodic structures are often called *diffraction gratings*. We refer the reader to the book [7] for a description of this and other mathematical problems which arise in these applications. A good introduction to the problem of electromagnetic diffraction through periodic structures, along with some numerical methods, can be found in Petit [13]. A complete account of the general theory of inverse scattering problems in general (nonperiodic) structures may be found in the book of Colton and Kress [4] and references therein.

2. The direct scattering problem. Let the scattering profile (object) be described by the curve $\Gamma = \{(x_1, x_2) : x_2 = f(x_1)\}$ with a periodic function f of period 2π. The function f is supposed to be sufficiently smooth, for example of twice continuously differentiable. The space below Γ is filled with some perfectly reflecting material. Let $\Omega = \{x \in I\!R^2 : x_2 > f(x_1), x_1 \in I\!R\}$ be filled with a material whose index of refraction is a constant k. In fact, $k = \omega c^{-1}\sqrt{\epsilon\mu}$, where ω is the angular frequency, c is the speed of light, μ is the magnetic permeability which is assumed to be one everywhere, and ϵ is the dielectric coefficient. In this work, k is assumed to be a fixed complex number with $Re(k) > 0$ and a positive imaginary part, *i.e.* $Im(k) > 0$ which accounts for energy absorption. Suppose that a plane wave given by $u_I = e^{i\alpha x_1 - i\beta x_2}$ is incident on Γ from the top. Here $\alpha = k sin\theta$, $\beta = k cos\theta$, and $-\pi/2 < \theta < \pi/2$ is the incident angle. The scattering of time harmonic electromagnetic waves in the TE (transverse electric) mode can then be modeled by the following two dimensional Helmholtz equation with a homogeneous Dirichlet boundary condition

$$(\Delta + k^2)u = 0, \quad \text{in } \Omega, \tag{1}$$

$$u|_\Gamma = 0 \ . \tag{2}$$

We seek for quasiperiodic solutions to this problem, *i.e.* the solution u such that $ue^{-i\alpha x_1}$ is 2π-periodic for every x_2. To completely specify the boundary value problem, we need to impose a radiation condition.

Since $ue^{-i\alpha x_1}$ is 2π-periodic, one can expand u in a Fourier series

$$u(x_1, x_2) = \sum_{n \in Z} u_n(x_2) e^{i\alpha_n x_1}, \quad \alpha_n = n + \alpha \ , \tag{3}$$

where $u_n = \frac{1}{2\pi}\int_0^{2\pi} u(x_1, x_2) e^{-i\alpha_n x_1} dx_1$. Substituting the expression of u into the equation (1), we have the following simple ordinary differential equation

$$u_n'' + (k^2 - \alpha_n^2) u_n = 0 \ . \tag{4}$$

The radiation condition that we impose is the boundedness of u as x_2 tends to infinity. More precisely, we insist that u is composed of bounded outgoing plane waves plus the incident wave u_I. Let

$$\beta_n = e^{\theta_n/2}|k^2 - \alpha_n|^{1/2},$$

with $\theta_n = arg(k^2 - \alpha_n^2)$, $0 \leq \theta_n < 2\pi$, and $\beta_0 = \beta$. The radiation condition yields

$$u(x_1, x_2) = \sum_{n \in Z} a_n e^{i\beta_n x_2 + i\alpha_n x_1} + u_I \ , \quad x_2 > max\{f(x_1)\} \tag{5}$$

where the coefficients $\{a_n\}$ are complex scalars. Note that since $Im(k) > 0$, $Im(\beta_n) > 0$ for all n.

For the direct scattering problem, questions on existence and uniqueness are well understood, see for example [3]. Basically the scattering problem (1), (2), and (5) specified above has a unique quasiperiodic solution. Note that when k is real, one has to exclude a discrete number of the frequencies or k in order to prove existence and uniqueness, see [3], [5], and [9]. It is our goal of this paper to study the inverse problem or the profile determination problem. More precisely, the inverse problem can be stated as follows: Let T be a fixed constant such that $T > max\{f(x_1)\}$. Suppose that u (quasiperiodic) solves the scattering problem (1), (2), and (5). Determine $f(x_1)$ by the knowledge of $u(x_1, T)$, *i.e.* the trace of u. In the next section, we shall prove a uniqueness theorem for the inverse problem.

The following theorem plays an important role in our proof of the uniqueness theorem for the inverse problem.

