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Mixed-Drove Spatio-Temporal Co-occurrence Pattern Mining

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Abstract

Mixed-drove spatio-temporal co-occurrence patterns (MDCOPs) represent subsets of two or more different object-types whose instances are often located in spatial and temporal proximity. Discovering MDCOPs is an important problem with many applications such as identifying tactics in battlefields, games, and predator-prey interactions. However, mining MDCOPs is computationally very expensive because the interest measures are computationally complex, datasets are larger due to the archival history, and the set of candidate patterns is exponential in the number of object-types. We propose a monotonic composite interest measure for discovering MDCOPs and novel MDCOP mining algorithms. Analytical results show that the proposed algorithms are correct and complete. Experimental results also show that the proposed methods are computationally more efficient than naïve alternatives.

Index Terms

Spatia-temporal Data Mining, Spatio-temporal Co-occurrence Pattern Mining, Composite Interest Measure, Mixed-drove Spatio-temporal Co-occurrence Pattern.

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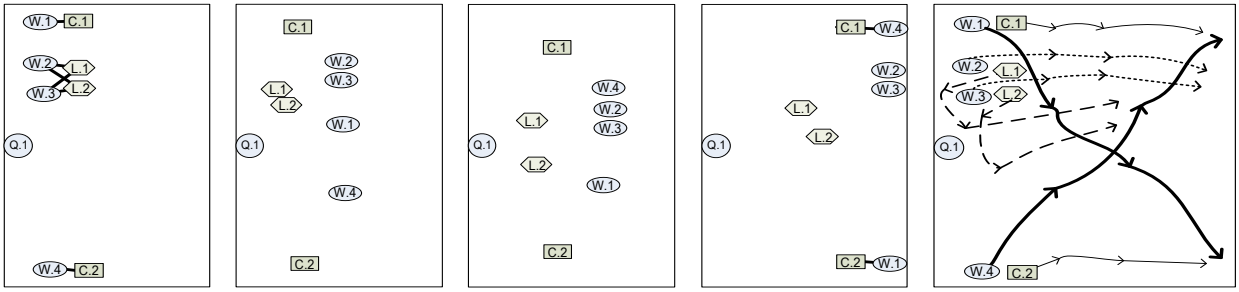
I. INTRODUCTION

As the volume of spatio-temporal data continues to increase significantly due to both the growth of database archives and the increasing number and resolution of spatio-temporal sensors, automated and semi-automated pattern analysis becomes more essential. As a result, spatio-temporal co-occurrence pattern mining has been the subject of recent research. Given a moving object database, our aim is to discover mixed-drove spatio-temporal co-occurrence patterns (MDCOPs) representing subsets of different object-types whose instances are located close together in geographic space for a significant fraction of time. Unlike the objectives of some other spatio-temporal co-occurrence pattern identification approaches where the pattern is the primary interest, in MDCOPs both the pattern and the nature of the different *object-types* are of interest.

A simple example of an MDCOP is in ecological predator-prey relationships. Patterns of movements of rabbits and foxes, for example, will tend to be co-located in many time-frames which may or may not be consecutive. Rabbits may attempt to move away from foxes, and the foxes may attempt to stay with the rabbits. Other factors such as available food and water may also affect the patterns.

More example MDCOPs may be illustrated in American football where two teams try to outscore each other by moving a football to the opponent's end of the field. Various complex interactions occur within one team and across teams to achieve this goal. These interactions involve intentional and accidental MDCOPs, the identification of which may help teams to study their opponent's tactics. In American football, object-types may be defined by the roles of the offensive and defensive players, such as quarterback, running back, wide receiver, kicker, holder, linebacker, and cornerback. An MDCOP is a subset of these different object-types (such as $\{$kicker, holder$\}$ or $\{$wide_receiver, cornerback$\}$) that occur frequently. One example MDCOP involves offensive wide receivers, defensive linebackers, and defensive cornerbacks, and is called a Hail Mary play. In this play, the objective of the offensive wide receivers is to outrun any linebackers and defensive backs and get behind them, catching an undefended pass while running untouched for a touchdown. This interaction creates an MDCOP between wide receivers and cornerbacks. An example Hail Mary play is given in Figure 1. It shows the positions of four offensive wide receivers (W.1, W.2, W.3, and W.4), two defensive cornerbacks (C.1 and C.2), two



(a) time slot t=0 (b) time slot t=1 (c) time slot t=2 (d) time slot t=3 (e) coach sketch

Fig. 1. An example Hail Mary play in American football

defensive linebackers (L.1 and L.2), and a quarterback (Q.1) in four time slots. The solid lines between the players show the neighboring players. The wide receivers W.1 and W.4 cross over each other and the wide receivers W.2 and W.3 run directly to the end zone of the field. Initially, the wide receivers W.1 and W.4 are co-located with cornerbacks C.1 and C.2 respectively and the wide receivers W2. and W.3 are co-located with linebackers L.1 and L.2 at time slot t=0 (Figure 1 (a)). In time slot t=1, the four wide receivers begin to run, while the linebackers run towards the quarterback and the cornerbacks remain in their original position, possibly due to a fake handoff from the quarterback to the running back (Figure 1 (b)). In time slot t=2, the wide receivers W.1 and W.4 cross over each other and try to drift further away from their respective cornerbacks (Figure 1 (c)). When the quarterback shows signs of throwing the football, both cornerbacks and linebackers run to their respective wide receivers (Figure 1(d)). The overall sketch of the game tactics can be seen in Figure 1(e). In this example, wide receivers and cornerbacks form an MDCOP since they are persistent over time and they occur 2 out of 4 time slots. However, wide receivers and linebackers do not form an MDCOP due to the lack of temporal persistence.

There are many applications for which discovering co-occurring patterns of specific combinations of object-types is important. Some of these include military (battlefield planning and strategy), ecology (tracking species and pollutant movements), homeland defense (looking for significant ”events”), and transportation (road and network planning) [10], [14].

However, discovering MDCOPs poses several non-trivial challenges. First, current interest measures (i.e. the spatial prevalence measure) are not sufficient to quantify such patterns, so

new composite interest measures must be created and formalized [12], [21]. Second, the set of candidate patterns grows exponentially with the number of object-types. Finally, since spatio-temporal datasets are huge, computationally efficient algorithms must be developed [22].

A. Contributions

This paper is an extended version of our paper published in the proceedings of the 6th IEEE International Conference on Data Mining (ICDM) [5], where we introduced an MDCOP mining problem, proposed a new monotonic composite interest measure, developed two MDCOP algorithms, and evaluated these using real datasets. This extended paper makes the following new contributions:

- It proposes a new and computationally efficient MDCOP mining algorithm (FastMDCOP-Miner)
- It compares the proposed algorithm with those in the ICDM paper [5].
- It presents additional experimental results with synthetic datasets for all MDCOP algorithms.
- It includes an expanded literature survey and a discussion of statistical spatio-temporal interest measures.
- It includes revised comparisons of approaches and experimental design.

B. Scope and Outline

This paper focuses on MDCOPs (typed collections of moving objects) by extending interest measures for spatial co-location patterns given a user-defined participation index threshold [12], [21]. The following issues are beyond the scope of this paper: (i) determining thresholds for MDCOP interest measures; (ii) similarity measures for tracking moving objects due to the focus on object-types rather than objects; (iii) indexing and query processing issues related to mining objects; (iv) discovering multisets (e.g.{A, A, B}).

The rest of the paper is organized as follows. Section II presents a discussion of related work. Section III presents basic concepts to provide a formal model of MDCOPs and the problem statement of mining MDCOPs. Section IV presents our proposed MDCOP mining algorithm. Analysis of the algorithm is given in Section V. Section VI presents the experimental evaluation and Section VII presents conclusions and future work.

II. Related Work

Data analysis can be broadly categorized into statistical approaches and data mining approaches. In statistical approaches, there are bodies of work in both spatial and temporal analysis. Spatial point patterns are often described by metrics such as the intensity function and Ripley's K [19], [20]. Other measures such as complete spatial randomness (CSR) and spatial covariance functions are used to describe the spatial relationships of adjacent areas and continuous variables as random fields [6]. Temporal patterns have been extensively studied in models such as moving averages, first and second order autoregression, integration, and periodic patterns such as seasonality [24]. Granger has looked at co-occurring temporal patterns under an assumption of cointegration [7]. There has also been some recent research in combining spatial and temporal analysis, such as Brix and Diggle's extended intensity function and the extended K(r,t) function [1], [17]. Most attempts to combine the fields suffer from limitations such as the inability to model space-time interactions, treating time as merely another dimension of space and assuming separability and independence between space and time [20]. Statistical research specifically focused on spatio-temporal co-occurrence patterns and their possible interactions has been limited.

Previous data mining studies for mining spatio-temporal co-occurrence patterns can be classified into two categories: mining of uniform groups of moving objects, and mining of mixed groups of moving objects.

To mine uniform groups of moving objects, the problems of discovering flock patterns [16], [9], [8] and moving clusters [13] are defined. A flock pattern is a moving group of the same kind of objects, such as a sheep flock or a bird flock. Gudmundsson et al. proposed algorithms for detection of the flock pattern in spatio-temporal datasets [9], [8]. Kalnis et al. defined the problem of discovering moving clusters and proposed clustering-based methods to mine such patterns [13]. In this approach, if there is a large enough number of common objects between clusters in consecutive time slots, such clusters are called moving clusters. These methods do not take object-types into account, and thus are not effective for mining MDCOPs [5]. To mine mixed groups of moving objects, the problems of discovering collocation episodes [4] and topological patterns [23] are important. Both generalize co-location patterns [12] (subsets of object-types that are frequently located together in space) to the spatio-temporal domain.

A collocation episode is a sequence of co-location patterns with some common object-types across consecutive time slots. However, if there is no common object-type in consecutive time slots, the proposed approach will not identify any pattern. For example, the collocation episodes algorithm will not be able to find any pattern from the dataset given in Figure 1 if the window length (which is used to find co-location patterns) is 2. For this case, the algorithm tries to find co-location patterns that are persistent in 2 consecutive time slots, but there is no such pattern in the dataset because wide receivers and cornerbacks are forming co-locations in time slots t=0 and t=3 and wide receivers and linebackers are forming co-locations in time slots t=0. Thus, there may not be any co-location patterns and collocation episodes identified in the dataset.

