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Dedication

To my parents Diana and Carlos, who made me the person I am today, and to my brother Diego, who has been my partner in this adventure since I was three.

Abstract

This dissertation consists of three chapters. The first chapter studies the impact of oil discoveries on default risk. I use data of giant oil field discoveries to estimate their effect on the spreads of 37 emerging economies and find that spreads increase by up to 530 basis points following a discovery of average size. I develop a quantitative sovereign default model with capital accumulation, production in three sectors, and oil discoveries. Following a discovery, investment and foreign borrowing increase. These choices have opposite effects on spreads: borrowing increases them and investment reduces them. The discovery also generates a reallocation of capital away from manufacturing and toward oil extraction and the non-traded sector, which is the so-called Dutch disease. This reallocation increases the volatility of tradable income used to finance foreign debt payments, which undermines the effect of investment on spreads.

The second chapter studies an environment in which the Dutch disease amplifies an inefficiency in private sectoral investment that affects the government's ability to borrow from abroad. The model features production in two intermediate sectors: tradable manufactures and a non-tradable good. In addition, the economy receives an endowment of a tradable commodity. The household makes investment decisions in both manufacturing and non-traded sectors and does not internalize how these choices affect the volatility of total tradable income. The government does not have enough instruments to fully manipulate the household's investment choices, which implies that investment allocations are inefficient from the point of view of a social planner.

The third chapter studies how the informativeness of the exchange rate affects the sensitivity of prices to nominal depreciations. In an environment with imperfect information, the exchange rate is a signal of the state of the economy and thus relevant for pricing decisions, even if it does not affect costs. This gives rise to a policy trade-off: a currency intervention that reduces the level of a depreciation also reduces exchange rate volatility and, therefore, increases its precision as a signal. This increases the elasticity of prices with respect to the exchange rate. Overall, the effect of such currency interventions on inflation is ambiguous: on one hand they reduce the magnitude of the shock and, on the

other, they increase the responsiveness of firms when adjusting their prices. I use policy changes in Mexico to identify an increase in the precision of information provided by the Central Bank. Holding everything else constant, this increase accounts for 4 out of a 12 point drop in the elasticity of prices with respect to the exchange rate.

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Chapter 1

The Sovereign Default Risk of Giant Oil Discoveries

1.1 Introduction

Between 1970 and 2012, sixty-four countries discovered at least one giant oil field, and fourteen of these countries had a default episode in the following ten years.[1] Considering all countries in the world, the unconditional probability of observing a country default in any given ten year period was 12%. Conditional on discovering a giant oil field, this probability was 23%.[2] This means that a country that just became richer also became more likely to default on its debt. This chapter studies how the discovery and exploitation of natural resources impact default risk. Following the sovereign default literature, I focus on emerging economies as they are more prone to default episodes.

I use data of giant oil field discoveries to document the effect of a sudden increase in available natural resources on sovereign interest rate spreads. I build on the work by Arezki, Ramey,

[1] A giant oil field contains at least 500 million barrels of ultimately recoverable oil. At 54 USD per barrel the value of this volume would be 27 billion USD (if the barrels were already out of the ground and ready for sale), which is around 1% of the GDP of France in 2018. "Ultimately recoverable reserves" is an estimate (at the time of the discovery) of the total amount of oil that could be recovered from a field.

[2] The data of default episodes are from Tomz and Wright [2007] for the years between 1970 and 2004. The default probability conditional on discovery is the probability that a country has a default episode in any of the ten years following a discovery.

and Sheng [2017], who work with datasets on giant oil discoveries in the world collected by Horn [2014] and the Global Energy Systems research group at Uppsala University. They use these data to calculate the net present value of potential future revenues from a discovery relative to the GDP of the country where it happened. I use this measure of size to estimate the effect of discoveries on the spreads of 37 emerging economies and find that the effect is large and positive: spreads increase by up to 530 basis points following a discovery of average size. I also estimate the effect of discoveries on the current account, investment, GDP, and consumption. Following a discovery, these countries run a current account deficit and GDP, investment, and consumption increase, which is consistent with the findings of Arezki, Ramey, and Sheng [2017] for a wider set of countries. In addition, I estimate the effects on sectoral investment and the real exchange rate and find evidence of the Dutch disease: the share of investment in the manufacturing sector decreases in favor of a higher share of investment in commodities and non-traded sectors.[3] This investment reallocation is accompanied by an appreciation of the real exchange rate. Arezki, Ramey, and Sheng [2017] find weak evidence of real exchange rate appreciation following oil discoveries for all countries in the world. In contrast, I find that the evidence is stronger if one focuses only on the 37 emerging economies considered in this chapter.

To reconcile these facts, I develop a small-open economy model of sovereign default with capital accumulation and production in three intermediate sectors: a non-traded sector, a traded "manufacturing" sector, and a traded "oil" sector. All three sectors use capital for production and the oil sector additionally requires an oil field, which I model as a fixed factor of production. The economy starts with a small oil field and receives unexpected news about the discovery of a larger one, which will become productive at a given time in the near future. This lag between discovery and production is important because the actions in between, along with uncertainty about the price of oil once production starts,

[3]The Dutch disease refers to the way an increase in natural resource exports induces a reallocation of production factors away from manufacturing. Higher revenues from the natural resource boom increase the demand for all consumption goods. This income effect raises the price of non-traded goods, which causes an appreciation of the real exchange rate. This appreciation makes imports of manufactures relatively cheaper and thus induces the reallocation of production factors away from this sector into the non-traded sector. The term was first used in 1977 by The Economist to describe this phenomenon in the Dutch economy after the discovery of natural gas reserves in 1959.

are what drive the increase in spreads. In the data, Arezki, Ramey, and Sheng [2017] find that the average waiting period between discovery and production is of 5.4 years.

After an oil discovery, investment increases so the economy can exploit the larger field when it becomes productive. The economy runs a current account deficit by issuing foreign debt to finance investment. Also, once exploitation of the larger field starts, there is a reallocation of capital away from manufacturing and toward the non-traded sector, which is the Dutch disease.

In the model, as in the data, the price of oil is relatively more volatile than the price of the other traded goods.[4] Higher investment decreases spreads and higher foreign borrowing increases them. The latter effect dominates because the reallocation of production factors implied by the Dutch disease makes tradable income more dependent on oil revenue and thus more volatile.

I calibrate the model to the Mexican economy, which is a typical small-open economy widely studied in the sovereign debt and emerging markets literature. Mexico did not have any giant oil field discoveries between 1993 and 2012, which is the period analyzed in this chapter.[5] This lack of discoveries is desirable because it allows me to discipline the parameters of the model with business cycle data that does not have any variation that could be driven by oil discoveries. I then validate the theory by contrasting the co-movement of model variables in response to unexpected oil discoveries with the responses estimated from the data.[6] Additionally, I use the oil discoveries data from Arezki, Ramey, and Sheng [2017] to discipline the size of discoveries in the model.

Under the benchmark calibration, the model explains 300 out of the 530 percentage points of the maximum increase in spreads following an oil discovery, out of which 200 are accounted for by the Dutch disease. There are other complementary forces that could also

[4]Commodities have always shown a higher price volatility than manufactures. Jacks, O'Rourke, and Williamson [2011] document this stylized fact using data that goes back to the 18th century.

[5]An interesting case of study would be the Mexican default in 1982, which was preceded by two giant oil field discoveries: one in 1977 and another in 1979, each with an estimated net present value of potential revenues of 50 percent of Mexico's GDP at the time. The main inconvenience is the lack of data on sovereign spreads, which are crucial to discipline the parameters in the model that control default incentives.

[6]The exercise of looking at model responses to unexpected news shocks is standard in the news-driven business cycle literature, see for example Jaimovich and Rebelo [2008], Jaimovich and Rebelo [2009], and Arezki, Ramey, and Sheng [2017].

make spreads increase that the model does not consider. For example, growth externalities in the manufacturing sector could make the Dutch disease inefficient if they are not internalized. This could hamper future growth and increase spreads in the present.[7] Also, deterioration of institutions following giant oil discoveries could also cause spreads to increase. For example, Lei and Michaels [2014] find evidence that giant oil field discoveries increase the incidence of internal armed conflicts.

I compare the results from the model under the benchmark calibration with those from a model in which the price of oil is not volatile; I call this the *no-price-volatility* case. This exercise illustrates the counterfactual response of all variables if the economy was able to effectively and costlessly hedge swings in the price of oil. Additionally, I compare the results to those from an economy in which there is no default risk; I call this the *patient* case. In both counterfactual cases, as well as in the benchmark, the economy increases foreign borrowing to invest and all three feature the Dutch disease. These are the co-movements that, together with the uncertainty about the price of oil, explain the increase in spreads in the benchmark case. However, spreads increase by less than one percentage point in the *no-price-volatility* case and by virtually nothing in the *patient* case. These results stress two important points. First, the frictions in this economy that explain high spreads are market incompleteness, the lack of commitment from the government, and its high relative impatience. In the absence of these frictions the incentives to borrow to invest in the larger oil field and the incentives that drive the reallocation of capital due to the Dutch disease are still present. Second, it is in the presence of these frictions that the volatility of the price of oil, the choice of borrowing to invest, and the the Dutch disease together generate a large increase in spreads following an oil discovery.

Related literature.—This chapter contributes to the literature that studies the role of news as drivers of business cycles.[8] Beaudry and Portier [2004] were the first to propose a model that generates an economic expansion in response to good news about the future. In a later paper, Beaudry and Portier [2007] characterize the class of models that are able to generate business cycles driven by news or changes in expectations. They find that most neo-classical business cycle models fail to do so unless they allow for a sufficiently rich description of the production technology. Jaimovich and Rebelo [2008] propose a version

[7] See Hevia, Neumeyer, and Nicolini [2013b] for an example of such an externality.

[8] See Beaudry and Portier [2014] for an extensive review of this literature and its future challenges.

of an open economy neoclassical growth model that generates co-movement in response to unexpected TFP news. They highlight weak wealth effects on labor supply and adjustment costs to labor and investment as key elements. In a later paper, Jaimovich and Rebelo [2009] do the same study for a closed economy and find three key elements for the model to generate news-driven business cycles: variable capital utilization, adjustment costs to investment, and preferences that feature weak wealth effects on the labor supply. The model in Section 2.2 builds on the work in these papers and contributes by connecting it with the sovereign default literature. To my knowledge, this is the first work to study the effect of news on business cycles and default risk in a general equilibrium model with endogenous default.[9]

This chapter also builds on the quantitative sovereign default literature following Aguiar and Gopinath [2006] and Arellano [2008].[10] They introduce sovereign default models that feature counter-cyclicality of net exports and interest rates, which are consistent with the data from emerging markets. Hatchondo and Martinez [2009] and Chatterjee and Eyigungor [2012] extend the baseline framework of quantitative models of sovereign default to include long-term debt. Their extensions allow the models to jointly account for the debt level, the level and volatility of spreads around default episodes, and other cyclical factors.

Gordon and Guerron-Quintana [2018] analyze the quantitative properties of sovereign default models with capital accumulation and long-term debt. They show that the model can fit cyclical properties of investment and GDP while also remaining consistent with other business cycle properties of emerging economies. They also find that capital has non-trivial effects on sovereign risk but that increased capital almost always reduces risk premia in equilibrium. The model in Section 2.2 is based on their framework and extends it to have production in different sectors, with one of them also using natural resources. Arellano, Bai, and Mihalache [2018] document how sovereign debt crises have disproportionately negative effects on non-traded sectors. They develop a model with capital, production in two sectors, and one period debt. In their model, default risk makes recessions more pronounced for non-traded sectors. This is because adverse productivity shocks limit capital inflows

[9] In a related paper, Gunn and Johri [2013] explore how changes in expectations about future default on government debt can generate recessions in an environment where default is exogenous.

[10] These papers extend the approach developed by Eaton and Gersovitz [1981].

and induce a capital reallocation toward the traded sector to support debt payments. The model in Section 2.2 contrasts by featuring two traded sectors and long-term debt. The effect of sovereign risk on the non-traded sector during recessions also depends on shocks to the international price of oil and on the current capacity of the oil field. Additionally, news about future sovereign risk affect current variables due to the long-term nature of the debt.

This chapter is closely related to Hamann, Mendoza, and Restrepo-Echavarria [2018]. They study the relation between oil exports, oil reserves, and sovereign risk. They use an Institutional Investor Index as a measure of sovereign risk and document that, for the 30 largest emerging market oil exporters, country risk increases as unexploited oil reserves increase. Subsection 1.2.3 documents complementary evidence using a different measure of risk and a similar set of countries: sovereign risk, measured by interest rate spreads, increases following giant oil discoveries. They also document that an oil exporting country is perceived as less risky if its oil production is high and that oil exports increase during default episodes. These observations motivate their hypothesis that idle oil reserves allow these countries to withstand the consequences of financial autarky and thus increase default risk by improving their outside option. My work contrasts with theirs in two ways. First, while they focus on how existing oil reserves are exploited, I highlight the effects of new discoveries on sovereign risk. Second, they develop a model in which the intensity of reserves exploitation interacts with sovereign risk by endogenously changing the government's outside option. In contrast, the model presented in Section 2.2 abstracts from this strategic motive. Instead, it focuses on the effects on sovereign risk of new oil discoveries through their implications for borrowing, investment, and the sectoral allocation of capital.

Finally, this chapter relates to the literature that studies the macroeconomic effects of commodity-related shocks and the Dutch disease. Pieschacon [2012] studies the role of fiscal policy as a transmission mechanism of oil price shocks to key macroeconomic variables. She documents evidence that the predictions of the Dutch disease hold for the case of Mexico but not for that of Norway, and argues that fiscal policy is a key determinant of this difference. These findings could be consistent with the predictions of the model in Section 2.2. Introducing fiscal policy to save oil revenues by purchasing foreign assets could eliminate the effects of the Dutch disease. Arezki and Ismail [2013] study the implications that changes in expenditure policy in oil-exporting countries have on real effective exchange rate movements. They find that the real exchange rate appreciates when the oil export unit value increases, but, asymmetrically, that the real exchange rate does not change much when this unit value decreases.

Hevia, Neumeyer, and Nicolini [2013a] analyze optimal policy in a New Keynesian model of a small-open economy with shocks to terms of trade that generate Dutch disease periods. Their model features complete markets and an externality in the manufacturing sector that makes the Dutch disease inefficient. In contrast, the model I present in Section 2.2 has incomplete markets but does not feature any externality in production. Thus, factor reallocation by itself may not be inefficient. Hevia and Nicolini [2015] analyze optimal monetary policy in a small-open economy that specializes in the production of commodities. They find that, due to price and wage nominal frictions, the Dutch disease generates inefficiencies and full price stability is not optimal.[11] Ayres, Hevia, and Nicolini [2019] argue that shocks to primary commodity prices account for a large fraction of the volatility of real exchange rates between developed economies and the US dollar. They suggest that considering trade in primary commodities could help models generate real exchange rate volatilities that are more in line with the data. The model in Section 2.2 can be used as a baseline to study the co-movement of sovereign risk and real exchange rates, which could point to questions regarding monetary policy in future work.

Layout.—Section 1.2 describes the data, documents the effect of giant oil discoveries on sovereign spreads and other macroeconomic aggregates, and discusses the evidence that motivates the theoretical framework. Section 2.2 presents the model and discusses the theoretical mechanism. Section 1.4 describes the calibration. Section 1.5 presents the quantitative results and Section 2.5 concludes.

[11] Hevia and Nicolini [2013] find that in an economy with only price frictions, domestic price stability is optimal, even if fiscal policy cannot respond to shocks. In other words, as they explain it, the Dutch disease is not a disease.

1.2 Giant oil discoveries in emerging economies

This section documents the effects of giant oil discoveries on 37 emerging economies considered in JP Morgan's Emerging Markets Bonds Index (EMBI).[12] Due to data availability I restrict the analysis in this chapter to these economies and the years between 1993 and 2012. I use a measure of the net present value (NPV) of oil discoveries as a percentage of GDP constructed by Arezki, Ramey, and Sheng [2017]. I follow their empirical strategy to estimate the effects of oil discoveries on investment, the current account, GDP, and consumption. As they do for a larger set of countries, I find evidence for the intertemporal approach to the current account (as developed by Obstfeld and Rogoff [1995]) and the permanent income hypothesis.

My contribution is to estimate the effect of giant oil discoveries on the sovereign spreads of these economies. I find that spreads increase by up to 530 basis points following a discovery of average size. In addition, I estimate the effect of discoveries on the real exchange rate and investment by sectors and find evidence of the Dutch disease. Subsection 1.2.1 describes the data and the empirical strategy. Subsections 1.2.2 through 1.2.4 present the main results and Appendix A.1 discusses additional details and robustness checks.

1.2.1 Data and empirical strategy

Giant oil discoveries are a measure of changes in the future availability and potential exploitation of natural resources. As Arezki, Ramey, and Sheng [2017] argue, giant oil discoveries have three unique features that allow the use of a quasi-natural experiment approach to identify their effect: they indicate significant increases in future production possibilities, the timing of discoveries is exogenous due to uncertainty around oil and gas exploration, and there is a time delay of 5.4 years on average between discovery and production.[13]

[12] The thirty-seven countries are: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Hungary, Indonesia, Iraq, Jamaica, Kazakhstan, Republic of Korea, Lebanon, Malaysia, Mexico, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Serbia, South Africa, Sri Lanka, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam.

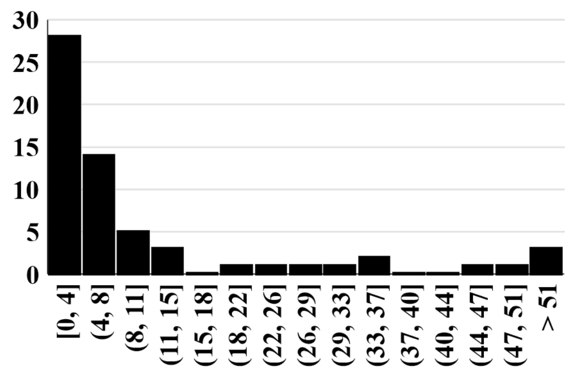
[13] Arezki, Ramey, and Sheng [2017] mention that experts' empirical estimates suggest that it takes between four and six years for a giant oil discovery to go from drilling to production. They also made their own calculation and found that the average delay between discovery and production is 5.4 years.

Arezki, Ramey, and Sheng [2017] construct a measure of the net present value (NPV) of giant oil discoveries as a percentage of GDP at the time of discovery as follows:[14]

$$NPV_{i,t} = \frac{[j = 5]J \sum \frac{q_{i,t+j}}{(1+r_i)^j}}{GDP_{i,t}} \times 100 \tag{1.1}$$

where $NPV_{i,t}$ is the discounted sum of gross revenue for country i at the year of discovery t, r_i is the annual discount rate in country i, and $GDP_{i,t}$ is annual GDP of country i at year t. The annual gross revenue $q_{i,t+j}$ is derived from an approximated production profile starting five years after the field discovery up to an exhaustion year J, which is greater than 50 years for a typical field of 500 million barrels of ultimately recoverable reserves.[15] Considering the thirty-seven economies in the EMBI and the years 1993–2012, there are 61 giant oil discoveries in 15 of the 37 countries. The average NPV of a discovery was 18 percent of GDP and the largest was 467 for a discovery in Kazakhstan in 2000. Figure 1.1 depicts the distribution of the NPV of these discoveries.

Figure 1.1: Distribution of NPV of giant oil discoveries



Percent of GDP, EMBI countries, 1993 –2012.

[14] They use the data on giant oil discoveries in the world collected by Horn [2014] and the Global Energy Systems research group at Uppsala University. For more details of the construction of the NPV see Section IV.B. in Arezki, Ramey, and Sheng [2017].

[15] It is important to mention that the gross revenue $q_{i,t+j}$ considers the same price of oil for subsequent years. Since the price of oil closely resembles a random walk, the current price is the best forecast of future prices. See Appendix B of Arezki, Ramey, and Sheng [2017] for a detailed explanation of the approximation of the production profile of giant oil discoveries.

I take investment, current account, GDP, and consumption data from the IMF [2013] and the World Bank [2013]. GDP and consumption are measured in constant prices in local currency units. Investment and the current account are measured as a percentage of GDP. Spreads data are from JP Morgan's Emerging Markets Bonds Index (EMBI) Global. The index tracks a value weighted portfolio of US dollar denominated debt instruments, with fixed and floating-rates, issued by emerging market sovereign and quasi-sovereign entities. Spreads are measured against comparable US government bonds.[16] The real exchange rate is calculated as $RER_{i,t} = \frac{e_{i,t} P_t^{US}}{P_t^i}$ where P_t^{US} and P_t^i are the US and country i's GDP deflators, respectively, and $e_{i,t}$ is the nominal exchange rate between country i's currency and the US dollar. These data are also from the IMF [2013]. Finally, the data on investment by sector is in terms of the share of total investment and is from the United Nations Statistics Division [2017].

Following Arezki, Ramey, and Sheng [2017], I estimate the effect of giant oil discoveries on different macroeconomic variables using a dynamic panel model with a distributed lag of giant oil discoveries:

$$y_{i,t} = \rho y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s} + \alpha_i + \mu_t + \sum_{s=1}^{10} \xi_s p_{oil,t} \mathbb{I}_{disc,i,t-s} + \epsilon_{i,t} \tag{1.2}$$

where $y_{i,t}$ is the dependent variable (the dependent variables I will consider are investment, the current account, log of real GDP, log of real consumption, sovereign spreads, log of the real exchange rate, and the share of investment by sector); $NPV_{i,t}$ is the NPV of a giant oil discovery in country i in year t; α_i controls for country fixed effects; μ_t are year fixed effects; $p_{oil,t}$ is the natural logarithm of the international price of oil at time t; $\mathbb{I}_{disc,i,t-s}$ is an indicator function of whether country i had an oil discovery in period $t-s$; and $\epsilon_{i,t}$ is the error term.[17] Country fixed effects control for any unobservable and time-invariant

[16] To be included, an instrument must have at least 2.5 years until maturity, a current face amount outstanding of at least 1 billion US dollars, and a sovereign credit rating of BB+ or lower. In addition, the issuing country's GNI per capita must be below a ceiling for three consecutive years. Currently, this is 19,708 US dollars.

[17] Also, as Arezki, Ramey, and Sheng [2017] do, I include country-specific quadratic trends for the regressions of variables $y_{i,t}$ that are non-stationary in the sample. These are GDP, consumption, the real

characteristics, while year fixed effects control for common shocks like world business cycles and the international price of oil.[18]

The interactions of the natural logarithm of the international price of oil with indicator functions of recent discoveries control for the fact that the reaction of the dependent variable to this common shock may differ conditional on having a recent discovery. As discussed in Appendix A.1.2, these control variables are only relevant for the estimations of the effects of discoveries on spreads and the real exchange rate. For consistency, the results presented in this section include these controls in all regressions. Appendix A.1.2 shows the results for the specifications without these controls.

As in Arezki, Ramey, and Sheng [2017]'s analysis, I exploit the dynamic feature of the panel regression and use impulse response functions to capture the dynamic effect of giant oil discoveries given by $\Delta y_{i,t} = \rho \Delta y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s}$.

1.2.2 Response of macroeconomic aggregates

Figure 1.2 shows the dynamic response of investment, the current account, GDP, and consumption to an oil discovery of average size, based on the estimated coefficients of equation (1.2).

The dotted lines are 90% confidence intervals based on a Driscoll and Kraay [1998] estimation of standard errors, which yields standard error estimates that are robust to general forms of spatial and temporal clustering.

Table A.1 in Appendix A.1.1 reports point estimates and their standard errors for the coefficients in equation 1.2.

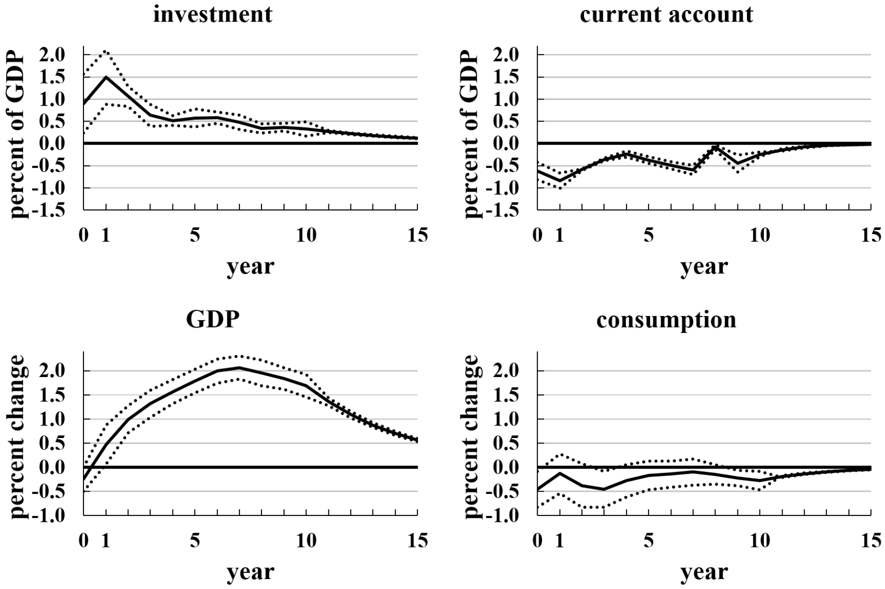
The top left panel shows that the investment-to-GDP ratio increases immediately after an oil discovery and continues to be higher in the subsequent years. The top right panel

exchange rate, and the spreads. For these variables the augmented Dickey-Fuller test fails to reject a unit root in all countries.

[18] As noted by Nickell [1981], estimates of a dynamic panel with fixed effects are inconsistent when the time span is small. He shows that this asymptotic bias is of the order $1/T$, which, in the case of the sample considered in this chapter, is 0.05. Arellano and Bond [1991] developed an efficient GMM estimator for dynamic panel data models with a small time span and large number of individuals. The results in this section are virtually unchanged using the Arellano-Bond estimator. Given the size of the Nickell bias and to keep the results comparable with those of Arezki, Ramey, and Sheng [2017] I use the above approach.

shows that oil discoveries have a negative effect on the current account-to-GDP ratio, which supports the hypothesis that these countries issue foreign debt to finance higher consumption and investment. The bottom panels show that both GDP and consumption increase after an oil discovery.

Figure 1.2: Impact of giant oil discoveries on macroeconomic aggregates

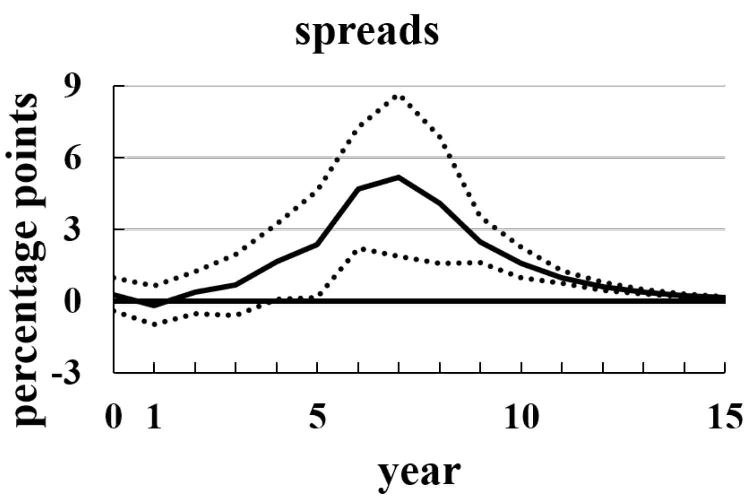


Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

1.2.3 Effect on sovereign spreads

This subsection presents the main empirical finding of this chapter. Figure 1.3 shows the dynamic response of the spreads. In the year of the discovery, this effect is small and not significantly different from zero. However, by the sixth year after the discovery is announced, spreads have increased by 5.3 percentage points on average.

Figure 1.3: Impact of giant oil discoveries on spreads



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

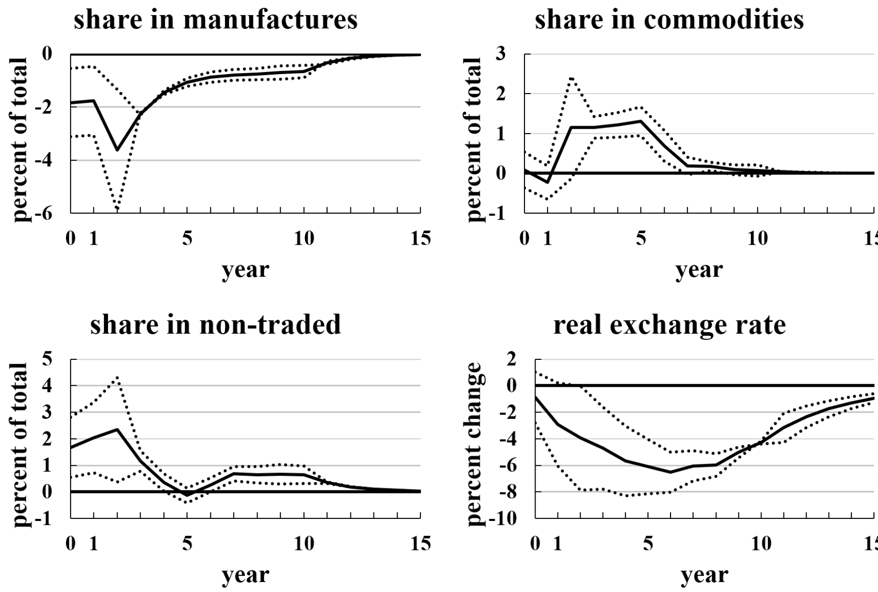
This result is striking since income increases during the years following the discovery as shown in the previous figure, which would indicate that the country has a higher ability to service its debt. The theoretical model in Section 2.2 shows how debt accumulation and the effects of the Dutch disease can reconcile these empirical observations.

1.2.4 The Dutch disease

Figure 1.4 shows the dynamic response of the real exchange rate, as well as the share of total investment in manufactures, commodities, and non-traded sectors.[19] Commodities comprise agricultural, fishing, mining and querying activities. The non-traded sector includes construction and wholesale, retail, and logistics services.

[19] The estimations for the shares of total investment consider a wider set of countries due to limited data availability for the 37 countries considered in this chapter. Their purpose is to support the evidence shown for the estimation of the effect of discoveries on the real exchange rate, which only considers the aforementioned 37 countries.

Figure 1.4: Impact of giant oil discoveries on sectoral investment and the RER



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

Following a discovery, the share of investment in the manufacturing sector decreases and the shares in both the commodities and the non-traded sectors increase. The real exchange rate appreciates, which is in line with the theoretical predictions of the Dutch disease: higher income from the commodity sector increases the consumption of non-traded goods. This in turn increases the price of non-traded goods and production factors are moved out of manufacturing into non-traded sectors and resource extraction. Arezki, Ramey, and Sheng [2017] also find (for a larger set of countries) that the real exchange rate appreciates during the five years following oil discoveries; however, their estimates are not significantly different from zero. Figure 1.4 shows that for the 37 countries studied in this chapter, the evidence of appreciation is more conclusive than when all countries were considered in the same regression, as in Arezki, Ramey, and Sheng [2017].

