Essays on Macroeconomics and Fiscal Policy

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Job Boerma

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Larry Jones, Ellen McGrattan

October, 2020

Acknowledgements

I am deeply grateful for the encouragement, help and continuous support of my thesis committee: Jonathan Heathcote, Larry Jones, Loukas Karabarbounis and Ellen McGrattan. I also thank Anmol Bhandari, V.V. Chari and Aleh Tsyvinski for many helpful suggestions and discussions. Finally, I am grateful to Carter Braxton and Guillaume Sublet for their friendship.

Dedication

To my parents, Harm and José, and my sister, Marleen

Contents

Chapter 1

Housing Policy Reform

Job Boerma

Introduction

Most high-income economies have adopted a large number of housing policies. Governments levy property taxes, subsidize rents, exempt imputed rents from asset income taxes, allow for the deduction of home mortgage interest payments for income taxation, and levy taxes on housing transactions. Housing policy affects the cost of housing services and the transaction costs on housing capital. It is naturally accompanied by a debate about the aggregate and distributional implications of these policies and of proposed reforms, such as a reduction in the home mortgage interest deduction.

The goal of this paper is to study the reform of housing policy in the Netherlands. I show that under current policy housing consumption is effectively subsidized for most households, while in any Pareto efficient allocation the housing consumption of every household is implicitly taxed when housing and non-market time are complements in home production. Further, while households currently pay a transaction tax when they buy a house, they face an implicit transaction tax or receive an implicit transaction subsidy in any efficient allocation.[1] I find an average effective subsidy of 8 percent on housing consumption for

[1] I use "efficiency" to refer to Pareto efficiency and the term "transaction tax" to refer to taxes paid when buying or selling a house. In the Netherlands, residential transactions are taxed at 6 percent of the property

homeowners, while the efficient average tax rate is 14 percent.

I develop my findings in an overlapping generations economy with incomplete markets which incorporates housing services that are produced by illiquid housing capital. During their members' working years, households face idiosyncratic wage risk in the labor market against which they can self-insure by adjusting their savings and their labor supply. Housing services differ from non-housing consumption because housing services are used together with non-market time in home production activities, such as cooking and gardening. Housing capital differs from other forms of savings because housing capital is illiquid owing to transaction costs. The complementarity in home production between housing and non-market time, and the presence of transaction costs, provide two distinct motives to tax housing, that is, to deviate from uniform commodity taxation (Atkinson and Stiglitz, 1976; Golosov, Kocherlakota, and Tsyvinski, 2003).

I quantify my findings using administrative records for households in the Netherlands. The government allows for the deduction of home mortgage interest payments for income taxation, subsidizes rents, levies property taxes, exempts housing from asset income taxes, and levies a transaction tax on home buyers. Given this prominent role of houses and mortgages in tax policy, the fiscal authority assesses the property value and the outstanding mortgage balance of both rental and owner-occupied housing every year. I use this data, combined with individual income records and employer-provided records on hours worked, to measure the implied subsidy to housing consumption and to calibrate the complementarity between non-market time and leisure in home production.

I argue for housing policy reform — moving from subsidizing to taxing housing services, and introducing state-dependent transaction subsidies — by comparing efficient outcomes with those under current policy. I develop the argument in four steps. First, I study the economy using a dynamic Mirrlees theory to isolate the margins for taxation in any efficient equilibrium with private information on labor productivity. Theory shows the use of implicit transaction taxes and subsidies and that efficient housing consumption taxes depend critically on the complementarity between housing services and non-market time.

value. In the United Kingdom, the marginal tax rate on transactions ranges between 0 and 12 percent. In the United States, the transaction tax is levied at the state and local level, with state-level rates as high as 2 percent (for example, in Delaware).

Second, to measure the effective housing consumption subsidy implied by current policy, I analyze my economy, together with administrative records, from a positive angle. Third, I use this quantitative positive model to estimate the elasticity of substitution between housing services and non-market time in home production. Fourth, I use the estimated model parameters to quantify efficient policy. Step 1 shows the role of implicit transaction subsidies; step 2 shows that housing consumption is currently subsidized; and steps 1, 3 and 4 together show that in efficient allocations housing consumption is taxed.

First, I analyze a planning problem to characterize efficient allocations. I show that when housing services and non-market time are complements in home production every household in any efficient allocation is implicitly taxed on its housing consumption. By taxing housing services, additional leisure is spent in a less desirable house, which provides additional incentives to work to productive households. When housing and non-market time are instead substitutes, the consumption of housing services is subsidized to encourage households to work.

I also use the dynamic Mirrlees theory to show the use of transaction taxes in efficient allocations. In the absence of any distortions, homeowners reside in a smaller residence than called for by an efficient plan because of private concerns over future transaction costs. An efficient allocation therefore calls for transaction taxes that reflect implicit subsidization of housing consumption. Specifically, efficient policy implicitly subsidizes transactions when households sell their houses after a negative shock and implicitly taxes when households sell their houses after a positive shock.

Theory thus shows that the consumption of housing is implicitly taxed when housing services and leisure are complements and that households may face a transaction tax or subsidy in any efficient allocation. To understand the scope for housing policy reform, I next measure that housing consumption is subsidized under current policy, while I estimate that housing services and non-market time are complements in home production.

Second, I measure the user's cost of housing capital across households to show that housing consumption is subsidized under current policy. By comparing the user's cost of housing capital under current policy with the user's cost without distortionary taxation, I obtain the effective subsidy to housing services under current policy for the distribution of

Dutch homeowners. I find an average effective subsidy of 7.5 percent to homeowners; the subsidy varies from 3.6 percent for old, low-income households to 14.1 percent for young, high-income households. This effective subsidy is driven by the home mortgage interest deduction and by the exclusion of housing capital from asset income taxation. The home mortgage interest deduction gives an average subsidy of 8.9 percent and especially benefits young households. The exemption of housing capital from asset income taxation, on the other hand, subsidizes retirees at 5.6 percent and does not benefit young homeowners.

Third, I use the positive model that incorporates current tax policy to discipline key model parameters. Importantly, by a "gap"-based indirect inference (Berger and Vavra, 2015), I find that housing services and leisure are complements in home production. The inferred complementarity between housing and non-market time in home production is based on the estimation of a standard intra-period optimality condition for a home production model without distortionary taxation and transaction costs, which requires households to produce more at home using their low cost input. I use the covariation between wages and home production inputs in the cross section of households to infer an elasticity of substitution between housing and non-market time of 0.95.

Fourth, using the calibrated parameters, I conclude that housing consumption is taxed in any efficient allocation by numerically solving for an efficient allocation. Given the calibrated parameters, holding the value added consumption tax constant at 13.4 percent, the average efficient housing consumption tax rate is 13.7 percent. The efficient housing consumption tax rate grows from 13.5 percent at age 25 to 13.8 percent at retirement. A simple reform that increases the property tax to mimic the efficient housing consumption tax generates a steady-state welfare improvement equivalent to 0.8 percent of steady-state non-housing consumption.

Related literature. The implications of tax policy for housing market outcomes have long been studied. Early work measures the user's cost of housing capital to quantify the implications of effective housing subsidies for housing market outcomes in the United States (Laidler, 1969; Aaron, 1970; Poterba, 1984, 1992). More recent work uses dynamic incomplete market models with heterogeneous households to study the effects of housing policy on housing market outcomes (Gervais, 2002; Chambers, Garriga, and Schlagenhauf, 2009; Floetotto, Kirker, and Stroebel, 2016; Sommer and Sullivan, 2018). These papers generally evaluate policy reforms that reduce the effective housing consumption subsidy for homeowners. They typically find that such reforms increase utilitarian welfare but are not Pareto improving.

I build on this literature by incorporating home production and by characterizing efficient policy reform. The complementarity in home production between housing services and non-market time provides a motive to distort housing consumption in any efficient allocation. Further, I show that households face implicit transaction taxes when they sell their house in efficient allocations.

I introduce non-separable preferences between housing services and labor supply by adopting a Beckerian view of home production (Becker, 1965; Ghez and Becker, 1975). In doing so, this paper relates to an extensive literature that studies how home production affects labor supply over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013), labor supply over the life cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007a; Dotsey, Li, and Yang, 2014), and welfare differences across households (Boerma and Karabarbounis, 2019a,c). My contribution is to characterize and quantify efficient policy for an environment with home production and incomplete asset markets and to estimate the elasticity of substitution between housing and non-market time in home production.

By studying efficient housing policy reform for an overlapping generations economy, my paper relates to an extensive literature on efficiency in public finance. Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2016) use a dynamic mechanism design approach to study efficient labor income and capital income taxation over the life cycle in a partial equilibrium framework with skill shocks. Stantcheva (2017) and Ndiaye (2018) extend their work to endogenize human capital accumulation and retirement, respectively. Hosseini and Shourideh (2019) study the efficient reform of labor income and asset taxes for an overlapping generations economy in which skills are deterministic. I introduce housing services consumption and illiquid housing capital to study efficient housing policy reform for an overlapping generations economy with life-cycle skill risk. I incorporate equilibrium responses in house prices by building on Negishi (1960) and Atkeson and Lucas (1992, 1995).

This paper adopts a Mirrlees approach to study policy reform, so the main determinant for distortions is a government's desire to insure households against skill shocks that are not directly observed. Without skill heterogeneity, an efficient allocation is attainable without distorting households' marginal choices. In related work, Olovsson (2015) studies optimal taxation for a representative household using a Ramsey framework with home production. When taxes are necessarily distortionary and given strong substitutability between market and home services, he finds an optimal tax rate for market services that is well below the tax rate on market consumption. For a heterogeneous household economy with incomplete asset markets, I find that an efficient tax rate for housing services is similar to the tax rate on non-housing consumption because I find the substitution elasticity between housing services and non-market time to be close to one.

In this paper, variable transaction costs and complementarity in home production present two motives to deviate from uniform commodity taxation in the presence of housing. In related work, Koehne (2018) shows that uniform commodity taxation is not efficient in presence of durable purchases with only fixed adjustment costs. By contrast, housing services are nondurable in my paper, and variable transaction costs determine the efficient transaction tax.

The remainder of this paper is organized as follows. In Section 1.1, I lay out the primitive environment. In Section 1.2, to characterize efficient allocations for this environment, I formulate a planning program. I present a characterization of efficient housing taxes in Section 1.3. In Section 1.4, after studying the forces that determine efficient housing taxes, I turn to study a positive economy by introducing current policy into the primitive environment. The positive economy is calibrated to the Netherlands in Section 1.5 and is used to quantitatively study policy reform in Section 1.6. Section 3.7 concludes.

1.1 Environment

Demographics. I study an infinite horizon economy populated by overlapping cohorts that live T years. Time is discrete and denoted by $j = 1, 2, \ldots$. A cohort is a continuum of households of mass one. Households work the first T_w years and are retired for the remaining years. Household age is denoted by t.

Preferences. Households derive utility from market consumption c, housing services d, and leisure ℓ, and maximize expected utility

$$\mathbb{E}\left(\sum_{t=1}^{T}\beta^{t-1}u\big(c_t, d_t, \ell_t\big)\right),$$

where β is the time discount factor.[2] The period utility function is separable in the utility from consumption $v(c)$, where v is increasing and strictly concave, and the flow utility from home production $h(d,\ell)$,

$$u(c,d,\ell) = v(c) + h\big(d,\ell\big). \tag{1.1}$$

Households have a unit of time each period to spent on work and leisure. When households are retired $\ell = 1$.

To fix ideas, I assume the home production technology is given by a CES aggregator over housing services and non-market time.[3] The home technology is parameterized by the elasticity of substitution, which I denote by σ, and by weight ω. Household preferences over the non-market good are captured by $\mathcal{H}$, that is,

$$h\big(d,\ell\big) = \mathcal{H}\left(\left(\omega d^{\frac{\sigma-1}{\sigma}} + (1-\omega)\ell^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right), \tag{1.2}$$

where $\sigma > 0$ and $\omega \in [0,1]$ are constant across households.

My specification of preferences and the home production technology are a special case

[2] My model features a single decision maker within each household. Hours worked across spouses are perfect substitutes and in the quantitative analysis I treat ℓ as average leisure time for both spouses.

[3] I use the CES specification in my quantitative analysis. The theory applies to more general preferences $h(d,\ell)$, for example, preferences in which housing services also generate utility separate from home production.

of the Beckerian model of home production (Becker, 1965; Ghez and Becker, 1975) where households have preferences over two goods. The first good is non-housing consumption, the second good is produced using housing services and time as inputs.[4] With $\omega = 0$, I obtain canonical life-cycle preferences, for which the efficient allocation is discussed in Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2016).

Skill Heterogeneity. Households are heterogenous with respect to their market productivity. The output y that a household produces is the product of their productivity θ and the hours they work $n = 1 - \ell$, that is, $y = \theta n$. Output, consumption and housing services consumption are publicly observed. Households are born at age 0, with ability θ_0 which is distributed following distribution $\pi^0(\theta_0)$, and enter the labor market at age 1. Skills evolve according to a first-order Markov process with an age-varying distribution function $\pi^t(\theta_t|\theta_{t-1})$ over a fixed set $\Theta \equiv \{\underline{\theta}_1, \ldots, \bar{\theta}_N\}$. The history of the skill shocks is given by θ^t. The probability density function for history $\theta^t \in \Theta^t$ is given by $\pi(\theta^t) \equiv \pi^t(\theta_t|\theta_{t-1}) \ldots \pi^1(\theta_1|\theta_0)\pi^0(\theta_0)$. The skill distribution is assumed independent across households within a cohort.

Technology. The economy is endowed with technologies to produce housing services and a general good. The economy is a small open economy with a domestic housing market, meaning that housing services have to be produced domestically. Non-housing goods and services can be traded across countries.

Housing Services. Households obtain housing services by living in a house. Houses differ in the flow services they provide, which are proportional to the house's capital value.[5] One unit of housing capital provides χ units of housing services. The total housing stock can be divided every period into individual houses without cost. The resource constraint for

[4]The home production literature adopts two classifications of time use. In my baseline model, time is allocated to work and non-market time following Becker (1965), Greenwood and Hercowitz (1991), and Boerma and Karabarbounis (2019c). The second approach differentiates non-market time from leisure (Gronau, 1977; Benhabib, Rogerson, and Wright, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Parente, Rogerson, and Wright, 2000; Karabarbounis, 2014; Boerma and Karabarbounis, 2019a). In Appendix 1.A, I show how the theory extends to the second class of home production models.

[5]In the data analysis I assume that the service flow of housing capital is proportional to the real market value of the housing unit. Housing services thus include the amenity values of the property.

housing services is thus:

$$D_j \le \chi H_j \ , \tag{1.3}$$

where D denotes aggregate housing services and H denotes housing capital.

When a household moves from consuming housing services d_{j-1} to consuming housing services d_j there is a technological transaction cost $\Phi(d_j, d_{j-1})$. This transaction cost is assumed continuously differentiable in both the current and the prior housing choice, with $\lim_{d_j \to d} \Phi_1(d_j, d) = 0$ and $\Phi_2(d_j, d) \ge 0$. This specification can capture both small variable home improvements costs as well as large moving costs such as rental costs of moving trucks and a brokerage technology.[6] Fixed technological transaction costs are thus approximated by a continuously differentiable function. The aggregate transaction costs for housing services are, with some abuse of notation, denoted by $\Phi_j \equiv \sum_{t,\theta^t} \Phi\big(d_j(\theta^t), d_{j-1}(\theta^{t-1})\big)$.

Construction. Time is required to build new houses, in the spirit of Kydland and Prescott (1982). Let $\iota \ge 0$ denote the time periods required to build new houses and let Q_j^H be building projects initiated in period j.[7] The law of motion for the housing capital stock is then

$$H_{j+1} = H_j + Q^H_{j+1-\iota} \ . \tag{1.4}$$

I do not restrict building projects to be nonnegative. For example, (1.4) allows housing capital to be converted into offices. Without loss, I assume that resources are only allocated to housing projects in their final stage.[8] In sum, building projects initiated in period j realize

[6] I implicitly specify the transaction cost as a function of a constant long-run equilibrium house price which is determined by the construction technology. Fluctuations in house prices do not enter the transaction costs function to ensure that endogenous house prices do not enter the aggregate resource constraints. In my model, households do not incur a time cost when adjusting their house, in line with the negligible and statistically insignificant time cost of durable adjustment in Berger and Vavra (2015).

[7] I model time to build in the housing sector to allow for different construction times for housing capital and business capital. The decision to incorporate time to build on housing rather than on business capital is suggested by the data. In the Netherlands, the production of housing units takes 23 months on average after a building permit is issued (see Section 1.5), while it takes only 175 days to build a warehouse valued at 50 times income per capita (World Bank's report on Doing Business).

[8] In Kydland and Prescott (1982) resources are allocated to time to build investment projects in all periods between the initial and the final stage, while in this paper resources are allocated only in the final stage. In my environment, these two formulations are isomorphic when the present value resource cost of new

in period $j + \iota$ and are financed in period $j + \iota - 1$. Time to build in construction implies that the housing supply is infinitely inelastic in the short run, and infinitely elastic in the long run. Alternatively, the housing supply only responds to government policy reform in the long run.

The housing stock depreciates at rate δ^H. Depreciation is exactly offset by required maintenance expenses in terms of the general good, such that housing capital does not deteriorate. Housing investment is the sum of resources allocated to building projects and required maintenance, $I_j^H = Q_{j+1-\iota}^H + \delta^H H_j$.

General Good. The technology for general goods $F(K, Y)$ is homogeneous of degree one in aggregate effective labor Y and business capital K and satisfies the Inada conditions. The general good can be consumed, invested in business capital and housing capital, used to pay government expenditures and transaction costs for housing services Φ_j, or exported to the rest of the world. A small open economy with positive net exports increases its net claims on foreign assets B with return R dictated by the world interest rate. The resource constraint for general goods is thus

$$C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} \leq F(K_j, Y_j) + RB_j, \tag{1.5}$$

where the business capital stock evolves according to $I_j^K = K_{j+1} - (1 - \delta^K)K_j$, where δ^K is the depreciation rate on business capital.

1.2 Efficiency

I study the efficiency properties of the overlapping generations economy. I define efficient allocations, which are necessarily incentive feasible and resource feasible, and I show how to characterize efficient allocations using a planner problem.

Identity and Resource Feasible. A household's identity is its birth year j and its productivity history θ^{t-1}. I use $i \equiv (j, \theta^{t-1})$ to denote a household.[9] The set of households

constructions is identical.

[9] Every household has only one identity. For every household born in future periods, the productivity

$\mathcal{I}$ is partitioned into households that are alive in the first period and households born in future periods, $\mathcal{I} \equiv \left\{ \{(0, \theta^{t-1})\}_{t=1}^{T}, \{(j, \theta_0)\}_{j=1}^{\infty} \right\}$.

An allocation for household i is a sequence of functions that specify non-housing consumption, housing services consumption and labor supply at age $t+v$ given the household's productivity history θ^{t+v}, $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t} = \left\{ \left(c_{j+v}(\theta^{t+v}), d_{j+v}(\theta^{t+v}), y_{j+v}(\theta^{t+v})\right) \right\}_{v=0}^{T-t}$. An allocation x specifies an allocation for every household i as well as aggregate quantities:

$$x \equiv \left\{ \{x(i)\}_{\mathcal{I}}, \left\{ \left(C_j, D_j, Y_j, B_{j+1}, H_{j+1}, K_{j+1}, I_j^H, I_j^K\right) \right\}_{j=1}^{\infty} \right\}.$$

An allocation is resource feasible if and only if the allocation satisfies the resource constraints (1.3)$-$(1.5) in all periods.

Incentive Feasible. Households know their own history θ^t up to age t, and the only possible source of information about this history are reports provided by the household itself.[10] By the revelation principle I can restrict the reporting space to be the type space without loss of generality. I use $\sigma_t(\theta^t)$ to denote the report that the household plans at date 1 to give about their date t shock when they experience θ^t. A reporting strategy, which specifies a report for every history, is denoted $\sigma \equiv \{\sigma_t(\theta^t)\}_{\Theta^t,t}$. A reporting strategy generates a corresponding report history $\sigma^t(\theta^t) = (\sigma_1(\theta^1), \ldots, \sigma_t(\theta^t))$. Denote by Σ the set of reporting strategies. The truthful reporting strategy is such that $\sigma^t(\theta^t) = (\theta_1, \ldots, \theta_t)$ for all t and all $\theta^t \in \Theta^t$.

Given a reporting strategy σ, the corresponding household allocation is given by $x^\sigma \equiv \{x_t(\sigma^t(\theta^t))\}_{\Theta^t,t} = \left\{ c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t)) \right\}_{\Theta^t,t}$. Given a reporting strategy σ and an allocation, the expected lifetime utility is

$$\mathcal{V}(x^\sigma) \equiv \sum_{t=1}^{T} \sum_{\theta^t} \beta^{t-1} \pi(\theta^t) u\left(c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t))/\theta_t\right). \tag{1.6}$$

history is a singleton, θ_0, and the household identity is hence (j, θ_0). In the initial period, households of all ages and productivity histories are alive, which is captured by identities $\{(t-1, \theta^{t-1})\}_{t=1}^{T}$.

[10] To simplify the exposition I describe incentive compatibility for a household born in the future and I suppress the identity. The corresponding definitions for households that are alive in the initial period naturally follow.

The continuation value after history θ^t, which is denoted by $V^\sigma(\theta^t)$, is given by:

$$V^\sigma(\theta^t) = u\left(c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t))/\theta_t\right) + \beta \sum_{\theta_{t+1}} \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V^\sigma\left(\theta^{t+1}\right),$$

for all $t = 1, \ldots, T$, with $V^\sigma(\theta^{T+1}) = 0$. The continuation value after history θ^t under a truthful reporting strategy thus solves:

$$V(\theta^t) = u\left(c_t(\theta^t), d_t(\theta^t), y_t(\theta^t)/\theta_t\right) + \beta \sum_{\theta_{t+1}} \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V\left(\theta^{t+1}\right), \tag{1.7}$$

for all $t = 1, \ldots, T$, with $V(\theta^{T+1}) = 0$.

An allocation is incentive feasible if and only if the truthful reporting strategy is an equilibrium reporting strategy given any history for every household i. An allocation is incentive compatible if and only if for all histories θ^t

$$V(\theta^t) \geq V^\sigma(\theta^t), \tag{1.8}$$

for all $\sigma \in \Sigma$. The set of incentive compatible allocations for household i is denoted $X_{IC}(i)$. An allocation x is thus incentive feasible if and only if the allocation for household i is incentive compatible for all households $i \in \mathcal{I}$. An allocation is feasible if and only if it is resource feasible and incentive feasible.

Efficiency. An allocation is efficient if and only if there exists no alternative feasible allocation that makes all households weakly better off and some households strictly better off. That is, there exists no alternative feasible allocation $\hat{x}$ such that:

$$\begin{aligned}
&\mathcal{V}_j\left(\hat{x}(i);\theta^{t-1}\right) \geq \mathcal{V}_j\left(x(i);\theta^{t-1}\right) \qquad \forall\, i \in \mathcal{I} \\
&\mathcal{V}_j\left(\hat{x}(i);\theta^{t-1}\right) > \mathcal{V}_j\left(x(i);\theta^{t-1}\right) \qquad \text{for some } i \in \mathcal{I},
\end{aligned}$$

I next formulate a planning program and show that this planning problem characterizes efficient allocations.[11]

[11]This definition requires that every household is strictly better off from an ex-ante perspective. It does not require that every household is strictly better off for any realization of future shocks.

Planning Problem. Given values $\{\mathcal{V}(i)\}_{\mathcal{I}}$, a capital endowment (B_1, H_1, K_1), housing allocations in period zero, a pipeline of building projects $\left\{Q^H_{1-\upsilon}\right\}_{\upsilon=1}^{\iota}$, and a government expenditures sequence $\{G_t\}$, the planning problem is to choose a feasible allocation that maximizes excess resources in the first period so that household values exceed $\mathcal{V}(i)$ for all $i \in \mathcal{I}$. Formally, the planning problem is:

$$\max_{x} \; F(K_1, Y_1) + RB_1 - C_1 - I_1^K - I_1^H - G_1 - \Phi_1 - B_2$$

subject to the housing services constraint in every period (1.3), the law of motion for housing capital (1.4), the resource constraints for general goods for period $j > 1$ (1.5), the incentive constraints for all households (1.8), and the promise keeping constraints for all households:

$$\mathcal{V}(i) \leq \mathcal{V}_j\big(x(i); \theta^{t-1}\big). \tag{1.9}$$

Proposition 1. Allocation x is efficient if and only if it solves the planner problem given $\mathcal{V}_j\big(x(i); \theta^{t-1}\big)$ for all $i \in \mathcal{I}$ with a maximum of zero.

The formal proof is in Appendix 1.B. If the allocation does not solve the planner problem there is an alternative allocation with excess resources that can be used to make a household strictly better off. If the allocation is not efficient, there exists a strict Pareto improvement with excess resources.

Proposition 1 provides a useful characterization of efficient allocations by combining ideas from Atkeson and Lucas (1992, 1995) and Negishi (1960). Atkeson and Lucas (1992, 1995) use prices to decentralize the problem of finding efficient allocations into component planner problems. They prove that an allocation is efficient if the allocation, finite prices, and a distribution of values solve the component planner problems (given prices and initial values) and satisfy the resource constraints. To connect to Negishi (1960), I refer to the allocation, prices, and the distribution of values as a component planner equilibrium. I characterize a component planner equilibrium using a planning formulation similar to Negishi (1960), who characterizes a competitive equilibrium by maximizing a linear social welfare function with appropriate weights subject to resource feasibility.

1.3 Efficient Housing Policy

I characterize efficient allocations and study its implications for housing policy.

Component Planner. I study the component planner problem to characterize the solution to the planning problem for a given household.[12] Given a household $i \in \mathcal{I}$ and a value $\mathcal{V}(i)$, the component planner chooses allocation $x(i)$ to maximize excess resources for household i subject to incentive constraints. To simplify the exposition I present the component planner problem of a household born in the future and I suppress notation on its identity.

Given a sequence of multipliers $\{p_j\}_{j=1}^{\infty}$ on the aggregate constraint for housing services (1.3), I define excess resources for household i as:

$$\Pi\left(x(i)\right) \equiv \sum_{t,\theta^t} \pi(\theta^t)\Big(wy(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi\left(d(\theta^t), d(\theta^{t-1})\right)\Big)\Big/ R^{t-1}$$

The component planning problem for household i given value $\mathcal{V}(i)$ is to solve:

$$\Pi(\mathcal{V}(i)) \equiv \max_{x(i)} \; \Pi\left(x(i)\right)$$

subject to

$$\mathcal{V}(i) \leq \mathcal{V}\left(x(i)\right)$$

$$x(i) \in X_{IC}(i)$$

I solve a relaxed version of this problem by using a local downward incentive constraints approach.

Local Downward Incentive Constraints. To solve the component planner problem for household i in a tractable manner, I assume only local downward incentive constraints bind at

[12]The Lagrange function corresponding to the planning problem is separable in the allocation of any household $x(i)$. Therefore, I can separately characterize the solution to the planning problem for any household. The corresponding allocation x is efficient when excess resources in the initial period are equal to zero.

the solution. Assuming that only local downward incentive constraints bind is a finite types analog for the first-order approach typically adopted in dynamic Mirrlees problems with a continuum of productivity types (Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Stantcheva, 2017). I replace the set of incentive compatibility allocations X_{IC} by a superset of allocations satisfying local downward incentive constraints for truthful reporting $X_{LD} \supset X_{IC}$.

Consider a one-shot deviation strategy from truthful reporting for a household with history θ^t. At age t the household reports a lie $l \neq \theta_t$ for a specific realization θ_t and reports truthfully in all other instances. The one-shot deviation strategy σ^l is thus,

$$\begin{aligned} \sigma_t^l(\theta^{t-1}, \tilde{\theta}) &= \tilde{\theta} \qquad\qquad \text{if } \tilde{\theta} \neq \theta_t \\ \sigma_t^l(\theta^{t-1}, \theta_t) &= l \,. \end{aligned}$$

The continuation value given one-shot deviation strategy σ^l is given by:

$$V^{\sigma^l}(\theta^t) = u\left(x_t(\theta^{t-1}, l); \theta_t\right) + \beta \sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V^{\sigma^l}\left(\theta^{t+1}\right), \tag{1.10}$$

for all $t = 1, \ldots, T$, with $V^{\sigma^l}(\theta^{T+1}) = 0$.

Given a first-order Markov process for labor productivity, there is no difference going forward between a household adopting reporting strategy σ^l with history θ^{t+1} that triggers lie l and a truth-telling household with history $(\theta^{t-1}, l, \theta_{t+1})$. Both households have identical reporting histories so there is no informational difference to distinguish them. Therefore, they necessarily receive the same continuation value:

$$V^{\sigma^l}(\theta^{t+1}) = V(\theta^{t-1}, l, \theta_{t+1}) \,. \tag{1.11}$$

By the one-shot deviation principle, incentive compatibility is equivalently formulated as:

$$\forall \theta^t \qquad V(\theta^t) = \max_l \; V^{\sigma^l}(\theta^t) \,, \tag{1.12}$$

for all σ^l. Substituting (1.10) and (1.11) into (1.12), I obtain that $\forall\theta^t$:

$$V(\theta^t) = \max_l \; u\left(x_t(\theta^{t-1}, l); \theta_t\right) + \beta \sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V(\theta^{t-1}, l, \theta_{t+1})\,. \tag{1.13}$$

This expression gives the following local downward incentive constraint:

$$u\left(x_t(\theta^t); \theta_t\right) + \beta \sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V(\theta^{t+1}) \geq u\left(x_t(\theta^{t-1}, \theta_t^-); \theta_t\right) + \beta \sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) V(\theta^{t-1}, \theta_t^-, \theta_{t+1}), \tag{1.14}$$

where θ_t^- is the productivity level right below θ_t. The set of allocations for household i that satisfy the local downward incentive constraints (1.14) is denoted $X_{LD}(i)$.

The relaxed component planner problem is formulated by replacing the set of constraints that ensure global incentive compatibility in the component planning problem, $X_{IC}(i)$, with the set of constraints that ensure the allocation satisfies all local downward incentive constraints, $X_{LD}(i)$. I write the relaxed component planner recursively and then characterize its solution.

Recursive Problem. To write the relaxed component planner problem recursively, it is useful to introduce the state variables continuation value $\mathcal{V}(\theta^t)$ and threat value $\tilde{\mathcal{V}}(\theta^t)$,

$$\mathcal{V}(\theta^t) \equiv \sum \pi^{t+1}(\theta_{t+1}|\theta_t) V(\theta^{t+1}) \tag{1.15}$$

$$\tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi^{t+1}(\theta_{t+1}|\theta_t^+) V(\theta^{t+1})\;. \tag{1.16}$$

The continuation value is the expected future value for a truth-telling individual with history θ^t. The threat value is the expected value using the probability distribution for an individual who experienced and reported an identical history until $t-1$, and who reports θ_t while being one level more skilled θ_t^+ at age t. In other words, the threat value is the continuation value for a one-time local deviation from truthful reporting for an individual with an identical history except for being more skilled at age t. Using these state variables, I rewrite the local downward incentive constraint (1.14) as:

$$u\left(x_t(\theta^t);\theta_t\right)+\beta\mathcal{V}(\theta^t)\geq u\left(x_t(\theta^{t-1},\theta_t^-);\theta_t\right)+\beta\tilde{\mathcal{V}}(\theta^{t-1},\theta_t^-)\,. \tag{1.17}$$

Using the continuation value and the threat value, I write the component planning problem recursively,

$$\Pi_t(\mathcal{V},\tilde{\mathcal{V}},d,\theta_-)\equiv\max_{x_t(\theta)}\sum\pi^t(\theta|\theta_-)\Big(wy_t(\theta)-c_t(\theta)-p_jd_t(\theta)-\Phi\left(d_t(\theta),d\right)+\Pi_{t+1}(\mathcal{V}_t(\theta),\tilde{\mathcal{V}}_t(\theta^+),d_t(\theta),\theta)/$$

where, with some abuse of notation, the choice variable is $x_t(\theta)=\{(c_t(\theta),d_t(\theta),y_t(\theta),\mathcal{V}_t(\theta),\tilde{\mathcal{V}}_t(\theta))\}$, and where maximization is subject to (1.15)−(1.17):

$$\mathcal{V}=\sum\pi^t(\theta|\theta_-)\big(u\left(c_t(\theta),d_t(\theta),y_t(\theta)/\theta\right)+\beta\mathcal{V}_t(\theta)\big) \tag{1.15}$$

$$\tilde{\mathcal{V}}=\sum\pi^t(\theta|\theta_-^+)\big(u\left(c_t(\theta),d_t(\theta),y_t(\theta)/\theta\right)+\beta\mathcal{V}_t(\theta)\big) \tag{1.16}$$

$$u\left(c_t(\theta),d_t(\theta),y_t(\theta)/\theta\right)+\beta\mathcal{V}_t(\theta)\geq u\left(c_t(\theta^-),d_t(\theta^-),y_t(\theta^-)/\theta\right)+\beta\tilde{\mathcal{V}}_t(\theta)\;. \tag{1.17}$$

This formulation has five state variables: continuation value $\mathcal{V}$, threat value $\tilde{\mathcal{V}}$, previous housing consumption d, lagged productivity θ_-, and age t. I solve the recursive component planner problem to characterize efficient policies for housing consumption and earnings as well as savings and housing wealth.

A solution to the relaxed component planner problem is a solution to the original component planner problem only if at optimum the local downward incentive constraints (1.17) are a sufficient condition for global incentive compatibility (1.8). I verify sufficiency of the local downward incentive constraints in the quantitative analyses.[13]

Implicit Taxes. Implicit taxes are distortions of households' marginal decisions under an efficient allocation. They provide information about efficient insurance when compared to a benchmark without intervention and similarly provide information about inefficiency in current policy when compared to distortions under current tax policy. In this section I

[13]This approach is common in the dynamic Mirrlees literature (Kapička, 2013; Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Stantcheva, 2017).

discuss four implicit taxes: a labor tax, a savings tax, a housing consumption tax, and a transaction tax.

The implicit housing services tax and the implicit transaction tax are distortions to households' marginal housing consumption decision, and are described by

$$p\left(1+\tau_d^c(\theta^t)\right)+\Phi_1\left(d_t(\theta^t), d_{t-1}(\theta^{t-1})\right)+\beta\mathbb{E}_t\left(\left(\Phi_2\left(d_{t+1}(\theta^{t+1}), d_t(\theta^t)\right)+\tau_d^t(\theta^{t+1})\right)\frac{u_c\left(\theta^{t+1}\right)}{u_c\left(\theta^t\right)}\right)=\frac{u_d\left(\theta^t\right)}{u_c\left(\theta^t\right)} \tag{1.18}$$

where $u_x(\theta^t) \equiv u_x\left(c_t(\theta^t), d_t(\theta^t), y_t(\theta^t); \theta_t\right)$ for $x \in \{c, d, y\}$. The implicit housing consumption tax, τ_d^c, is akin to a value-added tax on consumption, while the implicit transaction tax, τ_d^t, appears as a tax paid when household sell their house. Given the implicit taxes, households balance the marginal benefit of housing services with its marginal cost, which also consist of relative price p as well as current marginal transaction costs and changes in expected future transaction costs.

The implicit labor tax distorts between the marginal rate of substitution of consumption for labor and the marginal product of labor w:

$$1-\tau_{y,t}(\theta^t) \equiv -\frac{u_{y,t}(\theta^t)}{u_{c,t}(\theta^t)}\Big/ w. \tag{1.19}$$

The implicit savings tax is the distortion in the marginal rate of substitution between current consumption and expected consumption:

$$1-\tau_{s,t}(\theta^t) \equiv \frac{u_{c,t}(\theta^t)}{\beta R \sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) u_{c,t+1}(\theta^{t+1})}\,. \tag{1.20}$$

Efficient Taxes. I characterize the efficient taxes using the solution to the planner problem. Since efficient taxes depend on productivity history, I describe the efficient distortions as a function of current productivity θ_t, taking as given a history θ^{t-1}. To simplify notation I omit the explicit dependence on the productivity history, meaning that $x_t(\theta)$ denotes the value of a random variable x given history (θ^{t-1}, θ) and that $x_{t-1} = x_{t-1}(\theta^{t-1})$.

In discussing the efficient taxes I emphasize the housing consumption tax and the transaction tax, which are central to this paper. Before discussing efficient housing policy, I describe how the standard taxes, the implicit savings tax and the implicit labor tax, extend to my environment. The derivations are in Section 1.C.

Since household preferences are separable, increasing, and strictly concave in non-housing consumption, the inverse Euler equation holds (Rogerson, 1985; Golosov, Kocherlakota, and Tsyvinski, 2003). The solution to the planning problem thus features a positive savings wedge. In my environment, the savings wedge equally applies to housing and financial wealth. To implement this positive savings wedge, policy has to be proof to double deviations as discussed in Kocherlakota (2005), Albanesi and Sleet (2006), and Golosov and Tsyvinski (2006). The labor wedge in my model is similar to Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2016), and always positive.

Efficient Housing Consumption and Transaction Tax. I discuss the efficient housing consumption tax and the efficient transaction tax. To provide intuition, I discuss the forces that determine how the efficient tax is set. I show that housing services are taxed when housing services and leisure are complements in home production, and that the government efficiently subsidizes or taxes transaction tax depending on households' state when they sell their house. I first present general optimal tax formulas, and then illustrate the main forces using a two stage life-cycle problem with two productivity types.

Efficient taxes on housing services consumption are determined by a static and a dynamic component,

$$\tau_d^c(\theta) = \underbrace{\Delta h_d\left(d(\theta), 1 - y(\theta)/\theta^+\right)\frac{I(\theta)}{p\pi(\theta)}}_{\text{static component}} + \underbrace{\beta R \tau_{y,t-1}\frac{\pi_\Sigma(\theta) - \pi_\Sigma^+(\theta)}{\pi(\theta)}\frac{\Delta h_d\left(d(\theta), 1 - y(\theta)/\theta^+\right)}{\Delta h_y\left(d_{t-1}, 1 - y_{t-1}/\theta_{t-1}^+\right)}}_{\text{dynamic component}}, \tag{1.21}$$

where $\Delta h_d\left(d(\theta), 1 - y(\theta)/\theta^+\right) \equiv h_d\left(d(\theta), 1 - y(\theta)/\theta^+\right) - h_d\left(d(\theta), 1 - y(\theta)/\theta\right)$, and where $I(\theta)$ is the insurance value at type θ. As is standard in static optimal taxation models, such as Diamond (1998) and Saez (2001), the insurance value is the inverse marginal utility of consumption for households with skills above θ relative to its mean,

$$I(\theta) = \sum_{s=i+1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)} - (1 - \pi_\Sigma(\theta)) \sum_{s=1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)} \ . \tag{1.22}$$

Since the dynamic component and the insurance value are positive, the home technology determines whether housing consumption is taxed or subsidized.[14]

Housing consumption is taxed when housing services and leisure are complementary in home production. Holding constant a household's housing services and labor output, a more productive household enjoys more leisure. When housing services and leisure are complements, a productive household therefore has a higher marginal utility for housing services. By taxing the consumption of housing services, additional leisure time is spent in a less desirable house, which prevents productive households from working inefficiently few hours.

An efficient housing consumption tax balances the distortionary costs for a type θ against the benefit of relaxing the incentive constraints for all types above θ. Relaxing incentive constraints allows the planner to provide more insurance in period t by extracting resources from households more productive than θ, and by distributing these resources across all households with the same history, which is captured by $I(\theta)$. The inverse proportion $1/\pi(\theta)$ captures that the planner is willing to distort a household type more when there are fewer households of this type.

The dynamic component captures that a housing consumption tax in the current period relaxes incentive constraints in prior periods. When the Markov transition matrix for household skills is monotonic (Daley, 1968), implying $\pi_\Sigma(\theta) \geq \pi_\Sigma^+(\theta)$, a more productive household θ_-^+ is more likely to be affected by the housing services tax. The planner favors larger absolute distortions, all else equal, to exploit the dynamic incentive effect. The properties of the efficient housing consumption tax are summarized in Proposition 2. The proof is in Section 1.C.

[14]The efficient tax on housing consumption is equivalently written as $\tau_d^c(\theta) = q(\theta^+)\Delta h_d\left(d(\theta), 1 - y(\theta)/\theta^+\right)$, where $q(\theta^+)$ is the shadow value of relaxing the incentive constraint for type θ^+. This formulation of the housing consumption tax, which I derive in Section 1.C, directly shows that housing consumption is efficiently taxed or subsidized depending on the complementarity in the home production technology.

Proposition 2. *The housing services wedge is positive if and only if housing services and non-market time are complements in home production.*

Efficient transaction taxes implicitly subsidize households when they sell their residence after a negative shock, meaning that the household's consumption increased, and effectively taxes households when they sell their house after receiving a positive shock,

$$\tau_d^t(\theta_{t+1}) = \Phi_2\big(d(\theta_{t+1}), d(\theta)\big)\left(\frac{1}{\beta R}\frac{v_c(c(\theta))}{v_c(c(\theta_{t+1}))} - 1\right). \tag{1.23}$$

When the selling cost of a house increases in the value of the property that is sold, $\Phi_2 \geq 0$, households pay an implicit transaction tax when their marginal utility from consumption decreases. An efficient allocation provides insurance against transaction costs.

The planner uses transaction taxes to provide insurance against transaction costs when asset markets are incomplete. To understand why, consider the case where housing consumption is also separable from leisure so that the efficient housing consumption tax is zero, and let savings taxes be such that the intertemporal non-housing consumption choice is efficient. The planner's rate of transformation between housing services and non-housing consumption is $p + \Phi_1\big(d_t(\theta^t), d_{t-1}(\theta^{t-1})\big) + \frac{1}{R}\sum \pi^{t+1}(\theta_{t+1}|\theta^t)\Phi_2\left(d_{t+1}(\theta^{t+1}), d_t(\theta^t)\right)$ which reflects that the planner incurs increased transaction costs next period with certainty. Absent any transaction taxes, households substitute at $p + \Phi_1\big(d_t(\theta^t), d_{t-1}(\theta^{t-1})\big) + \beta\sum \pi^{t+1}(\theta_{t+1}|\theta^t)\Phi_2\left(d_{t+1}(\theta^{t+1}), d_t(\theta^t)\right)\frac{u_c(\theta^{t+1})}{u_c(\theta^t)}$. Households face uncertainty over future transaction costs, and evaluate marginal transaction costs by the corresponding marginal utility of consumption in each state. Relative to the planner, households overweight marginal transaction costs after negative shocks. Efficient transaction taxes correct for this, by subsidizing transaction costs after a negative shock. The efficient transaction tax is summarized by Proposition 3.

Proposition 3. *The efficient transaction tax is positive when households sell their house after a positive shock.*

Illustration with Two Periods and Two Types. I consider a two stage life-cycle to illustrate the motives for efficient taxes. Households are identical in the initial period but either have high productivity, θ_H, or low productivity, θ_L, in the final period, where $\theta_H > \theta_L > 0$. The downward incentive constraint prevents the high productivity household from mimicking the low productivity household.

The housing consumption tax in the final period prevents productive households from pretending to be unproductive. When a productive household pretends to be unproductive the benefit is additional leisure. When housing services and leisure are complements in home production, the increase in leisure is more valuable when the household also enjoys more housing services. To prevent households from pretending to be unproductive, the efficient allocation therefore depresses the consumption of housing services for low productivity households, which translates into a positive consumption tax. In sum, the efficient allocation discourages the productive households from misreporting by threatening them to enjoy their leisure in a less desirable house. The opposite mechanism applies when housing and leisure are substitutes in home production, which translates into a housing consumption subsidy. When housing services and leisure are neither complements nor substitutes, the housing consumption tax is zero, echoing the uniform commodity tax result of Atkinson and Stiglitz (1976). The sign on the housing consumption tax is determined by the degree of complementary in home production.

The efficient transaction tax is driven by the transaction cost technology and is independent of the home production technology. When choosing their housing services consumption in the initial period, households incorporate how their choice affects their transaction costs in the final period, weighting each state by its respective marginal utility of consumption, e.g. $v_c(c_H)/v_c(c_0)$. The planner insures transaction costs across productivity states by making households internalize the future marginal transaction costs at the marginal utility of consumption in the initial period instead, implying a unit weight for each productivity realization $\big(v_c(c_0)/v_c(c_0)\big)$. The efficient transaction tax for the high type is thus

$$\tau_d(\theta_H) = \Phi_2\big(d(\theta_H), d_0\big)\left(\frac{1}{\beta R}\frac{v_c(c_0)}{v_c(c_H)} - 1\right)$$

where $c_H > c_0 > c_L$. When households move in the second period they generally face a non-zero transaction tax. Specifically, when households move after a positive shock they efficiently face a positive transaction tax. This argument holds irrespective of the home production technology.

Having characterized the forces that drive efficient taxes, I turn to a positive description of the economy where I introduce current policy and instititions in the Netherlands. I study this economy to measure implicit subsidies on housing consumption under current policy and to infer parameters that quantitatively determine efficient taxes.

1.4 Positive Economy

I present a positive economy which I use to calibrate preferences and to obtain household-specific levels of lifetime utility under the current housing policy for the Netherlands.

Assets. Households enter each period with savings that they can allocate to three asset classes after they observe their labor productivity. Savings s can be held in the form of financial assets a, housing wealth $h \geq 0$, and mortgages $m \geq 0$. Uncollateralized lending is ruled out, $0 \leq s$. Mortgages are collateralized loans which can only be held by homeowners ($h > 0$). Households earn net interest r on their financial assets and pay the same interest rate on their mortgage debt.

Households can own or rent a house. The housing market is segmented into a rent and an owner-occupier segment. Households can purchase houses with capital levels above $\underline{h}$ at a price p^H per unit of housing capital. This cutoff is an entry barrier into homeownership for low-income households. Households can otherwise rent houses with capital levels below $\underline{h}$ from landlords at a rental price p per unit.[15] When households choose to be homeowners they incur a required maintenance costs δ^H per unit of housing capital, while landlords incur this maintenance costs on rental units.

Homeowners can finance their house by taking out a mortgage, which I model as non-defaultable debt.[16] The government dictates lending guidelines to financial intermediaries

[15]For analytical tractability I model a perfectly segmented housing market. When I instead allow for overlap in the available sizes of rental and owner-occupied houses, the efficient allocation features households living only in rental units, or the unit type with the lowest transaction costs, in the overlapping segment.

[16]I choose to not model bankruptcy given that the average number of bankruptcies in the Netherlands between 2006 and 2014 is only 163 for every 1 million individuals. Data from the American Bankruptcy Institute show that the bankruptcy rate in the United States is 25 times larger in this period (following the

that limit the size of the mortgage as a function of the value of the property, the household's labor earnings, and its age,

$$m \leq \kappa_t (h, wy). \tag{1.24}$$

The loan-to-value and loan-to-income requirements are a second entry barrier into homeownership. When households transition from being a homeowner to being a renter they repay their mortgage debt.

Current Housing and Income Tax Policy. The marginal tax rates that vary with income, housing, and mortgage choices are the income tax rate and the marginal tax rate on financial assets. The tax functions are calibrated in Section 1.5.

The income tax system is progressive and treats homeowners and renters differently. Taxable income $\tilde{y}$ is the sum of labor earnings wy, and imputed rental income $\tau_o p^H h$ minus home mortgage interest expenses rm,

$$\tilde{y} = wy + \tau_o p^H h - rm, \tag{1.25}$$

where τ_o is a policy variable that determines the fiscal rent-to-value. A distinguishing feature of the Dutch tax code is that homeowners add part of their imputed rental income to their taxable income.

Income is taxed at a progressive marginal rate τ_y which depends on the retirement status. The marginal income tax rate varies across $\mathcal{B}$ income brackets. For example, the marginal income tax rate for workers τ_y^w is given by the piecewise function

$$\tau_y^w = \begin{cases} \tau_{y,1}^w, & 0 \leq \tilde{y} < \underline{b}_1^w \\ \tau_{y,2}^w, & \underline{b}_1^w \leq \tilde{y} < \underline{b}_2^w \\ \vdots & \vdots \\ \tau_{y,\mathcal{B}}^w, & \underline{b}_{\mathcal{B}-1}^w \leq \tilde{y} < \underline{b}_{\mathcal{B}}^w. \end{cases}$$

approach of Livshits, MacGee, and Tertilt (2010)).

A household's total income tax, denoted by $T_t^y(\tilde{y})$, is the sum of its income tax across the brackets.

Finally, financial wealth is taxed. Households pay a tax τ_i on financial assets held in excess of cutoff $\underline{a}$. The financial wealth tax is captured by $T^a(a)$.

Household Problem. Households enter every period with accumulated savings s. At the beginning of the period households decide whether to rent or to buy a house and make their portfolio decision across financial assets, housing wealth, and mortgages. I next discuss the respective constraint sets and the decision to rent or buy.

Renter's Problem. Renters with net worth s can only hold their savings in the form of financial assets, so $s = a$. Working-age renters with net worth s that receive labor income $wy = \tilde{y}$ pay income taxes $T_t^y(\tilde{y})$ and financial wealth taxes $T^a(a)$. Renters can spent their after-tax income on consumption goods, housing services, and transaction costs, and save the remainder, knowing that the portfolio allocation is optimized at the beginning of next period. In summary, the sequential budget constraint for renters is

$$c_t+T^c(c_t)+p_jd_t+T_t^d(p_jd_t,\tilde{y}_t)+\Psi(d_t,d_{t-1})+s_{t+1} = wy_t-T_t^y(\tilde{y}_t)+Rs_t-T_t^a(s_t)+T_t, \quad (1.26)$$

where T_t is an age-dependent lump-sum transfer.

Renters maximize utility by choosing consumption, housing services, market hours, and savings subject to their budget constraint (1.26), the borrowing constraint $s_{t+1} \geq 0$, the time constraint $\ell_t + n_t = 1$, and the size limit for rental units $d \leq \underline{d} \equiv \chi\underline{h}$. The constraint set for renters is denoted $\Gamma_t^r(s_t, d_{t-1}; \theta_t)$. The value of being a renter with net worth s_t, having lived in dwelling d_{t-1} in the prior period, and with productivity θ_t is

$$V_t^r(s_t, d_{t-1}; \theta_t) = \max_{(c_t,d_t,n_t,s_{t+1})} u(c_t, d_t, \ell_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t)V_{t+1}(s_{t+1}, d_t; \theta_{t+1}) \ , \quad (1.27)$$

subject to $(c_t, d_t, n_t, s_{t+1}) \in \Gamma_t^r(s_t, d_{t-1}; \theta_t)$, and where $V_{t+1}(s_{t+1}, d_t; \theta_{t+1})$ is the value of entering the following period with savings s_{t+1} having lived in dwelling d_t. The future value is determined by the homeownership decision that is made at age $t + 1$ which I discuss below.

Homeowner's Problem. Homeowners hold their net worth as financial assets, housing wealth and mortgages, $s = a + p^H h - m$. Working-age homeowners with taxable income $\tilde{y} = wy + \tau_o p^H h - rm$ pay income taxes $T_t^y(\tilde{y})$ and financial asset taxes $T_t^a(a)$. Homeowners also pay a linear property tax τ_p on the market value of their house and incur required maintenance costs δ^H on their housing capital h. Homeowners allocate their disposable income towards consumption goods, transaction costs and savings. The sequential budget constraint for homeowners is

$$c_t + T^c(c_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = wy_t - T_t^y(\tilde{y}_t) + Ra_t - T_t^a(a_t) + \left(p_{j+1}^H - \tau_p p_j^H - \delta^H\right) h_t - Rm_t + T_t, \tag{1.28}$$

where Rm_t is the gross interest payment on the mortgage.

Homeowners maximize utility by choosing their asset portfolio, consumption, market hours, and savings subject to their budget constraint (1.28), the portfolio constraint $s = a + p^H h - m$, the borrowing constraint, the time constraint, and the house size restriction for homeowners $d_t \geq \underline{d}$. The constraint set for owners is $\Gamma_t^o(s_t, d_{t-1}, \theta_t)$. The value of being a homeowner is

$$V_t^o(s_t, d_{t-1}; \theta_t) = \max_{(c_t, d_t, n_t, a_t, h_t, m_t, s_{t+1})} u(c_t, d_t, \ell_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t) V_{t+1}(s_{t+1}, d_t; \theta_{t+1}) \,, \tag{1.29}$$

subject to the constraint that $(c_t, d_t, n_t, a_t, h_t, m_t, s_{t+1}) \in \Gamma_t^o(s_t, d_{t-1}; \theta_t)$.

Tenure Choice. At the beginning of every period households make their tenure choice, they decide whether to rent or to buy a house. Households choose to rent a house when the value of being a renter (1.27) exceeds the value of being an owner (1.29) given their state,

$$V_t(s_t, d_{t-1}; \theta_t) = \max\left(V_t^r(s_t, d_{t-1}; \theta_t), V_t^o(s_t, d_{t-1}; \theta_t)\right). \tag{1.30}$$

Production. The production side of the economy consists of three types of producers. Rental firms convert housing capital into housing services for renters, construction companies convert the general good into housing capital, and general good producers produce the numeraire good.

Rental firms operate in a competitive market using a technology that transforms one unit of housing capital into χ units of housing services.[17] Rental firms receive rent p_j per unit of housing services. They borrow funds at interest rate r to buy housing capital at the beginning of the period at unit price p_j^H, incur maintenance costs δ^H and can sell their housing capital at the end of the period at unit price p_{j+1}^H. Rental firms also pay property tax τ_p per unit of housing capital and receive a subsidy on interest payments τ_f. In equilibrium, rents are:

$$p_j = \frac{1}{\chi}\Big(r(1-\tau_f) + \tau_p + \hat{\delta}^H - \pi_{j+1}^H\Big)p_j^H, \tag{1.31}$$

where $\hat{\delta}^H \equiv \delta^H/p^H$.

Construction companies operate a time to build technology in a competitive market. In period $j+1-\iota$, a construction company commits to convert general goods into $Q_{j+1-\iota}^h$ units of housing capital using a one-to-one production technology in period j. In period j, general goods are converted into new housing units which are delivered at the end of the period and valued at price p_{j+1}^H. In the first period, the construction companies plan to deliver housing units in period ι. The house price for all periods $j > \iota$ is thus equal to unity since the supply of houses for all periods $j > \iota$ is perfectly elastic.

General good producers rent capital and hire workers to produce a numeraire good with a Cobb-Douglas technology using business capital and effective labor $F(K, Y)$. Given an interest rate on business capital r^K and a wage rate w, the firm chooses its inputs such that $r^K = F_K(K_j, Y_j)$ and $w = F_Y(K_j, Y_j)$.

Government. The government collects taxes on consumption, income, properties, financial

[17] In the Netherlands, only 14 percent of the rental supply is provided by households in 2018. I abstract from direct household rental supply by modeling rental firms providing the rental supply. Chambers, Garriga, and Schlagenhauf (2009) show that households are a prominent supplier of rental units in the United States.

wealth, and housing transactions. Tax revenues finance transfers, government expenditures, financing subsidies towards rental firms, and interest payments on the government's outstanding debt. The government issues debt when its expenses exceed its revenues.

Equilibrium. To conclude the model under current policy, I present a formal definition of equilibrium in Appendix 1.D. The housing market is local, and clears in equilibrium.

1.5 Calibration

The model is calibrated in three steps. In the first step, I calibrate demographic and technology parameters using aggregate data, and I calibrate policy parameters using the descriptions of the responsible government agencies. In the second step, I estimate the skill process using micro data on household wages. In the third step, three preference parameters are calibrated by matching model simulated moments to their empirical counterpart.

Calibration. I calibrate macro-parameters using aggregate data, such as the national income and product accounts, while I calibrate policy parameters using the description by the respective government agency. I use public data from Statistics Netherlands to calibrate the macro-parameters, and policy descriptions by the national tax office, unless stated otherwise. I use data for all years between 2006 and 2014. All amounts are denominated in 2015 euro.

Demographics. Households enter the labor market at 25 and participate for $T_w = 39$ years, which corresponds to the median retirement age of 63. Households live until 77, the median life expectancy for cohorts born between 1946 and 1960, conditional on surviving to age 25. Households enjoy $T_r = 13$ years of retirement. The demographic parameters are summarized in the top panel of Table 1.1.

Technology. I calibrate the interest rate to the average annual real mortgage rate on outstanding mortgages, which is 3.05 percent.[18] The time discount factor is set so that $\beta R = 1$,

[18] Data for the average nominal interest rate on outstanding mortgages is reported by financial intermediaries to the Dutch Central Bank. The interest rate is weighted by the outstanding mortgage balance, and deflated by the consumer price index.

Table 1.1: Exogenously Calibrated Parameters

	Parameter	Value	Data Target
		Demographics	
T	Length of life	53	Median life expectancy of 77
T_r	Retirement age	40	Median retirement age of 63
		Technology	
r	Interest rate	0.031	Mean interest rate on mortgage loans
α	Capital share	0.439	Capital income share
δ^K	Depreciation of capital	0.061	Depreciation rate of business capital
δ^H	Depreciation of housing	0.024	Depreciation rate of residential structures
χ	Housing services flow	0.055	Normalization of benchmark user's cost, $r + \delta^H$
ι	Time to build	2	Mean building time for new houses
ψ_b	Transaction cost, buyer	0.020	Mean broker fee, buyers
ψ_s	Transaction cost, seller	0.015	Mean broker fee, sellers

Table 1.1 presents the parameters calibrated exogenously.

or $\beta = 0.97$.

The general good is produced by a Cobb-Douglas technology with an output elasticity of business capital, α, set to 0.439, the capital income share in the Netherlands. The depreciation rate of business capital is $\delta^K = 0.061$, which corresponds to the depreciation rate of capital excluding housing, research and development, and software.

Housing services are proportional to the stock of housing capital, which depreciates at rate $\delta^H = 0.024$. The flow services per unit of housing capital, χ, is set such that the rental price in absence of government policy is 1, that is, $\chi = r + \delta^H$. The production of housing units takes 23 months on average after a building permit is issued, which is consistent with $\iota = 2$.[19] I calibrate the transaction cost function to real estate broker fees. The average broker fee for sellers in the Netherlands is equal to about 2 percent of the sales price (Gautier, Siegmann, and van Vuuren, 2018). The mean broker fee for buyers is approximately 1.5 percent of the transaction price. The technological parameters are summarized in the bottom panel of Table 1.1.

[19] Gomme, Kydland, and Rupert (2001) incorporate time to build for business capital into a business cycle model with housing capital, but they do not incorporate time to build in the housing sector. In this paper it takes one year to complete an investment in business capital, in line with Kydland and Prescott (1982) and Gomme, Kydland, and Rupert (2001), and the 175 days it takes to build a warehouse in the Netherlands (World Bank's report on Doing Business).

Table 1.2: Policy Parameters

	Policy Instrument	Value
	Housing	
τ_r	Transaction tax	0.060
τ_f	Financing subsidy	0.232
τ_o	Imputed rent tax	0.006
τ_p	Property tax	0.001
	Other	
τ_c	Consumption tax	0.134
T_b	Retirement benefit	18,140

Table 1.2 parameterizes affine policy functions. The specification of the nonlinear policy instruments, such as income taxes, asset taxes, and lending restrictions, is described in the text and presented in Figure 1.1.

Policy. I parameterize affine policy parameters, which I summarize in Table 1.2, and describe the nonlinear tax functions.

Buyers pay a transaction tax, τ_t, equal to 6 percent of the property value. The government indirectly subsidizes rental housing by guaranteeing loans of rental firms, which translates into an effective financing subsidy, τ_f, of 23.2 percent.[20] The statutory tax on imputed rent income τ_o is 0.6 percent of the property value. Since property taxes are levied at the local level in the data, I calibrate the model property tax τ_p to the value-weighted average of 0.1 percent.

The sales tax on consumption goods, τ_c, is 13.4 percent, which is the spending weighted average indirect tax on consumption goods. When retired, households receive public pension benefits equal to the minimum wage of full-time workers, or $T_b = 18,140$.

The tax instruments that drive differences in the user's cost across homeowners are labor income taxes and asset income taxes. The first two panels in Figure 1.1 plot the schedule for each of these nonlinear instruments.

The income tax schedule is progressive with marginal tax brackets of 34, 42 and 52 percent for working age households, with cutoff levels at 20 and 59 thousand euro. The

[20] Veenstra and van Ommeren (2017) estimate that explicit bailout clauses for Dutch housing corporations reduce their funding costs by 72 basis points. The authors use loan-level data covering approximately 44 percent of housing corporations' external funding between 1997 and 2013 to measure the interest rate differential between comparable guaranteed loans and non-guaranteed loans. In 2014, 95 percent of public housing corporation debt was guaranteed. I model all rental housing as indirectly subsidized.

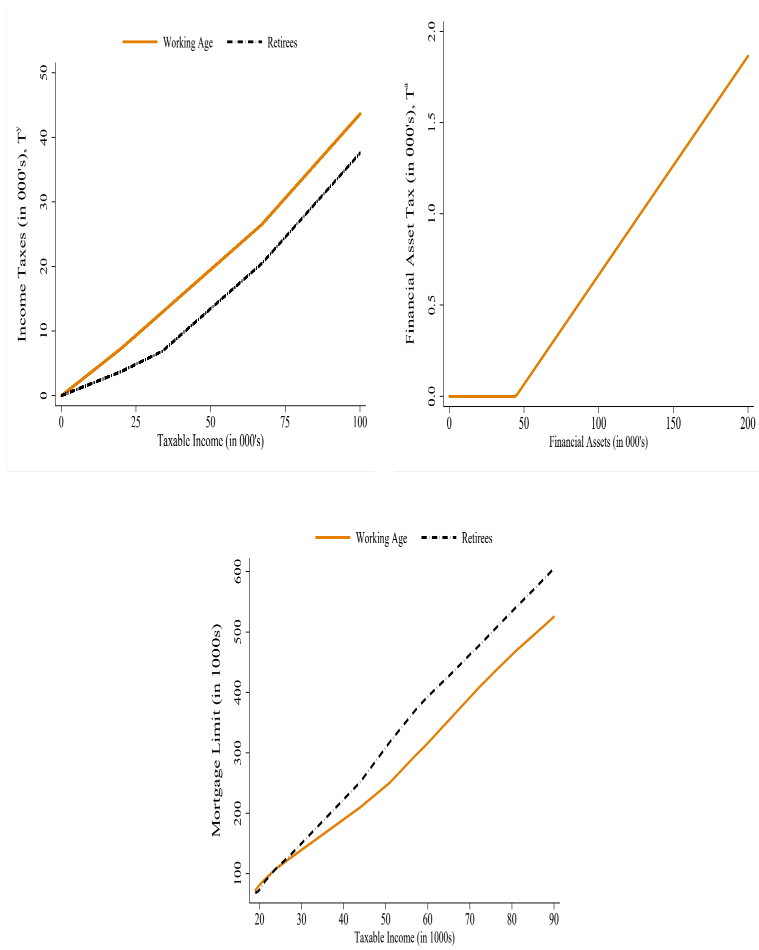


Figure 1.1: Income Tax, Asset Income Tax, and Mortgage Regulation

Figure 1.1 shows labor income taxation, asset income taxation, and mortgage regulation in the Netherlands. The left panel shows the income tax schedule for working age households (solid line) and retirees (dashed line). Financial holdings in excess of 46 thousand euro are taxed at 1.2 percent rate (middle). The right-hand panel displays the maximum loan-to-income guidelines that the government prescribes to financial intermediaries for working age households (solid line) and retirees (dashed line).

marginal tax rate is reduced for retirees with incomes below 35 thousand euro. The marginal tax rate is 17 percent below 20 thousand euro, and 24 percent below 35 thousand euro. The income tax schedule is shown in the left panel of Figure 1.1.

Financial wealth in excess of 46 thousand euro is taxed at a 1.2 percent rate. The government imputes an annual return of 4 percent for financial wealth, which it taxes at a 30 percent rate. The resulting asset income tax schedule is in presented in the middle panel of Figure 1.1.

The government prescribes guidelines to the financial sector that restrict the extension of home mortgages. The maximum mortgage loan that financial intermediaries can extend is determined by household income. The right-hand panel shows the maximum loan-to-income guideline between 2006 and 2014 for workers and retirees.[21] The extension of mortgages is further limited by a loan-to-value limit of 1.02 that was introduced in 2012. I translate both policies into a single mortgage limit that depends on household age and income as well as the value of the property, as in (1.24).[22]

Data. I use linked administrative records between 2006 and 2014 from Statistics Netherlands, the national statistics agency, to measure the user's cost of homeowners under current policy, to estimate a skill process for different education groups, and to calibrate household preferences.

I use a representative subsample of all Dutch households selected by Statistics Netherlands.[23] The sample consists of about 95 thousand households per year, roughly 1.3 percent of the population of households, covering a total of over 275 thousand individuals. For all analyses, I weight households with the provided sample weights. I consider all households with heads of household above age 25.

Labor Market. Income is measured by employer-provided earnings records. I construct an individual's annual taxable labor earnings, which includes the employer's health insurance contribution, by adding all earnings reports within a given calendar year. To construct an hourly wage rate, I divide taxable labor earnings by employer-reported hours worked. Because the model features a single decision maker for each household, I define the household wage rate for married and cohabitating households as the average individual wage rate weighted by the hours worked of each partner. For single households, the individual wage

[21] The mortgage guidelines are written by the National Institute for Budget Information. Starting in 2007, their prescriptions are adopted into a code of conduct for the financial sector. After the financial crisis, the guidelines have been incorporated into a binding legal arrangement. The methodology behind the mortgage guidelines is described in Warnaar and Bos (2017).

[22] The mortgage limit is the minimum of the maximum loan-to-income and the maximum loan-to-value. Households effectively choose between an annuity mortgage, a linear mortgage, and a balloon mortgage (of at most 50 percent of the property value, as prescribed by the mortgage guidelines). I model the maximum loan-to-value as the maximum outstanding mortgage balance under the three different contracts, under the assumption all households take out a 30 year mortgage at age 35, and extrapolating this function before age 35. By modeling the mortgage limit as a function of age, rather than the year in which the mortgage was extended, I contain the state space of the household problem.

[23] Specifically, I use the IPO subsample (Inkomenspanelonderzoek). To simplify the exposition, I omit the names of individual data sets that I link to this sample. If you are interested, contact me for more detail.

rate is the household wage rate. Household non-market time is given by average individual non-market time which is discretionary time minus individual hours worked. I set an individual's discretionary time equal to 16 hours a day for 365 days.

The measure of educational attainment for each individual is the highest degree they earned. I classify every degree as a low, a medium, or a high level of education. The low education level corresponds to a high school degree or a practical degree, the medium level is a degree from a university of applied sciences, while a high level of education is a university degree. I group households into six education bins, which are unordered pairs of the degree of each partner. Singles are grouped with couples in which both partners have obtained the same level of education.

Housing and Assets. To measure housing consumption for each household, I assume that the housing service flow is proportional to the real property value of the residence. For both renters and owner-occupiers, I measure housing services consumption using tax assessed property values. The fiscal authority assesses the market value of every property as of January 1 using transaction data of comparable units. To make property values comparable across time, I deflate property values by the regional house price index.

Households' financial assets and mortgage balances are obtained from a wealth registry which records households' financial position as of January 1. I combine the outstanding mortgage balance with the property value to measure the loan-to-value ratio for every household's primary residence. Financial assets are used to calculate the household's marginal tax rate on financial wealth.

I first use only the labor market data to estimate the household skill process. I then introduce the estimated skill process into the structural model to calibrate household preferences using both labor market and housing data. Finally, in Section 1.6 I use all data to measure the user's cost of housing for homeowners.

Skill Process. I parameterize the household skill process using estimates obtained outside the model using data on household wages.[24] I allow for heterogeneity between education groups in both the life-cycle profile and the idiosyncratic component of wages. For each household education bin I construct the life-cycle profile and I estimate a process for the residual wage. To the extent that wages follow different predictable life-cycle profiles across education groups, wage growth is accounted for by a difference in growth profiles rather than by being classified as idiosyncratic risk.

To obtain the life-cycle wage profile and the residual wage, I regress household wages on dummy variables to control for time and age effects within each education group.[25] The age effects capture the life-cycle profile, the residual is labeled wage risk. Let z_{ijt} be the residual wage for household i at time j with age t. I assume residual wages follow a first-order autoregressive process in logs with both persistent and transitory innovations,

$$\begin{aligned} \log z_{it} &= \log\theta_{it} + \varepsilon_{it} \qquad\qquad i = 1,\dots,N, \quad t = 1,\dots,T, \\ \log\theta_{it} &= \rho\log\theta_{it-1} + u_{it} \end{aligned}$$

where $u_{it} \sim \mathcal{N}(0,\sigma_u^2)$, $\varepsilon_{it} \sim \mathcal{N}(0,\sigma_\varepsilon^2)$ and $z_{i0} \sim \mathcal{N}(0,\sigma_{z_0}^2)$, and N is the number of households in the group.

