Essays on Fiscal Policies in Open Economies

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Dedication

To my parents, *Trần Tuệ Quang* and *Hồ Thị Hạnh*.

Abstract

This dissertation consists of three chapters. A unifying theme across all chapters is the interaction between government's motive for redistribution and its commitment to repay debt.

The first chapter studies optimal taxation in an open economy in which the government has a redistributive motive and faces self-enforcing debt constraints that arise from its limited commitment. Redistributive policies are proportional taxes on labor and domestic saving. Optimal labor taxes decrease over time and eventually converge to a non-zero limit, and the optimal capital tax is positive in the limit. The efficient contract features front-loading distortion and back-loading efficiency, allowing the government to borrow more in the future. The model's numerical exercise shows that a stronger redistributive motive requires greater tax distortions at the beginning of time as well as a higher external debt level in the long run.

The second chapter, in turn, proposes a theory of external debt sustainability based on the governments motive for redistribution. Given the endogenous debt constraints, the value of financial autarky determines the sustainable level of debt. Financial autarky is endogenously costly because redistribution requires high labor taxes, which distort labor supply and reduce the economy's efficiency. Having access to external financing allows the government to have more redistribution, measured as the differences in individual utilities than in financial autarky at the same level of efficiency cost. Quantitatively, the theory can account for the external debts recent buildup in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries and time periods. In response to a negative productivity shock, the optimal austerity policies are increasing external borrowing and redistribution while reducing redistribution to repay debt in the future. The magnitude of these responses varies with the underlying wage inequality.

The third chapter examines how income inequality affects the sovereign default risk. I study fiscal policies in a sovereign default model with heterogeneous agents and distortionary taxation. I quantify the model in the case of Spain and find that inequality worsens the debt crisis by increasing the government's incentive to default.

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Chapter 1

Optimal Redistributive Policies in Open Economies

1.1 Introduction

What are the optimal tax and debt policies for an economy that faces limited borrowing but has a motive for redistribution? Taxes are the main source of the government's revenue that affects the government's ability to issue or repay debt. At the same time, distortionary taxes and transfers redistribute resources across agents under social preferences, so preferences for greater redistribution significantly affect the government's willingness to raise revenue and influence its borrowing capacity. On the other hand, when borrowing is constrained, the government's ability to redistribute is limited.

Empirical works on the recent European debt crisis have demonstrated how the redistributive motive and debt accumulation are closely related. First, the rapid accumulation of external debt in the periods leading up to a crisis led to many countries facing such high costs of borrowing that they could not roll over their debt. Second, highly indebted countries, such as Greece, Portugal, and Spain, also experienced high levels of income dispersion. According to the European Union's statistics on Income and Living Conditions, the Gini coefficients and S80/S20 income quintile share ratio in these countries were both higher than the EU-27 country average. Countries proposed different policy strategies to tackle the problems of constrained borrowing while maintaining the desired level of redistribution.

Motivated by these observations, this chapter proposes a small open economy with heterogeneous agents and a government's lack of commitment. In each period, a benevolent government, without commitment, chooses both a level of debt going forward and a system of labor and capital income taxes. Borrowing is limited by the aggregate borrowing constraints. Taxes do not depend on income. The objective function of the government features a desire to redistribute. The government has the same discount factor as the domestic agents, which are less patient than the international lenders.

Borrowing constraints arise from a game between domestic agents, international lenders, and the government. When the government deviates from the contract, it faces a punishment imposed by domestic and international agents, which corresponds to a value of deviation. I assume the punishment to be financial autarky, where the government cannot have access to both domestic and international lending markets. In each period, the subgame perfect equilibrium can be characterized by self-enforcing constraints. These constraints are such that the future discounted welfare of the whole economy must be higher than or equal to the utility the government receives from deviation in each period. The constraints endogenously set the aggregate debt limits that the government faces when choosing the optimal policies.

Impatience implies that the intertemporal discounting rate of future utilities is lower than the international intertemporal price of resources. This assumption represents a weaker domestic financial market or political friction, so it provides an ex-ante motive for debt accumulation, and so the borrowing constraint will be binding in the future.

The redistributive concern is a natural rationale for distortionary taxation. For example, a government that wants to redistribute more toward lower-income agents will levy marginal labor taxes and lump-sum transfers. In this case, higher-income individuals will bear a higher tax burden.

Binding debt constraints impose an additional distortion on intratemporal substitution between consumption and leisure, encourage less accumulation of capital and less domestic borrowing. Redistributive concerns affect tax levels via the trade-off between equity and efficiency, whereas debt constraints affect tax dynamics. The optimal labor tax is constant during periods of unconstrained borrowing but falls permanently every time borrowing is limited, eventually converging to a non-zero limit that depends on the level of heterogeneity and social distribution. The contract features front-loading distortion and back-loading efficiency. This property allows the economy to increase its borrowing capacity even when debt constraints bind and thereby accumulate more debt. These results hold with separable isoelastic preference, impatience, and bounded deviation utility.

In addition, capital accumulation increases the level of capital stock that the government can expropriate when deviating from the contract ex post, which in turn increases the deviation utility that the government can receive and tightens the debt constraints.

Domestic borrowing, on the other hand, reduces the level of future utility, which also tightens the debt constraints. Therefore, in order to relax the debt constraints, taxes on capital and domestic borrowing are positive in the long run. The equilibrium level of capital is lower than the one would arise in the case with full commitment, featuring the problem of debt overhang.

Intuitively, nonbinding debt constraints entail no borrowing cost. In this case, the optimal choice for the government is to use debt to smooth finances over time and keep labor distortions equal to the level that maximizes the redistributive benefit. When debt constraints bind, there is an additional motive to increase social welfare and relax the debt constraints. The government then finds it optimal to adjust tax rates to lower the cost of delivering the promised utility. An increase in the current period's efficiency relaxes the debt constraints for all previous periods. Anticipating this effect, the government finds it beneficial to back-load the increase in efficiency, which implies a front-loading distortion.

In the long run, the efficient allocation minimizes the cost of delivering the deviation utility and maximizes the net payment to the international financial markets. The labor tax converges to a limit associated with the maximal sustainable debt level of the whole economy. Skill distribution and social preferences influence both sides of the debt constraints: the value of staying in the contract and the value of deviating. Therefore, they affect the endogenous level of the maximum aggregate debt, which in turn determines the tax limit. In this way, primitives determine the long-run steady state of the economy under optimal taxation.

With high elasticity of substitution and redistributive motive towards low-skilled agents, the optimal labor tax drifts downward over time as the debt constraints bind, i.e. the tax rates in any periods after the debt constraint binds are weakly lower than the tax rates in the periods before the debt constraint binds. This result implies that, when the willingness to lend of the international lenders is high (as the debt constraints do not bind), the government would like to redistribute via a high labor tax rate. When the willingness to lend decreases (as the debt constraints bind), the government reduces distortions (lower tax rates) to repay the debt.

The chapter applies the sub-market analysis in Werning (2007) to a small open economy with limited commitment. The competitive equilibrium generates an efficient assignment of individual allocation, which is captured by a set of market–Negishi weights that determine individual utility shares. This analysis provides a key insight of the redistributive effect on the optimal tax limit. Without heterogeneity or redistributive motive, the model collapses to the representative setting as in Aguiar and Amador (2016), which proved that the labor tax limit is zero. When there is heterogeneity, the labor tax is zero as long as the equilibrium Negishi weights are equal to the social weights. The redistributive motive affects optimal distortions only when it can deviate from the implied distribution of the competitive equilibrium.

The numerical exercise simulates the dynamics of allocation, taxes, and debt of the model. It shows that the government would find it optimal to front-load labor distortion when borrowing is not limited (using high positive labor tax rates). When the borrowing is tightened, the government discourages domestic borrowing by taxing it. It turns out that the higher-income agents borrow more over time comparing to the lower-income agents, so the borrowing taxes are redistributive. The government redistributes more via the borrowing taxes and less via labor taxes until it reaches the highest efficiency level where it does not face any distortionary cost. The limit represents the maximum debt capacity of the economy. The key feature is that the government dynamically substitutes between the intratemporal and intertemporal distortions.

The chapter provides a comparative static analysis of the tax rates and external debt with respect to the skill dispersion, measured as the ratio of individual labor productivity levels. Increasing in relative skill dispersion increases the labor tax when the debt constraints do not bind, and decreases labor tax limits. A higher skill-dispersion means a higher motive for redistribution, translating into a higher level of tax before the debt constraint binds. When reaching the debt constraints, the economy faces a higher distortion. In order to relax the debt constraints, the planner needs to reduce the higher efficiency cost. Consequently, the labor tax limit declines when the skill dispersion rises. When the debt constraints start binding, the return on domestic savings is higher the higher the skill dispersion is, but eventually converge to the steady-state non-zero rates. The external debt levels also change with respect to skill dispersion, in which for the higher the skill dispersion, the longer the periods of unconstrained borrowing, and the higher external debt positions in the long run. The main reason is that it is more costly for the higher dispersed economy to redistribute in financial autarky. Redistribution now leads to high debt accumulation before reaching the constraints, which leads to a higher debt later on to sustain the allocation.

Extending the analysis to separable preferences, a few results change. First, the labor tax is not constant even when the debt constraints do not bind. Instead, it changes with respect to the time-varying elasticities. The long-run convergence property of the labor tax is similar to the case of constant elasticities. If the economy converges to a steady state, the optimal labor tax also converges to a real constant. However, if steady-state allocation does not exist, the long-run optimal labor tax fluctuates in a bounded region. Similar to the case of separable isoelastic preference, this region corresponds to the region of the maximum debt capacity of the economy.

Related Literature. Several empirical papers have documented the correlation between the income dispersion and sovereign debt. Specifically, Berg and Sachs (1988) showed that income dispersion was a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) described a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Recently, Ferriere (2015) and Jeon and Kabukcuoglu (2018) also provided empirical evidence of rising income dispersion significantly increases sovereign default risk. This chapter provides a theoretical model that qualitatively accounts for the positive relationship between income dispersion and debt level.

This chapter finds optimal policy by characterizing the best allocation of any tax-distorted debt constrained equilibrium, i.e. the primal approach as in the public finance literature (Barro (1979), Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari et al. (2002), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this chapter, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders. The non-zero capital and domestic borrowing taxes are contrast to the zero convergence of the capital tax from Judd (1985), Chamley (1986), Chari, Christiano, and Kehoe (1994), and Straub and Werning (2014), because here the capital stock and domestic borrowing affect the debt constraints have an externality in influencing the debt constraints.

The chapter also contributes to the sovereign debt literature. Seminal papers studied sovereign debt in the lack of commitment environment include Eaton and Gersovitz (1981), Aguiar and Gopinath (2006a) Arellano (2008), Aguiar, Amador, and Gopinath (2009), and Aguiar and Amador (2011). There are a few recent works on optimal policies in the Eaton-Gersovitz-Arellano framework (Ferriere (2015) and Arellano and Bai (2016)), in which the government sometimes defaults in equilibrium and the spread of the sovereign debt depends on the probability of default. They quantitatively found that in the case of the government committing to the tax policy, fiscal austerity (in terms of raising tax rates), distortionary affect of the taxes would have made the country more likely to default, and hence it is not optimal. This chapter instead studies the self-enforcing debt contract as in

Aguiar and Amador (2011) and Aguiar and Amador (2014), and allows the government not to commit to any tax and debt policies.

The dynamic environment in this chapter is an extension to one in Aguiar and Amador (2016), adding heterogeneity, distribution motive, and allowing for richer tax systems. Aguiar and Amador (2016) found that labor tax must go to zero in the long run as a result of front-loading efficient consumption and leisure allocation. In this chapter, the tax limit can be any real value. More interestingly, when turning off the redistributive effect in the model, the limit of labor tax is zero, consistent with their findings. The chapter shows that redistributive consideration is the main source for the changes in optimal policies.

Work in optimal taxation with heterogeneity and redistributive motive includes Bhandari et al. (2017) and Werning (2007), which both found that redistribution had significant impact on optimal policies. This chapter instead explores redistribution in a small open economy setting and endogenous aggregate debt constraints. Werning (2007) developed the conditions for perfect tax smoothing, while Bhandari et al. (2017) emphasized the impact of the distribution of initial asset holdings on optimal allocation. The framework and analysis in this chapter are closely related to Werning (2007). The finding is that labor tax smoothing only occurs when the debt constraint does not bind. Long-run binding debt constraints then alters the dynamic of the labor tax, resulting in imperfect tax smoothing. Moreover, the initial distribution of after-tax net asset holdings matters in determining the private market shares across agents, which indirectly affect the labor tax limit.

Several recent chapters addressed the trade-off between redistribution and external debt. D'Erasmo and Mendoza (2016) focused on how redistributive incentives affected defaults on domestic debt. They asserted that equilibrium with debt could be supported only when the government was politically biased towards bond holders. Ferriere (2015) showed how modifying tax progressiveness could mitigate the cost of default. Dovis, Golosov, and Shourideh (2016) argued how the interaction between inequality and debt endogenized the dynamic cycles of debt, taxes, and transfers over time. Balke and Ravn (2016) studied time-consistent fiscal policy in a sovereign debt model la Eaton and Gersovitz (1981) with inequality through unemployment. They found that austerity policies were optimal during debt crises since they reduced default premium, which was correlated with debt issuance, and increased access to international lending market. This chapter instead emphasizes on the enforcement constraints arising from a self-enforcing contracting problem among the government, international lenders, and domestic agents. These constraints restrict the present value of future social welfare, which act like endogenous debt constraints. The chapter features the long-run binding debt constraints, in which austerity policies might not be optimal if they generate more distortion. The analysis here is more general with any distributive preference, instead of an utilitarian social welfare function as in these papers. The general redistributive motive determines both the level and dynamic of the taxes.

Outline. The chapter is organized as follows. Section 1.2 describes the environment, the competitive equilibrium, and the government's lack of commitment problem. Section 1.3 characterizes the equilibrium. Section 1.4 formulates the efficiency problem, while section 1.5 derives the main results of the optimal policies. Section 1.6 analyzes the effect of redistribution on optimal taxes. Section 1.7 provides the numerical exercise, explaining the dynamics of the efficient allocation as well as the comparative statics, and Section 1.8 generalizes the optimal tax formulas with separable preferences. Section 1.9 then concludes.

1.2 Model

1.2.1 Environment

A small open economy faces exogenous world interest rates $\{r_t^*\}_{t=0}^{\infty}$. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $\left(\theta^i\right)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $\left(\pi^i\right)_{i\in I}$ is normalized such that $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficient labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t U^i(c_t^i, l_t^i) \tag{1.1}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses both capital and labor to produce a single output good. The production function $F(K,L)$ is constant return to scale, where K and L are respectively the aggregate capital and labor. The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0}^{\infty}$. In each period, the government issues domestic and foreign bonds, imposes a lump-sum tax T_t, a marginal tax on labor income τ_t^n, a marginal tax on capital income τ_t^k, and sets the return on domestic bond r_t. Assume that only the government can borrow abroad.[1]

[1]This assumption is based on the empirical observation that domestic households often hold a very small amount of foreign assets.

1.2.2 Equilibrium

Consider the set of prices facing by agents and firm: w_t the labor wage, r_t^k the return on capital, and δ the capital depreciation rate.

Agents. Agent of type $i \in I$ faces the sequential budget constraint in period t,

$$c_t^i + k_{t+1}^i + b_{t+1}^{d,i} \leq (1-\tau_t^n) w_t l_t^i + \left[1 + (1-\tau_t^k) r_t^k - \delta\right] k_t^i + (1+r_t)\, b_t^{d,i} - T_t, \tag{1.2}$$

where $c_t^i, l_t^i, k_t^i, b_t^{d,i}$ denote consumption, effective labor, capital holding, and domestic bond holding of agent i in period t, respectively.

Moreover, no arbitrage exists such that the after-tax return is the same when investing in capital or in domestic bonds, i.e.

$$1 + (1-\tau_t^k) r_t^k - \delta = 1 + r_t,$$

which implies $(1-\tau_t^k) r_t^k = r_t + \delta$.

Representative Firm. The firm chooses capital and labor to maximize profit each period:

$$\max_{\{K_t, N_t\}} \quad F(K_t, L_t) - w_t L_t - r_t^k K_t,$$

which gives the following first-order conditions:

$$\begin{aligned} r_t^k &= F_K(K_t, L_t) \\ w_t &= F_L(K_t, L_t) \end{aligned} \tag{1.3}$$

The firm's profit is zero in equilibrium because of the constant return to scale production function.

Government. The government needs to finance an exogenous expenditure $\{G_t\}_{t=0}^{\infty}$. The government sells one-period bond B_t^d to domestic agents and B_{t+1} to international lenders at a price Q_{t+1}. The government's budget constraint in each period is

$$G_t + (1+r_t) B_t^d + B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t^k K_t + B_{t+1}^d + Q_{t+1} B_{t+1} + T_t,$$

where $L_t = \sum_{i \in I} \pi^i l_t^i$ is the aggregate labor, $K_t = \sum_{i \in I} \pi^i k_t^i$ is the aggregate capital, $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$ is the aggregate domestic bond, and B_t is the amount of the government's external debt. The government faces a no-Ponzi condition such that the present value of external debt is bounded below.

Define q_t as international price of a unit period-t consumption in terms of period-0 consumption units:

$$q_t = \Pi_{s=0}^{t} \frac{1}{1+r_s^*} \tag{1.4}$$

Optimality of the risk-neutral international lenders' problem gives $Q_t = \frac{1}{1+r_t^*}$. Using $\{q_t\}_{t=0}^{\infty}$ as the relevant intertemporal price, one can write the government's present-value budget constraint as

$$\sum_{t=0}^{\infty} q_t \left\{ G_t - \tau_t^n w_t L_t - \tau_t^k r_t^k K_t + (1+r_t) B_t^d - B_{t+1}^d - T_t \right\} \leq -B_0 \tag{1.5}$$

with normalizing $1 + r_0^* = 1$. [2]

Aggregate resource constraint. Using the agent's budget constraint and the government's budget constraint, one can obtain an aggregate resource constraint in terms of the intertemporal prices and the initial external debt:

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t \right] - B_0 \geq 0 \tag{1.6}$$

Competitive equilibrium. Given the above equations, the following defines a competitive equilibrium with taxes.

Definition 1.1. *Given initial external debt B_0 and individual wealth positions $\left(a_0^i\right)_{i\in I}$,*[3] *a competitive equilibrium with taxes for an open economy, is agent's allocation $z^{H,i} = \left\{c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{i,d}\right\}_{t=0}^{\infty}$, $\forall i \in I$, the firm's allocation $z^F = \{K_t, L_t\}_{t=0}^{\infty}$, prices $p = \left\{q_t, Q_t, w_t, r_t, r_t^k\right\}_{t=0}^{\infty}$, and government's policy $z^G = \left\{\tau_t^n, \tau_t^k, T_t, r_t, B_{t+1}^d, B_{t+1}\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves agent i's problem that maximizes (1.1) subject to (1.2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, which implies the first-order conditions (1.3), (iii) government budget constraint (1.5) holds, (iv) aggregate resource constraint (1.6) is satisfied, $\sum_{i\in I} b_t^{d,i} = B_t^d$, (iv) no arbitrage condition $(1-\tau_t^k)r_t^k = r_t + \delta$, and (v) p satisfies (1.4) given z^G.*

[2] This assumption is without loss of generality to fix the initial level of external debt.

[3] $a_0^i \equiv \left[1 + (1-\tau_0^k)r_0^k - \delta\right] k_0^i + (1+r_0)\, b_0^{d,i}$

1.2.3 Lack of commitment

Assume that the government is benevolent in that its objective is the weighted discounted utility of all agents in the economy, given by a set of social welfare weights $\boldsymbol{\lambda} = \left(\lambda^i\right)_{i \in I}$.[4] The government enters into contracts with private agents and foreign creditors that specify allocation of consumption, capital, labor, domestic and foreign bonds. Nevertheless, in every period, the government can drop external and domestic obligations, change the tax schedules, and expropriate all capital holdings. When deviating from the contracted allocation, the government faces the punishment from private agents and foreign lenders. The government then receives a deviation utility $\underline{U}_t(K_t)$ that depends on the current aggregate capital level that it can expropriate. The limited commitment implies that there exists a lower bound on future discounted aggregate utility. Specifically, for all $t \geq 0$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i \in I} \lambda^i \pi^i U^i \left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{1.7}$$

Following Chari and Kehoe (1990, 1993), this sustainability constraint is a characterization of a sub-game perfect equilibrium of a dynamic sovereign game between the government, private agents, and foreign creditors. The equilibrium sustains risk-free debt and no default on path. Appendix A.1 provides the details of the game and the equilibrium definition. In general, the set of sustainable equilibrium payoffs of this sovereign game can be supported by trigger strategies to the equilibrium that has the worst payoff. Due to the complication in characterizing the worst equilibrium of this dynamic game, this chapter uses autarky, in which there are no international and domestic financial markets, as the punishment for deviation. This assumption does not change the main results of the chapter. The reverting-to-autarky equilibrium is characterized by the constraint (1.7) where $\underline{U}_t(K_t)$ incorporates the autarkic value.[5] It is a constraint on aggregate allocation such that private agents do not directly take into account when solving their problems. The constraint imposes endogenous limits on the aggregate debt levels over time.

[4] For the relationship between the welfare weights and the utility frontier, see Section 1.4.

[5] Many papers have pointed out the problem in characterizing the worst equilibrium in this type of dynamic games as it might not be the repeated worst static Nash equilibrium. Instead, they made the same assumption of using autarky as the worst equilibrium (see Chari and Kehoe (1993) and Dovis, Golosov, and Shourideh (2016)).

1.3 Characterizing Equilibrium

In equilibrium, because all agents have the same preference, face the same tax rates, earn the same wage on their efficient labor units, and have the same return on savings, the intratemporal and intertemporal conditions are the same across agents. In each period t, for all i,

$$\begin{aligned}(1-\tau_t^n)w_t =& -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)} \\ 1+r_{t+1} =& \frac{U_c^i(c_t^i, l_t^i)}{\beta U_c^i(c_{t+1}^i, l_{t+1}^i)}\end{aligned}$$

Given the aggregate allocation (C_t, L_t) in every period, there is an efficient assignment of individual allocation $\left(c_t^i, l_t^i\right)_{i\in I}$ due to equal marginal rates of substitution between consumption and efficient labor. Moreover, because of equal marginal rates of substitution of future to current consumption, the efficient assignment needs to be the same across time. Any inefficiencies due to tax distortions are captured by the aggregate allocation. Werning (2007) incorporated these properties of the equilibrium allocation into first analyzing the static distortion problem, then looking at the dynamics in aggregate levels. This method allows for aggregation such that the competitive equilibrium allocation can be completely characterized in terms of aggregates and a static rule for individual allocation.

Sub-market analysis. For any equilibrium, there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual assignments solve a static problem

$$\begin{aligned}V(C, L; \boldsymbol{\varphi}) \equiv & \max_{(c^i, l^i)_{i\in I}} \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\ & s.t. \qquad \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L\end{aligned}$$

The market weights capture how individual allocation are determined given any aggregate allocation. This problem gives the policy functions for each agent i:

$$h^i(C, L; \boldsymbol{\varphi}) = \left(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\right)$$

A competitive equilibrium allocation must satisfy $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$, for all i and t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C, L; \varphi)$. Envelope conditions of the static problem give

$$(1-\tau_t^n)w_t = -\frac{V_L\left[h^i(C_t,L_t;\boldsymbol{\varphi})\right]}{V_C\left[h^i(C_t,L_t;\boldsymbol{\varphi})\right]} \tag{1.8}$$

$$1+r_{t+1} = \frac{V_C\left[h^i(C_t,L_t;\boldsymbol{\varphi})\right]}{\beta V_C\left[h^i(C_{t+1},L_{t+1};\boldsymbol{\varphi})\right]} \tag{1.9}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\sum_{t=0}^{\infty}\beta^t\left[V_C(C_t,L_t;\boldsymbol{\varphi})h^{i,c}(C_t,L_t;\boldsymbol{\varphi})+V_L(C_t,L_t;\boldsymbol{\varphi})h^{i,l}(C_t,L_t;\boldsymbol{\varphi})\right] = V_C(C_0,L_0;\boldsymbol{\varphi})\left(a_0^i - T\right), \tag{1.10}$$

where T is the present-value of lump-sum taxes,[6] and a_0^i is the individual initial after-tax wealth. Equation (1.10) is the individual implementability constraint.

Now one has the following characterization proposition.

Proposition 1.1. *Given initial individual wealth $\left\{a_0^i\right\}_{i\in I}$ and external debt B_0, an allocation $\{C_t,L_t,K_{t+1}\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if aggregate resource constraint (1.6) holds, and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$ and lump-sum tax T such that the implementability constraint (1.10) holds for all $i \in I$.*

1.4 Efficiency

This section formulates the planning problem in terms of a Ramsey's problem with the additional sustainability constraints induced by the limited commitment. It follows the primal approach in public finance to characterize the best equilibrium allocation and derive the optimal policies.

1.4.1 Planning problem

The set of equilibrium with limited commitment can be supported as a competitive equilibrium with taxes and the sustainability constraint (1.7). Define the set $\mathcal{U}$ of attainable utilities $\left\{u^i\right\}_{i\in I}$ such that $u^i = \sum_{t=0}^{\infty}\beta^t U^i\left(c_t^i, l_t^i\right)$ for any such equilibrium allocation. Given Proposition 1.1, $\left\{u^i\right\}_{i\in I}$ is the individual lifetime utilities for any allocation $\{C_t,L_t,K_t\}_{t=0}^{\infty}$ and a vector of market weights $\boldsymbol{\varphi}$ such that the aggregate

[6] $T \equiv \sum_{t=0}^{\infty}\beta^t \frac{V_C\left[h^i(C_t,L_t;\boldsymbol{\varphi})\right]}{V_C\left[h^i(C_0,L_0;\boldsymbol{\varphi})\right]}T_t$

resource constraint and the implementability constraint all $i \in I$ are satisfied. Specifically, $u^i = \sum_{t=0}^{\infty} \beta^t U^i \left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$. An efficient allocation is defined as one that reaches the northeastern frontier of $\mathcal{U}$, i.e. maximizing lifetime utility of one agent given that the utilities of other agents are above feasible thresholds. Necessary conditions can be derived by an alternative problem of maximizing a Pareto-weighted utility, where the Pareto weights are closely related to the feasible thresholds.[7]

Therefore, given the Pareto weights $\boldsymbol{\lambda} = \left\{\lambda^i\right\}_{i\in I}$ and exogenous international interest rates $\{r_t^*\}_{t=0}^{\infty}$, an efficient allocation maximizes the weighted utility subject to the aggregate resource constraint, the individual implementability constraints, and each-period sustainability constraint. The planning problem is formulated as

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} \quad & \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi})\right] \\
s.t. \qquad & \sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
\forall i, \; & \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
\forall t, \; & \sum_{s=t}^{\infty} \sum_{i\in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s; \boldsymbol{\varphi})\right] \geq \underline{U}_t(K_t)
\end{aligned}
$$

1.4.2 Characterizing efficient allocation

Let μ be the multiplier on the resource constraint, $\pi^i \eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t \gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = \left(\eta^i\right)_{i\in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints as

$$
\sum_{t=0}^{\infty} \beta^t W\left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(0; \boldsymbol{\varphi}) \sum_{i\in I} \pi^i \eta^i \left(a_0^i - T\right),
$$

where

$$
W\left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] \equiv \sum_{i\in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi})\right] + \sum_{i\in I} \pi^i \eta^i \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right]
$$

[7] As the set of attainable utilities $\mathcal{U}$ might not be convex, an allocation that solves (P) might not attain the utilities in $\mathcal{U}$. The analysis focuses on the necessary conditions, as they are enough to develop the properties of the optimal taxes. The set of optimal taxes is a subset of the set of taxes that implement any allocation satisfying the necessary conditions for efficiency. Therefore, optimal taxes also satisfy the attributes of taxes deriving from the necessary analysis. Park (2014) and Werning (2007) made the similar argument in their work.

The necessary conditions to characterize the set of efficient allocation are the first-order conditions of the planning problem, the aggregate resource constraint, the sustainability constraints, and the implementability constraints.

1.5 Optimal Taxation

This section provides results on optimal taxation. The work emphasizes on the case of separable isoelastic preferences. Optimal taxes implement the efficient allocation as an allocation of a competitive equilibrium with taxes. A key assumption throughout this section is the impatience of private agents with respect to the international intertemporal interest rates that the country faces when borrowing abroad, i.e.

Assumption 1.1 (Impatience)**.** *There exists $M > 0$ and T such that $\forall t > T,\ \beta(1 + r_t^*) < M < 1$.*

Consider the following functional form of the utility

Assumption 1.2 (Separable isoelastic preference)**.** *The utility function $U :\ \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ satisfies*

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \omega \frac{n^{1+\nu}}{1+\nu}$$

for $\sigma, \omega, \nu > 0$.

Given that the preference is separable and isoelastic, individual consumption and efficient labor supply are time-independently proportional to the aggregates

$$\begin{aligned} c_t^i &= h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) = \psi_c^i C_t \\ l_t^i &= h^{i,l}(C_t, L_t; \boldsymbol{\varphi}) = \psi_l^i L_t, \end{aligned} \tag{1.11}$$

where

$$\psi_c^i = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i \in I} \pi^i (\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}} \left(\varphi^i\right)^{-1/\nu}}{\sum_{i \in I} \pi^i \left(\theta^i\right)^{\frac{1+\nu}{\nu}} \left(\varphi^i\right)^{-1/\nu}} \tag{1.12}$$

Then V, W inherit the separable and isoelastic properties from U, i.e. $\forall t$,

$$\begin{aligned} V(C_t, L_t; \boldsymbol{\varphi}) &= \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu} \\ W\left[C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] &= \Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L_t^{1+\nu}}{1+\nu} \end{aligned}$$

and the objective is

$$\sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right),$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$, Φ_C^W, Φ_L^W depend on $\boldsymbol{\varphi}, \boldsymbol{\lambda}$, and $\boldsymbol{\eta}$, and Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix A.2.1).

