Spatial and Temporal Variability During Radiatively-Driven Convection in Lake Superior

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Abstract

In dimictic freshwater lakes, a period known as radiatively-driven convection can occur when the surface mixed layer or full water column is below the freshwater temperature of maximum density. During this period, incoming shortwave radiation warms either surface water or water beneath a layer of ice, generating density gradients and convective cells that then descend into the water column. Little is understood about the lateral variability during this period, and most previous studies have considered the process in ice-covered lakes. Using a unique two-dimensional mooring with both horizontal and vertical temperature resolution along with a nearby meteorological buoy and ADCP mooring from a 2019 deployment in Lake Superior, this analysis presents an in depth look at convective cell formation, propagation, and lateral scales. The convective cells observed were found to be dependent on shortwave radiation, wind speed, and thermal expansion coefficient, and to propagate along with background currents. The local time rate of change of temperature at fixed points was calculated and compared to the lateral advective term, which was calculated utilizing both the horizontal resolution of the 2D mooring and currents from the local ADCP; it was discovered that these two terms agreed strongly during periods when the calculation was possible, suggesting that both diffusion and heating of convective cells is insignificant on shorter time scales. This allowed for the application of a frozen field approximation, where convective cells were assumed to remain intact as they traveled. Results from this analysis agreed with correlation analysis results, suggesting horizontal scales of convective cells are on the order of 50 m, with varying structure geometry sometimes producing scales over 200 m. Additionally, vertical structure was found to be similar in nature, suggesting convective cells formed during this period remain intact both as they descend and propagate laterally.
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1.0 Introduction

Water motion is most easily visualized as resulting from mechanical forcing. Sloshing water in a tilted cup, ripples propagating in a puddle after a pebble is dropped in, and massive wakes trailing cargo ships are all straightforward enough examples to imagine, and many more examples are easily observable in everyday life. In large bodies of water, many complex motions occur due to the many different forms of forcing present. Convection, one significant type of fluid motion, typically results from the combined effects of density and gravity and is a significant fluid flow in Earth’s lakes, oceans, atmosphere, and even mantle. Large scale convective motions, with eddies on the order of 10 km or convective plumes on the order of 1 km, can occur in the ocean (Marshall and Schott 1999). Similarly, lakes experience various forms of convection, typically on much smaller scales, resulting from surface cooling, differential heating near sloping boundaries, under-ice penetrative heating, and sediment heat flux, among others (Bouffard and Wüest 2019).

Disregarding the effect of salinity, convection is driven in most freshwater by temperature induced density changes. Additionally, because of the freshwater temperature of maximum density ($T_{MD} = 3.98^\circ C$), convection in dimictic lakes can occur both from heating and cooling at the surface when the temperature is below or above $T_{MD}$, respectively. When surface water is below $T_{MD}$, incident solar radiation heats both the surface and near surface water during the day, generating radiatively-driven convection (RDC) due to the unstable density gradient developed. This process can occur in both ice-covered lakes when shortwave radiation penetrates the ice and reaches the water column (Kirillin et al. 2012), and ice-free lakes with surface mixed layers or full water columns below $T_{MD}$ (Cannon et al. 2019; Austin 2019).

Early interest in RDC in lakes was related to the effect on the ice cover, with speculations on the sinking of ice due to convection (Birge 1910) and melt patterns forming on the ice surface (Woodcock 1965). Church (1947) considered the various convection periods and effects on the thermal regime of Lake Michigan, in perhaps the
first known consideration of RDC in the Great Lakes. Other early examples of radiative convection analysis in lakes include Mortimer and Mackereth (1958) investigating ice-covered northern Swedish lakes and Ragotzkie and Likens (1964) determining the heat budget and difference in heating of two permanently covered Antarctic lakes undergoing RDC. The basis for many subsequent under ice and ice-free RDC analyses, Farmer (1975) investigated the thermal structure in deep (max depth ≈ 180 m) Babine Lake, British Columbia, focusing on the entrainment and kinetic energy of sinking convective cells and making the first estimates of a vertical velocity scale for RDC.

As with earlier work and the seminal work of Farmer (1975), most RDC studies have been concerned with ice-covered lakes. Bengtsson (1996) considered different mixing processes in ice-covered lakes, finding that RDC is much more efficient at heating the water column than sediment heat fluxes and only occurs in late winter when surface ice is exposed to solar radiation. An influential study for follow up work in Russia investigated penetrative convection in shallow, ice-covered lakes (Malm et al. 1997). Deardorff convective scales (Deardorff 1970) were considered for RDC, focusing on the volumetric forcing, rather than simply surface forcing, of solar radiation (Mironov and Terzhevik 2000). Following that, large-eddy simulations were performed for RDC in ice-covered lakes, and the resulting turbulence structure is contrasted between vertically distributed forcing and boundary forcing (Mironov et al. 2001). In addition to a thorough review of RDC studies up until that point, Mironov et al. (2002) compared previous observations, scaling laws, and large-eddy simulations and discovered agreement with turbulent kinetic energy (TKE) dissipation estimates. Jonas et al. (2003a) also analyzed the TKE balance in shallow, ice-covered Lake Vendyurskoe, Russia. Other studies, both in Lake Vendyurskoe and Lake Onego, Russia, have studied RDC under ice in terms of energetics (Volkov et al. 2019), internal wave generation (Bouffard et al. 2016), vertical velocities (Bogdanov et al. 2019), and vertical distribution of phytoplankton (Suarez et al. 2019). Additional study of under ice convection in relatively shallow lakes includes the work of Stefanovic and Stefan (2002) in Ryan Lake, Minnesota, and Pernica et al. (2017) investigating effects on phytoplankton in a few reservoirs in southern Saskatchewan and
Lake Simcoe in Ontario, Canada. Yang et al. (2017) also investigated Lake Simcoe and focused on temperature and dissolved oxygen variations.

Transitioning to slightly larger lakes, under ice RDC has also been documented in Lake Pääjärvi and Lake Päijänne, both in Finland. Salonen et al. (2014) investigated the circulation under ice driven by RDC, finding that horizontal convection resulting from a sloping lake bottom interacted with vertical motions and even dominated mixing in one of the years considered. Additionally, the impact on phytoplankton during RDC was described in these lakes (Kiili et al. 2009; Vehmaa and Salonen 2009). Forrest et al. (2008) used an autonomous underwater vehicle (AUV) to compare convective transport under ice in winter to a period in summer, estimating relatively short horizontal scales present during RDC under ice. Finally, theoretical development of the energy budget and pathways for RDC was performed by Winters et al. (2019) for shallow ice-covered lakes and Ulloa et al. (2018) for ice-covered lakes in general.

As mentioned previously, RDC also occurs in ice-free lakes with surface waters below $T_{MD}$, although it is typically limited to large lakes. While Farmer (1975) focused mainly on penetrative convection below ice, results also suggested the process continues following ice off in Babine Lake. The first known measurements of turbulent mixing from RDC were taken in Lake Michigan, indicating that this convection period plays a major role in driving spring vertical mixing (Cannon et al. 2019). Using both point moorings and autonomous underwater glider (AUG) transects in Lake Superior, Austin (2019) investigated both spatial and temporal scales of RDC, finding the bottom of the lake (~180 m depth) warms around 6 hours after sunrise and convection halts in the water column a few hours after sunset.

In contrast to the diurnal cycle of RDC in Lake Superior, the annual timing of onset is much more variable. Unlike the smaller ice-covered lakes investigated in previous studies, Lake Superior experiences minimal warming during periods of ice cover, especially in pelagic regions (Titze and Austin 2014). Thus, under ice RDC in Lake Superior is not as important to water column mixing as in other lakes. The ice
formation process differs as well, as ice was found to form only after the surface mixed layer (~60 m) drops to below 0.1°C (Titze and Austin 2016). In smaller lakes, ice forms much more quickly following surface water dropping below \( T_{MD} \). Additionally, it is unlikely that horizontal convection from littoral regions seen in smaller lakes (Horsch and Stefan 1988; Monismith et al. 1990; MacIntyre and Melack 1995) or sediment heat flux (Likens and Ragotzkie 1965; Bengtsson 1996; Bouffard and Wüst 2019) transport significant amounts of heat into the open waters of Lake Superior, leaving RDC as the sole heat source in deeper parts of the lake throughout spring.

Since the RDC period essentially begins in Lake Superior following ice off, the timing and duration is largely dependent on the winter ice cover. Additionally, the summer thermal conditions following RDC are directly related to the previous winter conditions in deep lakes like Lake Superior (Austin and Allen 2011). Trends towards lower ice cover have been observed in the Great Lakes (Assel et al. 2003; Wang et al. 2012), and dynamic modeling has suggested drastic changes in winter thermal structure for Lake Superior by the mid-21st century, specifically reduced ice cover and weaker stratification in the winter (Matsumoto et al. 2019). Similarly, Austin and Colman (2007) suggested Lake Superior could be mostly ice free by around 2040, and that it is warming more than the surrounding region, with most of the ice cover reduction occurring over the previous few decades (Austin and Colman 2008). The issue of ice-covered lakes warming significantly in recent years is not limited to Lake Superior, as worldwide trends show warming surface waters in seasonally ice-covered lakes (O’Reilly et al. 2015).

Historically, Lake Superior has had the lowest surface temperatures in the summer as well as annual mean among the Great Lakes (Bennett 1978), so it is especially susceptible to increasing temperatures and potential for reduced ice cover.

Directly related to the ice cover, the variability of “overturning” events, or periods where the water column is homogenous, has been investigated recently (Fichot et al. 2019). This study found the timing of these events, which are indicative of the onset of ice-free RDC, can vary by as much as a month in the spring. Additionally, it was found that Lake Michigan and Lake Ontario were susceptible to incomplete overturning events,
as the previous summer stratification never fully eroded over the winter. A final relevant observation from this study is that the period between overturning events can last almost two months in Lake Superior. A previous study of the thermal structure of Lake Superior found the isothermal RDC period lasted from 6 June through 7 July in 2007 (Austin and Allen 2011). In addition to ice cover and warming trends being largely important to the onset and timing, water clarity factors into the effectiveness of RDC.

Solar radiation warms the surface water and penetrates near surface waters; the extent of solar penetration depends on the water clarity and wavelength of incident light. While some previous RDC studies have incorporated multiple wavelengths into the analysis (e.g. Mironov et al. 2002), the source is typically considered monochromatic and a single attenuation coefficient is considered. Lower attenuation coefficients correspond to higher water clarity. Depending on the water column depth and attenuation coefficient, RDC may not occur in some small ice-covered lakes in the arctic (Cortés and MacIntyre 2020) and Russia (Malm et al. 1997). While the arctic lake may also have been influenced by non-negligible specific conductance of the water, low attenuation coefficients led to sunlight penetrating and warming the entire water column nearly uniformly.

In Lake Superior, specific conductance is not a significant factor for RDC, and the water column is deep enough to prevent sunlight from penetrating to the bottom. Lake Superior has the lowest levels of salinity in the Great Lakes, and no observed trend in specific conductance was seen in previous decades (Chapra et al. 2012). Additionally, water clarity has remained relatively constant lake-wide in Lake Superior, with some increased variance found in nearshore waters (Binding et al. 2015). In contrast, Lake Michigan and Lake Huron exhibited noticeable trends in increased clarity in the last couple decades due to dreissenid mussel invasions (Yousef et al. 2017), and mussel invasions have also had noticeable impacts on water clarity in Lake Erie and Lake Ontario (Binding et al. 2007). Barbiero et al. (2018) has also considered water clarity changes due to nutrients and food webs, finding increasing water clarity in Lake Michigan and Lake Huron. These increasing water clarities in most of the Great Lakes
suggest increased efficiency of RDC in the water columns due to higher levels of shortwave radiation penetration. However, the interplay between reduced ice cover and either increasing or constant water clarity in each of these lakes is not fully understood. Further investigation of RDC in these lakes will be crucial moving forward as continued warming and reduced ice cover occur.

In the spring, RDC drives vertical mixing in the water column (Cannon et al. 2019), and consequently plays an important role in biogeochemical processes. Convective mixing scales in ice-covered lakes suggest vertical velocities are greater than sinking rates of phytoplankton (Matthews and Heaney 1987). With thin snow cover on the ice, minimal sunlight penetrates the water column without driving extensive vertical mixing, and nonmotile phytoplankton can remain in the upper mixed layer and receive sufficient light for growth (Kelley 1997). As light penetration increases, vertical mixing begins and pushes phytoplankton into areas without sufficient light for photosynthesis (Pernica et al. 2017; Bouffard et al. 2019). Some motile cryptophytes have been found to remain near the surface and resist vertical mixing, giving them an advantage during this period (Kiili et al. 2009; Vehmaa and Salonen 2009). Sunlight then has a twofold effect on phytoplankton present under ice while RDC is ongoing: it provides the necessary light for growth yet also forces vertical transport out of sufficiently lit areas. Phytoplankton present in near shore areas were found to fare much better than those in deeper areas with much larger convectively mixed layers (Suarez et al. 2019). In ice-free Lake Superior, Austin (2019) noted the presence of anomalously low chlorophyll $a$ fluorescence in warm parcels of water at depth, indicating photoquenched organisms from near the surface were transported deeper in the water column. Besides simply transporting phytoplankton within the water column, convection also distributes nutrients and can cause the resuspension of sediments depending on the extent of mixing.

