

**FILTERED DIFFERENTIAL ALGEBRAS ARE  
COMPLETE INVARIANTS OF STATIC FEEDBACK**

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# Filtered Differential Algebras are Complete Invariants of Static Feedback

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## Abstract

To any control system

$$\dot{x} = f(x, u)$$

we associate an algebraic object which is a filtered differential algebra. We prove that two systems are globally static feedback equivalent if and only if the corresponding filtered differential algebras are isomorphic. As a consequence of this result it follows that a control system is globally static feedback linearizable if and only if its filtered differential algebra is free.

## Keywords

Static feedback, equivalence, nonlinear systems, linearization, differential algebras, filtrations

## 1 Introduction

In this paper we introduce a filtered differential algebra associated to any control system. Our main result says that two systems are globally static feedback equivalent if and only if their filtered differential algebras are isomorphic. As a consequence of this result we prove that a system is globally static feedback linearizable if and only if its filtered differential algebra is free. As another consequence one can easily show that global dynamic feedback linearizability implies global static feedback linearizability. (The local result was proved in [3] and [17].)

The feedback equivalence problem was studied since early 80-ties and the main early results concerned feedback linearizable systems (cf. [8], [14]) for references. Only special classes of nonlinear systems were classified so far (cf. [1], [2], [7], [11]). A Hamiltonian approach was proposed to construct covariants of static feedback [1] and microlocal feedback invariants [10]. This approach seems very promising. An alternative approach using Frenet formulas was proposed by I. Kupka [13], R. Gardner and G. Wilkens [ ].

In the present paper we propose a completely different approach to construct feedback invariants. This approach is inspired by the work of M. Fliess and his coworkers on differential fields in control theory (cf. e.g. [5], [6]). By changing differential fields to differential algebras, and purely algebraic constructions into geometric-algebraic constructions, we are able to construct a complete covariant of global static feedback equivalence. More precisely, our main result reduces the feedback classification problem to the problem of classification of isomorphism classes of a class of filtered differential algebras.

This paper follows the author's earlier work on dynamic feedback equivalence problem [9]. In this paper it was proved that, under mild assumptions, two systems are dynamic feedback equivalent if and only if their differential algebras are isomorphic. The additional structure needed for constructing complete invariants of static feedback is a natural filtration defined by each system.

We restrict our attention to the global problem. However, our results have natural counterparts for the local feedback equivalence, too.

## 2 Static feedback equivalence and static equivalence

Consider two control systems of the form

$$\Sigma : \quad \dot{x} = f(x, u), \quad x \in M, \quad u \in \Omega,$$

and

$$\tilde{\Sigma} : \quad \dot{\tilde{x}} = \tilde{f}(\tilde{x}, \tilde{u}), \quad \tilde{x} \in \tilde{M}, \quad \tilde{u} \in \tilde{\Omega}.$$

For simplicity we will assume that  $M = \mathbb{R}^n = \tilde{M}$  and  $\Omega = \mathbb{R}^m = \tilde{\Omega}$ , but our results are also valid on differential manifolds.

In our considerations we will fix a category  $k = pol$  (polynomial),  $\omega$  (real analytic), or  $\infty$  (smooth).

We will assume that  $f$  and  $\tilde{f}$  are of class  $C^k$ . For the aim of this paper we will assume that the controls belong to a class of admissible controls  $\mathcal{U}$ . We will assume that  $\mathcal{U}$  satisfies the following axioms:

- (a)  $\mathcal{U}$  contains all smooth functions  $u(\cdot) : I_u \rightarrow U$ , where  $I_u$  is any open subinterval of  $\mathbb{R}$  (depending on a function),
- (b)  $\mathcal{U}$  is contained in the class of all bounded, piecewise absolutely continuous functions  $u(\cdot) : I_u \rightarrow U$ ,  $I_u \subset \mathbb{R}$ .

By a trajectory of the system  $\Sigma$  we mean any pair  $(x(\cdot), u(\cdot))$  defined on a subinterval  $I_u$  which satisfies the system equations, with the control  $u(\cdot) \in \mathcal{U}$ .

The set of all trajectories of system  $\Sigma$  will be called the *behavior* of system  $\Sigma$  and will be denoted by  $\mathcal{B}_\Sigma$ . This terminology is in agreement with the one used by Willems [15] (in fact, his trajectories and behaviors are usually defined for all  $t \in \mathbb{R}$  which would be restrictive for us as it would exclude systems with escape to infinity in finite time).