The first-order conditions of the planning problem for any period $t \geq 1$ are

$$F_L(K_t, L_t) = \frac{\left\{ \Phi_L^W + \Phi_L^P \sum_{s=0}^t \gamma_s \right\} L_t^\nu}{\left\{ \Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s \right\} C_t^{-\sigma}} \tag{1.13}$$

$$F_K(K_t, L_t) = r_t^* + \delta + \frac{\beta^t}{q_t} \frac{\gamma_t}{\mu} \underline{U}_t'(K_t) \tag{1.14}$$

and

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} (1 + r_{t+1}^*) \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t+1} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s} \right] \tag{1.15}$$

From the competitive equilibrium's characterization, taxes on labor, capital and saving return must satisfy

$$(1 - \tau_t^n) F_L(K_t, L_t) = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}} \tag{1.16}$$

$$(1 - \tau_t^k) F_K(K_t, L_t) = r_t + \delta \tag{1.17}$$

$$C_t^{-\sigma} = \beta \left(1 + r_{t+1}\right) C_{t+1}^{-\sigma} \tag{1.18}$$

1.5.1 Labor income tax

Dividing equation (1.13) by equation (1.16) gives

$$\tau_t^n = 1 - \frac{\Phi_L^V}{\Phi_C^V} \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s}{\Phi_L^W + \Phi_L^P \sum_{s=0}^t \gamma_s} \right] \tag{1.19}$$

which is the optimal labor income tax. The time-variant component of the optimal labor tax is the sum of the Larange multipliers on the debt constraints, reflecting how limited borrowing influences the dynamic of taxes. When aggregate debt constraints do not bind, i.e. $\gamma_s = 0$, $\forall s \leq t$, the optimal labor tax becomes

$$\tau_t^n = 1 - \frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \equiv \bar{\tau}^n, \tag{1.20}$$

which is a time-independent constant. Thus, the optimal labor tax is constant when the debt constraint is not relevant. The intuition is that distortionary tax is a mechanism for redistribution. The distortion reflects the trade-off between the dispersion level, which is determined by the skill distribution, and the redistributive motive, which is from the social welfare weights. Because the skill distribution and welfare weights do not change, and borrowing is unconstrained, the government finds it optimal keep the intratemporal distortion constant and borrow as needed to finance expenditure. The unconstrained optimal level is formulated by (1.20), which indirectly depends on skill distribution and Pareto weights (see Appendix for the formulas of Φ's).

On the other hand, binding debt constraints limit the government's ability to borrow and smooth taxes over time. When the debt constraint binds, the cumulative sum of debt-constraint multipliers show up in the optimal labor tax formula. Given that private agents are impatient, the country's aggregate debt increases over time. In the environment without debt constraints, or debt constraints never bind, the Ramsey allocation features immiseration in the long run such that the marginal utility of consumption is growing without bound. Such scenario happens when the deviation utility is unbounded below, i.e. the value of deviating is low enough such that the government will always commit to the contract. However, if the deviation utility is bounded below, the full-commitment Ramsey allocation cannot be supported. There is no immiseration in the long run as the future utility is always bounded below. This no immiseration result is common in many models of limited commitment.

No immiseration also means that the debt constraint eventually binds, and the multiplier γ_t increases over time. In the long run, the cumulative sum of multipliers will diverge.[8] Given equation (1.19), it must be that $\lim_{t\to\infty} \tau_t^n = 1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$, or by substituting in the definitions,

$$\lim_{t\to\infty} \tau_t^n = 1 - \frac{\sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_c^i}{\sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_l^i}, \tag{1.21}$$

which is a different level from the unconstrained distortion level. The optimal tax limit similarly depends on redistributive preference, which comes from $\left\{\lambda^i\right\}_{i\in I}$, and inequality, captured by $\left\{\varphi^i\right\}_{i\in I}$ (utility shares),$\left\{\psi_c^i\right\}_{i\in I}$(consumption shares), and $\left\{\psi_l^i\right\}_{i\in I}$(labor shares).

Formally, consider the following assumption on deviation utility

Assumption 1.3. $\underline{U}_t(\cdot)$ *is bounded below, i.e. there exists a finite real* M_U *such that*

[8] Intuitively, optimal allocation must satisfies $\beta^t/q_t \left(\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s\right) C_t^{-\sigma} = \mu$. No immiseration implies that $C_t^{-\sigma}$ is bounded below. Since $\mu > 0$, as $\beta^t/q_t \to 0$, it must be that $\sum_{s=0}^t \gamma_s \to \infty$

$\inf_{K_t} \underline{U}_t(K_t) \geq M_U$.

Given this assumption, the consumption path is bounded below by zero in the long run, i.e.

Lemma 1.1 (No immiseration). *Suppose assumptions 1.2 and 1.3 hold, then for any efficient allocation* $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.

The following proposition characterizes the optimal labor tax in an economy facing debt constraints and distributive concern.

Proposition 1.2 (Optimal labor tax). *Given assumption 1.2, if an efficient allocation exists and debt constraint does not bind, there is constant labor tax given by (1.20). Moreover, if assumptions 1.1 and 1.3 also hold, and an interior efficient allocation exists, then the optimal labor tax converges to a real constant given by (1.21) that depends on skill distribution and redistributive preference. These results hold with or without the lump-sum transfers.*

Redistributive motive and limited borrowing

Both the redistributive motive and the limited borrowing determine the optimal level of distortion in the economy, expressed in equation (1.19). The intuition is best explained in the case with lump-sum transfers. The optimal tax then depends on the marginal benefit of redistribution and the marginal cost of distortion. When borrowing is not constrained, it is optimal to set the marginal cost equal to the marginal benefit, which is constant over time. Therefore, optimal tax rates do not change during the periods that debt constraints do not bind. As debt constraint binds, there is an additional benefit of relaxing the constraints. The marginal cost of distortion is equal to the net marginal benefit of redistribution and relaxing debt constraints. The following Proposition shows that this marginal cost of distortion is decreasing over time as the debt constraint binds, in an environment of only skill heterogeneity, a high consumption-inequality aversion, and a high redistributive motive towards low-skilled agents.

Proposition 1.3. *Given assumptions 1.1–1.3, and additionally suppose that there are (i) equal initial wealth distribution:* $a_0^i = a_0^j$, $\forall i, j \in I$, *(iii) high consumption-inequality aversion:* $\sigma \geq 1$, *and (iv) redistributive motive towards the low types:* $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, $\forall i, j \in I$, *then for any period* t *such that the debt constraint binds, the optimal labor tax decreases, i.e.* $\tau_t^n \leq \tau_{t-1}^n$. *Moreover,* $\tau_s^n \leq \tau_{t-1}^n$, $\forall s \geq t$.

This Proposition points out that it requires a lower labor tax rate not only at the period the debt constraint binds, but also at any periods afterwards. The optimal labor tax drifts downward over time as debt constraints bind, and given Proposition 1.2, it will eventually converge to a limit. As a result, the optimal labor tax before the debt constraint binds is at least as high as the optimal limit, i.e.

Corollary 1.1. *Given the assumptions of Proposition 1.3, $\bar{\tau}^n \geq \lim_{t\to\infty} \tau_t^n$.*

Intuitively, a government that has a high redistributive motive would like to keep a high labor tax. However, labor taxes weakly decrease whenever these constraints bind. A lower tax rate, in return, encourages more labor supply, output, and relaxes the debt constraints by increasing its borrowing capacity.

In general, this efficiency motive is a driver for the dynamic of optimal taxes. Consider the following expenditure minimization problem for each period t

$$
\begin{aligned}
(EM_t) \equiv \min_{C_s, L_s, K_{s+1}} & \sum_{s=t}^{\infty} q_s \left[C_s + G_s + K_{s+1} - F(K_s, L_s) - (1-\delta)K_s\right] \\
s.t. \quad & \sum_{s=t}^{\infty} \beta^{s-t} \left(\Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu}\right) = \underline{U}_t(K_t),
\end{aligned}
$$

which is the problem of minimizing the present value of resources needed to deliver $\underline{U}_t(K_t)$ as the planner's promised utility at period t. The solution to this minimization problem can be implemented with the labor tax $\tau_s^n = 1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$ for $s \geq t$. This equation is exactly the long-run limit of optimal labor taxes. The optimal labor tax formula (1.19) incorporates the solution to the Ramsey planner when debt is unconstrained (Φ^W's) and as debt constraint binds, a part of the solution to this expenditure minimization problem shows up (Φ^P's) with respect to the tightness of debt constraints ($\sum_{s\leq t} \gamma_s$). When there is unlimited borrowing, the planner should set the tax rate to achieve the most redistributive allocation, which is the allocation that the planner would choose if she never runs into debt constraints. However, when debt constraint binds, the planners want international lenders to continue the contract by offering an allocation such that it delivers the promised utility $\underline{U}_t(K_t)$ in a less costly way. As the debt constraint binds in the long run, the optimal allocation continues to lower the delivering cost and eventually reaches the allocation with minimal cost, which is the solution to (EM_∞). The labor tax starts of with the most redistributive level for the country, which is $1 - \frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W}$ and gradually converges to the most efficient level , i.e. $1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$.

In the case without lump-sum transfers, besides the redistributive benefit, the optimal tax rate is also set to meet the budgetary needs of the government. This additional need only changes the optimal tax levels, but not the dynamics. The limited borrowing still requires the distortion to be front-loaded.

1.5.2 Capital income and saving taxes

Combining equations (1.15) and (1.18), the optimal domestic return satisfies

$$1 + r_t = (1 + r_t^*) \frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t-1} \gamma_s} \tag{1.22}$$

When the sustainability constraint is not relevant, i.e. $\gamma_s = 0 \ \forall s \leq t$, it is optimal to set the domestic interest rate equal to the exogenous foreign interest rate. However, as the sustainability constraint binds, equation (1.22) implies that $r_t > r_t^*$, a saving subsidy. As the economy reaches its debt limits, shown as binding sustainability constraints, the government has an incentive to subsidize more on saving, or tax more on borrowing, to discourage private borrowing.

Moreover, the first-order conditions (1.14) and (1.17) give

$$\tau_t^k = 1 - \frac{r_t + \delta}{r_t^* + \delta + \frac{\beta^t}{q_t} \frac{\gamma_t}{\mu} \underline{U}_t'(K_t)} \tag{1.23}$$

If $\underline{U}_t'$ is positive, the above equation implies that there is a higher tax on capital income when debt constraints bind.[9] A greater capital tax reflects that there is a capital under-investment of the efficient allocation. Indeed, the first-order condition (1.14) shows that $F_K(K_t, L_t) > r_t^* + \delta$ when $\gamma_t > 0$, where $r_t^* + \delta$ is the first-best interest rate. Because the government expropriates more capital and receives higher utility from reneging, the optimal contract discourages capital accumulation, which can be implemented by imposing higher taxes on capital income.

1.6 Effect of Redistribution

This section analyzes how the government's redistributive motive affects the optimal labor tax. The redistribution not only influences tax levels from the trade-off between equity and efficiency, but also the tax dynamics from the interaction with debt constraints. Suppose that there is no heterogeneity, then the problem becomes the standard representative-agent

[9]The higher the amount of capital the government can expropriate, the higher the deviation utility. Proposition A.1 proves a case when it is true.

Ramsey's problem of a small open economy. If the government can use lump-sum taxes, there is no need for distortion, and the optimal labor tax will be zero. If the government can only impose distortionary taxes, the model collapses to the representative-agent setting as in Aguiar and Amador (2016), where the zero labor tax in the limit is optimal.

Proposition 1.4 (No heterogeneity)**.** *Suppose that $\theta^i = \theta^j, a_0^i = a_0^j$, $\forall i, j \in I$. Then there is zero labor tax in the long run. This result holds with or without lump-sum transfers.*

Proof. Follow from equation (1.21) with $\varphi^i = \psi_c^i = \psi_l^i = 1$, $\forall i \in I$. □

While equilibrium market weights determine how the competitive market chooses individual shares of utility, Pareto weights regulate social shares of utility. Any agent has an exogenous Pareto weight based on the government's distributive preference, and a market weight that depends on her relative skill and initial wealth. This is due to skill and initial wealth positions determine individual budget constraints, which gives individual implementability constraints in the planning problem. An interesting case is when the vector of market weights is equal to the vector of Pareto weights ($\boldsymbol{\psi} = \boldsymbol{\lambda}$). This property implies that there is no distributive effect, because the government, as a planner, distributes aggregate utility exactly the same way as the competitive market does. In this situation, (1.21) turns out to be $\lim_{t\to\infty} \tau_t^n = 0$, that is, the optimal labor tax converges to zero. The following Proposition summarizes these results.

Proposition 1.5. *There exists an efficient allocation $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ such that for all i, $\lambda^i = \varphi^{*i}$. Such an allocation can be implemented with a zero labor tax in the long run.*

Changing welfare weights, which represent the redistributive motive, affects the social utility, and so debt constraints.[10] Proposition 1.5 shows that the redistributive motive affects the level of optimal taxes only when it can deviate from the competitive equilibrium distribution. In the special case where welfare weights equal to inverse of marginal utilities, the efficient allocation is the non-distorted competitive equilibrium allocation. The redistributive motive, not heterogeneity, is the source of changes in optimal policies comparing to the representative-agent setting.

[10]The welfare weights determine the discounted future utility and the deviation utility, derived in the Appendix, so they influence both sides of the sustainability constraint.

1.7 Numerical Exercise

This section illustrates previous theoretical results by a numerical exercise. There is no capital. Production is linear in effective labor, i.e. $F(L_t) = L_t$. The deviation utility is a constant finite $\underline{U}$ so that it is consistent with Assumption 1.3.

Lemma 1.2. *If the sustainability constraint binds for some finite S , then it will bind for all $t > S$.*

Lemma 1.2 implies that the social welfare $\sum_{i \in I} \lambda^i \pi^i U^i[h^i(C_t, L_t; \boldsymbol{\varphi})]$ is equal to across all period $t > S$ that debt constraint binds. Combining this feature and the planner's first-order conditions gives the efficient allocation at each period after debt constraints bind, and then the allocation at period $t < S$ can be derived by solving backwards.

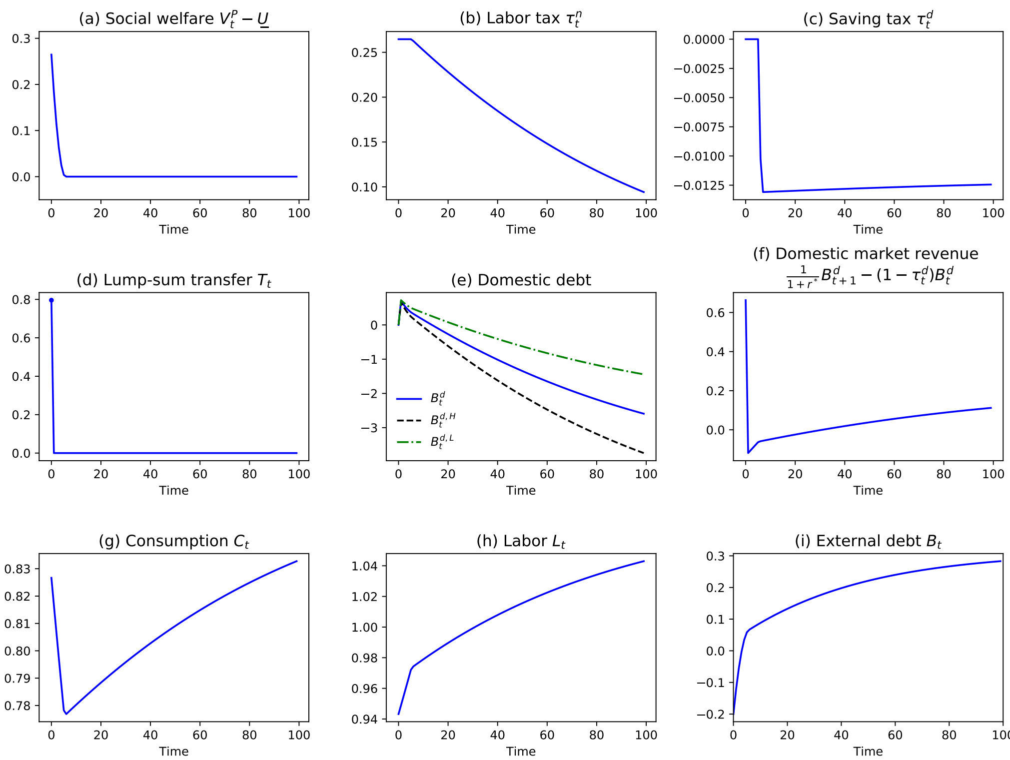
Consider a parametric economy consisting of two types of agents, denoted $I = \{H, L\}$, where $\theta^H \geq 1 \geq \theta^L$. Let $\pi^H = \pi^L = 0.5$, which implies that $\theta^H = 2 - \theta^L$. The two types have zero initial wealth positions, i.e. $a_0^i = 0$, $\forall i$. Let $U(c, n) = \log c - \omega \frac{n^{1+\nu}}{1+\nu}$, $\omega = 1$, $\nu = 2$ so that the Frisch elasticity of labor supply is 0.5. Assign $\beta = 0.94$ and $r_t^* = r^* = 0.05$ so that $\beta(1 + r^*) < 1$. The planner is utilitarian: $\lambda^H = \lambda^L = 1$. The government expenditure is constant across time and set to be 20 percent of the average productivity. The economy starts with an initial external asset position of 20%. $\underline{U}$ is the value of autarky, calculated as the maximal utility attained from a tax-distorted competitive equilibrium of the economy with no domestic and international credit markets.

1.7.1 Dynamics of policies and allocation

Figure 1.1 depicts the time paths of optimal policies and efficient allocation when the relative skill dispersion is such that $\theta^H = 2\theta^L$. Figure A.0 expands the time periods to show the long-run properties. Time is the horizontal axis. Panel (a), (b), and (c) plot the planner's utility relative to the deviation utility, the optimal labor tax, and the optimal domestic saving tax, respectively. The planner's utility is decreasing over time until it reaches the deviation utility and stays constant, as the debt constraint binds. When the debt constraint does not bind, the optimal labor tax starts at a positive level and constant, while the optimal saving tax is zero. As the debt constraint starts binding, optimal labor tax decreases while saving is optimally subsidized, implying a positive tax on borrowing. Figure A.0 shows that in the long run, there is labor subsidy and borrowing tax.

Due to Ricardian equivalence, I consider a particular implementation of the efficient allocation where the planner gives the present-value lump-sum transfer only in period 0. Panel (d) depicts the lump-sum transfer over time. Given this implementation, panel (e)

Figure 1.1: Time paths of economic aggregates when $\theta^H = 3\theta^L$



plots aggregate and individual domestic debt levels. Because of impatience, domestic agents borrow over time, and the planner acts as an intermediary that borrows abroad and lends to domestic agents. However, when debt constraints bind, the planner levies taxes on borrowing. In net, the planner collects revenue from the domestic market, as described in panel (f).

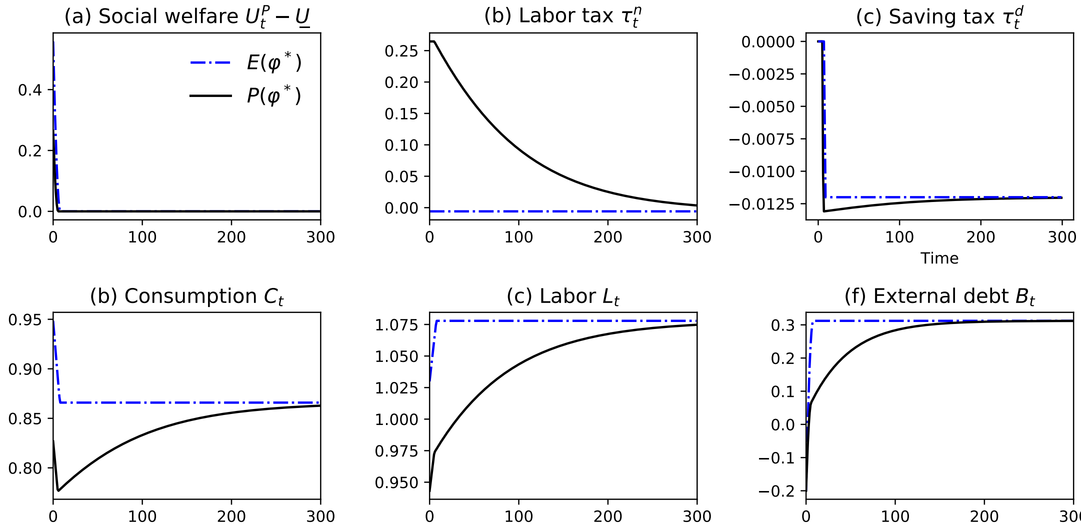
Panel (g) and (h) plot the dynamics of the aggregate consumption and labor. When debt constraints do not bind, there is front-loading consumption and leisure. When debt constraints bind, given the positive tax on borrowing and the decreasing labor tax, aggregate consumption and labor increase over time. Panel (f) shows the path of external debt B_t. The economy accumulates external debt quickly in the beginning of time. However, when the debt constraint binds, there is a slower accumulation of debt that eventually reaches its steady state, which is the maximum debt capacity of the economy.

An important point from panel (e) is that the higher-income agents borrow more over time. This is because all agents borrow at similar fractions of their income over time. Therefore, when the planner uses taxes on borrowing, she also redistributes more resources towards the lower-income agents, as the higher-income agents pay higher taxes on borrowing. When there is no cost of borrowing, the planner uses the labor distortion to redistribute. The high initial labor tax reflects the redistributive motive, and that the marginal benefit of redistribution is high. When hitting debt constraints, the planner can use borrowing taxes to redistribute. This additional redistributive tool allows a lower labor distortion until subsidy in the long run. In this case, the high-skilled agent is more productive than the average productivity. A lower labor tax encourages her to produce more output, which increases the economy's ability to repay.

1.7.2 Discussion of the optimal properties

Why do allocation, taxes, and debt change even when debt constraint binds? To answer this question, consider the following planning problem that does not incur any distortionary cost of redistribution but is subject to delivering the same distribution of

Figure 1.2: Time paths of benchmark and relaxed economic aggregates when $\theta^H = 3\theta^L$



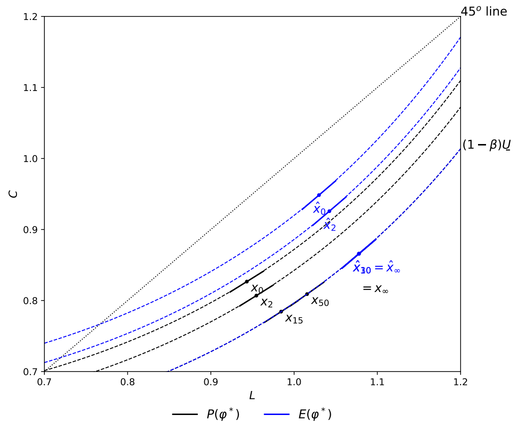
individual outcomes, which is fixing the optimal $\boldsymbol{\varphi}^*$,

$$
\begin{aligned}
E(\boldsymbol{\varphi}^*) \equiv \max_{\{C_t, L_t\}_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi}^*) \right] \\
s.t. \qquad & \sum_{t=0}^{\infty} q_t \left[L_t + -C_t - G_t \right] - B_0 \geq 0 \\
\forall t, \; & \sum_{s=t}^{\infty} \sum_{i \in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s; \boldsymbol{\varphi}^*) \right] \geq \underline{U}_t
\end{aligned}
$$

Figure 1.2 compares the dynamics of aggregates between the benchmark problem $P(\boldsymbol{\varphi}^*)$ and the relaxed problem $E(\boldsymbol{\varphi}^*)$. $E(\boldsymbol{\varphi}^*)$ delivers a higher ex-ante welfare (panel (a)). It takes longer for $E(\boldsymbol{\varphi}^*)$ to reach the debt constraint (see Figure **??** for a shorter time frame), but when the debt constraint binds, consumption, labor, taxes, and external debt stay constant. The implemented labor tax for the allocation of $E(\boldsymbol{\varphi}^*)$ is constant at the negative level. It is important to note that the solution of $P(\boldsymbol{\varphi}^*)$ converges to the solution of $E(\boldsymbol{\varphi}^*)$ in the long run. $E(\boldsymbol{\varphi}^*)$ is the most efficient way to deliver $\boldsymbol{\varphi}^*$, while $P(\boldsymbol{\varphi}^*)$ is the best way to deliver $\boldsymbol{\varphi}^*$ taking into account the distortionary cost of redistribution. The solution to $P(\boldsymbol{\varphi}^*)$ increases its efficiency every time the debt constraint binds and eventually reaches the most efficient outcome.

Figure 1.3 provides the trade-off path of the planning and relaxed allocation over time. The dash curves represent the intra-period indifference curve of the planning utility with respect to the aggregate consumption and labor. The planning and alternative allocation

Figure 1.3: The trade-off path of the efficient allocation when $\theta^H = 3\theta^L$

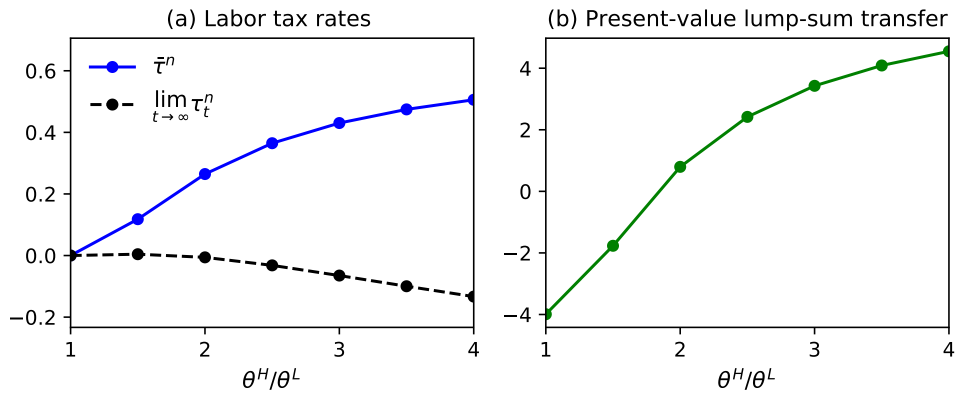


for each period t are indexed by x_t and $\hat{x}_t$, respectively. The slope of each associated line is the marginal rate of substitution at period t.[11] For the periods that debt constraints do not bind, the marginal rate of substitution remains constant, as the planning allocation drifts down utility indifference curves, decreasing consumption and increasing labor. The decline in consumption and leisure reflects the impatience of domestic agents, while the constant marginal rate of substitution comes from the tax smoothing. When the debt constraint starts binding, as illustrated before, it is not sustainable to stay at the same allocation. Therefore, the allocation moves along the autarkic utility indifference curve.

The efficient allocation moves up along the flow autarkic utility indifference curve. In the planning problem, the marginal rate of substitution between consumption and labor starts at a low level, as the slope of the indifference curve at x_0 is less than one. The argument is that it is always better for the planner to redistribute by distorting intratemporal decisions instead of intertemporal decisions. The marginal rate of substitution is then lower than one because of the distortionary cost of redistribution. On the other hand, the relaxed allocation does not have to take into account this distortionary cost, so its allocation ($\hat{x}_t$) always has a slope of one, in which the slope of the indifference curve equals to slope of the aggregate resource constraint. Tax smoothing implies that at the end of periods that debt constraints do not bind, the planning allocation's marginal rate of substitution has not changed and is less than one. Given the same promised utility, at the moment the debt constraint binds, suppose that the planner decreases one unit of labor, then she can only decrease consumption by less than one unit, implying that the planner will need to take

[11] The slope of the planning utility indifference curve is $\left.\frac{\Phi_C^P}{\Phi_L^P}\frac{C_t^{-\sigma}}{L_t^{\nu}}\right|_{u_t}$

Figure 1.4: Relative skill dispersion and tax rates



more debt. If the planner instead increases one unit of labor, she will only need to increase consumption by less than one unit. Then the planner can gain additional resources to pay back the existing debt. As a result, the planning allocation moves up along the indifference curve until it reaches the most efficient allocation with a slope of one.[12]

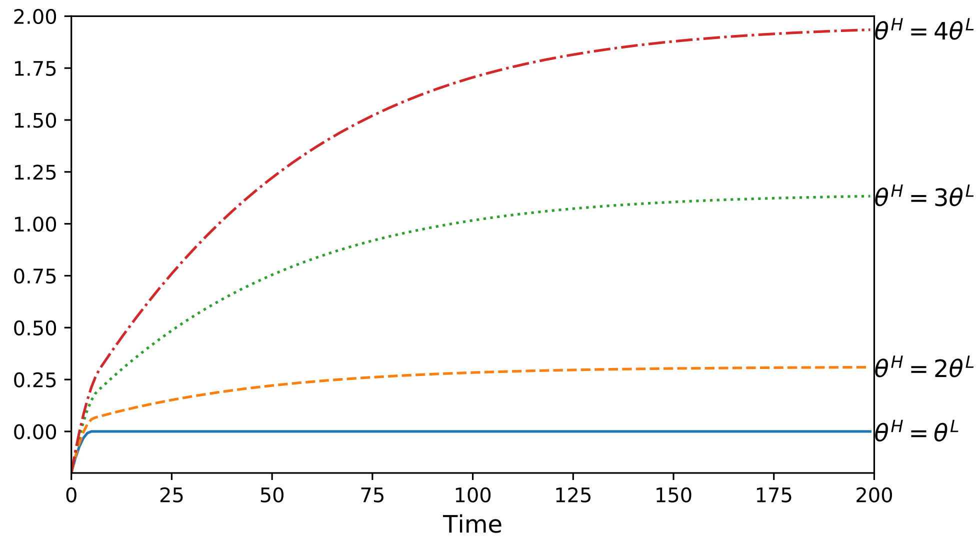
1.7.3 Comparative statics: skill dispersion

Figure 1.4 illustrates changes, with respect to the relative skill dispersion θ^H/θ^L, of the optimal labor and lump-sum taxes. Panel (a) plots the labor tax rate in periods where borrowing is unconstrained ($\bar{\tau}^n$) and at the limit ($\lim_{t\to\infty} \tau_t^n$), for the utilitarian planner. Panel (b) depicts the present-value of lump-sum tax. When there is no heterogeneity ($\theta^H = \theta^L$), the problem collapses to a Ramsey's problem of a representative-agent small open economy. Due to the presence of lump-sum tax, it is optimal to have zero labor distortion in all periods. As the skill dispersion increases, the government increases its motive for redistribution. While debt constraints do not bind, it is optimal to levy higher tax rates. Therefore, as shown in panel (a),$\bar{\tau}^n$ increases with θ^H/θ^L.

On the other hand, setting high tax rates during periods of non-binding debt constraints leads to a high cost of distortion when the economy first reaches the debt constraint. At a higher level of skill dispersion, the government wants to redistribute by increasing the marginal tax rates on labor income. Therefore, the marginal rate of substitution between consumption and labor in the planning utility starts at a lower level than before, remains constant during no-binding periods, and gradually increases to one as the debt

[12]In this example, one can show that $\frac{\Phi_C^W}{\Phi_L^W} > \frac{\Phi_C^P}{\Phi_L^P}$, which implies that $\frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^\nu} < \frac{\Phi_C^W C_t^{-\sigma}}{\Phi_L^W L_t^\nu} = 1$ for any period t such that the debt constraints have not binded before. In the long run, $\lim_{t\to\infty} \frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^\nu} = 1$.