I estimate the parameters that govern the residual wage process using the minimum distance estimator (Chamberlain, 1984). I minimize the sum of squared differences between empirical moments of the variance-covariance structure for residual wages and their analytical counterpart. Specifically, I target the residual wage variance and the first-order autocovariance at each age. To construct confidence intervals, I estimate the parameters for one thousand bootstrap samples.

Table 1.3 shows the estimated persistence ρ, and the variance of the permanent innovation σ_u^2, for each education group. The second and third column present the point estimate and confidence interval for the persistence, the fourth and fifth column present the point

[24] Since every competitive equilibrium is incentive compatible, estimating a productivity process using observed wages is not inconsistent with the assumption that the skill process is not observed by the government when designing policy reform.

[25] I estimate the household skill process using stable households, households for which the composition of adults as well as the employment status of the adults is stable over time. When an adult's employment status changes, this is thus not picked up as household skill risk.

Table 1.3: Estimated Wage Process Parameters

	Persistence, ρ		Variation of Innovation, σ_u^2	
Education Group	Point Estimate	Confidence Interval	Point Estimate	Confidence Interval
Low, Low	0.9542	(0.9515, 0.9575)	0.0096	(0.0093, 0.0102)
Low, Medium	0.9660	(0.9610, 0.9692)	0.0087	(0.0083, 0.0096)
Low, High	0.9673	(0.9628, 0.9710)	0.0162	(0.0153, 0.0176)
Medium, Medium	0.9570	(0.9536, 0.9612)	0.0099	(0.0091, 0.0103)
Medium, High	0.9616	(0.9520, 0.9782)	0.0109	(0.0082, 0.0124)
High, High	0.9564	(0.9501, 0.9582)	0.0172	(0.0164, 0.0184)

Table 1.3 shows the estimated wage parameters by education group. The second and third column show the estimates for the persistence of the residual wage, the fourth and fifth column present estimates for the variance of the persistent innovation. The 95 percent confidence intervals are constructed using one thousand bootstrap samples.

estimate and confidence interval for the variance of the persistent innovation. The persistence is similar across groups, ranging from 0.954 for households with lower education to 0.967 for households with a highly educated and a less educated spouse. The standard deviation of the innovation for high education households is 30 percent larger than the standard deviation of the innovation for low education households.

Calibration. After calibrating the demographic structure and the aggregate production technology, and estimating the skill process, I calibrate household preferences. I assume household flow utility is of the form

$$u(c, d, l) = \gamma \log c + (1-\gamma) \log \left(\omega d^{\frac{\sigma-1}{\sigma}} + (1-\omega) \ell^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1.32}$$

that is, I assume that both v and $\mathcal{H}$ in equations (1.1) and (1.2) are given by natural logarithm functions. The flow utility function is parameterized by a Cobb-Douglas weight on non-housing consumption γ, a weight on housing services in the home technology, ω, and the elasticity of substitution between housing services and leisure, σ. The three preferences parameters are chosen to minimize the squared difference between simulated moments from the model and their empirical counterpart. Table 1.4 displays the three preference parameters and shows that the model well approximates the empirical moments targeted in the

Table 1.4: Estimated Parameters

	Parameter	Value	Target	Data	Model
γ	Preference weight on consumption	0.343	Consumption to output ratio	0.64	0.66
ω	Housing share in home production	0.144	Housing share in consumption	0.17	0.16
σ	Elasticity of substitution	0.951	$\mathrm{Cov}(\ell/d, w)/\mathrm{Var}(w)$	-0.44	-0.43

Table 1.4 presents the preference parameters that are estimated within the model.

calibration.

The preference weight for consumption and the weight on housing services in the home technology are calibrated by targeting aggregate moments. The weight on consumption in preferences targets the aggregate consumption-output ratio, which is 0.642. The weight on housing services in the home technology targets the expenditure share of housing in consumption, which equals 0.174. The measurement of these two moments is discussed in Appendix 1.E.

To calibrate the elasticity of substitution between housing services and leisure in home production, I use the model without distortionary taxation and transaction costs as an auxiliary model. Specifically, I use the optimality conditions for housing consumption and leisure in the misspecified model to derive the regression equation

$$\log\left(\frac{\ell}{d}\right) = -\sigma \log\left(\frac{\omega}{1-\omega}\right) - \sigma \log w, \tag{1.33}$$

where w is the household wage rate.[26] I separately estimate (1.33) using actual data and simulated data. I choose the elasticity of substitution in the model such that the regression coefficient implied by the model is as close as possible to the regression coefficient in the data.[27]

[26] Note that v and $\mathcal{H}$ are not required to be natural logarithm functions to obtain this optimality condition. Further, note that the user's cost of housing capital is constant across households in the absence of distortionary taxes (see equation (1.34) below).

[27] The identification is akin to the "gap" based indirect inference used by Berger and Vavra (2015). Berger and Vavra (2015) minimize the gap between optimal consumption of durables if a household pays a fixed adjustment cost relative to actual durable consumption. To construct this gap in their data, they use a model-generated mapping from observables to the optimal choice after incurring fixed adjustment costs to impute the optimal choice. My gap is similar, yet more direct. I minimize the gap between the optimal ratio of home production inputs when a household does not face transaction costs and distortionary taxes relative to the observed home production input ratio. My model-generated mapping from my observable, the household wage, to the optimal choice is (1.33).

The elasticity of substitution between housing and leisure in the home production technology is identified by the covariation in the home production input ratio with the opportunity cost of time, similar to Rupert, Rogerson, and Wright (1995) and Aguiar and Hurst (2007a). The opportunity cost of time in my framework is the household wage rate. In absence of transaction costs and distortionary taxation, housing consumption increases one for one with wages without changing hours when the home technology is Cobb-Douglas ($\sigma = 1$). When housing services and non-market time are complements ($\sigma < 1$), the input mix instead decreases less than one for one with wage changes. Transaction costs change this relationship. As wages increase, households may not increase their housing consumption because of transaction costs, leading to a bias in the regression coefficient. In sum, the regression coefficient only indirectly informs the elasticity of substitution.

To align the administrative micro data and the data generated by the unitary household model, I measure non-market time ℓ in the micro data as average leisure time for adult household members, and the household wage rate as the annual hours-weighted average of hours worked by adult members. To obtain an equivalent measure of housing services consumption, I regress the value of the household's residence on dummy variables for the number of adults in the household.

I find that with an elasticity of substitution of $\sigma = 0.951$ the model matches the regression coefficient of -0.44 in the micro data. While the regression coefficient naively suggest strong complementarity between housing services and leisure, this regression coefficient is biased upward due to the transaction costs. My estimate is similar to the assumed elasticity of substitution of one in the business cycle analysis by in Greenwood and Hercowitz (1991).

Model Validation. Before using the model to study policy reform, I compare the model's predictions to a set of predictions that were not explicitly targeted in the calibration.

In Figure 1.2, I compare the homeownership rate and the loan-to-value ratio by household age in the model to the data. The left panel shows that homeownership increases between age 25 and age 45 and decreases in retirement. The loan-to-income and loan-to-value requirement restrict homeownership early in life. The minimum house size to own and the transaction costs act as an entry barrier into homeownership throughout life. While

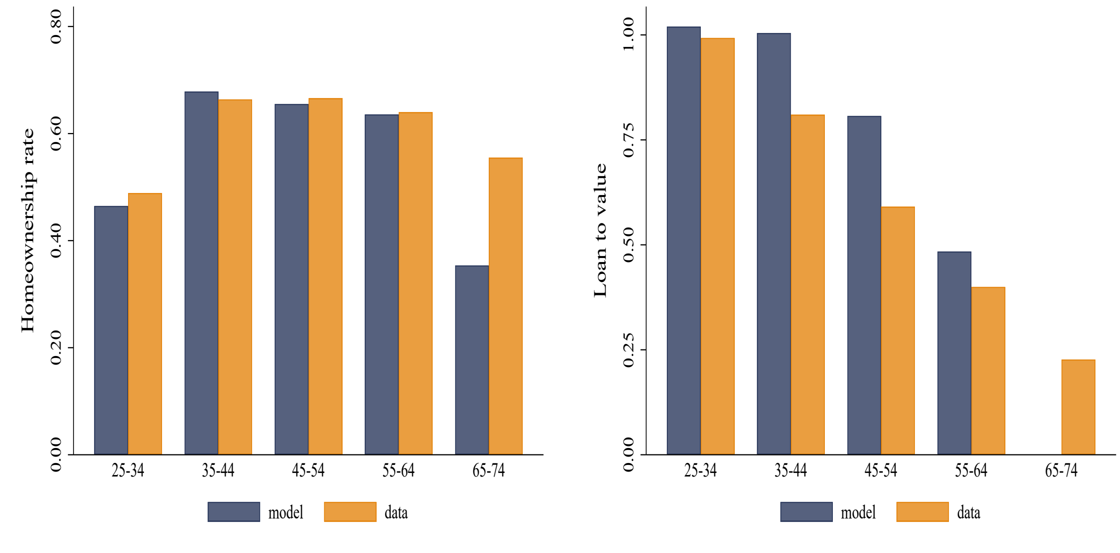


Figure 1.2: Homeownership and Loan to Value

Figure 1.2 compares the homeownership rate and the loan-to-value ratio by household age in the model to the data. The left-hand panel compares homeownership in the data (in orange) to homeownership in the model (in blue). The right-hand panel compares the loan-to-value ratio in the model and the data.

the model matches the homeownership rate of households until retirement, it predicts a low homeownership rate for retirees. Since households do not have preferences for leaving a bequest, and live until age 77 with certainty, they consume all savings in retirement. To smooth non-housing consumption households eventually sell their house and move to a rental unit, depressing the rate of homeownership for retirees in the model.

The right-hand panel shows the loan-to-value ratio in the model and the data. In the model, homeowners take out the maximum mortgage loan given their income and the value of their house. When they satisfy the income requirements, households take out the maximum size of their mortgage given the property value and their age. The household loan-to-value ratio by age reflects the maximum loan-to-value requirement as described in Footnote 22. In the model, households are only required to pay off their outstanding mortgage balance after age 35. In the data, households start reducing their outstanding mortgage balance earlier, and faster (for example, under a linear mortgage contract) explaining the gap between the loan-to-value ratio in the data and the model.

1.6 Quantitative Results

I quantify efficient policy reform by comparing efficient housing taxes with effective housing subsidies under current policy. I measure that the average homeowner receives an effective housing subsidy of 7.7 percent, which is declining in age. In contrast, I find an efficient average housing tax rate of 13.8 percent, which is almost constant with age, by computing an efficient allocation for the calibrated economy. Finally, I use the insights from the efficient allocation to inform simple policy reform.

User's Cost. To measure the effective subsidy on housing consumption under current policy I evaluate the user's cost of housing capital. User costs measure the marginal cost of housing services and are proportional to the static housing services wedge in an economy with proportional taxation of non-housing consumption. I measure the effective subsidy on housing consumption under current policy by comparing the user's cost under current policy to the user's cost in the absence of distortionary policy.[28]

Absent distortionary policy, households and firms can borrow at interest rate r to buy housing capital, on which they incur maintenance cost δ^H, and which they can sell with capital gain π^H. The user's cost in a laissez-faire economy, which I denote p^l, is therefore

$$p^l = r + \hat{\delta}^H - \pi^H, \tag{1.34}$$

where $\hat{\delta}^H \equiv \delta^H / p^H$. The laissez-faire user's cost increases in the cost of capital and the maintenance cost, and decreases with the capital gain. Given the calibrated values for the cost of capital and the depreciation rate of housing in Table 1.1, and an average real housing capital gain of minus 2.8 percent per year, I calculate a benchmark user's cost of 8.3 percent. For a house of 250 thousand euro, this implies a laissez-faire monthly rent of 1,725.

Homeowners. Housing policy changes the user's cost of homeowners by reducing their cost of

[28] In Appendix 1.F, I derive the user cost for renters and homeowners from their budget constraints. This user cost approach is similar to Laidler (1969), Aaron (1970), Dougherty and Van Order (1982), Poterba (1984, 1992), Himmelberg, Mayer, and Sinai (2005), Díaz and Luengo-Prado (2008) and Poterba and Sinai (2008). In contrast with the others, Díaz and Luengo-Prado (2008) incorporate selling costs into their user's cost. I consider transaction costs to measure an infrequent expenditure towards transaction services rather than a flow expenditure towards housing consumption.

capital and by increasing their expenses. The reduction in the cost of capital depends on the fraction of the property financed through debt, which I denote by $\varkappa_i$, where the subscript i indicates variation across households i. To the extent that a property is mortgage-financed, the borrowing cost reduces due to the deductibility of mortgage interest payments from taxable labor income. The value of the mortgage interest deduction thus depends on the household's marginal income tax rate $\hat{\tau}_{yi}$.[29] To the extent that a property is equity-financed, the borrowing cost is reduced due to the exclusion of housing capital from financial assets, which face a marginal rate $\hat{\tau}_{ai}$. The expenses on housing services increase due to property tax τ_p, which is not deductible for income taxation, and increase due to the imputation of rental income into taxable income. Fraction τ_o of imputed rental income is treated as taxable income, and thus faces a marginal tax rate $\hat{\tau}_{yi}$.[30] Combining the reduction in the cost of capital and the increase in expenditures, the user's cost for homeowners is

$$p^o = p^l - \hat{\tau}_{yi} r \varkappa_i - \hat{\tau}_{ai}(1 - \varkappa_i) + \tau_p + \hat{\tau}_{yi}\tau_o. \tag{1.35}$$

When households reduce their mortgage balance, they increase their user's cost by $\hat{\tau}_{yi} r - \hat{\tau}_{ai}$. The effective subsidy for homeowners is given by $p^o/p^l - 1$.

Renters. The user's cost of renters is reduced by indirect subsidies.[31] By the expression for the market rental rate (1.31), rents increase due to property taxes, and decrease due to financing subsidies towards rental firms, τ_f. Hence, the user cost for renters is

$$p^r = r(1 - \tau_f) - \pi^H + \hat{\delta}^H + \tau_p. \tag{1.36}$$

The effective subsidy for renters is $p^r/p^l - 1$. Given the calibrated policy parameters in Table 1.2, renters receive an effective subsidy of 7.5 percent.

[29] I use shorthand notation for the marginal income tax rate faced by household i at their income level $\tilde{y}_i = wy_i + \tau_o p^H h_i - rm_i$, that is, $\hat{\tau}_{yi} = T^y_{1t}(\tilde{y}_i)$.

[30] The Dutch tax system does not tax housing capital gains. Under an accrual system, the marginal tax rate on housing capital gains adds to the user's cost for homeowners, multiplied by the period housing capital gain. Instead, under a realization system the capital gains tax acts as a transaction tax.

[31] More generally, the user's cost of renters is also reduced through the marginal subsidy rate of direct subsidies. I abstract from direct rent subsidies because I found they only have a small effect on quantitative housing policy reform.

Table 1.5: House Values and Mortgage Balances

	Household Income (in thousand euro)					
Age of head	< 60	$60-80$	$80-120$	$120-200$	> 200	All
Panel A: Renters	Rental Property Value (in thousand euro)					
25−35	152.5	168.8	188.5	220.9	–	160.1
35−50	158.4	174.8	197.9	251.4	402.3	170.1
50−65	161.1	175.5	191.4	221.3	323.0	172.1
> 65	278.5	213.1	246.0	286.9	477.1	274.2
All	194.2	177.2	192.7	237.8	375.3	197.2
Panel B: Owners	Property Value (in thousand euro)					
25−35	167.1	185.1	210.8	256.5	321.6	190.6
35−50	212.7	223.5	255.5	324.9	433.4	255.3
50−65	233.6	245.8	269.4	325.7	425.0	274.4
> 65	255.0	314.0	348.9	395.4	507.2	285.7
All	223.0	229.5	260.0	325.4	431.4	255.4
Panel B: Owners	Loan-to-value ratio					
25−35	1.00	1.03	1.04	1.05	1.03	1.03
35−50	0.75	0.75	0.76	0.80	0.84	0.77
50−65	0.42	0.50	0.51	0.52	0.56	0.49
> 65	0.20	0.24	0.29	0.33	0.42	0.22
All	0.56	0.70	0.69	0.69	0.72	0.64

Table 1.5 summarizes main housing variables for all households in the Netherlands for owners and renters.

Effective Subsidy. Whether current policy implies an effective subsidy or tax on homeowners is a quantitative question. The user's cost expressions, (1.35) and (1.36), show that the effective subsidy varies in the cross section due to variation in marginal tax rates on income $\hat{\tau}_{yi}$, assets $\hat{\tau}_{ai}$, as well as variation in the loan-to-value ratio $\varkappa_i$. Quantitatively, the effective subsidy varies significantly between age and income groups, with young households financing homeownership through debt and high income households facing higher marginal tax rates.

Using tax records, I calculate the user's cost for the cross section of homeowners in the Netherlands. For homeowners, I use the variation in marginal tax rates on income, assets, and imputed rental income, as well as variation in the loan-to-value ratio. For every homeowner, in every time period, I evaluate (1.35). All variables indexed by i are household-specific, real house price inflation is specific to geographic regions, while all other parameters are common across households.

Table 1.6 shows the user's cost for homeowners between 2006 and 2014, averaged by age and income groups. The table shows that housing services are significantly subsidized under current policy with strong variation across age and income groups. Panel A shows the total subsidy, the bottom panels separately display the contribution of the home mortgage interest deduction and the exemption of housing capital from asset income taxation to the total subsidy.

The average effective housing subsidy for homeowners amounts to 7.5 percent. The average values range from 10.8 percent for young homeowners to 4.7 percent for old homeowners. The variation within age groups is driven by progressive income taxation given that leverage ratios are relatively constant within each age group. The subsidy increases with household income, reflecting that marginal tax rates on income and assets increase in their base. The subsidy decreases in age as homeowners reduce their mortgage balance while the benefit from the mortgage interest deduction exceeds the benefit of the exemption from asset income taxation.

Panel B and C show that housing is strongly subsidized through the home mortgage interest deduction and the exclusion of housing capital from asset income taxation. Young homeowners are subsidized through the mortgage interest deduction, which increases with income due to the progressive income tax schedule. Since they hold a small amount of financial assets, which thus face a zero marginal rate on asset income, young homeowners do not benefit from the exclusion of housing from asset income taxation. For old homeowners the opposite holds true. Old homeowners are mostly subsidized through the exclusion of housing from asset income taxation.

Table 1.6 suggests significant heterogeneity in the effects of policy reform. All else equal, Panel B suggests that eliminating the home mortgage interest deduction increases the average user's cost by 8.9 percent. This increase would be particularly strong for young homeowners with large mortgages. Eliminating the exclusion of housing from wealth taxation mostly affects old homeowners.

In sum, the user's cost shows that housing consumption of renters and homeowners is subsidized across the age and income distribution. To assess whether the current subsidies

Table 1.6: Effective Subsidy for Homeowners

	Household Income					
Age of head	< 60	60−80	80−120	120−200	> 200	All
Panel A	Effective Subsidy (in percent)					
25−35	9.8	10.7	11.6	12.6	14.1	10.8
35−50	7.1	7.4	8.3	10.2	11.4	8.2
50−65	5.1	6.0	6.8	8.2	10.0	6.7
> 65	3.6	6.4	7.2	7.7	9.5	4.7
All	6.1	7.6	8.1	9.4	11.0	7.5
Panel B	Home Mortgage Interest Deduction					
25−35	13.5	14.5	15.5	17.0	18.8	14.6
35−50	9.9	10.2	10.8	12.6	14.4	10.9
50−65	5.6	7.0	7.5	8.3	9.5	7.2
> 65	1.5	2.2	3.1	3.8	6.6	2.0
All	7.1	9.5	10.0	10.8	12.3	8.9
Panel C	Exclusion from Asset Income Taxation					
25−35	0.1	0.1	0.1	0.1	-0.1	0.1
35−50	0.8	1.0	1.3	1.8	1.9	1.2
50−65	3.2	2.9	3.3	4.2	5.0	3.5
> 65	4.7	7.2	7.4	7.4	7.4	5.6
All	2.4	1.8	2.1	2.9	3.3	2.4

Table 1.6 presents the effective housing subsidy for homeowners under status quo housing policy. Panel A summarizes the effective subsidy for various income and age groups. Panel B and Panel C respectively account for the effect of the home mortgage interest deduction and the exclusion of housing from wealth taxation.

are efficient, I compare the current policy to efficient policy.

Efficient Reform. To understand whether the current user's cost is close to efficient, I contrast the housing consumption wedge under current policy against the housing consumption wedge under an efficient reform. I use the estimated preference parameters and wage processes, and the technology parameters to calibrate the component planner's problem discussed in Section 1.3. I discuss the numerical algorithm in Section 1.G.[32]

[32] The numerical work of Farhi and Werning (2013), Golosov, Troshkin, and Tsyvinski (2016), Stantcheva (2017) and Ndiaye (2018) importantly relies on a random walk specification for the skill process and a preference specification which ensures that the recursive formulation of the component planning problem scales with the previous skill realization. In my paper, the estimated skill process does not follow a random walk and the general home production preferences do not scale with the previous skill realization. In

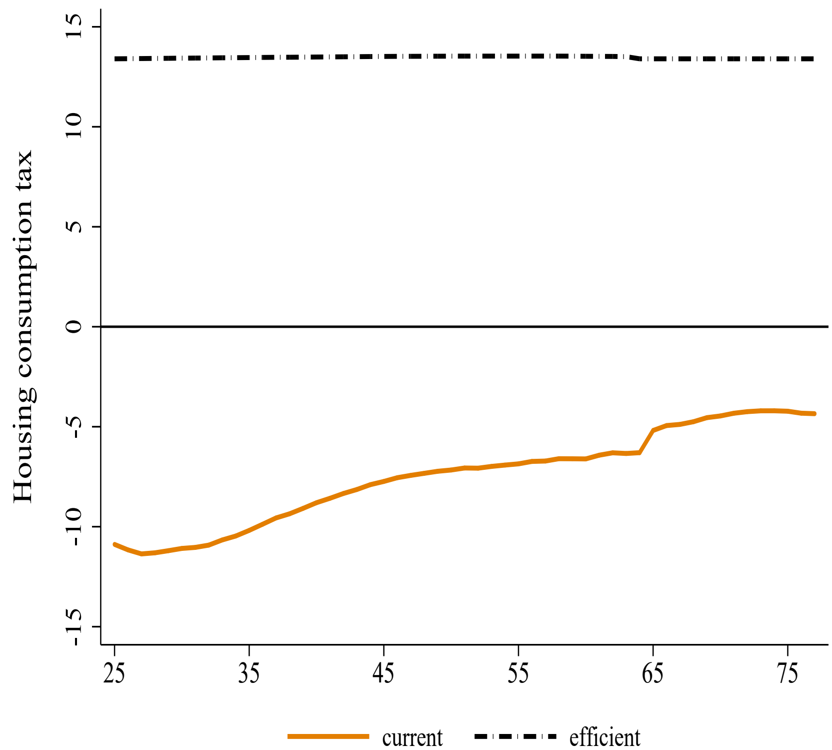


Figure 1.3: Efficient and Current Housing Policy

Figure 1.3 displays the average measured housing consumption subsidy under current policy (orange solid line) against an average efficient housing consumption tax (black dashed line) by household age.

Figure 1.3 shows that the efficient housing consumption wedge significantly differs from the user's cost under current policy. The orange solid line displays the current user's cost by household age as given by the data underlying Panel A in Table 1.6, while the black dashed line reports an average efficient housing consumption tax, which I obtain by evaluating a solution to the component planning problem. The average user's cost under current policy decreases from 12 percent to 5 percent over the life cycle, while the average efficient housing consumption tax is almost constant over the life-cycle, increasing from 13.5 percent to 13.8 percent.

The efficient user's cost is about 14 percent because the estimated elasticity of substitution between housing and leisure only implies a small complementarity. When the elasticity of substitution between housing and leisure equals one, the uniform commodity tax prescription of Atkinson and Stiglitz (1976) applies and so the efficient housing consumption

Section 1.G, I describe the algorithm that I use in detail.

Table 1.7: Simple Policy Reform

Implemented policies	Δc	Δf_h
Reduce transaction tax from 6 percent to 2 percent	2.48	2.03
Equalize mortgage interest deductability	0.23	0.30
Alternative proposals		
Increase imputed rent tax to 2%	0.18	0.35

Table 1.7 presents the steady-state welfare outcomes of simple policy reforms. The second column shows the lifetime non-housing consumption equivalent gain of simple policy reform; the third column displays the change in the homeownership rate.

wedge is equal to zero for every household in any efficient allocation. Holding constant the tax rate on non-housing consumption, the efficient housing consumption wedge equals zero only if the effective housing services tax is equal to the 13.4 percent tax on non-housing consumption. Quantitatively, I find that the difference from uniform commodity taxation is small, with an average efficient housing consumption wedge of 14 percent.

Simple Reforms. The implementation of an efficient reform requires cohort-specific and history-dependent taxes. I also use the model to simulate simple policy reforms to evaluate the long-run implications of policy reforms that were implemented by the Dutch government, and to design alternative simple reforms informed by the efficient reform.

Implemented Reform. In recent years, the government reduced both the transaction tax and the deductibility of home mortgage interest expenses. In 2011, during the recession, the transaction tax was lowered from 6 to 2 percent to spur the housing market. In 2014, the government started to reduce the deductibility of home mortgage payments. Specifically, it reduces the maximum rate at which mortgage interest payments can be deducted from 52 percent, the top income tax rate, to 37 percent, the lowest marginal income tax rate for workers, by 2023. The previous analysis indicates that these reforms move the effective tax rates on housing closer to the efficient tax rates. I use the model to evaluate the long-run effects of these policy changes on homeownership and household welfare by comparing steady states before and after policy changes. When I conduct these policy experiments, I hold constant the level of government debt and adjust the intercept of the income tax schedule to balance the government's budget.

Table 1.7 shows long-run consequences of simple policy reform. The second column shows the lifetime non-housing consumption equivalent gain of simple reform, while the third column displays the change in the homeownership rate. The first row shows that the welfare gain due to lowering the transaction tax is equal to 2.48 percent of lifetime non-housing consumption as households increase their consumption of housing services. A low transaction tax reduces the barrier to entry into homeownership with a small loss on public revenues. Transaction tax revenues on a given transaction fall, but this revenue loss is offset by increased property tax revenues as households live in larger houses, and increased transactions.

The second row of Table 1.7 shows that the reform of the mortgage interest deduction only generates a small increase in household welfare. Reducing the deductibility of mortgage interest expenses for high-income households increases welfare by 0.23 percent of lifetime consumption and slightly increases homeownership. This reform reduces the tax expenditure on high-income households, which is redistributed to households as a lump-sum transfer. The reduction in the home mortgage interest deduction hardly affects the decision rent or own or households that were previously homeowners, but allows marginal households to become homeowners.

Alternative Reform. I use the expression for the effective homeowner subsidy (1.35), together with the efficient average tax rate of 13.8 percent, to inform alternative policy reform. I approximate efficient average housing taxes by changing current tax parameters. I vary the imputed rent tax τ_o from 0.6 to 2 percent which only affects the housing wedge of homeowners.

The third row of Table 1.7 shows the consequences of increasing the imputed rent tax from 0.6 to 2 percent. The mechanism is similar to the reduction of the mortgage interest deduction. Increasing the imputed rent tax increases the user's cost for homeowners and tax revenues. The increased cost hardly affects the decision rent or own for the original homeowners, but the increased transfer allows some households close to the margin to become homeowners.

1.7 Conclusion

I the study efficient reform of housing policy in an overlapping generations economy with uninsurable wage risk, incomplete asset markets, home production, and housing transaction costs.

I use a dynamic Mirrlees theory to show that in any efficient allocation housing consumption of every household is taxed when housing consumption and non-market time are complements in home production. By taxing housing services additional non-market time is spent in a less desirable dwelling, which provides incentives to productive households to produce. I also use this theory to show that in any efficient allocation homeowners do not pay a transaction tax when they buy their house, but pay a tax or receive a subsidy when they sell their house. Specifically, the government subsidizes households when they sell their house after a bad skill realization, and taxes households when they sell their house after a good skill realization in order to prevent households from residing in a small residence because of private concerns over future transaction costs.

Using administrative records for all households in the Netherlands, I show that current policy effectively subsidizes housing consumption and taxes households when they buy their house. The average homeowner currently receives an 8 percent subsidy on their housing consumption, which decreases from 11 percent to 5 percent over the lifecycle, and faces a 6 percent transaction tax.

I quantify an efficient reform using the calibrated economy under current policy. I find that housing and non-market time are complements in home production, which translates into an average efficient housing consumption tax of 14 percent, which is almost constant over the lifecycle. A simple reform, which reduces the transaction tax from 6 to 2 percent, generates a welfare gain of 2.5 percent of steady-state consumption.

Housing Policy Reform

Appendix

Job Boerma

January 2019

Appendix 1.A Extensions of the Theory

In the main text I analyze a Beckerian framework in which goods and non-market time are inputs in the production of commodities that enter into household utility. In this appendix I show how the insights from the baseline analysis extend to a framework where time spent working in the market and at home directly enter into the household utility function as in Gronau (1977).

To show how the main insights extend, consider an economy with a market good c, a home commodity produced using housing services and non-market time $h(d, h_N)$, and leisure time $\ell = 1 - h_M - h_N$. I assume the planner observes the allocation of consumption c, housing services d, and labor supply y, but does not observe household skill θ, or time allocated to home production h_N. Household have preferences over market goods, the home commodity, and leisure. Preferences are continuous, strictly concave, and separable with respect to market consumption, and the home production technology is continuous and concave.

Given an allocation of market goods, housing services, and effective labor supply (c, d, y), household type θ chooses their non-market time $h_N \in [0, 1 - y/\theta)$ to maximize utility. By the maximum theorem, the value function is strictly concave, and the solution h_N is a continuous function in the allocation of housing services and effective labor supply (d, y). In sum, the value function is

$$v(c, d, y; \theta) = \max_{h_N} \; u\left(c, h(d, h_N), \ell\right) = u(c) + \tilde{h}(d, y). \tag{A.1}$$

Given (A.1), the analysis in the main text carries over to the framework where time spent working in the market and at home directly enter into the utility with the understanding that the indirect specification of the home technology in (A.1) differs from the direct specification of the home technology in (1.2).

Appendix 1.B Proof to Proposition 1

Proof. I show both directions by contradiction.

$\Rightarrow$ If an allocation x is efficient it solves the planner problem given $\mathcal{V}_j\big(x(j,\theta^{t-1});\theta^{t-1}\big)$ for all $i \in \mathcal{I}$ with a maximum of zero. Suppose x does not solve the planner problem and let $\hat{x}$ denote a solution to the planner problem. Because x is feasible, the allocation $\hat{x}$ generates strictly excess resources in the first period. Construct an alternative allocation $\tilde{x}$ identical to $\hat{x}$ but increase initial consumption such that the ICs are satisfied. The allocation $\tilde{x}$ strictly Pareto dominates x, which is a contradiction.

$\Leftarrow$ If an allocation x solves the planner problem given $\mathcal{V}_j\big(x(j,\theta^{t-1});\theta^{t-1}\big)$ for all $i \in \mathcal{I}$ with a zero maximum, then it is efficient. Suppose that x is not efficient, then there exists an alternative feasible allocation $\hat{x}$ such that all households are better off, with some household i strictly better off. Since allocation $\hat{x}$ is feasible and delivers at least $\mathcal{V}_j\big(x(j,\theta^{t-1});\theta^{t-1}\big)$ for all $i \in \mathcal{I}$, $\hat{x}$ is a candidate solution to the planner problem. Construct an alternative allocation $\tilde{x}$, which is equal to $\hat{x}$ but equally reduce initial consumption for household i that is strictly better off under $\hat{x}$ (such that the ICs are satisfied). Alternative allocation $\tilde{x}$ is feasible and generates excess resources in the initial period. This contradicts that x is a solution to the planner problem. ■

Appendix 1.C Derivation Wedges

I characterize the efficient labor and housing services wedge using the optimality conditions to the component planner problem. Recall that the component planner chooses $x_t(\theta) =$

$\{c_t(\theta), d_t(\theta), y_t(\theta), \mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta)\}$ to solve

$$\Pi_t(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) \equiv \max_{x_t(\theta)} \sum \pi^t(\theta|\theta_-)\Big(wy_t(\theta) - c_t(\theta) - p_j d_t(\theta) - \Psi\left(d_t(\theta), d\right) + \Pi_{t+1}(\mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta^+), d_t(\theta), \theta)/$$

where maximization is subject to (1.15)−(1.17):

$$\mathcal{V} = \sum \pi^t(\theta|\theta_-)\big(v\left(c_t(\theta)\right) + h\left(d_t(\theta), y_t(\theta)/\theta\right) + \beta\mathcal{V}_t(\theta)\big) \tag{1.15}$$

$$\tilde{\mathcal{V}} = \sum \pi^t(\theta|\theta_-^+)\big(v\left(c_t(\theta)\right) + h\left(d_t(\theta), y_t(\theta)/\theta\right) + \beta\mathcal{V}_t(\theta)\big) \tag{1.16}$$

$$v\left(c_t(\theta)\right) + h\left(d_t(\theta), y_t(\theta)/\theta\right) + \beta\mathcal{V}_t(\theta) \geq v\left(c_t(\theta^-)\right) + h\left(d_t(\theta^-), y_t(\theta^-)/\theta\right) + \beta\tilde{\mathcal{V}}_t(\theta)\ , \tag{1.17}$$

where I use that preferences are separable in consumption (1.1).

I denote the multiplier on the promise keeping constraint (1.15) by ν_t, the multiplier on the threat-keeping constraint (1.16) by μ_t, and multipliers on the downward incentive constraints (1.17) by $q_t(\theta_i)$, where θ_i denotes productivity realization $\theta_i \in (\theta_1, \ldots, \theta_N)$. The optimality conditions to the component planner problem for consumption, housing services, and effective hours are

$$[c_t(\theta_i)]\ \ \pi^t\left(\theta_i|\theta_-\right) = v_c(c_t(\theta_i))\left(\nu_t\pi^t\left(\theta_i|\theta_-\right) - \mu_t\pi^t\left(\theta_i|\theta_-^+\right) + q_t(\theta_i) - q_t(\theta_{i+1})\right) \tag{A.2}$$

$$\begin{aligned}[d_t(\theta_i)]\ \ \pi^t\left(\theta_i|\theta_-\right)p_j = &-\pi^t\left(\theta_i|\theta_-\right)\Psi_1\left(d_t(\theta_i), d\right) + h_d\big(d_t(\theta_i), y_t(\theta_i)/\theta_i\big)\left(\nu_t\pi^t\left(\theta_i|\theta_-\right) - \mu_t\pi^t\left(\theta_i|\theta_-^+\right) + q_t(\theta_i)\right. \\ &- h_d\big(d_t(\theta_i), y_t(\theta_i)/\theta_{i+1}\big)q_t(\theta_{i+1}) + \pi^t\left(\theta_i|\theta_-\right)\Pi_{t+1,3}(\mathcal{V}_t(\theta_i), \tilde{\mathcal{V}}_t(\theta_{i+1}), d_t(\theta_i), \theta_i)/R\end{aligned} \tag{A.3}$$

$$\begin{aligned}[y_t(\theta_i)]\ \ \pi^t\left(\theta_i|\theta_-\right)w = &-h_y\big(d_t(\theta_i), y_t(\theta_i)/\theta_i\big)\left(\nu_t\pi^t\left(\theta_i|\theta_-\right) - \mu_t\pi^t\left(\theta_i|\theta_-^+\right) + q_t(\theta_i)\right) \\ &+ h_y\big(d_t(\theta_i), y_t(\theta_i)/\theta_{i+1}\big)q_t(\theta_{i+1})\ .\end{aligned} \tag{A.4}$$

The optimality conditions for promised utility $\mathcal{V}_t(\theta_i)$ and threat utility $\tilde{\mathcal{V}}_t(\theta_i)$ are

$$[\mathcal{V}_t(\theta_i)]\ \ 0 = \pi^t\left(\theta_i|\theta_-\right)\Pi_{t+1,1}(\mathcal{V}_t(\theta_i), \tilde{\mathcal{V}}_t(\theta_{i+1}), d_t(\theta_i), \theta_i) + \beta R\left(\nu_t\pi^t\left(\theta_i|\theta_-\right) - \mu_t\pi^t\left(\theta_i|\theta_-^+\right) + q_t(\theta_i)\right) \tag{A.5}$$

$$[\tilde{\mathcal{V}}_t(\theta_i)]\ \ 0 = \pi^t\left(\theta_i|\theta_-\right)\Pi_{t+1,2}(\mathcal{V}_t(\theta_{i-1}), \tilde{\mathcal{V}}_t(\theta_i), d_t(\theta_{i-1}), \theta_{i-1}) - \beta R q_t(\theta_i). \tag{A.6}$$

The envelope conditions are

$$\Pi_{t,1}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = -\nu_t \tag{A.7}$$

$$\Pi_{t,2}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = \mu_t \tag{A.8}$$

$$\Pi_{t,3}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = -\sum \pi^t(\theta|\theta_-)\Psi_2\left(d_t(\theta), d\right). \tag{A.9}$$

It costs more resources to deliver a high promised utility, or excess resources decrease in the promised value, (A.7). It is cheap to stay below a high threat utility, or excess resources increase in the threat value, (A.8). Past housing services consumption decreases excess resources to the extent that current adjustment costs increase (A.9).

Housing Services Wedge and Labor Wedge. I obtain the housing services wedge and the labor wedge by manipulating the optimality conditions. I omit age script t when this does not cause confusion, and I use x_i to denote $x(\theta_i)$ for $x \in \{c, d, y\}$ and π_i and π_i^+ to abbreviate the conditional probability mass functions. The cumulative conditional probability mass function is abbreviated by $\pi_{\Sigma,i}$ and $\pi_{\Sigma,i}^+$.

Labor Wedge. To derive the labor wedge, I substitute the optimality condition for consumption (A.2) into the optimality condition for effective labor supply (A.4) to write

$$w = -\frac{h_y\big(d_i, y_i/\theta_i\big)}{v_c(c_i)} + \Delta h_y\big(d_i, y_i/\theta_{i+1}\big)\frac{q_{i+1}}{\pi_i}\ , \tag{A.10}$$

where $\Delta h_y\big(d_i, y_i/\theta_{i+1}\big)$ denotes the first difference in labor productivity. The efficient labor wedge is the distortion between the marginal rate of substitution of consumption for labor and the marginal product of labor (1.19), which thus satisfies

$$\tau_y = \Delta h_y\big(d_i, y_i/\theta_{i+1}\big)\frac{q_{i+1}}{w\pi_i}\ . \tag{A.11}$$

To simplify the labor wedge I note $q_{i+1} = -\sum\limits_{s=i+1}^{N}\big(q_{s+1} - q_s\big)$, where the difference between consecutive multipliers follows by rearranging the optimality condition for consumption (A.2),

$$q_{s+1} - q_s = (\nu - \mu)\,\pi_s - \mu\left(\pi_s^+ - \pi_s\right) - \pi_s \frac{1}{v_c(c_s)}\ . \tag{A.12}$$

Summing equation (A.12) over all labor productivity states, and by noting that $q_1 = q_{N+1} = 0$, this implies

$$\sum \pi_i \frac{1}{v_c(c_i)} = \nu - \mu. \tag{A.13}$$

To further characterize the labor wedge, I use the optimality condition for the threat value (A.6), the envelope condition for the threat value (A.8), and the expression for the labor wedge in (A.11), to write μ as

$$\mu = \beta R w \frac{\tau_{y,-}}{\Delta h_y\big(d_-, y_-/\theta_-^+\big)}\ . \tag{A.14}$$

The labor wedge is characterized by substituting (A.12), (A.13), and (A.14) into (A.11),

$$\tau_y = \Delta h_y\big(d_i, y_i/\theta_{i+1}\big)\frac{I_i}{w\pi_i} + \beta R \tau_{y,t-1}\frac{\pi_{\Sigma,i} - \pi_{\Sigma,i}^+}{\pi_i}\frac{\Delta h_y\big(d_i, y_i/\theta_{i+1}\big)}{\Delta h_y\big(d_-, y_-/\theta_-^+\big)}\ ,$$

where I_i is the insurance value (1.22). The labor wedge is the analog of the labor wedge in Golosov, Troshkin, and Tsyvinski (2016) for an economy with home production.

The efficient labor wedge is positive and balances the distortionary costs for type θ against the benefit of relaxing incentive constraints for all types above θ. By relaxing period t incentive constraints, a planner can provide additional insurance using resources extracted from households more productive than type θ. This insurance value of relaxing incentive constraint is given by I_i. The dynamic component captures that efficient labor wedges at age t relax incentive constraints at prior ages to the extent that decisions of a more productive household at a prior age are more likely to be distorted going forward.

Housing Services Wedge. To derive the housing wedge, I substitute the optimality condition for consumption (A.2), and the envelope condition for housing services (A.9), into the optimality condition for housing services consumption (A.3) to obtain

$$p_j + \Phi_1\left(d_i, d\right) = \frac{h_d\big(d_i, y_i/\theta_i\big)}{v_c(c_i)} - \Delta h_d\big(d_i, y_i/\theta_{i+1}\big)\frac{q_{i+1}}{\pi_i} - \sum \pi(\hat{\theta}|\theta_i)\Phi_2\big(d_{t+1}(\hat{\theta}), d_i)\big)/R\ .$$

The efficient housing services wedge (1.18) is thus

$$\tau_d = \frac{1}{p_j}\Delta h_d\big(d_i, y_i/\theta_{i+1}\big)\frac{q_{i+1}}{\pi_i} + \frac{1}{p_j}\sum \pi(\hat{\theta}|\theta_i)\Phi_2\big(d_{t+1}(\hat{\theta}), d_i\big)\left(\frac{1}{R} - \beta\frac{v_c(c_{t+1}(\hat{\theta}))}{v_c(c_i)}\right), \quad \text{(A.15)}$$

which by substituting (A.12), (A.13), and (A.14) is equivalent to,

$$\begin{aligned}\tau_d = {} & \Delta h_d\big(d_i, y_i/\theta_{i+1}\big)\frac{I_i}{\pi_i p_j} + \beta R \tau_{y,t-1}\frac{\pi_{\Sigma,i} - \pi^+_{\Sigma,i}}{\pi_i}\frac{w}{p_j}\frac{\Delta h_d\big(d_i, y_i/\theta_{i+1}\big)}{\Delta h_y\big(d_-, y_-/\theta^+_-\big)} \\ & + \frac{1}{p_j R}\sum \pi(\hat{\theta}|\theta_i)\Phi_2\big(d_{t+1}(\hat{\theta}), d_i\big)\left(1 - \beta R\frac{v_c(c_{t+1}(\hat{\theta}))}{v_c(c_i)}\right) \end{aligned} \quad \text{(A.16)}$$

Housing Capital and Business Capital Wedge. I characterize the efficient distortion on savings using a variational argument. The savings distortion applies to both business capital and housing capital and is obtained from the inverse Euler equation.

Consider an allocation $x(i)$ that solves the component planner problem for household i and fix a history θ^t. Consider the perturbed allocation $x^\delta(i) = (c^\delta(i), d^\delta(i), y^\delta(i))$, where the index $\delta > 0$ denotes the amount utility is decreased by at age $t+1$,

$$v(\underbrace{c(\theta^{t+1}) - \varepsilon(c(\theta^{t+1}), \delta)}_{\equiv c^\delta(\theta^{t+1})}) = v(c(\theta^{t+1})) - \delta$$

$$v(\underbrace{c(\theta^{t}) + \varepsilon(c(\theta^{t}), \delta)}_{\equiv c^\delta(\theta^{t})}) = v(c(\theta^{t})) + \beta\delta,$$

with $\big(d^\delta(\theta^s), y^\delta(\theta^s)\big) = \big(d(\theta^s), y(\theta^s)\big)$ for age t and age $t+1$. For every other history, the perturbed allocation is identical to the component planner solution.

The promise keeping constraint and the incentive constraints are satisfied under the perturbed allocation $x^\delta(i)$. The perturbed allocation increases utility at age t by $\beta\delta$ and decreases utility at $t+1$ by δ for histories passing through θ^t. Due to discounting the promise keeping constraint and the incentive constraints are both satisfied.

For small $\delta > 0$, I have $\varepsilon(c(\theta^{t+1}), \delta) = \delta / v_c(c(\theta^{t+1}))$ and $\varepsilon(c(\theta^t), \delta) = \beta\delta / v_c(c(\theta^t))$, and hence the change in excess resources given by

$$\pi(\theta^t)\left(\left(\frac{1}{R}\right)^{t-1}\frac{\beta\delta}{v_c(c(\theta^t))} - \left(\frac{1}{R}\right)^{t}\sum \pi^{t+1}(\theta_{t+1}|\theta_t)\frac{\delta}{v_c(c(\theta^{t+1}))}\right) . \tag{A.17}$$

At the solution to the component planner problem such a perturbation does not generate excess resources. In other words, the derivative of excess resources with respect to δ equals zero at the solution, which gives the inverse Euler equation

$$\frac{1}{v_c(c(\theta^t))} = \frac{1}{\beta R}\sum \pi^{t+1}(\theta_{t+1}|\theta_t)\frac{1}{v_c(c(\theta^{t+1}))} . \tag{A.18}$$

Given the definition of the savings wedge (1.20), the efficient intertemporal distortion is

$$1 - \tau_s(\theta^t) = \frac{\left(\sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right)\left(v_c(c(\theta^{t+1}))\right)^{-1}\right)^{-1}}{\sum \pi^{t+1}\left(\theta_{t+1}|\theta_t\right) v_c(c(\theta^{t+1}))}. \tag{A.19}$$

Because the utility from consumption v is strictly concave, the savings wedge is positive.

Appendix 1.D Definition of Equilibrium

Given a government expenditures sequence $\{G_j\}$, government policy, planned housing projects $\{P^H_{1-\upsilon}\}_{\upsilon=1}^{\iota}$, an initial savings distribution $\{s_1(i)\}_{\mathcal{I}}$, and initial assets $\{(B_0, H_0, K_0)\}$, an equilibrium consists of a price sequence $\{(w_j, r_j, p_j, p^H_j)\}$ and allocation $x^e \equiv \Big\{\{x^e(i)\}_{\mathcal{I}}, \{(A_j, B_j, C_j, D_j,$ where the equilibrium allocation for individual $i \in \mathcal{I}$ is

$$x^e(i) = \left\{\left(a_{j+v}(\theta^{t+v}), c_{j+v}(\theta^{t+v}), d_{j+v}(\theta^{t+v}), h_{j+v}(\theta^{t+v}), m_{j+v}(\theta^{t+v}), s_{j+v}(\theta^{t+v}), y_{j+v}(\theta^{t+v})\right)\right\}_{v=0}^{T-t} ,$$

such that:

1. Allocation functions $\{a_t, c_t, d_t, h_t, m_t, y_t, s_t\}$ solve the household maximization problem

2. Aggregate quantities are consistent with individual decision rules

$$A_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) a_j(\theta^t) \qquad H_j^o = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) h_j(\theta^t)$$

$$C_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) c_j(\theta^t) \qquad M_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) m_j(\theta^t)$$

$$D_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) d_j(\theta^t) \qquad S_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) s_j(\theta^t)$$

$$Y_j = \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) y_j(\theta^t)$$

3. Factor prices are consistent with the firm maximization problem

$$r_j = F_K(K_j, Y_j) - \delta^k$$

$$w_j = F_Y(K_j, Y_j)$$

4. The rental price of housing, p_j, is consistent with the rental firm's maximization problem
5. The house price, p_j^H, is consistent with the construction firm's maximization problem
6. The goods market and local housing market clear every period

$$C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} = F\left(K_j, Y_j\right) + RB_j$$

$$D_j = \chi H_j$$

where $I_j^K = K_{j+1} - (1-\delta^K)K_j$, $I_j^H = P_{j+1-\iota}^H + \delta^H H_j$.

7. The government budget constraint is satisfied, and $\lim_{j\to\infty} B_j/R^j \in [0, \infty)$.

Steady State Characterization. Given the equilibrium definition, I characterize a steady state.

1. By the firm's problem, the interest rate pins down the capital-labor ratio and the wage. The problem of the construction firm determines the house price, $p^H = 1$, and the landlord problem pins down rental price p.
2. Given prices and government policy, the household problem gives solution $\{a_t, c_t, d_t, h_t, m_t, y_t, s_t\}$.

Table A.1: National Income Product Accounts, 2006−2014

Total Adjusted Income	1.000
Labor Income	0.561
Compensation of Employees	0.501
Wages and Salary	0.395
Supplements to Wages and Salary	0.106
70% of prorietors' income	0.060
Capital Income	0.439
Profits	0.165
30% of prorietors' income	0.026
Indirect Business Taxes	0.102
Sales Tax	0.098
Consumption of Fixed Capital	0.168
Consumer Durable Depreciation	0.041
Imputed Capital Services	0.035
Consumer Durable Services	0.011
Government Capital Services	0.024

Table A.1 provides headline statistics for national income following the income approach. Author's calculations using aggregate data from Statistics Netherlands.

3. Use solution to household problem to obtain aggregate quantities $(A, B, C, D, H, H^o, M, Y, S, \Phi, K)$.
4. Total private savings is S, and the domestic housing stock is $H = D/\chi = H^o + H^r$. Domestic savings are the sum of private and public savings, and the domestic capital stock is the sum of business capital and housing capital. Given net foreign assets, public savings are determined.

Appendix 1.E Aggregate Data

I use data from the national income and product accounts to measure the aggregate capital income share, the consumption-output ratio, and the expenditure share of housing in total consumption. In Section 1.5, I use these moments to calibrate the capital share in production, α, the preference weight for consumption, γ, and the weight of housing services in home production, ω. I construct the moments using data that are publicly available through Statistics Netherlands' Statline.

Table A.2: National Income Product Accounts, 2006-2014

Total Adjusted Product	1.000
Consumption	0.642
Personal consumption expenditures	0.462
Less: Consumer durable goods	0.055
Less: Imputed sales tax, nondurables and services	0.087
Plus: Imputed capital services, durables	0.011
Government consumption expenditures, nondefense	0.246
Plus: Imputed capital services, government capital	0.024
Consumer durable depreciation	0.041
Tangible investments	0.343
Gross private domestic investments	0.166
Consumer durable goods	0.055
Less: Imputed sales tax, durables	0.011
Government gross investment	0.040
Net exports of goods and services	0.093
Defense spending	0.013

Table A.2 provides headline statistics for national income following the product approach. Author's calculations using aggregate data from Statistics Netherlands.

National Income and Product Accounts. I measure the capital share using the income accounts and the consumption-output ratio using the national product accounts. In Table A.1, I split national income between labor income and capital income. Labor income includes the compensation of employees and 70 percent of proprietors' income, while all other forms of income are categorized as capital income.

Capital income is adjusted to align my model with the data. First, I subtract sales taxes to measure production at producer prices rather than consumer prices. Second, I impute capital services for consumer durables and government capital. The imputed services are assumed to be 4 percent of the current-cost net stock of consumer durables and government fixed assets. Finally, I impute the depreciation of consumer durables. Since the depreciation rate of consumer durables is not available for the Netherlands, I assume the depreciation rate is equal to 5 percent which is the corresponding rate for the United States as calculated in McGrattan and Prescott (2017). I find a capital income share of 0.439, which is the value I choose for α.

On the production side, shown in Table A.2, I also adjust for sales taxes, capital services, and consumer durables depreciation. I assume that sales taxes primarily fall on personal consumption expenditures, and I allocate proportionally to durable goods, non-durable goods and services. Non-durable goods and services are consumption while durable goods are a tangible investment. Imputed capital services increase aggregate consumption, the sum of personal and government consumption from the national accounts. Consumption of consumer durables depreciates the outstanding stock, which motivates me to classify consumer durables depreciation as consumption. The consumption-output ratio equals 0.642. I use this number to calibrate the preference weight on consumption.

Expenditure Share on Housing. To calibrate the share on housing services in the home technology ω, I measure the expenditure share on housing as a fraction of total consumption. In Figure A.1, I show the expenditure share on housing between 1995 and 2015. The expenditure share on housing is relatively stable at 16 percent until the beginning of the housing crisis, but equals 18 percent on average after 2008. I target the average expenditure share between 2006 and 2014 of 17.4 percent.[33]

Appendix 1.F Household Problem

In this appendix I derive the user's cost for renters and homeowners and I characterize the solution to the household problem to obtain the estimation equation.

User's Cost. The user's cost, the cost of a marginal unit of housing, is obtained by differentiating the budget constraint with respect to housing capital. I assume that the household does not incur a transaction cost on the marginal unit of housing capital.

Homeowners. To derive the homeowner's user cost, it is useful to rewrite their budget constraint, (1.28), by adding and subtracting the gross market return on the investment in their house, $Rp^H h_t$. By recalling that homeowners hold their savings as financial assets, housing wealth and mortgages, $s_t = a_t + p^H h_t - m_t$, and by recalling the definition of

[33] The expenditure share in the Netherlands is close to the expenditure share on housing in the United States. Piazzesi and Schneider (2016) report a mean housing share of 17.8 percent between 1959 and 2014.

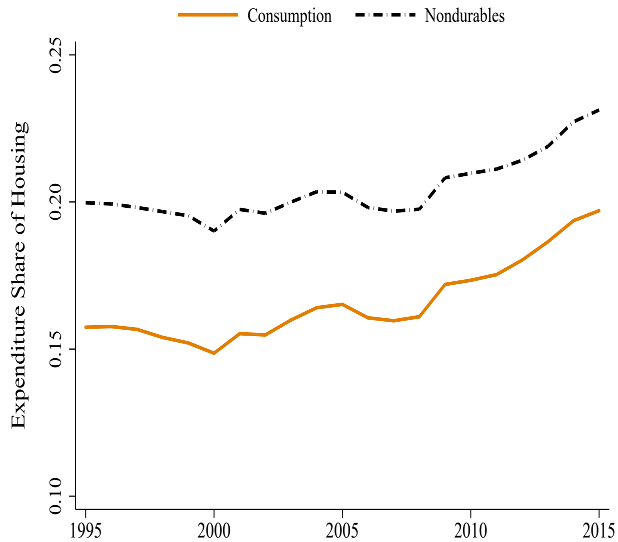


Figure A.1: Expenditure Share on Housing

Figure A.1 displays the expenditure share on housing in the Netherlands between 1995 and 2015. The solid orange line shows the expenditures on housing services in proportion to total consumption, the black dashed line shows the expenditures on housing services in proportion to the consumption of nondurables.

before-tax income (1.25), I write

$$c_t + T_t^c(c_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = wy_t - T_t^y(wy_t + b_t + \tau_o p^H h_t - rm_t) + Rs_t - T_t^a\left(s_t - p^H h_t + m_t\right) + T_t$$
$$+ \left(\Delta p^H - \tau_p p^H - \delta^H - p^H r\right) h_t.$$

I calculate the user's cost for homeowners holding constant the fraction of the property that is debt-financed, which I denote $\varkappa \equiv m/(p^H h)$. Furthermore, I denote the marginal income tax rate by $\hat{\tau}_y \equiv T_{1,t}^y\big(wy_t + b_t + \tau_o p^H h_t - rm_t\big)$ and the marginal tax rate on wealth by $\hat{\tau}_a \equiv T_{1,t}^a\big(s_t - p^H h_t - m_t\big)$. Hence, the user's cost for homeowners is

$$p_j^o = r + \tau_p + \delta^H - \pi_{j+1}^H - \hat{\tau}_y r \varkappa + \hat{\tau}_y \tau_o - \hat{\tau}_a(1 - \varkappa). \tag{1.35}$$

Renters. I obtain the user's cost for renters by using their budget constraint (1.26), and the market price for rental services (1.31). Using the budget constraint,

$$c_t + T^c(c_t) + pd_t + T^d_t(pd_t, \tilde{y}_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = \tilde{y}_t - T^y_t(\tilde{y}) + Rs_t - T^a_t(Rs_t) + T_t,$$

the marginal cost of housing services for non-moving renters is,

$$p^r = \big(r(1-\tau_r) - \pi^H + \hat{\tau}_p + \delta^H\big)\big(1 + \hat{\tau}_d\big), \tag{1.36}$$

where $\hat{\tau}_d \equiv T^d_{1,t}\big(pd_t, \tilde{y}_t\big)$.

Appendix 1.G Computation Component Planning Problem

I discuss the numerical approach to solving the planner problem. I scale the program to obtain a state space that is stable across ages, and transform the problem to a multiplier grid. To simplify notation, I omit age script t when this does not cause confusion. Further, I use x_i to denote $x(\theta_i)$ for $x \in \{c, d, y, \mathcal{V}, \tilde{\mathcal{V}}\}$ and π_i and π_i^+ to abbreviate the conditional probability mass functions.

Consider the profit maximization problem in state $(\mathcal{V}_-, \tilde{\mathcal{V}}_-, d_-, \theta_-, t)$:

$$\Pi_t(\mathcal{V}_-, \tilde{\mathcal{V}}_-, d_-, \theta_-) \equiv \max \sum \pi_i \Big(w_j y_{it} - c_{it} - p_j d_{it} - \Phi\big(d_{it}, d_-\big) + \Pi_{t+1}\big(\mathcal{V}_{it}, \tilde{\mathcal{V}}_{i+1t}, d_{it}, \theta_i\big)/R_j \Big)$$

where the choice variable is $x_{it} = \{c_{it}, d_{it}, y_{it}, \mathcal{V}_{it}, \tilde{\mathcal{V}}_{it}\}$, and maximization is subject to:

$$\begin{aligned}
& u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\mathcal{V}_{it} = u\left(c_{i-1t}, d_{i-1t}, y_{i-1t}/(\theta_i\zeta_t)\right) + \beta\tilde{\mathcal{V}}_{it} \qquad \forall\ i = 2, \ldots, N \\
& \mathcal{V}_- = \sum_i \pi_i \Big(u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\mathcal{V}_{it} \Big) \\
& \tilde{\mathcal{V}}_- = \sum_i \pi_i^+ \Big(u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\mathcal{V}_{it} \Big).
\end{aligned}$$

The deterministic age profile of productivity is captured by ζ_t.

To solve the life-cycle program, I ensure that the promised utility and the threat utility lie on a time-invariant grid by scaling remaining lifetime values by the geometric sum of current and future discount factors. I use $\beta_t \equiv 1+\beta+\cdots+\beta^{T-t}$ to denote the geometric sum of current and future discount factors at time t. The transformation ensures that promised utility and threat utility are measured in per period units rather than as remaining lifetime values. Formally, I define the scaled promised value by $\hat{\mathcal{V}}_{it} \equiv \mathcal{V}_{it}/\beta_{t+1}$ and the scaled threat value by $\hat{\tilde{\mathcal{V}}}_{it} \equiv \tilde{\mathcal{V}}_{it}/\beta_{t+1}$. These definitions imply $\hat{\mathcal{V}}_{-} = \mathcal{V}_{-}/\beta_t$ and $\hat{\tilde{\mathcal{V}}}_{-} = \tilde{\mathcal{V}}_{-}/\beta_t$. I scale the objective function in the same way, or $\hat{\Pi}_t(\hat{\mathcal{V}}_{-}, \hat{\tilde{\mathcal{V}}}_{-}, d_{-}, \theta_{-}) \equiv \Pi_t(\hat{\mathcal{V}}_{-}, \hat{\tilde{\mathcal{V}}}_{-}, d_{-}, \theta_{-})/\beta_t$.

Claim 1. The component planner problem is equivalent to the following scaled program:

$$\hat{\Pi}_t(\hat{\mathcal{V}}_{-}, \hat{\tilde{\mathcal{V}}}_{-}, d_{-}, \theta_{-}) \equiv \max \sum \pi_i \left(\frac{1}{\beta_t}\big(w_j y_{it} - c_{it} - p_j d_{it} - \Phi\big(d_{it}, d_{-}\big)\big) + \frac{\beta_{t+1}}{\beta_t}\hat{\Pi}_{t+1}\big(\hat{\mathcal{V}}_{it}, \hat{\tilde{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i\big)/R_j \right)$$

where the choice variable is $x_{it} = \{c_{it}, d_{it}, y_{it}, \hat{\mathcal{V}}_{it}, \hat{\tilde{\mathcal{V}}}_{it}\}$, and maximization is subject to:

$$\frac{1}{\beta_t} u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\frac{\beta_{t+1}}{\beta_t}\hat{\mathcal{V}}_{it} = \frac{1}{\beta_t} u\left(c_{i-1t}, d_{i-1t}, y_{i-1t}/(\theta_i\zeta_t)\right) + \beta\frac{\beta_{t+1}}{\beta_t}\hat{\tilde{\mathcal{V}}}_{it} \quad \forall\, i = 2, \ldots, N \quad (q_i) \tag{A.20}$$

$$\hat{\mathcal{V}}_{-} = \sum_i \pi_i \left(\frac{1}{\beta_t} u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\frac{\beta_{t+1}}{\beta_t}\hat{\mathcal{V}}_{it} \right) \quad (\nu_t) \tag{A.21}$$

$$\hat{\tilde{\mathcal{V}}}_{-} = \sum_i \pi_i^{+} \left(\frac{1}{\beta_t} u\left(c_{it}, d_{it}, y_{it}/(\theta_i\zeta_t)\right) + \beta\frac{\beta_{t+1}}{\beta_t}\hat{\mathcal{V}}_{it} \right) \quad (\mu_t) \tag{A.22}$$

Proof. Equivalence follows by dividing the objective function and all constraints of the component planner problem by β_t, and by multiplying and dividing by β_{t+1} the choices for the promised and the threat values, and the period $t+1$ objective function. ■

I characterize the solution to the program through its first-order optimality conditions. The optimality conditions with respect to consumption, housing services, and output are:

$$\pi_i = v_c(c_{it})\Big(\nu_t\pi_i - \mu_t\pi_i^{+} + q_i - q_{i+1}\Big) \tag{A.23}$$

$$w_j\pi_i = h_\ell\left(d_{it}, \ell_{it}\right)\frac{1}{\theta_i\zeta_t}\Big(\nu_t\pi_i - \mu_t\pi_i^{+} + q_i\Big) - h_\ell\left(d_{it}, \ell_{it}^{+}\right) q_{i+1}\frac{1}{\theta_{i+1}\zeta_t} \tag{A.24}$$

$$\begin{aligned} p_j\pi_i = {} & -\pi_i\Phi_1\left(d_{it}, d_{-}\right) + h_d\left(d_{it}, \ell_{it}\right)\Big(\nu_t\pi_i - \mu_t\pi_i^{+} + q_i\Big) - h_d\left(d_{it}, \ell_{it}^{+}\right) q_{i+1} \\ & + \beta_{t+1}\pi_i\hat{\Pi}_{t+1,3}(\hat{\mathcal{V}}_{it}, \hat{\tilde{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i)/R_j. \end{aligned} \tag{A.25}$$

The optimality conditions for the promise utility and the threat utility are:

$$
\begin{aligned}
0 &= \pi_i \hat{\Pi}_{t+1,1}(\hat{\mathcal{V}}_{it}, \hat{\hat{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i) + \beta R_j \left(\nu_t \pi_i - \mu_t \pi_i^+ + q_i\right) \\
0 &= \beta R_j q_i - \pi_{i-1} \hat{\Pi}_{t+1,\tilde{\mathcal{V}}}(\hat{\mathcal{V}}_{i-1t}, \hat{\hat{\mathcal{V}}}_{it}, d_{i-1t}, \theta_{i-1}) \ .
\end{aligned}
$$

Before I characterize the solution to the dynamic program over the life-cycle, I rewrite some optimality conditions in ways that are useful. First, I write the optimality condition for consumption as:

$$
\pi_i \left(\frac{1}{u_c(c_{it})} - \nu_t \right) = q_i - q_{i+1} - \mu_t \pi_i^+, \tag{A.26}
$$

Summing over all states at age t, and realizing that $q_1 = 0$ because the lowest type cannot pretend to be less productive, I obtain a restriction on the sum of the multipliers,

$$
\sum \pi_i \frac{1}{u_c(c_{it})} = \nu_t - \mu_t \ . \tag{A.27}
$$

Furthermore, it is useful to write the envelope conditions for promised utility and threat utility as:

$$
\begin{aligned}
\hat{\Pi}_{t,1}(\hat{\mathcal{V}}_-, \hat{\hat{\mathcal{V}}}_-, d_-, \theta_-) &= -\nu_t \\
\hat{\Pi}_{t,2}(\hat{\mathcal{V}}_-, \hat{\hat{\mathcal{V}}}_-, d_-, \theta_-) &= \mu_t \ .
\end{aligned}
$$

I use the envelope conditions to eliminate the derivates of the value function for the promised utility and the threat utility in the system of equations by incorporating the choice variables

$$
\begin{aligned}
-\nu_{it+1} &= \hat{\Pi}_{t+1,1}\big(\hat{\mathcal{V}}_{it}, \hat{\hat{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i\big) \\
\mu_{it+1} &= \hat{\Pi}_{t+1,2}\big(\hat{\mathcal{V}}_{it}, \hat{\hat{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i\big) \ .
\end{aligned}
$$

As a result, the optimality conditions for the promised utility and the threat utility are written as:

$$\nu_{it+1} = \beta R_j \left(\nu_t \pi_i - \mu_t \pi_i^+ + q_i\right) / \pi_i \tag{A.28}$$

$$\mu_{i-1t+1} = \beta R_j q_i / \pi_{i-1}. \tag{A.29}$$

I use this observation to solve the system of optimality conditions given states (ν_t, μ_t, d_-) instead of states $(\hat{\mathcal{V}}_-, \hat{\hat{\mathcal{V}}}_-, d_-)$. I note that (A.24), (A.25), (A.26), (A.28), (A.29), and the local incentive constraints (A.20) form a system of $6N-2$ equations and unknowns given state (ν_t, μ_t, d_-). I use the equations to characterize $6N-2$ unknowns $\{\{(c_{it}, d_{it}, y_{it}, \nu_{it+1})\}_{i=1}^{N}, \{(\mu_{it+1}, q_{i+1})\}_{i=1}^{N-1}\}$. After characterizing the unknowns, I evaluate the value of the profit function, and residually determine the implied promised and threat value by using the promise keeping condition (A.21) and the threat-keeping condition (A.22).

Final Work Period with Retirement. In the final work period no threat values are chosen since there is no difference in the productivity distribution next period which the government can exploit to distinguish productivity differences today. As a result, the incentive constraints feature only promised values. For period $t = T_W$, the planner problem is thus

$$\hat{\Pi}_t(\hat{\mathcal{V}}_-, \hat{\hat{\mathcal{V}}}_-, d_-, \theta_-) \equiv \max \sum \pi_i \left(\frac{1}{\beta_t} \big(w_j y_{it} - c_{it} - p_j d_{it} - \Phi\big(d_{it}, d_-\big)\big) + \frac{\beta_{t+1}}{\beta_t} \hat{\Pi}_{t+1}\big(\hat{\mathcal{V}}_{it}, d_{it}, \theta_i\big)/R_j \right),$$

where maximization is subject to

$$\frac{1}{\beta_t} u\left(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)\right) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{it} = \frac{1}{\beta_t} u\left(c_{i-1t}, d_{i-1t}, y_{i-1t}/(\theta_i \zeta_t)\right) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{i-1t}\ ,$$

the promise keeping condition (A.21) and the threat-keeping constraint (A.22). In this case, the first-order conditions are given by (A.23), (A.24), (A.25),

$$\nu_{it+1} = \big(\beta R_j \left(\nu_t \pi_i - \mu_t \pi_i^+\right) + q_i - q_{i+1}\big) / \pi_i, \tag{A.30}$$

the promise keeping condition (A.21), threat keeping condition (A.22), and the incentive constraints (A.20).

I compute the solution using the Newton-Raphson method over $2N-1$ variables. I provide a guess for the bottom $N-1$ elements of the consumption allocation c_{T_W} and I guess the housing services consumption vector d_{T_W}. Given guess $\{c_{iT_W}\}_{i=1}^{N-1}$ and states (ν_{T_W}, μ_{T_W}), (A.27) generates consumption at the top c_{NT_W}. Given consumption c_{T_W}, and state (ν_{T_W}, μ_{T_W}), I solve for q using the optimality condition for consumption (A.26), with $q_1 = 0$. Given q, I use the optimality condition with respect to the promised utility (A.30) to obtain the state for retirement ν_{iT_W+1}. Given housing services consumption d_{T_W}, and the state for retirement ν_{iT_W+1}, I obtain the implied promised value and profit function from equations (A.21) and (A.22).

I observe from the first-order optimality conditions that the high productivity type's marginal decisions are undistorted as $q_{N+1} = 0$, implying $\frac{h_\ell(d_{NT_W}, \ell_{NT_W})}{h_d(d_{NT_W}, \ell_{NT_W})} = \theta_N \zeta_{T_W} w/\tilde{p}$, and identifying ℓ_{NT_W}. Given all future values, I generate $\{\ell_{iT_W}, \ell_{iT_W}^+\}_{i=1}^{N-1}$ by backward iteration using the local incentive constraints

$$v\left(c_{it}\right) + h\left(d_{it}, \ell_{it}\right) + \beta\beta_{t+1}\hat{\mathcal{V}}_{it} = v\left(c_{i-1t}\right) + h\left(d_{i-1t}, \ell_{i-1t}^+\right) + \beta\beta_{t+1}\hat{\mathcal{V}}_{i-1t}\,.$$

I generated $3N$ unknowns, $\{c_{NT_W}, \{y_{iT_W}, \nu_{iT_W+1}\}_{i=1}^{N}, \{q_{i+1}\}_{i=1}^{N-1}\}$, which leaves $2N-1$ residual equations. I iterate until convergence using the optimality conditions for housing services (A.25), and the bottom $N-1$ optimality conditions for output (A.24). The promise-keeping condition (A.21) and threat-keeping constraint (A.22) are used to residually determine the promised value $\hat{\mathcal{V}}_{-}$, and the threat value $\hat{\hat{\mathcal{V}}}_{-}$.