We recall the usual definition of global static feedback equivalence.

**Definition 1** *Two control systems of class  $C^k$  are called static feedback equivalent if there exist transformations (called static feedback transformations) of class  $C^k$*

$$(SFT) : \quad x = \phi(\tilde{x}), \quad u = \psi(\tilde{x}, \tilde{u}),$$

$$(SFT) : \quad \tilde{x} = \tilde{\phi}(x), \quad \tilde{u} = \tilde{\psi}(x, u),$$

such that

(i)  $\chi = (\phi, \psi)$  and  $\tilde{\chi} = (\tilde{\phi}, \tilde{\psi})$  are mutually inverse one with respect to the other (and so they are global diffeomorphisms of  $M \times \Omega$  and  $\tilde{M} \times \tilde{\Omega}$ ),

(ii) the induced maps on pairs  $(x(\cdot), u(\cdot))$  and  $(\tilde{x}(\cdot), \tilde{u}(\cdot))$  preserve behaviors  $\mathcal{B}_\Sigma, \mathcal{B}_{\tilde{\Sigma}}$ .

We also introduce a definition of global static equivalence.

**Definition 2** *Two control systems of class  $C^k$  are called static equivalent if there exist transformations (called static transformations) of class  $C^k$*

$$(ST) : \quad x = \phi(\tilde{x}, \tilde{u}), \quad u = \psi(\tilde{x}, \tilde{u}),$$

$$(ST) : \quad \tilde{x} = \tilde{\phi}(x, u), \quad \tilde{u} = \tilde{\psi}(x, u),$$

such that

(i)  $\chi = (\phi, \psi)$  and  $\tilde{\chi} = (\tilde{\phi}, \tilde{\psi})$  are mutually inverse one with respect to the other (and so they are global diffeomorphisms of  $M \times \Omega$  and  $\tilde{M} \times \tilde{\Omega}$ ), and

(ii) the induced maps on pairs  $(x(\cdot), u(\cdot))$  and  $(\tilde{x}(\cdot), \tilde{u}(\cdot))$  preserve behaviors  $\mathcal{B}_\Sigma, \mathcal{B}_{\tilde{\Sigma}}$ .

As we see, the only difference is that in the second definition we allow a seemingly larger class of transformations (we will use this definition later). In fact, both definitions coincide.

**Proposition 1** *Two systems  $\Sigma$  and  $\tilde{\Sigma}$  are static feedback equivalent if and only if they are static equivalent.*

*Proof.* Only one implication requires a proof. Suppose that the two systems are static equivalent via the static transformations (ST). Differentiating the

relations (ST) and taking into account the system equations we obtain the relation

$$f(\phi(\tilde{x}, \tilde{u}), \psi(\tilde{x}, \tilde{u})) = \frac{\partial \phi}{\partial \tilde{x}}(\tilde{x}, \tilde{u})\tilde{f}(\tilde{x}, \tilde{u}) + \frac{\partial \phi}{\partial \tilde{u}}(\tilde{x}, \tilde{u})\dot{\tilde{u}}.$$

Let us fix the time instant  $t = 0$  and consider all trajectories at  $t = 0$ . As the derivative of the control does not appear on the left hand side of the above equality, while it can take arbitrary values on the right hand side, it follows that

$$\frac{\partial \phi}{\partial \tilde{u}}(\tilde{x}, \tilde{u}) \equiv 0.$$

This means that our static transformations are in fact static feedback transformations and so the systems are static feedback equivalent.  $\blacksquare$

### 3 Differential algebras

In this paper we mean by a differential algebra  $(A, D)$  a commutative and associative algebra  $A$  over  $\mathbb{R}$  (with identity) with a linear operator  $D : A \rightarrow A$ , called derivation, which satisfies the Leibnitz identity

$$D(ab) = (Da)b + a(Db).$$

A homomorphism of differential algebras  $h : A \rightarrow B$  is a homomorphism of algebras which commutes with the derivations,

$$D_B h = h D_A.$$

For more information on differential algebras we refer the reader to [12] and [16].

