The Influences of Executive Function and Relational Language on Number Relation Skills

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Dedication

This dissertation is dedicated to the memory of Ramsey Chan-Zhang, whose loving presence had provided me with much comfort and laughter throughout my graduate career.

Abstract

Executive function (EF) skills (i.e., inhibitory control, working memory, and cognitive flexibility) and relational language (e.g., *more, equal, before, between*) predict mathematical skills and may be particularly important for number relation skills, a component of early numeracy skills that involves the knowledge of cardinal (e.g., 5 is *more* than 4) and ordinal (e.g., 5 comes *after* 4) number relations. Specifically, comparing and making connections between numbers may require EF skills and relational language. I used a pretest – training – posttest paradigm to examine (a) whether EF skills and relational language influence number relation skills, (b) whether number relation skills mediate the reported relations between EF skills and mathematical skills, and between relational language and mathematical skills, (c) whether incorporating EF prompts and relational language instruction in number training has additional effects on number relation skills beyond number training alone, and (d) whether children's initial EF skills predict pretest to posttest gains in number relation skills beyond their initial number relation skills. I found that (a) EF skills and relational language separately predicted number relation skills, (b) number relation skills fully mediated the associations between EF skills and mathematical skills, and between relational language and mathematical skills, (c) incorporating EF prompts and relational language instruction in number training did not have additional effects on children's number relation skills, and (d) children's initial EF skills did not predict improvement in number relation skills beyond their initial number relation skills. The results extend previous findings on the influences of EF skills and relational language on mathematical skills and have implications for future research and educational practices.

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The Influences of Executive Function and Relational Language on Number Relation Skills

Early numeracy skills are paramount for later mathematics and academic achievements as well as problem solving and daily living skills (Butterworth, Varma, & Laurillard, 2011; Duncan et al., 2007). Research on the influences of early numeracy skills on later mathematical skills have revealed specific early numerical predictors (e.g., Geary, 1993; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Mazzocco & Thompson, 2005), and pathways through which these numerical predictors influence mathematical skills (Chu et al., 2015; Purpura, Baroody, & Lonigan, 2013). Non-numerical abilities, specifically executive function skills and knowledge of relational language, also influence mathematical skills (e.g., Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; Purpura & Logan, 2015; Purpura, Schmitt, & Ganley, 2017; Welsh, Nix, Blair, Bierman, & Nelson, 2010), and research examining the pathways through which these non-numerical abilities affect mathematical skills is underway (Attout, Noël, & Majerus, 2014; LeFevre et al., 2010). In the present study, I examined the influences of executive function skills and relational language on early numeracy skills by pursuing the following research aims. First, I investigated whether children's executive function skills and relational language predict their *number relation skills*, a component of early numeracy skills. Second, I tested whether number relation skills may be a pathway through which executive function skills or relational language influence mathematical skills. Third, I examined the potential causal influences of executive function skills and relational language on children's number relation skills by comparing the effects of number training with vs. without executive function prompts and relational language instruction on children's gains in number relation skills. Last, I explored whether children's initial executive function skills predict gains in their number relation skills from pretest to posttest.

Number Relation Skills

Number relation skills are a set of skills that involve the knowledge of how two or more numbers are related to each other in terms of their cardinal value, ordinal position, or spatial relation. *Cardinality* refers to the magnitude that a number represents and *ordinality* refers to the position of a number in the counting sequence. Numbers can be related to each other based on the cardinal values they represent (e.g., more, less) or their ordinal positions in the counting sequence (e.g., before, after). Although some numerical cognition studies have revealed differences between cardinality and ordinality, these constructs are intricately connected with each other (see Lyons, Vogel, & Ansari, 2016 for a review) in that numbers that are larger also come later in the counting sequence. In other words, the counting sequence provides an ordinal structure for numbers and organizes them from small to large based on their cardinal values. These cardinal and ordinal number relations can also be represented spatially on a number line in that the left to right arrangement of small to large numbers denotes ordinal number relations and the distance between numbers indicates cardinal number relations. Number relation skills are often measured with tasks such as number comparison (e.g., which number means more?), number ordering (e.g., are these numbers in ascending order from left to right?), and number line estimation (e.g., where does N go on a 0 - 100 number line?) in early childhood (Laski & Siegler, 2007; Lyons et al., 2014; Siegler & Booth, 2004; Vogel, Remark, & Ansari, 2015).

In addition to cardinal and ordinal number relations, adults with advanced expertise in mathematics are also likely to organize numbers based on more abstract relations, such as parity (i.e., odd and even), multiples (e.g., multiples of 2), and primes (i.e., numbers that can only be divided by 1 and themselves, e.g., 2, 3, 5, 7) (Rogers & Murphy, 2016; Shepard et al., 1975). In this study, I focus on cardinal and ordinal number relations because they emerge early in development, predict later mathematics achievement (Booth & Siegler, 2008; De Smedt, Verschaffel, & Ghesquière, 2009; Lyons, Price, Vaessen, Blomert, & Ansari, 2014), and do not require advanced knowledge in mathematics.

Number relation skills are an important foundation for mathematical skills. For instance, understanding the cardinal relations between numbers in the counting sequence (e.g., counting up from five to six also means adding one more to five) may provide an efficient strategy for solving story problems that involve addition or subtraction. Numerous studies also provide evidence for the concurrent association and predictive relation between number relation skills and aspects of mathematical skills. For instance, children's performance on number comparison is correlated with their concurrent arithmetic skills throughout elementary school (Lyons et al., 2014). Children's performance on number comparison in first grade predicts their mathematics achievement one year later (De Smedt et al., 2009). Similarly, children's performance on number ordering is associated with their concurrent arithmetic skills in Grades 3 to 6 (Lyons et al., 2014), and their performance on number ordering in Grade 1 predicts their mathematical skills in Grade 2 (Attout, Noël, et al., 2014). Children's performance on number line estimation in Grade 1 is correlated with their concurrent arithmetic performance and predictive of their arithmetic learning (Booth & Siegler, 2008). A meta-analysis further indicates that performance on number line estimation is correlated with overall mathematical competence in children from 4 to 14 years of age (Schneider et al., 2018). If number relation skills are important for mathematical skills, at issue is what factors influence number relation skills and whether number relation skills can be improved through training.

Factors Influencing Number Relation Skills

Counting/numbering skills. Counting/numbering skills are a set of skills that involve the knowledge of counting sequence, meanings of number words, and the mapping between different representations of numbers (e.g., Arabic numerals, numerosities, number words). Because cardinal and ordinal number relations are intuitively represented in the counting sequence, a logical developmental progression is that counting/numbering skills support number relation skills. Correlational and experimental studies on counting/numbering skills and number relation skills provide evidence to support this relation. For instance, Barth, Starr and Sullivan (2009) found that four- to six-year-olds who were able to recite the counting sequence up to 60 without error were more accurate at estimating sets of large numerosities. These children's estimates followed a linear function with a slope of .97 (e.g., provided 97 as an estimate for a set of 100 dots) and they produced larger number words for larger sets of numerosities, suggesting that they might have some understanding of the cardinal relations between number words in the counting sequence. Children who made errors in reciting the counting sequence tended to fail at producing larger number words for larger sets of numerosities, and the slope of their estimates was relatively flat (e.g., provided 50 as an estimate for sets of 20 and 70 dots) compared to the slope of estimates by children who did not make errors in verbal counting. Preschoolers who can correctly map number words onto their corresponding numerosities (e.g., number word "four" corresponds to a set of four things □□□□) are more accurate at inferring cardinal relations between two sets of objects compared to children who cannot do such mapping (Sarnecka & Carey, 2008). Furthermore, training on verbal counting and set counting improves preschoolers' performance on number ordering and their accuracy on number line estimation (Xu & LeFevre, 2016). Similar counting training also improves kindergarten students' performance on number comparison and number line estimation (Bos, Kroesbergen, & Luit, 2018).

The close relation between counting/numbering skills and number relation skills has led to questions concerning whether these skills are two distinct components of early numeracy. Jordan, Kaplan, Olah, and Locuniak (2006) found that six early numeracy tasks (counting, number mapping, numeral identification, number comparison, set relation, and number patterning) all loaded on one factor in kindergarten students, suggesting that counting/numbering and number relation may not be two distinct sets of skills. However, a different pattern of results emerged in other studies. Purpura and Lonigan (2013) conducted exploratory factor analyses with a battery of early numeracy tasks administered to preschoolers, and found that nine tasks that involved verbal counting, set counting, or number mapping loaded on one factor, whereas nine additional tasks that involved number comparison, number ordering, and set equivalence loaded on another factor. Aunio and colleagues conducted confirmatory factor analyses with 40 items from the Early Numeracy Test (Van Luit, Van de Rijt, Pennings, 1994) and found that a two-factor model (i.e., 20 items focusing on relations and 20 items focusing on counting) fitted the data better than a one-factor model (i.e., all 40 items together) for children from 4 to 7 years of age (Aunio et al., 2006). Although findings from factor analyses do not provide a clear picture of the distinction between counting/numbering skills and number relation skills, other studies show the dissociation between these two sets of skills, suggesting that they may be two distinct components of early numeracy skills.

Despite the close relation between counting/numbering skills and number relation skills, proficiency in counting/numbering skills does not guarantee number relation skills. Sarnecka and Carey (2008) found that 28% of three- and four-year-olds in their study were able to map number words to their corresponding numerosities, but still did not understand that adding one more item to a set means moving up one number in the counting sequence. Similarly Davidson, Eng, and Barner (2012) found that 20% of preschoolers in their study were able to map number words to their corresponding numerosities and correctly recite the counting sequence up to 30, but still did not know that the order of numbers in the counting sequence denotes magnitude relations. Berteletti and colleagues (2010) found that approximately 10% of five- and six-year-olds in their study were able to correctly recite the counting sequence up to 10, but did not place numbers 1 to 10 in order from left to right in the number line estimation task (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010). Furthermore, four-year-olds are able to count and figure out the number of objects in a set but fail to identify the ordinal position of a number in a horizontally arranged set (e.g., point to the third object in this row) (Colomé & Noël, 2012). To summarize, although knowledge of counting and number words is associated with performance on number relation tasks, proficiency in counting/numbering skills alone does not guarantee the understanding of number relations. Additional skills may be involved in thinking and learning about number relations, and the involvement of these additional skills may distinguish number relation skills from counting/numbering skills.

Executive function skills. *Executive function (EF)* skills are a set of skills that involve explicit and effortful control of one's attention, emotions, thoughts, and actions. They develop rapidly during early childhood and continue to develop into early adulthood (Carlson, Zelazo, & Faja, 2013; Zelazo & Carlson, 2012). These skills are distinct from processing speed (Clark et al., 2014) and general intelligence (Welsh, Pennington, & Groisser, 1991), and are typically conceptualized as comprised of three separate but related components: working memory, inhibition, and cognitive flexibility. *Working memory* refers to the ability to retain and manipulate information during a task. *Inhibition* refers to the ability to suppress or delay habitual or pre-potent response. *Cognitive flexibility* refers to the ability to consider multiple options simultaneously or shift attention among rules. The components of EF skills appear to emerge as a unitary construct in two- to six-year-olds (Wiebe, Espy, & Charak, 2008; Wiebe et al., 2011) and begin to differentiate into three separable components during elementary school years and into adulthood (Lee, Bull, & Ho, 2013; Miyake et al., 2000).

At the behavioral level, EF skills can be measured by using tasks that involve remembering some rules, shifting attention between these rules, and inhibiting habitual or pre-potent responses. The Dimensional Change Card Sort, Day Night, and Head Toes Knees Shoulders are commonly used to measure EF skills in early childhood (Carlson, 2005). In the Dimensional Change Card Sort task (Zelazo, 2006), children sort picture cards into two boxes based on the rules stated by the examiner. For instance, children are given some picture cards (e.g., blue rabbit), and have to put them in a box with a picture that matches the card on shape (e.g., pink rabbit) or a box with a picture that matches the card on color (e.g., blue boat). The examiner changes the sorting rules during the task and children have to shift between these rules flexibly. In the Day Night task, children see some pictures of a moon or a sun, and they have to say the opposite of what the picture shows. For instance, when they see a picture of a moon, they have to say "day" instead the habitual response, "night" (Gerstadt, Hong, & Diamond, 1994). Similar to the Day Night task, in the Head Toes Knees Shoulders task, children have to do something different from what the examiner says. For instance, when the examiner says, "touch your head", children have to inhibit touching their head, and instead, touch their toes. Although different tasks appear to tax certain components of EF more than others, the three components of EF skills are connected and are often all involved during these tasks.

Executive function skills, in additional to counting/numbering skills, may be involved in number relation skills. To understand the cardinal relation between numbers, such as four is *more* than three but *less* than five, children need to remember the sets of comparison, shift between these comparisons flexibly, and inhibit irrelevant comparisons. For instance, to compare four and five, children may need to remember the counting sequence, and focus on four and five while inhibiting irrelevant comparisons, such as four and three. This intuitive association between EF skills and number relation skills is supported by research studies. Specifically, children's ability to ignore irrelevant information while remembering relevant information is correlated with their concurrent performance on number comparison during preschool (Gashaj, Uehlinger, & Roebers, 2016; Purpura, Schmitt, & Ganley, 2017). Similarly, the ability to hold information in working memory and flexibly shift between rules is correlated with concurrent performance on number line estimation in four- to seven-year-olds (Bos, Kolkman, Kroesbergen, & Leseman, 2014; Gashaj et al., 2016). After six sessions of number training, children with high working memory capacity show more improvement on number line estimation compared to children with low working memory capacity (Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013). All three components of executive function skills are correlated with children's performance on a number ordering task during preschool (Purpura et al., 2017). Together, these studies show that EF skills are correlated with number relation skills, and predict growth in number relation skills. In the current study, I examine whether EF skills influence number relation skills beyond counting/numbering skills. If so, the additional influences of EF skills may account for the distinction between number relation skills and counting/numbering skills.

Relational language. Language is a tool for communication and also for reasoning and thought. Although the ability to form numerical representations, often measured by the accuracy on comparing sets of numerosities, is shared across species and is somewhat independent of language (Gelman & Butterworth, 2005), studies have revealed the influences of aspects of language on mathematical skills (e.g., Brooks, Pogue, & Barner, 2011; LeFevre et al., 2010; Negen & Sarnecka, 2012; Vukovic & Lesaux, 2013). As reviewed in the section on counting/numbering skills, the acquisition of number words and counting sequence supports mathematical skills (Chu et al., 2015). Language also influences mathematical skills through phonological awareness, syntactic structure, and general vocabulary knowledge. Children's *phonological awareness*, a linguistic process that involves detecting and segmenting phonemes, predicts their performance on counting in kindergarten (Krajewski & Schneider, 2009b) and their numeral reading and writing in Grades 2 to 4 (Lopes-Silva et al., 2016), suggesting that phonological awareness may influence children's ability to decode and produce words, including number words.

The syntactic constraints on number words in a sentence may also help children identify and interpret number words. Whereas other adjectives (e.g., *loud, big, bright*) may appear with modifiers (e.g., *very*), number words usually do not appear with modifiers, at least in English, and this may help children differentiate number words from other adjectives in a sentence (Bloom & Wynn, 1997). Number words may appear in a partitive frame (e.g., three *of* the cups), but other adjectives do not (e.g., yellow cups), and children may use this cue to differentiate number words from other adjectives when it appears in a partitive frame (Syrett, Musolino, & Gelman, 2012).

Children's general vocabulary knowledge, as measured using expressive (e.g., name objects) and receptive (e.g., point to pictures) vocabulary tasks, is correlated with their concurrent knowledge of number words in preschool (Negen & Sarnecka, 2012), and is predictive of their performance on numeral identification two years later (LeFevre et al., 2010), suggesting that having knowledge of general vocabulary may help children learn number words across contexts. When examining the pathways through which general vocabulary knowledge influences mathematical skills, studies revealed that children's knowledge of *relational language* fully mediated the association between general vocabulary knowledge and mathematical skills in preschool (Purpura & Reid,

2016) and in kindergarten (Toll & Luit, 2014). Together, these findings suggest that language may influence mathematical skills through various mechanisms. In this study, I focus on the influences of relational language on number relation skills.

Relational language (RL) is defined as the vocabulary for connecting and describing relations between items. The vocabulary can be used to denote relations in terms of quantity (e.g., *some, many, most, equal*), order (e.g., *before, after*), space (e.g., *between, left, far, below, on*), or magnitude or extent (*medium, wide, long, full*). Relational language is often assessed using expressive (i.e., manipulate or seriate objects) and receptive (i.e., point to pictures) tasks (Hassinger-Das, Jordan, & Dyson, 2015; Powell & Driver, 2014; Purpura & Reid, 2016). Although relational language can be applied to other mathematical concepts and is relevant to mathematical thinking, it may be specifically important for number relation skills. Relational language is used to describe and communicate the cardinal and ordinal relations between numbers. For instance, the cardinal relation between numbers can be described using terms such as *more, less, big,* and *small*. The ordinal and spatial relations between numbers can be described using terms such as *before, after, between, next*, and *first*. Understanding the meanings of relational language and using these terms accurately may be important skills for connecting numbers words in a meaningful and relational way. Thus, in addition to counting/numbering and executive function, relational language may be another set of skills required for number relation skills.

Studies with preschoolers support the notion that relational language is associated with early numeracy skills (Barner, Chow, & Yang, 2009; Purpura & Reid, 2016; Purpura & Logan, 2015; Toll & Van Luit, 2014). Preschoolers who have more knowledge of quantitative vocabulary also have better understanding of the cardinal value of a number word (Barner et al., 2009). Specifically, children who were able to identify and produce sets of objects described by quantitative terms (e.g., *some tokens*), were also more likely to correctly identify and produce sets described by exact number words (e.g., *five tokens*). In typically achieving preschoolers, knowledge of relational language predicts their performance on an early numeracy test that includes items on counting/numbering, number relation, and arithmetic, five months later (Purpura & Logan, 2015). When further exploring this relation in younger (three to four years of age) vs. older (four to five years of age) preschoolers, relational language emerges as a consistent predictor of early numeracy skills for both age groups, whereas the predictive value of executive function and non-symbolic number skills vary with age (Purpura, Day, Napoli, & Hart, 2017). Although these studies reveal the association between relational language and early numeracy skills, none of them explicitly focus on the influences of relational language on number relation skills. In this study, I examine whether relational language influences number relation skills, and whether it accounts for additional variance in number relation skills beyond counting/numbering skills.

Association between executive function and relational language. In the process of forming relations between items and using language to describe these relations, comparison between these items is required. Similar to the influences of EF skills on number relation skills, EF skills may be involved in comparing and ordering non-numerical items. Children may need to inhibit irrelevant information, shift attention between two or more items, make comparisons based on the relevant dimension, and incorporate information from the comparisons. For instance, to understand the meaning of *medium* and identify the medium item in a set of objects, children may need to conduct a series of pairwise comparisons and make connections between these comparisons to order the objects based on size. The process of learning relational language may also provide opportunities to practice EF skills. In fact, EF skills and relational language follow similar developmental trajectories during the same developmental period, and several studies suggest the association between the EF and relational language.

Both EF skills and relational language develop during preschool years. Many children perform poorly on EF tasks such as Day Night and DCCS at 3 years of age, but their performance improves overtime and they may demonstrate proficiency on these tasks at 5 or 6 years of age (Carlson, 2005; Carlson et al., 2013; Wiebe et al., 2008; Zelazo & Carlson, 2012). Similarly, children's performance on comparing and ordering items based on relational language improves throughout preschool years (Achenbach & Weisz, 1975; Ebeling & Gelman, 1994; Sera & Smith, 1987). Two-year-olds show emerging ability to use relational language flexibly when comparing size of objects (Sera & Smith, 1987). Specifically, two-year-olds are able to shift their size label for an object from "big" to "little" when the comparison changes (e.g., when a much bigger object is added and now the "big" object is "little" in this context), but only when the size difference between the objects is extreme. Three-year-olds often perseverate on their response to previous size comparisons (e.g., continue labeling an object as "big" even when the counterpart changes in the new comparison), whereas, four-year-olds are able to focus on the current comparison and adjust their size label for the target object (Gao, Zelazo, Sharpe, & Mashari, 2014). Furthermore, four-year-olds, but not younger children, are able to flexibly shift their judgment of size comparison based on normative (e.g., which object is bigger in the real world?) and perceptual (e.g., which object looks bigger in the picture?) contexts (Ebeling & Gelman, 1988, 1994; Sera & Smith, 1987). Together, the findings suggest that both EF skills and relational language may follow similar developmental trajectories during preschool.

Executive function and relational language are correlated with each other in early childhood. First graders' ability to hold information in working memory is correlated with their performance on seriating items based on size and sets based on quantity (Nunes et al., 2007). Direct manipulation of EF demands during relational language tasks may influence children's performance on these tasks. When the examiner reduces EF demands by reminding children the specific objects being compared, children are more likely to focus on the current comparison rather than perseverating on the old comparison (Gao et al., 2014). Similarly, preschoolers are able to seriate five objects based on size when the examiner reduces the EF demands by breaking down the task into iterative pairwise comparisons and prompting children to recall these comparisons (Bryant & Trabasso, 1971). These studies suggest that some relational language tasks may involve EF skills, and that EF skills may support the acquisition and use of relational language.

The focus of the current study is not investigating the relation between EF skills and relational language per se, but on the unique influences of EF skills and relational language on number relation skills. Given the association between EF skills and relational language, I statistically control for EF skills when examining the influences of relational language on number relation skills, and vice versa. By doing so, I can compare the amount of variance EF skills and relation language independently account for in number relation skills.

Number Relation Skills as a Mediator

Executive function and mathematical skills. Research studies have revealed both concurrent correlations and predictive relations between aspects of EF and aspects of mathematics skills (Best, Miller, & Naglieri, 2011; Bull & Scerif, 2001; Cragg & Gilmore, 2014; Fuhs, Nesbitt, Farran, & Dong, 2014; Geary, Hoard, & Nugent, 2012). For instance, preschoolers' EF skills, as measured by a broad battery of tasks that involve inhibition, working memory, and cognitive flexibility, predict their general mathematics ability nine months later (Clark et al., 2014). After controlling for concurrent mathematical skills, kindergarten students' working memory capacity remains a significant predictor of their mathematical skills nine months later (Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015), suggesting the unique influences of EF skills on later mathematical skills. Children's EF skills in first grade correlate with their concurrent mathematics performance, and predict their later mathematics achievement in third and fifth grades (Mazzocco & Kover, 2007). Children with poor EF skills show difficulty in mathematics and this relation is observed throughout elementary school years (e.g., Bull & Scerif, 2001; Geary et al., 2009; Geary, Hoard, Nugent, & Bailey, 2012; Wang, Georgiou, Li, & Tavouktsoglou, 2018). In addition to the correlational and longitudinal evidence on the relation between EF skills and mathematical skills, growth in the two domains is related to each other. Specifically, children's growth in EF skills during preschool is associated with their concurrent growth in mathematical skills (McClelland et al., 2007) and predicts their growth in mathematical skills during kindergarten (Welsh, Nix, Blair, Bierman, & Nelson, 2010). Together, these studies support the notion that EF skills influence mathematical skills and development in early childhood, but offer little if any evidence on the mechanisms underlying this relation.