Intermediate Period. I solve the program at each point in the state space (ν_t, μ_t, d_{t-1}), taking as given the value function for the next period.

The equations that characterize the solution are the optimality conditions with respect to consumption (A.23), output (A.24), housing services (A.25), promised utility (A.28) and threat utility (A.29), the promise keeping condition, the threat keeping condition, and the local downward ICs. I compute the solution using the Newton-Raphson method over $2N-1$ variables. I guess the first $N-1$ elements of c_t and the allocation of housing services d_t.

The guess is the solution at the state (ν_t, μ_t, d_{t-1}) in the following period. Given a guess $\{c_{it}\}_{i=1}^{N-1}$ and a state (ν_t, μ_t, d_{t-1}), (A.27) generates consumption at the top c_{Nt}. Given consumption c_t, and a state (ν_t, μ_t, d_{t-1}), I solve for q using the optimality condition for consumption (A.26), with $q_1 = 0$. Given d and q, we use the first-order condition with respect to the promised utility (A.28) and the threat utility (A.29) to obtain next period's states. Given the state for next period $(\nu_{it}, \mu_{it}, d_{it})$, and a realization for labor productivity θ_i, I use the implied promised value and threat value from the value function to obtain $(\hat{\mathcal{V}}_{it}, \hat{\hat{\mathcal{V}}}_{it}, \theta_i)$.

I observe from the first-order optimality conditions that the high productivity type's marginal decisions are undistorted as $q_{N+1} = 0$, implying $\frac{h_\ell(d_{Nt}, \ell_{Nt})}{h_d(d_{Nt}, \ell_{Nt})} = \theta_N \zeta_t w / \tilde{p}$, and identifying ℓ_{Nt}. Given all future values, I generate $\{\ell_{it}, \ell_{it}^+\}_{i=1}^{N-1}$ by backward iteration using the local incentive constraints

$$v(c_{it}) + h(d_{it}, \ell_{it}) + \beta\beta_{t+1}\hat{\mathcal{V}}_{it} = v(c_{i-1t}) + h\left(d_{i-1t}, \ell_{i-1t}^+\right) + \beta\beta_{t+1}\hat{\hat{\mathcal{V}}}_{it} .$$

I generated $4N-1$ unknowns, $\{c_{Nt}, \{y_{it}, \nu_{it+1}\}_{i=1}^{N}, \{(\mu_{it+1}, q_{i+1})\}_{i=1}^{N-1}\}$, which leaves $2N-1$ residual equations. I iterate until convergence using the first-order conditions for housing services (A.25), and the bottom $N-1$ optimality conditions for output (A.24). The promise-keeping condition (A.21) and threat-keeping constraint (A.22) are used to residually determine the promised value $\hat{\mathcal{V}}_-$, and the threat value $\hat{\hat{\mathcal{V}}}_-$.

First Period. The algorithm in the first period is identical to the algorithm in the middle period, where in the initial period $\mu_1 = 0$, and there are no adjustment costs.

Chapter 2

Inferring Inequality with Home Production

Job Boerma and Loukas Karabarbounis

2.1 Introduction

A substantial body of research examines the causes, welfare consequences, and policy implications of the pervasive dispersion across households in their labor market outcomes.[1] The literature trying to understand the dispersion in wages, hours worked, and consumption across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. It is well known, however, that households spend roughly half as much time in home production activities such as child care, shopping, and cooking as in the market.

While it is understood that home production of goods and services introduces, on average, a gap between household consumption recorded in official statistics and standards of living, little is known about how differences in home production across households affect inequality in standards of living. A priori there are good reasons why home production can

[1]See Heathcote, Perri, and Violante (2010) and Attanasio and Pistaferri (2016) for empirical regularities on household heterogeneity in labor market outcomes.

change the inferences economists draw from observing dispersion in labor market outcomes. To the extent that households are willing to substitute between market expenditures and time in the production of goods and services, home production will tend to compress welfare differences that originate in the market sector. However, to the extent that household differences in the home sector remain uninsurable and are large relative to the market sector, the home sector itself may emerge as an additional source of welfare differences across households.

We show that incorporating home production in a model with uninsurable risk and incomplete asset markets changes the inferred sources of heterogeneity across households, alters meaningfully the welfare consequences of dispersion, and leads to different policy conclusions. Surprisingly, we infer that inequality across households is larger than what one would infer without incorporating home production.[2] We reach this conclusion because, for households of all ages, the time input in home production does not covary negatively with consumption and wages in the cross section of households and production efficiency differences in the home sector are large. Thus, home production does not offset differences that originate in the market sector. Rather, home production amplifies these differences.

We develop our findings using a general equilibrium model with home production, heterogeneous households facing idiosyncratic risk, and incomplete asset markets. In the spirit of Ghez and Becker (1975), households produce goods with a technology which uses as inputs both expenditures and time. In the home sector, households are heterogeneous with respect to their disutility of work and their production efficiency. Home production is not tradeable and there are no assets households can purchase to explicitly insure against differences that originate in the home sector. In the market sector, households are also heterogeneous with respect to their disutility of work and their productivity. The structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences. We retain tractability and prove identification by extending the no-trade result with respect to certain assets for the one-sector model

[2] We use the term dispersion to refer to the variation in observed outcomes (such as time allocation, consumption expenditures, and wages) or inferred sources of heterogeneity (such as permanent or transitory productivity and taste shifters). We use the term inequality to refer to the mapping from dispersion to measures which capture welfare differences across households.

of Heathcote, Storesletten, and Violante (2014) to our model embedding multiple sectors. Therefore, we can characterize the allocations of time and consumption goods in closed form without simultaneously solving for the wealth distribution.

At the core of our approach lies an observational equivalence theorem which allows us to compare our model with home production to a nested model without home production. The observational equivalence theorem states that both models account perfectly for any given cross-sectional data on three observables: consumption expenditures, time spent working in the market, and market productivity (wages). However, the inferred sources of heterogeneity generating these data and inequality will in general differ between the two models. It is essential for our purposes that the two models are observationally equivalent because any differences between the two models is exclusively driven by structural factors and not by their ability to account for cross-sectional data on labor market outcomes.

We infer heterogeneity in market productivity and disutility of market work such that the allocations generated by the standard model without home production match the cross-sectional data on the three observables. Then, we infer the sources of heterogeneity such that the allocations generated by the model with home production match the same cross-sectional data and, additionally, time spent on home production activities. Separating disutility of work from production efficiency at home presents a challenge for home production models because, unlike expenditures and time inputs, the output of the home sector is not directly observable. Our solution to this identification problem is to pose that some of the cross-sectional differences in time spent working at home are driven by heterogeneity in production efficiency and the remaining differences are driven by heterogeneity in the disutility of work.

To quantify the role of home production for inequality, we use U.S. data between 1995 and 2016 on consumption expenditures, time spent on the market sector, and market productivity from the Consumption Expenditure Survey (CEX). The CEX does not contain information on time spent on home production. To overcome this problem, we use data from the American Time Use Survey (ATUS) to impute individuals' time spent on home production based on observables which are common between the two surveys. For our identification, we allow households to have different work disutility over some time activities such as cooking and cleaning because we find that these activities map closest to occupations which are intensive in manual skills. By contrast, other time activities such as child care and nursing are less intensive in manual skills and, so, we allow households to have different production efficiencies in them.

The key result of our analysis is that the world is more unequal than we thought when we take into account home production. We arrive at our conclusion using four ways to map dispersion in labor market outcomes into welfare-based measures of inequality. First, the standard deviation of equivalent variation across households is roughly 15 percent larger when we incorporate home production. Second, equalizing marginal utilities across households requires transfers with a standard deviation roughly 30 percent higher in the model with home production than in the model without home production. Third, an unborn household is willing to sacrifice 12 percent of lifetime consumption in order to eliminate heterogeneity in an environment with home production, compared to 6 percent in an environment without home production. Finally, taking into account home production, a utilitarian government would choose a more progressive tax system. For example, a household earning 200,000 dollars would face an average tax rate of 19 percent with home production, compared to 12 percent without home production. One way to understand our inequality result is in terms of the distinction between consumption and expenditures emphasized by Aguiar and Hurst (2005). We find that expenditures are less dispersed than the market value of total consumption which, in addition to expenditures, includes the market value of time spent on home production.

Heterogeneity in home production efficiency rather than disutility of work is essential in amplifying inequality across households. If there was only heterogeneity in the disutility of home work, then there would be no significant inequality gap between the model with and the model without home production. Our inference of home production efficiency is based on an intra-period optimality condition which requires households to consume more in their more efficient sector and implies a log-linear relationship between production efficiency and three observables (market expenditures, time spent on home production, and market productivity). Home production efficiency is dispersed as it cumulates the variances of these three observables while the covariation between them is small.

Our results are robust to a battery of sensitivity checks. First, our conclusions are robust to the estimated values of the elasticity of substitution across sectors, the parameter which governs the Frisch elasticity of labor supply, and the progressivity of the tax system. Second, our results apply separately within subgroups of households defined by their age, marital status, number of children, age of youngest child, the presence of a working spouse, and education levels. Third, our conclusions are robust to measures of expenditures that range from narrow (food) to broad (total spending including durables). Fourth, the inequality differences between the model with and the model without home production are robust to even large amounts of measurement error that may impact the dispersion in observables. Fifth, we examine four alternative datasets in which we do not need to impute home production time because they contain information on both expenditures and time use. We confirm our results in the Panel Study of Income Dynamics (PSID) with food expenditures, in a version of the PSID with expanded consumption categories, in a dataset from Japan, and in a dataset from the Netherlands.

There is an extensive literature which examines how non-separabilities and home production affect consumption and labor supply either over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013) or over the life cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007a; Dotsey, Li, and Yang, 2014). In these papers, home production provides a smoothing mechanism against differences that originate in the market sector if households are sufficiently willing to substitute expenditures with time. Our conclusions for the role of home production in understanding cross-sectional patterns differ from this literature because we find that time in home production is not negatively correlated with wages and expenditures in the cross section of households. By contrast, an assumption underlying the business cycle and life-cycle literatures is that decreases in the opportunity cost of time and in expenditures are associated with substantial increases in time spent on home production.

Even though the home production literature has emphasized shocks in the home sector in order to generate higher volatility in labor markets and labor wedges (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Karabarbounis, 2014), little is known about cross-sectional differences in shocks in the home sector. We develop a methodology to infer production efficiency and disutility of work heterogeneity in the home sector. Our conclusion is that these sources of heterogeneity are important in terms of generating cross-sectional and life-cycle patterns of expenditures and time allocation.

The literature on incomplete markets has started to incorporate home production and non-separabilities into models. Kaplan (2012) argues that involuntary unemployment and non-separable preferences allow an otherwise standard model with self-insurance to account for the variation of market hours over the life cycle. Blundell, Pistaferri, and Saporta-Eksten (2016) examine consumption inequality in a model in which shocks can also be insured within the family and preferences for hours are non-separable across spouses. Blundell, Pistaferri, and Saporta-Eksten (2018) incorporate child care into a life-cycle partial equilibrium model of consumption and family labor supply. Their paper aims to understand the responsiveness of consumption and time use to transitory and permanent wage shocks and, unlike our paper, it does not quantify the extent to which home production affects inequality.

Another related literature addresses consumption inequality. Earlier work (Deaton and Paxson, 1994; Gourinchas and Parker, 2002; Storesletten, Telmer, and Yaron, 2004; Aguiar and Hurst, 2013) has examined the drivers of life-cycle consumption inequality and their welfare consequences. More recent work focuses on the increase in consumption inequality (Krueger and Perri, 2006; Blundell, Pistaferri, and Preston, 2008; Aguiar and Bils, 2015) and the decline in leisure inequality (Attanasio, Hurst, and Pistaferri, 2015) over time. Our contribution is to introduce home production data into the inequality literature and show that they change the inferences we draw about welfare. Closest to the spirit of our exercise, Jones and Klenow (2016) map differences in consumption levels and dispersion, market hours, and mortality into welfare differences across countries and find that in some cases GDP per capita does not track welfare closely.

Finally, our paper relates to a strand of literature which uses no-trade theorems to derive analytical solutions for a certain class of models with incomplete markets and heterogeneous agents. Constantinides and Duffie (1996) first derived a no-trade theorem in an endowment economy. Krebs (2003) extends the theorem to an environment with capital, in which households invest a constant share of wealth in physical and human capital and total income follows a random walk in logs. Most relevant for us, Heathcote, Storesletten, and Violante (2014) extend the no-trade theorem by allowing for partial insurance of wage shocks and flexible labor supply. Our contribution is to extend the theorem when households can also direct their time to multiple sectors and face heterogeneity in both their home production efficiency and disutility of work.

2.2 Model

We first present the model and characterize its equilibrium in closed form. We then discuss the identification of the sources of heterogeneity across households.

2.2.1 Environment

Demographics. The economy features perpetual youth demographics. We denote by t the calendar year and by j the birth year of a household. Households face a constant probability of survival δ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

Technologies. We denote the good purchased in the market by c_M and the various goods produced at home by c_K where $K = 1, ..., \mathcal{K}$ indexes home produced goods. All goods are produced with labor. Hours worked in the market are h_M and hours worked in each home sector are h_K.

A household's technology in the market sector is characterized by its (pre-tax) earnings $y = z_M h_M$, where z_M denotes market productivity (wage) which varies across households and over time. Aggregate production of market goods is given by $\int_\iota z_M(\iota) h_M(\iota) \mathrm{d}\Phi(\iota)$, where ι identifies households and Φ denotes the cumulative distribution function of households. Goods and labor markets are perfectly competitive and the wage per efficiency unit of labor is one.

The government taxes labor income to finance (wasteful) public expenditures G of the market good. If $y = z_M h_M$ is pre-tax earnings, then $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$ is after-tax earnings, where τ_0 determines the level of taxes and τ_1 governs the progressivity of the tax system. When $\tau_1 = 0$ there is a flat tax rate. A higher τ_1 introduces a larger degree of progressivity into the tax system because it compresses after-tax earnings relative to pre-tax earnings.[3]

Households have access to $\mathcal{K}$ technologies in the home sector. Production of home goods is given by $c_K = z_K h_K$, where z_K denotes home productivity which varies across households and over time. Home production is not tradeable and not storable, meaning that in every instance home production must be consumed.

Preferences. Households order sequences of goods and time by $\mathbb{E}_j \sum_{t=j}^{\infty} (\beta\delta)^{t-j} U_t(c_t, h_{M,t}, h_{K,t})$, where c is the consumption aggregator and β is the discount factor. The period utility function is given by:

$$U = \frac{c^{1-\gamma}-1}{1-\gamma} - \frac{\left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \tag{1}$$

The curvature of the utility function with respect to consumption c is given by parameter $\gamma \geq 0$ and the curvature with respect to total effective hours by parameter $\eta > 0$. Hours are perfect substitutes across sectors. The disutility of work in the market sector is B and the disutility of work in each home sector is D_K. We allow B and D_K to vary across households and over time.[4]

Consumption is given by a CES aggregator of market and home goods, with an elasticity of substitution between any goods equal to $\phi > 0$:

$$c = \left(c_M^{\frac{\phi-1}{\phi}} + \sum \omega_K c_K^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}, \tag{2}$$

[3] Our tax schedule modifies the tax schedule considered, among others, by Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014) in that τ_1 is applied to market productivity z_M instead of earnings $z_M h_M$. We adopt the specification of after-tax earnings $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$ instead of $\tilde{y} = (1-\tau_0)(z_M h_M)^{1-\tau_1}$ because we can only prove the no-trade result in the home production model under the former specification. We argue that this modification does not matter for our results because market productivity z_M and hours h_M are relatively uncorrelated in the cross section of households and most of the cross-sectional variation in earnings $z_M h_M$ is accounted for by z_M. For this reason, our estimate of τ_1 in Section 2.3.2 is close to the estimates found in Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014).

[4] Our model features a single decision maker within each household. We model hours worked across spouses as perfect substitutes and in our quantitative results we define h_M and h_K as the sum of the respective hours worked across spouses. The perfect substitutability of hours (across sectors and spouses) is essential for the no-trade result. We can extend the model for separate disutility of work by spouse.

where ω_K denotes the consumption weight of home good K relative to market consumption. Similar to productivity z_K and disutility of work D_K, we allow ω_K to vary across households and over time.[5]

Our specification of preferences and technologies nests the standard model without home production when $\omega_K = 0$ for all K. For $\omega_K > 0$ our multi-sector model is a special case of the Beckerian model of home production in which expenditures and time combine to produce final utility (Becker, 1965; Ghez and Becker, 1975; Gronau, 1986).[6]

Home Production Efficiencies. In home production models it is essential to distinguish between consumption weights, ω_K, which transform consumption outputs c_K into utility and what we label production efficiencies, θ_K, which transform time inputs h_K into utility. To see where production efficiencies arise in households' problem, we substitute the consumption aggregator (2) and the technologies $c_K = z_K h_K$ into the utility function (2) to obtain the derived utility:

$$V = \frac{\left[\left(c_M^{\frac{\phi-1}{\phi}} + \sum(\theta_K h_K)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}\right]^{1-\gamma} - 1}{1-\gamma} - \frac{\left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \quad (3)$$

where $\theta_K \equiv \omega_K^{\frac{\phi}{\phi-1}} z_K$ is the production efficiency of hours in each sector, a convolution of the consumption weight, ω_K, and home productivity, z_K. Identifying separately ω_K from z_K is not feasible because home output c_K data are unavailable in common datasets. However, this lack of identification does not pose a challenge for our analyses. As we show below, all equilibrium allocations depend directly only on θ_K and not on its split between ω_K and z_K. Additionally, since equilibrium allocations and derived utility depend only on θ_K, this split does not affect any of our inequality results. In other words, even if one could separate ω_K from z_K, this split would not be informative for equilibrium allocations and welfare

[5] We normalize to unity an inessential constant multiplying c_M in equation (2). In our quantitative results with $\gamma = 1$, this constant becomes an additive term in utility which does not enter equilibrium allocations and, therefore, cannot be identified from data.

[6] We use the more common formulation of the home production model as in Gronau (1986) in which time spent working in the market and at home generate disutility. As we show in Boerma and Karabarbounis (2020), this version shares many predictions with the Beckerian framework in which expenditures and home time are inputs in the production of goods which enter into utility. Further, the no-trade result can be extended in the Beckerian version of the home production model with imperfect substitutability of hours.

analyses. For this reason, henceforth we focus our analysis on θ_K rather than ω_K and z_K.

Sources of Heterogeneity. Households are heterogeneous with respect to the disutilities of work B and D_K and efficiencies z_M and θ_K. For B and z_M we impose a random walk structure which is important for obtaining the no-trade result. Under certain parametric restrictions which we discuss below, we are able to obtain the no-trade result with minimal structure on the process governing home production efficiency θ_K and disutility of home work D_K.

Households' disutility of market work is described by a random walk process:

$$B_t^j = B_{t-1}^j + \upsilon_t^B. \tag{4}$$

Households' log market productivity $\log z_M$ is the sum of a permanent component α and a more transitory component ε:

$$\log z_{M,t}^j = \alpha_t^j + \varepsilon_t^j \ . \tag{5}$$

The permanent component follows a random walk, $\alpha_t^j = \alpha_{t-1}^j + \upsilon_t^\alpha$. The more transitory component, $\varepsilon_t^j = \kappa_t^j + \upsilon_t^\varepsilon$, is the sum of a random walk component, $\kappa_t^j = \kappa_{t-1}^j + \upsilon_t^\kappa$, and an innovation υ_t^ε. Finally, households are heterogeneous with respect to their production efficiency at home $\theta_{K,t}^j$ and disutility of work at home $D_{K,t}^j$. Our identification theorem below is based on cross-sectional data and does not restrict θ_K and D_K to a particular class of stochastic processes. We identify a household ι by a sequence $\{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$.

For any random walk, we use υ to denote innovations and Φ_{υ_t} to denote distributions of innovations. We allow distributions of innovations to vary over time t. We assume that $\theta_{K,t}^j$ and $D_{K,t}^j$ are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other. The distribution of initial conditions $(\theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$ can be non-degenerate across households born at j and can vary by birth year j.

Asset Markets. It is convenient to describe the restrictions on asset markets using the definition of an island in the spirit of Heathcote, Storesletten, and Violante (2014). Islands are capturing insurance mechanisms available to households for smoothing more transitory shocks in the market sector. Households are partitioned into islands, with each island consisting of a continuum of households who are identical in terms of their production efficiency at home θ_K, disutilities of work D_K and B, permanent component of market productivity α, and the initial condition of κ. More formally, household $\iota = \{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$ lives on island ℓ consisting of ι's with common initial state $(\theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$ and sequences $\{\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j\}_{t=j+1}^\infty$.

We now summarize the structure of asset markets. First, households cannot trade assets contingent on $\theta_{K,t}^j$ and $D_{K,t}^j$. Second, households can trade one-period bonds $b^\ell(s_{t+1}^j)$ which pay one unit of market consumption contingent on $s_t^j \equiv (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon)$ with households who live on their island ℓ. Third, households can trade economy-wide one-period bonds $x(\zeta_{t+1}^j)$ which pay one unit of market consumption contingent on $\zeta_t^j \equiv (\kappa_t^j, \upsilon_t^\varepsilon)$ with households who live on either their island or any other island.

To preview the implications of these assumptions, differences in $(\theta_K, D_K, B, \alpha)$ across households remain uninsurable by the no-trade result we will discuss below that yields $x(\zeta_{t+1}^j) = 0$ in equilibrium.[7] The more transitory component of productivity $\varepsilon_t^j = \kappa_t^j + \upsilon_t^\varepsilon$ becomes fully insurable because households on an island are only heterogeneous with respect to ζ_t^j and can trade bonds $b^\ell(\zeta_{t+1}^j)$. As a result, the island structure generates partial insurance with respect to market productivity differences. Anticipating these results, henceforth we call α the uninsurable permanent component of market productivity and $\varepsilon = \kappa + \upsilon^\varepsilon$ the insurable transitory component of market productivity. We offer some examples of the type of wage shocks accommodated by the framework. Aggregate changes in wages which load differently across households, such as the skill premium, may be more difficult to insure and are captured by α. By contrast, κ may be capturing persistent shocks such as disability and υ^ε may be capturing transitory shocks such as unemployment which are easier to insure using asset markets, family transfers, or government transfers.[8]

[7] Households still obtain implicit insurance by substituting time across sectors. A realization of θ_K which leads to low home-produced c_K can be offset by higher purchases in the market c_M if a household desires so. Similarly, households can offset realizations of α by reallocating their time across sectors.

[8] We refer the reader to Heathcote, Storesletten, and Violante (2008) for a more detailed discussion of how the partial insurance framework relates to frameworks with exogenously imposed incomplete markets or to frameworks in which incompleteness arises endogenously from informational frictions or limited commitment.

Household Optimization. We now describe the optimization problem of a particular household ι born in period j. The household chooses $\{c_{M,t}, h_{M,t}, h_{K,t}, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}_{t=j}^{\infty}$ to maximize the expected value of discounted flows of derived utilities in equation (3), subject to the sequential budget constraints:

$$c_{M,t} + \int_{s^j_{t+1}} q^{\ell}_b(s^j_{t+1}) b^{\ell}(s^j_{t+1}) \mathrm{d}s^j_{t+1} + \int_{\zeta^j_{t+1}} q_x(\zeta^j_{t+1}) x(\zeta^j_{t+1}) \mathrm{d}\zeta^j_{t+1} = \tilde{y}^j_t + b^{\ell}(s^j_t) + x(\zeta^j_t) \,. \quad (6)$$

The expenditure side of the budget constraint consists of market consumption $c_{M,t}$, island-level bonds $b^{\ell}(s^j_{t+1})$ at prices $q^{\ell}_b(s^j_{t+1})$, and economy-wide bonds $x(\zeta^j_{t+1})$ at prices $q_x(\zeta^j_{t+1})$. The income side of the budget constraint consists of after-tax labor income $\tilde{y}^j_t$ and bond payouts.

Equilibrium. Given tax parameters (τ_0, τ_1), an equilibrium consists of a sequence of allocations $\{c_{M,t}, h_{M,t}, h_{K,t}, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}_{\iota,t}$ and a sequence of prices $\{q^{\ell}_b(s^j_{t+1})\}_{\ell,t}$, $\{q_x(\zeta^j_{t+1})\}_t$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$\int_{\iota \in \ell} b^{\ell}(s^j_{t+1}; \iota) \mathrm{d}\Phi(\iota) = 0 \quad \forall \ell, s^j_{t+1}, \quad \text{and} \quad \int_{\iota} x(\zeta^j_{t+1}; \iota) \mathrm{d}\Phi(\iota) = 0 \quad \forall \zeta^j_{t+1}; \quad (7)$$

and (iii) the goods market clears:

$$\int_{\iota} c_{M,t}(\iota) \mathrm{d}\Phi(\iota) + G = \int_{\iota} z_{M,t}(\iota) h_{M,t}(\iota) \mathrm{d}\Phi(\iota), \quad (8)$$

where government expenditures are given by $G = \int_{\iota} \left[z_{M,t}(\iota) - (1-\tau_0) z_{M,t}(\iota)^{1-\tau_1} \right] h_{M,t}(\iota) \mathrm{d}\Phi(\iota)$.

2.2.2 Equilibrium Allocations

The model retains tractability because, under certain parametric restrictions, it features a no-trade result which allows us to solve equilibrium allocations in closed form. This section explains the logic underlying this result and Appendix 3.A presents the proof. Our proof follows very closely the proof presented in Heathcote, Storesletten, and Violante (2014). We extend their analysis along two dimensions. First, we prove the no-trade result in an environment with multiple sectors and heterogeneity in home production efficiency and disutility of home work. Second, we allow the disutility of market work B to be a random walk instead of a fixed effect.

We begin by guessing that the equilibrium features no trade across islands, that is $x(\zeta_{t+1}^j;\iota) = 0, \forall \iota, \zeta_{t+1}^j$. Further, we postulate that equilibrium allocations $\{c_{M,t}(\iota), h_{M,t}(\iota), h_{K,t}(\iota)\}$ solve a sequence of static island-level planning problems which maximize average utility within island, $\int_{\zeta_t^j} V\left(c_{M,t}(\iota), h_{M,t}(\iota), h_{K,t}(\iota); \iota\right) \mathrm{d}\Phi_t(\zeta_t^j)$, subject to island-level constraints equating aggregate market consumption to aggregate after-tax earnings $\int_{\zeta_t^j} c_{M,t}(\iota)\mathrm{d}\Phi_t(\zeta_t^j) = \int_{\zeta_t^j} \tilde{y}_t(\iota)\mathrm{d}\Phi_t(\zeta_t^j)$. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and all asset and goods markets clear.

We obtain the no-trade result in two nested versions of the model. The first nested model sets consumption weights in the home sector to $\omega_K = 0$ as in Heathcote, Storesletten, and Violante (2014). The second nested model sets the curvature of utility with respect to consumption to $\gamma = 1$ for any value of $\omega_K > 0$. The home production model nests the model without home production when $\gamma = 1$, which is the case we consider below in our quantitative results.

To understand the no-trade result, we begin with the observation that households on each island ℓ have the same marginal utility of market consumption because they are identical in terms of $(\theta_K, D_K, B, \alpha)$ and trade in state-contingent bonds allows them to perfectly insure against $(\kappa, \upsilon^\varepsilon)$. Considering first the model without home production ($\omega_K = 0$), the common marginal utility of market consumption $\mu(\ell)$ at the no-trade equilibrium is:

$$\mu(\ell) = \frac{1}{c_M^\gamma} = \left(\frac{\exp\left((1+\eta)(B - \log(1-\tau_0) - (1-\tau_1)\alpha)\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa + \upsilon^\varepsilon)\right) \mathrm{d}\Phi(\zeta)} \right)^{\frac{\gamma}{1+\eta\gamma}}, \tag{9}$$

where for simplicity we have dropped the time subscript from all variables. The no-trade result states that households do not trade bonds across islands, $x(\zeta_{t+1}^j) = 0$. Owing to the random walk assumptions on B and α, we see from equation (9) that the growth in marginal utility, μ_{t+1}/μ_t, does not depend on the state vector (B_t^j, α_t^j) which differentiates islands ℓ. As a result, all households value bonds traded across islands identically in equilibrium and there are no mutual benefits from trading $x(\zeta_{t+1}^j)$.

For the economy with home production and $\gamma = 1$, we obtain a marginal utility of market consumption:

$$\mu(\ell) = \frac{1}{c_M + \tilde{z}_M \sum \frac{\exp(D_K)}{\exp(B)} h_K} = \left(\frac{\exp\left((1+\eta)(B - \log(1-\tau_0) - (1-\tau_1)\alpha)\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa + \upsilon^\varepsilon)\right) \mathrm{d}\Phi(\zeta)} \right)^{\frac{1}{1+\eta}}. \quad (10)$$

The marginal utility in equation (10) has the same form as the marginal utility in equation (9) for $\gamma = 1$. Marginal utility growth does not depend on the state vector $(\theta^j_{K,t}, D^j_{K,t}, B^j_t, \alpha^j_t)$ which differentiates islands and the same logic explains why we obtain the no-trade result in the home production model. For this result, we note the importance of log preferences with respect to the consumption aggregator. Log preferences generate a separability between the marginal utility of market consumption and θ_K and D_K and, thus, the no-trade result holds irrespective of the value of the elasticity of substitution across sectors ϕ and further stochastic properties of θ_K and D_K.

The no-trade result allows us to derive equilibrium allocations for consumption and time using the sequence of planning problems described previously without solving simultaneously for the wealth distribution.[9] We summarize the equilibrium allocations for both models in Table 1. For convenience, we have dropped the household index ι from the table. The constant $\mathcal{C}_a$ is common across households and is proportional to a moment of the insurable component of market productivity ε. All sources of heterogeneity $(\theta_K, D_K, B, \alpha, \varepsilon)$ and allocations are ι-specific.

Starting with the model without home production, market consumption c_M depends positively on the tax-adjusted uninsurable permanent productivity component $(1-\tau_1)\alpha$ and negatively on the disutility of market work B. By contrast, c_M does not depend on the insurable component of market productivity ε because state-contingent assets insure against variation in ε. The final row shows that market hours h_M increase in the after-tax market productivity $\tilde{z}_M = (1-\tau_0) z_M^{1-\tau_1}$ with an elasticity η. This reflects the substitution effect on labor supply from variations in after-tax market productivity. Conditional on $\tilde{z}_M$, h_M decreases in $(1-\tau_1)\alpha$ which reflects the income effect from changes in the permanent

[9]The no-trade result applies to the bonds traded across islands $x(\zeta^j_{t+1}) = 0$ and not to the within-islands bonds $b^\ell(s^j_{t+1})$ which are traded in equilibrium. However, the bonds $b^\ell(s^j_{t+1})$ are state-contingent within each island and, therefore, solving for the equilibrium allocations amounts to solving a sequence of static planning problems.

Table 1: Equilibrium Allocations

Variable		No Home Production: $\omega_K = 0$	Home Production: $\gamma = 1$
1.	c_M	$\frac{\exp\left(\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathcal{C}_a^{\frac{1}{1+\eta\gamma}}$	$\frac{1}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{\frac{1}{1+\eta}}$
2.	h_K		$\frac{\theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{\frac{1}{1+\eta}}$
3.	h_M	$\tilde{z}_M^{\eta}\frac{\exp\left(-\eta\gamma\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathcal{C}_a^{-\frac{\eta\gamma}{1+\eta\gamma}}$	$\tilde{z}_M^{\eta}\frac{\exp(-\eta(1-\tau_1)\alpha)}{\exp(B)}\mathcal{C}_a^{-\frac{\eta}{1+\eta}} - \sum\frac{\exp(D_K)}{\exp(B)}h_K$

Table 1 presents the equilibrium allocation in the two models. Parameters γ, η, and ϕ and the constant $\mathcal{C}_a \equiv \int (1-\tau_0)^{1+\eta}\exp((1+\eta)(1-\tau_1)\varepsilon)\mathrm{d}\Phi_\zeta(\zeta)$ are the same across households. We define market productivity $z_M = \exp(\alpha+\varepsilon)$, after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$, and the rate of transformation $R \equiv 1 + \sum\left(\frac{\theta_K}{\tilde{z}_M}\frac{\exp(B)}{\exp(D_K)}\right)^{\phi-1}$.

component of market productivity. When $\gamma = 1$, substitution and income effects from variations in α cancel out and h_M depends positively only on the insurable component ε. Finally, h_M decreases in the disutility of market work B.

To understand the solutions in the home production model, households maximize their utility when the relative marginal rate of substitution between work and consumption across any two sectors, $\frac{\mathrm{MRS}_M}{\mathrm{MRS}_K} = \frac{\exp(B)}{\exp(D_K)}\frac{\omega_K c_M^{1/\phi}}{c_K^{1/\phi}}$, equals the ratio of after-tax productivities, $\frac{\tilde{z}_M}{z_K}$. Rearranging this optimality condition, we obtain:

$$\frac{h_K}{c_M} = \theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}. \tag{11}$$

The solution for c_M in the second column of Table 1 uses equation (11) to substitute out h_K from the marginal utility in equation (10). The solution for c_M has the same form as the solution in the model without home production under $\gamma = 1$ up to the rate of transformation $R \equiv 1 + \sum\left(\frac{\theta_K}{\tilde{z}_M}\frac{\exp(B)}{\exp(D_K)}\right)^{\phi-1}$. This rate describes the incentives of households to shift hours across sectors as a function of relative efficiencies and disutilities of work.

The second row shows that home hours h_K increase in home production efficiency θ_K when $\phi > 1$, in which case substitution effects from changes in θ_K dominate income effects. Hours h_K decrease in disutility of home work D_K for any value of ϕ. In the final row, we present the solution for hours in the market sector h_M. To understand this expression, we define effective total hours as $h_T = h_M + \sum \frac{\exp(D_K)}{\exp(B)} h_K$ and note that the solutions for h_T coincide in the two models under $\gamma = 1$. The expression for h_T shows that it does not depend on α because under $\gamma = 1$ substitution and income effects from permanent changes in wages cancel out. It also shows that h_T increases in ε with a Frisch elasticity of $(1 - \tau_1)\eta$.

2.2.3 Identification of Sources of Heterogeneity

We begin by explaining a fundamental identification challenge for home production models and then provide a solution to it. As seen in equation (11), home hours relative to market consumption h_K/c_M depend on both production efficiency θ_K and the disutility of work D_K. As a result, data on h_K/c_M are informative only for a combination of θ_K and D_K. More formally, the solutions for the home production model in Table 1 reveal that we have $3 + \mathcal{K}$ observed variables in the data (c_M, h_M, z_M, h_K) to inform $3+2\times\mathcal{K}$ sources of heterogeneity (α, ε, B, D_K, θ_K). The gap between observed variables and sources of heterogeneity, equal to $\mathcal{K}$, reflects the fact that both θ_K and D_K can equally well account for the behavior of h_K. This identification challenge is special to home production models and does not arise in the standard model without home production. As evidenced in Table 1, in the latter we have 3 observed variables (c_M, h_M, z_M) to inform 3 sources of heterogeneity (α, ε, B). The challenge for home production models arises because in common datasets we observe home inputs h_K but not home outputs c_K.

Our solution to the identification challenge is to impose additional structure on θ_K and D_K. We assume that home production can be disaggregated into two sectors, N and P. In sector N, households are heterogeneous in their production efficiency θ_N and their disutility of home work equals that in the market $D_N = B$. In sector P, households are identical in their production efficiency θ_P and heterogeneous in their disutility of work D_P which may differ from B. These assumptions reduce the number of sources of heterogeneity to 5 (α, ε, B, D_P, θ_N) which, as we show below, can be identified from 5 observed variables (c_M, h_M, z_M, h_N, h_P).

This identification assumption balances two polar views on the origins of household differences in home work time. It allows the model to attribute some of the observed differences to heterogeneity in home production efficiency while other differences to heterogeneity in the disutility of home work. To give some concrete examples from our quantitative results below, we think of time spent on activities such as child care and nursing as belonging to h_N because these activities are relatively less intensive in manual skills and efficiency differences across households are likely to be a significant source of dispersion in hours. We think of time spent on activities such as cooking and cleaning as belonging to h_P because these activities are more intensive in manual skills and differences in their disutility of work across households are likely to be more important than efficiency differences. Because the welfare consequences of home production depend on the origins of heterogeneity, we will also discuss below the two polar cases of all home time $h_N + h_P$ belonging either to the sector with heterogeneity in production efficiency or to the sector with heterogeneity in disutility of work.

We now demonstrate how to infer the sources of heterogeneity, $\{\alpha, \varepsilon, B, D_P, \theta_N\}_\iota$, such that the models with and without home production both account perfectly for given cross-sectional data on consumption expenditures, hours, and wages.

Observational Equivalence Theorem. Let $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ be some cross-sectional data. Then, for any given parameters $(\eta, \phi, \tau_0, \tau_1)$:

1. There exists unique $\{\alpha, \varepsilon, B\}_\iota$ such that $\{c_M, h_M, z_M\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ under $\omega_K = 0$ for any γ.
2. There exists unique $\{\alpha, \varepsilon, B, D_P, \theta_N\}_\iota$ such that $\{c_M, h_M, z_M, h_N, h_P\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ under $\gamma = 1$ for any $\omega_K > 0$.

The theorem uses the fact that, in each model, the equilibrium allocations presented in Table 1 can be uniquely inverted to obtain, up to a constant, the sources of heterogeneity which generate these allocations. The formal proof is presented in Appendix 3.B.

Table 2 presents the identified sources of heterogeneity which allow the model without home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ and the model with home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$. Henceforth, we drop the bar to indicate variables observed in the data since, by appropriate choices of the sources of heterogeneity, both models generate perfectly these data.

To understand how observables inform the sources of heterogeneity, in Table 2 we define

Table 2: Identified Sources of Heterogeneity

		No Home Production: $\omega_K = 0$
1.	α	$\frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
2.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
3.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta\gamma}{1+\eta}\log c_M - \frac{1}{1+\eta}\log h_M$
		Home Production: $\gamma = 1$
4.	α	$\frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
5.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathcal{C}_s\right]$
6.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta}{1+\eta}\log c_T - \frac{1}{1+\eta}\log h_T$
7.	D_P	$B + \frac{1}{\phi}\log\left(\frac{c_M}{h_P}\right) + \frac{\phi-1}{\phi}\log\theta_P - \log(1-\tau_0) - (1-\tau_1)\log z_M$
8.	θ_N	$(1-\tau_0)^{\frac{\phi}{\phi-1}} z_M^{\frac{(1-\tau_1)\phi}{\phi-1}} \left(\frac{h_N}{c_M}\right)^{\frac{1}{\phi-1}}$

Table 2 presents the inferred sources of heterogeneity for the economy without home production (upper panel) and for the economy with home production (lower panel). We define the market value of total consumption $c_T \equiv c_M + \tilde{z}_M\left(h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P\right)$, effective total hours $h_T \equiv h_M + h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P$, and the constant $\mathcal{C}_s \equiv \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon)\mathrm{d}\Phi_\zeta(\zeta)$.

effective total hours as:

$$h_T \equiv h_M + h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P = h_M + h_N + \frac{\exp(D_P)}{\exp(B)}h_P, \tag{12}$$

and the market value of total consumption as:

$$c_T \equiv c_M + \tilde{z}_M\left(h_N + \left(\frac{c_M}{\theta_P h_P}\right)^{\frac{1}{\phi}}\frac{\theta_P}{\tilde{z}_M}h_P\right) = c_M + \tilde{z}_M\left(h_N + \frac{\exp(D_P)}{\exp(B)}h_P\right). \tag{13}$$

These expressions first define total hours and consumption only in terms of observables and parameters. The equality uses the inferred sources of heterogeneity to express total hours and consumption in a more intuitive way. Specifically, total hours h_T are the sum of hours in the three sectors, adjusted for disutility differences across sectors. The market value of total consumption c_T is the sum of market consumption, consumption in sector N valued in terms of market goods with the exchange rate $\frac{\tilde{z}_M}{z_N}$, and consumption in sector P valued in terms of market goods with the exchange rate $\frac{\tilde{z}_M}{z_P}\frac{\exp(D_P)}{\exp(B)}$.

Rows 1 to 6 show that, for $\gamma = 1$, the inferred α, ε, and B have the same functional forms between the two models. The difference is that the hours and consumption informative for the sources of heterogeneity in the home production model are h_T in equation (12) and c_T in equation (13), whereas in the model without home production $h_T = h_M$ and $c_T = c_M$. The inferred α depends positively on the consumption-hours ratio c_T/h_T and market productivity z_M and the inferred ε is the difference between $\log z_M$ and α. The inferred B depends on the gap between market productivity $\log z_M$ and a combination of consumption $\log c_T$ and hours $\log h_T$.

The new sources of heterogeneity in the home production model are presented in rows 7 and 8. These are inferred by rearranging the optimality conditions (11) for $K = \{P, N\}$ and solving for D_P and θ_N respectively:

$$\frac{\exp(D_P)}{\exp(B)} = \left(\frac{\theta_P}{\tilde{z}_M}\right)^{\frac{\phi-1}{\phi}} \left(\frac{c_M}{\tilde{z}_M h_P}\right)^{\frac{1}{\phi}}, \quad \text{and} \quad \frac{\theta_N}{\tilde{z}_M} = \left(\frac{\tilde{z}_M h_N}{c_M}\right)^{\frac{1}{\phi-1}}. \tag{14}$$

These expressions show how relative disutilities and efficiencies are inferred from the market value of sectoral consumptions. Holding constant relative efficiencies $\theta_P/\tilde{z}_M$, higher market expenditures relative to the market value of producing at home $c_M/\tilde{z}_M h_P$ leads to higher inferred relative disutility at home $\exp(D_P)/\exp(B)$. When sectors are substitutes ($\phi > 1$), higher market value of producing at home relative to market expenditures $\tilde{z}_M h_N/c_M$ leads to higher inferred relative efficiency at home $\theta_N/\tilde{z}_M$.

A numerical example in Table 3 provides some insights for the mechanisms of the model and draws lessons from the observational equivalence theorem. The economy is populated by two households, there are no taxes, and preference parameters satisfy $\gamma = \eta = 1$. In the upper panel, the economist uses the model without home production to infer the sources of heterogeneity. Household 1 earns a wage $z_M = 20$, spends $c_M = 1,000$, and works $h_M = 60$. Household 2 also earns $z_M = 20$, but spends $c_M = 600$ and works $h_M = 40$. The analytical solutions in Table 2 show that households with a higher expenditures to hours ratio, c_M/h_M, or higher market productivity, z_M, have a higher uninsurable productivity component α. In

Table 3: Numerical Example

Household	z_M	c_M	h_M	h_N	h_P	α	ε	B	D_P	θ_N	T
1	20	1,000	60			2.90	0.09	-4.00			0
2	20	600	40			2.85	0.14	-3.54			399
1	20	1,000	60	10	50	2.95	0.04	-4.74	-4.74	6.07	0
2	20	600	40	50	30	2.95	0.04	-4.74	-4.74	29.20	-765

Table 3 presents an example with parameters $\tau_0 = \tau_1 = 0$ and $\gamma = \eta = 1$. The upper panel shows inference based on the model without home production and the lower panel shows inference based on the model with home production. For the home production model we use $\theta_P = 20$ and $\phi = 2.35$. The last column, labeled T, shows the equivalent variation to achieve the utility level of household 1.

Table 3 we thus infer that α is higher for household 1 than for household 2 (2.90 versus 2.85). Since both households have the same market productivity and $\alpha + \varepsilon$ add up to (log) market productivity, we infer that household 2 has a higher insurable productivity component ε than household 1. Finally, we infer that household 2 has a higher B because it spends less and works less than household 1 despite having the same market productivity.

In the lower panel the economist uses the home production model to infer the sources of heterogeneity. In addition to the same data on (c_M, h_M, z_M), now the economist observes that the first household works $h_N = 10$ and $h_P = 50$ hours and the second household works $h_N = 50$ and $h_P = 30$ hours in the two sectors. The inferred α now depends on the ratio of the market value of total consumption to total hours, c_T/h_T, rather than on the ratio of market expenditures to market hours c_M/h_M. Since both households have the same market value of total consumption, $c_T = 2,200$, and the same total hours, $h_T = 120$, the α's are equalized across households. Given the same market productivity, the ε's are also equalized. Given that the two households consume and work the same, the B's are also equalized. Equation (14) shows that D_P is also the same between the two households because they have the same value of home production in sector P relative to market expenditures $z_M h_P/c_M$. As Table 3 shows, all differences in observables between the two households are loaded into home production efficiency θ_N. We infer that θ_N is higher for household 2 because it has a higher value of production in sector N relative to market expenditures $z_M h_N/c_M$ and the sectors are substitutes ($\phi > 1$).

There are two lessons we draw from this example. First, home production efficiency θ_N is dispersed across households and absorbs dispersion one would attribute to (α, ε, B) in the absence of home production. This result generalizes in our quantitative application using U.S. data below in which we find that θ_N is significantly more dispersed than z_M and the dispersion in (α, ε, B) is smaller in the home production model.

The second lesson we draw is that a household's welfare ranking depends on whether the data has been generated by a model with or without home production. The last column of Table 3 shows equivalent variations T, equal to the transfers required for households to achieve a given level of utility if they re-optimize their consumption and hours choices. The reference utility level in Table 3 is the utility of household 1 and, thus, T for household 1 is always equal to zero. In the model without home production, T for household 2 equals 399. In the home production model, the two households are identical in terms of their $(\alpha, \varepsilon, B, D_P)$, but household 2 has a higher home production efficiency θ_N. Therefore, the welfare ranking changes and T becomes -765.

2.2.4 Discussion

Before proceeding to the quantitative results, we pause to make three comments. First, we emphasize the importance of developing an equilibrium model which expresses the arguments (c_M, h_M, h_N, h_P) of the utility function in terms of productivity and preference shifters and policy parameters. An alternative approach, followed by Krueger and Perri (2003) in their study of the welfare effects of increasing inequality in the United States and Jones and Klenow (2016) in their study of welfare and GDP differences across countries, is to plug what are endogenous variables in our framework into the utility function and conduct welfare experiments by essentially varying these variables. While our approach comes with additional complexity, it has the conceptual advantage of taking into account equilibrium responses when conducting welfare analyses with respect to changes in more primitive sources of heterogeneity and policies.

Second, we wish to highlight the merits of the Heathcote, Storesletten, and Violante (2014) framework used in our analysis compared to alternative frameworks. Standard general equilibrium models with uninsurable risk following Huggett (1993) and Aiyagari (1994) feature self-insurance via a risk-free bond. Solutions to these models are obtained computationally. While the present model also allows households to trade a risk-free bond (by setting $x(\zeta_t^j) = 1$ for all states ζ_t^j), the assumptions on asset markets, stochastic processes, and preferences allow us to derive a no-trade result and characterize equilibrium allocations in closed form. Owing to the analytical results, a major advantage of the framework is the transparency and generality of the identification.[10]

Third, our non-parametric approach to identifying the sources of heterogeneity is such that the model accounts perfectly for any given cross-sectional data on market consumption, hours, and wages. Conceptually, our approach is similar to Hsieh and Klenow (2009) who infer wedges in first-order conditions such that firm-level outcomes generated by their model match data analogs. Heathcote, Storesletten, and Violante (2014) also do not impose distributional assumptions on the sources of heterogeneity when estimating their model. A difference with Heathcote, Storesletten, and Violante (2014) is that they select moments in order to estimate parameters using the method of moments. Our approach, instead, does not require restrictions on which moments are more informative for the identification of the sources of heterogeneity.

2.3 Quantitative Results

We begin by describing the data sources and the parameterization of the model. We then present the inferred sources of heterogeneity.

2.3.1 Data Sources

For the baseline analyses we use data from the Consumer Expenditure Survey (CEX) and the American Time Use Survey (ATUS). We consider married and cohabiting households with heads between 25 and 65 years old who are not students. We drop observations for households with market productivity below 3 dollars per hour in 2010 dollars, working less

[10]Despite the wealth distribution not being an object of interest within this framework, a dynamic structure with non-labor income is still essential. In a framework without non-labor income, households would maximize derived utility subject to the budget constraint $c_M = z_M h_M$. Since market consumption to hours c_M/h_M equals market productivity z_M, any choice of (z_M, B, D_P, θ_N) is not sufficient to match data on $(c_M, z_M, h_M, h_N, h_P)$.

than 20 hours per week with market productivity above 300 dollars, with expenditures in the top and bottom one percent, and with respondents who indicated working more than 92 hours in the market or at home. In the ATUS we drop respondents during weekends and in the CEX we keep only households who completed all four interviews. The final sample from CEX/ATUS includes 32,993 households between 1995 and 2016. In all our results, we use sample weights provided by the surveys.

Data for market expenditures c_M, market productivity z_M, and market hours h_M come from CEX interview surveys collected between 1996 and 2017. Closest to the definition of Aguiar and Hurst (2013), for our baseline analyses c_M is annual non-durable consumption expenditures which include food and beverages, tobacco, personal care, apparel, utilities, household operations (including child care), public transportation, gasoline, reading material, and personal care. Non-durable consumption expenditures exclude health and education. We adjust consumption for household composition and size.

Our measure of income is the amount of wage and salary income before deductions earned over the past 12 months. Individual wages are defined as income divided by hours usually worked in a year, which is the product of weeks worked with usual hours worked per week. We define household market hours h_M as the sum of hours worked by spouses and market productivity z_M as the average of wages of individual members weighted by their market hours.

Data for home hours h_N and h_P come from the ATUS waves between 2003 and 2017. Randomly selected individuals from a group of households who completed their eight and final month interview for the Current Population Survey report their activities on a 24-hour time diary of the previous day. Similar to Aguiar, Hurst, and Karabarbounis (2013), total time spent on home production, $h_N + h_P$, includes housework, cooking, shopping, home and car maintenance, gardening, child care, and care for other household members.

To separate total home production time between h_N and h_P, we map disaggregated time uses into occupations and then classify in h_N all the time uses mapped into occupations performing tasks with low manual content and in h_P all time uses mapped into occupations performing tasks with high manual content. The logic underlying our approach is that time activities using the same skills as occupations with high manual content are less likely to display significant heterogeneity in terms of production efficiency. We use the mapping from time uses to occupations together with Occupational Information Network (O*NET) task measures for various activities described in Acemoglu and Autor (2011) to create an index of manual content for each disaggregated time use.[11] We classify activities in h_N if they have a manual skill index below the median and classify activities in h_P if they have an index above the median.

The CEX does not contain information on time spent on home production. To overcome this difficulty, we impute time use data from the ATUS into the CEX. Our imputation is based on an iterative procedure in which individuals in the CEX are allocated the mean home hours h_N and h_P of matched individuals from the ATUS based on group characteristics. We begin by matching individuals based on work status, race, gender, and age. We then proceed to improve these estimates by adding a host of additional characteristics, such as family status, education, disability status, geography, hours worked, and wages, and matching individuals based on these characteristics whenever possible. We first impute home hours to individuals and, similarly to market hours, then sum up these hours at the household level.

Our imputation accounts for approximately two-thirds of the variation in home hours h_N and h_P. In Appendix Table A.1 we confirm that our imputation does not introduce spurious correlations in the merged CEX/ATUS data by showing that the correlation of home hours with market hours and wages conditional on age is of similar magnitude between the ATUS sample of individuals and the merged CEX/ATUS sample of households. In Appendix Tables A.2 and A.3 we show that, conditional on age, married men, women, less educated, and more educated exhibit similar correlations between wages, market hours, and home hours in the ATUS. Further, in Appendix Tables A.4 and A.5 we show that the correlation of total home hours with market expenditures, market hours, and wages conditional on age is of similar magnitude between the CEX/ATUS and two PSID samples of households

[11] Because there are many such indices, we standardize the task measures to have mean zero and standard deviation of one and take the average across all manual tasks to create a single manual skill index. We list the mapping for the seven largest time use categories. Child care time is mapped to preschool teachers and child care workers; shopping time is mapped to cashiers; nursing time is mapped to registered nurses and nursing assistants; cooking is mapped to food preparation and serving workers; cleaning is mapped to maids and housekeeping cleaners; gardening is mapped to landscaping and groundskeeping workers; laundry is mapped to laundry and dry-cleaning workers.

Table 4: Summary Statistics of Time Allocation of Married Households

	Manual Skill Index	Hours per week		
		All	25-44	45-65
Market hours h_M		66.1	66.8	65.5
Home hours h_N		21.3	25.4	17.3
Child care	-0.73	10.8	14.9	6.7
Shopping	0.08	6.4	6.5	6.3
Nursing	-0.12	1.9	1.8	2.0
Home hours h_P		16.7	16.4	17.0
Cooking	0.41	7.5	7.4	7.5
Cleaning	0.43	3.7	3.7	3.6
Gardening	1.27	2.1	1.7	2.5
Laundry	0.89	2.0	2.1	1.9

Table 4 presents summary statistics of the time allocation of married households in the merged CEX/ATUS sample.

which do not require imputations since they contain information on home hours, market expenditures, market hours, and wages.

Table 4 presents summary statistics of the time allocation of married households in the CEX/ATUS sample along with the value of the manual skill index of occupations mapped to home production activities. Beginning with market hours h_M, we note a small decline over the life cycle. The three largest time uses classified in h_N are child care, shopping, and nursing. These are activities with lower manual content (and typically higher cognitive content) than activities such as cooking, cleaning, gardening, and laundry which we classify as h_P. The allocation of time between the two types of home production is relatively balanced, but there are noticeable differences over the life cycle. As expected, child care time declines significantly in the second half of working life which generates a decline in h_N over the life cycle. By contrast, h_P increases moderately over the life cycle.[12]

[12] Our life-cycle profiles are consistent with those reported in Cardia and Gomme (2018), who also embrace the view that child care has a different technology from other home production.

Table 5: Parameter Values

Parameter	$\omega_K = 0$	$\omega_K > 0$	Rationale
τ_1	0.12	0.12	$\log\left(\frac{\tilde{y}}{h_M}\right) = \mathcal{C}_\tau + (1-\tau_1)\log z_M$.
τ_0	-0.36	-0.36	Match $G/Y = 0.10$.
γ	1	1	Nesting of models.
η	0.90	0.50	Match $\beta = 0.54$ in $\log h_M = \mathcal{C}_\eta + \beta(\eta)\varepsilon$.
θ_P	—	4.64	$\theta_P = \left(\mathbb{E}\left(\frac{c_M}{\tilde{z}_M^\phi h_P}\right)^{\frac{1}{\phi}}\right)^{\frac{\phi}{1-\phi}}$.
ϕ	—	2.35	$\frac{\Delta_{65-25}\log(c_M/h_N)}{\Delta_{65-25}\log z_M} = \phi(1-\tau_1) = 2.07$.

Table 5 presents parameter values for the models without home production ($\omega_K = 0$) and with home production ($\omega_K > 0$).

2.3.2 Parameterization

Table 5 presents parameter values for our baseline analyses. We estimate the progressivity parameter τ_1 using data from the Annual Social and Economic Supplement of the Current Population Survey between 2005 and 2015. We use information on pre-tax personal income, tax liabilities at the federal and state level, Social Security payroll deductions, as well as usual hours and weeks worked. Our estimate of τ_1 comes from a regression of log after-tax market productivity on log market productivity before taxes. We estimate $\tau_1 = 0.12$ with a standard error below 0.01.[13] We choose $\tau_0 = -0.36$ to match an average tax rate on labor income equal to 0.10, which equals the average ratio of personal current taxes to income from the national income and product accounts.

For the home production model, we obtained the equilibrium allocations in closed form only under a curvature of the utility function with respect to consumption equal to $\gamma = 1$. We choose $\gamma = 1$ also for the model without home production. It is essential to nest the model without home production, so that welfare differences across the two models do not

[13]Our definitions of income and wage include the child care and earned income tax credits but exclude government transfers such as unemployment benefits, welfare, and food stamps because we think of fully insurable shocks ε as subsuming these transfers. Our estimated tax parameter is close to the estimate of 0.19 in Heathcote, Storesletten, and Violante (2014). Using their tax function $\log\tilde{y} = \text{constant} + (1-\tau_1)\log y$, we estimate $\tau_1 = 0.15$. We, therefore, argue that it is relatively inconsequential whether we apply the progressivity parameter $(1-\tau_1)$ to after-tax wages or after-tax labor income.

arise from different curvatures of the utility function with respect to consumption.[14]

Next, we estimate the parameter η for the curvature of the utility function with respect to hours. Our strategy is to choose η in each model such that a regression of log market hours $\log h_M$ on the insurable transitory component of market productivity ε yields a coefficient of 0.54. The target value of 0.54 comes from the meta analysis of estimates of the intensive margin Frisch elasticity from micro variation found in Chetty, Guren, Manoli, and Weber (2012). Consistent with the logic of Rupert, Rogerson, and Wright (2000) who argue that estimates of the Frisch elasticities are downward biased in the presence of home production, we estimate $\eta = 0.90$ in the model without home production and $\eta = 0.50$ in the model with home production.[15]

We now describe parameters specific to the home production model. To calibrate the constant level of production efficiency θ_P we use the optimality conditions (11) and take means over the population.[16] To estimate the elasticity of substitution ϕ, we again use the optimality conditions (11) to derive the regression:

$$\log\left(\frac{c_M}{h_N}\right) = \phi\log(1-\tau_0) + \phi(1-\tau_1)\log z_M - (\phi-1)\log\theta_N. \tag{15}$$

Estimation of ϕ using data on c_M/h_N and z_M would lead to biased estimates if z_M and θ_N are correlated. For this reason, we take changes over time in equation (15) and use a synthetic panel approach to estimate ϕ based on changes in c_M/h_N and changes in z_M between the beginning and the end of the life cycle. The identifying assumption is that changes in θ_N are uncorrelated with changes in z_M between the beginning and the end

[14] While welfare effects are sensitive to the value of γ in the model without home production, our inference of α and ε does not depend on γ as seen in Table 2.

[15] The Frisch elasticity for effective total hours h_T is $(1-\tau_1)\eta$ in both models. There are three reasons why η deviates from the targeted elasticity of 0.54. First, the progressivity of the tax system introduces the wedge $1-\tau_1$ between η and the Frisch elasticity for total hours h_T. Second, disutilities of work and home production efficiency are correlated with market wages. Third, even without such a correlation, the elasticities of market hours h_M differ between the two models because $h_M = h_T$ without home production whereas with home production h_M is negatively correlated to h_N and h_P. Our strategy is conservative in the sense that the inequality difference between the two models becomes larger when we set η to be equal between the two models.

[16] Rearranging this condition we obtain $\theta_P = \left(\mathbb{E}\left(\frac{c_M}{\bar{z}_M^{\phi} h_P}\right)^{\frac{1}{\phi}} / \,\mathbb{E}\left(\frac{\exp(D_P)}{\exp(B)}\right)\right)^{\frac{\phi}{1-\phi}}$. The level of θ_P is pinned down only relative to D_P/B. We normalize $\mathbb{E}\left(\frac{\exp(D_P)}{\exp(B)}\right) = 1$ because it standardizes the mean of θ_P in symmetric way to the mean of θ_N.

of the life cycle. This assumption is consistent with the assumptions underlying the no-trade result which requires $\theta_{N,t+1}$ to be independent of innovations to $z_{M,t+1}$. Both our estimation strategy and the no-trade theorem are consistent with a correlation of efficiency across sectors in levels.

We estimate that market and home goods are substitutes with an elasticity of $\phi = 2.35$. Our estimate of the elasticity of substitution is consistent with those found in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4 and Aguiar and Hurst (2007a) obtain estimates of around 2.

2.3.3 Inferred Sources of Heterogeneity

We extract the sources of heterogeneity using CEX/ATUS data on $(c_M, h_M, z_M, h_N, h_P)$ and our parameter values into the solutions of Table 2 for each household. In Figure 1 we present the age profiles of the means of $(\alpha, \varepsilon, B, D_P, \log \theta_N)$. To obtain these age profiles, we regress each source of heterogeneity on age dummies, cohort dummies, and normalized year dummies as in Deaton (1997).[17] We plot the coefficients on age dummies which give the mean of each source of heterogeneity by age relative to 25. To reduce noise in the figures, we present the fitted values from locally weighted regressions of the age dummies coefficients on age.

Recall from Table 2 that the insurable component of market productivity α grows over the life cycle when either the ratio of consumption to hours c_T/h_T grows or when wages z_M grow. The insurable component ε falls when the increase in c_T/h_T is large relative to the increase in z_M. The upper panels of Figure 1 show that the means of α and ε grow similarly until roughly 45 between the two models and diverge after that. The slower growth of α and the smaller decline in ε in the model with home production reflect the significant decline in home hours h_N in the later part of the life cycle which implies that c_T/h_T grows by less than c_M/h_M. In the lower panels we see that both models generate a relatively similar increase in the disutility of market work B, an increase reflecting the faster growth of z_M

[17]Results are similar when we extract the age effect in regressions which either control only for cohort dummies or only for year dummies.

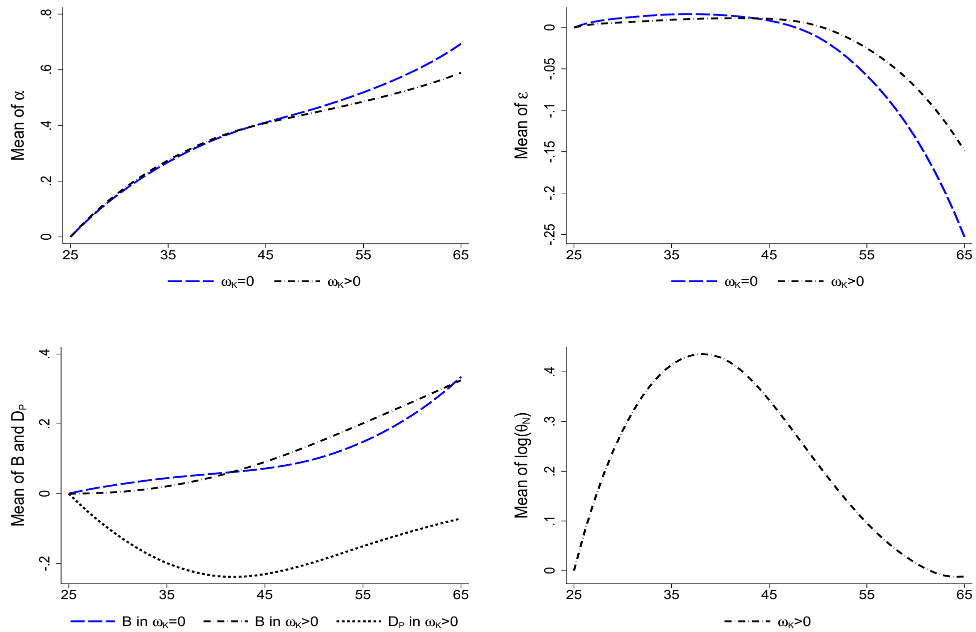


Figure 1: Means of Sources of Heterogeneity

Figure 1 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

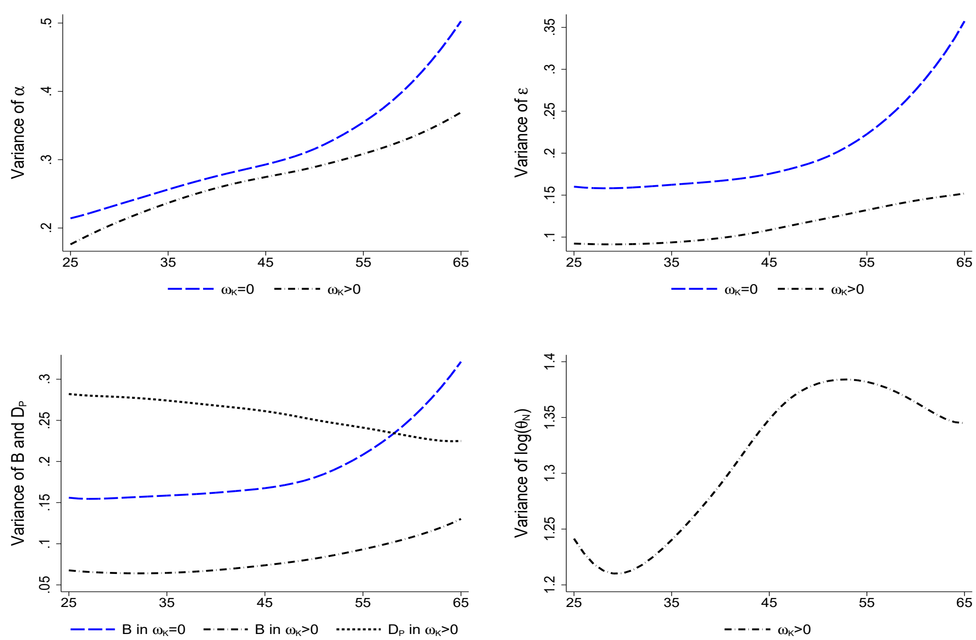


Figure 2: Variances of Sources of Heterogeneity

Figure 2 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

relative to c_T and h_T over the life cycle.[18]

The home production model generates a U-shaped profile of home disutility D_P which contrasts with the increasing profile of B. To understand this difference, recall from equation (14) that $\frac{\exp(D_P)}{\exp(B)} \propto \left(\frac{c_M}{h_P}\right)^{1/\phi} \frac{1}{\tilde{z}_M}$. To rationalize the faster growth of z_M relative to c_M/h_P during the earlier stages of the life cycle, the model requires a decline in D_P relative to B. As z_M and c_M/h_P comove more closely during the later stages of the life cycle, the profile of D_P slopes upward like the profile of B.

The model with home production generates a hump-shaped profile of home production efficiency θ_N. To understand this pattern, recall from equation (14) that $\theta_N = \tilde{z}_M^{\frac{\phi}{\phi-1}} \left(\frac{h_N}{c_M}\right)^{\frac{1}{\phi-1}}$. Until roughly 40, θ_N tracks market productivity z_M since $\phi > 1$. Despite z_M still rising, θ_N starts to decline after 40 and returns to its initial value by 65. This pattern is generated by the strong decline in hours h_N after 40. As shown in Table 4, child care is the subcategory of h_N responsible for this decline.

In Figure 2 we present the age profiles of cross-sectional variances of $(\alpha, \varepsilon, B, D_P, \log\theta_N)$, which equal the variances of the residuals for each age from a regression of each source of heterogeneity on age dummies, cohort dummies, and normalized year dummies. The home production model infers significantly smaller variances of α, ε, and B than the model without home production. From the solutions in Table 2, we observe that the increasing variance of α over the life cycle is driven by the increase in the variance of the consumption-hours ratio $\log(c_T/h_T)$ and the increase in the variance of wages $\log z_M$.[19] Because the variance of $\log(c_T/h_T)$ is lower than the variance of $\log(c_M/h_M)$, the home production model generates a lower variance of α. Given that both models match the same variance of $\log z_M$ but the

[18] The flexibility in terms of initial conditions allows the model to generate arbitrary inferred life-cycle profiles of heterogeneity without violating the random walk assumptions on the sources of heterogeneity which are essential for the no-trade result. For example, the mean of α_t^j is given by $\mathbb{E}\alpha_t^j = \mathbb{E}\alpha_j^j + \sum_{s=j+1}^{t} \mathbb{E}\upsilon_s^\alpha$, so the difference in the mean of α_t by age is $\mathbb{E}\alpha_t^{j-1} - \mathbb{E}\alpha_t^j = [\mathbb{E}\alpha_{j-1}^{j-1} - \mathbb{E}\alpha_j^j] + \mathbb{E}\upsilon_j^\alpha$, where the term in brackets is a cohort effect and the last term is a time effect. As a result, the inferred mean of α_t by age can appear to deviate from the mean of a random walk process with an innovation which grows constantly over the life cycle due to a combination of cohort and time effects which cannot be identified separately relative to age. Similarly, the change in the inferred variance of α_t is given by $\mathrm{Var}(\alpha_t^{j-1}) - \mathrm{Var}(\alpha_t^j) = [\mathrm{Var}(\alpha_{j-1}^{j-1}) - \mathrm{Var}(\alpha_j^j)] + \mathrm{Var}(\upsilon_j^\alpha)$ and can deviate from the change in the variance of a random walk process.

[19] To derive analytical solutions, we have not allowed for borrowing constraints which are important when thinking about the comovement of income with consumption at the bottom of the asset distribution. While the transmission mechanism is different than in our model, the presence of borrowing constraints generates comovement between income and consumption in a similar way to α.

home production model displays a larger covariance between α and ε than the model without home production (see Appendix Table A.6), ε turns out to be less dispersed in the home production model. The variance of B is also smaller in the home production model which reflects the smaller variance of a combination of $\log c_T$ and $\log h_T$ than a combination of $\log c_M$ and $\log h_M$.