Horizontal convection can occur in littoral regions due to density gradients and interactions with the sloping lake bottom (Monismith et al. 1990; MacIntyre and Melack 1995). Then, nutrients present in these regions and resuspended from the shallow sediments are transported out into deeper parts of a lake, creating a transfer between
littoral and pelagic regions (Horsch and Stefan 1988). Nutrients are much more scarce in deeper regions of lakes, and convective mixing has implications for biological oxygen demand in these regions (Bennett 1978). Oxygen concentrations throughout the water column are strongly correlated with the extent of vertical convective mixing, and intense mixing can increase dissolved oxygen availability at depth and allow for abundant algal growth (Yang et al. 2017).

While mixing during the RDC impacts the entire water column, Matsumoto et al. (2015) found that the interior of Lake Superior is not entirely ventilated in the spring, yet no parts of the water column are isolated from the surface for over 300 days. This ventilation can play a critical role in lake-atmosphere exchanges. Lakes and other inland waters were found to contribute to the global carbon cycle in a much larger capacity than previously estimated and at higher rates than land surrounding these regions, as carbon buried in the sediments can be transported to the surface and returned to the atmosphere (Cole et al. 2007; Tranvik et al. 2009). In this regard, an increased understanding of RDC in large lakes and its extent of vertical mixing relating to sediment resuspension could allow for better estimates of carbon exchanges with the atmosphere in dimictic lakes that exhibit this behavior.

Several previous studies have examined the horizontal scales of convective cells in lakes. Forrest et al. (2008) discovered a roughly 150 m wide region of penetrative convection under ice in Pavilion Lake, British Columbia. Similarly, Austin (2019) estimated horizontal scales on the order of 50 meters in ice-free Lake Superior. Analysis of vertical velocities using progressive-vector diagrams estimated horizontal scales on the order of tens of meters under ice in Lake Onego (Bogdanov et al. 2019). Horizontal resolution was also explored in Lake Geneva, where scales of convective plumes resulting from surface cooling were estimated around 5 m (Thorpe 1999). In contrast, convective plumes in the ocean have been found with horizontal resolutions of 350 m in the Mediterranean (Margirier et al. 2017), 1.4 km in the Golfe du Lion (Schott and Leaman 1991), and up to 10 km in the Greenland Sea (Wadhams et al. 2002). In addition
to horizontal resolution, the vertical progression and temporal scales of convective processes have also been explored.

Besides Austin (2019), who estimated vertical convective velocities around 1-2 cm/s and convective damping times of around 2 hours in ice-free Lake Superior, most investigations of vertical transport and convective decay have been conducted in ice-covered lakes. A range of under ice downward convective velocities have been observed between 1 mm/s and 7.5 mm/s (Bouffard et al. 2016; Bogdanov et al. 2019; Suarez et al. 2019). Additionally, downward plumes have been found to be more confined and larger in magnitude than upward plumes of colder water (Mironov et al. 2001; Jonas et al. 2003b; Bouffard et al. 2019). Under ice convective damping times have been estimated in various lakes: 1 hour in Lake Baikal (Kelley 1997), roughly 80 minutes in Pavilion Lake, British Columbia (Forrest et al. 2008), and between 4 and 37 minutes in several Canadian reservoirs/lakes (Pernica et al. 2017). Again, in contrast to estimates in lakes, convective velocities in the ocean are observed to be on the order of centimeters to tens of centimeters per second, with downward magnitudes larger than upward (Schott and Leaman 1991; Thorkildsen and Haugan 1999; Margirier et al. 2017).

Farmer (1975) noted the likely failure of a 1D array to fully capture the temperature profile of penetrative convection due to horizontal variability. Several analyses already mentioned make estimations of horizontal scales and variability. However, few have managed to capture horizontal features in lakes or oceans, as stationary 1D moorings are not suited for such measurements, and 2D moorings are less practical and difficult to deploy. Previous studies have obtained horizontal resolution through the use of underwater vehicles; autonomous underwater vehicles (Forrest et al. 2008) and gliders (Austin 2019) as well as submarines equipped with temperature sensors (Thorpe 1999) have been used to explore horizontal temperature structure in lakes. Towed vehicles and towed instrumented cables also provide horizontal resolution, as has been documented in oceanic studies (Klymak and Moum 2007a; b; Adams et al. 2019), 3D resolution was achieved by van Haren et al. (2016) through the use of a constructed thermistor array to study internal waves in the Atlantic Ocean. The mooring used to
collect the data considered in this thesis was based on the design of a 2D mooring designed and deployed in Massachusetts Bay to study internal waves (Trask et al. 1999; Grosenbaugh et al. 2002).

RDC in Lake Superior is the focus of this thesis, specifically horizontal variability during this period. Additionally, the observations discussed throughout are unique as they provide a 2D view of convective cell development and motion, made possible due to the coincidence of a nearby meteorological buoy measuring heat fluxes and wind speeds at the water surface and an ADCP measuring local currents. Observations of RDC by Austin (2019) in Lake Superior inspired the ongoing campaign to investigate horizontal convective scales during this period and potential implications for biological activity in the lake. Previous studies have mainly focused on RDC in ice-covered lakes, so a better understanding of the process in an ice-free lake, where the entire water column is susceptible to vertical convective mixing, is essential to determine differences in scales and features of convective cells developed during this unique and potentially significant period in large bodies of water. In addition to a manuscript in preparation (Austin et al. unpubl.), this thesis provides the first description and discussion of a 2D mooring deployment in 2019 examining the thermal structure of RDC in Lake Superior.
2.0 Background

2.1 Theoretical Description of Radiatively-Driven Convection

During RDC events in water columns, temperature anomalies generated by daytime near surface heating create buoyancy fluxes that result in downward vertical motion. For this problem, as with typical convection setups, an incompressible fluid is assumed. Additionally, the Boussinesq approximation is utilized, allowing for small density changes to be ignored in all terms except the gravity term. For this term, sometimes referred to as the buoyancy parameter, the density is defined as

\[ \rho = \rho_0 [1 - \alpha (T - T_0)] \]  

(2.1)

where \( \rho_0 \) is a reference or hydrostatic density (typically set to 1000 kg m\(^{-3}\)), \( T \) is the water temperature, \( T_0 \) is some reference or background temperature, and \( \alpha \) is the thermal expansion coefficient defined as

\[ \alpha = -\frac{1}{\rho_0} \frac{\partial \rho}{\partial T} \]  

(2.2)

with units of °C\(^{-1}\). The salinity dependence of density is not included in this analysis for Lake Superior, as it is negligible. The governing equations for open water RDC in Lake Superior, ignoring Coriolis effects, are then

\[ \vec{\nabla} \cdot \vec{u} = 0 \]  

(2.3)

\[ \frac{D\vec{u}}{Dt} = -\frac{\nabla p}{\rho_0} + \frac{g \rho}{\rho_0} \hat{k} + \nu \nabla^2 \vec{u} \]  

(2.4)

\[ \frac{D T}{Dt} = \kappa \nabla^2 T + S \]  

(2.5)

where the vertical direction \( \hat{k} \) is oriented positive downwards and terms are defined as the following:

- \( \vec{u} \) is the velocity vector [m s\(^{-1}\)]
- \( p \) is the water pressure [Pa]
- $g$ is the gravitational acceleration [m s\(^{-2}\)]
- $\nu$ is the kinematic viscosity [m\(^2\) s\(^{-1}\)]
- $\kappa$ is the thermal diffusivity [m\(^2\) s\(^{-1}\)]
- and $S$ is a heat source term [°C s\(^{-1}\)].

For RDC, the heat source is predominantly shortwave radiation with intensity given by

$$I = I_0 e^{-k_d z} \quad (2.6)$$

where $I_0$ is the intensity incident at the surface, $k_d$ is the light attenuation coefficient, and $z$ is again defined as positive downwards. The incident light is assumed to be monochromatic with a single attenuation coefficient, yet some analyses incorporate a range of wavelengths (e.g. Mironov et al. 2002; Jonas et al. 2003a; Bouffard et al. 2019). Then, the heat source term above can be calculated as

$$S = -\frac{1}{\rho_0 c_p} \frac{dI}{dz} = k_d I_0 \rho_0 c_p e^{-k_d z} \quad (2.7)$$

where $c_p$ is the specific heat capacity of water (4180 J kg\(^{-1}\) °C\(^{-1}\)) and other terms are defined as above. The problem setup can be simplified further using linear instability analysis.

Introducing a small perturbation into the background state gives the following expressions for horizontal velocity, vertical velocity, temperature, and pressure

$$u = U + \epsilon u' \quad (2.8)$$
$$v = V + \epsilon v' \quad (2.9)$$
$$w = W + \epsilon w' \quad (2.10)$$
$$T = \bar{T} + \epsilon T' \quad (2.11)$$
$$p = P + \epsilon p' \quad (2.12)$$

where $U$ and $V$ are background horizontal velocities; $W$ is a background vertical velocity; $\epsilon$ is a small perturbation parameter; $u'$, $v'$, and $w'$ are all perturbation velocities; $\bar{T}$ is a
background temperature; $T'$ is a perturbation temperature or small temperature anomaly; $P$ is a background or hydrostatic pressure; and $p'$ is a perturbation pressure. Considering an initially isothermal water column and no dynamic forcing other than RDC, the background vertical velocity $W$ is set to 0. Then, RDC-generated buoyancy fluxes result in vertical perturbation velocities that are effectively the downwelling and upwelling velocities of convective plumes. Inserting Equations 2.7-2.12 into the above equations, then using the hydrostatic approximation to relate the background, hydrostatic pressure to the background density and ignoring negligible $O(\epsilon^2)$ terms, the following perturbation equations are obtained

\[
\frac{\partial \vec{u}'}{\partial t} + \left( \vec{U} \cdot \vec{\nabla}_H \right) T' = -\frac{1}{\rho_0} \vec{\nabla} p' + g\alpha T' \hat{k} + \nu \nabla^2 \vec{u}'
\]

(2.13)

\[
\frac{\partial T'}{\partial t} + \left( \vec{U} \cdot \vec{\nabla}_H \right) T' = \kappa \nabla^2 T' + \frac{k d_0}{\rho_0 c_p} e^{-kd} \epsilon
\]

(2.14)

where all terms are defined as above ($\vec{U}$ implies $U \hat{i} + V \hat{j} + W \hat{k}$) and the material derivative on the left-hand side is expanded in Equations 2.13-2.14 to include the horizontal advection terms (vertical advection term disappears as background vertical velocity was assumed to be 0). The shortwave heat source term on the right-hand side of Equation 2.14 was assumed to be $O(\epsilon)$ along with the other terms present. This assumption is likely valid at sufficient depth (with an attenuation coefficient of 0.2 m$^{-1}$, the e-folding depth is only 5 m) but is not valid at the surface where the term is of a larger magnitude. This near surface radiative heating is one factor that sets the RDC process apart from more commonly studied convection problems.

2.2 Distinctions from Rayleigh-Bénard Convection

Radiatively-driven convection differs from Rayleigh-Bénard convection in several fundamental ways. The typical Rayleigh-Bénard setup consists of an enclosed fluid between two boundaries with a linearly decreasing temperature with height. The temperature gradient is achieved by setting the lower boundary to a higher temperature
than the upper boundary, resulting in warmer, less dense fluid sitting at the bottom of the container. Convection results, as the denser fluid above descends towards the bottom and is replaced by the warmer fluid rising to the top.

### 2.2.1 Freshwater Equation of State

Water near the temperature of maximum density exhibits interesting behavior; early studies by Townsend (1965) and Myrup et al. (1970) investigated “upside down” convection over ice, where cool ~0°C water near the ice sat below denser, warmer water above. Since the thermal expansion coefficient for freshwater is negative at temperature below 3.98°C, increasing temperatures in this range result in increasing density. Figure 1 shows the freshwater equation of state relating the temperature and density as well as the temperature dependence of $\alpha$, produced using the formulas derived by Chen and Millero (1986).

![Figure 1: Freshwater equation of state. Density (blue) is at a maximum at 3.98°C, indicated by the vertical dotted line. The thermal expansion coefficient (red) is negative below $T_{MD}$, with the horizontal dotted line highlighting the transition to positive values.](image)

The fundamental reason RDC occurs is the nonlinear relation between density and temperature below $T_{MD}$, which results in unstable water columns when surface layers are
either ice-covered or near 0°C. Additionally, the thermal expansion coefficient approaches zero as cool water approaches $T_{MD}$, so equivalent temperature increases in that range lead to progressively smaller density changes. Then, larger temperature increases are required to generate large enough density gradients to drive convection. While Rayleigh-Bénard convection has been investigated for freshwater near $T_{MD}$ (Large and Andereck 2014), the more common setup of a fluid with an inverse and approximately linear relationship between density and temperature is typically considered.

## 2.2.2 Penetrative Heating

The other main difference between RDC and a typical Rayleigh-Bénard setup is the nature of the heating applied to the fluid. As mentioned, Rayleigh-Bénard convection is often driven by boundary heating at the bottom, and the heat source does not penetrate the fluid. Alternatively, in RDC, the heat source warms both the surface and near surface, with the extent of penetration determined by the attenuation coefficient as described in Equation 2.6. This penetrative heating has been explored theoretically and experimentally.