A topological pattern [23] is a subset of object-types whose instances are close in space and time. An interest measure for a topological pattern {A,B} (e.g. participation index or support) is a spatio-temporal join of instances of A and instances of B [12]. This statistic may be high even if many instances of A and many instances of B are not spatially together for a moment in time. The semantics of topological patterns are not well-defined for moving objects. For example, this approach can not find an answer to the question of what fraction of time the pattern occurs. The answer of this approach may also be "'empty"' to the question of when (which time slots) the pattern occurs since there is no time slot notion. In the dataset given in Figure 1, this approach will discover the two patterns of {W, C} and {W, L}. Both patterns have the same support, but pattern {W, C} occurs in 2 time slots out of 4 (a persistent pattern) and pattern {W, L} occurs in 1 time slot out of 4 (a transient pattern) since tracks of objects are represented as spatio-temporal instances. The persistent pattern {W, C} occurs in time slots t=0 and t=3 and its instances {W1, C1} and {W4, C2} occur in time slot t=0 and {W1, C2} and {W4, C1} in time slot t=1. The transient pattern {W, L} occurs in time slot t=0 and its instances {W2, L1}, {W3, L1}, {W2, L2}, and {W3, L2} occur in time slot t=0.

In contrast, our proposed interest measure and algorithms will efficiently mine mixed groups of objects (e.g MDCOPs) which are close in space and persistent (but not necessarily close) in time. Unlike a number of the techniques just described, our approach will discover persistent patterns that co-occur in most but not all spatio-temporal intervals, so consecutive co-occurrences are not mandatory. For example, our proposed MDCOP mining approach will find the MDCOP {wide_receiver, cornerback} pattern in Figure 1, if the fraction of time slots where the pattern occurs over the total number of time slots is no less than a defined threshold, e.g., 0.5. It may

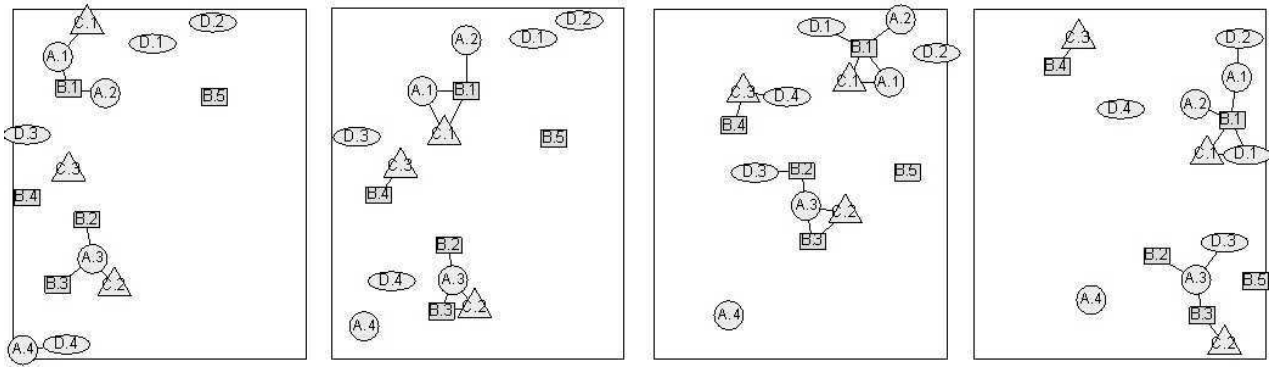
reject the pattern {W,L} in Figure 1 given the lack of time persistence of the {wide_receiver, linebacker} pattern. In fact, instances of MDCOP {wide_receiver, cornerback} are co-located in 2 time slots out of 4 and instances of {wide_receiver, linebacker} are co-located in 1 time slot out of 4. The instances of MDCOP {wide_receiver, cornerback} are {W.1, C.1} and {W.4, C.2} in time slot t=0, and {W.4, C.1} and {W.1, C.2} in time slot t=3.

III. Basic Concepts and Problem Definition

A. Spatial Prevalence Measure

The focus of this study is to discover mixed-drove spatio-temporal co-occurrence patterns (MDCOPs) over a spatio-temporal framework and a neighborhood relation *R*. First we explain the modeling of mixed groups of object-types in space, e.g., spatial co-locations [21]. In the next sections, we explain how we model MDCOPs by extending spatial co-location mining to include time information and then propose algorithms to mine these MDCOPs.

Spatial co-location mining algorithms are used to discover sets of mixed object-types that are frequently located together in a spatial framework for a given set of spatial object-types, their instances, and a spatial neighbor relationship *R* [12], [21]. For example, in Figure 2(a), in time slot t=0, {A.1, C.1} is an instance of a co-location if the distance between the objects is no more than a given neighborhood distance threshold. In Figure 2(a), the solid lines show the distance between the objects that satisfies the neighborhood distance threshold. The participation index is used to determine the strength of the co-location pattern, that is, whether the index is greater than or equal to a threshold [12], [21]. Such a co-location is called spatial prevalent. The participation index is defined as the minimum of the participation ratios (the fraction of the number of instances of object-types forming co-location instances to the total number of instances). For example, in Figure 2(a), {A, B} is a co-location in time slot t=0, and its instances are {A.1, B.1}, {A.2, B.1}, {A.3, B.2}, and {A.3, B.3}. In the dataset, object-type A has 4 instances and three of them (A.1, A.2, and A.3) are contributing to the co-location {A, B}, so the participation ratio of A is 3/4. The participation ratio of B is 3/5 since 3 out of 5 instances are contributing to the co-location {A, B}. The participation index of the co-location {A, B} is 3/5, which is the minimum of the participation ratios of object-types A and B. It has been shown that the participation index is anti-monotone in the size of co-locations [12], [21]. In other words, $participation\_index(P_j) \leq participation\_index(P_i)$ if P_i is a subset of P_j. In



(a) An input spatio-temporal dataset

Co-occurrence Patterns	Spatial prevalence index values				Time prevalence index values
	time slot 0	time slot 1	time slot 2	time slot 3	
A B	3/5	3/5	3/5	3/5	4/4
A C	2/4	2/4	2/4	0	3/4
B C	0	3/5	3/5	3/5	3/4
A B C	0	2/5	2/5	0	2/4

(b) A set of output mixed-drove spatio-temporal co-occurrence patterns

Fig. 2. An example spatio-temporal dataset

addition, [12], [21] show that the participation index has a spatial statistical interpretation as an upper bound on the cross-K function [6].

B. Modeling MDCOPs

Given a set of spatio-temporal mixed object-types and a set of their instances with a neighborhood relation R, an MDCOP is a subset of spatio-temporal mixed object-types whose instances are neighbors in space and time.

Definition 3.1: Given a spatio-temporal pattern and a set TF of time slots, such that $TF = [T_0, ..., T_{n-1}]$, the time prevalence or persistence measure of the pattern is the fraction of time slots where the pattern occurs over the total number of time slots.

For example, in Figure 2(a), the total number of time slots is 4 and pattern $\{$A, B$\}$ occurs in all 4 time slots, so its time prevalence is 4/4. Pattern $\{$A, C$\}$ occurs in 3 time slots, namely, time slots t=0, t=1, and t=2, and its time prevalence index is 3/4.

Definition 3.2: Given a spatio-temporal dataset of mixed object-types ST, and a spatial prevalence threshold θ_p, the mixed-drove prevalence measure of a spatio-temporal pattern P_i is a composition of the spatial prevalence and the time prevalence measures as shown below.

$$\text{P}rob_{t_m \in all\_time\_slot}(s\_prev(pattern\ P_i, time\_slot\ t_m) \geq \theta_p) \tag{1}$$

where $Prob$ stands for probability of overall prevalence time slots and s_prev stands for spatial prevalence, e.g., the participation index, described in Section III-A.

Definition 3.3: Given a spatio-temporal dataset of mixed object-types ST and a threshold pair (θ_p, θ_{time}), MDCOP P_i is a mixed-drove prevalent pattern if its mixed-drove prevalence measure satisfies the following.

$$\text{P}rob_{t_m \in all\_time\_slot}\left[s\_prev(pattern\ P_i, time\_slot\ t_m) \geq \theta_p\right] \geq \theta_{time} \tag{2}$$

where $Prob$ stands for probability of overall prevalence time slots, s_prev stands for spatial prevalence, θ_p is the spatial prevalence threshold, and θ_{time} is the time prevalence threshold.

For example, in Figure 2(a), $\{$A,B$\}$ is an MDCOP because it has mixed object-types, is spatial prevalent in time slots t=0, t=1, t=2, and t=3 since its participation indices are no less than the given threshold 0.4 in these time slots, and is time prevalent since its time prevalence index of 1 is above the time prevalence index threshold 0.5. In contrast, $\{$B,D$\}$ is not an MDCOP. Although it has mixed object-types and is spatial prevalent in time slot t=2, it is not time prevalent since its time prevalence index is no more than the given time prevalence index threshold 0.5.

C. Problem statement

Given:

- A set P of distinct Boolean spatio-temporal object-types over a common spatio-temporal framework STF.
- A neighbor relation R over locations.
- A spatial prevalence threshold, θ_P .
- A time prevalence threshold, θ_{time} .

Find: $\{P_i | P_i$ is a subset of P and P_i is a prevalent MDCOP as in Definition 3.3$\}$.

Objective: Minimize computation cost.

Constraints: To find a correct and complete set of MDCOPs.

Example: In American football, each play (e.g., Figure 1) may represent a spatio-temporal dataset and Boolean object-types may be identified by the role of the players (e.g., wide receiver, cornerback, and linebackers). Each object-type is considered as Boolean because we are interested in its presence or absence at any location and time. Figure 1(a)-(d) shows the position of the Boolean object-types for four time units. The straight lines between the players show the neighboring objects. The neighbor relation R may be defined by a distance less than one meter or an average arm's length. For example, in Figure 1(a), wide receiver W.1 is a neighbor of cornerback C.1. However, these players are not neighbors in Figure 1(b) since they are separated by more than a meter. In this example, $\{$wide_receiver, cornerback$\}$ forms a candidate MDCOP, given θ_p=0.5, and θ_{time}=0.5.

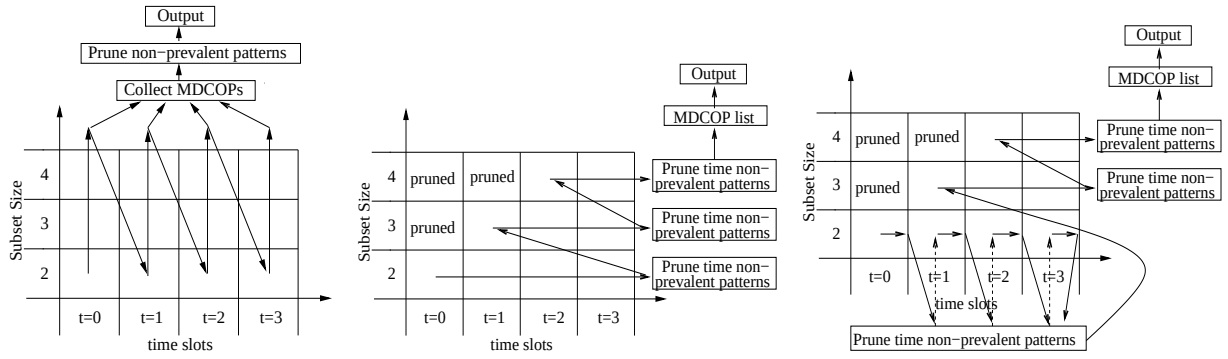
Threshold values selected for MDCOP interest measures (e.g. spatial prevalence measure and time prevalence measure) have important implications on the mining processes and results. Selection of a small interest measure threshold (close to 0) increases the computational complexity of the algorithms and the number of generated prevalent patterns. This may cause generation of insignificant patterns. Selection of a large interest measure threshold (close to 1) decreases the computational complexity of the algorithms and the number of prevalent patterns. This may cause pruning of some of the significant patterns. Nevertheless the selection of interest measure threshold values is dependent on the application and/or purpose of the analysis.