In the lower panels we observe that the dispersion in the disutility of home work D_P is lower than the dispersion in the disutility of market work B but that home production efficiency $\log \theta_N$ is significantly more dispersed than any other source of heterogeneity. To set a benchmark for $\log \theta_N$, we note that the variance of $\log z_M$ is 0.33 in the data. What explains the almost four times as large dispersion in $\log \theta_N$? From equation (14), inferred home production efficiency is:

$$\log \theta_N = \text{constant} + \left(\frac{1}{\phi - 1}\right)\left(\phi \log \tilde{z}_M + \log h_N - \log c_M\right). \tag{16}$$

Our result that home production efficiency is more dispersed than market productivity reflects the fact that $\log \theta_N$ cumulates the dispersions of three observables, $\log \tilde{z}_M$, $\log h_N$, and $\log c_M$, which are relatively uncorrelated with each other.[20] When ϕ tends to zero and the goods tend to become perfect complements, we obtain $\log \theta_N = \text{constant} + \log c_M - \log h_N$. In this case the variance of $\log \theta_N$ is roughly 1.3 because the variance of $\log c_M$ is roughly 0.3, the variance of $\log h_N$ is roughly 1, and the two variables are relatively uncorrelated in the cross-section of households. When ϕ tends to infinity and the goods tend to become perfect substitutes, we obtain $\log \theta_N = \text{constant} + \log \tilde{z}_M$. In that case, the variance of $\log \theta_N$ converges to the variance of $\log \tilde{z}_M$. When ϕ tends to one, the variance of $\log \theta_N$ tends to infinity. To summarize, for any value of ϕ, the variance of $\log \theta_N$ exceeds the variance of $\log \tilde{z}_M$.

Figure 3 summarizes the properties of production efficiencies.[21] The left panel shows

[20] Recall that home production efficiency is a convolution of productivity and consumption weights, $\theta_N = \omega_N^{\frac{\phi}{\phi-1}} z_N$. As a result, its dispersion reflects dispersion in both home productivity and consumption weight as well as their covariation, $\text{Var}(\log \theta_N) = \left(\frac{\phi}{\phi-1}\right)^2 \text{Var}(\log \omega_N) + \text{Var}(\log z_N) + 2\frac{\phi}{\phi-1}\text{Cov}\left(\log \omega_N, \log z_N\right)$. Under our estimated $\phi = 2.35$, the dispersion in θ_N is roughly four times as large as the dispersion in market productivity z_M when, for example, $\text{Var}(\log z_M) = \text{Var}(\log z_N) = \text{Var}(\log \omega_N)$ and $\text{Cov}\left(\log \omega_K, \log z_K\right) = 0$.

[21] Appendix Table A.6 presents the correlation matrix of all observables and sources of heterogeneity.

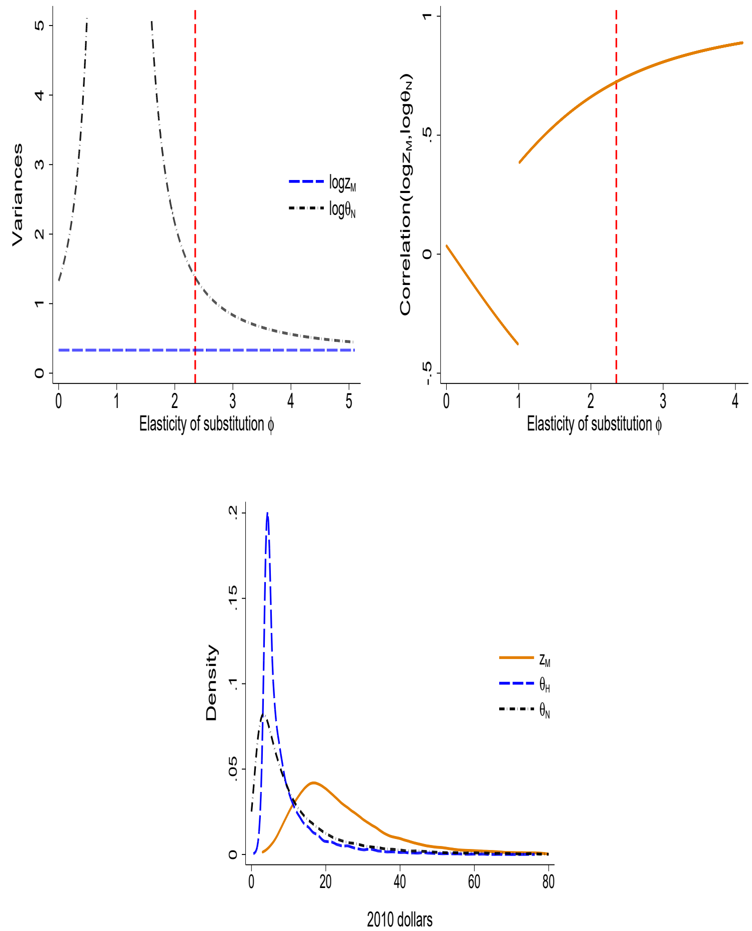


Figure 3: Production Efficiency Moments

The left panel of Figure 3 shows the variance of home production efficiency $\log \theta_N$ and market productivity $\log z_M$ and the middle panel shows the correlation between the two variables as a function of the elasticity of substitution across sectors ϕ. The dashed vertical line shows the variances and correlation at our estimated value of $\phi = 2.35$. The right panel plots estimates of the distributions of z_M, $\theta_H = \frac{h_N}{h_N+h_P}\theta_N + \frac{h_P}{h_N+h_P}\theta_P$, and θ_N at $\phi = 2.35$.

the variances of $\log \theta_N$ and $\log z_M$ and the middle panel shows the correlation of the two variables as function of the elasticity of substitution across sectors ϕ. The variance of $\log \theta_N$ is larger than the variance of $\log z_M$ for any value of $\phi < 5$ in the figure.[22] The correlation between the two variables changes sign with the value of ϕ. When goods are substitutes, $\phi > 1$ as suggested by our estimation, efficiency in the home sector is positively correlated

Appendix Figure A.1 shows estimates of the distributions of all other sources of heterogeneity.

[22]We note that the argument in the preceding paragraph referred to after-tax market productivity $\log \tilde{z}_M$ whereas in Figure 3 we use the more primitive pre-tax market productivity $\log z_M$. The former measure of productivity is roughly 77 percent as dispersed as the latter given our estimated tax progressivity parameter $\tau_1 = 0.12$.

with efficiency in the market sector. If goods were complements, $\phi < 1$, the correlation would typically have been negative.

The right panel of Figure 3 plots the distributions of production efficiencies under our estimated $\phi = 2.35$. We define effective home production efficiency $\theta_H = \frac{h_N}{h_N + h_P}\theta_N + \frac{h_P}{h_N + h_P}\theta_P$. Because θ_P is a constant, we find that effective efficiency at home θ_H is less dispersed than θ_N. The means of z_M, θ_H, and θ_N are 26.6, 10.9, and 14.3 dollars respectively. The fraction of households with efficiency exceeding 100 dollars per hour equals roughly 1 percent, 0.5, and 1.2 percent respectively.

2.4 Inequality and Home Production

We demonstrate that home production amplifies inequality across households and that heterogeneity in production efficiency rather than disutility of work is crucial in amplifying inequality.

2.4.1 Home Production Amplifies Inequality

We demonstrate that inequality across households is larger in the home production model than in the model without home production, despite both models generating the same data on market observables. By inequality, we mean a mapping from the dispersion in observed allocations and inferred sources of heterogeneity to measures capturing welfare differences across households. We acknowledge there are various such mappings and, therefore, present four inequality metrics.

Equivalent Variation

The equivalent variation, a broadly used metric in welfare economics, is the change in income required for a household to achieve a reference level of utility. Let $\hat{\iota}$ be a reference household with a derived utility $V(\hat{c}_{M,t}, \hat{h}_{M,t}, \hat{h}_{K,t}; \hat{\iota})$, and a value function $\hat{W}_t(\hat{\iota})$. For every household ι, we compute the income transfer $T_t(\iota)$ making it indifferent between being ι and being $\hat{\iota}$ in the current period, holding constant ι's expectation over all future allocations.

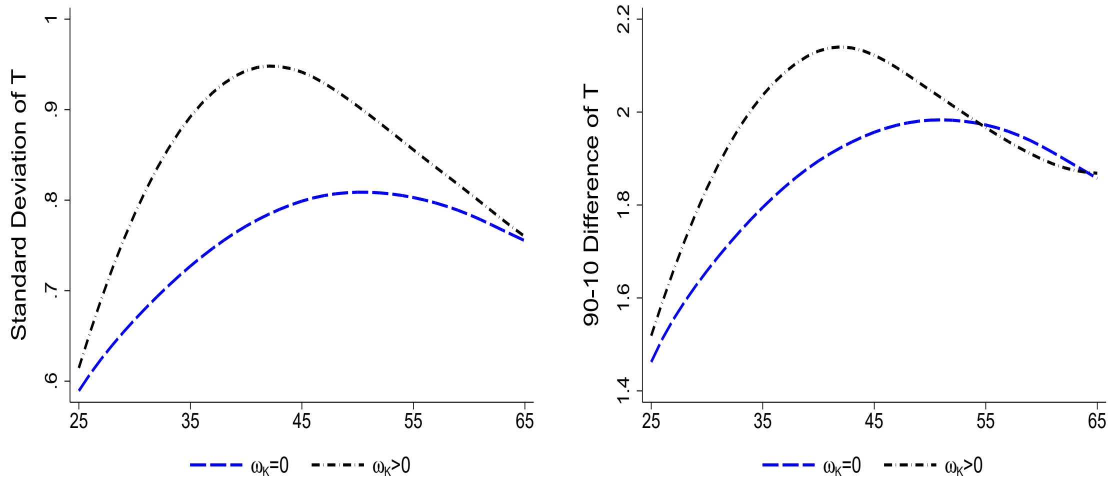


Figure 4: Dispersion in Equivalent Variation

Figure 4 shows the dispersion in equivalent variation T for the model without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age. The standard deviation of T is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

The equivalent variation $T_t(\iota)$ solves:

$$\hat{W}_t(\hat{\iota};\iota) = \max_{\{c_{M,t},h_{M,t},h_{K,t}\}} \left\{ V(c_{M,t}, h_{M,t}, h_{K,t}; \iota) + \beta\delta\mathbb{E}_t \left[W_{t+1}(\iota')|\iota \right] \right\}, \tag{17}$$

subject to the budget constraint:

$$c_{M,t} = \tilde{y}_t + T_t(\iota) + \overline{\mathrm{NA}}_t(\iota). \tag{18}$$

In the left-hand side of equation (17) we define $\hat{W}_t(\hat{\iota};\iota) \equiv V(\hat{c}_{M,t}, \hat{h}_{M,t}, \hat{h}_{K,t}; \hat{\iota}) + \beta\delta\mathbb{E}_t \left[W_{t+1}(\iota')|\iota \right]$ and in equation (18) we keep the net asset position $\overline{\mathrm{NA}}_t(\iota)$ constant at its value before the transfer $T_t(\iota)$ is given.

Figure 4 presents the cross-sectional dispersion in equivalent variation by age.[23] The left panel shows the standard deviation of equivalent variation, standardized by the mean value of market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages. The standard deviation is around 0.6 in both economies at 25. By 45, however, the standard

[23] For the equivalent variation in this figure, the reference household $\hat{\iota}$ is the household with the median utility in the sample. Our results are similar when $\hat{\iota}$ is the household with the mean utility in the sample, when the identity of $\hat{\iota}$ differs by age and is the household with the median utility for each age, and when the identity of $\hat{\iota}$ differs by age and is the household with the mean utility for each age.

deviation has increased to more than 0.9 in the home production model, as opposed to below 0.8 in the model without home production. Similarly, the right panel shows divergent patterns until 55 between the two models using the difference between the 90th and 10th percentile in equivalent variation. Both inequality metrics tend to converge across the two models for households older than 60.

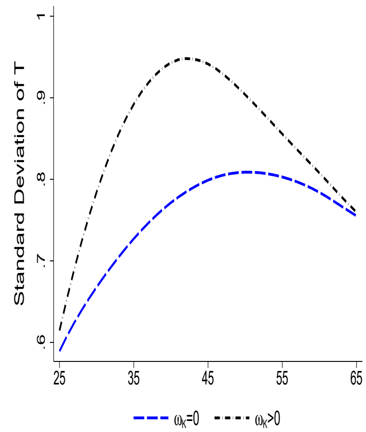
What drives our inference that inequality is higher with home production? An important feature of the data driving this inference is that home hours h_N are not negatively correlated with market consumption c_M and market productivity z_M in the cross section of households. We calculate that h_N has a correlation of 0.07 with $\log z_M$ and 0 with $\log c_M$. Thus, home production does not offset heterogeneity originating in the market sector. Instead, home production exacerbates inequality given the large dispersion in home production efficiency θ_N.[24]

To illustrate this point, in Figure 5 we repeat our analyses using a different correlation of home hours h_N with other observables in the data. The left panel repeats the age profile of the standard deviation in equivalent variation $T(\iota)$ shown in the left panel of Figure 4. In the other two panels we calculate the equivalent variation $T(\iota)$ when we repeat our inference of $(\alpha, \varepsilon, B, D_P, \theta_N)$ in counterfactual data in which the correlation of home hours h_N with market productivity $\log z_M$ and market expenditures $\log c_M$ is -0.8. The figure shows that if the data featured a significantly more negative correlation between h_N and either $\log z_M$ or $\log c_M$, then we would have concluded that inequality in the model with home production is actually lower.

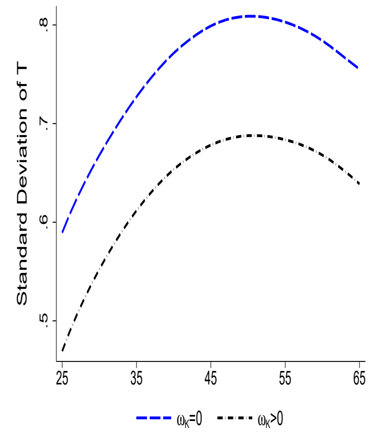
Redistributive Transfers

Our second measure of inequality is the cross-sectional dispersion in redistributive transfers which equalize marginal utilities. After households choose their allocations of consumption

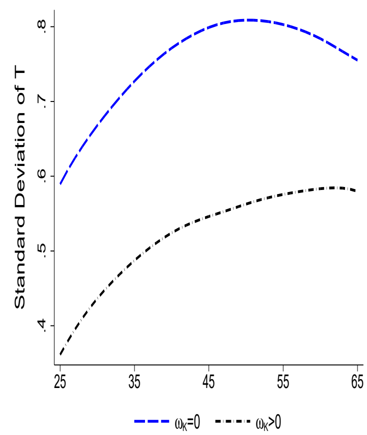
[24]We focus on h_N because its low correlation with c_M and z_M is more informative than the low correlations of h_P and further discuss the role efficiency and disutility heterogeneity in Section 2.4.2. Given that child care is the largest subcategory of h_N, our estimate of a weakly positive correlation between h_N and z_M is broadly consistent with the findings of Guryan, Hurst, and Kearney (2008) who document that higher educated and higher income parents tend to spend more time with their children. Appendix Tables A.1, A.2, and A.3 demonstrate that the lack of a negative correlation with wages is present both for individuals and households and is present within age, sex, and education groups. Appendix Tables A.4 and A.5 demonstrate that the correlation of home hours with both consumption and wages is broadly similar in magnitude between the CEX/ATUS sample and PSID samples in which home production time is not imputed.



(a) Baseline



(b) $\operatorname{corr}(h_N, \log z_M) = -0.8$



(c) $\operatorname{corr}(h_N, \log c_M) = -0.8$

Figure 5: Counterfactuals of Dispersion in Equivalent Variation

Figure 5 shows the dispersion in equivalent variation T for the model without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age in the baseline and in counterfactual datasets.

and hours, we allow a utilitarian planner to redistribute aggregate market consumption across households in order to maximize average household utility. The dispersion in these transfers captures the extent of redistribution required to maximize social welfare or, equivalently, to equalize marginal utilities of market consumption. Formally, the problem is to choose transfers $\{t(\iota)\}$ to maximize:

$$\int_{\iota} V(c_M(\iota) + t(\iota), h_M(\iota), h_N(\iota), h_P(\iota)) \mathrm{d}\Phi(\iota), \tag{19}$$

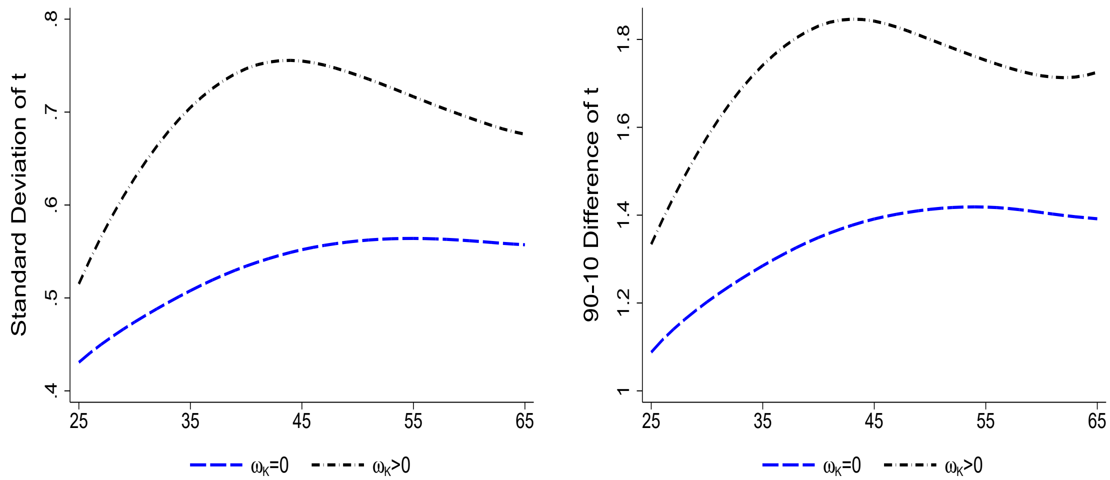


Figure 6: Dispersion in Redistributive Transfers

Figure 6 shows the dispersion in redistributive transfers t for the environment without ($\omega_K = 0$, blue dashed line) and with home production ($\omega_K > 0$, black dotted line) by age. The standard deviation of t is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

subject to aggregate transfers being equal to zero $\int_\iota t(\iota)\mathrm{d}\Phi(\iota) = 0$.

The optimal transfers equal the gap between average and individual market value of total consumption $c_T(\iota)$:[25]

$$t(\iota) = \int_\iota c_T(\iota)\mathrm{d}\Phi(\iota) - c_T(\iota). \tag{20}$$

The dispersion in redistributive transfers $t(\iota)$ differs from the dispersion in equivalent variation $T(\iota)$ in Section 2.4.1 because it leads to an equalization of marginal utilities instead of utility levels. An advantage of using the dispersion in $t(\iota)$ as a measure of inequality is that it depends transparently only on observables and estimated parameters.

The left panel of Figure 6 shows the age profiles of the cross-sectional standard deviation in redistributive transfers $t(\iota)$ for the two models, standardized again by the mean value of market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$. The standard deviation of $t(\iota)$ is larger and increases more over the life cycle in the model with home production. We obtain a similar result in the right panel which shows the difference between the 90th and 10th percentile in redistributive transfers $t(\iota)$.

[25] We remind the reader that the marginal utility of market consumption under an equilibrium allocation $(c_M + t, h_M, h_N, h_P)$ equals the inverse of the market value of total consumption c_T given in equation (13).

It is instructive to compare our findings to the those of Frazis and Stewart (2011) and Bridgman, Dugan, Lal, Osborne, and Villones (2012) who have embraced the view that home production decreases inequality. Their argument is that, since home hours do not correlate with income in the cross section of households, adding a constant value of home production across households results in a smaller dispersion in total income. Inspection of equation (13) for c_T reveals a fundamental difference in our logic. Home hours in our model are valued at their opportunity cost which varies across households. Using a constant opportunity cost does not take into account differences in the efficiency or disutility of home hours across households.[26]

Lifetime Welfare Cost of Heterogeneity

This section presents the lifetime welfare effects from heterogeneity across households. Our calculations contrast with our inequality metrics so far which ignore dynamic considerations. The lifetime welfare effect is the share of consumption in every period which a household is willing to sacrifice ex-ante to be indifferent between being born in the baseline environment with heterogeneity and allocations $\{c_t, h_{M,t}, h_{N,t}, h_{P,t}\}$ and a counterfactual environment in which dimensions of heterogeneity are shut down. The allocations in the counterfactual economy are denoted by $\{\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}, \hat{h}_{P,t}\}$ and are generated using the equations in Table 1 after shutting down particular dimensions of heterogeneity.[27]

The share of lifetime consumption that makes households indifferent between the actual and the counterfactual economy is given by the λ which solves:

$$\mathbb{E}_{j-1} W(\{c_t, h_{M,t}, h_{N,t}, h_{P,t}\}) = \mathbb{E}_{j-1} W(\{(1-\lambda)\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}, \hat{h}_{P,t}\}) \ , \tag{21}$$

[26] A reasonable concern using wages to value home hours is that some households or members of the household may be at a corner solution. In practice, we are not concerned that valuing home hours at its opportunity cost biases our results for three reasons. First, in our baseline CEX/ATUS sample of married households the fraction of households with either zero market hours or zero home hours per year is less than one percent. Further, sensitivity analyses presented in Section 2.5 confirm our inequality results in a sample of singles and in a subsample of married households with a working spouse for which valuation at market wages is less concerning. Finally, our notion of inequality in consumption allows for a wedge between the wage and the marginal value of home hours h_P arising from disutility differences across sectors.

[27] Consistent with our definition of equilibrium in which G is an endogenous variable, in these counterfactuals we keep constant the tax parameters (τ_0, τ_1) because we prefer to evaluate more direct welfare effects arising from heterogeneity rather than more nuanced effects arising from changes in the tax parameters in order to satisfy the government budget constraint. By contrast, when we calculate optimal taxes (τ_0, τ_1) in Section 2.4.1, we keep constant G to its initial equilibrium value.

Table 6: Lifetime Welfare Cost of Heterogeneity

	No Home Production: $\omega_K = 0$		Home Production: $\omega_K > 0$	
No dispersion in ...	λ_p	λ	λ_p	λ
z_M, θ_N, B, D_P	0.04	0.06	0.05	0.12
z_M, θ_N	0.04	0.07	0.05	0.16
z_M	0.04	0.07	0.05	0.11
θ_N	—	—	0.00	0.13

Table 6 shows changes in aggregate labor productivity λ_p and welfare λ for the environment without ($\omega_K = 0$) and with ($\omega_K > 0$) home production. Each row shuts down combinations of sources of heterogeneity.

where $c_t = \left(c_{M,t}^{\frac{\phi-1}{\phi}} + (\theta_{N,t}h_{N,t})^{\frac{\phi-1}{\phi}} + (\theta_{P,t}h_{P,t})^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$. When $\lambda > 0$, households prefer the counterfactual to the actual allocation. Benabou (2002) and Floden (2001) have emphasized that total welfare effects from eliminating heterogeneity arise both from level effects when aggregate allocations change and effects capturing changes in the dispersion in allocations across households. Therefore, alongside λ, we discuss how heterogeneity influences aggregate labor productivity $\int_\iota z_M(\iota)h_T(\iota)\mathrm{d}\Phi(\iota) / \int_\iota h_T(\iota)\mathrm{d}\Phi(\iota)$. We denote by λ_p the percent change in aggregate labor productivity between the counterfactual and the baseline allocation. Dispersion in market productivity z_M decreases aggregate labor productivity because h_T is negatively correlated with z_M in both models.

In the first row of Table 6, we shut down all sources of heterogeneity and both models collapse to a representative household economy. The welfare cost of heterogeneity λ is 12 percent in the model with home production as opposed to 6 percent in the model without home production. The difference between the two models reflects predominately the differential cost of dispersion in allocations rather than aggregate productivity changes λ_p which are relatively similar across models.[28]

The larger dispersion costs of heterogeneity in the home production model reflect the costs of dispersion in the efficiency of work rather than the disutility of work. To see this,

[28]The welfare effects in Table 6 reflect heterogeneity both within age and over the life cycle because each counterfactual imposes a constant value of the source of heterogeneity for households of all ages. We have repeated these exercises by shutting down only the within-age heterogeneity and allowing each source of heterogeneity to take its mean value by age as shown previously in Figure 1. Appendix Table A.7 shows similar welfare effects to those shown in Table 6 and, therefore, we conclude that the welfare effects predominately reflect the within-age component of heterogeneity.

in the second row we shut down heterogeneity in efficiencies, z_M and θ_N, while we maintain heterogeneity in disutilities of work B and D_P. We find even larger welfare effects than in the first row and, thus, conclude that heterogeneity in B and D_P is not important for the welfare effects of eliminating all heterogeneity. In the third row, we shut down only heterogeneity in market productivity z_M and find that eliminating this source of dispersion carries larger welfare gains in the model with home production than in the model without. In the fourth row, we shut down heterogeneity in home production efficiency θ_N only. The significant welfare effects illustrate again the key role of heterogeneity in θ_N for the welfare losses.

Optimal Tax Progressivity

This section contrasts the optimal progressivity of the tax system between the model with and without home production. Relative to our previous inequality metrics, this optimal taxation exercise mixes redistribution with efficiency concerns because the optimal progressivity of the tax system increases with redistributive motives and decreases with the efficiency losses from distorting labor allocations. However, this exercise allows us to more directly link our inequality result to policy.

Given government expenditures G fixed at its initial equilibrium level, the government chooses tax function parameters $\tau \equiv (\tau_0, \tau_1)$ to maximize utilitarian welfare:

$$\int_\iota V(c_M(\tau), h_M(\tau), h_N(\tau), h_P(\tau); \iota) \mathrm{d}\Phi(\iota) \ , \tag{22}$$

subject to the government budget constraint:

$$\int_\iota \left[z_M - (1-\tau_0) {z_M}^{1-\tau_1} \right] h_M(\tau) \mathrm{d}\Phi(\iota) = G. \tag{23}$$

In Figure 7 we plot the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$, both in thousands of 2010 dollars. The orange solid curve shows the relationship between y and $\tilde{y}$ under the parameter $\tau_1 = 0.12$ which we estimated in the data. The blue dashed and black dotted curves show this relationship under the optimal $\tau_1 = 0.06$ for

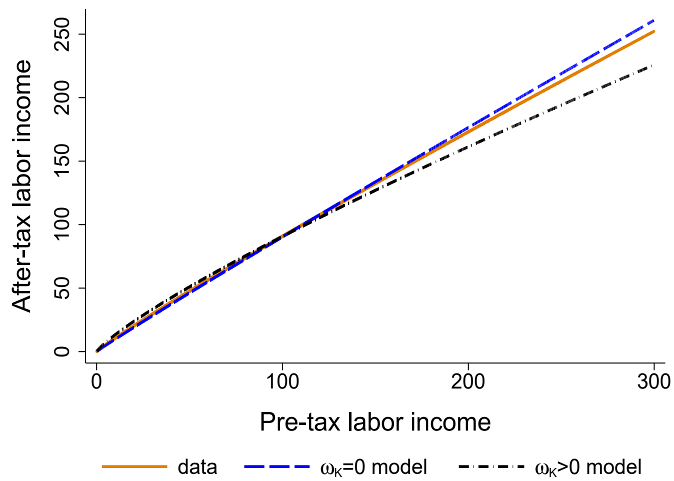


Figure 7: Optimal Tax Function

Figure 7 displays the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ under the parameters estimates for the United States (orange solid line), under the optimal tax function for the model without home production ($\omega_K = 0$, blue dashed line), and under the optimal tax function with home production ($\omega_K > 0$, black dotted line).

the model without home production and the optimal $\tau_1 = 0.24$ for the model with home production. The relationship between y and $\tilde{y}$ is significantly more concave in the model with home production. To give an example, consider a household earning 200 thousand dollars. Under the optimal tax schedule in the model without home production the household faces an average tax rate of 12 percent, while in the model with home production the average tax rate increases to 19 percent.

2.4.2 Heterogeneity in Home Efficiency versus Disutility of Work

Using four different metrics of inequality, we have demonstrated that home production amplifies inequality across households. In our baseline model heterogeneity in the home sector shows up in both the production efficiency of work θ_N and the disutility of work D_P because both sectors are operating in equilibrium ($\omega_N > 0$ and $\omega_P > 0$). Which source of heterogeneity is quantitatively more important in elevating inequality?

To quantify the importance of home production efficiency and disutility of home work, we consider the two polar cases of all home time $h_N + h_P$ belonging either to the sector with heterogeneity in production efficiency or to the sector with heterogeneity in disutility of work. When we set $\omega_P = 0$, then we obtain a two-sector model in which the disutility of

Table 7: The Role of Home Efficiency and Home Disutility in Amplifying Inequality

	No Home Production	Home Production		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13

Table 7 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. The estimated values for η are 0.90, 0.53, 0.50, and 0.57. The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work.

work B is equalized across sectors and the sectoral allocation of time depends on production efficiencies in the market z_M and at home θ_N. Instead, setting $\omega_N = 0$ yields a two-sector model in which market productivity z_M and differences in the disutility of work across sectors, B and D_P, determine the allocation of time.

Table 7 summarizes our results. The first column presents the four inequality metrics (averaged across all ages) in the model without home production and the last three columns present the metrics in the three versions of the home production model. In the home production model with only heterogeneity in home production efficiency, all inequality metrics are magnified relative to the baseline with heterogeneity in both efficiency and disutility. If there was only heterogeneity in disutility of home work, there would be no significant difference in inequality between the model with and the model without home production. We conclude that heterogeneity in home production efficiency rather than disutility of work is important in amplifying inequality across households.[29]

[29] We have also considered two additional cases of interest. In the first case there is heterogeneity in home production efficiency θ_N and θ_P in both sectors and no disutility differences across sectors, $B = D_N = D_P$. We obtain nearly identical results to the $\omega_P = 0$ case. In the second case there is heterogeneity in the disutility of home work D_N and D_P in both sectors and both θ_N and θ_P are constant across households. We obtain nearly identical results to the $\omega_N = 0$ case.

2.5 Sensitivity Analyses

In this section we present various sensitivity analyses with respect to the parameterization of the model, subsamples of the population, and measurement error in observables. Each row in Table 8 corresponds to a different sensitivity analysis. For both models, the columns show the standard deviation in equivalent variation T, the standard deviation in transfers required to equalize marginal utilities t, the ex-ante lifetime welfare loss from shutting down all heterogeneity λ, and the degree of progressivity in the optimal tax system τ_1. In each exercise, we repeat our analysis of identifying the sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \theta_N)$ and then calculate the inequality metrics. The first row of the table repeats these statistics for our baseline case.

Rows 2 to 9 vary parameters of the model. Relative to our estimated value $\tau_1 = 0.12$, changing the progressivity of the tax system to $\tau_1 = 0.06$ as in Guner, Kaygusuz, and Ventura (2014) or to $\tau_1 = 0.19$ as in Heathcote, Storesletten, and Violante (2014) does not alter significantly any result. We also obtain highly similar results when we change the target for the average labor income tax G/Y to 0.05 or 0.15. In rows 6 and 7 we change the target coefficient from the regression of $\log h_M$ on ε used to identify the curvature with respect to hours η. Raising η to target a coefficient of 0.70 as suggested by Pistaferri (2003) increases the level of three of the inequality metrics in both models, but in all cases inequality remains higher in the model with home production.

In rows 8 and 9, we vary the elasticity of substitution across goods ϕ. The Std(t) inequality metric is relatively insensitive to ϕ. When $\phi = 0.5$ and goods are complements, the Std(T), λ, and τ_1 metrics of inequality increase substantially relative to the baseline with $\phi = 2.35$. Intuitively, the complementarity between goods implies that home production amplifies differences in the market sector even more. When $\phi = 20$ and goods are almost perfect substitutes, we still find that inequality is higher with home production according to the Std(T), Std(t), and λ metrics but to a lesser extent than before. The main difference with our baseline arises in terms of the optimal progressivity which is significantly affected by the value of ϕ. Because a higher value of ϕ increases the efficiency losses from a progressive tax system, we obtain a lower τ_1 in the model with home production and $\phi = 20$ than in

Table 8: Sensitivity Analyses of Inequality Metrics

		No Home Production: $\omega_K = 0$				Home Production: $\omega_K > 0$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.06	0.90	0.73	0.12	0.24
	Parameter Values								
2.	$\tau_1 = 0.06$	0.78	0.55	0.06	0.12	0.93	0.74	0.14	0.27
3.	$\tau_1 = 0.19$	0.78	0.55	0.05	-0.04	0.88	0.72	0.10	0.20
4.	$G/Y = 0.05$	0.78	0.55	0.06	0.03	0.90	0.73	0.12	0.24
5.	$G/Y = 0.15$	0.78	0.55	0.06	0.09	0.90	0.73	0.12	0.25
6.	Target Frisch = 0.4	0.68	0.55	0.02	-0.74	0.80	0.73	0.10	0.06
7.	Target Frisch = 0.7	0.85	0.55	0.08	0.26	0.98	0.73	0.13	0.31
8.	$\phi = 0.5$	0.78	0.55	0.06	0.06	1.94	0.70	0.52	0.44
9.	$\phi = 20$	0.78	0.55	0.06	0.06	0.85	0.71	0.09	-0.80
	Marital, Employment, Family, and Education Groups								
10.	Singles	0.89	0.61	0.01	0.03	0.90	0.71	0.08	0.13
11.	Non-working spouse	0.80	0.55	0.10	0.22	1.34	1.07	0.21	0.33
12.	Working spouse	0.78	0.54	0.05	0.09	0.85	0.70	0.10	0.23
13.	No children	0.79	0.55	0.10	-0.06	0.81	0.67	0.18	0.13
14.	One child	0.78	0.55	0.07	0.10	0.85	0.72	0.11	0.27
15.	Two or more children	0.77	0.53	0.04	0.15	0.96	0.77	0.19	0.31
16.	Child younger than 5	0.77	0.54	0.01	0.15	1.02	0.82	0.24	0.34
17.	Less than college	0.78	0.55	0.02	-0.22	0.86	0.71	0.06	0.13
18.	College or more	0.76	0.52	0.06	-0.10	0.86	0.68	0.15	0.20
	Consumption Expenditures								
19.	Food expenditures	0.82	0.56	0.04	-0.05	0.92	0.75	0.11	0.21
20.	All expenditures	0.88	0.63	0.07	0.18	0.99	0.83	0.11	0.27
21.	Adjusted baseline	0.57	0.39	0.07	0.23	0.79	0.60	0.13	0.31
22.	Adjusted all	0.84	0.60	0.07	0.26	0.97	0.80	0.11	0.31

Table 8 presents each sensitivity analysis in a row. Columns show the four inequality metrics for the model without home production ($\omega_K = 0$) and the model with home production ($\omega_K > 0$).

the model without home production.

In rows 10 to 18 of Table 8 we repeat our analyses in subsamples of households defined along their marital status, employment status of the spouse of the head, number of children, age of youngest child, and education. Repeating our analyses for different samples allows us to explore whether our inequality results reflect within group inequality or inequality across groups. Additionally, verifying our results at the subgroup level is reassuring because it allows us to control for dimensions of heterogeneity which we did not model, such as spousal employment at the extensive margin, the presence of young children, or the number of children.

Our results are remarkably stable at the subgroup level, with the home production model always generating more inequality than the model without home production according to all four metrics. Row 10 shows the sample of singles, for which the inequality gap between models is generally smaller. Rows 11 and 12 show subsamples of married households according to whether the spouse is working or not. Reassuringly for the mechanisms we have stressed, we obtain a larger inequality gap between models in the group of households with non-working spouses for which we expect home production efficiency differences to be more important. In rows 13 to 15 we differentiate according to the number of children present in the household. We obtain larger inequality gaps between models in households with more children, which highlights the importance of time spent on child care for our results. Similarly, in row 16 we find even larger inequality gaps in households with a child younger than 5. Finally, rows 17 and 18 show results for married households with a head who has not completed college and with a head who has completed college or more. Our results are similar to the baseline with the exception of the optimal progressivity τ_1 which declines substantially in the model without home production.

In rows 19 and 20 we show that our results are robust under two alternative measures of market expenditures c_M. In row 19 we use food only whereas in row 20 we use all expenditures including health, education, and durables. The inequality metrics and the gap between the two models are generally similar to the baseline which used nondurable consumption excluding health and education. From the four metrics, the optimal progressivity τ_1 is the most sensitive to the measure of consumption.

A concern about our results is that the dispersion in reported consumption reflects measurement error which may affect inequality differentially across the two models. We now examine the robustness of our results to measurement error in consumption expenditures.

For each spending category comprising our aggregate household consumption measure, we use the elasticity of the spending category with respect to aggregate household consumption estimated by Aguiar and Bils (2015) to adjust households' spending category for measurement error. Aggregating across all spending categories produces measurement-error adjusted aggregate household consumption measures which we use to repeat our analyses.[30] We present results in row 21 for the baseline measure of nondurable consumption and in row 22 for all expenditures including health, education, and durables. We find that the model with home production still generates larger inequality than the model without home production.[31]

An alternative way to examine the sensitivity of our results to measurement error is to simulate the effects of reducing the dispersion in observables on the inequality metrics. We consider a classical measurement error model in which the reported value of variable x for household ι is:

$$\log x(\iota) = \log x^*(\iota) + m(\iota), \tag{24}$$

where x^* is the measurement-error adjusted value of variable x and m denotes a classical measurement error with variance σ_m^2.

In Table 9, rows 2 to 4 show results with measurement error in market consumption, rows 5 to 7 with measurement error in market hours, rows 8 to 10 with measurement error in home hours, and rows 11 to 13 with measurement error in all variables simultaneously. For each case we show measurement errors absorbing 20, 50, and 80 percent of the variance

[30] Let $x_{M,j}(\iota)$ be reported spending of household ι in category j, $x_M(\iota) = \sum_j x_{M,j}(\iota)$ be aggregate reported consumption of household ι, and β_j be the elasticity of spending $x_{M,j}$ with respect to aggregate household consumption x_M estimated by Aguiar and Bils (2015). We allocate aggregate spending over all households in each category for a particular year, $x_{M,j} = \sum_\iota x_{M,j}(\iota)$, to households in proportion to their predicted spending in that category based on their aggregate household consumption and the spending elasticity, $x_M(\iota)^{\beta_j}$. For each household ι we obtain $c_{M,j}(\iota) = \frac{x_M(\iota)^{\beta_j}}{\sum_\iota x_M(\iota)^{\beta_j}} x_{M,j}$ and define the measurement-error adjusted aggregate household consumption as $c_M(\iota) = \sum_j c_{M,j}(\iota)$.

[31] Measurement-error adjusted consumption is less dispersed than reported consumption and, therefore, inequality according to the Std(T) and Std(t) metrics decreases in both models relative to the baseline. The measurement error adjustment lowers consumption dispersion because low-elasticity categories (such as food) account for larger fractions of aggregate spending. We calculate that the expenditure-weighted average elasticity of spending categories is 0.71 with respect to nondurable consumption and 0.95 with respect to aggregate spending. We have confirmed that our results are similar when using NIPA expenditures instead of CEX expenditures.

Table 9: Inequality Metrics and Measurement Error

		No Home Production: $\omega_K = 0$				Home Production: $\omega_K > 0$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.06	0.90	0.73	0.12	0.24
	Consumption $x = c_M$								
2.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.74	0.51	0.05	0.10	0.87	0.69	0.12	0.26
3.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.62	0.41	0.04	0.15	0.80	0.60	0.12	0.27
4.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.45	0.26	0.03	0.18	0.70	0.47	0.12	0.29
	Market Hours $x = h_M$								
5.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.79	0.55	0.05	0.08	0.90	0.73	0.12	0.24
6.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.80	0.55	0.06	0.14	0.89	0.73	0.12	0.26
7.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.80	0.55	0.06	0.21	0.88	0.73	0.12	0.30
	Home Hours $x = \{h_N, h_P\}$								
8.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.78	0.55	0.06	0.06	0.92	0.74	0.12	0.24
9.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.78	0.55	0.06	0.06	0.88	0.73	0.13	0.24
10.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.78	0.55	0.06	0.06	0.78	0.70	0.14	0.25
	All Variables $x = \{c_M, h_M, h_N, h_P\}$								
11.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.74	0.51	0.05	0.12	0.89	0.70	0.12	0.26
12.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.63	0.41	0.05	0.21	0.77	0.59	0.13	0.29
13.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.46	0.26	0.05	0.31	0.51	0.40	0.14	0.33

Table 9 presents each sensitivity analysis in a row. Columns show the four inequality metrics for the model without home production ($\omega_K = 0$) and the model with home production ($\omega_K > 0$).

of the observed variable. Our process is to draw measurement error with variance σ_m^2 across households and then use simulated values x^*, which by construction display lower dispersion than reported values x, as the data input for the extraction of the sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \theta_N)$ and the measurement of inequality.

We find small differences relative to our baseline results. Inequality tends to decline with measurement error in consumption, but not differentially across the two models. For market hours, measurement error affects only the optimal progressivity τ_1, but we always find that progressivity is higher in the home production model. Finally, most of our results are robust to measurement error of up to 80 percent of the variance of home hours. At that level, the dispersion in equivalent variation in the model with home production is equal to the dispersion in the model without home production. We still obtain higher inequality with home production using the other three metrics of inequality.

2.6 Other Datasets and Countries

We show the similarity of the inequality results between the CEX/ATUS and three alternative datasets, the Panel Study of Income Dynamics (PSID), the Japanese Panel Survey of Consumers (JPSC), and the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS).

2.6.1 Comparison between CEX/ATUS and PSID

The PSID has two advantages relative to the CEX/ATUS. It has a panel dimension and contains information on both expenditures and time spent on home production. However, we prefer using the CEX/ATUS sample for our baseline analyses for three reasons. First, the PSID survey question covers aggregated time spent on home production, which does not allow us to separate credibly home hours h_N in the sector with efficiency heterogeneity from home hours h_P in the sector with disutility heterogeneity. Second, the PSID has lower quality of time use data as compared to the time diaries from the ATUS. In particular, it is not clear if respondents include activities such as child care and shopping in their reported home hours.[32] Third, food is the only measure of consumption which is consistently covered across surveys. Later surveys cover expanded categories but the sample size is significantly smaller than the CEX/ATUS sample.

We use two versions of the PSID. In the version in which c_M includes only expenditures on food, we have 69,951 observations between 1975 and 2014 for 10,992 households. In the version in which c_M includes food, utilities, child care expenses, clothing, home insurance, telecommunication, transportation, and home repairs, we have 13,626 observations between 2004 and 2014. PSID does not have information which allows us to disaggregate time spent on home production between h_N and h_P. To make the analyses as comparable as possible

[32] The survey question is "About how much time do you spend on housework in an average week? I mean time spent cooking, cleaning, and doing other work around the house."

to CEX/ATUS, we consider three cases. The first is when all home hours belong to h_N in the sector with efficiency heterogeneity. The second case, which is more comparable to our baseline in the CEX/ATUS, is that home hours are split equally between the two sectors. The third case is when all home hours belong to h_P in the sector with disutility heterogeneity.

Table 10 reassesses our conclusions regarding inequality.[33] The first panel repeats the findings of Table 7 in the CEX/ATUS for the four inequality metrics in the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. The second panel reports these statistics for the version of the PSID which includes an expanded set of consumption categories. The third and fourth panels report these statistics for the CEX/ATUS and PSID datasets when we restrict our measure of consumption to include only food.

Our conclusions regarding inequality and the prominent role of heterogeneity in home production efficiency are stable across the four datasets. First, the baseline model with home production generates higher inequality than the model without home production. Second, in the model with only efficiency heterogeneity, all inequality metrics are magnified relative to the baseline with both efficiency and disutility heterogeneity. Third, if there was only disutility heterogeneity, there would be no significant difference in inequality between the model with and the model without home production. The only significant change in the PSID relative to the CEX/ATUS is in the optimal progressivity τ_1 which displays a smaller difference between the two models.[34]

Our results using the PSID are particularly reassuring because we do not take a stance

[33] To isolate differences arising from samples rather than parameter values, we keep parameters fixed at their values shown in Table 5. We follow a similar strategy with the JPSC and the LISS datasets later. The exception is the constant level of production efficiency θ_P which we calibrate in each dataset to hit the same target as in the CEX/ATUS.

[34] Appendix Figures A.2 and A.3 display the age profiles of means and variances of sources of heterogeneity $(\alpha, \varepsilon, B, D_P, \log \theta_N)$ from the version of the PSID with food in the baseline case which splits home hours equally between h_N and h_P. The difference relative to the means and variances we extracted using the CEX/ATUS is that we obtain these age profiles by regressing each source of heterogeneity on age and year dummies and an individual fixed effect. Therefore, these profiles reflect the within-household evolution of the sources of heterogeneity. Despite this difference, most of age profiles in the PSID are quantitatively similar to the age profiles in the CEX/ATUS.

Table 10: Inequality and Home Production: CEX/ATUS and PSID

CEX All	No Home Production	Home Production		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
PSID All	**No Home Production**	**Home Production**		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.58	0.85	0.63	0.56
std(t)	0.40	0.61	0.51	0.45
λ	0.11	0.18	0.15	0.10
τ_1	0.33	0.36	0.33	0.29
CEX Food	**No Home Production**	**Home Production**		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.82	1.15	0.92	0.80
std(t)	0.56	0.84	0.75	0.67
λ	0.04	0.20	0.11	0.02
τ_1	-0.05	0.29	0.21	0.09
PSID Food	**No Home Production**	**Home Production**		
Statistics		Efficiency	Baseline	Disutility
std(T)	0.57	0.87	0.63	0.55
std(t)	0.40	0.62	0.51	0.45
λ	0.09	0.18	0.14	0.09
τ_1	0.28	0.33	0.29	0.24

Table 10 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work in the first panel; 4.01 and 6.68 in the second panel; 4.65 and 9.74 in the third panel; 3.80 and 6.31 in the fourth panel.

about the classification of time uses between h_N and h_P. Therefore, the result that inequality is higher with home production does not hinge on which activities are subject to efficiency heterogeneity and which activities are subject to disutility heterogeneity. What is important for this result is that some portion of home production time is subject to heterogeneity in production efficiency.

2.6.2 Comparison between US, Japan, and the Netherlands

In this section, we repeat our analyses using datasets from other countries. As in the PSID, these datasets have limited information to disaggregate time spent on home production between h_N and h_P. To make the analyses comparable to CEX/ATUS and PSID, we consider the three cases of all home hours belonging to h_N in the sector with efficiency heterogeneity, of splitting home hours equally between the two sectors, and of all home hours belonging to h_P in the sector with disutility heterogeneity. We apply the same sampling restrictions as in the CEX/ATUS and focus our analyses on married households.

The first dataset is the Japanese Panel Survey of Consumers (JPSC; see, for example, Lise and Yamada, 2018). The JPSC records information for time spent on commuting, working, studying, home production and child care, leisure, and sleeping, personal care and eating. For aggregate home hours $h_N + h_P$ we use the variable for home production and child care and for market hours we use hours worked. To calculate the home and market hours for a given week, we weight the time use on workdays and days off by the number of days worked. Our measure of consumption expenditures includes food, utilities, apparel, transport, culture and leisure, communication, trips and activities, house and land rent. The final dataset has 12,423 observations between 1998 and 2014. The second dataset is the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS; see, for example, Cherchye, Demuynck, De Rock, and Vermeulen, 2017), administered by CentERdata. The dataset is based on a representative sample of Dutch households who participate in monthly surveys. We use the three waves (2009, 2010, and 2012) which contain information on time use. Home production time includes household chores, child care, and administrative chores. Market hours are measured by time spent on paid work, which

Table 11: Inequality and Home Production: US, Japan, and the Netherlands

CEX/ATUS Statistics	No Home Production	Home Production Efficiency	Baseline	Disutility
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
JPSC Statistics	**No Home Production**	**Home Production Efficiency**	**Baseline**	**Disutility**
std(T)	0.66	0.98	0.76	0.67
std(t)	0.46	0.68	0.60	0.56
λ	0.04	0.11	0.07	0.02
τ_1	-0.15	0.19	0.11	0.03
LISS Statistics	**No Home Production**	**Home Production Efficiency**	**Baseline**	**Disutility**
std(T)	0.64	1.12	0.80	0.64
std(t)	0.45	0.77	0.63	0.54
λ	0.03	0.20	0.12	0.02
τ_1	-0.80	-0.12	-0.24	-0.80

Table 11 shows the four inequality metrics for the model without home production, the home production model with only efficiency heterogeneity, the baseline home production model with both efficiency and disutility heterogeneity, and the home production model with only disutility heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The estimated value of θ_P is 4.64 for the baseline home production model and 9.74 for the model with only heterogeneity in disutility of home work in the upper panel; 3.54 and 5.91 in the middle panel; 3.46 and 5.78 in the lower panel.

includes commuting time. Consumption expenditures include food, utilities, home maintenance, transportation, daycare, and child support. The final dataset has 978 observations.

Table 11 summarizes our results. The upper panel repeats our findings in the CEX/ATUS and the other panels show inequality statistics in the JPSC and the LISS. Our conclusions regarding inequality and the role of production efficiency heterogeneity are stable in other countries as well. Namely, the baseline model with home production always generates higher inequality than the model without home production. All inequality statistics are magnified in the home production model with only efficiency heterogeneity, whereas with only disutility heterogeneity there would be no significant difference in inequality between the models with and without home production.

2.7 Conclusion

The literature examining the causes, welfare consequences, and policy implications of the substantial labor market dispersion we observe across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. We revisit these issues taking into account that households spend a significant amount of their time in home production. Our model incorporates non-separable preferences between expenditures and time and home production efficiency and disutility of home work heterogeneity into a standard incomplete markets model with uninsurable risk.

We reach several substantial conclusions. We find that home production amplifies welfare-based differences across households and inequality is larger than we thought. Our result is surprising given that a priori one could expect home production to compress welfare differences originating in the market sector when households are sufficiently willing to substitute between market expenditures and time in the production of home goods. We show that home production efficiency is an important source of within-age and life-cycle differences in consumption expenditures and time allocation across households. Through the lens of the model we infer that home production does not offset differences that originate in the market sector because production efficiency differences in the home sector are significant and the time input in home production does not covary with consumption and wages in the cross section of households.

Inferring Inequality with Home Production

Online Appendix

Job Boerma and Loukas Karabarbounis

Appendix 2.A Proofs

In this appendix, we derive the equilibrium allocations presented in Table 1 in the main text and prove the observational equivalence theorem. We proceed in four steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 3.2 in the main text. Finally, we show how to invert the equilibrium allocations and identify the sources of heterogeneity leading to these allocations.

2.A.1 Preliminaries

In what follows, we define the following state vectors. The sources of heterogeneity differentiating households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can trade bonds within each island contingent on the vector s^j:

$$s_t^j = (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, we denote the history of all sources of heterogeneity up to period t with the vector:

$$\sigma_t^j = (\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, ..., \theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(\cdot|\cdot)$. For example, the probability that we observe σ_t^j conditional on σ_{t-1}^j is $f^{t,j}(\sigma_t^j|\sigma_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to the processes and Φ_υ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^B}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi^j_{\theta_{K,t}}, \Phi^j_{D_{K,t}}\}$, and the initial distributions to vary over cohorts j, $\{\Phi^j_{\alpha,j}, \Phi^j_{B,j}, \Phi^j_{\kappa,j}\}$. We assume that both $\theta_{K,t}^j$ and $D_{K,t}^j$ are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

2.A.2 Planner Problems

In every period t and in every island ℓ, the planner solves a static problem which consists of finding the allocations maximizing average utility for households on the island subject to an aggregate resource constraint. We omit t and ℓ from the notation for convenience.

No Home Production, $\omega_K = 0$

The planner chooses an allocation $\{c_M, h_M\}$ to maximize:

$$\int_Z \left[\frac{c_M^{1-\gamma} - 1}{1-\gamma} - \frac{(\exp(B) h_M)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta) \, , \tag{A.5}$$

subject to an island resource constraint for market goods:

$$\int_Z c_M \mathrm{d}\Phi_\zeta\left(\zeta\right) = \int_Z \tilde{z}_M h_M \mathrm{d}\Phi_\zeta\left(\zeta\right). \tag{A.6}$$

Denoting by $\mu(\alpha, B)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M]: \; c_M^{-\gamma} = \mu(\alpha, B), \tag{A.7}$$

$$[h_M]: \; \exp(B)^{1+\frac{1}{\eta}} h_M{}^{\frac{1}{\eta}} = \tilde{z}_M \mu(\alpha, B). \tag{A.8}$$

Equation (A.7) implies that market consumption is equalized for every ι on the island and, thus, there is full consumption insurance. Combining equations (A.6) to (A.8), we solve for market consumption and market hours for every ι:

$$c_M = \left[\frac{\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)}{\exp\left(\eta\left(1+\frac{1}{\eta}\right)B\right)} \right]^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}}, \tag{A.9}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}}{\exp\left(\left(1+\frac{1}{\eta}\right)B\right)^{\frac{1}{\eta}+\gamma}}. \tag{A.10}$$

Home Production, $\omega_K > 0$

The planner chooses $\{c_M, h_M, h_K\}$ to maximize:

$$\int_Z \left[\log c - \frac{\left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta), \tag{A.11}$$

where consumption is given by $c = \left(c_M{}^{\frac{\phi-1}{\phi}} + \sum(\theta_K h_K)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$ subject to the island market resource constraint (A.6).

Denoting by $\mu(\alpha, B, D_K, \theta_K)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M]: \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} c_M^{-\frac{1}{\phi}} = \mu(\alpha, B, D_K, \theta_K), \tag{A.12}$$

$$[h_M]: \left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{\frac{1}{\eta}} = \tilde{z}_M \frac{\mu(\alpha, B, D_K, \theta_K)}{\exp(B)}, \tag{A.13}$$

$$[h_K]: \left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{\frac{1}{\eta}} = \theta_K^{\frac{\phi-1}{\phi}} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} \frac{h_K^{-\frac{1}{\phi}}}{\exp(D_K)} \tag{A.14}$$

Combining equations (A.12) to (A.14), we solve for the ratio of home hours to consumption:

$$\frac{c_M}{h_K} = \left(\frac{\exp\left(D_K\right)}{\exp\left(B\right)/\tilde{z}_M}\right)^{\phi} \theta_K^{1-\phi}. \tag{A.15}$$

Substituting these ratios into equations (A.12) to (A.14), we derive:

$$c_M = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{1}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.16}$$

$$h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{\theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}. \tag{A.17}$$

These expressions yield solutions for $\{c_M, h_M, h_K\}$ given a multiplier $\mu(\alpha, B, D_K, \theta_K)$. The multiplier is equal to the inverse of the market value of total consumption:

$$c_M + \tilde{z}_M \sum \frac{\exp\left(D_K\right)}{\exp\left(B\right)} h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)}. \tag{A.18}$$

The equality follows from equations (A.16) to (A.17).

Substituting equation (A.9) into equation (A.6), we obtain the solution for $\mu(\alpha, B, D_K, \theta_K)$:

$$\mu(\alpha, B, D_K, \theta_K) = \frac{\exp(B)}{\left(\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right)^{\frac{1}{1+\eta}}}. \tag{A.19}$$

The denominator is an expectation independent of ζ. Therefore, μ is independent of ζ. We also note that $\mu(\alpha, B, D_K, \theta_K)$ in the model with home production equals $\mu(\alpha, B)$ in the model without home production under $\gamma = 1$. Given this solution for $\mu(\alpha, B, D_K, \theta_K)$, we obtain the solutions:

$$c_M = \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{1}{1+\sum \theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.20}$$

$$h_K = \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{\theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{1+\sum \theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.21}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\int_Z \tilde{z}_M^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{1}{1+\frac{1}{\eta}}}}{\exp(B)} - \sum \frac{\exp(D_K)}{\exp(B)} h_K.$$

2.A.3 Postulating Equilibrium Allocations and Prices

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade across islands, $x(\zeta_{t+1}^j;\iota) = 0, \forall \iota, \zeta_{t+1}^j$.

2. We postulate that the solutions $\{c_{M,t}, h_{M,t}\}$ for the model without home production and $\{c_{M,t}, h_{M,t}, h_{K,t}\}$ for the model with home production from the planner problems in Section 3.A.2 constitute components of the equilibrium for each model.

3. We use the sequential budget constraints to postulate equilibrium holdings for the bonds $b^\ell(s_t^j;\iota)$ which are traded within islands. For the models without home production these are given by:

$$b^\ell(s_t^j;\iota) = \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu_{t+n}(\alpha_{t+n}^j, B_{t+n}^j)}{\mu_t(\alpha_t^j, B_t^j)} (c_{M,t+n} - \tilde{y}_{t+n})\right], \tag{A.22}$$

   where $\tilde{y} = \tilde{z}_M h_M = (1-\tau_0) z_M^{1-\tau_1} h_M$ denotes after-tax labor income.

   For the model with home production, bonds $b^\ell(s_t^j;\iota)$ are given by the same expression but using the marginal utility $\mu(\alpha, B, D_K, \theta_K)$ instead of $\mu(\alpha, B)$. As shown above, the two marginal utilities are characterized by the same equation (A.19) under $\gamma = 1$.

4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s_{t+1}^j;\iota)$ and $x(\zeta_{t+1}^j;\iota)$. For the model without

home production, we obtain:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j), \tag{A.23}$$

$$q_x(Z_{t+1}) = \beta\delta \int \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}(\upsilon_{t+1}^B) \int \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}(\upsilon_{t+1}^\alpha)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \mathbb{P}\left(\left(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon\right) \in Z_{t+1}\right), \tag{A.24}$$

where $A \equiv (1+\eta)(1-\tau_1)$. For the model with home production, we obtain the same expressions under $\gamma = 1$.

2.A.4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section 3.A.3 constitutes an equilibrium by showing that the postulated equilibrium allocations solve the households' problem and that all markets clear.

Household Problem

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

No Home Production, $\omega_K = 0$. The optimality conditions are:

$$(\beta\delta)^{t-j}\, c_{M,t}^{-\gamma} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{\mu}_t, \tag{A.25}$$

$$(\beta\delta)^{t-j} \exp(B_t)^{1+\frac{1}{\eta}} \left(h_{M,t}\right)^{\frac{1}{\eta}} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{z}_{M,t}^j \tilde{\mu}_t, \tag{A.26}$$

$$q_b^\ell(s_{t+1}^j) = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}, \tag{A.27}$$

$$q_x(Z_{t+1}) = \int \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.28}$$

Comparing the planner solutions to the household solutions we verify that they coincide for market consumption and hours when the multipliers are related by:

$$\tilde{\mu}_t = (\beta\delta)^{t-j}\, f^{t,j}(\sigma_t^j|\sigma_j)\mu(\alpha_t^j, B_t^j). \tag{A.29}$$

Therefore, the Euler equations become:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \tag{A.30}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.31}$$

Home Production, $\omega_K > 0$. We denote total hours, taking into account the respective disutility, by $\tilde{h} = \exp(B)(h_M) + \sum \exp(D_K)(h_K)$. Using again the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and

$\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$, we write the optimality conditions as:

$$\frac{\tilde{z}_{M,t}}{\exp(B_t)}\left(c^{\frac{\phi-1}{\phi}}\right)^{-1} c_{M,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.32}$$

$$\frac{\theta_{K,t}^{\frac{\phi-1}{\phi}}}{\exp(D_{K,t})}\left(c^{\frac{\phi-1}{\phi}}\right)^{-1} h_{K,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.33}$$

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j, \tag{A.34}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha \mathrm{d}\theta_{K,t+1}^j \mathrm{d}D_{K,t+1}^j. \tag{A.35}$$

Euler Equations

We next verify that the Euler equations are satisfied at the postulated equilibrium allocations and prices.

No Home Production, $\omega_K = 0$. Using the marginal utility of market consumption of the planner problem $\mu(\alpha_t^j, B_t^j)$, we write the Euler equation for the bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \\ &= \beta\delta \frac{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_{t+1}^j\right)\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_t^j\right)\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \end{aligned} \tag{A.36}$$

where the second line follows from equations (A.7) and (A.9). Using that B_t^j follows a random walk-process with innovation υ_t^B we rewrite $q_b^\ell(s_{t+1}^j)$ as:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \frac{\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.37}$$

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$ we use that:

$$\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\left((1-\tau_1)\left(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right).$$

Given that $A = (1+\eta)(1-\tau_1)$, the expectation over the random variables in the numerator is given by:

$$\begin{aligned}
&\int \exp\left(A\left(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right) \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j) \\
=& \int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j) \int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)\,,
\end{aligned} \tag{A.38}$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa^j,t}(\kappa_t^j) \int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon). \tag{A.39}$$

As a result, the price $q_b^\ell(s_{t+1}^j)$ is:

$$\begin{aligned}
q_b^\ell(s_{t+1}^j) &= \beta\delta\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\exp\left(-(1-\tau_1)\,\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right) \\
&\quad\times\left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j),
\end{aligned} \tag{A.40}$$

where $f^{t+1,j}(s_{t+1}^j|s_t^j) = f(\upsilon_{t+1}^B)f(\upsilon_{t+1}^\alpha)f(\upsilon_{t+1}^\kappa)f(\upsilon_{t+1}^\varepsilon)$. This confirms our guess in equation (A.21). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in square brackets is independent of the state vector which differentiates islands ℓ. As a result, all islands ℓ have the same bond prices, $q_b^\ell(s_{t+1}^j) = Q_b\left(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha\right)$.

We next calculate the bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$:

$$q_b^\ell(\mathcal{V}_{t+1}) = \beta\delta \int_{\mathcal{V}^B} \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right) \int_{\mathcal{V}^\alpha} \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.41}$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim which does not depend on the realization of $(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha)$:

$$q_b^\ell(\mathbb{V}_{t+1}) = \beta\delta \int_{\mathbb{V}^B} \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right) \int_{\mathbb{V}^\alpha} \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.42}$$

All islands face the same price $q_b^\ell(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the prices of bonds x and b which are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and given by:

$$q_x(Z_{t+1}) = \mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right) Q_b(\mathbb{V}_{t+1}), \tag{A.43}$$

where $\mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right)$ denotes the probability of $(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon)$ being a member of Z_{t+1}. The expression for $q_x(Z_{t+1})$ confirms our guess in equation (A.22)

Home Production, $\omega_K > 0$. For the model with home production, we use the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$

to write the Euler equation for the bonds $b^{\ell}(s^j_{t+1})$ at the postulated equilibrium as:

$$q^{\ell}_b(s^j_{t+1}) = \beta\delta \int \frac{\mu(\alpha^j_{t+1}, B^j_{t+1}, D^j_{K,t+1}, \theta^j_{K,t+1})}{\mu(\alpha^j_t, B^j_t, D^j_{K,t}, \theta^j_{K,t})} f^{t+1,j}(\sigma^j_{t+1}|\sigma^j_t)\mathrm{d}\theta^j_{K,t+1}\mathrm{d}D^j_{K,t+1} \tag{A.44}$$

$$= \beta\delta \int \frac{\exp\left(B^j_{t+1}\right)\left[\int\left(\tilde{z}^j_{M,t+1}\right)^{1+\eta}\mathrm{d}\Phi_{\zeta^j_{t+1}}(\zeta^j_{t+1})\right]^{-\frac{1}{1+\eta}}}{\exp\left(B^j_t\right)\left[\int\left(\tilde{z}^j_{M,t}\right)^{1+\eta}\mathrm{d}\Phi_{\zeta^j_t}(\zeta^j_t)\right]^{-\frac{1}{1+\eta}}} f^{t+1,j}(\sigma^j_{t+1}|\sigma^j_t)\mathrm{d}\theta^j_{K,t+1}\mathrm{d}D^j_{K,t+1}.$$

where the second equality follows from equation (A.19). Using equations (A.29) and (A.30), and the fact that $\theta^j_{K,t+1}$ and $D^j_{K,t+1}$ are orthogonal to the innovations, the price $q^{\ell}_b(s^j_{t+1})$ simplifies to:

$$q^{\ell}_b(s^j_{t+1}) = \beta\delta\exp\left(\upsilon^B_{t+1} - (1-\tau_1)\,\upsilon^{\alpha}_{t+1}\right)$$
$$\times\left[\frac{\int\exp\left(A\upsilon^{\kappa}_{t+1}\right)\mathrm{d}\Phi_{\upsilon^{\kappa}_{t+1}}(\upsilon^{\kappa}_{t+1})\int\exp\left(A\upsilon^{\varepsilon}_{t+1}\right)\mathrm{d}\Phi_{\upsilon^{\varepsilon}_{t+1}}(\upsilon^{\varepsilon}_{t+1})}{\int\exp\left(A\upsilon^{\varepsilon}_t\right)\mathrm{d}\Phi_{\upsilon^{\varepsilon}_t}(\upsilon^{\varepsilon}_t)}\right]^{-\frac{1}{1+\eta}} f^{t+1,j}(s^j_{t+1}|s^j_t). \tag{A.45}$$

The price $q^{\ell}_b(s^j_{t+1})$ is identical to equation (A.40) for the model without home production under $\gamma = 1$. The remainder of the argument is identical to the argument for the model without home production.

Household's Budget Constraint

We now verify our guess for the bond positions $b^{\ell}_t(s^j_t)$ and confirm that the household budget constraint holds at the postulated equilibrium allocations. The proof to this claim is identical for both models. We define the deficit term by $d_t \equiv c_{M,t} - \tilde{y}_t$. Using the expression for the price $q^{\ell}_b(s^j_{t+1})$ in equation (A.30), the budget constraint at the no-trade equilibrium is given by:

$$b^{\ell}_t(s^j_t) = d_t + \beta\delta\int\frac{\mu(\alpha^j_{t+1}, B^j_{t+1}, D^j_{K,t+1}, \theta^j_{K,t+1})}{\mu(\alpha^j_t, B^j_t, D^j_{K,t}, \theta^j_{K,t})} b^{\ell}_{t+1}(s^j_{t+1}) f^{t+1}(\sigma^j_{t+1}|\sigma^j_t)\mathrm{d}s^j_{t+1}\mathrm{d}\theta^j_{K,t+1}\mathrm{d}D^j_{K,t+1}.$$

By substituting forward using equation (A.30), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.20) and show that the household budget constraint holds at the postulated equilibrium allocations.

Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations solving the planner problems satisfy the aggregate goods market clearing condition:

$$\int_\iota c_{M,t} \mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t} h_{M,t} \mathrm{d}\Phi(\iota). \tag{A.46}$$

Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota x(\zeta_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $x(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$, $\forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated bond holdings in equation (A.20) into the asset market clearing conditions:

$$\begin{aligned}\int b^\ell(s_t^j)\mathrm{d}\Phi(\iota) &= \int \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{K,t+n}^j, \theta_{K,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} d_{t+n}\right]\mathrm{d}\Phi(\iota)\\ &= \sum_{n=0}^{\infty}(\beta\delta)^n \int \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{K,t+n}^j, \theta_{K,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} d_{t+n} f(\sigma_{t+n}^j|\sigma_{t-1}^j)\mathrm{d}\sigma_{t+n}^j \mathrm{d}\Phi(\iota).\end{aligned}$$

For simplicity we omit conditioning on σ_{t-1}^j and write the density function as $f(\sigma_{t+n}^j|\sigma_{t-1}^j) = f(\{\upsilon_{t+n}^B\})f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{\theta_{K,t+n}\})f(\{D_{K,t+n}\})$. Further, the expression for the growth in marginal utility is identical between the two models and we denote it by $\mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^\alpha\right) \equiv \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{K,t+n}^j, \theta_{K,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)} = \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j)}{\mu(\alpha_t^j, B_t^j)}$. Hence, we write aggregate bond holdings $\int b^{\ell}(s_t^j)\mathrm{d}\Phi(\iota)$ as:

$$\begin{aligned}
&\sum_{n=0}^{\infty}(\beta\delta)^n \int\int \mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^{\alpha}\right) d_{t+n} f(\{\upsilon_{t+n}^B\}) f(\{\upsilon_{t+n}^{\alpha}\}) f(\{\upsilon_{t+n}^{\kappa}\}) f(\{\upsilon_{t+n}^{\varepsilon}\}) f(\{\theta_{K,t+n}\}) \ldots \\
&\qquad\qquad \ldots\ f(\{D_{K,t+n}\})\mathrm{d}\{\upsilon_{t+n}^B\}\mathrm{d}\{\upsilon_{t+n}^{\alpha}\}\mathrm{d}\{\upsilon_{t+n}^{\kappa}\}\mathrm{d}\{\upsilon_{t+n}^{\varepsilon}\}\mathrm{d}\{\theta_{K,t+n}^j\}\mathrm{d}\{D_{K,t+n}^j\}\mathrm{d}\Phi(\iota) \\
=&\sum_{n=0}^{\infty}(\beta\delta)^n \int d_{t+n} f(\{\upsilon_{t+n}^{\kappa}\}) f(\{\upsilon_{t+n}^{\varepsilon}\})\mathrm{d}\{\upsilon_{t+n}^{\kappa}\}\mathrm{d}\{\upsilon_{t+n}^{\varepsilon}\}\mathrm{d}\Phi(\iota) \\
\times&\int \mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^{\alpha}\right) f(\{\upsilon_{t+n}^B\}) f(\{\upsilon_{t+n}^{\alpha}\}) f(\{\theta_{K,t+n}\}) f(\{D_{K,t+n}\})\mathrm{d}\{\upsilon_{t+n}^B\}\mathrm{d}\{\upsilon_{t+n}^{\alpha}\}\mathrm{d}\{\theta_{K,t+n}\}\mathrm{d}\{D_{K,t+n}\}.
\end{aligned}$$

Recalling that the deficit terms equal $d_t = c_{M,t} - \tilde{y}_t$, the bond market clearing condition holds because the first term is zero by the island-level resource constraint.

