Evidence-Based Instructional Principles and Sequences for Effective Fraction Instruction

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Dedication

To my grandmother, Phyllis Running, who taught me to read and encouraged me to pursue a career in education.

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CHAPTER 1

Introduction

Fraction proficiency is essential for overall mathematic achievement. Elementary fraction knowledge uniquely predicts students' algebra readiness and high school general math outcomes (Booth & Newton, 2012), and is a better metric of future math performance than whole number knowledge (Siegler et al., 2012). Fraction competence is foundational for more complex mathematics, like proportional reasoning and algebra (National Mathematics Advisory Panel [NMAP], 2008). The relational aspect of fraction knowledge (i.e., a fraction, with its bipartite format $\frac{a}{b}$, represents the relationship between two integers) is particularly advantageous for proportional and analogical reasoning, even more so than decimal representations (DeWolf et al., 2015, 2016). Furthermore, fractions are often the first non-natural numbers that students learn (DeWolf et al., 2015), requiring students to use higher levels of cognitive abstraction compared to prerequisite skills (Obersteiner et al., 2018). It is unsurprising, then, that the NMAP (2008) listed fluency with fractions as a critical student benchmark for math development.

Fractions, while important for more advanced math, are exceedingly difficult for students to master. United States students show deficits in both conceptual and procedural fraction knowledge, as evidenced by data from the National Assessment of Education Progress (NAEP). In 1978, only 24% of eighth-grade students correctly estimated the closest whole number sum of $\frac{12}{13} + \frac{7}{8}$ from answer options of 1, 2, 19, 21, and "I don't know" on the NAEP. Lortie-Forgues et al. (2015) tested eighth-grade students with this question again in 2014. Only 27% of students answered it correctly; in over three decades, deficits in conceptual understanding of fraction magnitude have remained relatively constant.

United States students also need improvement with fraction procedures. Siegler and Pyke (2013) tested students with all four fraction operations and found that sixth-grade students correctly completed only 41% and eighth-grade students 57% of arithmetic problems. For reference, Common Core State Standards of Mathematics (CCSSM) expects United States students to be fluent in all fraction operations by the end of Grade 6 (NMAP, 2008; National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers, 2010). Siegler and Pyke's (2013) study also demonstrated that the procedural score gap between high and low-achieving students widens significantly between sixth and eighth grades, indicating a need for more effective core fraction instruction and early intervention for fraction skill deficits.

The high percentage of general education students who do not meet fraction proficiency standards indicates that there might be a problem with tier-one, core curriculum fraction instruction (Jitendra & Dupuis, 2015). Researchers need to evaluate whether core fraction lessons incorporate sufficient evidence-based instructional practices, as current assessment results reveal that students are not making adequate progress with fraction concepts or procedures. In addition to analyzing the presence of effective instructional principles in fraction instruction, the presentation order of mathematical concepts and procedures might affect student learning. In Study 1, this dissertation aimed to evaluate if core curriculum fraction lessons contain sufficient levels of evidence-based instructional strategies. In Study 2, this dissertation tested two instructional sequences of conceptual and procedural knowledge during a classwide fraction intervention.

Study 1: Evidence-Based Instructional Principles for Fraction Lessons

Previous studies have evaluated math textbooks for evidence-based practices (e.g., Bryant et al., 2008; Doabler et al., 2012; Sood & Jitendra, 2007), but none have reviewed the upper-elementary fraction lesson sequence. This dissertation's first study analyzed the use of evidence-based instructional principles in core curriculum fraction lessons. We first identified the most frequently recommended math curricula by systematically reviewing textbook adoption lists from state departments of education. We then evaluated third through fifth-grade fraction lessons from the four most commonly recommended curricula. We chose Grades 3 through 5 because the CCSSM has explicit fraction goals for those grade levels (NGACBP, 2010).

Fraction lessons were evaluated using a modified instructional principle rubric, initially developed by Doabler et al. (2012). This rubric was constructed based on research about effective intervention strategies for both struggling math learners and students with math-specific disabilities (Baker et al., 2002; Doabler et al., 2012; Gersten et al., 2008). Therefore, the instructional principles identified in the rubric should be appropriate for both students in general and special education.

Other reviews examining tier-one mathematics textbooks have found inconsistent use of evidence-based instructional strategies. Bryant et al.'s (2008) study examined math lessons from kindergarten, first, and second grades, and found that overall, textbooks did not adequately employ evidence-based strategies (e.g., instructional approach, provision of teacher examples, adequate practice opportunities, error correction, etc.). Notably, Bryant et al. replicated previous findings (e.g., Jitendra et al., 2005; Jitendra et al., 1999) that textbooks did not demonstrate an acceptable amount of explicit and strategic instruction. In another textbook analysis, Sood and Jitendra (2007) compared a first-grade reform-based textbook with three traditional math programs. They found that traditional textbooks had more explicit instruction than the reform-based one; the reform-based textbook more often relied on students discovering concepts and procedures based on teachers' sequenced questions (Sood & Jitendra, 2007). Still, the use of conspicuous instruction techniques varied both between and within the traditional textbooks, revealing a need for more evidence-based curricular materials (Sood & Jitendra, 2007).

Doabler et al. (2012) analyzed lessons from three math textbooks (Grades 2 and 4) for their use of instructional principles. Their rubric identified eight instructional principles: prerequisite skills, math vocabulary, explicit instruction, instructional examples, math models, practice opportunities and cumulative review, academic feedback, and formative feedback loops. A modified version of this rubric was used to evaluate fraction lessons in Study 1 of this dissertation. General results from Doabler et al.'s (2012) study indicated that the analyzed textbooks did not provide enough explicit instruction or practice opportunities for students to become proficient with the material.

Previous textbook analyses have offered suggestions about how to improve tier-one materials. As fractions are a challenging mathematical domain to teach, core curriculum materials should provide as much support to teachers as possible. This means that teachers' curricula should incorporate instructional principles that have proven effective for teaching general (Baker et al., 2002) and special education students (Gersten et al., 2009). Evaluating fraction lessons for the use of evidence-based strategies might provide insight into how we can make tier-one fraction instruction more effective.

In addition to evaluating core curricula for evidence-based instructional strategies, researchers and educators also might examine how instruction sequencing affects student performance. The order of conceptual and procedural instruction may impact students' ability to learn and generalize fraction skills.

Study 2: Evidence-Based Instruction Sequence for Fraction Interventions

This dissertation's second study aimed to determine the optimal instruction sequence of conceptual and procedural fraction knowledge. The CCSSM fraction progression generally reflects the concepts-first theory of development (i.e., that students should learn concepts before procedures) (NGACBP, 2010). However, there are relatively few empirical studies that examine the effectiveness of different instruction orders for conceptual and procedural knowledge (e.g., concepts-first, procedures-first, iterative). Study 2 reviews existing instruction sequencing studies and tests two instruction orders in a randomized control trial.

Conceptual and Procedural Knowledge

To understand how conceptual and procedural knowledge might interact and develop, we must first define them. This task has proven to be more difficult for conceptual knowledge than procedural knowledge. Crooks and Alibali's (2014) systematic literature review revealed six different definitions of mathematic conceptual knowledge throughout the research base, indicating that there is not a consensus about how to define conceptual knowledge. The different definitions could also result from the various facets of conceptual knowledge, as the nature of math concepts potentially changes depending on the skill domain (Crooks & Alibali, 2014). The six definitions identified by Crooks and Alibali (2014) were (a) *connection knowledge*, (b) *general principle knowledge*, (c) *category knowledge*, (d) *symbol knowledge*, (e) *domain structure knowledge,* and (f) *knowledge of principles underlying procedures*.

Connection knowledge was the most frequently cited type of conceptual knowledge and refers to the links between math ideas and numbers/symbols (Crooks & Alibali, 2014; Hiebert & Lefevre, 1986). *General principle knowledge* is the understanding of mathematical rules or principles that are constant throughout any given mathematical skillset (Crooks & Alibali, 2014; Rittle-Johnson & Alibali, 1999). *Category knowledge* refers to the taxonomy or classification of different mathematical principles (Crooks & Alibali, 2014). For example, a student might recognize a fraction to be different from a whole number and consequently realize the same principles may not apply to both types of numbers. This definition is similar to theories of concepts from the cognitive psychology literature (e.g., Murphy & Medin, 1985). *Symbol knowledge* is the ability to interpret and understand mathematical symbols and representations (Crooks & Alibali, 2014; Ploger & Hecht, 2009). This definition differs from connection knowledge in that conceptual understanding of symbols does not necessarily have a relationship with other knowledge within a domain or connect to a greater "web of knowledge" (Hiebert & Lefevre, 1986, p. 3). *Domain structure knowledge* is one's ability to understand or construct a mathematical schema (Crooks & Alibali, 2014).

The final type of conceptual knowledge, *knowledge of principles underlying procedures*, is knowing why specific procedures can or cannot work in a given context. Crooks and Alibali (2014) speculated that this type of knowledge creates critical thinking to safeguard against faulty procedures. A student with strong conceptual knowledge should realize it is incorrect to add the numerators and denominators together in a fraction addition problem, as the resulting sum would be less than or equal to one of the addends (Byrnes & Wasik, 1991; Crooks & Alibali, 2014). Crooks and Alibali categorized *knowledge of principles underlying procedures* as conceptual knowledge, yet they acknowledged it might also be considered a facet of deep procedural knowledge by some. Sure enough, Baroody et al. (2007) argued that understanding *why* procedures work might involve both procedural and conceptual knowledge.

There is more of a consensus about the definition of procedural knowledge. Math procedures are sequences of steps to solve problems (Hiebert & LeFevre, 1986; Rittle-Johnson & Schneider, 2015). Procedural knowledge has been described as "knowing how" to solve a problem and can incorporate the use of the standard algorithm in doing so (Byrnes & Wasik, 1991; Rittle-Johnson & Schneider, 2015).

Conceptual and Procedural Instruction

Some research studies have tested the effects of conceptual and procedural instruction. Distinctions between *learning* and *transfer* are relevant when reviewing the outcomes of these instruction studies. Learning refers to students' ability to demonstrate skills they have been explicitly taught (i.e., students perform better on a posttest that includes problems learned during the intervention). Alternatively, transfer refers to students' ability to generalize their skills to related, yet novel problems (Perry, 1991). Early proponents of concepts-based instruction predicted that conceptual instruction would improve students' ability to transfer skills to novel tasks better than procedural instruction (Hiebert & Lefevre, 1986).

This phenomenon was observed in Perry's first (1991) study. Students were assigned to receive a concept-based or procedure-based instruction session. Students in both instructional conditions improved similarly during the posttest (i.e., a measure of *learning*). However, students in the concept-based condition significantly outperformed the procedure-based condition on the transfer test. Transfer scores from Perry's second (1991) study, which compared the order of combined (i.e., concept-then-procedure vs. procedure-then-concept) instruction, were low for both conditions. This result led Perry to conclude that procedural instruction, even when combined with conceptual instruction, may inhibit a child's ability to generalize learning to novel problems.

Another study evaluating transfer scores appeared to replicate the results of Perry's (1991) study. Rittle-Johnson and Alibali's (1999) assigned students to receive conceptual or procedural instruction. Students receiving concept-only instruction scored significantly higher on the transfer assessment than those receiving procedure-only instruction. However, the instruction in both of these studies was brief, and not necessarily representative of a traditional school-based intervention. Longer, more comprehensive procedural instruction might result in better skill transfer.

In another instruction sequencing study, Rittle-Johnson and Koedinger (2009) tested concepts-first and iterative sequences during a longer, school-based decimal intervention. They found the iterative instruction sequence (i.e., introducing concepts and procedures in an interleaved, alternating order) resulted in better procedural learning and transfer than a concepts-first instruction sequence (i.e., introducing all concepts before procedures in massed lesson blocks). Both groups performed similarly on conceptual outcome measures.

This dissertation's second study tested these two instruction sequences (i.e., concepts-first and iterative) with a new content area, fractions, to determine if Rittle-Johnson and Koedinger's (2009) results replicate. We included generalization measures to determine if the fraction skills taught during the intervention transferred to novel problems. *Corrective Mathematics, Basic Fractions* (Engelmann & Steely, 2005), was chosen for the intervention lessons because it incorporates many of the evidence-based instructional principles identified in Study 1 (Doabler et al., 2012, 2018). Specifically, it involved the most rigorous type of explicit instruction, Direct Instruction, which has strong effect sizes on student learning (Ennis & Losinski, 2019; Flores & Kaylor, 2007; Stockard et al., 2018).

General Purpose of Studies

The persistence of low fraction performance in United States students across the past few decades reveals a need to improve both fraction instruction and intervention strategies. This dissertation aimed to address these needs by (a) examining the use of evidence-based instructional principles in core curriculum fraction lessons and (b) determining the most effective way to sequence conceptual and procedural lessons during fraction interventions. Study 1 analyzed the most frequently recommended curricula for the use of empirically validated instruction strategies. The results of this analysis revealed opportunities to strengthen tier-one fraction instruction. Study 2 compared concepts-first and iterative instruction sequences during a Direct Instruction fraction intervention (Engelmann & Steely, 2005). The results of this randomized control trial revealed opportunities to optimize fraction instruction based on the order of conceptual and procedural fraction lessons.

CHAPTER 2

A Review of Core Mathematics Curricula: Examining the Use of Instructional Principles in Fraction Lessons.

The NMAP (2008) identified fraction concepts and procedures as critical for students to learn as they predict later mathematics achievement. Fraction skills are longitudinal predictors of algebra readiness and high school general math achievement (Siegler et al., 2012). Despite the importance of fraction competence, United States students struggle to meet proficiency on fraction standards (NMAP, 2008). NAEP data have also shown that fraction knowledge deficits begin in elementary grades, as 40% of all fourth-grade students did not know that thirds were larger than fourths, fifths, and sixths on the 2013 NAEP test (Malone & Fuchs, 2017; National Center for Education Statistics, 2013).

Students with a history of low mathematics performance or a mathematics learning disability are at an even higher risk of low growth in fraction skills, which increases the risk for later math failure (Jordan et al., 2017). As both general and special education students are at risk for poor fraction performance in the United States, educators and interventionists have begun to critically evaluate fraction instruction and intervention techniques (Misquitta, 2011; Roesslein & Codding, 2019; Shin & Bryant, 2015). The pervasiveness of fraction skill deficits indicates a need for more rigorous research on whether tier-one, core curricula use evidence-based instructional principles to teach fraction concepts and procedures effectively.

Common Core State Standards of Mathematics

Researchers have evaluated curricula before for potential weaknesses (Bryant et al., 2008; Charalambous et al., 2010; Doabler et al., 2012; Sood & Jitendra, 2007). The Trends in International Mathematics and Science Study (TIMSS, 2002) provided evidence that United States students performed lower than students from other countries, leading to increased scrutiny of the United States' general education math curricula. One study determined that the discrepancy between the United States' performance and higher-performing countries could be due to the United States' relatively wide breadth of math topics in each grade level. The scope and sequence of math topics were not as focused in the United States as they were in higher-performing countries (Schmidt, 2002).

Common Core State Standards of Mathematics (CCSSM) were developed to synthesize existing state standards, teacher and content experts' opinions, and public feedback, to create ubiquitous learning goals for students in the United States (NGACBP, 2010). According to CCSSM, fraction instruction begins in third grade, when students should "develop an understanding of fractions as numbers" (NGACBP, 2010). This rational number sense development involves partitioning equal parts, understanding that fractions fall on the number line, and explaining equivalent fractions (NGACBP, 2010). Fourth-grade standards indicate that students will extend their understanding of fraction equivalence and ordering, build fractions from unit fractions, understand decimal notation for fractions, and compare decimal fractions (NGACBP, 2010). The fifth-grade standards are the last grade level in which CCSSM explicitly states Number and Operations goals for fractions (only one additional fraction goal under The Number System exists in Grade 6 standards, to divide fractions by fractions). By the end of fifth grade, students are expected to use equivalent fractions as a strategy to add and subtract fractions and also apply and extend previous understandings of multiplication and division of fractions (NGACBP, 2010). CCSSM provided fewer and more specific standards across the country. The scope and sequence of fraction instruction, moving from the conceptual foundation of equal partitioning to the procedures of fraction operations, is delineated in Grades 3 through 5 of Common Core.

Instructional Principles

Other studies have evaluated the quality of instructional principles used in core curricula. Sood and Jitendra's (2007) study looked at first-grade lessons from four United States curricula to determine early elementary math curricula's adherence to big ideas in number sense. The authors also sought to determine if there were differences in instructional principle adherence between traditional textbooks and a reform-based textbook. They found variation in the lesson adherence to number relations and real-world connections, with traditional textbooks providing more opportunities for number relationship tasks and the reform-based textbook providing more real-world connections to students. The authors also examined curricula for their use of conspicuous instruction, opportunities to respond, adequate distribution of review, and direct and explicit feedback. Results showed varied use of these instructional principles in both traditional textbooks and the reform-based textbook. Traditional textbooks utilized more explicit, direct instruction than the reform-based textbook. They also incorporated more opportunities for students to respond and more directions for giving students feedback. Conversely, the reform-based textbook had more opportunities to use mathematical models. Overall, this study called for better instructional principle adherence in both traditional and reform-based math curricula.

In yet another curriculum review, Doabler and colleagues (2012) examined elementary math programs in second and fourth grades for the use of eight evidence-based instructional principles. Doabler et al.'s (2012) study examined instructional techniques that have proven to be effective for both students with math disabilities (Gersten et al., 2009) and students at risk for low performance in mathematics (Baker et al., 2002). The eight instructional principles identified by this study included:

- teaching prerequisite skills
- defining math vocabulary
- using explicit instruction
- demonstrating instructional examples
- using effective math models
- providing opportunities to practice and review
- giving explicit academic feedback
- using formative feedback loops (Doabler et al., 2012)

Doabler et al. (2012) found that the use of instructional techniques varied between and within published curricula. The results also indicated that there is an overall need for more explicit instruction and opportunities to practice in United States math curricula.

These previous studies focused mainly on whole number knowledge in early school grades, and none specifically examined the presentation of all fraction instruction across Grades 3 through 5 in United States textbooks. The central instructional principles defined by Doabler et al. (2012) are generally appropriate for most math domains, but recently fraction-specific instructional strategies have also been summarized. Misquitta's (2011) systematic literature review of 10 empirical studies on effective fraction instruction demonstrated that three fraction instruction techniques yielded strong effect sizes for improving student outcomes: (a) graduated sequence, or using a Concrete-Representational-Abstract sequence of math models; (b) strategy instruction, or using heuristics during instruction; and (c) direct instruction, or sufficiently explaining concepts and procedures to students while incorporating modeling, guided practice, and independent practice of new skills.

Purpose of Current Study

The purpose of this review was to determine if United States curricula follow a consistent pattern in presenting fraction concepts and procedures from Grades 3 through 5 to students. These grades were chosen because they are the ones with explicit fraction goals in CCSSM (NGACBP, 2010). We adopted Doabler et al.'s (2012) rubric of evidence-based instructional techniques to determine if math curricula incorporated an adequate level of instruction strategies; the modifications to the instructional principle rubric reflect evidence-based instruction techniques for teaching fraction skills specifically (Fuchs et al., 2013; Misquitta et al., 2011; Roesslein & Codding, 2018). To evaluate instructional principles within fraction lessons, we first identified the math curricula most frequently recommended for United States school districts to adopt.