Recall that the incident plane wave is given by $u_I = e^{i\alpha x_1 - i\beta x_2}$, where $\alpha = ksin\theta$, $\beta = kcos\theta$, and $-\pi/2 < \theta < \pi/2$.

THEOREM 2.1. *Let $T > max\{f(x_1)\}$, and u be the quasiperiodic solution that solves the scattering problem (1), (2), and (5). Then there is a pseudodifferential operator B of order one such that*

$$\frac{\partial u}{\partial n}|_{x_2=T} = B(u|_{x_2=T}) - 2i\beta e^{-i\beta T + i\alpha x_1} \ . \tag{6}$$

Proof. From the expressions (3), (5), and the radiation condition, we can compute the Fourier components u_n explicitly. For $\Omega_T = \{x_2 \geq T > max f(x_1)\}$,

$$u_n(x_2) = \begin{cases} u_n(T)e^{i\beta_n(x_2-T)}, & n \neq 0, \text{ in } \Omega_T, \\ u_0(T)e^{i\beta_0(x_2-T)} + e^{-i\beta_0 x_2} - e^{i\beta_0(x_2-2T)}, & n = 0, \text{ in } \Omega_T. \end{cases} \tag{7}$$

Further

$$\frac{\partial u_n}{\partial n}|_{x_2=T} = \begin{cases} i\beta_n u_n(T), & n \neq 0, \\ i\beta_0 u_0(T) - 2i\beta_0 e^{-i\beta_0 T}, & n = 0. \end{cases} \tag{8}$$

Therefore

$$\frac{\partial u}{\partial n}|_{x_2=T} = \sum_{n \in Z} i\beta_n u_n(T) e^{i\alpha_n x_1} - 2i\beta_0 e^{-i\beta_0 T + i\alpha x_1} \ . \tag{9}$$

Define for $g \in H^{1/2}$

$$B(g) = \sum_{n \in Z} i\beta_n g_n e^{i\alpha_n x_1}, \tag{10}$$

where $g_n = \frac{1}{2\pi}\int_0^{2\pi} g(x_1, x_2) e^{-i\alpha_n x_1} dx_1$. Clearly, the "Dirichlet-Neumann" operator B is a pseudodifferential operator (in fact, it is a convolutional operator) of order one. The proof of Theorem 2.1 is completed. □

Remark on Theorem 2.1. The Sobolev continuity properties of pseudodifferential operators, see for example [14], yield $B : H^{1/2} \to H^{-1/2}$ continuously.

3. Uniqueness of the inverse problem. Recall once again that the incident wave is given by $u_I = e^{i\alpha x_1 - i\beta x_2}$, where $\alpha = k sin\theta$, $\beta = k cos\theta$, and $-\pi/2 < \theta < \pi/2$. Suppose that $u_j(x_1, x_2)$ $(j = 1, 2)$ are 2π-quasiperiodic and solve the scattering problem (1), (2), and (5) with respect to the profiles $f_j(x_1)$, where the functions f_j are 2π-periodic. Let $T > max\{f_1(x_1), f_2(x_1)\}$ be a fixed constant.

THEOREM 3.1. *Assume that $u_1(x_1, T) = u_2(x_1, T)$. Then $f_1(x_1) = f_2(x_1)$.*

Let us first prove the following lemmas which are crucial in the proof of Theorem 3.1.

LEMMA 3.2. *(Unique continuation) Suppose that $v \in C^2(\Omega) \cap C(\bar{\Omega})$ satisfies*

$$\triangle v + k^2 v = 0 , \quad in\ \Omega . \tag{11}$$

Let $I \in \Omega$ be a segment. Suppose also that $v|_I = 0$, $\frac{\partial v}{\partial \gamma}|_I = 0$ where γ is the normal to I. Then $v = 0$ in $\bar{\Omega}$.

Note that the problem may be viewed as a Cauchy problem. The proof of Lemma 3.1 may be given by a straightforward application of Holmgren's uniqueness theorem, see pp. 80-87 [8], and a simple continuation argument. We comment that since k has an imaginary part, one should consider the real and imaginary parts of (11) separately which form a Cauchy problem for a system of Helmhotz equations with real analytic coefficients. Thus Holmgren's theorem becomes applicable.

LEMMA 3.3. *Let D be a bounded domain in IR^2 and $v \in C^2(D) \cap C(\bar{D})$ satisfies*

$$\triangle v + k^2 v = 0 , \quad in\ D, \tag{12}$$

$$v|_{\partial D} = 0 . \tag{13}$$

Suppose that $Im(k) > 0$. Then $v = 0$ in $\bar{D}$.