Following early work by Kraichnan (1962) investigating different regimes of convection, the transition into the ultimate, or mixing length, regime has been suggested and explored. Zhu et al. (2018) found that the transition to the ultimate regime of convection, specifically in two dimensional RDC, occurs at Rayleigh numbers on the order of $10^{13}$. Using a laboratory setup where fluid was heated from below with a light source, Lepot et al. (2018) suggested that the RDC observed surpasses Rayleigh-Bénard convection and reaches the ultimate regime of mixing due to the heat penetration. Related work further examined the transition to and even beyond the ultimate regime, exploring relationships between the Prandtl, Nusselt, and Rayleigh numbers (Bouillaut et al. 2019; Miquel et al. 2019; Miquel et al. 2020).
2.3 Surface Heat Fluxes

While the shortwave radiation into the lake is the primary heat source during RDC in Lake Superior, all surface heat fluxes ultimately contribute. The total surface heat flux is a combination of four different terms: shortwave radiation, longwave radiation, latent heat flux, and sensible heat flux. Typically, the longwave radiation term is separated into a downward and upward term, so the expression for net heat flux into the lake surface is

\[ Q_{\text{net}} = Q_{SW} + Q_{LWd} - Q_{LWu} + Q_{LH} + Q_{SEn} \]  

(2.15)

where \( Q_{SW} \) is the shortwave radiation, \( Q_{LWd} \) is the downward longwave radiation, \( Q_{LWu} \) is the upward longwave radiation, \( Q_{LH} \) is the latent heat flux, and \( Q_{SEn} \) is the sensible heat flux. Heat fluxes into the lake are considered positive.

2.3.1 Shortwave Radiation

The shortwave radiation term in Equation 2.15 was previously introduced in Equation 2.6. Incident solar radiation warms the surface waters and penetrates a distance into the water column depending on water clarity. The amount of shortwave radiation reaching the surface is highly dependent on cloud cover, latitude, and time of year. Additionally, when there is ice cover, the ice albedo is largely important for determining the amount of heat absorbed that penetrates the water column below. A typical clear sky shortwave radiation for open Lake Superior peaks around local noon at approximately 1000 W m\(^{-2}\) during the summer.

2.3.2 Longwave Radiation

Longwave radiation is emitted from a body, including both the atmosphere and lake surface. Upward longwave radiation is emitted out of the lake and can be estimated using the Stefan-Boltzmann law for blackbody radiation

\[ Q_{LWu} = \varepsilon \sigma T_w^4 \]  

(2.16)
where $\varepsilon$ is the emissivity (roughly 1), $\sigma$ is the Stefan-Boltzmann constant (approximately $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$), and $T_w$ is the water temperature in K. Downward longwave radiation is typically emitted from the clouds or water vapor in the atmosphere and is largely dependent on the air temperature. Again, the Stefan-Boltzmann law can be utilized for a rough estimate, replacing the water temperature in Equation 2.16 with the air temperature. However, other factors, including humidity in the atmosphere, also factor into accurate estimates of downward longwave radiation. For the analysis in this thesis, the downward longwave radiation was measured directly by a meteorological buoy and can be assumed to be absorbed nearly entirely at the surface.

### 2.3.3 Latent Heat Flux

Latent heat flux is a turbulent flux at the surface related to the moisture of the air. A positive latent heat flux is condensation from the air above the water into the water, while evaporation is a negative latent heat flux process transferring heat from the water to the air. The total latent heat flux at the surface is

$$Q_{LH} = \rho_A L_v c_E (q_A - q_{W}) |u_A|$$

(2.17)

where $\rho_A$ is the air density, $L_v$ is the latent heat of vaporization, $c_E$ is a transfer coefficient (typically on the order of $10^{-3}$, see Fairall et al. 1996), $q_A$ is the specific humidity of air at some specified height above the water, $q_{W}$ is the specific humidity at the water surface, and $u_A$ is the wind speed or velocity of air above the water. In cases where the air velocity is small and water velocity is not negligible, $u_A$ can be calculated as the magnitude of the difference between the air and water velocities.

### 2.3.4 Sensible Heat Flux

Like the latent heat flux, the sensible heat flux is also a turbulent flux. Instead of being directly related to the humidity in the air, the sensible heat flux is more related to the transfer of heat between the water and atmosphere due to turbulent eddies formed from velocity shear. In a simplified sense, it can be thought of as conduction between the
air and water surface. As shown above for latent heat flux, a parameterization for sensible heat flux is given by

$$Q_{SEN} = \rho_A c_p c_H (T_A - T_W) |u_A|$$  \hspace{1cm} (2.18)

where $c_H$ is another transfer coefficient again on the order of $10^{-3}$ and the other terms are defined as above. Since this heat flux is directly related to velocity differences, $u_A$ can again be calculated as the magnitude of the difference between air and water velocities in cases where the velocities are of the same order.
3.0 Methods

3.1 Study Location in Lake Superior

Lake Superior is the largest freshwater body of water in the world in terms of surface area and one of the largest in terms of volume. It is also relatively easy to access, allowing for the study of large-scale processes in the lake that can be difficult to achieve elsewhere. While it has been warming considerably (Austin and Colman 2007; Austin and Colman 2008), Lake Superior is still the coldest of the Great Lakes, and its dimictic nature is less susceptible to change due to its “large thermal inertia” (Fichot et al. 2019). As such, Lake Superior is ideally suited for studying RDC, with less annual variability in stratification timing than the other Great Lakes, relatively large depth, and minimal wind shear during the springtime period of interest.

Multiple moorings were deployed in Lake Superior in May 2019 to investigate RDC, centered around the location indicated in Figure 2. The moorings were deployed in the western arm of the lake near a site with a temperature record dating back to 2005, as annual moorings are deployed there by the University of Minnesota Duluth’s Large Lakes Observatory. At this location, the water column is approximately 180 m deep, and the distance to both the Minnesota and Michigan shorelines to the northwest and southeast, respectively, is roughly 60 km. The data collected during this 2019 deployment is available on the Data Repository for University of Minnesota (DRUM) website (https://doi.org/10.13020/1XFM-MW95).
3.2 Data Description

3.2.1 Thermistor Data

A large, 2D array with both horizontal and vertical resolution was deployed at the location described in 3.1 and is the primary focus of this thesis and centerpiece of the deployment. The design of the mooring was inspired by previous work in Massachusetts Bay (Trask et al. 1999; Grosenbaugh et al. 2002). A schematic of the 2D mooring and nearby moorings deployed is shown in Figure 3. Winds to the southwest were present on the deployment date and pushed the R/V Blue Heron in that direction as individual parts of the mooring were dropped into the water. As a result, the 2D mooring was oriented in

![Figure 2: 2019 Lake Superior RDC Mooring Deployment Location. The center of the 2D mooring is marked by the asterisk in the western arm of Lake Superior.](image)
a southwest/northeast direction, roughly along the same orientation as the western arm of Lake Superior.

The 2D mooring consisted of two anchors (black squares in Figure 3) holding down diagonal lines supported by floats and connected to a 180 m long horizontal “headline”. This headline was positioned at roughly 34 m depth and remained nearly stationary for the entire deployment. Attached to the headline were 18 RBR TR-SOLO thermistors spaced 10 m apart and five vertical lines with RBR TR-SOLO thermistors spaced 30 m apart providing horizontal resolution at depth. The vertical lines included thermistors installed at depths of 44 m, 54 m, 74 m, 104 m, 134 m, and 164 m on each line. These thermistors provided horizontal resolution of 30 m at these depths in addition to the headline thermistors at 34 m depth with 10 m horizontal resolution. The TR-SOLO thermistors all recorded at 2Hz frequency with accuracies on the order of 0.002°C. Also included on the 2D mooring were RBR TR-Duet and TR-2050 sensors, measuring both temperature and pressure (depth) at various locations on the mooring, although temperatures recorded by the TR-2050 are not included in this analysis. Maximum variations found in the depth due to local currents disturbing the mooring were on the order of only 10 cm throughout the deployment (Austin et al. unpubl., manuscript in preparation).

**Figure 3: 2019 Lake Superior RDC moorings.** Lateral distance is used to show the scale of the horizontal mooring and is not to scale for separation of moorings. Vertical distances are given as heights above the bottom.

The 2D mooring consisted of two anchors (black squares in Figure 3) holding down diagonal lines supported by floats and connected to a 180 m long horizontal “headline”. This headline was positioned at roughly 34 m depth and remained nearly stationary for the entire deployment. Attached to the headline were 18 RBR TR-SOLO thermistors spaced 10 m apart and five vertical lines with RBR TR-SOLO thermistors spaced 30 m apart providing horizontal resolution at depth. The vertical lines included thermistors installed at depths of 44 m, 54 m, 74 m, 104 m, 134 m, and 164 m on each line. These thermistors provided horizontal resolution of 30 m at these depths in addition to the headline thermistors at 34 m depth with 10 m horizontal resolution. The TR-SOLO thermistors all recorded at 2Hz frequency with accuracies on the order of 0.002°C. Also included on the 2D mooring were RBR TR-Duet and TR-2050 sensors, measuring both temperature and pressure (depth) at various locations on the mooring, although temperatures recorded by the TR-2050 are not included in this analysis. Maximum variations found in the depth due to local currents disturbing the mooring were on the order of only 10 cm throughout the deployment (Austin et al. unpubl., manuscript in preparation).
An additional six TR-SOLO thermistors were included on a spar buoy located about 500 m northeast of the 2D mooring (shown directly to the right in Figure 3), measuring temperature at depths of 1 m, 3 m, 5 m, 10 m, 15 m, and 20 m. This buoy was included to provide near surface temperature records, which was not possible on the 2D mooring. Additionally, six more TR-SOLO thermistors were present on the acoustic Doppler current profiler (ADCP) mooring (described in the next subsection). These thermistors were installed at depths of 60 m, 80 m, 100 m, 120 m, 140 m, and 160 m.

Taking data from both the 2D mooring and spar buoy, Figure 4 shows the temperature record of thermistors from near surface down to over 160 m from throughout the deployment. The 2D mooring, along with most other the deployment with most other equipment included in this project, was deployed on 6 May 2019, and recovered on 16 July 2019.
A mooring consisting of an ADCP at 60 m depth and six thermistors at depths defined previously was deployed northwest of the 2D mooring (the far-right mooring shown in Figure 3). The ADCP on this mooring was a five-beam Nortek Signature 500 oriented upward, collecting data at 4Hz in burst intervals of 7 minutes on, 46 minutes off (53 minutes total) into 4 m vertical bins. Current velocity and magnitude for the entire deployment, as well as a five-day period of vertical velocities, are shown in Figure 5.

**Figure 4: Temperature record for 2019 Lake Superior RDC study.** The dashed vertical lines mark the end of negative stratification around 12 June and beginning of positive stratification around 7 July. Depths are displayed for the thermistor records plotted.

### 3.2.2 Acoustic Doppler Current Profiler Data
Currents were typically on the order of 2 cm s\(^{-1}\) (Figure 5a) and directed towards the southwest and along the 2D mooring (Figure 5b). Horizontal currents varied negligibly with depth and were depth averaged for the analyses presented in this thesis. The velocities shown in Figure 5c are intended to be representative of typical vertical velocities over the course of the study period. Since the ADCP was mounted at 60 m, there is no record of vertical velocities below that depth.

3.2.3 Vertical Microstructure Profiler Data
Vertical microstructure profiler (VMP) sampling was done on both the initial deployment cruise on 6 May 2019, and again on a mid-study cruise on 11-12 June 2019. The second cruise coincidentally lined up with the initiation of the RDC period at the study location in Lake Superior. The VMP was a Rockland VMP-250 with an average free-falling velocity of 0.65 m s\(^{-1}\). While the VMP collected a variety of data, only the temperature data is included in the scope of this thesis. Both FP07 and JAC temperature sensors were utilized, with respective sampling frequencies of 512Hz and 64Hz and accuracies/resolutions of 0.005°C/10\(^{-5}\)°C and 0.01°C/0.001°C. In total, 85 total profiles were collected on the two cruises. Data collected at depths above 5 m is discarded due to contamination from the boat wake, as profiles were collected while the R/V Blue Heron was in motion.

### 3.2.4 Meteorological Buoy Data

A meteorological buoy was also included in the 2019 RDC deployment and located approximately 600 m southwest of the 2D mooring. This buoy was equipped with Campbell sensors (CS100, CS107, CS215, CS310, and CS320) measuring barometric pressure, near surface water temperature, air temperature, relative humidity, and shortwave radiation. Additionally, an Apogee SL510 recorded downward longwave radiation and a Lufft WS200 recorded wind speed and direction; a MetSens MS600 included further measurements of wind, air temperature, relative humidity, and barometric pressure. Solar panels were also equipped on this buoy, charging the batteries that provided power to the sensors. Figure 6 includes most of the relevant data recorded by the meteorological buoy during the deployment.
Figure 6: Meteorological buoy data for 2019 RDC study. (a) Wind speed. (b) Wind direction (from). Dashed lines indicate the orientation of the 2D mooring. (c) Air temperature. (d) Relative humidity. (e) Barometric pressure
As seen in Figure 6a-b, winds were strongly aligned with the orientation of the 2D mooring for much of the period, with the strongest wind events coming from the northeast. Air temperatures (Figure 6c) increased progressively from May through July, with larger fluctuations appearing near the recovery date in mid-July. The shortwave radiation and downward longwave radiation measured by sensors on the meteorological buoy will be discussed further in a later section together with the other surface heat fluxes.
4.0 Results

4.1 Temperature Manipulation

4.1.1 Stratification Regimes

At the beginning of the measurement period in 2019, the water column in Lake Superior was negatively stratified, with the thermocline somewhere around 100 m depth. As can be seen in Figure 4, the temperature at 104 m oscillates between the range in the hypolimnion around 3.1°C and surface mixed layer, which warmed from roughly 1.6°C at the start of the deployment period until reaching the hypolimnion temperature, initiating the RDC period. This period began on 12 June 2019; the water column was nearly isothermal throughout for roughly a month until the onset of positive stratification on 7 July 2019. This RDC period, when the full water column is susceptible to vertical mixing, is the focus of remaining analysis in this thesis. The observable water column warmed uniformly from 3°C to 3.65°C when positive stratification started forming at the bottom of the lake as $T_{MD}$ was reached. At 164 m, the deepest point at which temperature was measured, $T_{MD}$ is roughly 3.65°C.