Figure 1.5: Time paths of external debt by relative skill dispersion



constraint binds (as shown in Figure 1.3). In the long-run, panel (a) shows that $\lim_{t\to\infty} \tau_t^n$ becomes negative and declines with respect to the skill dispersion. A higher skill dispersion implies that the highly productive agent becomes more productive than the average agent. Increasing the labor subsidy in the long run encourages a greater output to sustain the high level of debt. Although labor is subsidized in the long run, the government still achieves its redistributive purpose by combining the initial high tax rates and positive lump-sum transfer. Indeed, panel (b) shows that the lump-sum transfer increases with respect to the relative skill dispersion.

Figure 1.5 presents the dynamic of the government's external debt B_t for different levels of skill dispersion. While all economies start with the same initial external debt position, an economy with a higher skill dispersion accumulates higher debt over time. A highly-dispersed economy wants to redistribute more by levying a higher labor tax rate during the periods that debt constraints do not bind. The higher tax rate means that there is lower output, which is compensated by more borrowing.

The higher debt capacity of the economy corresponds to the need of stabilizing the higher debt level that the economy accumulates beforehand because of a higher redistributive motive. In addition, a higher skill dispersion is associated with a longer time of unconstrained borrowing. A highly-dispersed economy faces higher cost of redistribute during financial autarky. Therefore, it is optimal to prolong the periods that the debt constraint does not bind, in which the government can redistribute the most.

1.8 General Case: Separable Preference

This section extends the results of optimal labor taxation with separable preferences. As the elasticity of consumption intertemporal substitution and the elasticity of labor supply vary across time, the optimal labor tax fluctuates. In general, the labor tax is bounded in the long run. If the steady states exist, in the long run, the optimal tax goes to a real limit as in the case with separable isoelastic preferences. In either case, the distributive preference alters the level of optimal taxes in the long run. The results rely on the following assumptions of separability and boundedness.

Assumption 1.4 (Separable preference)**.** $U^i(c,l) = u(c) - v\left(l/\theta^i\right)$*, where* $u_c(\cdot) > 0$*,* $u_{cc}(\cdot) < 0$*,* $\lim_{c\to 0} u_c(c) = \infty$*,* $\lim_{c\to\infty} u_c(c) = 0$*,* $v_l(\cdot) > 0$*,* $v_{ll}(\cdot) > 0$*, and* $\lim_{l\to 0} v_l(l) = \infty$

Assumption 1.5 (Bounded elasticities)**.** *u and v are such that* $\forall c, n \in \mathbb{R}_+$*,* $0 < -\frac{u"(c)}{u'(c)}c < \infty$ *and* $0 < \frac{v"(n)}{v'(n)}n < \infty$

Since the preference is separable between consumption and leisure, individual consumption (labor) only depends on the aggregate consumption (labor). In addition, each agent's allocation is increasing with respect to the aggregates.

Lemma 1.3. *Given assumption 1.4, for any competitive equilibrium, there exist time-invariant functions* $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$*,* $\forall i$ *such that* $\forall i, \forall t$*,*

$$c_t^i = h^{i,c}(C_t;\boldsymbol{\varphi})$$
$$l_t^i = h^{i,l}(L_t;\boldsymbol{\varphi})$$

where $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ *are strictly increasing.*

The characterization of individual allocation from Lemma 1.3 and the bounded elasticities from assumption 1.5 give the long-run property of efficient allocation and optimal labor tax. Specifically, the efficient allocation also features no immiseration in the long run,

Lemma 1.4 (No immiseration)**.** *Suppose assumptions 1.3 and 1.4 hold, then for any efficient allocation* $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$*,* $\liminf_{t\to\infty} C_t^* > 0$*.*

and the long-run optimal labor tax is bounded.

Proposition 1.6 (Optimal labor tax in the long run)**.** *If assumptions 1.1,1.3,1.4, and 1.5 hold, and an interior efficient allocation* $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T$ *exists, then there exist*

$-\infty < \underline{\tau}, \bar{\tau} < \infty$ *such that* $\liminf_{t\to\infty} \tau_t^n = \underline{\tau}$ *and* $\limsup_{t\to\infty} \tau_t^n = \bar{\tau}$. *Moreover, if the steady states exist, then* $\lim_{t\to\infty} \tau_t^n$ *exists. These results hold with or without the lump-sum transfers.*

The results rely on the fact that the tax's long-run value relies on the marginal changes in individual allocation with respect to the aggregates in the long run. With constant elasticities, the individual allocation is linear in the aggregate allocation, as in equations (1.11), so the marginal change is constant over time, which means that the limit exists. However, when preferences are not isoelastic, the marginal change fluctuates over time. Therefore, the optimal labor tax does not necessary converge to a constant. Given the bounded elasticities, the marginal changes are bounded and so is the optimal labor tax. In case the steady state allocation exists, the marginal changes will converge to the steady state values, which implies the convergence to limit of the labor tax.

1.9 Conclusion

This chapter analyzes optimal taxation and debt management for a small open economy with impatient agents, endogenous debt constraints, and redistributive motive. Impatient agents borrow over time, which makes debt constraints relevant in the long run. Optimal labor taxes feature constant rates when borrowing is unconstrained, yet later a gradual convergence to non-zero values in the limit that are associated with the economy's aggregate debt limit. As debt constraints bind, it is optimal to increase the taxes on capital and domestic borrowing.

The government's redistributive motive significantly changes the implication for optimal fiscal policies. Specifically, it alters the long-run limit of taxes through interacting with the heterogeneity. It also changes both the inside and outside values of the contract, which indirectly determines the debt limits. Any country's optimal taxes and debt issuance crucially depend on its labor productivity distribution as well as its social distributional preference. On the other hand, debt constraints limit a government's ability to redistribute, in which the optimal policies decrease the redistribution and increase the efficiency whenever debt constraints bind.

The chapter also provides a mechanism explaining the relationship between sovereign debt accumulation and redistribution. A government with a high redistributive motive wants to set high tax rates and borrow more. When debt constraints bind, by lowering labor taxes and possibly subsidizing labor in the long run, the government can sustain this high debt position.

The main source of income heterogeneity in the model comes from the skill dispersion, which highlights the redistributive effect of labor taxes. Other sources of heterogeneity, such as capital income and wealth are worth being explored in future research. In the next chapter, I study how optimal policies response in this model with aggregate shock that leads to periods of fiscal and debt crises. The government will have an additional insurance motive against aggregate shocks. The trade-off between motive for insurance and for redistribution can result in interesting tax dynamics, as illustrated in Arellano and Bai (2016) and Balke and Ravn (2016). In addition, enriching the tax system to non-linear taxes can help study the optimal tax progressiveness in the presence of sovereign debt.

Chapter 2

Redistribution, Sovereign Debt, and Optimal Taxation

2.1 Introduction

The recent European debt crises have prompted intense policy debates on the design of fiscal policies during severe economic downturns, in which output is low and external debt rapidly increases until it is constrained by the country's inability to repay. Austerity policies such as increasing tax revenue or reducing government expenditure provide more resources to repay debt but have unequal effects on domestic residents.[1] Therefore, appropriate policy design requires understanding the interaction between a government's commitment to debt repayment and its commitment to maintain a level of redistribution. Three key questions arise: How does constrained borrowing affect a government's ability to redistribute? How does a government's redistributive goals affect its incentive to repay debt? Given the answers to the first two questions, how does one design optimal austerity policies taking their distributional consequences into account?

To address these questions, this chapter analyzes a small open economy model in which redistribution comes with an efficiency cost in terms of labor tax distortion and the government faces endogenous borrowing constraints due to its lack of commitment. Given

[1] The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension programs, and education programs. Monastiriotis (2011) argues that in Greece, the prolonged fiscal consolidation has exacerbated regional disparities and imbalances. Leventi and Matsaganis (2016) use a micro-simulation model to assess the distributional effects of austerity policies and find that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession. Brinca, Homem Ferreira, Franco, Holter, and Malafry (2019) show that fiscal consolidations are more recessive when income inequality is higher.

these constraints, the cost of financial autarky endogenously determines the sustainable level of external debt. I examine the theoretical properties of optimal taxation–the government's redistributive tools–in the presence of endogenous borrowing constraints, quantify the effect of the government's redistributive motive on external debt, and evaluate optimal responses of fiscal policies and redistribution to aggregate shocks.

The main result is a theory of external debt sustainability based on the government's motive for redistribution. Having access to external financing allows the government to have more redistribution, measured as the differences in individual utilities, than in financial autarky at the same level of efficiency cost. In fact, when borrowing is constrained, the government finds it optimal to lower labor taxes and levy taxes on domestic borrowing. These policy changes allow the economy to increase its efficiency and repayment capacity without sacrificing redistribution. In the long run, the labor tax needed to redistribute is lower when having access to external borrowing than in financial autarky. The government's need to redistribute endogenously makes financial autarky costly by introducing large efficiency losses from labor distortions, which in turn makes the government willing to repay debt. The model can quantitatively account for the recent buildup of external debt in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries and over time. The optimal austerity policies responding to a negative productivity shock are increasing external borrowing, decreasing average taxes, and increasing redistribution while raising average taxes and reducing redistribution to repay debt in the future. The magnitude of these responses varies with the underlying wage inequality.

As an empirical motivation, the chapter first documents the cross-country and time series properties of pre-tax income inequality and external debt using two multi-country panel data sets on inequality and balance of payments. First, highly indebted countries have also experienced high levels of pre-tax income inequality. Second, the increase in the country's net financial outflows has coincided with an increase in aggregate pre-tax income inequality. The regression estimation shows that this positive relationship is robust to changes in output levels and output growth.[2] These facts point to a connection between a government's redistributive goals and its external debt management.

The model features a continuum of domestic agents that are impatient and differ by labor productivity types. The aggregate shocks are in aggregate productivity and

[2] The end of this section provides an overview of other papers that document similar and related empirical patterns. These papers include Berg and Sachs (1988), Aizenman and Jinjarak (2012), Jeon and Kabukcuoglu (2018), and Ferriere (2015) which use different measures and estimation techniques.

government spending.[3] Domestic and external credit markets consist of state-contingent assets. The tax system has lump-sum taxes as well as marginal taxes that are distortionary to individual labor supply and saving decisions. The government cares about all domestic agents, assigning individual welfare weights that represent its distributional preference. The government lacks commitment in all tax and debt policies.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, the government redistributes resources by levying a positive labor tax alongside a lump-sum transfer. In this way, highly-skilled, high-income agents bear a larger tax burden than low-skilled, low-income ones. Alternatively, the government can use a tax on domestic borrowing and a lump-sum transfer, which implies that the highly indebted agents will pay more taxes than the less-indebted agents. In this environment, levels of tax distortions represent the cost of redistribution.[4]

The government's lack of commitment imposes endogenous limits on the economy's external borrowing. The government chooses its policies sequentially in a repeated game between the government, domestic agents, and international creditors. The contract is an ex ante set of policies such that if the government deviates from any of its policies, it triggers a punishment to financial autarky, in which permanent exclusions from domestic and external credit markets take place. For example, even if the government only defaults on external debt, it is still excluded from both domestic and external financing.[5] Domestic agents are still able to participate in the domestic credit market. One can characterize the subgame perfect equilibrium with self-enforcing constraints, in which the continuation value of staying in the contract has to be at least the value of financial autarky. These constraints act as endogenous borrowing constraints.

The impatience of the domestic agents means that they want to borrow. The domestic need for borrowing leads to the country running up debt and eventually hitting the borrowing constraints. However, an infinitesimal domestic agent does not internalize the fact that as she borrows more, the borrowing constraints become tighter. There is a shadow price of borrowing that does not enter into the individual problem. When the borrowing constraint binds, the government can set borrowing taxes such that domestic agents face the correct borrowing cost.

[3]Later on when I match the model to the data, government spending is measured as the government's final consumption of public goods, excluding spending on social welfare programs.

[4]This is because the government can raise lump-sum taxes to finance expenditures and debt repayment without distorting the domestic agents' decisions. Werning (2007) provides a similar intuition. The presence of lump-sum taxes removes the revenue purposes of distortionary taxation.

[5]This is equivalent to the assumption of nondiscriminatory defaults on domestic or external lenders. See, for example, D'Erasmo and Mendoza (2016) for a similar assumption. For an example on discriminatory defaults, see Gonzalez-Aguado (2018).

The theoretical results show that optimal labor taxes permanently decline and borrowing taxes are positive when borrowing constraints bind.[6] When the borrowing constraints do not bind, the government redistributes via constant labor taxes and zero domestic borrowing taxes. Intuitively, the optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. Since borrowing is not costly, the government can use debt to smooth the labor distortions.[7] On the other hand, when borrowing is limited, the decision to distort intertemporal margins (borrowing taxes) becomes beneficial because it aligns the intertemporal interests of the government with those of the domestic agents. These borrowing taxes turn out to be beneficial in redistribution since the high income agents are highly indebted. Therefore, the government can redistribute at a lower level of labor distortion. The decrease in efficiency cost allows for higher repayment capacity and external debt accumulation.

The optimal tax properties suggest that external borrowing is beneficial to efficiency and redistribution. Throughout this chapter, I use the utility difference across individuals as a measure of redistribution. In the contract, future declines in labor taxes improve the economy's efficiency by encouraging higher output and allowing the government to borrow more in the present. This additional unit of resource implies higher present consumption and so higher welfare than financial autarky. Furthermore, the contract exhibits a lower level of utility difference than in financial autarky at the same amount of labor tax in the long run. Having access to external financing allows the government to redistribute more than in financial autarky at the same level of efficiency cost.

The government's concern for redistribution affects its repayment incentive via influencing the value of financial autarky or the cost of default.[8] Default is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature, in contrast to the standard exogenous cost of default in terms of output or productivity losses.

The theory is quantitatively consistent with salient features of the data. Using Italy's data, I calibrate the model to match key macroeconomic statistics and average cross-sectional wage inequality and show that the model accounts for the average level

[6]See Tran Xuan (2019) for more results on optimal taxation in the deterministic case.

[7]See Lucas and Stokey (1983) and Werning (2007) for more details on the argument for labor tax smoothing. Both frameworks have state-contingent asset markets but are for a closed economy and do not feature the endgoneous borrowing constraints.

[8]In most of the literature, default often means not repaying debt. In my model, default means not honoring any terms of the contract, either in debt repayment or taxes.

and upward trend of the external debt-to-output ratio in Italy for the period 2002-2015, while also being consistent with key business cycle statistics. The simulation points out that the model can account for the positive association between pre-tax income inequality and external debt across countries. Specifically, I perform a regression analysis on a sample of simulated economies differentiated only by wage inequalities and find a statistically significant and positive correlation of pre-tax income Gini index and external debt-to-output. Moreover, a counterfactual exercise for Italy during the periods 1985-2001 and 2002-2015 exhibits that an increase in the underlying wage inequality that matches the increase in income inequality can account for 93% of the increase in the average debt-to-output ratio. These findings are consistent with the theory's prediction that a higher level of inequality, or a higher redistributive motive, corresponds to a higher cost of default, which results in a higher sustainable debt level.

Furthermore, I study optimal austerity through the lense of optimal policy response to shocks. Following a negative innovation of the productivity shock, external debt increases while utility differences among agents decline initially and increase in subsequent periods. More external borrowing allows higher transfers to individuals and more redistribution, while higher taxes and lower redistribution are needed in the future to repay debt. For a lower level of inequality, the magnitude of these responses to a negative productivity shock are large for both external debt and utility differences.

Endogenous borrowing constraints are essential to the theory because they link the government's external borrowing decisions to its concern for redistribution and tax distortions through the value of financial autarky. If the borrowing constraint is exogenous, given impatience, the only optimal policy is to borrow up to the exogenous debt limit, regardless of the level of redistributive motive or inequality.

The model builds on state-contingent financial markets and features no equilibrium default, in contrast to the sovereign default model.[9] However, the threat of default still affects the optimal allocation, as in Thomas and Worrall (1988) and Kehoe and Levine (1993). The endogenous borrowing constraints imply that there is imperfect insurance against aggregate risk. These constraints also endogenously determine the optimal debt portfolio, in contrast to the incomplete framework in which the the risk-free bond is the only financial asset. Furthermore, defaults in reality often accompany a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts.[10] This

[9] See, for example, Eaton and Gersovitz (1981).

[10] Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue. This definition covers both missed payments (breach of contract) and distressed debt restructurings that involve losses for creditors. This is the standard default definition used in the literature (e.g., Reinhart and Rogoff (2009)). See Ams, Baqir, Gelpern, and Trebesch

framework embeds part of the default procedure into self-enforcing borrowing constraints, instead of assuming zero net capital flows, as in the standard sovereign default model.[11]

I last show that heterogeneity, distortionary taxation, and impatience significantly affect the equilibrium sustainable level of external debt, whereas aggregate uncertainty and government expenditure have only a minor impact. Intuitively, the value of financial autarky decreases in the presence of distortionary taxes, inequality, and domestic agents' impatience, which increases the government's incentive to repay external debt.

Related Literature. This chapter builds on the sovereign debt literature of limited commitment and state-contingent asset market that follows Aguiar and Amador (2011, 2014). I present the quantitative predictions of this type of model, including highly volatile consumption and fiscal policies, which are similar with the findings in Kehoe and Perri (2002) for a two-country international real business cycle and Bauducco and Caprioli (2014) with two-sided limited commitment.

I introduce heterogeneity and redistributive effect of fiscal policies in the literature that studies the government's lack of commitment in both tax and debt policies. The volatile tax and government expenditures are similar to Cuadra, Sanchez, and Sapriza (2010). The theoretical finding of declining labor taxes when borrowing is tightened relate to the absence of tax smoothing in Pouzo and Presno (2015) and the quantitative result of Arellano and Bai (2016), in which higher tax distortion would make the country more likely to default. The government's incentive to front-loading tax distortion is also found in Karantounias (2018).

This chapter also contributes to the literature of endogenous cost of default beyond insurance motive. In Mendoza and Yue (2012), it is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. Balke (2017) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this chapter, distortionary taxation plays an important role in determining the cost of default. Default is costly because of the high and volatile labor distortions need to redistribute in financial autarky.

This chapter finds optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e. the primal approach as in the public finance literature (Barro (1979),Lucas and Stokey (1983) Chari, Christiano, and Kehoe (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari, Marcet, Sargent, and Seppälä

(2018) for a discussion on the pros and cons of different definitions of default.

[11] See Restrepo-Echavarria (2019) for a discussion on these issues.

(2002), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this chapter, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders.

This chapter relaxes the assumption on the government's commitment to policies in many papers that study trade-off between redistribution and debt management. I build upon the framework of optimal taxation with redistribution in Werning (2007), in which perfect tax smoothing occurs under the same assumptions. I establish that binding borrowing constraint due to lack of commitment alter the tax dynamics, resulting in imperfect tax smoothing. In addition, I show how the quantitatively high cost of default of redistribution lead to the high positive debt level in the long run, in contrast to Bhandari et al. (2016)'s finding that the average long-run on optimal public debt is not positive. Similar to Bhandari et al. (2017) which emphasize the importance of distribution of initial asset holdings, I find that it affects the long-run debt repayment capacity via the equilibrium level of redistribution.

Several recent papers addressed the trade-off between redistribution and external debt. This chapter extends the lack of commitment to both tax and debt policies, in contrast to Ferriere (2015) that assumes one-period commitment to tax progressivity, and finds a related result that a more progressive economy finds a higher cost of default because redistribution is more distortionary in autarky. My model features redistributive consequences of domestic defaults, as emphasized by D'Erasmo and Mendoza (2016). I incorporate this insight into linking redistributive incentives and the cost of default, which in turn affects the sustainable level of external debt. Balke and Ravn (2016) studies tax and debt policies from a Malkov perfect equilibrium with inequality through unemployment. This chapter allows for a more general framework of inequality and redistributive motive and focuses on the ex-ante welfare maximizing policies. My theoretical finding is consistent with their quantitative one, in which it's optimal to minimize tax distortions during crises.

Other papers have documented a positive relationship between income inequality and sovereign debt. Berg and Sachs (1988) show that income inequality is a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describe a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Jeon and Kabukcuoglu (2018) and Ferriere (2015) also provide evidence that rising income dispersion significantly increases sovereign default risk. This chapter extends the panel analysis to a more recent data set on income inequality and provides a theory that can collectively account for the increase in debt and income dispersion.

Outline. The chapter is organized as follows. Section 2.2 documents the relationship between income inequality and external debt. Section 2.3 describes the environment and sets up the competitive equilibrium. Section 2.4 formulates the planning problem and the main theoretical results. Section 2.5 provides a quantitative analysis and analyzes optimal austerity. Section 2.6 discusses assumptions and robustness. Section 2.7 then concludes.

2.2 Empirical Motivation

This section presents the empirical relationship between income inequality and external debt. I document that income inequality is positively correlated with external debt in both the cross section and time series. To measure a country's external indebtedness, I use the negative of the net foreign asset-to-GDP ratio from the External Wealth of Nations Database of Lane and Milesi-Ferretti (2018)[12]. The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2015. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of Solt (2019) that covers from 1960 to 2018.

2.2.1 Cross-Country Properties

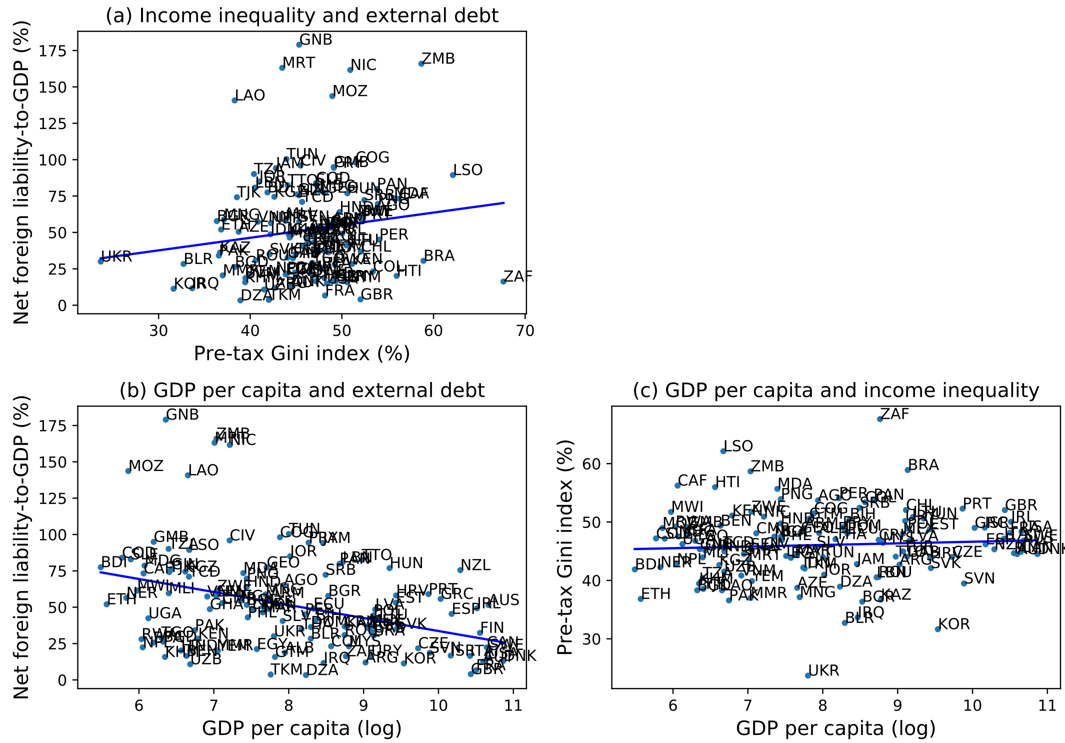
This subsection presents the cross-country properties in a panel data set of 120 countries over the time period of 1985-2015.[13]

Figure 2.1 plots averages across 1985-2015 of net foreign liability-to-GDP, pre-tax Gini index, and log GDP per capita in constant 2010 US Dollars. Panel (a) establishes a positive relationship between income inequality and external debt across countries. Panel (b) and (c) show the relationship of net-foreign liability-to-GDP and pre-tax Gini index with respect to GDP per capita. Across countries, the GDP per capita negatively associates with the net foreign liability-to-GDP, while it does not have a strong correlation with the pre-tax Gini index.

[12]The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates. A different measure of a country's external position is the net international investment position (NIIP), which is the difference between a country's stock of foreign assets and foreigner's stock of that country's assets. See Appendix B.1.2 for the estimation using NIIP.

[13]See Appendix B.6.2 for the list of all economies in the data set. I focus on countries that have the population in 1985 over one million people and the net foreign liability above zero. See Appendix B.1.1 for a similar analysis on a sub-sample of advanced and emerging market economies.

Figure 2.1: Income inequality, external debt, and GDP per capita across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for all economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

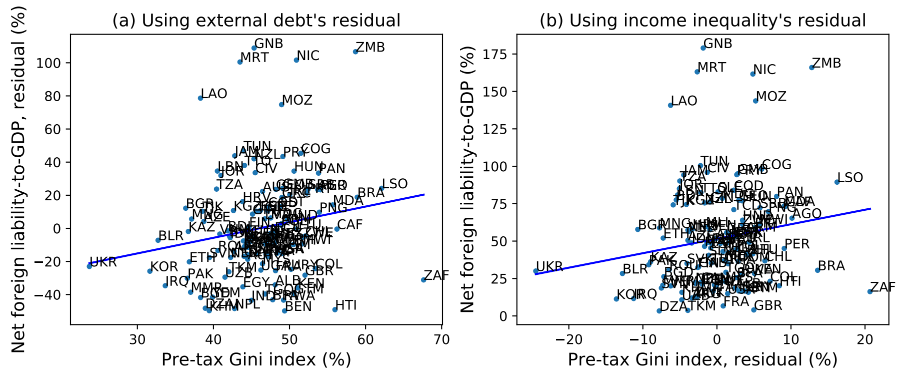
Both GDP per capita and GDP growth can account for the observed levels in income inequality and external debt across countries. However, the following exercises show that the positive correlation between inequality and debt is robust to such factors. Increases in the pre-tax Gini index account for increases in net foreign liability-to-GDP, excluding from changes in GDP per capita and GDP growth rates. The exercises are as follows. For a given country in the sample, I calculate average statistics across the time period of 1985-2015. I consider two regressions of net foreign liability-to-GDP and pre-tax Gini index on log GDP per capita and GDP growth rates:

$$\text{Net foreign liability-to-GDP}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{nfl} \quad (2.1)$$

$$\text{Pre-tax Gini Index}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{gini} \quad (2.2)$$

In Figure 2.2, Panel (a) of plots the residuals ϵ_i^{nfl} of equation (2.1) with respect to the pre-tax Gini index, and Panel (b) plots the net foreign liability-to-GDP with respect to the residuals ϵ_i^{gini} of equation (2.2). The positive trend in Panel (a) implies that a higher pre-tax Gini index is associated with a higher net foreign liability-to-GDP that do not come from GDP per capita or GDP growth rates. The positive trend in Panel (b) means that a higher pre-tax Gini index that is not due to different GDP per capita or GDP growth levels is correlated with a higher net foreign liability-to-GDP level.

Figure 2.2: Cross-country relationship between income inequality and external debt



Note: Averages across 1985-2015. Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (2.1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2.2). The sample is for all economies. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019)

Table 2.1 presents regression results of net foreign liability-to-GDP on pre-tax Gini index, reported as averages across 1985-2015. The results show a strong positive relationship between income inequality and external debt.

2.2.2 Time Series Properties

Income inequality has been rising over time across countries (Alvaredo et al. (2018)). At the same time, external debt is also increasing across many countries. The 2007-2009 financial crises contributed to the increase in borrowing across countries, particularly across European ones.[14] Figure 2.3 plots the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union. Over time, there are increasing trends in both income inequality and external debt.

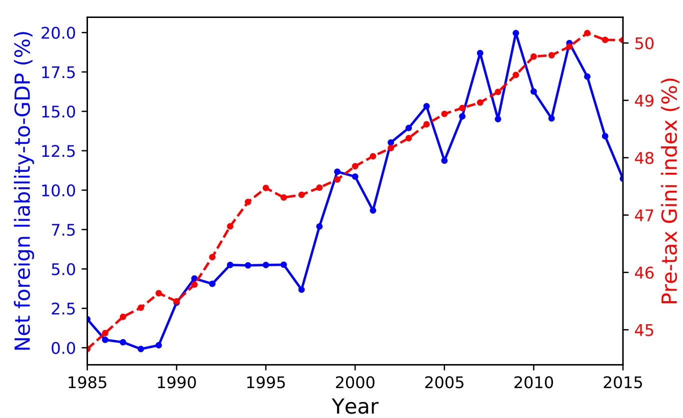
[14]Reinhart and Rogoff (2010) reported large increases in public debt across countries, especially in the period 2007-2009. External debt levels were particularly high among European countries.

Table 2.1: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%) Averages across 1985-2015 (1)	 (2)
Gini index, pre tax (%)	0.865*	0.968**
	(0.521)	(0.487)
GDP per capita (log)		-9.438***
		(2.19)
GDP per capita growth (%)		-0.477
		(1.68)
Inflation (%)		-0.0873*
		(0.047)
No. Countries	120	120

Note: The table describes the regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

Figure 2.3: Time series of income inequality and external debt



Note: The graph shows the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union from 1985 to 2015. Sources: Lane and Milesi-Ferretti (2018) and Solt (2019).

2.2.3 Estimation

To estimate the effect of income inequality on external debt, I use the following specification

$$\begin{aligned}\text{Net foreign liability-to-GDP}_{i,t} &= \alpha_0 + \alpha_1 \text{Gini}_{i,t} + \alpha_2 \text{GDP per capita}_{i,t} \\ &+ \alpha_3 \text{GDP growth}_{i,t} + \alpha_4 \text{Inflation}_{i,t} + u_i + z_t + \epsilon_{i,t}, \quad (2.3)\end{aligned}$$

where GDP per capita is the log of real GDP per capita series in constant 2010 US Dollars and inflation is calculated from GDP deflators. Both GDP per capita and inflation series are from the World Development Database.[15]

Table 2.2 shows results for the panel regression of all countries in the data set for the time period of 1985-2015. The first column presents estimations of equation (2.3) without control variables. The second column presents estimations with control variables. The clustered standard errors are in parentheses. The correlation between inequality and external debt is positive and statistically significant, robust to country and time fixed effects, GDP per capita, GDP per capita growth, and inflation. GDP per capita, GDP growth, and inflation are negatively correlated with net foreign liability-to-GDP, but the effect of GDP growth is not statistically significant[16].

2.3 Model

This section describes the main framework and sets up the competitive equilibrium given the government's policies. The competitive equilibrium can be characterized by a set of aggregate allocation and a distribution of marginal utility shares.

2.3.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. The exogenous risk-free international interest rate for borrowing is r^*. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $\left(\theta^i\right)_{i\in I}$, which are

[15] Appendix B.6.2 lists all countries considered in the regression.