In order to assign a differential algebra to our system we introduce the following notation. Let us denote by  $J(m)$  the space of infinite sequences

$$J(m) = \{ \{u^{(i)}\}_{i \geq 0}, u^{(i)} \in \mathbb{R}^m \}.$$

We denote

$$U = \{u^{(i)}\}_{i \geq 0}.$$

In the terminology of jets the space  $J(m)$  is the space of jets at  $t = 0$  of functions  $\mathbb{R} \rightarrow \mathbb{R}^m$  (in the usual jet notation  $J(m) = J_0^\infty(\mathbb{R}; \mathbb{R}^m)$ ), and the elements  $U \in J(m)$  are called (infinite) jets.

We introduce the algebra of real valued functions of  $x$  and  $U$  of class  $C^k$ ,

$$A^k(n, m) = C^k(\mathbb{R}^n \times J(m); \mathbb{R}),$$

where by definition each such function depends on a finite jet of  $u$  (the order of this jet depends on a function). In other words, any element  $a \in A^k$  is a function of class  $C^k$ ,

$$a = a(x, u, u^{(1)}, \dots, u^{(p)}),$$

where  $p \geq 0$  depends on  $a$ . In particular,  $x_s$  and  $u_j^{(i)}$  are elements of  $A^k(n, m)$  (treated as linear functions on  $\mathbb{R}^n \times J(m)$ ). Usually,  $n$  and  $m$  will be fixed and we will usually write  $A^k(n, m) = A^k$ .

We define the derivation of  $A^k$  corresponding to the system  $\Sigma$  by

$$D_\Sigma = \sum_{1 \leq s \leq n} f_s(x, u) \frac{\partial}{\partial x_s} + \sum_{i,j} u_j^{(i+1)} \frac{\partial}{\partial u_j^{(i)}},$$

where the second sum is treated as an infinite formal sum over all  $j = 1, \dots, m$  and  $i \geq 0$ . Even if this sum is infinite, there is no problem of defining  $D_\Sigma a$  for  $a \in A^k$  as  $a$  depends on a finite number of the variables  $u_j^{(i)}$  only.

The pair  $(A^k, D_\Sigma)$  will be called the *differential algebra of system*  $\Sigma$ . This algebra depends on the choice of the category  $k$  (we assume that this category is fixed during our considerations).

We define the following filtration of subalgebras of the algebra  $A$ . Let

$$\mathcal{F}_j = C^k(X \times J^j(m); \mathbb{R}),$$

where  $J^j(m)$  denotes the space of  $j$ -th jets of  $u$ , that is this is the space of finite sequences

$$\{u^{(i)}\}_{i=0, \dots, j} \in J^j(m).$$

In other words, elements  $a \in \mathcal{F}_j$  are all functions of class  $C^j$

$$a = a(x, u^{(0)}, \dots, u^{(j)}).$$

Clearly, we have that

$$\mathcal{F}_0 \subset \mathcal{F}_1 \subset \mathcal{F}_2 \cdots,$$

and

$$\bigcup_{j \geq 0} \mathcal{F}_j = A^k.$$

$\mathcal{F}_j$  are subalgebras, but not differential subalgebras, of the algebra  $A$ .

Note that

$$D_\Sigma \mathcal{F}_j \subset \mathcal{F}_{j+1}.$$

Let us denote by  $\mathcal{A}$  the following triple

$$\mathcal{A} = (A, \mathcal{F}, D_\Sigma).$$

We will call if the *filtered differential algebra* of system  $\Sigma$ .

A homomorphism (respectively, isomorphism) of filtered differential algebras  $(A, \mathcal{F}_A, D_A)$  and  $(B, \mathcal{F}_B, D_B)$  is a homomorphism (isomorphism) of differential algebras  $h : (A, D_A) \rightarrow (B, D_B)$  which preserves the filtrations, i.e. it satisfies the condition

$$h((\mathcal{F}_A)_j) \subset (\mathcal{F}_B)_j, \quad (\text{respectively, } h((\mathcal{F}_A)_j) = (\mathcal{F}_B)_j).$$

With our definition of the derivation corresponding to the system we can define jets of elements of our differential algebra. Namely, given an element  $a \in A^k(n, m)$  we define its jet as the infinite sequence  $Ja = (a, D_\Sigma a, D_\Sigma^2 a, \dots)$ .