1.3 Model

This section presents a dynamic small-open economy model in the Eaton and Gersovitz [1981] tradition with long-term debt and capital accumulation. I augment the model in Gordon and Guerron-Quintana [2018] to include production in different sectors and discovery of natural resources. There is a benevolent government that makes borrowing, investment, and production decisions and cannot commit to repay its debt.[20]

1.3.1 Environment

Goods and technology.—There is a final non-traded good used for consumption and capital accumulation. This good is produced with a constant elasticity of substitution (CES) technology using a bundle of an intermediate non-traded good $c_{N,t}$ and two intermediate traded goods: manufactures $c_{M,t}$ and oil, $c_{oil,t}$:

$$Y_t = \left[\omega_N^{\frac{1}{\eta}} \left(c_{N,t}\right)^{\frac{\eta-1}{\eta}} + \omega_M^{\frac{1}{\eta}} \left(c_{M,t}\right)^{\frac{\eta-1}{\eta}} + \omega_{oil}^{\frac{1}{\eta}} \left(c_{oil,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{1.3}$$

where η is the elasticity of substitution and ω_i are the weights of each intermediate good i in the production of the final good. Intermediate non-traded goods and manufactures are produced using capital k and decreasing returns to scale technologies:

$$y_{N,t} = z_t k_{N,t}^{\alpha_N} \tag{1.4}$$

$$y_{M,t} = z_t k_{M,t}^{\alpha_M} \tag{1.5}$$

where z_t is aggregate productivity in the economy and $0 < \alpha_N < 1$, $0 < \alpha_M < 1$.[21] There is a general stock of capital k_t that can be freely allocated in these two sectors within the same period such that $k_{N,t} + k_{M,t} = k_t$.[22] Each period the economy has access to an oil

[20] In Appendix A.2 I show how all the allocations and default decisions in this economy can be supported in a decentralized economy where households make consumption and investment decisions and firms demand capital in different sectors.

[21] Decreasing returns to scale captures the presence of a fixed factor, which in this case could be labor (immobile within sectors).

[22] The assumption about the free allocation of capital between the non-traded intermediate sector and manufacturing is made for simplicity. As it will become clear later, what is necessary for my results is that the capital to extract oil is sector specific. Having specific capital in all three sectors would add an

field with capacity n_t. To produce oil the economy uses the field's capacity n_t, capital $k_{oil,t}$ that is specific to the oil sector, and technology:

$$y_{oil,t} = z_t k_{oil,t}^{\alpha_{oil}(1-\zeta)} n_t^{\zeta} \tag{1.6}$$

where $\zeta \in (0,1)$ is the share of oil revenue that corresponds to the oil rent.
The resource constraint of the final non-traded good is:

$$c_t + i_{k,t} + i_{k_{oil},t} = Y_t + m_t, \tag{1.7}$$

where c_t is private consumption, $i_{k,t}$ is investment in general capital, $i_{k_{oil},t}$ is investment in capital for the oil sector, Y_t is production of the final non-traded good, and m_t is a small transitory income shock described below.[23] The laws of motion for the stocks of capital are:

$$k_{t+1} = (1-\delta)\, k_t + i_{k,t} - \Psi\left(k_{t+1}, k_t\right) \tag{1.8}$$

$$k_{oil,t+1} = (1-\delta)\, k_{oil,t} + i_{k_{oil},t} - \Psi\left(k_{oil,t+1}, k_{oil,t}\right) \tag{1.9}$$

where $i_{k,t}$ and $i_{k_{oil},t}$ are investment in general and oil capital, respectively; δ is the capital depreciation rate; and $\Psi\left(k_{t+1}, k_t\right) = \phi \left(\frac{k_{t+1}+k_t}{k_t}\right)^2 k_t$ is a capital adjustment cost function.[24] As discussed in Subsection 2.2.4, capital adjustment costs allow the model to reproduce the anticipation effect in investment observed in the data, that is, have the economy increase investment before production with the larger oil field starts.

Rest of the world and international prices of goods.—There is a rest of the world economy with which the small-open economy trades manufactures and oil. All prices are in terms of manufactures. I assume that the small-open economy is small enough so that

additional endogenous state variable, which would significantly complicate the computation of equilibrium without adding much to the informativeness of the model.

[23]The presence of this m_t shock facilitates the numerical computation of equilibrium. See Chatterjee and Eyigungor [2012] and Gordon and Guerron-Quintana [2018].

[24]Including capital adjustment costs is important in business cycle models to avoid investment being overly volatile; see Mendoza [1991] for a discussion of the case of small-open economies. Additionally, as Gordon and Guerron-Quintana [2018] show, sovereign default models with capital accumulation require capital adjustment costs to sustain positive levels of debt in equilibrium. Without adjustment costs, the cheapest way for the sovereign to hedge against fluctuations would be to reduce investment rather than borrowing from the rest of the world.

neither its actions nor its oil discoveries have an effect on the international price of oil. This price is pinned down in the rest of the world and for simplicity I assume it follows some exogenous stochastic process. As it will be discussed in Subsection 2.2.4, what is key for the results in this chapter is that the price of oil is relatively more volatile than the price of other traded goods. For a richer model of the international oil industry see Bornstein, Krusell, and Rebelo [2019].

Shocks and oil discoveries.—In each period the economy experiences one of finitely many events s_t that follow a Markov chain governed by transition matrix $\pi(s_{t+1}|s_t)$. The shock s_t determines aggregate productivity in the economy z_t and summarizes the shocks in the rest of the world that pin down the international price of oil $p_{oil,t}$. Additionally, in each period the economy receives a small transitory income shock $m_t \in [-\bar{m}, \bar{m}]$ drawn independently from a mean zero probability distribution with continuous CDF.[25]

The capacity of the oil field can take one of two values $n_t \in \{n_L, n_H\}$ with $0 \leq n_L < n_H$. The economy starts with $n_t = n_L$ and in some period τ receives *unexpected* news that its oil capacity will be larger six periods from then, that is $n_{\tau+6} = n_H$. The unexpected nature of the news is in line with the assumption made in Section 1.2 that, in the data, the timing of discoveries cannot be anticipated. Additionally, this is in line with the literature on news-driven business cycles, which models news shocks as one-time unexpected shifts (see, for example, Jaimovich and Rebelo [2008], Jaimovich and Rebelo [2009], and Arezki, Ramey, and Sheng [2017]). For simplicity I assume that n_t remains high forever.[26]

Preferences.—The government has preferences over private consumption c_t represented

[25]This i.i.d. income shock is included to make computation of the model possible. In the calibration, the parameter $\bar{m}$ is chosen so that this shock is relatively small (i.e. the right-hand side of equation (1.7) is always positive). See Chatterjee and Eyigungor [2012] for a detailed theoretical discussion in an exchange economy and for a discussion of the extension to production economies with capital accumulation see Gordon and Guerron-Quintana [2018].

[26]The average duration of a giant oil field is 50 years, longer than the time-span in the data in section 1.2.1. Moreover, as Arezki, Ramey, and Sheng [2017] document, the production rate is highest for the initial years after the field becomes productive and then decreases at a slow rate. A richer model of oil production would include details on the depletion of the reserves on the field through its exploitation. However, the focus of this chapter is on the effect of oil discoveries and the transition between discovery and production, rather than on the long life-cycle of oil fields.

by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right]$$

where $u\left(c,l\right)=\frac{c^{1-\sigma}-1}{1-\sigma}$ and β is the government's discount factor.

Debt structure.—As in Chatterjee and Eyigungor [2012] the government issues long-term bonds that mature probabilistically at a rate γ. Each period, the fraction $1-\gamma$ of bonds that did not mature pay a coupon κ. The law of motion of bonds is:

$$b_{t+1}=\left(1-\gamma\right)b_t+i_{b,t} \tag{1.10}$$

where b_t is the number of bonds due at the beginning of period t and $i_{b,t}$ is the amount of bonds issued in period t.[27] The bonds are denominated in terms of the numeraire good.

Default, repayment, and the balance of payments.—At the beginning of every period the government has the option to default. If the government defaults it gets excluded from international financial markets—although it can still trade in goods—for a stochastic number of periods; the government gets re-admitted to financial markets with probability θ and zero debt. While in default the transitory income shock is $-\bar{m}$ and productivity is $z_t^d\leq z_t$.[28] More specifically, I assume an asymmetric penalty to productivity so that $z_t^d=z_t-\max\left\{0,d_0z_t+d_1z_t^2\right\}$, where $d_0<0<d_1$. This implies that the productivity penalty is zero when $z_t\leq-\frac{d_0}{d_1}$ and rises more than proportionately when $z_t>-\frac{d_0}{d_1}$. This asymmetry in the default penalty is crucial in generating default dynamics that are in line with the data in this class of models (see the discussions in Arellano [2008] and Chatterjee and Eyigungor [2012]).

In default, the balance of payments is:

$$0=x_{M,t}+p_{oil,t}x_{oil,t} \tag{1.11}$$

[27] Hatchondo and Martinez [2009] and Arellano and Ramanarayanan [2012] have an alternative formulation with no coupon payments ($\kappa=0$). As Chatterjee and Eyigungor [2012] argue, including the parameter κ is advantageous because it allows the calibration to target data on maturity length and debt service separately.

[28] The transitory income shock is set to its minimal possible value to ease the computation of the equilibrium. All the results in this chapter are unchanged if this assumption was relaxed to have the transitory component of income to vary also while in default. For a discussion of the computational advantages of this formulation see Chatterjee and Eyigungor [2012].

where $x_{M,t} = y_{M,t} - c_{M,t}$ and $x_{oil,t} = y_{oil,t} - c_{oil,t}$ are net exports of manufactures and oil, respectively. Equation (1.11) implies that in default trade in goods has to be balanced; imports to increase consumption of a traded good have to be financed by exports of the other traded good.

If the government decides to pay its debt obligations then it has access to international financial markets and can issue new debt $i_{b,t}$. In this case, the balance of payments is:

$$[\gamma + (1-\gamma)\kappa]\, b_t = x_{M,t} + p_{oil,t}x_{oil,t} + q_t i_{b,t} \tag{1.12}$$

where q_t is the price of newly issued debt. Equation (1.12) shows how payments of debt obligations (left-hand side) are supported by net exports of goods and by the issuance of new debt.

Lenders.—The bonds issued by the government are purchased by a large number of risk-neutral foreign lenders. I assume these lenders have deep pockets (in the sense that an individual lender is always able to purchase all of the government debt) and behave competitively. Also, lenders have access to a one-period risk-free bond that pays a fixed interest rate $r^{\star}$.

1.3.2 Recursive formulation and timing

The state of the economy is the underlying stochastic variable s, the i.i.d. income shock m, the stock of general capital k, the stock of capital for the oil sector k_{oil}, the outstanding government debt b, and an indicator of whether the government is in default or not.

The government.—Let $V(s, m, k, k_{oil}, b)$ be the value of the government that starts the period not in default. I follow the Eaton and Gersovitz [1981] timing and assume that the government first chooses whether to repay its debt obligations, $d = 0$, or to default, $d = 1$:

$$V(s, m, k, k_{oil}, b) = \max_{d\in\{0,1\}} \left\{[1-d]\, V^P(s, m, k, k_{oil}, b) + dV^D(s, k, k_{oil})\right\}$$

where $V^P(s, m, k, k_{oil}, b)$ is the value of repaying and $V^D(s, k, k_{oil})$ is the value of default.[29] If the government decides to default then its debt obligations are erased and it gets excluded from financial markets. Then, the government simultaneously chooses the stocks of capital

[29] Alternative timing assumptions can give rise to multiplicity of equilibria like, for example, the one introduced by Cole and Kehoe [2000]. For detailed discussions and literature surveys on this topic see Aguiar and Amador [2014] and Aguiar, Chatterjee, Cole, and Stangebye [2016].

next period k' and k'_{oil}, static allocations of general capital in manufactures and the non-traded intermediate sector $K = \{k_N, k_M\}$, net exports of manufactures and oil $X = \{x_M, x_{oil}\}$, and consumption of final and intermediate goods $C = \{c, c_N, c_M, c_{oil}\}$ to solve:

$$V^D(s, k, k_{oil}) = \max_{\{k', k'_{oil}, C, K, X\}} \left\{ u(c) + \beta \mathbb{E}\left[\theta V\left(s', m', k', k'_{oil}, 0\right) + (1-\theta) V^D\left(s', k', k'_{oil}\right)\right]\right\}$$

subject to the resource constraint of the final good (1.7), the resource constraint of general capital $k_t = k_N + k_M$, the laws of motion of capital (1.8) and (1.9), the resource constraints of intermediate goods $c_N = y_N$, $c_M + x_M = y_M$ and $c_{oil} + x_{oil} = y_{oil}$, and the balance of payments under default (1.11). Note that the government can trade in goods, but trade has to be balanced since it cannot issue debt.

If the government decides to repay then it simultaneously chooses the stocks of capital k' and k'_{oil}, and debt b' in the next period, static allocations of general capital in manufactures and the non-traded intermediate sector $K = \{k_N, k_M\}$, net exports of manufactures and oil $X = \{x_M, x_{oil}\}$, and consumption of final and intermediate goods $C = \{c, c_N, c_M, c_{oil}\}$ to solve:

$$V^P(s, m, k, k_{oil}, b) = \max_{\{k', k'_{oil}, b', C, K, X\}} \left\{ u(c) + \beta \mathbb{E}\left[V\left(s', m', k', k'_{oil}, b'\right)\right]\right\}$$

subject to the resource constraint of the final good (1.7), the resource constraint of general capital $k_t = k_N + k_M$, the laws of motion of capital (1.8) and (1.9), the law of motion of bonds (1.10), the resource constraints of intermediate goods $c_N = y_N$, $c_M + x_M = y_M$ and $c_{oil} + x_{oil} = y_{oil}$, and the balance of payments under repayment (1.12).

Lenders.—In each period, if the government is in good financial standing it makes its borrowing and investment decisions simultaneously. Then, lenders observe these decisions and purchase the bonds. Since lenders behave competitively they make zero profits in expectation, which implies that they price the bonds issued by the government according to:

$$q\left(s, k', k'_{oil}, b'\right) = \frac{\mathbb{E}_{m', s'|s}\left\{\left[1 - d\left(s', m', k', k'_{oil}, b'\right)\right]\left[\gamma + (1-\gamma)\left(\kappa + q\left(s', k'', k''_{oil}, b''\right)\right)\right]\right\}}{1 + r^{\star}} \tag{1.13}$$

where k'', k''_{oil} and b'' are lenders' expectations about the government's investment and borrowing decisions in the following period. Note that, given the i.i.d. nature of the transitory income shock, the price schedule q does not depend on the current realization of m.

An important assumption in this environment is that all of the government's dynamic decisions are made simultaneously, in other words, both investment and indebtedness are contractible. This implies that capital is an argument of the price function in (1.13). In a recent paper Galli [2019] studies an environment in which investment is not contractible. In that case the price function does not depend on capital and multiple equilibria with high and low investment may arise.

1.3.3 Equilibrium

A Markov equilibrium is value functions $V(\cdot)$, $V^D(\cdot)$, and $V^P(\cdot)$; policy functions for capital in default $k^D(s,k,k_{oil})$ and $k^D_{oil}(s,k,k_{oil})$; policy functions for capital $k'(s,m,k,k_{oil},b)$ and $k'_{oil}(s,m,k,k_{oil},b)$ and debt issuance $b'(s,m,k,k_{oil},b)$ in repayment; a default policy function $d(s,m,k,k_{oil},b)$; policy functions for static allocations in repayment and in default; and a price schedule of bonds $q(s,k',k'_{oil},b')$ such that: (i) given the price schedule q, the value and policy functions solve the government's problem, (ii) the price schedule satisfies (1.13), and (iii) lenders have rational expectations about the government's future decisions, that is $k''=k'(s',m',k',k'_{oil},b')$, $k''_{oil}=k'(s',m',k',k'_{oil},b')$, and $b''=b'(s',m',k',k'_{oil},b')$ in equation (1.13).

1.3.4 Higher spreads and the Dutch disease

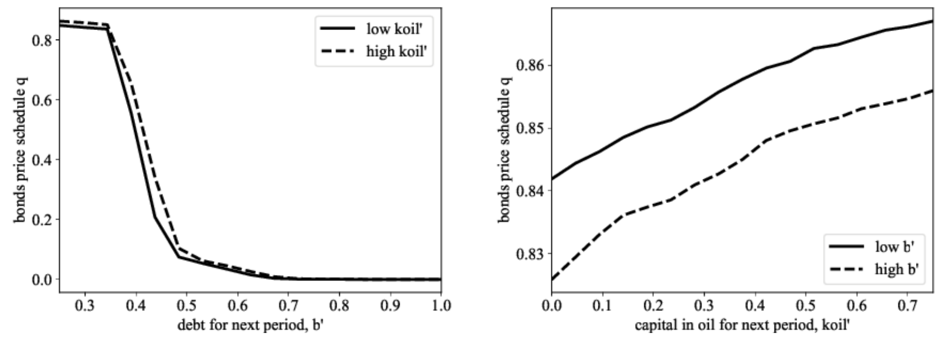
This subsection discusses in detail the mechanism through which spreads increase following an oil discovery, which can be summarized as follows. After an oil discovery, because of adjustment costs, the government borrows to invest in capital for the oil sector. Borrowing increases spreads and investment reduces them. The former effect dominates because once the large oil field is being exploited, capital is drawn away from the manufacturing sector. This reallocation makes tradable income—used to support debt payments—more dependent on oil revenue and thus, more volatile.

Borrowing to invest.—An oil discovery in period τ is news that the economy will have access to a larger oil field with capacity $n=n_H$ in period $\tau+6$. Thus, the government will want to have a higher level of capital for the oil sector k_{oil} in that period. Here is where capital adjustment costs in both laws of motion for capital play a role in generating the anticipation effect in investment. First, recall that in the data investment increases much earlier than a year before production in the newly discovered field starts. In the model, all the additional capital in the oil sector would be installed in period $\tau + 5$ in the absence of adjustment costs. The quadratic capital adjustment costs incentivizes the economy to smooth this investment through the preceding periods. Because of the adjustment costs for general capital, the government does not reallocate capital already installed for the other sectors to the oil sector. Instead, it borrows from the rest of the world in order to install new capital.

Borrowing increases spreads and investment, in general, reduces them.[30] Figure 1.5 illustrates this by showing the equilibrium price schedule of government bonds through two dimensions: bonds and capital in the oil sector chosen for the next period (using parameter values from the calibration in Section 1.4). The left panel shows how higher indebtedness reduces (increases) the market price of bonds (spreads), while the right panel shows how higher capital for the next period increases (reduces) it (them).

[30]For a detailed discussion of the effect of investment on spreads see Gordon and Guerron-Quintana [2018]. They show that investment has non-trivial effects on the equilibrium level of the price of new bonds. On one hand, more capital gives the sovereign the ability to avoid default in bad times by disinvesting to repay debt, which makes spreads decrease with investment; on the other, higher levels of capital increase the value of default in the future, which in turn increases the default set and spreads in the current period. They show that, given a high enough level of indebtedness, the former effect dominates the latter and, everything else constant, sovereign spreads decrease with investment.

Figure 1.5: Bonds price schedule



These graphs show the price function of bonds 1.13 using the parameter values from Section 1.4 evaluated at the mean of the productivity, income, and price of oil shocks. The left graph depicts the price of bonds as a function of debt in the next period b' for high and low values of capital in the oil sector k'_{oil} in the next period. The right graph shows the price of bonds as a function of capital in the oil sector k'_{oil} in the next period for high and low values of debt in the next period b'.

Note that all other state variables are kept fixed in Figure 1.5. The purpose of this figure is to illustrate how the two dynamic decisions (borrowing and investment) affect the price of bonds differently, effects which the government takes into account when making its decisions.

Dutch disease.—Now, I show how the Dutch disease operates. Recall that, within each period, general capital k can be freely allocated into the non-traded intermediate sector k_N and into the manufacturing sector k_M as long as $k_N + k_M = k$. Given the state of the economy, k_M is pinned down by:

$$\left(\frac{\alpha_M}{\alpha_N}\frac{(k-\boldsymbol{k_M})^{1-\alpha_N}}{(\boldsymbol{k_M})^{1-\alpha_M}}\right)^{\eta} f^N(z, k-\boldsymbol{k_M}) = \frac{\omega_N\left[f^M(z,\boldsymbol{k_M}) + p_{oil} f^{oil}(z, k_{oil}, n) - X\right]}{\omega_M + \omega_{oil}(p_{oil})^{1-\eta}} \tag{1.14}$$

where $X = [\gamma + (1-\gamma)\kappa]\, b - q(\cdot)\, i_b$ is payments of debt principal and interest net of new debt issuance. Note that the right-hand side is increasing in k_M and the left-hand side of equation 1.14 is decreasing. Thus, increasing n (while keeping k and k_{oil} fixed) lowers the equilibrium allocation of capital into the manufacturing sector.

An intuitive interpretation of the economic forces driving this reallocation can be drawn from the version of equation (1.14) in a decentralized economy:[31]

$$p_N f^N(z, k_N) = \frac{\omega_N (p_N)^{1-\eta} \left[f^M(z, k_M) + p_{oil} f^{oil}(z, k_{oil}, n) - X \right]}{\omega_M + \omega_{oil} (p_{oil})^{1-\eta}} \tag{1.15}$$

where p_N is the price of the non-traded intermediate good. Equation (1.15) shows that expenditure in the non-traded intermediate good (since $c_N = f^N(z, k_N)$) is a fraction of tradable income net of debt payments. Higher n implies higher income, so in order to increase consumption of the non-traded intermediate good the economy has to produce more of it—as opposed to consumption of manufactures, which can be increased by increasing imports. In the decentralized economy this higher production is supported by a higher p_N, which increases the marginal revenue of capital in that sector.

Higher volatility and spreads.—To highlight the role of volatility I borrow a simple example laid out in Arellano [2008]. Consider a small-open economy that each period receives a stochastic endowment of a tradable good $y \in Y = [\underline{y}, \bar{y}]$, which is iid across time and follows a cumulative distribution function F. There is an agent in the economy with preferences for lifetime consumption of the commodity $U(\{c_t\}_{t=0}^{\infty}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$ where u is strictly concave. The agent can issue one period non-contingent bonds b' and cannot commit to repay its debt. If the agent defaults on its debt it remains in autarky forever, which implies that the value of defaulting when income is y is $V^D(y) = u(y) + \frac{\beta}{1-\beta}\mathbb{E}[u(y')]$. If the agent repays then it chooses consumption and debt issuance to maximize its utility subject to its budget constraint $c + b \leq y + q(b')b'$. It can be shown that the sets of endowments $Y^D(b) \subseteq Y$ for which the agent decides to default given a debt level b can be characterized by an interval where only the upper bound is a function of assets $Y^D(b) = [\underline{y}, y^\star(b))$. The cutoff $y^\star(b)$ is the income level at which the agent is indifferent between repaying and defaulting $V^P(y^\star(b), b) = V^D(y^\star(b))$.[32] The debt of the agent is bought by a large number of risk-neutral competitive lenders with access to a risk free asset that pays interest rate r. Thus, the price of bonds b' in equilibrium is characterized by:

$$q(b') = \frac{1 - F(y^\star(b'))}{1 + r}$$

[31] Appendix A.2 shows an equivalence result between the centralized economy presented in this chapter and a decentralized economy with firms and a representative household.

[32] See Arellano [2008] for a proof of this result.

which is the probability of repayment in the next period discounted by the risk free interest rate. Now, consider an unexpected and permanent increase in the variance of y. Since u is strictly concave both V^P and V^D decrease. To highlight the role of volatility I assume preferences, the distribution F, and the change in volatility are such that the cutoffs $y^\star$ remain the same. With the same cutoffs the higher variance increases the probability of default, since the probability that $y < y^\star(b)$ is now higher. This decreases the price q at which lenders value the government debt and thus increases the spreads.

Going back to the model in this chapter, the reallocation of production factors once n is higher increases the volatility of traded income, as can be seen in the balance of payments equation:

$$\underbrace{\left[\gamma + (1-\gamma)\kappa\right] b - q(\cdot)\left[b' - (1-\gamma) b\right]}_{\text{net debt payments}} = \underbrace{\left[f^M(k_M) - c_M\right] + p_{oil}\left[f^{oil}(k_{oil}, n) - c_{oil}\right]}_{\text{traded income}}$$

where the right-hand side is more dependent on oil revenue with high n, which, by assumption, is more volatile than manufacturing revenue.

Slow adjustment of spreads.—Note that the reallocation of capital away from manufacturing is expected to happen in period $\tau+6$, which directly affects the price function of bonds from the perspective of period $\tau+5$. If the debt is long-term (i.e. $\gamma < 1$), a lower price of bonds in $\tau+5$ lowers the price of bonds in $\tau+4$, as can be seen in equation (1.13). This, along with the increase in borrowing following the discovery, increases spreads in all of the previous periods up until period τ, when the news about the oil discovery arrives. If the maturity of bonds was of one period then the reallocation of capital in period $\tau+6$ would only affect spreads in period $\tau+5$, which is at odds with the evidence from Figure 1.3.

1.4 Calibration

I calibrate the model to the Mexican economy using the period 1993–2012.[33] There are two reasons that make Mexico an ideal example for the purposes of this chapter: it is a typical small-open emerging economy that has been widely studied in the sovereign debt literature and it did not have any giant oil field discoveries during the period of study. This lack

[33] Except for the spreads data, which starts in 1998 for this country.

of giant oil discovery allows me to discipline the parameters of the model with business cycle data that does not include variation in endogenous variables induced by giant oil discoveries. I then validate the model by comparing the reaction of model variables to an oil field discovery with the estimates from Section 1.2.

A period in the model is one year, this is to be consistent with the empirical work from Section 1.2, which is limited to a yearly frequency since this is the scope of the oil discoveries data. There are two sets of parameters: the first (summarized in table 1.1) is calibrated directly from data and the second (summarized in table 1.2) is chosen so that moments generated by the model match their data counterparts. I set the capital shares to $\alpha_N = 0.32$ and $\alpha_M = 0.37$ following Valentinyi and Herrendorf [2008], who calculate labor shares for the U.S. for different sectors and aggregate them into tradable and non-tradable. I find it reasonable to use estimates for the U.S. given the assumption that in the model there are no technological differences between the small-open economy and the rest of the world. I set the share of oil rent to $\zeta = 0.38$ and the capital share in the oil sector to $\alpha_{oil} = 0.49$ as in Arezki et al. [2017]. I use data on sectoral GDP for Mexico between 1993 and 2012 to estimate the elasticity of substitution $\eta = 0.73$.[34] I set the weights $\omega_N = 0.79$, $\omega_M = 0.15$, and $\omega_{oil} = 0.06$ using aggregate consumption shares. I set the relative risk aversion parameter to $\sigma = 2$ and the risk free interest rate to $r^\star = 0.04$, which are standard values in the international macroeconomics literature.

I assume that the price of oil in the model follows an AR(1) process:

$$\log p_{oil,t} = (1 - \rho_{oil}) \log \bar{p}_{oil} + \rho_{oil} \log p_{oil,t-1} + \nu_p \epsilon_t \tag{1.16}$$

where ϵ_t are iid with a standard normal distribution and $\bar{p}_{oil}$ is the mean of the price of oil normalized to $\bar{p}_{oil} = 1$. Now, for the persistence and variance of the price of oil I take the cyclical component of the HP-Filtered series of the real price of oil and estimate:

$$\Delta \log p_{oil,t} = \rho_{oil} \Delta \log p_{oil,t-1} + \nu_p \Delta \epsilon_t. \tag{1.17}$$

[34] To estimate the elasticity of substitution I follow the methodology used by Stockman and Tesar [1995]. As discussed by Mendoza [2005] and Bianchi [2011], the range of estimates for the elasticity of substitution between tradables and non-tradables is between 0.40 and 0.83.

Table 1.1: Parameters calibrated directly from the data

Parameter		Value	Source
capital shares	α_N	0.32	Valentinyi and Herrendorf [2008]
	α_M	0.37	
	α_{oil}	0.49	Arezki et al. [2017]
oil rent	ζ	0.38	
elasticity of substitution	η	0.73	estimated for Mexico
intermediate output shares	ω_N	0.79	shares in aggregate consumption
	ω_M	0.15	
	ω_{oil}	0.06	
risk aversion	σ	2	standard values
risk free rate	r^*	0.04	
bonds maturity rate	γ	0.14	7 year average duration
bonds coupon rate	κ	0.03	Chatterjee and Eyigungor [2012]
probability of reentry	θ	0.40	2.5 years exclusion
support of i.i.d. shock	$\bar{m}$	0.018	Chatterjee and Eyigungor [2012]
standard deviation of i.i.d. shock	σ_m	0.009	bound is +/- 2 standard deviations
persistence of price of oil	ρ_{oil}	0.48	AR(1) estimation of innovations
volatility of price of oil	ν_p^2	0.23	in the real price of oil

I use data on the West Texas Intermediate price of oil divided by the US GDP deflator to calculate a real price and estimate that the persistence parameter in (1.17) is $\rho_{oil} = 0.48$ and the variance of the iid shock is $\nu_p^2 = 0.23$. Then I use these estimates to approximate the process with a finite state Markov-chain using the Rouwenhorst method.[35]

I set the probability of re-entry to financial markets to $\theta = 0.40$, so that the average duration of exclusion is 2.5 years, as documented for recent default episodes by Gelos, Sahay, and Sandleirs [2011]. I set $\gamma = 0.14$ so that the average duration of bonds is 7

[35]This method was first proposed by Rouwenhorst [1995] and it approximates the underlying AR(1) process better than that of Tauchen [1986] when the persistence ρ is close to 1. For a discussion on its properties see Kopecky and Suen [2010].

years, as documented for Mexico by Broner, Lorenzoni, and Schmukler [2013] and I set the coupon payments $\kappa = 0.03$ as in Gordon and Guerron-Quintana [2018]. For the transitory income shock m I follow Chatterjee and Eyigungor [2012] and assume $m \sim \text{trunc } N\left(0, \sigma_m^2\right)$ with points of truncation $-\bar{m}$ and $\bar{m}$. I set $\bar{m} = 0.018$ and $\sigma_m = 0.009$. For Chatterjee and Eyigungor [2012] these values are 0.006 and 0.003, respectively. I re-scale these values so that the size of the maximum transitory component of income relative to the average production of the final good remains the same.

The productivity shock is assumed to follow an AR(1) process:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \nu_t$$

where ν_t are iid with a standard normal distribution. The persistence ρ_z and variance σ_z are calibrated to match the persistence and volatility of the business cycle of the Mexican GDP (more details below).

Table 1.2 summarizes the parameters calibrated by simulating the model. This calibration is made in two steps: first, all parameters except n_H are chosen to match some business cycle moments for the Mexican economy in the period 1993–2012. This first step only considers simulated economies in their ergodic state with $n = n_L$. The second step introduces an unexpected oil discovery to these economies and calculates its net present value as a fraction of GDP, as explained below, to discipline n_H.

Table 1.2: Parameters calibrated simulating the model

Parameter		Value	Parameter		Value
discount factor	β	0.925	capital depreciation rate	δ	0.10
productivity	d_0	-0.40	persistence of productivity	ρ_z	0.95
default cost	d_1	0.412	volatility of productivity	σ_z	0.009
capital adjustment	ϕ	7.5	high oil for extraction	n_L	0.28
costs	ϕ_{oil}	7.5	high oil for extraction	n_H	1.12

Moment	Data	Model
Average spread	266	214
St. dev. of spreads	134	135
Debt-to-GDP ratio	0.14	0.09
Capital-to-GDP ratio	1.69	1.91
$\sigma_{inv}/\sigma_{GDP}$	3.4	1.8
ρ_{GDP}	0.30	0.42
σ_{GDP}	2.44	2.35
oil_{GDP}/GDP	0.07	0.07
NPV/GDP	18	22

For simplicity, I assume the capital adjustment cost functions are the same $\phi_{oil} = \phi$. Then, there is a total of nine parameters chosen to match nine moments from the data: the average and standard deviation of spreads, the debt-to-GDP ratio, the capital-to-GDP ratio, the relative volatility of investment to GDP, the persistence and variance of GDP, the average oil GDP to total GDP ratio, and the net present value of oil discoveries as a fraction of GDP.