Method

Curriculum Inclusion

We reviewed textbook adoption lists from the 20 states that require school districts to choose a mathematics curriculum from designated lists (Scudella, 2015). We did this to identify the most commonly recommended curricula from state departments of education, yielding 150 curricula. There were 69 unique programs left after removing duplicates. As we intended to review the most commonly recommended math curricula, we then excluded math curricula recommended by fewer than seven states, leaving five math programs. Of these five programs, four received “exemplary” status from Oregon’s department of education scoring rubric. Curricula had to meet specific criteria in (a) alignment to mathematical content, (b) alignment to mathematical practices, and (c) instructional supports to receive exemplary status in Oregon's rubric. We excluded the one curriculum that did not meet Oregon’s exemplary status as this review evaluated specific instructional principles, and Oregon’s evaluation had already determined there to be insufficient instructional supports in the math curriculum. The inclusion process is displayed in Figure 1.

The four programs included in the current review were *Everyday Mathematics*, *Math Expressions, EnVision Math,* and *GO Math!*. We rated the teacher editions of textbooks according to the instructional principle scoring rubric described above. All fraction lessons in Grades 3, 4, and 5 were intended to be included for review. We attempted to access these textbooks by utilizing Inter-Library Loan via our university’s library, asking publishers for desk copies of the curricula, and purchasing the textbooks. However, due to logistical constraints, we were unable to access all teacher edition textbooks for each grade level. Three textbooks were not accessed via ILL or publisher desk copies and had to be excluded from the review. Therefore, only three of the four included math programs have scores for each grade level.

After selecting the math programs to evaluate, we examined all lessons relating to fraction instruction. Every lesson of the math curricula was coded to assess adherence to effective instructional principles and the curriculum's overall structure.

Coding Criteria

Curriculum Structure

Each curriculum was first evaluated for its overall structure. Elements of the horizontal analysis of textbooks, the number and average length of lessons, from Charalambos et al.'s (2010) study were included in this review to compare the overall structure of fraction lessons in math curricula. Only lessons that included fractions as the target topic were included in the review.

Instructional Quality Indicators

Each curriculum was evaluated for instructional quality. We followed the rubric created by Doabler et al. (2012) because it was explicitly designed to evaluate math curricula. Some school districts have also adopted it when evaluating textbooks. All lessons received a discrete score ranging from 1 (the instructional principle was absent in the lesson) to 4 (the instructional principle was sufficiently present throughout the lesson), with scores of at least 3 meeting the acceptability criterion (Doabler et al., 2012). We then averaged all lesson scores for each grade level to calculate summative scores for each instructional principle.

While we preserved the eight evidence-based instructional principles from Doabler et al.'s rubric for this study (i.e., prerequisite skills, math vocabulary, explicit instruction, instructional examples, math models, practice opportunities and cumulative review, academic feedback, and formative feedback loops), we modified some of the codes to reflect the empirical literature base for fraction-specific instruction. Specifically, we separated the math model category into part-whole and measurement models. Previous studies have found that measurement models may be critical for fraction magnitude understanding (NMAP, 2008; Fuchs et al., 2013). We also separated the instructional examples and explicit instruction categories to examine fraction concepts and procedures. We did so because previous studies have emphasized the importance of teaching and modeling both concepts and procedures in fraction instruction (Misquitta, 2011; Rittle-Johnson et al., 2015). These changes resulted in 11 instructional quality indicators for the present study. Doabler et al.'s original (2012) rubric is included in Appendix A with fraction-specific adaptations italicized.

Prerequisite Skills. Explicitly teaching prerequisite skills before introducing new concepts can help students, even students with learning disabilities, perform better on new material (Munk et al., 2010). Math textbooks should connect new math material to previously learned skills. Including prerequisite skills in a warm-up activity at the beginning of a unit or lesson can prime students to use them throughout the lesson (Doabler et al., 2012). We rated whether or not the included math lessons gave opportunities to practice prerequisite skills before introducing new material.

Math Vocabulary. We rated each math curriculum for its use of fraction-specific math vocabulary (e.g., numerator, denominator, conversion, improper vs. proper fractions, etc.) Math programs received higher ratings if they explicitly defined relevant vocabulary terms and provided opportunities to practice using the math vocabulary. Students should be building precise definitions for math terms, and curricula that explicitly define new math vocabulary can help students make connections across the lessons (Hughes et al., 2016; Fuchs et al., 2013).

Explicit Instruction of Fraction Concepts. Baroody et al.'s (2007) article defined conceptual knowledge in mathematics to be "knowledge about facts, generalizations, and principles" (p. 107). Instruction about the concepts of equal sharing, fraction magnitude, fractions' relations to math models, and proportionality were all scored as conceptual knowledge in this review. Many empirical studies have shown that explicit instruction is one of the most effective ways to teach fraction concepts and procedures (Flores & Kaylor, 2007; Scarlato & Burr, 2002). Explicit instruction involves proactive, unambiguous teacher explanations (Doabler et al., 2012; Flores & Kaylor, 2007). Curricula that gave teachers detailed directions for explicit, scaffolded instruction about fraction concepts received higher scores on this instructional principle.

Explicit Instruction of Fraction Procedures. Just as math curricula should provide explicit instruction about math concepts to students, they should also explicitly teach math procedures. Misquitta's (2011) study indicated that procedural and conceptual understanding of fractions might require an iterative process of learning. Therefore, both concepts and procedures should be explicitly taught. Math curricula that included strong procedural instruction were scored higher on the rubric.

Rittle-Johnson & Schneider (2015) defined procedural knowledge of mathematics to be the "knowledge of the steps required to attain various goals", which can involve using algorithms or equation-solving steps with a precise sequence (p.1119). Fraction operations to convert, add, subtract, multiply, and divide fractions were all scored as procedural knowledge in this review.

Instructional Examples of Fraction Concepts. Curricula were also rated on the number of conceptual problem examples, as modeling is an essential aspect of effective fraction programs (Flores, 2007; Fuchs et al., 2013). Because fraction programs should provide instructional examples for both conceptual and procedural knowledge, we separated this rubric category to code for both types of knowledge.

Instructional Examples of Fraction Procedures. We coded instructional examples for fraction procedures, taking into consideration the graduated sequence of instructional examples. Students should have exposure to easier instructional examples (e.g., adding fractions with smaller least common denominators: $\frac{1}{2} + \frac{1}{4}$) before moving on to more complicated examples (e.g., adding fractions with larger least common denominators: $\frac{1}{3} + \frac{1}{8}$) (Doabler et al., 2012). Finally, we rated whether or not the types of instructional examples matched the difficulty of the student practice problems (Chard & Jungjohann, 2006; Doabler et al., 2012).

Part-whole Representation Math Models. Part-whole representations (e.g., fraction circles, area models, fraction arrays, etc.) reinforce conceptual understanding of fractions by bolstering symbol knowledge (Crooks & Alibali, 2014; Fuchs et al., 2013; Misquitta, 2011). Using math models within a graduated sequence (e.g., during Concrete-Representational-Abstract, or C-R-A instruction) has been proven to be an evidence-based instructional strategy at improving students' fraction knowledge (Misquitta, 2011; Watt & Therrien, 2016).

Measurement Representation Math Models. Measurement models (e.g., number lines, fractions strips, rulers, etc.) are critical for building students' fraction magnitude understanding (Jordan et al., 2017; Schumacher et al., 2018). Measurement models can also help develop students' understanding of fractions in relation to whole numbers. They demonstrate that fractions fall on the number line and that there is "an infinite density of fractions on any segment of the number line" (Fuchs et al., 2013, p. 684). Our coding criteria looked at math curricula's use of both part-whole and measurement models, as both models can help connect concepts and procedures.

Practice Opportunities and Cumulative Review. Curricula were rated on the number of opportunities for students to practice problems independently and as a group because opportunities to practice are an important aspect of effective fraction programs (Fuchs et al., 2014; Gearhart et al., 1999). These practice opportunities included single student verbalizations, choral responding, and written practice during class. Homework problems or optional differentiation activities were not included in review.

Academic Feedback. Immediate performance feedback is when the teacher corrects student errors immediately. This strategy can prevent student misconceptions and reinforce the correct use of fraction concepts and procedures. Error correction improves student performance, so core curricula should provide examples of common student misperceptions so teachers can quickly and efficiently address them (Hattie & Temperley, 2007). Math programs that preemptively described common student errors were scored higher on this category.

Formative Feedback Loops. Formative assessments throughout math units can help the teacher know if students are progressing at the expected rate. Teachers can use formative assessments to make data-based decisions about when to move on to new material (Doabler et al., 2012). Curricula received high scores if they included brief progress-monitoring assessments built into the math lessons.

Inter-Rater Reliability

To ensure accurate coding of the curricula using the instructional principle rubric, 20.17% of all lessons (47 lessons) were coded a second time by an independent scorer. The independent scorer was a fourth-year doctoral student in an educational psychology program. She had taken courses in cognition and student learning principles. The independent scorer was trained by the first author to use the rubric using example lessons. The independent scorer and main scorer came to agreement on 97.68% of scores. Academic feedback was the only principle with more than one disagreement; on this principle scorers sometimes disagreed about the relative ambiguity of reteaching strategies.

Results

Curriculum Structure

In total, 233 lessons across four separate math curricula were evaluated, including 45 lessons in Grade 3 programs, 94 lessons in Grade 4 programs, and 94 lessons in Grade 5 programs. *Everyday Mathematics'* Grade 3 teacher-edition textbook included 20 fraction-specific lessons, and each lesson averaged 6.30 pages. *Everyday Mathematics'* Grade 4 teacher-edition textbooks included 41 fraction-specific lessons, with lessons averaging 6.44 pages. *Math Expressions'* Grade 3 teacher-edition textbooks included 9 fraction-specific lessons, and each lesson averaged 8.22 pages. *Math Expressions'* Grade 4 teacher-edition textbooks included 23 fraction-specific lessons, with lessons averaging 8.17 pages. *Math Expressions'* Grade 5 teacher-edition textbooks included 40 fraction-specific lessons, and each lesson averaged 7.40 pages. *EnVision Math's* Grade 4 teacher-edition textbooks included 30 fraction-specific lessons and each lesson averaged 6 pages long. *EnVision Math's* Grade 5 teacher-edition textbooks included 29 fraction-specific lessons, with each lesson averaging 6 pages. *GO Math!'s* Grade 3 teacher-edition textbooks included 16 fraction-specific lessons, with each lesson averaging 8 pages. *GO Math's* Grade 5 teacher-edition textbooks included 25 fraction-specific lessons, with each lesson averaging 8 pages.

Instructional Principle Rubric Scores

The instructional principle summative rubric scores are described below by grade and curriculum. Results of average grade-level summative scores are in Tables 1, 2, and 3.

Grade 3

Third-grade fraction lessons in *Everyday Mathematics, Math Expressions,* and *GO Math!* were rated using the instructional principle rubric. Of the 33 summative scores for third-grade fraction lessons, 17 (51.51%) met Doabler et al.'s (2012) acceptability threshold. Prerequisite skills, instructional examples of fraction concepts, practice opportunities and cumulative review, and academic feedback were all strengths of the reviewed third-grade math curricula, as all three math programs scored above the acceptability standard on the rubric for these principles. *Everyday Mathematics* and *Math Expressions* received higher scores for explicit instruction of fraction concepts as compared to explicit instruction of fraction procedures, and all three curricula (*Everyday Mathematics, Math Expressions,* and *GO Math!*) yielded higher ratings for instructional examples of fraction concepts than for fraction procedures. *Math Expression* and *GO Math!* did not meet the acceptability criterion for explicit instruction of fraction concepts, while *Everyday Mathematics'* summative score met acceptability (3.2). None of the examined programs met the acceptability criterion for explicit instruction of fraction procedures or measurement math models; however, *Math Expressions* was relatively close to the acceptability threshold for measurement models (2.78).

Grade 4

Fourth-grade fraction lessons in *Everyday Mathematics, Math Expressions,* and *EnVision Math* were rated using the instructional principle rubric. Of the 33 summative scores for fourth-grade fraction lessons, 13 (39.39%) met Doabler et al.'s (2012) acceptability threshold. Prerequisite skills, instructional examples of fraction concepts, and practice opportunities and cumulative review were all strengths of the fourth-grade math curricula; all three math curricula received acceptable scores on the rubric for these principles.

Lessons in *Math Expressions* and *EnVision Math* met the acceptability criterion for measurement models, whereas *Everyday Mathematics'* lessons did not. None of the programs met the criterion for the use of part-whole models, though *Everyday Mathematics'* lessons were close to the threshold (2.95). None of the programs met the acceptability criterion for explicit instruction in fraction concepts or fraction procedures. *Math Expressions* scored the highest out of examined curricula in the explicit instruction categories for concepts (2.65), whereas *Everyday Math* had the most amount of explicit instruction for procedures (2.56). *EnVision Math* performed the lowest out of examined curricula, scoring 1 for explicit instruction of both concepts and procedures due to its highly guided discovery learning approach. This curriculum contained structured questioning to direct students' attention towards conceptual and procedural patterns. However, it did not have teachers explicitly explain concepts and procedures to children.

Explanations were provided only after students answered the questions incorrectly or incompletely, meaning its instructional approach resembles child-guided instruction (Baroody et al., 2015).

Grade 5

Fifth-grade math lessons from *EnVision Math, Math Expressions,* and *GO Math!* were rated using the instructional principle rubric. Of the 33 summative scores for fifth-grade fraction lessons, 15 (45.45%) met Doabler et al.'s (2012) acceptability threshold. Prerequisite skills and practice opportunities and cumulative review were strengths of the fifth-grade curricula, as all three curricula met the acceptability standard for these principles.

Lessons in *EnVision Math* and *GO Math!* met the acceptability criterion for instructional examples of fraction concepts, whereas *Math Expressions*' lessons fell below the criterion (2.88). Conversely, *Math Expressions* and *GO Math!* met acceptability for instructional examples of fraction procedures, while *EnVision Math* (2.86) did not. None of the three curricula met the acceptability criterion for math vocabulary, explicit instruction of concepts or procedures, or part-whole math models.

Summary Scores by Curriculum

Everyday Mathematics' fraction lessons in Grades 3 and 4 were rated in this review, and 11 (50%) of the 22 possible summary scores met the acceptability criterion as described in Doabler et al.'s (2012) study. *Math Expressions* fraction lessons in Grades 3, 4, and 5 were rated in this review, and 11 (33%) of the 33 possible summary scores met the acceptability standard. *EnVision Math's* fraction lessons in Grades 4 and 5 were rated in this review, and 11 (50%) of the 22 possible summary scores met the acceptability standard. *GO Math's* fraction lessons in Grades 3 and 5 were rated in this review, and 12 (54.54%) of the 22 possible summary scores met the acceptability standard.

Adherence to Common Core Standards

All curricula identified the lessons that adhered to CCSSM, and the number of lessons with target topics that mapped onto specific CCSSM are reported in Table 4. Some of the lessons adhered to multiple standards. There was variability in the number of lessons per standard between math programs, and some curricula had relatively few lessons target certain standards. For instance, *GO Math!* devoted only one lesson to the Grade 3 standard 3.NF.A.2 (represent a fraction on a number line) and *Math Expressions* had only one lesson that adhered to the Grade 4 standard 4.NF.C.5 (convert a fraction with a denominator of 10 to a fraction with a denominator of 100; add fractions with like denominators) (NGACBP, 2010). Across all examined curricula, relatively few lessons targeted 5.NF.B.3 (interpret a fraction as division of the numerator by the denominator and solve word problems involving division) compared to other Grade 5 standards (NGACBP, 2010). Finally, *Math Expressions* dedicated more lessons to Grade 5 standard 5.NF.B.6 (solve real world problems by multiplying fractions and mixed numbers) than *EnVision Math* or *GO Math!* (NGACBP, 2010).

Discussion

As longitudinal predictors of algebra readiness and general math achievement, fraction skills are critical for students to master (e.g., Siegler et al., 2012). Despite their importance, fraction proficiency deficits are pervasive in the U.S. (NMAP, 2008). The widespread underachievement in fraction concepts and procedures demonstrates a tier-one problem, and to address the pervasive need for more academic supports in fractions, tier-one curricula should be evaluated for evidence-based principles. The purpose of this review was to examine the most commonly recommended core mathematics curricula for their use of instructional principles in fraction lessons. Four math curricula in Grades 3, 4, and 5 were scored for their use of evidence-based instructional strategies. Results replicate some findings of previous curriculum reviews; specifically, there is a need for core curricula to incorporate more explicit instruction for both fraction concepts and procedures.

Instructional Principles

Practice opportunities and cumulative review was the highest scoring instructional principle in this review. All curricula and grade levels had strong scores for this category. This is perhaps a consequence of teacher textbooks providing many opportunities for varied types of student responses throughout lessons. Teachers were encouraged to elicit student responses during whole-class instruction (via either choral responding or individual student exemplars), practice with pairs of students, and independent written practice. Doabler et al.'s (2018) study indicated that successful lessons should include at least 10 opportunities for students to respond. Most lessons examined in this review met that criterion. One difference between this review and Doabler et al. (2012) is that the number of opportunities to respond dictated higher scores rather than discrimination practice opportunities. While the number of opportunities to respond in these programs was high, the number of discrimination opportunities was not (Doabler et al., 2012, 2018). Students practice discrimination when learning under what conditions and contexts to apply newly learned fraction concepts or procedures (Doabler et al., 2012).

Discrimination practice can be important for skill generalization (Stein et al., 2006), but this review coded practice opportunities and cumulative review according to Doabler et al.'s (2018) updated rubric, and therefore did not score for discrimination practice.

Prerequisite skills was another high-scoring category. Again, all grades and curricula met the acceptability threshold. This finding contrasts the result from Doabler et al. (2012), in which math lessons engaged prerequisite skills inconsistently. All of the programs we examined in the current review gave teachers defined warm-up activities, and many of the warm-up activities involved whole number fluency-building exercises. Because whole number fluency has been shown to be a prerequisite skill of all rational number skills (Hansen et al., 2017; Namkung et al., 2018), these lessons received high scores from the rubric.

Conversely, math vocabulary was inconsistently applied across both programs and lessons, with most programs and grades failing to meet the acceptability standard. Often, vocabulary words were provided at the beginning of each unit rather than throughout every lesson. There were generally few explicit opportunities in subsequent lessons to practice vocabulary words presented earlier in instruction, and none of the examined curricula directed teachers to prompt students to use previously learned vocabulary words. Using clear, accurate, and precise math vocabulary throughout instruction is important for students to understand math concepts (Hughes et al., 2016). Explicit instruction of math vocabulary may deepen students' understanding of math principles further (Hughes et al., 2016). It is important to note that three programs (*Math Expressions, EnVision Math* and *GO Math!*) offered teachers brief instructions for teaching relevant vocabulary words to English Language Learners (ELL). Increasing

ELL students' math vocabulary is important for their later mathematics success (Zhao & Lapuk, 2019); however, the quality of the ELL supports in textbooks was not assessed in this review.

Two programs, *GO Math!* and *EnVision Math*, received the lowest scores across both explicit instruction categories because these programs used a highly guided discovery learning approach to teach fraction concepts and procedures (Baroody et al., 2015). This type of child-guided instruction involves sequenced questions to students, rather than explicit, systematic instruction. Discovery learning approaches expect students to discover math patterns and strategies spontaneously or based on math materials and scaffolded questioning (Baroody et al., 2015; Sood & Jitendra, 2007). Minimally and moderately guided discovery learning involve even less scaffolding; stimuli or mathematical patterns may be presented to students, but without a particular graduated sequence of difficulty (Alfieri et al., 2011). Both *GO Math!* and *EnVision Math* provided teacher questions and sample answers in a graduated sequence of difficulty. The sample answers were provided for teachers to correct inaccurate or incomplete student answers. Although this is a structured version of discovery learning, which has shown to be more effective than minimally guided discovery learning (Alfieri et al., 2011), this approach was still not coded as explicit instruction. Explicit, systematic instruction places the obligation on the teacher, not the student, to proactively explain concepts and procedures using didactics, modeling, guided practice, and performance feedback (Doabler et al., 2012; Flores & Kaylor, 2007).