**Remark 1** *The algebra  $A^k$  is an integral domain, if  $k = \omega$ , or  $k = \text{pol}$ . This follows from the fact that the product of two nonzero analytic functions is a nonzero function. It follows that the field of quotients can be defined, corresponding to our algebra  $A^k$ , if  $k = \omega$ ,  $\text{pol}$ , and it will have a natural structure of a differential field, isomorphic to the field introduced in [4]. Our filtration  $\mathcal{F}$  induces a filtration of the differential field of quotients. In the case of  $k = \text{pol}$  this field coincides with the field used in the approach used by M. Fliess.*

## 4 Main result

Now we are in a position to state our main result.

**Theorem 1** *Two systems  $\Sigma$  and  $\tilde{\Sigma}$  of class  $C^k$ ,  $k = \text{pol}$ ,  $\omega$  or  $\infty$ , are static feedback equivalent if and only if their filtered differential algebras  $(A, \mathcal{F}, D_\Sigma)$  and  $(A, \mathcal{F}, D_{\tilde{\Sigma}})$  are isomorphic as filtered differential algebras.*

*Proof.* We will reduce the proof to the main result in [9].

“If” Suppose that  $(A, \mathcal{F}, D_\Sigma)$  and  $(A, \mathcal{F}, D_{\tilde{\Sigma}})$  are isomorphic as filtered differential algebras and  $h : A \rightarrow A$  is an isomorphism. Then  $h$  is also an isomorphism of differential algebras  $h : (A, D_\Sigma) \rightarrow (A, D_{\tilde{\Sigma}})$ . It follows that we can use the result in [9], Theorem 3, which says that there exists a map  $\eta = \eta(\tilde{x}, \tilde{U})$  with

$$\eta = (\eta^1, \eta^2), \quad \eta^1 = (\eta_1, \dots, \eta_n), \quad \eta^2 = (\eta_{n+1}, \dots, \eta_{n+m}),$$

such that

$$h = \hat{\eta}^*, \quad \text{where } \hat{\eta}^*(a) = a \circ \hat{\eta} \text{ and } \hat{\eta} = (\eta^1, J\eta^2).$$

Here  $J\eta^2$  denotes the jet extension of  $\eta^2$ ,

$$J\eta^2 = (\eta^2, D_{\tilde{\Sigma}}\eta^2, D_{\tilde{\Sigma}}^2\eta^2, \dots).$$

Additionally, from Theorem 3 in [9] we conclude that  $\eta$  satisfies the condition

$$D_{\tilde{\Sigma}}\eta^1 = f \circ \eta.$$

This implies that the induced map (by  $\eta$ ) transforms trajectories into trajectories (see the proof of Theorem 2 in [9]) (the same happens for the map induced by the inverse isomorphism  $h^{-1}$ ). Additionally, we have that

$$\eta_j = h(x_j), \quad \text{for } j = 1, \dots, n,$$

and

$$\eta_j = h(u_{j+n}), \quad \text{for } j = n+1, \dots, n+m.$$

From this and the fact that  $h$  preserves the filtrations we deduce that  $\eta_j \in \mathcal{F}_0$ , that is

$$\eta = \eta(\tilde{x}, \tilde{u}).$$

As the map induced by  $\eta$  on the trajectories preserves the behaviors, it follows that the transformation  $\chi = \eta$  establishes static equivalence of both systems.

“Only if” Suppose that both systems are static feedback equivalent. This means that there exist diffeomorphisms

$$\chi = (\phi, \psi), \quad \chi^{-1} = \tilde{\chi} = (\tilde{\phi}, \tilde{\psi})$$

which transform behaviors into behaviors. We will show that the maps  $\chi$  and  $\tilde{\chi}$  define homomorphisms of our filtered differential algebras (as they will be inverse one with respect to the other, they will be isomorphisms). It is enough to show this for  $\chi$ , only.

Let us define

$$\hat{\chi} = (\phi, J\psi),$$

where  $J\psi$  denotes the infinite jet of  $\psi$ . We define the map  $\hat{\chi}^* : A^k \rightarrow A^k$  as the pull-back

$$\hat{\chi}^*(a) = a \circ \hat{\chi}.$$

It is easy to see that this map is a homomorphism of algebras and preserves the filtrations.