The value of the discount factor β mainly determines the debt-to-GDP ratio. The average and standard deviation of spreads are mainly pinned down by the default cost parameters d_0 and d_1. The capital-to-GDP ratio and the relative volatility of investment are mostly determined, respectively, by the capital depreciation rate δ and the capital adjustment cost parameters ϕ and ϕ_{oil}. The values of ρ_{GDP} and σ_{GDP} are estimated using data of the cyclical component of Mexican GDP and GDP series generated by the model. Both are HP-filtered with a smoothing parameter of 100 for yearly data. These values for the data simulated by the model are pinned down by the persistence ρ_z and variance σ_z of the productivity shock. I choose n_L to match the average ratio of GDP in the oil sector to total GDP for Mexico between 1993 and 2012. Finally, I choose n_H to match the average net present value of oil discoveries as a fraction of GDP. These net present values are calculated as:

$$NPV_t = \sum_{s=6}^{\infty} \left(\frac{1}{1+r_t}\right)^s p_{oil,t} \left[f^{oil}\left(z_t, k_{oil,t}, n_H\right) - f^{oil}\left(z_t, k_{oil,t}, n_L\right)\right] \tag{1.18}$$

where r_t is the implied yield of the government bonds at the time of discovery t. This calculation is akin to the calculation made by Arezki, Ramey, and Sheng [2017] with actual data following equation (1.1).

Table 1.3 shows the performance of the model with non-targeted moments. The model does well with the over-volatility of consumption relative to output. The model also does a good job producing counter-cyclical spreads and trade balance, as well as predicting a lower correlation between investment and output relative to that of output and consumption. However, the magnitude of the correlation between trade balance and GDP in the model is much higher than in the data.

Table 1.3: Non-targeted moments

Moment	Data	Model
σ_c/σ_{GDP}	1.16	1.09
σ_{TB}	2.05	0.80
$Corr\left(c, GDP\right)$	0.87	0.98
$Corr\left(inv, GDP\right)$	0.86	0.37
$Corr\left(\frac{TB}{GDP}, GDP\right)$	-0.15	-0.26
$Corr\left(\text{spreads}, GDP\right)$	-0.37	-0.38

The following section shows the model's predictions after an oil discovery, with special focus on the model's ability to reproduce the responses documented from the data in Section 1.2.

1.5 Quantitative Results

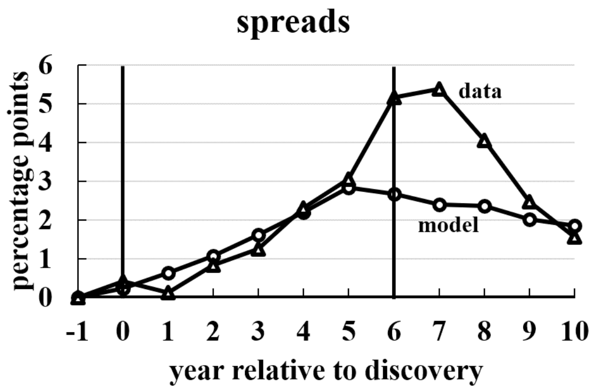
This section presents the main quantitative results. First, Subsection 1.5.1 compares the model predictions of the change in spreads and other macroeconomic variables to the estimates from the data laid out in Section 1.2. Then, Subsection 1.5.2 disentangles the effect that the Dutch disease has on the increase in spreads and the co-movement of other macroeconomic variables following an oil discovery in the model.

All the impulse-response functions from the model are computed as follows: (i) simulate 300 economies for 5001 periods without any oil discoveries, (ii) drop the first 5000 to eliminate any effect of initial conditions and take period 5001 as the starting point, (iii) make the economy experience an unexpected oil discovery in period 5002 and simulate 50 more periods, (iv) center all economies such that $t = 0$ is the period when the discovery is announced and calculate the average of all paths, (v) calculate the impulse-response function of variable x as the change with respect to its value before the oil discovery in period $t = -1$, $IR(x_t) = x_t - x_{-1}$.

1.5.1 Model vs data

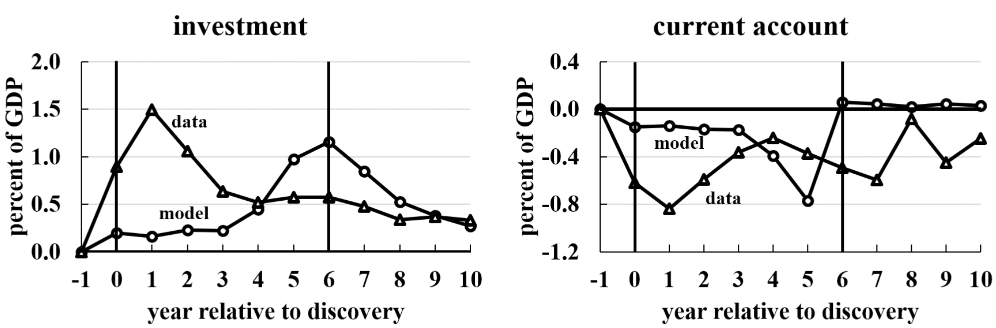
Figure 1.6 compares the impulse-response of spreads in the model to the estimates from Figure 1.3. In the data, spreads start increasing when the news of the discovery is realized and continue to increase until they peak in year 7, when they reach a maximum increase of 5.3 percentage points. In the model under the calibration from the previous section, spreads also increase when the news is realized and continue to do so until period 5, when they reach a maximum increase of almost 3 percentage points. The peak in the model happens exactly one period before the larger oil field is available.

Figure 1.6: Impulse-response to a discovery of average size



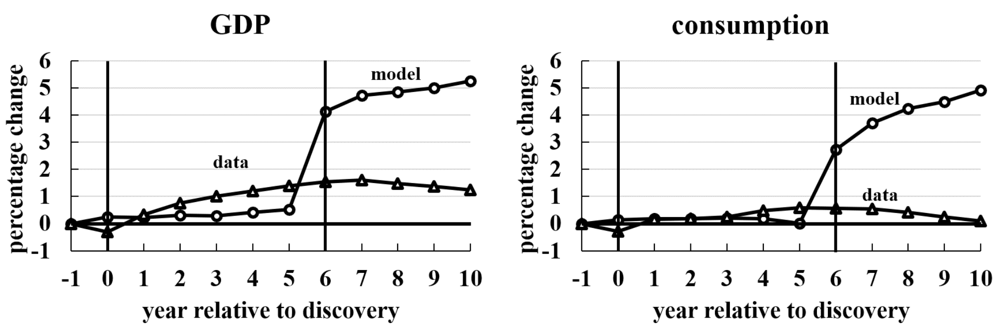
The model explains virtually all the increase up until period 5. The model also explains the subsequent decrease after spreads reach their peak. In the data, however, spreads continue to increase until period 7, after which they also start decreasing. One potential explanation is that the oil fields in the sample I consider took longer than average to start being productive. If I assumed the larger field in the model became available in year 8 rather than in year 6, the increase would continue until year 7, as in the data.
Figure 1.7 compares the impulse-response of investment and the current account in the model to the estimates from Figure 1.2.

Figure 1.7: Impulse-response to a discovery of average size



In the model, as in the data, investment increases and the current account goes into deficit between the announcement of the discovery and the start of production. The orders of magnitude of these changes are of around 1 percentage point of GDP. The changes in the model happen closer to when production starts, while in the data they happen closer to the announcement. This may be due to the timing assumption. In the model, the economy has to wait 6 years to access the oil in the field, while in the real world this waiting period depends on the intensity, speed, and efficiency of investment in the sector.
Figure 1.8 compares the impulse-response of investment and the current account in the model to the estimates from Figure 1.2. GDP and consumption increase both in the data and in the model. However, the increases in the model are more concentrated in the year when production starts.

Figure 1.8: Impulse-response to a discovery of average size



The government in the model cannot smooth consumption more because the debt level is already too high in the ergodic state. In other words, borrowing to consume is already too expensive. Regarding GDP, Arezki, Ramey, and Sheng [2017] find that, for a larger set of countries, GDP in the data also does not increase right away, which is consistent with standard models like the one they study and like the one laid out in Section 2.2. The fact that GDP increases right away for the sample of emerging economies considered in this chapter is puzling and a direction for future work.

1.5.2 The Dutch disease and the increase in spreads

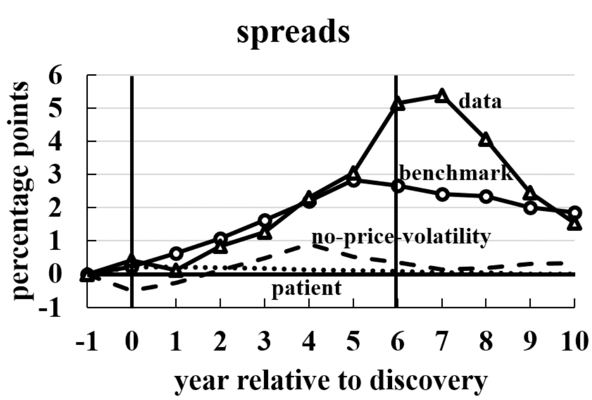
To decompose the effect of the Dutch disease I compare impulse-responses from the model under the benchmark calibration to those from a model in which the price of oil is fixed, the *no-price-volatility* case. To calculate these I recalibrate the parameters from Table 1.2 to match the same moments when the price of oil is fixed.

As discussed in Subsection 1.2.4, the reallocation of production factors implied by the Dutch disease increases the volatility of tradable income, which in turn undermines the effect of investment on spreads. The counterfactual case with no volatility in the price of oil shuts down this effect while still allowing for the reallocation of capital.

Additionally, as a reference, I also compare these responses to those from a model in which the government is as *patient* as the rest of the world, meaning $\beta = \frac{1}{1+r}$. If one assumes that the households are as patient as international markets, then $\beta < \frac{1}{1+r}$ can be interpreted as the government being more impatient than the households.[36] In this reference economy with a patient government default events are infrequent because it does not accumulate much debt.

Figure 1.9 shows the impulse-response of spreads for each of these cases. Spreads still increase in the model with no volatility in the price of oil, but not as much as in the benchmark case or in the data.

Figure 1.9: Impulse-response to a discovery of average size



[36]This lower discount factor for the government can be rationalized in political economy models where the government cares more about present consumption due to reelection incentives.

The increase peaks a little bit under 1 percentage point, which is one third of the peak under the benchmark calibration. The remaining two thirds of the increase is generated by the Dutch disease. In the economy with a patient government spreads barely change following an oil discovery. There are two reasons for this. First, the patient government accumulates lower levels of debt, so when news of an oil discovery arrive the increase in borrowing to invest does not increase spreads by much since the initial debt level was low. Second, the higher valuation of the future reduces default incentives for any state of the world and any level of borrowing *vis-a-vis* the economy with an impatient government, which also makes spreads smaller.

The frictions that make spreads high in the benchmark economy are market incompleteness, lack of commitment of the government, and impatience ($\beta < \frac{1}{1+r}$). The counterfactual case when the price of oil is fixed can be interpreted as reducing the intensity of market incompleteness (since one source of risk is taken away); while the counterfactual case of the patient government can be interpreted as eliminating the preference disagreement. Figure 1.9 shows that just eliminating the volatility of the price of oil generates almost the same response (or non-response) of spreads to oil discoveries as if the government was not relatively impatient.

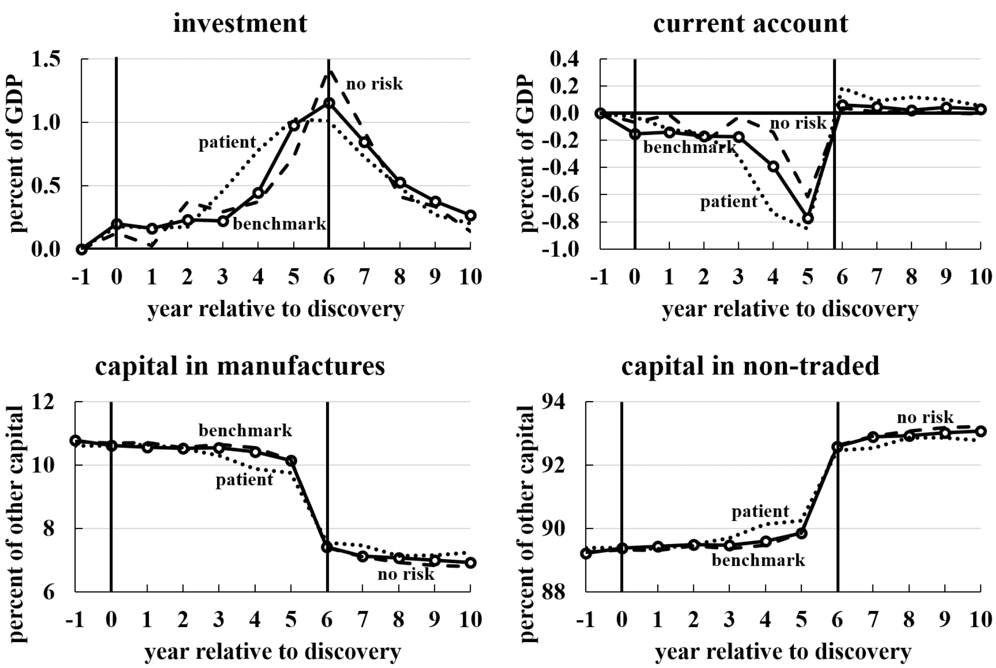
These results suggest that access to insurance against swings in the price of oil could eliminate most of the increase in spreads that follow giant oil discoveries.[37] In a recent paper Rebelo, Wang, and Yang [2019] study how financial development, defined as the extent to which countries can hedge in international capital markets, interacts with sovereign risk and debt accumulation. They find that the inability to hedge reduces debt capacity and increases credit spreads, consitent with the findings in this section. The exercise of eliminating the volatility in the price of oil are akin to giving the government the ability to hedge without any cost. A more realistic model of this would include the availability of contracts contingent on the price of oil. As Rebelo, Wang, and Yang [2019] argue, hedging is more cost effective than defaulting, so if these contracts were available the government

[37] There are multiple ways for an economy to hedge against the volatility of the price of oil, from simple financial instruments like sell options to more complicated institutional arrangements like the sovereign wealth funds in Norway and Chile (in this case, to hedge against the volatility of the price of copper).

would always take them.

Note that the reallocation of capital implied by the Dutch disease is not by itself the source of the increase in spreads. To illustrate this point, Figure 1.10 shows, for each of the three cases, the impulse responses of investment, the current account, and the share of general capital allocated to manufactures and the non-traded sector.

Figure 1.10: Impulse-response to a discovery of average size



The responses are virtually identical. In all three cases the government increases its foreign borrowing to invest by around the same share of GDP. These findings imply that the Dutch disease only increase spreads because of the high volatility of the price of oil. Furthermore, the capital reallocation seems to be the optimal choice for both a patient government or an impatient one that does not face the risk of swings in the price of oil.

1.6 Conclusion

In this paper, I documented the effect of giant oil field discoveries on sovereign spreads, the sectoral allocation of capital, and macroeconomic aggregates of emerging economies. Following a giant oil discovery, sovereign spreads increase by up to 530 basis points and the share of investment in manufacturing decreases in favor of investment in commodities and non-traded sectors. Countries run a current account deficit and GDP, investment, and consumption increase.

I developed a sovereign default model with production in three sectors, capital accumulation, and discovery of oil fields. The model accounts for most of the increase in spreads documented from the data. In the model, capital in the oil and non-traded sectors increase and capital in the manufacturing sector decreases. This shift—referred to in the literature as the Dutch disease—increases the volatility of tradable income that supports debt payments since the price of oil is more volatile than the price of manufactures. The Dutch disease accounts for two thirds of the increase in spreads in the model.

In the model presented in this paper, the frictions that explain high spreads are market incompleteness, the lack of commitment from the government, and its high relative impatience. In the absence of these frictions the incentives to borrow to invest in the larger oil field and the incentives that drive the reallocation of capital due to the Dutch disease are still present. In the presence of these frictions, the volatility of the price of oil, the choice of borrowing to invest, and the the Dutch disease together generate a large increase in spreads following an oil discovery. While eliminating the volatility of the price of oil reduces the increase in spreads, all other relevant responses remain virtually unchanged. This highlights the value of access to financial instruments or institutional arrangements that could allow governments to hedge against the volatility of the international price of oil.

Chapter 2

Sovereign Risk and Dutch Disease

2.1 Introduction

The Dutch disease refers to the way an increase in exports of natural resources induces a reallocation of production factors away from the manufacturing sector. [1] This chapter studies an environment in which the Dutch disease amplifies an inefficiency in private investment that affects the cost of foreign borrowing for the government. The model features production in two intermediate sectors: tradable manufactures and a non-tradable good. In addition, the economy receives an endowment of a tradable commodity. The international price of this tradable commodity follows an exogenous stochastic process. The international price of manufactures is fixed, but production in this sector is subject to productivity shocks. Given these processes, the volatility of total tradable income—which is used for consumption and to service foreign debt—depends on the size of the manufacturing sector relative to the endowment of the tradable commodity.

The household makes investment decisions in both manufacturing and non-traded sectors and does not internalize how these choices affect the relative size of traded sectors. In

[1] Higher revenue from the natural resource boom increases the demand for all consumption goods. This income effect raises the price of non-traded goods, which causes an appreciation of the real exchange rate. This appreciation makes imports of manufactures relatively cheaper and thus induces the reallocation of production factors away from this sector into the production of non-traded goods which cannot be imported. The term was first used in 1977 by The Economist to describe this phenomenon in the Dutch economy after the discovery of natural gas reserves in 1959.

particular, the household is miopic about how investment changes the volatility of total tradable income, which in turn affects the probability of future low income states in which the government would find it optimal to default. A higher probability of future default is costly for two reasons: it limits the economy's ability to borrow from the rest of the world through higher interest rates and because default events imply a dead-weight loss for the economy. Crucially, I assume that the government does not have enough instruments to fully manipulate the household's investment choices, which implies that investment allocations are inefficient from the point of view of a social planner.

Understanding the inefficiency laid out in this chapter will shed light on relevant policy trade-offs to be studied in future research. Policies aimed at correcting this inefficiency—such as differentiated investment taxes—should be guided by careful quantitative analyses of the marginal effects of such taxes on observable variables such as sovereign interest rate spreads, real exchange rates, and investment.

There is extensive literature that has highlighted frictions that make the Dutch disease inefficient, most of them related to economic growth.[2] Auty [1993] coined the term "resource curse" to describe the observation that countries with high natural resource wealth show poorer economic performance relative to those with lower. Sachs and Warner [2001] document that countries with large natural resource wealth grow more slowly. More recently, Alberola and Benigno [2017] study an environment in which the Dutch disease delays the economy's convergence to the world technological frontier because of the presence of dynamic productivity gains in the manufacturing sector.

Layout.—The remainder of the chapter is organized as follows. Section 2.2 presents the model and discusses the key friction. Section 2.3 presents a social planner's problem. Section 2.4 analyses how the Dutch disease interacts with the frictions in the model and how it makes the competitive allocation inefficient. Section 2.5 concludes.

2.2 Model

This section lays out the model that will be used to highlight the inefficiencies and their interaction. The model builds on the literature of quantitative sovereign default models

[2] For an extensive survey of this literature see Frankel [2012].

following Eaton and Gersovitz [1981] and augments the standard model with production in different sectors, sector specific capital accumulation, and an endowment of a tradable commodity. Subsection 2.2.1 lays out the physical environment, Subsection 2.2.2 characterizes production allocations and domestic prices, Subsection 2.2.3 sets up the problems of the government and the household recursively, and Subsection 2.2.4 discusses the trade-offs of the dynamic choices of households and their interaction with government policy.

2.2.1 Environment

There is a small open economy populated by a representative household, firms, and a government.

Production and technologies.—There are four goods in the economy: a final non-traded good, a non-traded intermediate good, a manufacturing good, and a commodity (these last two can be traded with the rest of the world). Each period, the household receives a fixed endowment of commodities $y_{C,t} = y_C \geq 0$. The non-traded and the manufacturing intermediate goods are produced by competitive firms using capital and decreasing returns to scale technologies:

$$y_{N,t} = z_t k_{N,t}^{\alpha_N} \tag{2.1}$$

$$y_{M,t} = z_t k_{M,t}^{\alpha_M} \tag{2.2}$$

were z is an aggregate productivity shock that affects both intermediate sectors equally and $0 < \alpha_N, \alpha_M < 1$.[3]

The final non-traded good is used for consumption and investment. This good is produced by a competitive firm with technology:

$$Y_t = A\left[\omega^{\frac{1}{\eta}}\left(c_{N,t}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}}\left(c_{T,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.3}$$

where $c_{N,t}$ is consumption of the intermediate non-traded good, $c_{T,t}$ is a composite of manufactures and commodities, ω is the weight of non-traded intermediates, η is the elasticity of substitution between traded and non-traded goods, and A is a normalization parameter. From the problem of a profit-maximizing firm we get the demands for the non-traded

[3] Decreasing returns to scale captures the presence of a fixed factor, which in this case could be labor (immobile within a sector).

intermediate and the composite traded good:

$$c_{N,t} = \left(\frac{P_t}{p_{N,t}}\right)^{\eta} \omega Y_t \tag{2.4}$$

$$c_{T,t} = \left(\frac{P_t}{p_{T,t}}\right)^{\eta} (1-\omega) Y_t \tag{2.5}$$

where $p_{N,t}$ is the relative price of the non-traded intermediate good, $p_{T,t}$ is the shadow price of the basket of intermediate traded goods, and P_t is the standard CES price index:

$$P_t = \left[\omega (p_{N,t})^{1-\eta} + (1-\omega)(p_{T,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.6}$$

which is consistent with the firm making zero profits in equilibrium. The intermediate traded goods are aggregated using technology:

$$c_{T,t} = \left[\gamma^{\frac{1}{\chi}} (c_{M,t})^{\frac{\chi-1}{\chi}} + (1-\gamma)^{\frac{1}{\chi}} (c_{C,t})^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

where $c_{M,t}$ is consumption of manufactures, $c_{C,t}$ is consumption of commodities, γ is the weight of manufactures, and χ is the elasticity of substitution between manufactures and commodities. From the problem of the final good firm we get the demands

$$c_{M,t} = \left(\frac{p_{T,t}}{p_{M,t}}\right)^{\chi} \gamma c_{T,t} \tag{2.7}$$

$$c_{C,t} = \left(\frac{p_{T,t}}{p_{C,t}}\right)^{\chi} (1-\gamma) c_{T,t} \tag{2.8}$$

for manufactures $c_{M,t}$ and commodities $c_{C,t}$ as functions of consumption of the demand for the composite traded good $c_{T,t}$ and the prices of manufactures $p_{M,t}$ and commodities $p_{C,t}$. Hereafter normalize all relative prices by the prices of manufactures so that $p_{M,t} = 1$. The shadow price of the basket of traded goods is $p_{T,t} = \left[\gamma + (1-\gamma)(p_{C,t})^{1-\chi}\right]^{\frac{1}{1-\chi}}$.

Shocks.—In each period the economy experiences one of finitely many events s_t that follow a markov chain. The shock s_t determines productivity in the non-traded and manufacturing intermediate sectors z_t and the international price of commodities $p_{C,t}$. Note that, since the economy is small relative to the rest of the world, it takes the realization of $p_{C,t}$ as given, however the relative price of non-traded intermediates $p_{N,t}$ is pinned down in equilibrium.

Households.—There is a representative household with preferences for consumption of the final good represented by $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The budget constraint of the household in each period t is:

$$P_t\left(c_t + i_{N,t} + i_{M,t}\right) \leq r_{N,t}k_{N,t} + r_{M,t}k_{M,t} + p_{C,t}y_{C,t} + \pi_{N,t} + \pi_{M,t} + \Pi_t + T_t \tag{2.9}$$

where $i_{N,t}$ and $i_{M,t}$ are investment in capital for non-traded and manufacturing production, respectively, $r_{N,t}$ and $r_{M,t}$ are the rental rates of each type of capital, $\pi_{N,t}$ and $\pi_{M,t}$ are profits from the non-traded and manufacturing intermediate good firms, respectively, Π_t are profits from the final good firm, and T_t are lump-sum transfers from the government. The household maximizes its lifetime expected utility $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta_H^t u\left(c_t\right)\right]$, where β_H is the household's discount factor, subject to its budget constraint (2.9) and the laws of motion of each stock of capital:

$$k_{N,t+1} = i_{N,t} + (1-\delta)\,k_{N,t} - \Phi\left(k_{N,t+1}, k_{N,t}\right) \tag{2.10}$$

$$k_{M,t+1} = i_{M,t} + (1-\delta)\,k_{M,t} - \Phi\left(k_{M,t+1}, k_{M,t}\right) \tag{2.11}$$

where δ is the depreciation rate of capital and $\Phi\left(k', k\right) = \frac{\phi}{2}\left(\frac{k'-k}{k}\right)^2 k$ is a capital adjustment cost function.[4]

Government and default.—The government can issue one period non-contingent debt in international financial markets and only has access to lump-sum transfers to tax the household. Government debt is purchased by risk-neutral competitive lenders with deep pockets.[5] If the government is in good financial standing its budget constraint is:

$$T_t + B_t = q_t B_{t+1} \tag{2.12}$$

where T_t is the lump-sum transfer to the household, B_t is outstanding debt due in period t, B_{t+1} is new debt issued due in period $t+1$ and q_t is the price of newly issued debt.

At the beginning of each period the government can decide to default on its debt obligations.[6] If the government defaults then it gets excluded from international financial markets for a stochastic number of periods; the government gets re-admitted to financial markets with probability θ and zero debt. While in default, productivity in the non-traded and

[4]Subsection (2.2.3) discusses the problem of the household in detail.

[5]The assumption of lenders with "deep pockets" implies that a single lender is always able to purchase all of the newly issued debt. This assumption allows to rule out self-fulfilling equilibria as introduced in Cole and Kehoe [2000]. Subsection (2.2.3) discusses in detail the behavior of the lenders.

[6]Subsection (2.2.3) discusses the timing assumptions of the model in detail.

manufacturing intermediate sectors is $z_{D,t} = z_t - \max\left\{0, d_0 + d_1 z_t + d_2 z_t^2\right\}$. With $d_0 < 0$ and $d_1 = d_2 = 0$ this default cost takes the form of the cost introduced by Arellano [2008] and with $d_0 = 0$ and $d_1 < 0 < d_2$ the default cost takes the form of the cost function used by Chatterjee and Eyigungor [2012] and Gordon and Guerron-Quintana [2018]. In either case, what is essential is that there is an asymmetry in the default penalty: it is low (or zero) for low realizations of z_t and high and increasing in z_t with higher realizations of z_t.[7] The government makes borrowing, taxing, and default decisions to maximize lifetime expected utility of the household $\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$, where I have assumed that the government's discount factor is the same as the household's. This assumption allows me to highlight that the inefficiency studied in this chapter is not a result of disagreement in preferences between the government and the household, but rather a consequence of the lack of instruments from the government to induce a different investment behavior from the household.

2.2.2 Domestic prices and firms allocations

From the problems of the firms and market clearing conditions we can express all the allocations of the firms and domestic prices $(p_{N,t}, r_{N,t}, r_{M,t})$ in terms of the aggregate capital stocks $K_t = (K_{N,t}, K_{M,t})$, the shock s_t, and the transfers from the government T_t:

$$c_{C,t}(s, K_t, T_t) = \frac{(1-\gamma)(p_{C,t})^{-\chi}}{\gamma + (1-\gamma)(p_{C,t})^{1-\chi}}\left[z_t K_{M,t}^{\alpha_M} + p_{C,t} y_C + T_t\right] \tag{2.13}$$

$$c_{M,t}(s, K_t, T_t) = \frac{\gamma}{\gamma + (1-\gamma)(p_{C,t})^{1-\chi}}\left[z K_{M,t}^{\alpha_M} + p_{C,t} y_C + T_t\right] \tag{2.14}$$

$$c_{N,t}(s, K_t) = z_t K_{N,t}^{\alpha_N} \tag{2.15}$$

$$p_{N,t}(s, K_t, T_t) = \left(\frac{c_{T,t}}{c_{N,t}} \frac{\omega}{1-\omega}\right)^{\frac{1}{\eta}} p_{T,t} \tag{2.16}$$

$$r_{M,t}(s, K_t, T_t) = \alpha_M z_t K_{M,t}^{\alpha_M - 1} \tag{2.17}$$

$$r_{N,t}(s, K_t, T_t) = p_{N,t} \alpha_N z_t K_{N,t}^{\alpha_N - 1} \tag{2.18}$$

where 2.13 and 2.14 imply that expenditures on commodities and manufactures by the final good firm are shares of aggregate tradable income net of government transfers, 2.17

[7] This asymmetry is crucial in generating high default rates and default events in "bad times" in this class of models, for a detailed discussion see Arellano [2008].

and 2.18 equate marginal revenue of capital to its marginal cost, and 2.16 expresses p_N as a function of demand of non-traded intermediates relative to the demand of traded goods. Similarly, the profits of the firms that produce intermediate goods are:

$$\pi_{N,t}\left(s, K_t, T_t\right) = p_{N,t} z_t K_{N,t}^{\alpha_N} - r_{N,t} K_{N,t} \tag{2.19}$$

$$\pi_{M,t}\left(s, K_t, T_t\right) = z_t K_{M,t}^{\alpha_M} - r_{M,t} K_{M,t} \tag{2.20}$$

since the technology to produce the final good has constant returns to scale I assume that the firm makes zero profits in equilibrium, which is consistent with the price index in 2.6.

2.2.3 Recursive formulation and equilibrium

The aggregate state of the economy is the exogenous state variable s, the aggregate capital stocks $K = (K_N, K_M)$, and the stock of government debt B. The timing of events within each period is as follows:

1. At the beginning of the period, the government observes the aggregate state (s, K, B) and decides whether to default or repay.

2. After the government decides to default or repay, then it chooses its debt issuance and transfers, lenders purchase the government debt, and the private sector makes their decisions. These events all happen **simultaneously**.

The government understands how its policy affects the price of debt and household's decisions. Lenders also understand this and take it into account when pricing the government debt. Lenders have access to a one period risk-free bond that pays a fixed interest rate $r^\star$ and purchase government debt as long as they break even in expected value, which implies the price of government debt is:

$$q = \frac{1 - \mathcal{D}}{1 + r^\star}$$

where $\mathcal{D}$ is the probability of the government defaulting in the next period. Given the timing assumption above we have that $\mathcal{D} = \mathbb{E}\left[d\left(s', K', B'\right)\right]$ where d is the government's policy that only depends on the aggregate state in the next period.[8] Thus, since s follows

[8] Since the auction of government debt and household investment happen simultaneously, the timing assumption rules out multiplicity of equilibria of the type studied in Galli [2019], in which expectations about household investment can imply two different self-fulfilling scenarios for how lenders price government debt.

a Markov process we can express the price of government debt as:

$$q\left(s, K', B'\right) = \frac{\mathbb{E}\left[1 - d\left(s', K', B'\right)|s\right]}{1 + r^{\star}}$$

To write down the problem of a representative household in recursive form it is crucial to distinguish between the household's individual capital holdings $k = (k_N, k_M)$ and the aggregate capital stocks in the economy $K = (K_N, K_M)$. The household takes the government's policy in the current period as given and has beliefs about the government's policy in the next period and about the law of motion of the aggregate state of the economy. If the government repays, then the value of the representative household is:

$$\begin{aligned}
H^P\left(s, k, K, B; T\right) = \max_{k'} & \left\{u\left(c\right) + \beta\mathbb{E}\left[\left(1 - d'\right) H^P\left(s', k', K', B'; T'\right)\right]\right. \\
& \left. + \beta\mathbb{E}\left[d' H^D\left(s', k', K'\right)\right]\right\} \\
s.t.\ P\left(s, K, T\right)\left(c + i_N + i_M\right) & \leq r_N\left(s, K, T\right) k_N + r_M\left(s, K, T\right) k_M + p_C y_C \\
& + \pi_N\left(s, K, T\right) + \pi_M\left(s, K, T\right) + \Pi\left(s, K, T\right) + T \\
i_N & = k'_N - \left(1 - \delta\right) k_N + \Phi\left(k'_N, k_N\right) \\
i_M & = k'_M - \left(1 - \delta\right) k_M + \Phi\left(k'_M, k_M\right) \\
K' & = \Gamma^P_K\left(s, K, B; T\right) \\
B' & = \Gamma_B\left(s, K, B\right) \\
d' & = \Gamma_d\left(s', K', B'\right) \\
T' & = q\left(s', \Gamma^P_K\left(s', K', B'\right), \Gamma_B\left(s', K', B'\right)\right)\Gamma_B\left(s', K', B'\right) - B'
\end{aligned}$$

where Γ^P_K is the household's belief about the law of motion for aggregate capital stocks when the government is in good financial standing, Γ_B is the household's belief about the law of motion of government debt, and Γ_d is the household's belief about the government's default decision in the next period. This problem yields policy functions $\boldsymbol{c}\left(s, k, K, B; T\right)$

and $\boldsymbol{k}(s,k,K,B;T)$. The value of the household if the government defaults is:

$$\begin{aligned} H^{D}(s,k,K) = \max_{k'} \Big\{ & u(c) + \beta\theta\mathbb{E}\left[\left(1-d'\right) H^{P}\left(s',k',K',0;T'\right)\right] \\ & +\beta\mathbb{E}\left[\left(1-\theta\left(1-d'\right)\right) H^{D}\left(s',k',K'\right)\right]\Big\} \\ s.t.\ P(s_D,K,0)(c+i_N+i_M) \leq\ & r_N(s_D,K,0)k_N + r_M(s_D,K,0)k_M + p_C y_C \\ & + \pi_N(s_D,K,0) + \pi_M(s_D,K,0) + \Pi(s_D,K,0) \\ i_N =\ & k'_N - (1-\delta)k_N + \Phi\left(k'_N,k_N\right) \\ i_M =\ & k'_M - (1-\delta)k_M + \Phi\left(k'_M,k_M\right) \\ K' =\ & \Gamma_K^D(s,K) \\ d' =\ & \Gamma_d\left(s',K',0\right) \\ T' =\ & q\left(s',\Gamma_K^P\left(s',K',0\right),\Gamma_B\left(s',K',0\right)\right)\Gamma_B\left(s',K',0\right) \end{aligned}$$

where Γ_K^D is the household's belief about the law of motion for aggregate capital stocks when the government is in bad financial standing and $s_D = (z_D, p_C)$. This problem yields policy functions $\boldsymbol{c^D}(s,k,K)$ and $\boldsymbol{k^D}(s,k,K)$.