Proof. Multiplying both sides of (12) by $\bar{v}$ and integrating over D give

$$\int_D \triangle v \bar{v} + k^2 \int_D v \bar{v} = 0.$$

Green's identity and the boundary condition of v yield

$$-\int_D |\nabla v|^2 + k^2 \int_D |v|^2 = 0 . \tag{14}$$

Now if $Re(k) \neq 0$, the imaginary part of the equation (14) gives $\int_D |v|^2 = 0$ hence $v = 0$. In the case where $Re(k) = 0$, since $Im(k) > 0$, we have

$$\int_D |\nabla v|^2 + Im(k)^2 \int_D |v|^2 = 0$$

therefore $v = 0$ in $\bar{D}$. □

We are ready to prove the main result of this paper, Theorem 3.1, a uniqueness theorem for the inverse diffraction problem.

Proof. Theorem 3.1 may be proved by contradiction. Let us assume that $f_1(x_1)$ and $f_2(x_1)$ are two different functions. Denote $v = u_1 - u_2$, $f(x_1) = max\{f_1(x_1), f_2(x_1)\}$, $\Omega_j = \{(x_1, x_2) : f_j(x_1) < x_2\}$, and $\Omega = \{(x_1, x_2) : f(x_1) < x_2\}$.

Then

$$v|_{\partial\Omega} = \begin{cases} 0, & \text{for } f_1(x_1) = f_2(x_1), \\ -u_2(x_1, f_1(x_1)), & f_1(x_1) > f_2(x_1), \\ u_1(x_1, f_2(x_1)), & f_1(x_1) < f_2(x_1)\ . \end{cases} \tag{15}$$

It follows from Theorem 2.1 and $u_1|_{x_2=T} = u_2|_{x_2=T}$ that

$$\frac{\partial u_1}{\partial n}|_{x_2=T} = \frac{\partial u_2}{\partial n}|_{x_2=T}$$

or

$$\frac{\partial v}{\partial n}|_{x_2=T} = 0.$$

Therefore,

$$\triangle v + k^2 v = 0, \text{ in } \Omega, \ v \in C^2(\Omega) \cap C(\bar{\Omega})\ , \tag{16}$$

$$v|_{x_2=T} = 0\ , \ \ \frac{\partial v}{\partial n}|_{x_2=T} = 0\ . \tag{17}$$

Applying Lemma 3.2 to the equation for v deduces

$$v = 0\ , \text{ in } \Omega\ .$$

In particular,

$$u_2(x_1, f_1(x_1)) = 0\ , \ \ u_1(x_1, f_2(x_1)) = 0\ . \tag{18}$$

W.l.o.g., assume that $f_1(x_1) \leq f_2(x_1)$ for some x_1. Let us denote the region between Ω and Ω_1 by $D_1 = \Omega_1 - \Omega$. According to the boundary condition of u_1 and (18)

$$\triangle u_1 + k^2 u_1 = 0, \ \text{ in } D_1, \ u_1 \in C^2(D_1) \cap C(\bar{D}_1)\ , \tag{19}$$

$$u_1|_{\partial D_1} = 0\ . \tag{20}$$

Lemma 3.3 implies that $u_1 = 0$ in D_1. Because of the hypothesis, we know that D_1 contains an open neighborhood. Thus using Lemma 3.2 again gives that

$$u_1 = 0\ , \ \text{ in } \Omega_1\ . \tag{21}$$

Since $v = 0$ in Ω, one can show that

$$u_2 = 0 \,, \quad \text{in } \Omega_2 \,. \tag{22}$$

But this contradicts the identity (6) since $\beta \neq 0$ and the assumption that $-\pi/2 < \theta < \pi/2$.

Hence

$$f_1(x_1) = f_2(x_1)$$

which concludes the proof of Theorem 3.1. □

Remarks on Theorem 3.1. When k is real, with single incident plane wave it is impossible to prove a global uniqueness theorem for the inverse problem. A simple counterexample may be constructed in the case of a flat surface with a normal incidence. Then the solution of the scattering problem can be written down explicitly. The nonuniqueness is obvious since the scattering fields will remain to be the same when one moves the flat surface up or down in certain multiples of the wavelength. Therefore in order to get uniqueness additional information must be provided. Our approach may be adopted to this situation. Different uniqueness theorems were given in [6] and [10].

An interesting future research topics is to establish some stability results, *i.e.* to characterize how the error in one's measurements influences the scatter profile determination.

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