[16] These results still hold for an alternative definition of external debt as the negative of net international investment position. See Appendix B.1.2 for more details.

Table 2.2: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%) Time periods: 1985-2015 (1)	 (2)
Gini index, pre tax (%)	2.36***	2.08***
	(0.365)	(0.389)
GDP per capita (log)		10.5**
		(4.91)
GDP per capita growth (%)		-0.925***
		(0.178)
Inflation (%)		-0.0069*
		(0.004)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No. Countries	120	120
No. Observations	2922	2922

Note: The table describes the panel regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

publicly observable. The fraction of agents with productivity θ^i is π^i, where $\left(\pi^i\right)_{i\in I}$ and $\left(\theta^i\right)_{i\in I}$ are normalized such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0 U^i(c_t^i, l_t^i) \tag{2.4}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is$F(L, s^t, t)$, constant return to scale, where L is the aggregate labor. The economy is subject to an exogenous sequence of government spending $\left\{G_t(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t, capturing deterministic changes such as growth, and the history s^t, capturing the uncertainty impact.

An allocation specifies consumption and labor in every period after every history: $\left\{c^i(s^t), l^i(s^t)\right\}$. The aggregates are denoted by $C(s^t) \equiv \sum_{i\in I}\pi^i c^i(s^t)$ and $L(s^t) \equiv \sum_{i\in I}\pi^i l^i(s^t)$.

Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q(s_{t+1}|s^t) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t, in units of consumption at history s^t. Similarly, $q(s^t) = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0. Let normalize $q(s^0) = 1$.[17] Note that $q(s^{t+1}) = Q(s_{t+1}|s^t)q(s^t)$. Assume only the government can borrow abroad.[18]

[17] This normalization is without loss of generality since the initial level of external debt is fixed.

[18] In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. In this environment, the set up is equivalent to the case where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.2.

2.3.2 Competitive Equilibrium

In every period and history s^t, the government issues both domestic and foreign bonds, imposes a lump-sum tax $T(s^t)$, a marginal tax on labor income $\tau^n(s^t)$ and levies a tax on the return of domestic saving $\tau^d(s^t)$. Both the firm and agents face the labor wage $w(s^t)$.

Domestic agent. Individual agent of type $i \in I$ faces the sequential budget constraint in period t and history s^t

$$c^i(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t) b^{d,i}(s^{t+1}) \leq (1-\tau^n(s^t))w(s^t)l^i(s^t) + (1-\tau^d(s^t))b^{d,i}(s^t) - T(s^t), \tag{2.5}$$

where $c^i(s^t), l^i(s^t), b^{d,i}(s^t)$ denote the consumption, labor, and domestic bond holding of agent i in period t, history s^t, respectively. $Q^d(s_{t+1}|s^t)$ is the price of one unit of domestic asset for realization s_{t+1} in period $t+1$ given history s^t.

Representative firm. The firm chooses capital and labor to maximize profit in each history node s^t

$$\max_{\{L(s^t)\}} \quad F\left(L(s^t), s^t, t\right) - w(s^t)L(s^t),$$

which gives the first-order condition

$$w(s^t) = F_L\left(L(s^t), s^t, t\right) \tag{2.6}$$

The firm makes zero profit in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\left\{G_t(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$. Given the one-period state-contingent domestic bond $B^d(s^t)$ and external bond $B(s^t)$, the government's budget constraint in each history node s^t is

$$\begin{aligned}&G(s^t) + (1-\tau^d(s^t))B^d(s^t) + B(s^t)\\ \leq&\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t),\end{aligned}$$

where $B^d(s^t) = \sum_{i\in I} \pi^i b^{d,i}(s^t)$ is the aggregate domestic bond, and $B(s^t)$ is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t\geq 0, s^t\in S^t} q(s^t)\Big\{G(s^t)-\tau^n(s^t)w(s^t)L(s^t)-T(s^t) \tag{2.7}$$
$$+\sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B^d(s^{t+1})-(1-\tau^d(s^t))B^d(s^t)\Big\}\leq B(s^0)$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the inter-temporal international prices and the initial external debt,

$$\sum_{t\geq 0, s^t\in S^t} q(s^t)\left[F(L(s^t),s^t,t)-G(s^t)-C(s^t)\right]\geq B(s^0) \tag{2.8}$$

Competitive equilibrium. The following defines a competitive equilibrium with taxes.

Definition 2.1. *Given initial external debt $B(s^0)$ and individual individual bond positions $\left(b^{i,d}(s^0)\right)_{i\in I}$, a competitive equilibrium with taxes for an open economy is individual agent's allocation $z^{H,i}=\left\{c^i(s^t),l^i(s^t),b^{i,d}(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, $\forall i\in I$, the representative firm's allocation $z^F=\left\{L(s^t)\right\}_{t=0,s^t}^{\infty}$, prices $p=\left\{q(s^t),w(s^t),Q^d(s_{t+1}|s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, and government's policy $z^G=\left\{\tau^n(s^t),\tau^d(s^t),\ T(s^t),B^d(s^t),B(s^t)\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (2.4) subject to (2.5) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (2.7) holds, (iv) the aggregate resource constraint (2.8) is satisfied, (iv) the domestic bond market clears $B^d(s^t)=\sum_{i\in I}\pi^i b^{d,i}(s^t)$, and (v) p satisfies $q(s^t)=\Pr(s^t)/(R^*)^t$ and equation (2.6) given z^G.*

2.3.3 Characterizing Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t, for any individual i,

$$(1-\tau^n(s^t))w(s^t)=-\frac{U_l^i(c^i(s^t),l^i(s^t))}{U_c^i(c^i(s^t),l^i(s^t))}$$
$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})}=\beta\Pr(s^{t+1}|s^t)\frac{U_c^i(c^i(s^{t+1}),l^i(s^{t+1}))}{U_c^i(c^i(s^t),l^i(s^t))}$$

Given the aggregate allocation $\left(C(s^t),L(s^t)\right)$, there is an efficient assignment of individual allocation $\left(c^i(s^t),l^i(s^t)\right)_{i\in I}$ due to the equal marginal rates of substitution between consumption and labor. Moreover, the efficient assignment needs to be the same across time because of the equal marginal rates of substitution of future to current consumption. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi (market) weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solve a static problem

$$
\begin{aligned}
V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} & \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L
\end{aligned}
$$

This problem gives the policy functions for each individual i

$$
h^i(C, L; \boldsymbol{\varphi}) = \Big(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\Big)
$$

A competitive equilibrium allocation must satisfy $\left(c^i(s^t), l^i(s^t)\right) = h^i(C(s^t), L(s^t); \boldsymbol{\varphi})$ for all i and s^t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C, L; \varphi)$. The envelope conditions of the static problem give

$$
(1 - \tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]} \tag{2.9}
$$

$$
\frac{Q^d(s_{t+1}|s^t)}{1 - \tau^d(s^{t+1})} = \beta \Pr(s^{t+1}|s^t) \frac{V_C\left[h^i(C(s^{t+1}), L(s^{t+1}); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]} \tag{2.10}
$$

Furthermore, the present-value budget constraint for individual i can be written as

$$
\begin{aligned}
\sum_{t\geq 0, s^t\in S^t} & \beta^t \Pr(s^t) \Big[V_C(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,c}(C(s^t), L(s^t); \boldsymbol{\varphi}) \\
& + V_L(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,l}(C(s^t), L(s^t); \boldsymbol{\varphi})\Big] = V_C(C(s^0), L(s^0); \boldsymbol{\varphi}) \left(b^i(s^0) - T\right)
\end{aligned} \tag{2.11}
$$

where T is the present-value of lump-sum taxes.[19] Equation (2.11) is the individual implementability constraint.

[19] $T \equiv \sum_{t=0}^{\infty} \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0), L(s_0); \boldsymbol{\varphi})\right]} T(s^t)$

One has the following characterization proposition.

Proposition 2.1. *Given the initial external debt $B(s^0)$ and individual bond holdings$\left\{b^i(s_0)\right\}_{i\in I}$,an allocation $\left\{C(s^t), L(s^t), K(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (2.8) holds, and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$ and lump-sum tax T such that the implementability constraint (2.11) holds for all $i \in I$.*

2.4 A Planning Problem

This section characterizes the planning problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The main theoretical result entails how limited borrowing affects optimal taxes and hence the government's redistributive ability.

2.4.1 Lack of Commitment

The government cares about all residents in the country, and its objective is the social welfare

$$\sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0 U^i\left(c^i(s^t), l^i(s^t)\right), \tag{2.12}$$

given by a set of social welfare weights $\boldsymbol{\lambda} = \left(\lambda^i\right)_{i\in I}$. In every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following 1990, 1993, the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The subgame perfect equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in Proposition 2.1 and the following self-enforcing constraint

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t,s^t\subseteq s^{t+j}}\beta^{k-t}\,\mathrm{Pr}(s^k|s^t)U^i(c^i(s^k), l^i(s^k)) \geq \underline{U}(s^t,t),\ \forall t, \forall s^t \tag{2.13}$$

where $\underline{U}(s^t,t)$ is the one-shot deviation value in which the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.[20] The government is then in financial autarky, in which it it has no access to external financial

[20]See Appendix B.2 for the formal set up of the sovereign game and its equilibrium characterization.

markets. $\underline{U}(s^t, t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of government's policies. If there is a positive net external debt, the government has an incentive to default externally in order to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and redistribute it.

The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

2.4.2 Efficient Allocation

Given the above set-up, an efficient allocation is defined as follows

Definition 2.2. *An efficient allocation* $\left\{C(s^t), L(s^t)\right\}, \boldsymbol{\varphi}$ *maximizes the social welfare function (2.12) and satisfies the conditions in Proposition 2.1 and the self-enforcing constraint (2.13)*

The efficient allocation is part of the solution to a planning problem

$$(P) \equiv \max_{\{C(s^t), L(s^t)\}, \boldsymbol{\varphi}, T} \sum_{i \in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[h^i(s^t; \boldsymbol{\varphi})\right]$$

$$\begin{aligned}
s.t. \qquad & \sum_{t \geq 0, s^t \in S^t} q(s^t)\left[F(L(s^t), s^t, t) - G(s^t) - C(s^t)\right] \geq B(s^0) \\
\forall i, \quad & \sum_{t \geq 0, s^t \in S^t} \beta^t \left[V_C(s^t; \boldsymbol{\varphi}) h^{i,c}(s^t; \boldsymbol{\varphi}) + V_L(s^t; \boldsymbol{\varphi}) h^{i,l}(s^t; \boldsymbol{\varphi})\right] = V_C(s_0; \boldsymbol{\varphi})\left(b^i(s^0) - T\right) \\
\forall t, \forall s^t, \; & \sum_{i \in I} \lambda^i \pi^i \sum_{k \geq t, s^t \subseteq s^{t+j}} \beta^{k-t} \Pr(s^k | s^t) U^i \left[h^i(s^k; \boldsymbol{\varphi})\right] \geq \underline{U}(s^t, t) \geq \underline{U}(s^t, t)
\end{aligned}$$

where $(s^t; \boldsymbol{\varphi}) \equiv (C(s^t), L(s^t); \boldsymbol{\varphi})$ for notation convenience.

The first constraint is the resource constraint. The second constraint is the implementability constraint that takes into account the distortionary effect of government's policies on individuals' decision. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies. Therefore, borrowing constraints indirectly affect domestic borrowing choices via the government's decision on saving taxes.

2.4.3 Optimal Taxation and Endogenous Borrowing Constraints

I derive optimal policies that implements the efficient allocation. Optimal labor and saving taxes reflect the trade-off between redistribution and efficiency. I show that this trade-off dynamically changes over time in the presence of borrowing constraints.

I first make the following assumptions

Assumption 2.1 (Separable isoelastic utility)**.** *The individual preference follows*

$$U^i\left(c,l\right)=\frac{c^{1-\sigma}}{1-\sigma}-\omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}$$

Assumption 2.2. *Welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistribution motive towards the low skills:* $\theta^i \leq \theta^j \iff \lambda^i \geq \lambda^j,\ \forall i,j \in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i \leq \theta^j \iff b^i(s^0) \leq b^j(s^0),\ \forall i,j \in I$
3. *High elasticity of substitution:* $\sigma \geq 1$

The first assumption implies that the individual preference is separable between consumption and labor, and the elasticities of substitution are constant across periods and histories. For the second set of assumptions, the first part is on welfare weights, meaning that the planner has a high motive of redistribution towards the lower skill, lower income individuals. The second part makes sure that the direction of inequality in skill is the same as in initial wealth, meaning that lower skill individuals start off with lower initial wealth. The last assumption implies that the intra-temporal elasticity of substitution is at least above log preference. This assumption determines the direction of change in the optimal tax rate in response to inter-temporal changes.

Define the implicit after-tax domestic interest rate to be $r^d(s^t) \equiv \frac{\Pr(s_t|s^{t-1})(1-\tau^d(s^t))}{Q^d(s_t|s^{t-1})} - 1$. I then establish the main theoretical result on the optimal taxes,

Proposition 2.2. *For any period* $\mathcal{T}$*, history* $s^{\mathcal{T}}$ *, and* $\forall s^{\mathcal{T}-1} \subseteq s^{\mathcal{T}}$*,*

1. *If the borrowing constraint does not bind at* $s^{\mathcal{T}}$*, the optimal labor tax does not change, i.e.* $\tau^n(s^{\mathcal{T}})=\tau^n(s^{\mathcal{T}-1})$*, and the optimal saving tax is zero, i.e.* $r^d(s^{\mathcal{T}}) = r^*$

2. *If the borrowing constraint binds at $s^{\mathcal{T}}$, the optimal labor tax is weakly decreasing in the future, i.e.* $\tau^n(s^t) \leq \tau^n(s^{\mathcal{T}-1})$, $\forall t \geq \mathcal{T}, \forall s^{\mathcal{T}} \subseteq s^t$, *and there is optimal saving subsidy, i.e.* $r^d(s^{\mathcal{T}}) > r^*$

The Proposition first implies that given non-binding borrowing constraints, optimal labor taxes do not change over time and across histories, and optimal saving taxes are zero. The labor tax result is related to the standard tax smoothing argument as in Lucas and Stokey (1983). Since the borrowing constraint does not bind, it is optimal to use debt to smooth out the distortionary cost across all states. Zero saving taxes, on the other hand, are related to the idea of no intertemporal distortion in Judd (1985) and Chamley (1986).

When the borrowing constraint binds, all future labor tax rates weakly decrease. The tax smoothing property implies that a one-time binding borrowing constraint does not decrease the labor tax in one current period but the reduction is spread out to all tax rates in subsequent periods. In addition, the government finds it optimal to use a saving subsidy to discourage the impatient domestic agents from over-borrowing.

Intuitively, distortionary taxation is a mechanism for redistribution. A positive marginal labor tax and lump-sum rebate to all agents imply that the higher skilled, higher income individuals pay more taxes than the lower skilled, lower income individuals. Therefore, a planner that cares about redistribution towards low income agents will find it necessary to levy a positive labor tax. Optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. The determinants of inequality and distributive motive do not vary over time and are independent of the aggregate shocks. When borrowing is not costly (non-binding borrowing constraints), both the distributive benefit and the distortionary cost do not vary, so it is optimal to keep labor taxes unchanged. When borrowing is limited (binding borrowing constraints), the distortion becomes more costly because it reduces the amount of resources that the economy can use to repay debt. Decreasing distortionary labor taxes is then optimal to increase the repaying capacity and relax borrowing constraints.

The forward-looking borrowing constraint induces a backward-looking effect on optimal policies. Suppose that the labor tax τ^n decreases in history node $s^{\mathcal{T}}$, then efficiency (e.g. aggregate output Y) is higher in node $s^{\mathcal{T}}$. Not only this increase in efficiency allows the economy to repay more debt in $s^{\mathcal{T}}$, but also to repay more debt in any previous histories $s^t \subseteq s^{\mathcal{T}}$ where $0 \leq t \leq \mathcal{T}$. As a result, a lower labor tax in the future relaxes all of the past borrowing constraints. Therefore, instead of a one-time large decrease in the tax rate when the borrowing constraint binds, a permanent small decrease in all future tax rates will relax more borrowing constraints.

Optimal saving taxes, on the other hand, relate to intertemporal distortions. When borrowing constraints do not bind, it is optimal not to distort the intertemporal margins.[21] When borrowing constraints bind, saving subsidies make the domestic agents internalize the additional cost of borrowing that comes from the binding constraints.

Formally, the proof relies on the property that the only component in the optimal taxes that varies across time periods and histories is the sum of all Larange multipliers on the borrowing constraints (γ) up to node $s^{\mathcal{T}}$, i.e. $\sum_{k=0.s^k \subseteq s^{\mathcal{T}}}^{\mathcal{T}} \gamma(s^k)$. If the borrowing constraint does not bind ($\gamma(s^{\mathcal{T}}) = 0$), the sum stays constant, and so labor taxes remain the same as before, and saving taxes are zero in $s^{\mathcal{T}}$. However, if borrowing constraints bind ($\gamma(s^{\mathcal{T}}) > 0$), the sum increases, which leads to a permanent decline in labor taxes. Differences in today's sum at $s^{\mathcal{T}}$ and yesterday's sum at $s^{\mathcal{T}-1}$ lead to a tax in domestic borrowing's return at $s^{\mathcal{T}}$.

Proposition 2.2 also implies how limited borrowing affect the government's redistributive policies. When borrowing constraints do not bind, it is optimal to distort the intratemporal margins and not the intertemporal margins. Hence, the government optimally redistribute using labor taxes and levies no borrowing taxes. However, when borrowing constraints bind, distorting the intertemporal margins is beneficial since it aligns the domestic agents' interests with the government's interests. It turns out that these domestic borrowing taxes also redistribute resources among agents. The government optimally redistributes via more borrowing taxes and less labor taxes, which reduces the overall cost of distortion in the economy.

2.5 Quantitative Analysis

The previous section provides a theoretical foundation on how borrowing constraints affect optimal taxation and the government's ability to redistribute. This section, in turn, shows the impact of the government's concern for redistribution on its incentive to repay debt and how that implies the positive correlation between income inequality and external debt in the cross section and over time periods. The benchmark calibration uses Italy's data. Lastly, the section presents optimal austerity policies, specifically the fiscal and redistributive implications of a negative productivity shock.

Throughout this section, there are the following assumptions on the domestic discount factor and deviation utility

Assumption 2.3 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *such that* $\beta R^* < \mathcal{M} < 1$.

[21] This is an insight from the public finance literature, found in Judd (1985), Chamley (1986), Chari and Kehoe (1999), Werning (2007), and many other papers.

Assumption 2.4. *$\underline{U}$ is bounded below, i.e. there exists a finite real M_U such that $\inf_{s^t,t} \underline{U}(s^t,t) \geq M_U$.*

2.5.1 Computation

Given the forward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019). Appendix B.5.2 provides more details on the computational algorithm. The key co-state variable is the discounted sum of the Larange multipliers on the borrowing constraints: $\Gamma(s^t) = (\beta R^*)^t \sum_{s^k \subseteq s^t} \gamma(s^k)$. Given that the domestic agents are impatient, the borrowing constraint will bind infinitely often in the long run. It turns out that Γ is bounded, i.e.

Proposition 2.3. *Suppose Assumptions 2.1, 2.3, and 2.4 hold, if an interior efficient allocation exists, then* $\lim_{t\to\infty} \Gamma(s^t) > 0$.

$\Gamma(s^t)$ reflects the marginal benefit of relaxing the current and previous borrowing constraints from increasing one unit of utility in period t and history s^t (either by giving more consumption or leisure). In the long run, Γ corresponds to the amount of external debt that can be sustained in equilibrium. Impatience implies that in the long run, there exists a positive component in the price of borrowing coming from the lack of commitment. If there exists an ergodic distribution of the efficient allocation, then Γ will also follows an ergodic distribution. This property allows the computational algorithm using Γ as one of the state variables to converge.

2.5.2 Calibration

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c,l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L,z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen's method with 31 evenly-spaced nodes. Let $s_t = z_t$, and the government expenditure is constant over time and across histories: $G(s^t, t) = \bar{g}$. Initial debt levels are $B(s^0) = 0$ and $b^{H,d}(s^0) = b^{L,d}(s^0) = 0$, where s^0 is the mean of the aggregate productivity distribution. The deviation utility $\underline{U}(s^t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(s^t, t)$ varies with respect to the realized shock $s_t = z_t$.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preference, β and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*.

A period in the model is one year. For output, I use the logged and linearly detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To calculate the wage ratio θ^H/θ^L, I use the data on cross-sectional inequality by Jappelli and Pistaferri (2010). For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.9475, which is the time-average of these wage ratios for the period 2002-2006. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015.[22] The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks.[23]

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments.[24]

Table 2.3 summarizes the parameter values and targets from the calibration exercise.

[22] These returns are secondary market returns, gross of tax, with around 10 years' residual maturity

[23] See Appendix B.6 for more data descriptions and sources.

[24] See Section 2.5.3 for the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

Table 2.3: Calibrated Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.9475	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the cross-sectional data set by Jappelli and Pistaferri (2010), covering the period of 2002-2006. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: Jappelli and Pistaferri (2010), Eurostat (2019), and The World Bank (2019)

2.5.3 Calibration Results

Table 2.4 shows results of the moment matching exercise. The first column reports statistics from the data for Italy in the period of 1985-2015. The second column reports statistics from simulating the model and taking the long-run averages.[25] The model successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

[25] All model statistics are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

Table 2.4: Targeted Statistics: Data and Model

Statistics	Data: 1985-2015	Model
Std. dev. log GDP	0.053	0.053
Avg. govt. consumption-to-GDP	0.19	0.19

Note: The table describes targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1985-2015. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Sources: The World Bank (2019)

Table 2.5 reports non-targeted statistics of the model comparing to the data. The first column is from the Italian data, and the second column is from the model. Key cyclical properties are the volatility and correlation with respect to output of consumption and net saving ratio. I consider net savings as the amount of output minus the total consumption. In the model, net saving is the net amount of resources used to repay external debt in every period.

Table 2.5: Non-targeted Statistics: Data and Model

Statistics	Data	Model
Cyclical property		
std (C) / std (Y)	1.0	1.2
std (NS/Y) / std (Y)	0.29	0.34
corr (C,Y)	0.97	0.94
corr (NS/Y,Y)	0.40	0.30
External debt property[a]		
Mean external debt/Y	0.24	0.21
Std. (external debt/Y)	0.027	0.022

[a]Sample period: 2002-2015.

Note: This table reports non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985-2015, unless specified. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Net saving (NS) is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended. Net saving and external debt ratio series is linear detrended.

Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving only has a volatility of more than a quarter of the volatility of output, and has a positive correlation with output that is around 40%.[26] The model correctly gets the qualitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

External debt. In the data, external debt is defined as the net foreign liability position, as reported by Lane and Milesi-Ferretti (2018)'s External Wealth of Nation Database. The model explains well both the first and second moments of external debt for Italy from 2002 to 2015. The model generates on average around 21% of external debt-to-output ratio, comparing to 24% of net foreign liability-to-output ratio in the data. This model feature is with a relatively high discount factor (0.969) with respect to the literature. The model also matches well the volatility of external debt-to-output ratio in the data.

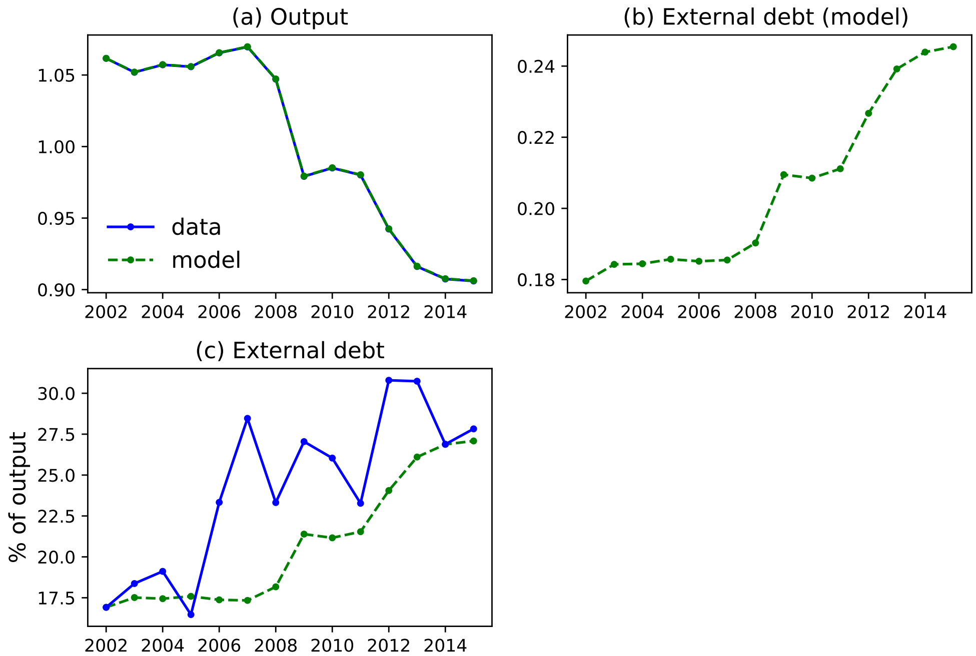
Event analysis. I now conduct an event analysis for Italy in period of 2002-2015. I feed into the model a sequence of productivity shock realizations such that the model's outputs matches ones in Italy from 2002 to 2015. I simulate a time path of external debt in the model given that the initial external debt-to-output is the data value in 2002 . I then compare the evolution of external debt-to-output in the data and in the model's simulation over time. Figure 2.4 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. From 2011 to 2015, Italy's output has dropped by 7.4% below trend, while external debt-to-output has increased by 4.6%. In the model's simulation, external debt has increased by 3.4%, which leads to a 5.5% increase in external debt-to-output.

2.5.4 Model Mechanics

This subsection explains the mechanism on how the government's concerns for redistribution affect its incentive to repay debt. The first part analyzes dynamics of

[26] Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

Figure 2.4: Italy's Recession: Data and Model



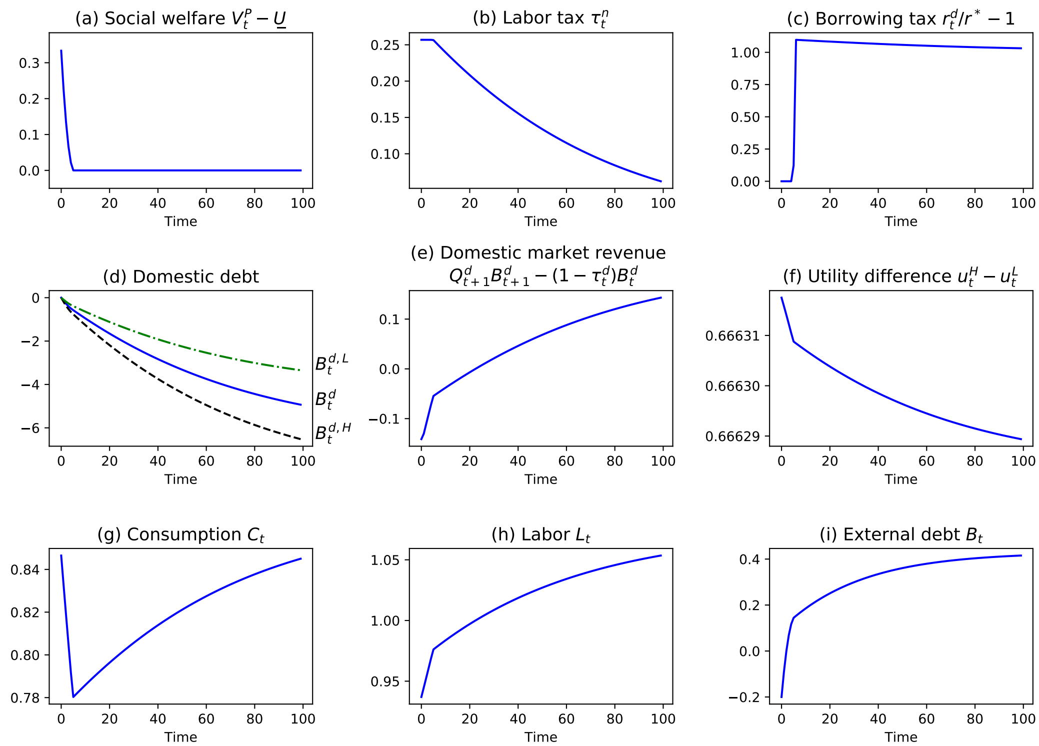
Note: The graph depicts time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output.The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. Data sources: Lane and Milesi-Ferretti (2018) and The World Bank (2019).

optimal policies, which provides insights on the properties of redistributive policies that allow the government to sustain debt. The second part studies the cost of default and its interaction with the government's redistributive motive. This interaction is the key explanation for the positive association between income inequality and external debt.

Dynamics of Optimal Policies

I first consider a special case of the model that faces a deterministic path of productivity: $z_t = \bar{z} = 1, \forall t$, and the economy starts with an initial external asset position: $B_0 = -0.2$. The other parameter values are the same as in Table 2.3. Without shocks, when the borrowing constraint binds, all future borrowing constraint will also bind. This framework can clearly show the effect of the binding borrowing constraint on optimal policies, in which there is a region where the borrowing constraint does not bind, and a region where it binds. The Ricardian equivalence implies that there is indeterminacy between lump-sum taxes and domestic debt holdings. Therefore, a particular implementation is that the government only levies the present-value of lump-sum taxes in the initial period. These lump-sum taxes determine the effective initial wealth positions of domestic agents.

Figure 2.5: Time paths of aggregates in the special case: $z_t = 1$ and $B_0 = -0.2$



Note: The graph plots deterministic time paths of optimal policies and aggregates from the planning problem in which $z_t = \bar{z} = 1, \forall t$ and $B_0 = -0.2$. The implementation is that lump-sum taxes only occur in period 0. Panel (a),(b), and (c) plot the difference between the social welfare and deviation utility, the optimal labor and saving taxes, respectively. Panel (d) plots the time paths of total and individual domestic debt. Panel (e) plots the net government's revenue of domestic market. Panel (f) depicts the utility difference between high and low income agents. Panel (g), (h), (i) plot aggregate consumption, labor, and external debt, respectively.