The government understands how its policies affect the household's decisions and the price of its debt. At the beginning of the period, the value of the government when it is in good financial standing is:

$$G(s,K,B) = \max_{d\in\{0,1\}} \left\{(1-d)G^P(s,K,B) + dG^D(s,K)\right\}$$

where $d = \boldsymbol{d}(s,K,B)$ is the government's default decision that solves the maximization problem. The value if the government decides to default is:

$$\begin{aligned} G^D(s,K) =\ & u\left(c^D(s,K,K)\right) + \beta_G\theta\mathbb{E}\left[G\left(s',K',0\right)\right] \\ & + \beta(1-\theta)\mathbb{E}\left[G^D\left(s',K'\right)\right] \\ s.t. \qquad & K' = \boldsymbol{k^D}(s,K,K) \end{aligned}$$

and the value if the government decides to pay its debt is:

$$\begin{aligned} G^P(s,K,B) = \max_{B',T} & \left\{u\left(c\left(s,K,K,B;T\right)\right) + \beta\mathbb{E}\left[G\left(s',K',B'\right)\right]\right\} \\ s.t. \qquad & T \leq q\left(s,K',B'\right)B' - B \\ & K' = \boldsymbol{k}(s,K,K,B;T) \end{aligned}$$

where $B' = \boldsymbol{b}(s, K, B)$ and $T = \boldsymbol{T}(s, K, B)$ are the government's decisions that solves the problem.

Equilibrium: An equilibrium is value functions for the household $H^P(\cdot)$, $H^D(\cdot)$, policy functions for the household $\boldsymbol{c}(\cdot)$, $\boldsymbol{k}(\cdot)$, $\boldsymbol{c^D}(\cdot)$ and $\boldsymbol{k^D}(\cdot)$, beliefs for the household $\Gamma_K(\cdot)$, $\Gamma_B(\cdot)$, $\Gamma_d(\cdot)$ and $\Gamma_K^D(\cdot)$, value functions for the government $G(\cdot)$, $G^P(\cdot)$ and $G^D(\cdot)$, policy functions for the government $\boldsymbol{d}(\cdot)$, $\boldsymbol{T}(\cdot)$ and $\boldsymbol{b}(\cdot)$, allocations for the firms, domestic prices, and a price schedule for bonds $q(\cdot)$ such that:

1. given the price of bonds and household's beliefs, the value and policy functions for the household solve the household's problem for each level of transfers T,

2. the beliefs of the household satisfy:

$$\begin{aligned}
\Gamma_B(s, K, B) &= \boldsymbol{b}(s, K, B) \\
\Gamma_K^P(s, K, B; T) &= \boldsymbol{k}(s, K, K, B; T) \\
\Gamma_d(s, K, B) &= \boldsymbol{d}(s, K, B) \\
\Gamma_K^D(s, K) &= \boldsymbol{k^D}(s, K, K)
\end{aligned}$$

3. given the the price of bonds and household's policy functions, the value and policy functions for the government solve the government's problem,

4. the price schedule of bonds satisfies:

$$q\left(s, K', B'\right) = \frac{\mathbb{E}\left[1 - \boldsymbol{d}(s', K', B') \,|s\right]}{1 + r^{\star}}$$

5. the allocations for firms and domestic prices satisfy equations 2.6 and 2.13 to 2.20.

2.2.4 Discussion

At the beginning of each period, given the realization of s and the capital stock in the manufacturing sector K_M, the government knows total tradable income is:

$$\text{Tradable Income} = zK_M^{\alpha_M} + p_C y_C$$

which in turn will be used for consumption and to service outstanding debt B. This model features the standard dynamics of borrowing and default as first analyzed in Eaton and

Gersovitz [1981] and, later, in Arellano [2008] and others.[9] In "good times", defined as realizations of shocks such that tradable income is high, the government faces favorable borrowing terms. This is because shocks are persistent, so "good times" are expected to continue in the following period and the government is expected to have little incentives to default. Since $\beta\left(1+r^{\star}\right)<1$ the government accumulates debt while borrowing is cheap. Then, in "bad times" borrowing to roll over the accumulated debt becomes expensive (since "bad times" are expected to continue in the next period) and increases the incentives the government has to default.

What is different in the environment laid out here is that the mean and variance of tradable income are endogenous since K_M is a choice, while these moments are typically exogenous and fixed in standard models. The household does not take into account the effects of investment choices on these moments and the effects these in turn have on default incentives. Moreover, since there are two types of capital, the government does not have enough instruments to fully manipulate the household's investment choices. The government can give lump-sum transfers/taxes to the household, which affect total savings but are not sufficient to distort the margins the household considers to choose its portfolio. This is the main friction analyzed in this chapter. To highlight how this friction results in inefficient allocations the following section lays out the problem of a social planner.

2.3 A social planner's problem

Now consider a social planner that makes all production, investment and borrowing decisions in the economy. The planner cares about household utility and is subject to the same frictions in the sense that has only access to one-period non-contingent debt and lacks commitment to repay.[10] The key difference with the decentralized equilibrium is that the planner will internalize the effect that investment decisions have on the probability of future default and, thus, on the price of foreign debt.

[9] See Aguiar and Amador [2014] for an exhaustive discussion and literature review.

[10] The planner does not care about the utility of foreign lenders.

2.3.1 Static allocations

Before defining the recursive problem of the planner, it is useful to define a feasibility function:

$$
\begin{aligned}
F(s,K,X) = \max_{c_N,c_T,c_M,c_C} & \; A\left[\omega^{\frac{1}{\eta}}\left(c_N\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}}\left(c_T\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
s.t. \quad c_N &= zK_N^{\alpha_N} \\
c_T &= \left[\gamma^{\frac{1}{\chi}}\left(c_M\right)^{\frac{\chi-1}{\chi}} + (1-\gamma)^{\frac{1}{\chi}}\left(c_C\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \\
c_M + p_C c_C &= zK_M^{\alpha_M} + p_C y_C + X
\end{aligned}
$$

which maximizes production of the final good for a given realization of the exogenous shock s, given stocks of capital $K = (K_N, K_M)$, and a value of net imports X subject to the production technologies and the budget constraint for traded goods. If the value of net imports is 0 then trade is balanced, however this does not necessarily mean that $c_M = zK_M^{\alpha_M}$ and $c_C = y_C$—the planner can choose, for example, $c_C > y_C$ as long as the required imports of commodities are financed by exports of manufactures. When the planner has access to foreign borrowing then it is feasible to have both $c_M > zK_M^{\alpha_M}$ and $c_C > y_C$ by using debt to finance the additional consumption.

It is easy to show that this feasibility function can be rewritten as:

$$
\begin{aligned}
F(s,K,X) &= A\left[\omega^{\frac{1}{\eta}}\left(c_N\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}}\left(c_T\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
s.t. \quad c_N &= zK_N^{\alpha_N} \\
c_T &= \left[\gamma^{\frac{1}{\chi}}\left(c_M\right)^{\frac{\chi-1}{\chi}} + (1-\gamma)^{\frac{1}{\chi}}\left(c_C\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \\
c_M &= \frac{\gamma}{\gamma + (1-\gamma)\left(p_C\right)^{1-\chi}}\left[zK_M^{\alpha_M} + p_C y_C + X\right] \\
c_C &= \frac{1}{p_C}\frac{(1-\gamma)\left(p_C\right)^{1-\chi}}{\gamma + (1-\gamma)\left(p_C\right)^{1-\chi}}\left[zK_M^{\alpha_M} + p_C y_C + X\right]
\end{aligned}
$$

where c_C and c_M are mixed as in the competitive equilibrium (equations 2.13 and 2.14) for any given level of capital and value of net imports. This is because there is no inefficiency in the way firms make their production decisions. The key difference with the competitive equilibrium will be in the investment decisions made by households in the previous period, which will give the level of installed capital in each sector.

2.3.2 Recursive problem

The state of the planner consists of the exogenous stochastic variable s, the aggregate capital stocks $K = (K_N, K_M)$, and the stock of foreign debt B. Since the planner makes all the decisions, I assume the timing of events is as in Eaton and Gersovitz [1981]: the planner makes investment and borrowing decisions, then lenders observe these decisions and purchase the bonds. If the planner is in good financial standing its value is:

$$V(s,K,B) = \max_{D\in\{0,1\}} \left\{(1-D)\,V^P(s,K,B) + DV^D(s,K)\right\}$$

where $D = \boldsymbol{D}(s,K,B)$ is the planners decision to default that solves the problem. If the planner defaults then its value is:

$$V^D(s,K) = \max_{c,K'} \left\{u(c) + \beta\theta\mathbb{E}\left[V\left(s',K',0\right)\right] + \beta(1-\theta)\,\mathbb{E}\left[V^D\left(s',K'\right)\right]\right\}$$

$$\begin{aligned} s.t. \qquad c + i_M + i_N &\leq F(s_D,K,0) \\ i_N &= K_N' - (1-\delta)\,K_N + \Phi\left(K_N',K_N\right) \\ i_M &= K_M' - (1-\delta)\,K_M + \Phi\left(K_M',K_M\right) \end{aligned}$$

where $s_D = (z_D, p_C)$. The solution to this problem gives the policy functions $\boldsymbol{K^D}(s,K)$ and $\boldsymbol{C^D}(s,K)$. If the planner repays then its value is:

$$V^P(s,K,B) = \max_{c,K',B',X} \left\{u(c) + \beta\mathbb{E}\left[V\left(s',K',B'\right)\right]\right\}$$

$$\begin{aligned} s.t. \qquad c + i_M + i_N &\leq F(s,K,X) \\ i_N &= K_N' - (1-\delta)\,K_N + \Phi\left(K_N',K_N\right) \\ i_M &= K_M' - (1-\delta)\,K_M + \Phi\left(K_M',K_M\right) \\ X &\leq Q\left(s,K',B'\right)B' - B \end{aligned}$$

where $Q(s,K',B')$ is the price schedule for foreign debt. The solution to this problem gives policy functions $\boldsymbol{C}(s,K,B)$, $\boldsymbol{X}(s,K,B)$, $\boldsymbol{K}(s,K,B)$ and $\boldsymbol{B}(s,K,B)$.

Equilibrium: An equilibrium is value functions $V(s,K,B)$, $V^P(s,K,B)$ and $V^D(s,K)$, policy functions $\boldsymbol{D}(s,K,B)$, $\boldsymbol{C}(s,K,B)$, $\boldsymbol{X}(s,K,B)$, $\boldsymbol{K}(s,K,B)$, $\boldsymbol{B}(s,K,B)$, $\boldsymbol{C^D}(s,K)$ and $\boldsymbol{K^D}(s,K)$, and a price schedule for bonds $Q(s,K',B')$ such that:

1. Given $Q(s,K',B')$, the value and policy functions solve the planner's problem

2. The price schedule of bonds satisfies:

$$Q\left(s, K', B'\right) = \frac{\mathbb{E}\left[1 - \boldsymbol{D}\left(s', K', B'\right)|s\right]}{1 + r^{\star}}$$

2.4 The Dutch Disease

This Section uses the environment and the problems laid out in Sections 2.2 and 2.3 to highlight the main inefficiency in the decentralized environment, which will be defined as the "Dutch Disease" in this model.

For exposition purposes I assume that all value functions, policy functions, and the bond price schedules are twice continuously differentiable. Also, to ease exposition, I will assume for now that there are no capital adjustment costs and summarize the aggregate state as $S = (s, K, B)$. Consider the planner's problem in repayment, the first order condition with respect to K_N' is:

$$u_c\left(c\right)\left[1 - F_X\left(\cdot\right) Q_{K_N}\left(\cdot\right) B'\right] = \beta \mathbb{E}\left[V_{K_N}\left(S'\right)\right] \tag{2.21}$$

where u_c is the marginal utility of consumption, F_X is the derivative of the feasibility function with respect to the value of imports, Q_{K_N} is the derivative of the price schedule of debt Q with respect to K_N, and V_{K_N} is the derivative of the value function with respect to capital in the non-traded sector. From the envelope theorem we know this derivative is:

$$\begin{aligned} V_{K_N}\left(S\right) = {} & \boldsymbol{D} u_c\left(\boldsymbol{C^D}\right)\left[F_{K_N}\left(s_D, K, 0\right) + (1 - \delta)\right] \\ & + \left(1 - \boldsymbol{D}\right) u_c\left(\boldsymbol{C}\right)\left[F_{K_N}\left(s, K, \boldsymbol{X}\right) + (1 - \delta)\right] \end{aligned} \tag{2.22}$$

where $\boldsymbol{D}$, $\boldsymbol{C^D}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ are the policy functions evaluated at S. Plugging into 2.21 we get the planner's Euler equation for capital in the non-traded sector:

$$\begin{aligned} \left[1 - F_X\left(\cdot\right) Q_{K_N}\left(\cdot\right) B'\right] = {} & \mathbb{E}\left[\beta \boldsymbol{D} \frac{u_c\left(\boldsymbol{C^D}\right)}{u_c\left(C\right)}\left[F_{K_N}\left(s_D', K', 0\right) + (1 - \delta)\right]\right] \\ & + \mathbb{E}\left[\beta\left(1 - \boldsymbol{D}\right)\frac{u_c\left(\boldsymbol{C}\right)}{u_c\left(C\right)}\left[F_{K_N}\left(s', K', \boldsymbol{X}\right) + (1 - \delta)\right]\right] \end{aligned} \tag{2.23}$$

where the policy functions in the right-hand side are evaluated at S'. Similarly, we can derive the household's Euler equation for k_N in the decentralized economy:

$$1 = \mathbb{E}\left[\beta \boldsymbol{d} \frac{u_c\left(\boldsymbol{c^D}\right)}{u_c\left(c\right)} \frac{\left[r_N\left(s'_D, K', 0\right) + P\left(s'_D, K', \boldsymbol{T}\right)\left(1-\delta\right)\right]}{P\left(s, K, T\right)}\right] + \mathbb{E}\left[\beta\left(1-\boldsymbol{d}\right) \frac{u_c\left(\boldsymbol{c}\right)}{u_c\left(c\right)} \frac{\left[r_N\left(s', K', \boldsymbol{T}\right) + P\left(s, K', \boldsymbol{T}\right)\left(1-\delta\right)\right]}{P\left(s, K, T\right)}\right]$$

where the policy functions in the right-hand side are evaluated at S'. It is easy to show that this equation can be rewritten as:

$$1 = \mathbb{E}\left[\beta \boldsymbol{d} \frac{u_c\left(\boldsymbol{c^D}\right)}{u_c\left(c\right)} \left[F_{K_N}\left(s'_D, K', 0\right) + \left(1-\delta\right)\right]\right] + \mathbb{E}\left[\beta\left(1-\boldsymbol{d}\right) \frac{u_c\left(\boldsymbol{c}\right)}{u_c\left(c\right)} \left[F_{K_N}\left(s', K', \boldsymbol{T}\right) + \left(1-\delta\right)\right]\right] \tag{2.24}$$

First, note that equation 2.23 has an additional term in the left-hand side compared to 2.24, $-F_X\left(\cdot\right) Q_{K_N}\left(\cdot\right) B'$. The planner takes into account that current investment in the non-traded sector affects her incentives to default in the next period, which in turn affects the price of borrowing in the current period. This term is absent in 2.24 because the household does not internalize how its investment decisions affect the borrowing ability of the government.

Similarly, we can derive the Euler equation for manufacturing capital K_M from the planner's problem:

$$\left[1 - F_X\left(\cdot\right) Q_{K_M}\left(\cdot\right) B'\right] = \mathbb{E}\left[\beta \boldsymbol{D} \frac{u_c\left(\boldsymbol{C^D}\right)}{u_c\left(C\right)} \left[F_{K_M}\left(s'_D, K', 0\right) + \left(1-\delta\right)\right]\right] + \mathbb{E}\left[\beta\left(1-\boldsymbol{D}\right) \frac{u_c\left(\boldsymbol{C}\right)}{u_c\left(C\right)} \left[F_{K_M}\left(s', K', \boldsymbol{X}\right) + \left(1-\delta\right)\right]\right] \tag{2.25}$$

and from the household's problem we get:

$$1 = \mathbb{E}\left[\beta \boldsymbol{d} \frac{u_c\left(\boldsymbol{c^D}\right)}{u_c\left(c\right)} \left[F_{K_M}\left(s'_D, K', 0\right) + \left(1-\delta\right)\right]\right] + \mathbb{E}\left[\beta\left(1-\boldsymbol{d}\right) \frac{u_c\left(\boldsymbol{c}\right)}{u_c\left(c\right)} \left[F_{K_M}\left(s', K', \boldsymbol{T}\right) + \left(1-\delta\right)\right]\right] \tag{2.26}$$

As in the case for K_N, the term $-F_X\left(\cdot\right) Q_{K_M}\left(\cdot\right) B'$ shows that the planner takes into account how investment in the manufacturing sector affects her incentives to default in the next period and, thus, borrowing costs in the current period. It is easy to show that $F_X > 0$, since a higher value of imports financed by debt allows the firms to consume more intermediate traded goods and, thus produce more of the final good. The way household capital allocations deffer from those chosen by the planner depends then entirely on how default incentives change with the mix of capital, which is captured by the sign of Q_{K_M} and Q_{K_N}.

Conjecture 1: As long as p_C is sufficiently volatile, then $Q_{K_M} > 0$ and $Q_{K_N} < 0$. This implies that, for a given realization of the shock s, capital in the non-traded sector increases default incentives and capital in the manufacturing sector reduces them.

The intuition behind **Conjecture 1** relies on the observation that future debt payments are supported by exports of traded goods. Income from production of manufactures is subject to swings in productivity z, while income from exports of commodities is subject to swings in the international price of commodities p_C. If the volatility of p_C is much higher than that of z, then a lower level of K_M has a double effect: on one hand it reduces total tradable income and, on the other, it makes total tradable income more volatile, which increases the likelihood of experiencing "bad" states in which tradable income is sufficiently low that servicing debt is no longer optimal.

If **Conjecture 1** is true, then equations 2.23 through 2.26 imply that the planner invests more in manufacturing than the household and invests less in the non-traded sector. This implication is true for any level of y_C.

Now, note that for higher levels of y_C total tradable income is higher, on average. With higher average income, demand for all goods is higher, in the case of the non-traded intermediate this income effect increases the average price of non-traded goods p_N. A higher level of p_N further increases the incentives of the household of investing in non-traded capital, while still ignoring the effect this has on the price of debt and the probability of future defaults.

Conjecture 2: As y_C increases, the differences in investment between the household and the planner become more pronounced, which takes the allocations from the decentralized equilibrium farther from those chosen by the planner inducing a higher welfare cost of the externality. This increasing welfare cost is the Dutch Disease.

2.5 Conclusion

This chapter presented an example of an environment in which the household's investment decisions can be inefficient because the household does not internalize how her portfolio choice affects the government's incentives to default. This inefficiency is augmented in economies with a higher endowment of commodities with a relatively volatile international price.

An important avenue for future work on this topic is to make quantitative analyses to understand how robust this inefficiency is under different parameter values and technology and preference specifications. This could inform future research about testable implications of the model and of how can the model be used to understand different patterns of industrialization and proneness to default of different countries.

Chapter 3

Pricing Following the Nominal Exchange Rate

3.1 Introduction

The relationship between the nominal exchange rate and inflation has been widely studied because of its potential implications for monetary policy. To do so, economists have focused on pass-through, which is the elasticity of prices with respect to the nominal exchange rate. This paper studies how pass-through changes with the informativeness of the exchange rate when agents use it as a signal of the state.

The main contribution of this paper is to introduce the *information channel* as a driver of the level of pass-through. The nominal exchange rate is observed more frequently and easily than other signals, such as inflation or economic activity indicators. Because of this informational feature, the nominal exchange rate is relevant for firms' individual pricing decisions, even if it does not affect production costs or the demand for their good.

Consider somebody operating a small barbershop whose only cost is a wage bill paid in domestic currency. Suppose she observes a nominal depreciation and knows that all domestic prices will remain the same. In this case she does not have any incentive to change her price as her nominal costs remain unchanged. Alternatively, suppose she does not know how all domestic prices will change. If she thinks the depreciation indicates domestic inflation,[1] then she will increase her prices in anticipation of a nominal wage increase.

Section 3 lays out an example of a closed economy that formalizes this pricing behavior. The exchange rate is defined as a noisy signal of the aggregate price level.[2] Pass-through is zero under perfect information and positive pass-through only arises with information frictions. Under incomplete information, agents also observe a noisy signal from the Central Bank about monetary policy. Signals are more or less relevant depending on how volatile they are (relative to the volatility of other signals). If the volatility of the nominal exchange rate is relatively high then it is a less informative signal and vice versa.

Policy interventions that affect the level of currency shocks have a second order effect on prices by reducing the volatility of the exchange rate. In an extreme case the intervention fixes the exchange rate and its volatility becomes zero. Suppose that the objective of an intervention is to reduce the magnitude of a depreciation in order to decrease its effect on inflation. Under incomplete information, such intervention increases the elasticity of prices with respect to the exchange rate and, hence, the net effect of the intervention on inflation is ambiguous. Hereafter I refer to this ambiguity as the *currency intervention trade-off*.

Section 5 extends the one period model to nest what this paper calls the *costs channel*, as discussed by Taylor [2000]. This channel focuses on the effect the nominal exchange rate has on prices directly through costs of imported inputs. If cost shocks are persistent then prices increase more and during more periods, which implies higher pass-through with higher persistence of nominal exchange rate shocks. This extension is used to compare the relevance of the information channel vis-à-vis the cost channel.

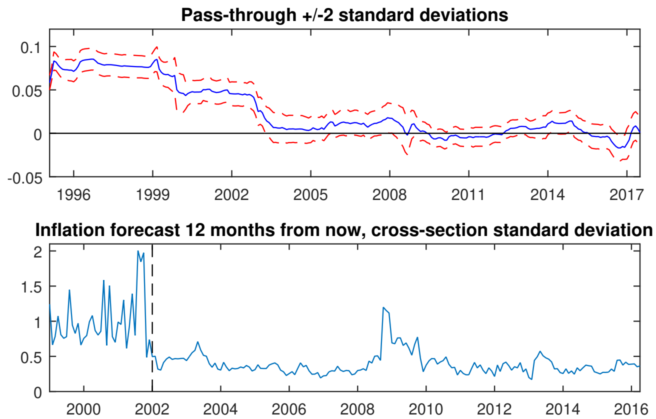
Section 6 calibrates the extended version of the model for the Mexican economy to analyze a decrease in pass-through in the early 2000's. This change has been documented by Capistran et al. [2011] and Cortes [2013], among others. The first chart in Figure 1 shows an estimate of pass-through for Mexico across time.[3]

[1]For example, if she believes the law of one price holds then a nominal depreciation could be due to domestic inflation or a decrease in the foreign price level.

[2]This is a strong assumption that allows to simplify the exposition of the information channel. It is discussed in detail in Section 2.

[3]By estimating the regression $\Delta_{12} \log CPI_t = \beta_0^s + \beta_1^s \Delta_{12} \log CPI_{t-1} + \beta_2^s \Delta_{12} \log ER_t + \epsilon_t$ considering a moving sample s of 4 years using monthly data. The first chart in Figure 1 plots the estimates for β_2^s.

Figure 3.1: Pass-through and inflation forecasts standard deviation



The second chart in Figure 1 illustrates how private inflation forecasts improved after three policy changes carried out by the Central Bank. The graph shows how the cross-sectional variance of private inflation forecasts decreased after 2002.[4] In the early 2000's the Central Bank carried out three policy changes which improved the information available to agents: it adopted an inflation-targeting regime with periodic announcements of its target, it increased the frequency of inflation reports from annual to quarterly and it increased the frequency of its objectives during open market operations.

Section 6 uses microprice data to estimate the average elasticity of prices (pass-through) with respect to foreign variations of the nominal exchange rate[5] for two periods (1995-2002 and 2003-2014); the estimated elasticities are 0.13 and 0.01, respectively. The calibration chooses the parameters in the extended model to target, among other moments, the estimated elasticity for the period 1995-2003. Finally, an improvement in the precision of the signal sent by the Central Bank in the model is identified with the data from the second chart of Figure 1.

Using the calibration for 1995-2003 but considering the policy changes (i.e. incresing the precision of the Central Bank's signal) the model yields an average elasticity of prices with respect to the exchange rate of 0.09. That is, the policy changes, through the information channel alone, explain 4 out of the 12 point drop in this elasticity. This is the main result

[4]A similar change can be observed in the mean absolute error of these forecasts.

[5]That is, controlling for domestic variables.

of the paper.
The aforementioned policy changes improved the information regarding monetary policy that the Central Bank communicates. Given a more precise signal from the Central Bank the nominal exchange rate is less relevant for pricing decisions. This translates into a lower elasticity of prices with respect to the exchange rate.

3.2 The informativeness of the nominal exchange rate

In order to simplify the exposition and restrict attention to the information channel, the two models in this paper abstract from modeling why the nominal exchange rate is volatile and how it relates to the aggregate price level. Instead, assume the nominal exchange rate is pinned down by the following general rule: $\log e_t = p_t + \eta_t$, where p_t is the log of the aggregate price level and η_t follows some stochastic process.
In a two country world with trade where the law of one price holds, $-\eta_t$ is the aggregate price level of the foreign country and the nominal exchange rate represents differences in inflation. The data shows that real exchange rates are volatile and persistent; thus define $\eta_t = \rho\eta_{t-1} - p_t^{\star} + \nu_t$. This is the interpretation η_t has in the remainder of this paper. Chari et al. [2002] show how sticky price models can produce the volatility and some of the persistence of real exchange rates observed in the data through trade of intermediate inputs.[6]

3.3 A one period model

There is an open economy with money and a nominal exchange rate pinned down by $\log e = p + \eta$, where $\eta \sim N\left(0, \frac{1}{\psi_\eta}\right)$. There is a subset of agents that consists of a household, a final good producer, and a continuum of intermediate goods producers who all take the rule that pins down the exchange rate as given. There is also a government that issues currency and an aggregate noisy signal about the currency's growth rate. Uncertainty

[6] Alternatively, η_t could represent changes in the risk premium as discussed in Alvarez et al. [2007]. In a similar paper, Alvarez et al. [2009] model exchange rate variations as a result of changes in the risk premium between domestic and foreign assets. Abstracting from trade and focusing on financial markets segmentation, they model changes in the risk premium driven by monetary policy.

arises due to productivity, monetary, information, and exchange rate shocks. Intermediate goods producers are monopolists that make pricing decisions under incomplete information. They observe the noisy signal of the monetary shock and a subset of prices, which includes the nominal exchange rate. All other agents have perfect information. Different levels of pass-through arise depending on the precision of the nominal exchange rate relative to the other signals available to monopolists.

Uncertainty, information and timing

The productivity shock affects intermediate producers. TFP for producer i is $\exp(A_i)$, which has an aggregate and an idiosyncratic component: $A_i = A + a_i$, $A \sim N\left(0, \frac{1}{\psi_A}\right)$, $a_i \sim N\left(0, \frac{1}{\psi_a}\right)$. There is a monetary shock such that the domestic money supply is $M = \exp(\mu) M_0$, where M_0 is some initial level of the money, $\mu = \bar{\mu} + \epsilon$ and $\epsilon \sim N\left(0, \frac{1}{\psi_\mu}\right)$. Intermediate producers cannot observe μ and instead observe a common noisy signal $\tilde{\mu} = \mu + \tilde{\epsilon}$, with $\tilde{\epsilon} \sim N\left(0, \frac{1}{\psi_{\tilde{\mu}}}\right)$. Finally, the external shock η, as defined previously, pins down the nominal exchange rate. All shocks are independent from each other.
The aggregate state of the economy is $s = \left(s^d, s^f\right)$. The domestic state

$$s^d = (A, \mu, \tilde{\mu}, \mathcal{F}(a_i))$$

, consists of the aggregate productivity shock, the aggregate monetary shock, the common signal of monetary policy and a distribution of individual productivity shocks. The foreign state consists of the external shock $s^f = \eta$.
Each intermediate producer i observes an incomplete information set

$$\mathcal{I}^i(s) = \{A_i, W_i(s), e(s), \tilde{\mu}\}$$

where W_i is the real wage monopolist i pays and $e(s)$ is the nominal exchange rate. Consumers and the final good producer observe the state and all prices. Intermediate producers satisfy the demand for their good given the price they set. At the beginning of the period shocks are realized and then all decisions are taken simultaneous.