2.A.5 Observational Equivalence Theorem

We derive the identified sources of heterogeneity presented in Table 2. Our strategy is to invert the equilibrium allocations presented in Table 1 and solve for the unique sources of heterogeneity leading to these allocations. The identification is defined up to a constant because the constant $\mathcal{C}_s$ appearing in the equations of Table 2 depends on the ε's.

No Home Production, $\omega_K = 0$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}\}_{\iota}$ and parameters $\gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t\}_{\iota}$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide the solution for c_M with the solution for h_M to obtain:

$$\frac{c_{M,t}}{h_{M,t}} = (1-\tau_0)\, z_{M,t}^{-\eta(1-\tau_1)} \exp((1-\tau_1)(1+\eta)\alpha_t) \int_{\zeta_t} \exp((1-\tau_1)(1+\eta)\varepsilon_t)\mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\,. \tag{A.47}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, ε_t is also uniquely determined. Finally, we can use the solution for $c_{M,t}$ or $h_{M,t}$ in Table 1 to solve for a unique value of B_t.

Home Production, $\omega_K > 0$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}, h_{N,t}, h_{P,t}\}_\iota$ and parameters $\phi, \gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t, \theta_{N,t}, D_{P,t}\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι.

Dividing the solution for h_N with the solution for c_M we obtain θ_N from the following equation:

$$\frac{h_{N,t}}{c_{M,t}} = \theta_{N,t}^{\phi-1} \tilde{z}_{M,t}^{-\phi} \,. \tag{A.48}$$

Next, we divide the solutions for h_P with the solution for h_N, we solve for the ratio of disutilities $\exp(D_P)/\exp(B)$:

$$\frac{h_{P,t}}{h_{N,t}} = \left(\frac{\theta_{P,t}}{\theta_{N,t}^j}\right)^{\phi-1} \left(\frac{\exp(B_t)}{\exp(D_{P,t})}\right)^{\phi} . \tag{A.49}$$

Next, we divide the solution for h_T with the solution for c_M and use equation (A.48) to obtain:

$$\begin{aligned} \frac{h_{M,t} + h_{N,t} + \frac{\exp(D_{P,t})}{\exp(B_t)} h_{P,t}}{c_{M,t}} = & \frac{z_{M,t}^{\eta(1-\tau_1)}}{1-\tau_0} \frac{\exp(-(1+\eta)(1-\tau_1)\alpha_t^j)}{\int_{Z_t} \exp((1+\eta)(1-\tau_1)\varepsilon_t) \mathrm{d}\Phi_{\zeta^j,t}(\zeta_t^j)} \\ & \times \left[1 + \left(\frac{\theta_{N,t}^j}{\tilde{z}_{M,t}^j}\right)^{\phi-1} + \left(\frac{\exp\left(B_t\right)/\tilde{z}_{M,t}}{\exp\left(D_{P,t}\right)/\theta_{P,t}}\right)^{\phi-1}\right] \end{aligned} \tag{A.50}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Next, we can identify B using the first-order conditions with respect to market consumption and equations (A.18), (A.48) and (A.49) to obtain:

$$\exp\big((1+\eta)\, B_t\big) = \frac{\left(\frac{\bar{c}_{M,t}}{\tilde{z}_{M,t}} + h_{N,t} + \left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}} \theta_{P,t}^{\frac{\phi-1}{\phi}} \frac{h_{P,t}}{\tilde{z}_{M,t}}\right)^{-\eta}}{\bar{h}_{M,t} + h_{N,t} + \left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}} \theta_{P,t}^{\frac{\phi-1}{\phi}} \frac{h_{P,t}}{\tilde{z}_{M,t}}} \,. \tag{A.51}$$

Finally, once we know B, we can solve for D_P from equation (A.49).

Appendix 2.B Additional Results

In this appendix we present summary statistics from various datasets and additional results and sensitivity analyses.

- Table A.1 shows summary statistics of wages and hours for married individuals in the ATUS and for married households in the CEX in which we have imputed home hours. The ATUS sample excludes respondents during weekends and, so, market hours are noticeably higher.
- Tables A.2 and A.3 show summary statistics of wages and hours for married individuals in the ATUS by sex and education.
- Tables A.4 and A.5 present summary statistics of wages, hours, and expenditures in the CEX and PSID samples.
- Table A.6 presents the correlation matrix of observables and sources of heterogeneity in the two models.
- Figure A.1 presents distributions of the sources of heterogeneity in the two models.
- Table A.7 presents the welfare effects of eliminating heterogeneity within age groups.
- Figures A.2 and A.3 present the life-cycle means and variances of the sources of heterogeneity in the version of the PSID with food expenditures. We obtain these age profiles by regressing each inferred source of heterogeneity on age and year dummies and an individual fixed effect. Therefore, these age profiles reflect the within-household evolution of the sources of heterogeneity.

Table A.1: ATUS (Raw) versus CEX (Imputed) Samples

	ATUS Married Individuals			CEX Married Households		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	42.1	41.9	42.2	66.1	66.8	65.5
Mean h_N	12.5	14.6	10.5	21.3	25.4	17.3
Mean h_P	10.6	10.7	10.5	16.7	16.4	17.0
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	-0.15	-0.14	-0.14
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	0.10	0.16	0.12
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	0.02	0.00	0.03
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.25	-0.36	-0.23
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.42	-0.42	-0.41
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.15	0.20	0.17

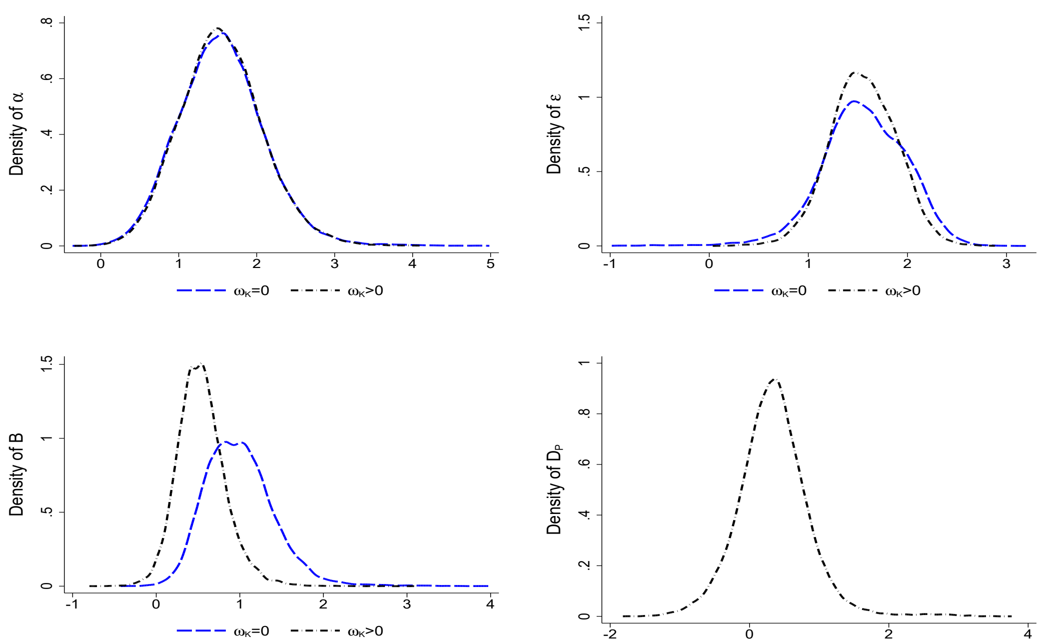


Figure A.1: Distributions of Sources of Heterogeneity

Table A.2: Correlations in ATUS Married by Sex

	ATUS All			ATUS Men			ATUS Women		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
corr(z_M, h_M)	0.06	0.03	0.08	0.02	0.00	0.04	0.04	0.02	0.06
corr(z_M, h_N)	0.01	0.04	-0.01	0.03	0.07	0.01	0.03	0.05	0.01
corr(z_M, h_P)	-0.08	-0.06	-0.09	-0.02	0.00	-0.04	-0.08	-0.08	-0.09
corr(h_M, h_N)	-0.44	-0.46	-0.42	-0.40	-0.41	-0.39	-0.44	-0.47	-0.43
corr(h_M, h_P)	-0.45	-0.44	-0.46	-0.39	-0.38	-0.41	-0.46	-0.44	-0.47
corr(h_N, h_P)	0.10	0.14	0.08	0.06	0.09	0.05	0.07	0.10	0.07

Table A.3: Correlations in ATUS Married by Education

	ATUS All			ATUS Less than College			ATUS College or More		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
corr(z_M, h_M)	0.06	0.03	0.08	0.05	0.04	0.03	0.05	0.02	0.07
corr(z_M, h_N)	0.01	0.04	-0.01	-0.01	0.01	-0.01	-0.02	0.02	-0.04
corr(z_M, h_P)	-0.08	-0.06	-0.09	-0.05	-0.03	-0.06	-0.07	-0.06	-0.09
corr(h_M, h_N)	-0.44	-0.46	-0.42	-0.42	-0.44	-0.41	-0.47	-0.50	-0.45
corr(h_M, h_P)	-0.45	-0.44	-0.46	-0.45	-0.43	-0.46	-0.45	-0.45	-0.45
corr(h_N, h_P)	0.10	0.14	0.08	0.08	0.12	0.06	0.14	0.17	0.13

Table A.4: CEX/ATUS (1995-2016) versus PSID (1975-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	67.8	65.3	70.3
Mean $h_N + h_P$	38.0	41.8	34.3	25.9	27.1	24.7
$\text{corr}(z_M, h_M)$	-0.15	-0.14	-0.14	-0.15	-0.15	-0.14
$\text{corr}(z_M, h_N + h_P)$	0.09	0.12	0.10	0.00	0.02	-0.02
$\text{corr}(z_M, c_M^{\text{food}})$	0.22	0.21	0.22	0.28	0.29	0.27
$\text{corr}(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.24	-0.28	-0.20
$\text{corr}(h_M, c_M^{\text{food}})$	0.10	0.09	0.12	0.06	0.06	0.08
$\text{corr}(h_N + h_P, c_M^{\text{food}})$	-0.03	-0.01	-0.02	0.01	0.03	-0.01

Table A.5: CEX/ATUS (1995-2016) versus PSID (2004-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	64.8	67.6	62.0
Mean $h_N + h_P$	38.0	41.8	34.3	24.3	24.1	24.6
$\text{corr}(z_M, h_M)$	-0.15	-0.14	-0.14	-0.09	-0.15	-0.06
$\text{corr}(z_M, h_N + h_P)$	0.09	0.12	0.10	-0.01	0.03	-0.03
$\text{corr}(z_M, c_M^{\text{nd}})$	0.25	0.25	0.25	0.26	0.29	0.25
$\text{corr}(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.23	-0.27	-0.20
$\text{corr}(h_M, c_M^{\text{nd}})$	0.14	0.16	0.13	0.20	0.21	0.20
$\text{corr}(h_N + h_P, c_M^{\text{nd}})$	-0.05	-0.04	-0.03	-0.03	-0.03	-0.03

Table A.6: Within-Age Correlations

$\omega_K = 0$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D_P	$\log \theta_N$
$\log z_M$	1.00	0.29	-0.07	—	—	0.70	0.42	0.42	—	—
$\log c_M$		1.00	0.13	—	—	0.69	-0.50	-0.55	—	—
$\log h_M$			1.00	—	—	-0.46	0.50	-0.71	—	—
$\log h_N$				—	—	—	—	—	—	—
$\log h_P$					—	—	—	—	—	—
α						1.00	-0.35	0.23	—	—
ε							1.00	0.26	—	—
B								1.00	—	—
D_P									—	—
$\log \theta_N$										—

$\omega_K > 0$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D_P	$\log \theta_N$
$\log z_M$	1.00	0.29	-0.07	0.07	-0.02	0.82	0.42	0.45	-0.58	0.69
$\log c_M$		1.00	0.13	0.00	-0.06	0.66	-0.54	-0.43	-0.01	-0.14
$\log h_M$			1.00	-0.17	-0.30	-0.32	0.38	-0.48	0.06	-0.20
$\log h_N$				1.00	0.18	0.13	-0.08	-0.29	-0.36	0.68
$\log h_P$					1.00	0.08	-0.15	-0.03	-0.70	0.12
α						1.00	-0.18	0.23	-0.41	0.46
ε							1.00	0.40	-0.34	0.46
B								1.00	-0.06	0.31
D_P									1.00	-0.66
$\log \theta_N$										1.00

Table A.7: Within-Age Heterogeneity and Lifetime Consumption Equivalence

No within-age dispersion in ...	$\omega_K = 0$ model	$\omega_K > 0$ model
z_M, θ_N, B, D_P	0.07	0.14
z_M, θ_N	0.07	0.16
z_M	0.07	0.11
θ_N	—	0.12

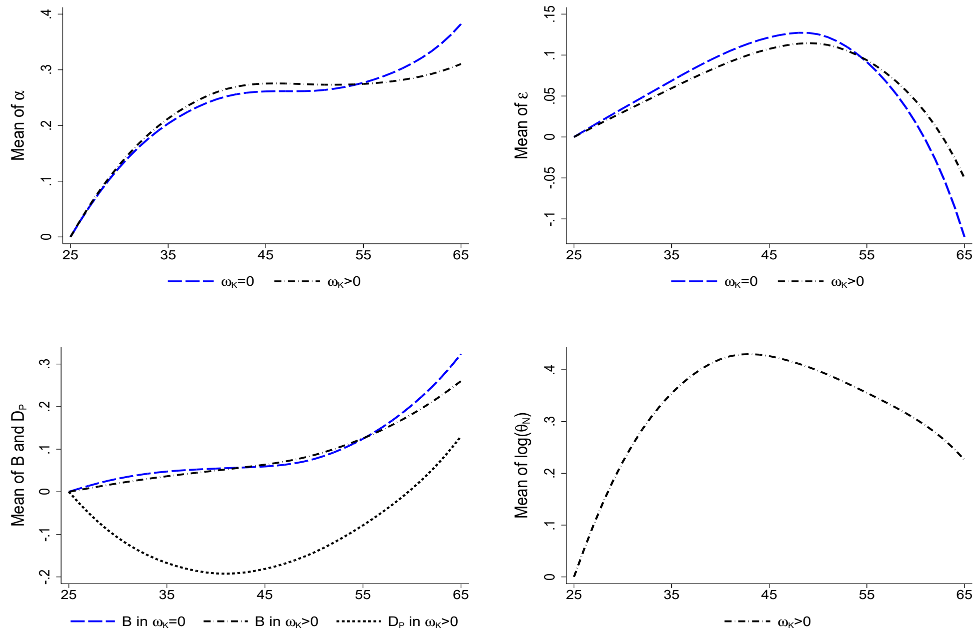


Figure A.2: Means of Sources of Heterogeneity (PSID Food)

Figure A.2 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

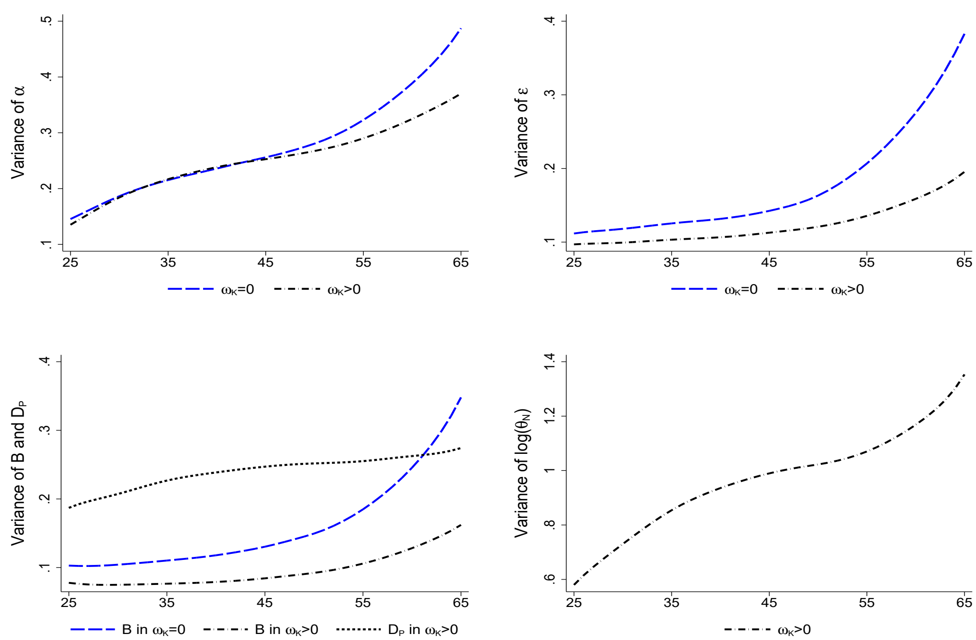


Figure A.3: Variances of Sources of Heterogeneity (PSID Food)

Figure A.3 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D_P, and home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, black dotted lines) and without home production ($\omega_K = 0$, blue dashed lines).

Chapter 3

Labor Market Trends and the Changing Value of Time

Job Boerma and Loukas Karabarbounis

3.1 Introduction

Wages and expenditures increased substantially for the average household during the past two decades. At the same time, these gains were not distributed equally across households.[1] The purpose of this paper is to develop a tractable framework that allows us to account quantitatively for the drivers of both average and divergent trends in labor market outcomes and to assess their welfare consequences.

Our framework is a general equilibrium model with incomplete asset markets and household heterogeneity in market and home technologies and preferences. Households have access to various home technologies that, following Ghez and Becker (1975), combine expenditures and time as inputs to produce final consumption goods. In the home sector,

[1]A large literature has documented the rise of the dispersion of wages, expenditures, and time allocation across households. For example, Autor, Katz, and Kearney (2008), Heathcote, Perri, and Violante (2010), and Attanasio, Hurst, and Pistaferri (2015) discuss several empirical facts underlying the evolution of heterogeneity in labor market outcomes.

households are heterogeneous with respect to their preferences across goods and their productivity of time. Home production is not tradeable and storable, meaning that in every instance home production must be consumed, and not insurable, meaning there are no assets that households can purchase to explicitly insure against differences that originate in the home sector. In the market sector, households are also heterogeneous with respect to their productivity. Following the approach of Heathcote, Storesletten, and Violante (2014), the structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences.

We apply our framework to married households surveyed by the Consumer Expenditure Survey (CEX) and the American Time Use Survey (ATUS) between 1995 and 2016. We split the home sector into a non-market sector in which expenditures and time are substitutes in production and a leisure sector in which expenditures and time are complements in production. The non-market sector includes expenditures such as food and household services and time uses such as housework and child care. The leisure sector includes expenditures such as telecommunication and entertainment and time uses such as television watching and other recreational activities.

An appealing feature of the framework is the transparency and generality of the identification of the sources of heterogeneity across households. The model retains tractability because it features a no-trade result with respect to certain assets. Therefore, we can characterize the allocations of expenditures and time across sectors in closed form. Following the same approach as in our earlier work (Boerma and Karabarbounis, 2019b), we use the analytical solutions to invert the equilibrium allocations and identify the sources of heterogeneity across households that perfectly account for the household-level data in any given point of time. Our exercise is to then shut off particular aspects of the evolution of the sources of heterogeneity over time. This allows us to assess the drivers of trends in sectoral expenditures and time allocation for the average household, the drivers of trends in the dispersion of sectoral expenditures and time allocation across households, and the welfare consequences of these trends.

We reach two main conclusions regarding the sources of heterogeneity that characterize households and their evolution of time. First, we infer that mean productivity of leisure time more than doubles between the beginning and the end of the sample. The key feature of the data leading to this inference is the dramatic increase in leisure expenditures relative to leisure time for the average household. The increase in expenditures relative to time is larger than the one predicted only by the decline in the relative price of leisure goods. Given that expenditures and time are complements in the production of leisure goods, we infer that the productivity of leisure time must have been increasing.

Second, the dispersion of the productivity of non-market and leisure time is larger than the dispersion of market productivity across households. Our inference of large uninsurable differences in home productivity follows from the observation that in the cross-section of households time spent either on the non-market or the leisure sector is weakly correlated with sectoral expenditures and market productivity. As a result, home productivity needs to be significantly dispersed in order to rationalize the variation of these observables. We document that the dispersion of the productivity of time inputs in home production has increased, paralleling the well-known increase in the dispersion of market productivity (wages) over time.

Our counterfactual analyses demonstrate the importance of market and home productivity and prices for the evolution of mean expenditures and market hours. Given the relative stability of market hours over time, the increase in mean market productivity accounts for most of the increase in mean expenditures over time. The increase in the relative price of non-market goods induces households to substitute away from non-market expenditures toward non-market time and the decline in the relative price of leisure goods induces households to complement leisure expenditures with rising leisure time. Changes in relative prices generate roughly 11 log points decline in market hours, with the majority of this decline accounted for by the increase in the relative price of non-market goods. This decline is offset by the rise of market and leisure productivities, which induce households to reallocate hours toward the market sector.

To assess the welfare effects of trends in labor market outcomes, we calculate consumption equivalent changes that arise from both changes in mean consumption and changes in the dispersion of consumption across households. By consumption we mean the final aggregator of the production process that involves aggregating sectoral goods produced with expenditures and time. A novel finding of our paper is to demonstrate that the rise of mean leisure productivity is quantitatively the most important driver of welfare changes over time. The increase in mean leisure productivity generates more than 30 log points increase in mean consumption over time. To put this number in context, the increase in mean market productivity contributes less than 10 log points increase in mean consumption. At the same time, the increase in mean leisure productivity, which affects all households equally, moderates the rise of consumption dispersion across households induced by changes in the variance of market and home productivities over time. The contribution of mean leisure productivity to welfare through the dispersion channel is roughly 10 log points of the consumption equivalent.

It is important to contrast our approach of assessing welfare effects through an equilibrium model to more descriptive approaches on the evolution of the dispersion of expenditures and time inputs.[2] Similar to the distinction emphasized by Aguiar and Hurst (2005), in developing our welfare metric we distinguish between expenditures, which serve as an input in the production of final goods, and consumption, which is the result of a production process involving expenditures, time, and productivity. This distinction matters for our conclusions. For example, we find that the increase in the variance of the permanent component of market productivity is the most important factor accounting for the increase in the dispersion of total expenditures over time. However, this factor contributes significantly less to the welfare costs of dispersion once we recognize that these welfare costs are linked more closely to the consumption aggregator than to total expenditures.

We examine trends in labor market outcomes through the lens of a structural model, complementing earlier attempts to measure welfare effects from changes in the dispersion of observables. Attanasio and Davis (1996) is an early study that links the divergence of group wages to the divergence of group expenditures and argues that this departure

[2]For example, Krueger and Perri (2006) and Aguiar and Bils (2015) measure the evolution of dispersion of expenditures over time, Aguiar and Hurst (2007b) document the rise of leisure inequality between the 1965 and the early 2000s, and Attanasio, Hurst, and Pistaferri (2015) and Han, Meyer, and Sullivan (2018) provide statistics of the evolution of dispersion of expenditures jointly with dispersion of time use. In their study of increasing inequality, Krueger and Perri (2003) conduct welfare experiments by essentially varying allocations that enter directly into the utility function. The difference with our approach is that we develop an equilibrium model that solves for arguments of the utility function as a function of more primitive productivity shifters, preference shifters, and policy parameters and, therefore, our counterfactuals account for equilibrium responses when conducting welfare analyses.

from full insurance carries significant welfare costs. Heathcote, Storesletten, and Violante (2013) discuss the merits of structural approaches relative to statistical approaches when calculating welfare effects and estimate that, in response to the observed changes in the structure of wages, the welfare gains in terms of average consumption and leisure dominate the losses arising from increased dispersion. Relative to these papers, our paper incorporates multiple time uses and highlights the primary role of changes in leisure productivity in terms of understanding the welfare effects of recent labor market trends.

An emerging literature examines the role of shifts originating in the leisure sector for labor supply trends. Vandenbroucke (2009) adopts a quantitative Beckerian framework to study the driving forces behind the decline in working hours and their increased concentration over the first half of the 20th century. Accounting for the decline in market hours, he finds a primary role for increasing skilled wages and a limited role for the declining price of leisure goods. Bridgman (2016) develops a model with non-separable preferences that is able to accommodate the rise of average leisure and leisure inequality during the second half of the 20th century and Boppart and Ngai (2019) lay out conditions under which these trends are consistent with a balanced growth path. Like these papers, we are also interested in accounting for the evolution of the allocation of time. An important point of departure from this literature is that we incorporate micro-level data into our analysis of the heterogeneity in labor market trends across households.

Closest to our conclusions, Aguiar, Bils, Charles, and Hurst (2018) infer a significant increase in the technological progress of recreational time of young men. Their inference comes from the observed increase in recreational computer time in excess of the predicted increase along a leisure demand system. Similar to them, we find a significant increase in leisure productivity over time. Under our maintained assumption that expenditures and time are complements in leisure technology, our inference comes from the observed increase in leisure expenditures relative to time in excess of the increase predicted by the decline in the relative price of leisure goods. Aguiar, Bils, Charles, and Hurst (2018) do not map changes in leisure productivity to changes in welfare, whereas we uncover significant welfare effects from the rise of mean leisure productivity reflecting both an increase in average consumption and a moderation of consumption inequality.

3.2 Model

Our model of time allocation and expenditures is Beckerian (Becker, 1965; Ghez and Becker, 1975) in the sense that expenditures and time combine as inputs to produce final utility. We embed the Beckerian household production model into the tractable framework of incomplete asset markets and household heterogeneity developed by Heathcote, Storesletten, and Violante (2014).[3] We first present the model and characterize its equilibrium in closed form. We then show how to infer the sources of heterogeneity across households such that the model accounts perfectly for cross-sectional data on sectoral expenditures and the allocation of time.

3.2.1 Environment

Demographics. The economy features perpetual youth demographics. We denote by t the calendar year and by j the birth year of a household. Households face a constant probability of survival δ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

Household Technologies. Vector $\mathbf{c}$ collects goods, $\mathbf{x}$ collects (real) expenditure inputs, and $\mathbf{h}$ collects time inputs. We denote the market good by c_M and home produced goods by c_K where K indexes different home goods. The difference between market and home goods is that the former are intensive in expenditures and do not use time as an input, $c_M = x_M$, whereas the latter use both expenditures and time to produce output, $c_K = c_K(x_K, h_K)$.

A household's technology in the market sector is characterized by its pre-tax earnings $y = z_M h_M$, where z_M denotes exogenous market productivity (wage) that varies across households and h_M denotes hours worked in the market sector. A household's after-tax earnings are given by $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$, where parameter τ_0 governs the level and parameter τ_1 governs the progressivity of the tax system. A higher τ_1 introduces more

[3] In Boerma and Karabarbounis (2019b) we also extended the framework of Heathcote, Storesletten, and Violante (2014) to a model with home production. The difference is that here we use the Beckerian framework in which expenditures and home time are inputs in the production of goods that enter into utility whereas in Boerma and Karabarbounis (2019b) time spent working at the market and at home generate disutility as in Gronau (1986). While the two versions of the household production model share many predictions, in this paper we prefer the former because it allows us to model more directly changes in the price of leisure expenditures and in the returns to leisure time.

progressivity into the tax system as it compresses after-tax earnings relative to pre-tax earnings.[4]

Home goods c_K are produced by combining expenditures x_K and time h_K inputs according to CES aggregators:

$$c_K = \left(x_K^{\frac{\sigma_K - 1}{\sigma_K}} + (z_K h_K)^{\frac{\sigma_K - 1}{\sigma_K}} \right)^{\frac{\sigma_K}{\sigma_K - 1}}, \tag{1}$$

where parameter $\sigma_K \geq 0$ denotes the elasticity of substitution between expenditures and time in the Kth home production technology and z_K denotes exogenous productivity of time use K (relative to expenditures) that varies across households. Home goods are consumed every period and cannot be stored or traded in a market. Households are endowed with one unit of time, $h_M + \sum h_K = 1$.

Household Preferences. Households born at period j order sequences of goods with expected discounted utility $\mathbb{E}_j \sum_{t=j}^{\infty} (\beta\delta)^{t-j} U(c_t)$, where β is the discount factor and c denotes a CES aggregator of goods. The period utility function is:

$$U(c) = \log \left(\omega_M x_M^{\frac{\phi-1}{\phi}} + \sum \omega_K \left(x_K^{\frac{\sigma_K - 1}{\sigma_K}} + (z_K h_K)^{\frac{\sigma_K - 1}{\sigma_K}} \right)^{\frac{\sigma_K}{\sigma_K - 1}\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, \tag{2}$$

where parameter $\phi \geq 0$ denotes the elasticity of substitution across goods and ω_M and ω_K govern preferences for goods that vary across households. We normalize the preference shifters such that $\omega_M + \sum \omega_K = 1$ for each household and henceforth carry over in our notation only the ω_K's.

Sources of Heterogeneity. Households are heterogeneous with respect to their market productivity z_M, home productivities z_K, and preferences over goods ω_K. Following Heathcote, Storesletten, and Violante (2014), we impose a random walk structure for market

[4] Our tax schedule modifies the tax schedule considered, among others, by Heathcote, Storesletten, and Violante (2014) in that τ_1 is applied to market productivity z_M instead of earnings $z_M h_M$. We adopt the specification of after-tax earnings $\tilde{y} = (1 - \tau_0) z_M^{1-\tau_1} h_M$ instead of $\tilde{y} = (1 - \tau_0) (z_M h_M)^{1-\tau_1}$ because we can only prove the no-trade result in the home production model under the former specification. In Boerma and Karabarbounis (2019b) we argued that this modification does not matter for the quantitative results because wages and market hours are relatively uncorrelated in the cross section of households. As a result, our estimate of τ_1 is close to the estimate of Heathcote, Storesletten, and Violante (2014).

productivity that is important for obtaining the no-trade result underlying the analytical solutions. Households' log market productivity $\log z_M$ is the sum of a permanent component α and a more transitory component ε:

$$\log z^j_{M,t} = \alpha^j_t + \varepsilon^j_t \ . \tag{3}$$

The permanent component follows a random walk, $\alpha^j_t = \alpha^j_{t-1} + \upsilon^\alpha_t$. The more transitory component, $\varepsilon^j_t = \kappa^j_t + \upsilon^\varepsilon_t$, equals the sum of a random walk component, $\kappa^j_t = \kappa^j_{t-1} + \upsilon^\kappa_t$, and an innovation υ^ε_t. For any random walk, we use υ to denote innovations and Φ_{υ_t} to denote distributions of innovations. We allow distributions of innovations to vary over time t.

Given the log preferences in equation (2), we obtain the no-trade result that guarantees analytical solutions with minimal structure on the processes that govern productivity and preferences in the home sectors. Home productivities follow $z^j_{K,t} \sim \Phi^j_{z_K,t}$ and preferences follow $\omega^j_{K,t} \sim \Phi^j_{\omega_K,t}$, where again we allow distributions to vary over time t. We assume that $z^j_{K,t}$ and $\omega^j_{K,t}$ are orthogonal to the innovations $\{\upsilon^\alpha_t, \upsilon^\kappa_t, \upsilon^\varepsilon_t\}$ and that all innovations are drawn independently from each other. The distribution of initial conditions of $(\omega^j_{K,j}, z^j_{K,j}, \alpha^j_j, \kappa^j_j)$ can be non-degenerate across households born at j and can vary by birth year j. From now on, we identify a household ι by a sequence $\{z^j_K, \omega^j_K, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$.

Asset Markets. We describe restrictions on asset markets using the definition of an island in the spirit of Heathcote, Storesletten, and Violante (2014). Islands capture insurance mechanisms available to households for smoothing more transitory shocks in the market sector. Households are partitioned into islands, with each island consisting of a continuum of households that are identical in terms of their productivity at home z_K, preferences ω_K, permanent component of market productivity α, and the initial condition of κ. More formally, household $\iota = \{z^j_K, \omega^j_K, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$ lives on island ℓ consisting of ι's with common initial state $(z^j_{K,j}, \omega^j_{K,j}, \alpha^j_j, \kappa^j_j)$ and sequences $\{z^j_{K,t}, \omega^j_{K,t}, \alpha^j_t\}^\infty_{t=j+1}$.

The structure of asset markets is as follows. Households cannot trade assets contingent on $z^j_{K,t}$ and $\omega^j_{K,t}$, but can trade one-period bonds $b^\ell(s^j_{t+1})$ that pay one unit of market consumption contingent on $s^j_t \equiv (\alpha^j_t, \kappa^j_t, \upsilon^\varepsilon_t)$ with households that live on their island ℓ. Across islands, households can trade economy-wide one-period bonds $a(\zeta^j_{t+1})$ that pay one unit of market consumption contingent on $\zeta_t^j \equiv (\kappa_t^j, \upsilon_t^\varepsilon)$.

As we discuss more formally below, differences in (z_K, ω_K, α) across households remain uninsurable by the no-trade result that generates $a(\zeta_{t+1}^j) = 0$ in equilibrium. The more transitory component of productivity ε_t^j becomes fully insurable because households on an island are only heterogeneous with respect to ζ_t^j and can trade state-contingent bonds $b^\ell(\zeta_{t+1}^j)$. As a result, the island structure generates partial insurance with respect to market productivity differences. Anticipating these results, henceforth we call α the uninsurable permanent component of market productivity and $\varepsilon = \kappa + \upsilon^\varepsilon$ the insurable transitory component of market productivity.[5]

Household Optimization. We now describe the optimization problem of a particular household ι born in period j. The household chooses $\{\mathbf{c}_t, \mathbf{x}_t, \mathbf{h}_t, b^\ell(s_{t+1}^j), a(\zeta_{t+1}^j)\}_{t=j}^\infty$ to maximize the expected value of discounted flows of utilities in equation (2), subject to the home production technologies in equation (1), the time endowment $h_M + \sum h_K = 1$, and the sequential budget constraints:

$$x_{M,t} + \sum p_{K,t} x_{K,t} + \int_{s_{t+1}^j} q_b^\ell(s_{t+1}^j) b^\ell(s_{t+1}^j) \mathrm{d}s_{t+1}^j + \int_{\zeta_{t+1}^j} q_a(\zeta_{t+1}^j) a(\zeta_{t+1}^j) \mathrm{d}\zeta_{t+1}^j = \tilde{y}_t^j + b^\ell(s_t^j) + a(\zeta_t^j) \,. \tag{4}$$

The market good x_M is the numeraire good with a price of one in all periods. Denoting by p_K the price of good x_K relative to market good, the left-hand side of the budget constraint equals total expenditures on goods $(px)_t = x_{M,t} + \sum p_{K,t} x_{K,t}$, island-level bonds $b^\ell(s_{t+1}^j)$ at prices $q_b^\ell(s_{t+1}^j)$, and economy-wide bonds $a(\zeta_{t+1}^j)$ at prices $q_a(\zeta_{t+1}^j)$. The right-hand side of the budget constraint consists of after-tax labor income $\tilde{y}_t^j$ and bond payouts.

Government. The government taxes labor income to finance public expenditures of market goods G. Its budget constraint is $G = \int_\iota \left[z_{M,t}(\iota) - (1-\tau_0) z_{M,t}(\iota)^{1-\tau_1} \right] h_{M,t}(\iota) \mathrm{d}\Phi(\iota)$, where Φ denotes the distribution function of households.

[5] The framework accommodates implicit insurance against α differences because households can substitute expenditures and time across sectors. Apart from explicit asset markets, some examples of mechanisms that insure ε shocks include family and government transfers.

Production. Aggregate production is given by $Y = \int_\iota z_M(\iota)h_M(\iota)\mathrm{d}\Phi(\iota)$. The markets for labor and goods are perfectly competitive and the wage per efficiency unit of labor is one. Production Y is transformed at a rate of one into market goods, $\int_\iota x_M(\iota)\mathrm{d}\Phi(\iota) + G$, and at rate A_K^{-1} into expenditures of home goods x_K. Therefore, relative prices equal $p_K = A_K^{-1}$.

Equilibrium. Given a tax function (τ_0, τ_1), an equilibrium consists of a sequence of allocations $\{\mathbf{c}_t, \mathbf{x}_t, \mathbf{h}_t, b^\ell(s_{t+1}^j), a(\zeta_{t+1}^j)\}_{\iota,t}$ and a sequence of prices $\{p_{K,t}\}_t$, $\{q_b^\ell(s_{t+1}^j)\}_{\ell,t}$, $\{q_a(\zeta_{t+1}^j)\}_t$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$\int_{\iota\in\ell} b^\ell(s_{t+1}^j;\iota)\mathrm{d}\Phi(\iota) = 0 \quad \forall \ell, s_{t+1}^j, \quad \text{and} \quad \int_\iota a(\zeta_{t+1}^j;\iota)\mathrm{d}\Phi(\iota) = 0 \quad \forall \zeta_{t+1}^j; \tag{5}$$

(iii) goods market clears:

$$\int_\iota \Big(x_{M,t}(\iota) + \sum p_{K,t}x_{K,t}(\iota)\Big)\,\mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t}(\iota)h_{M,t}(\iota)\mathrm{d}\Phi(\iota), \tag{6}$$

(iv) the government budget constraint holds $G = \int_\iota \left[z_{M,t}(\iota) - (1-\tau_0)z_{M,t}(\iota)^{1-\tau_1}\right] h_{M,t}(\iota)\mathrm{d}\Phi(\iota)$, and (v) relative prices are pinned down by the relative efficiency of transforming production $p_{K,t} = A_{K,t}^{-1}$.[6]

3.2.2 Equilibrium Allocations

The model retains tractability because it features a no-trade result. This section explains the logic and usefulness of this result and Appendix 3.A presents the proof. Our proof follows closely the proof presented by Heathcote, Storesletten, and Violante (2014) in an incomplete markets model with labor supply and further extended by Boerma and Karabarbounis (2019b) to an incomplete markets model with multiple time uses.

We begin by guessing that the equilibrium features no trade across islands, that is $a(\zeta_{t+1}^j;\iota) = 0, \forall \iota, \zeta_{t+1}^j$. Further, we postulate that an equilibrium allocation $\{\mathbf{c}_t(\iota), \mathbf{x}_t(\iota), \mathbf{h}_t(\iota)\}$ solves a sequence of static planning problems. The planner problems consist of maximizing

[6] We allow relative prices $p_{K,t}$ to vary over time. For the no-trade theorem, we do not need to impose restrictions on the stochastic processes of $A_{K,t}$. Henceforth, we treat the prices $p_{K,t}$ as exogenous with the understanding that there is a unique mapping from sectoral productivity to prices $p_{K,t} = A_{K,t}^{-1}$ that we could use to rationalize any path of prices we observe in the data. We also note that productivity changes in the market sector are implicitly subsumed into a common time component of $z_{M,t}$ across households.

average utility within each island, $\int_{\zeta_t^j} U\left(c_t(\iota);\iota\right) \mathrm{d}\Phi_t(\zeta_t^j)$, subject to households' home production technologies in equation (1), households' time endowment $h_{M,t}(\iota) + \sum h_{K,t}(\iota) = 1$, and the island-level resource constraint $\int_{\zeta_t^j} \left(x_{M,t}(\iota) + \sum p_{K,t} x_{K,t}(\iota)\right) \mathrm{d}\Phi_t(\zeta_t^j) = \int_{\zeta_t^j} \tilde{y}_t(\iota) \mathrm{d}\Phi_t(\zeta_t^j)$. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and all asset and goods markets clear.

To understand the no-trade result, we observe that households on each island ℓ have the same marginal utility of market consumption because they are identical in terms of (z_K, ω_K, α) and trade in state-contingent bonds allows them to perfectly insure against $(\kappa, \upsilon^{\varepsilon})$. The island-level marginal utility of market consumption $\mu(\ell)$ in the no-trade equilibrium is:

$$\mu(\ell) \;=\; \frac{1}{x_M + \sum p_K x_K + \tilde{z}_M \sum h_K} \;=\; \frac{1}{\int_{\zeta} \tilde{z}_M \mathrm{d}\Phi(\zeta)} \;=\; \frac{1}{\exp\left((1-\tau_1)\alpha\right)\mathbb{C}}, \tag{7}$$

where for simplicity we have dropped the time subscript from all variables and the constant $\mathbb{C} = \int_{\zeta} (1-\tau_0) \exp\left((1-\tau_1)(\varepsilon)\right) \mathrm{d}\Phi(\zeta)$ does not depend on ℓ and is common across all households. The no-trade result states that households do not trade bonds across islands, $a(\zeta_{t+1}^j) = 0$. Given the random walk assumption on α, equation (7) implies that the growth in marginal utility, μ_{t+1}/μ_t, does not depend on the state vector $(z_{K,t}^j, \omega_{K,t}^j, \alpha_t^j)$ that differentiates islands ℓ. As a result, all households value bonds traded across islands identically in equilibrium and hence there are no mutual benefits from trading $a(\zeta_{t+1}^j)$.[7]

Solutions to standard general equilibrium models with uninsurable risk and self-insurance via a risk-free bond, such as Huggett (1993) and Aiyagari (1994), are obtained numerically. While the present model also allows households to trade a risk-free bond (by setting $a(\zeta_t^j) = 1$ for all states ζ_t^j), the assumptions on asset markets, stochastic processes, and preferences allow us to characterize equilibrium allocations in closed form without solving

[7]For this result we note the importance of log preferences with respect to the consumption aggregator. Log preferences generate a separability between the marginal utility of market consumption and z_K and ω_K and, thus, the no-trade result holds irrespective of the value of the elasticity of substitution across sectors ϕ, the elasticity of substitution within sector σ_K, and further stochastic properties of z_K and ω_K. In Boerma and Karabarbounis (2019b) we show that the no-trade result holds in the Gronau (1986) version of the home production model when the disutility of total hours enter additively into the utility function and sectoral hours are perfect substitutes in the disutility.

simultaneously for the wealth distribution. Dropping the time index for notational simplicity, we summarize the equilibrium allocations in equations (8)-(11):

$$x_M = \exp\left((1-\tau_1)\alpha\right) \mathbb{C} \frac{1}{1+\sum \left(\frac{\omega_K}{\omega_M}\right)^{\phi} p_K^{1-\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}}, \tag{8}$$

$$x_K = \exp\left((1-\tau_1)\alpha\right) \mathbb{C} \frac{p_K^{-\phi} \left(\frac{\omega_K}{\omega_M}\right)^{\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-\sigma_K}{\sigma_K-1}}}{1+\sum \left(\frac{\omega_K}{\omega_M}\right)^{\phi} p_K^{1-\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}}, \tag{9}$$

$$h_K = \exp\left((1-\tau_1)\alpha\right) \mathbb{C} \frac{\left(\frac{p_K}{\tilde{z}_M}\right)^{\sigma_K} z_K^{\sigma_K-1} p_K^{-\phi} \left(\frac{\omega_K}{\omega_M}\right)^{\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-\sigma_K}{\sigma_K-1}}}{1+\sum \left(\frac{\omega_K}{\omega_M}\right)^{\phi} {p_K}^{1-\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}}, \tag{10}$$

$$h_M = 1 - \sum h_K, \tag{11}$$

where all allocations and sources of heterogeneity $(z_K, \omega_K, \alpha, \varepsilon)$ are household-specific, prices p_K are common across households, and the constant $\mathbb{C} = \int_\zeta (1-\tau_0) \exp\left((1-\tau_1)(\varepsilon)\right) \mathrm{d}\Phi(\zeta)$ is common across households.

Starting with expenditures in equations (8) and (9), we first note that, holding constant relative productivities $\frac{z_K}{\tilde{z}_M}$, an increase in the permanent uninsurable component α of market productivity increases both x_M and x_K because all goods are normal. Holding constant relative productivities $\frac{z_K}{\tilde{z}_M}$, expenditures do not depend on the transitory component of market productivity ε because state-contingent assets insure against variation in ε.

Dividing equation (9) with equation (8) sheds light on the allocation of expenditures across sectors:

$$\frac{x_K}{x_M} = p_K^{-\phi} \left(\frac{\omega_K}{\omega_M}\right)^{\phi} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{\phi-\sigma_K}{\sigma_K-1}}. \tag{12}$$

An increase in home productivity relative to the opportunity cost of time, $\frac{z_K}{\tilde{z}_M}$, has two effects on the allocation of expenditures. First, it makes good c_K cheaper to produce relative to good c_M, which tends to increase x_K relative to x_M. This effect is parameterized by the elasticity of substitution across goods ϕ. Second, it makes input x_K more expensive relative to input h_K in the production of good c_K, which tends to decrease x_K. This effect is parameterized by the home production elasticity σ_K. When $\phi > \sigma_K$, the first effect dominates and $\frac{x_K}{x_M}$ is increasing in $\frac{z_K}{\tilde{z}_M}$. By contrast, the effect of an increase in the price p_K is unambiguously negative because both the substitution away from good c_K toward good c_M and the substitution away from expenditures x_K toward time h_K work in the same direction.[8]

For the allocation of spending relative to time, we use equations (9) and (10) to obtain:

$$\frac{x_K}{h_K} = \left(\frac{\tilde{z}_M}{p_K}\right)^{\sigma_K} z_K^{1-\sigma_K}. \tag{13}$$

The first term shows that an increase in the opportunity cost of time $\tilde{z}_M$ relative to the price of expenditures p_K unambiguously increases expenditures relative to time in the production of good c_K. The second term shows that an increase in the relative productivity of time z_K increases expenditures relative to time when the two inputs are complements ($\sigma_K < 1$).

3.2.3 Identification of Sources of Heterogeneity

Building on the methodology introduced by Boerma and Karabarbounis (2019b), in this section we infer the sources of heterogeneity across households, $\{z_K, \omega_K, \alpha, \varepsilon\}_\iota$, such that the model accounts perfectly for any given cross-sectional data $\{x_M, x_K, h_M, h_K, z_M\}_\iota$ in any period. Given parameters $(\phi, \sigma_K, \tau_0, \tau_1)$, prices p_K, and cross-sectional data $\{x_M, x_K, h_M, h_K, z_M\}_\iota$, we invert the equilibrium allocations presented in equations (8)-(11) and the decomposition of market productivity into a permanent and transitory component, $\log z_M = \alpha + \varepsilon$, to obtain unique inferred sources of heterogeneity up to a constant (see Appendix 3.B for more

[8]The elasticity of $\frac{x_K}{x_M}$ with respect to p_K is a weighted average of the two substitution elasticities and equals $-\frac{1}{1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}}\phi - \frac{\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}}{1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}}\sigma_K$.

details):

$$z_K = \left(\frac{x_K}{h_K}\right)^{\frac{1}{1-\sigma_K}} \left(\frac{\tilde{z}_M}{p_K}\right)^{\frac{\sigma_K}{\sigma_K-1}}, \tag{14}$$

$$\omega_M = \frac{1}{1+\sum p_K \left(\frac{x_K}{x_M}\right)^{\frac{1}{\phi}} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{1}{\phi}\frac{\sigma_K-\phi}{\sigma_K-1}}}, \tag{15}$$

$$\omega_K = \frac{p_K \left(\frac{x_K}{x_M}\right)^{\frac{1}{\phi}} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{1}{\phi}\frac{\sigma_K-\phi}{\sigma_K-1}}}{1+\sum p_K \left(\frac{x_K}{x_M}\right)^{\frac{1}{\phi}} \left(1+\left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K-1}\right)^{\frac{1}{\phi}\frac{\sigma_K-\phi}{\sigma_K-1}}}, \tag{16}$$

$$\alpha = \frac{1}{1-\tau_1}\left[\log(x_M + \sum p_K x_K + \tilde{z}_M \sum h_K) - \log \mathbb{C}\right], \tag{17}$$

$$\varepsilon = \log z_M - \frac{1}{1-\tau_1}\left[\log(x_M + \sum p_K x_K + \tilde{z}_M \sum h_K) - \log \mathbb{C}\right]. \tag{18}$$

The solution for z_K in equation (14) comes from inverting equation (13) for the optimal allocation of expenditures and time inputs in the production of good K. Intuitively, when household expenditures x_K increase relative to the time input h_K and the two inputs are complements in production ($\sigma_K < 1$), it must be that household time is more productive in the production of good K. Given an inferred z_K, equations (15) and (16) show how relative preferences for goods are pinned down by relative expenditures, prices, and productivities. Up to a constant which is common across households in a given period, the permanent component of log market productivity α in equation (17) equals the market value of total consumption which consists of the sum of expenditures $px = x_M + \sum p_K x_K$ and the market value of time allocated to home production $\tilde{z}_M \sum h_K$. Finally, the transitory component of market productivity ε equals the gap between log market productivity and its permanent component.

3.3 Data

For our baseline analyses we combine data from the Consumer Expenditure Survey (CEX), the American Time Use Survey (ATUS), and the national income and product accounts (NIPA). We consider married and cohabiting households with heads between 25 and 65 years old who are not students. We drop observations with market productivity below 3 dollars per hour in 2010 dollars, with market productivity above 300 dollars but working less than 20 hours per week, with expenditures at the top and bottom one percent, and with more than 105 reported hours per week in any of the time use categories we consider. In the ATUS we drop respondents during weekends and in the CEX we keep households that completed all four interviews. The final sample from CEX/ATUS contains 34,775 households between 1995 and 2016. In all our results, we weight households with the sample weights provided by the surveys.

For our quantitative results, we specialize the general model with $K + 1$ goods to three goods. The market good $c_M = x_M$ is produced only with expenditures. The non-market good c_N is produced with non-market time h_N and non-market expenditures x_N. Finally, the leisure good c_L is produced with leisure time h_L and leisure expenditures x_L.

Data on expenditures come from CEX interview surveys. Our definition of expenditures is closest to the one in Krueger and Perri (2006). It covers both expenditures on non-durables and expenditures on durables such as housing, vehicles, and furniture. Our measure of consumption reflects the flow of services in a given period. For housing services we use rent paid if the household rents and a self-reported imputed rent for households that own. For services generated by vehicles and furnishings, we use the imputation approach of Cutler and Katz (1991) since there is no direct information on the value of the stock of vehicles and furniture.[9]

We split total expenditures px between market expenditures x_M, non-market expenditures $p_N x_N$, and leisure expenditures $p_L x_L$ by mapping 20 spending categories from the CEX to our three baskets. The logic underpinning our choice is to classify expenditures complementary to time as leisure (such as communication, entertainment, and reading), expenditures substitutable to time as non-market production (such as food, household services, and personal care), and expenditures that do not use a significant amount of time in

[9]For households that report spending on vehicles or furniture, we regress their durables spending on a quadratic in household expenditures (excluding vehicles and furniture), income, age, sex, and education of the household head. The imputed expenditure of vehicles or furniture is the predicted value of spending from this regression multiplied by the user cost of each durable (for vehicles we also multiply by the number of vehicles owned).

the production of commodities as market goods.[10]

To obtain quantities x_M, x_N, and x_L, we deflate expenditures in each category with their corresponding price index. We construct the Fisher price index for each basket using the price indices and aggregate spending for the 20 CEX spending categories as provided in NIPA Table 2.5.[11] For durable goods, we create corresponding price and spending series using user costs.[12]

From the CEX, we measure income as wage and salary income earned over the past 12 months and wages as income divided by hours usually worked in a year (the product of weeks worked with usual hours worked per week). Because we focus on married or cohabiting households, we define household market hours h_M as the sum of hours worked by spouses and market productivity z_M as the average of wages of individual members weighted by their market hours.

The market good is the numeraire good and we deflate the price of non-market goods p_N, the price of leisure goods p_L, and market productivity z_M with the price index for market goods. For consistency with the model in which aggregate expenditures $\int \left(x_{M,t}(\iota) + \sum p_{K,t} x_{K,t}(\iota)\right) \mathrm{d}\Phi(\iota) + G$ equal aggregate income $\int z_{M,t}(\iota) h_{M,t}(\iota) \mathrm{d}\Phi(\iota)$, for each household in the CEX we scale their quantities by a time-varying factor that aligns aggregate expenditures with aggregate income reported in the survey. The scaling factor is trending during the first years of our sample, reflecting an increasing gap between reported expenditures and income in the survey, but then stabilizes during the 2000s.

Data for non-market hours h_N come from the ATUS waves between 2003 and 2017. Our definition of time spent on non-market production follows the one in Aguiar, Hurst, and

[10] Market expenditures x_M include clothing and footwear, utilities and fuels, health, vehicles, public transport, motor vehicle operations, education, insurance, tobacco, and professional services. Non-market expenditures $p_N x_N$ include food and beverages (home and away), household services, and personal care. Leisure expenditures $p_L x_L$ include communication, entertainment, reading, and personal items. Housing, furniture and household equipment are allocated proportional to the expenditure shares of the three types of goods.

[11] Table 2.5.4 provides the price indices and Table 2.5.5 gives the corresponding aggregate spending levels. To illustrate our approach, we use the price index for communication in Table 2.5.4 as the price for the CEX category communication. In constructing the price index for leisure goods, we weight the price index for communication by aggregate spending on communication as documented in Table 2.5.5.

[12] To calculate the user cost for durable consumption goods we use the price index for the spending category from Table 2.5, the interest rate on the five-year constant maturity Treasury for the cost of capital, and NIPA fixed assets accounts to construct the depreciation rate. We calculate the depreciation rate as current-cost depreciation over the current-cost net stock plus half of investments using Tables 8.1, 8.4, and 8.7.

Karabarbounis (2013) and includes housework, cooking, shopping, home and car maintenance, gardening, child care, and care for other household members. The CEX does not contain information on time spent on non-market production. To overcome this difficulty, we follow Boerma and Karabarbounis (2019b) and impute time use data from the ATUS into the CEX. Our imputation procedure is to allocate to individuals in the CEX the mean non-market hours of matched individuals from the ATUS based on group characteristics that include work status, race, gender, age, family status, education, disability status, geography, hours worked, and wages.[13] We first impute non-market hours to individuals and, similarly to market hours, then sum up these hours at the household level. Finally, we measure leisure residually as total disposable time, which we set to 105 hours per week, minus market hours and time spent on non-market production, $h_L = 105 - h_M - h_N$.[14]

In Figure 1 we present the time evolution of relative prices, p_N and p_L, and means and variances of expenditure inputs x_M, x_N, and x_L, time inputs h_M, h_N, and h_L, total expenditures $px = x_M + p_N x_N + p_L x_L$, and market productivity z_M. To obtain the time profiles for the means of variables, we regress each variable at the household level on age and time fixed effects. The plotted means are the coefficients on the time dummies and, therefore, correspond to the mean value of each variable using within-age variation over time. The variances refer to the variances of the residuals in a given year from these regressions and, similarly, reflect changes in within-age variances over time.

The top panel of Figure 1 shows that the relative prices of non-market and leisure goods move in opposite direction over time, with the relative price of non-market goods increasing by roughly 20 percent and the relative price of leisure falling by roughly 45 percent. The

[13]In Boerma and Karabarbounis (2019b) we show that this imputation accounts for approximately three quarters of the variation in non-market hours. We also demonstrate that our imputation does not introduce spurious correlations in the CEX/ATUS merged data because the cross-sectional correlation of non-market hours with market hours and wages is similar in magnitude between the ATUS sample of individuals and the CEX/ATUS merged sample of households. Finally, we document that the cross-sectional correlation between non-market hours and market expenditures, market hours, and wages is similar in magnitude between the CEX/ATUS merged sample of households and two PSID samples of households with direct information on non-market hours, market expenditures, market hours, and wages.

[14]We define leisure residually to ensure that all households have the same endowment of time. The cross-sectional correlation between this definition of leisure and the direct ATUS measure of leisure defined in Aguiar, Hurst, and Karabarbounis (2013) is 0.5. Given the imperfect correlation, in our sensitivity analyses we reverse the definitions by using leisure directly from the ATUS and defining non-market hours residually as $h_N = 105 - h_M - h_L$. As reported in Section 3.6, our inferences on the role of leisure productivity and counterfactuals are not sensitive to this alternative measurement of time uses.

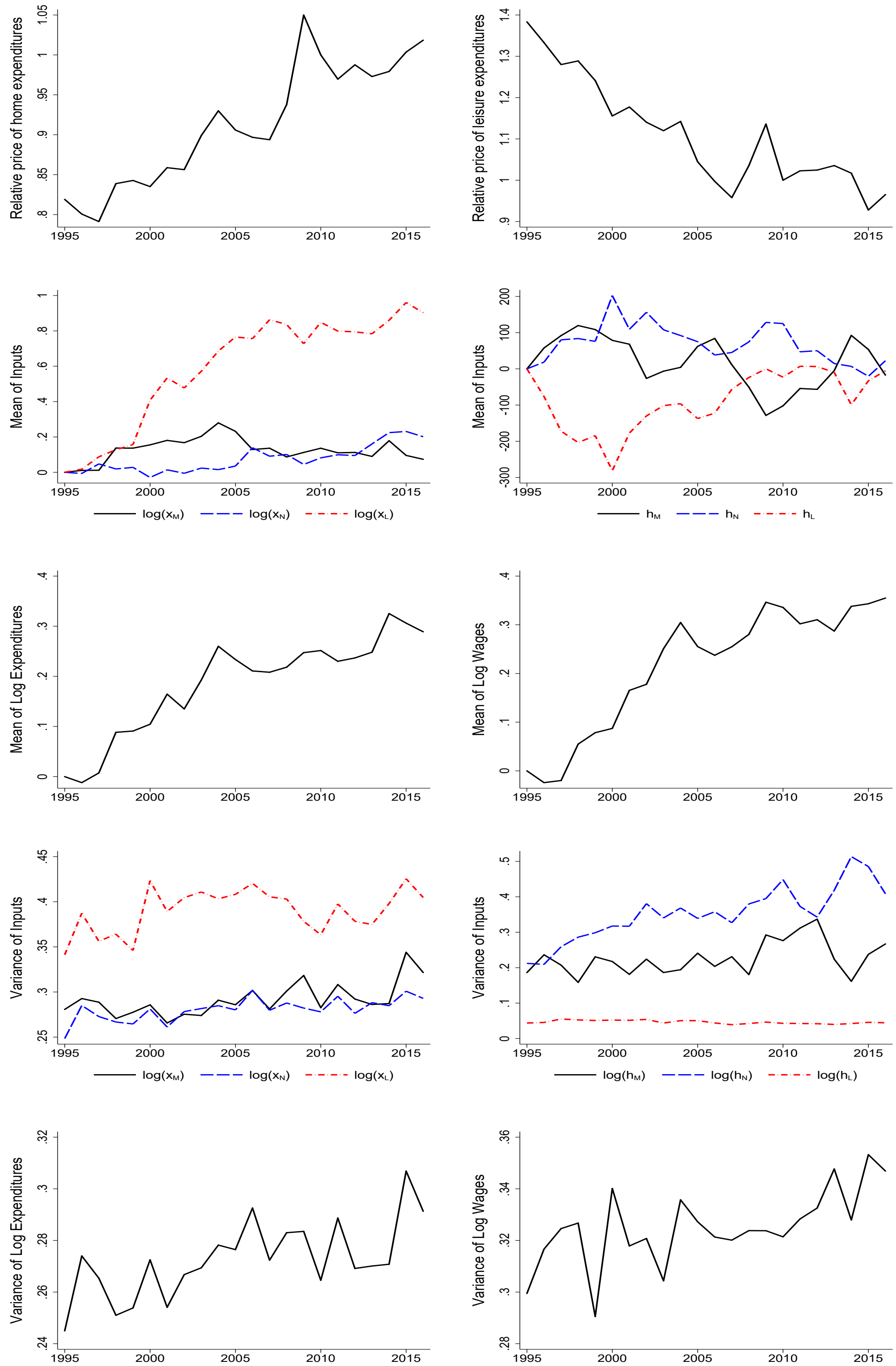


Figure 1: Means and Variances of Observables

Figure 1 plots the evolution of the relative prices p_N, and p_L, means and variances of expenditure inputs x_M, x_N, and x_L, time inputs h_M, h_N, and h_L, total expenditures $px = x_M + p_N x_N + p_L x_L$, and market productivity z_M.

Table 1: Unconditional Correlations Between Observables

	$\log z_M$	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
$\log z_M$	1.00	.	.	.	.	.	.	.
$\log(px)$	0.53	1.00	.	.	.	.	.	.
$\log x_M$	0.54	0.96	1.00	.	.	.	.	.
$\log x_N$	0.46	0.95	0.85	1.00	.	.	.	.
$\log x_L$	0.49	0.91	0.82	0.82	1.00	.	.	.
$\log h_M$	-0.07	0.23	0.27	0.18	0.17	1.00	.	.
$\log h_N$	0.04	-0.06	-0.08	-0.04	-0.04	-0.39	1.00	.
$\log h_L$	0.02	-0.18	-0.20	-0.15	-0.15	-0.47	-0.37	1.00

second row shows that while expenditures have increased for all three goods since the mid 1990s, the increase has been significantly larger in the leisure sector. Leisure time declines by almost 300 hours per year until the early 2000s, with this decline being offset by increases in both market and non-market hours. Since the 2000s, non-market time has declined whereas leisure time has returned to its 1995 level. These changes in the allocation of expenditures and time have been accompanied by a roughly 35 percent increase in average market productivity over time.

In the bottom rows of Figure 1, we document an increase in the variance of expenditures across households by roughly 5 log points over time. The increase in the dispersion of expenditures is apparent in all three expenditure categories. The variation in leisure hours has been stable over our sample period, while the variation in non-market hours doubled over the same period.[15] The variation in market hours has been relatively constant, with the exception of the period following the Great Recession. Finally, the variance of market productivity increases by roughly 4 log points over time.

Table 1 displays unconditional cross-sectional correlations between observables. These correlations are obtained after we absorb time and age fixed effects by regressing each

[15]This result is sensitive to identifying leisure time as the residual time given observed market and non-market hours. The direct measure of leisure from the ATUS (that includes activities such as television watching, socializing, exercising, playing sports, reading, computer time, and listening to music) displays an increasing dispersion over time. However, as reported in Section 3.6, our inferences on the role of leisure productivity and counterfactuals are not sensitive to this discrepancy.

observable on both age and time dummies. Market productivity is positively correlated with expenditures (with a correlation of 0.5) but uncorrelated with all time inputs. Households with high levels of total expenditures also tend to spend more in each sector. By contrast, within sector K expenditures x_K and time h_K are weakly correlated in the cross-section of households. Finally, higher time spent working in the market sector is offset by lower time in both the non-market and the leisure sector.

3.4 Quantitative Results

We begin by discussing the parameterization of the model. We estimate a progressivity parameter of $\tau_1 = 0.12$ based on a regression of log after-tax market productivity on log pre-tax market progressivity from the Current Population Survey between 2005 and 2015. We set the level parameter to $\tau_0 = -0.34$ such that the average tax rate on labor income equals 0.10 which is the average ratio of personal current taxes to income from the national income and product accounts. For our baseline analyses, we set the elasticity of substitution across goods to $\phi = 1$, the elasticity of substitution between expenditures and time in non-market technology to $\sigma_N = 2.5$, and the elasticity of substitution between expenditures and time in leisure technology to $\sigma_L = 0.5$. Our choice of $\sigma_N = 2.5$ is motivated by previous estimates in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4, Aguiar and Hurst (2007a) obtain estimates of 1.8, and Boerma and Karabarbounis (2019b) estimate a value of 2.3. The literature offers little guidance about the values of ϕ and σ_L. Consistent with our classification of expenditures and time in the three goods, we choose $\phi = 1$ and $\sigma_L = 0.5$ such that the elasticity of substitution across goods is larger than the elasticity of substitution between expenditures and time in leisure technology and smaller than the elasticity of substitution in non-market technology. Some of our quantitative results are sensitive to these values of elasticities, so in Section 3.6 we present several analyses under alternative values.

Given parameter values, we identify the sources of heterogeneity using equations (14) to (18). Figure 2 presents the evolution of means and variances for each source of heterogeneity. Similarly to the means of observables discussed previously in Figure 1, in the upper four

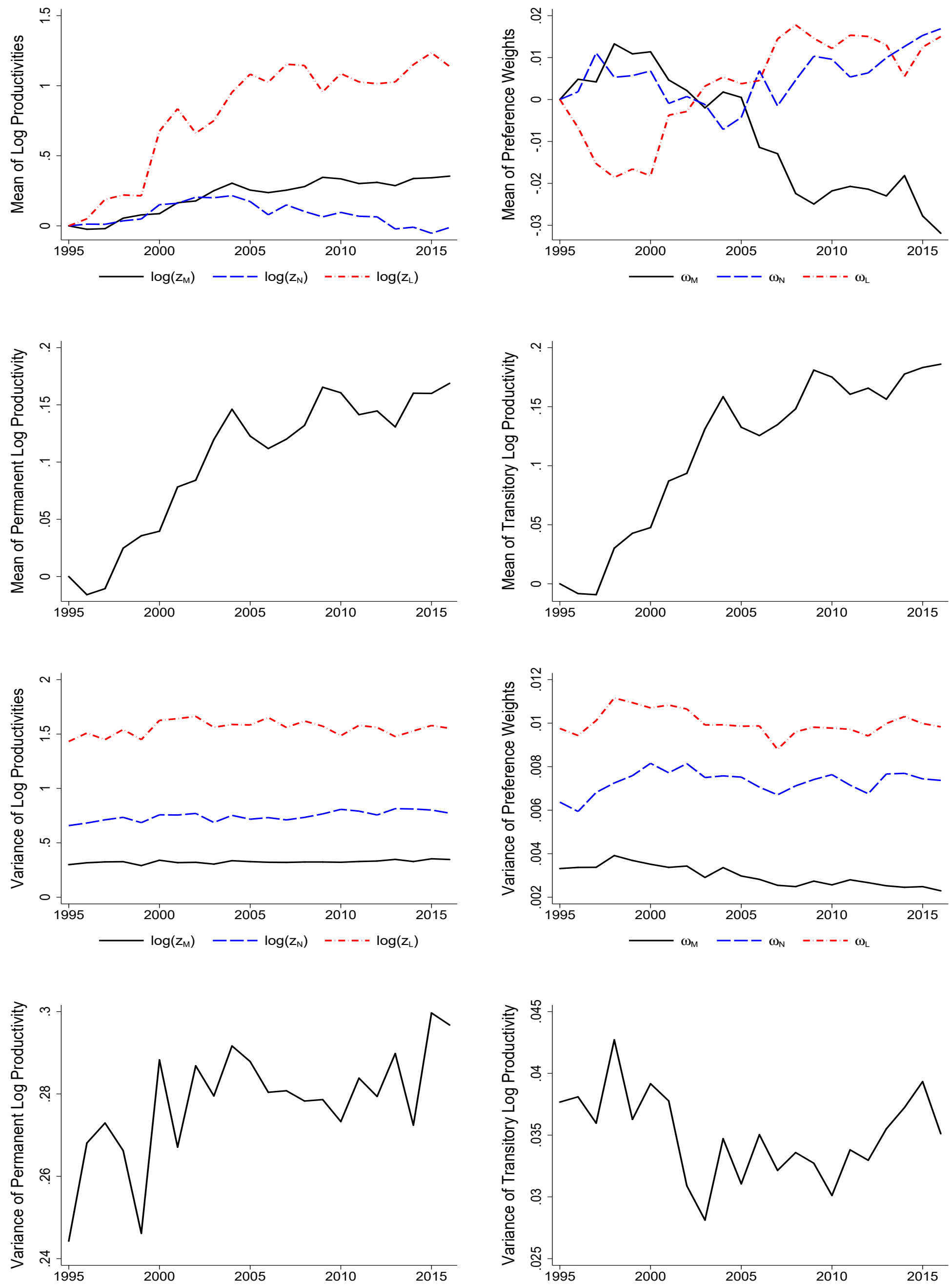


Figure 2: Means and Variances of Sources of Heterogeneity

Figure 2 plots the evolution of means and variances of productivities z_M, z_N, and z_L, preference weights ω_M, ω_N, and ω_L, the uninsurable permanent component of market productivity α, and the insurable transitory component of market productivity ε.

panels the plotted means are the coefficients on the time dummies from a regression of each source of heterogeneity on age and time fixed effects and the variances refer to the variances of the residuals in a given year from these regressions.