Arithmetic skills may be one of several potential pathways through which EF skills influence mathematical skills. Arithmetic skills involve recalling and holding arithmetic facts in working memory, inhibiting neighboring solutions or alternative operations, and shifting attention between operations and problem-solving strategies. The association between EF and arithmetic skills have been reported in preschool age children (Soto-Calvo, Simmons, Willis, & Adams, 2015), elementary school age students (Li, Zhang, Wang, Ding, & Si, 2018; Mabbott & Bisanz, 2008), and adults (Hubber, Gilmore, & Cragg, 2014). In kindergarten, students with high EF skills are more likely to use more sophisticated arithmetic problem-solving strategies, such as fact retrieval or decomposition of numbers, as opposed to less sophisticated strategies such as counting fingers (Geary, Hoard, & Nugent, 2012). Given the relation between EF and arithmetic skills, the influence of EF on mathematical skills may be through arithmetic skills. Cragg and colleagues tested the hypothesis by conducting a mediation analyses with arithmetic skills as a mediator between EF skills and mathematics achievement (Cragg et al., 2017). They found that arithmetic skills partially mediated the relation between EF skills and mathematics achievement in participants from 8 to 25 years of age. Fuhs and colleagues found that kindergarten students' ability to compose and decompose numbers fully mediated the relation between EF and mathematical skills (Fuhs, Hornburg, & McNeil, 2016). Together, these findings suggest that arithmetic skills may be a pathway through which EF skills influence mathematical skills.

Given the influences of EF skills on number relation skills (e.g., Bos et al., 2014; Gashaj et al., 2016; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Kolkman et al., 2013), number relation skills may be another pathway through which EF skills influence mathematical skills. Two studies have examined this pathway and found different results. Attout and Majerus (2017) found that performance on a number ordering task fully mediated the relation between working memory and arithmetic skills in children ages 7 to 9 years. However, Fuhs and colleagues (2016) found that although kindergarten students' EF skills significantly predicted their accuracy on number line estimation, performance on number line estimation did not mediate the relation between EF skills and mathematics achievement. In the current study, I include both number line estimation and number ordering in my battery of number relation tasks and examine whether number relation skills mediate the relation between EF and mathematical skills in kindergarten students.

Relational language and mathematical skills. Relational language can be used to describe relations between numbers but also other mathematical concepts. For instance, words such as *long, tall*, and *heavy* are often used for measurement; *near* and *far* are used to describe distance between two objects; and *parts* and *whole* may be involved in learning about fractions and proportion. In fact, relational language is recognized as an important component of mathematics curricula (Brunn, Diaz, & Dykes, 2015; Lansdell, 1999), and is embedded in research based mathematics curricula for preschoolers (Clements & Sarama, 2011) and kindergarten students (Chard, Baker, Clarke, Jungjohann, & Davis, 2008). Correlational, longitudinal, and intervention studies provide evidence to support the notion that relational language influence mathematical skills. For instance, relational language significantly predicts mathematical skills in preschool and kindergarten students (Purpura & Reid, 2016). Children's knowledge of relational language in fall semester predicts their numeracy skills in spring semester during preschool (Purpura et al., 2017). Kindergarten students with high knowledge of relational language show more growth in their mathematical skills in six months, compared to students with low knowledge of relational language (Toll & Van Luit, 2014). Furthermore, integrating relational language in instructional activities for measurement concepts increases fourth grade students' frequency of using relational language and their accuracy of applying relational language in mathematical contexts (Monroe & Pendergrass, 1997). Together, these studies suggest that relational language may influence mathematical skills in preschool and elementary school age children, and show that relational language may affect mathematical skills through measurement concepts in fourth grade students. The studies with preschool and kindergarten children do not, however, reveal potential mechanisms underlying the association between relational language and mathematical skills during early childhood. Given that relational language is a crucial tool for describing and communicating number relations, in the current study, I examine whether number relation skills may be a pathway through which relational language influences mathematical skills in kindergarten students.

Number Training

If skills in counting/numbering, executive function, and relational language influence number relation skills, promoting these foundational skills should improve number relation skills. Many researchers have conducted experimental studies to examine the effects of counting/numbering or number relation training on children's number relation skills (e.g., Bos, Kroesbergen, & Luit, 2018; Honoré & Noël, 2016; Siegler & Ramani, 2008; Xu & LeFevre, 2016). For instance, preschool children who played with a 1 - 10 number line board game that involved counting and moving tokens along the number line for four 15-minute sessions showed improvement on counting, numeral identification, number comparison and number line estimation, and the effects of training remained after two months (Ramani & Siegler, 2008). Similarly, kindergarten students who played with a virtual 1 – 100 number board game for ten 15-minute sessions showed improvement on counting and number line estimation, but they did not improve on arithmetic skills (Ramani, Jaeggi, Daubert, & Buschkuehl, 2017). Kindergarten students who practiced verbal counting and set counting for twelve 20-minute sessions showed improvement on counting and symbolic number line estimation, but they did not improve on non-symbolic number line estimation or number comparison (Bos et al., 2018). Preschool children who practiced counting with an examiner for 15 minutes showed improvement on number ordering and number line estimation (Xu & LeFevre, 2016). Together these studies reveal that short-term number training may be effective at improving the targeted and closely related skills such as counting and number line estimation, but the effects may not transfer to untrained skills. If executive function skills and relational language also influence number relation skills, perhaps promoting these non-numerical skills may improve number relation skills.

Effects of number + EF training. Although there is a consistent relation between EF and mathematical skills, findings on the effects of EF training or number + EF training on early numeracy skills are mixed. Ramani and colleagues (2017) provided ten 15-minute sessions of working memory training or number board game training, and compared the effects of these two training conditions on early numeracy skills. The working memory training involved recalling sequences of colors and the number board game training involved counting and number comparison. They found that both groups of kindergarten students improved on numeral identification and number line estimation, but students in the number board game training improved more on number line estimation compared to students in the working memory training. Kroesbergen and colleagues (2014) provided eight 30-mintue sessions of working memory training with either non-numerical stimuli or numerical stimuli, and compared their effects on early numeracy skills. The non-numerical working memory training involved recalling lists of everyday items and the numerical working memory training involved recalling numbers of items. They found that both groups of kindergarten students showed improvement on counting and the amount of improvement was comparable between the two groups. Kroesbergen and colleagues did not include a number only training, thus it was unclear whether numerical working memory training would be more effective than a number only training. Prager (2016) provided three 15-minute sessions of number only training, EF only training, or number + EF training to preschoolers and compared the training effects on early numeracy skills. The number only training involved counting sets of dots. The EF only training involved shifting attention and sorting pictures based on different dimensions. The number + EF training involved sets of items that varied in color and shape (e.g., 3 red rabbits, 2 blue rabbits, 2 red boats, and 1 blue boat), and children practiced counting these sets of items based on different dimensions (i.e., red things, rabbits). Children in the EF only training showed improvement on EF skills but not early numeracy skills. Children in number only or number + EF training showed comparable improvement on early numeracy skills. Children in the number + EF training did not outperform children in the number only training, however, the training was brief and it remained unclear whether more intensive training would lead to different findings. Together, these studies suggest that more extensive working memory training may lead to some improvement on early numeracy skills, but brief EF only training may not. Number + EF training may improve children's early numeracy skills but it is unclear whether number + EF training is more effective than number training alone.

Effects of number + RL training. Although relational language appears to be important for mathematical skills, the results on the effects of RL only training or number + RL training on mathematical skills are inconclusive. Jennings and colleagues (1992) examined whether using relational language during non-numerical book reading activities was more effective at improving kindergarten students' mathematics ability compared to a traditional mathematics curriculum. They found that after five months of intervention, children in the book reading + RL condition showed more improvement on mathematics ability and used more relational language during free play compared to children in the traditional mathematics curriculum condition. A different pattern of results emerged when a similar intervention was conducted with low achieving kindergarten students for two months (twenty-four 30-minute sessions). Hassinger-Das and colleagues (2015) found that using relational language during non-numerical book reading activities improved children's understanding of relational language but these children did not show improvement on mathematical skills. The RL only training in a non-numerical context may not improve children's mathematical skills, but number + RL training do affect children's early numeracy skills. Laski and Siegler (2007) provided number + RL training by prompting kindergarten students to categorize numbers as very small, small, medium, big and very big based on the cardinal values. After four sessions of number + RL training, students showed improvement on number line estimation but they did not improve on number comparison. This study did not include a number only training as a comparison, thus it was unclear whether number + RL training provided additional benefit on children's number line estimation beyond number training alone. Powell and Driver (2014) compared the effects of addition only training vs. addition + RL training in first grade students with mathematics difficulty. They found that after fifteen 10- to 15-minute training sessions, both groups of students showed improvement on their arithmetic skills, but students in the addition + RL training did not outperform students in the addition only training at posttest. Together, these studies reveal that extensive training on relational language improves children's knowledge of relational language, but the training effects on mathematical skills may depend on the dosage and children's initial mathematical skills at pretest. Number + RL training may improve children's early numeracy skills, but it is unclear whether number + RL training is more effective than number training alone.

Effects of children's initial executive function skills. Individuals vary in the amount of improvement they make from training sessions, and identifying factors that predict learning and growth is important for understanding what learning activities are most efficient and effective at improving an individual's outcome. Children's amount of improvement on mathematical skills may vary depending on their initial mathematical skills, but the patterns of results differ across studies. Children who perform poorly on number comparison and number line estimation at pretest show more improvement on these tasks after four number line training sessions, compared to children who perform well on these tasks at pretest (Ramani & Siegler, 2011). One interpretation of this finding is that perhaps children with low pretest scores have more room for improvement on these tasks. This is consistent with the findings on EF training. Children with low initial EF skills tend to benefit more from EF training programs compared to children with high initial EF skills (Diamond & Ling, 2016). However, other studies reveal different patterns of results. Swanson, Jerman, and Zheng (2008) found that children with high initial mathematics knowledge in first grade showed more growth in mathematical skills from Grade 1 to Grade 3, compared to children with low initial mathematical knowledge. Yet, Jordan and colleagues (2006) found no relation between children's initial numeracy skills and their growth in mathematical skills during kindergarten. Specifically, children with high or low initial numeracy skills progressed at the same rate in their growth of mathematical skills.

Given the evidence on the positive relation between EF skills and children's growth in mathematical skills (Best et al., 2011; Blair & Razza, 2007; McClelland et al., 2014; Welsh et al., 2010), EF skills may be another predictor of training effects on early numeracy skills. Kolkman and colleagues (2013) found that kindergarten students with high EF skills showed more improvement on number line estimation after six sessions of number training, compared to students with low EF skills. However, given the consistent correlation between EF and mathematical skills, it is possible that initial EF skills are a proxy of initial mathematical skills. Thus, it remains unclear whether children with high EF skills benefit more from the training sessions because (a) they have better behavioral regulation and attention control, (b) they have high initial mathematical skills, or (c) a combination of both.

Together, the training studies reveal that number training may be effective at improving children's early numeracy skills, but whether EF or RL training improves children's early numeracy skills may depend on the type, context, and dosage of the training. To systematically delineate the potential causal influences of EF skills and relational language on number relation skills, I examined whether number + EF + RL training were more effective at improving children's number relation skills compared to number training alone. By doing so, I experimentally tested whether executive function and relational language combined have additional influences on number relation skills beyond counting/numbering skills. I also examined whether initial EF skills predicted the amount of improvement children made on number relation skills, and tested the unique influences of initial EF skills on gains in number relation skills by controlling for children's initial number relation skills. Similarly, I explored whether children's initial knowledge in relational language predicted their gains in number relation skills.

Current Study

In the present study, I used a pretest – training – posttest research paradigm to examine the influences of executive function skills and relational language on number relation skills. My specific research questions are as follows:

a) Do executive function skills or relational language account for additional variance in number relation skills beyond counting/numbering skills?

b) Do number relation skills mediate the reported relation between executive function and mathematical skills, and the reported relation between relational language and mathematical skills?

c) Does number + EF + RL training improve number relation skills more than number only training?

d) Do children's initial executive function skills predict their pretest to posttest gains in number relation skills beyond their initial number relation skills?

Based on prior research, I hypothesize that (a) EF skills and relational language may account for additional variance in number relation skills beyond counting/numbering skills; (b) number relation skills may be a pathway through which EF skills and relational language influence mathematical skills; (c) children who receive number + EF + RL training may show more improvement on number relation skills from pretest to posttest compared to children who receive number only training; and (d) children's initial executive function skills may predict the pretest to posttest gains in number relation skills but this effect may not be significant after accounting for the correlation between EF and number relation skills at pretest.

Methods

At pretest, I measured children's general vocabulary knowledge, and their skills in mathematics, counting/numbering, number relation, EF, and relational language. Next, children were quasi-randomly assigned to one of three training conditions: (1) number, (2) number + EF + RL, or (3) alphabet, based on their EF skills so children with above vs. below sample median EF skills were equally represented in each of the three conditions. In the number training, children practiced counting and ordering numbers. In number + EF + RL training, children practiced counting and ordering numbers, reflected on their responses, generated an alternative problem-solving strategy, and received relational language instruction. In alphabet training, children practiced reciting and ordering alphabetic letters. All pretest measures were repeated at the post-test, except for general vocabulary knowledge and mathematical skills, to determine changes in these domains after training (Figure 1).

Participants

Participants were kindergarten students recruited from four public schools, in three school districts, near a metropolitan area in the Midwestern United States. The student body demographics were diverse, and the dominant racial category at each of these schools was either White (68%), Hispanic (52%), Asian (42%), or Black (82%). At each school, the percentage of the English Language Learners ranged from 9% to 57%, and the percentage of students eligible for free and reduced lunch (FRL) ranged from 22% to 89%.

Kindergarten students who spoke English, and who had no significant visual or auditory impairment or known developmental delay, were eligible and invited to participate in this study. English Language Learners who spoke English were included. Researchers invited parents to enroll their child in the study in person at school functions (e.g., parent teacher conference), or by sending parents enrollment materials through their child's teacher. If the researchers obtained consent in person, the lead researcher answered parents' questions about the study during consent process. If the researchers obtained consent via enrollment materials sent home to the parents, the lead researcher contacted parents to answer questions upon their request. On average, 50% of the invited families at the participating classrooms agreed to enroll their child in the study. Enrollment rates across schools ranged from 27 to 66%.

A total of 104 typically developing kindergarten students (46 boys) participated in the study. The participants were five or six years of age (*M* = 5.9 years, *SD* = 0.34), and were of White (38; including 2 Spanish/Hispanic/Latino), Black (26), Asian or Asian American (15), American Indian or Alaskan Native (2), or Other (23; including 20 Spanish/Hispanic/Latino) ethnicity. Approximately half of the participants were regularly exposed to languages other than English (22 Spanish, 14 Somali, 11 other), and the remaining participants were not (57). Five additional participants were excluded from the study due to family relocation ($n = 1$) or participants' request to end the sessions prior to study completion ($n = 4$). Power analysis revealed that a sample of 104 would have 80% power to detect medium to large effects of training condition ($f = .31$; Cohen (1992) suggests f values of .02, .15, and .35 represent small, medium, and large effect sizes) at $\alpha = .05$ level.

Procedure

All children were tested individually by a female examiner in the child's school, at a relatively quiet area away from other students. Most children completed the pretest measures in two sessions, which were followed by four training sessions, and one posttest session. Exceptions to this resulted in shorter, and one additional, pretest or posttest session due to breaks requested by children or time constraints set by teachers. All children completed all the study sessions over a five-week span (*M* = 36.09 days, *SD* = 10.78 days). Data collection was completed by the lead researcher and two trained research assistants. The lead researcher administered all training sessions, and approximately 61% of the pretest and posttest sessions. The two researcher assistants administered the remaining 39% of the pretest and posttest sessions. To maintain consistency of examiners, all children were tested by the lead researcher and only one of the two research assistants.

Pretest Sessions

Baseline measures. Children's verbal knowledge and mathematical skills were measured at pretest only, and included in select analyses as a covariate or outcome variable respectively.

Verbal knowledge. Almost half of the participants were regularly exposed to languages other than English, so baseline verbal knowledge was measured for all children, and statistically controlled for in select analyses. The Verbal Knowledge subtest of the Kaufman Brief Intelligence Test – 2nd Edition (KBIT-2; Kaufman & Kaufman, 2004) was used to measure children's receptive vocabulary knowledge in English. During this subtest, children viewed a set of six pictures on a single page of a testing easel placed on a table, while the examiner posed a question. Children were asked to identify which of six pictures corresponds to either the meaning of a spoken word (e.g., "point to, clock") or the answer to a question (e.g., "what lives in a forest?") posed by the examiner. KBIT raw scores reflect the number of correct picture selection responses, and are normed for participants from 4 to 90 years of age. In this study, I used the standard score based on an age-referenced mean of 100 and a standard deviation of 15 as the covariate. The internal-consistency reliability of the KBIT-2 Verbal Knowledge subtest is .87 (Kaufman & Kaufman, 2004).

Mathematical skills. The Test of Early Mathematics Ability – 3rd Edition (TEMA-3; Ginsburg & Baroody, 2003) was administered as a measure of children's general mathematical skills in order to determine whether number relation skills mediate the reported relations between EF skills and mathematical skills, and between relational language and mathematical skills. During this test, the examiner provided pictures, manipulatives, or paper and pencil, and children either pointed to a picture or gave a verbal or written response to the examiner's questions. For instance, the examiner showed a picture of three cats and asked children to respond either verbally or with paper and pencil to the question, "how many cats are there?" On problems involving manipulatives, the examiner placed the manipulatives on a table for children to use, and posed questions such as, "Amy has five tokens, and she gets two more. How many does she have altogether?" The TEMA-3 is a comprehensive assessment of children's formal and informal mathematical skills. It includes items such as verbal counting, set counting, story problems, and arithmetic fact retrieval. Similar types of problems were grouped for the ease of test administration but the beginning and the end of the test followed the standard procedure. All children began the test on the entry item for five-year-olds, and the test ended when they responded incorrectly on five consecutive items. The TEMA-3 raw scores representing the number of correct items are normed for children from 3 to 8 years of age. I used the standard score based on an age-referenced mean of 100 and a standard deviation of 15 as an indicator of children's general mathematical skills. The internal-consistency reliability of the TEMA-3 is .94 (Ginsburg & Baroody, 2003). Form A of the TEMA – 3 was administered in this study.

Measures of interest. Children's skills in counting/numbering, number relation, executive function, and relational language were measured at both pretest and posttest to examine changes in these skills following the training period.

Counting/numbering skills. Select TEMA-3 items that focused on counting sequence (counting aloud, counting backwards, and counting after), counting process (set counting), and numbers (numeral identification) were expanded to measure children's counting/numbering skills (see Appendix A for pretest and posttest items). A composite score for counting/numbering skills was created based on a confirmatory factor analysis with the following five tasks, and used for select analyses.

Counting aloud. To determine the highest number children were able to verbally count up to without error, children were asked to count as high as they could from 1. If children stopped counting, the examiner encouraged them to continue by prompting them with, "what comes next?" The task ended when children indicated that they did not know what came next or when they reached 130. The examiner recorded the highest number that the children counted to correctly. This TEMA-3 item was not expanded.

Counting backwards. The examiner first demonstrated counting backwards from three (i.e., three, two, one), then asked children to count backwards from 10, and then from 20. Children received a score based on the highest number they could count back from without error. For instance, they received a score of 20 if they could count backwards from 20 without error, a score of 12 if they could count backwards from 12, or a score of 0 if they could not count backwards at all. This TEMA-3 item was not expanded.

Counting after. Unlike counting aloud, this task examines whether children can continue a count sequence without starting from 1. The examiner said a number, and asked children to name the number that comes next. For instance, the examiner asked, "what comes next? Three and then comes..." The examiner waited for children to respond and recorded their response. The given number ranged from 1 to 99. This TEMA-3 item was expanded from nine trials to a total of 14 trials, and the examiner recorded the number of trials on which the children responded correctly.

Set Counting. The examiner showed children a page with a set of black dots and asked them to count the dots carefully with their fingers and tell the examiner how many there are. The set size ranged from 8 to 17. A total of four sets was presented, one at a time, and the examiner recorded the number of trials on which children counted the dots correctly. This TEMA-3 item was not expanded.

Numeral Identification. The examiner presented printed Arabic numerals in a random sequence, and asked children to name the number. The numbers ranged from 1 to 99. This TEMA-3 item was expanded from eight Arabic numerals to a total of 20 Arabic numerals and the examiner recorded how many numerals the children named correctly.

Number relation skills. Number relation tasks differ from counting/numbering tasks in that they require some knowledge on cardinal, ordinal, or spatial relations of numbers. Five tasks were administered to measure children's number relation skills with symbolic numbers (number comparison and number ordering), non-symbolic sets (set relation and numerosity estimation), and number-space mapping (number line estimation) (see Appendix B for pretest and posttest items). Although TEMA-3 also includes items similar to number comparison and numerosity estimation, these tasks were not expanded from TEMA-3. All, except number ordering, were adapted from experimental tasks found in research studies on mathematical development (e.g., De Smedt et al., 2009; Lipton & Spelke, 2005; Sarnecka & Carey, 2008; Siegler & Booth, 2004). Number ordering was a lab-developed task. A composite score for number relation skills was created based on a confirmatory factor analysis with the following five tasks, and used for select analyses.

Number Comparison. The examiner asked children which number means more, then stated two numbers. All number pairs differed by two (e.g., 3 vs. 5), and the numbers ranged from 1 to 99. A total of 16 trials was administered and the examiner recorded the number of trials on which children responded correctly. Variations of this task are widely used in other studies on mathematical development (e.g., De Smedt, Verschaffel, & Ghesquière, 2009; Gray & Reeve, 2016; Laski & Siegler, 2007; Lyons, Price, Vaessen, Blomert, & Ansari, 2014).

Number Ordering. The examiner presented three Arabic numeral cards, out of numerical order, on a table, and asked children to rearrange these numbers in order from smallest (left) to largest (right). The numerical distance between numbers was consistent within the triplets and was either one (e.g., 1, 2, 3) or two (e.g., 1, 3, 5) across triplets. The numbers ranged from 1 to 50. A total of 14 trials was administered and the examiner recorded the number of trials on which children correctly ordered the numerals from smallest to largest.

Set Relation. The examiner first showed children a small opaque bag and told them that there are N coins in the bag. Then the examiner added 1 or 2 coins, one at a time, and asked them whether there are N + 1 or N + 2 coins now. The first addend ranged from 1 to 20. A total of 8 trials was administered and the examiner recorded the number of trials on which children responded correctly. This task was adapted from Sarnecka and Carey's unit task (2008).

Numerosity Estimation. The examiner showed children sets of shapes on a 13-inch computer screen and asked them to estimate the quantity "really quickly without counting." Each set disappeared after 1.5 seconds, to prevent counting. To calibrate children's estimation, the examiner showed two sets of shapes, 20 and 100, and provided accurate numbers for these sets. All children were first calibrated to a set of 20 shapes, then estimated the quantity for 16 sets, one at a time. Next, they were calibrated to a set of 100 shapes, then estimated another 16 sets. The size of the individual shapes and the total filled area of the display were controlled for in both types of calibration, across trials. Individual shape size was consistent across the size-controlled trials such that larger numerosities also had greater total filled area. The total filled area was consistent across the area-controlled trials such that the individual shapes were smaller for larger numerosities. The numerosities presented ranged from 5 to 100. Because the range of numerical responses was not constrained, some children gave very large numbers, such as one million, when they saw ninety squares. To limit the influences of these large number responses on children's estimation accuracy, responses more than 10 times the calibration value (i.e., 20 or 100) were replaced with 200 or 1000, respectively. Percent absolute error ($|$estimate – target number$|$ / numerical range) was calculated for each trial. Children's performance on trials calibrated to 20 and to 100 were highly correlated with each other, *r*s > .47, *p*s < .01, thus the average percent absolute error was used as an indicator of children's performance on this task. Variations of this task are used in other studies on mathematical development (e.g., Barth, Starr, & Sullivan, 2009; Izard & Dehaene, 2008; Lipton & Spelke, 2005; Sullivan & Barner, 2014).