Preferences and technology

There is a representative household with preferences for consumption of a final good, real money holdings, and labor. The household splits into a shopper and a continuum of workers. Each worker goes to an industry i and supplies labor in a competitive local market in exchange for a wage. The shopper makes decisions about consumption and holdings of real money balances. The household has access to a complete information set $\mathcal{I}^H(s) = \{s\}$, owns all the firms in the economy and solves:

$$\max_{\{C(s), M^d(s)/P(s), L(s)\}} \left\{ \gamma \log C(s) + (1-\gamma) \log \left[M^d(s)/P(s) \right] - \Psi \int_0^1 \frac{L_i(s)^{1+\zeta}}{1+\zeta} \right\}$$

$$s.t. \quad P(s) C(s) + M^d(s) \leq P(s) \int_0^1 W_i(s) L_i(s) + \int_0^1 \pi_i(s) \, di + M_0 + T(s)$$

$$M_0 = 1$$

where $W_i(s)$ is the real wage in industry i, $\pi_i(s)$ are the nominal profits of intermediate producer i, and $T(s)$ are money transfers from the government such that $T(s) = (\exp(\mu) - 1) M_0$. From the first-order conditions of the household's problem we get:

$$\frac{1-\gamma}{\gamma} C(s) P(s) = M^d(s) \tag{3.1}$$

$$\Psi L_i(s)^{\zeta} = \gamma \frac{W_i(s)}{C(s)} \tag{3.2}$$

where (3.1) is a standard quantity equation and (3.2) is a continuum of intratemporal conditions that equate the marginal disutility for worker i of supplying one extra unit of labor to its marginal contribution to the household's utility. On the production side, there is a competitive final good producer that transforms intermediate goods $(Y_i)_{i \in [0,1]}$ into a final consumption good Y using the following technology:

$$Y = \left(\int_0^1 Y_i^{\theta} di \right)^{\frac{1}{\theta}}$$

This producer has access to a complete information set $\mathcal{I}^F(s) = \{s\}$, solves:

$$\max_{\{(Y_i(s))_{i \in [0,1]}\}} P(s) Y(s) - \int_0^1 P_i(s) Y_i(s) \, di$$

and makes zero profits. From this problem we get that the demand for intermediate good Y_i^d is:

$$Y_i^d(s) = \left(\frac{P(s)}{P_i(s)}\right)^{\frac{1}{1-\theta}} Y(s) \tag{3.3}$$

and from the technology and the zero profit condition we get the aggregate price level is:

$$P(s) = \left[\int_0^1 P_i(s)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}} \tag{3.4}$$

Intermediate goods are produced by monopolists with access to a linear technology:

$$Y_i = \exp(A_i) L_i \tag{3.5}$$

and to an incomplete information set $\mathcal{I}^i(s) = \{A_i, W_i(s), e(s), \tilde{\mu}\}$. Monopolist i chooses its price to maximize its expected profits given its available information:

$$\max_{P_i(\mathcal{I}^i)} \sum_s \Gamma\left(s|\mathcal{I}^i\right)\left[P_i\left(\mathcal{I}^i\right) Y_i^d(s) - P(s) W_i(s) L_i(s)\right]$$
$$s.t. \quad Y_i^d(s) = \exp(A_i) L_i(s)$$

where $\Gamma\left(s|\mathcal{I}^i\right)$ is the probability of state s conditional on observing information set $\mathcal{I}^i(s)$. From the solution of this problem we get that the price is:

$$P_i\left(\mathcal{I}^i(s)\right) = \frac{W_i(s)}{\exp(A_i)} \frac{\mathbb{E}\left[P(s)^{\frac{2-\theta}{1-\theta}} Y(s) | \mathcal{I}^i(s)\right]}{\theta \mathbb{E}\left[P(s)^{\frac{1}{1-\theta}} Y(s) | \mathcal{I}^i(s)\right]} \tag{3.6}$$

which is a markup ($\frac{1-\theta}{\theta}$) above the expected marginal cost given the available information $\mathcal{I}^i(s)$.[7]

In equilibrium, real wages and the aggregate price level depend on the aggregate productivity and monetary shocks. Hence, the nominal exchange rate depends on these same shocks plus the external shock. Each intermediate producer observes four signals $\{A_i, W_i(s), e(s), \tilde{\mu}\}$ and uses them to infer five unobserved state variables $(A, a_i, \mu, \tilde{\epsilon}, \eta)$.

[7]Note that, under perfect information, this is the standard monopolistic competition pricing policy $P_i(s) = \frac{P(s)W_i(s)}{\theta \exp(A_i)}$.

3.4 Equilibrium and pass-through

Definition 1. An *equilibrium* is allocations, prices, and cross-sectional distributions of intermediate goods and prices, $\mathcal{F}_p$ and $\mathcal{F}_y$, such that: (i) given the prices, the allocation of consumption, money holdings and labor solves the household's problem, (ii) given the prices, the allocation of final good production and intermediate inputs solves the final good firm's problem, (iii) given information set $\mathcal{I}^i(s)$, intermediate producer i's price solves the monopolist's problem, (iv) all markets clear, and (v) the aggregation of industry prices and production with $\mathcal{F}_p$ and $\mathcal{F}_y$ is consistent with the aggregate price level and product.

Definition 2. *Pass-through* in this model is the elasticity of a price with respect to the exchange rate given a foreign shock. In the case of the aggregate price level, pass-through is:

$$PT(s) = \frac{\Delta\%P}{\Delta\%e} = \frac{\frac{d}{ds^f}P(s)}{\frac{d}{ds^f}e(s)}\frac{e(s)}{P(s)}$$

In this paper, pass-through abstracts from any changes in the nominal exchange rate through changes in domestic prices caused by domestic shocks, such as a productivity shock. The reason for this simplification is to focus only on the cases when the exchange rate has a causal effect on domestic prices.[8]

Linear equilibrium

To solve for the equilibrium assume the intermediate producers' prior beliefs about A and μ are normally distributed, with means 0 and $\bar{\mu}$, and variances $\frac{1}{\psi_A}$ and $\frac{1}{\psi_\mu}$, respectively. Also, assume that their posterior beliefs about the distributions of A and μ are also normal. Denote the means of these distributions $\mathbb{E}_i[A]$ and $\mathbb{E}_i[\mu]$, and their variances $Var_i(A)$ and $Var_i(\mu)$, respectively. Finally, guess that the cross-sectional distributions of log prices and production, p_i and y_i, are normal.[9] Under these assumptions, the logs of equations (3.1) through (3.6) and the definition of the exchange rate are:

[8]Without solving for the equilibrium it is easy to show that the above definition using the differentials with respect to the domestic state is always equal to 1.

[9]Which is the case in a symmetric linear equilibrium as defined below.

$$\text{quantity equation:} \quad \log\left(\frac{\gamma}{1-\gamma}\right) + \mu = p + y \tag{3.7}$$

$$\text{intratemporal conditions:} \quad \log(\Psi) + \zeta l_i = \log(\gamma) + w_i - y \tag{3.8}$$

$$\text{intermediate demand:} \quad y_i = \frac{1}{1-\theta}p - \frac{1}{1-\theta}p_i + y \tag{3.9}$$

$$\text{aggregate price:} \quad p = \int_0^1 p_i + \frac{\theta}{\theta-1}\frac{1}{2}\left[\int_0^1 p_i^2 di - \left(\int_0^1 p_i di\right)^2\right] \tag{3.10}$$

$$\text{production technology:} \quad y_i = A + a_i + l_i \tag{3.11}$$

$$\begin{aligned} \text{optimal pricing:} \quad p_i = & \log\left(\frac{1}{\theta}\right) + w_i - A - a_i + \mathbb{E}_i[p] \\ & + \frac{1}{2}Var_i\left(\frac{2-\theta}{1-\theta}p + y\right) - \frac{1}{2}Var_i\left(\frac{1}{1-\theta}p + y\right) \end{aligned} \tag{3.12}$$

$$\text{definition of exchange rate} \quad \log e = p + \eta \tag{3.13}$$

Hereafter I restrict attention to a symmetric equilibrium.[10]

Definition 3. A *symmetric linear equilibrium* is allocations y, $(y_i, l_i)_{i\in[0,1]}$ and prices $\log e$, p, $(p_i, w_i)_{i\in[0,1]}$ such that: (i) the allocations and prices are linear functions of the state variables, (ii) posterior moments are conditional on all available information, meaning $\mathbb{E}_i[f(x)] = \mathbb{E}\left[f(x)\,|\mathcal{I}^i(s)\right]$, and (iii) the allocations and prices satisfy (3.7) through (3.13) (i.e. allocations are optimal and prices clear all markets).

Proposition 1. A *linear equilibrium* exists and is unique for a generic set of parameters.

This is an important feature of the model. Its main goal is to show how the equilibrium level of pass-through changes as the precision of exogenous information changes (i.e. as $\psi_{\tilde{\mu}}$ changes). **Proposition 1** guarantees that this comparison can be done for a generic set of $\psi_{\tilde{\mu}}$.

The proof of **Proposition 1** follows in three steps. The first is to note that the posterior moments of p and y in equilibrium are linear expressions of the posterior moments of μ and A (this follows from rearranging the above expressions and taking expectations). The second step is to show that the posterior moments of μ and A are linear expressions of the signals $\tilde{\mu}$, A_i, w_i and $\log e$ (this is briefly discussed below and follows from the assumption of Gaussian independent shocks). From the first two steps we get that equations (3.7) through

[10] As discussed in Amador and Weill [2010], not too many results have dealt with the existence of nonlinear equilibria in economies with asymmetric information.

(3.13) define a linear system of seven equations with seven unknowns. For a generic set of parameter values the system has a unique solution.[11] This completes the proof.

Perfect information benchmark

Under perfect information (3.12) can be written as $p_i = \log\left(\frac{1}{\theta}\right) + p + w_i - A - a_i$ and the aggregate price in equilibrium is $P(s) = \exp(\mu - A + \mathcal{K}_0)$. Pass-through as defined before is 0 since the price does not depend on the foreign state. This result follows directly from the assumption that the exchange rate is disconnected from all the agents in the model. As in the barbershop example highlighted in the introduction, a change in the foreign state moves the nominal exchange rate but is meaningless to all the agents in the model since it does not have cost or demand implications. Hence, under perfect information individual monopolists do not adjust their individual price due to a depreciation, which implies pass-through is zero.

Incomplete information

The characterization of the incomplete information equilibrium is exposed in detail in the Appendix. Here I will focus on two key intermediate steps that help give intuition about how agents use information and how it affects aggregates. First, rearrange (3.7) through (3.13) to get:

$$\begin{aligned}
w_i &= \mathcal{A}_0 + \mathcal{A}_1\mu + \mathcal{A}_2 A + \mathcal{A}_3 a_i + \mathcal{A}_4 \mathbb{E}\left[\mu|\mathcal{I}^i(s)\right] + \mathcal{A}_5 \mathbb{E}\left[A|\mathcal{I}^i(s)\right] \\
&\quad + \mathcal{A}_6 \int_0^1 \mathbb{E}\left[\mu|\mathcal{I}^j(s)\right] dj + \mathcal{A}_7 \int_0^1 \mathbb{E}\left[A|\mathcal{I}^j(s)\right] dj \\
p &= \mathcal{B}_0 + \mathcal{B}_1\mu + \mathcal{B}_2 A + \mathcal{B}_3 \int_0^1 \mathbb{E}\left[\mu|\mathcal{I}^j(s)\right] dj + \mathcal{B}_4 \int_0^1 \mathbb{E}\left[A|\mathcal{I}^j(s)\right] dj \\
\log e &= \mathcal{B}_0 + \mathcal{B}_1\mu + \mathcal{B}_2 A + \mathcal{B}_3 \int_0^1 \mathbb{E}\left[\mu|\mathcal{I}^j(s)\right] dj + \mathcal{B}_4 \int_0^1 \mathbb{E}\left[A|\mathcal{I}^j(s)\right] dj - \eta
\end{aligned}$$

Domestic prices do not depend directly on the foreign state, but only through the effect the nominal exchange rate has on expectations. Recall that monopolist i observes $\mathcal{I}^i(s) =$

[11] This comes from the fact that the constants in the system depend only on parameters and that the set of non-singular matrices is generic.

$\{A_i, w_i, \log e, \tilde{\mu}\}$. Rearrange the information set to get conditionally independent signals for μ:

$$\begin{aligned}
\tilde{\mu} &= \mu + \tilde{\epsilon} \\
\tilde{A}_{i,\mu} &= \mu + \frac{1}{\mathcal{D}_1} a_i \\
\tilde{w}_{i,\mu} &= \mu + \frac{1}{\mathcal{D}_2} A \\
\log \tilde{e}_\mu &= \mu + \frac{1}{\mathcal{D}_3} \epsilon^\star
\end{aligned}$$

where $\tilde{A}_{i,\mu}$, $\tilde{w}_{i,\mu}$ and $\log \tilde{e}_\mu$ are linear expressions of variables that are observable to monopolist i. Note that these include the its own posterior expectations as well as the aggregate posterior expectations of all other monopolists. In a symmetric equilibrium the pricing policy function of all monopolists is the same, so an individual monopolist i can, in equilibrium, correctly calculate the integral of all agents expectations.[12] Given the above rearrangement, the posterior expectation is a weighted sum of these signals and the posterior variance is a constant:

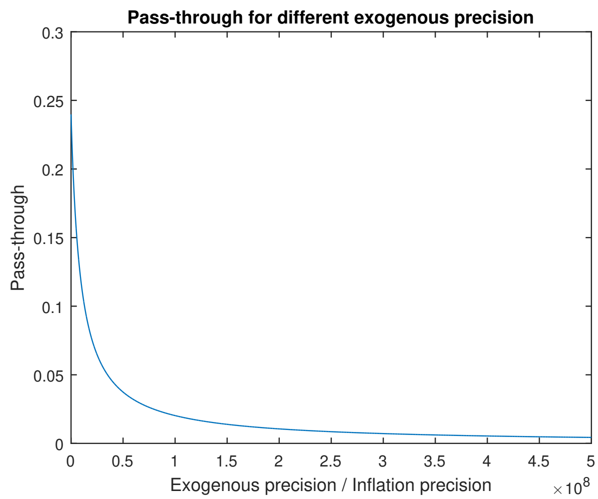
$$\begin{aligned}
\mathbb{E}\left[\mu | \mathcal{I}^i\left(s^t\right)\right] &= \frac{\psi_\mu \bar{\mu} + \psi_{\tilde{\mu}} \tilde{\mu} + \mathcal{D}_1^2 \psi_a \tilde{A}_{i,\mu} + \mathcal{D}_2^2 \psi_A \tilde{w}_{i,\mu} + \mathcal{D}_3^2 \psi_\eta \log \tilde{e}_\mu}{\psi_\mu + \psi_{\tilde{\mu}} + \mathcal{D}_1^2 \psi_a + \mathcal{D}_2^2 \psi_A + \mathcal{D}_3^2 \psi_\eta} \\
Var\left(\mu — \mathcal{I}^i\left(s^t\right)\right) &= \frac{1}{\psi_\mu + \psi_{\tilde{\mu}} + \mathcal{D}_1^2 \psi_a + \mathcal{D}_2^2 \psi_A + \mathcal{D}_3^2 \psi_\eta}
\end{aligned}$$

Note that the weights of these signals are increasing in their relative precision. For instance, if $\psi_{\tilde{\mu}}$ increases then the weight of $\tilde{\mu}$ increases, which means it becomes a more informative signal about μ. Note that if $\psi_{\tilde{\mu}} \to \infty$ then μ becomes observable and its weight converges to 1. If μ becomes observable then A becomes observable as well through wages and the nominal exchange rate becomes irrelevant to determine individual prices. Figure 2 illustrates the convergence of pass-through to the perfect information benchmark as $\psi_{\tilde{\mu}} \to$

[12] The rearranged signals $\tilde{w}_{i,\mu}$ and $\log \tilde{e}_\mu$ are linear expressions of expectations and aggregate expectations. Integrating over i, this posterior expectation (and doing the equivalent for A) defines a linear system on aggregate expectations $\int_0^1 \mathbb{E}\left[\mu | \mathcal{I}^j(s)\right] dj$ and $\int_0^1 \mathbb{E}\left[A | \mathcal{I}^j(s)\right] dj$. The solution of this linear system is aggregate expectations that depend linearly on $\tilde{\mu}$, w_i and $\log e$. Plugging back into the posterior expectations for monopolist i defines a linear system on individual expectations, the solution to this system is individual expectations that are also linear functions of $\tilde{\mu}$, w_i and $\log e$. These derivations are detailed in the Appendix on the proof of **Proposition 2**.

∞. On the horizontal axis the plot has the precision of the Central Bank's signal relative to the precision of the prior $\psi_{\tilde{\mu}}/\psi_{\mu}$ ("inflation precision").[13]

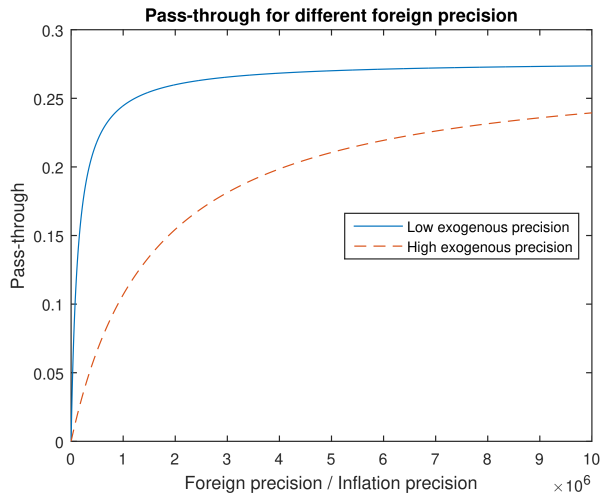
Figure 3.2: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$



Now, define ψ_{η} as the foreign precision of the nominal exchange rate. If this foreign precision increases (i.e. foreign exchange rate volatility decreases), then the weight of the nominal exchange rate increases, making it a more relevant signal for price decisions and increasing pass-through. Figure 3 illustrates this effect for different levels of the variance of the foreign shock relative to inflation variance.

[13] Figures 2 and 3 are generated using the parameter values from the calibration detailed in Section 6.

Figure 3.3: Pass-through for different relative variance



These two figures illustrate the information channel: the precision of the nominal exchange rate relative to the precision of all the other signals in the economy determines the level of pass-through. Figure 3 illustrate the currency intervention trade-off discussed in the introduction: pass-through is decreasing in the foreign volatility of the exchange rate.
As an example, suppose pass-through is 0.1 (i.e. ψ_η/ψ_μ is approximately 1 and $\psi_{\tilde{\mu}}$ is consistent with the dashed line) and that the Central Bank observes a 10% depreciation. Without intervention, the shock would increase inflation in 1%. If a currency intervention reduces the depreciation to 7% and increases ψ_η/ψ_μ to 6 then the net effect on inflation would be of 1.4%.

3.5 Infinite periods and persistent RER shocks

This section extends the model from Section 3 to infinite periods. The main purpose of this extension is to be able to compare the effects on pass-through of two channels: information and costs. To do so, I add three additional features: infinite periods, imported inputs for intermediate producers, and persistent real exchange rate shocks.
Time is discrete and runs forever. Define the nominal exchange rate such that the real exchange rate in period t is volatile and persistent. There are different trade frictions that

can deliver volatile and persistent real exchange rates.[14] For simplicity I abstract from modeling said frictions and instead take as given the following exogenous process for the real exchange rate: $RER_t = \exp(\eta_t)$, where $\eta_t = \rho\eta_{t-1} + \nu_t$, $0 < \rho < 1$, and $\nu_t \sim N\left(0, \frac{1}{\psi_\eta}\right)$. The other shocks are analogous to those in the one period model: TFP in period t for intermediate producer i is $\exp(A_{i,t})$ where $A_{i,t} = A_t + a_{i,t}$, $A_t \sim N\left(0, \frac{1}{\psi_A}\right)$, $a_{i,t} \sim N\left(0, \frac{1}{\psi_a}\right)$. The domestic money supply in period t is $M_t = \exp(\mu_t) M_{t-1}$ where $\mu_t = \bar{\mu} + \epsilon_t$ and $\epsilon_t \sim N\left(0, \frac{1}{\psi_\mu}\right)$. Intermediate producers observe the aggregate exogenous $\tilde{\mu}_t = \mu_t + \tilde{\epsilon}_t$, with $\tilde{\epsilon}_t \sim N\left(0, \frac{1}{\psi_{\tilde{\mu}}}\right)$. All shocks are independent from each other and across time.

The aggregate state of the economy in period t is now $s_t = \left(s_t^d, s_t^f\right)$. The domestic state is $s_t^d = (A_t, \mu_t, \tilde{\mu}_t, \eta_t, \mathcal{F}_t(a_{i,t}))$ and the foreign state is η_t. Let $s^t = \left(s_t, s^{t-1}\right)$ be the history of states up to period t.

At the beginning of each period all shocks are realized. Then all agents observe their information sets and make decisions simultaneously. Intermediate goods producers observe incomplete information set $\mathcal{I}^i\left(s^t\right)$ and make pricing decisions. Consumers and the final good producer observe the state and prices and make consumption and production decisions, respectively. At the end of the period all agents observe the state.[15]

The household's problem is now:

$$\max_{\{C, M^d/P, L\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Gamma\left(s^t\right) \left\{\gamma \log C\left(s^t\right) + (1-\gamma) \log\left[M^d\left(s^t\right)/P\left(s^t\right)\right] - \Psi \int_0^1 \frac{L_i\left(s^t\right)^{1+\zeta}}{1+\zeta} di\right\}$$

$$s.t. \quad P\left(s^t\right) C\left(s^t\right) + M^d\left(s^t\right) \leq P\left(s^t\right) \int_0^1 W_i\left(s^t\right) L_i\left(s^t\right) di + \int_0^1 \pi_i\left(s^t\right) di + M^d\left(s^{t-1}\right) + T\left(s^t\right)$$

$$M_0 = 1$$

where $0 < \beta < 1$ is the discount factor and $\Gamma\left(s^t\right)$ is the probability of the history of states s^t. From the first-order conditions of this problem we get the following Euler Equation for

[14] See Chari et al. [2002] for a detailed discussion of these frictions.

[15] This assumption simplifies the use of information for intermediate producers. As in the one period model, at each period t it is sufficient to characterize the posterior moments of η_t, A_t and μ_t. Without this assumption it would be also necessary to update the posterior moments of all previous realizations of η_t with the new information. Because η_t is persistent, signals at period t are informative about all previous realizations of the shock which are relevant for the current posterior distribution.

money and intratemporal condition:

$$\frac{\gamma}{1-\gamma}C\left(s^{t}\right)^{-1}P\left(s^{t}\right)^{-1}=M\left(s^{t}\right)^{-1}+\beta\mathbb{E}\left[\frac{\gamma}{1-\gamma}C\left(s^{t+1}\right)^{-1}P\left(s^{t+1}\right)^{-1}|s^{t}\right]$$

$$\Psi L_{i}\left(s^{t}\right)^{\zeta}=W_{i}\left(s^{t}\right)\gamma C\left(s^{t}\right)^{-1} \tag{3.14}$$

Iterating on the Euler Equation one can get:

$$\frac{\gamma}{1-\gamma}C\left(s^{t}\right)^{-1}P\left(s^{t}\right)^{-1}=\sum_{k=t}^{t+T}\beta^{k-t}\mathbb{E}\left[M^{d}\left(s^{k}\right)^{-1}|s^{t}\right]+\beta^{T}\mathbb{E}\left[M^{d}\left(s^{t+T}\right)^{-1}|s^{t}\right] \tag{3.15}$$

Proposition 2: If $\left\{M^{d}\left(s^{t+h}\right)\right\}_{h=0}^{\infty}$ belongs to an equilibrium allocation then, in the limit, we have $\lim_{T\to\infty}\beta^{T}\mathbb{E}\left[M^{d}\left(s^{t+T}\right)^{-1}|s^{t}\right]=0$.

The proof for **Proposition 2** can be found in the Appendix. It applies a similar argument as Obstfeld and Rogoff [1983] and Amador and Weill [2010] to rule out explosive solutions for money holdings. Given that $\mu_t-\bar{\mu}$ follows a white noise process, it can easily be shown that the limit of the right hand side of (3.15) as $T\to\infty$ is well defined.[16] So we can rewrite the equation as:

$$\frac{\gamma}{1-\gamma}Y\left(s^{t}\right)^{-1}P\left(s^{t}\right)^{-1}=M\left(s^{t}\right)^{-1}\frac{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_{\mu}}\right)}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_{\mu}}\right)-\beta} \tag{3.16}$$

for all t, s^t (here I also used the market clearing conditions for the final good and for money).

The problem of the final producer is static at each history of states s^t. The aggregation technology, demand equations and aggregate price are analogous:

$$Y\left(s^{t}\right)=\left(\int_{0}^{1}\left[Y_{i}\left(s^{t}\right)\right]^{\theta}di\right)^{\frac{1}{\theta}}$$

$$Y_{i}^{d}\left(s^{t}\right)=\left(\frac{P\left(s^{t}\right)}{P_{i}\left(\mathcal{I}^{i}\left(s^{t}\right)\right)}\right)^{\frac{1}{1-\theta}}Y\left(s^{t}\right) \tag{3.17}$$

$$P\left(s^{t}\right)=\left[\int_{0}^{1}P_{i}\left(s^{t}\right)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}} \tag{3.18}$$

[16] This structure for the process of μ_t is crucial for tractability of the solution. The model can be further extended to have μ_t follow a more general AR(1) process. One can still show that the infinite sum is well defined in this case; however it no longer has a closed form solution, so for exposition purposes, this paper does not consider this case.

The intermediate producers have access to a Cobb-Douglas technology:

$$Y_i = A_t Q_i^{\delta} L_i^{1-\delta}$$

where Q_i is an imported input that cannot be produced domestically. Denote the price of Q_i as P^Q and for simplicity assume it is constant. Now, intermediate monopolist i observes $\mathcal{I}^i\left(s^t\right) = \left\{s^{t-1}, W_i\left(s^t\right), P^Q, e\left(s^t\right), \tilde{\mu}_t\right\}$ and solves:

$$\max_{P_i(\mathcal{I}^i(s^t))} \sum_{s^t} \Gamma\left(s^t|\mathcal{I}^i\left(s^t\right)\right)\left[P_i\left(\mathcal{I}^i\left(s^t\right)\right) Y_i^d\left(s^t\right) - P\left(s^t\right) W\left(s^t\right) L_i\left(s^t\right) - e\left(s^t\right) P^Q Q_i\left(s^t\right)\right]$$

$$s.t. \quad Y_i^d\left(s^t\right) = A_t Q_i\left(s^t\right)^{\delta} L_i\left(s^t\right)^{1-\delta}$$

and thus the optimal price is:

$$P_i\left(\mathcal{I}^i\left(s^t\right)\right) = \frac{1}{\delta^{\delta}\left(1-\delta\right)^{1-\delta}}\left(\frac{P^T}{P^{\star}}\right)^{\delta} W_i\left(s^t\right)^{1-\delta} \frac{\mathbb{E}\left[\frac{\exp(\delta\eta_t)}{\exp(A_{i,t})} P\left(s^t\right)^{\frac{2-\theta}{1-\theta}} Y\left(s^t\right)|\mathcal{I}^i\left(s^t\right)\right]}{\theta\mathbb{E}\left[P\left(s^t\right)^{\frac{1}{1-\theta}} Y\left(s^t\right)|\mathcal{I}^i\left(s^t\right)\right]} \tag{3.19}$$

and the demand of labor and imported input:

$$L_i^d\left(s^t\right) = \frac{Y_i^d\left(s^t\right)}{\exp\left(A_{i,t}\right)}\left(\frac{e\left(s^t\right)P^Q}{P\left(s^t\right)W_i\left(s^t\right)}\frac{1-\delta}{\delta}\right)^{\delta} \tag{3.20}$$

$$Q_i^d\left(s^t\right) = \frac{Y_i^d\left(s^t\right)}{\exp\left(A_{i,t}\right)}\left(\frac{P\left(s^t\right)W_i\left(s^t\right)}{e\left(s^t\right)P^Q}\frac{\delta}{1-\delta}\right)^{1-\delta} \tag{3.21}$$

Note that if we take the limit as $\delta \to 0$ we get that the technology goes to the linear technology from (3.5) and the optimal price (3.14) approaches (3.6). Now the nominal exchange rate does affect costs directly, so even under perfect information an exogenous variation in the exchange rate (a change in η_t) will imply positive pass-through. Since it is a persistent shock, exogenous depreciations have an effect on contemporaneous and future price increases.

Finally, the nominal exchange rate is pinned down by:

$$\frac{e\left(s^t\right)P^{\star}}{P\left(s^t\right)} = \exp\left(\eta_t\right) \tag{3.22}$$

Linear equilibrium

Equations (3.14) and (3.16) through (3.22) define a static system and the definition of equilibrium is analogous to the one period version.

To characterize the equilibrium, as before, assume the prior beliefs that intermediate producers have about the distributions of A_t, η_t and μ_t are Normal with means 0, $\rho\eta_{t-1}$ and $\bar{\mu}$ and variances $\frac{1}{\psi_A}$, $\left(\frac{1}{1-\rho^2}\frac{1}{\psi_\eta}\right)$ and $\frac{1}{\psi_\mu}$, respectively. Also, assume that their posterior beliefs are normal with means $\mathbb{E}_i\left[A_t\right]$, $\mathbb{E}_i\left[\eta_t\right]$ and $\mathbb{E}_i\left[\mu_t\right]$ and variances $Var_i\left(A_t\right)$, $Var_i\left(\eta_t\right)$ and $Var_i\left(\mu_t\right)$, respectively. Finally, assume that the cross-sectional distributions of log prices and production, $p_{i,t}$ and $y_{i,t}$, are normal. Given these assumptions rewrite equations (3.14) and (3.16) through (3.22) in log linear form:

$$\text{quantity equation:} \quad \log\left(\frac{\gamma}{1-\gamma}\frac{1}{\tilde{\beta}}\right)+\mu_t = y_t + p_t \tag{3.23}$$

$$\text{intratemporal conditions:} \quad \log\left(\psi\right)+\zeta l_{i,t} = \log\left(\gamma\right)+w_{i,t}-y_t \tag{3.24}$$

$$\text{intermediate demand:} \quad y_{i,t} = \frac{1}{1-\theta}p_t - \frac{1}{1-\theta}p_{i,t}+y_t \tag{3.25}$$

$$\text{aggregate price:} \quad p_t = \int_0^1 p_{i,t} + \frac{\theta}{\theta-1}\frac{1}{2}\left[\int_0^1 p_{i,t}^2 di - \left(\int_0^1 p_{i,t} di\right)^2\right] \tag{3.26}$$

$$\text{labor demand:} \quad l_{i,t} = y_{i,t} - A_{i,t} + \delta\eta_t - \delta w_{i,t} + \delta\log\left(\frac{1-\delta}{\delta}\frac{P^Q}{P^\star}\right) \tag{3.27}$$

$$\text{imports demand:} \quad q_{i,t} = y_{i,t} - A_{i,t} - \left(1-\delta\right)\eta_t + \left(1-\delta\right)w_{i,t} + \left(1-\delta\right)\log\left(\frac{\delta}{1-\delta}\frac{P^\star}{P^Q}\right) \tag{3.28}$$

$$\begin{aligned} \text{optimal pricing:} \quad p_{i,t} &= \log\tilde{\theta} + \left(1-\delta\right)w_{i,t} - A_{i,t} + \mathbb{E}_i\left[\delta\eta_t + p_t\right] \\ &+\frac{1}{2}Var_i\left(\delta\eta_t + \frac{2-\theta}{1-\theta}p_t + y_t\right) - \frac{1}{2}Var_i\left(\frac{1}{1-\theta}p_t + y_t\right) \end{aligned} \tag{3.29}$$

$$\text{definition of exchange rate:} \quad \log e_t + p^\star = p_t + \eta_t \tag{3.30}$$

where $\tilde{\beta} = \frac{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)-\beta}$ and $\log\tilde{\theta} = \log\left(\frac{1}{\delta^\delta(1-\delta)^{1-\delta}}\frac{1}{\theta}\left(\frac{P^T}{P^\star}\right)^\delta\right)$. Now, the optimal pricing equation has additional terms related to the moments of the real exchange rate shock. Equations (3.23) through (3.30) define a static linear system for the equilibrium variables. **Definition 6** and **Proposition 3** below follow immediately:

Definition 4. A *symmetric linear equilibrium* is allocations $y_t, \left(y_{i,t}, l_{i,t}, q_{i,t}\right)_{i\in[0,1]}$ and prices

$\log e_t$, p_t, $(w_{i,t}, p_{i,t})_{i\in[0,1]}$ such that: (i) the allocations and prices are linear functions of the state variables, (ii) posterior moments are conditional on all available information, that is $\mathbb{E}_i [f(x)] = \mathbb{E}\left[f(x) | \mathcal{I}^i(s)\right]$, and (iii) the allocations and prices satisfy (3.23) through (3.30) (i.e. allocations are optimal and prices clear all markets).

Proposition 3. A *linear equilibrium* exists and is unique for a generic set of parameters.

The proof of **Proposition 3** is analogous to the proof of **Proposition 1**.

The nominal exchange rate affects prices in this economy through two channels. The first is the information channel discussed in the previous section. The second is the cost channel mentioned in the introduction: intermediate producers now use an imported input through which the exchange rate affects their costs. This implies that pass-through in the perfect information benchmark is no longer zero and instead is equal to δ, which is the expenditure share on imported inputs. Given the persistence of the exchange rate process, the effect of a one time shock lingers in subsequent periods through this cost channel.