The exact onset and timing of this period is variable (Fichot et al. 2019). The isothermal period lasted from early May through late June 2017 near this area in Lake Superior (Austin 2019), beginning significantly earlier than the 2019 onset of RDC. In contrast, Austin and Allen (2011) observed a similar onset and length of the RDC period in 2007 for Lake Superior (6 June through 7 July). After stratification initiated at the lake bottom, surface waters reached $T_{MD}$ within a few days, forming full summer stratification. With a positively stratified water column, heating surface water with positive thermal expansion coefficients no longer creates density instabilities in the water column, signaling the end of RDC for the year.

4.1.2 High-passing Temperature
To study horizontal variability and small temperature variations due to RDC, it is necessary to manipulate the temperature to focus only on rapid events. Since all thermistors at one depth will warm uniformly on longer time scales and include a diurnal signal, small events during the day are difficult to isolate from larger trends. The headline temperature throughout the RDC period is shown in Figure 7.

![Figure 7: 2D mooring headline temperature during the 2019 RDC period.](image)

Thermistors see roughly uniform heating and a strong diurnal signal.

To account for both the longer, roughly monthlong warming trend as well as the diurnal signal clearly present in Figure 7, a variety of different running mean windows were tested as high pass filters for temperature data. A 24-hour running mean accounts for the longer-term trend; however, it fails to filter out relatively significant events during daytime hours when all thermistors warm or cool uniformly. Eventually, a 2-hour running mean was selected. This window was chosen to be short enough to filter out sub-diurnal trends yet long enough to prevent most convective events from being smoothed over.
Previous estimates of horizontal scales were anywhere from 50 m in ice-free Lake Superior (Austin 2019) to around 150 m under ice in Pavilion Lake (Forrest et al. 2008). With average current speeds on the order of 2 cm s$^{-1}$ during the study period, water travels roughly 150 m over a 2-hour period, justifying the 2-hour window selected. Figure 8 includes the same range of temperature data as in Figure 7 and shows both the low-passed and high-passed signals.

![Temperature Data Graph](image)

**Figure 8: Headline temperature filtering.** (a) 2-hour running mean of headline temperature. (b) High-passed headline temperature with 2-hour running mean subtracted.

Now, the near month-long warming trend (Figure 8a) is removed from the temperature record, and a strong diurnal signal is present (Figure 8b). Temperature activity is high during the day and returns to ambient levels overnight. It is important to
note that the negative temperatures seen in Figure 8b are not truly negative water temperatures, but rather simply negative variations from the running mean. Now, the magnitudes seen in Figure 8b should represent the magnitudes of convective events that are the focus of this thesis. Negative values likely indicate the transition of a thermistor from anomalously warm water to cold, ambient water.

4.2 Convective Cell Observations

4.2.1 Initial Observations

Figure 8b suggests that the convective events of interest are on the order of 0.05-0.1°C. Picking out individual thermistor records and looking at shorter time scales allows for further investigation into the structure of convective cells. Figure 9 provides a zoomed in look at a single thermistor record, highlighting the nature of the convective cells present. Also, when inspecting the structure of individual cells, the high pass filter discussed in the previous section is not applied.
The temperature record in Figure 9 includes at least three separate events with sharp temperature gradients and magnitudes on the order of 0.05-0.1°C. Additionally, all three events observed remain detected by the thermistor for time spans on the order of 30 minutes or less. Next, the horizontal resolution provided by the headline on the 2D mooring can be utilized to examine the coincidence of these cells at different points in space. Figure 10 includes two separate 4-hour time periods, each with temperatures from three neighboring thermistors at the same depth. In both time periods shown in Figure 10, sudden temperature events are observed on each thermistor record. These thermistors were spaced 10 m apart, and the same events can often be observed on all three. The event showing up on the bottom-most thermistor in Figure 10a around 12:45 also shows
up on the next two thermistors at a delayed time and with slightly different structure. Other events in both Figure 10a and 10b also seem to occur on all three thermistor records at varying times, suggesting propagation of these cells along the 2D mooring. Currents measured by the ADCP at a nearby mooring can then be used to confirm if this propagation being observed is due to background flow at the 2D mooring.

4.2.2 Cell Propagation

Convective cell propagation along the headline can be observed in multiple ways. One such method is shown in Figure 11, which plots multiple thermistor records with an

![Figure 10: Zoomed-in temperature records for three neighboring headline thermistors.](image)

(a) Headline temperatures spaced 10 m apart with added temperature offset to emphasize separation on 13 June 2019 10:30-14:30, CST. (b) Same as (a) but for 4 July 2019 10:30-14:30, CST.
added temperature offset as in Figure 10, now including more thermistors to accentuate propagation along a larger distance.

![Figure 11](image)

**Figure 11: Headline thermistor records for 10 neighboring thermistors.** Thermistors are again spaced 10 m apart, so the total separation from the bottom thermistor to the top is 90 m. The dashed line represents the average current speed (3 cm s\(^{-1}\)) around that time on 16 June 2019.

Again, as in Figure 10, sudden temperature events are observed by the thermistors, with many being present on all ten thermistors plotted in Figure 11. The average current at 15:00 CST on 16 June 2019, roughly 3 cm s\(^{-1}\), was superimposed onto the temperature records to emphasize the propagation. The rate of temperature cell propagation roughly agrees with the slope of the current speed line. The larger event to the left of the dashed line shows up clearly on all thermistors, yet the smaller event to the right shows up briefly on the third bottom most thermistor and increases in size progressively as it reaches the rest of the thermistors. In this case, since the current was not directly along
the headline orientation, it is difficult to say if the lack of observation on the bottom two thermistors is due to the cell not being formed yet or if the cell was just pushed at an angle to the headline. This current was to the southwest as was most common during the study period (Figure 5b), and the thermistors shown are oriented southwest to northeast, top to bottom.

Another visualization of this propagation is shown in Figure 12. The diagonal structure represents propagation along the headline, with the warm convective events being observed first at the thermistor located at 180 m (northeast) and then progressively being observed at each thermistor until reaching the thermistor located at 10 m (southwest). While most of the observed anomalously warm events appear on all thermistor records, some are only observed by a few neighboring thermistors, such as the event around 16:30 CST observed by thermistors located at 10-80 m. The propagation speed is estimated using lag propagation analysis (described in section 4.5.1) as 4.9 cm s⁻¹ and superimposed on the plot as the red dashed line. This matches well with the ADCP-measured current, also 4.9 cm s⁻¹. Also of note, this current was calculated to be oriented only 6° away from the headline orientation, which is considered to be a fairly strong along headline current.
Alternatively, during periods with currents not oriented along the headline, the same propagation as in Figures 11-12 is not observed. Figure 13 shows a time period with

**Figure 12: Cell propagation along the headline on 4 July 2019.** Time is shown on the vertical axis, and lateral distance, or position along the headline, is shown on the horizontal axis. Then, the diagonal nature of the image indicates propagation of warm temperature events from one end of the headline to the other. The red dashed line represents the observed propagation speed calculated from lag propagation analysis, $4.9 \text{ cm s}^{-1}$. 

Alternatively, during periods with currents not oriented along the headline, the same propagation as in Figures 11-12 is not observed. Figure 13 shows a time period with
a stronger cross headline current component, plotted in the same way as Figure 12. During this time period, the current magnitude was 2.9 cm s\(^{-1}\) at an angle of 52° away from the headline orientation. While the clear propagation in the preceding figures is no longer apparent, the presence of convective cells is still observed. From around 15:30 through 16:45 this day, convective cells seem to be present at thermistors located at 10 m through 120 m.
The same activity shown in Figures 12-13 is also observed at themistors below the headline. However, the lower horizontal resolution at those depths makes the observations and visualizations much less clear. Since the current is roughly uniform with depth, it is assumed that the propagation of these convective cells is of the same nature.

Figure 13: Time period showing presence of convective cells without clear propagation. The averaged current during this period on 28 June 2019 was 2.9 cm s$^{-1}$ with an orientation of 52° away from the headline.
throughout the water column. Instead of recreating the figures and inspecting horizontal structure at different depths, the vertical thermal structure is investigated in the following subsection.

4.2.3 Vertical Structure

To get the best resolution (< 1 m) and view of vertical structure during the RDC period, profiles collected by the VMP are used. While the VMP is intended mainly as a turbulence profiler, the temperature recorded can also be useful. Here, examples of vertical profiles collected during a 11-12 June 2019 cruise are considered. Figures 14-16 show the various types of thermal structure observed: quiescent at depth, active at depth, and warm at depth with a cooler layer above, respectively.
Figure 14: Active near surface heating with quiescent water columns below. (a) 13:53 on 11 June 2019, CST. (b) 16:24 on 11 June 2019, CST. Both profiles exhibit near surface heating with temperature anomalies on the order of 0.1°C, yet no significant events occur at depths below 20 m.
Figure 15: Active near surface heating and events occurring at depth. (a) 12:25 on 11 June 2019, CST. (b) 10:04 on 12 June 2019, CST. Significant temperature events at depth surrounded by ambient, background temperature levels. Convective events appear on the order of 0.05°C.
Figure 14 shows two examples of profiles with relatively quiescent water columns during the day. Heating at the surface is clearly observed, yet no convective anomalies appear present at depth. In contrast, Figure 15 shows similar surface heating with clearly observed temperature events at depth. These sudden temperature fluctuations at depth are surrounded by an ambient, background temperature that appears constant with depth. A clear event is observed in Figure 15a at roughly 50 m with an anomalous temperature of roughly 0.05°C and height on the order of 5-10 m. Figure 15b features one clear event at depth, captured from roughly 45 m to 60 m depth, and a temperature fluctuation of approximately 0.03°C. The two profiles shown in Figure 16 show a much different

**Figure 16: Cooler upper water columns with warm region below.** (a) 15:14 on 11 June 2019, CST. (b) 8:28 on 12 June 2019, CST. Negligible surface heating is seen in (a) with small amounts seen in (b). Anomalously warm regions starting at ~70 m (a) and ~60 m (b) continue to the end of the measured depth.
vertical thermal structure. In both Figure 16a-b, a significantly warmer region of water exists below around 60 m, progressively warming with depth to about 80 m. Below that depth and to the end of the measured range, the temperature remains steady. The magnitude of these temperature increases at depth is significant, as the warmer region at the bottom of the observed water column in Figure 16b is around 0.1°C warmer than the ambient region directly above it. Although the exact nature is unknown, it is possible that the profile in Figure 16b, which was collected early morning before convective cells would reach these depths, is explained as warmed water heated the previous day remaining intact at depth without diffusing uniformly throughout the water column. Since these profiles were collected on 11 and 12 June, it is also likely that they were influenced by the initiation of the RDC period on 12 June following negative stratification, and it is possible that these two profiles were collected in areas where weak stratification was still present.

4.3 Meteorological Forcing

4.3.1 Surface Heat Fluxes

The role of currents in transporting convective cells has been discussed briefly, yet the background flow is not responsible for the creation of these temperature anomalies. To understand how the surface waters warm and cool, the surface heat fluxes must be considered. Using data from the meteorological buoy, with most of the relevant data shown in Figure 6, the surface heat and momentum fluxes were calculated using the algorithms developed by Fairall et al. (1996). Since the downward longwave radiation was recorded by the meteorological buoy, only the upward longwave radiation was calculated and combined to form a net longwave term. The three non-shortwave radiation heat fluxes are shown in Figure 17, while Figure 18 displays the total heat flux and dominance of the shortwave radiation.
Figure 17: Surface heat fluxes without shortwave. (a) Net longwave radiation. (b) Latent heat flux. (c) Sensible heat flux. Positive values indicate heat flux into the lake. Anomalous event observed on 30 June 2019 was recorded by multiple sensors and likely not an error.
The net longwave radiation is roughly centered at 0 W m$^{-2}$ and contributes the largest negative heat flux during the period (Figure 17a). Both the latent heat flux (Figure 17b) and sensible heat flux (Figure 17c) typically range from 0 through 50 W m$^{-2}$ but record anomalously high levels (>100 W m$^{-2}$) on 30 June 2019. This event corresponds to a large jump in wind speed from 5 to 15 m s$^{-1}$ (Figure 6) and is likely not an error, as it was observed by multiple sensors.

The springtime period examined here is a large contributor to Lake Superior’s annual heat budget, as roughly half of the annual heat income is used to warm the lake until surface waters reach $T_{MD}$ (Bennett 1978). The mostly positive contributions of both
the latent and sensible heat flux can be explained by RDC transporting warmer water to the bottom of the lake, especially during the full water column mixing period between 12 June and 7 July 2019. With warmer and denser water sinking throughout the period, cooler water remains present at the surface, and conductive heating is more efficient (Bennett 1978; Schertzer 1978).

Lofgren and Zhu (2000) also estimated low latent and sensible heat fluxes in the Great Lakes, representing a small percentage of the total heat flux into the lake. This is represented clearly by Figure 18a, showing the dominance of the shortwave radiation term throughout the measurement period. The only instances where these terms become significant include cloudy days with anomalously low shortwave radiation, such as 18-19 May 2019, 27 May 2019, or 24 June 2019. Heat flux out of the lake is limited to overnight periods when shortwave radiation is inactive (Figure 18b).