Number Line Estimation. The examiner presented a number line and a target number on a letter size paper, and asked children to estimate where the target number goes on the number line. The length of the line was 25.6 cm and two points were marked at 6.6 cm from each end of the line, resulting a length of 12.8cm in between two points. A total of 32 trials was administered, 16 trials with the points labeled as 0 and 20, and 16 trials with the points labeled as 0 and 100. On eight of the 0 – 20 trials, the number 10 was labelled at the midpoint, and on eight of the 0 – 100 trials the number 50 was labelled at the midpoint. These midpoint versions were included in view of the complexity of the number line task for young children. The target numbers ranged from 3 to 33 for 0 – 20 trials and 7 to 120 for 0 – 100 trials. As per Siegler and colleagues, percent absolute error (│estimate– target number│/ numerical range) was calculated for each trial (e.g., Siegler & Booth, 2004). Children's performance on these four types of trials were highly correlated with each other, *r*s > .42, *p*s < .01, thus the average percent absolute error was used as an indicator of children's performance on this task. Variations of this task are widely used in other studies on mathematical development (e.g., Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Friso-van den Bos et al., 2015; Lyons et al., 2014; Siegler & Booth, 2004).

Executive function skills. Two widely used tasks, the Head Toes Knees Shoulders and the Minnesota Executive Function Scale, were administered to measure aspects of children's EF skills. As described below, both tasks require children to remember rules, flexibly shift between different rules, and inhibit dominant responses.

An EF composite score was created based on the average z score of the two tasks, and used for select analyses.

Head Toes Knees Shoulders task (HTKS). Head Toes Knees Shoulders task (HTKS; Cameron Ponitz et al., 2008) is a standardized activity in which children are instructed to do something different from what the examiner says. For instance, children should touch their toes when the examiner says, “touch your head.” The task includes three levels presented in order of increasing difficulty that are designed to capture the range and variability of EF skills. Level 1 is comprised of one pair of rules: (a) “touch your head” means touch your toes, and (b) “touch your toes” means touch your head. Level 2 is similar to Level 1 but with an additional pair of rules: (a) “touch your knees” means touch your shoulders, and (b) “touch your shoulders” means touch your knees. Rules are changed in Level 3: (a) head is paired with knees, and (b) shoulders are paired with toes. The examiner provides practice trials with up to three reminders at each level. A total of 47 trials was administered (17 practice trials and 30 test trials), and children received 0 (incorrect), 1 (self-correct), or 2 (correct) points per trial, with a total possible score of 94. As per the task developer’s recommendation, the practice trials were included in the total score to increase its range and variability. The internal consistency reliability of the task ranges from .87 to .92 (Cameron Ponitz et al., 2008). Form A of the HTKS task was administered at pretest and an equivalent Form B was administered at posttest.

Minnesota Executive Function Scale (MEFS). Minnesota Executive Function Scale (MEFS; Carlson & Zelazo, 2014) is an adaptive iPad version of the Dimensional Change Card Sort task (Zelazo, 2006). Children sort virtual cards into two virtual boxes according to the stated rules. Children have to remember the rules, ignore conflicting features, and sort cards flexibly when the sorting rules change. For instance, the examiner reveals a virtual card with a picture of a green lion on the iPad, and children should drag the card to the box with a picture of a green monkey when they are playing the color game, but drag it to the box with a picture of an orange lion when they are playing the shape game. There are seven levels of difficulty and the entry level is determined by the participant's age. The difficulty increases with the level as the examiner introduces additional rules (in Levels 1 to 3), changes the rules (in Levels 4 and 5), or a combination of both (Levels 6 and 7). The examiner provides practice trials with feedback at the beginning of each level, and the examiner has children progress forward to the next level or backward to an easier level depending on children's performance. The total scores are computed based on highest level passed, highest level attempted, errors, and reaction time, and are normed for children from 2 to 13 years of age. A standard score based on an age-referenced mean of 100 and a standard deviation of 15 was used as the indicator of children's performance on this task. The test retest reliability for the MEFS is *ICC* = .93 (Beck, Schaefer, Pang, & Carlson, 2011). Form A of the MEFS was administered at pretest and an equivalent Form B was administered at posttest.

The HTKS total score at pretest was used to determine quasi-random assignment for training conditions, so that the proportion of children with high vs. low HTKS score, based on sample median, was comparable across the three training conditions. The HTKS score was used in training condition assignment process instead of the MEFS score because it had a wide range of scores in the sample (0 – 92). By contrast, MEFS scores were restricted, with over 40% of the sample had Level 4 as their highest level passed.

Moreover, the HTKS and the MEFS scores are highly correlated during kindergarten (*r* = .53 in McClelland et al., 2014; r_s = .44 in pretest and r_s = .52 at posttest in this study). The MEFS standard score at pretest confirmed that the distribution of children with high vs. low EF skills were comparable across training conditions (Table 1).

***Relational language**.* Boehm Test of Basic Concepts – 3rd Edition (here asfter Boehm Test; Boehm, 2001) was used to measure children's receptive knowledge of relational language. This is a standardized picture vocabulary task where the examiner presents an illustration with four or five options (e.g., four children at a drinking fountain) and asks children to point to the target option (e.g., the child who is *last* in line). Different from the KBIT-2 Verbal Knowledge subtest, Boehm Test focuses specifically on relational concepts (e.g., last, center, above, between). A total of 50 trials was included in this task, and the raw scores for the number of correct picture selection responses have been normed for children from 5 to 8 years of age. In this study, an age-referenced percentile was used as the indicator of children's knowledge in relational language. The internal consistency reliability of this task ranges from .88 to .90. Form E of the Boehm Test was administered at pretest and an equivalent Form F was administered at posttest.

Training Sessions

If number relation skills are influenced by not only counting/numbering skills but also EF skills and relational language, number training that incorporates EF prompts and RL instruction should improve children's number relation skills beyond number training alone. Thus, the purpose of the training was to test whether number + EF + RL training improve children's number relation skills more than number training alone.

The training session materials and procedures are based on an instructional activity, “X-ray vision,” derived from the Building Blocks Prekindergarten Mathematics curriculum (Clements & Sarama, 2007). During the “X-ray vision” activity, number cards are arranged in numerical order facedown, and children are asked to figure out what number is on each card. This activity was selected because it provides ample opportunities for children to practice counting and discuss number relations. It was modified and extended for this study, in order to create two training conditions (number, number + EF + RL) each with four 10- to 15-minute sessions. An additional alphabet training closely matching number training was created to serve as a control condition.

Training materials. A total of 20 cards was used during all four training sessions. To keep the stimuli engaging and to facilitate learning across different stimuli, the shape and color of the stimuli varied within and across sessions. In number and number + EF + RL training conditions, Arabic numerals 1 to 20 are each represented on a card with their corresponding numerosity. Within a session, the color of the numeral fonts, and the color and shape of the numerosity are consistent within each set of five cards (e.g., 1 to 5), but vary across four sets of five cards. For instance, numbers 1 to 5 are represented with orange numerals and sets of 1 to 5 brown circles, whereas numbers 6 to 10 are represented with gray numerals and sets of 6 to 10 purple squares in Training Session 1. Across sessions, the color of the numeral fonts, and the color and the shape of numerosities vary for each number. For instance, number 1 is represented by an orange numeral and one brown circle in Training Session 1, but is represented by a blue numeral and one orange square in Training Session 2. In the alphabet training condition, the upper- and lower-case versions of the first 20 alphabetic letters (Aa to Tt) were used as the stimuli instead of Arabic numerals 1 to 20 and their corresponding numerosities. Similar to the stimuli used in the other two training conditions, the color of the alphabetic letter fonts vary within and across sessions (see Appendix C).

Training activities and conditions. The examiner followed similar activity procedure for the three training conditions. The activity procedure for each training session are summarized in number training section, and the differences between number + EF + RL vs. number and alphabet vs. number are described in their respective sections.

Number training. The examiner practiced aspects of counting (e.g., one to one correspondence, counting from 1, counting from N) with children through introduction and feedback during all four training sessions in the number training condition. Details of the counting practice are described below (see Figure 2 for an illustration of the training sessions, and Appendix D for training scripts).

Training Session 1. The session began with the examiner introducing 1 to 20 by counting and placing the cards, one at a time in order, on a table. The numbers were arranged in two horizontal rows of ten, with numbers 1 to 10 in the top row, and numbers 11 to 20 in the bottom row. After the introduction, the examiner flipped all the cards facedown, and explained that there's a trick to figuring out what number was on each card without seeing it. To demonstrate the trick, the examiner asked the child to point to any card and the examiner counted quietly from 1 to figure out the target card. After the examiner announced her answer, she flipped over the card to reveal the number, then flipped the card facedown. After three demonstration trials, the roles changed and the examiner pointed to cards, one at a time, asked the child to figure out what numbers were on the cards, and recorded the child's response. The examiner provided feedback on each of the training trials. If children correctly identified the target number, the examiner confirmed by saying, “That’s right! It is N and it has N shapes on it.” If children responded incorrectly, the examiner practiced one to one correspondence and counting from one with them by saying, “Let’s count together” and pointed to each number card one at a time while counting.

Training Session 2. The steps and the feedback were the same as in Training Session 1, except that the examiner counted from the closest known number instead of from 1 during demonstration and feedback.

Training Session 3. The examiner and the child worked with five cards at a time, and the steps were repeated for each set of five cards. The session began with the examiner introducing 1 to 5 by counting and placing the cards, one at a time in order from left to right, on a table. After the introduction, the examiner hid one or two cards, and asked the child to figure out what number was missing, and where it should be in the sequence. After the child named the missing number, the examiner revealed the card. If children were correct, the examiner confirmed their answer and prompted them to put the card back where it belongs in the sequence. If children were incorrect, the examiner showed them the missing card, asked them to name it, counted the shapes together with them, then asked them to put the card back where it belonged in the sequence. If children inserted the target card in the correct location, the examiner provided feedback by confirming their answer. If children inserted the target card in an incorrect location, the examiner recited the counting sequence and moved the number card to the correct location.

Training Session 4. Similar to Training Session 3, the examiner and the child worked with five cards at a time. After introducing the five cards, the examiner mixed the cards so they were not in numerical order, and asked the child to put the cards back in order from the smallest number to the largest number. When children indicated that they were done, the examiner checked their work by pointing and naming the cards from left to right together with them. If children ordered the numbers correctly, the examiner provided confirmation. If children ordered the numbers incorrectly, the examiner helped them identify their error and correct the sequence by pointing at each card while reciting the counting sequence. For instance, if children ordered the numbers incorrectly (e.g., 1 2 4 3 5), the examiner said, "1... 2... 3... wait, this is not 3. This is 4, see 1, 2, 3, 4 (count the shapes). Which of these cards is 3 (point to the two remaining cards, 3 and 5)?" If children correctly identified 3, the examiner confirmed and put 3 in the correct location. If children misidentified 5 as 3, the examiner counted the five shapes on the card and said, "This is not 3. This is 5. This is 3 and 3 goes here (picked up 3 and put it in the correct location)."

Number + EF + RL training. The number cards and the activity procedure were the same as in the number training, except that the examiner provided EF prompts by asking children to reflect on their responses and to generate an alternative strategy. The reflection prompts were designed to provide opportunities for children to pause, recall and verbalize their problem-solving process, and the alternative strategy prompts were aimed to promote flexible thinking. The examiner also used relational language to elaborate on the cardinal and ordinal relations between numbers on each training trial.

The specific prompts are descried below (see Figure 3 for an illustration of the training sessions, and Appendix E for training scripts).

Training Session 1. After children said what they thought the number was on the target card, the examiner prompted them to reflect on their response by asking, “How do you know this is N?” prior to revealing the number. This is different from the reflection prompts used in other EF studies (Espinet, Anderson, & Zelazo, 2013; Kloo & Perner, 2003) where children are reminded of the rules of the card sorting activity when they make a mistake. The activities in the current study required counting thus the prompts were adapted to elicit reflection and verbalization of counting process instead of reciting rules for sorting cards. The reflection prompts were given on every trial to ensure that all children in this training condition practiced reflection regardless of their performance during the activity. After providing the same feedback as in Training Session 1 of the number training, the examiner used relational language to discuss the ordinal relations the target number has with its adjacent numbers by saying, “N is in *between* N – 1 and N + 1. N comes right *after* N – 1, and N comes right *before* N + 1.” On every three trials, the examiner prompted children to generate an alternative strategy by asking, “What is another way to figure out the answer?” If children generated a viable strategy, the examiner provided general encouragement and moved on to the next training trial. If children indicated that they could not think of another strategy or if their generated an incorrect strategy, the examiner demonstrated counting backwards as an alternative strategy to counting from 1.

Training Session 2. Similar to Training Session 1 in this condition, the examiner prompted children to reflect on their response by asking, “How do you know this is N?”

prior to revealing the number. After providing the same feedback as in Training Session 2 of the number training, the examiner used relational language to discuss cardinal relations the target number has with its adjacent numbers by saying, "N is in *between* N – 1 and N + 1. N has one *more* than N – 1, and N has one *less* than N + 1." The examiner prompted children to generate an alternative strategy and demonstrated counting backwards if needed on every three trials.

Training Session 3. The examiner prompted children to reflect on their responses by asking, "How do you know this is N?" after they responded to the identification question (i.e., what number is missing?). Similarly, the examiner prompted, "How do you know N goes here?" after children responded to the location question (i.e., where does it go?). After providing the same feedback as in Training Session 3 of the number training, the examiner discussed the ordinal and the cardinal relations between numbers by using the target number as an example and said, "N is in *between* N – 1, and N + 1. N comes *after* N – 1 and it has one *more* than N – 1. N comes right *before* N + 1 and it has one *less* than N + 1." The examiner prompted children to generate an alternative strategy and demonstrated counting backwards if needed on every two trials.

Training Session 4. The examiner prompted children to reflect on their responses by asking, "How do you know the numbers go in order like this?" after they indicated that they were done putting the numbers back in order. After giving the feedback that was the same as in Training Session 4 of the number training, the examiner discussed the ordinal and the cardinal relations of the five numbers by saying, for instance, "two comes *after* one, and two has one *more* than one. Three comes *after* two, and three has one *more* than two. Four comes *after* three, and four has one *more* than three. Five comes *after* four, and five has one *more* than four." The examiner prompted children to generate an alternative strategy and demonstrated counting backwards if needed on every two trials.

Alphabet training. This training condition was created to serve as a control condition, during which EF prompts and relational language instruction were not provided, and counting was never practiced. Instead, the training sessions were closely matched with those in number training, but with alphabetic letters. (see Figure 4 for an illustration of the training sessions, and Appendix F for training scripts).

Training Session 1. The session began with the examiner introducing the letters A to T (the first 20 alphabetic letters) in order. The letters were arranged in two horizontal rows of ten, with letters A to J in the top row and letters K to T in the bottom raw. Similar to the number training condition, the examiner flipped the cards face down and demonstrated how she figured out the letters by pointing to the cards one at a time and naming them from A. Children proceeded to the training trials after the demonstration. On each training trial, the examiner provided feedback by either confirming children's correct answer, or naming the letters from A to find the correct answer with them.

Training Session 2. The steps in Training Session 2 were similar to those in Training Session 1 in this condition, except that the examiner kept the cards facing up after they were identified. The examiner showed how to figure out the target letter from the closest known letter, instead of from A during the demonstration trials. On each training trial, the examiner provided feedback by either confirming children's correct answer, or naming the letters from the closest known letter to find the correct answer with them.

Training Session 3. The activity procedure was the same as in Training Session 3 of the number training, except that children interacted with alphabetic letter cards and the examiner provided feedback with alphabetic letters on each training trial. For instance, if children correctly identified the missing letter, the examiner confirmed their answer and prompted them to put the card back where it should be in the sequence. If they were incorrect, the examiner showed them the missing card, and asked them to name it and to put the card back where it should be in the sequence. If children inserted the target card in the correct location, the examiner confirmed children's correct response. If children inserted the target card in an incorrect location, the examiner recited the alphabet sequence and moved the letter to the correct location with children.

Training Session 4. The activity procedure and feedback were similar to those in Training Session 4 of the number training, except with alphabetic letters. After children were done ordering the cards, the examiner checked their work by naming the sequence together with them. The examiner provided confirmation, or helped them identify their error and correct the sequence by pointing at each card while reciting the alphabets. For instance, if children ordered the letters incorrectly (e.g., A B D C E), the examiner said, "A... B... C... wait, this is not C. This is D. Which of these cards is C (point to the two remaining cards, C and E)?" If they correctly identified C, the examiner provided confirmation and put C in the correct location. If they misidentified E as C, the examiner said, "this is not C, it is E (point to E). This is C and C goes here (picked up C and put it in the correct location)."

The order of the trials, the target cards (in Training Sessions 1 and 2), the missing cards (in Training Session 3), and the mixed sequences (in Training Session 4) were consistent across training conditions. All children completed at least 12 out of 20 trials in Training Sessions 1 and 2, at least 10 out of 12 trials in Training Session 3, and at least 8 out of 13 trials in Training Session 4. They completed more trials if time permitted. The duration and the number of trials completed for each training session by training condition are reported (Table 2).

Posttest Session

The measures of interest were re-administered during posttest to examine changes in counting/numbering, number relation, EF, and relational language following the training period. A different form of each task was used at posttest compared to pretest, but the posttest items closely mirrored the pretest items. The order of the tasks was consistent in pretest and in posttest except that the counting/numbering tasks were embedded in TEMA-3 during pretest.

Results

First, I conducted descriptive analyses to examine the distribution of the data and the correlations between the tasks, to support and inform my primary analyses plans. Two separate confirmatory factor analyses were conducted on the counting/numbering tasks and number relation tasks to create composite scores for counting/numbering skills and number relation skills for subsequent analyses. If number relation skills are distinct form counting/numbering skills, and involve EF skills and relational language, EF skills and relational language should predict number relation skills beyond counting/numbering skills. I compared one-factor vs. two-factor models with counting/numbering and number relation tasks to examine whether the two sets of skills are distinct, and used regression analyses to examine whether executive function score, a composite of HTKS and MEFS, or Boehm percentile significantly predict number relation score beyond counting/numbering score. To examine whether number relation skills were a pathway through which EF skills influence mathematical skills, I conducted mediation analyses with number relation score as a mediator between EF score and TEMA score. Similarly, mediation analyses were conducted with number relation score as a mediator between Boehm percentile and TEMA score to examine the pathway between relational language and mathematical skills. To examine whether children in number + EF + RL training improved more on aspects of number relation skills compared to children in number training, I conducted repeated measures MANOVAs with training condition as an independent variable and children's performance on number relation tasks at pretest and posttest as dependent variables. The MANOVAs with individual number relation tasks, rather than ANOVA with number relation composite score, were conducted in order to test the influences of training on specific aspects of number relation skills. Finally, to examine whether initial EF skills or relational language predicted gains in number relations skills, three separate MANOVAs were conducted with a median split of HTKS score, MEFS score, or Boehm percentile as an independent variable, and gains in number relation tasks as the dependent variables. P-values were adjusted with Bonferroni method for multiple comparisons.

Preliminary Analyses

Descriptive analyses and correlations. A review of the descriptive analyses showed that the standard scores for KBIT, TEMA, and MEFS conformed to the standardization (i.e., $M \approx 100$, $SD \approx 15$ appeared in Table 3). Based on the mean and range of scores on each task, none of the tasks was subject to ceiling or floor effects.

Correlation analyses were conducted to examine how performance on these tasks were related to each other at pretest and at posttest. Spearman correlations, instead of Pearson correlations, were conducted due to non-normal distributions of the scores on some tasks (e.g., counting backwards, counting aloud, Boehm Test). The correlations between scores from these tasks were moderate or strong ($|r_S| > .30$), with notable exceptions (see Table 4). Set counting and numerosity estimation were each only weakly correlated with many other tasks at both pretest and posttest. Together, the descriptive analyses and correlation analyses provide evidence for a wide range of performance levels in the sample, and relations across domains tasks.

Composite scores. Because children's counting/numbering skills and number relation skills were each assessed with five tasks, confirmatory factor analyses were conducted to (a) confirm whether these tasks measured aspects of their intended constructs, and (b) if suggested, reduce the data to a composite score for select further analyses. To aid comparison between estimation and other tasks, a negative sign was added to the percent absolute error for number line estimation and numerosity estimation tasks in factor analyses so that larger values represented better estimation compared to smaller values (e.g., -0.18 > -0.60). Confirmatory factor analyses were first conducted with pretest data, then with posttest data to examine the consistency of the findings.

Executive function skills were measured with only two tasks (i.e., HTKS and MEFS), thus factor analysis was not conducted. HTKS and MEFS scores were correlated at both pretest and posttest ($r_S > .44$), so the scores from these two tasks were transformed to a common z score scale, and averaged to create an EF score. Relational language was measured with one standard task, Boehm Test of Basic Concepts, thus no composite was created for relational language.

Counting/numbering skills. The correlation analyses revealed that children's performance on counting aloud, counting backward, counting after, and numeral identification were correlated with each other at pretest, r_ss ≥ .61, and posttest, r_ss ≥ .55. Set counting was only weakly correlated with other counting/numbering tasks, r_ss ≤ .31 (see Table 4). A confirmatory factor analysis revealed that, although all five tasks were significantly loaded on one factor, the loading for set counting was poor relative to the other tasks (see Table 5). Moreover, excluding set counting improved the fit indices at posttest. Because both the correlation and the factor analyses suggested that set counting did not conform with other counting/numbering tasks, it was excluded from the counting/numbering composite scores. The remaining four tasks were transformed to z scores and averaged to create a composite score for children's counting/numbering skills (here after CN score). The reliability of the CN score using these four tasks was good at pretest (α = .90) and posttest (α = .89), as indicated by Cronbach's alphas.

Number relation skills. Previous correlation analyses revealed that children's performance on number comparison, number ordering, set relation, and number line estimation were correlated with each other at pretest, $|r_s\text{s}| \geq .50$, ps < .001 and posttest, $|r_s\text{s}| \geq .40$, ps < .001. Although numerosity estimation also correlated with the four remaining number relation tasks, those correlations were weak ($|r_s\text{s}| \leq .33$) compared to the correlations between other tasks (see Table 4). Confirmatory factor analyses further revealed that only four number relation tasks loaded on one factor at pretest and at posttest (see Table 6). When numerosity estimation was excluded from the factor analysis models, the overall model fit improved. Because both the correlation and the factor analyses suggested that numerosity estimation task did not conform with other number relation tasks, it was excluded from the number relation composite. The remaining four tasks were scaled to z scores and then averaged to form a composite score for children's number relation skills (here after NR score). Cronbach's alphas indicated that the reliability of the NR score using these four tasks was good at pretest ($\alpha = 0.86$) and posttest ($\alpha = 0.84$).

In summary, the confirmatory factor analyses suggested that counting/numbering skills and number relation skills were each reliably measured by four tasks, and composite scores for counting/numbering skills and number relation skills were generated based on the factor analyses for select further analyses.

Primary Analyses

Influences of executive function and relational language on number relation skills. Prior to testing whether EF skills or relational language influence number relation skills and account for the distinction between counting/numbering skills and number relation skills, I first examined whether counting/numbering skills and number relation skills were two distinct components of early numeracy skills. I used the four counting/numbering tasks and the four number relation tasks from the earlier confirmatory factor analyses to examine whether two-factor structure was a better fit with the data compared to one-factor structure (all eight tasks).

Two confirmatory factor analyses were conducted to test whether number relation skills were distinct from counting/numbering skills. The first factor analysis was with all eight tasks in one factor, and the second factor analysis was with four counting/numbering tasks and four number relation tasks in two separate factors. A comparison of one-factor vs. two-factor solutions revealed that, although the two factors were correlated with each other (pretest: $r = .89$; posttest: $r = .86$), the two-factor model provided a stronger fit with the data than one-factor model (Table 7). In fact, the one-factor solution resulted in poor model fit, as indicated by RMSEA, in both pretest and posttest data. The findings replicated earlier reports that counting/numbering skills and number relation skills are two correlated but distinct components of early numeracy skills (Purpura & Lonigan, 2013: $r = .88$; Aunio et al., 2006: $r = .88$).