Pass-through

Having multiple periods allows one to analyze the following three concepts of pass-through:

Definition 5. *Short-term pass-through*:

$$PT\left(s^t\right) = \frac{\frac{d}{ds^f} P\left(s^t\right)}{\frac{d}{ds^f} e\left(s^t\right)} \frac{e\left(s^t\right)}{P\left(s^t\right)}$$

Definition 6. *Medium-term pass-through*:

$$PT\left(s^{t+h} | s^t\right) = \frac{\frac{d}{ds_t^f} P\left(s^{t+h}\right)}{\frac{d}{ds_t^f} e\left(s^t\right)} \frac{e\left(s^t\right)}{P\left(s^{t+h}\right)}$$

Definition 7. *Long-term accumulated pass-through*:

$$PT_\infty\left(s^t\right) = \sum_{h=0}^{\infty} PT\left(s^{t+h} | s^t\right)$$

Medium-term pass-through is the effect of the nominal exchange rate on future prices. These will be directly affected by the persistence of the real exchange rate shocks. Finally, the long-term accumulated pass-through measures the total effect of a single exogenous innovation on the exchange rate at a given period t. Having these temporal distinctions of pass-through helps disentangle the relevance of the information and the persistence channels. It also allows one to quantify their relative importance.

In general, it can be shown that medium-term and long-term pass-through are $\rho^h PT\left(s^t\right)$ and $\frac{1}{1-\rho}PT\left(s^t\right)$, respectively. Regardless of the presence of information frictions, the nominal exchange rate shocks affects future prices only through the persistence of the real exchange rate shock. In a richer version of the model in which monetary shocks were persistent as well, future prices would also be affected through the consumer's dynamic decisions about money holdings, since these would depend on present and future expected prices. In this case, both the persistence and information channels as defined in this paper would still be present, but their disentanglement would be more complicated to show.

Perfect information benchmark

Under perfect information, the aggregate price in equilibrium is

$$P\left(s^t\right) = \exp\left(\mu_t - \frac{1}{1-\delta}A_t + \frac{\delta}{1-\delta}\eta_t + \mathcal{K}_0\right)$$

. Short-term pass-through is δ, which is the optimal expenditure share on the imported good. This is an intuitive and expected result since under perfect information the information channel is not present and prices are just affected by the exchange rate through costs.

Incomplete information

As in the one period case, we can rearrange the equations that characterize the symmetric linear equilibrium to get a linear system for individual and aggregate expectations.[17] The most important difference is that now prices depend directly on the foreign state through costs and indirectly through expectations:

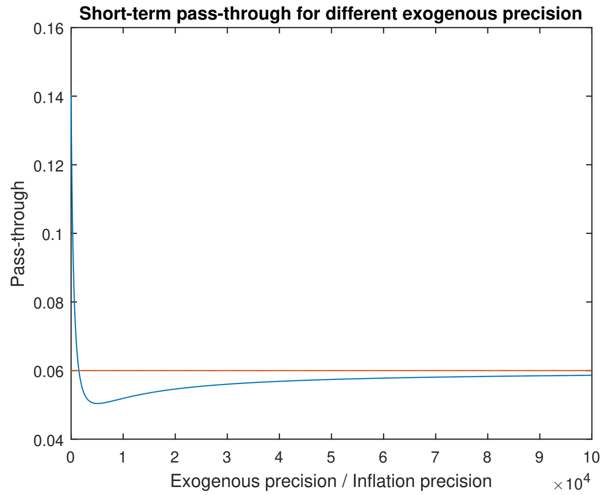
$$p_t = \mathcal{C}_0 + \mathcal{C}_1\mu_t + \mathcal{C}_2A_t + \mathcal{C}_3\eta_t + \mathcal{C}_4\int_0^1 \mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \mathcal{C}_5\int_0^1 \mathbb{E}\left[A_t|\mathcal{I}^j\right]dj + \mathcal{C}_6\int_0^1 \mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj$$

As in the one period model pass-through converges to the perfect information benchmark as $\psi_{\tilde{\mu}} \to \infty$. Figure 4 shows the convergence of equilibrium pass-through to the perfect information benchmark for $\delta = 0.06$.[18]

[17] See the appendix for the detailed derivation and solution of equillibriium.

[18] Figures 4 and 5 are generated using the parameter calibration detailed in Section 6.

Figure 3.4: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$

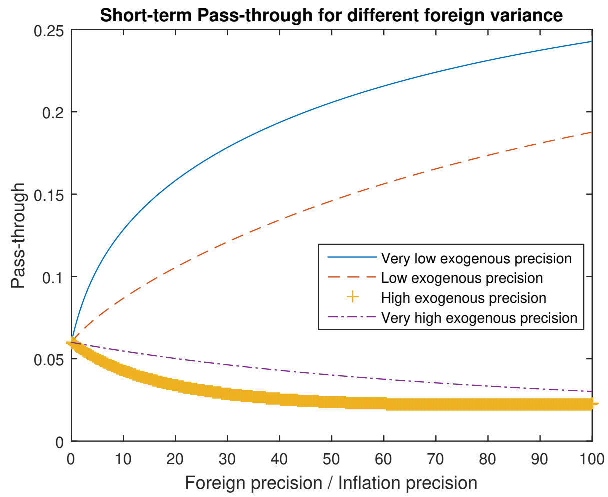


In this case, pass-through is lower than the benchmark before converging. This is because now the information friction poses a wedge on labor and imported input expenditure shares:

$$L^s = (1-\delta)\frac{1}{(1-\delta)+\delta\frac{\mathbb{E}[P(s^t)|\mathcal{I}^i(s^t)]}{P(s^t)}}$$
$$Q^s = \delta\frac{1}{\delta+(1-\delta)\frac{P(s^t)}{\mathbb{E}[P(s^t)|\mathcal{I}^i(s^t)]}}$$

This wedge depends on whether monopolists overestimate or underestimate prices, which in turn depends on how well informed they are. Thus, the information friction also affects the level of the direct channel discussed before, which explains why pass-through can be lower than the benchmark. The behavior of this wedge also affects the currency intervention trade-off, as Figure 5 illustrates:

Figure 3.5: Currency intervention trade-off



The trade-off exists if the precision of the signal from the Central Bank is too low (i.e. if it is in the decreasing region of Figure 4). For high enough precision the trade-off is no longer relevant and the elasticity becomes less responsive to changes in the exchange rate precision (hence the flatter curves).

3.6 Data, calibration and main results

Micro-price data and pass-through

Pass-through is estimated using confidential micro-price monthly data for the period 1995-2014. This is the same data used to construct the CPI in Mexico and is publicly available for 2011 onward. Each data point is the observed price for a generic product in a specific city. There are 282 generic products observed in 46 cities in Mexico. Using a similar specification as that in Kochen and Samano [2016], the estimation considers the following econometric model:

$$\Delta p_t^s = \beta_0 + \beta_1 \Delta_c e_t^s + \beta_2 \Delta_c \log y_t + u_t \tag{3.31}$$

where s denotes the generic product, Δp_t^s is the log price change of product s at time t, $\Delta_c e_t^s$ is the cumulative log change in the Mexican peso-US dollar nominal exchange rate during the same price spell and $\Delta_c \log y_t$ is the cumulative log change in the monthly economic

activity indicator IGAE. This last variable is used to control for variations caused by domestic factors. Under this specification, short-term pass-through (as defined previously in the model) is identified by β_1. Hence, long-term pass-through is $\frac{1}{1-\rho}\beta_1$, where ρ is the persistence of the process for η_t, which is estimated below.
The estimation is carried out for two separate periods: 1995-2002 and 2003-2014 with estimates $\beta_1^{95-02} = 0.13$ and $\beta_1^{03-14} = 0.01$. For both one can reject the null of $\beta = 0$ at the 1% confidence level.

Parameter calibration

Table 1 summarizes the calibration of the parameters in the model. All but two parameters are identified by moments other than the elasticity of prices estimated above. The two parameters set to match the level of pass-through are $\zeta = 1.95$ and $\theta = 0.29$, which imply labor elasticity of 0.58 and a markup of 240%.[19]

[19] As a reference for these numbers, Chari et al. [2002] set equivalent parameters to target a labor supply elasticity of 0.5 and a markup of 11% in the US.

Table 3.1: Calibrated parameters

Parameter	Value		Identification
	1995-2002	2003-2014	
β	0.94		Annual interest rate of 6%
γ	0.94		As in Chari, Kehoe and McGrattan (2002)
Ψ	10		
ζ	1.33		Set to match pass-through of 0.13 between 1995 and 2002
θ	0.38		
δ	0.06		$C_{imported}/C_{Total}$ for 1995 - 2002
ρ	0.89		$\Delta_{12}\log RER_t = \rho\Delta_{12}\log RER_{t-1} + \nu_t$ for 1995 - 2002
ψ_η	371		Variance of ν_t
$\psi_{\tilde{\mu}}$	8,656	57,986	Cross-sectional variance of inflation forecasts
ψ_μ	152		Annual inflation variance
ψ_A	1,088		Variance of annual GDP growth
ψ_a	0.045		Cross-sectional price variance
$\bar{\mu}$	0.18		monthly inflation for Mexico

The discount factor β is to be consistent with an annual interest rate of 6%. I set γ and Ψ as in Chari, Kehoe and McGrattan [2002]. I set δ to match the share of imported goods in consumption expenditure. I set $\bar{\mu}$ to match average annual inflation in 1995-2002. I choose ψ_μ to match an annual inflation standard deviation of 0.12. I choose ψ_A to match a variance of annual GDP growth of 0.04. Finally, I choose ψ_a to match the cross-sectional variance of log price changes observed in the data of 3.

To characterize the process for η_t, recall that it is defined as $\exp(\eta_t) = \frac{e_t P_t^\star}{P_t}$. I use monthly data for the nominal exchange rate (pesos/USD), the CPI for the U.S. and Mexico, to construct a time series for η_t and estimate the AR(1) model $\Delta_{12}\log RER_t = \rho\Delta_{12}\log RER_{t-1} + \nu_t$. This yields a value of $\rho = 0.89$ and a standard deviation of ν_t of 0.051.

Policy changes and main results

Table 2 summarizes three policies that affect the information available to agents. The column of communication shows the Central Bank's policy for reporting all of its activities to the public. The signaling column shows the Central Bank's daily operation policy, through which it sends signals to the financial markets. These three signaling policies are described in detail in the documents regarding the Central Bank's operational objectives.[20] This change can be summarized as going from less to more frequent short run updates regarding how the Central Bank will operate to achieve its targets. The objective column specifies the Central Bank's objective and term through which it aims to comply with its Constitutional mandate of price stability. Before 2001 the Central Bank targeted short term growth rates for the money supply. After 2001 the Central Bank adopted an inflation-targeting regime and started announcing multi-annual inflation targets as opposed to the shorter term money growth rate targets.

Table 3.2: Policy changes affecting agents' information

<table>
<tr><th>Period</th><th>Communication</th><th>Signaling</th><th>Objective</th></tr>
<tr><td>Jan 95 - Dec 99</td><td>Annual reports</td><td rowspan="3">Accumulated
Balances</td><td rowspan="2">Money supply growth,
short term</td></tr>
<tr><td>Jan 00 - Dec 00</td><td rowspan="4">Quarterly reports</td></tr>
<tr><td>Jan 01 - Sep 03</td><td rowspan="3">Inflation targeting,
long term</td></tr>
<tr><td>Oct 03 - Jan 08</td><td>Daily Balances</td></tr>
<tr><td>Feb 08 - present</td><td>Interest rate objective</td></tr>
</table>

To calibrate the precision of the aggregate signal, $\psi_{\tilde{\mu}}$, I use data from the Survey of Expectations of Specialists in Economics from the Private Sector[21] elaborated by the Bank of Mexico. This survey is conducted on a monthly basis since January 1999. On the first day of each month, 31 experts (on average) are asked about their expectations regarding current and future macroeconomic variables. I use data about their responses to the question "What will inflation be 12 months from now?" I set the precision parameter $\psi_{\tilde{\mu}}$ to match the average cross-sectional variance of annual inflation forecasts for the two periods

[20] Objetivos Operacionales del Banco de México.

[21] In Spanish is the Encuesta sobre las Expectativas de los Especialistas en Economía del Sector Privado (EEEESP)

of interest.
Table 3 has the main result of this paper. It compares the estimated short and long term pass-through with that implied by the model calibration. By construction, model pass-through is the same for the period 1995-2003. Given that, the calibration for 2003-2014 considers only an improvement in the available information (that is, it only changes the value of $\psi_{\tilde{\mu}}$). This means that the policy changes, through the information channel alone, explain approximately one third of the reduction in short and long term pass-through.

Table 3.3: Pass-through before and after policy changes

Period	Short-term		Long-term	
	Data	Model	Data	Model
1995 - 2003	0.13	0.13	0.65	0.65
2003 - 2014	0.01	0.09	0.05	0.45
Change	-0.12	-0.04	-0.60	-0.20

Currency intervention trade-off

The data is rich enough to estimate pass-through in (3.31) distinguishing by product, city, product-city and across time.[22] Monthly estimates of β_1 are used to test how relevant the trade-off for currency interventions is. Table 4 below shows the results of running the regression $\beta_{1,t} = \gamma_0 + \gamma_1 Var_t\,(\Delta_{1\text{ day}} \log ER) + v_t$ where $Var_t\,(\Delta_{1\text{ day}} \log ER)$ is the variance of the daily exchange rate log change during month t.

Table 3.4: Pass-through and ER volatility

1995-2002		
Coefficient	Estimate	p-value
γ_0	0.2	0.0000
γ_1	-6.7	0.0302
R^2	0.02	

2003-2016		
Coefficient	Estimate	p-value
γ_0	0.03	0.0722
γ_1	-6.7	0.0552
R^2	0.02	

[22] These estimations are part of a work in progress that would improve the robustness of the results in this paper.

A negative value of γ_1 means that prices are more sensitive to movements of the nominal exchange rate in months when the latter is less volatile, thus suggesting the existence of the currency intervention trade-off. During the period 1995-2016 the average of $Var_t\left(\Delta_{1\text{ day}} \log ER\right)$ was 0.00004 and its standard deviation was 0.00017. A decrease in monthly exchange rate variance of one standard deviation increases pass-through in 0.0012. Note from Table 1 that the ratio $\psi_{\tilde{\mu}}/\psi_{\mu}$ goes from 56 to 380, both values are in the region in which the model predicts that the currency intervention trade-off exists (that is, when pass-through is decreasing in the precision of the Central Bank's signal). Figure 6 illustrates this point, it is the same as Figure 4 but for values of $\psi_{\tilde{\mu}}/\psi_{\mu}$ less than 500.

Figure 3.6: Pass-through for different exogenous precision $\psi_{\tilde{\mu}}$

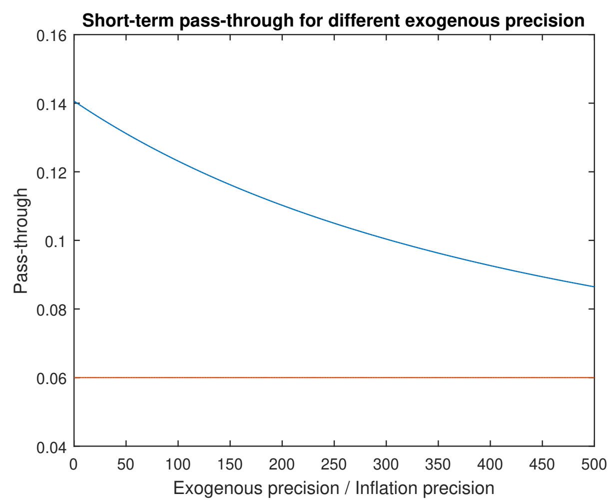
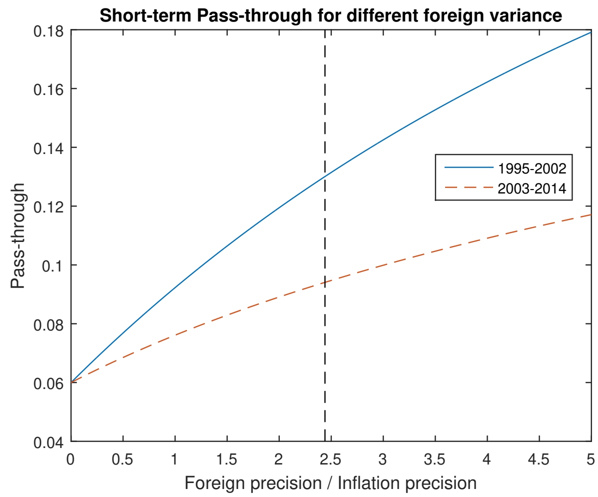


Figure 7 is the same as Figure 5 using the calibrated values for $\psi_{\tilde{\mu}}/\psi_{\mu}$ of 56 and 380 for the periods 1995-2002 and 2003-2014, respectively. The vertical line is at the calibrated value for ψ_{η}/ψ_{μ}. This Figure shows how the model predicts that the improvement in the available information implied by the policies is not enough by itself to eliminate the currency intervention trade-off, which is consistent with the implication of the estimates in Table 4.

Figure 3.7: Currency intervention trade-off



3.7 Conclusions

The informativeness of the nominal exchange rate affects the sensitivity of prices to the exchange rate, even if it does not affect demand and costs directly. This information channel implies a trade-off for currency interventions when the precision of other available information is relatively low. Given this trade-off, a currency intervention intended to decrease inflation after an exchange rate shock could end up increasing it depending on how much the depreciation reduces the volatility of the exchange rate.

Taking advantage of policy changes in Mexico as a case study, I find that the information channel explains approximately one third of the observed decrease in the average elasticity of prices. I also find that before and after the policy change, the policy intervention trade-off exists.

Further research on this topic would include a general equilibrium model in which the nominal exchange rate (as well as its relation to the price level) is modeled endogenously. Also, throughout this paper I assumed agents do not internalize the fact that the a change in the volatility of the exchange rate could be caused by an action by the Central Bank. Allowing agents internalize the strategic actions of the Central Bank is a theoretical extension that would enrich the results of this paper.

References

Mark Aguiar and Manuel Amador. Sovereign debt. In *Handbook of International Economics*, volume 4, pages 647–687, 2014.

Mark Aguiar and Gita Gopinath. Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69(1):64–83, 2006.

Mark Aguiar, Satyajit Chatterjee, Harold L. Cole, and Zachary Stangebye. Quantitative models of sovereign debt crises. In *Handbook of Macroeconomics*, 2016.

Enrique Alberola and Gianluca Benigno. Revisiting the commodity curse: A financial perspective. *Journal of International Economics*, 108(Supplement 1):S87–S106, May 2017.

Fernando Alvarez, Andrew Atkenson, and Patrick Kehoe. If exchange rates are random walks, then almost everything we say about monetary policy is wrong. *American Economic Review: Papers & Procedings*, 97(2):339–345, May 2007.

Fernando Alvarez, Adrew Atkenson, and Patrick Kehoe. Time-varying risk, interest rates, and exchange rates in general equilibrium. *Review of Economic Studies*, 76(3):851–878, July 2009.

Manuel Amador and Pierre-Oliver Weill. Learning from prices: Public communication and welfare. *Journal of Political Economy*, 118(5):866–907, 2010.

C. Arellano. Default risk and income fluctuations in emerging economies. *American Economic Review*, 2008.

C. Arellano and Ananth Ramanarayanan. Default and the maturity sturcture in sovereign bonds. *Journal of Political Economy*, 120(2):187–232, 2012.

Cristina Arellano, Yan Bai, and Gabriel Mihalache. Default risk, sectoral reallocation, and persistent recessions. *Journal of International Economics*, 112:182–199, March 2018.

Manuel Arellano and Stephen Bond. Some tests of specification for panel dada: Monte carlo evidence and an application to employment equations. *The Review of Economic Studies*, 58(2):277–297, April 1991.

Rabah Arezki and Kareem Ismail. Boom-bust cycle, asymmetrical fiscal response and the dutch disease. *Journal of Development Economics*, 101:256–267, November 2013.

Rabah Arezki, Valerie A. Ramey, and Liugang Sheng. News shocks in open economies: Evidence from giant oil discoveries. *Quarterly Journal of Economics*, pages 103–155, 2017.

Richard Auty. *Sustaining Development in Mineral Economies: The Resource Curse Thesis*. Oxford University Press, 1993.

Joao Ayres, Constantino Hevia, and Juan Pablo Nicolini. Real exchange rates and primary commodity prices. *Federal Reserve Bank of Minneapolis*, Staff Report(584), May 2019.

Paul Beaudry and Franck Portier. An exploration into pigou's theory of cycles. *Journal of Monetary Economics*, 51:1183–1216, 2004.

Paul Beaudry and Franck Portier. When can changes in expectations cause business cycle fluctuations in neo-classical settings? *Journal of Economic Theory*, 135:458–477, November 2007.

Paul Beaudry and Franck Portier. News-driven business cycles: Insights and callenges. *Journal of Economic Literature*, 52(4):993–1074, December 2014.

Javier Bianchi. Overborrowing and systemic externalities in the buisness cycle. *American Economic Review*, 101:3400–3426, December 2011.

Gideon Bornstein, Per Krusell, and Sergio Rebelo. Lags, costs, and shocks: Shocks: An equilibrium model of the oil industry. April 2019.

Fernando A. Broner, Guido Lorenzoni, and Sergio L. Schmukler. Why do emerging economies borrow short term? *Journal of the European Economic Association*, 11(1): 67–100, January 2013.

Carlos Capistran, Raul Ibarra-Ramirez, and Manuel Ramos-Francia. El traspaso de movimientos del tipo de cambio a los precios: Un analisis para la economia mexicana. *Banco de Mexico*, 1(12):1–25, 2011.

V. V. Chari, Patrick Kehoe, and Ellen McGrattan. Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies*, 69(3): 533–563, July 2002.

S. Chatterjee and B. Eyigungor. Maturity, indebtedness, and default risk. *American Economic Review*, 2012.

Harold Cole and Timothy Kehoe. Self-fulfilling debt crises. *The Review of Economic Studies*, 67:91–116, 2000.

Josue Cortes. Una estimacion del traspaso de las variaciones en el tipo de cambio a los precios en mexico. *Banco de Mexico*, 1(2):1–32, 2013.

United Nations Statistics Division. *National Accounts Official Country Data*. United Nations, New York, NY, 2017 edition, 2017.

John C. Driscoll and Aart C. Kraay. Consistent covariance matrix estimation with spatially dependent panel data. *Review of Economics and Statistics*, 80:549–560, 1998.

Jonathan Eaton and Mark Gersovitz. Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 1981.

Jeffrey A. Frankel. The natural resource curse: A survey of diagnoses and some prescriptions. *HKS Faculty Research Working Paper Series RWP12-014*, 2012.

International Monetary Fund. *World Economic Outlook (WEO)*. International Monetary Fund, Washington, DC, 2013 edition, 2013.

Carlo Galli. Self-fulfilling debt crises, fiscal policy and investment. Working Paper, February 2019.

R. Gaston Gelos, Ratna Sahay, and Guido Sandleirs. Sovereign borrowing by developing economies: What determines market access? *Journal of International Economics*, 83: 243–254, 2011.

Grey Gordon and Pablo A. Guerron-Quintana. Dynamics of investment, debt, and default. *Review of Economic Dynamics*, 28:71–95, 2018.

World Bank Group. *World Development Indicators (WDI)*. World Bank Group, Washington, DC, 2013 edition, 2013.

Christopher M. Gunn and Alok Johri. Fear of sovereign default, banks, and expectations-driven business cycles. Technical report, Carleton Economic Papers (CEP), May 2013.

Franza Hamann, Enrique G. Mendoza, and Paulina Restrepo-Echavarria. Resource curse or blessing? sovereign risk in resource-rich emerging economies:. *Federal Reserve Bank of Saint Louis, Working Paper Series*, (32A), October 2018.

J.C. Hatchondo and L. Martinez. Long-duration bonds and sovereign defaults. *Journal of International Economics*, 2009.

Constantino Hevia and Juan Pablo Nicolini. Optimal devaluations. *IMF Economic Review*, 61(1):22–51, 2013.

Constantino Hevia and Juan Pablo Nicolini. Monetary policy and dutch disease: The case of price and wage rigidity. *Federal Reserve Bank of Minneapolis*, Working Paper(726), June 2015.

Constantino Hevia, Pablo Andres Neumeyer, and Juan Pablo Nicolini. Optimal moneary and fiscal policy in a new keynesian model with a dutch disease: The case of complete markets. *Universidad di Tella Working Paper*, 2013a.

Constantino Hevia, Pablo Andres Neumeyer, and Juan Pablo Nicolini. Optimal monetary and fiscal policy in a new keynesian model with a dutch disease: The case of complete markets. *Universidad di Tella Working Paper*, 2013b.

Myron K. Horn. Giant oil and gas fields of the world. https://edx.netl.doe.gov/dataset/aapg-datapages-giant-oil-and-gas-fields-of-the-world, 2014.

David S. Jacks, Kevin H. O'Rourke, and Jeffrey G. Williamson. Commodity price volatility and world market integration since 1700. *Review of Economics and Statistics*, 93(3):800–813, August 2011.

Nir Jaimovich and Sergio Rebelo. News and business cycles in open economies. *Journal of Money, Credit and Banking*, 40(8):1699–1711, December 2008.

Nir Jaimovich and Sergio Rebelo. Can news about the future drive the business cycle? *American Economic Review*, 99(4):1097–1118, 2009.

Federico Kochen and Daniel Samano. Price-setting and exchange rate pass-through in the mexican economy: Evidence from cpi micro data. *Banco de Mexico*, 1(13):1–45, 2016.

Karen A. Kopecky and Richard M.H. Suen. Finite state markov-chain approximations to highly persistent processes. *Review of Economic Dynamics*, 13:701–714, March 2010.

Yu-Hsiang Lei and Guy Michaels. Do giant oilfield discoveries fuel international armed conflicts? *Journal of Development Economics*, 110:139–157, July 2014.

Enrique Mendoza. Real buisness cycles in a small open economy. *American Economic Review*, 81(4):797–818, September 1991.

Enrique G. Mendoza. Real exchange rate volatility and the price of nontradable goods in economies prone to sudden stops. *Economia: Journal of the Latin American and Caribbean Economic Association*, 6(1):103–135, 2005.

Stephen Nickell. Biases in dynamic models with fixed effects. *Econometrica*, 49(6):1417–1426, November 1981.

Maurice Obstfeld and Kenneth Rogoff. Speculative hyperinflations in maximizing models: Can we rule them out? *Journal of Political Economy*, 91(4):675–687, 1983.

Maurice Obstfeld and Kenneth Rogoff. The intertemporal approach to the current account. In Gene M. Grossman and Kenneth Rogoff, editors, *Handbook of International Economics*, volume 3, pages 1731–1799. Elsevier, 1995.

Anamaria Pieschacon. The value of fiscal discipline for oil-exporting countries. *Journal of Monetary Economics*, 59:250–268, March 2012.

Sergio Rebelo, Neng Wang, and Jinqiang Yang. Rare disasters, financial development, and sovereign debt. Working Paper, May 2019.

K. G. Rouwenhorst. *Frontiers of Buisness Cycle Research*, chapter Asset Pricing Implications of Equilibrium Buisness Cycle Models, pages 294–330. Princeton University Press, Princeton, NJ, 1995.

Jeffrey Sachs and Andrew Warner. The course of natural resources. *European Economic Review*, 45(4):827–838, May 2001.

Alan C. Stockman and Linda L. Tesar. Tastes and technology in a two-country model of the buisness cycle: Explaining international comovements. *American Economic Review*, 85(1):168–185, 1995.

George Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economic Letters*, 20(2):177–181, 1986.

John B. Taylor. Low inflation, pass-through, and the pricing power of firms. *European Economic Review*, 44(1):1389–1408, 2000.

Michael Tomz and Mark L. J. Wright. Do countries default in "bad times"? *Journal of the European Economic Association*, 5(2-3):352–360, April 2007.

Akos Valentinyi and Berthold Herrendorf. Measuring factor income shares at the sectoral level. *Review of Economic Dynamics*, 11:820–835, February 2008.

Appendix A

Appendix to Chapter 1

A.1 Data appendix

This Appendix supports the empirical work in Section 1.2.

A.1.1 Benchmark estimations

Tables A.1 and A.2 show the estimation results for equation (1.2). The estimated coefficients in Table A.1 are used to construct the impulse-response functions for spreads, investment, the current account, GDP, consumption, and the real exchange rate reported in Figures 1.2, 1.3, and 1.4.[1] Table A.2 presents the point estimates of the coefficients ξ_s related to the interaction between the natural logarithm of the price of oil $p_{oil,t}$ and the indicator of an oil discovery in $t - s$ for $s = 1...10$.

[1] Appendix A.1.3 shows the details about the estimation of the shares of investment in different sectors.

Table A.1: Estimation results of main variables, benchmark specification

	(1) spreads	(2) inv/GDP	(3) CA/GDP	(4) ln(GDP)	(5) ln(cons)	(6) ln(RER)
y_{t-1}	0.622***	0.818***	0.577***	0.807***	0.701***	0.741***
	(0.114)	(0.059)	(0.083)	(0.037)	(0.050)	(0.200)
NPV_t	1.616	4.009	-3.469***	0.029	-1.078	-7.596
	(2.879)	(2.534)	(0.640)	(0.535)	(1.030)	(7.769)
NPV_{t-1}	-1.934	4.208*	-2.675**	3.698*	0.996	-12.454
	(3.226)	(2.407)	(1.002)	(1.823)	(1.921)	(14.576)
NPV_{t-2}	2.608	-0.949*	-0.594	3.576***	-1.465	-10.191
	(4.043)	(0.520)	(0.440)	(1.079)	(2.013)	(20.275)
NPV_{t-3}	2.471	-1.318*	-0.112	3.007**	-0.900	-10.214
	(5.136)	(0.749)	(0.408)	(0.971)	(1.733)	(17.806)
NPV_{t-4}	6.884	0.021	-0.193	2.904***	0.264	-12.294
	(6.305)	(0.274)	(0.478)	(0.792)	(1.597)	(16.665)
NPV_{t-5}	7.347	0.849	-1.298***	3.005***	0.228	-10.611
	(8.270)	(0.697)	(0.432)	(0.699)	(1.382)	(15.277)
NPV_{t-6}	18.011*	0.607	-1.537***	3.163***	-0.079	-11.280
	(9.214)	(0.364)	(0.530)	(0.677)	(1.219)	(13.272)
NPV_{t-7}	12.428	0.028	-1.726**	2.604***	0.038	-6.809
	(11.364)	(0.519)	(0.674)	(0.618)	(1.223)	(12.179)
NPV_{t-8}	4.954	-0.298	1.455***	1.658**	-0.469	-8.367
	(7.577)	(0.274)	(0.498)	(0.716)	(0.913)	(10.377)
NPV_{t-9}	-0.435	0.498*	-2.242**	1.510**	-0.616	-3.344
	(1.080)	(0.255)	(0.851)	(0.563)	(0.743)	(8.435)
NPV_{t-10}	0.107	0.155	0.077	1.165*	-0.652	-3.108
	(0.852)	(0.579)	(0.442)	(0.648)	(0.866)	(5.120)
N	430	622	660	676	672	653
within R-squared	0.557	0.735	0.426	0.989	0.980	0.787

All regressions include country and year fixed effects as well as a constant. All regressions control for the interaction of the price of oil with an indicator for recent discoveries. Country specific quadratic trends are included for spreads, log real exchange rate, log GDP, and log consumption. Robust standard errors for panel regressions with cross-sectional dependence are in parenthesis. ***p¡0.01, **p¡0.05, *p¡0.1.