Beginning in the first panel, mean leisure productivity $\log z_L$ increases substantially over time and by the end of the sample reaches a level roughly 110 log points higher than its 1995 level. To understand this pattern, equation (14) shows that leisure productivity z_L increases in leisure expenditures relative to time x_L/h_L and decreases in the relative input price $\tilde{z}_M/p_L$ when expenditures and time are complements ($\sigma_L < 1$). Quantitatively, the substantial increase in x_L/h_L over time dominates the increase in $\tilde{z}_M/p_L$ and accounts for the increase in leisure productivity over time.[16] Mean non-market productivity $\log z_N$ tracks mean market productivity $\log z_M$ until the mid 2000s, reflecting the growth of the relative input price $\tilde{z}_M/p_N$ and the fact that expenditures and time are substitutes in the non-market technology ($\sigma_N > 1$). After the mid 2000s, mean non-market productivity starts to decline, reflecting the increase in non-market expenditures relative to time x_N/h_N and the flattening of $\tilde{z}_M/p_N$.

The second panel documents a decline in the mean preference for market goods ω_M relative to non-market and leisure goods. Given our choice of an elasticity of substitution $\phi = 1$ across goods, preference weights equal the cost share of each good in the market value of total consumption, $\omega_{j'} = \frac{p_{j'}x_{j'}+\tilde{z}_M h_{j'}}{\sum_j (p_j x_j + \tilde{z}_M h_j)}$ for each good $j, j' = \{M, N, L\}$. The decline in mean ω_M, therefore, reflects the decline in market expenditures x_M relative to the market value of total consumption $\sum_j (p_j x_j + \tilde{z}_M h_j)$. Finally, the third and fourth panels show an increase in the mean value of the uninsurable permanent component of log productivity α and the mean value of the insurable transitory component of productivity ε over time. These increases reflect the growth of the market value of total consumption and market productivity over time.[17]

[16] The inferred increase in mean leisure productivity becomes larger as σ_L increases toward one. For $\sigma_L = 0$, z_L equals x_L/h_L and grows by roughly 90 log points.

[17] In equation (17), $(1 - \tau_1)\alpha$ equals the difference between the market value of total consumption and a moment of the transitory component of productivity $\exp(\varepsilon)$ and, in equation (18), ε equals the difference between market productivity $\log z_M$ and α. As a result, the plotted means depend on an arbitrary choice of means in some initial period. We choose to attribute half of the level of $\log z_M$ to α and half to ε and, so, both rise by roughly the same amount over time. Our inferences of the other sources of heterogeneity, welfare effects, and our counterfactuals are not sensitive to this normalization.

Table 2: Unconditional Correlations Between Sources of Heterogeneity

	$\log z_M$	$\log z_N$	$\log z_L$	ω_M	ω_N	ω_L	α	ε
$\log z_M$	1.00	.	.	.	.	.	.	.
$\log z_N$	0.80	1.00	.	.	.	.	.	.
$\log z_L$	0.08	-0.17	1.00	.	.	.	.	.
ω_M	-0.33	-0.59	0.60	1.00	.	.	.	.
ω_N	-0.06	0.26	0.27	-0.03	1.00	.	.	.
ω_L	0.23	0.10	-0.56	-0.52	-0.84	1.00	.	.
α	0.95	0.71	0.23	-0.21	0.02	0.10	1.00	.
ε	0.38	0.42	-0.40	-0.43	-0.25	0.44	0.06	1.00

Moving to the bottom panels, we first observe that the (within-age) cross-sectional variances of non-market and leisure productivity are significantly larger than the variance of market productivity.[18] To understand this result it is useful to once more refer to equation (14) that relates $\log z_K$ to $\log \tilde{z}_M$, $\log x_K$, and $\log h_K$. As discussed in Table 1, time inputs are relatively uncorrelated with market productivity and expenditures in the cross-section of households and, as a result, the variance of $\log z_K$ cumulates the variances of these three observables and exceeds the variance of market productivity. The variance of market productivity rises somewhat over time. The variances of non-market and leisure productivity rise even more over time which, in addition to the increase in the variance of market productivity, reflects the increases in the variances of non-market production time $\log h_N$ and leisure expenditures $\log x_L$.

As Figure 2 shows, the cross-sectional variances of preference weights are relatively stable over time. The cross-sectional variance of the permanent component of market productivity α is large relative to the variance of the transitory component of market productivity ε. This reflects the fact that the cross-sectional variance of the market value of consumption is roughly equal to the variance of market productivity. The variance of α rises over time which reflects the increase in the cross-sectional variance of the market value of total consumption. By contrast, the variance of ε is stable over time.

[18] A similar finding is documented by Boerma and Karabarbounis (2019b) in the context of a model with a non-market technology only.

In Table 2 we present the cross-sectional correlations between sources of heterogeneity. Similar to the correlations of observables in Table 1, these correlations are obtained after absorbing time and age fixed effects in regressions of each source of heterogeneity on age and time fixed effects. We obtain a high and positive correlation between market productivity z_M and non-market productivity z_N which, quantitatively, reflects the fact that expenditures and time are substitutes in the non-market technology ($\sigma_N > 1$). Market productivity is relatively uncorrelated with leisure productivity z_L, reflecting roughly offsetting effects from a strong correlation between z_M and x_L in the cross-section of households and the complementarity between expenditures and time in leisure technology ($\sigma_L < 1$). The correlation between preference weights ω_K and productivities z_K are negative for the market and the leisure sector and positive for the non-market sector. Finally, the correlation between the two components of market productivity, α and ε, is essentially zero.

We conclude this section by presenting the evolution of welfare over time. Our measure of welfare is the consumption equivalent χ_t that leaves utilitarian welfare unchanged between the two allocations:

$$\sum \pi_t(\iota) \log((1-\chi_t)c_t(\iota)) = \sum \pi_0(\iota) \log(c_0(\iota)), \tag{19}$$

where $\pi_t(\iota)$ denote survey weights, the flow utility $\log(c_t(\iota))$ is given by equation (2), and the right-hand side of the equation denotes the baseline allocation in some period 0. A positive value for χ_t denotes an increase in welfare in period t relative to period 0.[19]

Following Benabou (2002) and Floden (2001) who have emphasized that total welfare effects arise from level effects when aggregate allocations change and effects capturing changes in the dispersion of allocations across households, we break χ_t into a level component χ_t^L and a dispersion component χ_t^D. We define the level component as:

$$\chi_t^L = 1 - \frac{\sum \pi_0(\iota)c_0(\iota)}{\sum \pi_t(\iota)c_t(\iota)}. \tag{20}$$

[19] We have confirmed that all our conclusions are similar if we consider a consumption equivalent that leaves an unborn household indifferent over its life-cycle between two allocations. The difference between the two welfare measures is that the life-cycle measure discounts future utilities more than the utilitarian measure.

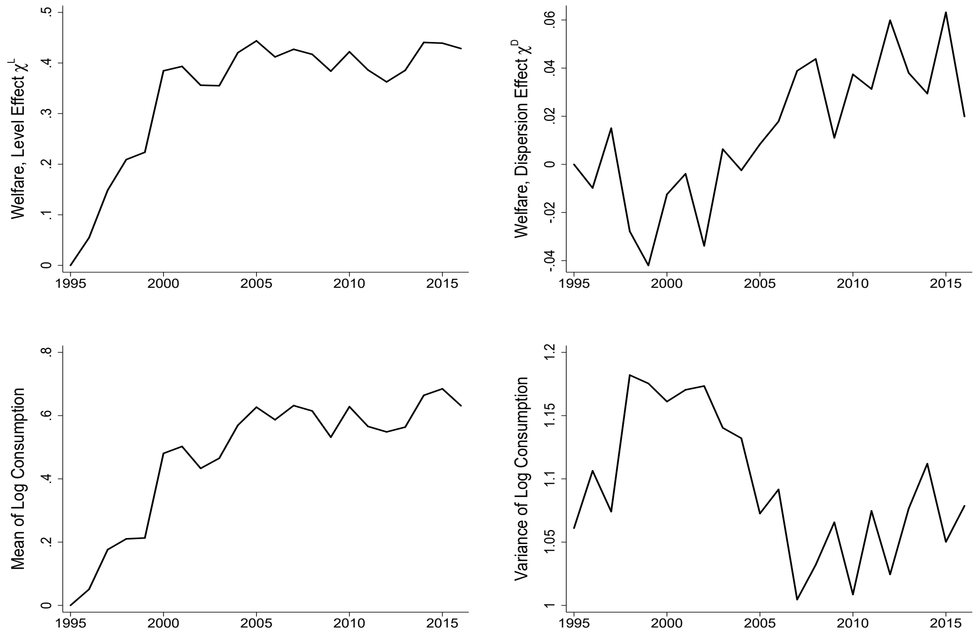


Figure 3: Welfare

Figure 3 plots the evolution of the level component of the consumption equivalent χ^L, the dispersion component of the consumption equivalent χ^D, the mean of $\log c$, and variance of $\log c$.

When the level component is positive, mean consumption is higher in the current allocation in period t than in the baseline allocation in period 0. Given this definition of χ_t^L we obtain the decomposition $\log(1-\chi_t) = \log(1-\chi_t^L) + \log(1-\chi_t^D)$, where the dispersion component is given by:

$$\log(1-\chi_t^D) = \sum \pi_0(\iota) \log\left(\frac{c_0(\iota)}{\sum \pi_0(\iota) c_0(\iota)}\right) - \sum \pi_t(\iota) \log\left(\frac{c_t(\iota)}{\sum \pi_t(\iota) c_t(\iota)}\right). \tag{21}$$

When the dispersion component is negative, the consumption dispersion around its mean in the current allocation in period t is higher than the consumption dispersion around its mean in the baseline allocation in period 0. As a result, dispersion contributes negatively to welfare.

The first panel of Figure 3 shows that the level component of welfare (relative to 1995) grows by roughly 40 log points until the mid 2000s and then stabilizes. In the second panel, we observe a roughly 4 log points decline in welfare due to the dispersion component χ^D until 2000. After 2000, χ^D starts to rise and by the end of the sample it roughly goes back to its 1995 level. The lower panels of the figure demonstrate that changes in welfare due to level and dispersion effects are closely related to the mean of $\log c_t$ and the variance of $\log c_t$ over time.[20]

3.5 Counterfactuals

In this section we present counterfactual exercises in which we shut off the evolution of driving forces in order to assess their contribution to the evolution of observables and welfare. We begin in Table 3 by assessing the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. The first row documents the change of means between the end of the sample (2012-2016) and the beginning of the sample (1995-1999) in the baseline model which replicates the data perfectly. For example, the change in mean total expenditures $px = x_M + p_N x_N + p_L x_L$ over that period is 24.6 log points. Each other row represents a different experiment in which we shut off either the evolution of the mean or the evolution of the variance of driving forces.[21] As an example, the second row shows that keeping the price of non-market goods p_N constant at their lower initial level would generate an increase of 31.1 log points in mean total expenditures. Because mean total expenditures increased by 24.6 log points in the baseline model which replicates the data perfectly, we conclude that the increase in p_N over time causes a 6.5 log points decline in total expenditures.

[20] We drop from the sample an additional 0.1 percent of observations (34 observations) with extreme levels of consumption c_t to improve the visibility of this figure. The mean of $\log c_t$ deviates slightly from the level effect χ_t^L since the mean of $\log c_t$ reflects within-age variation over time, whereas the level effect does not correct for differences in the age structure over time. We also observe a close (negative) association between the dispersion effect χ_t^D and the variance of log consumption. If log consumption follows a normal distribution, then χ_t^D and the variance of log consumption are related by $\log(1 - \chi_t^D) = -\mathrm{Var}(\log c_t)/2$. While log consumption is not exactly normally distributed in our economy, this equation still provides a useful approximation in thinking about the dispersion effect.

[21] Let $x_t(\iota)$ be the log of a source of heterogeneity in the baseline and $x_t^c(\iota)$ be the counterfactual which keeps either the mean or the variance of the source of heterogeneity constant at its 1995 value. When we shut off the evolution of the mean of a source of heterogeneity, we set $x_t^c(\iota) = x_t(\iota) - \mathbb{E}x_t(\iota) + \mathbb{E}x_{95}(\iota)$, so that in all periods we retain the same dispersion across households as in the baseline. When we shut off the evolution of the variance of a source of heterogeneity, we set $x_t^c(\iota) = \lambda_t^0 + \lambda_t^1 x_t(\iota)$ and solve for λ_t^0 and λ_t^1 such that in all periods the variance equals its 1995 value and in all periods we retain the same mean across households as in the baseline. In conducting counterfactuals with a particular preference weight, we adjust the other weights such that the weights sum up to one. For example, when we shut off the decline in the mean value of ω_M, we allocate proportionally to ω_N and ω_L the difference relative to the baseline path of ω_M.

Table 3: Means: Counterfactuals

	$100 \times$ Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.1	16.5	78.2	-1.8	-7.4	2.1
Mean p_N	31.1	5.1	50.7	78.2	6.1	-21.3	2.1
Mean p_L	27.0	5.1	16.5	62.9	1.2	-7.4	0.4
Mean $\alpha + \varepsilon$	-9.7	-22.4	-35.5	62.8	-9.3	9.3	0.5
Variance α	26.4	6.5	18.9	79.0	-1.3	-8.6	2.2
Variance ε	24.6	5.0	16.4	78.2	-1.7	-7.2	2.2
Mean $\log z_N$	23.2	5.1	12.9	78.2	-3.8	-4.4	2.1
Variance $\log z_N$	23.3	5.1	13.5	78.2	-3.5	-2.7	2.1
Mean $\log z_L$	18.2	5.1	16.5	35.1	-9.1	-7.4	6.3
Variance $\log z_L$	24.8	5.1	16.5	79.0	-1.6	-7.4	2.1
Mean ω_M	33.7	30.8	12.7	74.5	8.1	-11.1	-1.6
Variance ω_M	23.2	1.1	16.4	78.1	-2.4	-7.4	2.0
Mean ω_N	24.4	5.4	15.7	78.5	-2.1	-8.1	2.4
Variance ω_N	24.8	5.2	16.8	78.4	-1.6	-7.0	2.3
Mean ω_L	29.4	12.0	23.4	72.5	4.0	-0.5	-3.6
Variance ω_L	24.5	5.0	16.4	78.2	-1.8	-7.4	2.1

The most important driver of the rise of total expenditures px is the increase in mean log market productivity (row "Mean $\alpha + \varepsilon$"). The rise of market productivity is quantitatively important for the evolution of each expenditure input, x_M, x_N, and x_L. Among other driving forces, we note the role of the growth in the mean leisure productivity $\log z_L$, which accounts for a significant fraction of the increase in px and x_L over time.[22] The increase in the relative price of non-market goods p_N significantly depresses the quantity of non-market expenditures x_N and the decrease in the relative price of leisure goods p_L contributes modestly to the increase in the quantity of leisure expenditures x_L over time. The decline in the preference weight for market goods ω_M offsets the increase in market productivity and moderates the rise of market expenditures x_M over time.

[22] As can be seen from equations (9) and (10), with unitary elasticity $\phi = 1$ across goods cross-price effects are zero and z_K and p_K do not affect x_{-K} and h_{-K}.

Market hours $\log h_M$ fall moderately between the beginning and the end of the sample. Movements in the relative prices of goods, p_N and p_L, generate a significant decline in market hours over time. To understand this result, we refer to equation (10) which shows that an increase in p_N leads to an increase in h_N since expenditures and time are substitutes in non-market production and a decline in p_L leads to an increase in h_L since expenditures and time are complements in leisure production. The increase in the relative price of non-market goods generates 8 log points decline in market hours, whereas the decline in the relative price of leisure goods generates 3 log points decline in the market hours.[23] The other significant contributor to the decline in mean hours is the decline in the preference for market goods ω_M relative to non-market and leisure goods. As Table 3 shows, the decline in market hours generated by changes in relative prices and preference weights is offset by the rise of mean market productivity, $\alpha + \varepsilon$, and leisure productivity, $\log z_L$.

Next, Figure 4 assesses the welfare effects of shutting off the evolution of driving forces. In the left panels, we plot the time path of the level component χ^L in the baseline model and in various counterfactual exercises. In the right panels, we plot the time paths of the dispersion component χ^D. The main takeaway from Figure 4 is that the growth in mean leisure productivity $\log z_L$ is the most important driver for welfare and this influence is apparent in both the level and the dispersion components of welfare. The rise of mean leisure productivity generates more than 30 log points welfare gain in terms of mean consumption. To set a benchmark for comparisons, the rise of mean market productivity $\log z_M$ generates less than 10 log points gain. Further, mean leisure productivity moderates the rise of inequality over time. The increase of mean leisure productivity generates additional 10 log points of welfare gain in terms of lower consumption dispersion and offsets the negative welfare effects that arise from increases in the variances of market and leisure productivity over time.

To understand the importance of the rise of mean leisure productivity for welfare, we use the close relationship between χ^L and χ^D and the mean and variance of log consumption in Figure 3. Using our analytical solutions under the parametric restriction $\phi = 1$, we express

[23] Vandenbroucke (2009) emphasizes the small effects of the decline in leisure prices between 1900 and 1950 on market hours.

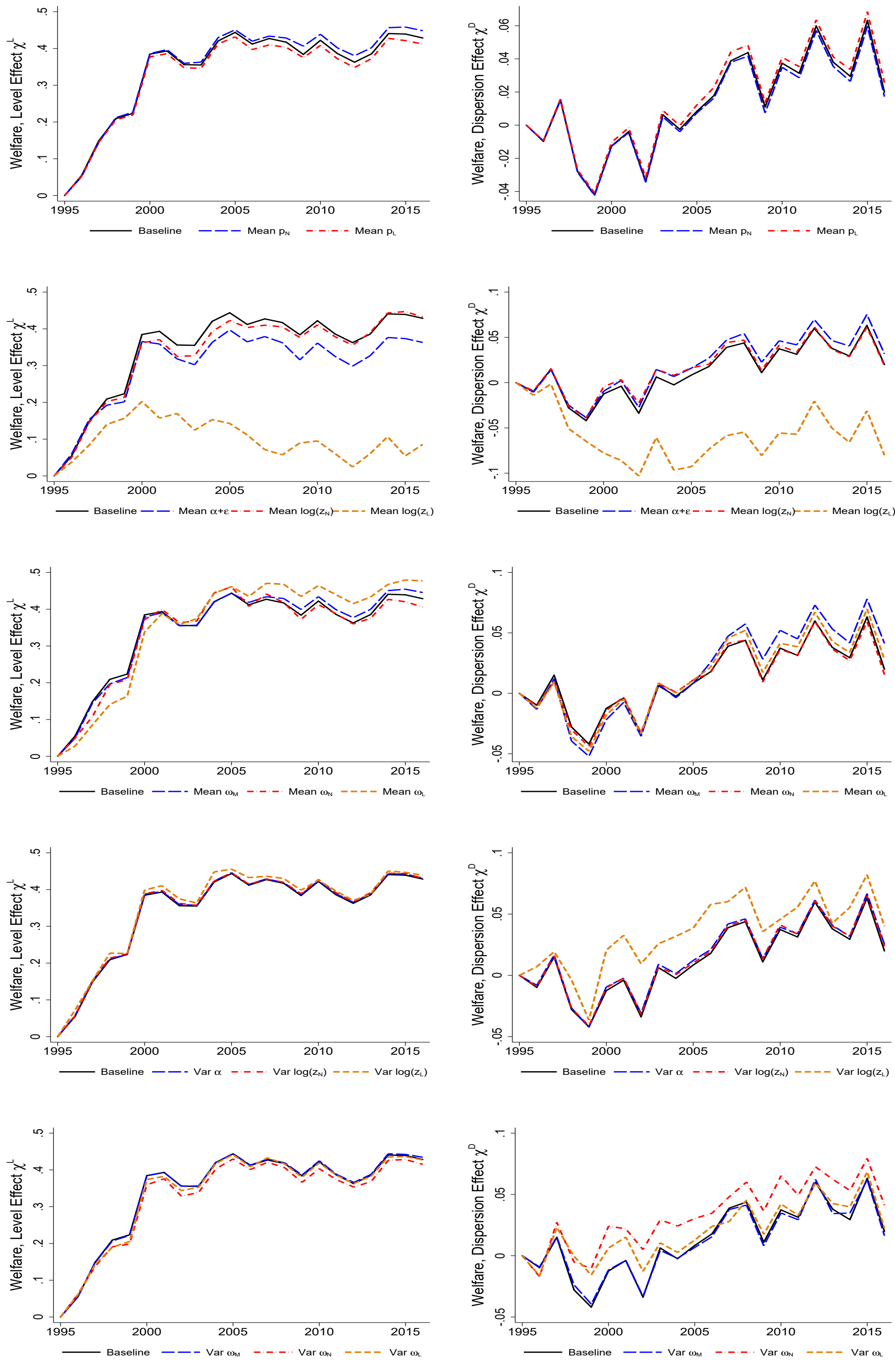


Figure 4: Welfare: Counterfactuals

Figure 4 plots the evolution of the level component of the consumption equivalent χ^L (left panels) and the dispersion component of the consumption equivalent χ^D (right panels). In each panel we present the evolution in the baseline path (solid line) together with the evolution in counterfactuals (dashed lines) in which we shut off particular aspects of the evolution of the heterogeneity across households.

log consumption for every household ι as a function of the primitive sources of heterogeneity:

$$\begin{aligned} \log c &= (1-\tau_1)\alpha + \log \mathbb{C} + \omega_M \log(\omega_M) \\ &+ \sum \omega_K \left[\log\left(\frac{\omega_K}{p_K}\right) + \frac{1}{\sigma_K - 1} \log\left(1 + \left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K - 1}\right)\right]. \end{aligned} \tag{22}$$

In the first two terms, a higher permanent component of market productivity α or aggregate transitory productivity encoded in the function $\mathbb{C} = \int_\zeta (1-\tau_0)\exp\left((1-\tau_1)(\varepsilon)\right) \mathrm{d}\Phi(\zeta)$ raise the consumption of all three goods, c_M, c_N, and c_L, and lead to higher $\log c$. As expected, higher price of expenditures p_K lowers $\log c$ and higher productivity of time z_K increase $\log c$. Given this result and the significant growth of mean z_L over time, it is not surprising that the level component of welfare χ^L increases significantly in response to the increase in mean leisure productivity.

To understand the result that higher mean leisure productivity lowers the welfare cost of dispersion, we parameterize leisure productivity as $z_L = \gamma_L \hat{z}_L$ where γ_L is the common component of z_L across households in each period and $\hat{z}_L$ is the idiosyncratic component. Next, we express the variance of log consumption in equation (22) as the sum of the variance of the term that involves γ_L, and other variances and covariances:

$$\mathrm{Var}(\log c) = \left(\frac{1}{1-\sigma_L}\right)^2 \mathrm{Var}\left(\log\left(1 + \gamma_L^{\sigma_L - 1}\left(\frac{\hat{z}_L p_L}{\tilde{z}_M}\right)^{\sigma_L - 1}\right)\right) + \mathrm{Var}(.) + ... + \mathrm{Cov}(.). \tag{23}$$

In Appendix 3.C we prove that, holding constant the other variances and covariances in equation (23), the variance of log consumption is decreasing in mean leisure productivity γ_L if and only if $\sigma_L < 1$.[24] The key term $1 + \gamma_L^{\sigma_L - 1}\left(\frac{\hat{z}_L p_L}{\tilde{z}_M}\right)^{\sigma_L - 1}$ that appears in equation (23) is related to the consumption of the leisure good c_L after factoring out the contribution of leisure expenditures x_L which is already accounted for through terms that involve $(1 - \tau_1)\alpha + \log \mathbb{C}$, ω_L, and p_L in equation (22). This key term equals the constant 1 and a term that denotes the contribution of the time input h_L to consumption c_L. When mean leisure

[24] To gain some insight of why this is true, consider a function $f(x;\gamma_L) = \log(1 + \gamma_L^{\sigma_L - 1}x)$. To a first-order approximation, we obtain $\mathrm{Var}(f(x;\gamma_L)) = \left(\frac{\gamma_L^{\sigma_L - 1}\bar{x}}{1+\gamma_L^{\sigma_L - 1}\bar{x}}\right)\mathrm{Var}(x/\bar{x})$, where $\bar{x}$ is an approximation point. This formula shows that the variance is decreasing in γ_L if and only if $\sigma_L < 1$. Our proof in the appendix does not rely on approximations.

productivity γ_L increases, the relative contribution of the time input to the sum becomes smaller given that $\sigma_L < 1$, the key term approaches the constant, and the variance of log consumption across households declines.

Before concluding this section, it is worth contrasting our welfare costs of dispersion to alternative approaches we discussed in the introduction that describe the dispersion of observables and its evolution over time. Figure 5 evaluates the effects of shutting off the evolution of driving forces on the variance of total expenditures $\log(px)$ (in the left panels) and the variance of market hours $\log h_M$ (in the right panels). An important difference between the welfare-based measures of dispersion shown in the right panels of Figure 4 and the variances of observables shown in Figure 5 is that the latter fail to capture the welfare effects of an increase in mean leisure productivity in terms of lowering consumption dispersion. In Figure 5, the increase in variance of the permanent component of market productivity α generates most of the increase in the variance of total expenditures and the decline in the mean preference for market goods ω_M generates most of the increase in the variance of market hours. However, as Figure 4 shows these factors are less important quantitatively than mean leisure productivity for the evolution of the welfare costs of dispersion. The welfare costs of dispersion are associated more closely with the dispersion of the consumption aggregator and less with the dispersion of expenditures or market hours.

3.6 Sensitivity Analyses

In this section we discuss sensitivity analyses. Here we summarize the most important results and present the detailed tables and figures underlying our analyses in Appendix 3.D. For each sensitivity analysis, we repeat the identification of the sources of heterogeneity as in Section 3.4 and then perform the same counterfactuals as in Section 3.5.

We begin by varying the elasticities of substitution between expenditures and time in each sector. Increasing σ_N from 2.5 in the baseline to 3.5 magnifies the negative impact of the price of non-market goods p_N on market hours h_M from 8 log points to 13 log points. Lowering σ_N to 1.5 mitigates the negative impact of p_N to 3 log points. Similarly, lowering σ_L from 0.5 in the baseline to 0.2 magnifies the negative impact of the price of leisure goods

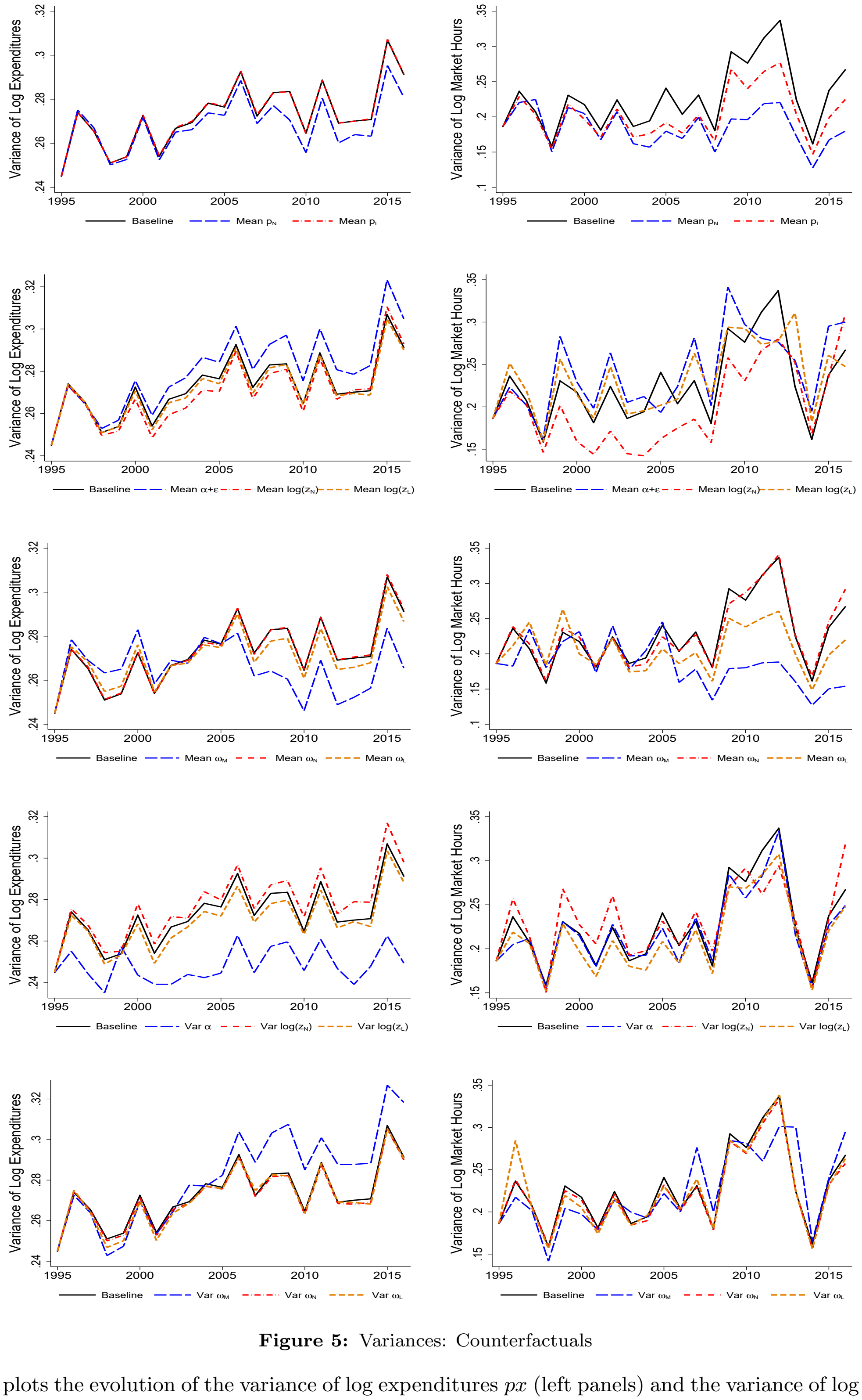


Figure 5: Variances: Counterfactuals

Figure 5 plots the evolution of the variance of log expenditures px (left panels) and the variance of log market hours h_M (right panels). In each panel we present the evolution in the baseline path (solid line) together with the evolution in counterfactuals (dashed lines) in which we shut off particular aspects of the evolution of the heterogeneity across households.

p_L on market hours to 5 log points and increasing σ_L to 0.8 mitigates the negative impact of p_L to 1 log point.

The impact of the increase in mean leisure productivity on welfare remains relatively robust across all these parameterizations. In the baseline parameterization, the increase in mean leisure productivity contributes to welfare 35 log points through an increase in mean consumption and 9 log points through a decline in consumption dispersion. Under $\sigma_N = 3.5$, we obtain contributions of 35 and 9 log points and under $\sigma_N = 1.5$ we obtain contributions of 36 and 10 log points. Under $\sigma_L = 0.2$, we obtain contributions of 31 and 5 log points and under $\sigma_L = 0.8$ we obtain contributions of 51 and 27 log points. In all cases mean leisure productivity is the most important factor driving welfare trends over time.[25]

Our quantitative results on the role of mean leisure productivity in increasing mean consumption and decreasing consumption dispersion are not sensitive to perturbations of the progressivity parameter τ_1 to 0.06 and to 0.18. By contrast, the results are sensitive to the value of the elasticity of substitution across goods ϕ. We have experimented with many values of ϕ and concluded that ϕ changes in a non-monotonic way the contributions of mean leisure productivity. In all cases, however, the contributions are positive both in terms of the level and the dispersion components of welfare.[26]

Next, we perform sensitivity analyses with respect to the measurement of key variables underlying our analysis. To address potential measurement error in expenditures in the CEX, for each of the 20 spending categories underlying the construction of our three baskets we use the estimated expenditure elasticity in Aguiar and Bils (2015) together with households' spending share in the cross section and construct an alternative measure of household spending.[27] Our results are almost unchanged using this alternative measure of

[25]It may appear surprising that mean leisure productivity becomes more important as σ_L approaches closer to one. Equation (14) shows that, for a given increase in leisure expenditures relative to time, the inferred increase in mean leisure productivity is larger as σ_L approaches one.

[26]For values of $\phi = \{0.2, 0.5, 0.7, 1.0, 1.3, 2.0, 3.0\}$ the contribution of mean leisure productivity to welfare through an increase in mean consumption is $\{57, 30, 21, 35, 20, 35, 67\}$ log points. The contribution to welfare through a decline in consumption dispersion is $\{1, 1, 2, 9, 3, 6, 14\}$ log points.

[27]Let β_K be the estimated elasticity in Aguiar and Bils (2015), $\bar{px}$ be mean total expenditures in the cross section, and $px(\iota)$ be total expenditures of household ι. We allocate total spending in each category to households in proportion to their spending share $p_K x_K(\iota) = \frac{(\frac{px(\iota)}{\bar{px}})^{\beta_K}}{\sum_\iota (\frac{px(\iota)}{\bar{px}})^{\beta_K}} p_K X_K$, where $p_K X_K$ is a cross-sectional measure of expenditures in category K.

expenditures. For example, the increase in the mean leisure productivity contributes to welfare 33 log points through an increase in mean consumption and 9 log points through a decline in consumption dispersion.

In our baseline analyses, we define non-market hours directly from the survey data and leisure residually as total disposable time minus market hours and time spent on non-market production, $h_L = 105 - h_M - h_N$. We examine the sensitivity of this choice by repeating our analyses when defining leisure hours directly from the survey data and non-market hours residually as $h_N = 105 - h_M - h_L$. We find that our welfare results and counterfactuals are robust to the measurements of non-market and leisure time. For example, using this alternative definition of leisure, the increase in the mean leisure productivity contributes to welfare 27 log points through an increase in mean consumption and 8 log points through a decline in consumption dispersion.

3.7 Conclusion

The purpose of this paper is to account for recent trends in labor market outcomes and understand their welfare consequences. To do so, we develop a model with incomplete asset markets and household heterogeneity in market and home technologies and preferences. Using micro data on expenditures and time use, we identify the sources of heterogeneity across households, document how these sources have changed over time, and perform counterfactual analyses.

Our most important finding is to document the substantial increase of leisure productivity over time. This follows from the observation that, for the average household, leisure expenditures relative to leisure time increases dramatically more than predicted from the decline in the relative price of leisure goods. We demonstrate that the increasing productivity of leisure time is associated with significant welfare gains. The increase in mean productivity of leisure time generates significantly larger gains in terms of mean consumption than the increase in mean wages. Additionally, the increase in mean leisure productivity induces significant welfare gains by lowering the dispersion of consumption across households.

Finally, we wish to highlight the importance of taking into account the allocation of time and expenditures across sectors in evaluating welfare effects of trends in labor market outcomes. We demonstrate that the distinction between expenditures and consumption matters for the conclusions one draws from trends in labor market outcomes. While the increase in the variance of the permanent component of wages is the most important factor accounting for the increase in the dispersion of expenditures over time, this factor contributes significantly less than leisure productivity to the welfare costs of dispersion. This is because these welfare costs are linked more closely to consumption than to expenditures.

Labor Market Trends and the Changing Value of Time

Online Appendix

Job Boerma and Loukas Karabarbounis

Appendix 3.A Equilibrium Allocations

In this appendix, we derive the equilibrium allocations presented in Section 3.2. We proceed in three steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 3.2.

3.A.1 Preliminaries

In what follows, we define the following state vectors. The idiosyncratic shifters that differentiate households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can trade bonds within each island contingent on the vector s^j:

$$s_t^j = (\alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{z_K^j, \omega_K^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, we denote the history of all sources of heterogeneity up to period t with the vector:

$$\theta_t^j = (z_{K,t}^j, \omega_{K,t}^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, ..., z_{K,j}^j, \omega_{K,j}^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(.|.)$. For example, the probability that we observe θ_t^j conditional on θ_{t-1}^j is $f^{t,j}(\theta_t^j|\theta_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to the processes and Φ_υ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi^j_{z_{K,t}}, \Phi^j_{\omega_{K,t}}\}$, and the initial distributions to vary over cohorts j, $\{\Phi^j_{\alpha,j}, \Phi^j_{\kappa,j}\}$. We assume that $z^j_{K,t}$ and $\omega^j_{K,t}$ are orthogonal to the innovations $\{\upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that these innovations are drawn independently from each other.

3.A.2 Planner Problem

In every period t and in every island ℓ, the planner solves a static problem that consists of finding the allocations that maximize average utility for households on the island subject to a resource constraint and household-specific home production technologies. We omit t and ℓ from the notation for convenience. The planner chooses $\{x_M(\iota), h_M(\iota), x_K(\iota), h_K(\iota)\}$ to maximize:

$$\int_Z \log\left(\omega_M(\iota)\, x_M(\iota)^{\frac{\phi-1}{\phi}} + \sum \omega_K(\iota)\left(x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}} + \left(z_K(\iota)\, h_K(\iota)\right)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} \mathrm{d}\Phi_\zeta(\zeta)\,,$$

subject to an island resource constraint for market goods:

$$\int_Z \left(x_M(\iota) + \sum p_K x_K(\iota)\right)\mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M(\iota)\, h_M(\iota)\, \mathrm{d}\Phi_\zeta(\zeta), \tag{A.5}$$

and individual-specific time constraints:

$$1 = \sum h_K(\iota) + h_M(\iota)\,. \tag{A.6}$$

Denoting by $\mu(z_K, \omega_K, \alpha)$ the multiplier on the island resource constraint and by $\chi(\iota)$ the multipliers on the household's time constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[x_M(\iota)]:\ \mu(z_K,\omega_K,\alpha)=\frac{1}{\mathcal{C}(\iota)}\omega_M(\iota)\,x_M(\iota)^{-\frac{1}{\phi}} \tag{A.7}$$

$$[x_K(\iota)]:\ \mu(z_K,\omega_K,\alpha)=\frac{1}{\mathcal{C}(\iota)}\frac{\omega_K(\iota)}{p_K}\left(x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}}+\left(z_K(\iota)\,h_K(\iota)\right)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K(\phi-1)}{(\sigma_K-1)\phi}-1}x_K(\iota)^{-\frac{1}{\sigma_K}} \tag{A.8}$$

$$[h_M(\iota)]:\ \chi(\iota)=\tilde{z}_M(\iota)\,\mu(z_K,\omega_K,\alpha) \tag{A.9}$$

$$[h_K(\iota)]:\ \chi(\iota)=\frac{z_K(\iota)}{\mathcal{C}(\iota)}\omega_K(\iota)\left(x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}}+\left(z_K(\iota)\,h_K(\iota)\right)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}-1}\left(z_K(\iota)\,h_K(\iota)\right)^{-\frac{1}{\sigma_K}} \tag{A.10}$$

where

$$\mathcal{C}(\iota)=\omega_M(\iota)\,x_M(\iota)^{\frac{\phi-1}{\phi}}+\sum\omega_K(\iota)\left(x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}}+\left(z_K(\iota)\,h_K(\iota)\right)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}}.$$

Combining (A.7) to (A.10), we obtain effective labor input relative to expenditures for each home sector:

$$\frac{z_K(\iota)\,h_K(\iota)}{x_K(\iota)}=\left(\frac{z_K(\iota)\,p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K}. \tag{A.11}$$

Using this equation, we note that the home production aggregator simplifies to:

$$x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}}+\left(z_K(\iota)\,h_K(\iota)\right)^{\frac{\sigma_K-1}{\sigma_K}}=x_K(\iota)^{\frac{\sigma_K-1}{\sigma_K}}\left(1+\left(\frac{z_K(\iota)\,p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right).$$

Using this expression, we relate home production expenditures to market expenditures by (*A.7*) and (*A.8*):

$$x_K(\iota)=\left(\frac{\tilde{\omega}_K(\iota)}{\omega_M(\iota)\,p_K}\right)^{\phi}x_M(\iota), \tag{A.12}$$

where the transformed preference shifter on good k is

$$\tilde{\omega}_K(\iota)\equiv\omega_K(\iota)\left(1+\left(\frac{z_K(\iota)\,p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}-1}.$$

Substituting into equation (A.7), we derive:

$$x_M(\iota) = \frac{1}{\mu(z_K,\omega_K,\alpha)} \frac{1}{1+\sum\left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} p_K^{1-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}} \,. \tag{A.13}$$

This expression, combined with the relation between home production and market expenditures (A.12), the relation between effective labor inputs and home production expenditures (A.11), and the household time constraint (A.6), yield solutions for $\{x_M(\iota), h_M(\iota), x_K(\iota), h_K(\iota)\}$ given a multiplier $\mu(z_K,\omega_K,\alpha)$.

The multiplier is equal to the inverse of the market value of consumption:

$$x_M(\iota) + \sum p_K x_K(\iota) + \tilde{z}_M(\iota)\sum h_K(\iota) = \frac{1}{\mu(z_K,\omega_K,\alpha)} \,, \tag{A.14}$$

which is derived by substituting the solutions given a multiplier $\mu(z_K,\omega_K,\alpha)$ into the expression on the left-hand side.

Substituting the individuals' time constraint into the island resource constraint to eliminate market hours, we write:

$$\int_Z \Big(x_M(\iota) + \sum p_K x_K(\iota) + \tilde{z}_M(\iota)\sum h_K(\iota)\Big)\mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M(\iota)\,\mathrm{d}\Phi_\zeta(\zeta) \,,$$

which by substitution of the expression for the multiplier $\mu(z_K,\omega_K,\alpha)$ in (A.14) yields a closed-form characterization of this multiplier:

$$\mu(z_K,\omega_K,\alpha) = \left(\int_Z \tilde{z}_M(\iota)\,\mathrm{d}\Phi_\zeta(\zeta)\right)^{-1} \tag{A.15}$$

The denominator is an expected value independent of ζ. Thus μ is independent of ζ. Note that $\mu(z_K,\omega_K,\alpha) = \mu(\alpha)$. The marginal utility from market spending is independent of non-market productivity and preference shifters. Given this solution for $\mu(z_K,\omega_K,\alpha)$, we obtain the solutions

$$x_M(\iota) = \left(\int_Z \tilde{z}_M(\iota)\,\mathrm{d}\Phi_\zeta(\zeta)\right) \frac{1}{1+\sum \left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} {p_K}^{1-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}} \tag{A.16}$$

$$x_K(\iota) = \left(\int_Z \tilde{z}_M(\iota)\,\mathrm{d}\Phi_\zeta(\zeta)\right) \frac{\left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} {p_K}^{1-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-\sigma_K}{\sigma_K-1}}}{1+\sum \left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} {p_K}^{1-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}} \frac{1}{p_K} \tag{A.17}$$

$$h_K(\iota) = \frac{\left(\int_Z \tilde{z}_M(\iota)\,\mathrm{d}\Phi_\zeta(\zeta)\right)}{z_K(\iota)} \frac{\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K}\left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} {p_K}^{-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-\sigma_K}{\sigma_K-1}}}{1+\sum \left(\frac{\omega_K(\iota)}{\omega_M(\iota)}\right)^{\phi} {p_K}^{1-\phi}\left(1+\left(\frac{z_K(\iota)p_K}{\tilde{z}_M(\iota)}\right)^{\sigma_K-1}\right)^{\frac{\phi-1}{\sigma_K-1}}} \frac{1}{p_K} \tag{A.18}$$

$$h_M(\iota) = 1 - \sum h_K(\iota) \tag{A.19}$$

3.A.3 Postulating Equilibrium Allocations and Prices

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade between islands, $a(\zeta_t^j;\iota)=0$.
2. We postulate that the solutions $\{x_M(\iota), h_M(\iota), x_K(\iota), h_K(\iota)\}$ to the planner problem in Section 3.A.2 constitute components of the equilibrium.
3. We use the sequential budget constraints to postulate equilibrium holdings for the bonds $b^\ell(s_t^j;\iota)$ that are traded within islands:

$$b^\ell(s_t^j;\iota) = \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n \frac{\mu_{t+n}(\alpha_{t+n}^j)}{\mu_t(\alpha_t^j)}\left(x_{M,t+n}(\iota)+\sum p_K x_{K,t+n}(\iota)-\tilde{y}_{t+n}(\iota)\right)\right] \tag{A.20}$$

where $\tilde{y} = \tilde{z}_M h_M = (1-\tau_0)z_M^{1-\tau_1}h_M$ denotes after-tax labor income.

4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s^j_{t+1};\iota)$ and $a(\zeta^j_{t+1};\iota)$:

$$q^\ell_b(s^j_{t+1}) = \beta\delta \exp\left(-\left(1-\tau_1\right)\upsilon^\alpha_{t+1}\right)$$
$$\times\left[\frac{\int \exp\left(A\upsilon^\varepsilon_t\right)\mathrm{d}\Phi_{\upsilon^\varepsilon_t}(\upsilon^\varepsilon_t)}{\int \exp\left(A\upsilon^\kappa_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\kappa_{t+1}}(\upsilon^\kappa_{t+1})\int \exp\left(A\upsilon^\varepsilon_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\varepsilon_{t+1}}(\upsilon^\varepsilon_{t+1})}\right] f^{t+1,j}(s^{t+1,j}|s^{t,j}) \tag{A.21}$$

$$q_a(Z_{t+1}) = \beta\delta\int \exp\left(-\left(1-\tau_1\right)\upsilon^\alpha_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\alpha_{t+1}}\left(\upsilon^\alpha_{t+1}\right)$$
$$\times\left[\frac{\int \exp\left(A\upsilon^\varepsilon_t\right)\mathrm{d}\Phi_{\upsilon^\varepsilon_t}(\upsilon^\varepsilon_t)}{\int \exp\left(A\upsilon^\kappa_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\kappa_{t+1}}(\upsilon^\kappa_{t+1})\int \exp\left(A\upsilon^\varepsilon_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\varepsilon_{t+1}}(\upsilon^\varepsilon_{t+1})}\right]\mathbb{P}\left(\left(\upsilon^\kappa_{t+1},\upsilon^\varepsilon_{t+1}\right)\in Z_{t+1}\right) \tag{A.22}$$

where $A \equiv 1-\tau_1$.

3.A.4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section 3.A.3 constitutes an equilibrium by showing that the postulated equilibrium allocations solve the households' problem and that all markets clear.

Household Problem

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

Using the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and $\mu(\alpha_t^j)$, we write the optimality conditions directly as:

$$\omega_M x_M^{-\frac{1}{\phi}} = \frac{\omega_K}{p_K}\left(x_K^{\frac{\sigma_K-1}{\sigma_K}} + (z_K h_K)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}-1} x_K^{-\frac{1}{\sigma_K}} \tag{A.23}$$

$$\omega_M x_M^{-\frac{1}{\phi}} = \omega_K \left(x_K^{\frac{\sigma_K-1}{\sigma_K}} + (z_K h_K)^{\frac{\sigma_K-1}{\sigma_K}}\right)^{\frac{\sigma_K}{\sigma_K-1}\frac{\phi-1}{\phi}-1} (z_K h_K)^{-\frac{1}{\sigma_K}} \frac{z_K}{\tilde{z}_M} \tag{A.24}$$

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j)}{\mu(\alpha_t^j)} f^{t+1,j}(\theta^{t+1,j}|\theta^{t,j})\mathrm{d}\omega_{K,t+1}\mathrm{d}z_{K,t+1} \tag{A.25}$$

$$q_a(\zeta_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j)}{\mu(\alpha_t^j)} f^{t+1,j}(\theta^{t+1,j}|\theta^{t,j})\mathrm{d}\upsilon_{t+1}^\alpha \mathrm{d}\omega_{K,t+1}\mathrm{d}z_{K,t+1} \tag{A.26}$$

Euler Equations

We next verify that the Euler equations are satisfied at the postulated equilibrium allocations and prices.

Using the marginal utility of market consumption of the planner problem $\mu(\alpha_t^j)$, we write the Euler equation for the bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta \int \frac{\mu_{t+1}(\alpha_{t+1}^j)}{\mu_t(\alpha_t^j)} f^{t+1,j}(\theta^{t+1,j}|\theta^{t,j})\mathrm{d}\omega_{K,t+1}\mathrm{d}z_{K,t+1} \\ &= \beta\delta \int \frac{\left(\int \tilde{z}_{M,t+1}^j \left(\alpha_{t+1}^j, \varepsilon_{t+1}\right)\mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right)^{-1}}{\left(\int \tilde{z}_{M,t}^j \left(\alpha_t^j, \varepsilon_t\right)\mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right)^{-1}} f^{t+1,j}(\theta^{t+1,j}|\theta^{t,j})\mathrm{d}\omega_{K,t+1}\mathrm{d}z_{K,t+1}\ , \end{aligned} \tag{A.27}$$

where the second line follows from equation (A.15).

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$ we use that:

$$\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\left((1-\tau_1)\left(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right). \tag{A.28}$$

Given $A = (1-\tau_1)$, the expectation over the random variables in the numerator is given by:

$$
\begin{aligned}
&\int \exp\left(A\left(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right) \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j) \\
&= \int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j) \int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon) \ , \quad \text{(A.29)}
\end{aligned}
$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$
\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa^j,t}(\kappa_t^j) \int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon) \ . \tag{A.30}
$$

As a result, the price $q_b^\ell(s_{t+1}^j)$ is:

$$
q_b^\ell(s_{t+1}^j) = \beta\delta \left(\frac{\exp\left(-\left(1-\tau_1\right)\upsilon_{t+1}^\alpha\right) \int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon^\varepsilon,t}(\upsilon_t^\varepsilon)}{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon^\kappa,t+1}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon^\varepsilon,t+1}(\upsilon_{t+1}^\varepsilon)} \right) f^{t+1,j}(s^{t+1,j}|s^{t,j}) \ ,
$$

where $f^{t+1,j}(s^{t+1,j}|s^{t,j}) = f(\upsilon_{t+1}^\alpha)f(\upsilon_{t+1}^\kappa)f(\upsilon_{t+1}^\varepsilon)$. This confirms our guess in equation (A.21). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in brackets is independent of the state vector that differentiates islands ℓ. As a result, all islands ℓ have the same bond prices, $q_b^\ell(s_{t+1}^j) = Q_b\left(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha\right)$.

We next calculate the bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$:

$$
\begin{aligned}
q_b^\ell(\mathcal{V}_{t+1}) = \beta\delta &\int_{\mathcal{V}^\alpha} \exp\left(-\left(1-\tau_1\right)\upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right) \\
&\times \left(\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)} \right)^{-1} .
\end{aligned}
$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim that does not depend on the realization of

$(\upsilon^{\alpha}_{t+1})$:

$$q^{\ell}_{b}(\mathbb{V}_{t+1}) = \beta\delta \int_{\mathbb{V}^{\alpha}} \exp\left(-\left(1-\tau_1\right)\upsilon^{\alpha}_{t+1}\right) \mathrm{d}\Phi_{\upsilon^{\alpha}_{t+1}}\left(\upsilon^{\alpha}_{t+1}\right)$$
$$\times \left(\frac{\int \exp\left(A\upsilon^{\kappa}_{t+1}\right) \mathrm{d}\Phi_{\upsilon^{\kappa}_{t+1}}(\upsilon^{\kappa}_{t+1}) \int \exp\left(A\upsilon^{\varepsilon}_{t+1}\right) \mathrm{d}\Phi_{\upsilon^{\varepsilon}_{t+1}}(\upsilon^{\varepsilon}_{t+1})}{\int \exp\left(A\upsilon^{\varepsilon}_{t}\right) \mathrm{d}\Phi_{\upsilon^{\varepsilon}_{t}}(\upsilon^{\varepsilon}_{t})} \right)^{-1}.$$

All islands face the same price $q^{\ell}_{b}(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the price of an inter-island claim equals the price of the same within-island claim. The price of a claim traded across islands for set Z gives:

$$q^{a}_{t+1}(Z; s^{t,j}) = \mathbb{P}\left((\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1}) \in Z\right) q^{\ell}_{b}(\mathbb{V}_{t+1})$$

This concludes the discussion on asset prices.

By no arbitrage, the prices of bonds a and b that are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and given by:

$$q_a(Z_{t+1}) = \mathbb{P}\left((\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1}) \in Z_{t+1}\right) Q_b(\mathbb{V}_{t+1}), \tag{A.31}$$

where $\mathbb{P}\left((\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1}) \in Z_{t+1}\right)$ denotes the probability of $(\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1})$ being a member of Z_{t+1}. The expression for $q_a(Z_{t+1})$ confirms our guess in equation (A.22).

Household's Budget Constraint

We now verify our guess for the bond positions $b^{\ell}_{t}(s^{j}_{t})$ and confirm that the household budget constraint holds at the equilibrium allocations that we postulated. We define the deficit term by $d_t \equiv x_{M,t} + \sum p_K x_{K,t} - \tilde{y}_t$. Using the expression for the price $q^{\ell}_{b}(s^{j}_{t+1})$ in equation (A.25), the budget constraint at the no-trade equilibrium is given by:

$$b^{\ell}_{t}(s^{j}_{t}) = d_t + \beta\delta \int\int\int \frac{\mu(\alpha^{j}_{t+1}, \omega^{j}_{K,t+1}, z^{j}_{K,t+1})}{\mu(\alpha^{j}_{t}, \omega^{j}_{K,t}, z^{j}_{K,t})} b^{\ell}_{t+1}(s^{j}_{t+1}) f^{t+1}(\sigma^{j}_{t+1}|\sigma^{j}_{t}) \mathrm{d}s^{j}_{t+1} \mathrm{d}z^{j}_{K,t+1} \mathrm{d}\omega^{j}_{K,t+1}.$$

By substituting forward using equation (A.25), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.20) and show that the household budget constraint holds at the postulated equilibrium allocations.

Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations that solve the planner problems satisfy the aggregate goods market clearing condition:

$$\int_\iota \Big(x_{M,t}(\iota) + \sum A_K^{-1} x_{K,t}(\iota) \Big) \mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t}(\iota) h_{M,t}(\iota) \mathrm{d}\Phi(\iota) \ .$$

Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota a(\zeta_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $a(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\mathrm{d}\Phi(\iota) = 0, \forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated bond holdings in equation (A.20) into the asset market clearing conditions:

$$\begin{aligned} \int b^\ell(s_t^j)\mathrm{d}\Phi(\iota) &= \int \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu(\alpha_{t+n}^j, \omega_{K,t+n}, z_{K,t+n}^j)}{\mu(\alpha_t^j, \omega_{K,t}, z_{K,t}^j)} d_{t+n} \right] \mathrm{d}\Phi(\iota) \\ &= \sum_{n=0}^{\infty} (\beta\delta)^n \int\int \frac{\mu(\alpha_{t+n}^j, \omega_{K,t+n}, z_{K,t+n}^j)}{\mu(\alpha_t^j, \omega_{K,t}, z_{K,t}^j)} d_{t+n} f(\theta_{t+n}^j|\theta_{t-1}^j)\mathrm{d}\theta_{t+n}^j \mathrm{d}\Phi(\iota). \end{aligned}$$

For simplicity we omit conditioning on θ_{t-1}^j and write the density function as $f(\theta_{t+n}^j|\theta_{t-1}^j) = f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{z_{K,t+n}, \omega_{K,t+n}\})$. Further, we denote the marginal utility growth by $\mathcal{Q}\left(\upsilon^{\alpha}_{t+n}\right) \equiv \frac{\mu(\alpha^{j}_{t+n})}{\mu(\alpha^{j}_{t})}$. Hence, we write aggregate bond holdings $\int b^{\ell}(s^{j}_{t})\mathrm{d}\Phi(\iota)$ as:

$$
\begin{aligned}
&\sum_{n=0}^{\infty}(\beta\delta)^{n}\int\int \mathcal{Q}\left(\upsilon^{\alpha}_{t+n}\right) d_{t+n} f(\{\upsilon^{\alpha}_{t+n}\}) f(\{\upsilon^{\kappa}_{t+n}\}) f(\{\upsilon^{\varepsilon}_{t+n}\}) f(\{z_{K,t+n},\omega_{K,t+n}\})\ldots \\
&\qquad\qquad \ldots\ \mathrm{d}\{\upsilon^{\alpha}_{t+n}\}\mathrm{d}\{\upsilon^{\kappa}_{t+n}\}\mathrm{d}\{\upsilon^{\varepsilon}_{t+n}\}\mathrm{d}\{z_{K,t+n},\omega_{K,t+n}\}\mathrm{d}\Phi(\iota) \\
=&\sum_{n=0}^{\infty}(\beta\delta)^{n}\int d_{t+n} f(\{\upsilon^{\kappa}_{t+n}\}) f(\{\upsilon^{\varepsilon}_{t+n}\})\mathrm{d}\{\upsilon^{\kappa}_{t+n}\}\mathrm{d}\{\upsilon^{\varepsilon}_{t+n}\}\mathrm{d}\Phi(\iota) \\
\times&\int \mathcal{Q}\left(\upsilon^{\alpha}_{t+n}\right) f(\{\upsilon^{\alpha}_{t+n}\}) f(\{z_{K,t+n},\omega_{K,t+n}\})\mathrm{d}\{\upsilon^{\alpha}_{t+n}\}\mathrm{d}\{z_{K,t+n},\omega_{K,t+n}\}.
\end{aligned}
$$

Recalling that the deficit terms equal $d_t = x_{M,t} + \sum p_K x_{K,t} - \tilde{y}_t$, the bond market clearing condition holds because the first term is zero by the island-level resource constraint.

Appendix 3.B Inference of Sources of Heterogeneity

In this appendix we show how to derive the sources of heterogeneity $\{z_{K,t}, \omega_{K,t}, \alpha_t, \varepsilon_t\}_{\iota}$ presented in Section 3.2.3. Our strategy is to invert the equilibrium allocations presented in Section 3.2.2 and solve for the unique sources of heterogeneity that lead to these allocations. We note that the identification is defined up to a constant because the constant $\mathbb{C}$ that appears in the equations of Section 3.2.3 depends on the ε's.

The solution for z_K in equation (14) comes from inverting equation (13) for the optimal allocation of expenditures and time inputs in the production of good K. Next, we use the solution for x_K in equation (9) together with the solution for x_M in equation (8) and invert these solutions to solve for the preference weight for good K relative to M:

$$
\frac{\omega_K}{\omega_M} = p_K \left(\frac{x_K}{x_M}\right)^{\frac{1}{\phi}} \left(1 + \left(\frac{z_K p_K}{\tilde{z}_M}\right)^{\sigma_K - 1}\right)^{\frac{\sigma_K - \phi}{\sigma_K - 1}\frac{1}{\phi}} .
$$

Using this equation together with the normalization $\omega_M + \sum \omega_K = 1$ yields solutions for ω_M in equation (15) and ω_K in equation (16) in the text. Finally, we infer the permanent component of market productivity α by inverting equation (7) in the text that defines the marginal utility of market consumption. The transitory part of labor productivity is then given by $\varepsilon = \log z_M - \alpha$.

Appendix 3.C Variance of Log Consumption

In this section we prove the statement that the variance of log consumption in equation (23)) is decreasing in γ_L. To proof the statement, let $f(x;\kappa,\gamma_L,\sigma_L) = \log\left(\kappa + \gamma_L^{\sigma_L - 1} x\right)$, where we restrict $\kappa, \gamma_L \geq 0$, $\sigma_L \geq 1$, and $x \geq 0$. This implies that the derivative of f is increasing in x, and increasingly so for larger values of γ_L,

$$f_x\left(x;\kappa,\gamma_L,\sigma_L\right) = \frac{1}{\kappa + \gamma_L^{\sigma_L-1}x}\gamma_L^{\sigma_L-1} = \left(\kappa\gamma_L^{1-\sigma_L} + x\right)^{-1} \geq 0\ ;$$
$$f_{x\gamma_L}\left(x;\kappa,\gamma_L,\sigma_L\right) = -\left(\kappa\gamma_L^{1-\sigma_L} + x\right)^{-2}\kappa\left(1-\sigma_L\right)\gamma_L^{-\sigma_L} \geq 0.$$

The cross-derivative is the key component of the proof.

To establish the result, it is useful to write the variance as:

$$\begin{aligned}\mathrm{Var}\left(f(x;\kappa,\gamma_L,\sigma_L)\right) &= \int \left(f(x;\kappa,\gamma_L,\sigma_L) - \mathbb{E}f(x;\kappa,\gamma_L,\sigma_L)\right)^2 f(x)\mathrm{d}x \\ &= \int \left(\int_{\bar{x}(\gamma_L)}^{x} f_x(x;\kappa,\gamma_L,\sigma_L)\right)^2 f(x)\mathrm{d}x\end{aligned}$$

where $\bar{x}(\gamma_L)$ is such that $f(\bar{x}(\gamma_L);\kappa,\gamma_L,\sigma_L) = \mathbb{E}f(x;\kappa,\gamma_L,\sigma_L)$.

Because f_x is increasing in γ_L, we know that for any $\tilde{\gamma}_L \leq \gamma_L$.

$$\int \left(\int_{\bar{x}(\gamma_L)}^{x} f_x(x;\kappa,\gamma_L,\sigma_L)\right)^2 f(x)\mathrm{d}x \geq \int \left(\int_{\bar{x}(\gamma_L)}^{x} f_x(x;\kappa,\tilde{\gamma}_L,\sigma_L)\right)^2 f(x)\mathrm{d}x$$

Second, we know that the mean minimizes the variance, that is,

$$\mathbb{E}f(x;\kappa,\gamma_L,\sigma_L) = \arg\min_\nu \int \left(f(x;\kappa,\gamma_L,\sigma_L) - \nu\right)^2 f(x)\mathrm{d}x. \tag{A.32}$$

We let $\bar{x}(\tilde{\gamma}_L)$ such that $f(\bar{x}(\tilde{\gamma}_L);\kappa,\tilde{\gamma}_L,\sigma_L) = \mathbb{E}f(x;\kappa,\tilde{\gamma}_L,\sigma_L)$.

As a result, we know that

$$\int \left(\int_{\bar{x}(\gamma_L)}^{x} f_x(x;\kappa,\tilde{\gamma}_L,\sigma_L)\right)^2 f(x)\mathrm{d}x \geq \int \left(\int_{\bar{x}(\tilde{\gamma}_L)}^{x} f_x(x;\kappa,\tilde{\gamma}_L,\sigma_L)\right)^2 f(x)\mathrm{d}x$$

and hence that

$$\int \left(\int_{\bar{x}(\gamma_L)}^{x} f_x(x; \kappa, \gamma_L, \sigma_L) \right)^2 f(x) \mathrm{d}x \geq \int \left(\int_{\bar{x}(\tilde{\gamma}_L)}^{x} f_x(x; \kappa, \tilde{\gamma}_L, \sigma_L) \right)^2 f(x) \mathrm{d}x \tag{A.33}$$

which is what we wanted to show.

Appendix 3.D Sensitivity Analyses

In this appendix we present the details underlying our sensitivity analyses. For every sensitivity analysis, we regenerate the sources of heterogeneity and then perform the counterfactual analyses.

- $\sigma_N = 3.5$: Table A.1 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.1 presents the welfare effects of shutting off the evolution of driving forces.

- $\sigma_N = 1.5$: Table A.2 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.2 presents the welfare effects of shutting off the evolution of driving forces.

- $\sigma_L = 0.2$: Table A.3 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.3 presents the welfare effects of shutting off the evolution of driving forces.

- $\sigma_L = 0.8$: Table A.4 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.4 presents the welfare effects of shutting off the evolution of driving forces.

- $\tau_1 = 0.06$: Table A.5 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.5 presents the welfare effects of shutting off the evolution of driving forces.

- $\tau_1 = 0.18$: Table A.6 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.6 presents the welfare effects of shutting off the evolution of driving forces.

- $\phi = 0.5$: Table A.7 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.7 presents the welfare effects of shutting off the evolution of driving forces.

- $\phi = 2.0$: Table A.8 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.8 presents the welfare effects of shutting off the evolution of driving forces.

- Adjusted consumption measures: Table A.9 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.9 presents the welfare effects of shutting off the evolution of driving forces.

- Singles: Table A.10 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.10 presents the welfare effects of shutting off the evolution of driving forces.

- Alternative measure of leisure: Table A.11 assesses the contribution of driving forces, presented in rows, to the mean values of observables, presented in columns. Figure A.11 presents the welfare effects of shutting off the evolution of driving forces.