To examine whether EF skills or relational language influence number relation skills and account for the distinction between counting/numbering skills and number relation skills, I conducted three separate regression analyses. The first regression analysis was with CN score and EF score as predictors of NR score to test whether EF skills predicted number relations skills beyond counting/numbering skills. The second regression analysis was with CN score and Boehm percentile as predictors of NR score to test whether relational language predicted number relations skills beyond counting/numbering skills. The third regression analysis included CN score, EF score, and Boehm percentile as predictors of NR score to test whether EF skills or relational language uniquely predicted number relations skills beyond counting/numbering skills.

The first regression revealed that EF score significantly predicted children's NR score ($\beta_{\text{EF}} = .29$, $t(94) = 4.42$, $p < .001$), and accounted for an additional 5.89 % of the variance in children's NR score beyond the variance accounted for by CN score (63%), $F(2, 96) = 19.50$, $p < .001$. The second regression revealed that Boehm percentile significantly predicted children's NR score ($\beta_{\text{Boehm}} = .32$, $t(94) = 6.42$ $p < .001$), and accounted for an additional 10.54% of the variance in children's NR score beyond the variance accounted for by CN score (63%), $F(2, 96)= 41.6$, $p < .001$). When both EF score and Boehm percentile were simultaneously entered in one regression model, only Boehm percentile remained a significant predictor of NR score ($\beta_{\text{Boehm}} = .27$, $t(94) = 4.56$, $p < .001$), and it accounted for an additional 11.8% of the variance in children's NR score beyond the variance accounted for by CN score (63%). The pattern of results was consistent with the posttest data.

The results replicated previous confirmatory factor analyses (Aunio et al., 2006; Purpura & Lonigan, 2013), and revealed that that EF skills and relational language separately predicted children's number relation skills beyond counting/numbering skills, but when combined, only relational language significantly predicted number relation skills beyond counting/numbering skills.

Number relation skills as a mediator. Based on the finding that EF skills predicted number relation skills, and the reported relations between EF skills and mathematical skills and between number relation skills and mathematical skills, next I examined whether number relation skills may be a pathway through which EF skills influence mathematical skills by conducting a mediation analysis with the NR score as a mediator between EF score and TEMA score. The KBIT score was entered in the mediation model as a covariate to control for the potential influences of general verbal knowledge on the relations between EF, number relation, and mathematical skills. To test whether number relation skills were a unique mediator of the relation between EF skills and mathematical skills, I replaced NR score with CN score in the mediation model to examine whether counting/numbering skills also mediated the relation between EF skills and mathematical skills. In view of the correlation between EF score and Boehm percentile (r_s = .72), Boehm percentile was added as an additional covariate to test the unique influences of EF skills on number relation skills, counting/numbering skills, and mathematical skills beyond relational language.

Similarly, I examined whether number relation skills are a unique pathway through which relational language influence mathematical skills by conducting two separate mediation analyses with NR score or CN score as a mediator between Boehm percentile and TEMA score. The mediation analyses were first conducted with KBIT score as a covariate, then repeated with KBIT score and EF score as covariates.

EF skills and mathematical skills. The first step of the mediation analysis confirmed that with the inclusion of KBIT score as a covariate, EF score significantly predicted TEMA score, $\beta = .35$, $t(101) = 3.52$, $p < .001$ (Figure 5a). Next, adding number relation score as a mediator of the relation between EF and TEMA scores revealed a full mediation. Specifically, EF predicted number relation, number relation predicted TEMA, and the relation between EF and TEMA was not significant, $p = .652$ (Figure 5b). A mediation analysis with the counting/numbering score as a mediator between EF and TEMA also revealed that counting/numbering fully mediated the relation between EF and TEMA (Figure 5c). The same mediation analyses with KBIT score and Boehm percentile as covariates revealed a similar pattern of results. Specifically, the path between EF and TEMA was marginally significant, $\beta = .21$, $t(100) = 1.70$, $p = .089$ (Figure 5d), and the relation between EF and TEMA was fully mediated by number relation (Figure 5e) or counting/numbering (Figure 5f).

In summary, number relation skills fully mediated the relation between EF and mathematical skills, but so did counting/numbering skills, suggesting that number relation skills were not a unique mediator of the relation between EF and mathematical skills. Although counting/numbering tasks were expanded from select TEMA-3 items, the path weight between counting/numbering and TEMA scores was not stronger than the path weight between number relation and TEMA scores, suggesting that the path weight between counting/numbering and TEMA scores was not inflated by the fact that counting/numbering tasks were an expanded subset of TEMA-3. Furthermore, the counting/numbering tasks are commonly included in other standardized assessments (e.g., Early Math Test, Number Knowledge Test, Research-based Early Math Assessment) and research studies (Barth et al., 2009; Bos et al., 2018; Brannon & Van de Walle, 2001; Purpura & Lonigan, 2013), and the standard TEMA instruction was followed during counting/number tasks simply for testing efficiency. Number relation skills or counting/number skills remained a full mediator of the relation between EF and mathematical skills when both verbal knowledge and relational language were controlled for. When relational language was included as a covariate, the path weights between EF and mathematical skills, between EF and number relation skills, and between EF and counting/numbering skills were smaller, suggesting that there was some shared variance between EF skills and relational language in children's mathematical skills, number relation skills, and counting/numbering skills. Nonetheless, these path weights remained significant suggesting that EF skills may have influences on children's skills in mathematics, number relation, and counting/numbering, independent of relational language.

Relational language and mathematical skills. The first step of the mediation analysis confirmed that with the inclusion of KBIT score as a covariate, Boehm percentile significantly predicted TEMA score, $\beta = .46$, $t(101) = 3.71$, $p < .001$ (Figure 6a). Next, adding number relation score as a mediator of the relation between Boehm percentile and TEMA score revealed a full mediation. Specifically, Boehm predicted number relation, number relation predicted TEMA, and the relation between Boehm and TEMA was not significant, $p = .377$ (Figure 6b). When replacing number relation score with counting/numbering score in the mediation model, counting/numbering also fully mediated the relation between Boehm and TEMA (Figure 6c). With the inclusion of KBIT score and EF score as covariates, the pattern of results remained. Specifically, the path between Boehm and TEMA was significant, $\beta = .33$, $t(100) = 2.04$, $p = .041$ (Figure 6d), and the relation between Boehm and TEMA was fully mediated by number relation (Figure 6e) or counting/numbering (Figure 6f).

In summary, number relation skills fully mediated the relation between relational language and mathematical skills, but so did counting/numbering skills, suggesting that number relation skills were not a unique mediator of the relation between relational language and mathematical skills. The full mediation of number relation skills or counting/numbering skills remained when both verbal knowledge and EF skills were controlled for. The path weights between relational language and mathematical skills, between relational language and number relation skills, and between relational language and counting/numbering skills were smaller, but remained significant, when EF skills were added as a covariate. This aligned with the earlier findings on the shared variance between EF skills and relational language in children's mathematical skills, number relation skills, and counting/number skills. The significant path weights suggested that relational language may have unique influences on children's skills in mathematics, number relation, and counting/numbering beyond EF skills.

Effects of training condition. Before testing the effects of training condition on children's number relation skills, I first examined whether children's performance on pretest measures was comparable between gender and their regular exposure to a non-English language, and across schools and training conditions. A multivariate multiple regression was conducted with KBIT, TEMA, counting/numbering, number relation, EF, and Boehm scores as the dependent variables, and gender, school, regular exposure to a non-English language, and training condition, as independent variables. The regression revealed that children's performance on pretest measures did not differ whether they were a boy vs. a girl, exposed vs. not exposed to a non-English language, or in number vs. number + EF + RL vs. alphabet training condition, *p*s > .10. Children's pretest performance significantly differed across schools, $F(3,96) = 23.16$, $p < .001$. Specifically, children from the school with the lowest rate of free and reduced lunch (FRL 22%) consistently outperformed children from the three remaining schools (FRL > 66%) on all pretest measures (see Table 8). In summary, children's performance on pretest measures was comparable between gender and whether they were regularly exposed to a non-English language, and across training conditions, but there were systematic differences between schools. Therefore, the effects gender and regular exposure to a non-English language were not further examined, and school (low FRL vs. other) was included as a variable in subsequent analyses examining the effects of training condition on children's number relation skills.

To address whether number + EF + RL training was more effective at improving children's *number relation skills* compared to number training, I conducted two separate repeated measures MANOVAs to examine (a) whether children showed improvement on individual number relation tasks from pretest to posttest, and (b) whether training condition affected children's performance on number relation tasks. Because the training conditions differed on their inclusion of counting practice, executive function prompts, and relational language instruction (number + EF + RL training included all the above; number training included counting only; alphabet training included none), separate MANOVAs (or ANOVAs) were conducted with children's performance on counting/numbering tasks, EF tasks, or Boehm Test scores as dependent variables to test the influences of training condition on pretest to posttest gains in counting/numbering skills, EF skills, and relational language, respectively.

Number relation skills. To examine whether children showed improvement on number relation tasks from pretest to posttest and whether the amount of improvement differed by school, a 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures MANOVA was conducted with the scores on number comparison, number ordering, set relation, number line estimation, and numerosity estimation as the dependent variables. In addition to the previously reported main effect of school, $F(5,96) = 7.41$, $p < .001$, $\eta^2 = .278$, there was a main effect of time, $F(5,96) = 4.66$, $p = .010$, $\eta^2 = .143$, but no effect of Time × School interaction, $p = .12$. Univariate ANOVAs further indicated that the effect of time was significant for the number ordering task, $F(1,100) = 5.69$, $p = .019$, $\eta^2 = .054$, and the number line estimation task, $F(1,100) = 4.88$, $p = .029$, $\eta^2 = .04$, whereby children improved on number ordering (pretest: $M = 9.53$, $SD = 4.66$;

posttest: $M = 10.39$, $SD = 4.10$), and made smaller estimation errors on number line estimation at posttest (pretest: $M = 0.29$, $SD = 0.12$; posttest: $M = 0.27$, $SD = 0.12$). The absence of a Time × School interaction indicated that the amount of improvement children made on number relation tasks did not significantly differ by school. Therefore, school was not included as a variable in the following MANOVA in which the effects of training condition on children's number relation skills were examined.

A 2 (Time: pretest vs. posttest) × 3 (Training Condition: number vs. number + EF + RL vs. alphabet) repeated measures MANOVA on the five number relation tasks revealed the previously reported main effect of time, $F(5,95) = 4.62$, $p = .001$, $\eta^2 = .196$. There were no effects training condition and no Time × Training Condition interaction, $ps \geq .410$.

In summary, children showed improvement on number relation skills, specifically on number ordering and number line estimation. The absence of a Time × Training Condition interaction suggested that the gains on number relation skills did not differ across training conditions. Children who received number + EF + RL training did not improve more on number relation skills compared to children who received number or alphabet training.

Counting/numbering skills. Similar to the analyses conducted with number relation tasks, I first conducted a 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures MANOVA on counting aloud, counting backwards, counting after, set counting, and numeral identification, then added Training Condition as an independent variable in the MANOVA model to examine the effects of training condition on children's counting/numbering skills.

A 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures MANOVA on the five counting/numbering tasks revealed the previously reported main effect of school, $F(5,98) = 3.33$, $p = .008$, $\eta^2 = .145$. The main effect of time was also significant, $F(5,98) = 3.76$, $p = .004$, $\eta^2 = .161$. Univariate ANOVAs further indicated that the effect of time was significant for the counting backwards task, $F(1,102) = 10.42$, $p = .002$, $\eta^2 = .093$, and the numeral identification task, $F(1,102) = 11.59$, $p = .001$, $\eta^2 = .102$, whereby children improved on counting backwards (pretest: $M = 12.95$, $SD = 6.74$; posttest: $M = 14.98$, $SD = 6.12$) and numeral identification (pretest: $M = 16.96$, $SD = 4.23$; posttest: $M = 17.84$, $SD = 3.53$). Because the Time × School interaction was not significant, school was not included as a variable in the following MANOVA in which I examined the effects of training condition on children's counting/numbering skills.

A 2 (Time: pretest vs. posttest) × 3 (Training Condition: number vs. number + EF + RL vs. alphabet) repeated measures MANOVA on the five counting/numbering tasks revealed the previously reported main effect of time, $F(5,97) = 5.87$, $p < .001$, $\eta^2 = .232$, and a Time × Training Condition interaction, $F(5,98) = 2.61$, $p = .005$, $\eta^2 = .118$. Univariate ANOVAs revealed that the Time × Training Condition interaction was significant for the counting backwards task, $F(2,101) = 7.18$, $p = .001$, $\eta^2 = .124$. Post hoc pairwise comparison revealed that children in number + EF + RL training improved on counting backwards, $p < .001$, but children in alphabet or number training did not, *ps* $> .221$ (Figure 7).

In summary, children showed improvement on counting/numbering skills, specifically on numeral identification and counting backwards. The Time × Training

Condition interaction emerged in counting backwards. Specifically, only children in number + EF + RL training improved on counting backwards.

EF skills. I first conducted a 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures MANOVA on HTKS total score and MEFS standard score to examine the effects of time and its potential interaction with school on children's EF skills. Next, Training Condition was entered in the MANOVA model as an independent variable to examine the effects of training condition on children's EF skills.

A 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures MANOVA on HTKS total score and MEFS standard score revealed the previously reported main effect of school, $F(2,101) = 22.84$, $p < .001$, $\eta^2 = .311$, and a main effect of time, $F(2,101) = 9.36$, $p < .001$, $\eta^2 = .156$. The Time × School interaction was not significant, $p = .33$. Univariate ANOVAs revealed that the effect of time was significant for both HTKS total score, $F(1,102) = 9.53$, $p = .003$, $\eta^2 = .085$, and MEFS standard score, $F(1,102) = 12.10$, $p = .001$, $\eta^2 = .106$. Children improved on both HTKS (pretest: $M = 60.59$, $SD = 26.13$; posttest: $M = 68.00$, $SD = 20.93$), and MEFS (pretest: $M = 100.40$, $SD = 9.88$; posttest: $M = 104.77$, $SD = 12.50$) from pretest to posttest. Because there was no Time × School interaction, school was not included as a variable in the following MANOVA in which I examined the effects of training condition on children's EF skills.

A 2 (Time: pretest vs. posttest) × 3 (Training Condition: number vs. number + EF + RL vs. alphabet) repeated measures MANOVA on the two EF tasks revealed the previously reported main effect of time, $F(2,100) = 13.48$, $p < .001$, $\eta^2 = .212$. There were no effects of training condition and no Time × Training Condition interaction, *p*s

> .196. In summary, children showed improvement on EF sills, but the improvement did not differ across training conditions.

Relational language. Because relational language was measured with one task in this study, ANOVAs, instead of MANOVAs, were conducted to examine the effects of school and training condition on children's improvement in relational language. A 2 (Time: pretest vs. posttest) × 2 (School: low FRL vs. other) repeated measures ANOVA on Boehm percentile revealed the previously reported main effect of school, $F(1,102) = 88.67$, $p < .001$, $\eta^2 = .465$, and a main effect of time, $F(1,102) = 7.91$, $p = .006$, $\eta^2 = .072$. Children improved on Boehm Test from pretest ($M = 34.55$, $SD = 29.89$) to posttest ($M = 38.15$, $SD = 31.87$). The Time × School interaction was not significant, $p = .065$. Because there was no Time × School interaction, school was not included as a variable in the following MANOVA in which I examined the effects of training condition on children's relational language.

A 2 (Time: pretest vs. posttest) × 3 (Training Condition: number vs. number + EF + RL vs. alphabet) repeated measures ANOVA on Boehm percentile revealed the previously reported main effect of time, $F(1,101) = 4.89$, $p = .029$, $\eta^2 = .046$. There were no effects of training condition and no Time × Training Condition interaction, ps > .117. In summary, children showed improvement on relational language from pretest to posttest, but the improvement did not differ across training conditions.

Overall, children showed improvement on aspects of number relation skills, counting/numbering skills, EF skills, and relational language from pretest to posttest. The gains differed across training conditions only for counting backwards. Children in number + EF + RL training improved on counting backwards, whereas children in number or alphabet training did not.

Effects of initial EF skills and relational language. To examine the effects of children's initial EF skills on their gains in number relation skills, I conducted a 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (HTKS: high vs. low) MANOVA on children's gains (posttest score – pretest score) in the five number relation tasks. In view of the correlations between scores on HTKS and number relation tasks, I added children's number relation score at pretest as a covariate to test whether initial HTKS level had unique influences on gains in number relation skills beyond children's initial number relation skills. To examine whether the pattern of results was consistent when using MEFS score, instead of HTKS score, as an indicator of children's initial EF skills, I replaced high vs. low HTKS level with high vs. low MEFS level, and repeated the MANOVA and MANCOVA on children's gains in number relation tasks. To explore the potential influences of children's initial relational language on their gains in number relation skills, the MANOVA and MANCOVA were conducted with high vs. low Boehm level as an independent variable. Because children also showed improvement on counting/numbering skills, the MANOVAs and MANCOVAs were conducted with gains in counting/numbering tasks as dependent variables, and median split HTKS, MEFS, or Boehm scores as an independent variable, to examine the influences of initial EF skills or relational language on children gains in counting/numbering skills.

Number relation skills.

Initial HTKS level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (HTKS: high vs. low) MANOVA on gains in number comparison, number ordering, set relation, number line estimation, and numerosity estimation revealed a main effect of HTKS level, $F(5,92) = 3.03$, $p = .014$, $\eta^2 = .141$. Univariate ANOVAs further indicated that the main effect of HTKS level was significant for the number ordering task, $F(1,96) = 8.16$, $p = .005$, $\eta^2 = .078$, and the numerosity estimation task, $F(1,96) = 5.72$, $p = .019$, $\eta^2 = .056$ (Table 9). Children with low HTKS score improved on number ordering, $p < .001$, whereas children with high HTKS score did not, $p = .608$. The pattern of results was different for numerosity estimation. Children with high HTKS score made smaller errors on numerosity estimation at posttest vs. pretest, $p = .049$, whereas children with low HTKS score did not, $p = .180$. A 3 × 2 MANCOVA with pretest number relation score as the covariate revealed no effect of HTKS level, $p = .241$. In summary, only children with low HTKS score showed improvement on the number ordering task and only children with high HTKS score showed improvement on the numerosity estimation task. However, the absence of the HTKS effect when controlling for children's pretest number relation score suggested that HTKS level did not predict children's gains in number relation skills beyond their initial number relation skills.

Initial MEFS level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (MEFS: high vs. low) MANOVA on gains in the five number relation tasks revealed no effect of Training Condition, no effect of MEFS level, and no Training Condition × MEFS interaction, ps > .192. Because the effect of MEFS level on gains in number relation tasks was not significant, the MANCOVA testing the unique effects of MEFS level on gains in number relation tasks was not conducted. This pattern of finding indicated that children's initial MEFS level did not predict their gains in number relation skills.

Initial Boehm level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (Boehm: high vs. low) MANOVA on gains in number relation tasks revealed a main effect of Boehm level, $F(5,92) = 2.41$, $p = .042$, $\eta^2 = .116$. Univariate ANOVAs further indicated that the effect of Boehm level was significant for the number ordering task, $F(1,96) = 9.64$, $p = .003$, $\eta^2 = .091$, whereby children with low Boehm percentile improved on the number ordering task, $p < .001$, but children with high Boehm percentile did not, $p = .761$ (Table 10). A 3 × 2 MANCOVA with pretest number relation score as the covariate revealed that the effect of Boehm level was not significant, $p = .792$. In summary, only children with low Boehm percentile showed improvement on number ordering, but this effect was not significant when controlling for children's pretest number relation score.

In summary, children with low HTKS score or low Boehm percentile showed improvement on the number ordering task, and children with high HTKS score showed improvement on the numerosity estimation task. However, the effects of HTKS level or Boehm level were not significant when controlling for children's pretest number relation score suggesting that children's initial EF skills or relational language did not predict gains in number relation skills beyond their initial number relation skills.

Counting/numbering skills.

Initial HTKS level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (HTKS: high vs. low) MANOVA on gains in counting aloud, counting backwards, counting after, set counting, and numeral identification revealed a main effect of training condition, $F(10,190) = 2.64$, $p = .005$, $\eta^2 = .122$,and a main effect of HTKS level, $F(5,94) = 3.19$, $p = .010$, $\eta^2 = .145$. Univariate ANOVAs further indicated that consistent with the previously reported Time × Training Condition interaction, the effect of training condition was significant for the counting backwards task, $F(2,98) = 7.90$, $p = .001$, $\eta^2 = .139$, whereby only children in number + EF + RL training showed improvement on counting backwards, $p < .001$. The effect of HTKS level was significant for numeral identification, $F(1,98) = 10.86$, $p = .001$, $\eta^2 = .100$, and counting backwards, $F(1,98) = 4.69$, $p = .033$, $\eta^2 = .046$. Children with low HTKS score showed improvement on numeral identification and counting backwards, *p*s < .001, whereas children with high HTKS score did not, *p*s > .216 (Table 11). A 3 × 2 MANCOVA with pretest counting/numbering score as a covariate revealed that the effect of HTKS level was not significant, $p = .822$. In summary, only children with low HTKS score showed improvement on numeral identification and counting backwards, but this effect was not significant when controlling for children's pretest counting/numbering score.

Initial MEFS level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (MEFS: high vs. low) MANOVA on gains in the five counting/numbering tasks revealed the previously reported effect of training condition, $F(10,190) = 2.60$, $p = .006$, $\eta^2 = .120$. There was no effect of MEFS level and no Training Condition × MEFS interaction, *p*s > .257. This pattern of finding indicated that children's initial MEFS level did not predict their gains in counting/numbering skills.

Initial Boehm level. A 3 (Training Condition: number vs. number + EF + RL vs. alphabet) × 2 (Boehm: high vs. low) MANOVA on gains in the five counting/numbering tasks revealed the previously reported effect of training condition, $F(10,190) = 2.54$, $p = .007$, $\eta^2 = .118$. The main effect of Boehm level was significant, $F\ (5,94) = 2.68$, $p = .026$, $\eta^2 = .125$. Univariate ANOVAs revealed that the effect of Boehm level was significant for numeral identification, $F(1,98) = 9.12$, $p = .003$, $\eta^2 = .085$, whereby children with low Boehm percentile improved on this task, $p < 0.01$, and children with high Boehm percentile did not, $p > .140$ (Table 12). A 3 × 2 MANCOVA with pretest counting/numbering score as a covariate revealed that the effect of Boehm level was not significant, $p = .882$. In summary, only children with low Boehm percentile showed improvement on the numeral identification task, but this effect was not significant when controlling for children's pretest counting/numbering score.

In summary, children with low HTKS score or low Boehm percentile showed improvement on the numeral identification task, and children with low HTKS also showed improvement on the counting backwards task. However, the effects of initial HTKS level or initial Boehm level were not significant when controlling for children's pretest counting/numbering score, suggesting that children's initial EF skills or relational language did not predict gains in counting/numbering skills beyond their initial counting/numbering skills.

Discussion

In the current study, I examined the influences of EF skills and relational language on number relation skills to pursue four research aims. First, I sought to test whether EF skills and relational language predict number relation skills beyond counting/numbering skills. Second, I tested whether number relation skills may be a pathway through which EF and relational language influence mathematical skills. Third, I sought to test for potential causal effects of EF and relational language on number relation skills by comparing the effects of number + EF + RL training vs. number training on children's number relation skills. Last, I sought to explore whether children's initial level of EF skills or relational language predict gains in number relation skills following training. Addressing these aims collectively contributes new findings to the growing literature on the influences of EF and relational language on mathematical skills.