It is important to note that none of the programs evaluated in this current review would qualify as the most rigorous type of explicit instruction, Direct Instruction

(Engelmann et al., 1988; Stockard et al., 2018). This type of instruction has proven to be effective at teaching most elementary subjects, including reading, writing and mathematics, to both struggling learners and special education students (Stockard et al., 2018). Direct Instruction has proven to be an effective technique for teaching fractions as well, specifically in tier-two and classwide interventions (Flores & Kaylor, 2007; Scarlato & Burr, 2002). Utilizing more structured, Direct Instruction techniques in core curricula may be the most effective way to teach struggling learners fraction concepts and procedures.

Explicit, systematic instruction is broader than Direct Instruction; curricula in this review were simply rated on their inclusion of teacher explanations and scaffolded instruction (Doabler et al., 2012). *Everyday Mathematics* and *Math Expressions* scored higher in explicit instruction of fraction concepts for Grade 3 than in explicit instruction of fraction procedures. This finding is perhaps expected, as the Institute of Education Science (IES) fraction guidelines recommend that earlier grades place greater emphasis on fraction concepts before introducing fraction procedures (Siegler et al., 2010). Only *Everyday Math* received an acceptable rating of explicit instruction of fraction concepts across all Grade 3 lessons. None of the other programs received acceptable ratings for explicit instruction of fraction concepts at any grade level. This reveals an opportunity for more explicit instruction in core mathematics curricula to adequately teach all students conceptual fraction knowledge.

While only Grade 3 lessons from one math program received an acceptable score for its use of explicit instruction of fraction concepts, none of the examined curricula received acceptable ratings for explicit instruction of fraction procedures at any grade level. The absence of sufficient explicit instruction for fraction procedures in tier-one programs may help explain the general lack of proficiency in students' fraction performance. Incorporating systematic, explicit instruction improves student outcomes for both struggling learners and students with math-specific learning disabilities (Gersten et al., 2009). If we want students to meet CCSSM for fraction procedures (i.e., master all four fraction operations by the end of sixth grade), explicit, systematic instruction should be better included in our tier-one programming (NGACBP, 2010).

Math curricula had strong scores across all grades for instructional examples of fraction concepts, with only *Math Expressions'* fifth-grade program falling just short of the acceptability threshold (2.88). Sufficient instructional examples of fraction concepts were present throughout most grades of the math programs, revealing a relative asset of the examined core curricula. Despite the importance of integrating instruction of procedures and concepts (Misquitta, 2011; Rittle-Johnson et al., 2015), instructional examples of fraction procedures were less consistently used than instructional examples of concepts. Overall, there were higher scores in Grades 4 and 5 compared to Grade 3 lessons, which follows traditional concepts-first instructional sequencing as proposed by IES and CCSSM guidelines (NGACBP, 2010; Siegler et al., 2010). *Everyday Mathematics'* Grade 4 and *Math Expressions'* and *GO Math!'s* Grade 5 lessons all had acceptable scores for instructional examples of procedures. Instructional examples of procedures produced relatively weaker scores compared to instructional examples of concepts, indicating that core curricula might benefit from more modeled examples of fraction procedures.

The use of part-whole math models was more mixed than expected. Previous research had hypothesized that United States curricula utilize part-whole math models more frequently than measurement models (Fuchs et al., 2014). However, the use of part-whole math models throughout fraction lessons was inconsistent. Two programs in Grade 3 (*Everyday Mathematics* and *GO Math!*) and one program in Grade 4 (*Everyday Mathematics*) met the acceptability criterion for use of part-whole math models. No Grade 3 programs met acceptability criterion for measurement models. This finding matches IES guidelines, which supports the use of part-whole math models in earlier grades and then the use of measurement models in later grades (Siegler et al., 2010).

The use of measurement models in core curricula was stronger than part-whole models. In Grades 4 and 5, more programs met acceptability for the use of measurement math models than part-whole math models. Measurement math models may be linked to improved student performance in fraction arithmetic (Fuchs et al., 2013; Fuchs et al., 2017), so this finding was promising. Overall, the structure of the adapted scoring rubric may have influenced the summative scores for both part-whole and measurement math models, as lessons were coded with 1s if they did not incorporate the respective type of math model. Thus, a lesson using only fraction circles would have been coded as a 1 for the measurement model category. This perhaps produced an underestimation of textbooks' true scores for use of math models generally.

The principle of academic feedback received strong scores for all programs across Grade 3 lessons but had varied results across programs in Grades 4 and 5. Most curricula included sections of each lesson that were meant to anticipate student errors or common misconceptions, but not all of these sections received high scores. For example, some lessons had less specific descriptions of potential student errors (e.g., “If a student has difficulty maintaining focus...”) and general correction procedures (e.g., “…stand near that student as you speak to the class”) (*Math Expressions*; Fuson, 2013, p. 224). Academic feedback and re-teaching strategies received low scores if they did not directly address *mathematical* misperceptions or common errors.

Formative feedback loops was the final instructional principle evaluated in the rubric. Formative assessments are intended to facilitate Data-Based Decision Making (DBDM). They are brief assessments that help teachers gauge overall student understanding and misconceptions so as to target instruction to meet student needs (Gersten et al., 2009). *Everyday Mathematics* and *Math Expressions* included sections in each lesson labeled “formative assessment”; however, these sections were brief verbal checks for student understanding rather than Curriculum Based Measures (CBMs) or more formal formative assessments. Because neither scoring nor DBDM procedures were described, these programs received relatively low scores for formative feedback loops. *GO Math!* and *EnVision Math* met criteria for higher scores in the formative feedback loop principle category because each program had clear scoring criteria. These criteria indicated when teachers could move on with instruction and linked re-teaching strategies for students who did not meet the specified criterion. However, these formative assessments still lacked psychometric evidence, like reliability and validity evidence, indicating that scores for these two programs (i.e., *GO Math!* and *EnVision* Math) may have been overestimated.

Previous Curricular Reviews

Previous curricular reviews have demonstrated a need to incorporate more explicit instruction with graduated instructional sequences (Bryant et al., 2008; Doabler et al., 2012; Sood & Jitendra, 2007). This review replicated these findings as there was generally insufficient explicit, systematic instruction of fraction procedures and concepts in the examined curricula. Given the consistency of the data illustrating the importance of explicit instruction, it is remarkable that reviews of commonly used textbooks continue to illustrate this persistent gap between evidence-based practices and textbook construction (Baker et al., 2002; Gersten et al., 2009; Hattie, 2008). Scores for prerequisite skills and practice opportunities and cumulative review were notably higher in this study than the examined programs in Doabler et al. (2012). This is likely due to the inclusion of whole number fluency practice, as well as modifications in the practice opportunities and cumulative review domain from Doabler et al.'s (2012) study to reflect the updated (2018) rubric. That is, the number of practice opportunities was scored rather than the presence of discrimination practice.

Limitations of Current Study

There were logistical limitations of the current study that prevented us from coding all four curricula for Grades 3 through 5. We had difficulty accessing all curricula, which limited the number of programs we were able to examine for each grade level.

There were also potential limitations in our interpretations of the scoring rubric that influenced program scores. For example, guided practice opportunities may have been coded as instructional examples if the teachers' textbooks did not clearly differentiate between teacher models and guided student practice. The formative feedback loop principle did not capture the quality of separate assessment guides, and the scores for this principle may have overestimated the formative feedback loops in *EnVision Math* and *GO Math!*. These two curricula incorporated acceptable criteria for moving on with instruction, but the formative assessments in each lesson might not meet current standards for linking formative assessments to best DBDM practices (Gersten et al., 2009). Scores for practice opportunities and cumulative may also have been inflated with our interpretation of the scoring rubric. Besides graduated sequence of practice opportunity difficulty, the scores in this principle domain generally reflect the quantity of practice opportunities as opposed to their quality.

Conclusions and Implications for Practice and Future Research

This review extends previous curricular research by examining fraction lessons in math curricula most frequently recommended by required state textbook adoption lists. It replicates some findings from previous literature; specifically, that there is a need for core math textbooks to use more explicit instruction of both concepts and procedures (Bryant et al., 2008; Doabler et al., 2012; Sood & Jitendra, 2007). We also found there to be opportunities for more explicit instruction and instructional examples for *procedural* fraction skills, in particular. Using more rigorous standards for explicit instruction may be one way to sufficiently address fraction skill deficits in students. Strengths of the math curricula we reviewed included practice opportunities and cumulative review, prerequisite skills, and the use of instructional examples of fraction concepts. However, to equitably reach all students, including those at-risk for low math performance or math disabilities, tier-one instruction should be utilizing all of the instructional principles we know to be effective for struggling learners (Baker et al., 2002; Doabler et al., 2012). Building a strong foundation in mathematics means implementing effective instructional strategies in our tier-one programming. This will strengthen all students' mathematical skills and allow for proper resource allocation in our school systems.

CHAPTER 3

Evidence-Based Instruction Sequencing: Comparing the Effects of Concepts-First and Iterative Fraction Instruction

In 1989, the National Council for Teachers for Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics*, a report which advocated for greater use of concepts-based math instruction. This report represented a shift in the math education field away from traditional math textbooks, which emphasized mostly procedural learning, to reform-based textbooks, which included more conceptual instruction. As math curricula moved towards more contextualized, conceptual instruction, the importance of procedural knowledge may have been inadvertently minimized (Baroody et al., 2007; Star, 2005). This was despite the fact that math education experts (e.g., Institute of Education Sciences Fraction Educator Guide, Siegler et al., 2010; National Research Council, Kilpatrick et al., 2001; National Mathematics Advisory Panel, 2008) continued to uphold the importance of both conceptual and procedural instruction.

As math educators and researchers scrutinized the respective value of conceptual and procedural instruction in math curricula, they also considered the presentation order of conceptual and procedural knowledge. Albeit small, the existing evidence evaluating conceptual and procedural instruction suggests there to be an iterative development of concepts and procedures (Canobi, 2009; Hecht & Vagi, 2010; Rittle-Johnson & Koedinger, 2009). Despite evidence for an iterative development, the current NCTM (2014) recommendations reflect a concepts-first theory, suggesting that math educators teach math concepts before math procedures. As there is a lack of theoretical clarity about the importance and role of conceptual and procedural instruction sequencing, more empirical evidence is needed.

Conceptual and Procedural Knowledge

There has been a long history of debate in the math education and cognitive psychology fields about the definitions and importance of conceptual and procedural knowledge (Schneider et al., 2011; Star, 2005). However, recent efforts have sought to explicitly define these two types of math knowledge (Crooks & Alibali, 2014; Rittle-Johnson & Schneider, 2015). Conceptual math knowledge, in particular, has proven difficult to define. The National Research Council defined conceptual understanding as the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 5). Various other definitions of conceptual knowledge exist within the math education field. In a systematic review of conceptual knowledge definitions, Crooks and Alibali (2014) identified two, *general principle knowledge* and *knowledge of principles underlying procedures*, to be the most valuable. *General principle knowledge* refers to the fundamental assumptions, or constant mathematical rules, that govern a given skillset (Crooks & Alibali, 2014; Prather & Alibali, 2009). For example, the concept, or general principle, of mathematical equivalence holds whether an equation contains a whole number addition problem (e.g., $3 + 5 = 8$) or a rational number multiplication problem (e.g., $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$). The general principle of equivalence is declarative knowledge about the rule of the equal sign (Crooks & Alibali, 2014; Tobias & Duffy, 2009). Thus, it is not specific to a problem or procedure.

Conversely, *principles underlying procedures* is conceptual understanding of particular procedures or contexts. For example, the conceptual understanding for natural number multiplication (e.g., $7 \times 3 = 21$) must be different than that for rational number multiplication (e.g., $\frac{7}{2} \times \frac{1}{3} = \frac{7}{6}$). A student might conceptualize natural number multiplication as repeated addition, understanding that the product must be larger than the multiplicand (Obersteiner et al., 2018). The principle underlying fraction multiplication, however, is more abstract. Fraction multiplication problems can result in larger or smaller products than the multiplicand, depending on if the fractions are proper or improper (Obersteiner et al., 2018). Crooks & Alibali (2014) preferred these two definitions (i.e., *general principle knowledge* and *principles underlying procedures*) of conceptual understanding because they capture general and procedure-specific concepts. Additionally, many of the other types of mathematical concepts can be incorporated within them (e.g., general principle knowledge typically requires connection, symbol, domain structure knowledge, etc.; Crooks & Alibali, 2014).

Compared to conceptual knowledge, there are fewer definitional variants of procedural knowledge. Rittle-Johnson et al. (2001) defined procedural knowledge as "the ability to execute action sequences to solve problems" (p. 346), and this definition is generally consistent with those from other math education and cognition researchers (e.g., Byrnes & Wasik, 1991; Canobi et al., 2003; Perry, 1991). Previous definitions have sometimes conflated knowledge type with knowledge quality. Concepts have been associated with deep, rich knowledge and procedures with rote, superficial knowledge (Star, 2005). Star (2005) argued for a renewed examination of procedural knowledge, one that acknowledges the potential for deep procedural understanding. The National Research Council's (2001) definition of procedural fluency as the flexible and efficient use of procedures complements Star's view. Students' procedural flexibility, or "the knowledge…and use of multiple [solution] strategies, particularly in order to select the most efficient strategy to solve a given problem" should be the goal of procedure-based instruction (DeCaro, 2016, p. 1138). Baroody et al. (2007) also advocated for a more integrated theory of math concepts and procedures and acknowledged that knowledge type and quality should not be confounded. Thus, the common consensus is that well-connected mathematical knowledge requires deep conceptual and procedural understanding (Baroody et al., 2007).

Theories of Conceptual and Procedural Knowledge Development

There are four opposing theories for conceptual and procedural knowledge development within mathematics education; these are, (a) concepts-first, (b) procedures-first (also referred to as "skills-first"), (c) simultaneous, and (d) iterative. Concepts-first theorists argue that conceptual understanding must be developed before procedural understanding can be formed (Baroody, 2003). For instance, a student must conceptually understand equivalence before they can accurately employ procedures to find a missing portion of an equation (Briars & Larkin, 1984; Riley et al., 1983). Alternatively, procedures-first theorists claim that students first learn through practicing procedures (often via modeling). Only then do students observe mathematical patterns and extrapolate the concepts (e.g., in early numeracy, the concept of magnitude may be formed through counting and matching set quantities; Baroody, 2003; Rittle-Johnson et al., 2015). The simultaneous view of development proposes that concepts and procedures are inextricably linked and must develop synchronously (Rittle-Johnson & Siegler, 1998). The final theory is iterative development. The iterative viewpoint claims that the development of concepts and procedures affect each other; a student's procedural practice might inform their conceptual framework, which would, in turn, improve their procedural understanding (Baroody, 2003; Rittle-Johnson et al., 2015).

Empirical evidence to uncover the progression of conceptual and procedural knowledge could inform instruction sequences that match skill growth (e.g., if concepts-first theorists are correct, math educators should teach concepts before introducing procedures). Although there are strong advocates, particularly in the math education field, for the concepts-first instruction sequence and development (e.g., NCTM, 1989, 2000, 2014), existing empirical studies examining the relationship between conceptual and procedural knowledge appear to support the iterative theory of development.

Conceptual and Procedural Sequencing Studies

While it is generally accepted that conceptual understanding supports procedural performance (Rittle-Johnson et al., 2015), previous research has also shown that procedural knowledge predicts conceptual knowledge. In correlation studies, the bidirectional relationship between conceptual and procedural knowledge has been demonstrated in several math domains, including counting, whole number addition and subtraction, equivalence, decimals, and fractions (Canobi, 2009; Jordan et al., 2009; LeFevre et al., 2006; Rittle-Johnson & Schneider, 2015; Vukovic et al., 2014).

Other empirical studies have also examined the experimental relationship between conceptual and procedural instructional sequences. Rittle-Johnson & Alibali's (1999) study tested the effects of conceptual and procedural instruction on learning and transfer. Students were randomly assigned to receive concept-only or procedure-only instruction about mathematical equivalence. Students in both conditions improved on conceptual and procedural assessments. This study provided causal evidence for the bidirectional relationship between concepts and procedures. Conceptual instruction improves procedural understanding, just as procedural instruction improves conceptual understanding. This causal bidirectional effect was also observed with another content domain, whole number arithmetic (Canobi, 2009; McNeil et al., 2012), providing more empirical support for the theory that math concepts and procedures develop iteratively (Rittle-Johnson et al., 2015).

While the empirical evidence for iterative development of concepts and procedures is compelling, these studies did not directly compare instruction sequences in which only the order of conceptual and procedural instruction was altered. This led Rittle-Johnson and Koedinger (2009) to test the concepts-first and iterative instructional sequences. During a six-lesson academic intervention about decimals, students either received conceptual and procedural lessons iteratively (i.e., in an interleaved, alternating sequence) or in a concepts-first sequence (i.e., all concepts were taught before procedures) (Rittle-Johnson & Koedinger, 2009). The iterative students ended up outperforming the concepts-first students on procedural measures of learning and transfer, while both groups showed comparable conceptual outcomes. These results revealed that procedural instruction could lead to meaningful knowledge transfer when interleaved with conceptual instruction. Moreover, it provided causal evidence that the iterative instruction sequence resulted in similar conceptual and better procedural outcomes than a concepts-first sequence. More research is needed to support this preliminary finding and determine if it replicates with different math domains. Testing this effect within fraction instruction appears to be a natural follow-up. Decimals and fractions both represent parts of integers yet require a different base of conceptual and procedural knowledge.

Rational Number Knowledge

Fraction knowledge has been shown to strongly predict later math skills, such as proportional reasoning and algebra (Booth et al., 2014; DeWolf et al., 2015). Fraction skills predict overall math achievement up to six years later, even after controlling for other confounding factors, like working memory and general intellectual ability (Siegler et al., 2012). Students who show low growth in fraction skills are also at higher risk for later math failure (Jordan et al., 2017), indicating that fraction mastery is essential.

Though fractions are crucial for strong overall math development, students in the United States (U.S.) struggle to master both fraction concepts and fraction arithmetic (NMAP, 2008; Siegler & Pyke, 2013). For example, nationally, 45% of Grade 4 students were unable to correctly add together three fractions with like denominators ($\frac{2}{5} + \frac{3}{5} + \frac{4}{5}$) on the 2013 NAEP test (National Center for Education Statistics, 2013). On the 2017 NAEP test, 68% of Grade 4 students could not correctly categorize six fractions, $\frac{1}{3}, \frac{2}{3}, \frac{2}{6}, \frac{4}{6}, \frac{2}{8}, \frac{4}{8}$, to be less than, equal to, or greater than one-half (National Center for Education Statistics, 2017). Therefore, determining the most effective instructional sequence for teaching fraction concepts and procedures is a goal of great practical importance.

Understanding fraction concepts and procedures in elementary school is critical because fraction achievement gaps widen in middle school; students with poor fraction performance in Grade 6 tend to stay low by Grade 8, despite receiving fraction instruction in general education math classes (Siegler & Pyke, 2013). This finding suggests efforts should be made in elementary school to strengthen early fraction understanding (i.e., interventions should target Grades 3 and 4 CCSSM). Additionally, whole number proficiency should be assessed before fraction interventions, as considerable evidence illustrates that sufficient mastery of whole number knowledge concepts and fluency with whole number operations predicts fraction performance (Hansen et al., 2017; Jordan et al., 2017; Namkung et al., 2018).

