The Differential Effects of Elaborated Task and Process Feedback on Multi-digit Multiplication

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Abstract

Given persistent low achievement in mathematics for students in the United States, researchers and practitioners have a vested interest in identifying effective intervention components. This study explored the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) when combined with a cover, copy, compare (CCC) intervention as compared to a repeated practice control condition on students' fluency and strategy use. The multi-digit multiplication class-wide intervention was implemented in 10-sessions with a sample of 101 students from two suburban schools in the Midwest. Due to an interest in the impact of feedback over time, hierarchical linear modeling (HLM) and hierarchical generalized linear modeling were used to examine changes in performance across the intervention. Despite an overall strong effect, the impact of feedback can vary by context, delivery, and purpose (Kluger & DeNisi, 1996). This study addressed gaps in the feedback literature by providing feedback on strategy use and testing the effects of feedback with elaboration to guide error correction. Non-significant effects were found for both types of feedback on fluency and strategy use. The observed increases in fluency over time across conditions provides additional support for the impact of deliberate, repeated practice in mathematics (e.g. Clarke et al., 2016; Fuchs et al., 2010). Implications of the bidirectional relationship observed between strategy use and fluency as well as the potential moderating effects of individual student characteristics are also explored; implications for practice and future research are discussed. Results underscore the importance of research on interventions targeting mathematics skills beyond single-digit computation.

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CHAPTER 1

Introduction

Thinking mathematically is a crucial skill for being quantitatively literate and understanding the scientific and technological issues faced in modern society (National Council of Teachers of Mathematics [NCTM], 2018). In the current job market, the demand for mathematics-intensive science and engineering jobs outpaces overall job growth (National Mathematics Advisory Panel [NMAP], 2008). However, on the 2019 National Assessment of Educational Progress [NAEP], only 41% of fourth grade students and 34% of eighth grade students performed at or above the proficient level in mathematics (National Center for Educational Statistics [NCES], 2019). In response to persistent low achievement, NMAP (2008) identified the need for research-based, high-quality mathematics instruction on conceptual understanding, procedural fluency, and automatic fact recall.

Procedural fluency and conceptual understanding develop together in an iterative process, with improvement in one skill leading to improvement in the other (Baroody, 2003; National Research Council [NRC], 2001; NCTM, 2014; Rittle-Johnson et al., 2001). To develop proficiency, students must be able to flexibly apply strategies to solve contextual problems, understand and explain their strategy selection, and efficiently produce accurate answers (NCTM, 2014). Fluency with procedures is necessary to execute an action sequence to solve the problem, and conceptual knowledge provides the flexibility to generalize that understanding to new problems (Rittle-Johnson et al., 2001).

Multi-digit Multiplication

Developing procedural fluency and conceptual understanding with whole number operations is a foundational skill in primary grades (NCTM, 2000). Fluency in whole-number operations facilitates performance in higher level mathematics including decimals, fractions, and algebra (NCTM, 2000). Students begin understanding number combinations through single-digit arithmetic (NCTM, 2000). Over time, they recognize arithmetic operations as they are embedded in number systems (Geary, 2006). Studies of children's strategy use with multi-digit multiplication have found that children follow a typical trajectory from modeling with individual units, to applying knowledge of base ten systems, and finally using symbolic mathematical models (e.g. Carpenter et al., 2015; Venkat & Mathews, 2019). Students have achieved procedural fluency when they are able to use methods for solving multi-digit computation problems which are efficient, accurate, generalizable, and demonstrate an understanding of place value and the properties of arithmetic operations (Fuson & Beckmann, 2012).

Students naturally develop strategies for learning mathematics facts given exposure and practice opportunities (e.g. Carpenter et al., 2015; Siegler, 2006; Woodward, 2006). Mathematics instruction aims to facilitate the understanding and fluent use of reliable algorithms for accurately and efficiently solving arithmetic problems (NCTM, 2000). Strategy instruction facilitates the organization of mathematics facts into coherent knowledge networks and improves long-term retention and recall (Isaacs & Carroll, 1999; Woodward, 2006). The Common Core State Standards [CCSS] stipulate the introduction of foundational concepts for multiplication by second grade, the formal operations for single-digit multiplication in third grade, and multi-digit multiplication in fourth grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

The strategies employed to solve multi-digit problems vary by level of efficiency and conformity to mathematical principles (Lampert, 1986). When initially learning a new skill, children tend to use less efficient strategies (Siegler, 2006). As children perceive higher level strategies to be more accurate and efficient, previously dominant strategies are weakened and replaced by increasingly sophisticated strategies (Carpenter et al., 2015; Siegler, 2006). This evolution is gradual, and children typically use a variety of strategies which coexist over time (Siegler, 2006; Zhang, Xin et al., 2014). Providing instruction, practice, and feedback targeted to students' strategy use has been identified as effective in facilitating the shift to more efficient strategies (e.g. Siegler, 2006; Zhang, Xin et al., 2014).

For multi-digit number computations, fluency relies on the understanding and transfer of strategies which are used for basic number combinations and extended facts (Fuson, 2003; Woodward, 2006). As with solving single-digit problems, students move from direct modeling strategies using counters to more symbolic procedures and may decompose numbers to simplify calculation (Carpenter et al., 2015). This progression generally follows from counting single units (unitary) to direct modeling with tens to invented algorithms (Carpenter et al., 2015). Additional studies have identified the development specific to multiplication as progressing from *unitary counting* to *skip* *counting*, *double counting*, *repeated addition*, and *decomposition*, before ending with *direct retrieval* of the relevant algorithm (e.g. Downton, 2008; Zhang, Xin et al., 2014).

Classwide Interventions to Promote Procedural Fluency

The pervasive low performance in mathematics (NCES, 2019) indicates the need for class-wide interventions to address mathematics skill deficiencies for students across the general school population (Hawkins, 2010). Class-wide interventions targeted to improving computation skills and concepts have demonstrated effectiveness for students across skill levels including students at risk for greater mathematics difficulties (Fuchs et al., 2014; Poncy et al., 2010; VanDerHeyden et al., 2012). Effective interventions for improving procedural fluency provide brief, repeated opportunities to practice across a variety of examples (e.g. Cozad & Riccomini, 2018; Daly et al., 2007).

Cover-copy-compare (CCC; Skinner et al., 1997) has been established as an effective class-wide intervention for procedural fluency (e.g. Ardoin et al., 2005; Codding, Chan-Iannetta et al., 2009; Poncy et al., 2010). CCC has been implemented with elementary and intermediate students representing a diverse demographic, students with and without disabilities, and across a variety of calculation skills (Joseph et al., 2011). Effects from the class-wide implementation of CCC ranged from small to large gains in computation skills (Codding et al., 2017) with maintenance up to two months after the intervention ended (Poncy et al., 2010). One study (Codding, Shiyko et al., 2007), which examined the relationship between pre-intervention skill level and the intervention, found that CCC was more effective for students with lower accuracy indicating that it is a better fit for students with high error rates. Student ratings of acceptability for CCC have ranged from moderately to highly acceptable (Codding et al., 2017).

Standard administration of CCC involves five steps: (a) looking at the math problem and answer, (b) covering the problem, (c) writing the problem and answer or the answer under a pre-printed copy of the problem, (d) uncovering the original problem with the answer, and (e) comparing the written response to the model (e.g. Codding et al., 2017; Skinner et al., 1997). In prior studies it has been implemented for 3-10 minutes, 2-5 times per week, for 2-6 weeks (Codding et al., 2017). CCC allows for efficient, productive practice by facilitating the completion of multiple learning trials in minimal time and, with the immediate self-evaluation component, preventing students from practicing inaccurate responses (Skinner et al., 1997). Variations to CCC include adding reinforcement components such as feedback or goal setting to encourage students to persist on the practice tasks (e.g. Codding, Chan-Iannetta, et al., 2009; Skinner et al., 1997).

Feedback

Feedback provided to students can both guide the correction of errors in understanding or procedure and increase motivation (Codding et al., 2017). In the instructional context, feedback is information provided to students regarding the performance or understanding of a task which can be used by the recipient to confirm, reject, or alter their prior knowledge (Hattie & Timperley, 2007; Fyfe et al., 2015).

Previous meta-analyses (Hattie, 1999, 2012) found large mean effects of feedback on student achievement (d = 0.79, 0.75 respectively), therefore classifying feedback as one of the top ten most effective influences on student achievement. However, prior research also indicates that the effect of feedback is highly variable with differential effects attributed to the specific feedback components including the type, specificity, and individual reception of feedback (e.g. Bangert-Drowns et al., 1991; Hattie, 2009, 2012; Kluger & DeNisi, 1996).

Elaborated Feedback

The specificity of feedback refers to the amount of information provided. In the simplest form, feedback may only inform students whether their response is correct. In contrast, elaborated feedback provides an explanation to correct misconceptions or procedural errors (e.g. Rakoczy et al., 2013; Shute, 2008). As students develop mathematical proficiency, errors in computation may indicate conceptual misunderstandings such that analysis of these errors can be used to target instruction and facilitate both conceptual understanding and procedural fluency (Schulz & Leuders, 2018). Elaborated feedback functions to correct inaccurate strategy use, procedural errors, or conceptual misunderstanding rather than simply reinforce correct answers (Harks et al., 2014; Mory, 2004). Prior studies have found that elaborated feedback led to greater gains in learning and, based on student reports, was perceived as more beneficial because it included specific suggestions for improvement (e.g. Bangert-Drowns et al., 1991; Black & Wiliam, 1998; Mory, 2004; Rakoczy et al., 2013). Additionally, Bangert-Drowns et al. (1991) found a moderate positive correlation (r = .48) between error rate and the size of effect for feedback, indicating that elaborated feedback had greater benefit for students who were making more errors.

Feedback Focus

The effectiveness of feedback may also be impacted based on what aspect of performance is focused on with the information provided (Hattie & Timperley, 2007; Kluger & DeNisi, 1996). Two foci of feedback examined in recent mathematics intervention studies are (a) feedback directed to the learner's performance on the task and (b) feedback directed to the process for how the task is completed (e.g. Duhon et al., 2015; Fyfe et al., 2015; Gersten, Chard, et al., 2009). Task feedback such as, "You got the right answer – *X* is the right answer" (Fyfe et al., 2012, p. 1097), considers how well a task was performed and the correctness of the answer. This type of feedback is often specific, directs the learner to find new or additional information, and may be particularly useful for novice learners (Hattie, 2012; Heubusch & Lloyd, 1998). In contrast, process feedback provides information on the behavioral processes used to complete the task or obtain the response (e.g. Dweck, 2008; Earley et al., 1990; Fyfe et al., 2015). In mathematics, this refers to the problem-solving procedures (Fyfe et al., 2015). For example, the statement, "That is one correct way to solve the problem," (Fyfe et al., 2012, p. 1097) provides process feedback for a student by addressing the strategy used for solving. Process feedback directs the learner's attention to the action taken to complete the task and may guide error correction or the use of more efficient problem-solving strategies (Earley, et al., 1990; Hattie & Gan, 2011).

A few studies (e.g. Fyfe et al., 2012; Fyfe et al., 2015; Narciss & Huth, 2006) have directly compared the effect of task and process feedback on students' mathematics performance. When studies used simple feedback in their comparison, conceptualizing task feedback as the accuracy of the answer and process feedback as the correctness of the strategy selected to solve a problem, no significant main effects were found for the focus of feedback (Fyfe, 2012; Fyfe et al., 2015) on procedural or conceptual outcomes. Recent research suggests that prior knowledge may moderate the effect of feedback such that students with lower levels of prior knowledge benefit more from corrective feedback (e.g. Fyfe & Rittle-Johnson, 2016, 2017). However, more research is needed to understand if the impacts of students' instructional level differs depending on the focus of the feedback.

The only identified study (Narciss & Huth, 2006) that compared task- and process-focused feedback with elaboration on errors occurred in the context of a computer-based subtraction task. Students were randomly assigned to receive (a) feedback on the correctness of their response with the presentation of the correct response and two opportunities to correct their response or (b) feedback on the correctness of their response with information on the location of errors or an explanation of incorrect strategy application and two opportunities to correct their response. Narciss and Huth (2006) found that students receiving the elaborated process feedback demonstrated greater growth and achieved a higher level of performance than students receiving the elaborated task feedback.

Goal Orientation and Self-Efficacy

Research has also theorized that students may respond differently to feedback based on their personal attributions of success (e.g. Black & Wiliam, 1998; Dweck, 1986). These attributions or goal orientations may influence how learners perceive their past performance and whether that performance is attributed to factors such as ability, effort, task difficulty, or luck (e.g. Pawlik & Rosenzweig, 2000; Pintrich, 2000; Schunk, 1983). By focusing the learner's attention on either the task or the process, feedback may influence which factors the learner attributes for their performance (Clore et al., 2013). Additionally, focusing feedback on malleable factors, such as strategy use, is theorized to increase self-efficacy with the task (e.g. Schunk 1983, 1984). However, prior research has not examined the differential impacts of task and process feedback on either goal orientations or self-efficacy in mathematics interventions.

Purpose

The purpose of the current study is to extend the literature on feedback in whole number interventions by examining the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) with a cover, copy, compare (CCC) intervention as compared to a performance in a control condition with repeated practice (RP) of mathematics facts but no feedback. The following hypotheses were generated: (a) students receiving CCC + feedback, regardless of type, would demonstrate greater fluency in their post-intervention scores than students in the RP condition; (b) students in the CCC + ETF condition would demonstrate higher final scores and greater growth in fluency rates than students in the other groups; and (c) students in the CCC + EPF condition would use more types of problem-solving strategies and would use efficient strategies more frequently than students in other groups. Differential performance of students based on their pre-intervention fluency was also assessed (Codding et al., 2007). Additionally, changes in student self-efficacy and achievement goal orientations were examined. Teacher and student acceptability data were collected post-intervention to provide evidence of social validity for the feedback conditions (Eckert & Hintze, 2000).

Research Questions

The following research questions guided this study:

1. What is the effect of condition (CCC + ETF, CCC + EPF, RP) on students' final scores and growth rates on a measure of multi-digit multiplication fluency?
2. Do the treatment effects depend on students' initial fluency?
3. Do treatment effects depend on the efficiency of strategy use?
4. What are the effects of CCC + ETF and CCC + EPF on the efficiency of strategy use?
5. What are the effects of CCC + ETF and CCC + EPF on the generalization of mathematics skills (WJ-IV calculation subtest and conceptual measure)?
6. What are the effects of CCC + ETF and CCC + EPF on students' goal orientations and self-efficacy of mathematics skills?

Definitions

Conceptual Understanding: An integrated framework for comprehending mathematical concepts and operations such that the underlying meaning of the problem is understood and can be applied in appropriate contexts (e.g. NRC, 2001; Robinson & LeFevre, 2012). Mathematics ideas are organized in a manner which facilitates recall and establishes connections between related concepts (NRC, 2001; Rittle-Johnson et al., 2001). Conceptual understanding may also be referred to as conceptual knowledge.

Elaborated Feedback: Feedback which provides details on how to improve the answer rather than indicating only the accuracy of the answer (Shute, 2008). Elaborated feedback functions to correct inaccurate strategy use, procedural errors, or conceptual misunderstanding rather than simply reinforcing correct answers (Harks et al., 2014; Mory, 2004)

Process Feedback: Feedback directed at the learning processes used to complete the task or obtain the response (e.g. Fyfe et al., 2015; Hattie & Gan, 2007). Process feedback may guide error correction or, in mathematics, cue the use of more efficient domain-specific problem-solving strategies (e.g. Earley, et al., 1990; Fyfe et al., 2015; Hattie & Gan, 2011).

Procedural Fluency: The skill to solve problems by executing an action sequence flexibly, accurately, and efficiently in the appropriate contexts (e.g. NRC, 2001; Rittle-Johnson et al., 2001). Procedural fluency provides for accuracy and efficiency in solving known problem types but may not be generalizable (Rittle-Johnson et al., 2001).

Procedural fluency may also be referred to as procedural knowledge or computational fluency (NCTM, 2014).

Task Feedback: Feedback directed at the task outcome, such as how well a task was performed and the correctness of the answer (e.g. Fyfe et al., 2012; Hattie & Timperley, 2007). Task feedback is often specific, directs the learner to find new or additional information, and may be particularly useful for novice learners (Hattie, 2012).

CHAPTER 2

Literature Review

Chapter 2 presents relevant literature on feedback and instruction for mathematics computation. The first section addresses the research base on feedback as an intervention component and factors which may influence the effectiveness of feedback on academic performance. The second section describes mathematics instruction and intervention on whole number computation, specifically focusing on problem-solving strategies in multi-digit multiplication. The third section examines the intersection of these concepts by presenting the scope of research on underrepresented components of feedback provided during whole number mathematics interventions.

Feedback as an Effective Intervention Component

In the instructional context, feedback is information provided to students on the performance or understanding of a task; it can be used by the recipient to confirm, reject, or alter their prior knowledge (Hattie & Timperley, 2007; Fyfe et al., 2015). Feedback is one of the top ten most effective influences on student achievement (d = 0.79, 0.75 respectively; Hattie, 1999, 2012). However, prior research also indicates that the effect of feedback is highly variable (e.g. Bangert-Drowns et al., 1991; Hattie, 2009, 2012; Kluger & DeNisi, 1996). In meta-analyses, effects have ranged from negligible to large (Bangert-Drowns et al., 1991, ES = -0.83 to 1.42; Hattie, 2012, d = 0.12 – 2.87; Kluger and DeNisi, 1996, d = -0.14 - 0.69). The variation may be due to a variety of feedback components such as the type, complexity, timing, delivery, or individual reception of feedback (e.g. Bangert-Drowns et al., 1991; Gersten, Chard et al., 2009; Hattie & Gan, 2011; Kluger & DeNisis, 1996).

Feedback improves instruction for learners by (a) correcting errors in understanding or process, (b) directing attention to gaps between current and desired performance, and (c) promoting feelings of competence and accomplishment through motivation or reinforcement (Harks et al., 2014; Hattie & Gan, 2011). As such, modern theories of feedback are multidimensional. In Kluger and DeNisi's (1996) seminal study, feedback was conceived to influence performance by directing the learner's attention to the specific comparison of behavior to a goal or standard. Mory (2004) and Hattie and Gan (2011) expanded on this theory to propose synthesis models of feedback which involved the self-regulation of learning such that learners evaluate the gaps in their knowledge, beliefs, motivation, and cognitive process. In this way, learners think about the information provided through feedback and then apply that information to the focal task and enhance their learning (Eckert et al., 2006).

Feedback appears to be beneficial because it contributes to mathematics learning as an active process in which learners revise their performance and understanding based on information provided by peers, adults, and self-reflection (Gersten, Chard, et al., 2009; NMAP, 2008). Timely and descriptive feedback regarding the demonstration of conceptual understanding, application of procedural strategies, or mathematical reasoning is proposed to enhance learning by providing reinforcement of correct answers, facilitating accurate revision of incorrect responses, and promoting the discovery of effective alternatives (e.g. Fyfe & Rittle-Johnson, 2017; NMAP, 2008; Skinner et al., 1992). By facilitating error correction, feedback can decrease the practice of incorrect conceptual or procedural understanding and increase the likelihood of producing a correct response in the future (e.g. Eckert et al., 2006; Skinner et al., 1992).

In addition, feedback as a motivational component of intervention is theorized to influence achievement through engagement and the positive reinforcement of effort (DiPerna et al., 2005; Gersten, Chard, et al., 2009; NMAP, 2008). By increasing the number of learning trials and improving the rate of performance, feedback can function as a contingent reward for students, thereby reinforcing the correct responses and leading to more opportunities to practice (Codding, Baglici et al., 2009). Reinforcement may lead to enhanced engagement and contribute to a productive disposition toward mathematics and a willingness to persist in solving challenging problems (NMAP, 2008; NRC, 2001).

Although previous meta-analyses of components of mathematics instruction (e.g. Codding et a., 2011; Codding, Hilt-Panahon et al., 2009; Swanson et al., 1999) have examined feedback as incorporated into instructional features such as *drill-repetition-practice-feedback, goal setting*, or *probing-reinforcement,* only two attempted to isolate the impact of feedback on mathematics outcomes (Baker et al., 2002; Gersten, Chard et al., 2009). Baker et al. (2002) found a moderate weighted effect on achievement (d = 0.57) across four studies in which students received feedback on either their performance or effort from a teacher or computer. Gersten, Chard, et al. (2009) yielded negligible to small Hedges g effects (-0.17 to 0.24) for six studies that examined the effects of performance feedback without the addition of goal setting and a moderate effect (d = 0.60) for one study (Schunk & Cox, 1986) which included feedback to students on their effort. Additionally, providing students with feedback on their progress by using graphs had a small to moderate mean effect (g = 0.23; Gersten, Chard et al., 2009). However, closer examination of the studies included in both reviews demonstrated that studies were included that provided feedback within the context of a larger intervention (e.g. direct instruction or peer tutoring) in addition to the studies which isolated the impact of feedback provided to learners. Therefore, while feedback appears to be an effective component of mathematics intervention, the effects of feedback alone may have been confounded with other intervention components.

Effective Mathematics Intervention

Manipulating feedback as an intervention component relies on the assumption that the base intervention uses effective mathematics instruction. National panels have identified evidence-based practices for universal instruction and supplemental intervention to support students who have difficulty with mathematics (Gersten, Beckmann, et al., 2009; NCTM, 2014; NMAP, 2008; NRC, 2001). Recommendations include utilizing mathematics instruction which develops conceptual understanding, procedural fluency, and automatic fact recall using various instruction components including problem solving, visual representations, explanations of students' thinking, and feedback (e.g. Gersten, Beckmann, et al., 2009; NCTM, 2014).

Conceptual understanding and procedural fluency are mutually dependent and develop together in an iterative process (Baroody, 2003; NRC, 2001; NCTM, 2014; Rittle-Johnson et al., 2001). Students who have developed conceptual understanding can demonstrate comprehension of the underlying meaning of the problem and apply these principles in appropriate contexts (e.g. NRC, 2001; Robinson & LeFevre, 2012). Conceptual understanding indicates mathematics ideas are organized in a coherent framework which facilitates recall and establishes connections between related concepts (NRC, 2001; Rittle-Johnson et al., 2001). Students who have developed procedural fluency demonstrate the ability to solve problems by executing an action sequence flexibly, accurately, and efficiently and can apply that knowledge in appropriate contexts (NRC, 2001; Rittle-Johnson et al, 2001). Procedural fluency provides for accuracy and efficiency in solving mathematics problems, and conceptual knowledge provides the ability to generalize knowledge to new problem types (Rittle-Johnson et al., 2001). Mathematics proficiency requires both understanding and fluency as students must be able to flexibly select and apply strategies across contexts, explain their strategy selection, and produce accurate answers in an efficient manner (NCTM, 2014).

Therefore, effective interventions promote procedural fluency and conceptual knowledge and may combine multiple instructional components which provide unique contributions (e.g. Gersten, Beckmann et al., 2009; Swanson & Hoskyn, 1998). Components which have been identified as effective for students who have difficulties with mathematics include strategy instruction, frequent opportunities to practice, student verbalizations, and reinforcement such as feedback (e.g. Codding et al., 2017; Gersten, Chard et al., 2009). Strategy instruction includes modeling various strategies for solving a computational problem and can incorporate both students' constructed knowledge and explicit instruction (e.g. Gersten, Chard et al., 2009; Carpenter et al., 1996). Instruction focused on strategies can assist students in developing their conceptualizations of whole number operations and understand the relations between numbers in a problem, thereby developing their conceptual understanding (e.g. Gersten, Chard et al., 2009; Carpenter et al., 1996). Frequent rehearsal and practice can facilitate long-term competency with mathematics skills and develop procedural fluency (Codding et al., 2017). Practice should be systematic, aligned with the student's instructional level, and provided in brief, frequent sessions (e.g. Codding et al., 2017; Gersten, Beckmann et al., 2009). Encouraging students to explain their mathematical reasoning has also been identified as an effective intervention component which supports the development of conceptual understanding (e.g. Gersten, Chard et al., 2009; Rittle-Johnson et al., 2017). As previously discussed, feedback as an instructional and motivational component may also enhance intervention effectiveness (e.g. Gersten, Chard et al., 2009; Hattie & Timperley, 2007). Combining multiple intervention components has been supported in meta-analytic research as providing unique contributions beyond that of the individual procedural or conceptual techniques (Gersten, Chard, et al., 2009; Swanson & Hoskyn, 1998).