Number Relation Skills

Given some evidence that number relation skills may be distinct from counting/numbering skills (Aunio et al., 2006; Purpura & Lonigan, 2013), it may be important to identify the factors that contribute to their differences. I replicated the previous findings that two-factor model fitted the data better than one-factor model for number relation tasks and counting/numbering tasks. This finding differed from Jordan and colleagues' study (2006) that revealed superior model fit for one factor solution for number relation tasks and counting/numbering tasks in kindergarten students. The discrepancy may be due to the number of tasks included in the factor analyses. At least three tasks are needed to represent a factor in factor analyses (Raubenheimer, 2004). Jordan and colleagues only included four counting/numbering tasks and two number relation tasks, thus, the discrepancy between studies may be due to insufficient tasks in their factor analyses.

The results from the factor analyses support the notion that number relation and counting/numbering may be two distinct, but highly correlated (in this study, *r*s = .86 - .89; and other studies, *r*s= .88), components of early numeracy skills in kindergarten students. The correlations between number relation skills and counting/numbering skills suggest the interdependence of these two sets of skills. The notion that counting/numbering skills may be a foundation for number relation skills is aligned with the developmental progression of number concepts (Siegel, 1971) and the learning sequence in mathematics curricula (e.g., Building Block, Everyday Math).

The high correlations between these two early numeracy skills combined with the similar pattern of results in the mediation analyses with counting/numbering skills or number relation skills raised questions regarding the theoretical and practical significance of distinguishing these two sets of early numeracy skills. As revealed in the mediation analyses, EF skills and relational language each predict number relation (NR) skills and counting/numbering (CN) skills. Although the path weights between EF and NR scores (β = .31), and between the EF and CN scores (β = .34), were comparable to each other, the path between Boehm percentile and NR score (β = .53) was stronger compared to the path between Boehm percentile and CN score (β = .37). The difference in the path weights was consistent with the finding that relational language predicts number relation skills beyond EF skills and counting/numbering skills, and suggested its unique influences on number relation skills. Additional regression analyses also revealed that EF skills and relational language separately had additional influences on number relation skills beyond counting/numbering skills. Together, these findings indicate that EF skills and relational language may account for some differences between number relation skills and counting/numbering skills. In addition to knowing the number words and the counting sequence, having better EF skills or more knowledge of relational language is associated with better number relation skills. Whereas counting/number skills may serve as a foundation for number relation skills, EF skills and relational language may provide additional support for number relation skills.

Different from previous studies that examined the influences of EF skills or relational language on early numeracy skills, this study included both factors in one context. When the influences of EF skills and relational language on number relation skills were tested simultaneously, only relational language significantly predicted number relation skills beyond counting/numbering skills. Although relational language, but not EF skills, was a significant predictor in this model, EF skills and relational language together accounted for more variance (11.8%) in number relation skills compared to the model in which relational language was the sole non-numerical predictor (10.5%). This does not imply that relational language is more important than EF skills for number relation skills. Instead, the finding suggests that there may be some shared variance between EF skills and relational language despite their individual unique influence on number relation skills. This notion is supported by results from the mediation models, wherein the path between EF and number relation skills decreased, but remained significant, when controlling for relational language. Similarly, the path between relational language and number relation skills decreased but remained significant when controlling for EF skills.

The current findings align with previous findings on the association between EF skills and relational language (Gao et al., 2014; Nunes et al., 2007) and have implications for future studies on their influences on mathematical skills. The variance shared between EF skills and relational language may represent the required EF skills for reasoning about relations described by relational language, and that may be the same set of skills required for understanding number relations. Although it may be important to consider the association between EF skills and relational language, they each may still have unique influences on early numeracy and mathematical skills. For instance, knowing the meaning of *equal* may help children understand the quantitative relations between numbers, equations, or sets of objects and this influence may be independent of EF skills. Studies delineating the relations between these numerical and non-numerical skills may be fruitful lines of inquiry that help researchers and educators to accurately identify the specific skills children have difficulty with and target those skills to maximize their learning outcomes.

Number Relation Skills as a Mediator

Relation between EF skills and mathematical skills. Number relation skills fully mediated the relation between EF and mathematical skills, suggesting that number relation skills may be a pathway through which EF skills influence general mathematical skills. Children's abilities to remember the pairs of numbers being compared, shift attention between these pairs of comparison, and inhibit or ignore irrelevant comparisons may require EF skills and these skills may be an important foundation for mathematical skills. The path between EF skills and number relation skills is consistent with the previous findings that EF skills are correlated with, and predictive of, aspects of number relation skills (e.g., Bos et al., 2014; Gashaj et al., 2016; Geary et al., 2008b), and the current finding that EF skills predict number relation skills beyond counting/number skills. Although the path between number relation skills and mathematical skills suggests that number relation skills influence mathematical skills, an alternative interpretation is that number relation tasks are developmentally appropriate measures of general mathematical skills in kindergarten students thus they are simply indicators, instead of predictors, of general mathematical skills. This alternative interpretation is unlikely because children’s performance on number line estimation in kindergarten predicts their mathematics achievement in Grade 1, even after controlling for their mathematics achievement in kindergarten (Friso-van den Bos et al., 2015), suggesting that number relation skills may have unique influences on mathematical skills, not simply an indicator of mathematical skills.

The mediation was not unique to number relation skills. Children’s counting/numbering skills also fully mediated the association between EF skills and mathematical skills. Children’s abilities to remember and update numbers in working memory during counting, shift attention between number words and numerosities during set counting, and inhibit habitual responses such as counting forward during backward counting may require EF skills. In fact, preschoolers’ EF skills are correlated with their performance on verbal counting, set counting, and numeral identification (Purpura et al., 2017), and predict aspects of counting/numbering skills (Mulder, Verhagen, Van der Ven, Slot, & Leseman, 2017). Counting/numbering skills predict later mathematical skills (Aunio & Niemivirta, 2010; Chu, VanMarle, & Geary, 2016) but also growth in mathematical skills (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004), suggesting that counting/numbering skills may be an important foundation for, not simply a concurrent indicator of, mathematical skills.

Overall, these results align with previous findings that specific numeracy skills mediate the relation between EF skills and mathematical skills (Cragg et al., 2017; Fuhs et al., 2016). Specifically, Cragg and colleagues (2017) found that arithmetic skills partially mediated the relation between EF skills and mathematical skills in participants from age 8 to 25 years; and Fuhs and colleagues (2016) found that kindergarten students’ ability to compose and decompose numbers fully mediated the relation between EF skills and mathematics achievement. The current study extended previous findings in that EF skills may influence mathematical skills not only through arithmetic skills but also number relation skills and counting/numbering skills. Different from the earlier studies, I included children's relational language, in addition to verbal knowledge, as a covariate in all paths, and found smaller, but still significant, path weights between EF skills and number relation skills, and between EF skills and counting/numbering skills. If the relation between EF skills and numeracy skills merely reflects the role of EF in general learning process, the influence of EF skills on numeracy skills should be similar to their influence on acquiring verbal knowledge and relational concepts. The reduced path weights after controlling for KBIT and Boehm scores echo the earlier finding on the shared variance between EF skills and relational language, and represents the potential support that EF provides to learning in general. The significant relation between EF and numeracy skills may represent their unique role in supporting number relation skills and counting/numbering skills, in addition to their influences on learning in general.

My findings are inconsistent with Hassinger-Das and colleagues' study (2014) in which EF skills partially mediate the relation between early numeracy skills in kindergarten and mathematics achievement in Grade 1. Unlike other studies where the predictor and the mediator were measured concurrently (e.g., Fuhs et al., 2016, Cragg et al., 2017, the current study), Hassinger-Das and colleagues assessed early numeracy skills in Fall semester and EF skills in Spring semester of kindergarten, therefore, the directionality of the influences was supported by the temporal sequence. The finding was interpreted as EF skills supporting mathematical learning from kindergarten to first grade and provided evidence for the bidirectional relation between EF and mathematical skills (Clements, Sarama, & Germeroth, 2016). Although the concurrent assessment precluded causal inferences in the current study, reverse mediations with EF skills as a mediator between number relation skills (or counting/numbering skills) and mathematical skills were not supported by the data. Specifically, the path from EF skills and mathematical skills was not significant, thus EF skills did not mediate the relation between number relation skills (or counting/numbering skills) and mathematical skills. Fuhs and colleagues (2016) also found no support for the reverse mediation with EF skills as a mediator between composition/decomposition and mathematics achievement. The relations between EF skills, early numeracy skills, and mathematical skills are likely to be complex, and the directionality of their relative influences may depend on context and development. The current study provides evidence of at least two pathways, number relation skills and counting/numbering skills, through which EF skills influence mathematical skills. Studies identifying the mechanisms underlying the unique relation between EF skills and numeracy skills, and the contexts in which EF skills influence mathematical skills and vice versa, may offer insights into ways for promoting mathematical development and potentially also EF skills in young children.

Relation between relational language and mathematical skills. Number relation skills, as well as counting/numbering skills, independently mediated the relation between relational language and mathematical skills. Children's knowledge of relational vocabulary such as *more, less, before*, and *between*, may influence their understanding of cardinal and ordinal number relations and their performance on number relation tasks. It is less intuitive how relational language may influence counting/numbering skills. The path between relational language and counting/numbering skills remained significant when controlling for verbal knowledge and EF skills, suggesting that the influence of relational language on counting/numbering skills was independent of these covariate abilities. One possibility is that relational language may be applied to the different instances of number word uses (e.g., five more cookies, two minutes after) and help enrich children's understandings of sets and number words, just as knowledge of relational language may help unify different instances spatial relations (e.g., above, below) leading to abstract spatial concepts (Scott & Sera, 2018). Another possibility is that relational language may influence children's skills in various types of verbal counting. For instance, knowing the meanings of *backwards* and *next* may be important for understanding and applying this knowledge on counting tasks such as reciting the counting sequence backwards, or continuing the counting sequence from a given number. Although the paths from relational language to number relation skills and to counting/numbering skills are both significant, the strength of the two relations differ. The path between relational language and number relation skills ($\beta = .53$) appeared stronger than the path between relational language and counting/numbering skills ($\beta = .37$) providing support for the close relation and the unique influences of relational language on number relation skills.

Without differentiating number relation skills and counting/numbering skills, previous studies showed that preschooler's knowledge and use of relational language correlated with their concurrent early numeracy skills (Purpura & Reid, 2016), and predicted their later mathematical skills (Purpura et al., 2017). Kindergarten students' knowledge in relational language also predicted their growth in mathematical skills (Toll

& Van Luit, 2014). Building upon the previous studies, the current study revealed that both number relation skills and counting/numbering skills may be pathways through which relational language influence mathematical skills in kindergarten students. Although the findings were based on concurrent assessment, reverse mediations with relational language as a mediator between number relation skills (or counting/numbering skills) and mathematical skills were not supported by the data. Specifically, the path between relational language and mathematical skills were not significant in these reverse mediations.

In summary, the current study revealed that number relation skills, as well as counting/numbering skills, may be pathways through which EF skills and relational language influence mathematical skills. Although EF skills and relational language separately predict children's mathematical skills (Clark, Sheffield, Wiebe, & Espy, 2013; McClelland et al., 2014; Purpura & Logan, 2015; Purpura & Reid, 2016), the findings on the full mediations suggest that they may indirectly influence mathematical skills through specific early numeracy skills. The findings do not imply that other numerical skills or EF and relational language are not important for mathematical skills. Instead, the findings provide potential mechanisms underlying the relations between EF skills and mathematical skills, and between relational language and mathematical skills.

Given the indirect influences of EF skills and relational language on mathematical skills, an implication of the current finding is that promoting EF skills or relational language alone may not lead to much improvement on general mathematical skills. Extensive training on relational language may improve children's knowledge of relational language, but the effect on mathematical skills is unclear (Hassinger-Das et al., 2015;

Jennings et al., 1992). Although Ramani and colleagues (2017) showed that ten 15-minute sessions of working memory training may improve kindergarten students' number relation skills, the effect was relatively small compared to the number board game training and it did not transfer to arithmetic skills. Preschool children who practiced counting with an examiner for just one 15-minute session showed improvement on number ordering and number line estimation (Xu & LeFevre, 2016). Preschool children who played with a number line board game for just four 15-minute sessions showed improvement on number comparison, number line estimation, counting, and numeral identification, and the gains remained nine weeks later (Ramani & Siegler, 2008). Although EF skills and relational language are predictive and may be important for mathematics achievement (Best et al., 2011; Toll & Luit, 2014), training specific numeracy skills may be more efficient and effective compared to training EF skills or relational language alone if the goal is to promote mathematical development.

Training Effects

Improvement from pretest to posttest.

Numeracy Tasks. Children in number training, number + EF + RL training, but also alphabet training, showed improvement on numeral identification, number ordering, and number line estimation from pretest to posttest. Overall, compared to pre-test scores, children correctly named one additional numeral, ordered one additional number triplet, and improved their accuracy of number line estimation by 2% after four brief training sessions. Although numerals were used in number and number + EF + RL training, the comparable improvement on numeral identification task compared to the alphabet training regardless of the presence of numerals suggests that improvement on numeral identification may be related to growth or learning in classroom, and independent of the training activities. Specifically, reading and writing numerals are standard instructions in kindergarten classrooms, and students often practice this skill during mathematics instruction so the improvement observed in this study may reflect children's learning in their classroom.

Children in all three training groups also showed comparable improvement on number ordering and number line estimation. One possibility is that comparing and ordering stimuli, either numbers or letters, may improve children's performance on the tasks that involved ordering numbers and estimating the location of a number on a number line. Previous studies showed that activities involving counting, and comparing and ordering numbers improved children's performance on counting, number line estimation, number comparison, and number ordering (e.g., Bos et al., 2018; Ramani et al., 2017; Xu & LeFevre, 2016). Ordering skills, even with non-numerical stimuli, are correlated with mathematical skills in children and adults. Children's performance on ordering sequence of familiar events at age five is correlated with their concurrent mathematical skills and predictive of their mathematical skills one year later (O'Connor, Morsanyi, & McCormack, 2018). The association between non-numerical ordering skills and mathematical skills is also observed in adults. Adults' performance on comparing and ordering months is correlated with their arithmetic skills (Morsanyi, O'Mahony, & McCormack, 2017). Furthermore, ordinal judgement of numbers and alphabets shares common neural substrates in adults (Attout, Fias, Salmon, & Majerus, 2014). Specifically, anterior intraparietal sulcus shows an increase in activation when the distance between numbers or letters decreases during an ordinal judgment task. Based on the association between non-numerical ordering skills and mathematical skills, perhaps activities involving order and comparison, whether with numbers or alphabets, promote children's number relation skills.

However, an alternative interpretation of the current finding is that the improvement reflects practice effects, maturation, or learning in classroom that would have taken place regardless of the training obtained in the present study. If the improvement was due to practice effects or maturation, children should show improvement on all tasks, not just the tasks that focused on ordinal and spatial relations between numbers. The training activities in which children recited and ordered numbers (or alphabets) from left to right may have specific influences on their performance on ordering numbers and estimating spatial location of numbers on a number line. Although number ordering and number line estimation were research-based tasks not often seen in common kindergarten classrooms, the classroom mathematics instruction, instead of the training activities, may have led to improvement on these tasks.

Executive function Tasks. Children showed improvement on HTKS and MEFS from pretest to posttest regardless of training condition. One possible interpretation is that the training activities, either with numbers or letters, may promote children's EF skills. EF skills are correlated with the ability to compare and order numbers in preschool age children (Purpura, Schmitt, & Ganley, 2017), school age students (Attout, Noël, et al., 2014; Krajewski & Schneider, 2009a) and adults (Lyons & Beilock, 2009). The association between EF skills and non-numerical ordering skills is also observed in six-year-olds (Nunes et al., 2007) and adults (Attout, Fias, et al., 2014). Although EF skills may influence children's performance on comparison and ordering tasks (e.g., Attout, Noël, et al., 2014; Lyons & Beilock, 2009; Purpura et al., 2017), this relation may not be unidirectional. Comparing and ordering numbers (or letters) may require children to hold the sequences in their working memory and shift attention between numbers (or letters), and these tasks may provide opportunities for children to practice and improve their EF skills. This is consistent with the finding that early numeracy skills predict EF skills (Hassinger-Das et al., 2014), and the notion that the relation between EF skills and mathematical skills may be bidirectional (e.g., Clements, Sarama, & Germeroth, 2016; Welsh, Nix, Blair, Bierman, & Nelson, 2010).

An alternative interpretation of the finding is that the improvement on the EF tasks reflects practice effects or maturation instead of training effects. In the current study, children showed an average of 7.8% increase in HTKS scores in just five weeks. In comparison, McClelland and colleagues (2014) revealed that, without any training or intervention, kindergarten students showed an average of 9.8% increase in HTKS scores from fall semester to spring semester. Children in the current study also improved their MEFS standard score from 100.4 to 104.8, corresponding to approximately one third of a standard deviation increase in MEFS performance. Because the MEFS standard scores used in the current study were normed based on children's age, and previous studies revealed that children do not show improvement on MEFS from simple retesting (Beck et al., 2011) or from set counting training (Prager, 2016), the EF improvement observed in the current study are unlikely to be a result of age-related improvement, simple practice, or non-EF related training. Although children's improvement on HTKS and MEFS appears to be greater than typical growth without intervention, an additional non-ordering control condition is needed to determine whether the improvement on the EF tasks is due to the ordering aspect of the training activities.

Relational language task. Overall, children improved from 34.6 percentile to 38.2 percentile on the Boehm Test from pretest to posttest. The standardization sample did not show improvement on this test when it was administered twice within two weeks (Boehm, 2001), suggesting that the improvement observed in the current study may not be due to practice effects. Although the training activities may provide opportunities for children to practice and improve their relational language, other potential interpretations such as maturation and learning in classroom cannot be ruled out in the current study. In fact, calendar, time, measurement comparison, and spatial relations are topics included in kindergarten mathematics curricula (e.g., Everyday Math, KinderMath, My Math), and the teachers in the current study reported teaching some of these topics in their classroom.

In summary, children in all three training conditions showed improvement on numeral identification, number ordering, number line estimation, MEFS, HTKS, and Boehm Test. Although it is possible that the training activities may have influences on these skills, other factors such as learning in classroom cannot be ruled out in the current study. In fact, counting/numbering skills, number relation skills, and relational language are a part of kindergarten mathematics standards (CCSSO/NGA, 2010) and most of the participants in this study (81%) were tested during the spring semester, thus, they were likely to have had practiced and acquired some of these skills prior to the pre-test and continued learning these skills in their classroom. Even though there was a wide range of skill levels captured by each of the numeracy tasks, the average proportion of correct responses ranged from 62 – 85% for these tasks at pre-test suggesting that many children had some knowledge in the targeted content prior to training. The combination of learning relevant concepts in classroom with relatively small room for growth may contribute to the findings that overall, children only made small improvements on very few tasks and effects of training condition only emerged in counting backwards. Future testing of a no-training control group or a non-ordering training group are needed to determine whether and how much children benefitted from the training activities.

Differences across training conditions.

Counting backwards. Children in the number + EF + RL training showed improvement on counting backwards. On average, they were able to count backwards from 13 at pretest and from 15 at posttest. In the number + EF + RL training, the examiner prompted children to generate an alternative problem-solving strategy to promote children EF skills, and encouraged children to count backwards. Although number + EF + RL training did not lead to condition-specific improvement on number relation skills, it did lead to improvement on counting backwards. Counting backwards can be challenging for children because they have to remember the counting sequence, inhibit the habitual response of counting forward, and instead say the number that comes before. Children not proficient at this task may have to hold the starting number in working memory, count from one to figure out the number that comes before, and repeat this process as they count backwards to reach one. If the improvement on this task was due to the overall EF support provided in number + EF + RL training, children in this training condition should also show greater improvement on the EF tasks compared to children in number and alphabet training conditions. Although the comparable EF improvement across the three training conditions suggests that the improvement on counting backwards may be due to the backward-counting practice as opposed to the EF prompts incorporated in number + EF + RL training, the finding shows that practice with counting backwards even just four to six times for four sessions during number ordering activities may be effective at improving performance on this task.

Executive Function prompts. Despite the longer duration of the number + EF + RL training (*M* = 16.47 minutes) compared to the number training (*M* = 10.46 minutes), children in the number + EF + RL training did not outperform children in the number training on any numeracy or EF tasks, except counting backwards, at posttest. Children in the number + EF + RL training received EF prompts to reflect on their problem-solving strategy and to generate an alternative strategy, however, they did not show additional improvement on EF skills compared to children in the number training or the alphabet training. One potential explanation for this finding is that the EF prompts embedded in the brief number training activities may not affect children's performance on other non-numerical activities such as MEFS and HTKS. Previous training studies revealed that children who were prompted to reflect on their responses during a brief picture-sorting training showed improvement on MFES, an EF measure that involved sorting pictures based on different dimensions (Espinet et al., 2013; Prager, 2016). However, children who were prompted to reflect on their responses during a brief number training did not show improvement on EF tasks from pretest to posttest (Prager, 2016). The results from the current study and Prager's study (2016) are consistent with and provide further evidence for the notion that whether and when the training is effective may depend on the skill of interest and the context (Diamond & Ling, 2016). The effect of training may be limited to the targeted skills and the training context, and may not transfer across domains or contexts. For instance, EF training may lead to improvement on EF skills tested in a similar context, however, it may not lead to improvement on mathematical skills or on EF skills tested in a different context.

The EF prompts did not have additional effects on children's number relation skills in the current study. This was consistent with the previous finding that preschoolers who received brief number training with reflection prompts did not show more improvement on early numeracy skills compared to children who received number training alone (Prager, 2016). Although prompting kindergarten children to simply reflect on their responses during number training did not seem to affect their number relation skills, studies with older students revealed that asking students to provide explanation for correct and incorrect responses improves their mathematical performance. Fourth grade students showed improvement on their fraction comparison after thirty-six 35-minute intervention sessions that involved examples of high quality explanations and prompts to generate explanations for problem solving procedures (Fuchs et al., 2016). Even without examples of high quality explanation and intensive intervention, third to fifth grade students still benefited from explaining why a response was correct or incorrect and showed improvement on arithmetic after one 40-minute training session (Rittle-Johnson, 2006). Future studies examining the variation of reflection and explanation prompts, and their relation with EF skills across development and context may be important for understanding whether and when these prompts influence children's mathematical thinking, and have implications for teaching in formal classroom and learning in informal contexts.

Relational language instruction. During the number + EF + RL training, the examiner used relational language to describe the cardinal and ordinal relations between numbers, however, children in this training condition did not show additional improvement on relational language or number relation skills. This was consistent with Powell and Driver's finding (2014) that first grade students who received addition + RL training did not show additional improvement on relational language or arithmetic skills compared to students who received addition only training. In the current study, the relational vocabulary used during the brief training sessions was limited to quantitative and ordinal relations, and that may not lead to an overall improvement on relational language. When the training activities involve a variety of relational language (e.g., spatial, magnitude, size, proportion) across different story book contexts over two to five months, kindergarten students do show improvement on overall relational language (Hassinger-Das et al., 2015; Jennings et al., 1992), suggesting that perhaps more extensive and broad training may be effective at improving relational language.