Table A.2: Point estimates of interaction between price of oil and indicators of recent discoveries

	(1) spreads	(2) inv/GDP	(3) CA/GDP	(4) ln(GDP)	(5) ln(cons)	(6) ln(RER)
$p_{oil,t}\mathbb{I}_{disc,i,t-1}$	-0.253*	0.000	0.001	0.001	0.003**	0.009
	(0.129)	(0.001)	(0.002)	(0.002)	(0.002)	(0.008)
$p_{oil,t}\mathbb{I}_{disc,i,t-2}$	-0.240	0.002	0.000	0.001	0.002**	0.018
	(0.169)	(0.001)	(0.001)	(0.001)	(0.001)	(0.011)
$p_{oil,t}\mathbb{I}_{disc,i,t-3}$	-0.143	0.001	0.000	-0.001	0.000	0.008
	(0.250)	(0.001)	(0.001)	(0.001)	(0.001)	(0.006)
$p_{oil,t}\mathbb{I}_{disc,i,t-4}$	-0.376*	-0.001	0.001	-0.002	0.002	0.010
	(0.207)	(0.001)	(0.001)	(0.001)	(0.001)	(0.008)
$p_{oil,t}\mathbb{I}_{disc,i,t-5}$	-0.142	0.001	0.001	-0.002	0.000	0.010
	(0.238)	(0.001)	(0.001)	(0.001)	(0.001)	(0.006)
$p_{oil,t}\mathbb{I}_{disc,i,t-6}$	0.245	-0.002*	0.004***	-0.002	-0.002	0.018
	(0.600)	(0.001)	(0.001)	(0.002)	(0.002)	(0.011)
$p_{oil,t}\mathbb{I}_{disc,i,t-7}$	0.043	-0.001	0.001	-0.001	0.000	0.008
	(0.190)	(0.001)	(0.001)	(0.001)	(0.001)	(0.009)
$p_{oil,t}\mathbb{I}_{disc,i,t-8}$	0.116	0.000	0.000	0.001	0.000	0.006
	(0.162)	(0.001)	(0.001)	(0.001)	(0.001)	(0.012)
$p_{oil,t}\mathbb{I}_{disc,i,t-9}$	0.120	0.000	0.001	0.001	0.000	0.004
	(0.157)	(0.001)	(0.001)	(0.001)	(0.001)	(0.006)
$p_{oil,t}\mathbb{I}_{disc,i,t-10}$	-0.430	0.001	-0.004***	0.002	0.000	0.003
	(0.322)	(0.001)	(0.001)	(0.001)	(0.001)	(0.004)

***p¡0.01, **p¡0.05, *p¡0.1.

Note that the coefficients in column (1) are three orders of magnitude larger than those in columns (2) through (5). Similarly, the coefficients in column (6) are also much larger than those in columns (2) through (5). As discussed in the following section, this difference shows how the inclusion of these control variables is relevant for the estimation of the effect of oil discoveries on spreads and the real exchange rate but not for their effect on the rest of the variables.

A.1.2 Estimations without interaction control variables

Table A.3 shows the estimation results for the following regression:

$$y_{i,t} = \rho y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s} + \alpha_i + \mu_t + \epsilon_{i,t}$$

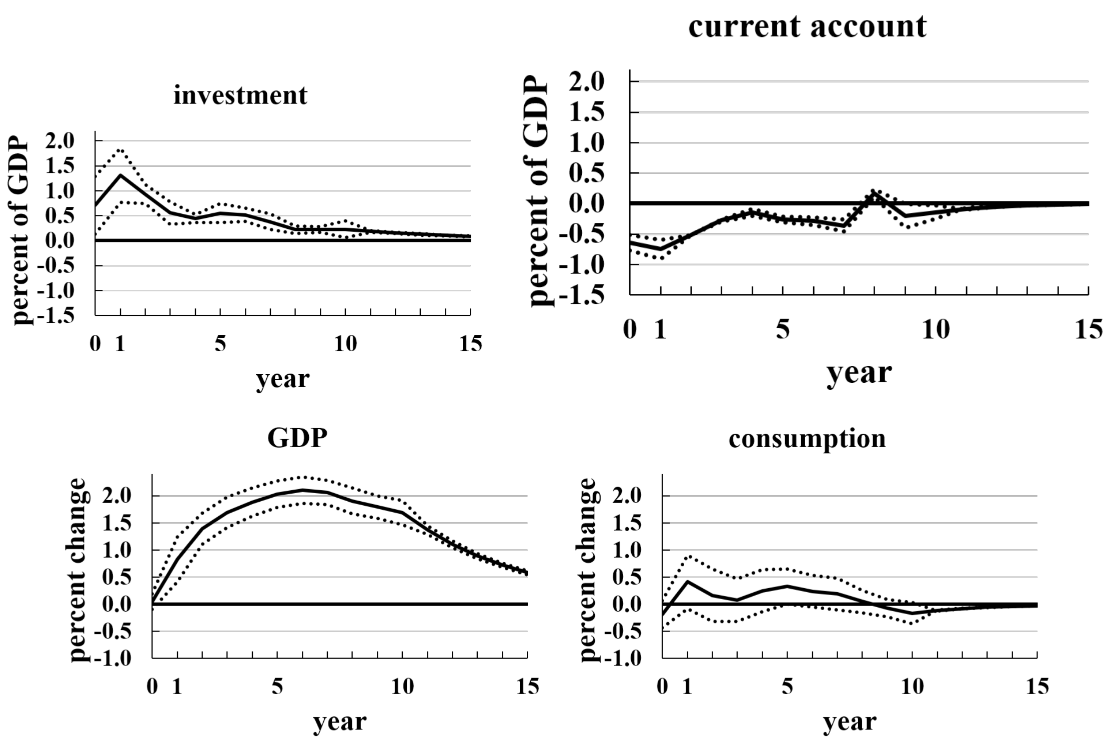
Table A.3: Estimation results of main variables, no interaction term

	(1) spreads	(2) inv/GDP	(3) CA/GDP	(4) ln(GDP)	(5) ln(cons)	(6) ln(RER)
y_{t-1}	0.621***	0.820***	0.582***	0.807***	0.703***	0.744***
	(0.118)	(0.060)	(0.084)	(0.036)	(0.049)	(0.197)
NPV_t	-1.491	3.937	-3.600***	0.262	-1.040	-8.304
	(2.799)	(2.479)	(0.551)	(0.620)	(1.072)	(7.972)
NPV_{t-1}	-7.769*	4.050*	-2.082**	4.394**	3.007	-6.185
	(4.155)	(2.110)	(0.962)	(1.780)	(2.193)	(10.852)
NPV_{t-2}	-6.075	-0.776*	-0.437	3.995***	-0.700	-2.295
	(4.680)	(0.410)	(0.357)	(1.066)	(1.985)	(15.110)
NPV_{t-3}	-5.349	-1.176*	0.135	3.183***	-0.229	-3.170
	(4.502)	(0.646)	(0.311)	(0.947)	(1.673)	(13.035)
NPV_{t-4}	-3.212	-0.044	0.066	2.878***	1.097	-5.286
	(5.341)	(0.157)	(0.374)	(0.781)	(1.659)	(12.029)
NPV_{t-5}	-1.386	1.022	-0.992***	2.833***	0.833	-3.368
	(6.427)	(0.682)	(0.267)	(0.671)	(1.337)	(10.805)
NPV_{t-6}	25.514*	0.363	-0.756*	2.574***	0.039	-4.525
	(13.036)	(0.398)	(0.390)	(0.657)	(1.172)	(9.186)
NPV_{t-7}	15.521**	-0.243	-1.071*	2.045***	0.120	-0.994
	(7.267)	(0.491)	(0.569)	(0.546)	(1.189)	(8.519)
NPV_{t-8}	4.411	-0.498**	2.107***	1.330**	-0.458	-3.264
	(6.384)	(0.190)	(0.434)	(0.629)	(0.859)	(6.231)
NPV_{t-9}	-0.975	0.245	-1.665**	1.421**	-0.618	0.151
	(1.131)	(0.171)	(0.763)	(0.519)	(0.682)	(5.719)
NPV_{t-10}	-0.457	0.237	-0.147	1.353*	-0.624	-1.228
	(0.522)	(0.634)	(0.567)	(0.617)	(0.873)	(3.235)
N	430	622	660	676	672	653
within R-squared	0.545	0.731	0.414	0.989	0.980	0.786

All regressions include country and year fixed effects as well as a constant. Country specific quadratic trends are included for spreads, log real exchange rate, log GDP, and log consumption. Robust standard errors for panel regressions with cross-sectional dependence are in parenthesis. ***p¡0.01, **p¡0.05, *p¡0.1.

That is, equation (1.2) without controlling for the interaction between the price of oil and indicators for recent discoveries. Comparing the results shown in Table A.3 with those from Table A.1 it is clear that the interaction controls are of very little consequence for all regressions except for those regarding spreads and the real exchange rate.
To illustrate this point even further, Figures A.1, A.2, and A.3 show the impulse-response functions constructed with the point estimates from Table A.3.

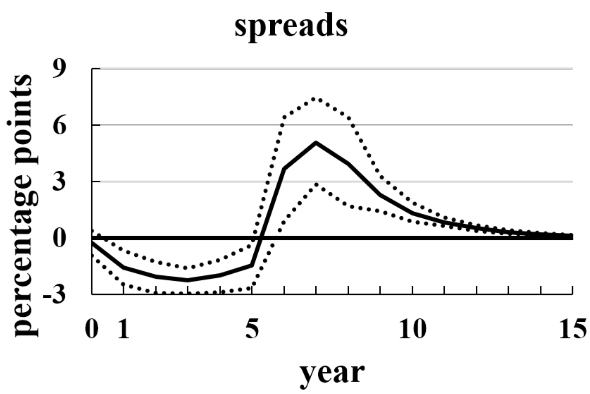
Figure A.1: Impact of giant oil discoveries on macroeconomic aggregates



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

As is clear from comparing Figure A.1 with Figure 1.2, the impulse-response functions of investment, the current account, GDP, and consumption remain virtually unchanged if we exclude the interaction controls. By comparing Figure A.2 with Figure 1.3, we can observe that the impact of oil discoveries on the dynamics of spreads is sensitive to the inclusion of these interaction controls.

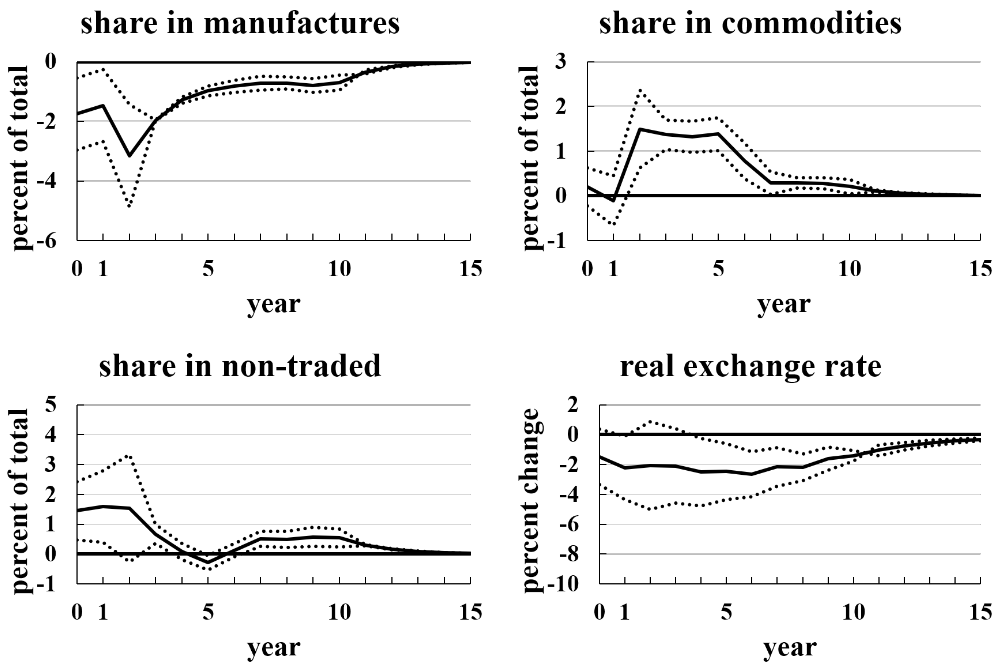
Figure A.2: Impact of giant oil discoveries on spreads



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

In both cases, with and without the interaction controls, the change in spreads peaks in the seventh year after a discovery at around 5 percentage points. However, in the benchmark specification spreads steadily increase in the years following a discovery, while in the specification that excludes the interaction controls spreads first decrease during the first five years and then increase. These differences are expected considering the sign of the coefficients reported in column (1) of Table A.2. These coefficients are negative for $p_{oil,t}\mathbb{I}_{disc,i,t-s}$ for $s = 1...5$, which implies that the coefficients of $NPV_{i,t-s}$ for $s = 1...5$ are biased downward when the interaction terms are omitted.

Figure A.3: Impact of giant oil discoveries on sectoral investment and the RER



Impulse response to an oil discovery with net present value equal to 18 percent of GDP. The dotted lines indicate 90 percent confidence intervals.

Figure A.3 presents the impulse-response functions of the real exchange rate and the shares of total investment that go into manufacturing, commodities, and non-traded sectors for the estimations that do not consider the interaction controls. As is clear by comparing Figure A.3 with Figure 1.4, only the response of the real exchange rate is affected by the omission.[2] Given the sign of the coefficients reported in column (6) of Table A.2, the coefficients of $NPV_{i,t-s}$ for $s = 1...10$ are biased upward when the interaction terms are omitted.

[2] Note how the coefficients in column (6) of Table A.2 are much larger than the coefficients reported in Table A.6.

A.1.3 The effect of oil discoveries on investment shares by sector

This Section provides details on the estimation of the effect of oil discoveries on the share of total investment in manufactures, commodities, and non-traded sectors. These estimates consider 47 countries for which sectoral investment data for the period 1993–2012 are available.[3]

The data of investment by sector are from Division [2017] following the International Standard Industrial Clasification, Revision 3 (ISIC Rev. 3). It considers investment per country for 11 sub-items. Table A.4 summarizes the sub-items and how I classify them into non-traded, manufacturing, and commodities.

Table A.4: Industry classification

sub-item	clasification
Agriculture, hunting, forestry; fishing (A+B)	commodities
Mining and quarrying (C)	commodities
Manufacturing (D)	manufacturing
Electricity, gas and water supply (E)	non-traded
Construction (F)	non-traded
Wholesale retail; hotels and restaurants (G+H)	non-traded
Transport, storage and communications (I)	non-traded
Financial intermediation; real estate (J+K)	non-traded
Public administration; compulsory social security (L)	non-traded
Education; health and social work; other (M+N+O)	non-traded
Private households with employed persons (P)	non-traded

[3] These countries are Armenia, Australia, Austria, Azerbaijan, Belarus, Belgium, Botswana, Canada, Cyprus, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Kuwait, Latvia, Lithuania, Luxembourg, Malta, Mauritius, Mexico, Namibia, Netherlands, New Zealand, Norway, Oman, Pakistan, Poland, Portugal, Qatar, Saudi Arabia, Slovenia, South Africa, Spain, Sweden, Syrian Arab Republic, Tunisia, Ukraine, United Arab Emirates, United Kingdom, United States, and Uruguay.

Tables A.5 and A.6 show the estimation results for equation (1.2). The estimated coefficients in Table A.5 are used to construct the impulse-response functions for the shares of total investment that go into manufacturing, commodities, and non-traded sectors reported in Figure 1.4.

Table A.5: Estimation results of investment shares, benchmark specification

	(1)	(2)	(3)
	non-traded	manufacturing	commodities
y_{t-1}	0.545***	0.499***	0.520***
	(0.037)	(0.071)	(0.113)
NPV_t	9.306*	-10.222	0.475
	(4.895)	(5.570)	(1.949)
NPV_{t-1}	6.289	-4.746	-1.529
	(5.362)	(6.327)	(1.772)
NPV_{t-2}	6.789	-15.227	7.059
	(8.062)	(10.547)	(5.725)
NPV_{t-3}	-0.594	-2.491**	3.065***
	(1.214)	(1.212)	(0.435)
NPV_{t-4}	-1.577	-1.854	3.431***
	(1.180)	(1.248)	(0.604)
NPV_{t-5}	-1.822	-1.883	3.758***
	(1.153)	(1.247)	(0.788)
NPV_{t-6}	1.887	-1.884	0.072
	(1.128)	(1.250)	(0.850)
NPV_{t-7}	2.983**	-2.014	-0.967*
	(1.151)	(1.214)	(0.534)
NPV_{t-8}	1.511	-1.984	0.407
	(1.232)	(1.235)	(0.319)
NPV_{t-9}	1.763	-1.827	0.014
	(1.445)	(1.394)	(0.407)
NPV_{t-10}	1.528	-1.750	0.152
	(1.272)	(1.261)	(0.564)
N	569	569	569
within R-squared	0.522	0.414	0.461

All regressions include country and year fixed effects as well as a constant. All regressions control for the interaction of the price of oil with an indicator for recent discoveries. Robust standard errors for panel regressions with cross-sectional dependence are in parenthesis. ***p¡0.01, **p¡0.05, *p¡0.1.

Table A.6 presents the point estimates of the coefficients ξ_s of the interaction between the natural logarithm of the price of oil $p_{oil,t}$ and the indicator of an oil discovery in $t-s$ for $s = 1...10$.

Table A.6: Point estimates of interaction between price of oil and indicators of recent discoveries

	(1) non-traded	(2) manufacturing	(3) commodities
$p_{oil,t}\mathbb{I}_{disc,i,t-1}$	-0.002*	0.002	0.000
	(0.003)	(0.002)	(0.002)
$p_{oil,t}\mathbb{I}_{disc,i,t-2}$	-0.001	0.001	0.000
	(0.001)	(0.002)	(0.002)
$p_{oil,t}\mathbb{I}_{disc,i,t-3}$	-0.002	0.003**	-0.001
	(0.003)	(0.001)	(0.003)
$p_{oil,t}\mathbb{I}_{disc,i,t-4}$	0.002	0.001	-0.003**
	(0.002)	(0.001)	(0.001)
$p_{oil,t}\mathbb{I}_{disc,i,t-5}$	0.002	0.000	-0.001
	(0.002)	(0.002)	(0.002)
$p_{oil,t}\mathbb{I}_{disc,i,t-6}$	-0.001	0.000	0.001
	(0.001)	(0.001)	(0.001)
$p_{oil,t}\mathbb{I}_{disc,i,t-7}$	-0.003**	0.003	0.001
	(0.002)	(0.004)	(0.002)
$p_{oil,t}\mathbb{I}_{disc,i,t-8}$	-0.001	0.000	0.002*
	(0.002)	(0.002)	(0.001)
$p_{oil,t}\mathbb{I}_{disc,i,t-9}$	0.001	-0.005***	0.004***
	(0.002)	(0.001)	(0.001)
$p_{oil,t}\mathbb{I}_{disc,i,t-10}$	-0.001	0.001	0.000
	(0.001)	(0.001)	(0.002)

***p¡0.01, **p¡0.05, *p¡0.1.

Finally, Table A.7 shows the estimation results for the following regression:

$$y_{i,t} = \rho y_{i,t-1} + \sum_{s=0}^{10} \psi_s NPV_{i,t-s} + \alpha_i + \mu_t + \epsilon_{i,t}$$

that is the same as equation (1.2) but without controlling for the interaction between the price of oil and indicators for recent discoveries.

Table A.7: Estimation results of investment shares, no interaction term

	(1)	(2)	(3)
	non-traded	manufacturing	commodities
y_{t-1}	0.551***	0.496***	0.533***
	(0.036)	(0.069)	(0.112)
NPV_t	8.047*	-9.735*	1.098
	(4.220)	(5.216)	(1.844)
NPV_{t-1}	4.418	-3.351	-1.191
	(4.909)	(5.912)	(2.252)
NPV_{t-2}	3.654	-13.469	8.607**
	(7.419)	(7.988)	(3.840)
NPV_{t-3}	-0.958	-2.228*	3.184***
	(1.087)	(1.254)	(0.483)
NPV_{t-4}	-1.598	-1.734	3.280***
	(1.052)	(1.233)	(0.638)
NPV_{t-5}	-1.868	-1.909	3.765***
	(1.024)	(1.234)	(0.763)
NPV_{t-6}	1.614	-1.871	0.264
	(1.009)	(1.247)	(0.874)
NPV_{t-7}	2.437**	-1.734	-0.744
	(1.057)	(1.302)	(0.618)
NPV_{t-8}	1.175	-2.000	0.757**
	(1.055)	(1.166)	(0.326)
NPV_{t-9}	1.683	-2.453*	0.720*
	(1.251)	(1.298)	(0.367)
NPV_{t-10}	1.268	-1.705	0.318
	(1.178)	(1.396)	(0.526)
N	569	569	569
within R-squared	0.514	0.398	0.449

All regressions include country and year fixed effects as well as a constant. Robust standard errors for panel regressions with cross-sectional dependence are in parenthesis. ***p¡0.01, **p¡0.05, *p¡0.1.

A.2 Decentralized economy

This Appendix shows how the allocations from the economy in Section 2.2 can be decentralized by an economy with a representative household, a government, and competitive firms. First I lay out the environment and then I prove an equivalence result that is akin to a first welfare theorem.

A.2.1 Environment

Final Good.—There is a competitive firm that assembles the final non-traded good Y_t from the intermediate non-traded good $c_{N,t}$, manufactures $c_{M,t}$, and oil $c_{oil,t}$ and sells it to the representative household at price P_t. The firm has access to the technology:

$$Y_t = f^Y\left(c_{N,t}, c_{M,t}, c_{oil,t}\right) = \left[\omega_N^{\frac{1}{\eta}}\left(c_{N,t}\right)^{\frac{\eta-1}{\eta}} + \omega_M^{\frac{1}{\eta}}\left(c_{M,t}\right)^{\frac{\eta-1}{\eta}} + \omega_{oil}^{\frac{1}{\eta}}\left(c_{oil,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution and ω_i are the weights of each intermediate good i in the production of the final good. The firm purchases manufactures and oil in international markets at prices p_M and $p_{oil,t}$ and purchases the intermediate non-traded good at price $p_{N,t}$ from a domestic producer. Cost minimization implies the demands for intermediate goods are:

$$c_{N,t} = \left(\frac{P_t}{p_{N,t}}\right)^{\eta} Y_t \omega_N$$
$$c_{M,t} = \left(\frac{P_t}{p_{M,t}}\right)^{\eta} Y_t \omega_M$$
$$c_{oil,t} = \left(\frac{P_t}{p_{oil,t}}\right)^{\eta} Y_t \omega_{oil}$$

and since the firm is competitive the price of the final good equals its marginal cost:

$$P_t = \left[\omega_N\left(p_{N,t}\right)^{1-\eta} + \omega_M\left(p_{M,t}\right)^{1-\eta} + \omega_{oil}\left(p_{oil_t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Intermediate Goods.—Manufactures $y_{M,t}$, oil $y_{oil,t}$, and the intermediate non-traded good $y_{N,t}$ are produced by competitive firms with access to technologies:

$$y_{N,t} = f^{N}\left(z_t, k_{N,t}\right)$$
$$y_{M,t} = f^{M}\left(z_t, k_{M,t}\right)$$
$$y_{oil,t} = f^{oil}\left(z_t, k_{oil,t}, n_t\right)$$

. Each period, these firms rent general capital $k_{n,t}$ and $k_{M,t}$ and capital for oil extraction $k_{oil,t}$ from the household in exchange for rental rates r_t and $r_{oil,t}$. The manufacturing and oil firms sell their product in international markets at prices $p_{M,t}$ and $p_{oil,t}$ and the non-traded firm sells its product to the domestic final good firm at price $p_{N,t}$. The representative household owns all the firms and gets the profits from the firms.

Households.—There is a representative household with preferences over consumption c_t represented by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right]$$

where $u\left(c\right) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and β is the discount factor. The household owns all the firms and faces a budget constraint and laws of motion for capital:

$$c_t + \left(1+\tau_{i,t}\right)i_{k,t} + \left(1+\tau_{i_{oil},t}\right)i_{k_{oil},t} \leq \frac{r_t}{P_t}k_t + \frac{r_{oil,t}}{P_t}k_{oil,t} + \frac{\pi_t^N + \pi_t^M + \pi_t^{oil}}{P_t} + m_t + T_t$$
$$k_{t+1} = \left(1-\delta\right)k_t + i_{k,t} - \Psi\left(k_{t+1}, k_t\right)$$
$$k_{oil,t+1} = \left(1-\delta\right)k_{oil,t} + i_{k_{oil},t} - \Psi_{oil}\left(k_{oil,t+1}, k_{oil,t}\right)$$

where $\tau_{i,t}$ and $\tau_{i_{oil},t}$ are distortionary taxes, T_t are transfers from the government, m_t is a small transitory income shock, and π_t^N, π_t^M and π_t^{oil} are profits from the intermediate goods firms. The household takes taxes and prices as given and maximizes its lifetime utility subject to its budget constraint and the laws of motion of capital.

Government.—There is a benevolent government that can issue long term debt in international financial markets and lacks commitment to repay. The law of motion for debt is:

$$b_{t+1} = \left(1-\gamma\right)b_t + i_{b,t}$$

where γ is the fraction of debt that matures each period, b_t is the stock of debt in period t and $i_{b,t}$ is new debt issuances. At the beginning of each period the government chooses whether to default or not. If the government defaults then productivity in the economy is $\tilde{z}_t = z_t - \max\left\{d_0 z_t + d_1 z_t^2\right\}$. After default, the government is excluded from financial markets and is readmitted with probability θ. If the government repays then it can issue new debt. Regardless of default or repayment, the government has access to distortionary taxes $\tau_{i,t}$ and $\tau_{i_{oil},t}$ and lump-sum transfers T_t to influence the decisions of the households. The government maximizes the representative household's utility subject to its budget constraint and to an implementability constraint that restricts the allocations that the government chooses for the household to be a solution to the household's problem given the taxes. If the government is in good financial standing then its budget constraint is:

$$P_t \tau_{i,t} i_{k,t} + P_t \tau_{i_{oil},t} i_{k_{oil},t} + q_t(\cdot)\left[b_{t+1} - (1-\gamma) b_t\right] = P_t T_t + \left[\gamma + (1-\gamma)\kappa\right] b_t$$

where $(1-\gamma)\kappa b_t$ are the coupon payments for the outstanding debt. If the government decides to default then its budget constraint is:

$$P_t \tau_{i,t} i_{k,t} + P_t \tau_{i_{oil},t} i_{k_{oil},t} = P_t T_t$$

and gets readmitted to financial markets with probability θ and zero debt.

A.2.2 Equivalence result

In this subsection I prove that the allocations that characterize the equilibrium of the economy in Section 2.2 can be decentralized by the market economy described above. I do this in two steps: first, I show that, given the state and the dynamic decisions, the static allocations in each period are the same. Then I show that the dynamic problems are the same.

The recursive formulation of the problem of the government in Section 2.2 is:

$$V(s, m, k, k_{oil}, b) = \max_{d \in \{0,1\}} \left\{[1-d] V^P(s, m, k, k_{oil}, b) + d V^D(s, k, k_{oil})\right\}$$

where the value in repayment is:

$$
\begin{aligned}
V^P(s,m,k,b) &= \max_{\left\{k',b',l,\overrightarrow{C},\overrightarrow{L},\overrightarrow{K},\overrightarrow{X}\right\}} \left\{u(c) + \beta\mathbb{E}\left[V\left(s',m',k',b'\right)\right]\right\} \\
s.t. \qquad c + i_k + i_{k_{oil}} &\leq Y + m \\
k' &= (1-\delta)k + i_k - \Psi\left(k',k\right) \\
k'_{oil} &= (1-\delta)k_{oil} + i_{k_{oil}} - \Psi_{oil}\left(k'_{oil},k_{oil}\right) \\
Y &= f^Y(c_N, c_M, c_{oil}) \\
c_N &= f^N(z, k_N) \\
c_M &= f^M(z, k_M) - x_M \\
c_{oil} &= f^{oil}(z, k_{oil}, n) - x_{oil} \\
q\left(s,k',b'\right)\left[b' - (1-\gamma)b\right] &= p_M x_M + p_{oil} x_{oil} + [\gamma + \kappa(1-\gamma)]b \\
k &= k_N + k_M
\end{aligned}
$$

and the value in default is:

$$
\begin{aligned}
V^D(s,k,k_{oil}) &= \max_{\left\{k',k'_{oil},\overrightarrow{C},\overrightarrow{K},\overrightarrow{X}\right\}} \left\{u(c) + \beta\mathbb{E}\left[\theta V\left(s',m',k',k'_{oil},0\right) + (1-\theta)V^D\left(s',k',k'_{oil}\right)\right]\right\} \\
s.t. \qquad c + i_k + i_{k_{oil}} &\leq Y - \bar{m} \\
k' &= (1-\delta)k + i_k - \Psi\left(k',k\right) \\
k'_{oil} &= (1-\delta)k_{oil} + i_{k_{oil}} - \Psi_{oil}\left(k'_{oil},k_{oil}\right) \\
Y &= f^Y(c_N, c_M, c_{oil}) \\
c_N &= f^N(\tilde{z}, k_N) \\
c_M &= f^M(\tilde{z}, k_M) - x_M \\
c_{oil} &= f^{oil}(\tilde{z}, k_{oil}, n) - x_{oil} \\
0 &= p_M x_M + p_{oil} x_{oil} \\
k &= k_N + k_M
\end{aligned}
$$

These problems can be rewritten as:

$$
\begin{aligned}
V^P(s,m,k,b) &= \max_{\left\{k',b',l,\overrightarrow{C},\overrightarrow{L},\overrightarrow{K},\overrightarrow{X}\right\}} \left\{u(c) + \beta\mathbb{E}\left[V\left(s',m',k',b'\right)\right]\right\} \\
s.t. \qquad c + i_k + i_{k_{oil}} &\leq F(s,k,k_{oil},X) + (1-\delta)k + m \\
k' &= (1-\delta)k + i_k - \Psi\left(k',k\right) \\
k'_{oil} &= (1-\delta)k_{oil} + i_{k_{oil}} - \Psi_{oil}\left(k'_{oil},k_{oil}\right) \\
X &= q\left(s,k',b'\right)\left[b' - (1-\gamma)b\right] - [\gamma + \kappa(1-\gamma)]b
\end{aligned}
$$

and:

$$
\begin{aligned}
V^D(s,k) = & \max_{\left\{k',l,\vec{C},\vec{L},\vec{K},\vec{X}\right\}} \left\{u(c,l) + \beta\mathbb{E}\left[\theta V\left(s',m',k',0\right) + (1-\theta)V^D\left(s',k'\right)\right]\right\} \\
s.t. \qquad & c + i_k + i_{k_{oil}} \leq F^D(s,k,k_{oil}) + (1-\delta)k - \bar{m} \\
& k' = (1-\delta)k + i_k - \Psi\left(k',k\right) \\
& k'_{oil} = (1-\delta)k_{oil} + i_{k_{oil}} - \Psi_{oil}\left(k'_{oil},k_{oil}\right)
\end{aligned}
$$

where $F(s,k,k_{oil},X)$ and $F^D(s,k,k_{oil})$ summarize all the static allocations given the state and the choices of (k',k'_{oil},b'). In repayment F is defined as:

$$
\begin{aligned}
F(s,k,k_{oil},X) = & \max_{c_N,c_M,c_{oil},k_N,k_M,k_{oil},x_{oil},x_M} f^Y(c_N,c_M,c_{oil}) \\
s.t. \qquad & c_N = f^N(z,k_N) \\
& c_M = f^M(z,k_M) - x_M \\
& c_{oil} = f^{oil}(z,k_{oil},n) - x_{oil} \\
& X = p_M x_M + p_{oil} x_{oil} \\
& k = k_N + k_M
\end{aligned}
$$

and in default F^D is defined as:

$$
\begin{aligned}
F^D(s,k,k_{oil}) = & \max_{c_N,c_M,c_{oil},k_N,k_M,k_{oil},x_{oil},x_M} f^Y(c_N,c_M,c_{oil}) \\
s.t. \qquad & c_N = f^N(\tilde{z},k_N) \\
& c_M = f^M(\tilde{z},k_M) - x_M \\
& c_{oil} = f^{oil}(\tilde{z},k_{oil},n) - x_{oil} \\
& 0 = p_M x_M + p_{oil} x_{oil} \\
& k = k_N + k_M
\end{aligned}
$$

In repayment, the first-order conditions that characterize the static allocations are:

$$f^{Y}_{c_N}(c_N, c_M, c_{oil}) = \lambda_{C_N} \tag{A.1}$$

$$f^{Y}_{c_M}(c_N, c_M, c_{oil}) = \lambda_{C_M} \tag{A.2}$$

$$f^{Y}_{c_{oil}}(c_N, c_M, c_{oil}) = \lambda_{C_{oil}} \tag{A.3}$$

$$f^{N}_{k}(z, k_N) = \frac{\lambda_k}{\lambda_{C_N}} \tag{A.4}$$

$$f^{M}_{k}(z, k_M) = \frac{\lambda_k}{\lambda_{C_M}} \tag{A.5}$$

$$f^{oil}_{k}(z, k_{oil}, n) = \frac{\lambda_k}{\lambda_{C_{oil}}} \tag{A.6}$$

$$\lambda_{C_{oil}} = \frac{p_{oil}}{p_M}\lambda_{C_M} \tag{A.7}$$

$$\lambda_{BoP} = \frac{\lambda_{C_M}}{p_M} \tag{A.8}$$

where λ_{C_N}, λ_{C_M}, $\lambda_{C_{oil}}$, λ_k, and λ_{BoP} are the multipliers of the market clearing constraints for intermediate goods, for capital, and for the balance of payments, respectively. Note that equations (A.7) and (A.8) already pin down $\lambda_{C_{oil}}$ and λ_{BoP} in terms of λ_{C_M} and the international prices p_M and p_{oil}. Thus, we are left with a system of 6 first-order conditions plus 5 constraints to solve for 8 static allocations c_N, c_M, c_{oil}, k_N, k_M, k_{oil}, x_{oil}, and x_M and 3 multipliers λ_{C_N}, λ_{C_M}, and λ_k.