IV. Mining MDCOPs

In this section, we discuss a naïve approach and then propose two novel MDCOP mining algorithms - MDCOP-Miner and FastMDCOP-Miner - to mine MDCOPs. We also give execution traces of these algorithms.

A. Naïve approach

A naïve approach can use a spatial co-location mining algorithm for each time slot to find spatial prevalent co-locations and then apply a post-processing step to discover MDCOPs by checking their time prevalence. To mine co-locations, Huang, Shekhar and Xiong proposed a



(a) Naïve Approach (b) MDCOP-Miner (c) FastMDCOP-Miner

Fig. 3. Comparison of Naïve Approach, MDCOP-Miner, and FastMDCOP-Miner

join-based approach, Yoo, Shekhar and Celik proposed a partial join-based approach and a join-less approach, and Zhang et al. proposed a multi-way spatial join-based approach [3], [12], [21], [26], [27], [28], [29]. This study will be based on the join-based spatial co-location pattern mining algorithm proposed by Huang et al., but it is also possible to use other approaches. The naïve approach will generate size $k+1$ candidate co-locations for each time slot using spatial prevalent size k subclasses until there are no more candidates. After finding all size spatial prevalent co-locations in each time slot, a post-processing step can be used to discover MDCOPs by pruning out time non-prevalent co-locations. Even though this approach will prune out spatial non-prevalent co-locations early, it will not prune out time non-prevalent MDCOPs before the post-processing step (Figure (3(a)). This leads to unnecessary computational cost.

B. MDCOP-Miner

To eliminate the drawbacks of the Naïve approach, we propose an MDCOP mining algorithm (MDCOP-Miner) to discover MDCOPs by incorporating a time-prevalence based filtering step in each iteration of the algorithm. The algorithm, first, will discover all size k spatial prevalent MDCOPs and then will apply a time-prevalence based filering to discover MDCOPS. Finally, the algorithm will generate size $k+1$ candidate MDCOPs using size k MDCOPs (Figure 3(b)). The participation index is used as a spatial prevalence interest measure to check if the pattern is spatial prevalent at a time slot [12]. The time prevalence (i.e., persistence measure in definition

3.1) is used as a time prevalence interest measure. First we give the pseudo code of the algorithm, and then we provide an execution trace of it using the dataset from Figure 2(a). Algorithm 1 gives the pseudo code of the MDCOP-Miner algorithm. This pseudo code is used to explain two algorithms: MDCOP-Miner and FastMDCOP-Miner. FastMDCOP-Miner will be discussed in the next section. The choice of the algorithm is provided by the user. The inputs are algorithm choice alg_choice with value *MDCOP-Miner*, a set of distinct spatial object-types E, a spatio-temporal dataset ST, a spatial neighborhood relationship R, and thresholds of interest measures, i.e. spatial prevalence and time prevalence; the output is a set of MDCOPs. In the algorithm, steps 1 include initialization of the parameters, steps 2 through 14 give an iterative process to mine MDCOPs, and step 15 gives a union of the results. Steps 2 through 14 continue until there are no candidate MDCOPs to be generated. The functions of the algorithm are explained below.

Generating candidate co-occurrence patterns (step 3): This function uses an apriori-based approach to generate size $k+1$ candidate co-locations C_{k+1} for each time slot, using all size k mixed-drove co-occurrence patterns MDP_k [2].

Generating spatial co-occurrence instances (step 5): The instances of candidate C_{k+1} are generated by joining neighbor instances of size k MDCOPs for each time slot. This is similar to the instance generation step of the co-location miner algorithm [12].

Finding spatial prevalent co-occurrence patterns (step 6): All spatial prevalent size $k+1$ patterns SP_{k+1} are found by pruning the ones whose spatial prevalence indices, i.e., participation indices, are less than a given threshold for each time slot. Computation of the participation indices follows the same algorithmic ideas as those in the co-location mining algorithm [12].

In the for loop, the algorithm finds size $k+1$ spatial prevalent co-location for each time slot. MDCOP-Miner skips steps 8, 9, and 10 which are activated using the FastMDCOP-Miner.

Forming a time prevalence table (steps 8 and 12): In steps 8 and 12, the time prevalence indices of the mined spatial prevalent patterns are calculated in FastMDCOP-Miner and MDCOP-Miner algorithms respectively. The time prevalence index of a spatial prevalent co-location is the fraction of the number of time slots where the pattern occurs over the total number of the time slots. Step 8 is activated in FastMDCOP-Miner algorithm and it is used to calculate the time prevalence index of the size k patterns before generating size k patterns of next time slot. Step 12 is activated in MDCOP-Miner and it is used to calculate time prevalence indices of patterns after size k patterns of all time slots are generated .

Algorithm 1 MDCOP-Miner and FastMDCOP-Miner

Inputs:

alg_choice: Choice of algorithm, "MDCOP-Miner" or "FastMDCOP-Miner"

E: a set of distinct spatial object-types

ST: a spatio-temporal dataset <object_type, object_id, x, y, time slot

R: spatial neighborhood relationship

TF: a time slot frame $\{t_0, \ldots, t_{n-1}\}$

θ_p: a spatial prevalence threshold

θ_{time}: a time prevalence threshold

Output: MDCOPs whose spatial prevalence indices, i.e., participation indices, are no less than θ_p, for time prevalence indices are no less than θ_{time}

Variables:

k: co-occurrence size

t: time slots $(0, \ldots, n-1)$

T_k: set of instances size k co-occurrences

C_k: set of candidate size k co-occurrences

SP_k: set of spatial prevalent size k co-occurrences

TP_k: set of time prevalent size k co-occurrences

MDP_k: set of mixed-drove size k co-occurrences

Algorithm:

(1) initialization : co-occurrence size $k = 1, C_k = E, MDP_k(0) = ST$
(2) while (not empty MDP_k) {
(3) $C_{k+1}(0) = gen\_candidate\_co - occur(C_k, MDP_k)$
(4) for each time_slot t in $(0, \ldots, n-1)$ {
(5) $T_{k+1}(t) = gen\_co - occur\_instance(C_{k+1}(t), T_k(t), R)$
(6) $SP_{k+1}(t) = find\_spatial - prevalent\_co - occur(T_{k+1}(t), C_{k+1}(t), \theta_p)$
(7) If (alg_choice ==**"FastMDCOP-Miner"**) {
(8) $TP_{k+1}(t) = find\_time\_prevalence\_index(SP_{k+1}(t))$
(9) $MDP_{k+1}(t) = find\_time - prevalent\_co - occur(TP_{k+1}(t), \theta_{time})$
(10) $C_{k+1}(t) = MDP_{k+1}(t)$ } }
(11) If alg_choice=="MDCOP-Miner" {
(12) $TP_{k+1} = find\_time\_prevalence\_index(SP_{k+1})$
(13) $MDP_{k+1} = find\_time - prevalent\_co - occur(TP_{k+1}, \theta_{time})$ }
(14) $k = k + 1$ }
(15) return union ($MDP_2, \ldots, MDP_{k+1}$)

Finding mixed-drove co-occurrence patterns (step 9 and step 13): These steps discover MDCOPs by checking the time prevalence indices of the spatial prevalent co-locations if they

are no less than a given time prevalence threshold θ_{time} . The patterns whose time prevalence indices do not satisfy the given threshold are pruned at this stage. The remaining patterns will be MDCOPs and will be used to generate candidate supersets of the MDCOPs in step 3. In step 13, MDCOP-miner prunes time non-prevalent patterns after all size k patterns in all time slots are generated. In step 9, FastMDCOP-Miner prunes time non-prevalent patterns before generating size k patterns in the next time slot.

The algorithm will run iteratively until there are no more candidate MDCOPs to be generated. The algorithm outputs the union of all size MDCOPs.

An Execution Trace of MDCOP-Miner: The execution trace of the MDCOP-Miner is given in Figure 4 using the dataset given in Figure 2. This dataset contains four object-types A, B, C, and D and their instances in four time slots. A has 4 instances, B has 5 instances, C has 3 instances, and D has 4 instances. The instances of each object-type have a unique identifier, such as A.1. Some of the patterns of these object-types form an MDCOP. To discover MDCOPs we propose a monotonic composite interest measure (the mixed-drove prevalence measure) which is a composition of the spatial prevalence and time prevalence measures applied to mixed object-types. The spatial prevalence measure (participation index) shows the strength of the spatial co-location when the index is greater than or equal to a given threshold [12], [21]. The time prevalence measure (time prevalence index) shows the frequency of the pattern over time.

In Figure 4(a), in step 1, candidate pairs of the distinct object-types and their instances are generated for each time slot. The spatial co-locations whose participation indices are less than a given threshold are then pruned. A spatial non-prevalent pair {A,D} is pruned in time slot t=0, {C, D} is pruned in time slots t=2 and t=3, and {B,D} is pruned in time slots t=3, because their participation indices are less than the given threshold 0.4. A time prevalence table of pairs of spatial prevalent co-locations is then formed by entering a 1 if the participation index of the corresponding pattern satisfies a given participation index threshold. Time-prevalence indices are then found. For example, in the time prevalence table (step 2 in Figure 4(b)), spatial prevalent pattern {A,B} is persistent for all time slots and its time prevalence index is 4/4, and spatial prevalent pattern {A,C} is persistent in time slots t=0, t=1, and t=2 and its time prevalence index is 3/4. The MDCOPs whose time prevalence indices are no less than a given threshold are selected for generating superset candidate MDCOPs.