Figure 2.5 depicts aggregate time paths of this special case. The borrowing constraint does not bind when $V^P > \underline{U}$ and binds when $V^P = \underline{U}$. Due to impatience, social welfare decreases over time until it reaches the deviation utility value. Optimal taxes follow the properties in Proposition 2.2. Optimal labor taxes are positive and constant, and saving taxes are zero when borrowing constraints do not bind. However, when the borrowing constraint starts binding, optimal labor taxes permanently decrease , and saving taxes become negative, imply taxes on domestic borrowing. In present value terms, the planner gives a small lump-sum transfer (negative lump-sum tax) to all residents. Domestic debt decreases over time, implying that agents are borrowing. Since all agents borrow at the same fraction of their income, high-income agents borrow more than low-income agents and are net debtors. The government acts as an financial intermediary between the domestic agents and international lenders. Panel (f) describes the net resources that the government receives from the domestic credit market. In the beginning of time, the government gives resources to domestic agents. The government collects taxes on labor income in return. When borrowing constraints bind, the government uses borrowing taxes and start collecting net revenue from the domestic credit market. As labor taxes decline over time, there is less labor tax revenue. Impatience implies that the planner front loads consumption and leisure when borrowing constraints do not bind. When borrowing constraints bind, declining labor taxes encourage increases in labor and output, while saving subsidies encourage back-loading consumption. In the long run, the government continues to redistribute, as the utility difference declines over time, by using claims and taxes in the domestic credit market and less labor taxes.

Figure 2.6 depicts the long-run time paths of the aggregates and policies of the baseline model for a simulation. In this case, the borrowing constraint is occasionally binding, generating the fluctuations in the efficient allocation and policies. Both the social welfare V^P and aggregate consumption C are highly correlated with the productivity shock z. Over time, since the borrowing constraint binds more often, both aggregate consumption and labor go up, as predicted by the deterministic case. The labor tax declines over time as the borrowing constraint occasionally binds. The external debt follows an ergodic distribution that fluctuates with respect to the aggregate productivity shock. An increase in external borrowing coincides with an increase in redistribution as the utility difference between the two agents declines.

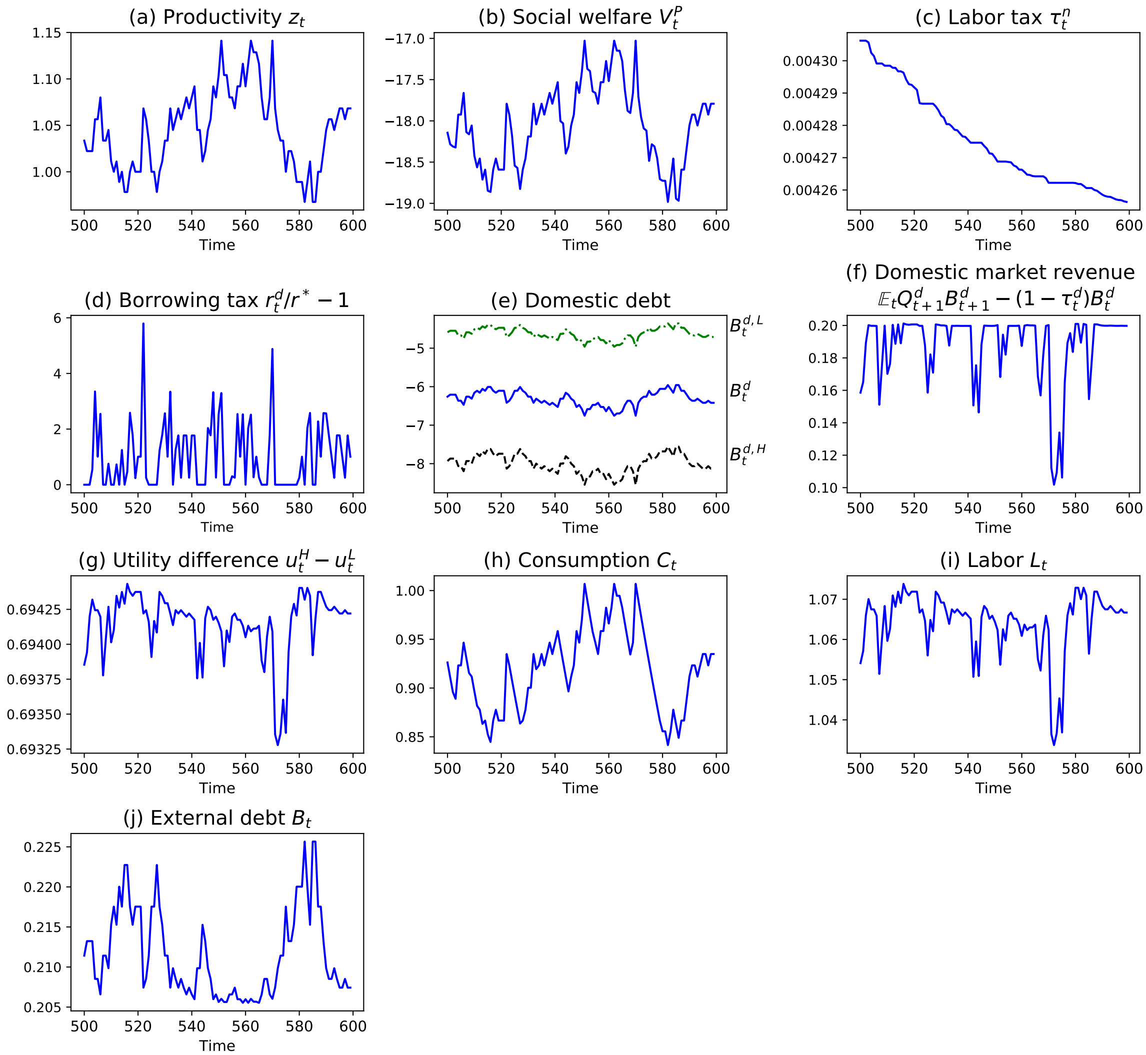
Figure 2.6 points out the changes in the government's redistributive policies over time. Since high-income agents borrow more than low-income ones, borrowing taxes act as an additional redistributive policy when borrowing constraints bind. The government redistributes via taxes on borrowing instead of labor taxes, which increases the efficiency gain and allows the government to sustain the existing level of debt.

Redistribution and Cost of Default

The cost of default determines the equilibrium level of external debt. The more costly is to default and goes into financial autarky, the more likely that the government is willing to repay debt, and so the higher level of external debt is.

In this framework, this cost consists of two main components: insurance and redistribution. The former comes from the fact that without credit markets, the government cannot insure itself against aggregate fluctuations. The insurance cost is present in many representative-agent models in the literature (Eaton and Gersovitz (1981),

Figure 2.6: Long-run time paths of aggregates in the baseline model



Note: The graph plots long-run time paths of optimal policies and aggregates of the planning problem for the baseline model. The implementation is that lump-sum taxes only occur in period 0. Panel (a) and (b) plot realized productivity path and social welfare, respectively. Panel (c) and (d) depict optimal labor and saving taxes, respectively. Panel (e) plots time paths of total and individual domestic debt. Panel (f) plots net government's revenue of domestic market. Panel (g) depicts the utility difference between high and low income agents. Panel (h), (i), (j) plot aggregate consumption, labor, and external debt, respectively.

Aguiar and Gopinath (2006a), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). This chapter introduces the redistributive cost, which is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract.

The government's redistributive motive affects its repayment incentive via changing the cost of default. This an endogenous component of the cost of default that is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract, so the government is more willing to repay its debt.

What entails the redistributive cost of default, or the benefit of repayment? First, note that if the government defaults, it does so both domestically and externally. Domestic default creates an adverse distributive effect. Figure 2.6 shows that the high-income agents are net domestic debtors in the long run. Domestic default erases the distribution of domestic wealth and so implicitly transfer more resources to the high-income agents. Furthermore, the labor distortion needed for redistribution in financial autarky is higher than the labor distortion in the contract. When the country can borrow externally, the government uses taxes on domestic borrowing to redistribute and collect revenue to repay external debt, which means it relies less on labor taxes.

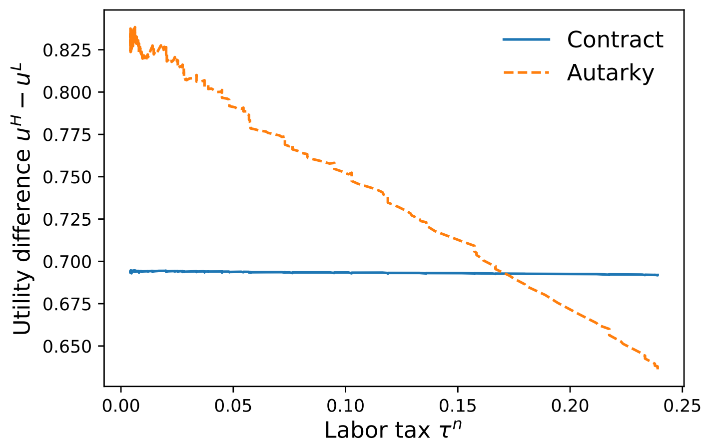
The previous argument relies on a particular decentralization of the efficient allocation. Formally, I show that having access to external financing is more beneficial in terms of efficiency and redistribution than financial autarky. The following Lemma shows that financial autarky is not optimal because there exists a more effficient deviation that allows for higher welfare.

Lemma 2.1. *The financial autarkic allocation is not optimal.*

The proof relies on the following intuition. There is labor distortion in financial autarky. In the future periods, lowering the labor distortion without changing the continuation value allows the economy to produce more than its consumption, which implies that it can borrow today and repay the debt in the future. The extra unit of borrowing allows for more consumption and welfare today.

Furthermore, external financing also allows for more redistribution. Figure 2.7 plots the individual utility difference with respect to a given level of labor tax in the contract and in financial autarky after the government defaults in period 500. Initially, when labor tax is high, the utility difference in the contract is higher than in autarky. However, in the long run, when the optimal labor tax is low, the utility inequality in the contract is lower than in autarky. It means that in the long run, to achieve the same level of redistribution, financial autarky requires a higher cost of efficiency.

Figure 2.7: Labor tax and utility difference



Note: The graph depicts the level of utility difference $u^H - u^L$ for a given level of labor taxes τ^n in the contract and in financial autarky after the government defaults in period 500. The contract line comes from the simulated data for 1500 periods. The autarky line comes from solving the autarkic allocation given a fixed level of labor tax.

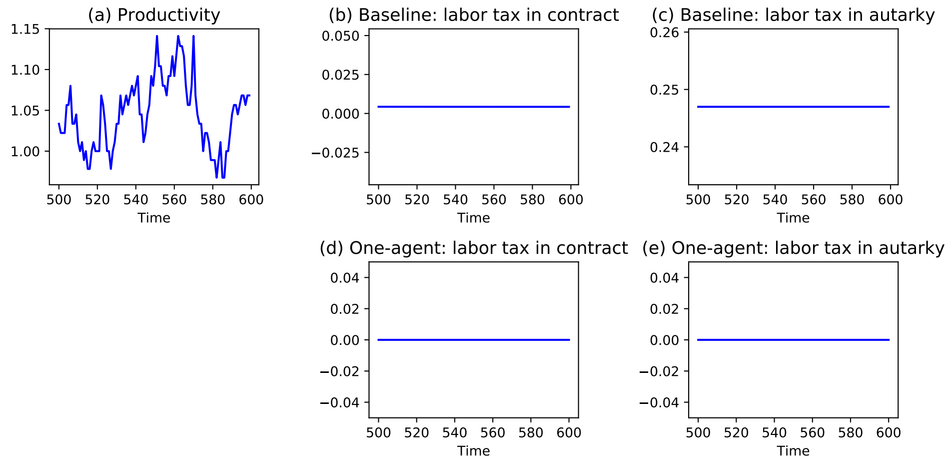
Figure 2.8 shows optimal labor taxes in different scenarios. Panel (a) plots the path of productivity shock in the long run, starting at period 500. Panel (b) and (c) plot the baseline simulation of optimal labor taxes in contract and in autarky, respectively. Panel (d) and (e) are the analogs of panel (b) and (c) for the one-agent's model.

In the one-agent model, there is no need for redistribution, so the labor taxes are zero across time periods and histories. If the government defaults and goes into financial autarky, labor taxes remain zero. However, in the baseline model with heterogeneous agents, there are differences in labor taxes between the contract and autarky. The labor tax in autarky is higher than the labor tax in the contract. These properties lead to financial autarky, or default, be more costly than repayment.

Quantitatively, the welfare cost of insurance is trivial, so the amount of external debt that the one-agent model can sustain is quantitatively small.[27] When the government has concerns for redistribution, the redistributive cost of default arise endogenously. This cost turns out to be quantitatively large enough to account for the observed external debt levels. Given the calibrated parameters, the long-run average external debt-to-output is 2.8% in the one-agent model and 21% in the heterogeneous-agent model. These results imply that the insurance cost accounts for 12% , and the redistributive cost accounts for

[27] Lucas (1987) pointed out that the cost of eliminating business cycles is quantitatively small for the standard neoclassical growth model. In the sovereign debt framework, Aguiar and Gopinath (2006a), Arellano (2008), and Chatterjee and Eyigungor (2012) have shown that in order to match the observed debt levels in the data, they need to impose additional output losses in default.

Figure 2.8: Labor distortion in contract and in autarky



Note: The graph describes time paths of productivity shock and optimal labor taxes in contract and in financial autarky for the baseline model and the one-agent model. The autarky case is when the government defaults at period 500 and is permanently excluded from all credit markets.

88% of the long-run average external debt-to-output. Appendix B.5.3 provides a measure of the redistributive cost of default in terms of productivity loss.

Higher income inequality is correlated with higher external debt because of the higher redistributive cost of default. Given the government's redistributive preference towards low-income agents, a higher income inequality implies a higher motive for redistribution and a larger labor tax distortion in financial autarky, making it more costly to default. Therefore, the government is willing to sustain a higher amount of external debt.

In the next subsections, I estimate the effect of income inequality on external debt in the cross section and over time.

2.5.5 Cross-Country Estimation

Table 2.6 shows estimation results of the correlation between pre-tax Gini index and net foreign liability-to-GDP from the model and the data. Data values are from the second column of Table 2.2, robust to country, time, and other controls. Model values come from the regression using the model's simulated data. Given the calibrated parameters, I solve different versions of the model differentiated only by wage inequality and compute pre-tax Gini indices, long-run averages of external debt-to-output ratios, and output per capita. I then estimate the regression $\text{NFL}_i = \beta_0 + \beta_1 \text{Gini}_i + \beta_2 \log \text{GDP per capita}_i + \epsilon_i$ and report $\hat{\beta}_1$ and its standard error.[28]

[28] In the model, average output growth rates and inflation are zero, so I omit them as control variables.

Table 2.6: Cross-country estimation: Data and Model

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Data: 1985-2015	Model
Gini index, pre tax (%)	0.968**	2.42***
	(0.487)	(0.0674)
Controls	Yes	Yes
No. Observations	120	30

Note: The table describes cross-country estimations of the coefficient of pre-tax Gini index (%) on net foreign liability-to-GDP (%) in the data and in the model. Details on data estimation are from Table 2.2. The model estimation comes from simulated data of 30 different versions of the model that are differentiated by wage ratios.

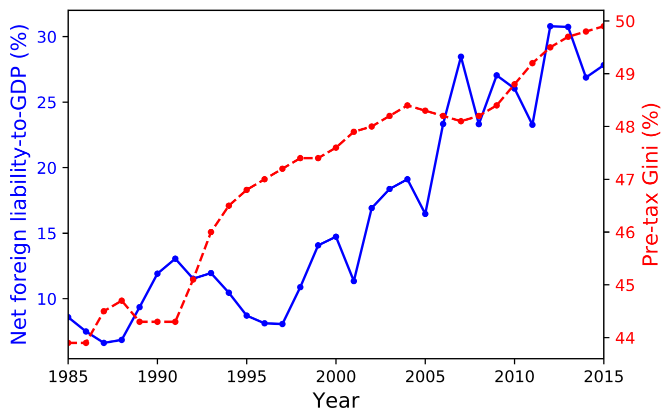
The model produces a positive and statistically significant coefficient of the pre-tax Gini index. The coefficients imply that a one percent increase in the pre-tax Gini index corresponds to a 0.968% increase in net foreign liability-to-GDP in the data, comparing to a 2.4233% increase in the model.

2.5.6 Comparative Statics Exercise

This subsection estimates the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001 and 2002-2015. Figure 2.9 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 2.7 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001 to 2002-2015, the model can account for 93% of the increase in the external debt-to-output ratio.

Figure 2.9: Income inequality and external debt in Italy



Notes: The graph shows time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

Table 2.7: Comparative statics results for periods 1985-2001 and 2002-2015

Statistics	Data	Model
Targeted		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted		
Δ Extenal debt/Y	14%	13%

Notes: The table reports results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.9475 minus the same statistic of the model with the wage ratio equal to 1.73.

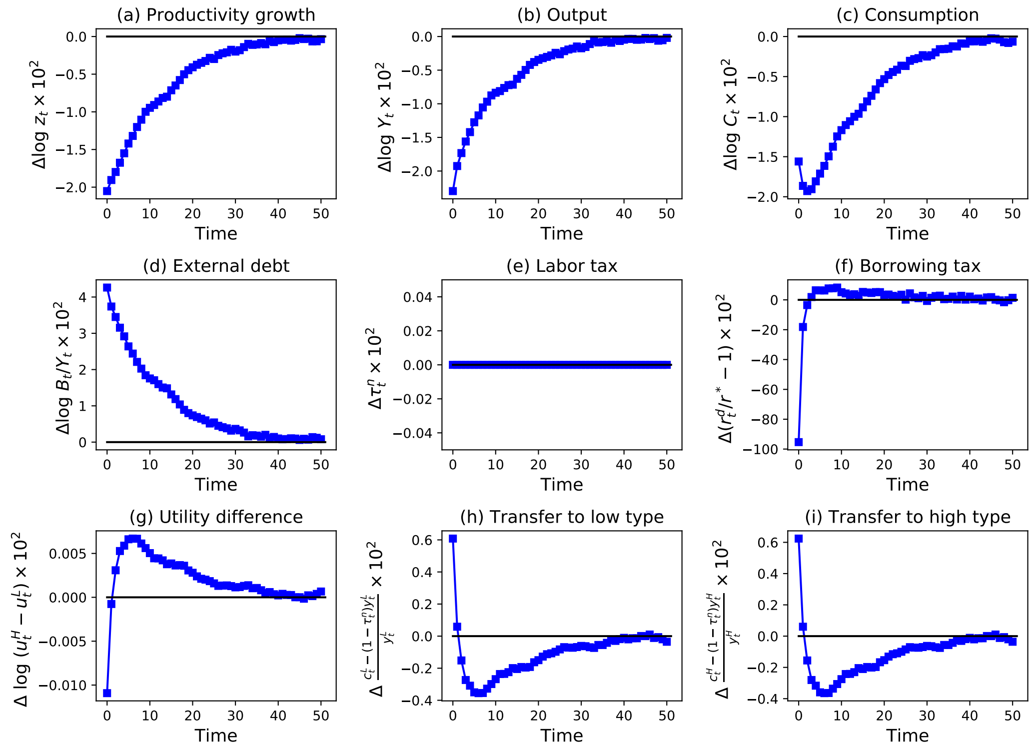
2.5.7 Optimal Austerity Policies

This section evaluates responses of optimal austerity policies to a negative productivity shock in the presence of inequality.

Figure 2.10 plots impulse response functions of aggregate variables and tax policies with respect to a one standard deviation decline in productivity growth, computed via local projection.[29] Panel (a) plots the path of productivity growth given the negative

[29]The approach to calculate impulse respond functions is econometrically equivalent to the approach of Jordà (2005). I simulate the economy for 10500 periods with aggregate productivity shocks and exclude the first 500 periods. I then calculate the realized time series of shocks to productivity, ϵ_t^z. To compute the response of a variable X to the shock ϵ_t^z, I perform the OLS regressions $\Delta \log X_t = \alpha + \beta_k \epsilon_{t+k}^z + \eta_t$ to get the estimated $\hat{\beta}_k$. The horizontal τ IRF is then $IRF_\tau = \sum_{k=0}^{\tau} \hat{\beta}_t$. The effect of one standard deviation shock to ϵ_t^z is the responses $\sigma_z \times IRF_\tau$. Since labor and saving taxes can be zero or negative, the dependent

Figure 2.10: Benchmark impulse response functions to a negative productivity shock



Notes: The graph shows impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005). Panel (a) plots the productivity growth response. Panel (b) and (c) plot the responses of output and consumption, respectively. Panel (d), (e), and (f) plot the responses of fiscal policies: external debt, labor, and saving taxes, respectively. Panel (g), (h), and (i) show responses of redistribution: variance of log utilities and average tax-to-income ratios across agents.

innovation shock occurring in period 0. There are three groups of responses: aggregates, fiscal policies, and redistribution. Panel (b) and (c) plot the first group of responses of output and consumption, respectively. For fiscal policies, Panel (d), (e), and (f) plot the responses of external debt, labor taxes, and borrowing taxes. Lastly, Panel (g), (h), and (i) plot the responses related to redistribution: utility differences and transfer rates across individuals.[30]

A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. Labor taxes remains unchanged, while there is a sharp decrease in borrowing taxes in the first period, accompanying with a decline. Optimal borrowing taxes are positive in the long

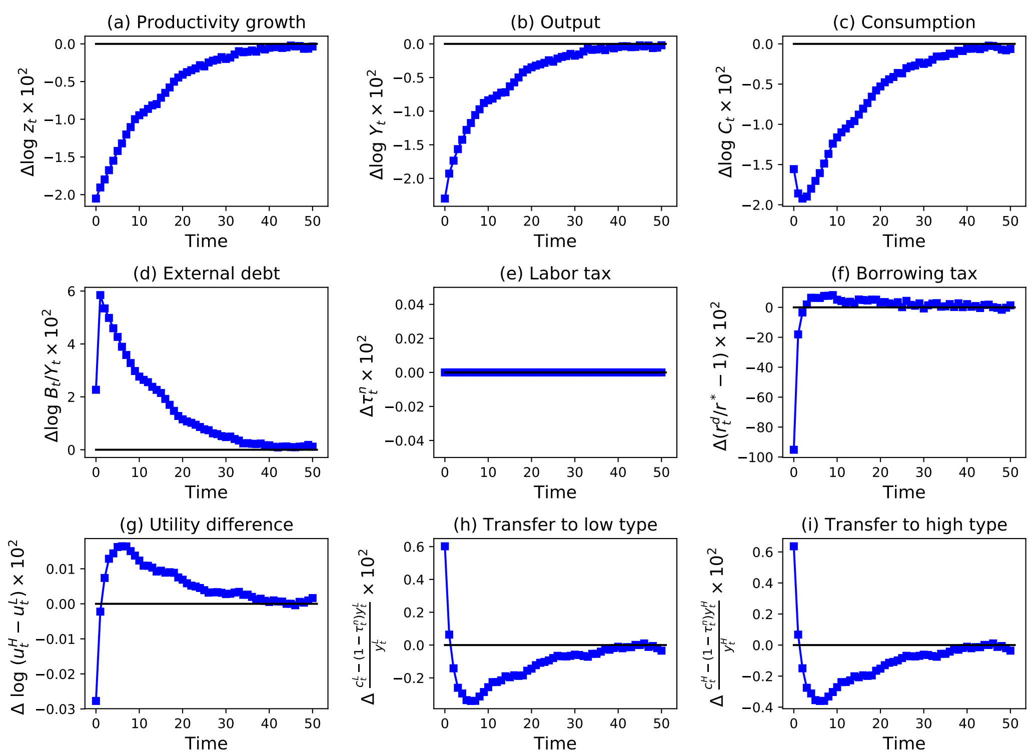
variable in the OLS regressions are ΔX_t instead of $\Delta \log X_t$. To my best knowledge, Mongey (2019) is the first chapter that applies this computational technique in calculating impulse responses.

[30]The transfer rate is defined as amount of resources an individual receives excluding labor taxes over the individual income, equal to $\frac{(1-\tau^n)y^i - c^i}{y^i}$ for an individual i with income y^i and consumption c^i and faces labor tax τ^n.

run. The initial decrease in borrowing taxes comes from non-binding borrowing constraints in the first few periods after a negative shock.[31] However, as borrowing constraints bind in the future, it is then optimal to increase borrowing taxes. In terms of redistribution, the utility difference initially decreases then increases subsequently. Similarly, transfer rates increase for both agents, following by declines which are larger for higher-skilled agents.

Intuitively, a negative shock leads to a reduction in the deviation utility, and so the borrowing constraint becomes non-binding. The non-binding constraint allows the government to accumulate external debt, temporarily decreases average tax rates, and increases redistribution. In future periods, the government raises taxes to repay the debt and reduces redistribution.

Figure 2.11: Counterfactual impulse response functions to a negative productivity shock



Notes: The graph shows the impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005) for the model with a wage ratio equal to 1.7

Policy responses are non-linear with respect to the underlying inequality level. Figure 2.11 plots responses to a negative productivity shock given a lower level of wage inequality than the baseline value. Comparing to the baseline case, consumption, output, labor and

[31]Proposition 2.2 shows that $\tau^d = 0$ when borrowing constraints do not bind. So τ^d goes from a negative number to zero initially.

Table 2.8: Role of heterogeneity

θ^H/θ^L	1	1.7	1.9475 Baseline	2.5
Long-run statistics				
Mean C/Y	0.82	0.82	0.82	0.81
Mean B/Y	0.028	0.073	0.21	0.63
Tax policies				
Initial τ_0^n	0.0	0.15	0.25	0.36
$\lim_{t\to\infty} \tau_t^n$	0.0	0.0077	0.0042	-0.016
Lump-sum tax T_0/Y_0	5.8	2.5	0.87	-1.7

Notes: The table reports long-run statistics and policies for different levels of wage inequality, measured as θ^H/θ^L. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

saving taxes respond similarly to a negative shock, but increases in external debt and changes in utility differences are larger.

2.6 Discussions

This section discusses how optimal policies respond to different model ingredients: heterogeneity/redistributive motive, distortionary taxation, discount factor, aggregate uncertainty, and government expenditure. Throughout this section, I consider a particular implementation in which the government only uses lump-sum taxes in the initial period.[32]

2.6.1 Role of Heterogeneity and Redistributive Motive

The previous section has shown that the presence of heterogeneity or concern for redistribution plays a role in debt sustainability. Here, I provide more details on their impact on the optimal allocation, debt, and tax policies. Table 2.8 describes the long-run statistics and optimal policies for different levels of wage inequality, measured as the ratio of high-to-low productivity levels.[33] The all columns report the statistics and the optimal tax policies from the model simulation.

When the wage inequality increases, the average consumption-to-output ratio remains around 82% (average government spending-to-output is roughly 19%), while the average external debt-to-output increases, as shown in the previous sections. The tax policies to

[32]This tax is the present value of all lump-sum taxes

[33]In this environment, the average productivity is normalized to one, so increases in θ^H/θ^L imply the mean preserving spreads

Table 2.9: Role of Distortionary Taxation

	Baseline	Skill-dependent lump-sum tax
Long-run statistics		
Mean C/Y	0.82	0.82
Mean B/Y	0.21	0.029
Tax policies		
Initial τ_0^n	0.25	0.0
$\lim_{t\to\infty} \tau_t^n$	0.0042	0.0
Lump-sum tax T_0/Y_0	0.88	5.7, high type = 20, low type $=-8.6$

Notes: The table reports long-run statistics and tax policies for the baseline and the case in which the government has access to fully skill-dependent lump-sum taxes, $T^i, \forall i \in I$. The last row and column reports the average lump-sum tax, as well as the individual lump-sum taxes.

finance external debt are such that, rising wage inequality or motive for redistribution implies a higher initial labor tax, a lower labor tax in the limit, accompanying with a lower lump-sum tax. Intuitively, the government with a higher redistributive motive redistributes via a higher labor tax and a higher lump-sum transfer. The government accumulates more external debt to finance such policies. In the long run, in order to repay debt, the government reduces the labor distortion and uses claims and taxes in the domestic credit market to redistribute. The lower tax distortion in the limit encourages more output to repay.

2.6.2 Role of Distortionary Taxation

The previous sections argue how the redistributive tax policy comes with a cost of distortion, and by front-loading the distortion, the economy sustains a high debt. This subsection relaxes the assumption of distortionary taxation by considering the case in which the planner has access to skill-specific lump-sum transfer, so the planner achieves perfect redistribution without generating any distortionary cost. The need to use distortionary taxation as a redistributive tool makes the government be willing to sustain highly positive debt in the long run. Table 2.9 reports external debt and tax policies of the baseline model using linear taxes comparing to the alternative framework in which the lump-sum transfer/tax depends on individual income.

While both cases give the same average consumption-to-output ratio in the long run, the alternative case quantitatively generates a much smaller amount of debt with a higher volatility than the baseline case. Across all periods, the labor tax is zero, since all of the redistribution is done via the type-dependent lump-sum taxes. In present-value terms, the government taxes the high-income agents and transfers to the low-income agents.

2.6.3 Role of Discount Factor

Table 2.10 considers the optimal allocation and policies under different values of the domestic discount factor. When domestic agents become more impatient (their discount factor is declining), the average consumption-to-output ratio roughly remains the same (with a slight decrease), while the average external debt-to-output ratio declines. Lower external debt levels also coincide with lower labor taxes and lump-sum taxes. A higher discount factor implies a lower value of autarky and a longer time period it takes for the economy to reach the borrowing constraint. Therefore, it allows the government to accumulate and sustain a higher external debt level.

Table 2.10: Role of discount factor

β	0.9	0.95	0.967 Baseline
Long-run statistics			
Mean C/Y	0.81	0.81	0.82
Mean B/Y	0.085	0.17	0.21
Tax policies			
Initial τ_0^n	0.246	0.250	0.252
$\lim_{t\to\infty} \tau_t^n$	0.0066	0.005	0.0042
Lump-sum tax T_0/Y_0	0.46	0.70	0.87

Notes: The table reports long-run statistics and policies for different levels of discount factor β. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

2.6.4 Role of Aggregate Uncertainty

Table 2.11 reports statistics and policies for different levels of the shock variance. The first column reports the deterministic case, where $\sigma_z = 0$. The other columns report scenarios under different levels of uncertainty. A higher variance of the productivity shock implies a lower external debt-to-output, a higher consumption-to-output, a lower initial labor tax, and higher labor taxes in the limit and positive lump-sum taxes. In this environment, a higher uncertainty leads to more fluctuations, resulting in more time periods that the borrowing constraint does not bind in the long run. In fact, in the deterministic case, one can show that if the borrowing constraint binds, it will find

Table 2.11: Role of aggregate uncertainty

σ_z	0	0.01	0.0205 Baseline	0.022
Long-run statistics				
Mean C/Y	0.81	0.81	0.82	0.82
Mean B/Y	0.25	0.245	0.21	0.21
Tax Policies				
Initial τ_0^n	0.25	0.25	0.25	0.24
Mean LR $\lim_{t\to\infty} \tau_\infty^n$	0.0029	0.0031	0.0042	0.0043
Lump-sum tax T_0/Y_0	0.6	0.65	0.87	0.9

Notes: The table reports long-run statistics and policies for different levels of the shock variance σ_z. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

forever.[34] This property leads to lower external debt levels in the long run. In addition, a higher uncertainty implies a higher precautionary motive, which helps explain the lower external debt accumulation.