The RL prompts in number + EF + RL training also did not have additional effects on children's number relation skills compared to number training alone. In the current study, the relational language instruction consisted of statements about the ordinal and cardinal relations between numbers. Without requiring children to respond, the instruction may not actively engage children in applying their knowledge of relational language in number domain (Greenwood, Charles, Horton, Betty, and Utley, 2002). Laski and Siegler (2007) found that engaging children in active categorization of numbers using relational language (i.e., small, medium, big) led to significant improvement on number line estimation. However, Laski and Siegler did not include a number only training for comparison to test the unique effects of relational language. Powell and Driver (2014) included an addition only training, and found comparable improvement between children who received addition training vs. addition + RL training. Importantly, Powell and Driver's addition + RL training involved actively engaging students by asking them to define relational vocabulary and using manipulatives or gestures to demonstrate these relational terms, but there were still no effects of relational language after fifteen 10- to 15-minute training sessions. The finding suggests that the non-interactive nature of the relational language instruction in the current study may not fully account for the absence of the effects of relational language instruction.

In summary, the effects of training condition emerged in counting backwards, but they did not appear to be linked to the efforts in promoting EF skills or relational language per se. Instead, the improvement on counting backwards seemed to be due to the backward-counting practice embedded in the alternative strategy prompt. Although the examiner provided the reflection prompt and relational language instruction on every trial during the training sessions, children only received a total of one-hour training in the context of number card activities over 2 – 3 weeks period. With the brief training over a short period of time in a relatively limited context, children may not acquire or apply the EF and relational language skills from the training to other number relation tasks. In other words, more EF prompts and relational language instruction over a longer period of time across more diverse contexts may be required to observe their effects on number relation skills. Alternatively, all the prompts may simply be too much for kindergarten students. During the training sessions, children not only have to count and figure out an unknown number, but also reflect on their responses, attend to the relational language instruction, and sometimes generate an alternative problem-solving strategy. With their EF and language skills still developing, the layers of instruction embedded in the training activities may be too overwhelming for kindergarten students to benefit from these additional prompts. Studies investigating these two competing hypotheses may have important implications for instructional designs and practices in kindergarten classroom.

Effects of initial executive function skills and relational language.

Executive function skills. The effects of children's initial EF skills on their gains in number relation skills and counting/numbering skills emerged when HTKS score was used as an indicator of children's initial EF skills. Children with low HTKS score showed improvement on number ordering, counting backwards, and numeral identification, but children with high HTKS score did not. Children with high HTKS score showed improvement on numerosity estimation but children with low HTKS score did not. However, when children's pretest scores on these early numeracy tasks were controlled for in the analyses, the effect of initial HTKS level was not significant, indicating that children's initial EF skills did not predict gains in early numeracy skills beyond children's initial early numeracy skills. Given the consistent findings on the correlation between EF skills and mathematical skills across studies (e.g., Geary, Hoard, & Nugent, 2012; Mazzocco & Kover, 2007; McClelland et al., 2014; Welsh et al., 2010), the effects of initial EF skills on gains in early numeracy skills may be an artifact of the correlation between EF skills and early numeracy skills at pretest, at least in this study. Specifically, children with low HTKS score performed poorly on the number ordering, counting backwards, and numeral identification tasks at pretest, thus they had more room for improvement and actually improved more compared to children with high scores on

HTKS and these early numeracy tasks. This finding is consistent with a previous study where children with low vs. high initial early numeracy skills benefit more from playing a number line board game and show more improvement on number comparison and number line estimation (Ramani & Siegler, 2011). This is also consistent with the general findings on the effects of EF training programs on children EF skills where children with low initial EF skills benefit more from the training programs compared to children with high initial EF skills (Diamond, 2012). Although the pattern of results was different for the numerosity estimation task where only children with high HTKS score showed improvement on this task, children's initial HTKS score still did not predict their gains in this task beyond their pretest performance. Kolkman and colleagues (2013) found that children with high initial EF skills showed more improvement on number line estimation after six number training sessions compared to children with low initial EF skills. However, Kolkman and colleagues did not control for children's pretest performance on number line estimation, therefore, it remained unclear whether the effect of initial EF skills was independent of pretest performance on number line estimation in their study.

Unlike the findings with the initial HTKS score, children's initial MEFS score did not predict their gains in the early numeracy tasks. One possibility is that EF skills may be differentiating during kindergarten, and HTKS and MEFS may tax the components of EF skills differently. Although EF skills appear to be an unitary construct in early childhood (Wiebe et al., 2008, 2011), Lee and colleagues (2013) reveal that EF skills slowly differentiate into three components from kindergarten to ninth grade. In some research studies with preschoolers, HTKS is used as a measure of inhibition (e.g., Fuhs & McNeil, 2013) whereas DCCS (table-top version of MEFS) is used as a measure of cognitive flexibility (e.g., Purpura et al., 2017). In the current study, HTKS was more strongly correlated with the early numeracy tasks (except for numerosity estimation at pretest and counting aloud at posttest) compared to MEFS, and this may account for the diverging results between initial HTKS score and initial MEFS score. The finding from the current study suggests that early numeracy skills may be more closely related to the component of EF skills taxed by HTKS vs. MEFS, at least in kindergarten students. This aligns with the notion that inhibition may be especially important for mathematical skills during preschool and early elementary years because children may need to suppress less sophisticated strategies (e.g., counting from 1) as they acquire new concepts and more sophisticated strategies (e.g., counting from the first addend; Cragg & Gilmore, 2014). The finding is also consistent with Purpura and colleagues' finding (2017) that inhibition, as measured by the Day Night task, is correlated with more early numeracy tasks compared to cognitive flexibility, as measured by DCCS, in preschool age children.

An alternative possibility for the diverging patterns of results may be related to the distribution of HTKS and MEFS scores. Because of the standardization of the MEFS scores, most of the MEFS scores clustered around the mean leading to a narrower distribution compared to the HTKS scores. The wider range of scores for HTKS may have allowed more room for significant correlations. Thus, the different patterns of results between MEFS and HTKS may be an artifact of the data distribution.

Relational language. Similar to the findings on the effects of children's initial HTKS score, children with low Boehm percentile showed more improvement on the number ordering task and the numeral identification task from pretest to posttest, compared to children with high Boehm percentile. However, the effects of initial Boehm level were not significant when controlling for children's scores on these early numeracy tasks at pretest, suggesting that children's initial knowledge of relational language may not predict gains in early numeracy skills beyond their initial early numeracy skills. Given the correlation between relational language and early numeracy skills in other studies (e.g., Purpura et al., 2017; Purpura & Logan, 2015; Purpura & Reid, 2016) and the current study ($|r_s\text{s}| > .30$), the effects of initial Boehm level may be a product of the correlation between Boehm percentile and children performance on the early numeracy tasks at pretest. The result is consistent with the general findings that children with low initial skill level improve more on that skill after training compared to children with high initial skill level (e.g., Ramani & Siegler, 2011; Diamond, 2012). The finding is inconsistent with Toll and Van Luit's study (2014) in which they find that kindergarten students with high knowledge of relational language show more growth in early numeracy skills compared to students with low knowledge of relational language, however, it is unclear whether the effect of relational language observed in Toll and Van Luit's study (2014) is independent of children's initial early numeracy skills.

In summary, children's initial EF skills and relational language were related to their growth in early numeracy skills, however, these relations were not independent of, and not significant after controlling for, their initial early numeracy skills. In fact, children who showed improvement on early numeracy tasks had lower scores on early numeracy as well as EF and relational language at pre-test compared to children who did not show improvement. This provides further evidence on the close relations between EF, relational language, and early numeracy skills but also raises questions on the unique influences of initial EF skills and relational language on gains in early numeracy skills.

The two measures of the EF skills, HTKS and MEFS, varied in their strength of correlations with early numeracy tasks. The different patterns of results may be due to their score distribution, but may also reflect more fundamental differences between these two EF tasks. The findings from this study have two implications for future research. First, although it remains unclear how EF skills, relational language and early numeracy skills interact and influence the development of mathematical skills, the findings do suggest the importance of accounting for and delineating the relations between these three sets of skills. Doing so will allow us to accurately identify the predictors of improvement and to inform teaching and practice. Second, it may be important to delineate the relations between components of EF skills and aspects of mathematical skills across development to provide insights into the cognitive processes involved in various mathematical tasks and inform assessment and intervention.

Limitations

This study had several limitations. First, the participants were kindergarten students recruited from four different schools in the same metropolitan area and may not be representative of the overall demographic of the state or the country. Although limiting to kindergarten students precluded the possibility of examining the relations between the skills of interest over time, kindergarten was an appropriate developmental stage to examine these skills (e.g., Bos et al., 2018; Kolkman et al., 2013; Kroesbergen et al., 2014; Purpura & Reid, 2016; Ramani et al., 2017). Even though the sample was not representative of the population, the participants were from diverse background and the sample included a wide range of skill levels across tasks allowing for the investigation of the concurrent relations between the skills of interest. Second, set counting and numerosity estimation were excluded from their respective factor and composite score during the factor analyses process. One potential explanation for set counting loading poorly on counting/numbering factor was that it involved pointing and counting sets of dots whereas the remaining four tasks were verbal counting or naming numerals. The differences in the task demands between set counting and the remaining four counting/numbering tasks may account for its low factor loading. Numerosity estimation did not load on number relation factor and it may be because children did not perceive numerosity estimation as a relational task. The sets of numerosities were presented individually and without explicit instruction, children may not compare the numerosities to 20 or 100 (the calibration) during estimation. Still, I was able to identify two distinct factors with the remaining tasks and revealed the influences of EF skills and relational language on number relation skills. Third, the mediation analyses were based on concurrent data thus the directionality of the influences could not be inferred. However, I demonstrated that the reverse mediations were not supported by the data. Last, in this study, I did not include a no-training control condition to account for maturation or learning in classroom. Future studies would benefit from recruiting a more representative sample, including a broader range of counting/numbering tasks and number relation tasks, collecting longitudinal data, and including additional control conditions to accounting for maturation and experience in classroom.

Conclusions and Future Directions

In conclusion, the current study contributes new findings to the literature on the influences of EF skills and relational language on early numeracy skills. First, I extended previous findings on the distinction between number relation skills and counting/numbering skills, and found that EF skills and relational language separately predicted number relation skills beyond counting/numbering skills. Second, I found that number relation skills, as well as counting/numbering skills, fully mediated the relations between EF skills and mathematical skills, and between relational language and mathematical skills in kindergarten students. Third, I found that incorporating EF prompts and relational language instruction in brief number training did not have additional effects on children's early numeracy skills beyond number training alone. Last, children's initial EF skills or relational language did not predict improvement on early numeracy skills beyond their initial early numeracy skills.

There are implications from this study relevant to future research and practice. If EF skills and relational language have influences on number relation skills, it may be important to identify the underlying mechanisms that account for their influences. The association between EF skills and relational language suggests that this relation should be further examined and accounted for when delineating their independent influences on mathematical skills. If EF skills and relational language only indirectly influence mathematical skills through early numeracy skills, efforts in developing mathematics instructions and interventions perhaps should prioritize promoting early numeracy skills over EF skills or relational language. The results from the training suggest that potential starting points from which to launch further inquiries include studies of whether EF prompts and relational language instructions embedded in number training have immediate effects on learning during training, whether these prompts and instructions are effective at improving early numeracy with sufficient dosage, and whether interventions focusing on numerical skills are equally or more effective at improving early numeracy compared to interventions enriched with non-numerical prompts for children this age.

Table 1

Number of children with high vs. low EF skills in each training condition based on sample median of HTKS total score or MEFS standard score.

	HTKS Total Score		MEFS Standard Score	
Training condition	High	Low	High	Low
Number (n = 35)	16	19	16	19
Number + EF + RL (n = 36)	19	17	18	18
Alphabet (n = 33)	15	18	17	16

Table 2.

Duration (in minutes) and the number of trials completed during each training session by training condition.

Training condition	Number		Number + EF + RL		Alphabet	
Training session	Duration *M* (*SD*)	Total Trials *M* (*SD*)	Duration *M* (*SD*)	Total Trials *M* (*SD*)	Duration *M* (*SD*)	Total Trials *M* (*SD*)
1	9.16 (3.00)	18.03 (2.87)	16.00 (4.08)	14.25 (1.50)	10.80 (5.01)	17.21 (3.16)
2	8.32 (2.83)	18.87 (2.33)	16.02 (4.24)	15.06 (2.51)	9.31 (3.76)	18.85 (2.28)
3	12.14 (3.10)	11.29 (1.47)	17.32 (3.96)	10.53 (0.88)	12.08 (4.07)	11.18 (1.57)
4	12.23 (4.81)	9.46 (1.40)	16.55 (5.07)	9.08 (1.40)	11.91 (5.72)	9.58 (1.50)

Table 3

Mean, standard deviation, and range of scores for each task at pretest and posttest.

	Pretest (*N* = 104)			Posttest (*N* = 104)		
Tasks	Mean	SD	Range	Mean	SD	Range
Baseline measures						
KBIT standard score	98.18	15.57	55 – 125	--	--	--
TEMA standard score	99.59	13.69	68 – 134	--	--	--
Counting/numbering tasks						
Counting aloud	80.05	42.65	6 – 130	81.85	38.27	12 – 130
Counting backward	12.95	6.74	0 – 20	14.98	6.12	0 – 20
Counting after	10.52	3.95	0 – 14	10.98	3.66	0 – 14
Set counting	2.86	0.99	0 – 4	2.78	1.01	0 – 4
Numeral identification	16.96	4.23	3 – 20	17.84	3.53	6 – 20
Number relations tasks						
Number comparison	12.27	2.89	7 – 16	12.61	2.97	6 – 16
Number ordering	9.53	4.66	0 – 14	10.39	4.10	0 – 14
Set relation	6.59	1.54	2 – 8	6.36	1.66	2 – 8
Numerosity estimation	1.12	1.09	0.19 – 7.35	1.09	0.90	0.18 – 4.06
Number line estimation	0.29	0.12	0.07 – 0.62	0.27	0.12	0.06 – 0.63
Executive function tasks						
HTKS total score	60.59	26.13	0 – 92	68.00	20.93	0 – 93
MEFS standard score	100.40	9.88	71.5 – 127	104.77	12.50	65.5 – 127
Relational language task						
Boehm percentile	34.55	29.89	1 – 99	38.15	31.87	1 – 99

Abbreviations: KBIT, Kaufman Brief Intelligence Test- Verbal Knowledge subtest; TEMA, Test of Mathematics Ability; HTKS, Head Toes Knees Shoulders task; MEFS, Minnesota Executive Function Scale

Table 4

Spearman correlations between all tasks at pretest (upper triangle) and posttest (lower triangle).

	KBIT	TEMA	Caloud	Cback	Cafter	SCount	NID	NCom	NOrd	SR	NE	NLE	HTKS	MEFS	Boehm
KBIT	--	.41***	.34***	.24*	.37***	.22*	.26**	.36***	.39***	.51***	-.32**	-.42***	.40***	.53***	.74***
TEMA	--	--	.62***	.65***	.76***	.36***	.75***	.64***	.69***	.65***	-.38***	-.57***	.53***	.38***	.51***
Caloud	.31**	.53***	--	.61***	.64***	.27**	.62***	.60***	.55***	.50***	-.28**	-.52***	.49***	.33***	.42***
Cback	.24*	.58***	.60***	--	.69***	.31**	.67***	.63***	.64***	.48***	-.20*	-.48***	.52***	.29**	.42***
Cafter	.34***	.70***	.55***	.68***	--	.21*	.77***	.70***	.78***	.63***	-.19^{+}	-.59***	.57***	.28**	.52***
SCount	.24*	.16	.14	.31**	.24*	--	.30**	.25*	.37***	.30**	-.35***	-.33***	.28**	.22*	.30**
NID	.25*	.73***	.62***	.67***	.78***	.14	--	.68***	.73***	.52***	-.13	-.51***	.60***	.28**	.43***
NCom	.38***	.69***	.53***	.65***	.73***	.22*	.71***	--	.77***	.60***	-.30**	-.60***	.57***	.45***	.55***
NOrd	.33***	.64***	.48***	.53***	.67***	.16	.64***	.74***	--	.60***	-.26**	-.67***	.61***	.45***	.64***
SR	.33***	.59***	.39***	.54***	.63***	.23*	.60***	.60***	.51***	--	-.27**	-.50***	.45***	.40***	.53***
NE	-.20*	-.28**	-.21*	-.30**	-.26**	-.28**	-.16	-.28**	-.18^{+}	-.19^{+}	--	.32***	-.15	-.28**	-.32***
NLE	-.49***	-.60***	-.45***	-.46***	-.67***	-.38***	-.56***	-.61***	-.63***	-.40***	.33***	--	-.52***	-.50***	-.63***
HTKS	.42***	.56***	.39***	.51***	.64***	.27**	.57***	.58***	.56***	.49***	-.17^{+}	-.52***	--	.44***	.62***
MEFS	.40***	.48***	.39***	.39***	.47***	.24*	.41***	.47***	.49***	.37***	-.11	-.51***	.52***	--	.65***
Boehm	.77***	.59***	.38***	.38***	.55***	.31**	.46***	.58***	.57***	.47***	-.29**	-.71***	.64***	.57***	--

$^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$; $^{+}p < .1$

Note. Correlations among groups of tasks for counting/numbering skills, number relation skills, and executive function skills are outlined in boxes. Weak correlations ($|r_s| < .30$) are in gray font.

Abbreviations: KBIT, Kaufman Brief Intelligence Test- Verbal Knowledge subtest; TEMA, Test of Mathematics Ability; Caloud, counting aloud; Cback, counting backward; Cafter, counting after; SCount, set counting; NID, numeral identification; NCom, number comparison; NOrd, number ordering; SR, set relation; NE, numerosity estimation; NLE, number line estimation; HTKS, Head Toes Knees Shoulders task; MEFS, Minnesota Executive Function Scale; Boehm, Boehm Test of Basic Concepts.

Table 5

Factor loadings for the counting/numbering tasks and the model fit indices for the factor.

	Include set counting		Exclude set counting	
	Pretest	Posttest	Pretest	Posttest
Counting/Numbering Tasks				
Counting aloud	.766***	.678***	.763***	.667***
Counting back	.793***	.791***	.789***	.788***
Counting after	.903***	.903***	.910***	.900***
Set counting	.371***	.295**	--	--
Numeral Identification	.856***	.922***	.853***	.926***
Model Fit Indices				
χ^2	274.91	278.93	257.41	266.634
df	10	10	6	6
p	< .01	< .01	< .01	< .01
CFI (> .90)	1.000	0.991	1.000	0.995
TLI (> .90)	1.000	0.982	1.000	0.985
AIC	2981.97	2886.39	2699.87	2594.04
BIC	3008.42	2912.83	2721.02	2615.20
RMSEA (< .08)	0.000	0.000	0.000	0.000
SRMR (< .08)	0.022	0.029	0.012	0.014

*** $p < .001$; ** $p < .01$

Abbreviations: CLI, Comparative Fit Index; TLI, Tucker-Lewis Index; AIC, Akaike Information Criterion; BIC, Bayesian Information Criterion; RMSEA, root mean square error of approximation; SRMR, standardized root mean square residual.

Note. Numbers in parentheses represent the conventional cutoffs for good model fit.

Table 6

Factor loadings for the counting/numbering tasks and the model fit indices for the factor.

	Include numerosity estimation		Exclude numerosity estimation	
	Pretest	Posttest	Pretest	Posttest
Counting/Numbering Tasks				
Number comparison	.853***	.909***	.851***	.908***
Number ordering	.904***	.820***	.908***	.822***
Set relation	.639***	.628***	.636***	.629***
Numerosity estimation	.078	.180	--	--
Number line estimation	.713***	.682***	.712***	.680***
Model Fit Indices				
χ^2	214.01	187.01	203.48	182.54
df	10	10	6	6
p	<0.01	<0.01	<0.01	<0.01
CFI (> .90)	0.961	1.000	1.000	1.000
TLI (> .90)	0.923	1.000	1.000	1.000
AIC	1484.51	1480.42	1181.09	1208.32
BIC	1510.86	1506.76	1202.24	1229.40
RMSEA (< .08)	0.128	0.000	0.000	0.000
SRMR (< .08)	0.050	0.025	0.011	0.017

*** $p < .001$

Abbreviations: CLI, Comparative Fit Index; TLI, Tucker-Lewis Index; AIC, Akaike Information Criterion; BIC, Bayesian Information Criterion; RMSEA, root mean square error of approximation; SRMR, standardized root mean square residual.

Note. Numbers in parentheses represent the conventional cutoffs for good model fit.

Table 7

Fit indices for the one-factor and two-factor models at pretest and posttest.

	χ^2	*df*	CFI (> .90)	TLI (> .90)	AIC	BIC	RMSEA (< .08)	SRMR (< .08)
Pretest								
One Factor	589.68	28	.960	.944	3792.52	3834.83	.104	.035
Two Factor: CN, NR	589.68	28	.997	.996	3772.79	3817.74	.028	.024
Posttest								
One Factor	558.51	28	.946	.925	3703.34	3745.49	.118	.048
Two Factor: CN, NR	558.51	28	.993	.989	3679.57	3724.36	.044	.038

Abbreviations: CN, counting/numbering; NR, number relation; CLI, Comparative Fit Index; TLI, Tucker-Lewis Index; AIC, Akaike Information Criterion; BIC, Bayesian Information Criterion; RMSEA, root mean square error of approximation; SRMR, standardized root mean square residual.
Note. Numbers in parentheses represent the conventional cutoffs for good model fit.

Table 8

Mean and standard deviation of scores at pretest by schools

	low FRL school (n = 31)		other schools (n = 73)	
Pretest Measures	*M*	SD	*M*	*SD*
KBIT standard score*	110.65	7.16	92.89	15.17
TEMA standard score*	108.74	10.71	95.70	13.00
Counting/numbering score*	0.46	0.58	-0.20	0.89
Number relation score*	0.63	0.70	-0.27	0.74
Executive function score*	0.69	0.61	-0.29	0.74
Boehm percentile*	63.26	22.51	22.36	23.69

* $p < .05$.

Abbreviations: KBIT, Kaufman Brief Intelligence Test- Verbal Knowledge subtest; TEMA, Test of Mathematics Ability.

Table 9

Mean and standard deviation of scores on the number ordering task and the numerosity estimation task by high vs. low HTKS score at pretest and posttest.

Tasks	pretest		posttest		gain (posttest – pretest)	
	M	*SD*	*M*	*SD*	*M*	*SD*
Number Ordering						
low HTKS score*	7.08	4.54	8.58	4.16	1.50	2.36
high HTKS score	12.08	3.21	12.28	3.08	0.20	2.42
Numerosity Estimation						
low HTKS score	1.00	0.75	1.19	0.95	0.22	1.11
high HTKS score*	1.26	1.35	0.98	0.86	-0.24	0.80

* The gain is significant at $p < .05$.

Abbreviations: HTKS, Head Toes Knees Shoulders task

Table 10

Mean and standard deviation of scores on the number ordering task by high vs. low Boehm percentile at pretest and posttest.

	pretest		posttest		gain (posttest – pretest)	
Task	*M*	*SD*	*M*	*SD*	*M*	*SD*
Number Ordering						
low Boehm percentile *	6.49	4.27	8.12	4.14	1.63	3.12
high Boehm percentile	12.57	2.60	12.67	2.49	0.10	1.19

* The gain is significant at $p < .05$.

Table 11

Mean and standard deviation of scores on the numeral identification task and the counting backwards task by high vs. low HTKS score at pretest and posttest.

	pretest		posttest		gain (posttest – pretest)	
Tasks	*M*	*SD*	*M*	*SD*	*M*	*SD*
Numeral identification						
low HTKS score*	14.80	4.75	16.24	4.18	1.44	2.41
high HTKS score	19.30	1.58	19.56	1.20	0.26	0.97
Counting backwards						
low HTKS score*	9.98	6.93	12.93	6.75	2.94	5.78
high HTKS score	16.16	4.82	17.20	4.44	1.04	4.76

* The gain is significant at $p < .05$.