Step 1: Generate pairs and find participation indices

	time slot t=0						time slot t=1						time slot t=2						time slot t=3					
Co-occurrence patterns	A B	A C	A D	B C	B D	C D	A B	A C	A D	B C	B D	C D	A B	A C	A D	B C	B D	C D	A B	A C	A D	B C	B D	C D
Co-occurrence pattern instances	A.1 B.1	A.1 C.1	A.4 D.4				A.1 B.1	A.1 C.1		B.1 C.1			A.1 B.1	A.1 C.1		B.1 C.1	B.1 D.1	C.3 D.4	A.1 B.1		A.1 D.2	B.1 C.1	B.1 D.1	C.1 D.1
	A.2 B.1	A.3 C.2					A.2 B.1	A.3 C.2		B.3 C.2			A.2 B.1	A.3 C.2		B.3 C.2	B.2 D.3		A.2 B.1		A.3 D.3	B.3 C.2		
	A.3 B.2						A.3 B.2			B.4 C.3			A.3 B.2			B.4 C.3			A.3 B.2			B.4 C.3		
	A.3 B.3						A.3 B.3						A.3 B.3						A.3 B.3					
P. ratio	3/4 3/5	2/4 2/3	1/4 1/4				3/4 3/5	2/4 2/3		3/5 3/3			3/4 3/5	2/4 2/3		3/5 3/3	2/5 2/4	1/3 1/4	3/4 3/5		2/4 2/3	3/5 3/3	1/5 1/4	1/3 1/4
P. index	3/5	2/4	1/4				3/5	2/4		3/5			3/5	2/4		3/5	2/5	1/4	3/5		2/4	3/5	1/5	1/4
If PI threshold is 0.4	• {A,D} is pruned												• {C,D} is pruned						• {B,D} and {C,D} are pruned					

(a) Step 1

Step 2: Form time prevalence table

	time slot t=0	time slot t=1	time slot t=2	time slot t=3	time prevalence index
A B	1	1	1	1	4/4
A C	1	1	1	0	3/4
A D	0	0	0	1	1/4
B C	0	1	1	1	3/4
B D	0	0	1	0	1/4
C D	0	0	0	0	0

Step 3 Generate superset patterns (triplets)

	time slot t=0	time slot t=1	time slot t=2	time slot t=3
	A B C	A B C	A B C	A B C
		A.1 B.1 C.1	A.1 B.1 C.1	
		A.3 B.3 C.2	A.3 B.3 C.2	
PR		2/4 2/5 2/3	2/4 2/5 2/3	
PI		2/5	2/5	

Step 4: Find mixed-drove co-occurrence patterns

	time slot t=0	time slot t=1	time slot t=2	time slot t=3	time prevalence index
A B C	-	1	1	-	2/4

- {A, B, C} is mixed-drove co-occurrence pattern

- If time prevalence index threshold 0.5 (50%) then prune {A,D} and {B,D}
- {A,B}, {A,C}, {B,C} are mixed-drove co-occurrence patterns

(b) Steps 2, 3, and 4

Fig. 4. Execution trace of the MDCOP-Miner algorithm

Spatial prevalent patterns {A,B}, {A,C}, and {B,C} are selected as MDCOPs since they are also time prevalent (their time prevalence indices satisfy the given time prevalence index threshold 0.5). In contrast, spatial prevalent patterns {A,D}, {B,D}, and {C,D} are pruned since they are time non-prevalent. Using MDCOPs {A,B}, {A,C}, and {B,C}, the next candidate MDCOP {A,B,C} is generated. The next step is to generate instances of candidate {A,B,C} in time slots where its subsets exist and to check its participation indices in these time slots. Since all subsets of MDCOP {A,B,C} are MDCOPs and exist in time slots t=1 and t=2, there is no need to generate instances of them for time slots t=0 and t=3. In step 3 (Figure 4(b)), the instances of candidate MDCOP {A,B,C} are generated and participation indices are found which are 2/5 for time slots t=1 and t=2. In step 4 (Figure 4(b)), the time prevalence table is formed for pattern {A, B, C} and its time prevalence index is checked to see if it satisfies the time prevalence threshold. Candidate MDCOP {A, B, C} is an MDCOP since its time prevalence index 0.5 is equal to the time prevalence threshold 0.5. Since there are not enough subsets to generate the next superset patterns, the algorithm stops at this stage and outputs the union of all size MDCOPs, i.e., {A, B}, {A, C}, {B, C}, and {A, B, C}.

C. Modified MDCOP-Miner (FastMDCOP-Miner)

In this section, we propose a new algorithm, called FastMDCOP-Miner, which improves the computational efficiency of the MDCOP-Miner discussed in Section IV-B. As can be seen in Figure 3(b) and in Algorithm 1, MDCOP-Miner waits to prune time non-prevalent patterns until all size k spatial prevalent patterns are generated for all time slots and then prunes time non-prevalent patterns to discover MDCOPs. However, it is possible to optimize the MDCOP-Miner. We propose to prune time-non prevalent patterns as early as possible by moving "prune non-prevalent patterns" between the time slots shown in Figure 3(c) where candidate size 2 pattern generation is illustrated. The pseudo-code of the FastMDCOP-Miner is given in Algorithm 1. When the FastMDCOP-Miner is chosen, the algorithm will activate steps 8, 9, and 10 and deactivate steps 12 and 13. This will allow the algorithm to check the time prevalence of a pattern after every time slot is processed. The functions of the algorithm are as described in Section IV-B. In step 8, FastMDCOP-Miner checks whether the time prevalence indices of size k patterns (size 2 patterns in Figure 3(c)) satisfy the time prevalence index threshold before generating size k patterns for the next time slot. Early discovered time non-prevalent patterns are pruned in Step 9 and time prevalent patterns are used as candidate co-occurrences (Step 10) in the next time slot. For example, assume that there are 10 time slots and the time prevalence index threshold is 0.5. In this case, a size k pattern should be present for at least 5 time slots to satisfy the threshold. If the time prevalence index of a pattern is 0 for the first (or any) 6 time slots, there is no need to generate this pattern and check the prevalence of it for the rest of the time slots. Even if it is time persistent for the remaining 4 time slots, it will not be able to satisfy the given time prevalence index threshold.

An Execution Trace of FastMDCOP-Miner: The execution trace of the FastDCOP-Miner is given in Figure 5 using the dataset given in Figure 2, which has four time slots. Assume that the spatial prevalence index threshold is 0.4 and the time prevalence index threshold is 0.75. If a pattern is not consistent in more than 1 out of 4 time slots, it can be pruned whenever it is discovered. In Step 1(a) pairs and their instances are generated. Pattern $\{$A,D$\}$ is pruned at this step since it is spatial non-prevalent. Based on the outcomes of Step 1(a), the prevalence table is updated by entering a 1 for spatial prevalent patterns (Step 1(b)). The time prevalence table

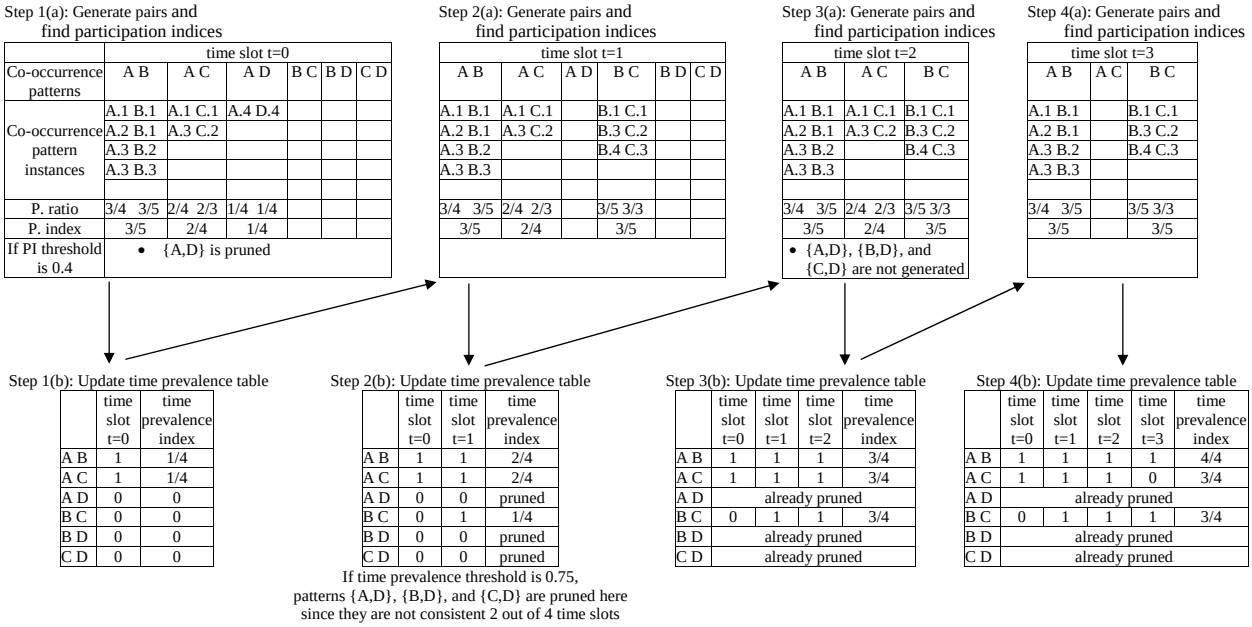


Fig. 5. Execution trace of the FastMDCOP-Miner algorithm

initially contains all possible pairs of subsets of object-types. The algorithm checks if time non-prevalent patterns can be discovered at this step. Since the result of one time slot are not enough to make a decision, instances for patterns given in the time prevalence table are generated for time slot t=2 (Step 2(a)) and the time prevalence table is updated (Step 2(b)). Patterns {A,D}, {B,D}, and {C,D} are pruned in Step 2(b), since they are not time consistent in time slot t=0 and t=1, (they will not be time prevalent even if they are spatial prevalent in the remaining time slots t=2 and t=3). These patterns are not present in 2 or more of the 4 time slots. In Step 3(a) instances are generated for patterns {A,B}, {A,C}, and {B,C} which are candidate MDCOPs and their time prevalence indices are updated (Step 3(b)). At this step no pattern is pruned since there is no possible time non-prevalent pattern. Similary, in Step 4(a) instances are generated for patterns {A,B}, {A,C}, and {B,C} which are candidate MDCOPs and their time prevalence indices are updated (Step 4(b)). At this step no pattern is pruned since there is no possible time non-prevalent pattern and algorithm outputs MDCOPs {A,B}, {A,C}, and {B,C}. Next, the algorithm continues to discover possible MDCOPs by generating candidate triple patterns.

V. Analysis of the MDCOP-Miner

This section gives the analysis of the mixed-drove prevalence index measure, and correctness and completeness derviations for the MDCOP mining algorihtms.

A. The Mixed-Drove Prevalence Index Measure is Monotonic

Lemma 5.1: Spatial prevalence measure participation index and participation ratio are monotonically non-increasing in the size of the MDCOPs at each time slot [12], [21].

Proof: The participation ratio pr is monotonically non-increasing because an instance of a spatial object-type that is contributing to a co-location c_i is also contributing to a co-location c_j where $c_j \subseteq c_i$. The spatial prevalence measure participation index pi is also monotonic because 1) participation ratio is monotonic and 2)

$$pi(c \bigcup f_{k+1}) = min_{i=1}^{k+1}\{pr(c \bigcup f_{k+1}, f_i)\}$$
$$\leq min_{i=1}^{k}\{pr(c \bigcup f_{k+1}, f_i)\} \leq min_{i=1}^{k}\{pr(c \bigcup f_i)\} = pi(c)$$ ■

Lemma 5.2: A mixed-drove prevalence index measure is monotonically non-increasing with the size of MDCOP over space and time. In other words, it is monotically non-increasing, if MDCOP P_i is a subset of MDCOP P_j and

$Prob_{t_m \in all\_time\_slot}(s\_prev(\ P_i, t_m) \geq \theta_p)$, and $Prob_{t_m \in all\_time\_slot}(s\_prev(\ P_j, t_m) \geq \theta_p)$,

where $Prob$ stands for the probability of overall prevalence time units, s_prev stands for spatial prevalence, θ_p is the spatial prevalence threshold, and t_m is the time slot.