Conceptual and Procedural Assessment

Measuring conceptual and procedural knowledge separately from one another is challenging, and this reality has contributed to the debates about the role and development of conceptual and procedural knowledge (Rittle-Johnson et al., 2015; Schneider & Stern, 2010). As deep mathematical knowledge is theorized to require both conceptual and procedural understanding, it might be difficult to uniquely capture each type of knowledge in a controlled way (Baroody et al., 2007). Rittle-Johnson and Schneider (2015) suggested that conceptual math knowledge can be measured implicitly or explicitly. Explicit measurement requires students to provide written or verbal explanations of general or procedure-specific principles (Canobi & Bethune, 2008; Rittle-Johnson & Schneider, 2015). Explicit measures of conceptual understanding are resource-intensive to collect, so other studies have also utilized implicit measures. Examples of implicit conceptual tasks include, but are not limited to, magnitude comparison tasks, quantity translation between different representational systems, and the evaluation of novel procedures (Byrnes & Wasik, 1991; Rittle-Johnson & Schneider, 2015). There is less variation in the measurement of procedural knowledge, as it is typically assessed using accuracy scores from problem-solving assessments.

Purpose of Current Study

To expand the empirical literature comparing the presentation order of conceptual and procedural lessons, this study compared the efficacies of concepts-first and iterative instruction sequences with a previously unexamined math domain, fractions. These two instructional sequences were selected for comparison because previous research has indicated an iterative instruction sequence may be optimal for student learning (Rittle-Johnson & Koedinger, 2009). However, math education experts recommend a concepts-first approach (e.g., NCTM 1989, 2000, 2014).

This study determined if a concepts-first or iterative instruction sequence resulted in differential student learning of two fraction concepts and two fraction procedures that align with third and fourth-grade goals in Common Core State Standards (2010):

- writing standard notation from math models (concept)
- comparing fraction magnitudes to a benchmark whole number (concept)
- adding and subtracting fractions with like denominators (procedure)
- multiplying fractions (procedure)

We chose these two fraction concepts because Rittle-Johnson and Schneider (2015) defined "translating quantities between representational systems" and "comparing quantities" to be implicit conceptual tasks (p. 1121). Conversely, arithmetic operations (i.e., fraction addition, subtraction, and multiplication) are often considered math procedures, as they require students to know how and why to carry out a series of action steps (Baroody et al., 2007; Rittle-Johnson et al., 2001).

The instruction was delivered in the form of a classwide fraction intervention using a Direct Instruction fraction program, *Corrective Mathematics, Basic Fractions*

(Engelmann & Steely, 2005). This program was selected because it incorporated high levels of explicit, systematic instruction (Flores & Kaylor, 2007), an instructional principle that has proven to have substantial effects on student learning (Stockard et al., 2018). Both instructional sequence groups (i.e., concepts-first and iterative) were compared to a control to ensure that this fraction instruction program improved students' overall fraction outcomes. This study addressed the following research questions:

1. Were there differences between instructional sequence groups (i.e., concepts-first, iterative, and control) on proximal outcome measures of fraction concepts and procedures?
2. Were there differences in transfer between instructional sequence groups on distal fraction outcome measures?
3. Did whole number knowledge predict performance on rational number assessments?
4. Did the outcome measures capture multiple dimensions of rational number knowledge or a single underlying dimension?

Method

Participants and Setting

All participants were recruited from two suburban elementary schools in the Midwest region of the United States. The two schools were comprised of 12.6 and 7.6% Hispanic/Latino; 31.7 and 5.7% Black/African American; 27.4 and 78.7% White/Caucasian; 10.3 and 6.6% two or more races; 17.8 and 0.7% Asian/Asian American; and 0.3 and 0.7% American Indian/Alaskan Native students. The two participating schools also reported that, of their total students, 43.2 and 13.5% qualified for free/reduced-priced meals, 0 and 1.2% of students were homeless or highly mobile, 16.6 and 14.7% of students qualified for an Individualized Education Plan (IEP) in Special Education, and 16.8 and 6.1% of students were English Language Learners (ELLs).

There were 114 participating Grade 4 students (52.63% female) across five classrooms. A power analysis, using G*Power, was conducted to determine an adequate sample size. Previous research has indicated large effect sizes for fraction interventions using explicit instruction (Ennis & Losinski, 2019; Rittle-Johnson & Koedinger, 2009). Given this, a power analysis was conducted to detect a large effect size ($f = .40$) at 0.80 power and 0.05 alpha probability, which indicated that this study required 64 participants. To be conservative, we also ran a power analysis to detect a medium effect size ($f = .25$) at 0.80 power and 0.05 alpha probability, which indicated that this study required 158 participants. Therefore, the range of participants needed for adequate power was 64 to 158.

Recruitment and Consent Procedures

All recruitment and study procedures were approved by the Institutional Review Board (STUDY00007711). All participating students had an equal opportunity to participate in the intervention. Passive parental consent forms went home with students, and parents could return the permission form to opt their child out of the study. The intervention did not begin until at least a week after consent forms were sent home, and parents could withdraw their child from the study at any point in the study's duration. One parent opted their child out of the research study. Teachers were also able to opt students out of the study if they felt the instructional match was inappropriate for the students' skill level. The general education teachers opted four students out of the study for this reason. Verbal student assent to participate was obtained for all 114 participants, and an alternative academic activity was provided for the 13 students who chose not to participate. After the initial assent protocol, no students withdrew from the study due to opting out or transferring schools. The 114 participants were randomly assigned to one of three groups: (a) concepts-first, (b) iterative, and (c) control groups; there were 38 participants in each group (respectively, 55.26, 42.11, and 60.53% female).

Intervention Setting and Core Instruction

Students randomly assigned to the intervention groups (i.e., concepts-first or iterative) received the math intervention during 20-min of their general education math block. Depending on space availability, the 12 intervention lessons were taught in the school libraries, cafeterias, or available, empty classrooms. Throughout the study's duration, all participating students continued to receive their general education core math lessons from the *GO Math!* curriculum for an average of 90-min per day. The general education math teachers supplemented this curriculum with explicit instruction. The intervention did not interfere with the didactic portion of the core curriculum, as students were only pulled from class after the teachers' explicit instruction. The first author, a school psychology doctoral student, administered all intervention sessions.

Dependent Variables

Students' fraction skills were assessed with proximal and distal fraction measures before and after the intervention. The primary outcome variable for this study was the number of problems correct on the conceptual and procedural proximal fraction assessments. The secondary outcome variable for this study was the number of problems correct on generalization measures.

Proximal Fraction Measures

Similar to Flores and Kaylor's (2007) study, the proximal fraction measures were curriculum-based assessments from the *Corrective Mathematics, Basic Fractions* program (Engelmann & Steely, 2005). We constructed these assessments using CBM-mathematics recommendations from Shinn (1989). We selected 20 student practice problems from the *Basic Fractions* curriculum to measure each conceptual (i.e., standard notation from math models and magnitude comparison) and procedural (i.e., fraction addition/subtraction with like denominators and fraction multiplication) skill in the intervention. Doing so resulted in a total of 80 fraction problems across both the fraction concepts and procedures assessments. Students had 10-min to complete assessments for each of the four fraction skills, resulting in approximately 40-min of total assessment time for each set of proximal fraction measures.

Fraction Concepts Assessment with Modeled Examples. The fraction concepts assessment with modeled examples (Concepts Ex) included 40 total items. It tested students' ability to (a) write fractions and fraction equations in standard notation based on part-whole math models and (b) compare fraction magnitudes to a benchmark whole number. These skills were chosen because Grades 3 and 4 CCSSM emphasize the importance of interpreting visual fraction models and extending students' understanding of fraction magnitude comparison (NGACBP, 2010). For 10 items, students wrote the standard notation of fractions based on two shaded fraction circles. For example, an item displayed two fraction circles, each divided into sixths. Four pieces of the first fraction circle were shaded. Students wrote the standard notation of the fraction, $\frac{4}{6}$, to receive credit. Instructions at the top of this section said, “Each fraction circle represents one whole. Write the fraction of shaded pieces in each item.” These items tested students’ ability to write five proper and five improper fractions based on part-whole representations.

Students were also tested on their ability to write fraction equations in standard notation. For 10 items, students wrote like-denominator addition equations based on two shaded fraction circles. For example, an item displayed two fraction circles, each divided into fifths. Three pieces of the first fraction circle were shaded in a solid pattern, and four pieces of the first and second fraction circles were shaded in a dotted pattern. Students wrote the standard notation of the fraction equation, $\frac{3+4}{5}$ or $\frac{3}{5}+\frac{4}{5}$, to receive credit. Students did *not* solve the fraction addition equations on the conceptual assessment. Instructions at the top of this section said, “Each fraction circle represents one whole. Write an equation to add the fraction of solid pieces to the fraction of dotted pieces in each item. Only write the equation; you do not have to solve the equation for an answer.”

The last conceptual skill assessed was magnitude comparison to a benchmark whole number. Students determined if 20 different fractions were more than, less than, or equal to one. For example, an item displayed $\frac{5}{7}$ on one side of an item space and “1” on the opposite side. In between the two numbers, students circled “more”, “=”, or “less”. Instructions at the top of this section said, “For each item, circle ‘more’, ‘=’, or ‘less’ to indicate if the fraction is more than, equals to, or less than the number 1.”

For each type of math problem on the fraction concepts assessment, there were two worked examples at the top of the page. These modeled examples were included to reduce confusion about item formatting. However, the pretest data indicated that the modeled examples inflated student scores. Students may have been able to inductively reason the fraction concepts from the examples and apply them to the remaining items. For this reason, we added the same assessments *without* the examples at the posttest. Because we still wanted to evaluate student growth from pre to posttest, we also administered the assessments with modeled examples during the posttest.

Fraction Procedures Assessment with Modeled Examples. The fraction procedures assessment (Procedures Ex) included 40 total items testing students' ability to (a) add and subtract fractions with like denominators and (b) multiply fractions. These skills were chosen because Grade 4 CCSSM emphasizes the importance of fraction addition and subtraction with like denominators and fraction multiplication (NGACBP, 2010). For 10 items, students solved for missing numerator addends in fraction addition and subtraction equations (e.g., an item displayed the equation, $1 = \frac{9 - \square}{5} = \frac{\square}{\square}$, and students filled in the missing numerator addend and solved the equation to receive credit, $1 = \frac{9-4}{5} = \frac{5}{5}$). Instructions at the top of this section said, "Fill in the missing number in each numerator and solve each addition or subtraction problem." There were five addition and five subtraction problems.

Students were also tested on their ability to solve like-denominator addition and subtraction equations for 10 items (e.g., an item displayed the equation, $\frac{5}{2} + \frac{4}{2} = \frac{\square}{\square}$, and students wrote $\frac{9}{2}$ to receive credit). There were three items in this section that displayed addition or subtraction equations with unlike denominators; therefore, students had to demonstrate they could discriminate between the types of problems they could and could not solve. Instructions at the top of this section said, “Solve the addition and subtraction problems with like denominators. If an item shows an addition or subtraction problem with unlike denominators, put an ‘X’ through it and continue to the next problem.”

Finally, students were tested on their ability to multiply fractions. Students completed 20 multiplication problems; 10 problems had students multiply two fractions together, and 10 problems had students multiply a fraction by a whole number (e.g., two items displayed the equations, $\frac{1}{5} \times 2 = \frac{\square}{\square}$ and $\frac{7}{6} \times \frac{1}{5} = \frac{\square}{\square}$, and students wrote $\frac{2}{5}$ and $\frac{7}{30}$ respectively to receive credit). Instructions for this section said, “Solve each multiplication problem.”

This fraction procedures assessment included two worked examples for each type of math problem at the top of each page. Like the examples in the fraction concepts assessment, the worked examples in the fraction procedures assessment were intended to clarify item formatting. However, relatively high student scores on the pretest indicated that the examples might have inflated student scores. Consequently, we added the same fraction procedures assessment *without* examples to the posttest. We still administered the fraction procedure assessments with examples at the posttest so that we could directly evaluate student growth from pre to posttest.

Fraction Concepts and Procedures Assessments *without* Modeled Examples. The fraction concepts assessment without modeled examples (Concepts) and procedures assessment without modeled examples (Procedures) were administered at the posttest only. These assessments included the same items as the ones with modeled examples; the only difference between them was the omission of the worked examples. We removed the examples to make the assessments more difficult, thereby allowing us to measure a greater skill range. The assessments without modeled examples were the first assessments administered in the posttest and were administered on a separate day than the assessments with modeled examples.

Generalization Measures

To determine how well the fraction skills taught in this intervention transferred to new measurement probes, participants completed three measures of generalization during pre and posttesting. Previous studies have used both the *Vanderbilt Fraction Battery* assessments (e.g., Fuchs et al., 2017; Fuchs et al., 2013; Fuchs et al., 2020) and questions from prior NAEP tests (e.g., Fuchs et al., 2020; Malone & Fuchs, 2014; Schumacher et al., 2018) to assess generalization of intervention skills.

Vanderbilt Fraction Battery. Students completed two probes from the Vanderbilt Fraction Battery during pre and posttest, a Grade 3 Super Challenge CBM and a Grade 4 Super Challenge CBM (Fuchs et al., 2020). Each probe was timed for 7-min and included 20 items. These assessments tested students' ability to place fractions on number lines, order three fractions from least to greatest, compare fraction magnitudes, solve for a missing numerator in equivalent fractions, solve fraction addition and subtraction problems with like denominators, and multiply whole numbers.

NAEP Assessment. As their final generalization probe, students completed a compilation of released fraction questions from the 1990 to 2017 Grade 4 NAEP tests. This generalization assessment included 21 easy, medium, and hard fraction items, as classified by NAEP. The types of questions on this assessment included fraction word problems (e.g., "Jim has $\frac{3}{4}$ of a yard of string, which he wishes to divide into pieces, each $\frac{1}{8}$ of a yard long. How many pieces will he have?"), magnitude comparisons (e.g., students determine if six fractions are greater than, equal to, or less than $\frac{1}{2}$), math model representations (e.g., students determine the standard notation of a fraction shaded on a fraction strip), fraction addition and subtraction with like denominators, and fraction multiplication. This assessment was scored per the NAEP scoring rules and answer key, so students could receive partial credit for six questions. The maximum score for this assessment was 21, and students had 15-min to complete this assessment.

Whole Number Assessments

Prerequisite skills in whole number knowledge were tested during the pretest. Previous research has indicated whole number knowledge to be highly predictive of rational number performance (Jordan et al., 2017). Therefore, we wanted to control for variation in whole number skills and determine if whole number knowledge predicted rational number outcomes for this intervention.

AIMSweb Mathematics Concepts and Applications (MCAP). The fourth grade MCAP is an 8-min test that assesses general mathematics concepts and applications (e.g., number sense, operations, patterns & relationships, measurement, geometry, data and probability) (Shinn, 2004). There are 30 items on this assessment, and students can be awarded up to 49 total points. Correct responses are scored between 1 to 3 points and incorrect responses are scored 0 points. One fourth-grade screening probe was administered to participating students at the pretest.

AIMSweb Multiplication. The AIMSweb Single-Skill Math Fact Probe for Multiplication is a 2-min test consisting of 84 whole-number multiplication problems (Shinn, 2004). It includes multiplication facts for numerals 0 through 12 and is scored by counting the number of correct digits. This assessment is a measure of whole-number multiplication fluency, and only one math fact probe was administered to participating students at the pretest.

AIMSweb Addition/Subtraction. The AIMSweb Mixed-Skill Addition and Subtraction Math Fact Probe is a 2-min test consisting of 84 whole-number addition and subtraction problems, with numerals 0 through 12 (Shinn, 2004). This probe is scored by counting the number of correct digits. This math fact probe is a measure of whole-number addition and subtraction fluency, and a single probe was administered to the participating students during pretest.

Intervention Groups

This study used a randomized block design to assign students to instructional groups. Students were stratified based on pretest proximal assessment scores and then randomly assigned to the (a) concepts-first, (b) iterative, or (c) control group. The blocking was done to ensure relatively equal fraction skill levels across all three groups (Konstantopoulos, 2013; Wilk, 1955). An equal number of students in each classroom was assigned to each of the three instructional groups so that intervention sizes were similar across each instructional group. Students in the concepts-first and iterative groups received the same intervention exercises and practice opportunities. The only difference between these instructional groups was the sequencing of the intervention exercises.

Concepts-First Group

In the concepts-first group, all of the conceptual and procedural exercises were massed into separate lessons. The students in this group received all conceptual fraction lessons before learning procedural fraction skills. The first six lessons of the intervention taught only concepts, and the second six lessons of the intervention taught only procedures.

Iterative Group

The students in the iterative group completed both conceptual and procedural exercises in each lesson. Conceptual and procedural exercises were alternated, so each iterative lesson contained either one procedural and two conceptual exercises or one conceptual and two procedural exercises.

Control Group

The students in the control group did not participate in the Direct Instruction fraction lessons. Instead, they remained in their general education math class with their teacher during the intervention time. Students in the control group completed additional rational number activities from the *GO Math!* curriculum, as the general education math teachers taught a fraction unit, including instruction about writing standard notation, fraction magnitude comparison, and fraction addition and subtraction with like denominators, during the intervention phase. The general education math teachers did not teach fraction multiplication, the second procedural skill targeted in the intervention.

Intervention Procedures

The first author, a fourth-year school psychology graduate student, instructed students in the concepts-first and iterative groups in a pull-out group format. Intervention groups ranged from 8 to 17 students at a given time, depending on the classroom teachers' math block schedules. Students in the control group worked on math with their general education teachers during the intervention time.

Intervention Curriculum

Intervention lessons consisted of exercises from a Direct Instruction program, *Corrective Mathematics, Basic Fractions* (Engelmann & Steely, 2005). The *Basic Fractions* program includes all of the core components of Direct Instruction (i.e., systematic, explicit instruction, teacher modeling, guided student practice, academic performance feedback, etc.), which have large effect sizes on student learning (Stockard et al., 2018). We made 12 intervention lessons, each containing three exercises. Each lesson required approximately 20-min to teach. Half of the 36 total exercises taught fraction concepts and half taught fraction procedures. Each of the 12 lessons included explicit instruction, interventionist modeling, guided practice, student practice, and immediate error correction.

Conceptual Exercises. There were two conceptual fraction skills taught in this intervention: (a) writing fractions and fraction equations in standard notation based on math model representations and (b) comparing fraction magnitudes to a benchmark whole number (see Appendix B). There were nine exercises in which students learned to write fractions and fraction addition equations in standard notation. Students first learned fraction rules for interpreting part-whole models, including how numerators and denominators can be represented on a fraction circle. They then learned to write fraction addition equations based on the visual representations.

The remaining nine exercises taught fraction magnitude comparison. Students first learned to determine if a shaded fraction circle represented one whole. They then learned to determine if a fraction written in standard notation represented one whole. Finally, students learned to compare a fraction (written in standard notation) to 1 to determine if the fraction was more than, equal to, or less than the benchmark whole number.

Procedural Exercises. There were two procedural fraction skills taught in this intervention: (a) addition and subtraction with like denominators and (b) fraction multiplication (see Appendix C). Students first learned to discriminate between like and unlike denominators to correctly apply the appropriate fraction addition or subtraction algorithms. Students then practiced solving the addition and subtraction equations. Explicit instruction of addition and subtraction with like denominators comprised 11 exercises. The second procedure was fraction multiplication. Students first learned the algorithm for multiplying two fractions together. Students moved on to learn how to multiply a fraction by a whole number. Fraction multiplication comprised the remaining seven procedural exercises.

Behavior Management

A behavior management component was introduced during the intervention phase to maintain on-task behavior during the lessons. For one school, the behavior management component began during the eighth intervention lesson, and for the other school, it began during the sixth lesson. Interdependent group contingency systems have been shown to increase academic engagement and reduce disruptive behaviors during classwide interventions (Little et al., 2015). We implemented a modified version of the "You/Me Game" as an interdependent group contingency in this study (see Codding et al., 2019 for full You/Me Game procedures). During this game, students received points on a scoreboard for their on-task behavior (e.g., answering questions, sitting in seats, listening to the instruction). The interventionist received points if students exhibited offtask behavior. At the end of each intervention lesson, students received a sticker if they outscored the interventionist by less than five points or a small prize (e.g., erasers, pencil sharpeners, wristbands) if they outscored the interventionist by more than five points. The You/Me Game rules were reviewed with the students at the beginning of each intervention lesson to remind them of behavioral expectations.