Proficiency in Multiplication

Developing proficiency with whole numbers is a foundational skill for students in elementary school which facilitates acquiring more advanced mathematics skills in later grades (e.g. NRC, 2001; Poncy et al., 2010). Proficiency allows students to operate more effectively within the constraints of information-processing resources (e.g. working memory, attention; Mabbott & Bisanz, 2003). Although research has demonstrated that children can intuit multiplication problems before formal instruction of the operations (e.g. Carpenter et al., 2015; Lampert, 1986), the development of procedural knowledge and proficiency with whole numbers begins with understanding number combinations in single-digit arithmetic (NCTM, 2000). Over time, students come to recognize arithmetic operations as embedded in number systems (Geary, 2006). While similarities in the development of strategies have been observed between solving addition and subtraction problems and solving multiplication problems (e.g. Carpenter et al., 2015; Geary, 2006), research also demonstrates that multiplication requires a significant change in thinking (e.g. Barmby et al., 2009; Nunes & Bryant, 1996). Whereas additive problems involve the joining of sets, multiplicative problems are about replication with two distinct inputs: the size of the set and the number of replications (Anghileri, 2000; Barmby, et al., 2009).

Studies of children's strategy use with multi-digit multiplication have found that children follow a typical trajectory beginning with modeling with individual units, then applying knowledge of base ten systems, and finally using symbolic mathematical models (e.g. Carpenter et al., 2015; Venkat & Mathews, 2019). Students have achieved procedural fluency when they are able to (a) use methods for solving multi-digit computation problems which are efficient, accurate, and generalizable and (b) demonstrate an understanding of place value and the properties of arithmetic operations (e.g. Clark et al., 2016; Fuson & Beckmann, 2012).

Students naturally develop strategies for learning mathematics facts given exposure and practice opportunities (e.g. Carpenter et al., 2015; Siegler, 2006; Woodward, 2006). Mathematics instruction aims to facilitate the understanding and fluent use of reliable algorithms for accurately and efficiently solving arithmetic problems (NCTM, 2000). While multiplication fluency with whole numbers is a crucial skill, understanding the underlying reasons for using algorithms allows for generalization to rational numbers and advanced mathematics (Fuson & Beckman, 2012). With this goal, strategy instruction facilitates the organization of mathematics facts into coherent knowledge networks and facilitates long-term retention and recall (Isaacs & Carroll, 1999; Woodward, 2006). In order to scaffold instruction for multiplication proficiency, the CCSS stipulate the introduction of foundational concepts for multiplication by second grade, the formal operations for single-digit multiplication in third grade, and multi-digit multiplication in fourth grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Multi-digit Problem-Solving Strategies

The strategies employed to solve multi-digit problems vary by level of sophistication. When initially learning a new skill, children tend to use less advanced strategies which are gradually replaced with more efficient strategies (Siegler, 2006). As children perceive higher level strategies to be more accurate and efficient, previously dominant strategies are weakened and replaced (Carpenter et al., 2015; Siegler, 2006). Additionally, as they develop flexibility in applying more abstract problem-solving strategies, their speed and accuracy are expected to improve (e.g. Carpenter et al., 2015; Geary, 2006; Siegler, 2006). However, children typically use a variety of strategies which coexist over time (Siegler, 2006; Zhang, Xin, et al., 2014). Models of the cognitive processes used for selecting and applying strategies indicate that less efficient strategies will not be completely abandoned, but they should be less frequently selected (Siegler, 2006). Flexible strategy use is an indication of conceptual understanding as students select the strategy perceived to be most efficient for each problem (Carpenter et al., 2015). Therefore, monitoring the type of problem-solving strategy most frequently selected can provide a measure of the students' proficiency.

For multi-digit number computations, fluency relies on the understanding and transfer of strategies which are used for basic number combinations and extended facts (Fuson, 2003; Woodward, 2006). As with solving single-digit number problems, students move from direct modeling strategies using counters to more symbolic procedures and may decompose numbers to simplify calculation (Carpenter et al., 2015). This progression generally follows from *counting single units (unitary)* to *direct modeling with tens* to *invented algorithms* (Carpenter et al., 2015). Additional studies have specified the developmental trajectory for multiplication as progressing from *direct modeling and counting* strategies, to *repeated addition*, then *decomposition*, before ending with *direct retrieval* of the relevant algorithm (e.g. Downton, 2008; Zhang, Ding, et al., 2014). While not typically intuited by children, using *array models* has also been identified as an effective multiplication strategy (Barmby et al., 2009).

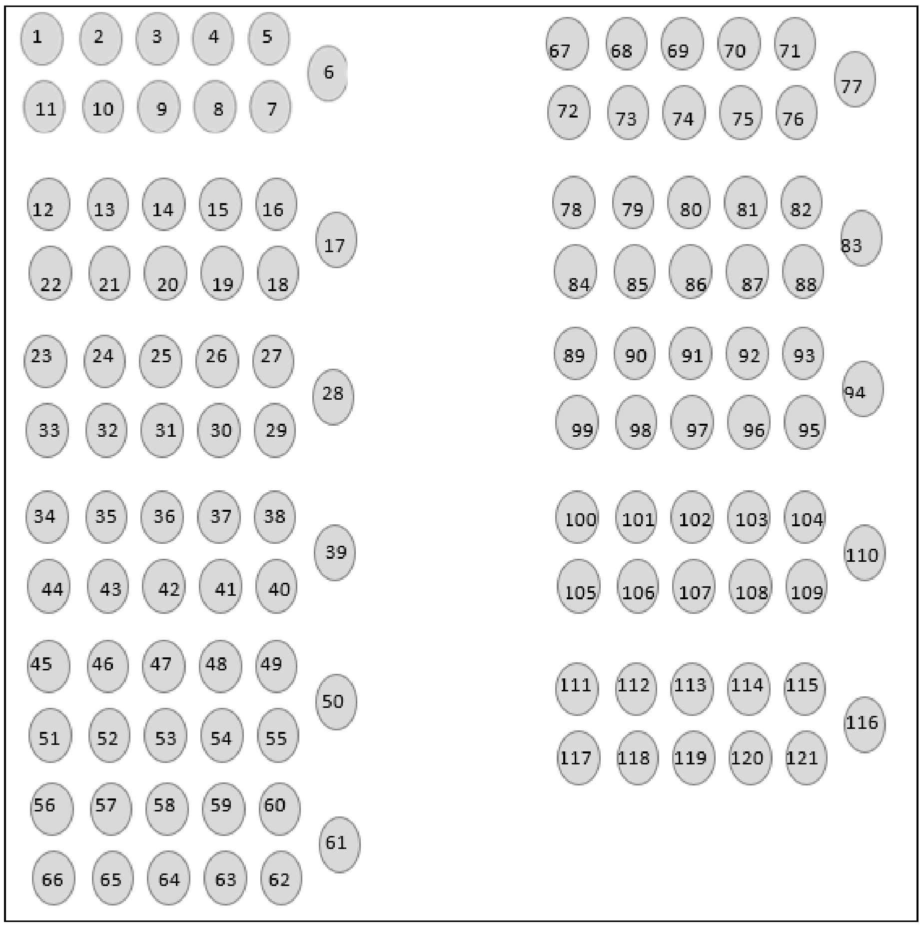
Direct Modeling

As novices with any operation, students begin developing understanding by using direct modeling and unitary counting strategies to solve (Carpenter et al., 2015). When students directly model multiplication problems with either concrete manipulatives or representational diagrams, the total quantity can be depicted as a number of groups with set number of members per group (Lampert, 1986). Students can then find the product by counting the total from the model, as depicted in Figure 1. Students as young as kindergarten have been able to solve multiplication problems using direct modeling (Downton, 2008). Unitary counting involves counting the objects in the model without any obvious reference to the multiplicative structure used to arrange the model (Mulligan & Mitchelmore, 1997). While direct modeling and unitary counting provide methods for students to find the solution, they do not represent a conceptual understanding of the meaning of multiplication.

Figure 1

Example of Direct Modeling to Solve a Multi-digit Multiplication Problem

$$\begin{array}{r} 11 \\ \underline{\times\ 11} \end{array}$$



Repeated Addition

Repeated addition strategies can include rhythmic counting, skip counting, or additive calculation since all are based on the principle of using additive thinking to combine sets using the structure of the problem (Mulligan & Mitchelmore, 1997). Rhythmic counting involves counting such that the units in each group are counted and then the counting sequence is extended to include additional groups; simultaneously, a second tally is used to keep track of the number of groups (Anghileri, 1989; Mulligan & Mitchelmore, 1997). In this way the student is demonstrating some ability to monitor the correspondence of the number of sets and the number of units per set (Anghileri, 1989). Skip counting involves counting multiples such as 5, 10, 15, 20. Repeated addition uses the same conceptual understanding of adding sets. For example, in additive thinking a student solves 5 x 4 by adding 5 + 5 more, then 10 + 5 more, and finally 15 + 5 more (Clark & Kamii, 1996). Utilizing a repeated addition strategy facilitates the understanding of equal groups as a foundational multiplication concept and aligns with an implicit understanding of multiplication (Clark & Kamii, 1996). However, repeated addition becomes increasing inefficient as the factors increase in magnitude and does not generalize for problems involving rational or irrational numbers (Barmby et al., 2009).

Decomposition

Invented algorithms with decomposition may involve additive and multiplicative relationships as students partition, manipulate, and recombine numbers (Lampert, 1986; Young-Loveridge & Mills, 2009). Decomposition requires a flexible understanding of related derived facts and the principles of multiplication (Carpenter et al., 1996; Lampert, 1986). While a variety of decomposition procedures may be proposed to solve any given problem, the procedures must decompose the factors without violating mathematical principles

(Lampert, 1986). With multi-digit multiplication, these fundamental principles include an understanding of the base-ten place value system and the distributive property (e.g. Kilpatrick et al., 2001; Lampert, 1986). Additive decomposition solutions often involve operating on the tens and units separately before applying the distributive property to recombine (Carpenter et al., 1996). For example, a student must understand that 52 could be decomposed in multiple ways (e.g. 50 + 2, 25 + 25 + 1 + 1, 26 + 26) without affecting the total quantity represented. Each of these elements can then be operated on to create partial products before recomposing (Lampert, 1986). Therefore, 52 x 8 could be solved by (a) decomposing 52 = 50 + 2, (b) multiplying 50 x 8 = 400 and 2 x 8 = 16, and (c) recomposing 400 + 16 = 416. Solving through additive decomposition facilitates the application of the distributive property of multiplication over addition (Lampert, 1986). Decomposition also facilitates the application of the associativity and commutativity of multiplication when students decompose multiplicatively into factors (e.g. Empson & Junk, 2004; Lampert, 1986). For example, 52 x 8 can be factored into (13 x 2 x 2) x (2 x 2 x 2). Using the associative property, this fact could alternatively be presented as 13 x 32, 26 x 16, 52 x 8, 104 x 4, 208 x 2, or 416 x 1 without changing the final product.

Array Models

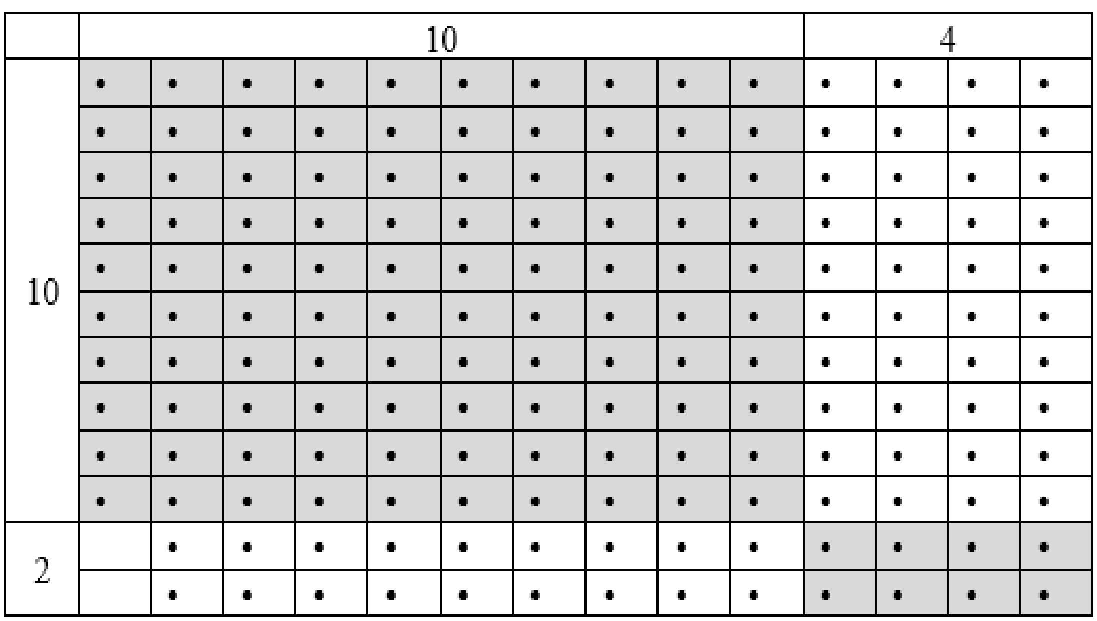
Array models provide a visual representation of multiplication by depicting the factors as vertical and horizontal sets (see Figure 2). While students do not naturally develop array representations for solving multiplication, they provide a useful and efficient problem-solving strategy by facilitating the understanding of the commutative property and facilitating generalization to rational numbers (Barmby et al., 2009;

Carpenter et al., 1996). The only difference between two representations of a calculation (e.g. 5 x 4 and 4 x 5) is the orientation of the array (Barmby et al., 2009). Additionally, unlike the additive strategies which rely on a single level of inclusion with groups combined successively, array models facilitate the multiplicative thinking using the inclusion of simultaneous relationships (Clark & Kamii, 1996). In multiplicative thinking, the student solves by creating 4 sets and 5 within each set. Therefore, the student must recognize that 5 individual units were combined to make each set and simultaneously recognize that those 4 sets of 5 units are combined (Clark & Kamii, 1996).

Figure 2

Example of an Array Model to Solve a Multi-digit Multiplication Problem

$$\begin{array}{r} 14 \\ \underline{\times\ 12} \end{array}$$



Direct Retrieval

Retrieving single-digit multiplication facts from long-tern memory is a focus of elementary school instruction with the intent that fluent retrieval of single-digit facts will facilitate the calculation of more complex problems (Geary, 2006). Although individuals may achieve retrieval through the application of a variety of underlying strategies (Sherin & Fuson, 2005), the ability to quickly state the product of two factors is related to increased fluency and accuracy in single-digit multiplication and minimized working memory demands for more complex problems (e.g. Geary, 2006; Zhang, Xin et al., 2014). While direct retrieval would not be expected to be a commonly used strategy for multi-digit multiplication, cognitive models suggest that students can acquire the memorization of facts given sufficient exposure and practice (Siegler, 1988). Specific multi-digit number combinations (e.g. 11 x 12, multiplying by 10, or perfect squares) may be retrieved by some learners.

Standard Algorithm

Mathematics algorithms are standardized procedures for solving a variety of problems which involve different numbers (NRC, 2001). Algorithms can simplify and consolidate the written steps or notations used to solve a problem. Surveys of mathematics instruction have identified a variety of standardized algorithms around the world (Fuson & Li, 2009). The standard algorithm for multi-digit multiplication in the United States uses a columnar procedure based on base-ten notation (Lampert, 1986). This algorithm allows multi-digit computation to be solved using a series of single-digit computations (Fuson & Beckman, 2012). Using the standard algorithm requires students to follow a set of procedural rules to solve and understand which operation to use, on which digits, in what order, and how the place value of the digit affects the answer (Fuson & Beckman, 2012; Lampert, 1986). As with decomposition, the distributive property supports the separate multiplication of parts of the multiplicand (Raveh et al., 2016). Correctly applying the standard algorithm in multi-digit multiplication both relies on and helps students develop an understanding of the base-ten place value system and regrouping of the multiplicand (e.g. Kilpatrick et al., 2001; Raveh et al., 2016). It also requires the use of derived facts in order to be applied successfully (Zhang, Xin et al., 2014). This combination of conceptual understanding and procedural fluency makes the standard algorithm an effective strategy (e.g. Carpenter et al., 1996; Fuson & Beckman, 2012). Additionally, while empirical evidence was not found comparing the efficiency of multi-digit multiplication strategies with children, the columnar standard algorithm approach procedure was demonstrated to be the fastest approach for adults (Geary et al., 1986).

Research has demonstrated that, while less efficient strategies should be less frequently selected as students cognitive understanding and procedural fluency develop, children typically use multiple strategies (Siegler, 2006; Zhang, Xin, et al., 2014). Many of the strategies facilitate the application of multiplication principles. The selection of strategies may be specific to factors of the problem (e.g. multiples of 10) or related to the comfort of an individual student with a specific strategy (e.g. Siegler, 1988). The flexible use and efficient application of strategies is a element of mathematics fluency and, therefore, an important component of fluency interventions (e.g. Clarke et al., 2016; Gersten, Chard et al., 2009).

Feedback in Whole Number Interventions

Feedback is an effective addition to interventions which provide systematic, intense practice of a mathematics skill as a method for guiding the correction of errors and increasing motivation (Codding et al., 2017). If feedback is used to guide recipients to alter their behavior or understanding based on the information provided, then components of feedback may direct the learners' attention (e.g. toward outcomes, errors, strategies, rate) and alter what aspect of performance they change (Kluger & DeNisi, 1996). Prior meta-analyses and reviews (e.g. Bangert-Drowns et al., 1991; Hattie & Gan, 2011; Kluger & DeNisi, 1996; Shute, 2008) have identified various components of feedback which could influence the effectiveness of feedback in an intervention. Based on a systematic review of feedback components, two were identified for additional study: focus and specificity.

Focus

Four main categories of feedback focus have been identified based on theories proposed in Kluger and DeNisis (1996) and expanded in Hattie (2009, 2012). First, task feedback considers how well a task was performed and the correctness of the answer. For example, "Good job! You got the right answer – *X* is the correct answer" or "Good try, but you did not get the right answer – *X* is the correct answer" (Fyfe et al., 2012, p. 1097) provide task feedback to the learner. This type of feedback is often specific, directs the learner to find new or additional information, and can be particularly useful for novice learners (Hattie, 2012; Heubusch & Lloyd, 1998). Second, process feedback provides information on the behavioral processes used to complete the task, such as strategy, effort, or perseverance (Dweck, 2008; Earley, et al., 1990). Hattie (2012) differentiates process feedback as a broader concept defining how a task is accomplished, rather than the knowledge of results provided in task feedback. Therefore, process feedback can include multiple problem-solving processes including strategies “That is one correct way to solve the problem” (Fyfe et al., 2012, p. 1097) or effort “You’ve been working hard” (Schunk, 1983, p. 851). Process feedback directs the learner’s attention to the action taken to complete the task and may guide error correction or the use of more efficient problem-solving strategies (Earley, et al., 1990; Hattie & Gan, 2011).

Third, self-regulation feedback refers to the self-monitoring needed to identify next steps and direct progress toward completing tasks (Hattie, 2012). Self-regulation feedback is used to increase internal feedback and self-assessment and may include a learner recording their own performance or progress (Hattie & Gan, 2011). Fourth, feedback directed to the self, addresses a learners’ ability or other individual characteristic. It includes comments such as “You’re good at this” (Schunk, 1983, p. 851). Prior research on self-focused feedback demonstrated negligible effects on performance (d = 0.09, Kluger & DeNisi, 1996). This feedback may lack the specificity or task-relevance needed to provide effective reinforcement since it is more general than the other types (e.g. Hattie, 2012; Henderlong Corpus & Lepper, 2007).

In a systematic review of intervention literature using whole numbers, feedback was overwhelmingly focused on the task (82%; Edmunds, 2018). Only 37% of feedback addressed the process, 11% addressed the self, and no feedback was directed at self-regulation. The prevalence of task feedback fits with the finding by Hattie and Timperley (2007) of task feedback as the most common of the four foci. Hattie and Timperley (2007) suggested that feedback focused on the task most closely aligns with the conceptualization of feedback typically held by teachers and students. Research indicates that task feedback helps learners identify errors and correct inaccurate information and is most impactful for novice learners who make frequent errors (e.g. Eckert et al., 2006; Phye & Bender, 1989). Therefore, negative task feedback (providing information regarding incorrect answers) may be particularly effective for improving task accuracy. Based on these theoretical recommendations and the fact that, in the reviewed studies, educators provided most of the feedback, it is unsurprising that the majority of the studies providing task feedback focused on incorrect answers.

Slightly more than one third of studies included process feedback. Within the category of process feedback, slightly more than half of the studies reviewed focused on effort and slightly less than half on the strategies used by students to solve math problems. Research on effort feedback stems from attributional theories regarding the influence of self-efficacy on achievement (e.g. Bandura & Schunk 1981; Kamins & Dweck, 1999; Schunk, 1983). This theory proposes that providing feedback on effort encourages future performance, increases the rate of problem solving, and leads to enhanced self-efficacy (Schunk, 1983). Similar to task feedback, process feedback regarding strategy use is proposed to increase student performance by guiding learners in identifying effective strategies and rejecting erroneous hypotheses (e.g. Earley et al., 1990; Hattie & Timperley, 2007). Feedback on strategies ranged from simple statements of “That is one correct way to solve the problem” (Fyfe et al., 2012, p. 1097) to detailed procedural hints regarding the location of an error and description of systematic errors, and modeling of worked-out examples (Narciss & Huth, 2006).

It is notable that no studies after 1984 included feedback directed at the self. Research has confirmed that this type of feedback is not effective and may lead to lower engagement and effort (e.g. Hattie & Timperley, 2007; Henderlong Corpus & Lepper, 2007; Kamins & Dweck, 1999; Kluger & DeNisi, 1996). Additionally, none of the studies reviewed included self-regulation feedback. If included, self-regulation feedback may have included feedback on students’ review of their own abilities, use of strategies, planning, correcting mistakes or assessment of their performance in comparison to a goal or others’ performance (Hattie & Timperley, 2007). This may be due to the criteria used in this review for feedback to be provided as an isolated intervention component. Feedback is an inherent aspect of self-regulation (Zimmerman, 2008), but feedback on students’ demonstration of self-evaluation skills may not be frequently studied in isolation.

Based on this review, both negative task feedback and process feedback focused on strategies applied to mathematics problem-solving are hypothesized to be effective for error correction. Empirical research is needed to compare how feedback impacts student performance and to which aspects of the task are recipient’s attention directed with each type. Based on theoretical models of feedback (e.g. Kluger & DeNisi, 1996), it is hypothesized that task feedback will direct attention toward achieving the correct answer and process feedback will direct attention toward the correct application of the problem-solving strategies

Specificity

Previous studies have explored the complexity of feedback by comparing simple and elaborated feedback. Simple feedback focuses on the correctness of the response or problem-solving process, whereas elaborated feedback provides an explanation to correct misconceptions or procedural errors (Shute, 2008). Simple feedback included a symbol such as a light turning on for correct answers (e.g. Barling, 1980), a statement on the accuracy of an answer or procedure (e.g. Fyfe et al., 2012), or a statement regarding an overall score (e.g. Codding et al., 2007). Elaborated feedback included additional information such as:

> Sorry, there is an error. Perhaps I can help you: it seems that you have made a carry when computing the digits in the right column. However, if you subtract same numbers the result is always 0 and the carry is not necessary! Try again! (Narciss & Huth, 2006)

Current evidence suggests that specificity alone may not determine the effectiveness of feedback (Harks, et al., 2014; Mory, 2004). However, targeting the specificity of feedback to the learner's skill level may matter. Hattie (2012) found that, overall, feedback was most effective when the student has not yet mastered the content. Shute (2008) suggested that low-achieving or novice learners may need more elaborated feedback whereas simple feedback alone may be sufficient for high-achieving or more proficient learners.