Proof: The basic proof sketch follows. Let $TS(P_j, \theta_p) = \{t_m | pi(P_j, t_m) \geq \theta_p\}$.

Lemma 5.1 implies that participation index $pi(P_j, t_m) \geq \theta_p$ for all $t_m \in TS(P_j, \theta_p)$, since P_i is a subset of P_j. Thus, $Prob_{t_m \in all\_time\_slot}\,[s\_prev(\ P_i, t_m) \geq \theta_p)] \in \theta_{time}$, where θ_{time} is the time prevalence threshold. ■

B. Correctness and Completeness

Theorem 5.1: The FastMDCOP-Miner, MDCOP-Miner, and naive approach are complete.

Proof: The FastMDCOP-Miner, MDCOP-Miner, and naive approach are complete if they find all MDCOPs that satisfy a given participation index threshold and time prevalence threshold. We can show this by proving that none of the functions of the algorithm miss any patterns, i.e., filter out a prevalent MDCOP.

The *gen_candidate_co-occur* function does not miss any patterns given the anti-monotone nature of the MDCOP interest measure. The input of this function is size k MDCOPs and the output is candidate size $k+1$ MDCOPs. If $c_1 = \{f_1, \ldots, f_k\}$ and $c_2 = \{f_1, \ldots, f_{k-1}, f_{k+1}\}$ are size k MDCOPs, candidate size $k+1$ pattern $C_{k+1} = \{f_1, \ldots, f_{k-1}, f_k, f_{k+1}\}$ will be produced by joining size k MDCOPs.

The *gen_co-occur_instanance* function does not miss any patterns. This function generates instances of candidate size $k+1$ MDCOPs by joining instances of size k MDCOPs if they are in the neighborhood distance and forming a clique.

The *find_spatial-prevalent_co-oc* function does not miss any patterns. It finds spatial prevalent patterns whose participation indices satisfy a given threshold.

The *find_time_prevalence_index* function does not miss any patterns. This function calculates time prevalence indices of the patterns found in steps 4 through 8 and does not do any pruning.

The *find_time-prevalent_co-occur* function does not miss any MDCOPs. The function finds all the MDCOPs whose time prevalence indices are no less than a given threshold. ■

Theorem 5.2: The FastMDCOP-Miner, MDCOP-Miner, and naive approach are correct. In other words, if a MDCOP pattern P is returned by MDCOP-Miner and FastMDCOP-Miner algorithms, then P is a prevalent MDCOP.

Proof: The proof is easy to establish due to the pruning steps of *find_spatial_prevalent_co-occur*, and *find_time_prevalent_co-occur*, which weed out candidates not meeting the given thresholds. ■

C. Algebraic Cost Model

In this section, we give the algebraic cost models of the MDCOP-Miner and the FastMDCOP-Miner algorithms. The cost model of the naive approach is not given since it is the worst case of the MDCOP-Miner and FastMDCOP-Miner and applies the pruning strategy in a post processing step. Let T_{MDCOP}, and T_{Fast} represent the total computational costs of the MDCOP-Miner and the FastMDCOP-Miner respectively. The total respective cost functions will be

$$T_{MDCOP} = \sum_{k>1} T_{MDCOP}(k, \theta_p, \theta_{time}, TF, S_{instance})$$
$$T_{Fast} = \sum_{k>1} T_{Fast}(k, \theta_p, \theta_{time}, TF, S_{instance}) \quad (3)$$

where $T_{MDCOP}(k, \theta_p, \theta_{time}, TF, S_{instance})$ and $T_{Fast}(k, \theta_p, \theta_{time}, TF, S_{instance})$ represent the generation of the total cost of size k ($k > 1$) MDCOPs for parameters θ_p and θ_{time}, time slots TF, and the average number of co-occurrence instances $S_{instance}$.

$$\begin{aligned} T_{MDCOP}(k, \theta_p, \theta_{time}, TF, S_{instance}) &= T_{gen\_candi}(MDP_{k-1}^{MDCOP}, TF, S_{instance}) \\ &+ T_{prune\_sp\_co-occ}(C\_SP_k^{MDCOP}, \theta_p, TF, S_{instance}) \\ &+ T_{prune\_time\_co-occ}(SP_k^{MDCOP}, \theta_{time}, TF, S_{instance}) \\ \approx T_{gen\_candi}(MDP_{k-1}^{MDCOP}, TF, S_{instance}) &+ T_{prune\_sp\_co-occ}(C\_SP_k^{MDCOP}, \theta_p, TF, S_{instance}) \end{aligned}$$

$$\begin{aligned} T_{Fast}(k, \theta_p, \theta_{time}, TF, S_{instance}) &= T_{gen\_candi}(MDP_{k-1}^{Fast}, TF, S_{instance}) \\ &+ T_{prune\_sp\_co-occ}(C\_SP_k^{Fast}, \theta_p, TF, S_{instance}) \\ &+ T_{prune\_time\_co-occ}(SP_k^{Fast}, \theta_{time}, TF, S_{instance}) \\ \approx T_{gen\_candi}(MDP_{k-1}^{Fast}, TF, S_{instance}) &+ T_{prune\_sp\_co-occ}(C\_SP_k^{Fast}, \theta_p, TF, S_{instance}) \quad (4) \end{aligned}$$

MDP_{k-1} is the size $k-1$ MDCOP sets. T_{gen_candi} represents the total cost of generating candidate MDCOPs for all time slots TF. C_SP_k represents the number of candidate size k MDCOPs. $T_{prune\_sp\_co-occ}$ represents the total cost of pruning candidate MDCOPs. SP_k represents the number of size k spatial co-location patterns of MDCOP mining algorithms. $T_{prune\_time\_co-occ}$ represents the total cost of pruning time non-prevalent patterns. MDCOP-Miner will generate all size k spatial prevalent patterns and then it will prune size k time non-prevalent patterns. In contrast, FastMDCOP-Miner will apply time non-prevalent pattern pruning as early as possible to eliminate generating instances of some size k time non-prevalent patterns. The cost of generating size k patterns of FastMDCOP-Miner will be no more than that of MDCOP-Miner. The FastMDCOP-Miner approach will run steps for the number of time slots in the dataset to find all size k MDCOPs. The cost of this process is negligible since it only checks the count of the

patterns whenever a new pattern is processed. If there is no pattern to be pruned early, the cost of the both approaches will be same. As a result, the cost of MDP_{k-1}^{Fast} will be no more than that of MDP_{k-1}^{MDCOP} and the cosf of $C\_SP_2^{Fast}$ will be no more then that of $C\_SP_k^{MDCOP}$.

Lemma 5.3: The total cost of time based pruning is negligible with respect to total cost of spatial prevalence based pruning, such that,

$$T_{MDCOPorFast} = T_{gen\_candi} + T_{prune\_sp\_co-occ} + T_{prune\_time\_co-occ} \approx T_{gen\_candi} + T_{prune\_sp\_co-occ} \quad (5)$$

Proof: The total cost $T_{gen\_candi} + T_{prune\_sp\_co-occ}$ includes the cost of generating all candidate patterns and their instances and the cost of calculating the participation index and ratio values of the generated patterns and pruning the non-prevalent patterns, respectively. This process is computationally complex due to the spatial join operation to generate the candidate patterns and their instances in each time slot. That is, the bulk of time is consumed in generating instances and pruning spatial non-prevalent MDCOPs.

In contrast, the total cost of $T_{prune\_time\_co-occ}$ includes the cost of calculating the time prevalence indices of the patterns for all time slots. This process is computationally very cheap due to the count of the existence of the patterns in whole dataset. ■

Lemma 5.4: The total cost of generating candidate MDCOPs of the proposed FastMDCOP-Miner is no more than the total cost of generating MDCOPs of the MDCOP-Miner, assuming the cost of the time prevalence based pruning is negligible (Lemma 5.3), such that,

$$T_{gen\_candi}(MDP_{k-1}^{Fast}, TF, S_{instance}) \leq T_{gen\_candi}(MDP_{k-1}^{MDCOP}, TF, S_{instance}) \quad (6)$$

Proof: Due to the early pruning of irrelevant candidates (steps 8 and 9 of Algorithm 1), the number of sets of size k MDCOPs generated by the FastMDCOP-Miner will be no more than that of the MDCOP-Miner size k patterns, e.g., $MDP_{k-1}^{Fast} \leq MDP_{k-1}^{MDCOP}$, since size k patterns of the FastMDCOP-Miner will prune time non-prevalent patterns earlier than MDCOP-Miner.

The other parameters affecting equation 6 are the number of time slots TF and average number of co-occurrence instances $S_{instance}$. If the number of time slots increases, the cost of the MDCOP-Miner will increase due to the increasing unnecessary generation of time non-prevalent patterns. Similarly, for greater average numbers of co-occurrence instances, the cost of the MDCOP-Miner increases. In the worst case, the number of sets of size k patterns generated

will be equal for both algorithms, if there is no early time prevalence based pruning. In that case, equation 6 will be true. ■

Lemma 5.5: The total cost of pruning the spatial non-prevalent candidate patterns of the FastMDCOP-Miner will be no more than that of MDCOP-Miner, assuming the cost of the time prevalence based pruning is negligible (Lemma 5.3), such that

$$T_{prune\_sp\_co-occ}(C\_SP_k^{Fast}, \theta_p, TF, S_{instance}) \leq T_{prune\_sp\_co-occ}(C\_SP_k^{MDCOP}, \theta_p, TF, S_{instance}) \tag{7}$$

Proof: Due to early pruning of irrelevant candidates (step 6 of Algorithm 1), the number of size $k+1$ candidate pattern sets generated by the FastMDCOP-Miner will be no more than that of the MDCOP-Miner e.g., $C\_SP_k^{Fast} \leq C\_SP_k^{MDCOP}$, and so equation 7 will be true. ■

Lemma 5.6: The total cost of pruning time non-prevalent candidate MDCOPs of the proposed FastMDCOP-Miner algorithm is no more than than the total cost of pruning time non-prevalent candidate patterns of the MDCOP-Miner, assuming the cost of the time prevalence based pruning is negligible (Lemma 5.3), such that

$$T_{prune\_time\_co-occ}(SP_k^{Fast}, \theta_{time}, TF, S_{instance}) \leq T_{NA}^{post}(SP_k^{MDCOP}, \theta_{time}, TF, S_{instance}) \tag{8}$$

Proof: Due to early pruning of irrelevant candidates (step 10 of Algorithm 1), the number of size k spatial prevalent patterns generated by the FastMDCOP-Miner will be no more than that of the MDCOP-Miner, e.g., $SP_k^{Fast} \leq SP_k^{MDCOP}$ and so equation 8 will be true. ■

Theorem 5.3: The total cost of the FastMDCOP-Miner is no more than the total cost of the MDCOP-Miner assuming the cost of the time prevalence based pruning is negligible.