Now, in the market economy the final good firm solves:

$$\min_{c_N, c_M, c_{oil}} p_N c_N + p_M c_M + p_{oil} c_{oil}$$
$$s.t. \quad Y \leq f^{Y}(c_N, c_M, c_{oil})$$

and the intermediate goods firms solve:

$$\max_{k_N} f^{N}(z, k_N) - r k_N$$
$$\max_{k_M} f^{M}(z, k_M) - r k_M$$
$$\max_{k_{oil}} f^{oil}(z, k_{oil}, n) - r_{oil} k_{oil}$$

The 8 static allocations c_N, c_M, c_{oil}, k_N, k_M, k_{oil}, x_{oil} and x_M, 3 endogenous prices p_N, r, and r_{oil}, and the multiplier μ^Y of the constraint in the minimization problem of the final good firm are pinned down by the 6 F.O.C.s of these problems:

$$f^Y_{c_N}(c_N, c_M, c_{oil}) = \frac{p_N}{\mu^Y} \tag{A.9}$$
$$f^Y_{c_M}(c_N, c_M, c_{oil}) = \frac{p_M}{\mu^Y} \tag{A.10}$$
$$f^Y_{c_{oil}}(c_N, c_M, c_{oil}) = \frac{p_{oil}}{\mu^Y} \tag{A.11}$$
$$f^N_k(z, k_N) = r \tag{A.12}$$
$$f^M_k(z, k_M) = r \tag{A.13}$$
$$f^{oil}_k(z, k_{oil}) = r_{oil} \tag{A.14}$$

the balance of payments, the market clearing conditions and the constraint:

$$c_N = f^N(z, k_N)$$
$$c_M = f^M(z, k_M) - x_M$$
$$c_{oil} = f^{oil}(z, k_{oil}, n) - x_{oil}$$
$$X = p_M x_M + p_{oil} x_{oil}$$
$$k_N + k_M = k$$

where, recall, $X = q(s, k', b')[b' - (1-\gamma)b] - [\gamma + \kappa(1-\gamma)]b$, k', b', and l are given. Note that if $\mu^Y = \frac{p_M}{\lambda_{C_M}}$, $p_N = \mu^Y \frac{\lambda_{C_N}}{p_M}$, $r = \mu^Y \frac{\lambda_k}{p_M}$, and $w = \mu^Y \frac{\lambda_l}{p_M}$ then the two systems of equations are the same and, thus, the allocations that satisfy them are the same.

Finally, for the dynamic allocations note that the government in the market economy has three instruments τ_k, $\tau_{k_{oil}}$, and T to pin down the households decisions for labor, capital in the next period, and consumption. Thus, with the correct choices of capital and transfers the two problems are equivalent.

Appendix B

Appendix to Chapter 3

Ruling out explosive equilibria and simplifying the Euler Equation

Recall that from F.O.C.s we have:

$$\frac{\gamma}{1-\gamma} C\left(s^t\right)^{-1} P\left(s^t\right)^{-1} = \sum_{k=t}^{t+T} \beta^{k-t} \mathbb{E}\left[M\left(s^k\right)^{-1} | s^t\right] + \beta^T \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^t\right]$$

First, note that it must be the case that:

$$\beta^T \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^t\right] \geq 0$$

for all t, T. Hence, we have:

$$\lim_{T\to\infty} \beta^T \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^t\right] \geq 0$$

Now, note that given the process for μ_t we can exactly calculate for any $t \geq 0,\ k > 0$:

$$\begin{aligned}
\mathbb{E}\left[M\left(s^{t+k}\right)^{-1}|s^t\right] &= \mathbb{E}\left[\left(\left\{\prod_{h=1}^{k}\exp\left(\mu_{t+k}\right)\right\}M\left(s^t\right)\right)^{-1}|s^t\right] \\
&= \mathbb{E}\left[\left(\left\{\exp\left(\sum_{h=1}^{k}\left[\bar{\mu}+\epsilon_{t+k}\right]\right)\right\}M\left(s^t\right)\right)^{-1}|s^t\right] \\
&= \mathbb{E}\left[\exp\left(-\sum_{h=1}^{k}\epsilon_{t+k}\right)|s^t\right]\exp\left(k\bar{\mu}\right)^{-1}M\left(s^t\right)^{-1} \\
&= \exp\left(-\sum_{h=1}^{k}\left[\mathbb{E}\left[\epsilon_{t+k}|s^t\right]+\frac{1}{2}Var\left(\epsilon_{t+k}|s^t\right)\right]\right)\exp\left(k\bar{\mu}\right)M\left(s^t\right)^{-1} \\
&= \exp\left(\frac{1}{2}\frac{1}{\psi_\mu}k\right)^{-1}\exp\left(k\bar{\mu}\right)M\left(s^t\right)^{-1} \\
&= \exp\left(k\left[\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right]\right)^{-1}M\left(s^t\right)^{-1}
\end{aligned}$$

So, we can use this to show that the following sum is well defined:

$$\begin{aligned}
\sum_{k=t}^{\infty}\beta^{k-t}\mathbb{E}\left[M\left(s^k\right)^{-1}|s^t\right] &= \sum_{h=0}^{\infty}\beta^h\mathbb{E}\left[M\left(s^{t+h}\right)^{-1}|s^t\right] \\
&= \sum_{h=0}^{\infty}\beta^h\exp\left(h\left[\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right]\right)^{-1}M\left(s^t\right)^{-1} \\
&= M\left(s^t\right)^{-1}\sum_{h=0}^{\infty}\left(\frac{\beta}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}\right)^h \\
&= M\left(s^t\right)^{-1}\frac{1}{1-\frac{\beta}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}} \\
&= M\left(s^t\right)^{-1}\frac{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu}+\frac{1}{2}\frac{1}{\psi_\mu}\right)-\beta}
\end{aligned}$$

Now, consider an arbitrary history s^t for which, in equilibrium, the state is such that:

$$\lim_{T\to\infty}\beta^T\mathbb{E}\left[M\left(s^{t+T}\right)^{-1}|s^t\right] > 0$$

Then we have that:

$$\frac{\gamma}{1-\gamma} C\left(s^t\right)^{-1} P\left(s^t\right)^{-1} > \sum_{k=t}^{\infty} \beta^{k-t} \mathbb{E}\left[M\left(s^k\right)^{-1} | s^t\right]$$

$$\iff u_c\left(C\left(s^t\right)\right) P\left(s^t\right)^{-1} > \sum_{k=t}^{\infty} \beta^{k-t} \mathbb{E}\left[v_m\left(M\left(s^k\right)/P\left(s^k\right)\right)/P\left(s^k\right)|s^t\right]$$

where we are using $u(C) = \gamma \log C$ and $v(M/P) = (1-\gamma)\log(M/P)$. Note that this cannot be an equilibrium since there is some positive number η such that it is feasible for the household to increase consumption of the good at time t by η and reduce money holdings by a fraction of η in all subsequent periods. This deviation would be strictly preferred given the above inequality, so in equilibrium it must be the case that:

$$\lim_{T\to\infty} \beta^T \mathbb{E}\left[M\left(s^{t+T}\right)^{-1} | s^t\right] = 0$$

and thus:

$$\begin{aligned}\frac{\gamma}{1-\gamma} C\left(s^t\right)^{-1} P\left(s^t\right)^{-1} &= \sum_{k=0}^{\infty} \beta^{k} \mathbb{E}\left[M\left(s^{t+k}\right)^{-1} | s^t\right] \\ &= M\left(s^t\right)^{-1} \frac{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_\mu}\right)}{\exp\left(\bar{\mu} + \frac{1}{2}\frac{1}{\psi_\mu}\right) - \beta}\end{aligned}$$

Characterization of the linear equilibrium

All derivations shown here are for the more general model presented in Section 4. Taking logs from the equations that characterize the equilibrium with asymmetric information we get:

$$\log\left(\frac{\gamma}{1-\gamma}\frac{1}{\tilde{\beta}}\right)+m_t=c_t+p_t$$

$$\log e_t=\eta_t+p_t-p_t^\star$$

$$\log\left(\frac{\psi}{\gamma}\right)+\zeta l_{i,t}=w_{i,t}-c_t$$

$$y_{i,t}=\frac{1}{1-\theta}p_t-\frac{1}{1-\theta}p_{i,t}+y_t$$

$$y_t=\int_0^1 y_{i,t}di+\frac{1}{2}\theta\left[\int_0^1 y_{i,t}^2di-\left(\int_0^1 y_{i,t}di\right)^2\right]$$

$$l_{i,t}=y_{i,t}-A_{i,t}+\delta\eta_t-\delta w_{i,t}+\delta\log\left(\frac{1-\delta}{\delta}\right)$$

$$q_{i,t}=y_{i,t}-A_{i,t}-(1-\delta)\,\eta_t+(1-\delta)\,w_{i,t}+(1-\delta)\log\left(\frac{\delta}{1-\delta}\right)$$

$$p_{i,t}=\log\left(\frac{1}{\delta^\delta(1-\delta)^{1-\delta}}\frac{1}{\theta}\right)+(1-\delta)\,w_{i,t}+\mathbb{E}\left[\delta\eta_t-\log A_t+p_t|\mathcal{I}^i\left(s^t\right)\right]$$
$$+\frac{1}{2}Var\left(\delta\eta_t-\log A_t+\frac{2-\theta}{1-\theta}p_t+y_t|\mathcal{I}^i\left(s^t\right)\right)-\frac{1}{2}Var\left(\frac{1}{1-\theta}p_t+y_t|\mathcal{I}^i\left(s^t\right)\right)$$

$$p_t=\int_0^1 p_{i,t}di+\frac{1}{2}\frac{\theta}{\theta-1}\left[\int_0^1 p_{i,t}^2di-\left(\int_0^1 p_{i,t}di\right)^2\right]$$

$$\log e_t=p_t-p_t^\star$$

Assume the solution is linear functions of the state, which yields that the variances are constants:

$$p_{i,t}=\mathcal{A}_{-1}+(1-\delta)\,w_{i,t}-\mathbb{E}\left[\log A_t|\mathcal{I}^i\left(s^t\right)\right]+\delta\mathbb{E}\left[\eta_t|\mathcal{I}^i\left(s^t\right)\right]+\mathbb{E}\left[p_t|\mathcal{I}^i\left(s^t\right)\right]$$

Rearranging we can rewrite local wages and the aggregate price level as linear functions of state variables and posterior expectations:

$$p_t=\mathcal{G}_0+\mathcal{G}_1\mu_t+\mathcal{G}_2A_t+\mathcal{G}_3\eta_t+\mathcal{G}_4\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj+\mathcal{G}_5\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj+\mathcal{G}_6\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj$$

$$w_{i,t}=\mathcal{J}_0+\mathcal{J}_1\mu_t+\mathcal{J}_2A_t+\mathcal{J}_3a_{i,t}+\mathcal{J}_4\eta_t+\mathcal{J}_5\mathbb{E}\left[\mu_t|\mathcal{I}^i\right]+\mathcal{J}_6\mathbb{E}\left[\log A_t|\mathcal{I}^i\right]+\mathcal{J}_7\mathbb{E}\left[\eta_t|\mathcal{I}^i\right]$$
$$+\mathcal{J}_8\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj+\mathcal{J}_9\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj+\mathcal{J}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj$$

Conditional expectations

The information set of agent i is:

$$\begin{aligned}
\tilde{\mu}_t &= \mu_t + \tilde{\epsilon}_t \\
A_{i,t} &= A_t + a_{i,t} \\
w_{i,t} &= \mathcal{J}_0 + \mathcal{J}_1 \mu_t + \mathcal{J}_2 A_t + \mathcal{J}_3 a_{i,t} + \mathcal{J}_4 \eta_t + \mathcal{J}_5 \mathbb{E}\left[\mu_t | \mathcal{I}^i\right] + \mathcal{J}_6 \mathbb{E}\left[\log A_t | \mathcal{I}^i\right] + \mathcal{J}_7 \mathbb{E}\left[\eta_t | \mathcal{I}^i\right] \\
&\quad + \mathcal{J}_8 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj + \mathcal{J}_9 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj + \mathcal{J}_{10} \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj \\
e_t &= \mathcal{G}_0 + \mathcal{G}_1 \mu_t + \mathcal{G}_2 A_t + (1 + \mathcal{G}_3)\, \eta_t + \mathcal{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj + \mathcal{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj + \mathcal{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj - p^\star
\end{aligned}$$

Rearranging we get observed variables on LHSs and state variables on RHSs:

$$\begin{aligned}
\tilde{\mu}_t &= \mu_t + \tilde{\epsilon}_t \\
A_{i,t} &= A_t + a_{i,t} \\
\tilde{w}_i = -\mathcal{J}_0 + w_{i,t} - \mathcal{J}_5 \mathbb{E}\left[\mu_t | \mathcal{I}^i\right] - \mathcal{J}_6 \mathbb{E}\left[\log A_t | \mathcal{I}^i\right] - \mathcal{J}_7 \mathbb{E}\left[\eta_t | \mathcal{I}^i\right] & \\
-\mathcal{J}_8 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj - \mathcal{J}_9 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj - \mathcal{J}_{10} \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj &= \mathcal{J}_1 \mu_t + \mathcal{J}_2 A_t + \mathcal{J}_3 a_{i,t} + \mathcal{J}_4 \eta_t \\
\log \tilde{e} = \left[\log p^\star - \mathcal{G}_0\right] + \log e_t - \mathcal{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\left(s^t\right)\right] dj & \\
-\mathcal{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj - \mathcal{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj &= \mathcal{G}_1 \mu_t + \mathcal{G}_2 A_t + (1 + \mathcal{G}_3)\, \eta_t
\end{aligned}$$

- Rearrange to get conditionally independent signals for μ_t:

$$\begin{aligned}
\tilde{\mu}_t &= \mu_t + \tilde{\epsilon}_t \\
\tilde{A}_{i,\mu} &= \mu_t - \frac{(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2}{\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1} a_{i,t} \\
\tilde{w}_{i,\mu} &= \mu_t + \frac{(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2}{\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1} A_t \\
\log \tilde{e}_\mu &= \mu_t + \frac{\left[(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2\right](1 + \mathcal{G}_3)}{\mathcal{G}_1 \left[(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2\right] - \mathcal{G}_2 \left[\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1\right]} \eta_t
\end{aligned}$$

  so the conditional expectation is standard for normal conjugate priors.

- Do the same for A_t:

$$\begin{aligned}
\tilde{\mu}_A &= A_t - \frac{\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1}{(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2} \tilde{\epsilon}_t \\
A_{i,t} &= A_t + a_{i,t} \\
\tilde{w}_{i,A} &= A_t + \frac{\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1}{(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2} \epsilon_t \\
\log \tilde{e}_A &= A_t + \frac{\left[\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1\right](1 + \mathcal{G}_3)}{\mathcal{G}_2 \left[\mathcal{J}_1 (1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_1\right] - \mathcal{G}_1 \left[(\mathcal{J}_2 - \mathcal{J}_3)(1 + \mathcal{G}_3) - \mathcal{J}_4 \mathcal{G}_2\right]} \eta_t
\end{aligned}$$

- Do the same for η_t:

$$\begin{aligned}
\tilde{\mu}_\eta &= \eta_t - \frac{\left(\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2\right)\mathcal{G}_1}{\left(1+\mathcal{G}_3\right)\left\{\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2\right\} - \mathcal{G}_2\left[\mathcal{J}_4\mathcal{G}_1 - \mathcal{J}_1\left(1+\mathcal{G}_3\right)\right]}\tilde{\epsilon}_t \\
\tilde{A}_{i,\eta} &= \eta_t - \frac{\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2}{\mathcal{J}_4\mathcal{G}_1 - \mathcal{J}_1\left(1+\mathcal{G}_3\right)}a_{i,t} \\
\tilde{w}_{i,\eta} &= \eta_t + \frac{\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2}{\mathcal{J}_4\mathcal{G}_1 - \mathcal{J}_1\left(1+\mathcal{G}_3\right)}A_t \\
\log\tilde{e}_\eta &= \eta_t + \frac{\left(\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2\right)\mathcal{G}_1}{\left(1+\mathcal{G}_3\right)\left\{\left[\mathcal{J}_2 - \mathcal{J}_3\right]\mathcal{G}_1 - \mathcal{J}_1\mathcal{G}_2\right\} - \mathcal{G}_2\left[\mathcal{J}_4\mathcal{G}_1 - \mathcal{J}_1\left(1+\mathcal{G}_3\right)\right]}\epsilon_t
\end{aligned}$$

Rename the weights to get:

$$\begin{aligned}
\mathbb{E}\left[\mu_t|\mathcal{I}^i\left(s^t\right)\right] &= \alpha_{0,\mu}\bar{\mu} + \alpha_{1,\mu}\tilde{\mu}_t + \alpha_{2,\mu}\tilde{A}_{i,\mu} + \alpha_{3,\mu}\tilde{w}_{i,\mu} + \alpha_{4,\mu}\log\tilde{e}_\mu \\
\mathbb{E}\left[A_t|\mathcal{I}^i\left(s^t\right)\right] &= \alpha_{1,A}\tilde{\mu}_A + \alpha_{2,A}A_{i,t} + \alpha_{3,A}\tilde{w}_{i,A} + \alpha_{4,A}\log\tilde{e}_A \\
\mathbb{E}\left[\eta_t|\mathcal{I}^i\left(s^t\right)\right] &= \alpha_{0,\eta}\rho\eta_{t-1} + \alpha_{1,\eta}\tilde{\mu}_\eta + \alpha_{2,\eta}\tilde{A}_{i,\eta} + \alpha_{3,\eta}\tilde{w}_{i,\eta} + \alpha_{4,\eta}\log\tilde{e}_\eta
\end{aligned}$$

Rearranging and renaming constants we get the following system:

$$\begin{aligned}
\mathbb{E}\left[\mu_t|\mathcal{I}^i\left(s^t\right)\right] &= \mathcal{Y}_0 + \mathcal{Y}_1\tilde{\mu}_t + \mathcal{Y}_2 A_{i,t} + \mathcal{Y}_3 w_{i,t} + \mathcal{Y}_4\log e_t + \mathcal{Y}_5\mathbb{E}\left[\mu_t|\mathcal{I}^i\right] + \mathcal{Y}_6\mathbb{E}\left[\log A_t|\mathcal{I}^i\right] \\
&\quad + \mathcal{Y}_7\mathbb{E}\left[\eta_t|\mathcal{I}^i\right] + \mathcal{Y}_8\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \mathcal{Y}_9\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj + \mathcal{Y}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj \\
\mathbb{E}\left[A_t|\mathcal{I}^i\right] &= \mathcal{Z}_0 + \mathcal{Z}_1\tilde{\mu}_t + \mathcal{Z}_2 A_{i,t} + \mathcal{Z}_3 w_{i,t} + \mathcal{Z}_4\log e_t + \mathcal{Z}_5\mathbb{E}\left[\mu_t|\mathcal{I}^i\right] + \mathcal{Z}_6\mathbb{E}\left[\log A_t|\mathcal{I}^i\right] \\
&\quad + \mathcal{Z}_7\mathbb{E}\left[\eta_t|\mathcal{I}^i\right] + \mathcal{Z}_8\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \mathcal{Z}_9\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj + \mathcal{Z}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj \\
\mathbb{E}\left[\eta_t|\mathcal{I}^i\right] &= \tilde{\mathcal{A}}_0 + \tilde{\mathcal{A}}_1\tilde{\mu}_t + \tilde{\mathcal{A}}_2 A_{i,t} + \tilde{\mathcal{A}}_3 w_{i,t} + \tilde{\mathcal{A}}_4\log e_t + \tilde{\mathcal{A}}_5\mathbb{E}\left[\mu_t|\mathcal{I}^i\right] + \tilde{\mathcal{A}}_6\mathbb{E}\left[\log A_t|\mathcal{I}^i\right] \\
&\quad + \tilde{\mathcal{A}}_7\mathbb{E}\left[\eta_t|\mathcal{I}^i\right] + \tilde{\mathcal{A}}_8\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \tilde{\mathcal{A}}_9\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj + \tilde{\mathcal{A}}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj
\end{aligned}$$

Integrate, rearrange and rename constants to get the system on aggregate expectations:

$$\begin{aligned}
\int_0^1\mathbb{E}\left[\mu_t|\cdot\right]dj &= \tilde{\mathcal{B}}_0 + \tilde{\mathcal{B}}_1\tilde{\mu}_t + \tilde{\mathcal{B}}_2 A_t + \tilde{\mathcal{B}}_3\int_0^1 w_{j,t}dj + \tilde{\mathcal{B}}_4\log e_t + \tilde{\mathcal{B}}_5\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj + \tilde{\mathcal{B}}_6\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj \\
\int_0^1\mathbb{E}\left[A_t|\cdot\right]dj &= \tilde{\mathcal{C}}_0 + \tilde{\mathcal{C}}_1\tilde{\mu}_t + \tilde{\mathcal{C}}_2 A_t + \tilde{\mathcal{C}}_3\int_0^1 w_{j,t}dj + \tilde{\mathcal{C}}_4\log e_t + \tilde{\mathcal{C}}_5\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \tilde{\mathcal{C}}_6\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\right]dj \\
\int_0^1\mathbb{E}\left[\eta_t|\cdot\right]dj &= \tilde{\mathcal{D}}_0 + \tilde{\mathcal{D}}_1\tilde{\mu}_t + \tilde{\mathcal{D}}_2 A_t + \tilde{\mathcal{D}}_3\int_0^1 w_{j,t}dj + \tilde{\mathcal{D}}_4\log e_t + \tilde{\mathcal{D}}_5\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\right]dj + \tilde{\mathcal{D}}_6\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\right]dj
\end{aligned}$$

Recall wages are:

$$\begin{aligned}
w_{i,t} &= \mathcal{J}_0 + \mathcal{J}_1\mu_t + \mathcal{J}_2 A_t + \mathcal{J}_3 a_{i,t} + \mathcal{J}_4\eta_t + \mathcal{J}_5\mathbb{E}\left[\mu_t|\mathcal{I}^i\left(s^t\right)\right] + \mathcal{J}_6\mathbb{E}\left[\log A_t|\mathcal{I}^i\left(s^t\right)\right] + \mathcal{J}_7\mathbb{E}\left[\eta_t|\mathcal{I}^i\left(s^t\right)\right] \\
&\quad + \mathcal{J}_8\int_0^1\mathbb{E}\left[\mu_t|\mathcal{I}^j\left(s^t\right)\right]dj + \mathcal{J}_9\int_0^1\mathbb{E}\left[A_t|\mathcal{I}^j\left(s^t\right)\right]dj + \mathcal{J}_{10}\int_0^1\mathbb{E}\left[\eta_t|\mathcal{I}^j\left(s^t\right)\right]dj
\end{aligned}$$

The integral is then:

$$\int_0^1 w_{j,t} dj = \mathcal{J}_0 + \mathcal{J}_1 \mu_t + \mathcal{J}_2 A_t + \mathcal{J}_4 \eta_t$$
$$+ [\mathcal{J}_5 + \mathcal{J}_8] \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj + [\mathcal{J}_6 + \mathcal{J}_9] \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj + [\mathcal{J}_7 + \mathcal{J}_{10}] \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj$$

Plugging in and renaming the constants:

$$\int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj = \tilde{\mathcal{E}}_0 + \tilde{\mathcal{E}}_1 \mu_t + \tilde{\mathcal{E}}_2 \tilde{\mu}_t + \tilde{\mathcal{E}}_3 A_t + \tilde{\mathcal{E}}_4 \eta_t + \tilde{\mathcal{E}}_5 \log e_t + \tilde{\mathcal{E}}_6 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj + \tilde{\mathcal{E}}_7 \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj$$
$$\int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj = \tilde{\mathcal{F}}_0 + \tilde{\mathcal{F}}_1 \mu_t + \tilde{\mathcal{F}}_2 \tilde{\mu}_t + \tilde{\mathcal{F}}_3 A_t + \tilde{\mathcal{F}}_4 \eta_t + \tilde{\mathcal{F}}_5 \log e_t + \tilde{\mathcal{F}}_6 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj + \tilde{\mathcal{F}}_7 \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj$$
$$\int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\right] dj = \tilde{\mathcal{G}}_0 + \tilde{\mathcal{G}}_1 \mu_t + \tilde{\mathcal{G}}_2 \tilde{\mu}_t + \tilde{\mathcal{G}}_3 A_t + \tilde{\mathcal{G}}_4 \eta_t + \tilde{\mathcal{G}}_5 \log e_t + \tilde{\mathcal{G}}_6 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\right] dj + \tilde{\mathcal{G}}_7 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\right] dj$$

Solving the system:

$$\int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\left(s^t\right)\right] dj = \tilde{\mathcal{K}}_0 + \tilde{\mathcal{K}}_1 \mu_t + \tilde{\mathcal{K}}_2 \tilde{\mu}_t + \tilde{\mathcal{K}}_3 A_t + \tilde{\mathcal{K}}_4 \eta_t + \tilde{\mathcal{K}}_5 \log e_t$$
$$\int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\left(s^t\right)\right] dj = \tilde{\mathcal{L}}_0 + \tilde{\mathcal{L}}_1 \mu_t + \tilde{\mathcal{L}}_2 \tilde{\mu}_t + \tilde{\mathcal{L}}_3 A_t + \tilde{\mathcal{L}}_4 \eta_t + \tilde{\mathcal{L}}_5 \log e_t$$
$$\int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\left(s^t\right)\right] dj = \tilde{\mathcal{M}}_0 + \tilde{\mathcal{M}}_1 \mu_t + \tilde{\mathcal{M}}_2 \tilde{\mu}_t + \tilde{\mathcal{M}}_3 A_t + \tilde{\mathcal{M}}_4 \eta_t + \tilde{\mathcal{M}}_5 \log e_t$$

Which can be plugged in to solve the system for individual expectations.
Recall the aggregate price only depends on aggregate expectations:

$$p_t = \mathcal{G}_0 + \mathcal{G}_1 \mu_t + \mathcal{G}_2 A_t + \mathcal{G}_3 \eta_t + \mathcal{G}_4 \int_0^1 \mathbb{E}\left[\mu_t | \mathcal{I}^j\left(s^t\right)\right] dj + \mathcal{G}_5 \int_0^1 \mathbb{E}\left[A_t | \mathcal{I}^j\left(s^t\right)\right] dj + \mathcal{G}_6 \int_0^1 \mathbb{E}\left[\eta_t | \mathcal{I}^j\left(s^t\right)\right] dj$$

Plugging in:

$$\begin{aligned} p_t = &\left[\mathcal{G}_0 + \mathcal{G}_4 \tilde{\mathcal{K}}_0 + \mathcal{G}_5 \tilde{\mathcal{L}}_0 + \mathcal{G}_6 \tilde{\mathcal{M}}_0\right] + \left[\mathcal{G}_1 + \mathcal{G}_4 \tilde{\mathcal{K}}_1 + \mathcal{G}_5 \tilde{\mathcal{L}}_1 + \mathcal{G}_6 \tilde{\mathcal{M}}_1\right] \mu_t + \left[\mathcal{G}_4 \tilde{\mathcal{K}}_2 + \mathcal{G}_5 \tilde{\mathcal{L}}_2 + \mathcal{G}_6 \tilde{\mathcal{M}}_2\right] \tilde{\mu}_t \\ &+ \left[\mathcal{G}_2 + \mathcal{G}_4 \tilde{\mathcal{K}}_3 + \mathcal{G}_5 \tilde{\mathcal{L}}_3 + \mathcal{G}_6 \tilde{\mathcal{M}}_3\right] A_t + \left[\mathcal{G}_3 + \mathcal{G}_4 \tilde{\mathcal{K}}_4 + \mathcal{G}_5 \tilde{\mathcal{L}}_4 + \mathcal{G}_6 \tilde{\mathcal{M}}_4\right] \eta_t + \left[\mathcal{G}_4 \tilde{\mathcal{K}}_5 + \mathcal{G}_5 \tilde{\mathcal{L}}_5 + \mathcal{G}_6 \tilde{\mathcal{M}}_5\right] \log e_t \\ &= \tilde{\mathcal{N}}_0 + \tilde{\mathcal{N}}_1 \mu_t + \tilde{\mathcal{N}}_2 \tilde{\mu}_t + \tilde{\mathcal{N}}_3 A_t + \tilde{\mathcal{N}}_4 \eta_t + \tilde{\mathcal{N}}_5 \log e_t \end{aligned}$$

Plugging in for $\log e_t = \eta_t + p_t - p^\star$,

$$\begin{aligned} p_t &= \tilde{\mathcal{N}}_0 + \tilde{\mathcal{N}}_1 \mu_t + \tilde{\mathcal{N}}_2 \tilde{\mu}_t + \tilde{\mathcal{N}}_3 A_t + \tilde{\mathcal{N}}_4 \eta_t + \tilde{\mathcal{N}}_5 \log e_t \\ p_t &= \frac{\tilde{\mathcal{N}}_0 - \tilde{\mathcal{N}}_5 p^\star}{1 - \tilde{\mathcal{N}}_5} + \frac{\tilde{\mathcal{N}}_1}{1 - \tilde{\mathcal{N}}_5} \mu_t + \frac{\tilde{\mathcal{N}}_2}{1 - \tilde{\mathcal{N}}_5} \tilde{\mu}_t + \frac{\tilde{\mathcal{N}}_3}{1 - \tilde{\mathcal{N}}_5} A_t + \frac{\tilde{\mathcal{N}}_4 + \tilde{\mathcal{N}}_5}{1 - \tilde{\mathcal{N}}_5} \eta_t \\ &= \tilde{\mathcal{O}}_0 + \tilde{\mathcal{O}}_1 \mu_t + \tilde{\mathcal{O}}_2 \tilde{\mu}_t + \tilde{\mathcal{O}}_3 A_t + \tilde{\mathcal{O}}_4 \eta_t \end{aligned}$$