Like the focus component, specificity was unevenly represented in the literature of whole number interventions (Edmunds, 2018). Nearly all studies (92%) provided simple feedback and only four (11%) provided students with elaboration. This may reflect the prevailing conceptualization of feedback as verification of the correct answer or process (e.g. Hattie & Timperley, 2007; Skinner et al., 1992). Additionally, Fyfe et al. (2015) explained their decision to control the amount of information provided to students due to concerns regarding the potential cognitive load required by elaborated feedback. Theories of feedback hypothesize that elaborated feedback will lead to better outcomes for students (e.g. Harks et al, 2014; Mory, 2004; Shute, 2008). Bangert-Drowns et al. (1991) found larger effects for elaborated feedback (*ES* = 0.05 to 1.24) than simple feedback (*ES* = -0.58 to 0.38) in their review of feedback in test-like conditions. More research is needed to empirically test this premise in mathematics interventions.

Summary

In reviewing 38 studies which examined the impact of feedback as provided in whole number interventions, feedback has been examined in conjunction with computation interventions using each of the four operations as well as with interventions focused on solving equivalence problems and using the order of operations. Multiple theoretically supported components of feedback were underrepresented in the empirical literature. Of the four foci of feedback proposed in theoretical models (Hattie, 2009, 2012; Kluger & DeNisi, 1996), only task feedback was represented in a majority of the studies. This contrasts with the suggestions provided in mathematics educational literature to direct learners' attention to the processes and strategies used while solving problems, rather than the outcomes (e.g. Black & Wiliam, 1998; Dweck, 2008; Woodward, 2006). Additionally, feedback is theorized to be more effective when it provides information to guide students in error correction (e.g. Bangert-Drowns et al., 1991; Black & Wiliam, 1998; Shute, 2008). However, in the review only four studies (11%) provided students with elaboration.

Purpose

The purpose of the current study is to extend the literature on feedback in whole number interventions by examining the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) with a cover, copy, compare (CCC) intervention as compared to a performance in a control condition with repeated practice of mathematics facts but no feedback (RP). The following hypotheses were generated: (a) students receiving CCC + feedback, regardless of type, would demonstrate greater fluency in their post-intervention scores than students in the RP condition; (b) students in the CCC + ETF group would demonstrate higher final scores and steeper slopes than students in the other groups; and (c) students in the CCC + EPF group would use more types of problem solving strategies and would use efficient strategies more frequently than students in other groups. Differential performance of students based on their pre-intervention skill level was also assessed (Codding et al., 2007). Additionally, changes in student self-efficacy and achievement goal orientations were examined. Teacher and student acceptability data were collected post-intervention to provide evidence of social validity for the feedback conditions (Eckert & Hintze, 2000).

CHAPTER 3

Method

The present study utilized a randomized controlled trial to examine the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) when combined with a cover, copy, compare (CCC) intervention as compared to a control condition. In the control condition, students also received repeated practice (RP) with multi-digit multiplication but did not receive any feedback on their performance. Differential performance between conditions was assessed based on pre-intervention fluency. The current study's participants, materials, and procedures are described in this section.

Participants and Setting

An a priori power analysis was conducted using the G*Power 3.1 computer program (Faul, Erdfelder, Lang, & Buchner, 2007). For an analysis of covariance (ANCOVA) with a small effect size (η_p^2 = .09, Codding et al., 2009), a desired power of .80, and an alpha level of .05, a total sample size of 101 students was required.

Setting

Students were recruited from fourth-grade classrooms at two suburban public schools in the Midwest region of the United States. Fourth grade was targeted for participation in the study to align with the grade-level emphasis on multi-digit multiplication by the Common Core State Standards Initiative (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Based on enrollment information from the state department of education for 2018-2019, the year the study was conducted, School 1 had 565 students enrolled in kindergarten through 5th grade. Across the entire school, 54.9% of students were male and 45.1% were female with racial demographics of 62.5% of students identifying as White, 11.0% as Asian, 10.3% as Black or African American, 8.7% as Two or More Race, 8.1% as Hispanic, and less than 1% as American Indian. Additionally, 18.4% of the student population qualified for free or reduced-price lunch, 14.5% received special education services, and 9.6% were identified as English Learners. In the spring of 2018, 80.0% of students in 3rd grade met or exceeded standards on the state standardized assessment.

School 2 had 883 students enrolled in early childhood classes through 5th grade. Across the entire school, 51.1% of students were male and 48.9% were female with racial demographics of 62.5% of students identifying as White, 19.4% as Hispanic, 7.4% as Two or More Race, 7.1% as Black, 3.2% as Asian, and less than 1% as American Indian. Additionally, 46.3% of the student population qualified for free or reduced-price lunch, 16.2% received special education services, and 4.9% were identified as English Learners. In the spring of 2018, 57.8% of students in 3rd grade met or exceeded standards on the state standardized assessment.

Curriculum. At both schools, students received 60 minutes of mathematics instruction per day. The primary mathematics curriculum at School 1 was *enVision MATH* (Foresman & Wesley, 2007) a Common Core-aligned mathematics curriculum which meets the “promising” level of evidentiary support established by the Every

Student Succeeds Act (ESSA; U.S. Department of Education, 2016). However, while this study was being implemented, the class was working on instructional standards related to fraction understanding and computation using the *Rational Number Project: Initial Fraction Ideas* (Cramer et al., 2009). School 2 did not have a designated mathematics curriculum. The intervention was implemented during the last month of the school year and the core instruction during this time covered a variety of review topics including multiplication and division, fractions, and measurement.

Teachers. After obtaining approval from the university's institutional review board and district research offices, teachers were recruited for participation. All three fourth grade teachers in School 1 agreed to participate and one fourth grade teacher in School 2. Three teachers were female, one was male. All four teachers identified as White and reported more than five years of teaching experience. Three of the four teachers held a master's degree in an education-related field.

Approaches to Instruction. All participating teachers completed four scales from the Patterns of Adaptive Learning Scales-Teacher (PALS; Midgley et al., 2000) on which they rated their classroom environment for mathematics regarding mastery and performance approaches to teaching and learning. Mastery approaches emphasize that the purpose of academic work is to develop competence whereas performance approaches emphasize that the purpose is to demonstrate competence. Two scales addressed mastery learning through (a) mastery goal structure for students and (b) mastery approaches to instruction; two scales addressed performance learning through (c) performance goal structure for students and (d) performance approaches to instruction.

In School 1, the three participating teachers endorsed a mastery goal structure (mean = 4.1, range 3.7 - 4.4.) and somewhat endorsed a mastery approach to instruction (mean = 3.2, range 2.75 - 3.5). For example, the teachers said that it is “very true” that students are told it is okay to make mistakes as long as they are learning and that it was “true” or “somewhat true” that they considered how much students had improved when providing grades. They did not endorse a performance goal structure for students (mean = 2.5, range 2.2 – 2.7) or performance approaches to instruction (mean = 2.1, range 2.0 – 2.2). For example, they responded that it was “not at all true” or “not true” that students are encouraged to compete with each other nor that they told students how their work compared to other students. In School 2, the participating teacher strongly endorsed a mastery goal structure and mastery approach to instruction (scale scores of 4.7 and 4.0 respectively). She did not endorse a performance goal structure or performance approach to instruction (scale scores of 2.0 and 1.0 respectively).

Teacher Self-efficacy. Teachers also completed one scale from the PALS-Teacher (Midgley et al., 2000) regarding their personal belief that they are significantly contributing to their students’ academic progress and that they are effective in teaching all students. Teachers in School 1 indicated moderate levels of personal efficacy (mean = 3.4, range 3.0 – 3.9). The teacher in School 2 indicated higher personal efficacy (scaled score = 4.0). For example, all teachers indicated that it was "true" that they make a difference in the lives of their students and that they could impact students learning.

Participants

All students who received their mathematics instruction in the participating classrooms were invited to participate in the study. Parental consent and student assent were obtained from 88% of students in School 1 and 100% in School 2, resulting in a final sample of 101 students. Demographic information was reported by each school with gender, race/ethnicity, and special education status reported for 98% of the sample (n = 99). One school declined to provide information identifying which students were identified as English Learners and neither school provided information regarding family income or eligibility for free or reduced-price lunch. Based on the data reported, students in the sample were primarily White (62%), male (54%), and did not receive special education (87%) or English language (69%) services (see Table 1). Data on the specific special education category was not reported; therefore, it is unknown whether any of the students were receiving services under a specific learning disability related to mathematics.

Table 1

Demographic Data in Number of Participants by Condition

	CCC + EPF	CCC + ETF	Vocabulary	Total
Total participants	33	34	34	101
Teacher				
0	8	7	7	22
1	9	10	10	29
2	7	8	7	22
3	9	9	10	28
Gender				
Male	16	19	20	55
Female	16	14	14	44
Missing	1	1	0	2
Race/Ethnicity				
White	20	22	21	63
Black/African American	2	7	3	12
Asian	6	1	4	11
Hispanic	4	1	4	9
Two or More	0	2	2	4
Missing	1	1	0	2
Receives English learner services				
No	21	24	24	69
Yes	4	2	3	9
Missing	8	8	7	23
Receives special education services				
No	29	30	29	88
Yes	3	3	5	11
Missing	1	1	0	2

Note: CCC + EPF = cover, copy, compare with elaborated process feedback; CCC+ ETF = cover, copy, compare with elaborated task feedback.

Interventionists

The first author and a fifth-year doctoral student served as the primary interventionists. Additionally, one fourth-year doctoral student and two third-year doctoral students assisted in delivering pre- and post-intervention assessments and substituted for two intervention sessions. The interventionists were all White women with prior experience working in schools. All interventionists had completed coursework on academic assessment and intervention, including the implementation of curriculum-based measures in mathematics and the Cover-Copy-Compare (CCC) intervention for mathematics skills through coursework in a school psychology training program. Prior to implementation of the study, the interventionists received training on the standardized administration of the vocabulary and CCC intervention with *review, practice,* and *sprint* components. Training consisted of observing the delivery of the intervention and role-play practice following a script. Implementation mastery was determined when each interventionist could implement each step of the protocol independently with 100% accuracy.

Treatment Conditions

Student performance was compared across three conditions: cover-copy-compare with elaborated task feedback (CCC + ETF), cover-copy-compare with elaborated process feedback (CCC + EPF), and a control condition with repeated practice of the target skill but no feedback (RP). All conditions utilized a three-part intervention of *review, practice,* and *sprint.* The *review* components provided the feedback portion of the current study and were differentiated by condition. The *practice* component provided the CCC practice of multi-digit multiplication and was consistent between treatment groups but differed for the repeated practice condition. The *sprint* component provided repeated practice of the target skill and was consistent across conditions. Condition-specific intervention packets were created by the first author. Example materials by condition are included in Appendix A.

Cover-Copy-Compare with Elaborated Task Feedback (CCC + ETF)

Task feedback is intended to direct students' attention to the outcome. Therefore, feedback in this condition was provided regarding (a) overall performance in the number of correct digits and (b) errors in the answer for individual items.

Review. During the review component, students were presented an individualized bar graph showing the number of correct digits in previous sessions. The graph was accompanied by the statement "This graph shows how many digits you have answered correctly in each session." Below the graph was an example which depicted (a) a completed problem, (b) the problem with the correct answer, and (c) a typed response indicating how a student may respond when asked to explain their thinking about why the answer is correct.

This was followed by the review problems. For each problem, in the first column on the left side of the page was the statement, "This was your answer last time," accompanied by a copy of the student's work and answer for a problem that the student had previously completed. During the first session, the four problems presented were selected from the pre-intervention single-skill measure (SSM) assessment. If the student completed four or more problems incorrectly on this assessment, four problems were randomly selected for presentation on the *review* sheet. If the student completed fewer than four problems incorrectly, the incorrect problems and additional randomly selected problems were included to provide a total of four problems to review and practice. After the first session the same procedure was used to select problems from the *sprint* completed during the previous session.

In the second column was the statement, "Compare your answer to the correct answer," accompanied by the same problem with the correct answer presented horizontally, center-aligned in the top third of the box. In the third column, the student was asked to explain their thinking. Following the guidance from Siegler (2002), written prompts were provided to encourage self-explanation. If the student had answered the initial problem incorrectly, the written prompt stated, "Why is the answer **in box 2** correct? Why was **your** answer incorrect?" If the students had provided the correct answer to the initial problem, the written prompt stated, "Why is this answer correct?"

Practice. The *practice* component for the treatment conditions (CCC + ETF and CCC + EPF) followed the recommendations for standard Cover, Copy, and Compare (CCC; Skinner et al., 1997). At the top of the page was a statement reviewing standardized CCC instructions (Skinner et al., 1997). Initially each CCC probe included six items based on rates of responding when the materials were piloted with a class of fifth grade students. However, six additional items were added as students' rate of responding increased. For each item, the stimulus problem was presented on the left side of the paper. These items were randomly selected problems of the target skill. Each stimulus problem modeled solving the problem using one of three strategies which had been identified by the participating teachers as the most familiar for their students: decomposition of the factors by place value, decomposition of the factors presented using a variation referred to in class as the “box” or “window” method, or the standard algorithm (see Appendix A). Each strategy was represented with equal frequency, and the order the strategies were presented was randomly selected.

Students were informed that they could solve the problem using any solution; they were not required to solve using the strategy modeled in the stimulus item. In this manner, students in both treatment conditions had repeated exposure to multiple problem-solving strategies throughout the intervention. Next to each stimulus item, presented in the center and the right of the paper were two additional copies of the same problem presented horizontally with no answer provided. Students were given an index card to use to cover the stimulus while implementing the CCC procedures. The *practice* materials were identical for the CCC + ETF and CCC + EPF treatment conditions.

Cover-Copy-Compare with Elaborated Process Feedback (CCC + EPF)

Process feedback is intended to direct students’ attention to the strategies used to solve the problem. Therefore, feedback in this condition was provided regarding (a) the variety of strategies used to solve problems and (b) errors in the application of a specific problem-solving strategy for individual items.

Review. During the review component, students were presented an individualized bar graph showing the number of problems attempted in previous sessions using direct retrieval or the four targeted problem-solving strategies: repeated addition, decomposition, array, or standard algorithm. The graph was accompanied by the statement “This graph shows how many times you used each strategy in each session.” The graph was also accompanied by a legend to help students identify the color associated with each strategy type.

Below the graph was an example which depicted (a) a completed problem with the strategy used to solve identified and highlighted, (b) the same problem accompanied by an example of solving the problem with the same strategy but no answer, and (c) a typed response indicating how a student may respond when asked to explain their thinking about why the modeled strategy would work to solve the problem. This was followed by problems selected using the same procedures as in the CCC + ETF condition. For each problem, in the first column on the left side of the page was a statement identifying the strategy the student used to solve the problem. This was followed by a copy of the student’s work and answer for a problem that the student had previously completed. During the first session, the four problems presented were selected from the pre-intervention single-skill measure (SSM) assessment. If the student completed four or more problems incorrectly on this assessment, four problems were randomly selected for presentation on the *review* sheet. If the student completed fewer than four problems incorrectly, the incorrect problems and additional randomly selected problems were included to provide a total of four problems to review and practice. After the first session the same procedure was used to select problems from the *sprint* completed during the previous session.

In the second column, the same problem was presented with a model of the correct application of that problem-solving strategy but with no answer provided. Above the worked example was the statement "Compare your work to the *X* strategy used in this example." For example, if the student solved using the standard algorithm, the statement would read, "Compare your work to the STANDARD ALGORITHM strategy used in this example." followed by a copy of solving the problem using the standard algorithm. Each of these worked problems was handwritten by the first author to match the student's attempted strategy, copied, and printed on the page. In both columns, the name of the strategy used was printed in all caps and highlighted yellow.

In the third column, the student was asked to explain their thinking. Following the guidance from Siegler (2002), written prompts were provided to encourage self-explanation. If the student had answered the initial problem incorrectly, the written prompt stated, "Why did the strategy in **box 2** work? How would you get **your** strategy to work?" If the students' previous work was correct, the statement was "Why did this strategy work?" In this manner the statements in the CCC + ETF and CCC + EPF conditions were formatted similarly but cued the student to either attend to their answer or their strategy use.

Practice. The *practice* component was consistent between the CCC + ETF and CCC + EPF treatment conditions so that the participants received the same exposure and practice with the CCC procedures and problem-solving strategies.

Repeated Practice (RP)

This study used an alternate intervention rather than a business-as-usual control to examine the additive effect of feedback and cover-copy-compare to a repeated practice intervention. To maintain equivalence in the amount of intervention time, students in the RP condition practiced mathematics vocabulary during the *review* and *practice* components. The *sprint* component was consistent across all conditions and served to provide all students with repeated practice of the target skill.

Review. During the review component, students were initially presented a list of four mathematics vocabulary terms each with a drawing or graphic depicting the meaning of the word and the definition. Students were instructed to draw a line to match each definition with the corresponding graphic. The words and graphics were randomly selected from a list of CCSS mathematics vocabulary words for grades 3 and 4 (Granite School District Math Department, 2018). Definitions for each word were selected from the *enVision MATH* (Foresman & Wesley, 2007) textbook. Each definition was tested for readability using the Lexile Framework for Reading using the Lexile Analyzer (MetaMetrics, 2018). Definitions which exceeded a third-grade equivalent score (i.e. above 760 Lexile) were simplified without changing the core concepts in order to make definitions accessible for independent reading and comprehension. Vocabulary items were then randomly assigned to each probe.

Practice. The tasks in the *practice* component of the RP condition were designed based on recommendations from Marzano (2004) for high-quality vocabulary instruction and practice. First, students were again presented with each vocabulary word and definition presented in the left column. In the second column an example sentence was provided demonstrating the use of the vocabulary word in context. These sentences were all written at a third-grade or lower reading level based on the Lexile score. Students were prompted to write their own sentence using the vocabulary word. In the third column, students were prompted to draw a picture of the vocabulary word.

Measures

Single-skill Measures (SSM)

Based on teacher recommendations and CCSS for fourth grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), two calculation skills were assessed through a survey-level assessment (a) 2 x 2-digit multiplication without regrouping and (b) 2 x 2-digit multiplication with regrouping to determine the students’ pre-intervention instructional level. Using procedures recommended by Shapiro (2011) and criteria from Poncy and Duhon (2017), a target skill was selected for the intervention. Given similar performance class-wide on the two skills, the problems were interleaved at a ratio of three problems without regrouping to one problem with regrouping. Eleven parallel SSM forms of 36 total problems were developed by randomly selecting 27 problems without regrouping and nine problems with regrouping, organizing the problems into sets of four (three problems without regrouping and one with regrouping), and randomly assigning the order of the problems within each set. One SSM was administered for the pre-intervention testing, and 10 were administered during the *sprint* portion of the intervention sessions.

SSM probes were created according to the procedures described by Shapiro (2011) and administered following standard directions (Shinn, 1989). The 36 problems were presented on four pages with nine problems per page (three rows of three problems). Each problem was presented horizontally, center-aligned, and in the top fourth of the box.

Standardization and scoring procedures (Shapiro, 2011) specified administering the SSM for five minutes and scoring according to the number of correct (CD) and incorrect (ID) digits. The appropriate digits in the proper place value column were scored as correct. Inappropriate digits, digits written in the wrong place value column, or digits skipped were scored as incorrect. Research has supported that scores from SSMs provide valid and reliable data (test-retest r = .79, parallel forms r = .61-.79; Foegen et al., 2007) for decisions on instructional adaptation and detecting intervention effects (Christ et al., 2008).

Pre- and Post-intervention Generalization Measures

Two measures of mathematical performance were administered pre- and post-intervention to assess the differential impact of feedback on the generalization of mathematics skills.

Woodcock-Johnson Test of Achievement-IV (WJ-IV; Schrank, Mather, & McGrew, 2014). The calculation subtest of the WJ-IV was administered pre- and post-intervention as a standardized, normed measure of mathematics computation skills. Form A was administered for the pre-intervention assessment and the alternate, parallel Form B was administered at the post-intervention assessment (McGrew et al., 2014). The WJ-IV calculation subtest consisted of mathematical computation tasks ranging from simple addition to calculus operations. Following standardized procedures, students were instructed to solve the problems in order until they were no longer able to answer the questions. This was an untimed measure. The WJ-IV mathematics calculation subtest has high reliability (r = .91) for children ages 9-10 (Schrank et al., 2014).

Conceptual Measure. A 20-item conceptual understanding measure (Burns et al., 2018) using single-digit whole-number multiplication problems which addressed representation, reversibility, flexibility, generalization, associative property, and commutative property was administered pre- and post-intervention. Previous research demonstrated that the conceptual measure was correlated with measures of concepts, applications, and calculation and did not correlate with a measure of fact fluency (Burns et al., 2018).

Patterns of Adaptive Learning Survey – Student

The *Patterns of Adaptive Learning Survey – Student* (PALS; Midgley et al., 2000) assesses motivation using an achievement goal theory framework. Five scales were administered post-intervention to assess (a) academic self-efficacy, (b) mastery goal orientations, (c) performance-approach goal orientations, (d) perceptions of classroom mastery goal structure, and (e) perceptions of classroom performance-approach goal structure. Items were answered using a 5-point rating scale anchored at 1 (*not at all true*), 3 (*somewhat true*), and 5 (*very true*). The PALS has been shown to have high concurrent, construct, and discriminant validity (Midgley et al., 1998).

The survey was administered to all students in paper-pencil format. Following standard administration procedures, the survey directions and items were read to students (Midgley et al., 2000). The average score for each scale was calculated with higher scores indicating greater endorsement of the construct.

Academic Self-efficacy. One scale from the PALS-Student (Midgley et al., 2000) was administered to assess students' self-efficacy for learning (five items). These items were adapted to be specific to students' self-efficacy in mathematics and measured the extent to which the student anticipated mastering the skills they were taught in math class (Friedel et al., 2010). This scale has been shown to have adequate reliability (α = .78 - .89; Friedel et al., 2010; Midgley et al., 2000).

Personal Achievement Goal Orientations. Two scales from the PALS-Student (Midgley et al., 2000) were administered to address mastery and performance goal orientations. These items were adapted to be specific to the context of mathematic classwork (Friedel et al., 2010). The mastery goal orientation scale assessed perceptions regarding the importance of understanding math, learning as much math as possible, and improving math skills (five items). The performance-approach goal orientation scale assessed perceptions regarding the importance of looking smart in math compared to other students, showing others that they were good in math, and demonstrating that math is easy (five items). Both scales have been shown to have adequate internal consistency (α =.82 - .89 and r = .86-.90, respectively; Friedel et al., 2010; Midgley et al., 2000).

Perception of Classroom Goal Structures. Two scales from the PALS-Student (Midgley et al., 2000) were adapted to be specific to the mathematics context and administered to address students' perceptions of the mastery and performance goal structure in their classroom. The classroom mastery goal structure scale assessed perceptions of the emphasis in mathematics class on trying hard, understanding the material, and the acceptability of making mistakes (six items). The classroom performance-approach goal structure scale assessed perceptions regarding the emphasis in mathematics class on getting the right answer and getting good grades (three items). Both scales have been shown to have adequate internal consistency (α =.76 and α = .70, respectively; Midgley et al., 2000).

Patterns of Adaptive Learning Survey – Teacher

The *Patterns of Adaptive Learning Survey – Teacher* (PALS; Midgley et al., 2000) consists of five scales regarding (a) personal teaching efficacy, (b) mastery goal structure for students, (c) performance goal structure for students, (d) mastery approaches to instruction, and (e) performance approaches to instruction. Items were answered using a 5-point rating scale anchored at 1 (*not at all true*), 3 (*somewhat true*), and 5 (*very true*). The survey was administered in paper-pencil format to each classroom teacher during the pre-intervention data collection as a measure of the classroom environment. The average score for each scale was calculated with higher scores indicating greater endorsement of the construct (Midgley et al., 2000).

Personal Teaching Efficacy. One scale from the PALS-Teacher (Midgley et al., 2000) was administered to assess teachers' efficacy for contributing to their students' learning and their ability to effectively teach all students (seven items). This scale has been shown to have adequate internal consistency (α =.74; Midgley et al., 2000).