Data Analysis Plan

To compare the effects of the intervention between the three intervention groups, analyses of variance (ANOVAs) and analyses of covariance (ANCOVAs) were conducted for the outcome measures. ANOVAs were used to evaluate the proximal fraction assessments without examples. ANCOVAs were used to compare groups after controlling for pretest performance as the covariate. ANCOVAs were run on the proximal fraction assessments with examples and the generalization assessments. Additionally, separate ANCOVAs were run with the whole number assessments as covariates to determine if whole number proficiency significantly predicted performance on the posttest. Correction tests were run post-hoc to detect significance within the ANOVA and ANCOVA models.

Principal Component Analyses (PCAs) were also conducted to evaluate the functioning of the assessments themselves. Three PCAs evaluated the rational number pretests, rational and whole number pretests, and the posttests. The purpose of the PCAs was to determine the number of unique dimensions measured in each set of assessments.

Inter-Rater Reliability

To ensure accurate scoring of fraction assessments, 20% of all fraction probes were scored a second time by a research team member separate from the interventionist.

The research team member was a fourth-year graduate student in the school psychology program. This individual had completed school psychology assessment and intervention coursework. The independent scorer was provided with answer keys for the proximal and distal fraction assessments (Engelmann & Steely, 2005; Fuchs et al., 2020; Schumacher et al., 2013). Because the NAEP assessment included some items with partial credit options, the independent scorer demonstrated readiness by correctly scoring a partial-credit item independently. The agreement between scorers was 99.57%.

Treatment Adherence

All intervention sessions were audio tape-recorded. Of the 72 total audio recordings, 20 (27.78%) were randomly sampled and independently scored by a research team member to calculate treatment adherence. The research team member was a second-year graduate student in a school psychology program. This individual had completed school psychology assessment and intervention coursework. The independent scorer was given a sample intervention lesson and taught to identify each step of the treatment adherence checklist with a sample recording. The independent scorer then demonstrated readiness to score when she reached scoring agreement with this example recording. The 7-step treatment adherence checklist (see Appendix D) included essential components of the intervention. The interventionist adhered to 95.71% of the checklist steps in the sampled intervention sessions.

General Procedures

This study consisted of three phases: (a) pretesting, (b) intervention, and (c) posttesting. Each classroom had 20 total study sessions, including three sessions of pretesting, 12 sessions of intervention lessons, three sessions of posttesting, and two sessions of makeup testing.

Pretesting

On the first day of pretesting, students verbally assented to the study and completed the three whole number assessments, AIMSweb MCAP (8-min), AIMSweb multiplication (2-min), AIMSweb addition/subtraction (2-min). Students then completed the proximal and generalization rational number assessments over two additional days. The second day of testing was designated for a proximal assessment (Concepts Ex, 20-min) and the two generalization assessments from the *Vanderbilt Fraction Battery* (Super Challenge Grades 3 & 4, 14-min for both). The third day of testing was designated for the remaining proximal fraction assessment (Procedures Ex, 20-min) and the NAEP assessment (15-min). There was one additional day of testing designated for student makeups, as needed. The assessment administration order was held constant for each classroom. The assessments were administered in a group format, with no assessment period taking longer than 40-min with instructions. Pretesting across the five classrooms lasted three weeks due to classroom scheduling.

Intervention

The intervention phase lasted six weeks, and intervention sessions were delivered one to three times a week, depending on classroom schedules. Each of the 12 intervention sessions lasted approximately 20-min. Throughout the study duration, students in the concepts-first and iterative treatment groups received a total of four hours of Direct Instruction about fraction concepts and procedures. Students in the control group received four hours of rational number instruction from their general education math curriculum, *GO Math!*.

Posttesting

During posttesting, students completed the same rational number assessments as they did in pretesting. Additionally, they completed the proximal fraction assessments without modeled examples. Posttesting occurred over three testing sessions. The first day of testing was devoted to the two proximal fraction assessments without modeled examples (Concepts and Procedures, 40-min for both). The second day of testing was designated for a proximal assessment (Concepts Ex, 20-min) and the two generalization assessments from the *Vanderbilt Fraction Battery* (Super Challenge Grades 3 & 4, 14-min for both). The final day of testing was designated for the remaining proximal assessment (Procedures Ex, 20-min) and the NAEP assessment (15-min). One additional day of testing was held for student makeups, as needed. The assessment administration order was consistent for each classroom. The assessments were administered in group format, with no assessment period taking longer than 45-min with instructions. Posttesting across the five classes lasted two weeks.

Results

To analyze the results from this study, we conducted three types of statistical models: (a) Principal Component Analyses (PCAs), (b) Analyses of Variance (ANOVAs), and (c) Analyses of Covariance (ANCOVAs). The PCAs revealed the number of distinct components measured from the rational and whole number assessments. The ANOVAs compared instructional groups at the posttest to determine group differences on the proximal assessments without modeled examples. Finally, ANCOVAs compared instructional groups at the posttest to assess group differences on the proximal assessments with modeled examples and the generalization measures after controlling for students' pretest performance. All analyses were conducted using IBM SPSS Statistics Version 26.

Principal Component Analyses

Principal Component Analyses (PCAs) modeled student scores to determine the number of factors measured by the assessments. The primary research question was whether each set of assessments measured a single rational number factor or separable conceptual, procedural, and whole number factors. We conducted three PCAs using the pre and posttest assessment data from this study. Beforehand, we examined the appropriateness of using PCAs by testing the statistical assumptions. For each of the following PCAs, the Kaiser-Meyer-Olkin value was greater than the 0.6 criterion (Kaiser, 1970, 1974), and the Tests of Sphericity were all statistically significant (p-value < 0.05; Bartlett, 1954). Correlation matrices showed all assessments to have coefficients greater than 0.3. Therefore, PCAs were appropriate to use with this dataset.

Pretest Rational Number Assessments

The first PCA modeled the rational number assessments from the pretest, including the two proximal and three generalization measures. This resulted in seven variables, as each of the proximal measures were comprised of two sub-scores. The goal of this PCA was to determine if the pretest proximal assessments we developed from the fraction curriculum accounted for similar student variance as the published generalization measures.

The first component of the analysis was the only factor to have an eigenvalue greater than 1, and it explained 58.01% of the variability amongst the five variables; see Table 5. A scree plot also showed a one-factor extraction to be appropriate, as the inflection point of the graph occurred after the first component.

The component matrix revealed all pretests to have strong loadings on the first component, indicating all pretests are represented in this first construct. From the content of these assessments, the construct is likely general fraction knowledge. Thus, it appears that the proximal fraction assessments accounted for similar student variability as the generalization assessments before students received the rational number intervention.

Pretest Rational and Whole Number Assessments

The second PCA included both the rational and whole number assessments at the pretest to determine if the rational number assessments uniquely accounted for variance beyond whole-number assessments. The eigenvalues from two factors met Kaiser's criterion of greater than 1, with these two factors accounting for 65.99% of the variability amongst all assessments; see Table 5. The scree plot shows two inflection points in the graph, with a sharper drop occurring after the first component and a more gradual decline after the second component. Consistent with the prior PCA, most rational number assessments strongly loaded on the first component. The only rational number assessment to show a weaker loading on this component was the fraction multiplication sub-score from the fraction procedures assessment. The two whole-number computation assessments – AIMSweb Basic Multiplication Facts and AIMSweb Basic Addition and Subtraction Facts – as well as the fraction multiplication sub-score from the fraction procedures assessment had strong positive loadings on the second component.

Thus, it appears that the rational number assessments (which mostly loaded on the first component) may be measuring something unique from whole number fluency (which only loaded on the second component).

Posttest Rational Number Assessments

The final PCA was conducted to determine if the posttest component structure changed from the pretest. It analyzed the number of measured factors from all posttest rational number assessments, including the proximal and generalization assessments. The posttest versions of the same seven variables from the first PCA were included in this last model. In addition to the original seven variables, we included four sub-scores from the proximal assessments without modeled examples, resulting in 11 total variables. We added these assessments to determine if the two sets of proximal assessments – those with and without modeled examples – loaded similarly onto the same components. The goal of this PCA was to evaluate if additional components met the inclusion threshold following the intervention, which could indicate the assessments measured multiple student dimensions (perhaps separate conceptual and procedural skills) after the students were exposed to fraction instruction. The alternative hypothesis is that the posttest assessments were only measuring a single rational number construct, as they were in the pretest.

This PCA revealed two factors that surpassed an eigenvalue of 1, capturing 69.32% of the observed variance; see Table 6. The scree plot showed two breaks in the data: the steepest slope occurred after the first component, and a more gradual slope occurred after the second component.

The rotated component matrix showed that all fraction procedure sub-scores (i.e., those with and without modeled examples) loaded strongly onto only the first component. The fraction concept sub-scores were split more evenly between the two components. One fraction concept sub-score, magnitude comparison, showed a slightly higher loading on the first component (with examples: .59; without examples: .67) than it did on the second component (with examples: .49; without examples: .49). The other fraction concept sub-score, standard notation, showed a slightly higher loading on the second component (with examples: .55; without examples: .53) than it did on the first (with examples: .41; without examples: .43). While both fraction concept sub-scores had moderate loadings on the two components, the three posttest generalization measures all loaded strongly onto only the second component.

The magnitudes of the loadings were comparable for the proximal fraction sub-scores both with and without examples. This shows that we only changed the assessment difficulty when we removed the modeled examples from the assessments, *not* the underlying measured construct. This PCA's results also indicate that the fraction procedure assessments measured a distinct construct from the generalization measures because each loaded strongly onto separate components. Finally, the magnitudes of the loadings from the fraction concepts assessments were different from the fraction procedures assessments, as seen in Table 6. The different loading strengths on each of the two components suggest that the conceptual and procedural assessments captured separate student variance. Overall, this is preliminary empirical evidence that it may be possible to disentangle aspects of fraction concepts and procedures and separately measure them in future studies.

Analyses of Variance

The means of the three instructional groups (i.e., control, concepts-first, and iterative) were compared at the posttest using one-way Analyses of Variance (ANOVAs) to analyze the fraction concepts and procedures assessments without modeled examples; see Table 7. To test for homogeneity of variance, we conducted the Levene's Test of Equality of Variances. This test was not significant for the fraction concepts assessment and therefore met the homogeneity of variance assumption. Levene's Test was significant for the fraction procedures assessment ($p < .001$); however, because the three groups each had an equal number of participants, the homogeneity of variance assumption should be robust enough to protect against this bias (Stevens, 1996). Histograms were used to test for normal distribution of data. Both the fraction concepts and procedures assessment violated the assumption of normality, as ceiling effects were present in both datasets. We continued with the statistical plan to conduct ANOVAs because the normality assumption has been shown to be robust enough to protect against biases when there are large enough sample sizes in each group ($n > 30$) (Feir-Walsh & Toothaker, 1974).

Fraction Concepts Assessment without Modeled Examples

There were statistically significant differences between groups in posttest scores on the fraction concepts assessment without modeled examples, F (2, 113) = 7.10, p = .001. The Bonferonni procedure for correcting the family-wise error rate was used because it has been shown to be appropriate for social and behavioral science research (Frane, 2015). The Šidák correction for the Bonferonni method was applied because it improves upon the accuracy of the simple Bonferonni method in the case of independent test statistics (Shaffer, 1995). Post-hoc comparisons using the Šidák correction showed that students in the concepts-first (M = 33.13, SD = 9.63) and iterative groups (M = 33.42, SD = 8.32) correctly completed significantly more problems than students in the control group (M = 26.08, SD = 10.76); see Table 8. When compared to the control group, medium effect sizes were yielded for both the concepts-first (p = 0.005, d = 0.69) and the iterative (p = 0.004, d = 0.76) groups (Sawilowksy, 2009). There was no difference between concepts-first and iterative students on this assessment.

Figure 2 displays the average performance of each instructional group on the fraction concepts assessment without modeled examples. It shows that both the concepts-first and iterative groups outperformed the control. It also reveals no difference in average conceptual performance between these two intervention groups.

Fraction Procedures Assessment without Modeled Examples

On the fraction procedures assessment without modeled examples, there were also statistically significant differences between groups, F (2, 113) = 28.91, p < .001. We again conducted Šidák post-hoc comparisons to determine which groups differed, as seen in Table 8. The concepts-first (M = 33.18, SD = 9.52) and iterative groups (M = 37.50, SD = 4.48) significantly outperformed the control group (M = 23.37, SD = 9.80), yielding very large effect sizes (Sawilowksy, 2009). Statistically significant differences were not found between the concepts-first and iterative groups (p = 0.07); however, the effect size for the iterative group compared to control (p < 0.001, d = 1.85) was larger than that for the concepts-first group compared to control (p < 0.001, d = 1.02). While there was no statistically significant effect, there appeared to be practically significant ones, with the iterative group answering, on average, 4.32 more questions correct than the concepts-first group and 14.13 more questions correct than the control.

Figure 3 shows the average performance of each instructional group on the fraction procedures assessments without modeled examples. The bar graph shows a clear differentiation between the control and the intervention groups (i.e., concepts-first and iterative). Though it was not statistically significant, iterative students also tended to perform higher than concepts-first students.

Analyses of Covariance

We compared average posttest performance between groups for the five rational number assessments after controlling for the corresponding rational number pretest using analyses of covariance (ANCOVAs); see Table 7. We used Levene's Test of Equality of Variances to determine if the data met the assumption of homogeneity of variance. The three generalization measures and the fraction concepts assessment with examples did not have significant results on this test and therefore met the assumption. The fraction procedures assessment with examples did show a significant result ($p = 0.03$), which violated the assumption. As each group's size was relatively equal, below the 1.5 smallest to largest ratio criterion, the homogeneity of variance assumption should be robust enough to continue with the ANCOVA (Stevens, 1996, p. 249). The fraction concepts and procedures assessments with modeled examples had prominent ceiling effects in both datasets as indicated by histograms, while the generalization assessments (i.e., the NAEP, Super Challenge Grades 3 and 4) showed less skew. Even so, this assumption is relatively robust, and the sample sizes of each group ($n > 30$) should be enough to protect against non-normality (Feir-Walsh & Toothaker, 1974).

The main ANCOVAs reported here have only one covariate in each model, the respective pretest scores for each assessment. In addition to these ANCOVAs, we also ran the same models with additional covariates controlling for students' whole number pretest performance. Due to larger amounts of missing data from the whole number assessments, these ANCOVAs are reported after the ANCOVAs with a single covariate. By adding the whole number assessments into each model as covariates, the number of participants falls from 109 to 98.

Fraction Concepts Assessment with Modeled Examples

A one-way ANCOVA revealed significant differences between groups on the fraction concepts assessment with modeled examples, after controlling for students' pretest performance on the same assessment, F (2, 109) = 3.31, p = .04, η_p^2 = 0.06. Unsurprisingly, pretest performance was also a significant predictor of posttest performance, F (1, 109) = 42.95, p < .001, η_p^2 = 0.29. As shown in Table 8, post-hoc comparisons using the Šidák correction revealed it was the iterative group (M = 36.86, SD = 4.28) that significantly outperformed the control group, with (M = 33.50, SD = 8.45), with a medium effect size, p = .04, d = 0.50 (Sawilowksy, 2009). There was no statistically significant difference between the concepts-first group (M = 36.06, SD = 7.53) and either iterative or control.

Fraction Procedures Assessment with Modeled Examples

A one-way ANCOVA showed significant differences between groups on the fraction procedures assessment with modeled examples after controlling for students' pretest performance on the assessment, F (2, 107) = 4.91, p = .009, η_p^2 = 0.09. Pretest performance was also a significant predictor of posttest performance, F (1, 107) = 53.804, p < .001, η_p^2 = 0.34. The Šidák post-hoc comparisons (Table 8) again revealed the iterative group (M = 37.86, SD = 3.16) significantly outperformed the control group

(M = 32.94, SD = 6.47), with a large effect size, p = 0.007, d = 0.96 (Sawilowksy, 2009). There were no statistically significant differences between the concepts-first group (M = 35.26, SD = 9.71) and iterative or control.

Generalization Assessments

There were no differences between instructional groups on any of the three generalization assessments, the NAEP, Super Challenge Grade 3, and Super Challenge Grade 4. Instead, pretest performance on each assessment was the only significant predictor of posttest scores (NAEP: F (1, 109) = 210.62, $p < 0.001$, $\eta_p^2 = 0.67$; Super Challenge Grade 3: F (1, 109) = 76.61, $p < 0.001$, $\eta_p^2 = 0.42$; Super Challenge Grade 4: F (1, 107) = 128.15, $p < 0.001$, $\eta_p^2 = 0.55$). These results indicate this intervention did not produce generalization of fraction skills to distal assessments.

Though there were no between-group differences observed for any of the three generalization measures, there were significant improvements overall from pre to posttest. Paired samples t-tests of assessment growth demonstrated significant improvement for the NAEP, $t(113) = 10.39$, $p < .001$, Super Challenge Grade 3, $t(113) = 10.08$, $p < .001$, and Super Challenge Grade 4, $t(113) = 7.64$, $p < .001$, assessments. These improvements likely reflect learning from the general education math curriculum, as students completed a fraction unit in their math classes during the intervention phase.

ANCOVAs with Whole Number Assessments

The pretest whole number assessments were included as a covariate into the ANCOVA models to determine if proficiency with prerequisite skills explained variance on posttest outcome measures. However, the rational number assessments were prioritized during data collection, leaving the whole number pretests with more missing cases. The ANCOVAs were run first with just the rational number pretest as a covariate in the models to retain as many participants as possible. The whole number assessments were added in as covariates to determine if whole number proficiency was a significant predictor of posttest rational number performance and see if they changed any of the ANCOVA results. While we administered three whole number assessments at pretest, only two were included as covariates in the ANCOVAs to reduce multicollinearity. The AIMSweb Basic Multiplication Facts and Basic Addition and Subtraction Facts were highly correlated with one another ($r = 0.78$), so only the Multiplication Facts assessment was used to represent whole number fluency. The AIMSweb MCAP was also incorporated within the models to assess broader types of problem-solving. Adding in whole number assessments as a covariate did not change whether the instructional group or rational number pretest was significant in any of the models.

Proximal Assessments. In addition to the significant instructional group and rational number pretest performance, the MCAP assessment also significantly predicted posttest performance on the fraction concepts assessments with modeled examples, $F\ (1, 97) = 7.19$, $p = 0.009$. The AIMSweb Basic Multiplication Facts did not significantly account for additional variance on the conceptual fraction assessment. On the fraction procedures assessment with modeled examples, neither the MCAP nor the AIMSweb Basic Multiplication Facts assessment significantly accounted for additional variance. The instructional group and rational number pretest remained significant.

Generalization Assessments. Both the MCAP, $F\ (1, 98) = 16.40$, $p < 0.001$, and the AIMSweb Basic Multiplication Facts, $F\ (1, 98) = 4.38$, $p = 0.039$, were significant predictors of posttest NAEP performance. The pretest NAEP assessment remained significant while the instructional group remained non-significant. The MCAP, $F(1, 97) = 27.52$, $p < 0.001$, and the Multiplication Facts assessment, $F(1, 97) = 5.66$, $p = 0.019$, also accounted for significant variance on the Super Challenge Grade 3 assessment. Finally, the MCAP, $F(1, 97) = 16.06$, $p < 0.001$, and the Multiplication Facts assessment, $F(1, 97) = 4.03$, $p = 0.048$, significantly predicted the last generalization assessment, Super Challenge Grade 4. On both Super Challenge probes, pretest performance stayed significant, and the instructional group remained non-significant.