2.6.5 Role of Government Spending

This subsection shows the role of government expenditure, $\bar{g}$ on the efficient allocation and optimal policies. Table 2.12 reports results for different values of $\bar{g}$. An increase in government expenditure implies a lower fraction of aggregate consumption-to-output in the long run, while external debt-to-output ratio remains around the same value.[35] Both labor taxes and lump-sum taxes increase in response to a higher government expenditure. In this environment, lump-sum taxes allow the government to finance its expenditure without incurring any distortionary cost. The distortionary cost only comes from the need for redistribution. Therefore, the level of government expenditure does not significantly affect the sustainable level of external debt.

2.7 Conclusion

This chapter proposes a theory of external debt sustainability that comes from the motive for redistribution of the government. I introduce the government's redistributive

[34]See Tran Xuan, 2019

[35]In details, the external debt level slightly increases with respect to the increasing government expenditure, yet output can be increasing or decreasing, resulting in the non-linear response of external debt-to-output to government expenditure.

Table 2.12: Role of government spending

$\bar{g}$	0	0.1	0.205 Baseline	0.3
Long-run statistics				
Mean C/Y	1.0	0.9	0.82	0.73
Mean B/Y	0.20	0.21	0.21	0.21
Tax Policies				
Initial τ_0^n	0.21	0.23	0.25	0.25
$\lim_{t\to\infty} \tau_t^n$	-0.016	-0.0069	0.0042	0.017
Lump-sum tax T_0/Y_0	-4.4	-1.7	0.87	3.1

Notes: The table reports long-run statistics and policies for different levels of government expenditure $\bar{g}$. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

concern and distortionary taxation into a sovereign debt framework. I analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to the domestic agents' impatience. Redistribution comes with the cost of tax distortions.

The chapter's theoretical contribution is the effect of borrowing constraints on optimal taxation in the presence of inequality. While the magnitude of inequality determines the optimal level of taxes and redistribution, the borrowing constraints determine the optimal dynamics. The main conclusion is that labor taxes are unchanged, and borrowing taxes are zero when borrowing constraints do not bind. Binding borrowing constraints lead to permanent declines in labor taxes and positive borrowing taxes to discourage domestic borrowing. Borrowing taxes have a redistributive benefit, which allows the government to distort labor less and increases the economy's efficiency.

The quantitative contribution is showing that the government's redistributive motive plays an important role in determining the equilibrium level of external debt. This channel comes from the additional cost of redistribution during financial autarky. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The redistributive cost of default quantitatively accounts for 87% of the long-run average external debt-to-output, while the insurance cost of default only accounts for 13%. Furthermore, the theory can account for the cross-country and time-series relationship between income inequality and external debt in the data.

The model has implications on optimal austerity policies in the presence of inequality.

Estimations from the model's simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. The average tax-to-income ratio initially decreases for all agents and more for high-income agents, and utility difference decreases. In the future, the government raises average taxes and reduces redistribution to repay debt.

Future research includes allowing for equilibrium defaults and incorporating various types of debt crises. In the next project, I examine the interaction between inequality and default risks and their implications for austerity policies.

Chapter 3

Sovereign Default and Inequality

3.1 Introduction

The first two chapters focus on how income inequality affects the government's debt issuance absent of default in equilibrium. In this chapter, I introduce a sovereign default framework with heterogeneous agents and quantify the effect of income inequality on sovereign default risk.

I incorporate heterogeneous labor productivity, endogenous labor supply, and distortionary taxation into the canonical sovereign default model. The small open economy faces an aggregate productivity shock. The government redistributes resources via affine taxes (marginal labor tax and lump-sum tax) and borrows internationally with state-uncontingent bonds. The government can default on its bond under the cost of productivity losses and some period of exclusion from the international financial market. The bond prices incorporate a risk premium to compensate international lenders for their default losses.

The government's debt choices interact with inequality via its redistributive goals. Conditional on repayment, the higher the debt is, the higher marginal taxes the government needs to levy in order to repay its debt. Therefore, the benefit of default is to increase the amount of resources transferred to private workers and reduce the marginal labor tax, which in turn lowers the distortion. On the other hand, the benefit of repayment is using international financial markets to smooth out aggregate fluctuations and distortions. Absent of inequality, default does not reduce distortion, so the benefit of default is low.

I quantify the model using Spanish data and examine how inequality contributes to the sovereign debt crisis. I calibrate the model to match key macroeconomic statistics of Spain. To evaluate the role of inequality/redistributive motive, I compare the benchmark model to the model with no inequality.

The findings are as follows. First, inequality worsens the crisis by increasing the default probability. Both average and volatility of spreads are higher in the benchmark model than the no-inequality model. Second, default risk generates a deep and long decline in output. The government's motive for redistribution mitigates the debt crisis with smaller drop in output and lower sovereign spread at the cost of higher labor taxes. Lastly, average sovereign debt and spread vary non-linearly to income inequality.

Related Literature. The model builds on the sovereign default model developed by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006b), and Arellano (2008). The contribution is a framework that highlights the interaction between government's default, debt, and redistributive policies. In contrast to an endowment economy like most work in the literature, labor supply is endogenous, which generates the distortionary cost of policies.

This research also contributes to the public finance literature that studies the trade-off between debt management and redistribution such as Werning (2007), Bhandari, Evans, Golosov, and Sargent (2016), and Bhandari, Evans, Golosov, and Sargent (2017) by introducing strategic defaults and quantifying the relationship between inequality and sovereign spread.

Outline. The chapter is organized as follows. Section 3.2 describes a model of sovereign default and inequality. Section 3.3 defines the recursive equilibrium of the government. Section 3.4 presents the quantitative analysis. Lastly, Section 3.5 concludes.

3.2 Model of Sovereign Default and Inequality

This section describes the model of sovereign default, income inequality, and distortionary taxation. I consider a small open economy with a continuum of workers that are differentiated by their labor productivities, a production technology, and a benevolent government. The government borrows noncontingent bonds internationally and can default with punishment of lower productivity and temporary exclusion from the international markets. This model departs from the canonical sovereign default model by introducing worker heterogeneity, endogenous labor supply, and distortionary taxes that act as redistributive policies.

3.2.1 Environment

A small open economy faces publicly observed aggregate productivity shocks $z_t \in S$ in period t, where Z is some finite set. The exogenous risk-free international interest rate for borrowing is r^*. A history of shock is $z^t = (s_0, s_1, ..., z_t)$. Allocation, government policies, and prices depend on histories of shock. For convenience, I suppress the notation z^t.

Workers. The worker's productivities are$\left(\theta^i\right)_{i \in I}$, which are publicly observable. The fraction of workers with productivity θ^i is π^i, where $\left(\pi^i\right)_{i \in I}$ and $\left(\theta^i\right)_{i \in I}$ are normalized such that $\sum_{i \in I} \pi^i = 1$ and $\sum_{i \in I} \pi^i \theta^i = 1$. All workers have the same discount factor β and the static utility $U(c, n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i) \tag{3.1}$$

where $U^i(c, l) = U\left(c, \frac{l}{\theta^i}\right)$.

Firm. There is a representative firm that uses labor to produce a single final good. The production function is$F(z, L) = \bar{z}(z)L$, where L is the aggregate labor supply and $\bar{z}$ is the realized productivity that the economy receives.

Government. The benevolent government maximizes the social welfare

$$\mathbb{E}_0 \sum_{i \in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t U^i\left(c_t^i, l_t^i\right),$$

where $\left\{\lambda^i\right\}_{i \in I}$ is the set of welfare weights ($\lambda^i > 0, \sum_{i \in I} \lambda^i = 1$). In every period, the government can levy a marginal tax rate on labor income τ^n, a lump-sum tax T, and borrows state-uncontingent B internationally. The government can default on these obligations. Denote d_t the default policy, where $d_t = 0$ implies default and $d_t = 1$ implies no default. The punishment for default includes a drop in productivity $z_d(z) \leq z$ and exclusion from international financial markets for some periods. With probability ψ, the country regains access to international financial markets with zero debt and is no longer subject to productivity loss. Following the sovereign default literature, I assume that only the government can participate in the international financial markets and redistribute the proceeds back to individuals in a lump-sum fashion.

International lenders. The international lenders are competitive and risk neutral, willing to lend to the government at the break-even price q_t that internalizes the government default decision .

Allocation. An allocation specifies consumption and labor: $\left\{c^i, l^i\right\}$. The aggregates are denoted by $C \equiv \sum_{i\in I} \pi^i c^i$ and $L \equiv \sum_{i\in I} \pi^i l^i$.

3.2.2 Competitive Equilibrium

I formally define competitive equilibrium given the government policies

Workers. Given the government's policies, the individual worker of type $i \in I$ maximizes their utility subject to the following budget constraint for every period t

$$c_t^i = (1-\tau_t^n) w_t l_t^i - T_t \tag{3.2}$$

Firm. The representative firm pays each unit of efficiency-unit labor at a wage

$$w_t = \bar{z}_t = (1-d_t) z_t + d_t z_d(z_t), \forall t \tag{3.3}$$

Government The government's budget constraint is

$$(1-d_t) B_t \leq \tau_t^n w_t L_t + T_t + q_t B_{t+1} \tag{3.4}$$

Resource Constraint The resource constraint of the economy in every period is

$$C_t + (1-d_t) B_t \leq F(z_t, L_t) + q_t B_{t+1} \tag{3.5}$$

Definition 3.1. *A competitive equilibrium is competitive equilibrium with taxes for an open economy is individual worker's allocation* $z^{H,i} = \left\{c_t^i, l_t^i\right\}$, $\forall i \in I$, *the representative firm's allocation* $z^F = \{L_t\}$, *prices* $p = \{q_t, w_t\}_t$, *and government's policy* $z^G = \{\tau_t^n, T_t, B_{t+1}\}$ *such that*

(i) Given policies and prices, $z^{H,i}$ *solves individual* i*'s problem that maximizes (3.1) subject to (3.2) and* z^F *solves firm's problem*

(ii) The government budget constraint (3.4) and the resource constraint (3.5) are satisfied

(iii) p *satisfies the international break-even rule and equation (3.3) given* z^G

3.2.3 Characterizing Competitive Equilibrium

For any equilibrium, there exist a set of Neghishi (market) weights $\boldsymbol{\varphi}_t = \left(\varphi_t^i\right)_{i \in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solve a static problem

$$
\begin{aligned}
V(C_t, L_t; \boldsymbol{\varphi}_t) \equiv \max_{(c^i, l^i)_{i \in I}} & \sum_{i \in I} \varphi_t^i \pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i \in I} \pi^i c^i = C; \quad \sum_{i \in I} \pi^i l^i = L
\end{aligned}
$$

This problem gives the policy functions for each individual i

$$
h^i(C_t, L_t; \boldsymbol{\varphi}) = \Big(h^{i,c}(C_t, L_t; \boldsymbol{\varphi}),\ h^{i,l}(C_t, L_t; \boldsymbol{\varphi}) \Big)
$$

A competitive equilibrium allocation must satisfy $\left(c_t^i, l_t^i\right) = h^i(C_t, L_t; \boldsymbol{\varphi}_t)$ for all i. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C_t, L_t; \boldsymbol{\varphi}_t)$. The envelope conditions of the static problem give

$$
(1 - \tau_t^n) w_t = -\frac{V_L\left[h^i(C_t, L_t; \boldsymbol{\varphi}_t)\right]}{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}
$$

Furthermore, the budget constraint for individual i in period t can be written as

$$
V_C(C_t, L_t; \boldsymbol{\varphi}_t) h^{i,c}(C_t, L_t; \boldsymbol{\varphi}_t) + V_L(C_t, L_t; \boldsymbol{\varphi}_t) h^{i,l}(C_t, L_t; \boldsymbol{\varphi}_t) + V_C(C_t, L_t; \boldsymbol{\varphi}_t) T_t = 0 \quad (3.6)
$$

Equation (3.6) is the individual implementability constraint.

One has the following characterization proposition.

Proposition 3.1. *An allocation $\{C_t, L_t\}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (3.5) holds, and there exist market weights $\boldsymbol{\varphi}_t = \left(\varphi^i\right)_{i \in I}$ and lump-sum tax T_t such that the implementability constraints (3.6) hold for all $i \in I$.*

Proposition 3.1 implies that instead of choosing the policies and letting the private sector responds optimally, the government can directly choose the aggregate allocation and the individual allocating rule (the set of market weights) that take into account the distortionary cost of policies, as captured by the implementability constraints.

The social welfare function for every period t becomes

$$W(C_t, L_t; \boldsymbol{\varphi}_t) = \sum_{i \in I} \lambda^i \pi^i U^i \left(h^i(C_t, L_t; \boldsymbol{\varphi}_t) \right)$$

3.3 Recursive Equilibrium

In each period, the aggregate state of the economy consists of a level of aggregate productivity z and public debt B. Denote the government value conditional on repayment by the function $V^R(z, B)$, its value conditional on default by the function $V^D(z)$, and its optimal value by the function $V(z, B)$.

The repayment value is

$$\begin{aligned} V^R(z, B) = & \max_{C, L, B', \boldsymbol{\varphi}, T} W(C, L; \boldsymbol{\varphi}) + \beta \mathbb{E}_z V(z', B') \\ & s.t. \qquad C + B = zL + q(z, B')B' \\ & \qquad\qquad C, L, \boldsymbol{\varphi}, T \text{ satisfy implementability constraints} \end{aligned}$$

The default value is

$$\begin{aligned} V^D(z) = & \max_{C, L, B', \boldsymbol{\varphi}, T} W(C, L; \boldsymbol{\varphi}) + \beta \left\{ \psi \mathbb{E}_z V(z', 0) + (1 - \psi) \mathbb{E}_z V^D(z') \right\} \\ & s.t. \qquad C + B = z_d(z)L \\ & \qquad\qquad C, L, \boldsymbol{\varphi}, T \text{ satisfy implementability constraints} \end{aligned}$$

Finally, the government chooses between repayment and default in every period, i.e.

$$V(z, B) = \max \left\{ V^R(z, B), V^D(z) \right\}$$

I assume that the government repays if it is indifferent between repayment and default. Therefore, the government defaults if and only if $V^D(z) > V^R(z, B)$. The default decision rule is $d(z, B)$ and the borrowing decision rule when repayment happens is $B'(z, B)$.

Equilibrium bond price. Since the international financial market is competitive, the unit price of debt is consistent with zero profits adjusting for the probability of default, i.e.

$$q(z, B) = \frac{\mathbb{E}_z \left[1 - d(z', B'(z, B)) \right]}{1 + r^*}$$

I formally define the recursive equilibrium of the government:

Definition 3.2. *A Markov recursive equilibrium consists of the value functions* $V(z,B), V^R(z,B)$, $V^D(z)$, *the policy functions* $C(z,B), L(z,B), T(z,B), \boldsymbol{\varphi}(z,B)$, $d(z,B), B^{'}(z,B)$ *and the bond price* $q(z,B)$ *such that*

(i) Given the bond price function, the policy functions and value functions satisfy the government's optimization problem.

(ii) Given the policy functions, the bond price reflects the government's default probabilities and are consistent with expected zero profits of the international lenders.

3.4 Quantitative Analysis

This section presents the quantitative properties of the model. I calibrate the model to the Spanish economy. I illustrate how tax and debt policies respond to different levels of aggregate productivity in the presence of income inequality. I then analyze the impulse responses to a negative productivity shock for the benchmark model and the alternative model where there is no income inequality. Lastly, I consider how average taxes, public debt, and spread respond to different levels of income inequality.

3.4.1 Calibration

I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. I consider the following parametric form of the utility

$$U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\nu}}{1+\nu},$$

where $\sigma > 0$ is the risk aversion, and $\nu > 0$ is the inverse labor elasticity. I assume that the productivity shock z follows a logged first-order autoregressive process:

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen's method with 21 evenly-spaced nodes. The loss in the productivity during default takes a form as in Chatterjee and Eyigungor (2012), i.e. $z_d(z) = z - \max\left\{d_0 z + d_1 z^2, 0\right\}$ with $d_0 < 0 < d_1$.

A period is one year in the model, and we parametrize it to match the key properties of the Spanish economy from 1980 to 2017. I consider the cyclical properties relative to a long-run trend (HP smoothing parameter of 10^5) in annual data.

Table 3.1 reports the parameter values and targets from the calibration exercise. The first group of parameters are externally calibrated. I set the risk-free rate r^* at 4% and the risk aversion σ at 2, which are standard values in the sovereign default literature. The labor elasticity is 0.5, which implies that $\nu = 2$. The wage ratio θ^H/θ^L is set to match the wage Gini of 28% from the cross-sectional data set by Pijoan-Mas and Sanchez-Marcos (2010). The persistence of the productivity shock is set to be 0.9, similar to values set by other international business cycle studies. The recovery rate is 0.25, which implies that the defaulting countries are excluded from international financial markets for four years on average, consistent with the finding in Gelos, Sahay, and Sandleris (2011).

Table 3.1: Calibrated Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.04	Standard literature value
σ	Risk aversion	2	Standard literature value
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	3.51	Average wage Gini
ρ_z	Auto-corr. of prod.	0.9	Standard literature value
ψ	Recovery rate	0.25	Standard literature value
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.048	Std. dev. log GDP
β	Discount factor	0.91	Std. dev. trade balance/GDP
d_0	Default cost parameter	-0.4	Average spread
d_1	Default cost parameter	0.5	Std. dev. spread

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The wage ratio θ^H/θ^L is set to match the wage Gini, which is the author's calculation from the cross-sectional data set by Pijoan-Mas and Sanchez-Marcos (2010). The recovery parameter is chosen to be consistent with the finding in Gelos, Sahay, and Sandleris (2011). The auto-correlation and standard deviation of GDP cover the period of 1980-2017.

The second group of parameters are internally calibrated to match key business cycle statistics of Spain. The standard deviation σ_z in the productivity process, the discount factor β, and the default cost parameters d_0 and d_1 are set to jointly target the volatility of aggregate GDP of 6.6%, the volatility of trade-balance-to-GDP of 3.4%, and the average and volatility of spreads of 1% and 1.2%.

3.4.2 Findings

Table 3.2 shows the moment matching exercise of the benchmark and the data. The first column reports the statistics from the Spanish data in the period of 1980-2017. The second and third column reports the statistics from simulating the models and taking the long-run averages.[1] The benchmark statistics are close to the data statistics. In the alternative model without inequality, trade balance volatility is higher, while average and volatility of spreads are lower than the benchmark values.

Table 3.2: Targeted Statistics: Data and Models

Statistics (%)	Data	Benchmark	No inequality
Std. dev. log GDP	6.62	6.49	6.49
Std. dev. trade balance/GDP	3.44	3.44	3.70
Average spread	1.02	1.00	0.97
Std. dev. spread	1.20	1.20	1.01

Note: The table describes the targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1980-2017. The second and third columns report the statistics derived from the model simulation for 10000 periods and excluding the first 500 periods. Column "No inequality" corresponds to the model in which $\theta^H = \theta^L$.

The next exercise evaluates the model's performance for non-targeted moments. Table 3.3 reports the non-targeted statistics for data, benchmark model, and the model with no inequality. The first column is from the Spanish data, and the second column is from the benchmark simulation. The third column reports the statistics from the model without inequality.

[1] All model statistics are long-run averages of simulating the economy for 10000 periods and discarding the first 500 periods.

Table 3.3: Non-targeted Statistics: Data and Models

Statistics	Data	Benchmark	No inequality
Average external debt/GDP (%)	54	18	18
Std. consumption / std. GDP	1.02	1.00	1.17
Corr. spread & GDP (%)	−83	−41	−40
Corr. trade Balance/GDP & GDP (%)	−67	−9	−1
Corr. trade Balance/GDP & spread (%)	72	30	31

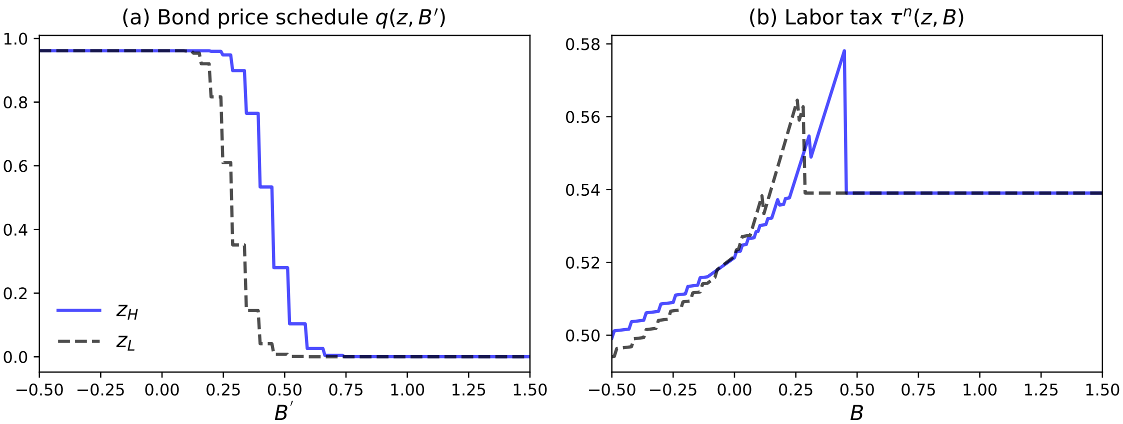
Note: The table describes the targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1980-2017. The second and third columns report the statistics deriving simulations for 10000 periods and excluding the first 500 periods. Column "No inequality" corresponds to the model in which $\theta^H = \theta^L$.

The benchmark model qualitatively match the moments in the data with positive debt, high consumption volatility, negative correlations of spread and trade balance to output, and the positive correlation between trade balance and spread.

3.4.3 Policy Functions

Figure 3.1 plots the bond price schedule $q(z, B^{'})$ and labor tax $\tau^n(z, B)$ across different levels of borrowing for high and low values of aggregate productivity (5% above and below the mean productivity, respectively). For a given level of debt $B^{'}$, higher aggregate productivity implies a better price q since the expected default probability is lower. The optimal labor taxes is increasing in the initial debt level until the government defaults and sets it constant. If the government starts the period with positive assets (B is negative), higher aggregate productivity implies higher labor taxes. If the initial debt is low, lower productivity leads to higher taxes until the government defaults.

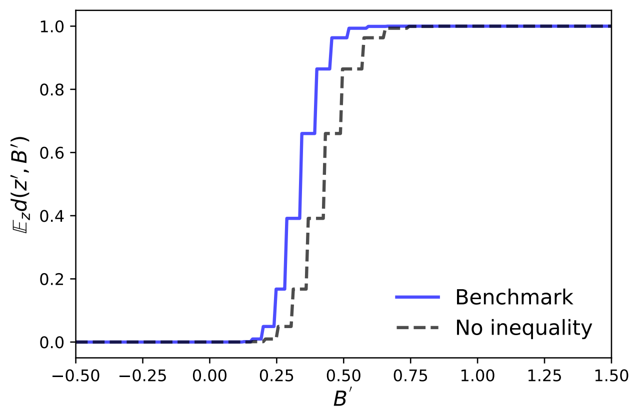
Figure 3.1: Bond price schedule and labor tax



Note: The figure plots the policy functions of the recursive equilibrium for high and low values of aggregate productivity z, which are 5% above and below its mean, respectively. Panel (a) plots the bond price schedule $q(z, \cdot)$ over the values of tomorrow's borrowing B'. Panel (b) plots the labor tax $\tau^n(z, \cdot)$ over the values of today's debt obligation B.

Exposing to inequality increases the government's incentive to default. Figure 3.2 plots the expected default probability at the average aggregate productivity with respect to next period borrowing for the benchmark model and the no-inequality model. For a given level of borrowing, the government is more likely to default when it faces inequality than when it does not. Intuitively, higher the debt is, the higher marginal taxes the government needs to levy in order to repay its debt. Therefore, the benefit of default is to increase the amount of resources transferred to private workers and reduce the marginal labor tax, which in turn lowers the distortion. Absent of inequality, default does not reduce distortion, so the benefit of default is low, and the government is more likely to repay the debt.

Figure 3.2: Expected default probability and debt



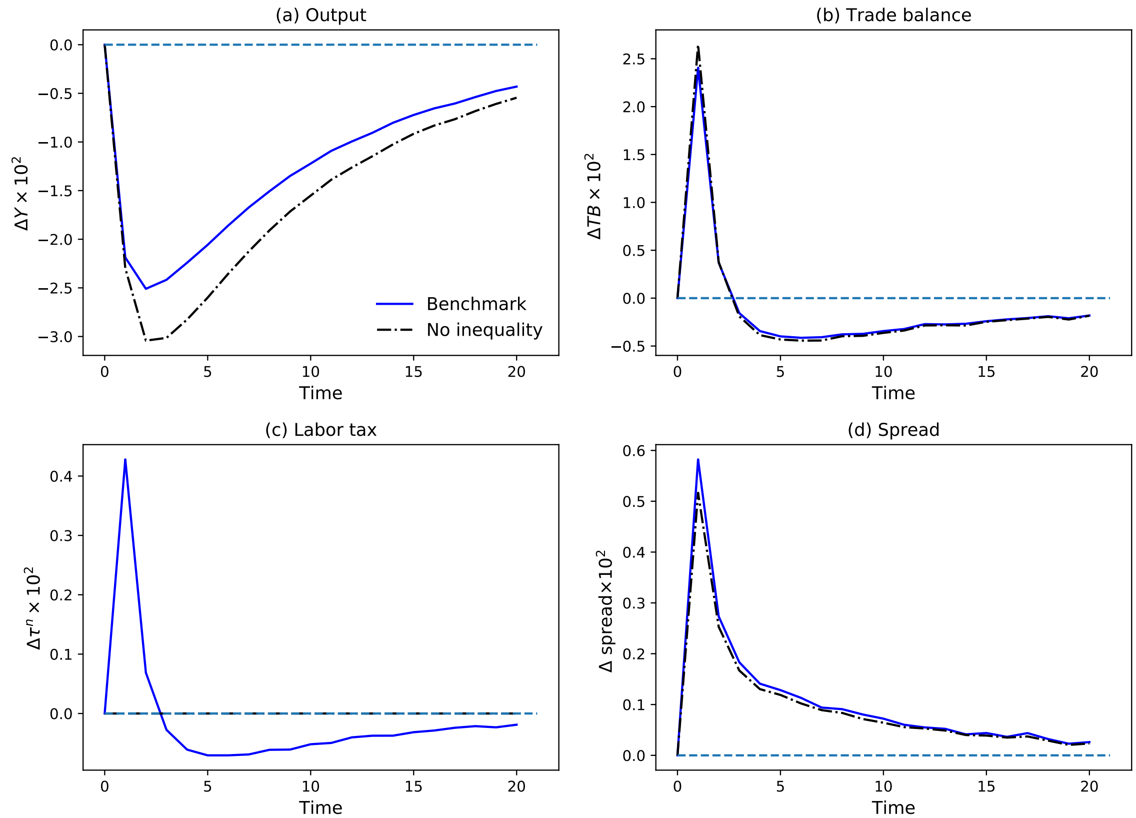
Note: The figure plots the expected default probability at the average aggregate productivity $\bar{z}$ with respect to next period borrowing $B^{'}$. "No inequality" corresponds to the model in which $\theta^H = \theta^L$.

3.4.4 Impulse Response Functions

Figure 3.3 plots the impulse response functions of aggregate variables and tax policies with respect to a one standard deviation decline in productivity growth for both the benchmark and no inequality models. The impulse respond functions are computed via the local projection method.[2] Default risk generates a deep and long decline in output, accompanying with increasing in sovereign spreads and labor taxes. The no-inequality model has a lower drop in output and lower sovereign spreads. Since there is no inequality, there is no need for the government to levy distortionary labor taxes to redistribute, so labor taxes are constant at zero. The government's motive for redistribution mitigates the debt crisis with smaller drop in output and lower sovereign spread at the cost of higher labor taxes.

[2]The approach to calculate impulse respond functions is econometrically equivalent to the approach of Jordà (2005). I simulate the economy for 10000 periods with aggregate productivity shocks and exclude the first 500 periods. I then calculate the realized time series of shocks to productivity, ϵ_t^z. To compute the response of a variable X to the shock ϵ_t^z, I perform the OLS regressions $\Delta \log X_t = \alpha + \beta_k \epsilon_{t+k}^z + \eta_t$ to get the estimated $\hat{\beta}_k$. The horizontal τ IRF is then $IRF_\tau = \sum_{k=0}^{\tau} \hat{\beta}_t$. The effect of one standard deviation shock to ϵ_t^z is the responses $\sigma_z \times IRF_\tau$. Since labor and saving taxes can be zero or negative, the dependent variable in the OLS regressions are ΔX_t instead of $\Delta \log X_t$.

Figure 3.3: Impulse Response Functions to a Negative Productivity Shock

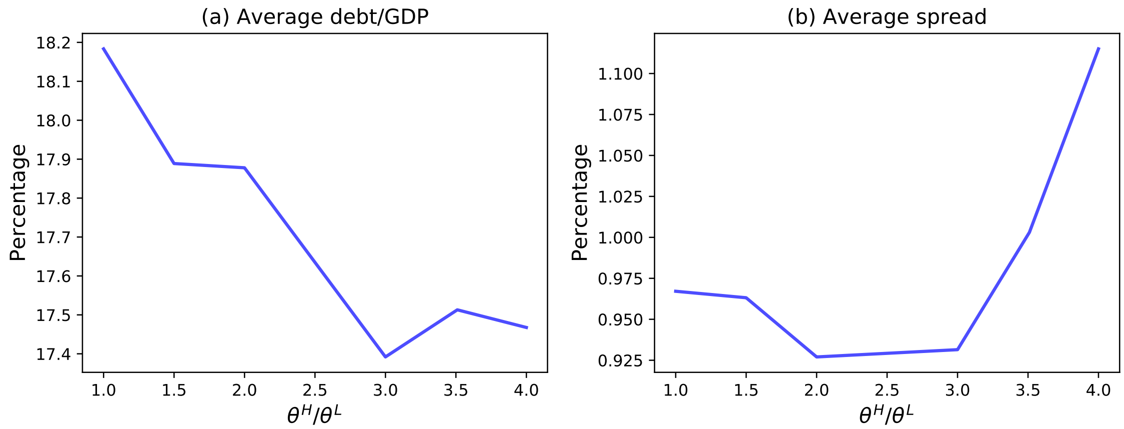


Note: The figure depicts the impulse respond functions (IRFs) of aggregates and policies to a one standard deviation decline in productivity growth. The IRFs are calculated using the local projection method as in Jordà (2005)

3.4.5 Effect of Inequality

Figure 3.4 plots the average debt/GDP and the average spread for economies with different levels of inequality. While the economy with no inequality has higher debt and lower spread than an economy with inequality on average, as income inequality increases, the relationship becomes non-linear.