Table A.1: Means: Counterfactuals ($\sigma_N = 3.5$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.0	16.5	78.2	-1.8	-7.4	2.1
Mean p_N	35.3	5.0	59.7	78.2	10.9	-31.6	2.1
Mean p_L	27.0	5.0	16.5	62.9	1.2	-7.4	0.4
Mean $\alpha + \varepsilon$	-15.6	-22.4	-54.0	62.8	-16.8	18.3	0.5
Variance α	26.7	6.5	19.5	79.0	-0.5	-9.4	2.2
Variance ε	24.5	5.0	16.4	78.2	-1.8	-7.1	2.2
Mean $\log z_N$	25.0	5.0	17.3	78.2	-1.5	-8.2	2.1
Variance $\log z_N$	23.5	5.0	14.0	78.2	-3.6	-3.3	2.1
Mean $\log z_L$	18.2	5.0	16.5	35.1	-9.1	-7.4	6.3
Variance $\log z_L$	24.8	5.0	16.5	79.0	-1.6	-7.4	2.1
Mean ω_M	33.7	30.8	12.7	74.4	8.1	-11.1	-1.6
Variance ω_M	23.2	1.1	16.4	78.1	-2.4	-7.5	2.0
Mean ω_N	24.3	5.4	15.7	78.5	-2.1	-8.1	2.4
Variance ω_N	24.8	5.2	16.8	78.4	-1.6	-7.0	2.3
Mean ω_L	29.4	11.9	23.3	72.5	4.0	-0.5	-3.6
Variance ω_L	24.5	5.0	16.4	78.2	-1.8	-7.4	2.1

Table A.2: Means: Counterfactuals ($\sigma_N = 1.5$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.1	16.5	78.3	-1.8	-7.4	2.1
Mean p_N	26.8	5.1	41.0	78.3	1.1	-11.8	2.1
Mean p_L	27.0	5.1	16.5	62.9	1.3	-7.4	0.4
Mean $\alpha + \varepsilon$	-3.6	-22.4	-18.7	62.9	-2.3	-1.4	0.5
Variance α	26.1	6.5	18.3	79.1	-1.7	-7.8	2.2
Variance ε	24.6	5.1	16.5	78.3	-1.6	-7.3	2.2
Mean log z_N	21.3	5.1	8.3	78.3	-5.6	-0.9	2.1
Variance log z_N	21.7	5.1	9.7	78.3	-5.7	0.8	2.1
Mean log z_L	18.3	5.1	16.5	35.2	-9.0	-7.4	6.3
Variance log z_L	24.8	5.1	16.5	79.0	-1.5	-7.4	2.0
Mean ω_M	33.7	30.8	12.8	74.5	8.1	-11.1	-1.6
Variance ω_M	23.2	1.2	16.4	78.2	-2.4	-7.5	2.0
Mean ω_N	24.4	5.4	15.8	78.6	-2.0	-8.2	2.4
Variance ω_N	24.9	5.3	16.9	78.4	-1.6	-7.1	2.3
Mean ω_L	29.5	12.0	23.4	72.5	4.0	-0.5	-3.6
Variance ω_L	24.6	5.0	16.5	78.2	-1.7	-7.5	2.1

Table A.3: Means: Counterfactuals ($\sigma_L = 0.2$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.1	16.5	78.2	-1.8	-7.4	2.1
Mean p_N	31.1	5.1	50.8	78.2	6.2	-21.3	2.1
Mean p_L	28.6	5.1	16.5	69.9	3.1	-7.4	-0.8
Mean $\alpha + \varepsilon$	-8.0	-22.4	-35.5	70.0	-7.4	9.3	-0.7
Variance α	26.3	6.5	18.9	78.7	-1.4	-8.6	2.3
Variance ε	24.6	5.1	16.4	78.3	-1.7	-7.2	2.1
Mean $\log z_N$	23.2	5.1	12.9	78.2	-3.8	-4.4	2.1
Variance $\log z_N$	23.3	5.1	13.5	78.2	-3.5	-2.7	2.1
Mean $\log z_L$	16.6	5.1	16.5	19.7	-11.0	-7.4	7.3
Variance $\log z_L$	24.9	5.1	16.5	79.6	-1.4	-7.4	2.0
Mean ω_M	33.7	30.8	12.8	74.5	8.1	-11.1	-1.6
Variance ω_M	23.2	1.1	16.4	78.2	-2.4	-7.5	2.0
Mean ω_N	24.4	5.4	15.7	78.6	-2.0	-8.1	2.4
Variance ω_N	24.8	5.3	16.8	78.4	-1.6	-7.0	2.3
Mean ω_L	29.5	12.0	23.4	72.5	4.0	-0.5	-3.6
Variance ω_L	24.6	5.0	16.4	78.2	-1.7	-7.4	2.1

Table A.4: Means: Counterfactuals ($\sigma_L = 0.8$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.1	16.5	78.2	-1.8	-7.4	2.1
Mean p_N	31.1	5.1	50.8	78.2	6.1	-21.3	2.1
Mean p_L	25.5	5.1	16.5	55.8	-0.6	-7.4	1.5
Mean $\alpha + \varepsilon$	-11.3	-22.4	-35.5	55.6	-11.2	9.3	1.5
Variance α	26.5	6.5	18.9	79.4	-1.3	-8.6	2.1
Variance ε	24.6	5.0	16.4	78.2	-1.7	-7.2	2.2
Mean $\log z_N$	23.2	5.1	12.9	78.2	-3.8	-4.4	2.1
Variance $\log z_N$	23.3	5.1	13.5	78.2	-3.5	-2.7	2.1
Mean $\log z_L$	20.2	5.1	16.5	50.3	-6.5	-7.4	5.1
Variance $\log z_L$	24.7	5.1	16.5	78.1	-1.7	-7.4	2.1
Mean ω_M	33.7	30.8	12.8	74.5	8.1	-11.1	-1.6
Variance ω_M	23.2	1.1	16.4	78.1	-2.4	-7.5	2.0
Mean ω_N	24.4	5.4	15.7	78.5	-2.1	-8.1	2.4
Variance ω_N	24.8	5.2	16.9	78.4	-1.6	-7.0	2.3
Mean ω_L	29.5	11.9	23.4	72.5	4.0	-0.5	-3.6
Variance ω_L	24.5	5.0	16.4	78.2	-1.7	-7.4	2.1

Table A.5: Means: Counterfactuals ($\tau_1 = 0.06$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	26.7	7.2	18.6	80.4	-1.8	-7.4	2.1
Mean p_N	33.2	7.2	52.8	80.4	6.2	-21.4	2.1
Mean p_L	29.1	7.2	18.6	65.0	1.3	-7.4	0.4
Mean $\alpha + \varepsilon$	-9.8	-22.2	-36.9	63.9	-10.1	10.5	0.3
Variance α	28.6	8.7	21.1	81.2	-1.3	-8.7	2.2
Variance ε	26.7	7.2	18.5	80.4	-1.9	-7.2	2.2
Mean $\log z_N$	25.9	7.2	16.5	80.4	-3.0	-5.6	2.1
Variance $\log z_N$	25.3	7.2	15.4	80.4	-3.9	-2.4	2.1
Mean $\log z_L$	20.2	7.2	18.6	36.3	-9.5	-7.4	6.5
Variance $\log z_L$	26.9	7.2	18.6	81.0	-1.6	-7.4	2.1
Mean ω_M	35.8	32.8	14.9	76.6	8.2	-11.1	-1.6
Variance ω_M	25.3	2.9	18.5	80.3	-2.4	-7.5	2.0
Mean ω_N	26.5	7.6	17.8	80.7	-2.1	-8.2	2.5
Variance ω_N	27.0	7.4	19.0	80.5	-1.6	-7.0	2.3
Mean ω_L	31.5	14.0	25.4	74.6	4.1	-0.6	-3.6
Variance ω_L	26.7	7.1	18.6	80.3	-1.7	-7.4	2.1

Table A.6: Means: Counterfactuals ($\tau_1 = 0.18$)

	100 $\times$ Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	22.4	2.9	14.3	76.1	-1.8	-7.3	2.1
Mean p_N	29.0	2.9	48.7	76.1	6.1	-21.2	2.1
Mean p_L	24.8	2.9	14.3	60.7	1.2	-7.3	0.4
Mean $\alpha + \varepsilon$	-9.5	-22.6	-33.9	61.8	-8.7	8.1	0.6
Variance α	24.1	4.2	16.5	76.8	-1.4	-8.4	2.2
Variance ε	22.4	2.9	14.3	76.1	-1.8	-7.2	2.1
Mean $\log z_N$	20.3	2.9	9.1	76.1	-4.3	-3.2	2.1
Variance $\log z_N$	21.2	2.9	11.6	76.1	-3.4	-2.9	2.1
Mean $\log z_L$	16.2	2.9	14.3	34.0	-8.5	-7.3	6.1
Variance $\log z_L$	22.6	2.9	14.3	77.0	-1.5	-7.3	2.0
Mean ω_M	31.5	28.7	10.6	72.3	8.1	-11.1	-1.6
Variance ω_M	21.1	-0.9	14.2	76.0	-2.5	-7.4	2.0
Mean ω_N	22.2	3.2	13.6	76.3	-2.1	-8.0	2.4
Variance ω_N	22.6	3.1	14.6	76.2	-1.6	-7.0	2.3
Mean ω_L	27.3	9.8	21.2	70.3	4.1	-0.4	-3.6
Variance ω_L	22.4	2.8	14.2	76.0	-1.8	-7.4	2.1

Table A.7: Means: Counterfactuals ($\phi = 0.5$)

	100 $\times$ Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.6	5.1	16.5	78.2	-1.8	-7.4	2.1
Mean p_N	30.4	6.4	47.5	79.6	5.3	-24.6	3.5
Mean p_L	26.5	4.2	15.6	63.7	0.6	-8.3	1.2
Mean $\alpha + \varepsilon$	-5.7	-13.0	-34.3	60.1	-5.2	10.5	-2.2
Variance α	26.2	6.0	18.7	79.2	-1.6	-8.7	2.4
Variance ε	24.6	5.0	16.4	78.2	-1.7	-7.2	2.2
Mean $\log z_N$	23.1	5.4	12.1	78.6	-4.0	-5.3	2.5
Variance $\log z_N$	23.2	5.4	12.8	78.5	-3.6	-3.4	2.4
Mean $\log z_L$	-2.3	-21.9	-10.4	51.3	-32.2	-34.3	22.5
Variance $\log z_L$	25.1	5.6	17.0	78.7	-1.3	-6.9	1.8
Mean ω_M	29.3	21.1	12.9	74.5	2.5	-6.9	-0.3
Variance ω_M	23.9	2.9	17.3	79.1	-2.6	-7.7	2.4
Mean ω_N	24.6	5.6	15.5	79.0	-1.5	-8.0	2.4
Variance ω_N	23.8	6.2	13.3	79.4	-2.5	-9.6	3.9
Mean ω_L	60.7	47.7	58.2	71.5	35.6	-11.6	-27.5
Variance ω_L	19.3	2.7	13.6	63.6	-2.0	9.1	-1.8

Table A.8: Means: Counterfactuals ($\phi = 2.0$)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.2	4.6	16.2	77.6	-1.9	-7.3	2.1
Mean p_N	32.2	1.7	56.8	74.7	7.8	-14.9	-0.8
Mean p_L	27.6	6.4	17.9	60.7	2.5	-5.5	-1.2
Mean $\alpha + \varepsilon$	-17.9	-42.1	-38.8	67.1	-18.2	6.4	5.4
Variance α	26.4	7.0	18.8	78.1	-0.8	-8.2	1.9
Variance ε	24.2	4.6	16.1	77.7	-1.9	-7.2	2.2
Mean $\log z_N$	23.0	3.9	14.2	76.9	-3.6	-2.7	1.4
Variance $\log z_N$	23.1	4.0	14.5	77.0	-3.4	-1.3	1.5
Mean $\log z_L$	48.6	41.7	53.2	-14.6	24.1	29.8	-42.8
Variance $\log z_L$	23.7	3.6	15.1	78.9	-1.7	-8.4	2.6
Mean ω_M	19.8	-34.4	17.8	79.3	-6.9	-5.6	3.8
Variance ω_M	23.3	4.8	16.7	78.2	-2.9	-6.8	2.7
Mean ω_N	17.4	18.5	-81.9	91.5	-9.0	-105.3	16.0
Variance ω_N	27.5	-1.2	22.9	71.8	0.8	-0.6	-3.7
Mean ω_L	-3.0	-59.8	-48.3	94.0	-27.4	-71.7	18.5
Variance ω_L	27.2	8.1	19.6	74.6	-0.5	-3.9	-0.9

Table A.9: Means: Counterfactuals (Adjusted Consumption Measures)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.1	6.1	15.1	78.3	-1.8	-7.4	2.1
Mean p_N	30.6	6.1	49.3	78.3	6.2	-21.4	2.1
Mean p_L	26.4	6.1	15.1	63.0	1.2	-7.4	0.4
Mean $\alpha + \varepsilon$	-10.3	-21.4	-36.8	62.9	-9.3	9.4	0.5
Variance α	26.1	7.7	17.8	79.2	-1.2	-8.7	2.2
Variance ε	24.0	6.0	15.1	78.3	-1.7	-7.2	2.1
Mean $\log z_N$	23.0	6.1	12.5	78.3	-3.2	-5.1	2.1
Variance $\log z_N$	22.7	6.1	12.0	78.3	-3.6	-2.5	2.1
Mean $\log z_L$	18.0	6.1	15.1	35.1	-8.7	-7.4	6.3
Variance $\log z_L$	24.4	6.1	15.1	79.9	-1.4	-7.4	2.0
Mean ω_M	32.8	29.8	11.4	74.6	8.0	-11.0	-1.6
Variance ω_M	22.3	1.9	15.0	78.2	-2.1	-7.4	2.0
Mean ω_N	23.9	6.2	14.8	78.4	-2.0	-7.6	2.2
Variance ω_N	24.3	6.2	15.3	78.5	-1.5	-7.1	2.3
Mean ω_L	29.1	13.0	22.1	72.4	4.3	-0.4	-3.8
Variance ω_L	24.0	6.0	15.0	78.2	-1.8	-7.5	2.1

Table A.10: Means: Counterfactuals (Singles)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	17.6	-1.3	9.7	66.8	-3.0	-18.2	8.2
Mean p_N	22.2	-1.3	39.7	66.8	3.2	-36.5	8.2
Mean p_L	20.0	-1.3	9.7	51.3	0.4	-18.2	6.4
Mean $\alpha + \varepsilon$	-10.8	-25.7	-30.6	52.9	-7.3	2.4	6.6
Variance α	20.5	1.2	13.4	68.2	-2.5	-20.7	8.3
Variance ε	17.7	-1.3	9.7	66.8	-4.4	-17.9	8.3
Mean $\log z_N$	14.9	-1.3	3.1	66.8	-6.1	-8.6	8.2
Variance $\log z_N$	17.5	-1.3	9.5	66.8	-3.3	-17.3	8.2
Mean $\log z_L$	13.1	-1.3	9.7	37.5	-8.2	-18.2	11.5
Variance $\log z_L$	14.2	-1.3	9.7	45.6	-7.6	-18.2	12.8
Mean ω_M	27.2	26.2	5.2	62.2	7.1	-22.7	3.7
Variance ω_M	16.0	-5.8	9.6	66.7	-2.4	-18.3	8.1
Mean ω_N	18.9	-3.2	15.5	64.9	-1.5	-12.4	6.3
Variance ω_N	16.9	-1.6	8.3	66.6	-3.9	-19.6	8.0
Mean ω_L	27.1	11.8	22.9	57.0	8.6	-5.1	-1.4
Variance ω_L	16.5	-2.5	8.6	65.8	-3.8	-19.3	7.2

Table A.11: Means: Counterfactuals (Alternative Measure of Leisure)

	100 × Difference between 2012-2016 and 1995-1999						
Case	$\log(px)$	$\log x_M$	$\log x_N$	$\log x_L$	$\log h_M$	$\log h_N$	$\log h_L$
Baseline	24.5	4.9	16.4	78.1	-1.8	12.0	-4.0
Mean p_N	32.6	4.9	54.2	78.1	8.0	1.6	-4.0
Mean p_L	26.8	4.9	16.4	62.3	1.2	12.0	-6.2
Mean $\alpha + \varepsilon$	-11.6	-22.5	-40.5	62.3	-11.4	23.8	-6.1
Variance α	26.4	6.4	19.1	78.9	-1.1	11.1	-3.9
Variance ε	24.4	4.9	16.3	78.1	-1.7	12.1	-4.0
Mean $\log z_N$	28.5	4.9	25.5	78.1	2.9	7.2	-4.0
Variance $\log z_N$	27.9	4.9	23.6	78.1	3.4	5.0	-4.0
Mean $\log z_L$	17.6	4.9	16.4	30.8	-9.5	12.0	1.8
Variance $\log z_L$	29.8	4.9	16.4	97.6	4.2	12.0	-9.5
Mean ω_M	33.6	30.6	12.7	74.3	8.1	8.3	-7.7
Variance ω_M	23.1	1.1	16.3	78.0	-2.4	11.9	-4.1
Mean ω_N	24.5	9.2	10.1	82.4	-1.7	5.7	0.3
Variance ω_N	24.4	4.9	16.2	78.0	-1.8	11.8	-4.1
Mean ω_L	25.4	6.3	17.7	76.4	-0.6	13.3	-5.7
Variance ω_L	24.5	4.9	16.4	78.3	-1.7	12.0	-3.8

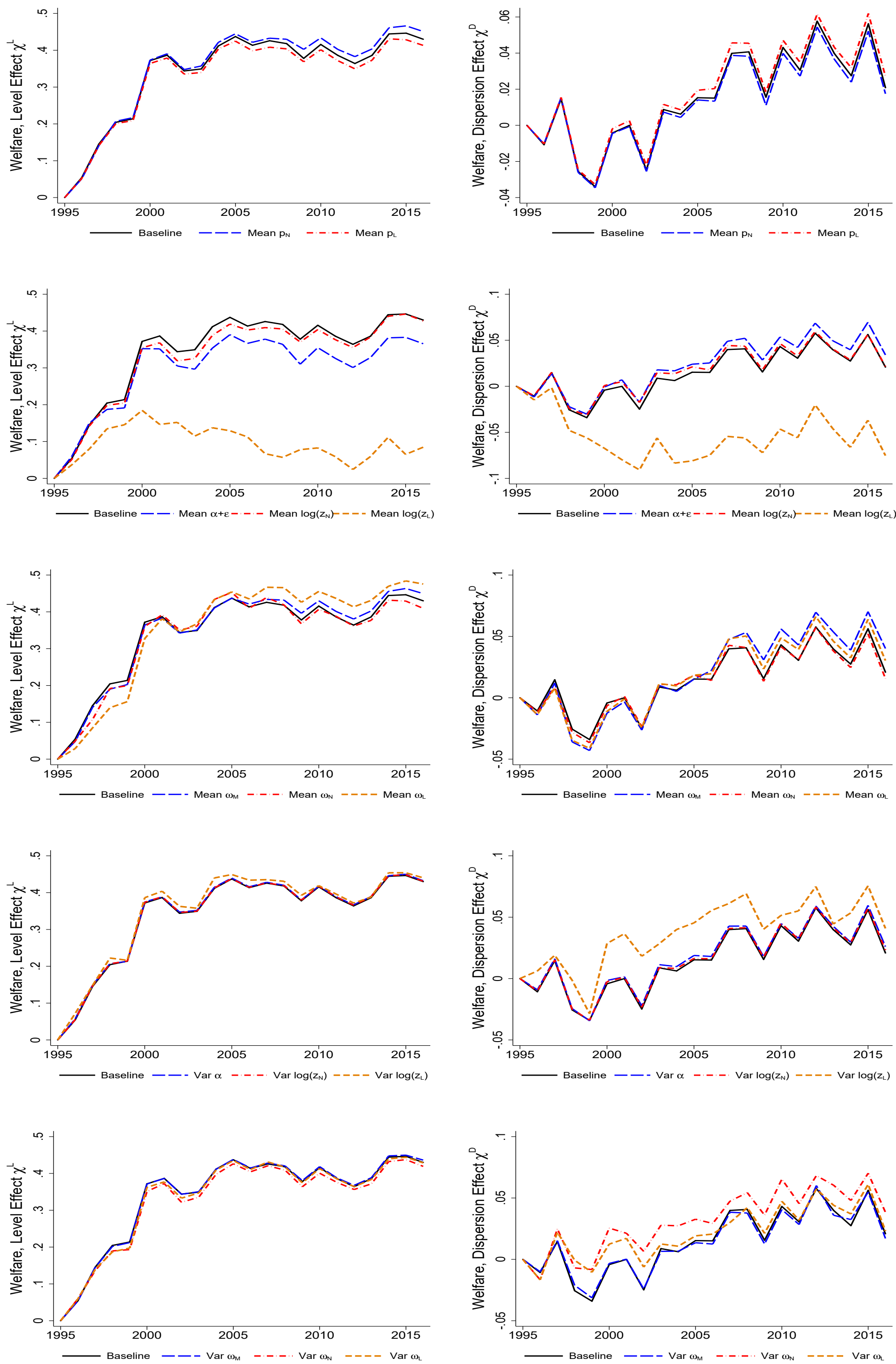


Figure A.1: Welfare: Counterfactuals ($\sigma_N = 3.5$)

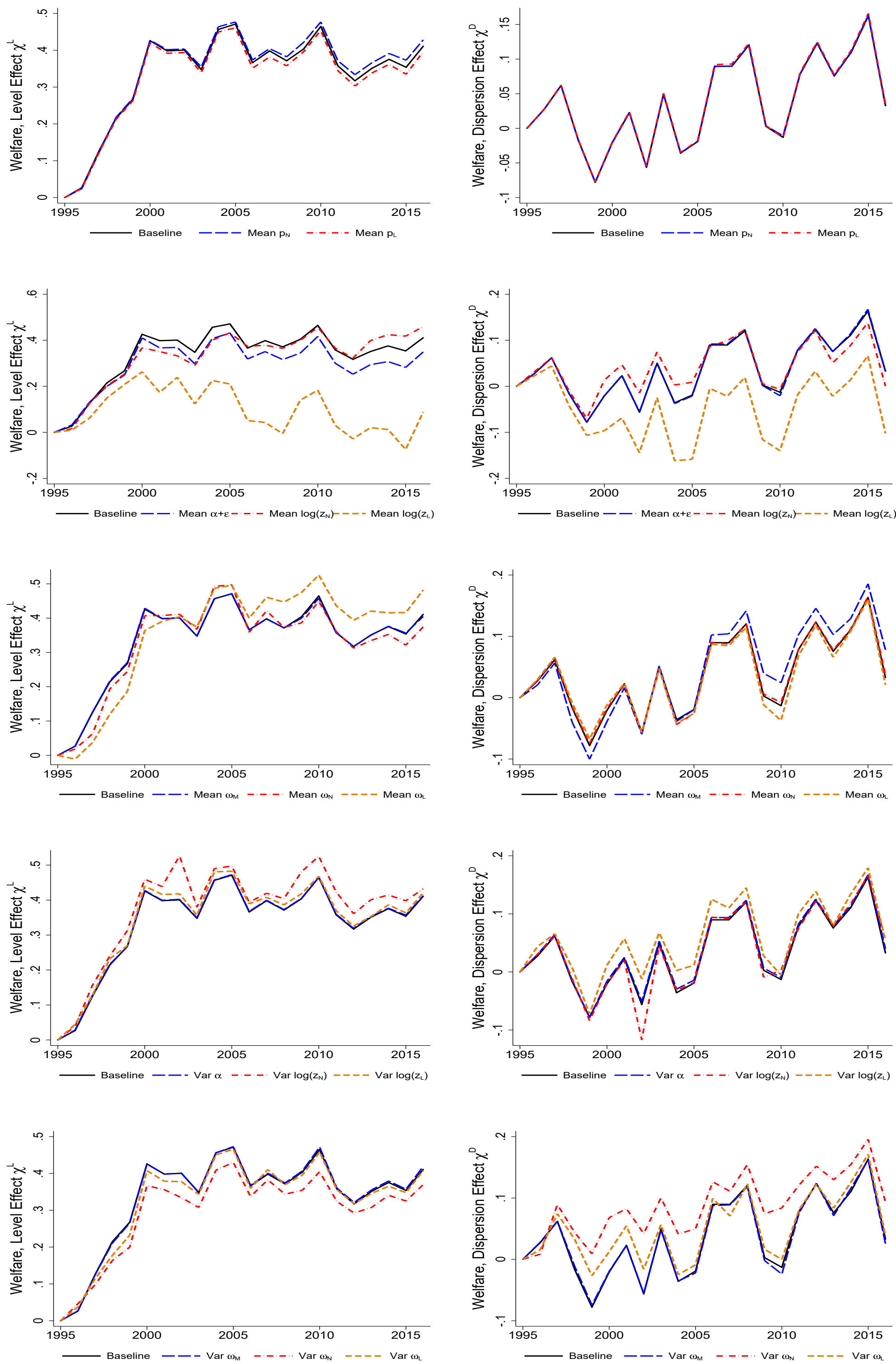


Figure A.2: Welfare: Counterfactuals ($\sigma_N = 1.5$)

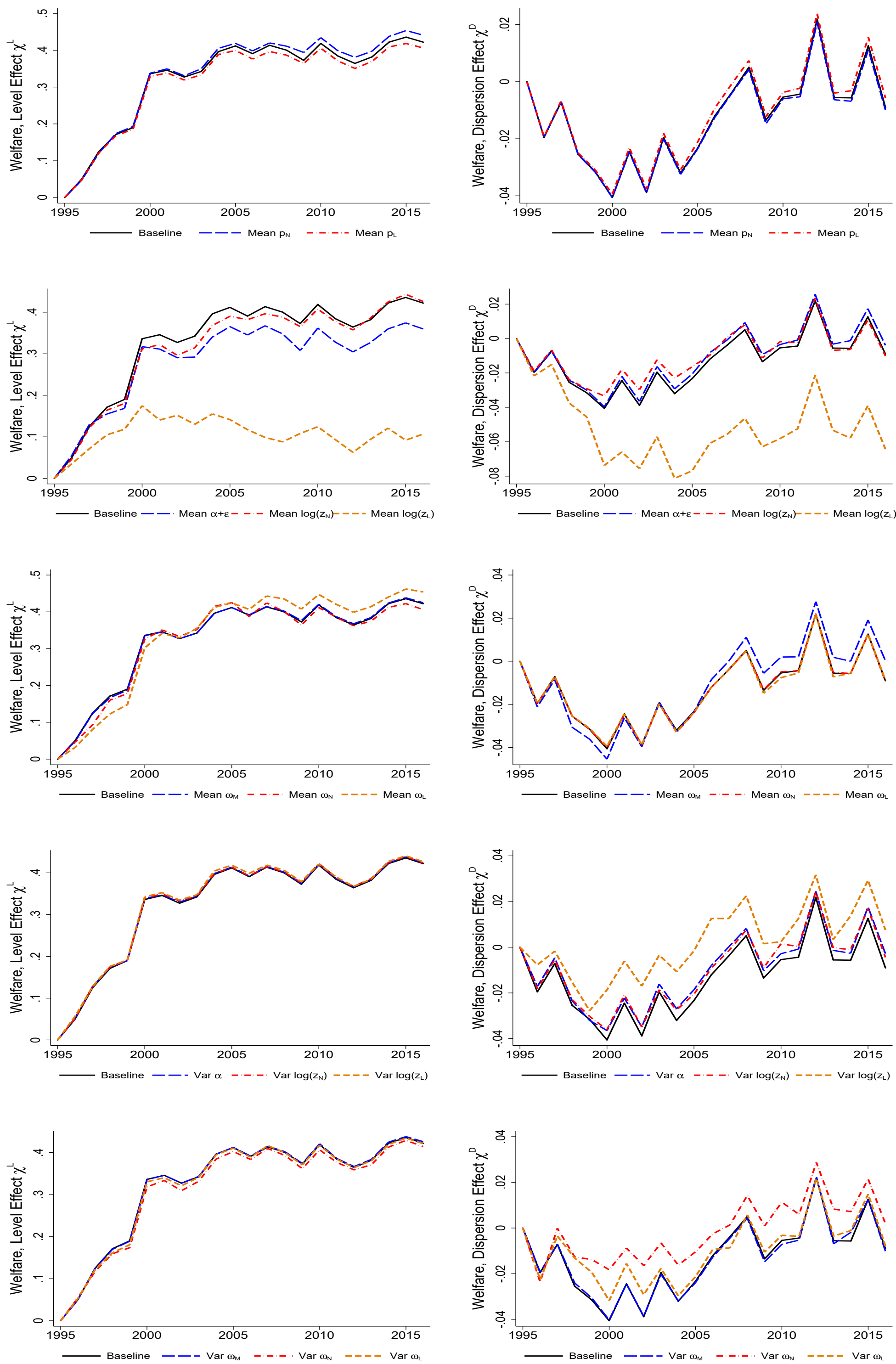


Figure A.3: Welfare: Counterfactuals ($\sigma_L = 0.2$)

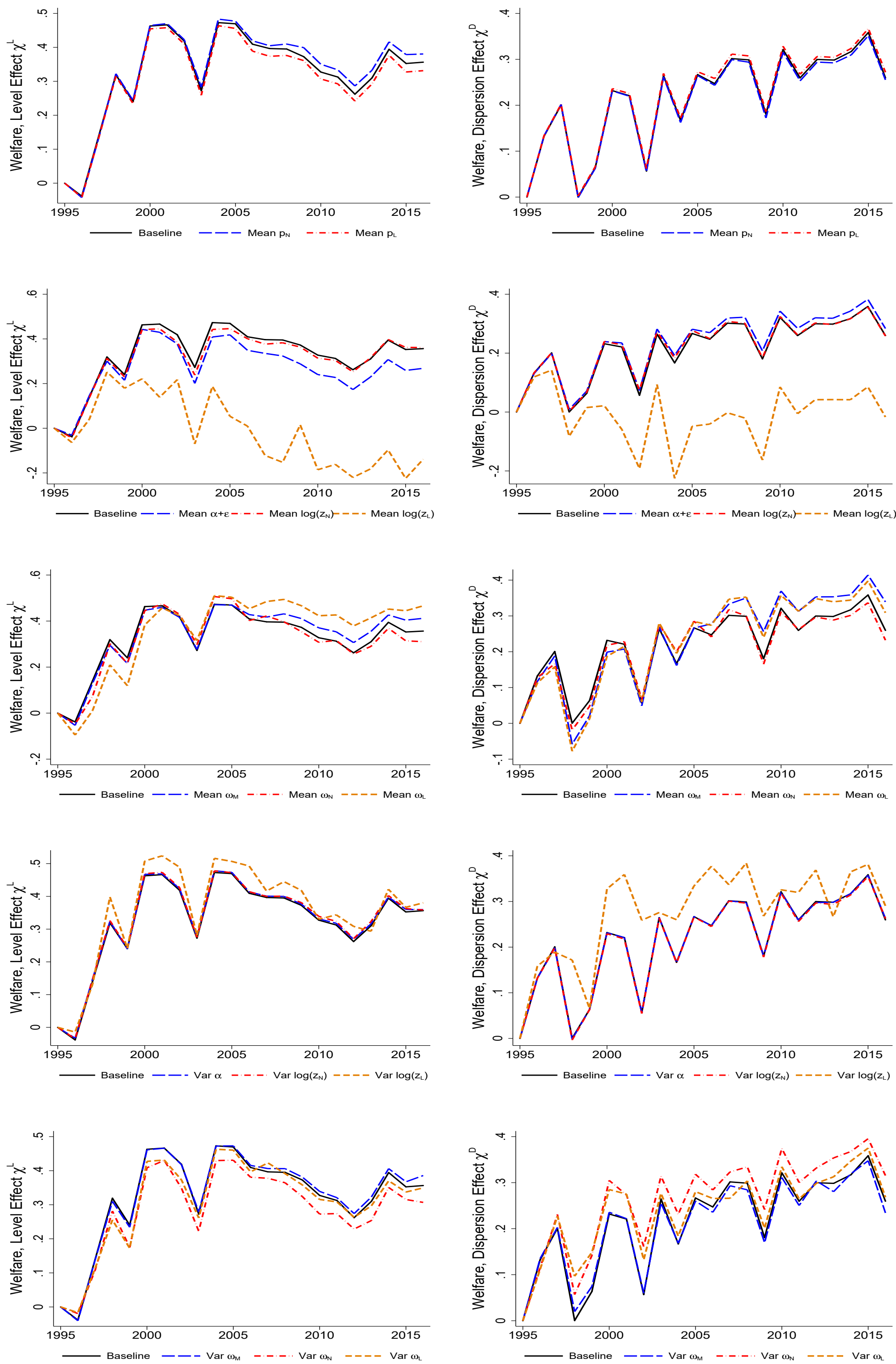


Figure A.4: Welfare: Counterfactuals ($\sigma_L = 0.8$)

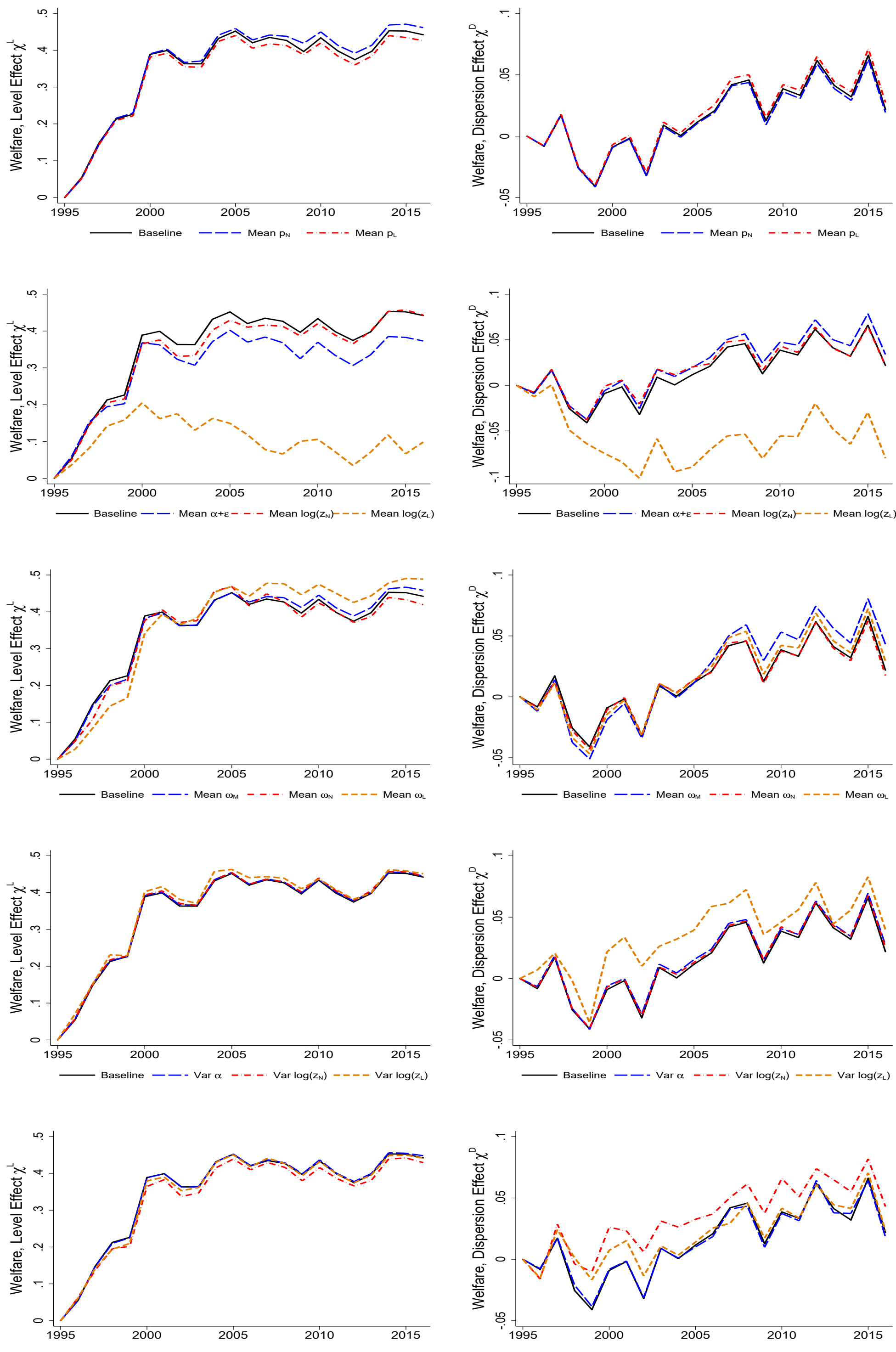


Figure A.5: Welfare: Counterfactuals ($\tau_1 = 0.06$)

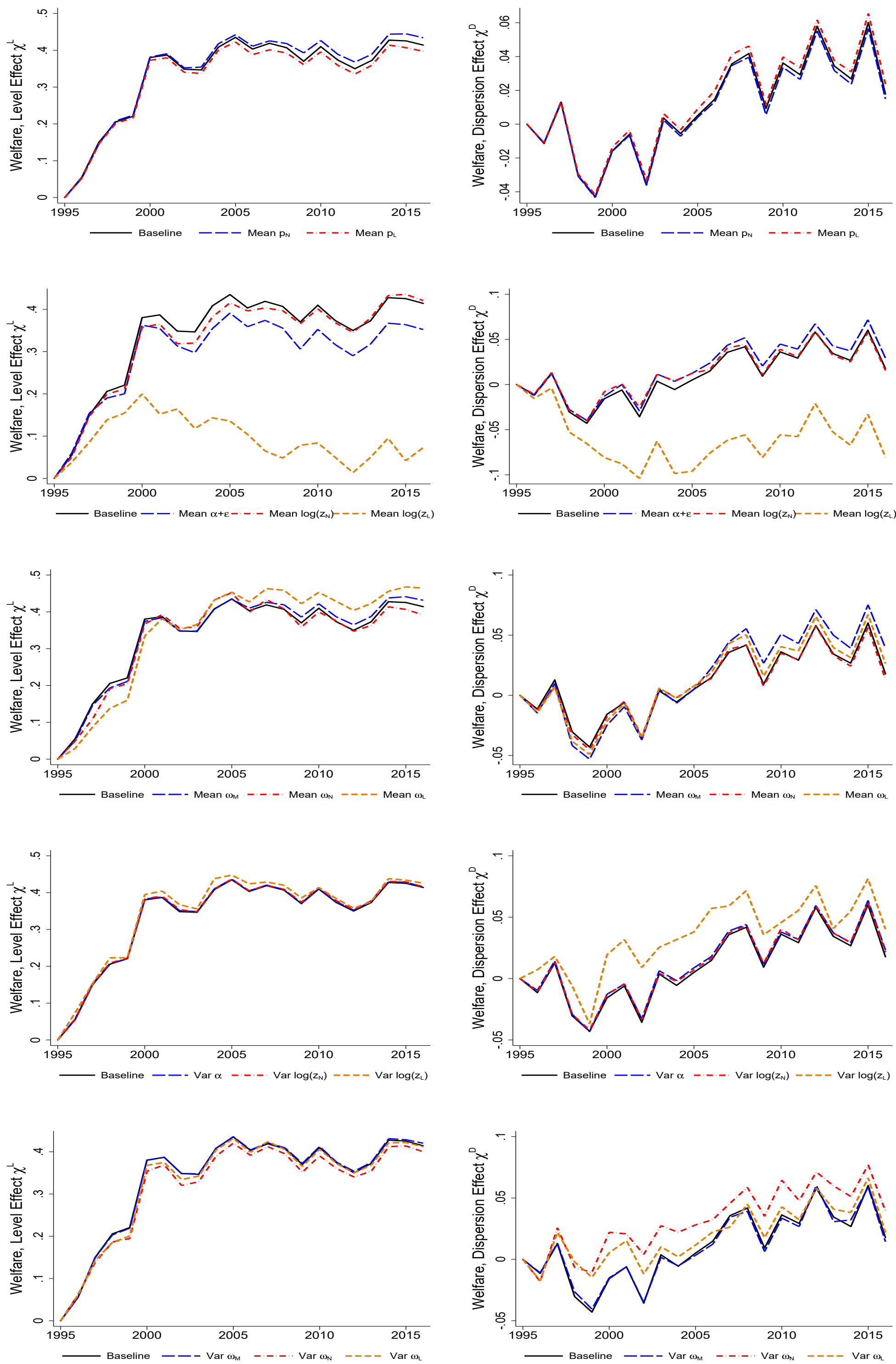


Figure A.6: Welfare: Counterfactuals ($\tau_1 = 0.18$)

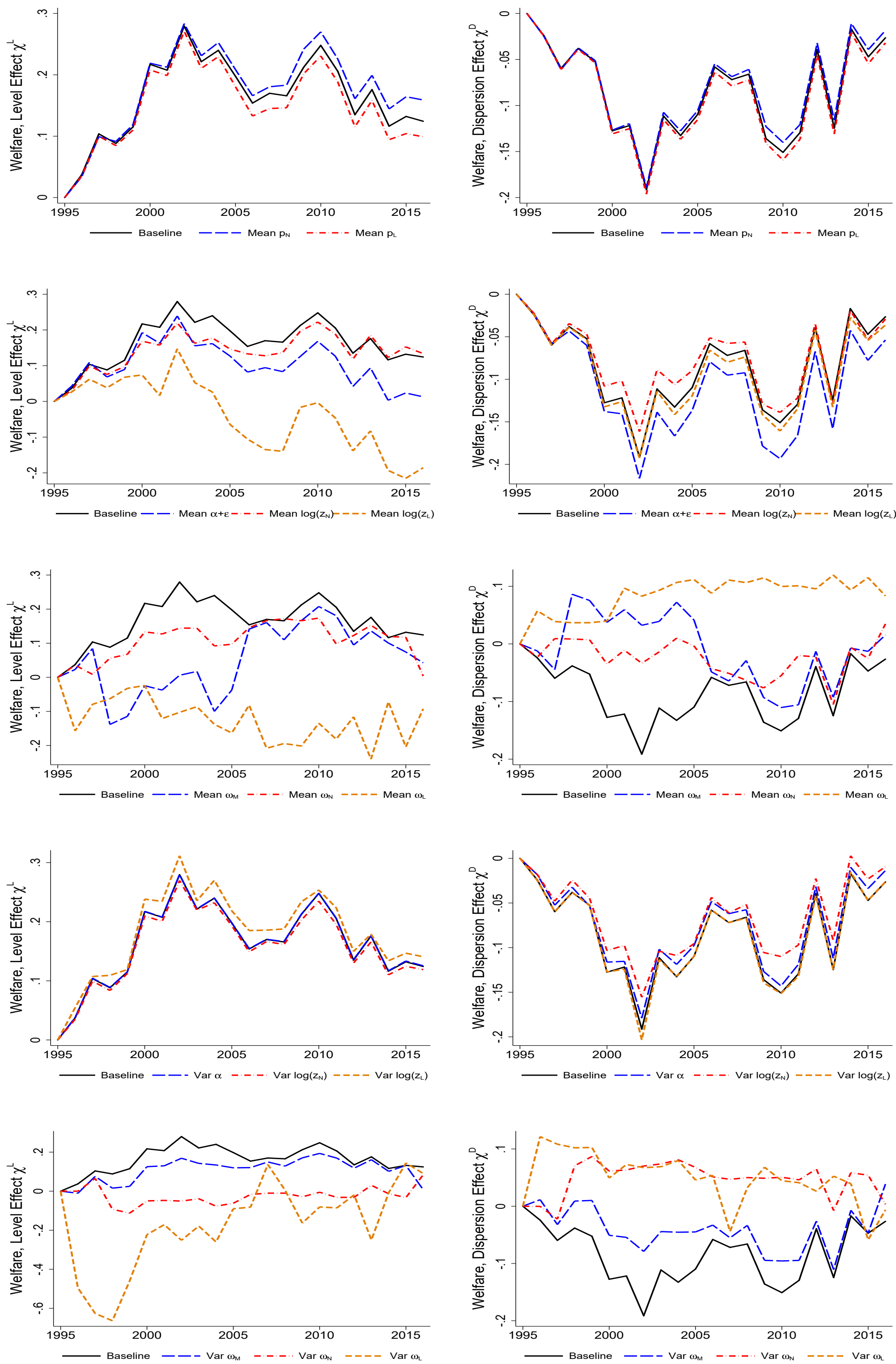


Figure A.7: Welfare: Counterfactuals ($\phi = 0.5$)

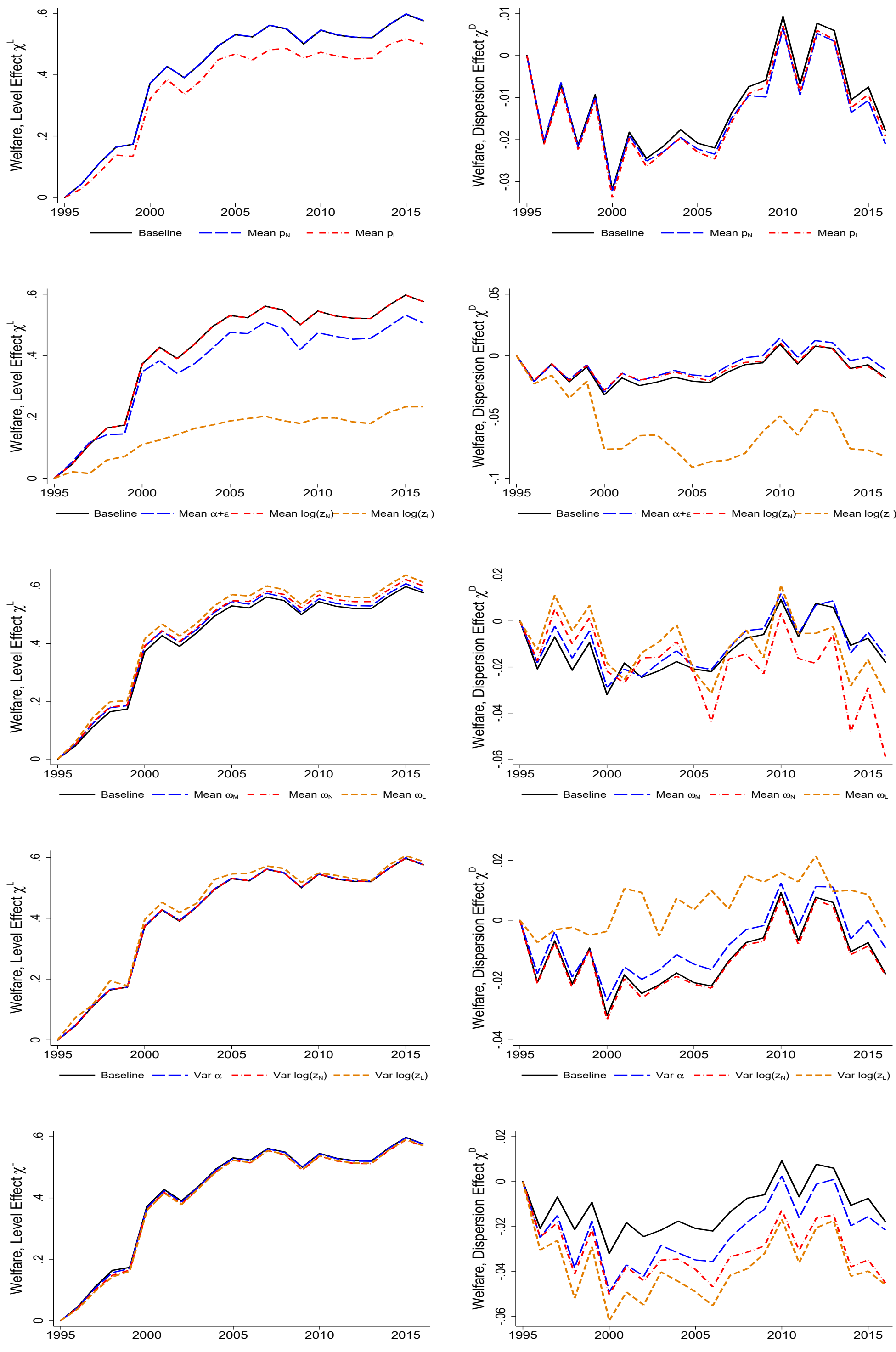


Figure A.8: Welfare: Counterfactuals ($\phi = 2.0$)

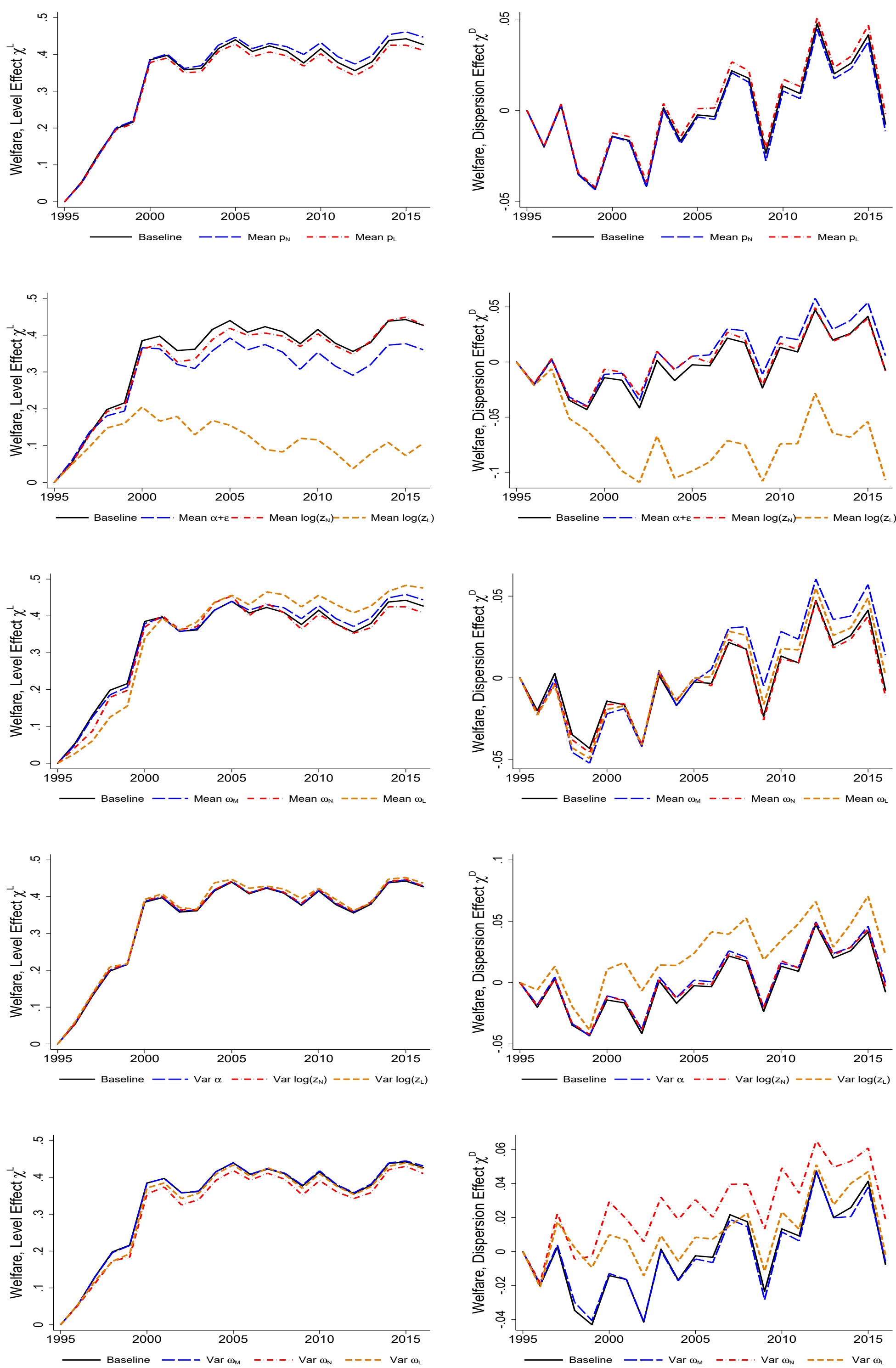


Figure A.9: Welfare: Counterfactuals (Adjusted Consumption Measures)

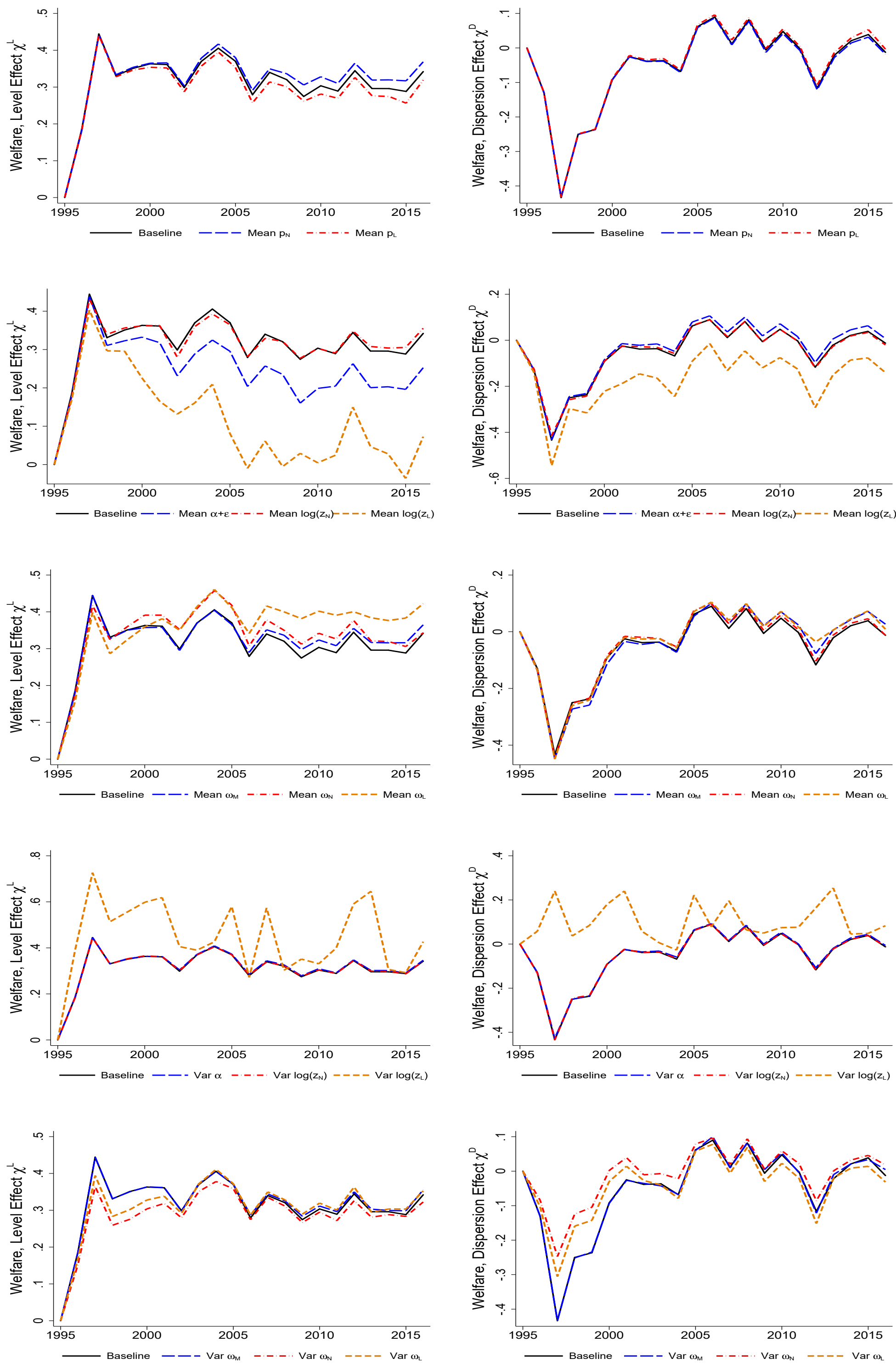


Figure A.10: Welfare: Counterfactuals (Singles)

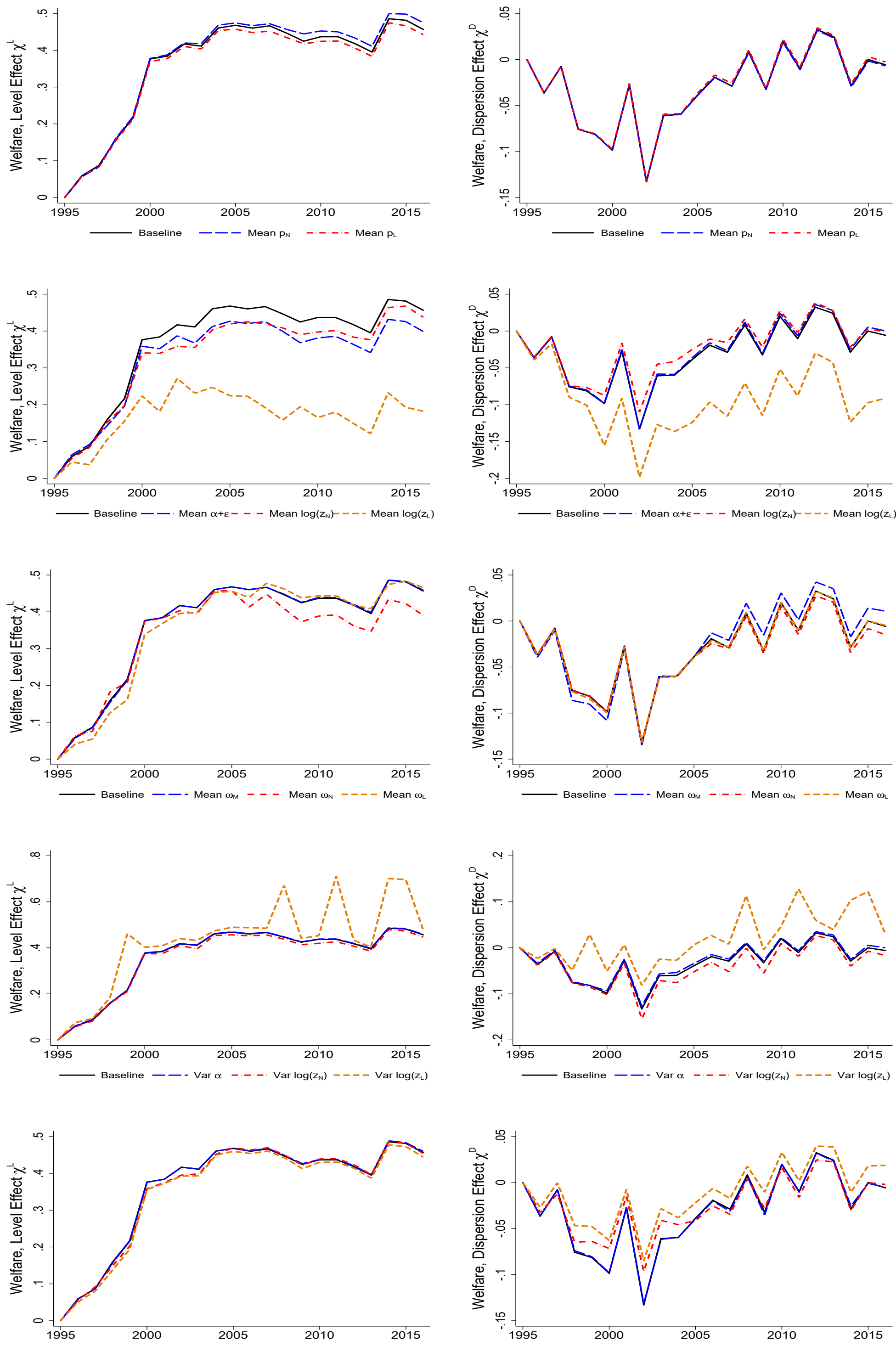


Figure A.11: Welfare: Counterfactuals (Alternative Measure of Leisure)

Chapter 4

Quantifying Efficient Tax Reform

Job Boerma and Ellen McGrattan

4.1 Introduction

This paper quantifies welfare gains from Pareto reforms in an overlapping generations framework with policies constrained due to private information about shocks to household labor productivity. We use administrative panel data for the Netherlands to first estimate key parameters under status quo policies for households in different education groups. We then solve for Pareto optimal reforms and decompose the source of welfare changes into gains from level effects and gains from improved insurance.

To model the Netherlands, we use a small open economy framework with overlapping generations and households that are heterogeneous in age, education, and productivity. Fiscal policy in this economy is summarized by tax schedules on incomes and assets and a tax rate on consumption. We compute values under current policy and use them—along with estimates for preferences, technologies, and wage processes—as inputs to our reform problem. In the reform problem, we compute the maximum consumption equivalent gain, which is restricted to be the same for all households.

For our baseline parameterization, we find large welfare gains, on the order of 17 percent of lifetime consumption. Optimal consumption allocations are higher and more smooth than allocations under current policy, while leisure allocations are lower and more volatile. To investigate this further, we decompose the total gain into contributions for level effects and contributions for improved insurance—for consumption and leisure. Increasing mean consumption is by far the largest source of gain, although some education groups with high variability in wages also have significant gains in lowering consumption dispersion.

We also show that the welfare decomposition is quantitatively sensitive to estimates of wage profiles and processes governing shocks to labor productivity. We explore two variations of the baseline model. First, we turn off growth in wages over the life cycle. In this case, the gains from smoothing consumption are close to zero, even for households with significant variation in their labor productivity shocks. Second, we lower the variances of shocks for all households. Here again, we find a significant effect on estimates of the gains for lowering dispersion in allocations.

This paper is related to the literature on optimal income taxation. We extend Kapička (2013); Farhi and Werning (2013), and Golosov, Troshkin, and Tsyvinski (2016) to compute Pareto reforms using a baseline matched to the Netherlands with a more general productivity process. Like Hosseini and Shourideh (2019), we compute the set of Pareto improving policy reforms, but we allow for stochastic productivity shocks. We find that allowing for stochastic shocks is quantitatively important for our welfare decomposition.

4.2 Theory

In this section, we describe the positive economy that is our baseline for estimating key parameters of preferences, technologies, and current fiscal policies of an actual economy. We then describe the associated planning problem used to quantify Pareto reforms of the original OLG economy.

4.2.1 Positive Economy

In this section, we describe the model economy that will be matched up to administrative data for the Netherlands. The environment is relatively standard with the exception of country-specific fiscal policies. There are a large number of households facing uninsurable productivity risks and perfectly competitive firms with constant-returns-to-scale technologies.

Households differ by age j, assets a, and productivity ϵ. They solve the following dynamic program:

$$v_j(a, \epsilon; \Omega) = \max_{c,n,a'} \left\{ U(c, \ell) + \beta E[v_{j+1}(a', \epsilon'; \Omega)|\epsilon] \right\}$$

subject to the budget constraint

$$a' = (1 + r)a - T^a(ra) + w\epsilon n - T^n(j, w\epsilon n) - (1 + \tau_c)c$$

and a lower bound on asset holdings: $a' \geq 0$. The aggregate state vector contains prices and policies:

$$\Omega = \{r, w, G, B, T^a, T^n, \tau_c\},$$

where r is the interest rate, w is the wage rate, G is government consumption, B is external debt, $T^a(\cdot)$ is the tax schedule for financial assets, $T^n(\cdot)$ is the tax schedule for labor income less transfers, and τ_c is the tax rate on consumption.

We assume the economy is small and open with interest rates r set in international markets. Firm technologies are constant-returns-to-scale functions in capital K and labor N with output Y given by:

$$Y = F(K, N).$$

Thus, knowing r, we also know the aggregate capital-labor ratio K/N and the wage rate w from the firm's optimality conditions. For computations below, we assume that prices and policies are fixed.[1] In a competitive equilibrium, the resource constraint must also hold in all periods:

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t + B_{t+1} - RB_t = Y_t.$$

The key outputs obtained from computing equilibria for the positive economy are the

[1]This assumption that policies are fixed can easily be relaxed without adding much computational burden as shown by Nishiyama and Smetters (2014).

values under current policy, namely

$$\vartheta(\epsilon^{j-1}) = E[v_j(a, \epsilon; \Omega)|\epsilon^{j-1}]$$

or, in the case of future generations $\vartheta(\epsilon_0) = E[v_1(a, \epsilon; \Omega)|\epsilon_0]$. We want to Pareto improve on these value, ideally by shifting the allocations to the efficient frontier. As an example, consider the two-person case drawn in Figure 1. The allocation for the Netherlands (point NL) is the result of calculating the equations above. In the next section, we compute a reform problem that puts households on the efficient frontier, at a point with their consumption levels higher by the same percentage (say, Δ).

4.2.2 Reform Problem

In this section, we describe the planning problem that we solve to compute Pareto reforms given the initial valuations from the positive economy.

As in the positive economy, the interest rate r is given as we are working with a small open economy. Given the production function $F(K, N)$, we can determine the optimal capital-labor ratio K/N and, hence, the marginal product of labor w. We also assume that the planner must finance government spending $\{G_t\}$ and takes the initial assets $B_0 + K_0$ as given.

Given the initial values for all households, the planning problem is to choose a feasible allocation that maximizes excess initial resources so that remaining lifetime values exceed their initial values for all households. Formally, the planning problem is:

$$\max\ F(K_0, N_0) + RB_0 - C_0 - K_1 + (1-\delta)K_0 - G_0 - B_1$$

subject to the laws of motion for capital and the resource constraints for all periods, along with incentive constraints that ensure truthful reporting of households private productivity, and a condition such that lifetime value exceeds the given initial value. In the appendix, we prove that an allocation is Pareto efficient if and only if it solves the planning problem, taking as given initial values that are generated by this allocation.

It turns out that the Lagrange function for the planning problem is separable in the allocation of each household and, therefore, we can separately characterize the solution to the planning problem for each household. The planner problem for each household is to choose a household allocation to maximize excess resources subject to the household's incentive constraints. To make this tractable, we assume that only local downward incentive constraints bind at the solution. Assuming that only the local downward incentive constraints bind is a finite type analog for the first-order approach typically adopted in dynamic Mirrlees problems with a continuum of productivity types. (See, for example, Kapička (2013); Farhi and Werning (2013), and Golosov, Troshkin, and Tsyvinski (2016).) The relaxed component planner problem is formulated by replacing the set of constraints that ensure global incentive compatibility in the component planning problem with the set of constraints that ensure the allocation satisfies all local downward incentive constraints. We write the relaxed component planner problem recursively and then characterize its solution.

This relaxed recursive problem can be formalized as follows. The planner chooses sequences of consumption $c_j(\epsilon)$, labor $n_j(\epsilon)$, promised values $V_j(\epsilon)$ for telling the truth about the productivity type, and threat values $\tilde{V}_j(\epsilon)$ for reporting a productivity type of ϵ while being one level more skilled, which we denote by ϵ^+. The recursive planning problem is given by:

$$\Pi_j(V_-, \tilde{V}_-, \epsilon_-) \equiv \max \sum_{\epsilon_i \in \mathcal{E}} \pi_j(\epsilon_i|\epsilon_-)\Big(w_t \epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R\Big)$$

subject to:

$$U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)$$
$$\geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i),\ \textit{for}\ i = 2, \ldots, N \tag{1}$$
$$V_- = \sum_{\epsilon_i \in \mathcal{E}} \pi_j(\epsilon_i|\epsilon_-)\Big(U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\Big) \tag{2}$$
$$\tilde{V}_- = \sum_{\epsilon_i \in \mathcal{E}} \pi_j(\epsilon_i|\epsilon_-^+)\Big(U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)\Big), \tag{3}$$

where $\pi_j(\epsilon_i|\epsilon_-^+)$ is the conditional probability over current states ϵ_i for households that were one level more productive in the previous period ϵ_{-}^{+}. The first set of constraints in 1 ensures that utility is higher under truth-telling, with the leisure arguments given by:

$$\ell_j(\epsilon_i) = 1 - n_j(\epsilon_i) \tag{4}$$

$$\ell_j^{+}(\epsilon_{i-1}) = 1 - n_j(\epsilon_{i-1})\epsilon_{i-1}/\epsilon_i. \tag{5}$$

When calculating the welfare of efficient reform, we replace V_{-} in the problem above at the initial age with $\vartheta(\epsilon_0) + \vartheta_\Delta$, where $\vartheta(\epsilon_0)$ is the initial value for future generations—that is, $E[v_1(0, \epsilon; \Omega)|\epsilon_0]$ in the positive economy—and the ϑ_Δ is the value corresponding to giving Δ more consumption to households.

4.3 Data and Estimation

In this section, we discuss the administrative data from the Netherlands and estimation methods used to parameterize the model. We start with aggregated data from the national accounts, flow of funds, population censuses, and tax authorities. We then discuss the micro data on earnings, hours, and education.

4.3.1 Aggregate Data

The main data source for the aggregate data is the Dutch Bureau of Statistics. These data are publicly available.

National accounts

The primary data source for national income and product accounts is the *nationale rekeningen*. Table 1 splits national income by factor of production. Labor income includes compensation of employees and 70% of proprietors' income. All other income is categorized as capital income, which we adjust in three ways. First, we subtract product-specific taxes as measured in the government's income and expenditure accounts. We make this correction because we are interested in production at producer prices rather than at consumer prices.

Second, we impute capital services for consumer durables—which we treat as investment—and government capital. The imputed services are assessed to be 4 percent of the current-cost net stock of consumer durables and government fixed assets. Government fixed assets as well as consumer durables are recorded as non-financial balances. Finally, we impute depreciation of consumer durables. Since our data do not include the equivalent of the United States flow of funds, we assume the ratio of consumer durable depreciation to consumer durable goods to be identical to the United States.[2] This implies consumer durable depreciation of 5 percent.

On the product side, revisions must also be made with regard to sales taxes, capital services and consumer durables depreciation. The sales taxes are assumed to primarily fall on personal consumption expenditures. We assume pro rata shares when assessing how much of the taxes are on durables, non-durables and services. We include nondurables and services with consumption and durable goods with tangible investment. Therefore, we subtract sales taxes from both product categories. Imputed capital services only affect our consumption measure, which combines personal and government consumption from the national accounts. The consumption of consumer durables depreciates the outstanding stock of durables, which motivates us to classify consumer durables depreciation as consumption.

Fixed assets and other capital stocks used in our analysis are shown in Table 2 with averages for 2000-2010. As in the case of national accounts, we divide all estimates by adjusted GDP. We add the stock of consumer durables. The data are separated for businesses, households, and the government. We also include the value of land, which is much higher than estimates reported by McGrattan and Prescott (2017) for the United States. In fact, the data show that the value of residential land exceeds the value of structures by roughly 12 percent, likely due to strict land-use regulation. Since the oil and gas sector is so significant for the Netherlands, we include reserves. Related to fixed assets are the valuations in flow of funds data which we report in Table 3. Here, we report estimates for household net worth and government debt relative to GDP averaged over the sample 2000-2010.

Finally, in Table 4, we report aggregates on population and hours, which we use to parameterize preferences and to check aggregated micro data. Averaging data between 2000

[2]See Table 1 in McGrattan and Prescott (2017).

and 2010, we estimate that the Dutch population worked 12,243 million hours, implying average annual hours of 1,135 for every individual between ages 16 and 64.

The data from the national accounts and population census are used to parameterize the discount factor, the capital share, the depreciation rate, the weight on leisure in preferences, the length of working life, and the length of retirement.

Fiscal Policy

In Figure 2, we plot the income tax schedule for the Netherlands during our sample period. The figure shows three marginal tax rates, namely, 34, 42 and 52 percent for working age households, with cutoff levels of 20,000 and 59,000 Euro. Marginal tax rates are reduced for retirees with incomes below 35,000 Euro. Specifically, the marginal tax rate is 17 percent for incomes below 20,000 Euro and 24 percent for incomes below 35,000 Euro. In Figure 3, we show the tax schedule for financial assets. Below 46,000 Euro, the tax rate is 0. Above this level, the rate of taxation is 1.2 percent. Finally, we assume an effective tax rate on consumption of 13.4 percent, which is the weighted average VAT for a basket of goods in the Netherlands. These schedules and rates are used to parameterize T^a, T^n, and τ_c.

4.3.2 Micro Data

We use linked administrative records between 2006 and 2014 from Statistics Netherlands for the information on education, earnings, and hours—series that we need to estimate productivity processes $\{\epsilon\}$ and wage profiles $\{\zeta\}$ over the life cycle for different education groups.

Merged datasets

We start with a representative subsample of all Dutch households selected by Statistics Netherlands. The sample consists of roughly 95 thousand households per year, which is 1.3 percent of the population of households, covering a total of over 275 thousand individuals. For all analyses, we weight households with the provided sample weights. We consider all households with heads of household above age 25. Income is measured by employer-provided earnings records. We construct an individual's annual taxable labor earnings, which includes the employer's health insurance contribution, by adding all earnings reports within a given calendar year. To construct an hourly wage rate, we merge the earnings dataset with a dataset on employer-reported hours worked, dividing taxable labor earnings by hours of work. Because the model features a single decision maker for each household, we define the household wage rate for married and cohabitating households as the average individual wage rate weighted by the hours worked of each partner. For single households, the individual wage rate is the household wage rate. Household non-market time is given by average individual non-market time which is discretionary time minus individual hours worked. We set an individual's discretionary time equal to 16 hours a day for 365 days.

We merge the datasets for earnings and hours with another that provides education levels for our sample. We need this information because we assume that there is ex-ante heterogeneity in productivity and wage profiles based on the highest educational degree earned. We classify every degree as a low, a medium, or a high level of education. The low education level is a high school degree or a practical degree, the medium level is a degree from a university of applied sciences, and the high level is a university degree. We group households into six education bins, which are unordered pairs of the degree of each partner. Singles are grouped with couples in which both partners have obtained the same level of education.[3]

We should note here that there are significant advantages to the merged data available in the Netherlands relative to what is available in most other countries. For example, in the case of the United States, we only have administrative data for earnings whereas in the Netherlands we have earnings and hours linked and available for all members of the household. We also have detailed data on education, which is not available in the United States.

[3] In our sensitivity analysis, we also explore conditioning on head of household, which is more common in the literature.

Estimated wage processes

We estimate the parameters that govern the residual wage process using the minimum distance estimator (Chamberlain (1984)). We first regress logarithmic wages on as follows:

$$\log W_{ijt} = A_t + X_{ijt} + \omega_{ijt},$$

where the household index is i, age is j, and the period is t. The right-hand-side variables are time effects A_t and household observables X_{ijt}. The observables include a set of dummy variables for the age of the household head, the coefficient of which is our estimate of the lifecycle profile ζ_j.