Overall, whole number performance at pretest significantly predicted performance on all three generalization measures above the variance explained by the rational number pretests. In contrast, only the MCAP significantly predicted performance on the fraction concepts assessment. Pretest whole number performance did not predict posttest fraction procedure performance beyond the variance already explained by the fraction procedure pretest and instructional group.

Discussion

Math educators and researchers have debated the role of and optimal order for conceptual and procedural instruction. Yet, we are aware of no prior studies that have examined conceptual and procedural instructional sequences for fractions. This study compared two instruction sequences, concepts-first (i.e., all conceptual instruction precedes procedural instruction) and iterative (i.e., conceptual and procedural instruction is alternated), with a control group to evaluate each sequence's relative effectiveness during a classwide fraction intervention. Outcomes were evaluated across four proximal measures intended to assess both conceptual and procedural fraction knowledge. Three generalization fraction outcome measures were also evaluated to determine skill transfer.

The primary findings suggested that the iterative instruction sequence resulted in better student learning outcomes, consistent with hypotheses. Outcomes did not transfer to more general and comprehensive fraction measures for any instructional group. As expected, pretest whole number performance significantly predicted posttest performance on most rational number outcome measures. The fraction procedure assessment was the only rational number measure for which whole number assessments did not account for additional student variance beyond the respective rational number pretest and instructional group.

A secondary research question examined whether the researcher-developed measures included in the study measured multiple aspects of rational number knowledge (such as separate fraction concepts and procedures), or a single component, likely general rational number knowledge. These analyses suggested that only a single dimension of rational number knowledge was measured at pretest, but it was unique from whole number fluency. An additional component was extracted during posttest, indicating that the assessment functioning changed after students experienced fraction instruction. At posttest, proximal fraction procedure assessments measured something unique from generalization probes. Additionally, different component loadings from the fraction concepts and procedures assessments provided preliminary empirical evidence that it may be possible to measure aspects of fraction concepts and procedures separately.

Instructional Sequences for Fractions

The primary outcome measures for this study were the proximal fraction assessments without modeled examples. This tool most closely aligned with the curriculum and subsequently is the most sensitive measure of instructional effect. The proximal assessments without modeled examples had less prominent ceiling effects than the ones with modeled examples because they required students to discriminate which fraction concepts or procedures to apply to solve each fraction problem.

Both the concepts-first and iterative groups significantly outperformed the control group, with effect sizes falling in the medium ($d = 0.69$ to 0.76) range, respectively, on the fraction concepts assessment without examples. There were no practical or statistical differences between instructional groups. This result demonstrates that the intervention lessons were effective at improving students' conceptual knowledge. It also shows that the order of conceptual and procedural instruction, either massing all conceptual lessons before procedural ones or interleaving concepts and procedures within the same lesson, may not make a difference in how well students learn fraction concepts. This finding converges with a past research study, which found concepts-first and iterative groups performed similarly on conceptual outcome measures (Rittle-Johnson & Koedinger, 2009).

On fraction procedure outcomes, however, students in the iterative instructional group tended to perform better. Like the fraction concepts assessment, both instructional groups (i.e., concepts-first and iterative) significantly outperformed the control group on the fraction procedures assessment. On average, students in the iterative group correctly answered 4.32 more questions (10.80% of the procedural assessment items) than students in the concepts-first group. The difference between instructional groups did not meet the threshold for statistical significance ($p = .074$). Still, the iterative group ($d = 1.85$) showed a larger effect size compared to control than the concepts-first group ($d = 1.02$) did. These results were likely constrained by the assessment's ceiling effect and the study's relatively small sample size. With a larger sample size to increase power and a more difficult assessment to capture the full range of students' abilities, future studies may find statistically significant differences between these instructional groups. Again, this result replicated a finding from Rittle-Johnson and Koedinger's (2009) study, where the iterative group showed better procedural performance than the concepts-first group.

The proximal assessments with modeled examples also showed between-group differences. After controlling for pretest performance as a covariate in the model, students in the iterative group significantly outperformed students in the control on both the conceptual and procedural fraction assessments with examples. There were no statistically significant differences between iterative and concepts-first students or concepts-first and control students. The ceiling effects on these assessments likely restricted the measurement of student variability, even though improved performance favored the iterative instruction sequence.

Overall, the iterative group performed similarly to or better than the concepts-first group on the proximal fraction assessments. This study extends findings from a previous study evaluating a decimal intervention, as it replicated the overall results with a new math content domain, fractions (Rittle-Johnson & Koedinger, 2009). The iterative sequence improved students' procedural learning and resulted in comparable conceptual learning to the concepts-first sequence. Thus, the calls from CCSSM and NCTM for math concepts to be taught before procedures appear to be inconsistent with existing data, and the assertion that "conceptual understanding…establishes the foundation, and is necessary, for developing procedural fluency" may be misleading (NCTM, 2014, p. 7).

Reasons for Improved Student Outcomes in the Iterative Sequence

Cognitive and learning theories may explain why the iterative instruction sequence resulted in improved student outcomes, most reliably in improved procedural student outcomes. The spacing effect is one such explanation. The spacing effect is a well-documented phenomenon in which distributed practice over time improves learning more than massed practice (Dempster, 1988). This finding has been replicated across multiple content domains (Dempster, 1988) and demonstrated by researchers of different disciplines (Varma & Schleisman, 2014). For the iterative group, conceptual and procedural instruction was interleaved within each lesson, so students learned and practiced the concepts and procedures across more sessions. Conversely, the first half of the concepts-first group lessons were devoted entirely to concepts and the second half solely to procedures. Thus, the iterative instruction sequence capitalized on the spacing effect. Though all students had the same cumulative instruction time, iterative students' practice was more distributed.

While the spacing effect could explain the improved procedural outcomes, it remains unclear why we did not observe a similar improvement in conceptual outcomes. The proximal fraction concepts assessment without examples revealed equivalent performance between the iterative and concepts-first groups. This finding replicated Rittle-Johnson and Koedinger's (2009) study, where they found comparable conceptual performance between groups. However, the proximal fraction concept assessments with modeled examples showed only students in the iterative group outperformed the control, suggesting the iterative instruction sequence may yield improved conceptual understanding. Future studies may aim to examine the validity evidence of conceptual assessments, as perhaps we did not comprehensively measure conceptual knowledge.

Alternatively, procedural knowledge may simply be more responsive to the effects of spaced practice than conceptual knowledge.

Another possible explanation for the improved performance from the iterative sequence is that interleaving conceptual and procedural instruction results in a stronger link between math concepts and procedures. Baroody and colleagues (2007) argued that "linking procedural to conceptual knowledge can make learning facts and procedures easier, provide computational shortcuts, ensure fewer errors, and reduce forgetting (i.e., promote efficiency)" (p. 127). By introducing procedures alongside concepts, the iterative instruction sequence may be promoting a stronger connection between and deeper learning of math concepts and procedures (Star, 2005).

Promoting rich, connected knowledge (Star, 2005) is the ultimate goal of math instruction. However, this link, or connectedness, between conceptual and procedural knowledge may be more salient in some math domains than others. LeFevre et al. (2006) assessed conceptual and procedural growth trajectories of counting in early elementary grades (kindergarten through second grade), finding the growth curves of conceptual and procedural knowledge to be markedly dissimilar. The authors concluded that, for some types of math knowledge, the interrelationship between concepts and procedures might not be as straightforward as some bidirectional studies (e.g., Rittle-Johnson et al., 2001) have implied. While the connection between fraction concepts and procedures appeared important for the current study's targeted skills (as evidenced by improved outcomes in the iterative group), further research may be warranted to examine the developmental trajectories of procedural and conceptual knowledge. There may be variation in how concepts and procedures develop for different math skills, and consequently, the importance of the link between concepts and procedures may also be content-dependent. However, this current study provides evidence for the theory of an iterative development of concepts and procedures.

Generalization Outcomes

There were no differences between groups on any of the three generalization probes used at the pre and posttest, meaning the students did not transfer skills to novel problems on distal measures. The Super Challenge probes were considered more generalized measures in this study, as they assessed multiple skills within each assessment (Fuchs et al., 2020). Generalized assessments are typically used to measure longer-term skill growth connected to broader curriculum goals (Fuchs & Deno, 1991). They are less sensitive to short-term change (Ysseldyke et al., 2010), so it is unsurprising that this relatively brief (12, 20-min lessons) and targeted intervention focusing exclusively on four fraction skills (i.e., writing standard notation from math models, comparing fraction magnitudes to a benchmark whole number, adding and subtracting fractions with like denominators, and multiplying fractions) did not affect students' generalization scores. Generalization assessments are critical for a holistic measurement of skill trajectories, but the proximal assessments are likely a more accurate indication of specific skill growth (Fuchs & Deno, 1991).

Whole Number Knowledge

As math knowledge is often cumulative, we wanted to control for students' whole number skills when evaluating the rational number outcome measures. As predicted (Namkung et al., 2018), whole number performance at pretest did significantly account for student variance on most rational number posttests, even beyond what was captured by the corresponding rational number pretest and instructional group. The only outcome measure that did not show whole number performance to be a statistically significant covariate was the fraction procedures assessment with modeled examples. The fraction procedures assessment had the largest ceiling effect out of all of the posttests, which helps explain why whole number performance did not account for additional student variability beyond the instructional group and the procedural pretest. Likely, whole number knowledge is still related to fraction procedures, and the lower amount of variability in the posttest is obscuring their real relationship.

Ceiling Effects

The proximal assessments, constructed from items in the intervention curriculum, all showed varying degrees of ceiling effects. The ceiling effects are likely a consequence of using specific subskill mastery measurement (SSMM) probes to gauge student growth. By their nature, SSMMs are highly sensitive to student skill growth (Ysseldyke et al., 2010). They are closely connected to the instruction, thereby providing strong instructional utility (Fuchs & Deno, 1991; Yalow & Popham, 1983). As this academic intervention was relatively brief and short-term, SSMM probes were the most appropriate assessments to monitor specific fraction skill changes. Traditionally, teachers use a relatively high mastery benchmark on SSMM probes before moving on to more challenging content (Fuchs & Deno, 1991).

The modeled examples at pretest also inflated student scores. Students appeared to inductively reason the relevant concepts and procedures to complete the items, even when they had not yet received the fraction instruction. We added the same assessments without modeled examples at the posttest to reduce the ceiling effect, and these assessments were our primary outcome measure. The posttest PCA helped confirm that the proximal assessments with and without examples loaded similarly onto components, suggesting we only changed the assessment's difficulty when we removed the examples rather than the underlying measured dimension.

Another possible explanation for the presence of ceiling effects in the data is the power of Direct Instruction. Direct Instruction programs consistently have large effect sizes on student learning (Stockard et al., 2018), even for students with learning difficulties or disabilities (Ennis & Losinski, 2019). The combination of the powerful instruction program and the assessments' sensitivity led to overall high student scores on the proximal fraction assessments. While high assessment scores show strong student mastery of the material, the ceiling effects in the data make it harder to see between-group differences, as they truncate student variability. Despite these prominent ceiling effects, we still observed significant differences between groups on all proximal fraction assessments.

Implications for Conceptual and Procedural Fraction Assessment

While this study's primary focus was on the effectiveness of the math intervention, the functioning of the pre and posttest assessments was also of interest. The exploratory PCAs revealed preliminary empirical evidence for the number of components measured by the pre and posttest assessments. At pretest, the rational and whole number assessments loaded onto separate components, indicating that the rational number assessments captured student variance unique from whole number fluency probes. However, the proximal and generalization fraction assessments loaded onto a single component. Before students received the fraction intervention, the proximal and generalization assessments measured one underlying dimension of student performance, which was likely general fraction knowledge.

The number of components measured from the assessments changed from pre to posttest, as there were two unique dimensions measured in the posttest assessments. At posttest, the generalization assessments were distinct from the fraction procedures assessment, while the fraction concepts assessment loaded onto both components. This finding provided preliminary empirical evidence that aspects of procedural and conceptual knowledge may be measured separately in assessments. The first conceptual skill, *writing standard notation from math models*, clustered more strongly with the generalization assessments on the second component. The second conceptual skill, *magnitude comparison*, grouped more strongly with the procedural skills on the first component. This result shows the heterogeneity of how researchers define and assess conceptual knowledge (Rittle-Johnson et al., 2015). It should be emphasized that the fraction concepts and procedures assessments were not measuring completely separate dimensions at the posttest, as there was some overlap in the variance accounted for by each assessment.

Theories of mathematical concepts and procedures indicate that it may be difficult, if not impossible, to isolate the measurement of math concepts and procedures completely (Baroody et al., 2007; Star, 2005). There is causal evidence that math concepts and procedures are interdependent and have a bidirectional relationship (Canobi, 2009; Rittle-Johnson et al., 2015). The intertwining of concepts and procedures likely intensifies as students reach more in-depth and advanced math knowledge (Kilpatrick et al., 2001). Baroody et al. (2007) argue that deep conceptual knowledge necessitates deep procedural knowledge and vice versa. While we have strong theoretical content validity arguments for the conceptual and procedural assessments, future assessment studies may be warranted to better understand how conceptual and procedural assessments are functioning empirically. Conceptual understanding may be especially difficult to measure in implicit tasks (LeFerve et al., 2006), particularly when there is not yet a consensus about how to define it (Crooks & Alibali, 2014).

Limitations and Future Research

There are both clear limitations of this study and avenues for future research. The ceiling effects observed in the researcher-created proximal assessments serve as a type of measurement error. Ideally, future studies would utilize assessments that capture the full range of student abilities, by either incorporating items of varying difficulties or by shortening the test time limits. Although progress monitoring was not feasible for this study, future studies should consider including ongoing monitoring if resources allow. Frequent progress-monitoring will alert researchers or educators to a ceiling or floor effect by showing changes (or lack thereof) in student performance so that an intervention's lesson content or assessment difficulty can be adjusted accordingly. If students reach high benchmarks in progress monitoring of SSMMs, future researchers might consider limiting the test time so fluency metrics can differentiate skill levels. This study's ceiling effects truncated the upper range of student variability, so the proximal assessments are likely underestimating group differences. Even so, we still saw significant effects in favor of an iterative instruction sequence.

Another notable limitation of this study was that outcomes did not result in the generalization of student skills. Though the Direct Instruction intervention program showed substantial effects on the proximal assessments, it was not powerful enough to affect students' abilities to transfer the knowledge to novel problems. Students did grow significantly on all generalization measures from pre to posttest, which is likely due to the fraction instruction they received simultaneously from the core math curriculum. The generalization measures more closely aligned with broad goals from the fourth grade *GO Math!* fraction curriculum, making it unsurprising that they did not capture differences between the intervention groups. Future studies may use transfer assessments that include novel math problems, but still test the intervention's targeted skills (e.g., testing students' ability to transfer fraction addition skills to fraction addition word problems).

Finally, we could not compare all instruction sequences (i.e., procedures-first instruction) in this study because of limited resources. Future research studies may evaluate the effect of a procedures-first sequence compared to iterative and concepts-first sequences. It would also be valuable to administer a midpoint assessment after comparing concepts-first and procedures-first sequences. Researchers could then evaluate the differences between concepts-only and procedures-only instruction in the context of a longer academic intervention. Previous research studies have typically compared concepts-only and procedures-only instruction with relatively short instructional time (e.g., Perry, 1991; Rittle-Johnson et al., 2016). Additionally, different instruction orders may be optimal for different math domains (e.g., early numeracy skills may respond better to a different instructional sequence than fraction skills). As relatively few instruction sequencing studies exist (Rittle-Johnson et al., 2015), there are several productive avenues for future research in this area.

Conclusion

Overall, this study provided support for introducing fraction procedures early within the instructional sequence, alongside fraction concepts, rather than massing instruction about concepts before procedures. The iterative instruction sequence resulted in equivalent or improved student outcomes on proximal assessments compared to a concepts-first sequence, which contradicts the recommendation from some math education experts (e.g., NCTM, 2014) to provide a conceptual understanding of math before introducing procedures. This study replicated the general finding of another experimental study that showed an iterative sequence might be the optimal instructional order (Rittle-Johnson & Koedinger, 2009), thereby extending previous research by testing instruction orders with a new math domain, fractions. These findings support the iterative theory of conceptual and procedural math development, providing some empirical support for interleaving conceptual and procedural math instruction.

CHAPTER 4

Synthesis and General Discussion

Proficiency with fractions is a critical milestone for students to reach. Fraction performance predicts proportional reasoning, high school algebra readiness, and general math achievement (Booth & Newton, 2012; DeWolf et al., 2015, 2016). Fraction assessment scores even predict later math performance after controlling for other predictive factors, such as general intelligence, socio-economic status, and prerequisite math skills (i.e., whole number performance; Booth et al., 2014; Siegler et al., 2012). Fraction competence is also essential for entering and succeeding in the workplace (Geary et al., 2012), as approximately two-thirds of Americans report using rational numbers in their work (Handel, 2016). Because fraction skills are highly predictive of, and likely foundational for, later math outcomes, the National Mathematics Advisory Panel (NMAP, 2008) placed a high priority on improving the fraction performance of United States students.

A considerable number of United States students fail to meet fraction proficiency standards despite the extensive time devoted to fraction instruction across the upper elementary and middle school grades (NGACBP, 2010). For example, on the 2013 NAEP test, 65% of all eighth-grade students did not know that $\frac{1}{2} + \frac{3}{8} + \frac{3}{8}$ was larger than one (Malone & Fuchs, 2016). On the 2017 NAEP test, 68% of all fourth-grade students were unable to correctly categorize six fractions, $\frac{1}{3}, \frac{2}{3}, \frac{2}{6}, \frac{4}{6}, \frac{2}{8}, \frac{4}{8}$, to be less than, equal to, or greater than one-half (National Center for Education Statistics, 2017). For both elementary and middle school students, fraction proficiency is low. These percentages surpass the prevalence rate of a mathematics learning disability, which ranges from approximately 6 to 14% (Barbaresi et al., 2005), and suggest that the presence of learning disabilities alone cannot explain the overall low rational number performance of United States students (Mazzocco & Devlin, 2008). Instead, researchers should critically evaluate the overall effectiveness of our fraction instruction.

The two studies in this dissertation aimed to address this need for improved fraction instruction by (a) reviewing the use of evidence-based instructional principles in tier-one fraction lessons and (b) testing two instruction sequences during a classwide fraction intervention. Study 1 examined required state textbook adoption lists to determine the four most commonly recommended math curricula from state departments of education. Third, fourth, and fifth-grade fraction lessons of these curricula were then evaluated for their use of evidence-based instructional principles using a modified instructional principle rubric (Doabler et al., 2012, 2018). Study 2 tested two sequences of conceptual and procedural instruction (i.e., concepts-first and iterative) against a control group to determine if instruction sequence affected student learning and transfer of two fraction concepts and two fraction procedures. The two targeted fraction concepts were *writing standard notation from math models* and *comparing fraction magnitudes to a benchmark whole number*. The two targeted fraction procedures were *fraction addition/subtraction with like denominators* and *fraction multiplication*. The findings from these two studies replicated aspects of previous textbook analyses and instruction sequencing studies.

General Findings from Study 1

Fraction lessons from four math curricula, *Everyday Math*, *Math Expressions*, *GO Math!*, and *EnVision Math,* were evaluated in Study 1 for their use of evidence-based instructional principles. Of the instructional principles, the curricula scored highest in practice opportunities and cumulative review. All textbooks provided a sufficient number of written and verbal practice opportunities for students. However, a limitation of this review is that the rubric may not have captured the quality of these practice opportunities (e.g., problem discrimination practice, transfer practice). Another notable strength of all the curricula was prerequisite skills. All of the textbooks included lesson warmup activities, often incorporating whole number fluency practice, which has shown to be a prerequisite for rational number knowledge (Jordan et al., 2017). Some textbooks also included foundational lessons at the beginning of fraction chapters so that teachers could address gaps in student knowledge before moving on to the fraction instruction.