4.3.2 Meteorological Forcing Effect on Temperature Variability

Positive heat flux into the lake warms the water column and generates temperature anomalies at the surface. The abundance of temperature anomalies can be estimated by calculating the standard deviation of temperature, hereafter referred to as the temperature variability. Since temperature anomalies are sudden events and disturb ambient water, abundance and magnitude of these convective cells should be proportional to the high-passed temperature variability. As apparent from Figure 18, shortwave radiation is the largest contributor to the heat flux and is thus considered the sole mechanism behind RDC generation. To illustrate a typical shortwave signal, the average diurnal shortwave radiation profile during the RDC period was calculated and is shown in Figure 19.
As expected, the shortwave radiation peaks at local noon (Figure 19). This signal was averaged over roughly a month, so variable sunrise and sunset times were incorporated into the averaging. The average peak shortwave radiation was around 800 W m\(^{-2}\), yet typical maximum shortwave radiation values during the period were closer to 1000 W m\(^{-2}\). Besides considering shortwave radiation as the only significant meteorological factor influencing RDC, wind speed must also be addressed. The relationship between these forcing parameters and temperature was investigated by comparing the daytime temperature variability, the daily integrated shortwave radiation, and the daily averaged wind speed. To get a single measurement for temperature variability each day, the standard deviations of all headline thermistors were averaged. The results are shown in Figure 20. The analysis used a daily time window of 7AM-9PM

**Figure 19: Averaged diurnal shortwave radiation profile.** Daily data was averaged over the RDC period from 12 June through 7 July 2019. The vertical dashed line represents local noon CST, coincident with the daily shortwave radiation peak.
with the intent of ensuring both meteorological forcing and temperature variability were equally active over the entire period.

**Figure 20: Temperature response to meteorological forcing.** (a) Daytime temperature variability as a function of the daily-integrated shortwave radiation. (b) Daytime temperature variability as a function of the daily-averaged wind speed. Pearson correlation coefficient and \( p \) values are reported for both relationships.

While Figure 20a shows a reasonably strong correlation between temperature variability and shortwave radiation (Pearson correlation coefficient = 0.443, \( p \) value = 0.026), Figure 20b suggests that there is no significant correlation between temperature variability and wind speed, stated by the relatively large \( p \) value of 0.373. It is unknown exactly how these two factors impact the thermal structure during RDC, but it is assumed that shortwave radiation builds up the temperature anomalies by providing heat and higher wind speeds prevent buildup of larger anomalies by generating surface waves that disturb convective cell formation. To investigate the combined effect of solar radiation and wind speed on the temperature variability, a multiple linear regression analysis was performed. Using the same data as described for the analysis in Figure 20, the regression analysis produced a \( R^2 \) value of 0.238 and \( p \) value of 0.05. This calculated \( p \) value is just within the typical significance level of 0.05, indicating that the temperature variability is dependent upon some combined effect of shortwave radiation and wind speed. Other surface heat fluxes likely play a negligible role relative to shortwave radiation on the
generation of convective cells, yet the interdependence of factors and nonlinear nature is difficult to determine.

4.4 Convective Cell Temporal Properties

Following the discussion of the meteorological forcing during the RDC period, an analysis of convective cell generation resulting from the forcing as well as eventual decay is considered. First, the lake phase response to the incoming shortwave radiation signal is investigated, followed by several estimates of velocity scales and convective decay rates. This analysis provides more information on the extent and effectiveness of shortwave radiation during a period where it is the sole significant heat source and mechanism driving vertical mixing.

4.4.1 Phase Analysis

As first utilized by Farmer (1975) for RDC in an ice-covered lake, a spectral analysis is a useful technique for investigating the diurnal nature of RDC and the phase delay throughout the water of shortwave radiation incident at the surface as well as the magnitude of the signal at depth. This analysis was recreated for Lake Superior for 2017 RDC period (Austin 2019), and the same technique is applied here for the 2019 RDC period. The periodic diurnal signal is the only frequency considered, and the temperature record for each thermistor can then be represented by a Fourier series with amplitude

\[ F_\omega = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} T(t) e^{-i\omega t} dt \]  

(4.1)

where \( t_2 \) and \( t_1 \) represent the start and end of the RDC period (12 June and 7 July, respectively), the difference \( t_2 - t_1 \) is the integer number of days, \( T(t) \) is the temperature record for a given thermistor, and \( \omega \) is the diurnal frequency, \( \frac{2\pi}{24\text{hr}} \). Additionally, the phase angle can be calculated and converted from radians to hours, resulting in a phase lag defined as
\[ \tau_{\text{lag}} = \frac{1}{\omega} \tan^{-1} \left( \frac{b_\omega}{a_\omega} \right) \] (4.2)

where \( b_\omega \) and \( a_\omega \) represent the sine and cosine coefficients in the diurnal Fourier series, respectively. Data from both the 2D mooring and nearby spar buoy were included in this harmonic analysis to get as complete an image as possible over the entire water column, and the results are displayed in Figure 21.

![Figure 21: Phase analysis for 2019 RDC period. (a) Magnitude of phase with depth. (b) Phase lag in hours with depth, including a linear fit for depths > 50 m. In both (a) and (b), the orange points indicate thermistor data from the spar buoy, and blue points indicate thermistor data from the 2D mooring.](image)

As expected, due to the exponential heating from incident shortwave radiation, Figure 21a shows the magnitude of the phase decreasing with depth exponentially. The magnitude is at a maximum near the surface, and by roughly 80 m depth, the magnitude remains uniform throughout the observed water column. The phase lag shown in Figure 21b is mostly linear with depth. To get an estimate of the phase propagation speed, a
linear fit was applied to the lag values below 50 m, as this range was the most linear. The slope of this fit line represented a propagation speed of 3.9 mm s\(^{-1}\). This value agrees well with results found by Austin (2019) for multiple moorings in western Lake Superior. This speed represents the downward propagation of the incident shortwave signal at the surface and is not the same as downward velocities generated by individual convective cells.

### 4.4.2 Convective Velocity and Decay

The velocity of individual convective cells or plumes can be estimated with the Deardorff velocity scale (Deardorff 1970), defined here as

\[
w_D = (Bh)^{1/3}
\]  

(4.3)

where \(B\) is the buoyancy flux and \(h\) is the depth of the convectively mixed layer. During the RDC period, the convectively mixed layer is essentially the entire water column, so \(h\) is set to the water depth at the mooring location, 180 m. The parameter responsible for the convective instabilities formed during RDC, buoyancy flux [m\(^2\) s\(^{-3}\)], is given by

\[
B = -g \alpha I_k
\]  

(4.4)

where \(I_k\) is the kinematic heat flux [\(°C \cdot m \cdot s^{-1}\)] due to shortwave radiation, found by dividing the shortwave radiation heat flux defined in Equation 2.6 by the density and specific heat capacity of water \(I_k = \frac{I(z)}{\rho_0 c_p}\). The buoyancy flux during the RDC period is positive, or pointed downward into the water column, as the thermal expansion coefficient is negative for temperatures below \(T_{MD}\). The convective velocity varies throughout the measurement period, as both the magnitude of the buoyancy flux and thermal expansion coefficient decrease approaching \(T_{MD}\).

The vertical velocity scale defined in Equation 4.4 was estimated for several different surface temperatures. The incident shortwave radiation at the surface was assumed to be 800 W m\(^{-2}\), roughly the average daily peak signal during the measurement
period (Figure 19). Then, the buoyancy flux and velocity scale were calculated for various scenarios, and results are summarized in Table 1.

<table>
<thead>
<tr>
<th>Temp., °C</th>
<th>( \alpha, ^\circ C^{-1} )</th>
<th>( B, m^2 s^{-3} )</th>
<th>( w_D, \text{cm s}^{-1} )</th>
<th>( \tau_D, \text{hours} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>-3.28 \times 10^{-5}</td>
<td>6.15 \times 10^{-8}</td>
<td>1.83</td>
<td>0.91</td>
</tr>
<tr>
<td>3.0</td>
<td>-1.60 \times 10^{-5}</td>
<td>3.00 \times 10^{-8}</td>
<td>1.76</td>
<td>1.71</td>
</tr>
<tr>
<td>3.4</td>
<td>-9.46 \times 10^{-6}</td>
<td>1.77 \times 10^{-8}</td>
<td>1.47</td>
<td>2.03</td>
</tr>
<tr>
<td>3.9</td>
<td>-1.38 \times 10^{-6}</td>
<td>2.58 \times 10^{-9}</td>
<td>0.77</td>
<td>3.87</td>
</tr>
</tbody>
</table>

**Table 1: Convective velocity scale and decay estimates.** Thermal expansion coefficients, buoyancy fluxes, Deardorff velocity scales, and convective decay times are calculated for several surface temperatures.

The four temperatures considered in Table 1 are intended to cover the entire measurement period, with the first representing the negatively stratified period when the convectively mixed layer was roughly 100 m. The second temperature is approximately the surface temperature at the beginning of the RDC period, while the third is a temperature roughly during the middle of the period. The final temperature signals the end of the RDC period and positive stratification. With increasing temperatures, and thus decreasing thermal expansion coefficients and buoyancy fluxes, the convective velocities were found to decrease throughout the measurement period. During the RDC period, typical peak convective velocities during the daytime were on the order of 1.5 cm s\(^{-1}\).

Again following in the steps of Austin (2019) in Lake Superior, the convective damping time, or time required for convective cells to decay, was calculated. The convective damping time is estimated by

\[
\tau_D = 0.6 \left( \frac{h^2}{B} \right)^{1/3}
\]  

where 0.6 is a previously determined constant (Lombardo and Gregg 1989; Kelley 1997), and the other two terms are previously defined. The convective damping times calculated range from 0.9 hours in May before RDC initiation to 3.9 hours at the end of the RDC period (Table 1). Typical convective damping times during the RDC period for peak
shortwave heat flux were on the order of 2 hours. With a general understanding of the generation and movement of convective cells, it is then possible to investigate the geometry of these cells.

4.5 Correlation Analysis

Following the initial observations of convective cells presented in 4.2, a more statistical approach was considered to capture the horizontal scales present. After removing longer term temperature trends, a correlation analysis was completed for thermistors on the 2D mooring. The intent of this correlation analysis is to capture the coincidence of anomalous temperature events like those that appear in Figures 9-11. However, the direction of the current needs to be considered, as periods with a strong along-headline current show clear propagation with all thermistors seeing roughly the same event with a delay (Figure 12); no clear propagation is observed when the cross-headline current is stronger, yet neighboring thermistors that see the same event may provide information on lateral scales (Figure 13).

To prevent results from being skewed by combining periods with both cross and along-headline currents, the correlation analysis was done separately for each scenario. As with the previously discussed analysis investigating meteorological forcing, only daytime hours are included in the analysis. Here, considering active daytime periods only is especially important, as correlating temperature records between thermistors overnight when the water column is inactive would produce high correlation values and not provide the desired information from the analysis.

Daytime hours were chosen to be 8AM-8PM CST, which roughly corresponded to typical start and end times for anomalous temperature activity each day. Then, the 12-hour daytime period was divided into 3-hour windows to determine whether a period should be included in either along headline current or cross headline current correlation analyses, and this determination was made based on the calculated current angle. The
current direction cutoffs for whether a time period was considered to have along or cross headline current were less than 15° and greater than 45°, respectively. This determination was both based on observations of temperature anomaly propagation along headline thermistors and to ensure that both data sets analyzed were roughly the same length (each included around 30 3-hour time periods out of 96 total). All possible pairs of headline thermistors were included in the analysis, and the resulting correlations were then averaged for each separation distance (10 m – 170 m). Since the intent is to obtain a statistical description of a large amount of data, error propagation is incorporated into this analysis. The error on the mean was used as the uncertainty on the final correlation value for each separation distance except for the correlation value for 170 m separation, which was estimated to be the average standard deviation of all correlations.

4.5.1 Along-Headline Current

Before considering the described correlation analysis for periods with along headline current, the nature of convective cell propagation is first investigated. To highlight the importance of the flow direction on the resulting correlation and the delayed observation of the same temperature anomaly at all headline thermistors during periods of along-headline flow, cross (lag) correlation between neighboring headline thermistors was attempted for various periods with primarily along-headline flow. Figure 2 displays lag correlation results for 4 July 2019, when the current was oriented less than 10° away from the headline. This analysis was highly sensitive to current direction and thermistor separation distance. Since the current always had some cross-headline component, thermistors spaced further apart were less likely to see the same temperature anomaly even on days with strong along-headline currents. The period analyzed for Figure 22 only includes the lag correlation results for 10 neighboring thermistors, or a total separation distance of 100 m.
Clear evidence of propagation is observed in Figure 22. This lag correlation introduces lags, or time shifts, into each of the headline thermistor records. Figure 22a shows the maximum cross-correlation value among all lag values as a function of separation distance along the headline. With increasing separation, the maximum correlation value drops. Even though all thermistors are likely seeing the same convective cell, there is enough temporal variation and perhaps even enough deflection away from the headline due to current orientation to decrease the correlation as the cell progresses. The clear cell propagation on the headline is best illustrated in Figure 22b, showing the lag value to maximum correlation (in minutes) for separation distances from 10 m through 100 m. This observed linear response is expected for periods with primarily along-headline flow.