Perception of School Goal Structure for Students. Two scales from the PALS-Teacher (Midgley et al., 2000) were administered to reflect teachers' perceptions of the mastery and performance goal structure communicated to students from the school. The mastery goal structure for students scale assessed teacher perceptions that the school conveyed the purpose of school work was to develop competence and emphasized recognizing effort (seven items). The performance goal structure for students scale assessed teacher perceptions that the school conveys the purpose of school work was to demonstrate competence and emphasized obtaining high grades (six items). Both scales have been shown to have adequate internal consistency (α = .81 and α = .70, respectively; Midgely et al., 2000).

Approaches to Instruction. Two scales from the PALS-Teacher (Midgley et al., 2000) were administered to reflect teachers use of strategies that conveyed mastery and performance goal structures. The mastery approaches scale assessed teachers use of strategies that emphasized individual progress (four items). The performance approaches scale assessed teachers use of strategies that emphasized competition and the recognition of the highest performing students (five items). Both scales have been shown to have minimally acceptable internal consistency (α = .69 and α = .69, respectively; Midgely et al., 2000).

Social Validity

Two measures of social validity were administered to measure both student and teacher perceptions of the acceptability and usefulness of the intervention.

Student Perceptions. The Kids Intervention Profile (KIP; Eckert et al., 2017) was administered following the final intervention session as an assessment of students' perceptions of the social validity of the intervention. Following administration procedures used by Eckert et al. (2017), the directions, items, and response options were read aloud. The KIP consists of eight items and was answered using a five-point rating scale ranging from *not at all* to *very, very much* or from *never* to *many, many times*. Instead of using a numbered rating scale, the KIP utilizes a graphic of boxes with increasing sizes (Eckert et al., 2017). For scoring these responses were coded 1-5 with the largest box scored as a 5. Two items used reverse-worded statements and were recoded during scoring. Item scores were summed to create a total score. Possible total scores ranged from 8 to 40. Higher scores on the KIP indicates greater social validity, with a total score greater than 24 indicating an acceptable rating for the intervention (Eckert et al., 2017).

Teacher Perceptions. The Intervention Rating Profile-15 (IRP-15; Martens et al., 1985) was administered as a measure of teacher perceptions of the social validity of the two types of feedback tested in this study. The measure was adapted to refer to classwide academic skills. Teachers received two copies of the IRP-15 post-intervention. Each copy was printed on a different color of the paper and accompanied by a cover sheet which included a description of the relevant treatment condition and instructions to complete the survey based on their observations of the feedback. The IRP-15 uses a 6-item rating scale ranged from *strongly disagree* to *strongly agree*. Scores can range from 15-90 with higher scores indicating greater acceptability. The IRP-15 has a one-factor structure with high internal consistency (α = .98; Martens et al., 1985).

Procedures

Pilot Testing of Materials

Prior to the start of the intervention, the review and practice materials were distributed to a classroom of fifth grade students. The classroom teacher wanted to informally assess her students' knowledge of 2 x 2-digit multiplication and agreed to incorporate these intervention materials into her lesson. The students were discretely timed as they completed each set of materials. This information was used to determine the number of items initially included in the review and practice materials. The classroom teacher retained all information regarding the students answers and performance.

Survey Level Assessment

Six weeks prior to the start of the intervention, all fourth-grade students in the School 1 (n = 82) completed the survey-level assessment using procedures recommended by Shapiro (2011) to determine the appropriate target skill. Two multi-digit multiplication skills were assessed using SSMs: 2 x 2-digit multiplication without regrouping and 2 x 2-digit multiplication with regrouping. Given similar performance class-wide on the two skills, the problems were interleaved at a ratio of three problems without regrouping to one problem with regrouping to great the target skill SSM. Data from this class-wide assessment was provided to teachers for use in instructional planning.

Consent and Assent Obtained

One month prior to the start of the intervention, parental consent and student assent were obtained for participation. Then, a block randomization procedure was used to assign students to one of three treatment conditions such that the conditions were equally represented in each classroom (Imbens & Rubin, 2015). Across the four classrooms, 101 students were enrolled in the study and randomly assigned to the treatment groups of CCC + EPF (n = 33) and CCC + ETF (n = 34) or the comparison group of RP (n = 34). A one-way analysis of variance (ANOVA) indicated baseline equivalence in fluency with no significant differences between groups on the single-skill multi-digit multiplication measure in fluency ($F(2, 99) = 2.19$, $p = .12$) or accuracy ($F(2,99) = 2.02$, $p = .14$). Additionally, no significant differences were found across demographic variables.

Pre-intervention Testing

One week prior to the start of the intervention, participating students completed the three assessments of mathematics skill: SSM of 2 x 2-digit multiplication problems with and without regrouping, the WJ-IV calculation subtest, and the conceptual measure over three testing sessions. At this time, teachers completed the selected scales of the PALS-Teacher to assess their achievement goal orientations.

Instruction

Prior to the first intervention session, all students were provided one lesson on the four multiplication problem-solving strategies. The instructional lesson began with a classroom number talk (Parrish, 2011) in which a 2 x 1-digit multiplication problem was posed to students. Students were encouraged to try to solve the problem using multiple methods and then volunteers shared their answers. As each student explained how they solved the problem, the first author recorded their responses. After multiple responses were shared, the first author identified each strategy by name and modeled problem-solving strategies which had not been identified by the class to ensure that the strategies of (a) repeated addition, (b) array models, (c) decomposing, and (d) the standard algorithm were each represented at least once (Carpenter et al., 2015; Zhang, Xin, et al., 2014). Each strategy was then reviewed following a direct instruction format. First, the first author demonstrated solving a problem using the identified strategy while recording and narrating her steps for solving. Then, the class solved a problem together with students narrating the steps as the first author recorded them. Finally, students were asked to solve the third problem on their own or with a partner. This process was completed with each strategy using a total of twelve 2 x 1-digit multiplication problems.

Students also completed one lesson on condition-specific procedures. Students received instruction from the primary interventionists in small groups based on condition assignment. The interventionists modeled using the condition-specific procedures for the *review* and *practice* portions of the intervention using 2 x 1-digit multiplication problems. First, students were shown how to read and understand their review packet. Students in the treatment conditions practiced comparing work from a fake student to the model (e.g. correct answer for ETF or strategy for EPF) and writing sentences to explain their thinking. It was explained to students that during the intervention they would be seeing their own work. Then, students in the treatment conditions were introduced to the practice portion and taught standard CCC procedures. Standard administration of CCC involves students (a) looking at the math problem and answer, (b) covering the problem, (c) writing the problem and answer or the answer under a pre-printed copy of the problem, (d) uncovering the original problem with the answer, and (e) comparing their written response to the model (e.g. Codding et al., 2017; Skinner et al., 1997). The first problem was modeled for students demonstrating how to use CCC. Then students participated in solving the second problem. Finally, students practiced independently. All students were told that they could solve the problem using any strategy they wanted; they did not have to solve the problem using the strategy shown in the model.

For the repeated practice (RP) condition, students were presented with an example *review* and *practice* packet. Instructions during this practice condition were same as used in the intervention. Students were told to review the mathematics vocabulary and match the definition to the picture. They were then taught and practiced writing sentences using the vocabulary word in context and drawing their own picture.

Intervention

The intervention consisted of 10 sessions. In School 1, the intervention was spread over six weeks due to days off school. Four weeks contained two sessions per week and two weeks contained only one session (the third and tenth session). In School 2, intervention sessions occurred twice per week for five weeks. Each session followed the *review, practice,* and *sprint* procedures with variations as described for each condition. All assessment and treatment sessions were provided in the students' classrooms. Sessions were standardized so that students in all treatment conditions completed the intervention at the same time.

Review and Practice. Individualized packets containing the review and practice components of the intervention were distributed to all students. Students had 10 minutes to complete the review and practice components and were allowed to work at their own pace within that time limit.

During this time, students in the RP condition reviewed mathematics vocabulary by (a) matching the vocabulary word and definition to an image, (b) reviewed an example sentence using the vocabulary word in context, (c) wrote their own sentence, and (d) drew a picture representing the vocabulary word.

Students in the CCC + ETF condition reviewed the individualized bar graph showing the number of digits previously completed correctly and compared their answer on four problems with the correct answer. Students were asked to explain their thinking regarding why this answer was correct and, if applicable, why their answer was incorrect. Students then practiced additional problems using CCC.

Students in the CCC + EPF condition reviewed the individualized bar graph showing the number of problems that they had solved using each of the target strategies. They were then presented with four problems. They were asked to compare the way that they solved the problem with a model demonstrating solving the problem using the same strategy. Students were asked to explain their thinking regarding why this strategy worked, and if applicable, why their method did not. Students then practiced additional problems using CCC.

Sprint. During the *sprint* component, all students were provided with a four-page SSM practice packet containing 36 total problems of the target skill. Following the recommended administration from Shapiro (2011) students were told they would have five minutes to answer as many problems as they could while showing their work. Following administration procedures adapted from Shinn (1989), they were told that there were more problems in the packet than they could complete, and they should not expect to finish all the problems. If they made a mistake, they could cross out the answer and write a new one, and they could skip questions if they did not know how to complete the problem. After they were told to begin working, students were discretely timed. After five minutes, students were instructed to lay down their pencil and all *sprint* packets were collected.

Scoring

After each session, the SSMs were scored for the accuracy and fluency of the completed items. Additionally, the problem-solving strategies used were coded to identify the efficiency of strategy use. Problems with at least one digit of the answer written were considered complete and were scored. Problems that were not attempted or were started but did not have any digits in the answer written were considered incomplete and not scored.

Fluency of Problem Solving. A fluency score was calculated as the number of correct digits (CD) in completed problems. The appropriate digits in the proper place value column in the answer were scored as correct. Inappropriate digits, digits written in the wrong place value column, or digits omitted in the answer were scored as incorrect digits (ID; Shapiro, 2011).

Accuracy of Problem Solving. Accuracy was calculated as the total number of CD in completed problems divided by the total number of CD and ID multiplied by 100 to create a percentage.

Coding of Strategy Use. The strategies that students used to solve each multiplication were assigned an efficiency score based on the rubric developed by Zhang, Xin et al. (2014). The original rubric included five categories (a) the use of incorrect operations (e.g. 12 x 14 = 26), (b) unitary counting (i.e. directly modeling the problem and then counting to get the total), (c) double counting or repeated addition (e.g. 8 + 8 + 8 + 8), (d) decomposition (e.g. 97 x 10 is solved as 90 x 10 and 7 x 10), and (e) direct retrieval or multiplication algorithm (i.e. following a standard sequence of operations to multiply the multiplicand by each digit in the multiplier and then adding the results).

Since the application of this standard multiplication algorithm requires the memorization of facts, the algorithm was categorized with direct retrieval (Zhang, Xin et al., 2014). Since this coding rubric closely aligns with the categories utilized the CGI framework of problem-solving strategies and research regarding the developmental sequence for multi-digit multiplication and division (e.g. Carpenter et al., 2015, Downton, 2008), it was applied in this study. Some categories were renamed for closer alignment with the CGI framework and the use of an array model was included (Barmby et al., 2009). The final rubric for this study identified strategies as (a) incorrect operation; (b) direct modeling or counting, (c) adding strategies, (d) decomposition, or array models, and (e) direct retrieval or the standard algorithm.

Each strategy was assigned a value ranging from 1 to 5 indicating the level of underlying conceptual understanding and efficiency represented by the category (e.g. Carpenter et al., 2015; Siegler, 2006; Zhang, Xin et al. 2014). The first three values were assigned to strategies which represent no or less advanced understanding of multiplication. Incorrect operations were assigned a value of 1. Next, direct modeling or counting and adding strategies were assigned values of 2 and 3 respectively. Both strategies rely on an additive understanding of the relationship between numbers which becomes increasingly inefficient as the factors increase in magnitude and is limited in its generalizability (e.g. Barmby et al., 2009; Clark & Kamii, 1996).

In contrast, the use of an array, decomposition, or the standard algorithm for multiplication represent an understanding of multiple mathematical concepts including place-value, multiplicative reasoning, and the distributive and commutative properties of multiplication (e.g. Barmby et al., 2009; Clark & Kamii, 1996; Raveh et al., 2016). Decomposition incorporates a variety of algorithms that may rely on additive and multiplicative relationships as students partition, manipulate, and recombine numbers (Lampert, 1986; Young-Loveridge & Mills, 2009). The application of a decomposition strategy both relies on and supports the understanding of the base-ten place value system and the distributive property (e.g. Kilpatrick et al., 2001; Lampert, 1986). Using an array strategy provides a visual representation of the commutative property as the directionality of the array does not change the total value (Barmby, 2009). If an array utilizes a grid depiction, it can support the use of derived facts and the application of the distributive property. As with decomposition, an array relies on multiplicative reasoning by simultaneously accounting for the size of the set and the replications of that set (e.g. Barmby et al., 2009; Clark & Kamii, 1996).

The standard algorithm provides another example of the interconnections between procedures and conceptual knowledge. As with decomposition, the distributive property supports the separate multiplication of parts of the multiplicand (Raveh et al., 2016). Correctly applying the standard algorithm in multi-digit multiplication both relies on and helps students develop an understanding of the base-ten place value system and regrouping of the multiplicand (e.g. Kilpatrick et al., 2001; Raveh et al., 2016). The standard algorithm also requires the use of derived facts in order to be applied successfully (Zhang, Xin et al., 2014). While direct retrieval would not be expected to be a commonly used strategy for multi-digit multiplication, cognitive models suggest that students can acquire the memorization of facts given sufficient exposure and practice (Siegler, 1988). Although individuals may achieve retrieval through the application of a variety of underlying strategies (Sherin & Fuson, 2005), the ability to quickly state the product of two factors is related to increased fluency and accuracy in single-digit multiplication and minimizing working memory demands for more complex problems (e.g. Geary, 2006; Zhang, Xin et al., 2014).

Given the multifaceted conceptual understanding applicable for all these strategies, the difference in assigning the value of 4 to decomposition and array strategies and a value of 5 to direct retrieval and the standard algorithm relied on differentiation due to efficiency. The ability to apply a strategy which is generalizable and efficient is important for mathematics proficiency (Kilpatrick et al., 2001). Research has demonstrated that, other than for specific number combinations (e.g. 11 x 12, numbers x 10), retrieval or the standard algorithm are the most efficient problem-solving approaches for adults (Geary et al., 1986). Table 2 provides additional information and examples of each strategy.

Table 2

Coding Rubric for Multi-digit Multiplication Strategies

Coding value	Strategy type	Definition
1	Incorrect operation	Response is an erroneous application of an operation (e.g. 28 x 15 = 43 or 28 x 15 = 2815).
2	Counting strategies	The factors in the problem are represented using diagrams/tallies which are counted to solve the problem. Figure 1 demonstrates solving with direct modeling.
3	Adding strategies	One number is decomposed into groups, each represented by a numeral, and added together such that one factor is used to coordinate the counting sequence for the other (e.g. 4 x 12 = 12 + 12 +12 + 12).
4	Array models	The problem is visually represented as vertical and horizontal sets. Figure 2 demonstrates solving with an array model.
4	Decomposition	One or both factors are broken down and solved using knowledge of a derived fact, tens and ones, or a doubling strategy creating a series of simpler multiplication problems to solve (e.g. 14 x 3 is solved using 10 x 3 = 30 and 4 x 3 = 12 with 30 + 12 = 42 or 8 x 7 = (4 x 7) x 2).
5	Standard algorithm	The problem is written as a vertical set and solved using the standard columnar algorithm of multiplying the multiplicand by the multiplier and then adding the proper values.
5	Direct retrieval	The student provides the answer through memorization of the multi-digit multiplication fact without any need to write down steps of the problem.

Note: Adapted from Zhang, Xin, et al. (2014) and Carpenter et al. (2015).

Average Efficiency of Strategy Use. Strategies were coded from 1 to 5, with 1 as the least efficient strategy (i.e. incorrect operations) and 5 as the most efficient strategy (i.e. direct response/standard algorithm). Models of the cognitive processes used for selecting and applying strategies suggest that students use multiple strategies concurrently (Siegler, 2006). Flexibility in strategy use can be an indicator of conceptual understanding if students are varying the selected strategy based on the specific factors in the multiplication problem (e.g. Carpenter et al., 2015). Therefore, students were expected to use multiple strategies within and across sessions. In order to represent strategy use by session, an average efficiency score was calculated by weighting the frequency of each strategy use by the coding value and dividing by the number of problems completed (Kanive, 2016). The maximum average efficiency score was when a student used the direct retrieval or standard algorithm for all completed problems. For example, if student completed 20 items, used an incorrect operation twice, did not use direct modeling, counting or an array, attempted to use an addition strategy six times, used decomposition eight times, and the standard algorithm three times, the average score would be 3.6 ([((2 x 1) + (6 x 3) + (8 x 4) + (4 x 5)]/20 = 3.6). The average efficiency score was calculated for each session and used to represent strategy use. In the descriptive analyses and when included in the hierarchical linear models as a predictor, this variable was treated as continuous.

Preliminary analyses demonstrated that the distribution of the average efficiency score was highly skewed and multimodal with scores primarily clustered at 5.0. Across students, the average efficiency score was 5.0 in 642 sessions (60.7%) indicating that students frequently used the direct retrieval or standard algorithm strategies for all completed problems. Therefore, for the fourth research question regarding the effect of feedback on strategy use, the efficiency of strategy use was treated as a dichotomous variable. The variable represented that students earned an average efficiency score of 5.0 indicating that they used the direct retrieval or standard algorithm strategies for all completed problems in the session (1) or that they earned an average efficiency score below 5.0 by using any other strategy at least once (0).

Post-intervention Testing

After the last sessions of the treatment condition, students completed the KIP to assess social validity of the intervention. At this time, teachers also completed the IRP-15 to assess their perceptions of the intervention. In subsequent sessions, students completed the WJ-IV calculation subtest and conceptual understanding assessments as measures of their post-intervention skill level and the selected scales of the PALS-Student to measure their post-intervention self-efficacy and achievement goal orientations.

Missing Data

Missing data was assessed for each outcome variable. Most students (n = 63, 62.4%) completed all progress monitoring sessions. One student missed six sessions due to leaving the school in the middle of the study. Additionally, data was missing due to student absences one student missed three sessions, eight students missed two sessions, and 28 students missed one session. Overall, this resulted in 4.8% of cases missing data on the progress monitoring variable. By individual session, the percentage of missing data ranged from 0% on the pre-test to 6.9% (*n* = 7) students absent from the first, fifth, sixth, and eighth sessions. A missing values analysis indicated that Little's (1988) test of Missing Completely at Random (MCAR) was not significant, $\chi^2 = 205.8364$, DF = 195, $p = 0.28$. A significant result on Little's test suggests that the hypothesis that the data are MCAR can be rejected. Given the non-significant result and the documentation that the reasons for missing data were students absence from class or transferring schools, there is no evidence to suggest that the data were not MCAR. All progress monitoring data was dependent in that missing a session resulted in missing data on all outcome variables of growth. As such, listwise deletion was used for the growth model. The final growth model included 1058 cases nested within 101 students.

For the WJ-Calculation variables, all students completed the test pre-intervention and the only missing data post-intervention was from the student who transferred out of the school. For the measure of conceptual understanding (Burns et al., 2018), one student did not complete the test pre-intervention and two students did not complete the test post-intervention. Missing values analyses indicated that Little's (1988) test was not significant for either the WJ-Calculation variables, $\chi^2 = 1.75$, DF = 2, $p = 0.42$, or the conceptual variables, $\chi^2 = 1.38$, DF = 3, $p = 0.71$. Given the non-significant result and the documentation that the reasons for missing data were students absence from class or transferring schools, there is no evidence to suggest that the data were not MCAR. As such, pairwise deletion was used for the tests of covariance.

Data Analysis

Longitudinal Modeling

Hierarchical linear modeling (HLM; Raudenbush & Bryk, 2002) was used to investigate the effect of condition assignment on students' mathematics growth. A two-level model was used with individuals as the Level-2 unit and repeated measures over time as the Level-1 unit (Figure 3). HLM was selected for analysis to partition variance based on (a) individual student progress over time with repeated measures nested within students and (b) differences in performance as a function of treatment group membership. Additionally, HLM maximizes the use of collected data despite missing values which increases power and accuracy of parameter estimation (Singer & Willett, 2003). The fluency outcome was the number of correct digits (CD) on the SSMs from the ten intervention sessions. Through the model building process, the relationship between treatment condition (RP, CCC + ETF, CCC + EPF) and the fluency outcome were examined when controlling for demographic characteristics, accuracy, pre-intervention fluency, growth rate, the effect of the average pre-intervention fluency of students' peers in the classroom, and the efficiency of strategy use. Data was centered on scores from the final intervention session so that the intercept could be interpreted for scores in the final session. Accuracy and strategy use in level-1 were group centered and the corresponding average scores were included at level-2 in order to differentiate change over time for each student and differences between students (e.g. Wu & Wooldridge, 2005). The continuous variables at level-2 were grand centered to allow for interpretation of the intercept as an adjusted mean (Raudenbush & Bryk, 2002).

Figure 3

Hierarchical Linear Model Building Process for Predicting Fluency in Digits Correct

Step 1: Unconditional model

$$DigitsCorrect_{ij} = \beta_{00} + r_{0i} + e_{ti}$$

Step 2: Effect of condition on fluency across sessions

$$DigitsCorrect_{ij} = \beta_{00} + \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + r_{0i} + r_{1i}(\text{Session}_{ti}) + e_{ti}$$

Step 3: Effect of condition on fluency when controlling for the average pre-intervention fluency of peers in the individual's class and individual demographic variables

$$DigitsCorrect_{ij} = \beta_{00} + \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + r_{0i} + r_{1i}(\text{Session}_{ti}) + e_{ti}$$

Step 4. Effect of condition on fluency when controlling for accuracy and previously identified controls

$$DigitsCorrect_{ij} = \beta_{00} + \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{07}(\text{Individual's Average Accuracy}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + \beta_{20}(\text{Accruacy}_{ti}) + r_{0i} + r_{1i}(\text{Session}_{ti}) + e_{ti}$$

Step 5: Effect of condition on fluency when controlling for the individual's pre-intervention fluency, relevant interactions, and previously identified controls

$$DigitsCorrect_{ij} = \beta_{00} + \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{07}(\text{Individual's Average Accuracy}_i)$$

$$+ \beta_{08}(\text{Individual's Pre-intervention Fluency}_i)$$

$$+ \beta_{09}(\text{Pre-intervention Fluency*Condition\_EPF}_i) + \beta_{010}(\text{Pre-intervention}$$

$$\text{Fluency*Condition\_ETF}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + \beta_{11}(\text{Pre-intervention Fluency}_i\text{*Session}_{ti})$$

$$+ \beta_{20}(\text{Accruacy}_{ti})$$

$$+ r_{0i} + r_{1i}(\text{Session}_{ti}) + e_{ti}$$

Step 6: Effect of condition on fluency when controlling for the individual's strategy use, relevant interactions, and previously identified controls

$$DigitsCorrect_{ij} = \beta_{00} + \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{07}(\text{Individual's Average Accuracy}_i)$$

$$+ \beta_{08}(\text{Individual's Pre-intervention Fluency}_i)$$

$$+ \beta_{09}(\text{Pre-intervention Fluency*Condition\_EPF}_i) + \beta_{010}(\text{Pre-intervention}$$

$$\text{Fluency*Condition\_ETF}_i)$$

$$+ \beta_{011}(\text{Individual's Average Strategy Efficiency Score}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + \beta_{11}(\text{Pre-intervention Fluency}_i\text{*Session}_{ti})$$

$$+ \beta_{20}(\text{Accruacy}_{ti})$$

$$+ \beta_{30}(\text{Strategy Use Efficiency Score}_{ti})$$

$$+ r_{0i} + r_{1i}(\text{Session}_{ti}) + e_{ti}$$

Note: For Step 3 and subsequent steps, Class Average Pre-intervention Fluency was centered around the grand mean. For Step 4 and subsequent steps, Accuracy by Session was centered around the group mean and the Individual's Average Accuracy were centered around the grand mean. For Step 5 and subsequent steps, the Individual's Pre-intervention Fluency, and Pre-intervention Fluency by Condition interactions were centered around the grand mean. For Step 6, the Strategy Use Efficiency Score by Session was centered around the group mean and the Individual's Average Strategy Efficiency were centered around the grand mean.