Proof: Based on Lemmas Lemma 5.3, 5.4, 5.5, and 5.6 the total cost of the FastMDCOP-Miner algorithm will be no more than the total cost of the MDCOP-Miner. ■

VI. Experimental Evaluation

In this section, we present our experimental evaluations of several design decisions and workload parameters on our MDCOP mining algorithms. We used a real-world training dataset and synthetic datasets. We evaluated the behavior of the FastMDCOP-Miner, MDCOP-Miner and naïve approach to answer the following questions:

- What is the effect of the number of timeslots?
- What is the effect of the number of object-types?
- What is the effect of the spatial prevalence index threshold?
- What is the effect of the time prevalence index threshold?
- What is the effect of the number of noise instances?
- What is the effect of the average number of instances?

Figure 6 shows the experimental setup to evaluate the impact of design decisions on the performance on the three algorithms. Experiments were conducted on an IBM Netfinity Linux Cluster, 2.6 GHz Intel Pentium 4 with 1.5 GB of RAM.

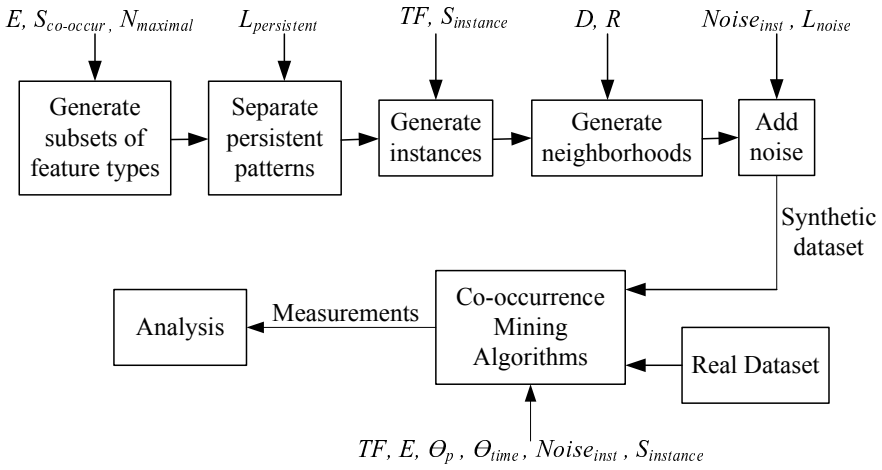


Fig. 6. Experimental setup and design

A. Datasets

1) Real Dataset: The real dataset contains the location and time information of moving objects. It includes 15 time snapshots and 22 distinct vehicle types and their instances. The minimum instance number is 2, the maximum instance number is 78, and the average number of instances is 19.

2) Synthetic Dataset Generation: To evaluate the performance of the algorithms, spatio-temporal datasets were generated based on the spatial data generator proposed by Huang et. al. [12]. Synthetic datasets were generated for spatial frame size DxD (the first part of Figure 6).

For simplicity, the datasets were divided into regular grids whose side lengths had neighborhood relationship R. First, subsets of object-types were generated using the parameters average co-occurrence size $S_{co-occur}$ and number of maximal patterns $N_{maximal}$. Object-types and sizes of each pattern were chosen randomly. The generated patterns were then divided into two categories - persistent patterns and transitory patterns - using the persistent pattern ratio $L_{persistent}$. Persistent patterns are ones whose time prevalences are strong over time, while transitory patterns are ones whose spatial prevalences are strong at a specific time slot. Next, instances of the patterns were generated based on the average number of co-occurrence instances $S_{instance}$. Instances were chosen at randomly located grid cells. This process was applied for each time slot TF. Finally, using the parameters number of noise instances $Noise_{inst}$ and ratio of noise objects over number of features L_{noise}, noise object and their instances were generated and added to the dataset.

The parameters of the synthetic dataset generator and their definitions are listed in Table I.

TABLE I

SYNTHETIC DATASET GENERATION PARAMETERS

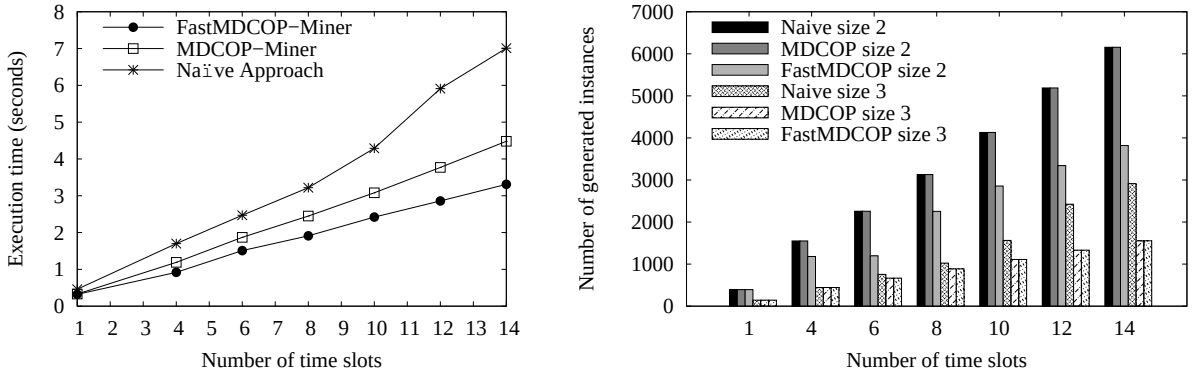
Parameter	Definition	Experiments					
		Syn-1	Syn-2	Syn-3	Syn-4	Syn-5	Syn-6
E	Number of object types	200	100-400	200			
$S_{instance}$	Average number of co-occurrence instances*	10					5-20
$N_{maximal}$	Number of maximal co-occurrence patterns*	10					
$S_{co-occur}$	Minimum co-occurrence pattern size*	4					
$L_{persistent}$	Ratio of persistent patterns over transitory patterns*	0.5	0.8	0.5	0.8		
L_{noise}	Ratio of noise object-types over number of object-types	0.25					
$Noise_{inst}$	Number of noise instances	1000				1000-5000	1000
TF	Number of timeslots	10-50	20	50		20	
D	Spatial framework size (DXD)	10^6	10^3	10^6		10^3	
R	Spatial Neighborhood relationship	10					

* : For initial co-occurrence patterns

B. Experiment Results for Real Datasets

1) Effect of Number of Time Slots: In the first experiment, we evaluated the effect of the number of time slots on the execution time of the MDCOP algorithms using the real dataset. The participation index, time prevalence index, and distance were set at 0.2, 0.8, and 150m respectively. Experiments were run for a minimum of 1 time slot and a maximum of 14 time

slots. As can be seen in Figure 7(a), the FastMDCOP-Miner requires less execution time than the MDCOP-Miner and naïve approaches, since it prunes out time non-prevalent MDCOPs as early as possible. It can also be seen that as the number of time slots increases, the ratio of the increase in execution time is smaller for FastMDCOP-Miner than for the MDCOP-Miner and naïve approaches. Figure 7(b) shows the number of generated size 2 and size 3 instances for algorithms. The FastMDCOP-Miner generates fewer patterns due to its early pruning of time non-prevalent patterns. The MDCOP-Miner and naïve approaches generate the same number of size 2 instances. The MDCOP-Miner applies time pruning after it generates all possible size 2 patterns and the naïve approach applies time pruning in the post-processing step.



(a) Execution time of MDCOP algorithms

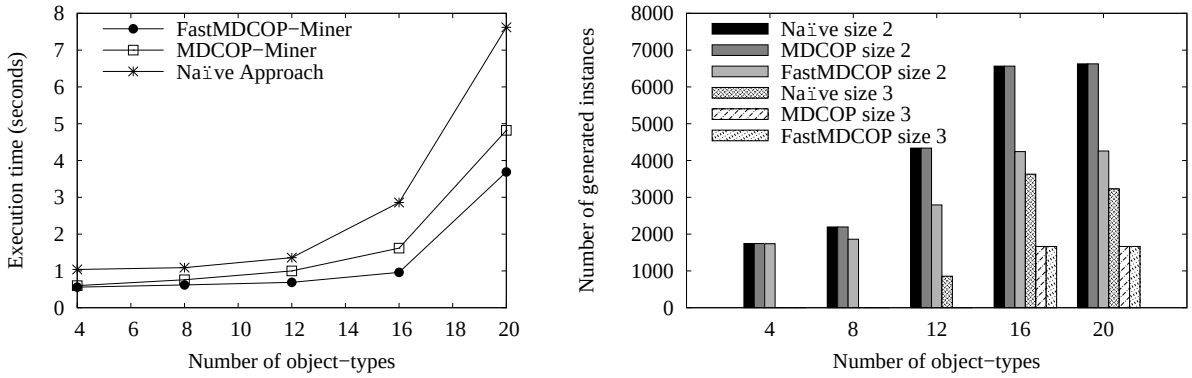
(b) Number of generated instances

Fig. 7. Effect of number of time slots in MDCOP mining algorithms using real dataset

As discussed in Section V-C, the number of time slots TF is one of the parameters that affects the cost of the algorithms. As predicted in equation 3, the cost of the algorithms increases as the number of time slots increases and the FastMDCOP-Miner outperforms the other approaches due to the early pruning strategy (Theorem 5.3). The FastMDCOP-Miner algorithm examines fewer instances than the other approaches since it deals with the MDCOP prevalent patterns as early as possible (Lemmas 5.4, 5.5, and 5.6).

2) Effect of Number of Object-types: In the second experiment, we evaluated the effect of the number of object-types on the execution time of the algorithms using the real dataset. The participation index, time prevalence index, number of time slots and distance were set at 0.2,

0.8, 15, and 150m respectively. The FastMDCOP-Miner outperforms the other approaches as the number of object-types increases (Figure 8(a)-(b)). It is observed that the increase in execution time for the naïve approach is bigger than that of the MDCOP-Miner and the FastMDCOP-Miner as the number of object-types increases for datasets.