Instructional examples of fraction concepts were also well embedded in most curricula, with only the Grade 5 *Math Expressions* lessons just missing the acceptability criterion. In contrast, fewer programs met the acceptability standard for instructional examples of fraction procedures. The addition of more fraction procedure modeling in tier-one may improve students' procedural understanding and performance.

While there were some notable strengths of the examined textbooks, Study 1 also revealed deficiencies of the math curricula. None of the textbooks at any grade level incorporated an acceptable level of math vocabulary. Many programs only included vocabulary at the beginning of chapters rather than embedding the vocabulary throughout individual lessons. Academic feedback and formative feedback loops were also inconsistently employed both within and between curricula.

The inadequate amount of explicit instruction was the most noticeable weakness of the math curricula. Two of the included programs, *GO Math!* and *EnVision Math*, resembled a highly guided discovery learning framework, meaning the lesson plans did not contain proactive, explicit instruction. *Everyday Math* Grade 3 lessons were the only ones to reach the acceptability criterion for explicit instruction of fraction concepts in the instructional principle rubric. No other program or grade level met the standard. Furthermore, none of the included programs met the threshold for an adequate amount of explicit instruction of fraction procedures.

The main finding from Study 1 was that textbooks lacked adequate levels of explicit fraction instruction. This finding replicated past textbook analyses (e.g., Bryant et al., 2008; Doabler et al., 2012; Sood & Jitendra, 2007), which similarly found math curricula to have insufficient levels of explicit or conspicuous instruction. There is a substantial amount of empirical evidence that supports the use of explicit instruction, especially for students considered to be at-risk for low performance in math (Stockard et al., 2018). For low fraction performance in the United States to be addressed, textbooks should provide more opportunities for explicit, systematic instruction on fraction concepts and procedures.

General Findings from Study 2

In Study 2, we conducted a classwide fraction intervention using the most rigorous type of explicit instruction, Direct Instruction. We constructed lessons from a Direct Instruction fraction program, *Corrective Mathematics, Basic Fractions*, to follow a concepts-first and iterative instruction sequence for conceptual and procedural knowledge (Engelmann & Steely, 2005). The concepts-first (i.e., conceptual instruction precedes procedural instruction) sequence was selected because the NCTM (2014) explicitly recommends math educators instruct with a concepts-first approach. However, a previous study showed that the iterative (i.e., conceptual and procedural instruction are interleaved in alternating order) sequence led to better procedural outcomes than the concepts-first sequence (Rittle-Johnson & Koedinger, 2009). Thus, concepts-first and iterative sequences were compared in Study 2. The lesson content in Study 2 was the same for the concepts-first and iterative intervention groups; only the order of the content differed. These two groups were compared to a control group to determine the fraction intervention's overall efficacy.

On the primary outcome measures that most closely aligned with the fraction intervention curriculum (the proximal fraction assessments without modeled examples), both intervention groups outperformed the control group, indicating the Direct Instruction lessons were effective at improving conceptual and procedural fraction skills. The concepts-first and iterative groups performed similarly on the conceptual assessment. On the procedural assessment, the iterative group demonstrated a larger effect size compared to control than the concepts-first group. While the difference between the concepts-first and iterative groups on the procedural assessment just missed statistical significance, it appeared to be practically significant. The iterative group correctly answered 10% more assessment items than the concepts-first group. These findings replicated the general results from Rittle-Johnson and Koedinger's (2009) study, which found comparable conceptual scores for concepts-first and iterative groups and improved procedural scores for the iterative group during a decimal intervention. Further research is warranted to determine if this effect holds for other math domains.

On the other proximal outcome measures, the fraction assessments with modeled examples, the iterative group again performed best. The iterative group significantly outperformed control for both the conceptual and procedural assessments with examples. There were no other significant differences between groups. These effects are influenced by the assessments' ceiling effects, likely underestimating the between-group differences. Despite these ceiling effects, significant differences favoring the iterative sequence were observed. This suggests that the current recommendation to provide math instruction in a concepts-first sequence (NCTM, 2014) is inconsistent with existing empirical data (Rittle-Johnson & Koedinger, 2009).

The pre and posttest assessments used in Study 2 were examined using Principal Component Analyses (PCAs), which helped determine the number of unique dimensions measured by each assessment set. Only one component was extracted in the rational number pretest, meaning the pretests were likely measuring one underlying skill of fraction knowledge, as opposed to separate conceptual and procedural skills. However, this rational number pretest component was separate from whole number fluency probes. The posttest PCA extracted a second component, showing that the proximal fraction procedures assessments measured a separate dimension from the generalization probes. Both extracted components captured variance from the proximal fraction concepts assessments. Future assessment studies may aim to separate the measurement of conceptual and procedural skills. The last finding was that Study 2 did not result in the generalization or transfer of fraction skills for any instructional group.

Overall, Study 2 provided evidence for using an iterative instruction sequence during fraction instruction. The iterative group performed similarly to (conceptual outcomes) or better than (procedural outcomes) the concepts-first group on our primary outcome measures. This study replicated the principal finding from Rittle-Johnson &

Koedinger (2009) and extended the literature base by testing the sequences with a new math domain, fractions.

Practice and Research Implications

The outcomes of this dissertation reveal clear implications for future research and practice. In Study 1, we found textbooks lacked appropriate explicit instruction. Ultimately, textbook publishers should incorporate higher levels of explicit instruction in teacher lesson plans to make fraction skills accessible for all students, especially students at-risk for low math performance. Math educators may also choose to supplement their curriculum with explicit instruction of fraction concepts and procedures. Doing so will allow more students to access the lessons.

There were also opportunities for other instructional principles to be incorporated more fully into the fraction lessons. Doabler et al. (2012, 2018) designed their instructional principle rubric to be used by teachers and school administrators during curricula evaluation. Teachers or other school staff may be inclined to use the rubric to check the instructional quality of their math curricula and ensure evidence-based instructional strategies are adequately employed. Study 1 replicated previous findings that evidence-based instructional principles are variable both within and across math curricula (Doabler et al., 2012; Sood & Jitendra, 2007).

Study 2 demonstrated that the iterative instruction sequence results in similar or improved student outcomes compared to a concepts-first approach when using a Direct Instruction fraction curriculum. These data suggest that introducing procedures alongside concepts may be the optimal way of sequencing fraction instruction. Though more research is needed to determine if this effect replicates with other math domains, math educators and researchers may consider sequencing instruction iteratively to space procedural and conceptual instruction. Preliminary analyses of the fraction measures suggest that the study assessed aspects of procedural and conceptual fraction knowledge, unique from whole number fluency probes. Future researchers may also examine the psychometric properties of conceptual and procedural assessments to determine if they capture separate dimensions or a single underlying construct.

Conclusion

This dissertation aimed to address the need for more effective fraction instruction in tier-one materials. The first study reviewed commonly recommended math curricula in the United States for its adherence to research-based instructional strategies. Several strengths and limitations of the core curricula were reviewed, but the overall finding was that there was an inadequate amount of explicit instruction in the fraction lessons. To improve students' fraction knowledge, we should provide our teachers with effective instructional strategies within curricula. As none of the reviewed math curricula met the standards for an adequate amount of explicit instruction for concepts and procedures, we instead chose a Direct Instruction program to intervene on fraction concepts and procedures in this dissertation's second study. Results from the second study revealed that the order of fraction instruction might affect student learning. Specifically, an iterative instruction sequence outperformed concepts-first and control groups, as it resulted in improved procedural outcomes for participating students. Employing evidence-based instructional strategies and sequences in fraction lessons will help advance students' fraction knowledge and allow more students, even those at-risk for low math performance or with math-specific learning disabilities, to engage with the math curricula.

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Table 1

Average Grade 3 Scores for Instructional Principle Scoring Rubric

Instructional Principle	Everyday Math	Math Expressions	GO Math!
Prerequisite Skills	**4.00**	**4.00**	**3.81**
Math Vocabulary	**3.50**	2.89	2.63
Explicit Instruction of Fraction Concepts	**3.20**	2.44	1.00
Explicit Instruction of Fraction Procedures	2.05	1.33	1.00
Instructional Examples of Fraction Concepts	**3.60**	**3.33**	**3.81**
Instructional Examples of Fraction Procedures	2.50	1.33	1.25
Measurement Math Models	2.25	2.78	2.38
Part-Whole Math Models	**3.00**	1.78	**3.69**
Practice Opportunities and Cumulative Review	**3.95**	**3.56**	**4.00**
Academic Feedback	**3.25**	**3.00**	**3.81**
Formative Feedback Loops	2.00	2.00	**4.00**

Note. The acceptability criterion for all instructional principles is 3. Average scores at 3 or above are in bold.

Table 2

Average Grade 4 Scores for Instructional Principle Scoring Rubric

Instructional Principle	Everyday Math	Math Expressions	EnVision Math
Prerequisite Skills	**4.00**	**4.00**	**3.00**
Math Vocabulary	2.73	1.83	1.60
Explicit Instruction of Fraction Concepts	2.59	2.65	1.00
Explicit Instruction of Fraction Procedures	2.56	2.39	1.00
Instructional Examples of Fraction Concepts	**3.10**	**3.26**	**3.33**
Instructional Examples of Fraction Procedures	**3.12**	2.70	2.80
Measurement Math Models	2.51	**3.00**	**3.10**
Part-Whole Math Models	2.95	1.65	1.80
Practice Opportunities and Cumulative Review	**3.98**	**3.78**	**4.00**
Academic Feedback	2.85	2.70	2.97
Formative Feedback Loops	2.15	2.00	**4.00**

Note. The acceptability criterion for all instructional principles is 3. Average scores at 3 or above are in bold.

Table 3

Average Grade 5 Scores for Instructional Principle Scoring Rubric

Instructional Principle	EnVision Math	Math Expressions	GO Math!
Prerequisite Skills	**3.00**	**3.93**	**4.00**
Math Vocabulary	1.41	2.78	2.16
Explicit Instruction of Fraction Concepts	1.00	2.40	1.00
Explicit Instruction of Fraction Procedures	1.00	2.38	1.00
Instructional Examples of Fraction Concepts	**3.34**	2.88	**3.24**
Instructional Examples of Fraction Procedures	2.86	**3.28**	**3.20**
Measurement Math Models	**3.07**	2.48	2.08
Part-Whole Math Models	2.24	1.68	2.56
Practice Opportunities and Cumulative Review	**4.00**	**3.75**	**4.00**
Academic Feedback	**3.07**	2.70	**3.80**
Formative Feedback Loops	**4.00**	2.00	**4.00**

Note. The acceptability criterion for all instructional principles is 3. Average scores at 3 or above are in bold.

Table 4

Adherence to Common Core State Standards of Mathematics (CCSSM)

CCSSM	Everyday Math	Math Expressions	EnVision Math	GO Math!
Grade 3 Total Lessons	20	9	-	16
3.NF.A.1	13	3	-	7
3.NF.A.2	7	4	-	1
3.NF.A.3	13	6	-	8
Grade 4 Total Lessons	41	23	30	-
4.NF.A.1	10	4	7	-
4.NF.B.2	11	8	7	-
4.NF.A.3	24	7	17	-
4.NF.B.4	16	4	17	-
4.NF.C.5	4	1	6	-
4.NF.C.6	10	5	6	-
4.NF.C.7	5	3	6	-
Grade 5 Total Lessons	-	40	29	25
5.NF.A.1	-	15	11	7
5.NF.A.2	-	18	12	4
5.NF.B.3	-	5	2	1
5.NF.B.4	-	13	6	7
5.NF.B.5	-	10	3	3
5.NF.B.6	-	17	4	2
5.NF.B.7	-	5	5	4

Note. This table displays the number of lessons from each curriculum devoted to each fraction-specific Common Core State Standard.

The empty cells indicate that the grade level for that curriculum was not reviewed in this study.

Table 5

Pretest PCA Factor Loadings

Assessment Type	Measure	Pretest Rational Number	Pretest Rational & Whole Number	
		Component 1	Component 1	Component 2
Whole Number	AIMSweb MCAP	--	**.63**	**.50**
	AIMSweb Mult	--	.27	**.88**
	AIMSweb Add/Sub	--	.21	**.90**
Proximal	Concepts Ex Standard Notation	**.61**	**.56**	.21
	Concepts Ex Magnitude Comparison	**.62**	**.54**	.23
	Procedures Ex Add/Sub	**.77**	**.69**	.33
	Procedures Ex Mult	**.63**	.41	**.62**
Generalization	NAEP	**.89**	**.85**	.33
	Super Challenge Grade 3	**.87**	**.85**	.21
	Super Challenge Grade 4	**.88**	**.89**	.20

Note. $N = 108$ for the Pretest Rational Number PCA and $N = 97$ for the Pretest Rational & Whole Number PCA. The extraction method was a principal component analysis with orthogonal (Varimax with Kaiser normalization) rotation. Factor loadings at or above .50 are in bold.

Table 6

Posttest PCA Factor Loadings

Assessment Type	Measure	Posttest Rational Number	
		Component 1	Component 2
Proximal	Concepts Ex Standard Notation	.41	**.55**
	Concepts Ex Magnitude Comparison	**.59**	.49
	Concepts Standard Notation	.43	**.53**
	Concepts Magnitude Comparison	**.67**	.49
Proximal	Procedures Ex Add/Sub	**.71**	.36
	Procedures Ex Mult	**.87**	.17
	Procedures Add/Sub	**.77**	.39
	Procedures Mult	**.86**	.03
Generalization	NAEP	.19	**.89**
	Super Challenge Grade 3	.15	**.90**
	Super Challenge Grade 4	.24	**.90**

Note. N = 107 for the Posttest Rational Number PCA. The extraction method was a principal component analysis with orthogonal (Varimax with Kaiser normalization) rotation. Factor loadings at or above .50 are in bold.

Table 7

Posttest Means and Standard Deviations by Instructional Group

Assessment Type	Measure	Concepts-First			Iterative			Control		
		n	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*
Proximal	Concepts	38	33.13	9.63	38	33.42	8.32	38	26.08	10.76
	Procedures	38	33.18	9.52	38	37.50	4.48	38	23.37	9.80
	Concepts Ex	36	36.06	7.53	37	36.86	4.28	36	33.50	8.45
	Procedures Ex	35	35.26	9.71	37	37.86	3.16	35	32.94	6.47
Generalization	NAEP	36	10.81	5.75	37	10.74	5.21	36	11.43	5.47
	Super Challenge Grade 3	36	11.81	5.47	37	13.32	4.49	36	12.69	4.41
	Super Challenge Grade 4	35	11.74	5.21	36	12.58	4.87	36	12.25	5.06

Note. The Concepts and Procedures assessments refer to the proximal fraction assessments without modeled examples delivered at posttest. The Concepts Ex and Procedures Ex assessments refer to the proximal fraction assessments with modeled examples delivered at pre and posttest.

Table 8

Planned Post-Hoc Comparisons of Significant One-Way ANOVA and ANCOVA Statistics

					Šidák Comparisons *p*-value	
Proximal Measure	Group	*n*	Mean	*SD*	Concepts-First	Iterative
Concepts	Concepts-First	38	33.13	9.63	--	--
	Iterative	38	33.42	8.32	.999	--
	Control	38	26.08	10.76	.005*	.004*
Procedures	Concepts-First	38	33.18	9.52	--	--
	Iterative	38	37.50	4.48	.074	--
	Control	38	23.37	9.80	<.001*	<.001*
Concepts Ex	Concepts-First	36	36.06	7.53	--	--
	Iterative	37	36.86	4.28	.790	--
	Control	36	33.50	8.45	.261	.038*
Procedures Ex	Concepts-First	35	35.26	9.71	--	--
	Iterative	37	37.86	3.16	.432	--
	Control	35	32.94	6.47	.236	.007*

Note. The Concepts and Procedures assessments refer to the proximal fraction assessments without modeled examples delivered at posttest. The Concepts Ex and Procedures Ex assessments refer to the proximal fraction assessments with modeled examples delivered at pre and posttest. Values denoted with * indicate statistical significance at $p < .05$.

Figure 1

Inclusion of Math Curricula in Study 1 Review

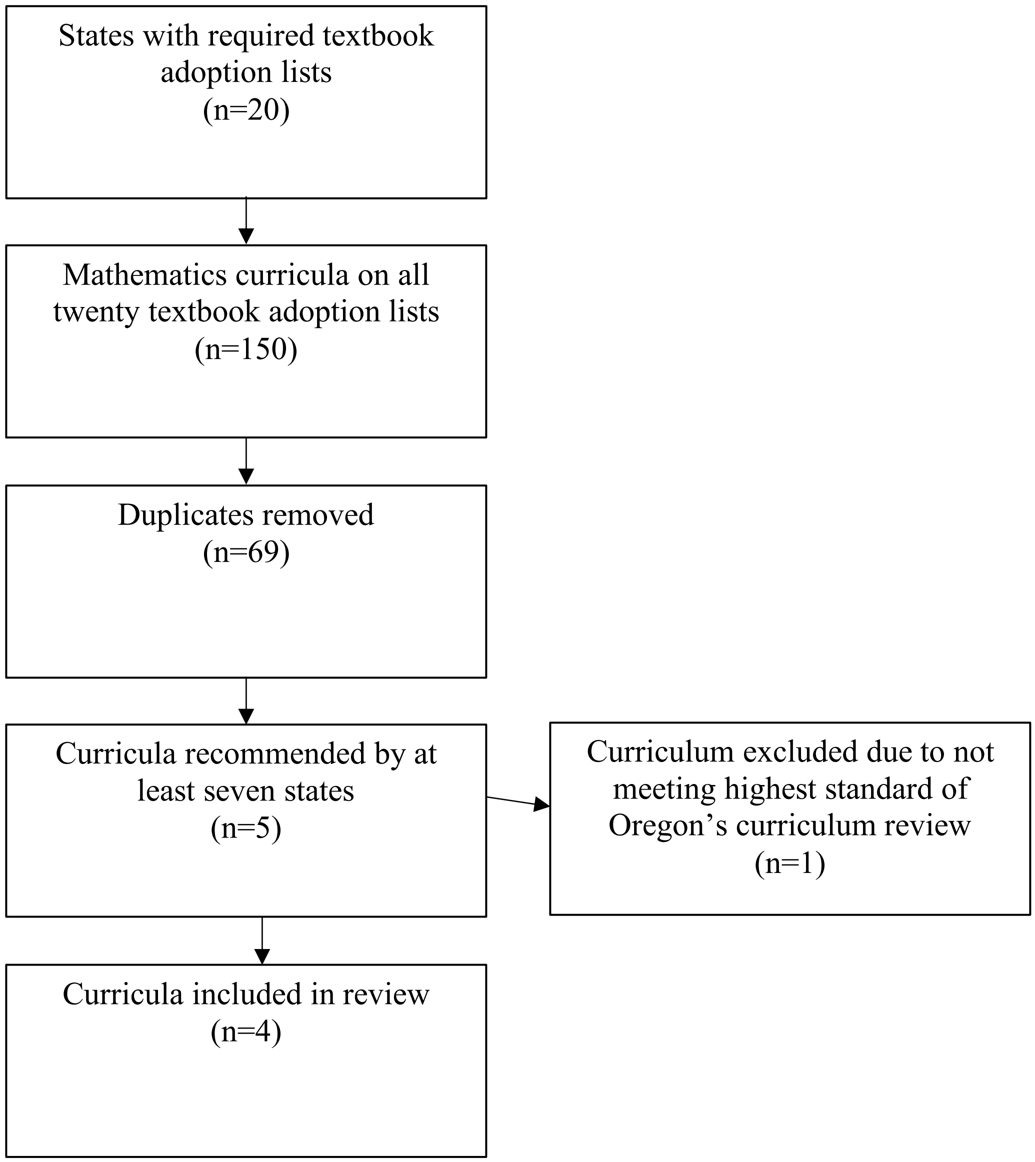
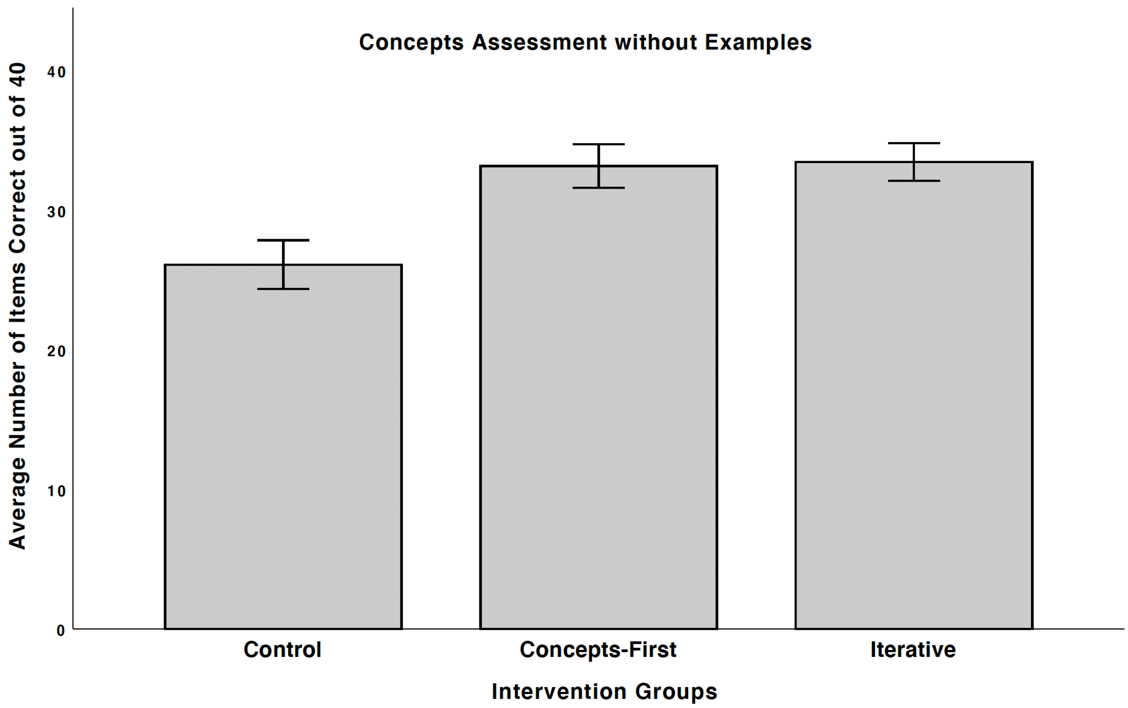