A second hierarchical model was tested to examine the longitudinal effects of feedback on strategy use (Figure 4). As previously described, due to the homogeneity in students' strategy use, the average efficiency of strategy use was treated as a dichotomous variable and a hierarchical generalized linear model (HGLM; Raudenbush & Bryk, 2002) was used with a two-level logistic regression to examine the relationship between the treatment conditions and the odds of only using the most efficient strategies (i.e. direct retrieval or the standard algorithm). Through the model building process, this relationship was tested when controlling for student demographic characteristics, the effect of the average fluency of the students' peers in the classroom, accuracy, fluency, and growth rate. Pre-intervention fluency was also tested but was removed for parsimony given non-significant contributions to the model. Data was centered on scores from the final intervention session. Using the HLM 8 software, HGLM involves Bernoulli sampling model and a logit function (Raudenbush et al., 2019):

$$\eta_{ij} = \log \frac{\Phi_{ij}}{(1 - \Phi_{ij})}$$

such that η_{ij} represents the log of the odds of earning an average efficiency score of 5.0 and Φ_{ij} represents the probability of success at .5. If the probability of a 5.0 efficiency score is < .5, then η_{ij} will be negative, and if the probability is > .5, it will be positive. Therefore, odds ratios are reported for this model in addition to the coefficients. Odds ratios are appropriately interpreted in comparison to 1.0, with an odds ratio > 1.0 indicating a greater likelihood of a result of 1 (i.e. used only the standard algorithm or direct retrieval strategies) and a odds ration < 1.0 indicating a decreased likelihood.

Figure 4

Hierarchical Generalized Linear Model Building Process for Predicting Strategy Use

Step 1: Unconditional model

$$\textit{Log Odds of Strategy Efficiency Score of } 5.0_{ij} = \beta_{00} + r_{0i}$$

Step 2: Effect of condition on strategy use across sessions

$$\textit{Log Odds of Strategy Efficiency Score of } 5.0e_{ij} = \beta_{00}$$

$$+ \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + r_{0i} + r_{1i}(\text{Session}_{ti})$$

Step 3: Effect of condition on strategy use when controlling for individual demographic variables and the average pre-intervention fluency of peers in the individual's class,

$$\textit{Log Odds of Strategy Efficiency Score of } 5.0e_{ij} = \beta_{00}$$

$$+ \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + r_{0i} + r_{1i}(\text{Session}_{ti})$$

Step 4: Effect of condition on strategy use when controlling for accuracy and previously identified controls

$$\textit{Log Odds of Strategy Efficiency Score of 5.0}e_{ij} = \beta_{00}$$

$$+ \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{07}(\text{Individual's Average Accuracy}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + \beta_{20}(\text{Accuracy}_{ti})$$

$$+ r_{0i} + r_{1i}(\text{Session}_{ti})$$

Step 5: Effect of condition on strategy use when controlling for fluency and previously identified controls

$$\textit{Log Odds of Strategy Efficiency Score of 5.0}e_{ij} = \beta_{00}$$

$$+ \beta_{01}(\text{Condition\_EPF}_i) + \beta_{02}(\text{Condition\_ETF}_i)$$

$$+ \beta_{03}(\text{Male}_i) + \beta_{04}(\text{Person of Color}_i) + \beta_{05}(\text{Receives Special Education Services}_i)$$

$$+ \beta_{06}(\text{Class Average Pre-intervention Fluency}_i)$$

$$+ \beta_{07}(\text{Individual's Average Accuracy}_i)$$

$$+ \beta_{08}(\text{Individual's Average Fluency}_i)$$

$$+ \beta_{10}(\text{Session}_{ti}) + \beta_{20}(\text{Accuracy}_{ti})$$

$$+ \beta_{30}(\text{Fluency}_{ti})$$

$$+ r_{0i} + r_{1i}(\text{Session}_{ti})$$

Note: For Step 3 and subsequent steps, the average Pre-intervention Fluency for the class was centered around the grand mean. For Step 4 and subsequent steps, Accuracy by Session was centered around the group mean. The Individual's Average Accuracy was centered around the grand mean. For Step 5, Fluency by Session were centered around the group mean and the Average Fluency were centered around the grand mean.

Given the multiple research questions posed in this study, multiple comparisons are required. The application of multiple tests creates a risk for identifying statistically significant effects due to Type 1 error. Traditional approaches require the application of a Bonferroni correct to adjust the *p*-value based on the number of tests performed. Although a Bonferroni correction limits chance significance due to multiple comparisons (i.e. Type 1 error), it does so at the expense of increase the risk for a false-negative result (i.e. Type 2 error) and can reduce the power for detecting an effect. Within a Bayesian multilevel modeling framework, a robust model can provide a more reliable point estimate through the process of partial pooling and more effectively minimize the effects of both Type 1 and Type 2 errors (Gelman et al., 2012). Therefore, a multilevel estimate is appropriately more conservative despite the inclusion of multiple fixed effects and comparisons within the multilevel model are valid without an adjustment of *p*-values (Gelman et al., 2012). No adjustments were made to the *p*-values within each longitudinal model. However, this approach does not account for the risk from applying two independent longitudinal models. Therefore, a Bonferroni correction was applied to control for chance significance across models by applying the $\alpha = .05/2$ and comparing *p*-

values to the standard of .025 to identify significant effects for the two longitudinal outcomes (Howell, 2013).

Post-test Comparisons

Differences in performance post-intervention on the WJ-IV calculation subtest and conceptual measure by condition were assessed using analyses of covariance (ANCOVA) while controlling for pre-intervention scores. Additionally, differences in student goal orientations and self-efficacy as measured by the PALS-Student were assessed post-intervention using analyses of variance (ANOVA). Given the increased risk for chance significance given the multiple post-test comparisons and the use of independent tests of variance, a Bonferroni correction was applied with $\alpha = .05/4$ and the standard of .0125 for *p*-value comparisons (Howell, 2013). Effect sizes for post-test comparisons were as eta squared, η^2, (Richardson, 2011).

Treatment Integrity

One graduate student that did not implement the intervention was trained on the intervention procedures and observed two intervention sessions (20%) with each group. Adherence to the treatment implementation was assessed using a checklist describing the procedural steps. Fidelity was calculated by dividing the total number of correctly implemented steps by the sum of the number of correctly and incorrectly implemented steps and multiplying by 100 to create a percentage. Across sessions and interventionists, adherence to the intervention procedures average 95% with a range of 85% - 100% of components completed. Feedback was provided to interventionists regarding missed components to increase adherence in future sessions.

Inter-Scorer Agreement

Independent scorers were graduate students enrolled in a school psychology program. Each was trained on the scoring and coding scheme with practice materials. Twenty percent of randomly selected SSMs (*n* = 212) and pre- and post-intervention generalization measures (*n* = 84) were scored by independent scorers. Comparisons were made between the first author's scores on the worksheets and the scores computed by the independent scorer. Inter-scorer agreement was computed by dividing the number of digits in agreement by the sum of digits in agreement and disagreement and multiplying by 100 to obtain a percentage. Across outcome measures agreement ranged from 96% - 99%.

CHAPTER 4

Results

Purpose and Research Questions

The purpose of the current study is to extend the literature on feedback in whole number interventions by examining the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) with a cover, copy, compare (CCC) intervention as compared to a performance in a control condition with repeated practice of mathematics facts but no feedback (RP). Due to an interest in the impact of feedback over time, hierarchical linear modeling (HLM) and hierarchical generalized linear modeling (HGLM; Raudenbush & Bryk, 2002) were used to answer four research questions guiding this study:

1. What is the effect of condition (CCC + ETF, CCC + EPF, RP) on students' final scores and growth rates on a measure of multi-digit multiplication fluency?
2. Do the treatment effects depend on students' initial fluency?
3. Do treatment effects depend on the efficiency of strategy use?
4. What is the effect of condition on the efficiency of strategy use?

Two additional questions were answered using analyses of covariance (ANCOVA) and analyses of variance (ANOVA).

5. What are the effects of CCC + ETF and CCC + EPF on the generalization of mathematics skills (WJ-IV calculation subtest and conceptual understanding)?
6. What are the effects of CCC + ETF and CCC + EPF on students' goal orientations and self-efficacy of mathematics skills?

Descriptive Analyses

Descriptive statistics by condition were reported for all achievement measures (see Table 3). Baseline equivalence was established between groups in fluency ($F(2, 99) = 2.19$, $p = .12$) and accuracy ($F(2,99) = 2.02$, $p = .14$) on the single-skill multi-digit multiplication measure. No significant differences between the three conditions were identified for the demographic descriptors for gender, race/ethnicity, or receiving special education services.

Table 3

Student Performance by Time, Condition, and Measure

	CCC + EPF		CCC + ETF		Vocabulary		Total			
Measures	*n*	*Mean (SD)*	*N*	*Mean (SD)*	*n*	*Mean (SD)*	*n*	*Mean (SD)*	*Skew*	*Kurtosis*
Pre-intervention Fluency	33	17.48 (10.94)	34	15.65 (9.78)	34	21.50 (14.39)	101	18.22 (12.01)	0.92	1.08
Sprint										
Average Fluency	33	24.03 (14.08)	34	19.88 (10.87)	34	25.19 (14.94)	101	23.03 (13.47)	1.51	-1.02
Average Accuracy	33	0.76 (0.19)	34	0.75 (0.18)	34	0.82 (0.15)	101	0.78 (0.18)	-1.33	-1.54
Average Efficiency	33	4.75 (0.39)	34	4.64 (0.39)	34	4.63 (0.44)	101	4.67 (0.41)	-0.79	1.20
Generalization										
WJ-IV Calculation										
Pretest	33	28.67 (3.68)	32	27.78 (3.47)	34	29.12 (3.27)	99	28.54 (3.49)	-0.43	-0.03
Posttest	33	30.85 (4.40)	32	30.03 (4.24)	34	31.56 (3.30)	99	30.83 (4.01)	-0.69	1.18
Conceptual Understanding										
Pretest	33	11.73 (3.98)	32	11.81 (4.35)	34	13.26 (3.64)	99	12.28 (4.02)	-0.75	-0.45
Posttest	33	12.61 (3.51)	32	12.28 (4.37)	34	13.88 (2.97)	99	12.94 (3.68)	-1.00	0.36

	CCC + EPF		CCC + ETF		Vocabulary		Total			
Measures	*n*	*Mean (SD)*	*N*	*Mean (SD)*	*n*	*Mean (SD)*	*n*	*Mean (SD)*	*Skew*	*Kurtosis*
PALS – Student										
MG	26	4.22 (0.65)	24	4.47 (0.50)	24	4.34 (0.63)	74	4.34 (0.60)	-0.95	-0.02
PG	26	2.14 (0.79)	24	2.38 (1.13)	24	2.15 (1.24)	74	2.22 (1.06)	0.75	-0.59
CM	26	4.27 (0.56)	24	4.49 (0.51)	24	4.30 (0.60)	74	4.35 (0.56)	-0.91	0.24
CP	26	2.78 (1.01)	24	2.97 (1.35)	24	3.27 (0.95)	74	3.00 (1.12)	-0.01	-0.95
AE	33	3.87 (0.64)	28	4.11 (0.75)	32	4.20 (0.74)	93	4.06 (0.71)	-1.11	1.38
Social Validity										
KIP	33	26.48 (6.15)	30	27.30 (6.10)	32	28.03 (4.49)	95	27.26 (5.61)	-0.41	0.08

Note: CCC + EPF = cover, copy, compare with elaborated process feedback; CCC+ ETF = cover, copy, compare with elaborated task feedback; WJ-IV = Woodcock-Johnson Test of Achievement-IV; PALS = Patterns of Adaptive Learning Survey; MG = mastery goal orientation; PG = performance-approach goal orientation; CM = perceptions of classroom mastery goal structure; CP = perceptions of classroom performance-approach goal structure; AE = academic self-efficacy; KIP = Kids Intervention Profile.

Hierarchical Linear Analyses

Impact of Feedback Over Time on Fluency

The first three research questions examined the effect of feedback on students' fluency in digits correct and the differential effects of initial fluency and efficiency of strategy use. All three questions were answered by fitting successive hierarchical linear models (HLM) with full-maximum likelihood estimation using HLM 8 (Raudenbush et al., 2019). First, an unconditional model (Step 1) with no level 1 or 2 predictors (β_{00} = 23.03) was tested to determine whether multilevel modeling was appropriate (see Table 4). Based on the intraclass correlation coefficient (ICC) $\rho = .74$, the use of a two-level model was supported (Raudenbush & Bryk, 2002). Due to the potential nesting of students within classrooms, a three-level model was tested but not supported ($\rho < .001$).

Additional variables were added in five stages to evaluate the impact of feedback on fluency scores (see Figure 3). The second stage (Step 2) included only the longitudinal variable at level-1 and the treatment conditions at level-2. Then, in Step 3, student demographic characteristics of gender, race/ethnicity, and whether the student was receiving special education services were incorporated into the model. The average pre-intervention fluency of peers in the classroom was also added to control for the effects of differences in multi-digit multiplication fluency across the four classes. Other variables were tested to control for differences by classroom, but the class-wide mean fluency score contributed to the best model fit (Raudenbush & Bryk, 2002). Due to missing demographic data, two students were removed from the model in Step 3 and subsequent steps. Next, in Step 4, the individual's group-centered accuracy by session and average accuracy for the intervention were included. Accuracy was added to the model as a centered first-level predictor and an averaged second-level predictor to disentangle the session-level and individual level effects of accuracy (Wu & Wooldridge, 2005). In addition to affecting student performance within each session, the student's overall accuracy with multi-digit multiplication may have a contextual effect on their fluency and their response to feedback. Then, in Step 5, the individual student's initial fluency was controlled for by adding in the pre-intervention fluency as a main effect, the interactions of pre-intervention fluency with the conditions, and an interaction effect with growth over time. Finally, the effects of strategy use were added in Step 6 as a centered first-level predictor, averaged second-level predictor, and a first-level interaction with pre-intervention fluency rate. This created the final model. The steps of the model-building process are presented in Figure 4; the results of the model are summarized in Table 4.

Table 4

Predicting Fluency Rates by Condition, Accuracy, Pre-intervention Fluency Level, and Strategy Use

	Step 1 n = 101	Step 2 n = 101	Step 3 n = 99	Step 4 n = 99	Step 5 n = 99	Step 6 n = 99
Parameters	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)
Fixed effects						
Intercept, β_{00}	23.03 (1.33)	29.85*** (2.26)	36.04*** (2.67)	32.12*** (2.34)	27.02*** (1.98)	27.14*** (1.96)
Treatment: CCC + EPF, β_{01}		-2.37 (2.60)	-3.08 (2.44)	-1.71 (1.99)	0.72 (2.29)	0.52 (2.27)
Treatment: CCC + ETF, β_{02}		-4.00 (2.57)	-4.51 (2.42)	-2.75 (1.99)	0.88 (2.24)	0.56 (2.22)
Male, β_{03}			-5.53** (2.04)	-3.65 (1.68)	-0.70 (1.04)	-0.62 (1.03)
POC, β_{04}			-6.33** (2.08)	-4.00* (1.71)	-0.51 (1.05)	-0.63 (1.04)
SPED, β_{05}			-3.43 (3.24)	-2.27 (2.65)	-1.04 (1.62)	-0.81 (1.60)
Class average PIF, β_{06}			-0.00 (0.36)	-0.03 (0.29)	-0.57** (0.18)	-0.63*** (0.18)
Average accuracy, β_{07}				28.64*** (4.98)	9.83** (3.42)	10.36** (3.40)
PIF, β_{08}					0.99*** (0.12)	0.96*** (0.12)
PIF by CCC + EPF interaction, β_{09}					-0.00 (0.10)	-0.00 (0.10)
PIF by CCC + ETF interaction, β_{010}					-0.05 (0.11)	-.04 (0.10)

Parameters	Step 1 n = 101 β (SE)	Step 2 n = 101 β (SE)	Step 3 n = 99 β (SE)	Step 4 n = 99 β (SE)	Step 5 n = 99 β (SE)	Step 6 n = 99 β (SE)
Average strategy use, β_{011}						1.51 (1.40)
Time varying						
Time, β_{10}		-0.94*** (0.11)	-0.94*** (0.11)	-0.79*** (0.11)	-0.79*** (0.11)	-0.78*** (0.10)
PIF on time, β_{11}					-0.03** (0.01)	-0.03** (0.01)
Accuracy, β_{20}				19.73*** (1.19)	19.09*** (1.16)	18.81*** (1.16)
Strategy use, β_{30}						1.51** (0.54)
Random Effects						
Intercept, r_0	173.82*** (13.18)	281.40*** (16.78)	283.28*** (16.83)	237.00*** (15.39)	143.21*** (11.97)	140.18*** (11.84)
Slope, r_1		0.86*** (0.93)	0.87*** (0.93)	0.89*** (0.94)	0.79*** (0.89)	0.77*** (0.88)
Residual, e	61.03 (7.81)	41.53 (6.44)	41.81 (6.47)	32.32 (5.68)	32.29 (5.68)	32.14 (5.67)
Model Summary						
Deviance statistic	7698.14	7398.31	7282.89	7018.31	6907.63	6898.76
Estimated parameters	3	8	12	14	18	20

Note: PIF = pre-intervention fluency; CCC + EPF = cover, copy, compare with elaborated process feedback; CCC+ ETF = cover, copy, compare with elaborated task feedback; POC = person of color; SPED = receiving special education services

*p <.025, **p < .01, *** p <.001

Effects on Fluency (Digits Correct). This model predicted the effects of participating in the intervention on the number of digits correct on the single-skill measure (SSM). The intercept represents the digits correct in the final intervention session for a White, female student in the control group (Repeated Practice; RP) who did not receive special education services, was in the class with average pre-intervention fluency, had scores at the mean for the sample on pre-intervention fluency, overall accuracy, and efficiency in strategy use, given that individual student's average accuracy and strategy use across the intervention. The model indicates that this student would have approximately 27 digits correct in the final session.

In the fully specified model, controlling for the other predictors, the outcome of digits correct in the final session was significantly impacted by (a) the individual's growth over time, (b) the class average pre-intervention fluency, (c) the individual's pre-intervention fluency, (d) the individual's average accuracy, (e) the individual's accuracy over time, and (f) the individual's strategy use over time. Additionally, pre-intervention fluency was a significant covariate for the slope. These significant effects are contextualized below. Non-significant effects were found for both treatment conditions, the interactions of treatment condition and pre-intervention fluency, each of the demographic variables, and student's average strategy use.

The significant effect of time indicates growth of nearly one digit correct (β = -0.78) for each session of the intervention controlling for the additional predictors in the model. This coefficient is negative due to the coding of the sessions such that the final session was the 0 value.

The class average pre-intervention fluency was included in the model to control for differences across the four classrooms (β = -0.63). This was an inverse effect in that for approximately each two-digit increase in the average pre-intervention fluency for the class, there was a one-digit decrease in digits correct at the end of the intervention, controlling for the additional predictors in the model.

The individual' pre-intervention fluency also had a significant main effect on digits correct with a one-digit difference in score on the pre-intervention assessment corresponding with approximately a one-digit faster rate after completing the intervention (β = 0.96). Additionally, pre-intervention fluency had small but significant impact on the rate of change in fluency over time with higher initial fluency increasing the rate of change (β = -0.03). The negative coefficient is again related to the coding of the time variable.

Controlling for the other predictors in the model, a significant effect was found for accuracy both within- and across-students (β = 18.81, 10.36 respectively). Therefore, both increases in student's accuracy (compared to their own performance) and higher average accuracy (compared to other students in the sample) corresponded to more digits correct in the final session. Given that accuracy was presented as a decimal ranging from 0-1, the coefficients represent a very small change in fluency for each increase in one percentage point of accuracy.

Controlling for the other predictors in the model, a significant effect was found for strategy use only within-students (β = 1.51). This indicates that a one-unit change in average efficiency of strategy use (e.g. using all repeated addition to using all decomposition), compared to the student's typical strategy use, corresponded with approximately 1.5 more digits correct in the final session.

Variance. The significant random effects for both the intercept and slope indicate significant variance in both students' final digits correct and their growth rates. Comparisons of variance estimates across the model building steps indicates the final model represents the best fit.

Assumptions. Following the recommendations for hierarchical linear modeling (Raudenbush & Bryk, 2002), the final model was assessed for violations of the assumptions of independent level-1 errors with constant variance and a level-2 multivariate normal distribution of random effects. The model converged within acceptable iteration limits. Assessments of distributions of the level-1 predictor (i.e. performance across sessions), the level-2 intercepts and slopes as indicated by the Empirical Bayes and fitted values estimates, and the cross-level residual variance based on ordinary least squares estimation indicated no violations of independence. The evaluation of the residual variance based on the final fitted fixed effects indicated no violation of the assumptions for normality and homogeneity of standard errors at level-1. At level-2, the distribution of the Mahalanobis distance indicated a positive skew; however, when examined in conjunction with the expected values, the distribution of random effects indicated sufficient normality to meet assumptions.

Impact of Feedback Over Time on Strategy Use

The fourth research question examined the effect of feedback on students' efficiency of strategy use and the differential effects of fluency and initial fluency. An average efficiency score was calculated for the strategy use in each session. This score represented the number of items attempted using each strategy type weighted by the coding value and divided by the number of problems attempted (Kanive, 2016). Strategies were coded from 1 to 5, with 1 as the least efficient strategy (i.e. incorrect operation) and 5 as the most efficient strategy (i.e. direct response/standard algorithm; Kanive, 2016). Due to the homogeneity in students' strategy use, the efficiency of strategy use was treated as a dichotomous variable for this analysis to represent that students earned an average efficiency score of 5.0 indicating that they used the direct retrieval or standard algorithm strategies for all completed problems in the session (1) or that they earned an average efficiency score below 5.0 by using any other strategy at least once (0).

This research question was answered by fitting successive hierarchical generalized linear models (HGLM) with penalized quasi-likelihood (PQL) estimation of the unit-specific model using HLM 8 (Raudenbush et al., 2019). First, an unconditional model (Model 1) with no level 1 or 2 predictors (β_{00} = 0.75) was fitted to determine whether multilevel modeling was appropriate (see Table 5). The ICC was estimated using the random intercept logistic model ρ = .73 (Wu et al., 2012) and the use of a two-level model was supported (Raudenbush & Bryk, 2002).

Additional variables were added in five stages to evaluate the impact of feedback on the odds ratios of using only the direct retrieval or standard algorithm strategies. First, the longitudinal variable at level-1 and the treatment conditions at level-2 were added. Next, in Step 3, the student demographic characteristics of gender, race/ethnicity, and whether the student was receiving special education services were included in the model. Also at this step, differences in initial skill level by classroom were controlled through the inclusion of the average digits correct for the class as a grand-centered variable. Other variables were tested to control for differences by classroom, but the classwide mean fluency score contributed to the best model fit (Raudenbush & Bryk, 2002). Due to missing demographic data, two students were removed from the model at Step 3 and subsequent steps. In Step 4, the individual's group-centered accuracy by session and average accuracy for the intervention were included. As in the hierarchical linear model, accuracy was added to the model as a centered first-level predictor and an averaged second-level predictor to disentangle the session-level and individual level effects of accuracy (Wu & Wooldridge, 2005). Finally, in Step 5, the individual's group-centered fluency by session and average fluency for the intervention were included. This created the final model. The steps of the model-building process are presented in Figure 4; the results of the model are summarized in Table 5.