Abbreviations: HTKS, Head Toes Knees Shoulders task

Table 12

Mean and standard deviation of scores on the numeral identification task by high vs. low Boehm percentile at pretest and posttest.

	pretest		posttest		gain (posttest – pretest)	
Task	*M*	*SD*	*M*	*SD*	*M*	*SD*
Numeral identification						
low Boehm percentile*	15.08	4.85	16.50	4.14	1.42	2.26
high Boehm percentile	18.85	2.32	19.17	2.09	0.40	1.30

* The gain is significant at $p < .05$.

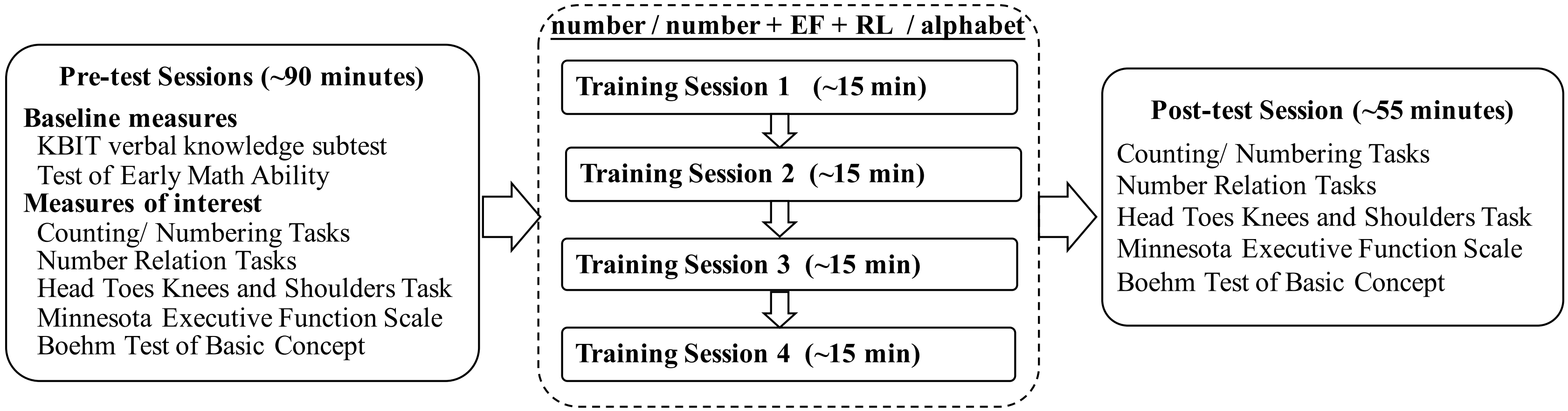
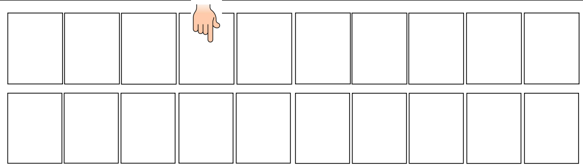


Figure 1. Study design. All children completed the pretest measures in two or three sessions, which were followed by four training sessions in either number, number + EF + RL, or alphabet condition, and one or two posttest sessions.

Training Session 1. Identify

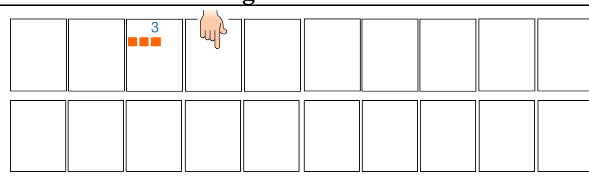


What do you think this card is?

If correct: That's right! It's 4. It also has 4 circles on it.

If incorrect: It is 4. Let's count together (*point to each card while counting from 1*). It also has 4 circles on it.

Training Session 2. Relate

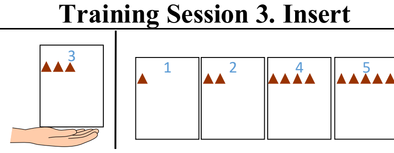


What do you think this card is?

If correct: That's right! It's 4. It also has 4 circles on it.

If incorrect: It is 4. Let's count together (*point to each card while counting from 3*). It also has 4 squares on it.

Training Session 3. Insert



What number is missing?

If correct: That's right! It's 3. It has 3 triangles on it.

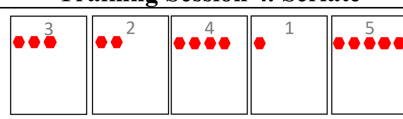
If incorrect: It is 3. Let's count together (*point to each circle while counting*). It has 3 triangles on it.

Where do you think it goes?

If correct: 1, 2, 3, 4, 5. That's right! 3 goes there.

If incorrect: 1, 2... 3... 4, 5 so 3 goes here (*put the cards in order*).

Training Session 4. Seriate



Please put these back in order.

If correct: 1, 2, 3, 4, 5. That's right!

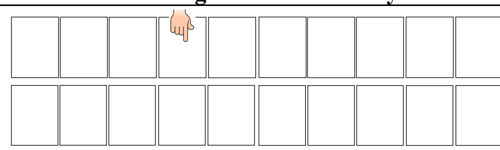
If incorrect (1, 2, 4, 3, 5): 1... 2... 3... wait, this is not 3. This is 4, see 1, 2, 3, 4 (*count the shapes*). Which of these cards is 3 *(point to the two remaining cards, 3 and 5)*?

- If correct: That's right! This is 3 and 3 goes here (*put 3 in the correct location*).
- If incorrect: This is not 3. This is 5. This is 3 and 3 goes here (*pick up 3 and put it in the correct location*).

Figure 2.

Illustration of the training activity, sample prompts, and feedback for each training session in number training. The examiner's actions are italicized in parentheses.

Training Session 1. Identify



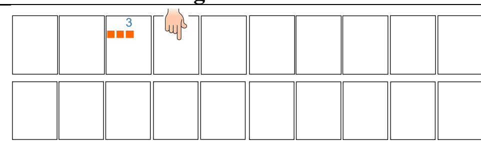
What do you think this card is? **How do you know this is N?**

If correct: That's right! It's 4. It also has 4 circles on it.

If incorrect: It is 4. Let's count together (*point to each card while counting from 1*). It also has 4 circles on it.

Four is in between 3 and 5. 4 comes right after 3 and 4 comes right before 5.

Training Session 2. Relate



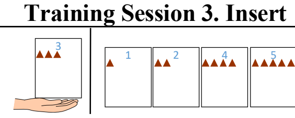
What do you think this card is? **How do you know this is N?**

If correct: That's right! It's 4. It also has 4 squares on it.

If incorrect: It is 4. Let's count together (*point to each card while counting from 3*). It also has 4 squares on it.

Four is in between 3 and 5. 4 is one more than 3 and 4 is one less than 5.

Training Session 3. Insert



What number is missing? **How do you know this is N?**

If correct: That's right! It's 3. It has 3 triangles on it.

If incorrect: It is 3. Let's count together (*point to each triangle while counting*). It has 3 triangles on it.

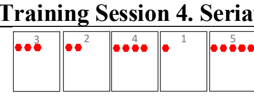
Where do you think it goes? **How do you know N goes here?**

If correct: 1, 2, 3, 4, 5. That's right! 3 goes there.

If incorrect: 1, 2... 3... 4, 5 so 3 goes here (*put the cards in order*).

Three is in between 2 and 4. 3 comes right after 2 and 3 has 1 more than 2. 3 comes right before 4 and it has 1 less than 4.

Training Session 4. Seriate



Please put these back in order. **How do you know the numbers go in order like this?**

If correct: 1, 2, 3, 4, 5. That's right!

If incorrect: 1... 2... 3... wait, this is not 3. This is 4, see 1, 2, 3, 4 (*count the shapes*). Which of these cards is 3 *(point to the two remaining cards, 3 and 5)*?

If correct: That's right! This is 3 and 3 goes here (*put 3 in the correct location*).

If incorrect (1, 2, 4, 3, 5): This is not 3. This is 5. This is 3 and 3 goes here (*pick up 3 and put it in the correct location*).

Two comes after 1 and 2 has 1 more than 1. 3 comes after 2 and 3 has 1 more than 2. 4 comes after 3 and 4 has 1 more than 3. 5 comes after 4 and 5 has 1 more than 4.

Generate an alternative strategy(every two or three trials)

What is another way to figure it out?

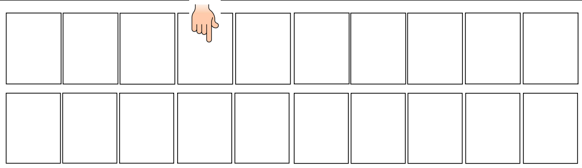
If correct: Great Job!

If incorrect: **We can figure it out by counting the cards backwards (*count backwards*).**

Figure 3.

Illustration of the training activity, sample prompts, and feedback for each training session in number + EF + RL training. The examiner's actions are italicized in parentheses. The EF questions and the relational language instruction are bolded.

Training session 1. Identify

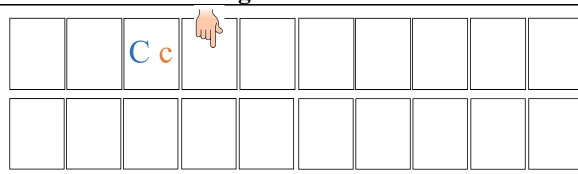


What do you think this card is?

If correct: That's right! It's D.

If incorrect: It is D. Let's say the letters together (*point to each card while saying from A*).

Training session 2. Relate

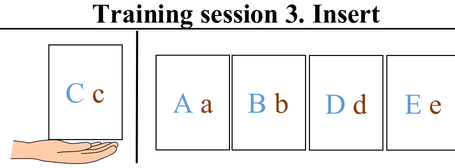


What do you think this card is?

If correct: That's right! It's D.

If incorrect: It is D. Let's say the letters together (*point to each card while saying from C*).

Training session 3. Insert



What letter is missing?

If correct: That's right! It's C.

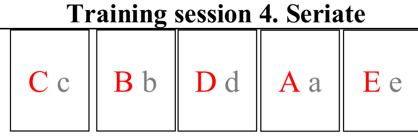
If incorrect: It is C.

Where do you think it goes?

If correct: A, B, C, D, E. That's right! C goes there.

If incorrect: A, B... C... D, E so C goes here.

Training session 4. Seriate



Please put these back in order.

If correct: A, B, C, D, E. That's right!

If incorrect: A... B... C... wait, this is not C. This is D. Which of these cards is C *(point to the two remaining cards, 3 and 5)*?

If correct: That's right! This is C and C goes here (*put C in the right location*).

If incorrect: This is not C. This is E. This is C and C goes here (*pick up C and put it in the right location*).

Figure 4.

Illustration of the training activity, sample prompts, and feedback for each training session in Alphabetic training. The examiner's actions are italicized in parentheses.

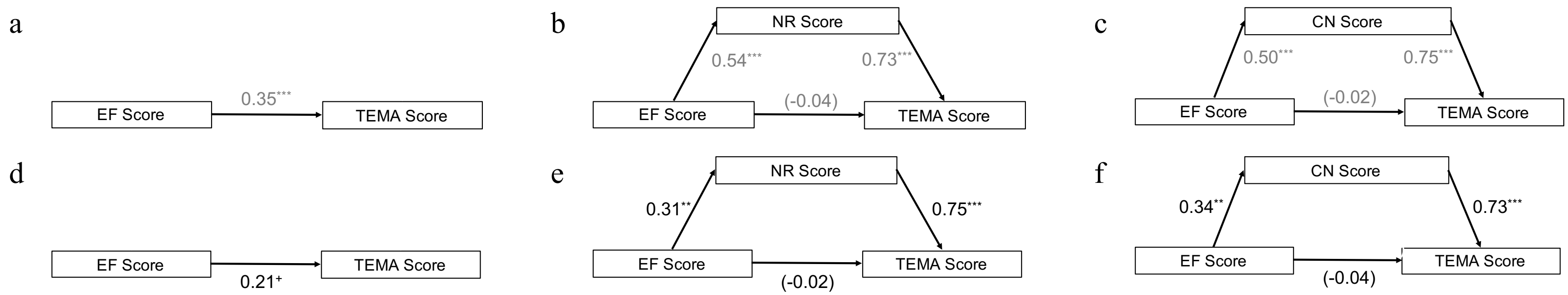


Figure 5.

Indirect effects of the association between EF skills (EF score) and mathematical skills (TEMA score) through number relation skills (NR score; b, e) or counting/numbering skills (CN score; c, f) with verbal knowledge (KBIT score; a, b, c) or verbal knowledge and relational language (KBIT score and Boehm percentile; d, e, f) as covariates.
Abbreviations: EF, executive function; TEMA, Test of Early Math Ability; NR, number relation; CN, counting/numbering.
The significant pathways are indicated with asterisks, $^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$; $^{+}p < .1$.
All models fitted well with the data, CFIs = 1.00, TLIs = 1.00, RMSEAs = .00.

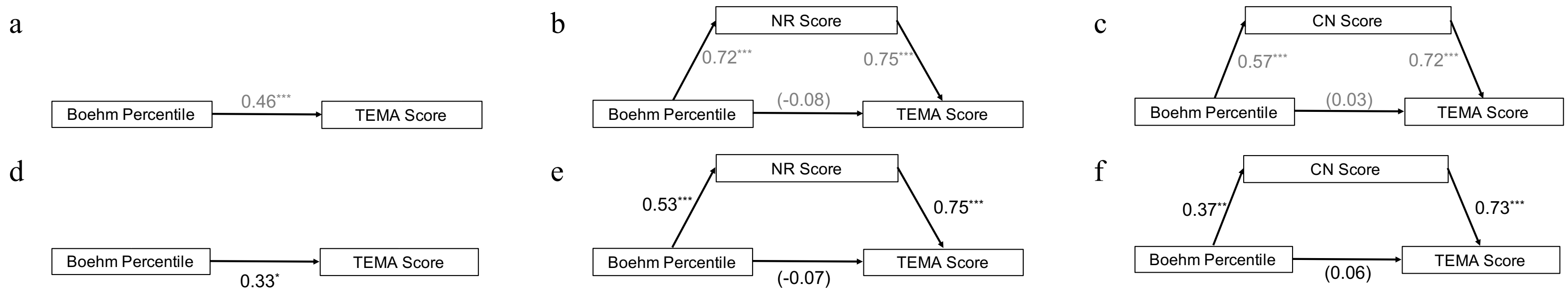


Figure 6.

Indirect effects of the association between relational language (Boehm percentile) and mathematical skills (TEMA score) through number relation skills (NR score; b, e) or counting/numbering skills (CN score; c, f) with verbal knowledge (KBIT score; a, b, c) or verbal knowledge and EF skills (KBIT score and EF score; d, e, f) as covariates.
Abbreviations: TEMA, Test of Early Math Ability; NR, number relation; CN, counting/numbering.
The significant pathways are indicated with asterisks, $^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$.
All models fitted well with the data, CFIs = 1.00, TLIs = 1.00, RMSEAs = .00.

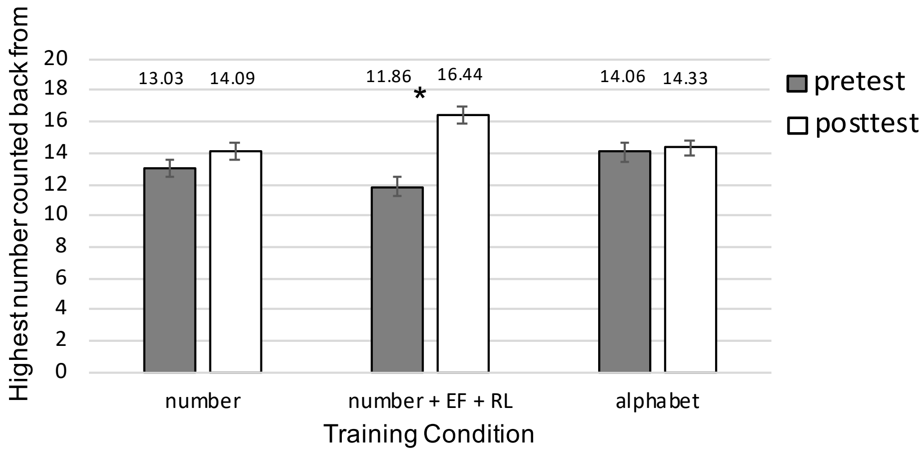


Figure 7.

Time and training condition interaction on children's counting backward performance. Error bars represent standard errors.
* $p < .05$.

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APPENDIX A
Counting and Numbering Tasks

PRETEST

Counting Aloud *Count to* _______

I'd like you to count out loud for me. I'll tell you when to stop

If child is silent: Count out loud like this with me. 1, 2, 3 ... you keep going now by yourself and count up as high as you can.
If child stops: what comes next?
If child reaches 130: okay, you can stop there.

Counting Backwards *Count from* _____

Now I want you to count backwards, like when a rocket blasts off. For instance, 3, 2, 1, blast-off. Now you count backwards starting from 10.

10 9 8 7 6 5 4 3 2 1

Now you count backwards starting from 20.

20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Counting After *Total Correct:* ______ */ 14*

Trial	Prompt	Response	Trial	Prompt	Response
1	Count with me, 2, 3, 4 and then comes...		8	11 and then comes	
2	What number comes next; 3 and then comes		9	19 and then comes	
3	9 and then comes		10	16 and then comes	
4	5 and then comes		11	24 and then comes	
5	7 and then comes		12	33 and then comes	
6	13 and then comes		13	29 and then comes	
7	8 and then comes		14	49 and then comes	

Set Counting *Total Correct:* ______ */ 4*

Count these dots with your finger and tell me how many there are. Do it carefully and make sure you touch each dot as you count.

Trial	Prompt	Correct
1	9 dots	Y / N
2	10 dots	Y / N
3	14 dots	Y / N
4	16 dots	Y / N

Numeral Identification *Total Correct:* ______ */ 20*

Trial		Response	Trial		Response	Trial		Response	Trial		Response
1	2		6	4		11	14		16	13	
2	5		7	8		12	20		17	16	
3	6		8	9		13	11		18	28	
4	3		9	12		14	15		19	47	
5	9		10	19		15	10		20	90	

POSTTEST

Counting Aloud *Count to* _______

I'd like you to count out loud for me. I'll tell you when to stop

If child is silent: Count out loud like this with me. 1, 2, 3 ... you keep going now by yourself and count up as high as you can.

If child stops: what comes next?

If child reaches 130: okay, you can stop there.

Counting Backwards *Count from* _____

Now I want you to count backwards, like when a rocket blasts off. For instance, 3, 2, 1, blast-off. Now you count backwards starting from 10.

10 9 8 7 6 5 4 3 2 1

Now you count backwards starting from 20.

20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Counting After *Total Correct:* ______ / *14*

Trial	Prompt	Response	Trial	Prompt	Response
1	Count with me, 1,2,3 and then comes...		8	11 and then comes	
2	What number comes next; 4 and then comes		9	19 and then comes	
3	8 and then comes		10	16 and then comes	
4	6 and then comes		11	25 and then comes	
5	9 and then comes		12	34 and then comes	
6	13 and then comes		13	39 and then comes	
7	7 and then comes		14	59 and then comes	

Set Counting *Total Correct:* ______ / *4*

Count these dots with your finger and tell me how many there are. Do it carefully and make sure you touch each dot as you count.

Trial	Prompt	Correct
1	8 dots	Y / N
2	9 dots	Y / N
3	13 dots	Y / N
4	17 dots	Y / N

Numeral Identification *Total Correct:* ______ / *20*

Trial		Response	Trial		Response	Trial		Response	Trial		Response
1	3		6	4		11	13		16	14	
2	7		7	8		12	20		17	17	
3	9		8	6		13	10		18	27	
4	2		9	12		14	15		19	46	
5	5		10	18		15	11		20	80	

APPENDIX B
Number Relation Tasks

PRETEST

Number Comparison *Total Correct:* ______ / *16*
I am going to say two numbers. Please tell me which number means more.
For example, which is more ___ or ____

Trial	Prompt	Response	Trial	Prompt	Response	Trial	Prompt	Response
1	3 or 5		7	69 or 71		13	15 or 17	
2	11 or 13		8	17 or 15		14	57 or 59	
3	20 or 18		9	8 or 10		15	42 or 40	
4	40 or 42		10	13 or 11		16	18 or 20	
5	59 or 57		11	5 or 3				
6	10 or 8		12	71 or 69				

Number Ordering *Total Correct:* ______ / *14*
Now I'm going to show you three numbers and please put them in order from the smallest number (*point left*) to the largest number (*point right*).

Trial	Prompt	Response	Trial	Prompt	Response
1	4 2 6		8	47 49 48	
2	14 16 15		9	5 3 4	
3	7 8 6		10	13 17 15	
4	35 33 34		11	9 7 5	
5	23 21 19		12	10 8 9	
6	9 13 11		13	17 18 16	
7	20 22 21		14	11 9 10	

Set Relation *Total Correct:* ______ / *8*
I have N coins in this backpack. How many coins are in the backpack?
If incorrect: Oops! Let's try this again (repeat)
If correct: Good! Now watch! (*add 1 or 2 coins*)
Now are there N +1 or N+2 coins in the backpack?

Trial	Prompt	Response	Trial	Prompt	Response
1	5 + 2		5	13 + 2	
2	11 + 1		6	4 + 1	
3	7 + 2		7	9 + 1	
4	16 + 1		8	19 + 2	

Numerosity Estimation
Here is 1 square. Here are 20 squares. That's too many to count very quickly. I am going to show you some squares. Please tell me how many you think there are really quickly without counting. How many are there?

Calibrate to 20											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	11		5	17		9	11		13	40	
2	40		6	9		10	9		14	17	
3	14		7	7		11	14		15	7	
4	5		8	60		12	5		16	60	

Now here is 1 circle. Here are 100 circles. That's too many to count very quickly. I am going to show you some circles. Please tell me how many you think there are really quickly without counting. How many are there?

Calibrate to 100											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	19		5	95		9	19		13	48	
2	10		6	35		10	60		14	84	
3	84		7	10		11	35		15	60	
4	48		8	72		12	72		16	95	

Number Line Estimation

This is a 0 to 20 number line (*point to 0 and 20*). A number line is a line with numbers across it. The numbers on the number line go in order from the smallest number to the largest number, so each number has its very own spot on the number line (*trace number line*).

If 0 goes here, and 20 goes here (*point*), where do you think ____ goes on this number line?

0 – 20 without midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	3		3	11		5	33		7	6	
2	14		4	8		6	19		8	22	
0 – 20 with midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	16		3	7		5	17		7	24	
2	13		4	4		6	31		8	9	

This is a 0 to 100 number line (*point to 0 and 100*). The numbers on the number line still go in order from the smallest number to the largest number, and each number has its very own spot on the number line (*trace number line*).

If 0 goes here, and 100 goes here (*point*), where do you think ____ goes on this number line?

0 – 100 without midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	28		3	7		5	65		7	105	
2	15		4	53		6	76		8	91	
0 – 100 with midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	51		3	16		5	77		7	89	
2	9		4	26		6	64		8	120	

POSTTEST

Number Comparison *Total Correct:* ______ / 16

I am going to say two numbers. Please tell me which number means more.
For example, which is more ___ or ____

Trial	Prompt	Response	Trial	Prompt	Response	Trial	Prompt	Response
1	7 or 5		7	14 or 12		13	21 or 19	
2	54 or 56		8	19 or 21		14	78 or 80	
3	38 or 36		9	5 or 7		15	17 or 19	
4	19 or 17		10	12 or 14		16	36 or 38	
5	80 or 78		11	11 or 9				
6	9 or 11		12	56 or 54				

Number Ordering *Total Correct:* ______ / 14

Now I'm going to show you three numbers and please put them in order from the smallest number (*point left*) to the largest number (*point right*).