Figure 3.4: Bond price schedule and labor tax



Note: Panel (a) plots the average debt/GDP from model simulations given the value of θ^H/θ^L. Similarly, panel (b) plots the average spread.

3.5 Conclusion

This chapter studies the impact of income inequality on sovereign default risk. The model is a sovereign default framework embed with heterogeneous agents, endogenous labor supply, and distortionary taxes. I quantify the model with Spanish data and find that inequality makes default more likely. When the government faces distortionary cost of redistribution, default can increase the amount of resources transferred to private workers and lowers the distortion needed to redistribute. Future research incorporates long-term debt and analyzes the interaction between inequality and debt dilution.

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Appendix A

Appendix to Chapter 1

A.1 Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{d_t\}_{t=0}^{\infty}$, where $d_t \in \{0,1\}$ and $d_t = 0$ implies default[1]. The government's budget constraint becomes

$$G_t + (1+r_t)B_t^d + d_t B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + B_{t+1}^d + Q_{t+1}B_{t+1} + T_t$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t, \left(k_t^i, b_t^{i,d}\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^k, T_t, d_t, r_t, B_{t+1}, B_{t+1}^d\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{d,i}\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = (K_t, L_t)$, and the international lenders choose holdings of government's bonds $z_t^* = (B_{t+1})$ given the price Q_{t+1}.

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period

[1] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

t. The government strategy is $\sigma_t^G : H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i}$: $H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$. The prices are determined by the pricing rule: $p : H_p^t \to \mathbb{R}_+$

Definition A.1 (Sustainable equilibrium)**.** *A sustainable equilibrium is* $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ *such that (i) for all* h^{t-1}*, the policy* z_t^G *induced by the government strategy maximizes the socially weighted utility given* λ *subject to the government's budget constraint (1.5) (ii) for all* h_p^t*, the strategy induced policy* $\left\{z_t^G\right\}_{t=0}^{\infty}$*, allocation* $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$*, and prices* $\{Q_t\}_{t=0}^{\infty}$ *constitute a competitive equilibrium with taxes.*

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition A.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t*, there exists* $\underline{U}_t(\cdot)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{1.7}$$

Furthermore, $\underline{U}_t(\cdot)$ *is increasing.*

Proof. Define $\underline{U}_t(K_t)$ as the maximum discounted weighted utility for the agents in period t when the government deviates. In period t, the agents save and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to financial autarky where the agents do not save and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that starts at period t and in which distortionary taxes and savings are only in the initial period. Then it is true that the higher the initial capital stock (in this case K_t), the higher utility that the agents receive. Hence, $\underline{U}_t(\cdot)$ is increasing.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $s > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (1.7). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U_t^d(K_t)$ in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility at K_t, the right-hand side of (1.7) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. □

A.2 Formulas and Proofs

A.2.1 Formulas for separable isoelastic preference

Given the formulas for ψ_c^i and ψ_l^i in (1.12),

$$\begin{aligned}
\Phi_C^V &= \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^\sigma; & \Phi_L^V &= \omega\left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu}\right]^{-\nu} \\
\Phi_C^W &= \Phi_C^V \sum_{i\in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; & \Phi_L^W &= \Phi_L^V \sum_{i\in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right] \\
\Phi_C^P &= \Phi_C^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_c^i; & \Phi_L^P &= \Phi_L^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_l^i
\end{aligned}$$

A.2.2 Proof of Proposition 1.1

Proof. ($\Rightarrow$) Let $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{C_t, L_t, K_t\}_{t=0}^\infty$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ satisfies

$$\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})} \\
1 + r_{t+1} &= \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{\beta V_C\left[h^i(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})\right]}
\end{aligned}$$

Substituting for w_t and r_t into the budget constraint (1.2), and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\left\{w_t, r_t^k\right\}_{t=0}^\infty$ using firm's first-order conditions (1.3). $\{\tau_t^n\}_{t=0}^\infty$ can be calculated using the intratemporal condition (1.8), while one can choose $\{r_t\}_{t=0}^\infty$ that satisfy the intertemporal constraint (1.9). The tax on capital $\left\{\tau_t^k\right\}_{t=0}^\infty$ can be derived from $(1-\tau_t^k)r_t^k = r_t + \delta$. Define $\{q_t\}_{t=0}^\infty$ by (1.4).

Rewriting the aggregate resource constraint using $F(K, L) = wL + rK$ gives

$$\sum_{t=0}^{\infty} q_t \left\{ C_t + K_{t+1} - (1-\tau_t^n) w_t L_t - \left[1 + (1-\tau_t^k) r_t^k - \delta \right] K_t + T_t \right\}$$
$$+ \sum_{t=0}^{\infty} q_t \left[G_t - \tau_t^k r_t K_t - \tau_t^n w_t L_t - T_t \right] \leq -\delta_0 B_0 \qquad \text{(A.1)}$$

Aggregating up the agent's budget constraints implies

$$C_t + K_{t+1} + B_{t+1}^d = (1-\tau_t^n) w_t L_t + \left[1 + (1-\tau_t^k) r_t^k - \delta \right] K_t + (1+r_t) B_t^d - T_t$$

or

$$C_t + K_{t+1} - (1-\tau_t^n) w_t L_t - \left[1 + (1-\tau_t^k) r_t^k - \delta \right] K_t + T_t = (1+r_t) B_t^d - B_{t+1}^d$$

Substituting the last equation into (A.1) gives the government's budget constraint (1.5). Thus, $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. □

A.2.3 Proof of Lemma 1.4

Proof. Given an efficient allocation $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$, suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*$, $\forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \leq 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity,

$$\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{(C_s^*)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
\leq& M_U \\
\leq& \underline{U}_{t_0}(K_{t_0}^*)
\end{aligned}$$

which contradicts the aggregate debt constraint at t_0. □

A.2.4 Proof of Proposition 1.2

Proof. The first statement directly follows from equations (1.19) and (1.20). Let $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$A_C = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{A.2}$$

where ψ_c^i, ψ_l^i are defined by equations (1.12) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma A.1. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0$, $\forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (1.12), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i \in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0$, $\forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i \in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i$, $\pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. □

Define (P^M) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*), \forall t > M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$ is a solution to (P^M), and (P^M) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\left\{r^M, \mu^M, \eta^{M,1}, \ldots, \eta^{M,I}, \gamma_0^M, \ldots, \gamma_M^M\right\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t} \left\{ r^M A_C + \sum_i \pi^i \eta^{M,i} (1 - \sigma) \psi_c^i + \sum_{s=0}^{t} \gamma_s^M A_C \right\} \Phi_C^V C_t^{-\sigma} = \mu^M \tag{A.3}$$

$$\frac{\beta^t}{q_t} \left\{ r^M A_L + \sum_i \pi^i \eta^{M,i} (1 + \nu) \psi_l^i + \sum_{s=0}^{t} \gamma_s^M A_L \right\} \Phi_L^V L_t^{\nu} = \mu^M F_L(K_t, L_t) \tag{A.4}$$

Since the allocation is interior and $A_C, A_L > 0$, one can rewrite the first-order conditions as

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^V C_t^{-\sigma} = \mu^M$$

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i \frac{A_C}{A_L} + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^V C_t^{-\sigma} = \mu^M \frac{A_C}{A_L} F_L(K_t, L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}$$

Subtracting the first from the second line gives

$$\frac{\beta^t}{q_t}\left\{\Phi_C^V \sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\right\} C_t^{-\sigma} = \mu^M \left[\frac{A_C}{A_L} F_L(K_t, L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}} - 1\right] \tag{A.5}$$

The following lemma shows that the resource constraint binds for any sub-problem (P^M) and $M \geq 1$.

Lemma A.2. *In any sub-problem (P^M) with $M \geq 1$, $\mu^M > 0$*

Proof. Suppose, by contradiction, that $\mu^M = 0$ so the resource constraint does not bind. Consider allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ which is the solution to (P^M). Then there exists $\epsilon > 0$ such that

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 - \epsilon \geq 0$$

Define $\left\{\hat{L}_t\right\}_{t=0}^{\infty}$ where $\hat{L}_1 < L_1$ such that $F(K_1, \hat{L}_1) = F(K_1, L_1) - \epsilon/q_1$, and $\hat{L}_t = L_t$, $\forall t > 1$. The allocation $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, \hat{L}_t; \boldsymbol{\varphi})\right] > \sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$$

which contradicts $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ being optimal solution for (P^M). □

By Lemma A.5 and interior allocation, we can rewrite equation (A.5) as

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right] = \frac{q_t}{\beta^t} C_t^{\sigma}\left[\frac{A_C}{A_L} F_L(K_t, L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}} - 1\right]$$

Specifically, for any $M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right]=\frac{q_1}{\beta}\left(C_1^*\right)^{\sigma}\left[\frac{A_C}{A_L}F_L(1)\frac{\Phi_C^V\left(C_1^*\right)^{-\sigma}}{\Phi_L^V\left(L_1^*\right)^{\nu}}-1\right]$$

Note that the left-hand side is a function of (C_1^*, L_1^*, K_1^*) , which implies that there exists a constant κ such that $\forall M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right]=\kappa$$

Hence, (A.5) can be rewritten as

$$\frac{\beta^t}{q_t}C_t^{-\sigma}\kappa=\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}-1$$

Note that $\lim_{t\to\infty}\beta^t/q_t=0$ and $C_t^{-\sigma}$ is bounded by Lemma 1.4, so taking the limit on both sides gives

$$\lim_{t\to\infty}\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}=1$$

Hence, given the definition of τ_t^n and the fact that A_C,A_L are bounded,

$$\lim_{t\to\infty}\tau_t^n=\lim_{t\to\infty}\left[1-\frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}}\frac{1}{F_L(K_t,L_t)}\right]=1-\frac{A_C}{A_L}$$

In addition, the above argument does not rely on the existence of lump-sum transfers.

□

A.2.5 Proof of Proposition 1.3

Proof. Rewrite the optimal labor tax formulas as

$$\tau_t^n=1-\frac{\Phi_L^V\Phi_C^W+\Phi_L^V\Phi_C^P\sum_{s=0}^t\gamma_s}{\Phi_C^V\Phi_L^W+\Phi_C^V\Phi_L^P\sum_{s=0}^t\gamma_s} \tag{A.6}$$

By definitions,

$$\frac{\Phi_L^V\Phi_C^W}{\Phi_C^V\Phi_L^W}=\frac{\sum_i\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i}+(1-\sigma)\eta^i\right]}{\sum_i\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i}+(1+\nu)\eta^i\right]}=\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]+\sigma\text{cov}\left(\psi_c^i,\frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]-\nu\text{cov}\left(\psi_l^i,\frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \operatorname{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \operatorname{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j/\varphi^j - \lambda^i/\varphi^i$, and the definitions $\mathbb{E}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\operatorname{cov}(x^i, y^i) \equiv \mathbb{E}\left[x^i y^i\right] - \mathbb{E}\left[x^i\right]\mathbb{E}\left[y^i\right]$.

Lemma A.3. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. Given that $a_0^i = A_0, \forall i \in I$, the individual implementability constraints can be rewritten as

$$\psi_c^i \Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu} = \Phi_C^V C_0^{-\sigma}\left(A_0 - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu}}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma}\left(A_0 - T\right)}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}}$$

which implies that $\psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

The next step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\operatorname{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\operatorname{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. □

Lemma A.3 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$. Suppose that the debt constraint binds at period t, then $\gamma_t > 0$, which leads to $\sum_{s=0}^{t} \gamma_s > \sum_{s=0}^{t-1} \gamma_s$. Applying equation (A.6) gives $\tau_t^n \leq \tau_{t-1}^n$. □

A.2.6 Proof of Proposition 1.5

Proof. $\lambda^i = \boldsymbol{\varphi}^{*i}$, $\forall i \in I$ implies that $A_C = 1$ and $A_L = 1$. Therefore, $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves

$$\begin{aligned}
\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} \quad & \sum_{t=0}^{\infty} \beta^t V(C_t, L_t; \boldsymbol{\varphi}) \\
s.t. \quad & \sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
& \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
& \sum_{s=t}^{\infty} \beta^{s-t} V(C_t, L_t; \boldsymbol{\varphi}) \geq \underline{U}_t(K_t)
\end{aligned}$$

To implement $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ given the specified tax system, by (1.21), it must be that

$$\lim_{t \to \infty} \tau_t^n = 0$$

□

A.2.7 Proof of Lemma 1.3

Proof. For any competitive equilibrium, there exists market weight $\boldsymbol{\varphi} = \left\{\varphi^i\right\}_{i \in I}$ such that $\forall t$, given C_t, L_t, individual assignment $\left\{c_t^i, l_t^i\right\}_{i \in I}$ solves

$$\begin{aligned}
V(C_t, L_t; \boldsymbol{\varphi}) = \max_{(c^i, l^i)_{i \in I}} & \sum_i \varphi^i \pi^i \left[u(c^i) - v\left(l^i/\theta^i\right)\right] \\
s.t. \quad & \sum_i \pi^i c^i = C_t; \ \sum_i \pi^i l^i = L_t
\end{aligned}$$

Let μ^m and η^m be the Lagrange multipliers on the consumption and labor constraints. The first-order conditions for interior solutions are

$$c_t^i = u_c^{-1}\left(\mu^m/\varphi^i\right) \tag{A.7}$$

$$l_t^i = \theta^i v_l^{-1}\left(\theta^i \eta^m/\varphi^i\right) \tag{A.8}$$

Substituting for c_t^i and l_t^i in the constraints gives

$$\sum_{i\in I}\pi^i u_c^{-1}\left(\mu^m/\varphi^i\right) = C_t$$
$$\sum_{i\in I}\pi^i\theta^i v_l^{-1}\left(\theta^i\eta^m/\varphi^i\right) = L_t$$

These equations imply functions $\mu^m(C_t)$ and $\eta^m(L_t)$. Substituting in (A.7) and (A.8), for all i implies that

$$c_t^i = u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right)$$
$$l_t^i = \theta^i v_l^{-1}\left(\theta^i\eta^m(L_t)/\varphi^i\right)$$

Thus, the time-invariant functions $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ are

$$h^{i,c}(C_t;\boldsymbol{\varphi}) = u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right)$$
$$h^{i,l}(L_t;\boldsymbol{\varphi}) = \theta^i v_l^{-1}\left(\theta^i\eta^m(L_t)/\varphi^i\right)$$

Note that $u_c(\cdot)$ is strictly decreasing, so $u_c^{-1}(\cdot)$ is strictly decreasing. This implies that $\mu^m(\cdot)$ is strictly decreasing. Then $h^{i,c}(\cdot;\boldsymbol{\varphi})$ must be strictly increasing. Similarly, one can argue that $h^{i,l}(\cdot;\boldsymbol{\varphi})$ is also strictly increasing. $\square$

A.2.8 Proof of Lemma 1.4

Proof. Given an efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, suppose that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty}\beta^{s-t}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*$, $\forall s \geq t$. Such ϵ exists since the utility function is unbounded. Furthermore, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then since $u(\cdot)$ and $h^{i,c}(\cdot;\boldsymbol{\varphi})$ are strictly increasing,

$$
\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \sum_{i\in I} \lambda^i \pi^i \left[u\left(h^{i,c}(C_s^*;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \sum_{i\in I} \lambda^i \pi^i \left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \\
\leq& M_U \\
\leq& \underline{U}_t(K_t^*)
\end{aligned}
$$

which is a contradiction. □

A.2.9 Proof of Proposition 1.6

Proof. The proof follows a similar structure of the proof of Proposition 1.2. Let $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exist $\boldsymbol{\lambda}$ such that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). For any interior allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T$ from problem (P), define the followings

$$
A_C(t) = \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} \tag{A.9}
$$

$$
A_L(t) = \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} \tag{A.10}
$$

Then the following lemma holds.

Lemma A.4. *Given an interior allocation, for all t, $0 < \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$, and so* $0 < A_C(t),\ A_L(t) < \infty$

Proof. First, it must be that $\varphi^i > 0$, $\forall i$. Suppose there exists an i such that $\varphi^i = 0$. Then from the static sub-problem, it is optimal to set $c_t^i = 0$ for all t, which contradicts the assumption of interior allocation.

Note that from the proof of Lemma 1.3, using implicit function derivatives, one has

$$
\begin{aligned}
\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} &= \frac{\frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}}{\sum_i \pi^i \frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}} \\
\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} &= \frac{\frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}{\sum_i \pi^i \frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}
\end{aligned}
$$

Given $u_{cc}(\cdot) < 0$, $v_{ll}(\cdot) > 0$ by assumption 1.4, and $\varphi^i > 0$, $\forall i$, it must be that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} > 0$. Moreover, $\sum_{i\in I} \pi^i \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} = \sum_{i\in I} \pi^i \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} = 1 < \infty$ implies that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$.

Since all the terms are positive and bounded, by definition, $A_C(t)$ and $A_L(t)$ are positive and bounded. $\square$

Define (P^T) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*)\,, \forall t > T$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$ is a solution to (P^T), and (P^T) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\left\{r^T, \mu^T, \eta^{T,1}, \ldots, \eta^{T,I}, \gamma_0^T, \ldots, \gamma_T^T\right\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} h^{i,c}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_C(t)\right\} * V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{A.11}$$

$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} h^{i,l}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_L(t)\right\} * V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{A.12}$$

Using the Envelope conditions of the static sub-problem, one can show that

$$\begin{aligned}
\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} h^{i,c}(t;\boldsymbol{\varphi}) &= \frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]} h^{i,c}(t;\boldsymbol{\varphi}) \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} \\
\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} h^{i,l}(t;\boldsymbol{\varphi}) &= \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]} h^{i,l}(t;\boldsymbol{\varphi}) \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}
\end{aligned}$$

Define $\sigma_t^i = -\frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]} h^{i,c}(t;\boldsymbol{\varphi})$ and $\nu_t^i = \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]} h^{i,l}(t;\boldsymbol{\varphi})$, then equations (A.11) and (A.12) become

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i \eta^{T,i}(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} + \sum_{s=0}^{t} \gamma_s^T A_C(t)\right\} V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{A.13}$$

$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i \eta^{T,i}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} + \sum_{s=0}^{t} \gamma_s^T A_L(t)\right\} V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{A.14}$$

Since the allocation is interior and $A_C(t),\ A_L(t) > 0$ by Lemma A.4, one can combine equations (A.13) and (A.14) to get

$$\frac{\beta^t}{q_t}\left\{\sum_i \pi^i\eta^{T,i}(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)}\sum_i \pi^i\eta^{T,i}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right\}V_C(t;\boldsymbol{\varphi}) = \mu^T\left[1+F_L(K_t,L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right] \tag{A.15}$$

Lemma A.5. *In any subproblem (P^T) with $T \geq 1$, $\mu^T > 0$, i.e. the resource constraint binds.*

Proof. Follows directly from the proof of Lemma A.5. □

Given Lemma A.5 and interior allocation, (A.15) becomes

$$\frac{1}{\mu^T}\left\{\sum_i \pi^i\eta^{T,i}\left[(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right]\right\}V_C(t;\boldsymbol{\varphi}) = \frac{q_t}{\beta^t}\frac{1}{V_C(t;\boldsymbol{\varphi})}\left[1+F_L(K_t,L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]$$

Define the left-hand side of the above equation as $\kappa(t)$, then the following lemma gives an important property of $\kappa(t)$.

Lemma A.6. *For any sub-problem (P^T) with $T \geq 1$,$\kappa(t)$ is bounded $\forall t \geq 1$.*

Proof. Note that $\forall t, \forall i$, by assumption 1.5, σ_t^i and ν_t^i are bounded.

Any sub-problem (P^T) with $T \geq 1$ has

$$\sum_i \frac{\eta^{T,i}}{\mu^T}\pi^i\left[(1-\sigma_s^i)\frac{\partial h^{*i,c}(s;\boldsymbol{\varphi})}{\partial C_s^*} - \frac{A_C^*(s)}{A_L^*(s)}(1+\nu_1^i)\frac{\partial h^{*i,l}(s;\boldsymbol{\varphi})}{\partial L_s^*}\right] = \frac{q_s}{\beta^s}\frac{1}{V_C^*(s;\boldsymbol{\varphi})}\left[1+F_L^*(s)\frac{V_C^*(s;\boldsymbol{\varphi})}{V_L^*(s;\boldsymbol{\varphi})}\frac{A_C^*(s)}{A_L^*(s)}\right]$$

for $s = 1, ..., \|I\|$.

The above equations formulate a linear system with respect to $\|I\|$ variables $\left\{\frac{\eta^{T,i}}{\mu^T}\right\}_{i\in I}$. By Lemma A.4 and interior allocation, the right-hand sides and the coefficients are bounded. Therefore, for any T, $\left\{\frac{\eta^{T,i}}{\mu^T}\right\}_{i\in I}$ are functions of $\{C_s^*, L_s^*, K_s^*\}_{s=0}^{\|I\|}$, $\boldsymbol{\varphi}^*$and bounded.

So $\forall t \geq 1$,

$$\kappa(t) = \sum_i \frac{\eta^{T,i}}{\mu^T}\pi^i\left[(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)}(1+\nu_1^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right]$$

is bounded. □

Substituting for $\kappa(t)$ into equation (A.15) provides

$$\frac{\beta^t}{q_t}\kappa(t)V_C(t;\boldsymbol{\varphi}) = \left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]$$

Assumption 1.1 implies that $\lim_{t\to\infty}\beta^t/q_t = 0$. By Lemma A.6, $\kappa(t)$ is bounded. Since $\liminf_{t\to\infty} C_t > 0$ from Lemma 1.4, $V_C(t;\boldsymbol{\varphi}) = \varphi^i u_c(h^{i,c}(t;\boldsymbol{\varphi}))$ is bounded. Then taking the limit as $t \to \infty$ on both sides of the above equation gives

$$\lim_{t\to\infty}\left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right] = 0$$

From Lemma A.4, it must be true that as t approaches infinity, we have that $0 < A_C(t), A_L(t) < \infty$, so $-\infty < \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)}$, $\limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)} < \infty$. Define $\underline{\tau} = 1 - \limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)}$ and $\bar{\tau} = 1 - \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)}$. Then using the definition of τ_t^n gives

$$\begin{aligned}
\liminf_{t\to\infty}\tau_t^n &= \liminf_{t\to\infty}\left[1 + \frac{1}{F_L(K_t, L_t)}\frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})}\right] = 1 - \limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)} = \underline{\tau} \\
\limsup_{t\to\infty}\tau_t^n &= \limsup_{t\to\infty}\left[1 + \frac{1}{F_L(K_t, L_t)}\frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})}\right] = 1 - \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)} = \bar{\tau}
\end{aligned}$$

In the case of steady states, it must be true that $A_C(\infty), A_L(\infty)$ exist and that $0 < A_C(\infty), A_L(\infty) < \infty$. Hence, $\lim_{t\to\infty}\tau_t^n = 1 - \frac{A_C(\infty)}{A_L(\infty)}$.

Similarly, the argument of the proof does not rely on lump-sum transfers. □

A.2.10 Proof of Lemma 1.2

Proof. Note that the sustainability constraint is rewritten as $\forall t$,

$$\sum_{s=t}^{\infty}\beta^{s-t}\left\{\Phi_C^P\frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L_s^{1+\nu}}{1+\nu}\right\} \geq \underline{U}$$

Define $u_t = \Phi_C^P\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L_t^{1+\nu}}{1+\nu}$. Then the proof is similar to Lemma 2 in Aguiar and Amador (2016). □

A.3 Numerical Appendix

This section explains the numerical algorithm that is implemented in Section 1.7 for a simple environment with no capital, and additional plots.

A.3.1 Deviation Utility

The deviation utility $\underline{U}$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where the government does not issue external debt. Given that output is equal to the total effective labor supply, one has

$$\begin{aligned}
\underline{U} \equiv & \max_{c_t^i, l_t^i, \tau_t^n, T_t} \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\
s.t. \quad & c_t^i + b_{t+1}^{i,d} = (1-\tau_t^n) l_t^i - T_t + (1+r_t) b_t^{i,d} \\
& G_t + (1+r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d
\end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ such that

$$\begin{aligned}
\underline{U} \equiv & \max_{C_t, L_t, \hat{\boldsymbol{\varphi}}, T} \sum_{t=0}^{\infty} \beta^t \left[\hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\
s.t. \quad & C_t + G_t \leq L_t
\end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$.

A.3.2 Algorithm

State variables: μ, Γ

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess of the expectation $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right]$.

   (b) Assume the constraint does not bind in t: $\gamma_t = 0$. Solve for the allocation C_t, L_t using FOCs

$$\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] C_t{}^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] L_t{}^{\nu} &= \mu
\end{aligned}$$

   (c) Compute $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$\begin{aligned}
A_t &= \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right] \\
&= \Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t)
\end{aligned}$$

(d) Check if $A_t \geq \underline{U}_t$. If it is, proceed to the next step. If not, solve for C_t, L_t, γ_t using these optimality equations

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] {C_t}^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] {L_t}^{\nu} = \mu$$
$$\Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t)) = \underline{U}_t$$

(e) Given C_t, L_t, γ_t (γ_t can be zero or not), compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find μ and φ

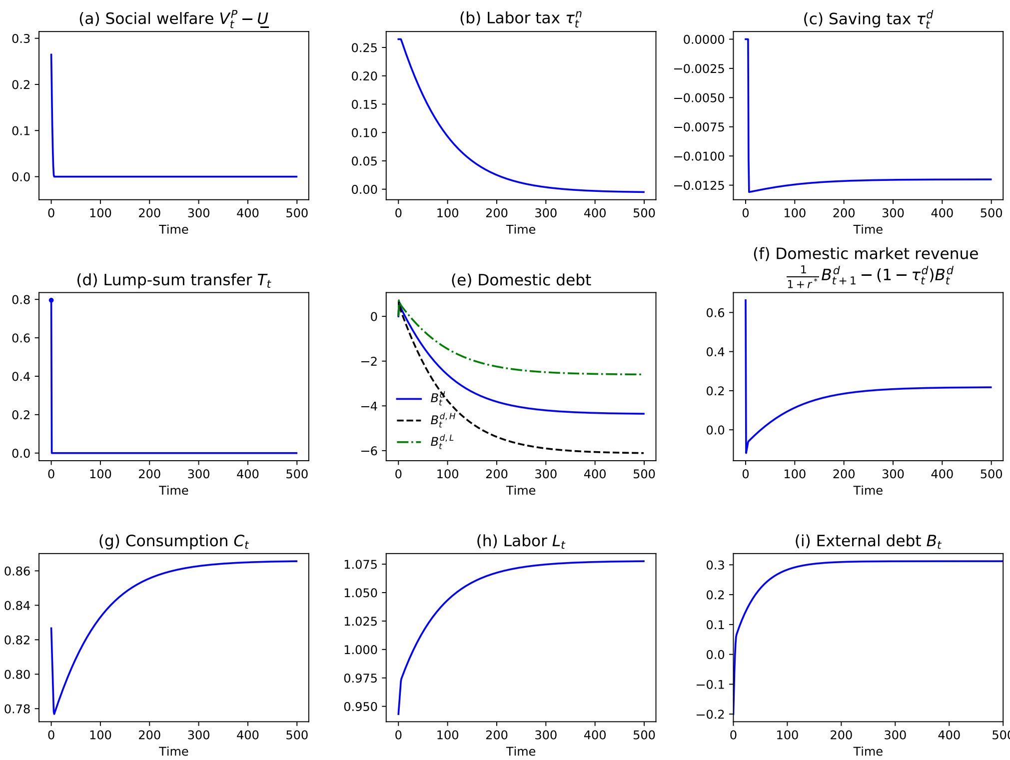
$$r^{\mu} = \sum_{t=0}^{\infty} q_t \left[L_t - G_t - C_t\right] - B_0$$
$$r_{ij}^{\varphi} = \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) {C_t}^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) {L_t}^{1+\nu}\right]$$

$$r = (r^{\mu})^2 + \sum_{i,j} (r_{ij}^{\varphi})^2 \tag{A.16}$$

3. Find μ and $\boldsymbol{\varphi}$ such that (A.16) is minimized using a Nelder-Mead algorithm.

A.3.3 Additional figures

Figure A.0: Time paths of economic aggregates when $\theta^H = 2\theta^L$



Appendix B

Appendix to Chapter 2

B.1 Empirical Appendix

B.1.1 Figures

This subsection presents analogs of Figure 2.1 and Figure 2.2 for a sub-sample that includes all advanced and emerging market economies.

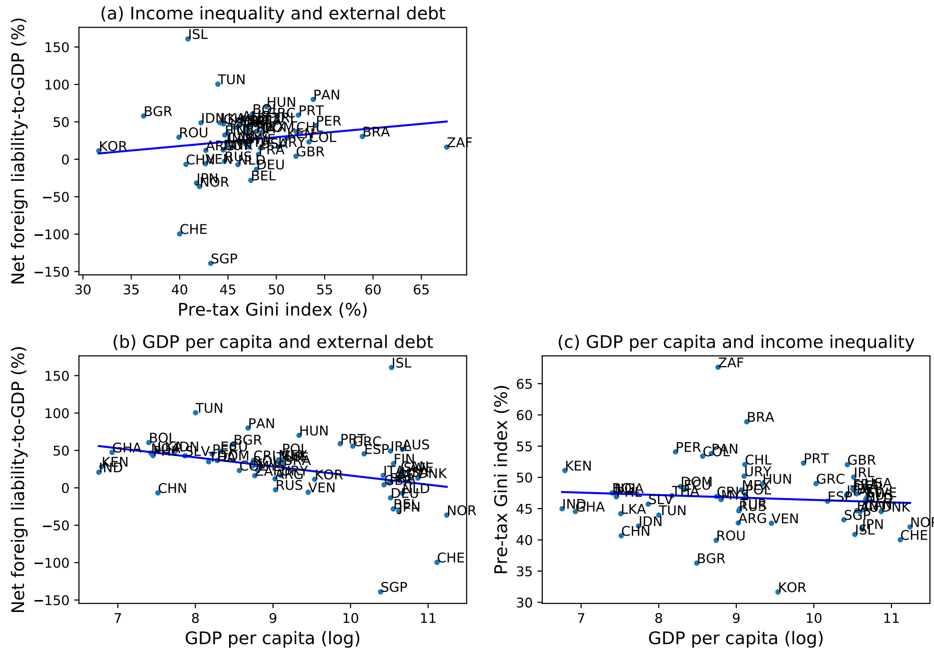
Figure B.0 shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars across advanced and emerging market economies. The pre-tax Gini index is positively correlated with the net foreign liability-to-GDP. The GDP per capita negatively associates with the net foreign liability-to-GDP, while it does not have a strong correlation with the pre-tax Gini index.

Figure B.0 shows the cross-country relationship between income inequality and external debt controlling for other common factors, for all countries in the sample. Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (2.1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2.2). Both panels show a positive correlation between the two main variables in the cross section.