Table 5

Predicting Strategy Use by Condition, Accuracy, Fluency, and Pre-intervention Fluency Level

	Step 1 n = 101		Step 2 n = 101		Step 3 n = 99		Step 4 n = 99		Step 5 n = 99	
Parameters	β (SE)	OR	β (SE)	OR	β (SE)	OR	β (SE)	OR	β (SE)	OR
Fixed Effects										
Intercept, β_{00}	0.78* (0.32)	2.19	1.10 (0.61)	2.99	1.62 (0.76)	5.04	1.46 (0.79)	4.32	0.95 (0.78)	2.59
Treatment: CCC + EPF, β_{01}			0.45 (0.78)	1.57	0.30 (0.76)	1.35	0.39 (0.76)	1.47	0.33 (0.74)	1.39
Treatment: CCC + EPF, β_{02}			-0.17 (0.77)	0.84	-0.19 (0.74)	0.83	-0.08 (0.76)	0.92	0.19 (0.74)	1.20
Male, β_{03}					-0.72 (0.63)	0.49	-0.64 (0.66)	0.53	-0.32 (0.63)	0.73
POC, β_{04}					0.10 (0.64)	1.10	0.21 (0.66)	1.24	0.46 (0.64)	1.58
SPED, β_{05}					-1.44 (0.98)	0.24	-1.30 (0.99)	0.27	-1.49 (0.96)	0.23
Class average PIF, β_{06}					0.38*** (0.11)	1.47	0.37** (0.11)	1.45	0.36** (0.11)	1.43
Average accuracy, β_{07}							1.44 (1.83)	4.24	-1.58 (2.05)	0.21
Average fluency rate, β_{08}									0.09** (0.03)	1.09
Time varying										
Time, β_{10}			-0.07 (0.04)	0.93	-0.07 (0.04)	0.93	-0.07 (0.04)	0.93	-0.03 (0.04)	0.97
Accuracy, β_{20}							-0.67 (0.63)	0.51	-1.62* (0.70)	0.20

	Step 1 n = 101		Step 2 n = 101		Step 3 n = 99		Step 4 n = 99		Step 5 n = 99	
Parameters	β (SE)	OR	β (SE)	OR	β (SE)	OR	β (SE)	OR	β (SE)	OR
Digits correct, β_{30}									0.06** (0.02)	1.06
Random Effects										
Intercept, r_0	8.56*** (2.93)		11.36*** (3.37)		11.06*** (3.33)		10.73*** (3.28)		10.38*** (3.22)	
Slope, r_1			0.03 (0.17)		0.03 (0.17)		0.03 (0.16)		0.02 (0.15)	

Note: PIF = pre-intervention fluency; CCC + EPF = cover, copy, compare with elaborated process feedback; CCC+ ETF = cover, copy, compare with elaborated task feedback; POC = person of color; SPED = receiving special education services; OR = odds ratio

*p <.025, **p < .01, *** p <.001

Multivariate tests for comparing model fit based on deviance are not available for the model predicting strategy use because the use of PQL in HGLM is based on quasi-likelihood estimation, not maximum-likelihood estimation (Raudenbush et al., 2019). Therefore, model fit was assessed in the reduction of variance across the stages of model-building.

Effects on Strategy Use. This model predicted the effects of participating in the intervention on the likelihood of using the most efficient strategy types (i.e. having a 5.0 average strategy use score by using only the standard algorithm or direct retrieval strategies). For this model, the intercept represents the likelihood that the outcome equals 1 (i.e. having a 5.0 average strategy use score) rather than 0 (i.e. having an average strategy use score below 5.0) for the final intervention session for a White, female student in the control group (Repeated Practice; RP) who did not receive special education services, was in the class with average pre-intervention fluency, had scores at the mean for the sample for overall accuracy and fluency, given that individual student's average accuracy and fluency across the intervention. In the final model, the intercept was not significant indicating that the null hypothesis that the intercept may equal zero cannot be rejected.

In the fully specified model, controlling for the other predictors, the likelihood of using only the standard algorithm or direct retrieval strategies was significantly impacted by (a) the class average pre-intervention fluency, (b) the individual's accuracy over time, (c) the individual's average fluency, and (d) the individual's fluency in digits correct over time. These significant effects are contextualized below. Non-significant effects were found for both treatment conditions, each of the demographic variables, and the effect of time. The non-significant longitudinal effect indicates that the progression of time across the intervention did not change the likelihood that students would use the direct retrieval or standard algorithm strategies.

The average pre-intervention fluency of peers in the classroom was included in the model to control for differences across the four classrooms. This significant effect indicated that after controlling for other variables in the model, students in the classrooms with higher pre-intervention fluency had an increased likelihood of using only the standard algorithm and the direct retrieval strategies.

After controlling for the over variables in the model, within-student accuracy had a significant effect on the likelihood of using only the most efficient strategies (β = -1.62). A one unit increase in the student's accuracy across time compared to their own performance corresponded to an odds ratio of 0.20 times as likely to have a score of 5.0. In other words, compared to students' own performance and controlling for other variables in the model, lower accuracy corresponded to an increased likelihood of using only the standard algorithm and direct retrieval strategies. Accuracy across-students (i.e. compared to other students in the sample) was not significant.

Finally, controlling for other variables, fluency in the number of digits correct was significant both within- and across-student (β = 0.06, 0.09 respectively). Both increases in fluency at the session level (compared to their own average performance) and higher average fluency (compared to other students in the sample) corresponded with an increased likelihood of using only the standard algorithm and the direct retrieval strategies. Although significant, both effects are small with a one digit increase in fluency corresponding to odds ratios of 1.06 and 1.09 times as likely to have a score of 5.0.

Variance. Random effects for the intercept indicated significant variance in students' likelihood of using only the direct retrieval or standard algorithm strategies in the final session. Non-significant random effects for the slope indicated that there was not significant variance in how students' strategy use changed over time. Decreases in the intercept variance across the model building steps were used to assess the fit of the final model given that deviance statistics were not available in HGLM (Raudenbush et al., 2019).

Assumptions. Due to the dichotomous nature of the outcome variable, hierarchical logistic models do not meet the same assumptions of normality and homogeneity of variance with the level-1 residuals (Raudenbush & Bryk, 2002). The final model was assessed for violations of the assumptions of independence. The model converged within acceptable iteration limits. Assessments of distributions of the level-1 predictor (i.e. performance across sessions), the level-2 intercepts and slopes as indicated by the Empirical Bayes and fitted values estimates, and the cross-level residual variance based on ordinary least squares estimation indicated sufficient independence.

Generalization of Effects

The final research questions addressed the impact of feedback for generalizing mathematics skills and attitudes toward mathematics.

Effect of Feedback on Calculation Skills

The calculation subtest of the Woodcock-Johnson Test of Achievement-IV (WJ-IV; Schrank et al., 2014) was administered pre- and post-intervention as a standardized measure of computation skills. Data from 99 participants were included in the analysis with missing data for one participant and one outlier removed from the analysis. The average pre- and post-intervention scores are presented in Table 3. The distributions on both the pre- and post-intervention assessments were negatively skewed indicating that student performance was similarly clustered around higher scores on both measures. Given that analysis of covariance (ANCOVA) has been shown to be robust to violations of the assumption of normality (Blanca et al., 2018), the non-normal distributions were acceptable. No other violations to assumptions were noted.

ANCOVA was performed in R using the car package (Fox & Weisber, 2019) to examine the effects of feedback type on post-intervention scores with scores on the pre-intervention assessment serving as a covariate. The main effect of pre-intervention calculation score was a significant predictor of student performance on the post-intervention test ($F(704, 1) = 84.13$, $p < .01$) with a large effect $\eta^2 = .49$. Neither the main effect for feedback type nor the interaction effect with the pre-intervention scores were significant ($F(29,2) = 1.75$, $p = .18$; $F(27, 2) = 1.63$, $p = .20$, respectively).

Effect of Feedback on Conceptual Understanding

The conceptual understanding measure (Burns et al., 2018) was administered pre- and post-intervention to assess students' understanding of multiplication concepts of representation, reversibility, flexibility, generalization, associative property, and commutative property. Data from 99 participants were included in the analysis due to missing data for two participants. The average pre- and post-intervention scores are presented in Table 3. The distributions on both the pre- and post-intervention assessments were negatively skewed indicating that student performance was similarly clustered around higher scores on both measures. Given that analysis of covariance (ANCOVA) has been shown to be robust to violations of the assumption of normality (Blanca et al., 2017), the non-normal distributions were acceptable. No other violations to assumptions were noted.

ANCOVA was performed in R using the car package (Fox & Weisber, 2019) to examine the effects of feedback type on post-intervention scores with scores on the pre-intervention assessment serving as a covariate. The main effect of pre-intervention conceptual understanding score was a significant predictor of student performance on the post-intervention test (F (619, 1) = 91.37, p <..01) with a large effect η^2 = .52. Neither the main effect for feedback type nor the interaction effect with the pre-intervention scores were significant (F (9,2) = 0.66, p = .52; F (5, 2) = 0.39, p = .68, respectively).

Effect of Feedback on Attitudes About Mathematics

Motivation. After the completion of the intervention, students completed four scales from the *Patterns of Adaptive Learning Survey – Student* (PALS; Midgley et al., 2000) which assessed motivation using an achievement goal theory framework: (a) mastery goal orientations, (b) performance-approach goal orientations, (c) perceptions of classroom mastery goal structure, and (d) perceptions of classroom performance-approach goal structure. The survey was administered class-wide and, following standard administration procedures, the survey directions and items were read to students

(Midgley et al., 2000). The survey was printed with shading to help students differentiate the item responses for each question. Despite this guidance, students appeared to have difficulty following the survey directions to select one response for each item. Three students had missing data for the entire survey; 24 students had missing data for at least one scale due to errors in completing one or more items in the scale (i.e. selecting no responses or more than one response for the item).

Only students with complete data across scales were included resulting in a final sample of 74 students. The average post-intervention scores for each scale are presented in Table 3. The distributions for the individual mastery orientation and class mastery orientation scales were negatively skewed indicating that students similarly endorsed that these items were true. The distributions for the individual performance orientation was positively skewed indicating that students similarly endorsed that these items were not true, but the distribution for the class performance orientation was normally distributed indicating that students had varied responses to these items. Given that analysis of variance (ANOVA) has been shown to be robust to violations of the assumption of normality (Blanca et al., 2017), the non-normal distributions were acceptable. No other violations to assumptions were noted. A multivariate analysis of variance (MANOVA) was performed in R to examine the effects of feedback type on post-intervention perceptions of mathematics across PALS scales with no significant effects found ($F(71, 2) = 0.87$, $p = .54$).

Academic Self-efficacy. After the completion of the intervention, students completed one scale from the *Patterns of Adaptive Learning Survey – Student* (PALS;

Midgley et al., 2000) which assessed students' self-efficacy in mathematics and measured the extent to which the student anticipated mastering the skills they were taught in math class (Friedel et al., 2010). Three students had missing data for the entire survey and five students had missing data for the academic efficacy scale resulting in a final sample of 93 students. The average post-intervention scores for each scale are presented in Table 3. The distribution for academic efficacy was negatively skewed indicating that students similarly endorsed that they believed that they could master their mathematics content. Given that ANOVA has been shown to be robust to violations of the assumption of normality (Blanca et al., 2017), the non-normal distribution was acceptable. No other violations to assumptions were noted. An ANOVA was performed in R to examine the effects of feedback type on post-intervention self-efficacy with no significant effects found ($F(90, 2) = 1.86$, $p = .16$).

Social Validity

Two measures of social validity were administered to measure both student and teacher perceptions of the acceptability and usefulness of the intervention.

Student Perceptions. The Kids Intervention Profile (KIP; Eckert et al., 2017) was administered following the final intervention session as an assessment of students' perceptions of the social validity of the intervention. Following administration procedures used by Eckert et al. (2017), the directions, items, and response options were read aloud.

Possible total scores on the KIP ranged from 8 to 40. Higher scores on the KIP indicates greater social validity, with a total score greater than 24 indicating an acceptable rating for the intervention (Eckert et al., 2017). Six students had missing data resulting final sample of 95 students. The average social validity scores are presented in Table 3. The distribution for students' total rating of the intervention met assumptions of normality and homogeneity of variance indicating that students' perceptions of the intervention varied. There were no significant differences between groups ($F(92,2) = 0.61$, $p = .54$). Overall, 71% of students ($n = 67$) provided an acceptable rating for the intervention.

Teacher Perceptions. The Intervention Rating Profile-15 (IRP-15; Martens et al., 1985) was administered as a measure of teacher perceptions of the social validity of the intervention conditions. The measure was adapted to refer to classwide academic skills. Teachers received a copy of the IRP-15 for each of the treatment conditions (CCC + EPT and CCC + ETF) each accompanied by a cover sheet which included a description of the relevant condition and instructions to complete the survey based on their observations of the relevant condition. They returned the survey anonymously. Scores on the IRP-15 can range from 15-90 with higher scores indicating greater acceptability. Teachers provided similar moderate ratings of acceptability for both conditions with an average rating of 63 for CCC + EPF (range 57 – 69) and of 60 for CCC + ETF (range 48 – 66). One teacher indicated a strong preference for the process feedback condition rating it 17 points higher than task feedback. The other three teachers had differences less than 10 points between conditions.

Two teachers also provided written feedback regarding the intervention. One teacher suggested that the intervention could be strengthened by having students discuss their thinking with their peers and including additional teacher-led number talks (Parrish, 2011) to support student thinking. The other teacher commented that the first step of the process-feedback condition (i.e. *Review*) reinforced reflective learning but that the second step (i.e. the CCC *Practice*) was not well understood by her students. This teacher also commented that the task-feedback condition appeared to motivate the students with stronger multiplication skills but that students with more average skills became discouraged. It was suggested that the intervention might be more effective in a small group setting.

CHAPTER 5

Discussion

The objective of this chapter is to contextualize the results of this study within the extant literature. The chapter begins with a review of the purpose of the study. Next, the findings for each research question are presented. Then, potential implications for future practice and research are included. Lastly, the chapter concludes with strengths and limitations of the current study.

Purpose of the Study

The purpose of the current study is to extend the literature on the differential effects of specific feedback elements. To address an underrepresented area of mathematics research, this study targeted multi-digit multiplication computation. Specifically, this study examined the effects of combining elaborated task feedback (ETF) or elaborated process feedback (EPF) with a cover, copy, compare (CCC) intervention on multi-digit multiplication fluency and strategy use. Elaborated feedback provides information to help the recipient address misconceptions or correct errors (e.g. Harks et al., 2014; Shute, 2008). ETF provides information to address misconceptions and errors regarding the answer to a problem; EPF provides information to correct misconceptions and errors regarding the process used to complete the problem (e.g. Hattie & Timperley, 2007). Students in the comparison group received repeated practice (RP) of mathematics facts but no feedback in order to control for practice effects. Longitudinal models were used to examine the impact of the treatments over time. Additionally, the study aimed to contribute to research regarding the moderating effects of initial skill level on intervention effectiveness and the interaction of strategy use and fluency rates. Finally, this study examined the effects of feedback on the generalization of mathematics skills, goal-orientations, and self-efficacy.

Summary of Findings

Across measures, the impact of the treatment conditions (CCC + EPF and CCC + ETF) did not have significant effects on student performance. In both longitudinal models, the demographic characteristics of gender, race, and receiving special education services were not significant. However, the control variable of the average pre-intervention fluency of peers in the classroom was significant for both fluency and strategy use indicating that there were differences between classrooms. This is consistent with prior research demonstrating that instructional practices and teacher characteristics influence student achievement (e.g. Strong et al., 2011).

Results of the first longitudinal model indicated that initial skill level, accuracy, and strategy use were significant predictors of participants' fluency in digits correct. Across conditions, students increased their rate of responding with more digits correct over the course of the intervention. Additionally, higher fluency pre-intervention, greater accuracy during the intervention, and changes to using more efficient strategies all corresponded with completing more digits correct in the final session. Pre-intervention fluency also corresponded with faster growth.

Results of the second longitudinal model indicated that increases in fluency compared to the students' own performance and higher average fluency in the intervention both corresponded to an small increased likelihood of using only the standard algorithm or direct retrieval strategies. The effects of accuracy were more nuanced in impacting strategy use. Increases in accuracy compared to the student's own performance corresponded with decreased likelihood of using only the standard algorithm or direct retrieval strategies, but differences between students based on overall accuracy was not significant. These results are contextualized below.

Relationship Between Feedback and Fluency

The first two research questions examined the effect of each type of feedback + CCC on fluency and the moderating effect of initial skill level.

Research Question 1: What is the Effect of Condition on Final Scores and Growth Rates on a Measure of Multi-digit Multiplication Fluency?

The first research question examined the effect of feedback on students' fluency. Results indicated that while students across conditions demonstrated increased fluency over the course of the intervention at a rate of nearly one additional digit correct for each session (β = -0.78, $p < .001$), fluency scores were not impacted by either treatment condition of CCC combined with feedback. These results do not replicate the findings in previous research that multicomponent fluency interventions which combine repeated practice, correction, and motivational components enhance the effect of the intervention on fluency rates (e.g. Duhon et al., 2015; Joseph et al., 2012) and are contrary to research demonstrating the positive effects of feedback in mathematics interventions (e.g. Duhon et al., 2015; Fyfe et al., 2015). However, these results are consistent with meta-analyses demonstrating that the effect of feedback is highly variable (e.g. Gersten, Chard et al., 2009; Kluger & DeNisi, 1996). The effect of feedback may be moderated by components of the feedback, underlying intervention, or participant characteristics (e.g. Hattie & Gan, 2011; Kluger & DeNisis, 1996).

Additionally, the fluency growth across conditions reinforces the impact of deliberate practice in mathematics (e.g. Fuchs et al., 2010). Despite research indicating that students benefit from frequent, repeated practice through discrete learning trials (e.g. Clarke et al., 2016), few mathematics textbooks and instructional sequences provide sufficient opportunities to practice computational skills (Doabler et al., 2012). In this study, students in all conditions received five-minutes of deliberate, skill-specific multiplication practice through the sprint component of the intervention. Core instruction in the participating schools did not include this type of discrete learning trial fluency-building activities. Previous studies have demonstrated that intensive skill-specific practice may be the most effective intervention component in mathematics and that adding additional intervention components does not reliably improve performance beyond the effects of practice (e.g. Codding et al., 2007; Fuchs et al., 2010; Powell et al., 2009). The effectiveness of repeated practice in increasing students' fluency may have outweighed any additional benefit incurred from the modeling and feedback provided in the treatment conditions.

Research Question 2: Do the Treatment Effects Depend on Students' Initial Skill Level?

The effect of treatment was insignificant despite controlling for initial skill level in the fluency model. However, consistent with previous research (e.g. Clark et al., 2019; Burns et al., 2015), students' initial skill level had a significant impact on students' fluency ($\beta = 0.96$, $p < .001$) and their rate of improvement ($\beta = -0.03$, $p < .01$). This result indicated that students with higher initial fluency demonstrated both more growth over the course of the intervention and higher fluency post-intervention.

Timed practice has been supported as an effective fluency-building intervention (e.g. Clark et al., 2016; Codding et al., 2011). Consistent with research on the skill-by-treatment interaction (Burns et al., 2010), students with higher fluency rates were expected to be highly responsive to that practice. This was reflected in that students with higher initial fluency demonstrated more improvement from the intervention across conditions.

Research on the skill-by-treatment interaction (Burns et al., 2010) would also suggest that receiving an acquisition intervention such as CCC should result in large effects for students with frustration level skills. However, participating in the CCC intervention in the treatment groups did not appear to benefit students with lower initial fluency as compared to the control group. Overall, students in the intervention had low initial rates of responding with an average 18.22 digits correct in five minutes or the equivalent of 3.64 digits correct per minute (dcpm). Although empirical research has not provided guidelines for differentiating student skill-level in multi-digit multiplication, extrapolating from research on single-digit computation (Burns et al., 2006) which identified scores below 24 dcpm indicative of performance in the frustrational range for fourth grade students, the average participant in this sample had rates of responding far below the frustrational skill level. Despite selecting multi-digit multiplication as the target for the intervention based on alignment with Common Core State Standards

(CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), the low average fluency indicates that many students continued to have skill deficits in this area. Increasing the intensity of the intervention through increasing the amount of instruction or the number or frequency of intervention sessions may have been necessary to demonstrate significant improvements and detect differences (e.g. Burns et al., 2015; Doabler et al., 2019; Duhon et al., 2009).

Relationships Between Feedback, Fluency, and Strategy Use

The third and fourth research questions examined the relationships between feedback, fluency, and strategy use. Computational fluency involves the quick, accurate recall of facts and the flexible, efficient application of strategies (Baroody, 2011). Therefore, a hypothesized bidirectional relationship between fluency and strategy use was examined across models. First, strategy use was added as a predictor to the fluency model (third research question). Then, fluency was examined as a moderator of the effect of feedback on strategy use (fourth research question).

Research Question 3: Do the Treatment Effects Depend on the Efficiency of Strategy Use?

Contrary to the hypothesis that the efficiency of strategy use would moderate the effectiveness of feedback such that students using less efficient strategies would benefit more from process-based feedback, the effect of feedback type was insignificant despite the inclusion of strategy use in the fluency model. No prior studies were identified which examined the impact of strategy use in moderating the effect of feedback. The lack of differentiation in the current study could result from the underlying skill deficits discussed previously or that other characteristics of the intervention and learners were more salient in moderating the effects of feedback (e.g. Kluger & DeNisi, 1996).

Research Question 4: What is the Effect of Condition on the Efficiency of Strategy Use?

In the second longitudinal model which examined the impact of feedback on strategy use, there were no significant differences between groups on strategy use. This is contrary to the hypothesis that students in the CCC + EPF group would use more types of problem-solving strategies and would use efficient strategies more frequently than students in other groups. Alibali (1999) found that feedback alone was not enough to change students' strategy use; significant effects were found only in conditions which included explicit instruction on strategy use. Although all students in this study received one lesson on problem-solving strategies from the interventionist and their teachers reported that they had been taught all of the targeted strategies in their core mathematics instruction, the feedback and CCC intervention may not have been sufficient to motivate changes in strategy use. Cognitive models suggest that students use a new strategy when that strategy is perceived as more accurate and efficient (e.g. Carpenter et al., 2015; Siegler, 2006). The elaborated feedback provided in the current study focused on the accuracy of strategy application and may not have sufficiently cued students to consider differences in the efficiency of strategy use.

Bi-directional Relationship of Fluency and Strategy Use

Although the effect of feedback was not significant in predicting either fluency or strategy use, other variables were significant. When controlling for the other variables in the model, strategy use by session had a significant effect corresponding to more than a one-digit increase in the final fluency score (β = 1.51, p < .01), and both within-student changes in fluency (Odds ratio = 1.06, β = 0.06, p < .01) and across-student differences in average fluency (Odds ratio = 1.09, β = 0.09, p < .01) corresponded with a slight increased likelihood of using only the direct retrieval or standard algorithm. These effects suggest a bidirectional relationship between higher fluency scores, which are indicative of more advanced multiplication skills, and the use of more efficient strategies. This finding is consistent with both theoretical (Siegler, 1988, 2006) and empirical research (e.g. Rittle-Johnson et al., 2001; Zhang et al., 2014) that math achievement and efficient strategy use are developed iteratively.

The use of fluent, accurate, and efficient problem-solving strategies is theorized to facilitate the organization of mathematics facts into coherent knowledge networks, streamline recall, and further increase overall fluency (e.g. Siegler, 1988, 2006; Woodward, 2006). As such, the use of efficient strategies precipitates greater fluency and vice versa. The bidirectional relationship between fluency and efficiency in strategy use also suggests that students who used more efficient strategies were able to answer problems more quickly and therefore, complete more learning trials. Increasing the number of learning trials or opportunities to respond is a critical instructional component for increasing intervention effectiveness (e.g. Codding et al., 2019). Therefore, the current results further support recommendations to include independent practice on targeted skills with a high number of opportunities to respond to improve fluency as a component of class-wide mathematics instruction (e.g. Clarke et al., 2016; Gersten, Beckmann et al., 2009).