In order to see that this is an isomorphism of differential algebras we use the chain condition and check that the homomorphism  $h = \hat{\chi}^*$  commutes with the derivations. We first prove that

$$\begin{aligned} \hat{\chi}^*(D_{\Sigma}a) &= (D_{\Sigma}a) \circ \hat{\chi} = \left( \sum_s f_s \frac{\partial a}{\partial x_s} \right) \circ \hat{\chi} + \left( \sum_{i,j} u_j^{(i+1)} \frac{\partial a}{\partial u_j^{(i)}} \right) \circ \hat{\chi} \\ &= \sum_s f_s \circ \chi \frac{\partial a}{\partial x_s} \circ \hat{\chi} + \sum_{i,j} D_{\tilde{\Sigma}}^{i+1}(\chi_{n+j}) \frac{\partial a}{\partial u_j^{(i)}} \circ \hat{\chi}. \end{aligned}$$

From the fact the map  $\chi$  preserves the trajectories it follows that

$$D_{\tilde{\Sigma}}\phi = \frac{\partial \tilde{\phi}}{\partial \tilde{x}} \tilde{f} = f \circ \chi.$$

Thus, we can replace the first sum in the expression for  $\hat{\chi}^*(D_{\Sigma}a)$  and obtain

$$\begin{aligned} \hat{\chi}^*(D_{\Sigma}a) &= \sum_s D_{\tilde{\Sigma}}(\chi_s) \frac{\partial a}{\partial x_s} \circ \hat{\chi} + \sum_{i,j} D_{\tilde{\Sigma}}(D_{\tilde{\Sigma}}^i \chi_{n+j}) \frac{\partial a}{\partial u_j^{(i)}} \circ \hat{\chi} \\ &= D_{\tilde{\Sigma}}(a \circ \hat{\chi}) = D_{\tilde{\Sigma}}(\hat{\chi}^*(a)). \end{aligned}$$



It follows that  $\chi^*$  is a homomorphism differential algebras and so the proof is complete.  $\blacksquare$

## 5 Static feedback linearization

We end the paper with a formulation of a result on static feedback linearization which is a consequence of our results in the preceding sections. We call a system  $\Sigma$  *static feedback linearizable* if it is (globally) static feedback equivalent to a controllable linear system

$$\Lambda : \quad \dot{z} = Az + Bv.$$

In order to formulate a criterion for global static feedback linearizability we introduce the following definition.

**Definition 3** *A differential algebra  $(A, D)$  of real valued functions is called free (in  $C^k$  category) if there exists a finite number of elements  $w_1, \dots, w_r \in A$  such that the following two conditions are satisfied (we denote  $w = (w_1, \dots, w_r)$  and  $W = Jw = \{D^i w\}_{i \geq 0}$ ).*

(a) *For any function  $h \in C^k(J(r); \mathbb{R})$  (depending on a finite jet only) we have that*

$$h \circ W \equiv 0 \implies h \equiv 0.$$

(b) *For any element  $a \in A$  there exists a function  $h \in C^k(J(r); \mathbb{R})$  (depending on a finite jet only) such that*

$$a = h \circ W.$$

*A filtered differential algebra  $(A, \mathcal{F}, D)$  is called free if the differential algebra  $(A, D)$  is free, with a set of free generators  $w_1, \dots, w_r$ , and the filtration  $\mathcal{F}$  is given by*

$$\mathcal{F}_j = \{ a \in A \mid \exists g \in C^k(J(r); \mathbb{R}) \text{ such that } a = g \circ W^j \},$$

where

$$W^j = \{D^i w_s\}_{s=1, \dots, r, i=1, \dots, j}.$$

A filtration  $\mathcal{G}_0, \mathcal{G}_1, \mathcal{G}_2, \dots$  of  $A$  is called a *refinement* of a filtration  $\mathcal{F}$  if

$$\mathcal{G}_j \subset \mathcal{F}_j.$$

**Theorem 2** *A system  $\Sigma$  of class  $C^k$ ,  $k = \text{pol}, \omega$ , or  $\infty$ , is static feedback linearizable if and only if there is a refinement  $\mathcal{G}$  of the natural filtration  $\mathcal{F}$  of the system  $\Sigma$  such that the filtered differential algebra  $(A^k, \mathcal{G}, D_\Sigma)$  is free (in the  $C^k$  category).*

This theorem follows from Theorem 1, and the following lemma.

**Lemma 1** *For a controllable linear system  $\Lambda$  there exists a refinement  $\mathcal{G}$  of the natural filtration of  $\Lambda$  such that the corresponding filtered differential algebra  $(A^k, \mathcal{G}, D_\Sigma)$  is free (in each of the categories  $k = \text{pol}, \omega, \infty$ ).*

A more detailed description of this problem will be provided in another paper.

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