It is then possible to use the slope value to calculate a propagation speed along the headline for comparison with the ADCP-measured current. Error bars were chosen based on the resulting linear fit and chi-square ($\chi^2$) since a linear relationship was expected. Here, the standard form of $\chi^2$ is used, defined as

\begin{figure}[h]
    \centering
    \includegraphics[width=\textwidth]{figure22.png}
    \caption{Lag correlation for 4 July 2019. (a) Maximum cross correlation value calculated for each separation distance. (b) Lag time to maximum cross correlation observation in minutes for each separation distance including slope of fit line to data.}
\end{figure}
\( \chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} \)  

where the summation is over all data points, \( O_i \) is the observed temperature data and \( E_i \) is the expected data from the fit line. The slope of the fit, shown in Figure 21b, was inverted to calculate the propagation speed implied by the lag correlation results: \( 4.9 \pm 0.1 \text{ cm s}^{-1} \). The magnitude of the average ADCP-measured current during this time window was \( 4.8 \pm 0.2 \text{ cm/s} \). Error for this current was estimated by averaging current measurements at different depths as well as averaging throughout the day. These two currents, one directly measured by an ADCP and the other estimated by lag correlation, show significant agreement and strongly imply that the convective cells present propagate along the headline. Time periods with stronger cross-headline currents do not show the same, clean linear relationships as in Figure 22, emphasizing the importance of accounting for current direction when performing correlation analysis and determining convective cell sizes.

The same technique used to calculate the propagation speed for a single day was then applied to the entire RDC period and compared with the ADCP-measured currents. Instead of performing the analysis over an entire day, an hourly lag correlation was calculated between headline thermistor pairs, and the lag value to maximum correlation was recorded. Then, the propagation speed estimated from the lag correlation was calculated by dividing the separation distance by the lag value. Figure 23 shows the results of this analysis and includes periods with current angles less than 45° away from the headline, and the ADCP-measured along-headline velocity is included for comparison. Throughout the period, there is significant agreement between the estimated propagation speeds from lag correlation and along-headline currents, especially later in the period from late June through early July. The lag propagation speed outliers are likely due to either the cross-headline current component being significant or the lag correlation not successfully capturing the propagation of cells, but rather just some common background signal between thermistors.
Following this lag correlation analysis, the straightforward linear correlation was completed, as described at the beginning of 4.4. To reiterate, only daytime hours throughout the RDC period are considered, and only 3-hour windows with primarily along headline current (<15° from headline) are included in this analysis. The results of the correlation analysis for all along headline flow periods is shown in Figure 24.

**Figure 23: Lag propagation speeds compared to ADCP-measured currents.** The small x’s represent the actual data included in the comparison, and the orange lines connecting them indicate consecutive estimates during periods of consistent along-headline flow.
All p values reported from the correlation calculations were lower than the typical 0.05 level. While correlation values are quite small for most separation distances after 30 or 40 m, the amount of data included (2Hz sampling frequency, over 600,000 points) was likely enough to mask any significant decorrelation distance estimation resulting from a higher p value suggesting data sets are independent. Additionally, even after high-passing and removal of nighttime periods from the temperature records, it is reasonable to assume some correlation between thermistors at the same depth on this < 200 m lateral scale.

Figure 24 shows the highest correlation values at the smallest separation distance of 10 m.
before dropping rapidly and reaching a negative minimum around 100 m. Following this, correlation coefficient values increase back to 0.

As already examined earlier in this subsection and illustrated in Figures 11-12, thermistors frequently observe the same convective cells propagating along the headline during periods with along-headline current. Consequently, straightforward correlation analyses like this may not be entirely appropriate for along headline flow periods, as the delayed coincidence of these convective cells at neighboring thermistors is unaccounted for. Alternatively, if the current is primarily across the headline, only neighboring thermistors will detect a single convective cell and may provide a better estimate of horizontal dimensions of these cells.

4.5.2 Cross-Headline Current

The same correlation as completed for Figure 24 is recreated here, now for periods with primarily cross-headline flow. The cutoff angle for a period to be considered cross-headline was 45° away from the headline orientation. The results from the linear correlation analysis for this setup are shown in Figure 25. Correlation coefficients drop off rapidly with separation distance, reaching an initial minimum here at 50 m. Error bars are significantly larger than those seen in Figure 24, as standard deviations of the correlations are much higher for cross headline periods due to the same anomalous temperature cells not passing all thermistors. When the current is along the headline, correlations vary less since each thermistor pair observes roughly the same temperature fluctuations as cells propagate along all thermistors, leading to a lower standard deviation and error on the mean. While most p values are all significantly lower than the typical 0.05 level and do not suggest independence of thermistor pairs, the p values for 120 m and 140 m separation were 0.155 and 0.131, respectively. These results are not entirely surprising, as it is expected to see lower correlations and thus larger p values during these periods where thermistors are unlikely to observe the same temperature fluctuations.
Since single point moorings are only able to record temperature at a fixed location in a Eulerian frame, it is not possible to estimate the temperature rate of change of a parcel of water as it is moving. Alternatively, sensors placed on a drifter free to move in a non-zero background flow are able to measure the rate of change of a parcel as it moves in a Lagrangian frame, yet are not able to estimate the temporal change at fixed points. The mooring setup for the RDC investigation considered in this thesis allows for an estimation of temperature change in both Eulerian and Lagrangian frames.

Figure 25: Headline thermistor correlation during periods of cross-headline flow. The same description as stated for Figure 24 applies here.

4.6 Material Derivative Analysis

Since single point moorings are only able to record temperature at a fixed location in a Eulerian frame, it is not possible to estimate the temperature rate of change of a parcel of water as it is moving. Alternatively, sensors placed on a drifter free to move in a non-zero background flow are able to measure the rate of change of a parcel as it moves in a Lagrangian frame, yet are not able to estimate the temporal change at fixed points. The mooring setup for the RDC investigation considered in this thesis allows for an estimation of temperature change in both Eulerian and Lagrangian frames.
To understand how this is done, it is first necessary to start with the definition of the material, or Lagrangian, derivative of temperature in this system, defined in Equation 2.5. The material derivative term on the left-hand side can be expanded into a local time rate of change term (Eulerian frame) and advective terms in all three spatial dimensions, given by

\[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \quad (4.7)
\]

where terms are the same as previously defined, and here the x, y, and z directions are oriented parallel to the headline, perpendicular to the headline, and downward into the water column, respectively. The first term, \(\frac{\partial T}{\partial t}\), is estimated by simply taking a first difference of temperature data and dividing by the time step, or \(\frac{\Delta T}{\Delta t}\). The 2D mooring only has 1D lateral resolution, and considering the axes orientation, \(u \frac{\partial T}{\partial x}\) can be estimated while \(v \frac{\partial T}{\partial y}\) cannot. During periods with primarily along headline flow, \(v \approx 0\), and the vertical advective term is assumed to be relatively small (justified by ADCP measurements), so the material derivative reduces to the first two terms on the right-hand side of Equation 4.7 and can be estimated by thermistor records on the 2D mooring. This is possible due to the relatively high lateral resolution of the headline thermistors (10 m) and the coincidence of a local ADCP. The raw east/north oriented ADCP data was rotated to the headline orientation (roughly 30° from due east) to provide both \(u\) and \(v\). Then, \(\frac{\partial T}{\Delta x}\) was calculated by selecting thermistors along the headline and dividing the first difference of temperature by the separation distance.

Three neighboring thermistors were selected for this analysis, and the local time rate of change was calculated for the middle thermistor. Then, the two outside thermistors were used for the spatial derivative, so \(\Delta x\) was 20 m. Before performing the calculation, some consideration of the resolution is required. Here, the local rate of change term has 2Hz resolution, while the spatial derivative is limited by 10 m horizontal resolution. Considering a typical current during this period of 2 cm s\(^{-1}\), this spatial resolution
corresponds to roughly 0.002Hz resolution. The time rate of change derivative is therefore low pass filtered to allow for comparable resolutions between the two terms. To illustrate the agreement in magnitude between these two terms during a period with along-headline flow, Figure 26 provides estimates of both the local rate of change term and advective term for 4 July 2019, when the average current direction was within 6° of the headline orientation.

Figure 26: Material derivative estimation for 4 July 2019. (a) Inverted local time rate of change term in blue and advective term in orange for 4 July 2019. (b) Two-hour window of same values as in (a), centered on noon CST on 4 July 2019.
Significant agreement is observed for the two terms throughout the day (Figure 26a; rho = 0.71, p value \(\approx 0\)), where the local time rate of change term is inverted to highlight the similarity. Figure 26b provides a zoomed-in look at a 2-hour period during the same day, further emphasizing the similar magnitudes and fluctuations observed. This smaller window displays a period near peak heating and convective cell activity during the day, and the same rapid fluctuations observed are captured by both terms. Alternatively, during periods with considerable cross-headline flow, this balance does not exist.

Figure 27 shows the same results for 14 June and 22 June 2019, during which average current angles relative to the headline orientation were roughly 56° and 86°, respectively. In contrast to the results in Figure 26, the material derivative terms for the two days shown in Figure 27 show no significant agreement. Few events are observed by both terms, with the advective term exhibiting inactivity during extended periods while the local time rate of change term is not. During these days, the cross-headline velocity, \(v\), is no longer negligible, so it is not possible to estimate the entire advective contribution as it no longer is only due to a single term.
Figure 27: Material derivative estimation for 14 June and 22 June 2019. (a) Inverted local time rate of change term in blue and advective term in orange for 14 June 2019. (b) Same as (a) except here for 22 June 2019.
5.0 Discussion

5.1 Convective Cell Formation and Lateral Scale Estimates

Evidence of convective cells and their propagation has clearly been observed from the collected 2D mooring, VMP, and ADCP data. The nature of these cells is presented in Figure 9, showing the presence of sudden events with temperature anomalies on the order of 0.05-0.1°C. This scale is roughly the same throughout the RDC period, with more significant events occasionally reaching anomalous temperature values over 0.1°C. These estimates agree well with previous observations in an ice-covered lake (Forrest et al. 2008) and ice-free Lake Superior (Austin 2019). While slightly larger, these anomalous temperatures are at least on the same scale as those observed during surface cooling in a small lake (~0.02°C), where the convective plumes represent cooler regions than the surrounding, ambient water (Jonas et al. 2003b).

The size of temperature anomalies is dependent upon multiple factors. Temperature variability was shown to respond significantly to shortwave radiation (Figure 20a), yet the relationship to wind speed was not as clear but still suggestive of an inverse relationship (Figure 20b). Naturally, higher rates of solar heating at the surface will lead to larger temperature increases in the water column. However, larger temperature increases do not necessarily imply that convective cells become progressively larger, as the thermal expansion coefficient must also be considered. In addition to the wind potentially breaking up the formation of convective cells at the surface, the buoyancy flux plays a role in determining the size of convective cells as they sink. The thermal expansion coefficient approaches 0 °C⁻¹ as water temperatures rise to $T_{MD}$, so larger temperature gradients are required to produce enough buoyancy flux, or a large enough density gradient, to drive downward motion of convective cells forming at the surface. Figure 28 shows the results of a similar analysis as done in Figure 20, yet now for the temperature variability and thermal expansion coefficient.
A clear relationship is observed between the temperature variability and thermal expansion coefficient in Figure 28. The temperature variability was calculated the same way as earlier for Figure 20. The thermal expansion coefficient was calculated by taking the average daytime temperature at the headline depth and using the formula derived by Chen and Millero (1986). These two parameters show much more significant correlation than either set in Figure 20, with a reported Pearson correlation coefficient of 0.660 and \( p \) value of 0.0003. This \( p \) value is much lower than those reported earlier, indicating significant correlation between temperature variability and \( \alpha \). While not the same as magnitude of convective cell temperature anomalies, the temperature variability is likely a good representation. Then, these results suggest that the size of convective cell temperature anomalies increase progressively with \( \alpha \), which increases roughly linearly.
with temperature at values below $T_{MD}$ (Figure 1). However, since shortwave radiation, wind speed, and $\alpha$ all show trends throughout the RDC period, it is difficult to fully distinguish the effects of all the different forcing mechanisms present.

In addition to forming near the surface and sinking, these cells have been shown to propagate along with background currents. Similar cell propagation was observed for convective cells resulting from surface cooling in Pavilion Lake (Forrest et al. 2008). The propagation is apparent in Figures 11-12, as ADCP-measured currents show strong agreement with propagation speeds calculated from lag correlation analysis at the headline thermistors spaced 10 m apart. In contrast, the same behavior is not observed in Figure 13 when the cross-headline current component is more significant. During this period, the propagating convective cells are only observed at neighboring thermistors at separation distances less than the lateral size of a cell. Then, after confirming the presence of propagating cells and dependence upon the current direction, the correlation analysis provides an estimate of these lateral sizes.

Since the current direction was confirmed to be responsible for the propagation of convective cells and temperature observations along the headline differ considerably depending on the direction relative to the headline (Figures 12-13), it was expected to produce distinctly different looking correlation vs. separation plots for each of the two scenarios. As seen in Figures 24-25, both correlation analyses show similar behavior in correlation decreasing with distance yet differences in decorrelation length estimation (where correlation drops to or below 0) and correlation at larger separation distances. Figure 24 suggests minimum correlation between thermistors at separation distances around 100 m, but these results are undoubtedly skewed by the nature of propagation along the headline. If RDC is responsible for forming regularly shaped convective cells (e.g. circles or ovals) that descend into the water column, it would be reasonable to assume that correlation analysis of anomalous temperature records of laterally spaced thermistors on this scale would reveal one of the cell’s lateral dimensions, especially during time periods with cross-headline current. As a cell moves across the headline,
thermistors that observe the warm cell will produce higher correlation with each other than others further away and outside of the convective cell.