Results of both models also indicated that accuracy had a significant effect on fluency and strategy use. The positive relationship between accuracy and fluency are consistent with previous research that students improve fluency and accuracy with additional practice (e.g. Clark et al., 2019; Poncy et al., 2007). For strategy use, the effect of accuracy was more nuanced. Students' overall accuracy did not have a significant effect on the likelihood of using the most efficient strategies, but increases in accuracy compared to the student's own performance corresponded with decreased likelihood of using only the standard algorithm or direct retrieval strategies. This finding may be explained by contextualizing it with the overall consistency in strategy use indicated by the non-significant time variable. Students with mathematics difficulties or lower achievement tend to use less efficient strategies and to be less flexible in their strategy use, meaning that they use the same strategy despite differences in the characteristics of the problem (e.g. Lenmaire & Siegler, 1995; Zhang et al., 2014). Given the low initial skill level for many students in the sample, the overall consistency in strategy use indicated by the non-significant time variable is consistent with this prior research. Therefore, the finding that variability in accuracy across sessions corresponded with a negative relationship with the use of the standard algorithm and direct retrieval may indicate that students with variable accuracy were less prone to making procedural errors (e.g. forgetting to carry or incomplete factorization of partial products) when using a decomposition strategy and more prone to errors with the standard algorithm or direct retrieval. This would be consistent with research indicating that the most common multiplication errors are procedural, especially for students with lower skill levels (e.g. Hickendorff et al., 2019). Future research could include an error analysis to understand the relationships between strategy use, characteristics of the problem, and the types of errors that contributed to lower accuracy.

Distal Effects on Mathematics Skills and Beliefs

The final research questions examined the impact of feedback on enhancing the generalization of mathematics skills, affecting goal orientations, and increasing self-efficacy.

Research Question 5: What are the Effects of the Treatment Conditions on the Generalization of Mathematics Skills?

For the generalization measures of broad calculation skills and conceptual understanding, students' pre-intervention score was significant but participation in the treatment conditions was not. This result is consistent with research that treatment effects are specific to the skill targeted by the intervention with limited transfer observed for related mathematics skills (e.g. Bryant et al., 2019; Clarke et al., 2014; Fuchs et al., 2008). This intervention narrowly targeted computation with multi-digit multiplication. Prior research demonstrates that improved fluency generalizes to related skills (e.g. single- to multi-digit addition; VanDerHeyden & Burns, 2009) but not across mathematics skills unless students are taught a linking strategy (Poncy et al., 2010). In this study, the generalization measures assessed broad calculation skills (e.g. problems ranging from one-digit addition to calculus) and conceptual understanding of foundational multiplication principles (e.g. the associative and commutative properties with single-digit multiplication). These concepts were not directly targeted in the intervention; therefore, only prior knowledge predicted performance on either measure.

Research Question 6: What are the Effects of the Treatment Conditions on Goal Orientations and Self-efficacy of Mathematics Skills?

Goal Orientations. Students' individual goal orientations (e.g. their personal motivation for learning) and their perceptions of the goal orientations in their classrooms (e.g. students' perceptions of the emphasis from their teacher) were assessed post-intervention. Contrary to the hypothesis that process feedback may be more aligned to mastery goal orientations (Rakoczy et al., 2013), no significant effects were found by condition. Students across conditions endorsed both individual and classroom mastery goal orientations.

Mastery goal orientations are aligned with intrinsic motivation and a focus on learning as the goal; performance goal orientations indicate a focus on comparing performance to others and being perceived as smart or capable (Pintrich, 2000). While prior research demonstrated a positive relationship between mastery goal orientations and higher self-efficacy, effort, and achievement, the impact of performance orientations been mixed with studies demonstrating positive, negative, and null impacts on academic performance (e.g. Linnenbrink & Pintrich, 2002; Pintrich, 2000; Rakoczy et al., 2013; Skaalvik, 2018). Motivational theories of feedback suggest that process-oriented feedback will be more aligned with mastery orientations by connecting process components such as effort or problem-solving strategies to the attainment of learning goals (e.g. Dweck, 2008; Rakoczy et al., 2013; Schunk, 1995). However, across conditions most students endorsed being personally motived to learn and try even the most challenging tasks and that the focus in their mathematics classes was on developing an understanding of math, improving math skills, and accepting the importance of making mistakes (i.e. mastery orientation). Most students did not endorse that they felt a need to compete with other students or demonstrate that they were smarter than their classmates (i.e. performance orientation). Similarly, all four teachers reported that they emphasized a mastery orientation over a performance orientation in their instruction.

Research indicates that environmental factors and teachers' goal orientations influence students' goal orientations, especially in elementary school (Friedel et al., 2007; Zheng et al., 2019). Therefore, the strong focus on developing a mastery orientation in the core mathematics instruction at both schools may explain the personal mastery orientation endorsed by most students and the lack of differentiation by condition.

Self-efficacy. Given the instructional match of the CCC intervention to the low fluency rates demonstrated by most students (Codding et al., 2007) and previous research indicating that feedback on malleable factors (e.g. fluency or strategy use) has a positive effect on performance (e.g. Schunk 1983, 1984), students in the treatment groups were expected to demonstrate higher levels of self-efficacy. Contrary to this hypothesis, differences between groups were not significant. Although research has demonstrated a positive relationship between self-efficacy and student effort and engagement in classes (e.g. Martin & Rimm-Kaufman, 2015; Sakiz et al., 2012), the relationship with academic skill growth is less clear. Previous studies demonstrated both positive and null effects of higher self-efficacy on achievement (e.g. Cleary & Kitsantas, 2017; Marsh & O'Mara, 2008; Mercer et al., 2011). As Mercer et al. (2011) suggested, the effects of self-efficacy may only be detected when examined over longer time intervals; the short time frame of this study may limit the ability to identify differentiation. Additionally, previous research has demonstrated that self-efficacy is related to the adoption of a mastery goal orientation (e.g. Fadlelmula et al., 2015; Fast et al., 2010). Given that, across conditions, most students endorsed a high level of mathematics self-efficacy, it is possible that the strong focus on a mastery orientation reported across classes increased students' self-efficacy over and above the support of the intervention.

Implications

The results of the current study contribute to the research on feedback, class-wide mathematics interventions, and strategy use with multi-digit multiplication. Implications for areas of practice as well as future research are discussed. Potential implications are presented as considerations for research and practice and should be interpreted with caution.

Potential Implications for Practice

Given pervasive low performance in mathematics in the United Stated (NCES, 2019) and the limitations of time and resources for providing supplemental interventions, class-wide evidence-based interventions are important to foster mathematics achievement (e.g. Codding et al., 2019; VanDerHeyden et al., 2019). In the current study, multi-digit multiplication was selected as the target skill for alignment with the Common Core State Standards for fourth grade (National Governors Association Center for Best Practices &

Council of Chief State School Officers, 2010). Despite being a grade-level standard, the average rate of responding was below levels indicating mastery (Burns et al., 2006). Following recommendations for effective mathematics instruction (Gerten, Beckmann et al., 2009), class-wide intervention should include ten-minutes of practice of basic arithmetic facts. In this intervention, all students participated in five-minutes of deliberate, skill-specific multiplication practice; neither school included targeted, timed fact fluency practice as part of core instruction. Consistent with previous research, (e.g. Codding et al., 2007; Fuchs et al., 2010; Powell et al., 2009), results indicated that repeated, deliberate practice, even without the addition of modeling and feedback, contributed to improved performance. Elementary teachers can replicate that practice by including targeted practice for fluent fact retrieval as one component of core mathematics instruction.

Multiple effective class-wide procedures for developing fluency have been identified which provide frequent opportunities to practice in a short period of time (e.g. Fuchs et al., 1997; VanDerHeyden & Codding, 2015). Aligning student practice with their instructional level has been supported by meta-analyses of a skill-by-treatment interaction (Burns et al., 2010; Burns et al., 2014), suggesting the use of differentiated skill-based practice aligned to individual instructional levels or the use of the median level of class performance to target a class-wide instructional skill (Shapiro, 2011).

Potential Implications for Future Research

The current study contributes to the variability in extant literature regarding the effects of feedback. Meta-analyses of feedback have found that the effect of feedback is moderated by specific feedback components (e.g. Hattie & Timperley, 2007; Kluger & DeNisi, 1996). While this study controlled for multiple facets of feedback to compare task-based and process-based feedback it may be that other feedback components are more salient. Given the paucity of research on elaborated feedback, future studies should examine the effects of elaborated feedback when provided immediately on an item-by-item basis (e.g. Kulik & Kulik, 1988). Given the increasing prevalence of technology-based interventions, individualized feedback could be feasibly incorporated into a class-wide intervention when delivered in this medium.

Additionally, this study demonstrated the limitations in current research on multi-digit multiplication, the applicability of the instructional hierarchy for complex computation, and the relationship between problem type and strategy use. Intervention research has primarily focused on single-digit fact fluency as a foundational skill for later mathematics achievement (Codding et al., 2011). More research is needed to understand how fluency is most effectively facilitated in the context of complex computation such as multi-digit multiplication. No empirical studies were identified which provided decision points for identifying the appropriate instructional level for 2 x 2-digit multiplication. Given the additional procedural demands required for multi-digit computation, adapting the procedures used to empirically derive fluency criteria for single-digit computation (e.g. Burns et al., 2006) to apply to multi-digit computation is necessary to guide future decision-making and instructional match. While Zhang et al. (2014) used teacher ratings of high-, average-, and low-achieving students to compare the accuracy and flexibility of strategy use among elementary students with a limited sample of multiplication problems, additional research is needed to understand how students apply problem-solving strategies and whether strategies are differentially efficient depending on the factors in the problem (e.g. are students more efficient using a decomposition strategy for problems include a factor which is a multiple of 10).

Future research could extend the results of this study by varying the treatment intensity and frequency of intervention. This intervention was implemented twice per week. Previous research has suggested that effective mathematics interventions should be implemented four times per week (Codding et al., 2016). Increasing the dosage or frequency may have increased both the impact on students' math achievement and the differential effectiveness of the feedback.

Limitations

The results of the current study should be considered within the context of its limitations. First, given the paucity of research on multi-digit multiplication, estimations were made for determining instructional match and ratio of easier and more challenging problems to interleave. This may have resulted in student exposure to problems that were more or less difficult than advisable for effective skill-by-treatment implementation (e.g. Burns et al., 2010). The students' skill level could also impact the effectiveness of the feedback. Research indicates that the effects of feedback are variable, and some studies have found that feedback is less likely to be effective for learners with high levels of prior knowledge (e.g. Fyfe & Rittle-Johnson, 2016; Gielen et al., 2010). While pre-intervention fluency rates were considered in these models, the impact of feedback may have been impacted by the intervention to skill alignment.

Although primarily manipulating the type of elaborated feedback when added to a CCC intervention, this intervention included multiple components in the *Review, Practice, Sprint* format. Without a component analysis, effects cannot be attributed to one part of the intervention in isolation. Although multiple components of feedback were controlled in this study including the specificity, timing, and mode of delivery, students were required to read and interpret the feedback independently. Variation in how students processed the feedback may have impacted the effect. Some research has suggested that students need instruction on how to use feedback to improve their learning and that feedback which provides extraneous information may distract the learner from the task (e.g. Burke, 2009; Fyfe & Rittle-Johnson, 2017). Manipulating the delivery format through one-on-one verbal delivery, peer tutoring, or via technology may alter how students respond to the feedback.

The a priori power analysis indicated that a sample of 101 students was required to determine an effect. Although 101 students participated in the study, due to data loss in the pre- and post-intervention measures the final samples for the WJ-Calculation and Conceptual Understanding measures included 99 students. The use of a smaller sample could increase the risk of a false-negative result and decreases the ability to detect an effect of the intervention.

This study was implemented in two suburban school districts in the Midwest with a primarily White student population. The results of this study should be replicated in different settings and with other student populations to examine how individual characteristics may moderate the effects of the intervention.

Conclusion

The present study examined the differential effects of elaborated task feedback (ETF) and elaborated process feedback (EPF) when combined with a cover, copy, compare (CCC) intervention as compared to a control condition. Results support the importance of aligning interventions with students' instructional level given that initial skill level and accuracy both impacted fluency. Additionally, results support a bidirectional relationship of fluency and strategy use with students who were more fluent using the most efficient strategies (i.e. standard algorithm and direct retrieval). Regarding the effects of feedback, neither treatment condition resulted in significant differences in fluency or strategy use as compared to the control condition. This result contributes to the growing field of research examining how feedback components moderate the overall effect of feedback on student performance by indicating a null effect of written, elaborated feedback provided on complex computation.

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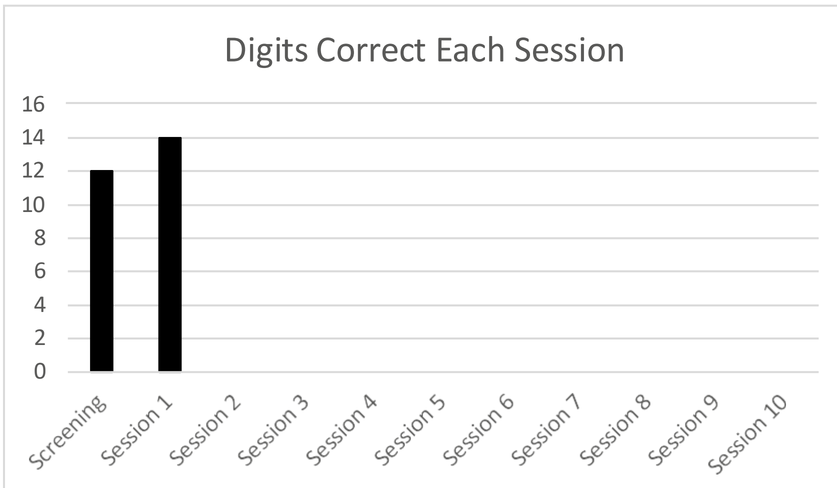
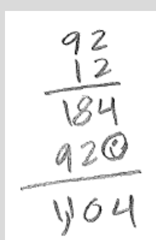
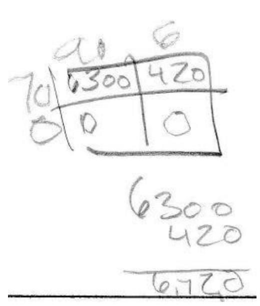
Appendix A. Example Intervention Packets

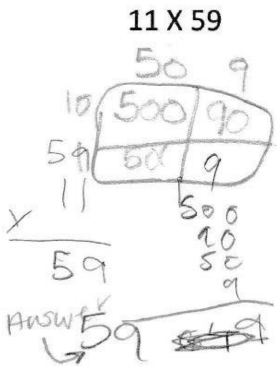
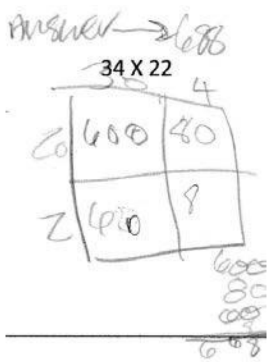
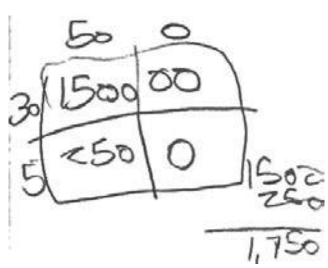
Cover – Copy - Compare with Elaborated Task Feedback (CCC + ETF)

Session 2
Review

This graph shows how many digits you have answered correctly in each session.



<u>EXAMPLE:</u>	This was your answer last time. 	Compare your answer to the correct answer. **92 x 12 = 1,104**	Why is this the correct answer? I know that the algorithm is a short way of seeing 12 as 10 + 2. I can multiply 2 x 92 and get 184. Then I multiply 10 x 92 and get 920. I added them together to get **1,104**.
This was your answer last time. 		Compare your answer to the correct answer. **70 X 96 = 6,720**	Why is this answer correct?

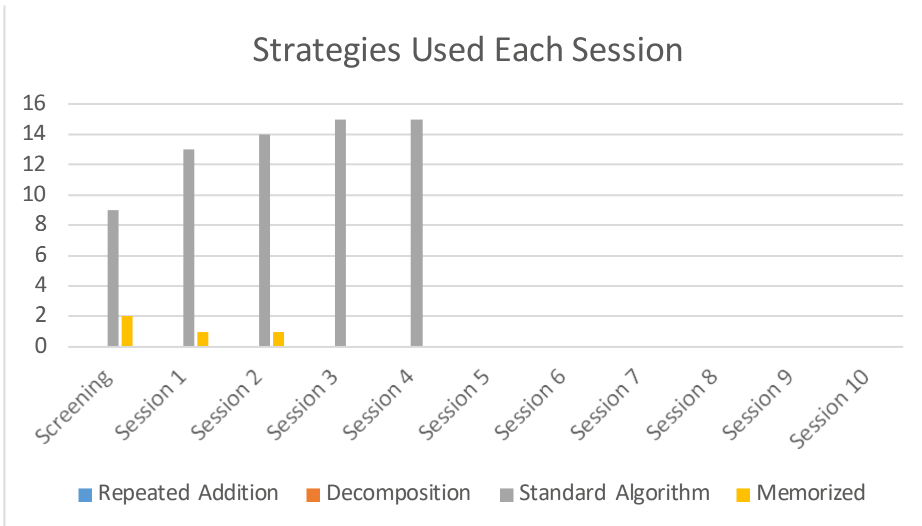
This was your answer last time. 	Compare your answer to the correct answer. **11 X 59 = 649**	Why is the answer **<u>in box 2</u>** correct? Why was **<u>your</u>** answer incorrect?
This was your answer last time. 	Compare your answer to the correct answer. **34 X 22 = 748**	Why is the answer **<u>in box 2</u>** correct? Why was **<u>your</u>** answer incorrect?
This was your answer last time. 	Compare your answer to the correct answer. **50 X 35 = 1,750**	Why is this answer correct?

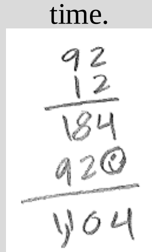
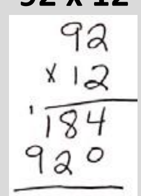
Cover – Copy - Compare with Elaborated Process Feedback (CCC + EPF)

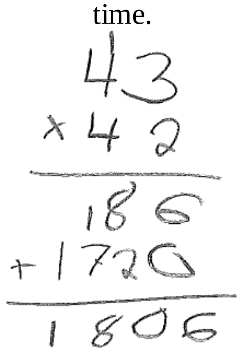
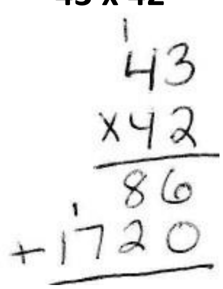
Session 5

Review

This graph shows how many times you used each strategy in each session.



<u>EXAMPLE</u>:	You used the STANDARD ALGORITHM strategy last time. 	Compare your work to the STANDARD ALGORITHM strategy used in this example. **92 x 12** 	Why did this strategy work? I know that the **standard algorithm** is a short way of seeing 12 as 10 + 2. I can multiply 2 x 92 and get 184. Then I multiply 10 x 92 and get 920. I added them together to get 1,104.

You used the STANDARD ALGORITHM strategy last time. 	Compare your work to the STANDARD ALGORITHM strategy used in this example. **43 x 42** 	Why did this strategy work?

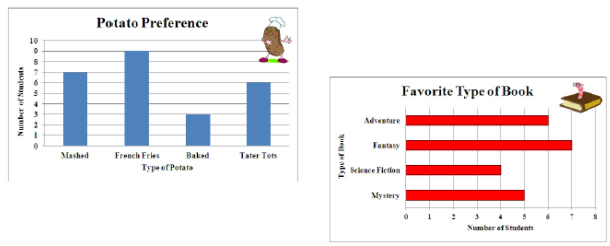
You used the STANDARD ALGORITHM strategy last time. 17 38 × 29 342 + 760	Compare your work to the STANDARD ALGORITHM strategy used in this example. **38 x 29** 1 7 38 x 29 1 342 + 760	Why did the strategy in **<u>box 2</u>** work? Why did **<u>your</u>** strategy not work?
You used the STANDARD ALGORITHM strategy last time. 97 × 11 197 970 1067	Compare your work to the STANDARD ALGORITHM strategy used in this example. **97 x 11** 97 x 11 97 1 + 970	Why did this strategy work?
You used the STANDARD ALGORITHM strategy last time. 13 × 13 39 130 199	Compare your work to the STANDARD ALGORITHM strategy used in this example. **13 x 13** 13 x 13 39 + 130	Why did the strategy in **<u>box 2</u>** work? Why did **<u>your</u>** strategy not work?

Repeated Practice (RP)

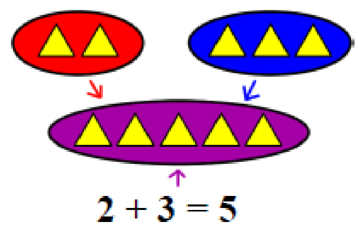
Session 7
Review

Draw a line to match the definition to the correct picture.

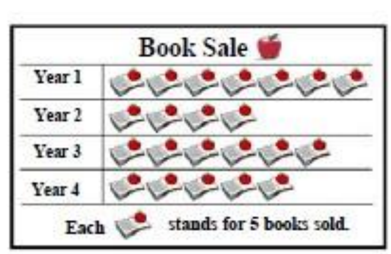
1. **Greater than:** Compares two numbers when the first number is larger than the second.

A. 

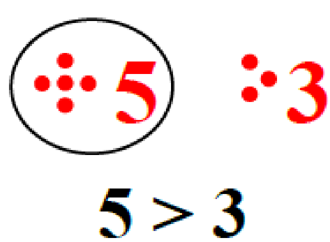
2. **Bar graph**: A graph drawn using bars to show data.

B. 

3. **Pictograph**: A graph that uses symbols and pictures to show data.

C. 

4. **Add**: An operation to solve a math problem. You combine two or more numbers.

D. 

Greater than: Compares two numbers when the first number is larger than the second.	Look at this example sentence: Because it has a higher value, 7 is **greater than** 3. I could also write this as 7 > 3. Write your own sentence using the word **greater than.**	Draw a picture of **greater than.**
Bar graph: A graph drawn using bars to show data.	Look at this example sentence: I could make a **bar graph** to show how many problems I solve each day. On Tuesday, I answered 10 problems. I draw a bar up to the number 10 on the graph. Write your own sentence using the word **bar graph.**	Draw a picture of **bar graph.**

Pictograph: A graph that uses symbols and pictures to show data.	Look at this example sentence: I made a graph and drew six apples for Monday. That **pictograph** shows how many apples I ate each day. Write your own sentence using the word **pictograph.** ________________ ________________ ________________	Draw a picture of **pictograph.**
Add: An operation to solve a math problem. You combine two or more numbers.	Look at this example sentence: I **add** 7 + 6 and the sum is 13. Write your own sentence using the word **add.** ________________ ________________ ________________ ________________	Draw a picture of **add.**

Appendix B. Instruction Lesson Protocols

CCC + ETF Instruction Protocol

Materials

❑ Student *Review & Practice* Training Packets ❑ Protocol

❑ Stopwatch ❑ Extra Pencils ❑ Index Cards

Procedures

Preparation

❑ 1. (Rebecca) Each *review* sheet includes an example graph and example problems for the CCC + ETF. Each *review* sheet is attached to a CCC *practice* packet.

❑ 2. Interventionist coordinates with the classroom teacher to meet with all students in the CCC + EFT condition in a small group.

Implementation (in small groups with only students assigned to CCC + ETF condition)

❑ 3. Interventionist introduces herself and the intervention. **(1 minute)**

Session #1:

- "Hello, my name is … I am from the University of Minnesota and I'm working with your teacher to do a study on how students learn math."
- "I will be coming to your room to practice math facts. Today, we will do our first practice. In the future, we will practice with the whole class. The activities we do in class will be the same as we do today."
- "Each time we practice we will have 3 steps. First, we will *review*, then we will *practice*, and finally, we will solve *math facts*."

❑ 4. Review procedures. **(10 minutes)**

- Distribute review & practice packets face down.
- "Leave your packet face down until I tell you to turn it over. Does anyone need a pencil? Ok, turn your packet over to see your *review* sheet. Every time we do this activity we will start by looking at the review."
- Introduce the graph and have students identify the meaning and value of each bar.
- "At the top of your review sheet there will be a graph that looks like this. This one is an example. The graph tells us how many digits we answered correctly in the previous session. If the problem has 4 numbers in the answer that problem has 4 digits. For example, in this problem the answer is *51*. There are 2 numbers so that is 2 digits. What does the first bar in our graph mean?"
    - Have students identify how many digits the example student got right each session. Point out that: "Our goal is to solve more problems and have the bars increase over the sessions. That will mean we are increasing our fluency or getting faster and more accurate at solving problems."