B.1.2 Net International Investment Position

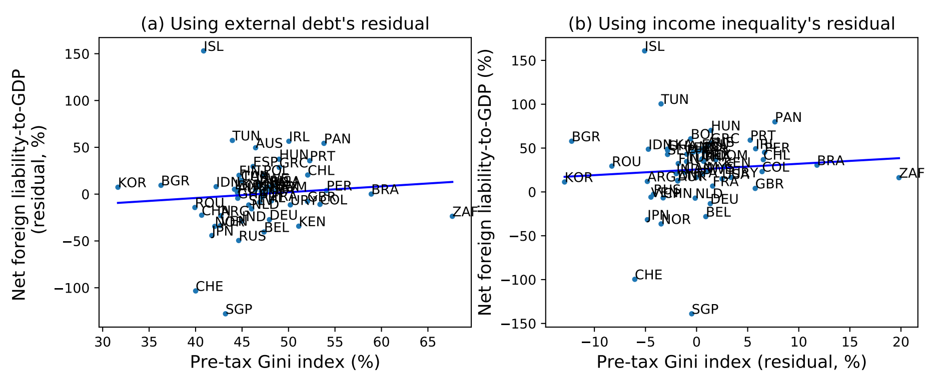
This subsection provides the estimation using negative net international investment position an alternative definition of a country's external indebtedness. Table B.1 reports the regression results. High income inequality levels are correlated with high external debt positions, though the coefficients are not statistically significant. Fewer countries and observations lead to differences in the results between this estimation and the ones

Figure B.0: Income inequality, external debt, and GDP per capita across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for advanced and emerging market economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: Lane and Milesi-Ferretti, 2018, Solt, 2019, and The World Bank, 2019.

Figure B.0: Cross-country relationship between income inequality and external debt



Note: Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (2.1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2.2). The sample includes all advanced and emerging market economies in the dataset. Sources: Lane and Milesi-Ferretti, 2018, Solt, 2019, and The World Bank, 2019.

presented in the main text.

Table B.1: Regression analysis of income inequality and external debt

	Dependent Variable: Net international liability position-to-GDP (%) Time periods: 1985-2015 (1)	(2)
Gini index, pre tax (%)	0.2681	0.1755
	(0.7528)	(0.7707)
GDP per capita (log)		20.793***
		(7.5818)
GDP growth (%)		-1.0290**
		(0.4211)
Inflation (%)		0.0787**
		(0.0386)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No. Countries	137	137
No. Observations	2028	2028

Note: The table describes the panel regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net international liability position-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti, 2018, Solt, 2019, and The World Bank, 2019.

B.2 Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default.[1] The government's budget constraint becomes

$$\begin{aligned}&G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t)\\ \leq&\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t)\end{aligned}$$

[1]Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

The price of international debt takes into account the probability of default is

$$Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1+r^*}$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period and history, the state variable for the game is $\left\{B(s^t), \left(b^{i,d}(s^t)\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t)\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t)\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i}\right)_{i\in I}, z_{t-1}^F, z_t^*, s_t\right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^t \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** *A sustainable equilibrium is* $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ *such that (i) for all* h^t*, the policy* z_t^G *induced by the government strategy maximizes the socially weighted utility given* λ *subject to the government's budget constraint (2.7) (ii) for all* h_p^t*, the strategy induced policy* $\left\{z_t^G\right\}_{t=0}^\infty$*, allocation* $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^\infty$*, and prices* $\{Q_t,\}_{t=0}^\infty$ *constitute a competitive equilibrium with taxes.*

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G, p \right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t *and any* s^t*, there exists* $\underline{U}(s^t,t)$ *such that* $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ *satisfies the constraint*

$$\sum_{i\in I} \lambda^i \pi^i \sum_{k\geq t, s^t\subseteq s^k} \beta^{k-t} \Pr(s^k|s^t) U^i(c^i(s^k), l^i(s^k)) \geq \underline{U}(s^t,t) \tag{2.13}$$

Proof. Define $\underline{U}(s^t,t)$ as the maximum discounted weighted utility for the agents in period t, history s^t, when the government deviates. At period t and history s^t, the government taxes all domestic wealth ($\tau^d(s^t) = 1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k > t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t, distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (2.13). So $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ satisfies condition (ii).

Next, suppose $\left\{ \left(z^{H,i}\right)_{i\in I}, z^F, z^G \right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t. Since

$\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U^d(s_t,t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k>t$. Because the plan is constructed to maximize the utility in period t, the right-hand side of (2.13) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in Chari and Kehoe, 1990.

B.3 Model Analysis for Separable Isoelastic Preference

This section provides details on the characterization of the equilibrium, efficient allocation, and optimal policies given that the individual utility is

$$U^i\left(c,l\right)=\frac{c^{1-\sigma}}{1-\sigma}-\omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}$$

Section B.4 will use this analysis to prove the propositions in the main text.[2]

B.3.1 Equilibrium Characterization

Individual consumption and efficient labor supply are time- and history-independently proportional to the aggregates:

$$\begin{aligned} c^i(s^t)&=h^{i,c}(C(s^t),L(s^t);\boldsymbol{\varphi})=\psi_c^iC(s^t) \\ l^i(s^t)&=h^{i,l}(C(s^t),L(s^t);\boldsymbol{\varphi})=\psi_l^iL(s^t) \end{aligned} \tag{B.1}$$

where

$$\psi_c^i=\frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}},\quad \psi_l^i=\frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i\left(\theta^i\right)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}} \tag{B.2}$$

B.3.2 Planning Problem

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\Pr(s^t)\gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta}=\left(\eta^i\right)_{i\in I}$ and rewrite the Larangian of the

[2]This analysis is an extension of Tran Xuan, 2019 to a model with aggregate uncertainty.

planning problem with a new pseudo-utility function that incorporates the implementability constraints:

$$\sum_{t=0}^{\infty} \beta^t W\left[s^t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(s_0; \boldsymbol{\varphi}) \sum_{i \in I} \pi^i \eta^i \left(b^i(s^0) - T\right)$$

where

$$W\left[s^t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] \equiv \sum_{i \in I} \lambda^i \pi^i U^i\left[h^i(s^t; \boldsymbol{\varphi})\right] + \sum_{i \in I} \pi^i \eta^i \left[V_C(s^t; \boldsymbol{\varphi}) h^{i,c}(s^t; \boldsymbol{\varphi}) + V_L(s^t; \boldsymbol{\varphi}) h^{i,l}(s^t; \boldsymbol{\varphi})\right]$$

Then V and W inherit the separable and isoelastic properties from U, i.e. $\forall t, \forall s^t$,

$$\begin{aligned} V(C(s^t), L(s^t); \boldsymbol{\varphi}) &= \Phi_C^V \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L(s^t)^{1+\nu}}{1+\nu} \\ W\left[C(s^t), L(s^t); \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] &= \Phi_C^W \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L(s^t)^{1+\nu}}{1+\nu} \end{aligned}$$

and the social welfare is

$$\sum_{t \geq 0, s^t \in S^t} \beta^t \Pr(s^t) \left(\Phi_C^P \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^t)^{1+\nu}}{1+\nu} \right)$$

where

$$\begin{aligned} \Phi_C^V &= \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^{\sigma}; & \Phi_L^V &= \omega \left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu}\right]^{-\nu} \\ \Phi_C^W &= \Phi_C^V \sum_{i \in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; & \Phi_L^W &= \Phi_L^V \sum_{i \in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right] \\ \Phi_C^P &= \Phi_C^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_c^i; & \Phi_L^P &= \Phi_L^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_l^i \end{aligned}$$

B.3.3 Optimal Taxation

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(s^t, t) = \frac{\left\{\Phi_L^W + \Phi_L^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)\right\} L(s^t)^{\nu}}{\left\{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)\right\} C(s^t)^{-\sigma}} \tag{B.3}$$

and

$$Q(s_{t+1}|s^t) = \beta \Pr(s^{t+1}|s^t) \frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}} \frac{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^{t+1}}^{t+1} \gamma(s^k)}{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)} \tag{B.4}$$

The optimal taxation follows

$$\tau^n(s^t) = 1 - \frac{1}{F_L(s^t,t)} \frac{\Phi_L^V L(s^t)^\nu}{\Phi_C^V C(s^t)^{-\sigma}} \tag{B.5}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta \Pr(s^{t+1}|s^t) \frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}} \tag{B.6}$$

B.4 Proofs

B.4.1 Proof of Proposition 2.1

Proof. ($\Rightarrow$) Let $\left\{C(s^t), L(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\left\{C(s^t), L(s^t)\right\}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\left\{C(s^t), L(s^t)\right\}$ satisfies

$$(1-\tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta \Pr(s^{t+1}|s^t) \frac{V_C\left[h^i(C(s^{t+1}), L(s^{t+1}); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}$$

Substituting for $w(s^t)$ into the budget constraint (2.5), and using $(c^i(s^t), l^i(s^t)) = h^i(C(s^t), L(s^t); \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\left\{C(s^t), L(s^t)\right\}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\left\{w(s^t)\right\}$ using the firm's first-order condition (2.6). $\left\{\tau^n(s^t)\right\}$ can be calculated using the intra-temporal condition (2.9), and choosing $\left\{Q^d(s^t)\right\}$ to satisfy the inter-temporal constraint (2.10). Define $\left\{q(s^t)\right\}$ by $q(s^t) = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$\begin{aligned}
&\sum_{t\geq 0, s^t\in S^t} q(s^t)\left\{C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t)\right\} \\
&+ \sum_{t\geq 0, s^t\in S^t} q(s^t)\left[G(s^t,t) - \tau^n(s^t)w(s^t)L(s^t) - T(s^t)\right] \leq -B(s^0)
\end{aligned} \tag{B.7}$$

Aggregating up the agent's budget constraints implies

$$C(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) = (1-\tau^n(s^t))w(s^t)L(s^t) + (1-\tau^d(s^t))B^d(s^t) - T(s^t)$$

or

$$C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t) = (1-\tau^d(s^t))B^d(s^t) - \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1})$$

Substituting the last equation into (B.7) gives the government's budget constraint (2.7). Thus, $\{C(s^t), L(s^t)\}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. □

B.4.2 Proof of Proposition 2.2

Proof. Given equations (B.3) and (B.5), the optimal labor tax is

$$\tau^n(s^t) = 1 - \frac{\Phi_L^V\Phi_C^W + \Phi_L^V\Phi_C^P \sum_{k=0,s^k\subseteq s^t}^{t}\gamma(s^k)}{\Phi_C^V\Phi_L^W + \Phi_C^V\Phi_L^P \sum_{k=0,s^k\subseteq s^t}^{t}\gamma(s^k)} \tag{B.8}$$

Given equations (B.4) and (B.6), the optimal saving return is[3]

$$1 + r^d(s^t) = \frac{\Phi_C^W + \Phi_C^P \sum_{k=0,s^k\subseteq s^t}^{t}\gamma(s^k)}{\Phi_C^W + \Phi_C^P \sum_{k=0,s^k\subseteq s^{t-1}}^{t-1}\gamma(s^k)}(1+r^*) \tag{B.9}$$

Suppose that the borrowing constraint does not bind at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) = 0$, which implies $\tau^n(s^{\mathcal{T}}) = \tau^n(s^{\mathcal{T}-1})$, and $r^d(s^t) = r^*$

To prove the second part of the proposition, I first show that $\frac{\Phi_L^V\Phi_C^W}{\Phi_C^V\Phi_L^W} \leq \frac{\Phi_L^V\Phi_C^P}{\Phi_C^V\Phi_L^P}$. By definitions,

$$\frac{\Phi_L^V\Phi_C^W}{\Phi_C^V\Phi_L^W} = \frac{\sum_i \pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma\mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu\mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V\Phi_C^P}{\Phi_C^V\Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

[3] There is an indeterminacy between Q^d and τ^d. Here I assume a particular implementation where $Q^d(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)}{1+r^*} = Q(s_{t+1}|s^t)$, that is the price of the domestic debt is the same as the price of the external debt. The government uses τ^d to manipulate the domestic stochastic discount factor.

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j/\varphi^j - \lambda^i/\varphi^i$, and the definitions $\mathbb{E}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\mathrm{cov}(x^i, y^i) \equiv \mathbb{E}\left[x^i y^i\right] - \mathbb{E}\left[x^i\right]\mathbb{E}\left[y^i\right]$.

Lemma B.1. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. The first step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma}\left(a^i(s_0) - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma}\left(a^i(s_0) - T\right)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}$$

By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j$, and by assumption, $\theta^i \geq \theta^j \iff a^i(s_0) \geq a^j(s_0)$, which implies that $\theta^i \geq \theta^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. □

Lemma B.1 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$.

Suppose that the borrowing constraint binds at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) > 0$, which leads to $\sum_{\tau=0,s^\tau}^{\mathcal{T}} \gamma(s^\tau) > \sum_{\tau=0,s^\tau}^{\mathcal{T}-1} \gamma(s^\tau)$. Applying equation (B.8) gives $\tau^n(s^{\mathcal{T}}) \leq \tau^n(s^{\mathcal{T}-1})$. In addition, equation (B.9) implies that $r^d(s^{\mathcal{T}}) > r^*$. □

B.4.3 Proof of Proposition 2.3

Proof. Let $\left\{C^*(s^t), L^*(s^t)\right\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\left\{C^*(s^t), L^*(s^t)\right\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$A_C = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{B.10}$$

where ψ_c^i, ψ_l^i are defined by equations (B.2) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma B.2. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0$, $\forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (B.2), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0$, $\forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i$, $\pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. □

For any M and s^M, define (P^{s^M}) the same problem as (P) with the restriction that $\left(C(s^t), L(s^t)\right) = \left(C^*(s^t), L^*(s^t)\right), \forall t > M,\ s^t \supset s^M,\ \boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, and $T = T^*, \forall t$. Note that $\left\{C^*(s^t), L^*(s^t)\right\}$ is a solution to (P^{s^M}), and (P^{s^M}) has a finite number of constraints. By a Lagrangian theorem in Luenberger, 1969, there exists non-negative, not-identically zero vector $\left\{r^{s^M}, \mu^{s^M}, \eta^{s^M,1}, \ldots, \eta^{s^M,I}, \gamma^{s^M}(s^0), \ldots, \gamma^{s^M}(s^M)\right\}$ such that the first-order and complementarity conditions hold for $t \in \{1, .., M\}, s^t \subseteq s^M$, i.e.

$$(\beta R^*)^t \left\{ r^{s^M} A_C + \sum_i \pi^i \eta^{s^M,i} (1-\sigma) \psi_c^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_C \right\} \Phi_C^V C(s^t)^{-\sigma} = \mu^{s^M} \tag{B.11}$$

$$(\beta R^*)^t \left\{ r^{s^M} A_L + \sum_i \pi^i \eta^{s^M,i} (1+\nu) \psi_l^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_L \right\} \Phi_L^V L(s^t)^{\nu} = \mu^{s^M} F_L(s^t) \tag{B.12}$$

Equation (B.11) can be rewritten as

$$(\beta R^*)^t \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) = \frac{\mu^{s^M}}{A_C \Phi_C^V C(s^t)^{-\sigma}} - (\beta R^*)^t \left[r^{s^M} + \frac{1}{A_C} \sum_i \pi^i \eta^{s^M,i} (1-\sigma) \psi_c^i \right] \tag{B.13}$$

The following lemma shows that μ^{s^M} and $C(s^t)^{-\sigma}$ are always positive for the sub-problem (P^{s^M}) for any $M \geq 1$ and any s^M.

Lemma B.3. *In the sub-problem (P^{s^M}) for any $M \geq 1$ and s^M, $\mu^{s^M} > 0$*

Proof. Suppose, by contradiction, that $\mu^{s^M} = 0$ so the resource constraint does not bind. Consider allocation $\{C(s^t), L(s^t), K(s^t)\}$ which is the solution to (P^{s^M}). Then there exists $\epsilon > 0$ such that

$$\sum_{t \geq 0, s^t} q(s^t) \left[F(L(s^t), s^t, t) - G(s^t, t) - C(s^t) \right] - B(s^0) - \epsilon \geq 0$$

Define $\left\{\hat{L}(s^t)\right\}$ such that for a fixed s^1, $\hat{L}(s^1) < L(s^1)$ such that $F(\hat{L}(s^1), s^1, 1) = F(L(s^1), s^1, 1) - \epsilon / q(s^1)$, and $\hat{L}(s^t) = L(s^t)$, $\forall t > 1, \forall s^t$. The allocation $\left\{C(s^t), \hat{L}(s^t)\right\}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C(s^t), \hat{L}(s^t)\right\}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{i \in I, t \geq 0, s^t} \lambda^i \pi^i \beta^t \Pr(s^t) U^i \left[h^i(C(s^t), \hat{L}(s^t); \varphi) \right] > \sum_{i \in I, t \geq 0, s^t} \lambda^i \pi^i \beta^t \Pr(s^t) U^i \left[h^i(C(s^t), L(s^t); \varphi) \right]$$

which contradicts $\left\{ \left(C(s^t), L(s^t) \right)_{s^t} \right\}_{t=0}^{\infty}$ being optimal solution for (P^{s^M}). □

The consumption path is bounded below by zero in the long run, i.e.

Lemma B.4 (No immiseration)**.** *Suppose Assumptions 2.1 and 2.4 hold, then for any efficient allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}$, $\liminf_{t \to \infty} C_t^* > 0$.*

Proof. Given an efficient allocation $\{C^*(s^t), L^*(s^t)\}$, suppose, by contradiction that for a sequence of shocks $\{s_0, ..., s_t, ...\}$, $\liminf_{t \to \infty} C^*(s^t) \leq 0$. Find $\epsilon > 0$ such that $\forall t, \forall s^t$,

$$\sum_{k=t}^{\infty} \beta^{\tau - t} \sum_{s^t \subseteq s^k} Pr(s^\tau) \left[\Phi_C^V \frac{C(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu} \right] \leq M_U$$

with $C(s^t) = \epsilon$ and $C(s^k) = C^*(s^k),\ \forall k > t,\ s^t \subset s^k$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C^*(s^t) \le 0$, there exists a t_0 such that $C^*(s^{t_0}) < \epsilon$. Then by monotonicity,

$$
\begin{aligned}
&\sum_{k=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0}\subseteq s^k} \Pr(s^k)\left[\Phi_C^V \frac{C^*(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu}\right] \\
<&\sum_{k=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0}\subseteq k} \Pr(s^k)\left[\Phi_C^V \frac{C(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu}\right] \\
\le& M_U \\
\le& \underline{U}(s^{t_0}, t_0)
\end{aligned}
$$

which contradicts the aggregate debt constraint at s^{t_0}. □

Taking the limit on both sides of equation (B.13) gives

$$
\lim_{t\to\infty} (\beta R^*)^t \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) = \lim_{t\to\infty} \frac{\mu^{s^M}}{A_C \Phi_C^V C(s^t)^{-\sigma}} > 0
$$

□

B.5 Quantitative Appendix

This section provides additional details that is implemented in Section 2.5 .

B.5.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$
\underline{U}(z) \equiv \max_{c^i(s^t), l^i(s^t), \tau^n(s^t), T(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i \mathbb{E}_z U^i(c^i(s^t), l^i(s^t))
$$

$$
\begin{aligned}
s.t. \quad & C(s^t) + G = z(s^t)L(s^t) \\
& c^i(s^t) + \sum_{s_{t+1}} Q(s_{t+1}|s^t) b^{d,i}(s^{t+1}) = (1-\tau^n(s^t))z(s^t)l^i(s^t) + b^{d,i}(s^t) - T(s^t) \\
& (1-\tau^n(s^t))z(s^t) = -\frac{U_l^i(c^i(s^t), l^i(s^t)}{U_c^i(c^i(s^t), l^i(s^t)} \\
& Q(s_{t+1}|s^t) = \beta \Pr(s^{t+1}|s^t) \frac{U_c^i(c^i(s^{t+1}), l^i(s^{t+1}))}{U_c^i(c^i(s^t), l^i(s^t))} \\
& b^{d,i}(s^0) = b^{d,j}(s^0) \\
& z(s^0) = z
\end{aligned}
$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ that satisfies the conditions in Proposition 2.1 such that

$$
\begin{aligned}
\underline{U}(z) \equiv \max_{C(s^t), L(s^t), \boldsymbol{\varphi}, T} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_z & \left[\hat{\Phi}_C^W \log C(s^t) - \hat{\Phi}_L^W \frac{L(s^t)^{1+\nu}}{1+\nu} \right] \\
s.t. \quad & C(s^t) + G = z(s^t)L(s^t) \\
& z(s^0) = z
\end{aligned}
$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$ (see Appendix B.3 for the formulas).

B.5.2 Computational Algorithm

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ. Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j \geq 0, s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu} \right]$

   (b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$
\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] C(s_t)^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] L(s_t)^{\nu} &= \mu F_L(s_t)
\end{aligned}
$$

   (c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$
\begin{aligned}
A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu} \right] \\
&= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\
&+ \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})
\end{aligned}
$$

(d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$
\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] C(s_t)^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] L(s_t)^{\nu} &= \mu F_L(s_t) \\
\Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} & \\
+\beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) &= \underline{U}(s_t)
\end{aligned}
$$

(e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$
\begin{aligned}
V^{n+1}(s_t, \Gamma_{t-1}) &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\
&+ \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right)
\end{aligned}
$$

2. Compute residuals to find μ and φ

$$
\begin{aligned}
r^\mu &= \sum_{t \geq 0, s^t \in S^t} q^*(s^t) \left[F(L(s^t), s^t) - G(s^t) - C(s^t)\right] - B(s^0) \\
r^\varphi &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) C(s^t)^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) L(s^t)^{1+\nu}\right]
\end{aligned}
$$

3. Find μ and φ such that $r^\mu = 0$ and $r^\varphi = 0$.

B.5.3 Measuring Redistributive Cost of Default

This subsection performs a counterfactual exercise that measures the equivalent productivity loss of the distributive component of the cost of default. I consider a representative-agent model in which default not only leads to exclusions from financial markets, but also a permanent loss of productivity. I model a proportional loss of productivity in default as in Aguiar and Gopinath, 2006a,[4] i.e. $z^{default} = (1-\kappa)z$, for $0 \leq \kappa \leq 1$. I calibrate κ such that the representative-agent model generates the same amount of external debt-to-output as the hetereogenous-agent model.

Table B.2: Measuring the distributive cost of default

Description	Parameter	Value	Target	Baseline	One agent & exog. prod. loss
Fraction of prod. loss in default	κ	0.3%	Mean B/Y	21%	21%

Notes: The table reports the value of κ, fraction of productivity that is lost in default such that the long-run average external debt-to-output of the baseline model is the same as the one-agent model. The statistics come from simulations of the models for 10500 periods, excluding the first 500 periods.

Table B.2 reports the results of the exercise. The equivalent loss of productivity for the distributive component is 0.3%

B.5.4 Strategy for Cross-Country Estimation

I estimate the correlation between income inequality and external debt in the model in the following steps:

1. Construct a 31-point evenly spaced grid of θ^H/θ^L, which measures wage inequality. The grid values are between 1.5 and 3[5]

2. For each value of wage inequality, indexed by j,

   (a) Solve and simulate the model for 10500 periods

   (b) Calculate the following averages from the simulation sample, excluding the first 500 periods

   - Pre-tax income Gini Index ($y^H/y^L - 1/2$)

[4] For non-linear default costs that are widely used in the literature, see Arellano, 2008 and Chatterjee and Eyigungor, 2012.

[5] The result is robust to different ranges of wage inequality.

- External debt-to-output (B/Y)
- Log GDP per capita ($\log Y$)

3. Perform the following regression

$$\text{External debt-to-output}_j = \alpha_0 + \alpha_1 \text{Pre-tax Gini Index}_j + \alpha_2 \text{GDP per capita (log)}_j + \epsilon_j$$

The estimation for α_1 is reported in the second column of Table 2.6.

B.6 Data

B.6.1 Data Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti, 2018
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, Lane and Milesi-Ferretti, 2018
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt, 2019.
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, The World Bank, 2019
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, The World Bank, 2019
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, The World Bank, 2019
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, The World Bank, 2019
- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for converenge purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.

- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, The World Bank, 2019
- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri, 2010 using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, The World Bank, 2019
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, The World Bank, 2019

B.6.2 Lists of Countries

- List of all countries in the data set:

Albania, Algeria, Angola, Argentina, Armenia, Australia, Austria, Azerbaijan, Bangladesh, Belarus, Benin, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, Colombia, Congo, Costa Rica, Cte d'Ivoire, Croatia, Czech Republic, Dem. Rep. Congo, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Finland, France, Gabon, Gambia, Georgia, Ghana, Greece, Guatemala, Guinea, Guinea-Bissau, Haiti, Honduras, Hungary, India, Indonesia, Iraq, Ireland, Israel, Italy, Jamaica, Jordan, Kazakhstan, Kenya, Korea, Kyrgyz Republic, Lao, Latvia, Lebanon, Lesotho, Lithuania, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mexico, Moldova, Mongolia, Morocco, Mozambique, Myanmar, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Rwanda, Senegal, Serbia, Sierra Leone, Slovakia, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Tajikistan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Kingdom, United States, Uruguay, Uzbekistan, Vietnam, Yemen, Zambia, Zimbabwe.

- List of advanced and emerging market economies:

Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Denmark, Dominican Republic, Ecuador, El Salvador, Finland, France, Germany, Ghana, Greece, Hungary, Iceland, India, Indonesia, Ireland, Italy, Japan, Kenya, Korea, Malaysia, Mexico, Netherlands, Nigeria, Norway, Panama, Peru, Philippines, Poland, Portugal, Romania, Russian Federation, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Tunisia, Turkey, United Kingdom, United States, Uruguay, Venezuela.

Appendix C

Appendix to Chapter 3

C.1 Proof

C.1.1 Proof of Proposition 3.1

Proof. If $\{C_t, L_t\}$ is an aggregate allocation of an open economy competitive equilibrium with taxes, then it satisfies the resource constraint (3.5). Given any market weights $\boldsymbol{\varphi}$, $\{C_t, L_t\}$ satisfies

$$(1-\tau_t^n)w_t = -\frac{V_L\left[h^i(C_t, L_t; \boldsymbol{\varphi}_t)\right]}{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}$$

Substituting for w_t in the individual worker's budget constraint gives the implementability constraint (3.6). $\boldsymbol{\varphi}$ is such that (3.6) holds $\forall i \in I$.

Suppose that there exist $\boldsymbol{\varphi}$ and T such that $\{C_t, L_t\}, \boldsymbol{\varphi}, T$ satisfy (3.5) and (3.6). Then construct $\tau_t^n = 1 + \frac{1}{\bar{z}_t}\frac{V_L\left[h^i(C_t, L_t; \boldsymbol{\varphi}_t)\right]}{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}$ and substitute in (3.6) gives the individual worker's budget constraint (3.2). Aggregating up (3.2) gives

$$C_t = (1-\tau_t^n)w_t L_t - T_t$$

Subtracting this equation from the resource constraint (3.5) gives the government's budget constraint (3.4) □

C.2 Separable Isoelastic Preference

C.2.1 Parametric forms

The individual allocation rule is $c_t^i = \psi_{c,t}^i C_t$, $l_t^i = \psi_{l,t}^i L_t$, where

$$\psi_{c,t}^{i}=\frac{\left(\varphi_{t}^{i}\right)^{1/\sigma}}{\sum_{i}\pi^{i}(\varphi_{t}^{i})^{1/\sigma}};\quad \psi_{l,t}^{i}=\frac{(\theta_{t}^{i})^{1/\nu+1}\left(\varphi_{t}^{i}\right)^{-1/\nu}}{\sum_{i}\pi^{i}(\theta_{t}^{i})^{1/\nu+1}\left(\varphi_{t}^{i}\right)^{-1/\nu}}$$

Furthermore, V is separable and isoelastic, i.e.,

$$V(C_t,L_t;\varphi_t)=\Phi_{C,t}^{V}\frac{C_t^{1-\sigma}}{1-\sigma}-\Phi_{L,t}^{V}\frac{L_t^{1+\nu}}{1+\nu}$$

where

$$\Phi_{C,t}^{V}=\left[\sum_{i}\pi^{i}(\varphi_{t}^{i})^{1/\sigma}\right]^{\sigma};\Phi_{L,t}^{V}=\omega\left[\sum_{i}\pi^{i}\left(\varphi_{t}^{i}\right)^{-1/\nu}(\theta_{t}^{i})^{(1+\nu)/\nu}\right]^{-\nu}$$

The implementability constraint for worker i at time t becomes

$$\psi_{c,t}^{i}\Phi_{C,t}^{V}C_t^{1-\sigma}-\psi_{l,t}^{i}\Phi_{L,t}^{V}L_t^{1+\nu}+\Phi_{C,t}^{V}C_t^{-\sigma}T_t=0 \tag{C.1}$$

Social wefare function is

$$W(C_t,L_t;\boldsymbol{\varphi}_t)=\Phi_{C,t}^{W}\frac{C^{1-\sigma}}{1-\sigma}-\Phi_{L,t}^{W}\frac{L^{1+\nu}}{1+\nu},$$

where

$$\Phi_{C,t}^{W}=\Phi_{C,t}^{V}\sum_{i\in I}\pi^{i}\frac{\lambda^{i}}{\varphi_{t}^{i}}\psi_{c,t}^{i};\quad \Phi_{L,t}^{W}=\Phi_{L,t}^{V}\sum_{i\in I}\pi^{i}\frac{\lambda^{i}}{\varphi_{t}^{i}}\psi_{l,t}^{i}$$

C.2.2 Recursive Formulation

The repayment value is

$$\begin{aligned} V^{R}(z,B)=\max_{C,L,B',\boldsymbol{\varphi},T}\ &\Phi_{C}^{W}\frac{C^{1-\sigma}}{1-\sigma}-\Phi_{L}^{W}\frac{L^{1+\nu}}{1+\nu}+\beta\mathbb{E}_{z}V(z',B')\\ s.t.\qquad &C+B=zL+q(z,B')B'\\ &C,L,\boldsymbol{\varphi},T\text{ satisfy }(C.1),\forall i\in I \end{aligned}$$

The default value is

$$\begin{aligned} V^{D}(z)=\max_{C,L,B',\boldsymbol{\varphi},T}\ &\Phi_{C}^{W}\frac{C^{1-\sigma}}{1-\sigma}-\Phi_{L}^{W}\frac{L^{1+\nu}}{1+\nu}+\beta\left\{\psi\mathbb{E}_{z}V(z',0)+(1-\psi)\mathbb{E}_{z}V^{D}(z')\right\}\\ s.t.\qquad &C+B=z_{d}(z)L\\ &C,L,\boldsymbol{\varphi},T\text{ satisfy }(C.1),\forall i\in I \end{aligned}$$