(a) Execution time of the algorithms

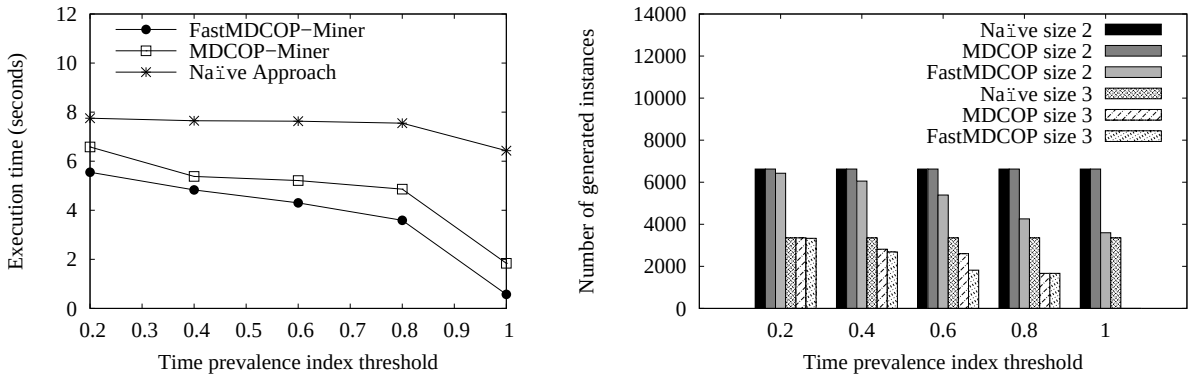
(b) Number of generated instances

Fig. 8. Effect of number of object-types in MDCOP mining algorithms using real dataset

The trends are consistent with the algebraic cost models given in equation 3. The cost of the algorithms increases as the number of the object-types increases due to the increase in the number of join operations. The MDCOP-Miner and naïve approaches generate the same number of size 2 instances. MDCOP-Miner applies time pruning after it generates all possible size 2 patterns and naïve approach applies time pruning in the post-processing step (Figure 8(b)). In contrast, The FastMDCOP-Miner generates patterns by pruning non-prevalent patterns as early as possible (Lemmas 5.4, 5.5, and 5.6). Because of the early pruning strategy of FastMDCOP-Miner, its cost is no more than that of the MDCOP-Miner and naïve approaches as shown in Figure 8(a) (Theorem 5.3).

3) Effect of the Time Prevalence Index Threshold: In the third experiment, we evaluated the effect of the time prevalence index threshold on the execution times of the MDCOP mining algorithms for the real dataset. The fixed parameters were participation index, number of time slots, and distance, and their values were 0.2, 15, and 150m respectively. For the naïve approach, the effective cost in execution time to generate spatial prevalent co-locations will be constant since

it generates the same number of spatial prevalent patterns as the time prevalence index increases. In that case, the cost of the post-processing step will reflect the trend of the naïve approach. Experimental results show that the FastMDCOP-Miner is more computationally efficient than the other approaches because of the early pruning strategy (Figure 9(a)). The execution times of the FastMDCOP-Miner and MDCOP-Miner decrease as the time prevalence index threshold increases. It is also observed that the naïve approach is computationally more expensive as the time prevalence index threshold decreases because of the increase in the number of MDCOPs to be discovered.



(a) Execution time of the algorithms

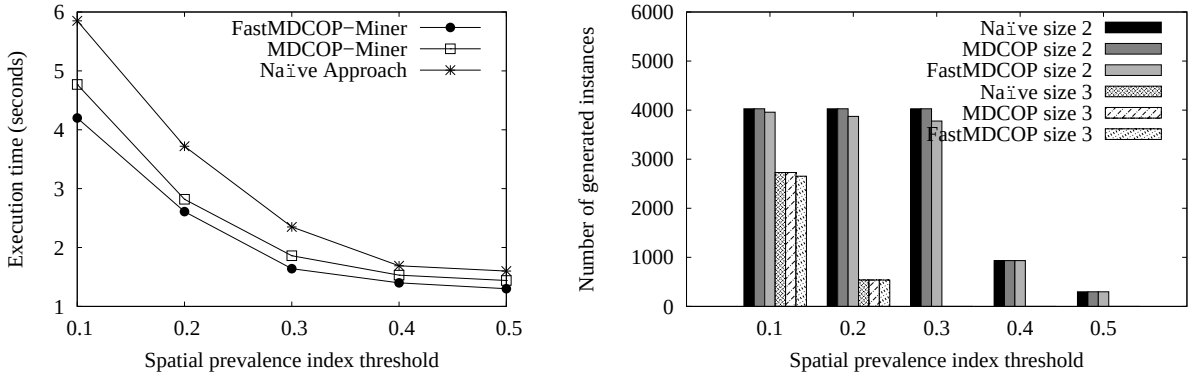
(b) Number of generated instances

Fig. 9. Effect of the time prevalence index threshold in MDCOP mining algorithms using real dataset

The trends are consistent with the algebraic cost models given in equation 3. The cost of FastMDCOP-Miner is no more than that of MDCOP-Miner (Lemma 5.6 and Theorem 5.3). The naïve approach is not sensitive to the time prevalence index threshold (Lemma 5.6). Without the post-processing step, the cost of the naïve approach is constant. The trend of the naïve approach in Figure 9 is characterized by the cost of the post-processing step.

4) Effect of the Spatial Prevalence Index Threshold: In the fourth experiment, we evaluated the effect of the spatial prevalence index threshold on the execution times of the MDCOP algorithms. The fixed parameters were time prevalence index, number of time slots, and distance, with values of 0.2, 15, and 100m respectively. Figure 10(a) shows the execution times of the algorithms and Figure 10(b) shows the number of generated size 2 and 3 instances for the

algorithms. FastMDCOP-Miner and MDCOP-Miner do not generate more than size 3 instances for a spatial prevalence index threshold of greater than 0.2. The FastMDCOP-Miner outperforms the MDCOP-Miner and naïve approaches as the spatial prevalence index threshold increases (Figure 10(a)-(b)). The cost of the naïve approach will be higher than that of the FastMDCOP-Miner and MDCOP-Miner for low values of the spatial prevalence index threshold.



(a) Execution time of the algorithms

(b) Number of generated instances

Fig. 10. Effect of the spatial prevalence index threshold in MDCOP mining algorithms using real dataset

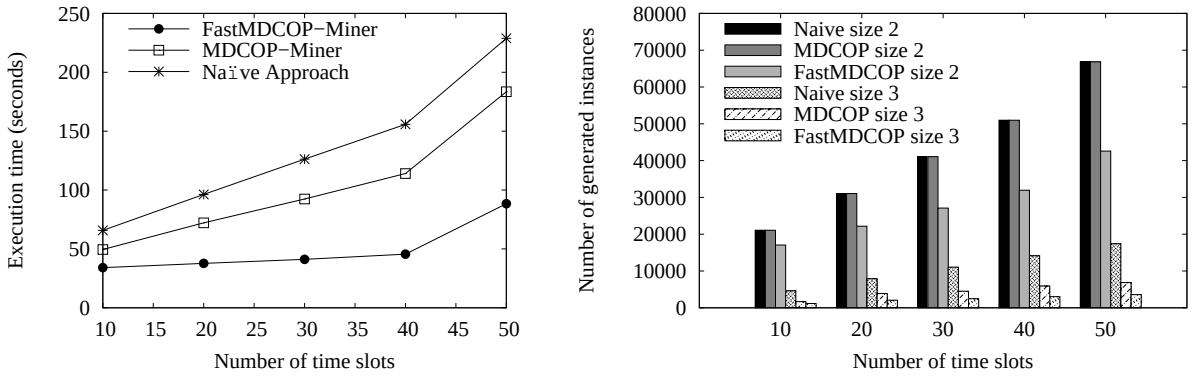
The trends are consistent with the algebraic cost models given in equation 3. The algorithms are sensitive to the spatial prevalence index threshold (Lemma 5.5).

C. Experiment Results for Synthetic Datasets

1) Effect of Number of Time slots: In this experiment, we evaluated the effect of the number of time slots on the execution time of the algorithms using synthetic datasets. To generate the datasets, we used a framework size of $10^6 x 10^6$, a square proximity neighborhood size of 10 x 10, a noise feature ratio of 0.25, a noise instance number of 1000, an average number of co-occurrence instances of 10, and a maximal co-occurrence pattern number of 10 (Table I, column syn-1). In the experiments, the participation index, time prevalence index, and distance were set at 0.3, 0.9, and 10m respectively. Experiments were for a minimum of 10 time slots and a maximum of 50 time slots. The results showed that the FastMDCOP-Miner requires less execution time than the other approaches, since it prunes out time non-prevalent MDCOPs as early as possible

(Figure 11(a)). The generated size 2 and size 3 instances are given in Figure 11(b). The naive approach generated up to size 7 spatial prevalent subsets before the post-processing step. In contrast, FastMDCOP-Miner and MDCOP-Miner generated up to size 4 subsets.

The trends are consistent with the algebraic cost models given in equation 3. The cost of the algorithms increases as the number of time slots increases and the FastMDCOP-Miner outperforms the other approaches due to its early pruning strategy ((Theorem 5.3). As can be seen in Figure 11(b), the FastMDCOP-Miner algorithm examines fewer size 2 and 3 instances than the other approaches due to the early pruning strategy (Lemmas 5.4, 5.5, and 5.6).



(a) Execution time of the algorithms

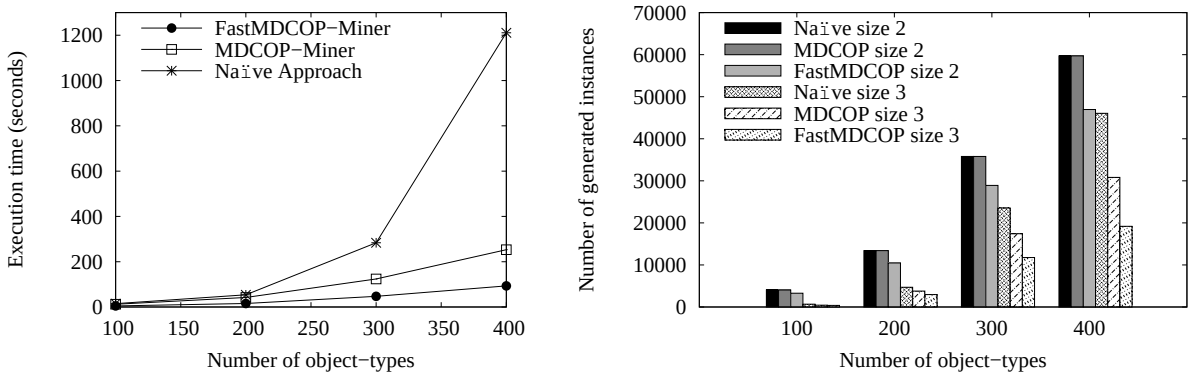
(b) Number of generated instances

Fig. 11. Effect of number of time slots in MDCOP mining algorithms using synthetic dataset

2) Effect of Number of Object-types: We evaluated the effect of the number of object-types on the execution time of the algorithms for synthetic datasets. The parameters used to generate the datasets are given in Table I, column syn-2. The participation index, time prevalence index, number of time slots, and distance were set at 0.3, 0.8, 20, and 10m respectively. The FastMDCOP-Miner outperforms the MDCOP-Miner and naive approaches as the number of object-types increases (Figure 12(a)-(b)). The ratio of the increase in the execution time of the naïve approach is greater than that of the MDCOP-Miner and FastMDCOP-Miner as the number of object-types increases. Figure 12(b) shows the number of generated size 2 and 3 instances for the algorithms.