The results presented in Figure 25 for periods with cross-headline current suggest that the lateral scales of these convective cells are on the order of 50 m, since this separation distance corresponds to the initial minimum of correlation around or below 0. This finding agrees well with results found by Austin (2019), who estimated lateral scales of convective cells to be around 50 m in Lake Superior. After roughly 50 m separation, the correlation values remain nearly constant around 0 within uncertainty estimates. Convective cells observed by thermistors near the ends of the headline were likely not fully captured by the correlation analysis. Additionally, the across-headline current analysis was limited by the lateral resolution of 10 m, although this impact may not be as significant if lateral scales are on the order of 50 m or higher. For both scenarios, but especially for periods with along-headline current when convective cells do not necessarily pass centered with the headline, it is likely that the true lateral scales of convective cells are being underestimated rather than overestimated. Then, the reported 50 m lateral scale likely represents the lower end of possible convective cell sizes during the RDC period.

5.2 Frozen Field Approximation

The observed cell propagation and difference in temperatures depending on current direction was also considered for its effect on the time rate of change observed at a thermistor. Expanding the material derivative of temperature, as shown in Equation 4.7, produces a local time rate of change and three advective terms for a moving parcel of water. Then, using the spatial resolution allowed by the 2D mooring, the advective term along the headline orientation was calculated. If limiting the analysis to only periods with primarily along-headline current and orienting the axes along the headline, the total contribution of advection is roughly estimated by a single term: \((\mathbf{u} \cdot \nabla)T \approx u \frac{\partial T}{\partial x}\). This
term was calculated along with a local time rate of change term, and results were plotted in Figures 26-27. Figure 26 represents a typical day with currents strongly aligned along the headline, which shows that the two terms are roughly equal in magnitude but opposite in sign throughout the day. A similar balance is not observed in Figure 27, a day with a stronger cross-headline current component. These findings suggest that the local time rate of change term and advective terms cancel each other, at least on hourly time scales and at the headline depth of 34 m. Then, the right-hand side of Equation 2.5 is approximately 0, and

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \approx 0
\]

(5.1)

where the other horizontal and vertical advective terms are ignored as \( v \approx 0 \) and \( w \) is assumed to be much smaller than lateral currents. It can further be assumed that the balance between the local time rate of change term and advective terms exists even during periods with a non-negligible cross-headline current \( \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \approx 0 \right) \) as current direction does not ultimately impact the nature of convective cell formation or propagation.

While these results suggest that all other days during the study showed similar behavior depending on the current orientation, there were multiple instances of days showing significant disagreement between the two differential terms even with primarily along-headline currents. Naturally, there is always some cross-headline current component that is not considered in the analysis, perhaps explaining some of the disagreement. However, it is likely that these periods without the significant agreement seen in Figure 26 are also attributable to inability to estimate the vertical advection of temperature. It is possible that more significant vertical velocities are present during extended periods either due to convective cell velocity or some other process and combined with relatively large temperature gradients, and the total advective term cannot be estimated by a single component.

The alternative to the disagreement being due to poor estimation of advection is
that the terms on the right-hand side of Equation 2.5 are relatively large. This would either be due to more vigorous diffusion during certain periods or higher rates of radiative heating at the headline depth. Cannon et al. (2019) found average daytime diffusivities of 4.9×10^{-2} m^2 s^{-1} in Lake Michigan during the RDC period, with values occasionally exceeding 10^{-1} m^2 s^{-1}. Diffusivities were not estimated in the scope of this thesis, but it is assumed that similar values apply in Lake Superior. In this case, during periods of turbulent mixing and thus elevated diffusivities, the diffusion term may be more considerable and of the same order of magnitude as other terms present in Equation 2.5.

While the radiative heating term is unlikely to change significantly over the entire RDC period at the headline depth of 34 m, as the attenuation coefficient and peak rates of shortwave radiation remain the same throughout, it is not an insignificant term at the surface. As seen in Figures 26-27, the magnitudes of the local time rate of change and temperature advection are on the order of 5×10^{-5} °C s^{-1} with peak values around 2×10^{-4} °C s^{-1}. The heating term due to peak shortwave radiation of 1000 W m^{-2}, calculated using Equation 2.7, is only 5×10^{-8} °C s^{-1} at the headline depth. However, near the surface, this term is approximately 5×10^{-5} °C s^{-1} and of the same order as the local time rate of change and advective terms. Ultimately, shortwave radiation is the primary heat source throughout the RDC period and always provides a positive input to Equation 2.5, while the local time rate of change and advective terms will average to 0 °C s^{-1}.

Furthermore, estimates of convective damping times can be used to determine whether the approximation in Equation 5.1 is appropriate on desired time scales. This convective damping time is variable throughout RDC as it is dependent on the varying buoyancy flux (Equation 4.5), but typical damping times resulting from peak shortwave radiation at the surface were on the order of 2 hours. This same estimate was made previously for the RDC period in Lake Superior (Austin 2019). Convective damping time estimates for ice-covered lakes also are on the same order of magnitude, although typically slightly less due to smaller mixed layer depths (Kelley 1997; Forrest et al. 2008; Pernica et al. 2017). Additionally, the convective damping time of 2 hours agrees well
with previous microstructure analyses, as TKE dissipation was found to lag convective cell generation by 1-1.5 hours (Jonas et al. 2003a; Volkov et al. 2019).

Using the findings implied in Equation 5.1 and the estimated convective damping times, a “frozen field” approximation is made. This is similar to previous findings in both ice-covered lakes (Bogdanov et al. 2019) and oceans (Schott and Leaman 1991), where convective cells were seemingly frozen without exhibiting significant temperature diffusion or shear as they moved along with uniform background currents. A similar definition is implied here: RDC-generated convective cells are assumed to remain approximately constant in terms of both lateral size and temperature magnitude on time scales of 1-2 hours. On these time scales, current directions will remain relatively uniform, and convective cells are not likely to diffuse significantly, or temperature variability of a moving parcel of water is insignificant relative to the spatial variability present.

5.2.1 Mapping Temperature into 2D

Applying the frozen field approximation then allows for more direct interpretation of spatial variability during the RDC period than was possible with the correlation analysis. The position of a moving parcel of water at a fixed point in time is determined using

\[ \bar{x} = \bar{x}_0 + \int_{t_0}^{t} \bar{u} \, dt' \]  

(5.2)

where \( \bar{x}_0 \) are the fixed headline thermistor positions, \( t_0 \) is a fixed point in time, and \( \bar{u} \) is the ADCP-measured horizontal current velocity. In this way, positions for parcels of water present at each headline thermistor can be determined for each interval in a specified time window. Combining time-varying position estimate with temperatures measured at the headline but assumed to remain constant on 1-2 hour time scales (i.e. the frozen field approximation) then allows for translating temperature measurements from time into space.
Figure 29 shows this analysis for two separate windows during the RDC period. The distances shown are a function of the time interval and current speed and vary between the two plots.

![Graphs](image)

**Figure 29:** Converting time into space for thermistor temperature records. (a) Temperature as a function of lateral distance on 19 June 2019, 10:50-12:00 CST. (b) Same as for (a) but time window is 3 July 2019, 10:20-12:00 CST.

In both Figure 29a and 29b, the same magnitude of temperature anomalies as previously noted are seen (~0.1°C), yet now the lateral size and gradients of these events are observable. Spatial gradients at the edge of these temperature events vary from 0.5 °C m⁻¹ up to values approaching 2.0 °C m⁻¹. The first event seen in Figure 29a is roughly 60 m wide, while others seen during the rest of this period and that in Figure 29b seem to be closer to 20-30 m. It is important to note here that these estimates are not representative
of the full lateral size of convective cells. These records are for single thermistors, and it is unlikely that the center of convective cells is propagating directly past the thermistor during either period. As such, these lateral scales are most likely underestimates of the true lateral size of the convective cells observed.

Employing this same procedure for all headline thermistors allows for the production of 2D “maps” of horizontal temperature fields at the headline depth during specified time windows. Selecting a point in time to serve as a central pivot point, Equation 5.2 can be applied both forward and backward in time, providing a larger area of temperature coverage yet maintaining a small enough time window to justify the frozen field approximation. Periods with stronger across-headline current are selected to produce the most informative temperature maps, as this analysis will then generate spatial resolution in a more perpendicular direction to the headline rather than along it. If the current is mostly along the headline, the lateral resolution being mapped is in the same direction, and meaningful interpretations of lateral temperature variability are difficult. These interpretations are much simpler when a larger area is resolved by the analysis, and better estimates of convective cell lateral scales are possible than those obtained from a single thermistor as in Figure 29. To ensure similar resolution in both the along and cross-headline directions, the calculated positions are low pass filtered; the resolution along the headline is limited by the thermistor spacing of 10 m, while considering typical current speeds of 2-3 cm s⁻¹ and sampling frequency of 2Hz results in spatial resolutions of roughly 5 cm for directions more perpendicular to the headline. A good example of this 2D temperature analysis is shown in Figure 30.
After applying a low-pass filter to the positions calculated, the temperature data was interpolated between headline thermistor positions to produce a more coherent figure. The period shown in Figure 30 is a 4-hour window centered around 10:00 CST on 08 June 2019. This period is slightly before the initiation of the RDC period, yet the same convective processes are occurring at the headline depth, which is well above the weakened thermocline on this date (>100 m). The sinuosity seen in the plotted positions is due to the current direction varying over the 4 hours considered. Clear, distinctive temperature events are observed throughout the plotted region with magnitudes at or above 0.02°C. The temperatures plotted are high-passed, anomalous temperatures, so the values indicate the fluctuations away from the running mean. Nighttime periods show

Figure 30: Using the frozen field approximation to map temperature into 2D. The two axes indicate lateral distance to the east (x) and north (y). The small black markers near the center of the plot indicate the headline positions.
little activity relative to Figure 30, representing a mostly quiescent lateral layer of water following sundown and convective cell decay.

The same blob-like or circular structure of convective cells observed in Figure 30 is also present in Figure 31, which shows progressive 2-hour temperature windows on 29 June 2019. The 2D temperature maps range from windows centered on 08:00 CST (Figure 31a) through 18:00 CST (Figure 31f). Similar cell structure and magnitudes are observed, with lateral sizes estimated to be $O(10)$ m up to $O(100)$ m and temperature anomalies typically greater than 0.02°C. Additionally, by presenting progressive time windows as in Figure 31, the propagation of individual convective cells is observable. In this case, the temperatures plotted for positions to the left and above the headline then appear in the following window to the right and below the headline position. A mostly quiescent layer is present in Figure 31a, followed by multiple convective cells passing the headline and present on either side (Figure 31b-d). Then, the magnitude of the convective cells seems to decrease at the headline depth in Figure 31e-f.
In contrast to the blob-like or circular structure of convective cells observed in Figures 30-31, other daytime periods show evidence of streaky or linear looking...
structure. Figure 32 shows the same analysis for 15 June 2019.

![Diagram of convective cell presence and propagation near the headline on 15 June 2019.](image)

**Figure 32: Convective cell presence and propagation near the headline on 15 June 2019.**
Each plot is a 2-hour window centered on (a) 08:00 CST, (b) 10:00 CST, (c) 12:00 CST, and (d) 14:00 CST on 15 June 2019. Temperatures are represented by the same color scales on all plots as defined by the scale in (b).

While similar temperature anomaly magnitudes and propagation across the headline is observed in Figure 32 as in Figures 30-31, the lateral structure of convective cells is significantly different. On 15 June 2019, the cells show a much more linear structure,
with lateral spans or lengths over 200 m. The width of the cells is closer to the previous estimates of lateral scales, on the order of 50 m. The nature of these streaky cells is currently unknown; analysis of a presumed connection to either wind direction or current direction yielded no meaningful results. Based on this, it seems that the streakiness of these cells is not due to any shearing at the surface from wind or at depth as current uniformity with depth is roughly the same for the periods in both Figure 31 and 32.

The lateral scales estimated from the 2D temperature analysis agree with the correlation analysis and estimates by Austin (2019) for Lake Superior. The circular structures consistently appear on scales this size, while the linear structures have lengths on the order of hundreds of meters. Forrest et al. (2008) observed regions of elevated temperatures surrounded by ambient water with approximate lateral scales of 150 m, in rough agreement with both estimates found here. These scales are much larger than the approximately 5 m horizontal scales observed for convective plumes resulting from surface cooling (Thorpe 1999; Jonas et al. 2003b; Forrest et al. 2008).

5.2.2 Comparison of Horizontal and Vertical Thermal Structure

Now that temperature is mapped from time into space by the frozen field approximation, it is possible to compare spatial structure between the horizontal results presented in 5.2.1 and vertical temperature profiles collected by the VMP and presented in 4.2.3. With an average fall rate of 0.65 m s\(^{-1}\) and sampling rate of 512Hz, the spatial resolution of VMP temperature data is roughly 1.3 mm. For a more direct comparison, this data is low pass filtered to a spatial resolution closer to the resolution of horizontal data, approximately 1-2 cm. For ease and to provide estimates of the vertical temperature gradient, the profile shown in Figure 15a is reproduced in Figure 33, now rotated so that temperature is on the vertical axis. Then, the depth is shown on the horizontal axis, and temperature fluctuations with respect to distances can be compared directly with those seen in Figures 29-32.
The temperature structure in Figure 33a is remarkably similar to structure seen in the horizontal direction. Additionally, similar vertical structure was observed during the RDC period in Lake Michigan (Cannon et al. 2019, Figure 5e). Sharp temperature events occur with anomalous temperatures on the order of 0.05-0.1°C, surrounded by a constant background temperature representing ambient water. The most distinctive event observed in Figure 33a is centered around 50 m depth, and the constant background temperature is apparent on either side. Higher activity is observed at smaller distances (closer to the surface) than deeper in the water column.