Trial	Prompt	Response	Trial	Prompt	Response
1	4 8 6		8	49 47 48	
2	14 12 13		9	3 5 4	
3	7 6 8		10	16 12 14	
4	33 35 34		11	9 11 7	
5	21 23 19		12	13 15 11	
6	12 8 10		13	17 19 18	
7	24 22 23		14	10 11 9	

Set Relation *Total Correct:* ______ / 8

I have N coins in this backpack. How many coins are in the backpack?

If incorrect: Oops! Let's try this again (repeat)

If correct*:* Good! Now watch! (*add 1 or 2 coins*)

Now are there N +1 or N+2 coins in the backpack?

Trial	Prompt	Response	Trial	Prompt	Response
1	5 + 1		5	13 + 1	
2	11 + 2		6	4 + 2	
3	7 + 1		7	9 + 2	
4	16 + 2		8	19 + 1	

Numerosity Estimation

Here is 1 square. Here are 20 squares. That's too many to count very quickly. I am going to show you some squares. Please tell me how many you think there are really quickly without counting. How many are there?

Calibrate to 20											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	13		5	18		9	13		13	38	
2	38		6	8		10	8		14	18	
3	5		7	60		11	5		15	60	
4	14		8	7		12	14		16	7	

Now here is 1 circle. Here are 100 circles. That's too many to count very quickly. I am going to show you some circles. Please tell me how many you think there are really quickly without counting. How many are there?

Calibrate to 100											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	23		5	91		9	23		13	52	
2	52		6	68		10	68		14	91	
3	10		7	10		11	35		15	84	
4	84		8	35		12	60		16	60	

Number Line Estimation

This is a 0 to 20 number line (*point to 0 and 20*). A number line is a line with numbers across it. The numbers on the number line go in order from the smallest number to the largest number, so each number has its very own spot on the number line (*trace number line*).

If 0 goes here, and 20 goes here (*point*), where do you think _____ goes on this number line?

0 – 20 without midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	16		3	7		5	17		7	24	
2	13		4	4		6	31		8	9	
0 – 20 with midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	3		3	11		5	33		7	6	
2	14		4	8		6	19		8	22	

This is a 0 to 100 number line (*point to 0 and 100*). The numbers on the number line still go in order from the smallest number to the largest number, and each number has its very own spot on the number line (*trace number line*).

If 0 goes here, and 100 goes here (*point*), where do you think _____ goes on this number line?

0 – 100 without midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	51		3	16		5	77		7	89	
2	9		4	26		6	64		8	120	
0 – 100 with midpoint											
Trial		Estimate	Trial		Estimate	Trial		Estimate	Trial		Estimate
1	28		3	7		5	65		7	105	
2	15		4	53		6	76		8	91	

APPENDIX C

Training materials by training condition and training session

Training session	Number training and Number + EF + RL training	Alphabet training
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	A a, B b, C c, D d, E e, F f, G g, H h, I i, J j K k, L l, M m, N n, O o, P p, Q q, R r, S s, T t
2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	A a, B b, C c, D d, E e, F f, G g, H h, I i, J j K k, L l, M m, N n, O o, P p, Q q, R r, S s, T t
3	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	A a, B b, C c, D d, E e, F f, G g, H h, I i, J j K k, L l, M m, N n, O o, P p, Q q, R r, S s, T t
4	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	A a, B b, C c, D d, E e, F f, G g, H h, I i, J j K k, L l, M m, N n, O o, P p, Q q, R r, S s, T t

APPENDIX D
Number Training Script

Session 1
We are going to do a card activity today. There are different numbers on these cards. Let's put them in order *(count while putting down 1 to 20)*.
Now we'll turn them over (*says the numbers when flipping them*) and I'll show you a trick. I will tell you what is on any card without even seeing it. Let's try it!

PRACTICE TRIALS (3 TIMES)
Please point to a (or another) card. (*secretly count from 1*) This is… N.
Now, please flip over the card and see if I am right?
I figured it out by counting. See *(count cards)*. This card also has N shapes. A way to check if I'm right is by counting the shapes (*count*).
Now please flip the card back over.

TEST TRIALS AND FEEDBACK
Now it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card.
What do you think this card is?
If correct: You are right! It is N *(reveal card)*. It also has N shapes on it.
If IDK or incorrect: It is N. Let's count together! *(count from 1)*. It also has N shapes on it.

Trial	Target	Response	Trial	Target	Response
1	2		11	16	
2	5		12	19	
3	3		13	20	
4	6		14	17	
5	8		15	1	
6	9		16	14	
7	13		17	4	
8	15		18	7	
9	12		19	10	
10	11		20	18	
Total Correct ________ / __________					

Session 2

Remember the card activity that we did last time? Today we are doing it a little differently.

Let's put them in order *(count while putting down 1 to 20)*.

Now we'll turn them over (*says the numbers when flipping them*)

Let me show you how we do the activity today.

PRACTICE TRIALS (3 TIMES)

Please point to a (or another) card. (*secretly count from 1 or closest N*) This is… N.

Now, please flip over the card and see if I am right?

I figured it out by counting. See *(count from 1 or closest N)*. This card also has N shapes.

A way to check if I'm right is by counting the shapes (*count*).

Last time we flipped the card back over, but this time, we'll keep it face up.

TEST TRIALS AND FEEDBACK

Now, let's flip over the cards and it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card. Remember, we'll keep the cards face up after they are asked.

What do you think this card is?

> If correct: You are right! It is N *(reveal card)*. It also has N shapes on it.
>
> If IDK or incorrect: It is N. Let's count together**!** *(count from closest N)*. It also has N shapes on it *(count dots)*.

Trial	Target	Response	Trial	Target	Response
1	3		11	20	
2	4		12	16	
3	2		13	19	
4	7		14	18	
5	6		15	1	
6	9		16	15	
7	13		17	5	
8	12		18	10	
9	14		19	8	
10	15		20	11	
Total Correct _______ / _________					

Session 3

Today we are doing the card activity differently. Let me show you how we do the activity today.

TEST TRIALS AND FEEDBACK

First, let's put them in order on this black space *(count while putting down 1 to 5)*.
Close your eyes *(take one card)*. Okay, now open your eyes.
What number is missing?

If correct: You are right! It is N *(reveal card)*. It has N shapes on this card.

If IDK or incorrect: Let's count together! *(count shapes)* It is N and it has N shapes.

Where do you think it goes?

If correct: That's right! N goes right here.

If IDK or incorrect: Let's count together! *(count cards)* N goes right here.

Now we are done with 1 to 5, let's put them here (1-5 board) and try the same thing with 6 to 10 (11 to 15, 16 to 20).

Trial	Target	Response	Order
1	3		___ 1 __ 2 __ 4 __ 5 ___
2	2, 4		___ 1 ___ 3 ___ 5 ____
3	9		___ 6 __ 7 __ 8 __ 10 ___
4	6, 8		___ 7 ___ 9 ___ 10 ____
5	12		__ 11 __ 13 __ 14 __ 15 __
6	15		__ 11 __ 12 __ 13 __ 14 ___
7	11, 14		__ 12 __ 13 __ 15 ___
8	18		__ 16 __ 17 ___ 19 __ 20 ___
9	16		__ 17 __ 18 __ 19 __ 20 ___
10	17, 20		___ 16 __ 18 __ 19 ___
Now we are putting all the numbers together. I am going to hide 4 cards this time.			
11	5, 10, 13, 19		___ 1 ___ 2 ___ 3 ___ 4 ___6 ___ 7 ___ 8 ___ 9 ___ ___ 11 __ 12 __ 14 __ 15 ___16 __ 17 __ 18 __ 20 ___
12	4, 6, 14, 17		___ 1 ___ 2 ___ 3 ___ 5 ___ 7 ___ 8 ___ 9 ___ 10 ___ ___ 11 __ 12 __ 13 __ 15 ___16 __ 18 __ 19 __ 20 ___
ID Correct _______ / _________ Order Correct _______ / _________			

Session 4

Today we are doing the card activity differently. I'm going to mix the cards and it's your job to put them back in order. Let's try it!

TEST TRIALS AND FEEDBACK

When child is done…

If correct: 1… 2… 3… 4… 5… That's right! Great job.

If incorrect (example if child respond 1,2,4,5,3):

This is 1 and 1 goes here… *(point to 1)* This is 2 and 2 goes here… (*point to 2*) This is not 3… 1, 2, 3, 4 (*count shapes*) this is 4. Which of these cards is 3 (*point to 5 and 3*)?

If correctly identified 3: That's right! This is 3 and 3 goes here.

If incorrect: 1… 2… 3… 4… 5… (*count shapes*) This is not 3, it's 5. This is 3 (*point to 3*) and 3 goes right here

Now we are done with 1 to 5, let's put them here (1-5 board) and try the same thing with 6 to 10.

Trial	Sequence	Response
1	1, 3, 2, 4, 5	___ ___ ___ ___ ___
2	2, 1, 4, 3, 5	___ ___ ___ ___ ___
3	4, 3, 5, 1, 2	___ ___ ___ ___ ___
4	6, 9, 7, 8, 10	___ ___ ___ ___ ___
5	8, 9, 10, 6, 7	___ ___ ___ ___ ___
6	7, 9, 10, 8, 6	___ ___ ___ ___ ___
7	11,12,14,13,15	___ ___ ___ ___ ___
8	13,12,11,14,15	___ ___ ___ ___ ___
9	15,12,14,13,11	___ ___ ___ ___ ___
10	16,19,18,17,20	___ ___ ___ ___ ___
11	19,18,17,16,20	___ ___ ___ ___ ___
12	16,18,17,20,19	___ ___ ___ ___ ___
Now I am mixing all the numbers together and you put them back in order from smallest to largest.		
13	Mix 1 to 20	___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___
Total Correct _______ / ________		

Note: Every third trial was skipped if the child responded previous two trials correctly.

APPENDIX E
Number + EF + RL Training Script
Executive function and relational language prompts are in bold.

Session 1
We are going to do a card activity today. There are different numbers on these cards. Let's put them in order *(count while putting down 1 to 20)*.
Now we'll turn them over (*says the numbers when flipping them*) and I'll show you a trick. I will tell you what is on any card without even seeing it. Let's try it!

PRACTICE TRIALS (3 TIMES)
Please point to a (or another) card. (*secretly count from 1*) This is… N.
Now, please flip over the card and see if I am right?
I figured it out by counting. See *(count cards)*. This card also has N shapes. A way to check if I'm right is by counting the shapes (*count*). **N is in between N - 1 and N + 1. It's right after N - 1 and right before N + 1** *(points cards)***.**
Now please flip the card back over.

TEST TRIALS AND FEEDBACK
Now it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card.
What do you think this card is? **How do you know?**

> If correct: You are right! It is N *(reveal card)*. It also has N shapes on it. **N is in between N - 1 and N + 1. It's right after N - 1 and right before N + 1**
> If IDK or incorrect: It is N *(reveal cards)*. Let's count together! *(count from 1)*. It also has N shapes on it. **N is in between N - 1 and N + 1. It's right after N - 1 and right before N + 1**

What is another way to figure it out? (on trial 3, 6, 9, 12, 15, 18)

> If correct: Great Job!
> If IDK or incorrect: **We can figure it out by counting the cards backwards *(count backwards)*.**

Trial	Target	Response	Trial	Target	Response
1	2		11	16	
2	5		12	19	
3	3		13	20	
4	6		14	17	
5	8		15	1	
6	9		16	14	
7	13		17	4	
8	15		18	7	
9	12		19	10	
10	11		20	18	
Total Correct _______ / _________					

<u>Session 2</u>
Remember the card activity that we did last time? Today we are doing it a little differently.
Let's put them in order *(count while putting down 1 to 20)*.
Now we'll turn them over (*says the numbers when flipping them*)
Let me show you how we do the activity today.

PRACTICE TRIALS (3 TIMES)
Please point to a (or another) card. (*secretly count from 1 or closest N*) This is… N.
Now, please flip over the card and see if I am right?
I figured it out by counting. See *(count from 1 or closest N)*. This card also has N shapes.
A way to check if I'm right is by counting the shapes (*count*). **N is in between N – 1 and N + 1. It's one more than N – 1 and one less than N + 1** *(point)*.
Last time we flipped the card back over, but this time, we'll keep it face up.

TEST TRIALS AND FEEDBACK
Now, let's flip over the cards and it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card. Remember, we'll keep the cards face up after they are asked.
What do you think this card is? **How do you know?**

> <u>If correct</u>: You are right! It is N *(reveal card)*. It also has N shapes on it. **N is in between N – 1 and N + 1. It's one more than N – 1 and one less than N + 1** *(point)*.
> <u>If IDK or incorrect</u>: It is N *(reveal cards)*. Let's count together**!** *(count from closest N)*. It also has N shapes on it (*count shapes*). **N is in between N – 1 and N + 1. It's one more than N – 1 and one less than N + 1** *(point)*.

What is another way to figure it out? (on trial 3, 6, 9, 12, 15, 18)

> <u>If correct</u>: Great Job!
> <u>If IDK or incorrect</u>: **We can figure it out by counting the cards backwards (*count backwards*).**

Trial	Target	Response	Trial	Target	Response
1	3		11	20	
2	4		12	16	
3	2		13	19	
4	7		14	18	
5	6		15	1	
6	9		16	15	
7	13		17	5	
8	12		18	10	
9	14		19	8	
10	15		20	11	
Total Correct _______ / _________					

Session 3

Today we are doing the card activity differently. Let me show you how we do the activity today.

TEST TRIALS AND FEEDBACK

First, let's put them in order on this black space *(count while putting down 1 to 5)*.
Close your eyes *(take one card)*. Okay, now open your eyes.
What number is missing? **How do you know?**

If correct: You are right! It is N *(reveal card)*. It has N shapes on this card.

If IDK or incorrect: Let's count together! *(count shapes)* It is N and it has N shapes.

Where do you think it goes? **How do you know?**

If correct: That's right! N goes right here. **N is in between N – 1 and N + 1. It is right after N – 1 and right before N + 1** *(point)*. **N is one more than N – 1 but one less than N + 1** *(point)*.

If IDK or incorrect: Let's count together! *(count cards)…* N goes right here. **N is in between N – 1 and N + 1. It is right after N – 1 and right before N + 1** *(point)*. **N is one more than N – 1 but one less than N + 1** *(point)*.

What is another way to figure it out? (on trial 2, 4, 6, 8, 10)

If correct: Great Job!

If IDK or incorrect: **We can figure it out by counting the cards backwards (***count backwards***).**

Now we are done with 1 to 5, let's put them here (1-5 board) and try the same thing with 6 to 10 (11 to 15, 16 to 20).

Trial	Target	Response	Order
1	3		___ 1 __ 2 __ 4 __ 5 ___
2	2, 4		___ 1 ___ 3 ___ 5 ____
3	9		___ 6 __ 7 __ 8 __ 10 ___
4	6, 8		___ 7 ___ 9 ___ 10 ____
5	12		__ 11 __ 13 __ 14 __ 15 __
6	15		__ 11 __ 12 __ 13 __ 14 ___
7	11, 14		__ 12 __ 13 __ 15 ___
8	18		__ 16 __ 17 ___ 19 __ 20 ___
9	16		__ 17 __ 18 __ 19 __ 20 ___
10	17, 20		___ 16 __ 18 __ 19 ___
Now we are putting all the numbers together. I am going to hide 4 cards this time.			
11	5, 10, 13, 19		___ 1 ___ 2 ___ 3 ___ 4 ___6 ___ 7 ___ 8 ___ 9 ___ ___ 11 __ 12 __ 14 __ 15 ___16 __ 17 ___ 18 __ 20 ___
12	4, 6, 14, 17		___ 1 ___ 2 ___ 3 ___ 5 ___ 7 ___ 8 ___ 9 ___ 10 ___ ___ 11 __ 12 __ 13 __ 15 ___ 16 __ 18 ___ 19 __ 20 ___
ID Correct _______ / _________ Order Correct _______ / _________			

Session 4

Today we are doing the card activity differently. I'm going to mix the cards and it's your job to put them back in order. Let's try it!

TEST TRIALS AND FEEDBACK

When child is done… **How do you know?**

If correct: 1… 2… 3… 4… 5… That's right! Great job. **2 is one more than 1 so it comes after 1. 3 is one more than 2 so it comes after 2. 4 is one more than 3 so it comes after 3. 5 is one more than 4 so it comes after 4.**

If incorrect (example if child respond 1,2,4,5,3):

This is 1 and 1 goes here… *(point to 1)* This is 2 and 2 goes here… (*point to 2*) This is not 3… 1, 2, 3, 4 (*count dots*) this is 4. Which of these cards is 3 (*point to 5 and 3*)? **How do you know?**

If correctly identified 3: That's right! This is 3 and 3 goes here.

If incorrect: **1… 2… 3… 4… 5…** (*count dots*) **This is not 3, it's 5.** This is 3 (*point to 3*). **3 is one more than 2 but one less than 4** *(point)* so 3 goes right here **between 2 and 4**.

What is another way to figure it out? (on trial 2, 4, 6, 8, 10)

If correct: Great Job!

If IDK or incorrect: **We can figure it out by counting the cards backwards** ***(count backwards)*.**

Now we are done with 1 to 5, let's put them here (1-5 board) and try the same thing with 6 to 10 (11 to 15, 16 to 20).

Trial	Sequence	Response
1	1, 3, 2, 4, 5	___ ___ ___ ___ ___
2	2, 1, 4, 3, 5	___ ___ ___ ___ ___
3	4, 3, 5, 1, 2	___ ___ ___ ___ ___
4	6, 9, 7, 8, 10	___ ___ ___ ___ ___
5	8, 9, 10, 6, 7	___ ___ ___ ___ ___
6	7, 9, 10, 8, 6	___ ___ ___ ___ ___
7	11,12,14,13,15	___ ___ ___ ___ ___
8	13,12,11,14,15	___ ___ ___ ___ ___
9	15,12,14,13,11	___ ___ ___ ___ ___
10	16,19,18,17,20	___ ___ ___ ___ ___
11	19,18,17,16,20	___ ___ ___ ___ ___
12	16,18,17,20,19	___ ___ ___ ___ ___
Now I am mixing all the numbers together and you put them back in order from smallest to largest.		
13	Mix 1 to 20	___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___
Total Correct _______ / ________		

Note: Every third trial was skipped if the child responded previous two trials correctly.

APPENDIX F
Alphabet Training Script

Session 1
We are going to do a card activity today. There are different letters on these cards. Let's put them in order *(name while putting down A to T)*.

Now we'll turn them over (*says the letters when flipping them*) and I'll show you a trick. I will tell you what is on any card without even seeing it.
Let's try it!

PRACTICE TRIALS (3 TIMES)
Please point to a (or another) card.
(*secretly count from A)* This is… X.
Now, please flip over the card and see if I am right?
I figured it out by saying the letters in order. See *(name cards)*.
Now please flip the card back over.

TEST TRIALS AND FEEDBACK
Now it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card.
What do you think this card is?

If child was correct: You are right! It is X *(reveal card)*.

If IDK or incorrect: It is X. Let's say them together! *(name from A)*.

Trial	Target	Response	Trial	Target	Response
1	B		11	P	
2	E		12	S	
3	C		13	T	
4	F		14	Q	
5	H		15	A	
6	I		16	N	
7	M		17	D	
8	O		18	G	
9	L		19	J	
10	K		20	R	
Total Correct ________ / __________					

Session 2

Remember the card activity that we did last time? Today we are doing it a little differently.
Let's put them in order *(count while putting down A to T)*.

Now we'll turn them over (*says the letters when flipping them*)
Let me show you how we do the activity today.

PRACTICE TRIALS (3 TIMES)

Please point to a (or another) card.
(*secretly count from A or closest X*) This is… X.
Now, please flip over the card and see if I am right?
I figured it out by saying the letters in order. See *(name cards)*.
Last time we flipped the card back over, but this time, we'll keep it face up.

TEST TRIALS AND FEEDBACK

Now, let's flip over the cards and it's your turn to figure out what are on these cards. I'll point to a card and you tell me what you think is on the card. Remember, we'll keep the cards face up after they are asked.
What do you think this card is?

If correct: You are right! It is X *(reveal card)*.

If IDK or incorrect: It is X *(reveal card)*. Let's say them together! *(name from closest to X)*.

Trial	Target	Response	Trial	Target	Response
1	C		11	T	
2	D		12	P	
3	B		13	S	
4	G		14	R	
5	F		15	A	
6	I		16	Q	
7	M		17	E	
8	L		18	J	
9	N		19	H	
10	O		20	K	
Total Correct ________ / __________					

Session 3

Today we are doing the card activity differently. Let me show you how we do the activity today.

TEST TRIALS AND FEEDBACK

First, let's put them in order on this black space *(count while putting down A to E)*. Close your eyes *(take one card from each row)*. Okay, now open your eyes. What letter is missing?

If correct: You are right! It is X *(reveal card)*.

If IDK or incorrect: It is X *(reveal card)*.

Where do you think it goes?

If correct: That's right! X goes right here.

If IDK or incorrect: Let's say them together! *(name cards)*… X goes right here.

Now we are done with A to E, let's put them here (A-E board) and try the same thing with F to J (K to O, P to T).

Trial	Target	Response	Order
1	C		___ A __ B __ D __ E ___
2	B,D		___ A ___ C ___ E ____
3	I		___ F __ G __ H __ J ___
4	F,H		___ G ___ I ___ J ____
5	L		__ K __ M __ N __ O __
6	O		__ K __ L __ M __ N ___
7	K,N		__ L __ M __ O ___
8	R		__ P __ Q ___ S __ T ___
9	P		__ Q __ R __ S __ T ___
10	Q,T		___ P __ R __ S ___
Now we are putting all the numbers together. I am going to hide 4 cards this time.			
11	E, J, M, S		__ A ___ B ___ C ___ D ___F ___ G ___ H ___ I ___ __ K ___ L ____ N ___ O ___ P ___ Q __ R __ T ___
12	D, F, N, Q		__ A ___ B ___ C ___ E ___ G ___ H ___ I ___ J ___ __ K ___ L ___ M ___ O ____ P ___ R ____ S __ T ___
ID Correct _______ / _________ Order Correct _______ / _________			

Session 4

Today we are doing the card activity differently. I'm going to mix the cards and it's your job to put them back in order. Let's try it!

TEST TRIALS AND FEEDBACK

When child is done…

If correct: A… B… C… D… E… That's right! Great job.

If IDK or incorrect (example if child respond A, B, D, E, C):

This is A and A goes here… *(point to A)* This is B and B goes here… (*point to B*) This is not C, this is D. Which of these cards is C (*point to E and C*)?

If correctly identified E: That's right! This is C and C goes here.

If IDK or incorrect: This is not C, it's E. This is C (*point to C*) and C goes here.

Now we are done with A to E, let's put them here (A-E board) and try the same thing with F to J (K to O, P to T).

Trial	Sequence	Response
1	A, C, B, D, E	___ ___ ___ ___ ___
2	B, A, D, C, E	___ ___ ___ ___ ___
3	D, C, E, A, B	___ ___ ___ ___ ___
4	F, I, G, H, J	___ ___ ___ ___ ___
5	H, I, J, F, G	___ ___ ___ ___ ___
6	G, I, J, H, F	___ ___ ___ ___ ___
7	K, L, N, M, O	___ ___ ___ ___ ___
8	M, L, K, N, O	___ ___ ___ ___ ___
9	O, L, N, M, K	___ ___ ___ ___ ___
10	P, S, R, Q, T	___ ___ ___ ___ ___
11	S, R, Q, P, T	___ ___ ___ ___ ___
12	P, R, Q, T, S	___ ___ ___ ___ ___
Now I am mixing all the numbers together and you put them back in order from smallest to largest.		
13	Mix A to T	___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___
Total Correct ________ / _________		

Note: Every third trial was skipped if the child responded previous two trials correctly.