- Introduce the review problems and have student practice comparing and writing a sentence to explain their thinking about why their answer is correct or incorrect.
- "Next on your review sheet there will be 4 problems like this. These are examples. In the future, these problems will be ones that you have worked on in the previous session. It will show how you solved the problem. The second box will show you the correct answer."
- Instruct (I Do)
- "Let's look at the first one. In the first column we see example problem. This student solved and got the answer *51*. Let's look at the second column. It tells us that 51 is the correct answer. In the third box, it asks me to explain why 51 is the correct answer. I would write (write as you talk – student's do not need to write this one with you, but can if they wish).
    - "I know that 17 x 3 is 51 because I multiplied 3 times 7 and got 21. Then I carried the 2. I multiplied 3 by 1 and added the 2 to get 5. This gave me the answer of 51"
- Now let's look at the 2nd problem. In this problem, I got the answer of 60. The second box tells me that the correct answer is 69. The third box asks me to explain why the answer of 69 is correct *and* my answer is incorrect.
- What could I say?" Prompt students to provide an answer. If they don't, supply an explanation.
    - "When I solved, I decomposed the 23 into 20 plus 3. Then I multiplied 20 x 3 which gave me 60. I forgot to multiply the 3 x 3. That is why my answer was wrong. If I added that 9 (from the 3 x3) to the 60, I would have gotten the right answer of 69."
    - Point out: See how I explained ***Both*** why the other answer was correct and why mine was incorrect.
- Model (We Do)
- "Now, let's look at the third problem. When I look at box 1 and box 2 I can see that I got the wrong answer. In box 3 it asks why the answer in box 2 is correct and mine is incorrect" (*Note*: in this problem it's set up correctly but there was an addition error at the end)
- "Who has an idea of what we can write to explain our thinking?"
- Practice (You Do)
- "Now you practice by looking at the fourth problem. See what I did in the first box, the answer in the second box and explain your thinking about why the answer is box 2 is correct and mine is not."

❑ 5. Practice procedures. **(5 minutes)**

- "Flip to the next page in your packet. The second activity that we will do is to *practice*. For these problems we are going to use a strategy called Cover – Copy – Compare."
- Give everyone an index card.
- Instruct (I Do) demonstrate reading the CCC instructions at the top of the page and solving the first problem using each step.
    - "Let's look at the first one. In the first column we see an example problem with the answer. In the second and third columns there is the same problem but no answer. Look at the directions at the top of the page. First, I'm going to study the problem and how they solved it. *48 x 3 = 144*. Then, I'm going to cover it up with my index card and I'm going to solve the problem by showing my work in the second column. I don't need to solve the same way they solved in the

problem. (Emphasize this – they can use any strategy to solve). This is only one way to solve the problem. I can solve it anyway I want. Ok, so I looked at the problem and answer. Now I am covering the problem with my index card and writing the answer – including my work – on the next column.

  - Solve using *Standard Algorithm* – intentionally write the wrong answer.
  - Now the instructions tell me to uncover and compare my answer. Look, I wrote the wrong answer. I need to study the problem and answer in the first column again, cover them up, and write the correct answer in the third column. Now I uncover and compare. It's the right answer! I can go to the next problem.
- Model (We Do) Model with student participation solving the second problem.
  - "What's my first step? (We look at the problem.) And next? (cover and copy in the second box). Let's all write it down. (Solve using *Standard Algorithm* to show that they can use a different strategy – get the answer right).
  - What's next? Remember that you can look at the top of the sheet if you forget what to do next! Let's compare, do our answers match?"
  - Have students identify each step of CCC, correcting is a step is missed. Demonstrate completing each step and have students complete the problem on their paper.
- Practice (You Do)
  - "Now you practice solving the third and fourth the problems. Make sure to follow all of the steps."
  - Monitor and correct students if they are not following the CCC procedures.
- After students finish or 5 minutes – whichever comes first:
- "Stop. Pencils down."
- "I will take your index card. When we do this in class, we will have 10 minutes to do the *review and practice*. That might not be enough time to finish all of the problems, but that's ok. You'll just do as much as you can. Do you have questions about what we will be doing? Ok. In class we will also do 5 minutes of multiplication practice after we review and practice. We will not do that today, because you know how to do that.
- Collect the review & practice packets. If there are no more questions, escort students back to class.

CCC + EPF Instruction Protocol

Materials

❑ Student *Review & Practice* Training Packets ❑ Protocol

❑ Stopwatch ❑ Extra Pencils ❑ Index Cards

Procedures

Preparation

❑ 1. (Rebecca) Each *review* sheet includes an example graph and example problems for the CCC + EPF. Each *review* sheet is attached to a CCC *practice* packet. *Sprint* packets are prepared in advance with the school and date of the intervention session.

❑ 2. Interventionist coordinates with the classroom teacher to meet with all students in the CCC + EFF condition in a small group.

Implementation (in small groups with only students assigned to CCC + EPF condition)

❑ 3. Interventionist introduces herself and the intervention.

Session #1:

- "Hello, my name is … I am from the University of Minnesota and I'm working with your teacher to do a study on how students learn math."
- "I will be coming to your room to practice math facts. Today, we will do our first practice. In the future, we will practice with the whole class. The activities we do in class will be the same as we do today."
- "Each time we practice we will have 3 steps. First we will *review*, then we will *practice,* and finally, we will solve *math facts*."

❑ 4. Review procedures.

- Distribute review & practice packets face down.
- "Leave your packet face down until I tell you to turn it over. Does anyone need a pencil? Ok, turn your packet over to see your *review* sheet. Every time we do this activity we will start by looking at the review."
- Introduce the graph and have students identify the meaning and value of each bar.
- "At the top of your review sheet there will be a graph that looks like this. This one is an example. Do you remember the strategies that we practiced in class? Who can tell me one of the strategies for multiplication (standard algorithm, decomposition (including window and array), repeated addition)? The graph tells us how many times we used each strategy in the previous session. What does the first blue bar mean?"
    - Identify each strategy and point how the key is used to help read the graph.
- Introduce the review problems and have student practice comparing and writing a sentence to explain their thinking about why the strategy worked and why their answer didn't work.

- "Next on your review sheet there will be 4 problems like this. These are examples. In the future, these problems will be ones that you have worked on. It will show how you solved the problem. The second box will show how somebody else solved the problem."
- Instruct (I Do)
- "Let's look at the first one. In the first column we see example problem. This student used the *decomposition* strategy. Let's look at the second column. the second column the example problem also used the *decomposition* strategy, but they solved it differently than I did in box one. In the third box, it asks me to compare these strategies and explain why the strategy in box 2 worked and why my strategy did not work. I would write (write as you talk – student's do not need to write this one with you, but can if they wish).
    - "In box two, they decomposed the 23 into 20 plus 3. Then they multiplied 20 x 3 and 3 x 3. This added together made 69. That's why their strategy worked. I forgot to multiply the 3 x 3 when I decomposed. That is why may strategy didn't work"
    - Point out: See how I explained ***Both*** why the other strategy worked and why mine didn't work.
- "Let's look at the next one. In the first column we see example problem. In this problem, I used the *algorithm* strategy. It looks like the 2nd box and my answer were solved in the same way. The third box only asks me to explain why the strategy worked. What could I say? " Prompt students to provide an answer. If they don't, supply an explanation.
    - "I know that 17 x 3 is 51 because I used the *standard algorithm* to solve. I multiplied 3 times 7 and got 21. Then I carried the 2. I multiplied 3 by 1 and added the 2 to get 5. This gave me the answer of 51."
- Model (We Do)
- "Now, let's look at the third problem. I used *decomposition* again – the window method. Look at box 1 and box 2. In box 3 it asks why the strategy in the second box worked and my strategy didn't." (*Note*: in this problem it's set up correctly but there was an addition error at the end)
- "Who has an idea of what we can write to explain our thinking?"
- Practice (You Do)
- "Now you practice by looking at the fourth problem with *repeated addition*. See what I did in the first box, the example in the second box and explain your thinking about why the example worked and mine didn't."

☐ 5. Practice procedures. **(5 minutes)**

- "Flip to the next page in your packet. The second activity that we will do is to *practice.* For these problems we are going to use a strategy called Cover – Copy – Compare."
- Give everyone an index card.
- Instruct (I Do) demonstrate reading the CCC instructions at the top of the page and solving the first problem using each step.
    - "Let's look at the first one. In the first column we see an example problem with the answer. In the second and third columns there is the same problem but no answer. Look at the directions at the top of the page. First, I'm going to study the problem and how they solved it. *48 x 3 = 144*. Then, I'm going to cover it up with my index card and I'm going to solve the problem by showing my work in the second column. I don't need to solve the same way they solved in the problem. (Emphasize this – they can use any strategy to solve). This is only one way to solve the problem. I can solve it anyway I want. Ok, so I looked at the problem and answer. Now I am covering the problem with my index card and writing the answer – including my work – on the next column.
    - <u>Solve using *Standard Algorithm* – intentionally write the wrong answer.</u>
    - Now the instructions tell me to uncover and compare my answer. Look, I wrote the wrong answer. I need to study the problem and answer in the first column again, cover them up, and write the correct answer in the third column. Now I uncover and compare. It's the right answer! I can go to the next problem.
- Model (We Do) Model with student participation solving the second problem.
    - "What's my first step? (We look at the problem). And next? (cover and copy in the second box). Let's all write it down. (Solve using *Standard Algorithm* to show that they can use a different strategy – get the answer right).
    - What's next? Remember that you can look at the top of the sheet if you forget what to do next! Let's compare, do our answers match?"
    - Have students identify each step of CCC, correcting is a step is missed. Demonstrate completing each step and have students complete the problem on their paper.
- Practice (You Do)
    - "Now you practice solving the third and fourth the problems. Make sure to follow all of the steps."
    - Monitor and correct students if they are not following the CCC procedures.
- After students finish or 5 minutes – whichever comes first:
- "Stop. Pencils down."
- "I will take your index card. When we do this in class, we will have 10 minutes to do the *review and practice*. That might not be enough time to finish all of the problems, but that's ok. You'll just do as much as you can. Do you have questions about what we will be doing? Ok. In class we will also do 5 minutes of multiplication practice after we review and practice. We will not do that today, because you know how to do that.
- Collect the review & practice packets. If there are no more questions, escort students back to class.

Repeated Practice/Vocab Instruction Protocol

Materials

❑ Student *Review & Practice* Training Packets ❑ Protocol

❑ Stopwatch ❑ Extra Pencils ❑ Extra Index Cards

Procedures

Preparation

❑ 1. (Rebecca) Each *review* sheet includes vocabulary words, definitions, and images. Each *review* sheet is attached to a *practice* packet.

❑ 2. Interventionist coordinates with the classroom teacher to meet with all students in the RP condition in a small group.

Implementation (in small groups with only students assigned to RP/Vocab condition)

❑ 3. Interventionist introduces herself and the intervention.

Session #1:

- "Hello, my name is … I am from the University of Minnesota and I'm working with your teacher to do a study on how students learn math."
- "I will be coming to your room to practice math facts. Today, we will do our first practice. In the future, we will practice with the whole class. The activities we do in class will be the same as we do today."
- "Each time we practice we will have 3 steps. First we will *review*, then we will *practice*, and finally, we will solve *math facts*."

❑ 4. Review procedures.

- Distribute review & practice packets face down.
- "Leave your packet face down until I tell you to turn it over. Does anyone need a pencil? Ok, turn your packet over to see your *review* sheet. Every time we do this activity we will start by looking at the review."
- Introduce the review vocabulary activity.
- "Our *review* and *practice* activities will help us learn math vocabulary."
- "The first column has math vocabulary words and definitions. The second column has the pictures, drawings, or symbols that shows the meaning of the word."
    - My first job is to look at the definitions and find the picture in the second column that matches. Who can read the first vocab word and definition. Who can identify one match? Let's draw a line between them.
    - Continue until all vocab terms and pictures are matched.

❑ 5. Practice procedures.

- "After we do this *review*, the second activity that we will do is to *practice* the vocab words. Flip to the next page in your packet."
- Distribute index cards.

- “In the first column is the vocab word and definition. The second column has an example sentence and asks me to write a sentence. Let’s read the first one.”
- Instruct (I Do)
- “First, I will read the word and the definition for *sum* again. Then, I will read the example sentence for *sum*. Now, I’m going to write a sentence using this word. I would write (write as you talk – student’s do not need to write this one with you, but can if they wish).
    - My example might be: “When I add two numbers together the answer is the sum.”
- Next, in the third column, it asks me to draw a picture of sum. I can draw a model of an equation with three circles. In the first circle, I put 3 dots. In the second I put 4 dots. I put a plus sign between them and an equal sign before the last circle. In the last circle I would need 7 dots to represent the sum. I am drawing an arrow to point to the sum.
- Model (We Do)
- “Now, let’s look at the second one. Who can read the vocab word? Who can read the definition? Who will read the example sentence? Does someone have an example that we could write for our own sentence using *transformation*? We can also look back at the picture, if that helps.”
- “Now, who has an idea of a picture we could draw?”
- Practice (You Do)
- “Now you practice by looking at the third and fourth problem. Read the vocabulary word and definition. Then look at the example sentence and write your own sentence. Finally, draw a picture.”
- Monitor that students are correctly writing sentences and drawing pictures. Remind them to use the definition, example sentence, and pictures from the review to help.
- After students finish or 5 minutes – whichever comes first:
- “Stop. Pencils down.”
- “You can put your index card away. When we do this in class, we will have 10 minutes to do the *review and practice*. That might not be enough time to finish all of the problems, but that’s ok. You’ll just do as much as you can. Do you have questions about what we will be doing? Ok. In class we will also do 5 minutes of multiplication practice after we review and practice. We will not do that today, because you know how to do that.
- Collect the review & practice packets. If there are no more questions, escort students back to class.

Intervention Protocol

Materials

❑ Student *Review & Practice* Packets ❑ Student *Sprint* Packets

❑ Protocol ❑ Stopwatch ❑ Extra Pencils ❑ Extra Index Cards

Procedures

Preparation

❑ 1. (Rebecca) Each *review* sheet is individualized by student and condition and is labeled with the student name, school, and date of the intervention session. Each *review* sheet is attached to a *practice* packet by condition (CCC or vocab). *Sprint* packets are prepared in advance with the school and date of the intervention session.

❑ 2. Before entering the class, the interventionist verifies that they have all correct packets and materials.

Implementation

❑ 3. Interventionist introduces herself and the intervention.

Session #1:

- "Hello, my name is … I am from the University of Minnesota and I'm working with your teacher to do a study on how students learn math."
- "We are going to do math practice 2 times per week for 5 weeks. That means we'll do this practice 10 times. We are going to do the intervention like you practiced in your small groups."
- "First we will look at our *review* problems. Then we will do our *practice* packet, and finally we will do our *math facts* packet.

Sessions #2-10 :

- "We have another day of practice! This is session [#]. Remember that today we will first look at our review problems. Then we will do our *practice* packet, and finally we will do our *math facts* packet.

❑ 4. Review & practice procedures.

- Distribute review & practice packets face down and index cards.
- "Take out your pencil and your index card. If you need a pencil or an index card, raise your hand and I will give you one. I am passing out your review and practice packets. Set your index card to the side. Leave them face down until I tell you to turn them over."
- After all students have their *review* & *practice* packet say:
- "When I say to begin, you can turn your packet over. You will have 10 minutes to review the information on the packet and practice. Read the directions silently. If you have any questions, raise your hand and I will come answer them."
- "Ready. Begin."
- *Discretely* time for 10 minutes.

❑ 6. Sprint procedures.

- After 10 minutes tell students:
- "Stop. Pencils down."
- "Set your index card to the side. Keep your pencil. I will collect your *review* and *practice* packet and give you your *math facts* packet. Write your name on the back of your packet. Leave your *math facts* packet face down until I tell you to turn them over."
- Collect the review & practice packets and distribute the math facts (these the same for all students). While distributing, monitor that students are writing their name on the back of the packet.
- "You will have five minutes to answer as many problems as you can. Make sure you show all of your work so that I can see how you are solving the problem! There are more problems in this packet than you can answer. Do not worry if you do not finish the problems. If you make a mistake, cross it out and write the correct answer. You can skip problems if you do not know how to do it. Just do your best and make sure you always show your work!"
- "Pencil in the air." (Wait for the student to hold their pencil in the air.)
- "Ready. Begin."
- *Discretely* time for five minutes. After five minutes tell the students
- "Stop. Pencils down and turn your packet over and check that you wrote your name on the back. Then hold your packet in the air so I can collect them."
- Collect math fact packets. Monitor that students are not continuing to solve problems and that they have written their name on the back.

Treatment Integrity Checklist

- [ ] Interventionist has all materials prepared

- [ ] Preparation: Each *review* sheet is individualized by student and condition and is labeled with the student name, school, and date of the intervention session. Each *review* sheet is attached to a *practice* packet by condition (CCC or vocab). *Sprint* packets are prepared in advance with the school and date of the intervention session.

- [ ] Introduction: Interventionist reminds students of 3 parts (says *review, practice, math facts)*

- [ ] Review & Practice:

  Review & practice packets are distributed face down.
  Makes sure all students have a pencil.
  States that students will have 10 minutes to complete the review.
  Discretely times for 10 minutes.
  After 10 minutes tell students to put pencils down.

- [ ] Math Facts:

  Collects review & practice packet and distributes math facts practice face down.
  Has students write their name on the back of the packet.
  States that students will have 5 minutes to complete the math facts.
  Reminds students to show their work
  States that there are more problems than they can answer and not to worry if they do not finish all problems.
  Reminds students they can skip questions
  Has all students begin on cue (may ask students to hold pencils in the air before beginning or use another technique to make sure students are not starting early).

Discretely times for 5 minutes.

After 5 minutes tell students to put pencils down and uses some technique to make sure students are not continuing to answer (e.g. have student hold paper in the air).

❑ Closure: Collects all packets.

Appendix C: Consent and Assent

STUDY INFORMATION SHEET

Dear Parent or Guardian:

We are sending this letter for your review because your child was invited to be in a research study looking at how children respond to feedback on their performance or the strategies they use while completing mathematics problems. All children in your child's class are invited to participate in this study. Your child's participation is voluntary. Please read this form and ask any questions you may have about your child's participation. All questions can be directed to your child's teacher or myself (Rebecca Edmunds).

Title of Research Study: Study on the Effects of Elaborated Task and Process Feedback on Multi-digit Multiplication

Researchers: Rebecca Edmunds is a graduate student at the University of Minnesota in the School Psychology program. This study is in partial fulfillment of her doctoral degree.

Study Purpose: The purpose of this study is to determine if students' multi-digit multiplication improves more if they receive: a) elaborated feedback on task outcomes (errors in the answer), b) elaborated feedback on process strategies (errors in the application of strategies used to solve the problem), or c) no feedback.

Procedures: If you agree to have your child participate in this study, as part of your typical education practice your child will

1. Take three pre-tests to identify their current math skills on a) multi-digit multiplication fact fluency, b) general computation, and c) conceptual understanding. Answer a survey about their opinions and beliefs about math. These activities will each take about 10 minutes to complete for a total time of 45-50 minutes and will occur over multiple days.
2. Complete one hour-long classwide instructional session on multiplication strategies and one 15-minute small-group lesson on the intervention procedures with Rebecca Edmunds.
3. Participate in a 15-minute classwide math facts practice with a trained graduate assistant two days per week for five weeks.
4. Students will be randomly assigned to one of three groups. Students in Group 1 will receive feedback on errors in task outcomes (errors in the answer). Students in Group 2 will receive feedback on errors in process strategies (errors in the application of strategies used to solve the problem). Students in Group 3 will practice math facts but will receive no feedback.
5. Take four post-tests to see if their math skills improved on general computation and conceptual understanding. These activities will each take about 15 minutes to complete for a total time of 30 minutes and will occur over multiple days.

6. Fill out a survey on their opinions about the intervention and a survey about their opinions and beliefs about math. These surveys will each take about 5 minutes to complete.

Participation in this study will be part of the general instruction offered by the school to all students in the class.

Benefits and Risks of the Study

The benefits are

- Your child will be given a math practice that targets their multiplication skills.
- Your child may be given feedback on their math performance and strategies throughout the program.
- We will monitor your child's improvement in math. This information will be given to their teacher at the end of the study

The possible risks are

- Your child may feel uncomfortable for participating in the math program. The risk of discomfort is expected to be small because all students in the class will be offered the chance to participate. The program will be scheduled during times of independent practice and not during direct instruction.
- Your child may become anxious about their performance. This anxiety is expected to be comparable to anxiety experienced when taking any school assessment. If this happens your child will be asked if they would like to stop the session and if they would like to talk to a teacher about these feelings. Your child has a right to discontinue a session or choose not participate in a session at any time.
- If your child does not improve as expected with the math program, their teacher will be notified and your child may be included in a different math program that is already used at the school. I expect children will improve their math skills using the math program we are providing.

Confidentiality: The records of this study will be kept private. In any report we might publish, we will not include any information that will make it possible to identify your child or the school. Records will be stored securely and only researchers will have access to the records. Records will be encrypted according to University policy for protection of confidentiality.

Voluntary: Participation in this study is voluntary. You are free withdraw your child at any time by signing and returning this form. Your decision to participate or not will not affect your relationships with the school or the University of Minnesota.

Contacts and Questions: The researcher conducting this study is: Rebecca Edmunds. The principal investigator is Dr. Robin Codding. If you have any questions, **you are encouraged** to contact Rebecca at 608-323-0827 or by email: edmun050@umn.edu or Dr. Codding at 612-625-8656 or by email: rcodding@umn.edu.

This research has been reviewed and approved by an IRB within the Human Research Protections Program (HRPP). To share feedback privately with the HRPP about your research experience, call the Research Participants' Advocate Line at 612-625-1650 or go to https://research.umn.edu/units/hrpp/research-participants/questionsconcerns. You **are encouraged** to contact the HRPP if: ● Your questions, concerns, or complaints are not being answered by the research team. ● You cannot reach the research team. ● You want to talk to someone besides the research team. ● You have questions about your rights as a research participant. ● You want to get information or provide input about this research.

Please sign and return this form if you DO NOT grant permission for your child to participate.

☐ **I DO NOT grant permission for my child to participate in this research project as a participant. Child's name: ___________________________**

Parent/Guardian Signature: ___________________________________ Date: ______________

To be completed by the researcher:

Signature of Investigator:______________________________________Date: ______________

ASSENT FORM

Effects of Elaborated Task and Process Feedback on Multi-digit Multiplication

Dear Student,

My name is Rebecca Edmunds, and I am a School Psychology graduate student at the University of Minnesota. I am working with your school to study how students do on mathematics problems when given different types of feedback.

I am asking if you are willing to be a part of a math program to practice math facts and be given feedback on your work. I hope that the feedback will help students who need extra practice with multi-digit multiplication, but we won't know if it works until we try it.

If you agree to be in this study, you will practice your multiplication facts twice per week for six weeks. We will do other math activities once before and once after the study to see if this math fact practice and feedback helps improve your performance. At the end of the study, I will also ask you questions about how you like the activities from the study.

Sometimes working on math facts can be frustrating and getting feedback can make you worried about your performance. The extra math practice and feedback may not make you do better on your math. If it does, it may help you do better in math class in the future.

Your teacher and your parents have agreed for you to be in the program, but you can make the choice. If you say no to being in this study, you will do a different activity while your classmates do this math practice. Being in this study is your choice, and no one will be mad at you if you don't want to do it. You can change your mind about being in this study at any time. You can ask any questions that you have about this study. If you have a question later that you didn't think of now, you can ask me later.

Signing here means that you have read this paper or had it read to you and that you are willing to be in this study. If you don't want to be in this study, don't sign. Remember, being in this study is up to you, and no one will be mad at you if you don't sign this or even if you change your mind later.

Student Signature: ___________________________________ Date: ______________

<u>To be completed by the researcher:</u>

Signature of Investigator:________________________________Date: _______________