The trends are consistent with the algebraic cost models given in equation 3. The cost of

the algorithms increases as the number of the object-types increases due to the increase in the number of join operations.



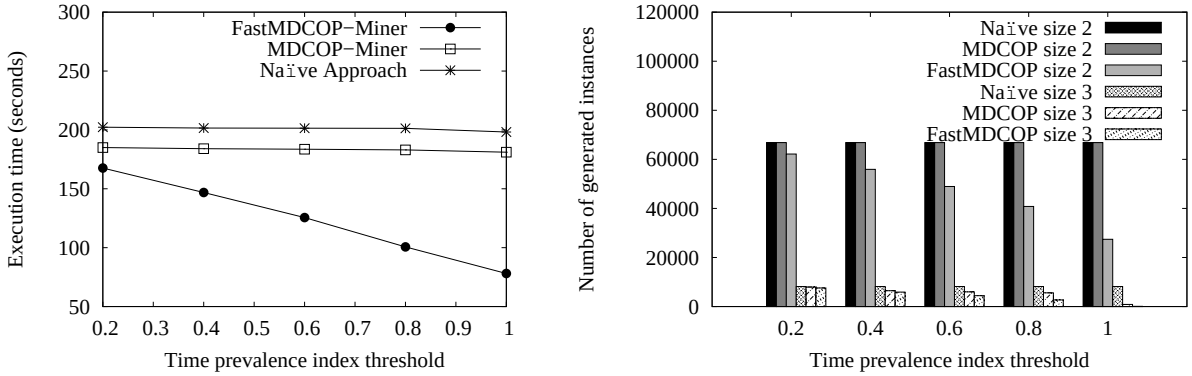
(a) Execution time of the algorithms

(b) Number of generated instances

Fig. 12. Effect of number of object-types in MDCOP mining algorithms using synthetic dataset

3) Effect of the Time Prevalence Index Threshold: We evaluated the effect of the time prevalence index threshold on the execution times of the algorithms for synthetic datasets. The parameters used to generate the datasets are given in Table I column syn-3. The fixed parameters were participation index, distance, and number of time slots, and their values were 0.4, 10m, and 50 respectively. Experimental results show that the FastMDCOP-Miner is more computationally efficient than the MDCOP-miner and naïve approaches because of the early pruning strategy (Figure 13(a)). The execution time of the FastMDCOP-Miner decreases as the time prevalence index threshold increases. The execution time of the naïve approach is almost constant since it does not prune time non-prevalent pattern before the post-processing step and it is computationally more expensive as the time prevalence index threshold decreases because of the increase in the number of MDCOPs to be generated (Figure 13(b)).

The naïve approach is not sensitive to the time prevalence index threshold (Lemma 5.6) which causes the increase of its cost. Without the post-processing step, the cost of the naïve approach is constant. The trend of the naïve approach in Figure 9 is characterized by the cost of the post-processing step (Figure 13(a)).



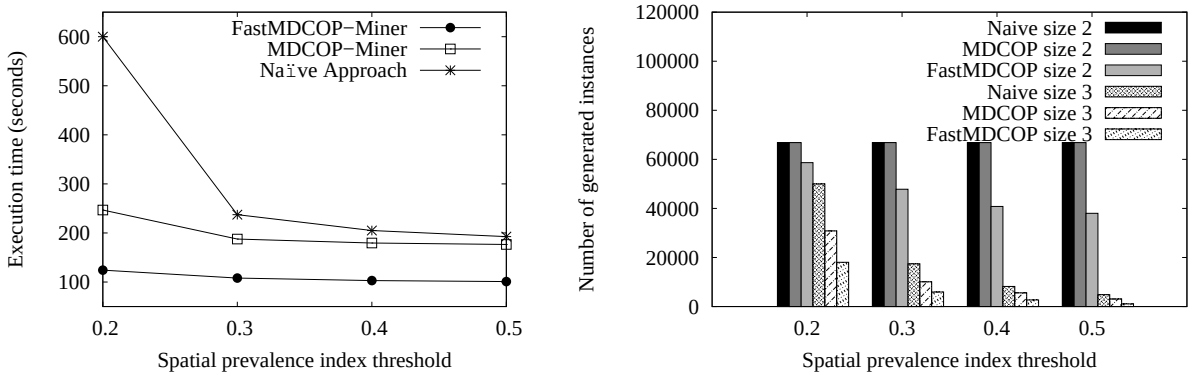
(a) Execution time of the algorithms

(b) Number of generated instances

Fig. 13. Effect of the time prevalence index threshold in MDCOP mining algorithms using synthetic dataset

4) Effect of the Spatial Prevalence Index Threshold: We evaluated the effect of the spatial prevalence index threshold on the execution times of MDCOP mining algorithms. To generate the dataset, we used a spatial framework size of $10^6 x 10^6$, a square proximity neighborhood size of 10 x 10, an average number of co-occurrence instances of 10, a noise feature ratio of 0.25, a noise instance number of 1000, a maximal co-occurrence pattern number of 10, and a time slot number of 50 (Table I, column syn-4). In the experiments, the value of the time prevalence index was 0.8. The FastMDCOP-Miner outperforms the other approaches (Figure 14(a)-(b)). The cost of the naïve approach will be higher than the MDCOP-Miner and FastMDCOP-Miner for low values of the spatial prevalence index threshold since the naïve approach tends to generate MDCOP non-prevalent patterns. For high values of the spatial prevalence index the cost of the algorithms are closer. The generated size 2 and size 3 instances are given in Figure 14(b). The naive approach generates up to size 8 spatial prevalent subsets for spatial prevalence thresholds 0.2 and 0.3, before the post-processing step. In contrast, the FastMDCOP-Miner and MDCOP-Miner generate up to size 4 MDCOP prevalent subsets.

The trends are consistent with the algebraic cost models given in equation 3. The algorithms are sensitive to the spatial prevalence index threshold (Lemma 5.5) but due to the early pruning strategy of FastMDCOP-Miner, its cost is no more than that of the MDCOP-Miner and naïve approach, as shown in Figure 14(a) (Lemma 5.5 and Theorem 5.3).



(a) Execution time of the algorithms

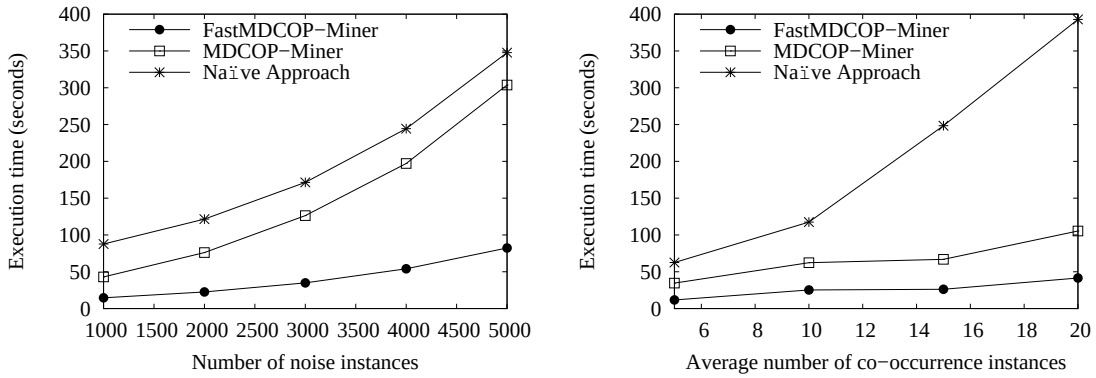
(b) Number of generated instances

Fig. 14. Effect of the spatial prevalence index threshold in MDCOP mining algorithms using synthetic dataset

5) Effect of the Number of Noise Instances: We evaluated the effect of the number of noise instances on the execution times of the MDCOP mining algorithms using the synthetic datasets. The parameters for data generation are listed in column syn-5 of Table I. The time prevalence index threshold, the spatial prevalence index threshold, and distance were 0.3, 0.8, and 10m respectively. The FastMDCOP-Miner is more robust than the MDCOP-Miner and naïve approaches as the number of noise features increases (Figure 15(a)).

The trends are consistent with the algebraic cost models given in equation 3. The FastMDCOP-Miner is more robust than that of the other approaches (Lemmas 5.6 and 5.5 and Theorem 5.3). In other words, the naïve approach is more sensitive to the noise features, which causes both its cost and the number of generated instances to increase (Figure 15(a)).

6) Effect of the Average Number of Instances: We evaluated the average number of total instances on the execution times of the MDCOP mining algorithms using synthetic datasets. The parameters for data generation are listed in column syn-6 of Table I. The time prevalence index threshold, the spatial prevalence index threshold, and distance were 0.3, 0.8, and 10m respectively. The FastMDCOP-Miner algorithm outperformed the other approaches as the average number of total instances increases (Figure 15(b)).



(a) Effect of noise features

(b) Effect of number of instances

Fig. 15. Effect of noise and average number of instances in MDCOP mining algorithms using synthetic dataset

VII. Conclusions and Future Work

We defined mixed-drove spatio-temporal co-occurrence patterns (MDCOPs) and the MDCOP mining problem and proposed a new monotonic composite interest measure which is the composition of distinct object-types, spatial prevalence measures, and time prevalence measures. We presented a novel and computationally efficient algorithm (the MDCOP-Miner) for mining these patterns. We also presented an improved MDCOP-Miner algorithm (the FastMDCOP-Miner) which prunes time non-prevalent patterns at an early stage and offers even greater computational efficiency than the MDCOP-Miner algorithm. We compared our algorithms with a naïve approach, which runs the spatial co-location mining algorithm at each time slot and then discovers MDCOPs using a post-processing step. We proved that the proposed algorithms are correct and complete in finding mixed-drove prevalent (i.e., spatial prevalent and time prevalent) MDCOPs. Our experimental results using a real and synthetic datasets provide further evidence of the viability of our approach.

For future work, we would like to explore the relationship between the proposed MDCOP interest measures and spatio-temporal statistical measures of interaction [3]. Another problem of interest is the characterization of the probability distribution of the proposed interest measure to help in making the choice of thresholds in the proposed measures. We plan to explore other potential interest measures for MDCOPs by evaluating similarity measures for tracks of moving

objects. We plan to investigate new monotonic composite interest measures and develop other new computationally efficient algorithms for mining MDCOPs.

In the literature, there are also other studies that have focused on defining spatio-temporal patterns and algorithms [9], [11], [13], [15], [18], [25]. Laube et al. defined several spatio-temporal patterns, such as leadership and convergence [16]. Query processing algorithms have been proposed to extract such patterns [16]. We hope to extend our algorithm to mine these patterns.

VIII. Acknowledgements

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