Figure 2

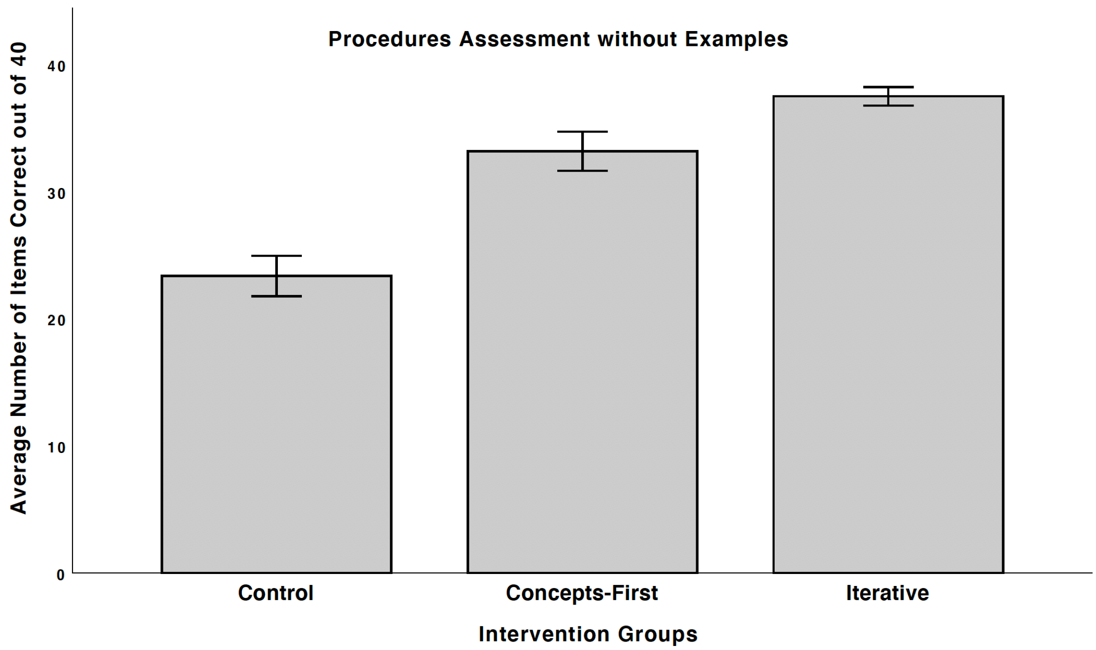
Posttest Intervention Group Differences for the Proximal Concepts Assessment without Examples



Note. Figure 2 displays the average number of items correct for each intervention group on the posttest fraction concepts assessment. Error bars represent standard errors.

Figure 3

Posttest Intervention Group Differences for the Proximal Procedures Assessment without Examples



Note. Figure 3 displays the average number of items correct for each intervention group on the posttest fraction procedures assessment. Error bars represent standard errors.

Appendices

Appendix A

Coding Rubric. Adapted from Doabler et al.'s 2012 study, with adaptations italicized.

Instructional quality indicator	1	2	3	4
Prerequisite skills	Does not identify prerequisite skills or provide warm-up activities.	Identifies prerequisite skills but provides warm-up activities unrelated to target topic.	Identifies prerequisite skills; however, offers limited warm-up activities to engage students' background skills.	Clearly identifies prerequisite skills and provides sufficient warm-up activities to engage students' background skills.
Math vocabulary	Does not identify vocabulary words.	Identifies key vocabulary but at the beginning of the chapter or unit rather than individual lessons.	Identifies and directly embeds (e.g., underlines) key vocabulary. Does not provide opportunities for students to use vocabulary.	Identifies and directly embeds (e.g., underlines) key vocabulary within lesson. Also, offers opportunities for students to use vocabulary.
Explicit instruction-*concepts*	Does not provide opportunities for teacher-led instruction on *concepts*. Instructional activities use a discovery learning approach.	Provides limited opportunities for teacher-led instruction on *concepts*.	Provides opportunities for teacher-led instruction on *concepts*, but limited directions on how to teach the target topic.	Systematically provides opportunities for teacher-led instruction on *concepts*. Offers explicit directions on how to directly teach the target topic.
Explicit instruction-*procedures*	Does not provide opportunities for teacher-led instruction on *procedures*. Instructional activities use a discovery learning approach.	Provides limited opportunities for teacher-led instruction on *procedures*.	Provides opportunities for teacher-led instruction on *procedures*, but limited directions on how to teach the target topic.	Systematically provides opportunities for teacher-led instruction on *procedures*. Offers explicit directions on how to directly teach the target topic.
Instructional examples-*concepts*	Does not provide teaching examples for *concepts*.	Provides poorly selected teaching examples for *concepts*. Examples are irrelevant to student practice.	Provides clear teaching examples for *concepts*; however, examples are limited in number (i.e., only two examples).	Provides clear and sufficient number of teaching examples for *concepts*. Also, examples are at least as complex as student practice.
Instructional examples-*procedures*	Does not provide teaching examples for *procedures*.	Provides poorly selected teaching examples for *procedures*. Examples are irrelevant to student practice.	Provides clear teaching examples for *procedures*; however, examples are limited in number (i.e., only two examples).	Provides clear and sufficient number of teaching examples for *procedures*. Also, examples are at least as complex as student practice.

Math models-*Part-whole representation models (e.g., fraction circles, fraction array models, etc.)*	Does not incorporate *part-whole* math models.	Incorporates *part-whole* math models for student use only. Does not incorporate *part-whole* models during teacher-led instruction.	Incorporates *part-whole* math models for teacher and student use. Provides limited directions on how teachers should use *part-whole* models during teacher-led instruction.	Appropriately incorporated *part-whole* models throughout the lessons (e.g., C-R-A approach). Provides clear directions on how to teach with the models.
Math models-*Measurement representation models (e.g., number lines, fraction strips, etc.)*	Does not incorporate *measurement* math models.	Incorporates *measurement* math models for student use only. Does not incorporate *measurement* models during teacher-led instruction.	Incorporates *measurement* math models for teacher and student use. Provides limited directions on how teachers should use *measurement* models during teacher-led instruction.	Appropriately incorporated *measurement* models throughout the lessons (e.g., C-R-A approach). Provides clear directions on how to teach with the models.
Practice opportunities and cumulative review	Does not provide practice and review opportunities.	Provides opportunities for math verbalizations but those offered are limited in structure (e.g., peer discourse). Includes limited opportunities for written exercises.	Provides opportunities for math verbalizations but are limited in number. Includes opportunities for written exercises.	Provides sufficient opportunities for math verbalizations, including occasions for group and individual responses, as well as sufficient written exercises. *Lesson contains at least 10 opportunities for verbalization practice (Doabler et al., 2018).*
Academic feedback	Does not provide academic feedback or correction procedures.	Includes only brief hints for anticipating student errors and misconceptions.	Provides hints to anticipate student errors and misconceptions. Procedures for correcting errors and reteaching are limited.	Provides hints to anticipate student errors and misconceptions. Provides procedures for academic and corrective feedback. Also, offers reteaching strategies.
Formative feedback loops	Does not provide assessments or opportunities to check for student understanding.	Provides end-of-unit assessments only.	Provides opportunities to check for student understanding and identifies an acceptable criterion for moving on with instruction.	Provides frequent opportunities to check for student understanding and identifies and acceptable criterion for moving on with instruction. Also, it has procedures to link test results.

Appendix B

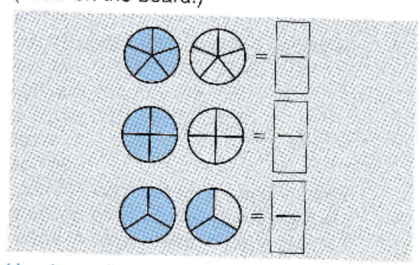
Conceptual exercise.

EXERCISE 4

Equals 1—With Pictures

a. (Draw on the board:)

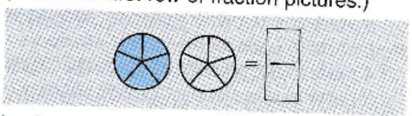
★



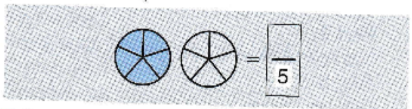
b. Here's a rule about fractions that equal 1. A fraction equals 1 when you use the same number of parts that are in each whole. Listen again. A fraction equals 1 when you use the same number of parts that are in each whole.

Lesson 8

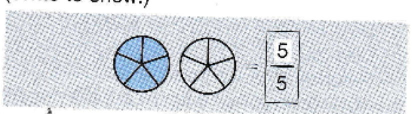
c. (Touch the first row of fraction pictures.)



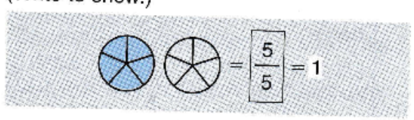
- Look at this picture. Tell me, how many parts are in each whole? (Pause.) Get ready. (Signal.) *5.*
- (Write to show:)



d. Tell me how many parts are used. (Pause.) Get ready. (Signal.) *5.*
- (Write to show:)

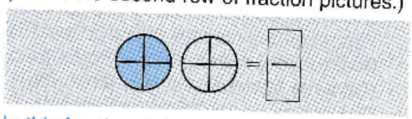


- You use the same number of parts that are in each whole. The fraction equals 1 whole.
- (Write to show:)



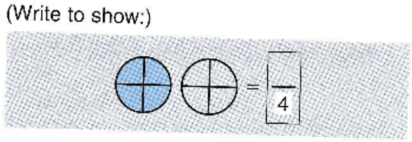
New Problem

a. (Touch the second row of fraction pictures.)

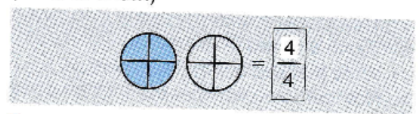


- In this fraction, tell me how many parts are in each whole. (Pause.) Get ready. (Signal.) *4.*

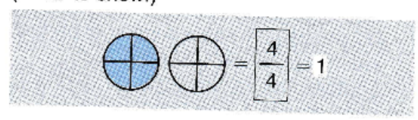
b. (Write to show:)



c. Tell me how many parts are used. (Pause.) Get ready. (Signal.) *4.*
- (Write to show:)

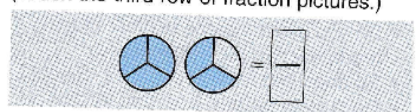


- Do you use the same number of parts that are in each whole? (Signal.) *Yes.*
- So the fraction equals 1.
- (Write to show:)



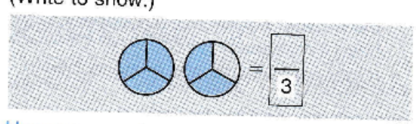
New Problem

a. (Touch the third row of fraction pictures.)

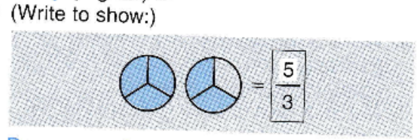


- In this picture, how many parts are in each whole? (Pause.) Get ready. (Signal.) *3.*

b. (Write to show:)



c. How many parts are used? (Pause.) Get ready. (Signal.) *5.*
- (Write to show:)



- Do you use the same number of parts that are in each whole? (Signal.) *No.*
- This fraction does not equal 1.

Workbook Practice

a. Look at Part 3 in your workbook.
- The first problem is already worked. There are 5 parts in each whole. How many parts are used? (Signal.) *5.*

Lesson 8

- Do you use the same number of parts that are in each whole? (Signal.) *Yes.*
- So the fraction equals 1. It equals 1 whole. That's what's written. **Equals 1.**

b. Touch the next problem.

- Write the fraction for the picture. ✔

c. Now we'll see whether the fraction equals 1. Remember, a fraction equals 1 when you use the same number of parts that are in each whole.

- In this picture, how many parts are in each whole? (Signal.) *7.*
- Do you use the same number of parts that are in each whole? (Signal.) *Yes.*
- So does that fraction equal 1? (Signal.) *Yes.*
- Write **equals 1** after that fraction. ✔

d. Now write the fraction for the next picture. ✔

- How many parts are in each whole? (Signal.) *7.*
- Do you use the same number of parts that are in each whole? (Signal.) *No.*
- So does that fraction equal 1? (Signal.) *No.*
- Don't write **equals 1** for that fraction. ✔

e. Work the rest of the problems in Part 3. First write the fraction for the picture. Then figure out whether the fraction equals 1. If the fraction equals 1, write **equals 1.** You have 4 minutes.

- (Observe students and give feedback.)

Appendix C
Procedural exercise.

Lesson 23

EXERCISE 3

Multiplication

a. (Write on the board:)

★

$$\frac{4}{5} \times \frac{3}{4} = \boxed{\frac{\ }{\ }}$$

- What does the sign say to do? (Signal.) *Multiply.*

b. Here's how we multiply fractions.
- We multiply top times the top and bottom times the bottom. How do we multiply fractions? (Signal.) *Top times the top and bottom times the bottom.*
- (Repeat until firm.)

c. Read the top. (Touch each part as the students read.) *4 times 3.*
- Tell me what the top equals. Get ready. (Signal.) *12.*
- The top equals 12. We write it on the top. Where do we write the 12? (Signal.) *On the top.*
- (Write to show:)

$$\frac{4}{5} \times \frac{3}{4} = \boxed{\frac{12}{\ }}$$

d. Read the bottom. (Touch each part as the students read.) *5 times 4.*
- Tell me what the bottom equals. Get ready. (Signal.) *20.*

- Where do we write the 20? (Signal.) *On the bottom.*
- (Write to show:)

$$\frac{4}{5} \times \frac{3}{4} = \boxed{\frac{12}{20}}$$

- 4 fifths times 3 fourths equals 12 twentieths. It sounds funny, but that's the answer.

e. (Write on the board:)

$$\frac{3}{2} \times \frac{6}{5} = \boxed{\frac{}{}}$$

- What does the sign say to do? (Signal.) *Multiply.*
- *How do we multiply fractions? (Signal.) Top times the top and bottom times the bottom.*

f. Read the top. (Signal.) *3 times 6.*
- Tell me what the top equals. Get ready. (Signal.) *18.*
- Where do I write the 18? (Signal.) *On the top.*
- (Write to show:)

$$\frac{3}{2} \times \frac{6}{5} = \boxed{\frac{18}{}}$$

g. Read the bottom. *2 times 5.*
- Tell me what the bottom equals. Get ready. (Signal.) *10.*
- Where do I write the ten? (Signal.) *On the bottom.*
- (Write to show:)

$$\frac{3}{2} \times \frac{6}{5} = \boxed{\frac{18}{10}}$$

- 3 halves times 6 fifths.
- Read the answer. (Signal.) *18 tenths.*

h. Turn to Part 5 in your workbook. Touch the first problem.
- What does the sign say to do? (Signal.) *Multiply.*
- How do you multiply fractions? (Signal.) *Top times the top and bottom times the bottom.*
- Touch the top.
- Figure out the answer when you multiply. (Pause.)
- Tell me what the top equals. Get ready. (Signal.) *6.*
- Write it in the answer box. ✔
- Touch the bottom.
- Figure out the answer when you multiply. (Pause.)
- What does the bottom equal? (Signal.) *12.*
- Write it in the answer box. ✔
- 3 sixths times 2 halves. What's the answer? (Signal.) *6 twelfths.*
- Right, 6 twelfths.

i. Touch the next problem.
- *What does the sign say to do? (Signal.) Multiply.*
- *How do you multiply fractions? (Signal.) Top times the top and bottom times the bottom.*
- Touch the top.
- Figure out the answer when you multiply. (Pause.)
- What does the top equal? (Signal.) *18.*
- Write it in the answer box. ✔
- Touch the bottom.
- Figure out the answer when you multiply.
- What does the bottom equal? (Signal.) *28.*
- Write it in the answer box. ✔
- 3 fourths times 6 sevenths. What's the answer? (Signal.) *18/28ths.*
- Right, 18/28ths.

j. Do the rest of the problems in Part 5. Remember, you multiply top times the top and bottom times the bottom. You have 4 minutes.
- (Observe students and give feedback.)

Appendix D
Treatment Adherence Checklist.

Treatment Adherence Steps for *Corrective Mathematics, Basic Fractions* Intervention	**Check if completed**
1. Interventionist administers exercise one, adhering to explicit instruction, modeling, and/or guided practice as written in instruction script.	
2. Interventionist administers exercise two, adhering to explicit instruction, modeling, and/or guided practice as written in instruction script.	
3. Interventionist administers exercise three, adhering to explicit instruction, modeling, and/or guided practice as written in instruction script.	
4. Interventionist gives time for students to practice the fraction concepts or procedures on their independent worksheets.	
5. Interventionist provides academic feedback during student practice.	
6. Interventionist reads aloud correct answers to student practice problems at the end of the intervention so that students can check their work.	
7. Interventionist praises students throughout the intervention session for their effort and performance.	