The second step is to estimate components of the residual wage after pooling across cohorts. More specifically, we use the method of simulated moments approach to estimate parameters ρ, σ_u^2, σ_η^2, $\sigma_{\epsilon 0}^2$ for the standard permanent-transitory process:

$$\begin{aligned} \omega_{ij} &= \epsilon_{ij} + \eta_{ij} \\ \epsilon_{ij} &= \rho\epsilon_{ij-1} + u_{ij} \end{aligned}$$

with the persistent component of the residual wages given by ϵ_{ij} and the transitory component given by η_{ij}. The error processes and initial conditions are assumed to be distributed normally, that is $\eta_{ij} \sim \mathcal{N}(0, \sigma_\eta^2)$, $u_{ij} \sim \mathcal{N}(0, \sigma_u^2)$, and $\epsilon_{i0} \sim \mathcal{N}(0, \sigma_{\epsilon 0}^2)$.

The moments we use to identify the parameters are the variances and first-order autocovariances. These moments can be written in closed form as follows:

$$\begin{aligned} \mathrm{var}(\omega_{ij}) &= \rho^{2j}\sigma_{\epsilon 0}^2 + \frac{1-\rho^{2j}}{1-\rho^2}\sigma_u^2 + \sigma_\eta^2 \\ \mathrm{cov}(\omega_{ij}, \omega_{ij-k}) &= \rho^k \frac{1-\rho^{2(j-k)}}{1-\rho^2}\sigma_u^2 + \rho^{2j-k}\sigma_{\epsilon 0}^2. \end{aligned}$$

These expressions are functions of (j, k) and the four parameters.

The estimation of the wage process uses the minimum distance estimator introduced by Chamberlain (1984), which minimizes a weighted squared sum of the differences between each moment in the model and its data counterpart. Let $m(\Lambda)$ be the vector of theoretical covariances and Λ be the parameter vector. The data counterpart is given by $\hat{m}$. In this case, the estimator solves:

$$\min_{\Lambda} (\hat{m} - m(\Lambda))' W (\hat{m} - m(\Lambda)),$$

where W is a weighting matrix. For our baseline parameterization, we use the identity matrix for W. To compute confidence intervals, we bootstrap using 1,000 replications. Given the closed form expressions for the theoretical moments, the objective function is efficiently evaluated.

We use the estimated parameters ρ and σ_u^2 to parameterize the residual wage process in the model.[4] The results of our estimation procedure are reported in Table 5. We find the parameters are precisely estimated with estimates for $\hat{\rho}$ in the range of 0.95 to 0.97 across education groups. If we construct estimates of variation for the residual wages, that is, $\hat{\sigma}_u^2/(1-\hat{\rho}^2)$, we find that households with a low and high member and those with two high members are close to twice as variable as the others.

In Figure 4, we report the life-cycle wage profiles (ζ_j) for the 6 education groups. The right side of the figure shows the population for each group. For example, the low-low group is the largest with 43 percent of the working population. We have normalized these estimates by dividing each profile using the average wage rate for the entire population. Not surprising, we find a steep rise between ages 25 and 45 for all groups, with the lowest higher by roughly 40 percent and the highest by roughly 200 percent.

4.3.3 Computation

When we compute equilibria for our positive economy and our reform problem, we approximate labor productivity shocks by a Markov chain with 20 types. For both problems we assume that baseline preferences are logarithmic, that is,

$$U(c, \ell) = \gamma \log c + (1-\gamma) \log \ell.$$

[4] We assume η is a shock that households can insure against, and we use the ergodic distribution based on ρ and σ_u^2 to parameterize the initial distribution of productivities.

Both problems are parallelizable and thus we can solve them quickly on most modern computer clusters. As a check on these choices, we will recompute results for a 40-type case and for different preferences.

4.4 Results

In this section, we report our main findings for the baseline model and several alternative parameterizations, namely, cases with double the number of types, alternative preferecnes, no wage growth, and lower variances in the shock processes.

4.4.1 Baseline

The main deliverables for our baseline model are labor wedges and welfare gains. We compute labor wedges for each education group. These wedges represent distortions used by the planner to incentivize individuals and to provide insurance across time and across types. We then report consumption-equivalent welfare gains and their decomposition into gains from increasing the level of consumption, gains from reducing dispersion in consumption, gains from increasing the level of leisure, and gains from reducing the dispersion in leisure.

The labor wedges are defined as follows:

$$\tau_n(\epsilon^j) = 1 - \frac{1}{w}\frac{U_\ell(c(\epsilon^j), \ell(\epsilon^j))}{U_c(c(\epsilon^j), \ell(\epsilon^j))}$$

and computed for each education group. Equation 4.4.1 tells us that in the optimal allocation there is a wedge between the wage rate w and the marginal rate of substitution between consumption and leisure. We report these wedges for the baseline model in Figure 5. The highest wedge is not that of the high-high group, but rather the low-high group. The reason is that the low-high group has the most variable wage process. The greater the dispersion in productivities, the greater are gains from insurance and higher is $\tau_n(\epsilon^j)$. In fact, if we were to take averages, we would find a positive correlation between the total variance $\hat{\sigma}_u^2/(1-\hat{\rho}^2)$ and the wedge across education groups. This information is useful for the reform of current policy.

In Table 6, we report the welfare gains and its decomposition for our baseline parameterization. We find a total gain of 17 percent for an efficient reform in which all individuals are made better off by the same percentage. Building on Floden (2001) and Benabou (2002), we decompose this total gain into the gain from increasing consumption, the gain from smoothing consumption, the gain from increasing leisure, and the gain from smoothing leisure. That is, we take the total consumption-equivalent gain Δ and compute:

$$\begin{aligned}\log(1-\Delta) &= \log((1-\Delta_c^L)(1-\Delta_c^D)) \\ &\quad + (1-\gamma)\log((1-\Delta_\ell^L)(1-\Delta_\ell^D))/\gamma.\end{aligned}$$

Let x be the allocation in the planner problem and $\hat{x}$ be the allocation in the positive economy. Then we define the gain due to a level increase in $x = \{c, \ell\}$ as

$$1-\Delta_x^L = \frac{\sum \pi(\epsilon^j)\hat{x}(\epsilon^j)}{\sum \pi(\epsilon^j)x(\epsilon^j)}.$$

We define the gain due to a reduction in dispersion in $x = \{c, \ell\}$ as:

$$\begin{aligned}1-\Delta_x^D &= \sum \beta^j \pi(\epsilon^j)\log\left(\frac{\hat{x}(\epsilon^j)}{\sum \pi(\epsilon^j)\hat{x}(\epsilon^j)}\right) \\ &\quad - \sum \beta^j \pi(\epsilon^j)\log\left(\frac{x(\epsilon^j)}{\sum \pi(\epsilon^j)x(\epsilon^j)}\right)\end{aligned}$$

The results of the decomposition are shown in Table 6. First, note that the summing across rows yields the total gain of 17 percent (and may be off because of rounding). Second, note that there are large gains for increasing and smoothing consumption, but the optimal plan calls for lower and more dispersed leisure than in the positive economy. The gains from increasing consumption are the most significant. For the low-high and high-high groups, there are also significant gains from reducing dispersion since the variances of wages are largest for these groups. The planner lowers leisure the most for the most productive. However, in terms of leisure dispersion, the most noteworthy group is high-high that face significantly more leisure dispersion under the optimal plan relative to the positive economy.

If we consider these results in light of more simple models—say, static models with and without insurance—we find that our results are in line with the simpler models. For example, consider the case in which there is no insurance and households maximize 4.3.3 subject to a budget constraint that consumption is less than or equal to after-tax labor earnings. The optimal plan in that case calls for variation in consumption but constant leisure. If instead there was full insurance, a planning problem would call for constant consumption and variation in leisure. The positive economy is closer to the no-insurance case and the reform problem is closer to the full-insurance case.

In Figure 6, we show the allocations of (log) consumption and leisure along with their variances for those in the low-low group. In the upper left panel of the figure, we have plotted consumption for ages 25 to 64. We see from the figure that the planner can completely smooth mean consumption, which is not possible under current policy due to the borrowing constraint. In the upper right panel of the figure, we have plotted the variance of consumption. Dispersion is lowered in early years, but is higher than the positive economy later in life. In the lower panels, we plot the results for leisure. As predicted, leisure is lower in the reform problem than the positive economy for most years. while the variance is higher.

4.4.2 Sensitivity

We recompute total consumption-equivalent gains and their decomposition in three alternate specifications. First, we double the number of types from 20 to 40. Second, we consider alternative preferences with a lower Frisch elasticity. Third, we set the wage profiles in Figure 4 to 1 for all types and all ages. Fourth, we lower the variance σ_u^2 of the labor productivity shocks for all education groups. In each case, we compare results for the levels and dispersion of allocations across ages for the low-low group to the baseline model.

The first set of results are shown in Table 7. In this case, we find a slightly lower overall welfare gain of 16 percent in the case with 40 types (and again 16 percent if we continue increasing the number of types). If we compare the decomposition of gains with the baseline model, we find only small differences. Similarly, if we compute statistics over the life cycle, we find that the differences are not large. This is evident in Figure 7 where we compare the allocations—both levels and variances—for the two models. We see almost no change in levels and small changes in the variances of the allocations.

In Table 8, we report results for an alterative specification of preferences, namely,

$$U(c,\ell) = \gamma \log c - \kappa \frac{(1-\ell)^{\alpha}}{\alpha}$$

which has a constant Frisch elasticity that we set equal to 1/2 (by setting $\alpha = 3$. In this case, the overall welare gain is 14 percent and the decomposition, not surprisingly, shows a smaller contribution from lowering dispersion in leisure. Figure 8 shows the comparison of the baseline statistics with those for these alternative preferences for the low-low group. Consistent with the welfare decomposition, we find little difference between means and variances of leisure in the positive and reform economies. In the case of consumption, there are some differences in the levels of means and variances, but the shifts are similar in the case of both economies.

If we set wage profiles equal to 1 for all ages and groups, we find a lower overall gain of 14 percent when compared to the 17 percent gain in the baseline. In Table 9, we see that the main difference between these models is the attribution of gains for smoothing consumption. If the profiles are flat, there are no gains to smoothing consumption: they are already smooth. In the case of the low-low group, we see in Figure 9 that the positive economy consumption and leisure allocations are already quite flat.

Finally, we compare the baseline model to one with the labor productivity variance σ_u^2 lowered by two-thirds. The overall gain in the latter case is 19 percent, higher than the baseline and, not surprising, the gains for reducing consumption and leisure dispersion are less significant than gains from changing levels. These results are reported in Table 10. In Figure 10, we again compare allocations to the baseline model for the low-low group. Here, we find no difference in the levels, but lower variances for consumption and leisure in the alternative model when compared to the baseline.

In all experiments, we find large gains to the Pareto reforms, but the sources of the gains do depend on the estimated processes for labor productivities.

4.5 Conclusion

In this paper, we computed efficiency gains of Pareto reforms in an environment with policies constrained due to private information about shocks to household labor productivity. Using administrative data for the Netherlands, we found the gains to be large but also found that quantifying the sources of these gains depends importantly on having precise estimates for household wage profiles and shock processes for labor productivity.

Appendix

In the main text we present a planning problem and discuss how we characterize its solution using relaxed and recursive household planning problems. In this appendix we describe the intermediate steps. We start with a general specification of the planning problem to be solved and our notion of efficiency. We then describe steps to be taken to compute a solution. Our discussion closely follows Boerma (2020).

The problem for the planner is to maximize the present value of aggregate resources subject to incentive constraints and meeting minimum bounds on lifetime utility for all households. A household i is identified by a birth year b and a history of shocks to productivities ϵ^{j-1} at the time of the reform, that is, $i \equiv (b, \epsilon^{j-1})$. The set of households considered by the planner at the time of the reform includes those currently alive and those that will be born in future periods. In period 1, this set is:

$$\mathcal{I} \equiv \left\{ \{(1-j, \epsilon^{j-1})\}_{j=1}^{J}, \{(b, \epsilon_0)\}_{t=1}^{\infty} \right\}.$$

We use the notation $x(i)$ for household $i = (t, \epsilon^{j-1})$'s allocation of consumption and labor:

$$x(i) \equiv \{x_{b+s}(\epsilon^s)\}_{s=1}^{J},$$

where $x_{b+s}(\epsilon^s) = (c_{b+s}(\epsilon^s, n_{b+s}(\epsilon^s)$. We use the notation x to summarize both the allocations of all households and the aggregate quantities of consumption C_t, hours N_t, end-of-period holdings of foreign assets B_{t+1}, and end-of-period capital K_{t+1}:

$$x \equiv \left\{ \{x(i)\}_{\mathcal{I}}, \{(C_t, N_t, B_{t+1}, K_{t+1})\}_{t=1}^{\infty} \right\}.$$

We study a small open economy and assume the returns on assets are given by rate R and that capital depreciates at rate δ, with the aggregate resource constraint given by:

$$C_t + K_{t+1} - (1-\delta)K_t + G_t + B_{t+1} \leq F(K_t, N_t) + RB_t.$$

The aggregate consumption and hours can be written as:

$$
\begin{aligned}
C_t &= \sum_{j=1}^{J} \int_{\mathcal{E}^j} c_t(\epsilon^j) \mathrm{d}\mu_j(\epsilon^j) \\
N_t &= \sum_{j=1}^{J} \int_{\mathcal{E}^j} \epsilon_j \zeta_j n_t(\epsilon^j) \mathrm{d}\mu_j(\epsilon^j)
\end{aligned}
$$

where ζ_j is the deterministic age profile of productivity.

Households have private information about the history of productivity shocks, so we need to specify the information sets and the reporting strategies of households. Here, we assume that households make a report $m_j \in \mathcal{E}_j$ at age j about their type ϵ_j, where $\mathcal{E}_j$ is the set of all possible productivities at age j. A strategy specifies a report for every productivity history, $\sigma = \{\sigma_j(\epsilon^j)\}_{\epsilon^j, j}$, where σ_j maps the history of shocks to the current message, that is, $\sigma_j : \epsilon^j \to m_j$. The history of reports is summarized as:

$$
\sigma^j(\epsilon^j) = (\sigma_1(\epsilon^1), \ldots, \sigma_j(\epsilon^j)).
$$

The maximization problem for the planner is:

$$
\max_x \sum_{t=1}^{T} R^{-t}(wN_t - C_t) + K_0 + B_0 - \sum_{t=1}^{T} G_t / R^t
$$

subject to the following constraints for all i, j_i, and all reporting strategies σ:

$$
\sum_{j=j_i}^{J} \beta^{j-j_i} \int_{\mathcal{E}^j} U(c_t(\epsilon^j), n_t(\epsilon^j)) \geq \sum_{j=j_i}^{J} \beta^{j-j_i} \int_{\mathcal{E}^j} U(c_t(\sigma^j(\epsilon^j)), n_t(\sigma^j(\epsilon^j))) \tag{6}
$$

$$
\sum_{j=j_i}^{J} \beta^{j-j_i} \int_{\mathcal{E}^j} U(c_{b+j}(\epsilon^j), n_{b+j}(\epsilon^j)) \mathrm{d}\mu_j(\epsilon^j | \epsilon^{j_i}) \geq v_0(i), \tag{7}
$$

where the marginal product of labor w and promised values $v_0(i)$, all i, are given, and j_i is the age for currently-alive households i at the time of the reform and j_i is 1 for all future households.

We are interested in allocations that are efficient in the sense that there is no alternative incentive and resource feasible allocation that makes all households weakly better off and some strictly better off.

Proposition. Let $v_0(i)$ be:

$$v_0(i) = \sum_{j_i}^{J} \beta^{j-1} \int_{\mathcal{E}^j} U(c_{b+j}(\epsilon^j), n_{b+j}(\epsilon^j)) \mathrm{d}\mu(\epsilon^j | \epsilon^{j_i}),$$

where the c and n are consumption and hours in allocation x. Then, allocation x is efficient if and only if it solves the planner problem with a maximum of zero, given the promised values in 4.5.

Proof. We show both directions by contradiction. $\Rightarrow$ If an allocation x is efficient it solves the planner problem given $v_0(i)$ for all $i \in \mathcal{I}$ with a maximum of zero. Suppose x does not solve the planner problem and let $\hat{x}$ denote a solution to the planner problem. Because x is feasible, the allocation $\hat{x}$ generates strictly excess resources in the first period. Construct an alternative allocation $\tilde{x}$ identical to $\hat{x}$ but increase initial consumption such that the incentive constraints in 6 are satisfied. The allocation $\tilde{x}$ strictly Pareto dominates x, which is a contradiction.

$\Leftarrow$ If an allocation x solves the planner problem given $v_0(i)$ for all $i \in \mathcal{I}$ with a zero maximum, then it is efficient. Suppose that x is not efficient, then there exists an alternative feasible allocation $\hat{x}$ such that all households are better off, with some household i strictly better off. Since $\hat{x}$ is feasible and delivers at least $v_0(i)$ for all $i \in \mathcal{I}$, $\hat{x}$ is a candidate solution to the planner problem. Construct an alternative allocation $\tilde{x}$, which is equal to $\hat{x}$ but reduce initial consumption for household i that is strictly better off under $\hat{x}$ (such that the incentive constraints are satisfied). Alternative allocation $\tilde{x}$ is feasible and generates excess resources in the initial period. This contradicts that x is a solution to the planner problem. ■

Figure 1. Pareto Efficient Frontier

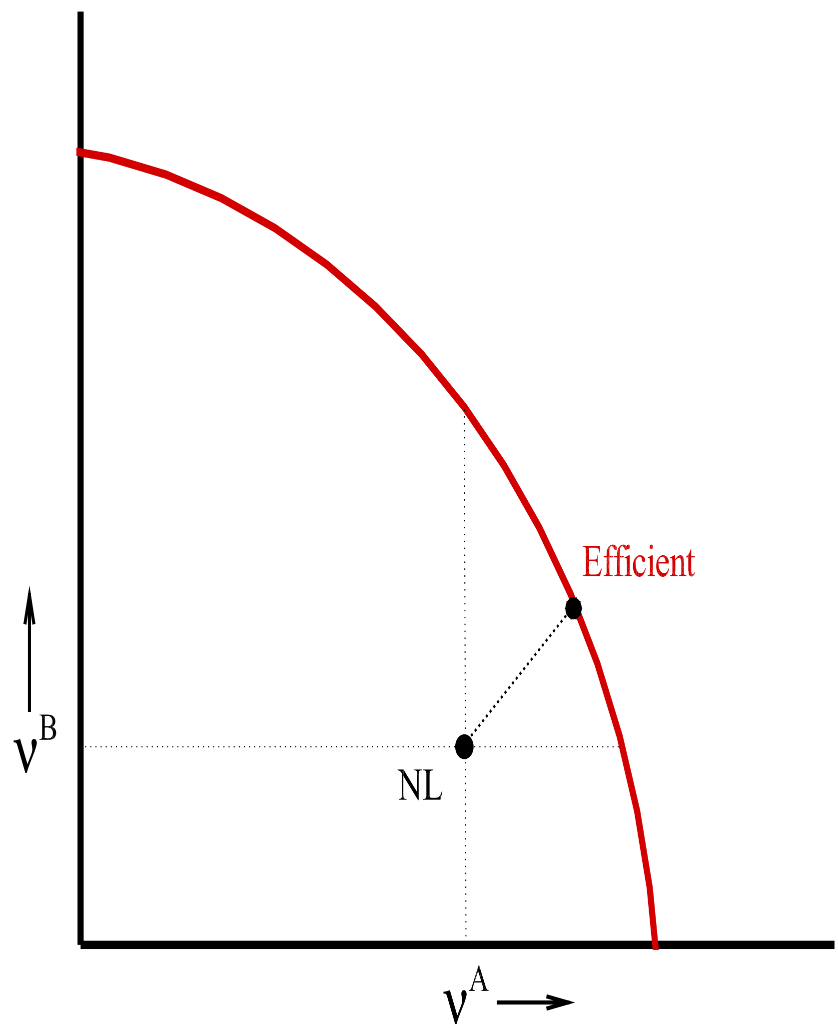


Figure 2. Income Tax Schedule

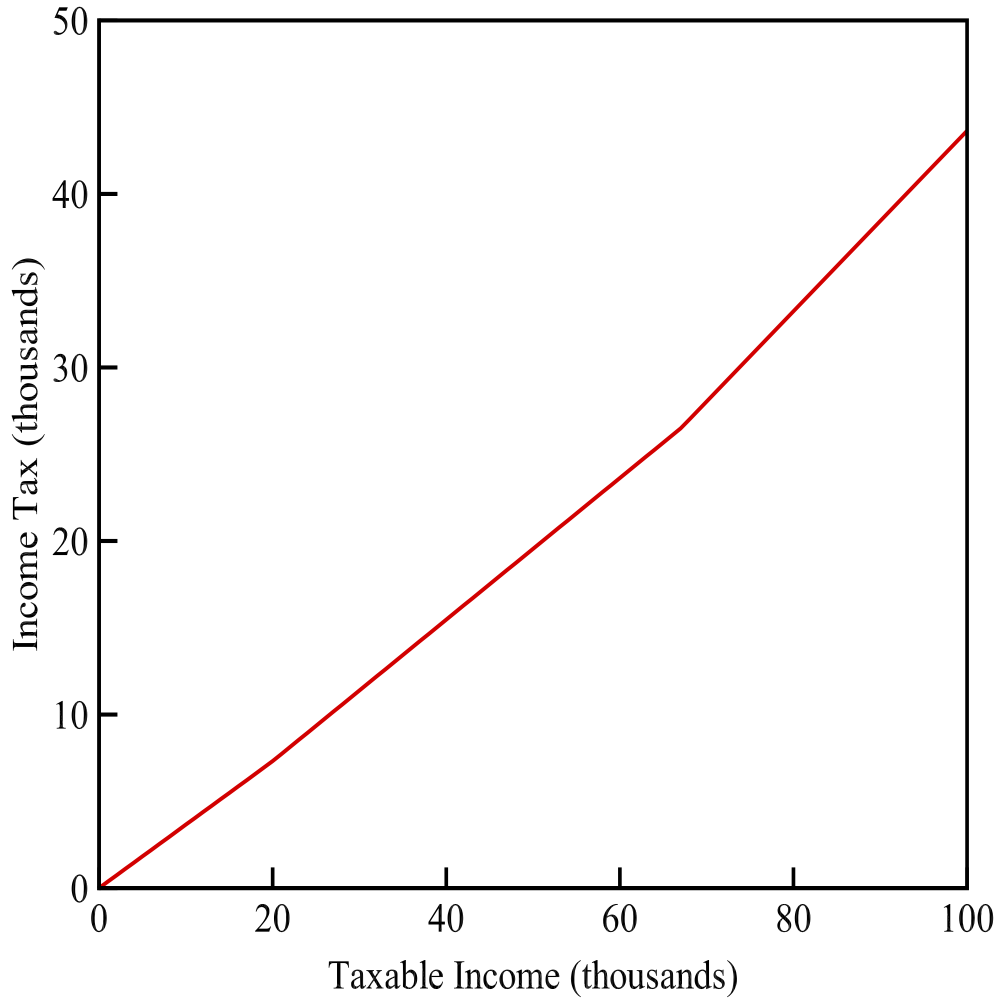


Figure 3. Financial Asset Tax Schedule

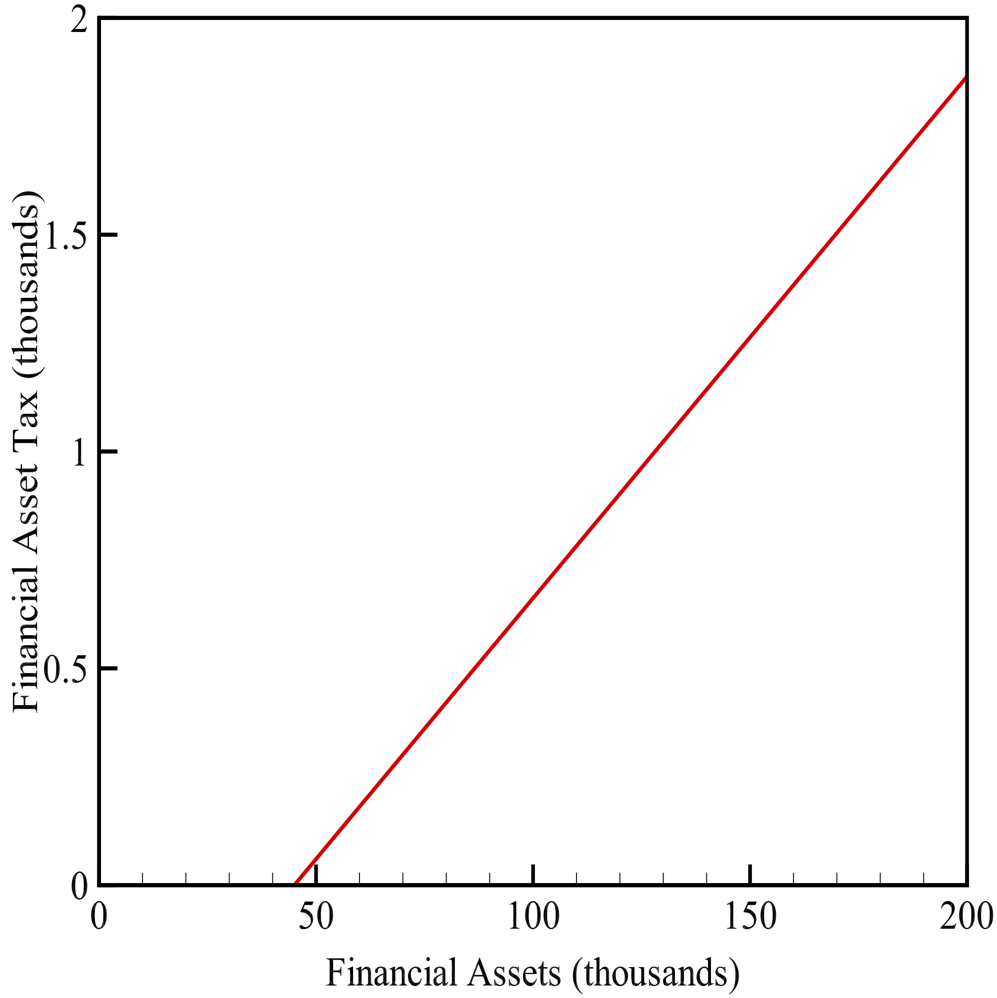


Figure 4. Wage Profiles

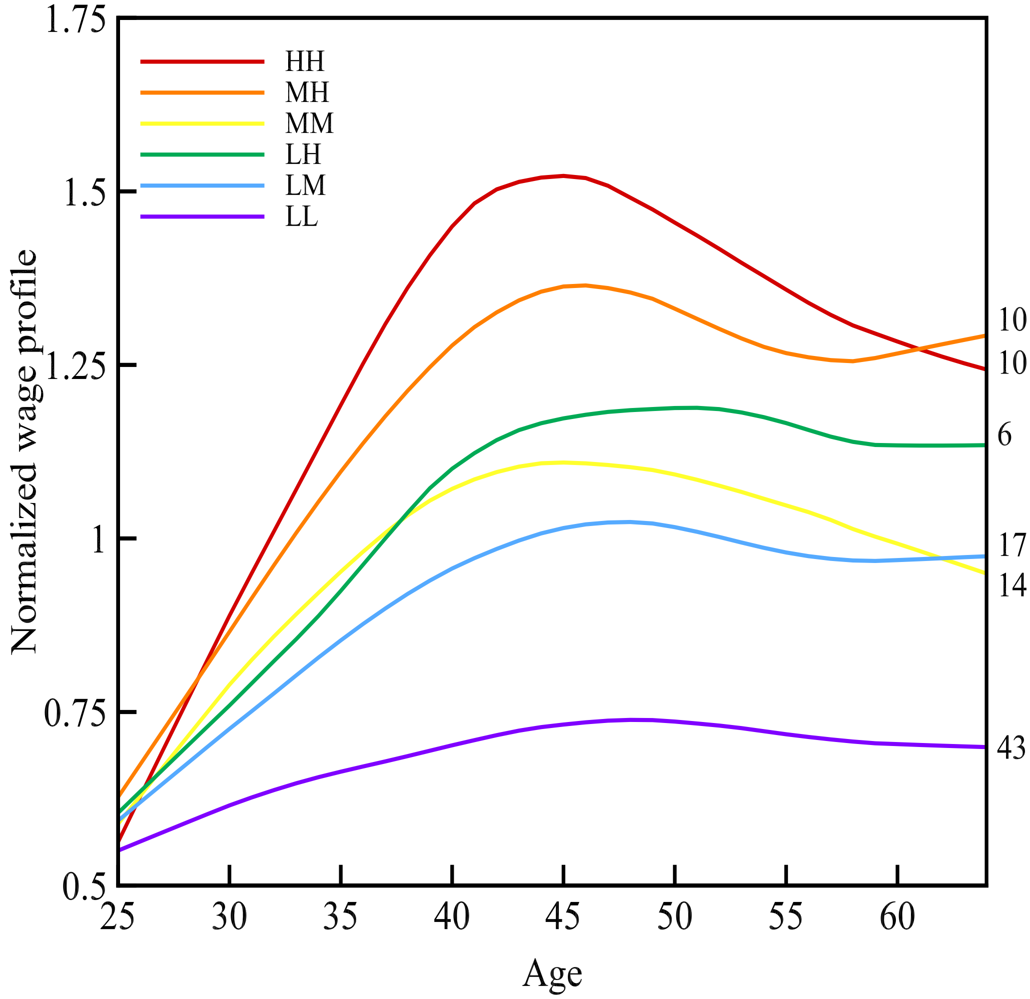


Figure 5. Labor Wedges: Baseline Model

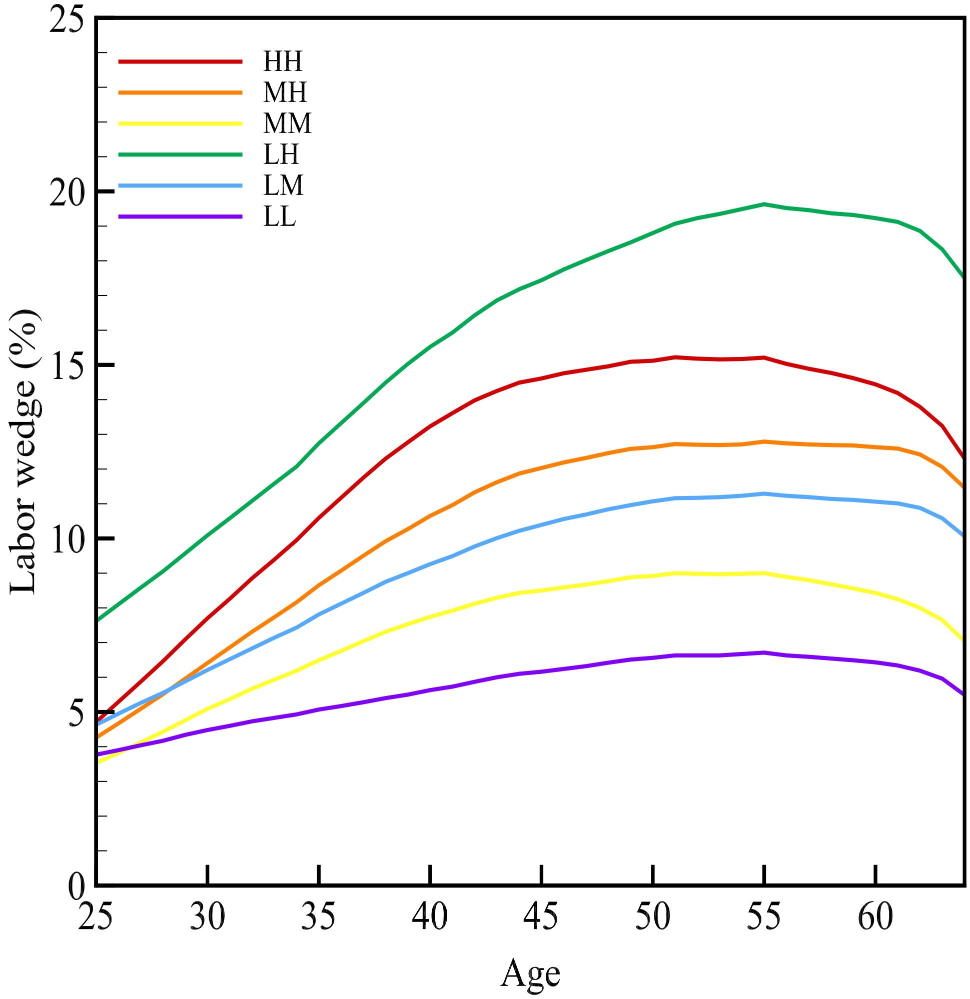
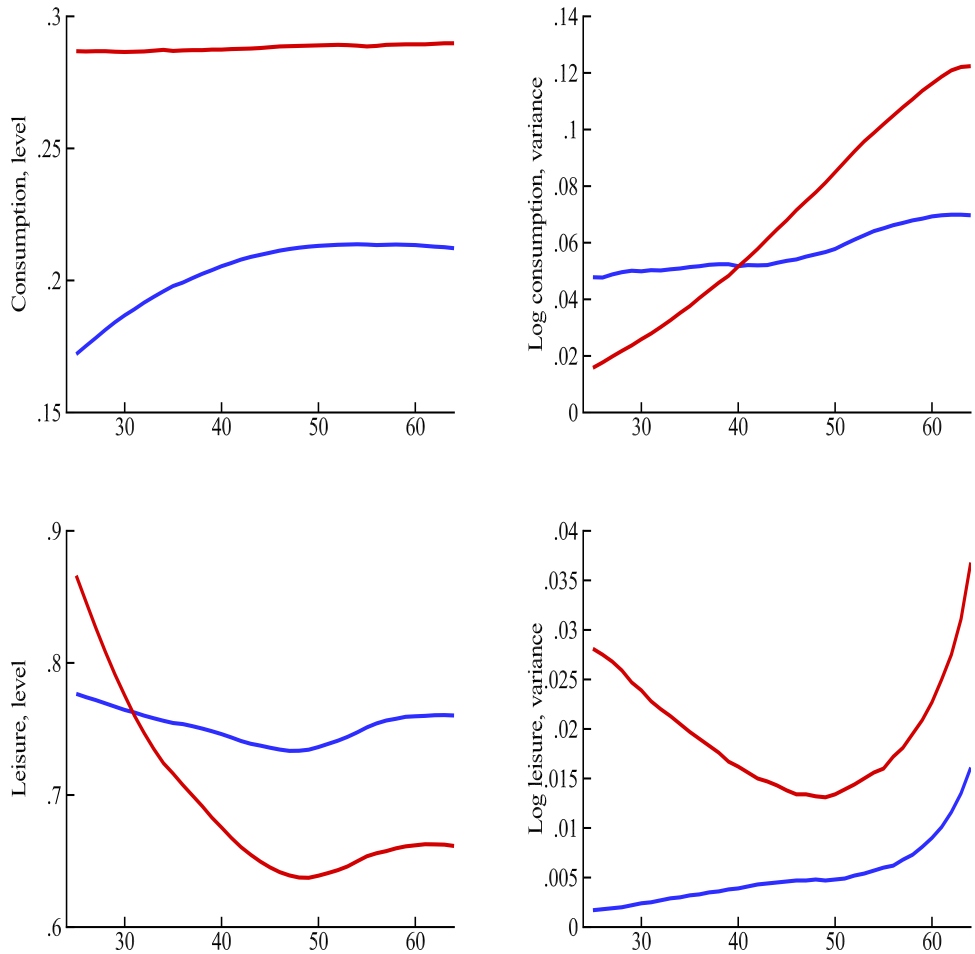
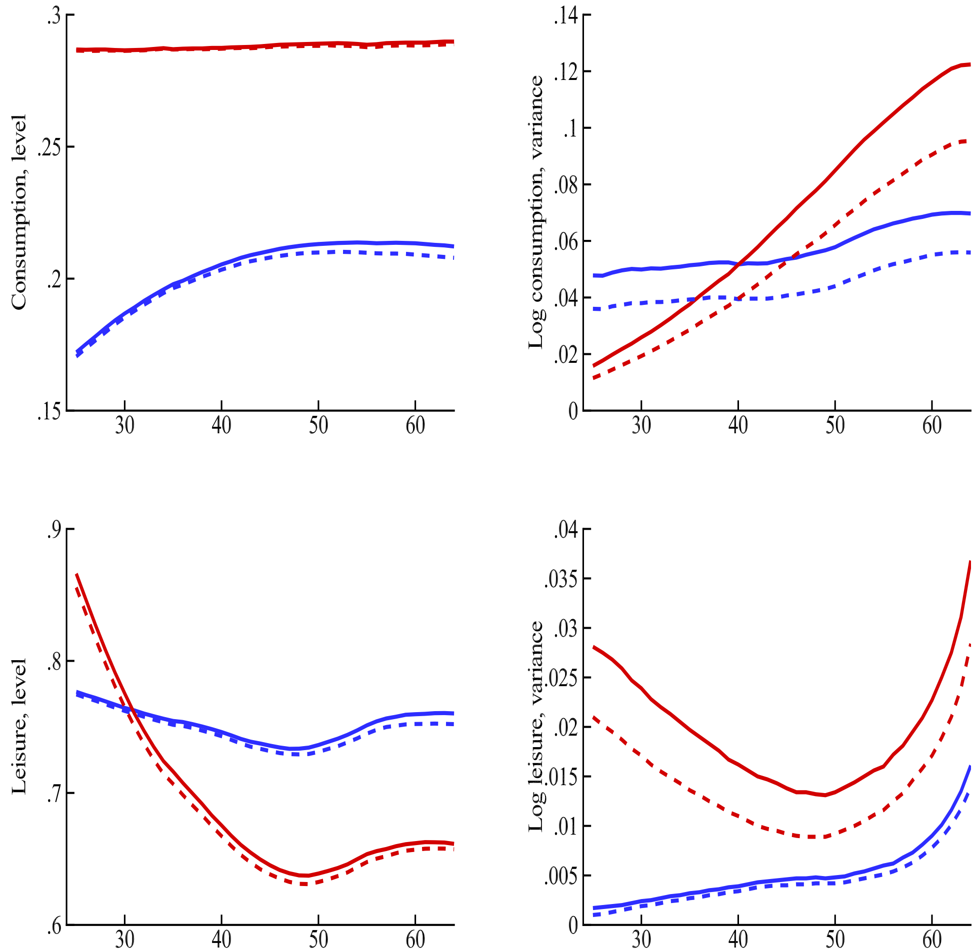


Figure 6. Allocation Levels and Dispersion for LL: Baseline Model



Note: Results for the positive economy are shown in blue and results for the reform problem are shown in red.

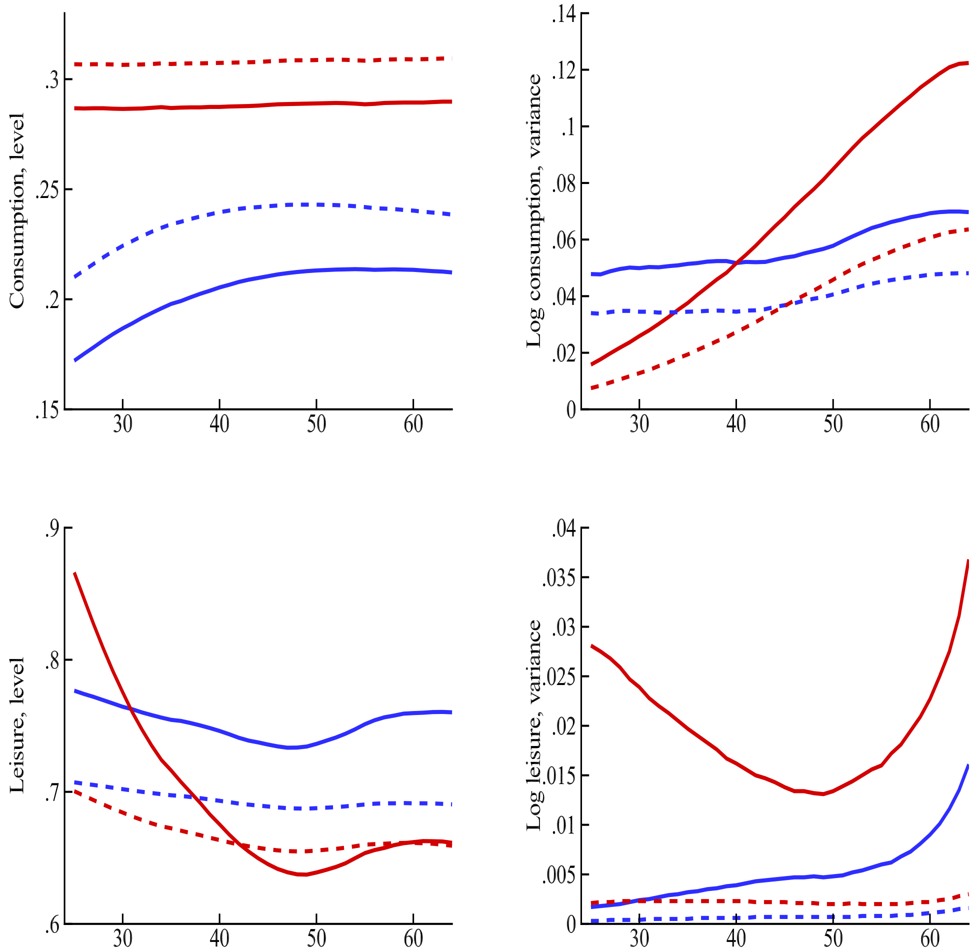
Figure 7. Allocation Levels and Dispersion for Group Low-Low
Comparison of Baseline and 40-Types Model



Note: Results for the positive economy are shown in blue and results for the reform problem are shown in red. The solid lines are the baseline model and the dashed lines are the 40-type model.

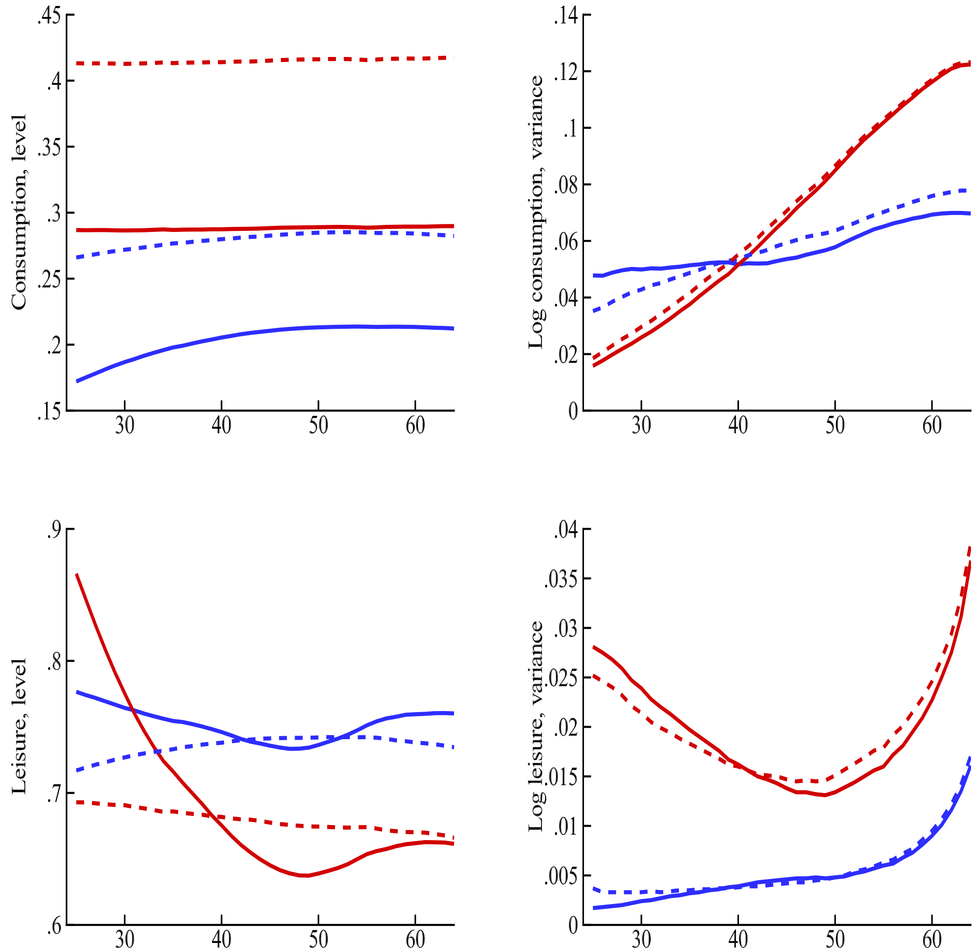
Figure 8. Allocation Levels and Dispersion for Group Low-Low

Comparison of Baseline and Alternative Preferences



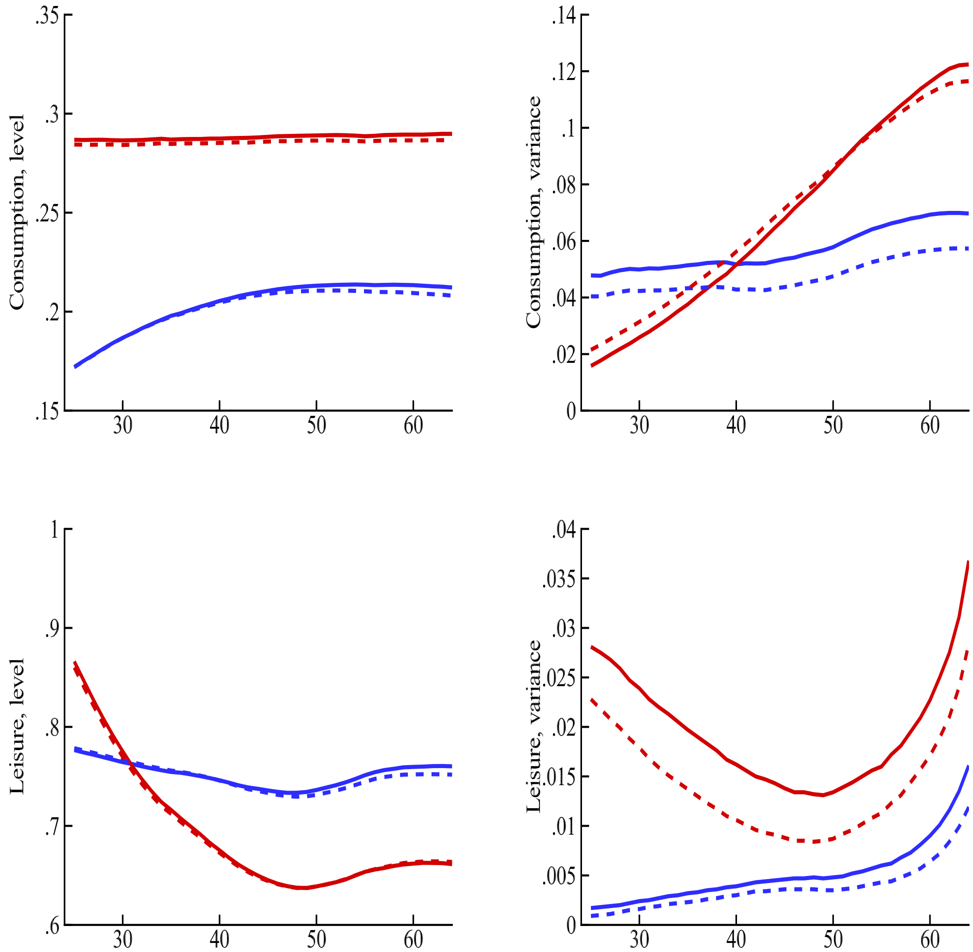
Note: Results for the positive economy are shown in blue and results for the reform problem are shown in red. The solid lines are the baseline model and the dashed lines are the model with constant Frisch elasticity preferences.

Figure 9. Allocation Levels and Dispersion for Group Low-Low
Comparison of Baseline and No Wage Growth Model



Note: Results for the positive economy are shown in blue and results for the reform problem are shown in red. The solid lines are the baseline model and the dashed lines are the no-wage-growth model.

Figure 10. Allocation Levels and Dispersion for Group Low-Low
Comparison of Baseline and Lower Shock Variance Model



Note: Results for the positive economy are shown in blue and results for the reform problem are shown in red. The solid lines are the baseline model and the dashed lines are the lower-shock-variance model.

Table 1. Revised National Income and Product Accounts
Averages Relative to Adjusted GDP, 2000–2010

Total Adjusted Income	1.000
Labor Income	.566
Compensation of employees	.502
Wages and salary accruals	.397
Supplements to wages and salaries	.105
70% of proprietors' income	.064
Capital Income	.434
Profits	.156
30% of proprietors' income	.027
Indirect business taxes	.105
Less: Sales tax	.103
Consumption of fixed capital	.165
Consumer durable depreciation	.050
Imputed capital services	.035
Consumer durable services	.012
Government capital services	.023

See footnotes at the end of the table.

Table 1. Revised National Income and Product Accounts Averages Relative to Adjusted GDP, 2000–2010 (Cont.)

Total Adjusted Product	1.000
Consumption	.635
Personal consumption expenditures	.484
Less: Consumer durable goods	.068
Less: Imputed sales tax, nondurables and services	.088
Plus: Imputed capital services, durables	.012
Government consumption expenditures, nondefense	.222
Plus: Imputed capital services, government capital	.023
Consumer durable depreciation	.050
Tangible investment	.351
Gross private domestic investment	.177
Consumer durable goods	.068
Less: Imputed sales tax, durables	.014
Government gross investment, nondefense	.041
Net exports of goods and services	.079
Defense spending	.014

Note: The data source for national income statistics is the Dutch Bureau of Statistics. Imputed capital services are equal to 4 percent times the current-cost net stock of government fixed assets and consumer durable goods.

Table 2. Revised Fixed Asset Tables with Stocks End of Period, Averages Relative to Adjusted GDP, 2000–2010

Total Capital	5.657
Fixed assets	3.068
Businesses	1.261
Government	0.571
Households	1.236
Consumer durables	.301
Inventories	.142
Businesses	.129
Households	.013
Land	1.905
Agricultural and productive land	.420
Residential land	1.485
Oil and gas	.241

Note: The data source for national income statistics is the Dutch Bureau of Statistics.

Table 3. Household Net Worth and Government Debt Averages Relative to Adjusted GDP, 2000–2010

Household Net Worth, end of period	3.895
Assets	5.130
Tangible	2.466
Financial	2.664
Liabilities	1.236
Government Debt, end of period	.556

Note: The data source for national income statistics is the Dutch Bureau of Statistics.

Table 4. Population, Employment, and Hours Averages, 2000–2010

Population in millions	
All ages	16.3
Ages 16 to 64	10.8
Population growth (%)	
All ages	0.5
Ages 16 to 64	0.3
Annual hours per population 16-64	1,135

Note: The data source for national income statistics is the Dutch Bureau of Statistics.

Table 5. Estimated Wage Process Parameters

	Persistence		Innovation Variance	
Education Group	$\hat{\rho}$	Confidence	$\hat{\sigma}_u^2$	Confidence
Low, Low	.9542	(.9515,.9575)	.0096	(.0093,.0102)
Low, Medium	.9660	(.9610,.9692)	.0087	(.0083,.0096)
Low, High	.9673	(.9628,.9710)	.0162	(.0153,.0176)
Medium, Medium	.9570	(.9536,.9612)	.0099	(.0091,.0103)
Medium, High	.9616	(.9520,.9782)	.0109	(.0082,.0124)
High, High	.9564	(.9501,.9582)	.0172	(.0164,.0184)

Table 6. Welfare Gain Decomposition: Baseline Model
Total Welfare Gain of 17%

	Consumption		Leisure	
Education group	Δ_c^L	Δ_c^D	Δ_ℓ^L	Δ_ℓ^D
Low, Low	22	2	−11	4
Low, Medium	21	3	−12	5
Low, High	16	5	−10	6
Medium, Medium	21	3	−13	5
Medium, High	19	5	−14	6
High, High	17	8	−15	7

Table 7. Welfare Gain Decomposition: 40 Types Model
Total Welfare Gain of 16%

	Consumption		Leisure	
Education group	Δ_c^L	Δ_c^D	Δ_ℓ^L	Δ_ℓ^D
Low, Low	22	1	−11	4
Low, Medium	21	3	−12	6
Low, High	17	5	−13	7
Medium, Medium	21	3	−13	5
Medium, High	20	5	−15	7
High, High	17	6	−15	7

Table 8. Welfare Gain Decomposition: Alternative Preferences
Total Welfare Gain of 14%

	Consumption		Leisure	
Education group	Δ_c^L	Δ_c^D	Δ_ℓ^L	Δ_ℓ^D
Low, Low	17	1	−4	1
Low, Medium	15	2	−4	1
Low, High	13	4	−4	0
Medium, Medium	16	2	−5	1
Medium, High	15	3	−5	1
High, High	13	6	−6	1

Table 9. Welfare Gain Decomposition: No Growth Model
Total Welfare Gain of 14%

	Consumption		Leisure	
Education group	Δ_c^L	Δ_c^D	Δ_ℓ^L	Δ_ℓ^D
Low, Low	28	0	−9	1
Low, Medium	27	0	−7	1
Low, High	25	0	−5	1
Medium, Medium	28	0	−8	1
Medium, High	27	0	−7	1
High, High	25	1	−6	1

Table 10. Welfare Gain Decomposition: Lower Variance Model
Total Welfare Gain of 19%

	Consumption		Leisure	
Education group	Δ_c^L	Δ_c^D	Δ_ℓ^L	Δ_ℓ^D
Low, Low	26	1	−12	4
Low, Medium	27	3	−19	7
Low, High	25	6	−21	8
Medium, Medium	26	3	−17	7
Medium, High	26	6	−21	8
High, High	21	7	−17	8

References

AARON, H. (1970): "Income Taxes and Housing," *American Economic Review*, 60(5), 789–806.

ACEMOGLU, D., AND D. AUTOR (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, pp. 1043–171. Elsevier.

AGUIAR, M., AND M. BILS (2015): "Has Consumption Inequality Mirrored Income Inequality?," *American Economic Review*, 105(9), 2725–2756.

AGUIAR, M., M. BILS, K. CHARLES, AND E. HURST (2018): "Leisure Luxuries and the Labor Supply of Young Men," NBER Working Paper No. 23552.

AGUIAR, M., AND E. HURST (2005): "Consumption versus Expenditure," *Journal of Political Economy*, 113(5), 919–948.

——— (2007a): "Life-Cycle Prices and Production," *American Economic Review*, 97(5), 1533–1559.

——— (2007b): "Measuring Trends in Leisure: The Allocation of Time over Five Decades," *Quarterly Journal of Economics*, 122, 969–1006.

——— (2013): "Deconstructing Life Cycle Expenditure," *Journal of Political Economy*, 121(3), 437–492.

AGUIAR, M., E. HURST, AND L. KARABARBOUNIS (2013): "Time Use During the Great Recession," *American Economic Review*, 103(5), 1664–1696.

Aiyagari, R. (1994): “Uninsured Idiosyncratic Risk and Aggregate Savings,” *Quarterly Journal of Economics*, 109(3), 659–684.

Albanesi, S., and C. Sleet (2006): “Dynamic Optimal Taxation with Private Information,” *Review of Economic Studies*, 73(1), 1–30.

Atkeson, A., and R. E. Lucas (1992): “On Efficient Distribution with Private Information,” *Review of Economic Studies*, 59(3), 427–453.

——— (1995): “Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance,” *Journal of Economic Theory*, 66(1), 64–88.

Atkinson, A. B., and J. E. Stiglitz (1976): “The Design of Tax Structure: Direct versus Indirect Taxation,” *Journal of Public Economics*, 6(1), 55–75.

Attanasio, O., and S. Davis (1996): “Relative Wage Movements and the Distribution of Consumption,” *Journal of Political Economy*, 104, 1227–1262.

Attanasio, O., E. Hurst, and L. Pistaferri (2015): “The Evolution of Income, Consumption, and Leisure Inequality in the United States, 1980-2010,” in *Improving the Measurement of Consumer Expenditures*, pp. 100–140. NBER Chapters.

Attanasio, O., and L. Pistaferri (2016): “Consumption Inequality,” *Journal of Economic Perspectives*, 30(2), 3–28.

Autor, D., L. Katz, and M. Kearney (2008): “Trends in U.S. Wage Inequality: Revising the Revisionists,” *Review of Economics and Statistics*, 90, 300–323.

Baxter, M., and U. J. Jermann (1999): “Household Production and the Excess Sensitivity of Consumption to Current Income,” *American Economic Review*, 89(4), 902–920.

Becker, G. S. (1965): “A Theory of the Allocation of Time,” *Economic Journal*, 75(299), 493–517.

Benabou, R. (2002): “Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?,” *Econometrica*, 70(2), 481–517.

Benhabib, J., R. Rogerson, and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99(6), 1166–1187.

Berger, D., and J. Vavra (2015): "Consumption Dynamics During Recessions," *Econometrica*, 83(1), 101–154.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98(5), 1887–1921.

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2016): "Consumption Inequality and Family Labor Supply," *American Economic Review*, 106(2), 387–435.

——— (2018): "Children, Time Allocation and Consumption Insurance," *Journal of Political Economy*, 126(1), 73–115.

Boerma, J. (2020): "Housing Policy Reform," University of Minnesota Working Paper.

Boerma, J., and L. Karabarbounis (2019a): "Inferring Inequality with Home Production," NBER Working Paper No. 24166.

——— (2019b): "Inferring Inequality with Home Production," NBER Working Paper No. 24166.

——— (2019c): "Labor Market Trends and the Changing Value of Time," NBER Working Paper No. 26301.

——— (2020): "Labor Market Trends and the Changing Value of Time," *Journal of Economic Dynamics and Control*, 21, 729–54.

Boppart, T., and R. Ngai (2019): "Rising Inequality and Trends in Leisure," London School of Economics Working Paper.

Bridgman, B. (2016): "Engines of Leisure," Bureau of Economic Analysis Working Paper.

Bridgman, B., A. Dugan, M. Lal, M. Osborne, and S. Villones (2012): "Accounting for Household Production in the National Accounts: An Update, 1965-2014," *Survey of Current Business*, 92, 23–36.

Cardia, E., and P. Gomme (2018): "Market Work, Housework and Childcare: A Time Use Approach," *Review of Economic Dynamics*, 29, 1–24.

Chamberlain, G. (1984): "Chapter 22 Panel Data," vol. 2 of *Handbook of Econometrics*, pp. 1247–1318.

Chambers, M., C. Garriga, and D. E. Schlagenhauf (2009): "Housing Policy and the Progressivity of Income Taxation," *Journal of Monetary Economics*, 56(8), 1116–1134.

Cherchye, L., T. Demuynck, B. De Rock, and F. Vermeulen (2017): "Household Consumption When the Marriage is Stable," *American Economic Review*, 107(6), 1507–1534.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2012): "Does Indivisible Labor Explain the Difference Between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities," *NBER Macroeconomics Annual*, 27, 1–56.

Constantinides, G. M., and D. Duffie (1996): "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, 104(2), 219–240.

Cutler, D. M., and L. F. Katz (1991): "Macroeconomic Performance and the Disadvantaged," *Brookings Papers on Economic Activity*, 1991(2), 1–74.

Daley, D. J. (1968): "Stochastically Monotone Markov Chains," *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 10(4), 305–317.

Deaton, A. (1997): "The Analysis of Household Surveys: A Microeconometric Approach to Development Policy," World Bank.

Deaton, A., and C. Paxson (1994): "Intertemporal Choice and Inequality," *Journal of Political Economy*, 102(3), 437–467.

Diamond, P. A. (1998): "Optimal Income Taxation: An Example with a U-shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88(1), 83–95.

Díaz, A., and M. J. Luengo-Prado (2008): "On the User Cost and Homeownership," *Review of Economic Dynamics*, 11(3), 584–613.

Dotsey, M., W. Li, and F. Yang (2014): "Consumption and Time Use Over the Life Cycle," *International Economic Review*, 55(3), 665–692.

Dougherty, A., and R. Van Order (1982): "Inflation, Housing Costs, and the Consumer Price Index," *American Economic Review*, 72(1), 154–164.

Farhi, E., and I. Werning (2013): "Insurance and Taxation over the Life Cycle," *Review of Economic Studies*, 80(2), 596–635.

Floden, M. (2001): "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, 48(1), 81–108.

Floetotto, M., M. Kirker, and J. Stroebel (2016): "Government Intervention in the Housing Market: Who Wins, Who Loses?," *Journal of Monetary Economics*, 80, 106–123.

Frazis, H., and J. Stewart (2011): "How Does Household Production Affect Measured Income Inequality?," *Journal of Population Economics*, 24, 3–22.

Gautier, P. A., A. Siegmann, and A. van Vuuren (2018): "Real-Estate Agent Commission Structure and Sales Performance," CEPR Working Paper 12587.

Gervais, M. (2002): "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49(7), 1461–1489.

Ghez, G., and G. S. Becker (1975): *The Allocation of Time and Goods Over the Life Cycle*. New York: Columbia University Press.

Golosov, M., N. Kocherlakota, and A. Tsyvinski (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70(3), 569–587.

Golosov, M., M. Troshkin, and A. Tsyvinski (2016): "Redistribution and Social Insurance," *American Economic Review*, 106(2), 359–386.

Golosov, M., and A. Tsyvinski (2006): "Designing Optimal Disability Insurance: A Case for Asset Testing," *Journal of Political Economy*, 114(2), 257–279.

Gomme, P., F. E. Kydland, and P. Rupert (2001): "Home Production Meets Time to

Build," *Journal of Political Economy*, 109(5), 1115–1131.

Gourinchas, P.-O., and J. A. Parker (2002): "Consumption Over the Life Cycle," *Econometrica*, 70(1), 47–89.

Greenwood, J., and Z. Hercowitz (1991): "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy*, 99(6), 1188–1214.

Gronau, R. (1977): "Leisure, Home Production, and Work − the Theory of the Allocation of Time Revisited," *Journal of Political Economy*, 85(6), 1099–1123.

——— (1986): "Home Production: A Survey," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and R. Layard, pp. 274–304. Elsevier.

Guner, N., R. Kaygusuz, and G. Ventura (2014): "Income Taxation of U.S. Households: Facts and Parametric Estimates," *Review of Economic Dynamics*, 17(4), 559–581.

Guryan, J., E. Hurst, and M. Kearney (2008): "Parental Education and Parental Time with Children," *Journal of Economic Perspectives*, 22(3), 23–46.

Han, J., B. Meyer, and Sullivan (2018): "Inequality in the Joint Distribution of Consumption and Time Use," NBER Working Paper No. 25199.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006," *Review of Economic Dynamics*, 13, 15–51.

Heathcote, J., K. Storesletten, and G. Violante (2013): "From Wages to Welfare: Decomposing Gains and Losses from Rising Inequality," *Advances in Economics and Econometrics: Tenth World Congress*, 2, 235–280.

Heathcote, J., K. Storesletten, and G. L. Violante (2008): "Insurance and Opportunities: A Welfare Analysis of Labor Market Risk," *Journal of Monetary Economics*, 55(3), 501–525.

——— (2014): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *American Economic Review*, 104(7), 2075–2126.

Himmelberg, C., C. Mayer, and T. Sinai (2005): "Assessing High House Prices: Bubbles, Fundamentals and Misperceptions," *Journal of Economic Perspectives*, 19(4), 67–92.

Hosseini, R., and A. Shourideh (2019): "Retirement Financing: An Optimal Reform Approach," *Econometrica*, 87(4), 1205–1265.

Hsieh, C.-T., and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124(4), 1403–1448.

Huggett, M. (1993): "The Risk Free Rate in Heterogeneous-Agents Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17(5–6), 953–969.

Jones, C. I., and P. J. Klenow (2016): "Beyond GDP? Welfare Across Countries and Time," *American Economic Review*, 106(9), 2426–2457.

Kapička, M. (2013): "Efficient Allocations in Dynamic Private Information Economies with Persistent Shocks: A First-Order Approach," *Review of Economic Studies*, 80(3), 1027–1054.

Kaplan, G. (2012): "Inequality and the Life Cycle," *Quantitative Economics*, 3(3), 471–525.

Karabarbounis, L. (2014): "Home Production, Labor Wedges, and International Business Cycles," *Journal of Monetary Economics*, 64, 68–84.

Kocherlakota, N. R. (2005): "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," *Econometrica*, 73(5), 1587–1621.

Koehne, S. (2018): "On the Taxation of Durable Goods," *International Economic Review*, 59(2), 825–857.

Krebs, T. (2003): "Growth and Welfare Effects of Business Cycles in Economies with Idiosyncratic Human Capital Risk," *Review of Economic Dynamics*, 6(4), 846–868.

Krueger, D., and F. Perri (2003): "On the Welfare Consequences of the Increase in Inequality in the United States," *NBER Macroeconomics Annual*, 18, 83–123.

——— (2006): "Does Income Inequality Lead to Consumption Inequality? Evidence and

Theory," *Review of Economic Studies*, 73(1), 163–193.

Kydland, F. E., and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

Laidler, D. (1969): "Income Tax Incentives for Owner-Occupied Housing," in *The Taxation of Income from Capital*, ed. by A. C. Harberger, and M. J. Bailey, pp. 50–76.

Lise, J., and K. Yamada (2018): "Household Sharing and Commitment: Evidence from Panel Data on Individual Expenditures and Time Use," University of Minnesota Working Paper.

Livshits, I., J. MacGee, and M. Tertilt (2010): "Accounting for the Rise in Consumer Bankruptcies," *American Economic Journal: Macroeconomics*, 2(2), 165–93.

McGrattan, E., and E. C. Prescott (2017): "On Financing Retirement with an Aging Population," *Quantitative Economics*, 8(1), 75–115.

McGrattan, E., R. Rogerson, and R. Wright (1997): "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review*, 38(2), 267–290.

Ndiaye, A. (2018): "Flexible Retirement and Optimal Taxation," Chicago Fed Working Paper No. 2018−18.

Negishi, T. (1960): "Welfare Economics and Existence of An Equilibrium for a Competitive Economy," *Metroeconomica*, 12(2-3), 92–97.

Nishiyama, S., and K. Smetters (2014): "Analyzing Fiscal Policies in a Heterogeneous-Agent Overlapping-Generations Economy," in *Handbook of Computational Economics*, vol. 3, pp. 117–160.

Olovsson, C. (2015): "Optimal Taxation with Home Production," *Journal of Monetary Economics*, 70, 39–50.

Parente, S. L., R. Rogerson, and R. Wright (2000): "Homework in Development

Economics: Household Production and the Wealth of Nations," *Journal of Political Economy*, 108(4), 680–687.

Piazzesi, M., and M. Schneider (2016): "Housing and Macroeconomics," in *Handbook of Macroeconomics*, vol. 2, pp. 1547–1640.

Pistaferri, L. (2003): "Anticipated and Unanticipated Wage Changes, Wage Risk, and Intertemporal Labor Supply," *Journal of Labor Economics*, 21, 729–754.

Poterba, J. M. (1984): "Tax Subsidies to Owner-Occupied Housing: an Asset-Market Approach," *Quarterly Journal of Economics*, 99(4), 729–752.

——— (1992): "Taxation and Housing: Old Questions, New Answers," *American Economic Review Papers & Proceedings*, 82(2), 237–42.

Poterba, J. M., and T. Sinai (2008): "Tax Expenditures for Owner-Occupied Housing: Deductions for Property Taxes and Mortgage Interest and the Exclusion of Imputed Rental Income," *American Economic Review Papers & Proceedings*, 98(2), 84–89.

Rios-Rull, J.-V. (1993): "Working in the Market, Working at Home, and the Acquisition of Skills: A General-Equilibrium Approach," *American Economic Review*, 83(4), 893–907.

Rogerson, W. P. (1985): "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53(6), 1357–1367.

Rupert, P., R. Rogerson, and R. Wright (1995): "Estimating Substitution Elasticities in Household Production Models," *Economic Theory*, 6(1), 179–193.

——— (2000): "Homework in Labor Economics: Household Production and Intertemporal Substitution," *Journal of Monetary Economics*, 46(3), 557–579.

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 68(1), 205–229.

Sommer, K., and P. Sullivan (2018): "Implications of US Tax Policy for House Prices, Rents, and Homeownership," *American Economic Review*, 108(2), 241–74.

Stantcheva, S. (2017): "Optimal Taxation and Human Capital Policies over the Life

Cycle," *Journal of Political Economy*, 125(6), 1931–1990.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): "Consumption and Risk Sharing Over the Life Cycle," *Journal of Monetary Economics*, 51(3), 609–633.

Vandenbroucke, G. (2009): "Trends in Hours: The U.S. from 1900 to 1950," *Journal of Economic Dynamics and Control*, 33, 237–249.

Veenstra, J., and B. van Ommeren (2017): "Bailout Clauses and the Price of Credit: The Dutch Experience for Housing Corporations," *De Economist*, 165(3), 295–320.

Warnaar, M., and J. Bos (2017): "Inkomensnormen voor Hypotheken," *Economisch Statistische Berichten*, 102(4794), 34–37.