To verify if the frozen field approximation also applies to the vertical temperature structure, the convective velocity scale and decay times can be used to determine if the...
cells diffuse significantly as they sink. The profile in Figure 33 was collected on 11 June 2019, and as reported in Table 1, a representative convective velocity for peak shortwave heating during this period is $1.76 \text{ cm s}^{-1}$. Then, convective cells could reach the maximum 100 m depth observed in approximately 95 minutes. This is slightly less than the estimated convective decay time of 2 hours typical of the RDC period, so the frozen field approximation is likely appropriate in this case.

Spatial temperature gradients also show agreement between the vertical and horizontal directions, as Figure 33b shows typical convective cell temperature gradients ranging from $0.5 \, ^\circ\text{C m}^{-1}$ up to $2.0 \, ^\circ\text{C m}^{-1}$. All VMP profiles collected on 11-12 June during the daytime and a single headline thermistor record for a five-day period during daytime hours were incorporated into a calculation of temperature gradients to further investigate the comparison between horizontal and vertical structure. The shortened period considered for the horizontal gradient calculated was selected to ensure roughly equal amounts of data in each analysis. An algorithm was developed to find local minima and maxima, then calculate the temperature jump from a minimum to a maximum. Using the distance between the two points calculated utilizing the frozen field approximation for headline data and VMP-measured depth for the vertical profiles, the temperature gradient terms were calculated. The current direction was not considered, so horizontal temperature gradients are not aligned along a specific direction and are defined as $\frac{\partial T}{\partial x}$, while the vertical gradient is $\frac{\partial T}{\partial z}$. The results of this analysis are reported in Figure 34.
Data was binned and plotted on a logarithmic scale in Figure 34 to prevent the abundance of smaller temperature events from concealing data of interest. Most of the data captured by this analysis were small, insignificant temperature jumps. The notable events that represent gradients on the edges of convective cells are represented by larger temperature increments. These temperature increments do not necessarily represent the true temperature anomalies related to convective cells, as the calculation is susceptible to the rapid fluctuations present in 2Hz data. More scatter is present in temperature gradient estimations in the horizontal direction (Figure 34a) than in the vertical direction (Figure 34b). While they differ slightly, temperature gradients are similar in both directions. It is likely that the rough cutoff value between abundant and sparse data points around 0.5 °C m⁻¹ is a rough estimation of typical convective cell gradients. Convective cells were found to be abundant during certain daytime periods, especially in the horizontal direction, so it is expected that this analysis would capture many instances of these events. Values above the 0.5 °C m⁻¹ cutoff observed are also likely the result of convective cells, with these points representing even sharper gradients along the edge of cells. Since this analysis only focused on finding temperature jumps from an ambient to
Elevated temperature, it is likely that some of the significant gradients observed in Figure 33 or estimated from Figure 29 were filtered out. Instead of calculating precise temperature gradient estimates in both the horizontal and vertical direction, the intent of this analysis was to roughly compare magnitudes in the two directions.

Furthermore, an estimation of the boundary width of convective cells is possible by inverting the slopes inferred by the fit lines (95% confidence bounds) shown in Figure 34. This boundary width value is an estimate of the transition region from ambient water into warm, convective cells. The results shown in Figure 34 suggest horizontal boundary widths of $0.09 \pm 0.02$ m at the headline depth of 34 m and vertical boundary widths of $0.056 \pm 0.006$ m. The same analysis was completed for thermistors at each depth on the 2D mooring, now for the entire RDC period, and the results are summarized in Table 2.

<table>
<thead>
<tr>
<th>Depth, m</th>
<th>Fit Slope, m$^{-1}$</th>
<th>Boundary Width, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>$10.7 \pm 0.9$</td>
<td>$0.094 \pm 0.008$</td>
</tr>
<tr>
<td>44</td>
<td>$8 \pm 1$</td>
<td>$0.12 \pm 0.02$</td>
</tr>
<tr>
<td>54</td>
<td>$10 \pm 1$</td>
<td>$0.10 \pm 0.01$</td>
</tr>
<tr>
<td>74</td>
<td>$9 \pm 1$</td>
<td>$0.12 \pm 0.02$</td>
</tr>
<tr>
<td>104</td>
<td>$9 \pm 2$</td>
<td>$0.11 \pm 0.02$</td>
</tr>
<tr>
<td>134</td>
<td>$8 \pm 2$</td>
<td>$0.12 \pm 0.03$</td>
</tr>
<tr>
<td>164</td>
<td>$7 \pm 2$</td>
<td>$0.14 \pm 0.04$</td>
</tr>
</tbody>
</table>

**Table 2: Estimated convective cell boundary widths.** The same analysis as shown in Figure 34 is completed for each depth on the 2D mooring over the entire RDC period, and boundary widths are estimated from fit slopes to gradient data.

Within error estimates, all estimated boundary widths of convective cells agree throughout the measured depths. Temperature gradient data exhibited more scatter with increasing depth and is reflected by the error estimates in Table 2. Minimum estimated boundary widths are approximately $0.09$ m, while the temperature gradients measured at 164 m depth suggest widths as large as $0.18$ m within one standard deviation. All horizontal boundary widths are significantly higher than the measured vertical boundary width estimate. While this could be due to an actual difference in convective cell
boundary widths in the two directions, it is more likely that the discrepancy is the result of the different measurement techniques, as temperature gradients were measured directly by the VMP and indirectly for stationary thermistors on the mooring. However, the boundary widths estimated in both directions, ranging from 0.05 to 0.18 m, suggest a sharp transition into convective cells and cannot be resolved directly by the 10 m spacing along the headline of the 2D mooring.

Comparisons of both temperature vs. distance as shown vertically in Figure 33 and horizontally in 5.2.1 and temperature gradients as seen in Figure 34 suggest that thermal structure is similar in both directions. Additionally, the boundary width of convective cells remains nearly uniform with depth (Table 2). These findings provide more justification for the frozen field approximation utilized as well as the convective decay times estimated. Rather than convective “chimneys” forming, where temperature anomalies build up at the surface and descend into the water column, diffusing along the way, the results presented here suggest that convective cells formed during the RDC period remain intact as they travel both horizontally and vertically on time scales of 1-2 hours.
6.0 Conclusion

The analysis completed and discussed in this thesis provides an in depth look at the radiatively-driven convection (RDC) period in Lake Superior. This is a relatively understudied process, especially in ice-free large lakes, and little is known of the horizontal variability and scales present. The uniqueness of the mooring used for this analysis sets it apart from previous studies, providing a level of pure horizontal resolution that is not possible with either a single point mooring or an autonomous underwater vehicle or glider. Additionally, the coincidence of both a meteorological buoy and ADCP mooring nearby distinguish the data set considered here, allowing for a more detailed understanding of the forcing involved during this period.

Using data collected from a 2019 deployment, which consisted of a 2D mooring outfitted with thermistors providing both horizontal and vertical resolution as well as nearby moorings/buoys providing additional information, the thermal structure of western Lake Superior was investigated beginning prior to the end of negative stratification and lasting until summer stratification was fully formed in July. Between the two stratification regimes, the water column is isothermal and below T_{MD}, so surface heating generates buoyancy fluxes and thus convection, driving circulation throughout the water column. As the primary heat source in the open lake is shortwave radiation, this process is referred to as radiatively-driven convection; this process has also been well documented under ice in lakes when sunlight penetrates through any present layers of the ice and snow.

The main purpose of the analysis considered in this thesis is to investigate the horizontal variability of RDC in a large lake and the forcing responsible for that variability. Since this process effectively circulates the full water column for a varying duration each year, it could potentially have large scale effects on the lake, especially in its relation to both winter and summer thermal structures. Opportune observations of relatively small horizontal scales made by Austin (2019) were the basis for this RDC investigation, and results presented here show similar scales.
Various methods of data analysis were utilized, yet all suggested similar horizontal scales of convective cells on the order of 50 m. Depending on the geometry of these cells, lateral scales were also found to extend well beyond 200 m. Convective cells formed following sunup during the day and diffused with surrounding water overnight, and multiple forcing parameters had varying effects on the production and presence of these cells. While shortwave radiation is the source of convection during this period and found to have a significant relation to temperature variability, the thermal expansion coefficient is also an important factor as it relates to the magnitude of buoyancy flux. Towards the end of the RDC period, a larger temperature increase, and thus more shortwave radiation input, is required to generate enough buoyancy force to circulate convective cells into the water column than earlier in the period.

Prior to the completion of this investigation, it was speculated that convective cells form a trail, or chimney, as they descend, yet vertical temperature profiles suggest a different scenario. Vertical temperature structures were found to be nearly the same as horizontal structures, and similar temperature gradients were observed in both directions. The convective cells generated at the surface were found to descend into the water column and propagate along with background currents. Since the headline of the 2D mooring was fixed at 34 m depth and little instrumentation was located closer to the surface, the formation process of the convective cells throughout the RDC period is not entirely clear. Cells form near the surface before generating a large enough buoyancy flux to sink, yet it is unknown if the shape and uniformity of the cells is well defined initially or as they descend to the observed headline depth. However, results presented in this thesis suggest that cells remain virtually intact after reaching the headline depth and traveling with background currents without significant diffusion on time scales of 1-2 hours. A comparison of different terms in the material derivative of temperature provided further evidence of minimal diffusion or heating as convective cells were transported.

This reported finding allowed for the application of a frozen field approximation, or the assumption that convective cells remain constant in magnitude and size relative to surrounding water as they travel on times scales on the order of hours. Then, elapsed time
in the thermistor temperature records was converted into space using the ADCP-measured currents, and 2D lateral maps of temperature were produced at the mooring headline depth of 34 m. These maps provided a detailed look at both convective cell structure and propagation during daytime, active hours. Significant convective cell estimates resulting from the application of the frozen field approximation and other analyses described in this thesis are summarized in Table 3.

<table>
<thead>
<tr>
<th>Convective Cell Property</th>
<th>Estimate (previous studies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature magnitude, °C</td>
<td>0.05-0.1 (Forrest et al. 2008; Austin 2019)</td>
</tr>
<tr>
<td>Boundary temp. gradient, °C m⁻¹</td>
<td>0.5-2.0</td>
</tr>
<tr>
<td>Boundary width, m</td>
<td>0.05-0.18</td>
</tr>
<tr>
<td>Convective velocity, cm s⁻¹</td>
<td>1.5 (Austin 2019)</td>
</tr>
<tr>
<td>Convective damping, hours</td>
<td>2 (Austin 2019)</td>
</tr>
<tr>
<td>Lateral scale/width, m</td>
<td>50 (Austin 2019; Bogdanov et al. 2019)</td>
</tr>
<tr>
<td>Length (for linear cells), m</td>
<td>&gt;200</td>
</tr>
</tbody>
</table>

Table 3: **Summary of estimated convective cell properties.** Calculated estimates for the values are presented in the second column, and previous studies with similar findings are referenced.

As the 2D maps were the highlight of this analysis, the follow-up campaign to investigate RDC in Lake Superior was designed to provide the best resolution possible. The data collected by the 2D mooring in 2019 was oriented along the primary current direction, meaning that most time periods were unsuitable for the 2D temperature map analysis, as increased resolution was only available in the same direction as the mooring orientation. The 2021 RDC deployment took place in April 2021; this deployment included a higher resolution horizontal mooring equipped with over 30 thermistors spaced 5 m apart, a nearby high resolution vertical mooring equipped with over 90 thermistors spaced either 1.5 m or 2 m apart depending on depth, and finally a nearby meteorological buoy as in 2019. The vertical mooring also included both an upward and downward facing ADCP, and the intent is to produce similar 2D temperature maps, now with both vertical and lateral axes. This data, along with the higher resolution horizontal
temperature data, will provide much more detail surrounding the RDC process in Lake Superior.

A more in depth look at the vertical structure of RDC will provide further information on convective cell properties during this period, and the closely spaced thermistors higher in the water column should provide some more detail on cell formation as they descend. Additionally, future analyses will include a consideration of microstructure measurements, providing finer scale detail of the nature of the cells forming than possible in this analysis. Combining turbulence measurements with larger scale convective cell property estimates will provide a nearly complete view of convective structure during RDC in Lake Superior, a process with direct implications for organisms present in the water column and applications for other fluid environments.

The findings discussed in this thesis are significant for providing both a better understanding of RDC specifically as well as general convective processes. While RDC does not occur in the ocean as waters are typically warmer and highly saline, similar convective events occur due to surface cooling. Convection can be studied on much smaller scales in Lake Superior without as significant wind or other meteorological events that require consideration for oceanic studies. Additionally, further understanding is needed of RDC in terms of its impact on biological communities present and nutrient redistribution from sediments. Ongoing work is considering the effect on diurnal vertical migration of zooplankton, as initial results suggested a distinct difference in behavior following the onset of the RDC period (Austin pers. comm.). The exact effect the convective cells discussed in this thesis has on organisms in the water column is unknown. For instance, it would be interesting to determine whether organisms present in a convective cell are trapped inside as they propagate or if organisms are free to move in and out of these cells. An investigation into the turbulence structure at the edge of cells should provide some information on this subject. Finally, since this period is directly dependent on the previous winter’s thermal structure and the duration may be largely variable moving into the future (Fichot et al. 2019), it is important to gain a better
understanding of RDC both for its lake-wide effects throughout the year and implications of a varying duration.
7.0 Bibliography


