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Wireless Sensor Networks with Energy Efficient Organization

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Abstract

A critical aspect of applications with wireless sensor networks is network lifetime. Battery-powered sensors are usable as long as they can communicate captured data to a processing node. Sensing and communications consume energy, therefore judicious power management and scheduling can effectively extend the operational time. One important class of wireless sensor applications consists of deployment of large number of sensors in an area for environmental monitoring. The data collected by the sensors is sent to a central node for processing. In this paper we propose an efficient method to achieve energy savings by organizing the sensor nodes into a maximum number of disjoint dominating sets (DDS) which are activated successively. Only the sensors from the active set are responsible for monitoring the target area and for disseminating the collected data. All other nodes are into a sleep mode, characterized by a low energy consumption. We define the maximum disjoint dominating sets problem and we design a heuristic that computes the sets. Theoretical analysis and performance evaluation results are presented to verify our approach.

Key Words: wireless sensor networks, energy efficiency, node organization, disjoint dominating sets.

1 Introduction

Recent improvements in affordability and efficiency of electronic devices have had a considerable impact on development of radio-frequency wireless communication equipment. Thus, it became feasible to build small sensors with wireless communication capabilities. New

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applications for surveillance, environmental control and defense are possible by deploying a large number of sensors in a target area and processing the information gathered from them (e.g. video, acoustic, seismic). An important issue with sensor networks is power limitation, which could significantly affect the performance of the application. In most cases the sensors are battery powered, and they can stay active for a limited time before the battery resources are depleted. For sensor network applications, a significant part of the system design deals with improving the network lifetime. Wireless communications is a main area where power savings can be achieved through judicious allocation of resources, power-aware network design and transmission scheduling methods.

In this paper we present a method for improving the lifetime of a wireless sensor network for a broad class of applications. We consider that a large number of sensors dispersed in an area must send the information they capture to a central processing node. In one scenario a large number of sensors could be randomly dispersed from an aircraft. Once they are deployed, they activate their sensing hardware and start generating data that must be sent to a collecting node that could be on an unmanned aircraft.

We assume that all sensors are dispersed inside the area to be monitored and that every sensor is able to monitor a disc area around its location and all have the same sensing and communication capabilities. Since the sensors cannot be placed at precise positions, in order to improve sensing coverage, applications usually deploy more than the optimal number of nodes.

One method to reduce the energy consumption is to organize the sensor nodes into disjoint sets, with only one set performing environmental monitoring at any moment. Scheduling and grouping of sensors into disjoint sets is done by the central controller node, which informs every sensor of the time intervals to be activated.

Generally, all sensor nodes which are part of the active set are in the *active* state, whereas all other nodes are into a low energy, *sleep* state, where the CPU is in a low power mode and transmission and reception are disabled. The ratio of energy consumed between the active state (i.e., when the CPU operates at full energy) and the sleep state is generally on the order of 100 or more (see [3]). These sensor sets are activated in turn for the same time interval, into a round-robin fashion, as long as the network has enough power to operate. Our goal is to maximize the number of disjoint sets, so that the time between two activations for any sensor is longer. Thus the network lifetime is extended and the spatial density of active nodes is decreased.

A relevant result to our approach is that batteries have approximately twice as long a lifetime if they are discharged in short bursts with significant off-time as compared to a continuous mode of operation (see [2]). Therefore, a mode of operation that alternates active and inactive battery states extends network lifetime.

We design every set as a dominating set such that every network node is either part of the set or has a neighbor, called a dominator, in its communication range which belongs to the set. By designing the sets as dominating sets, minimum lapses of coverage in the sensed area may occur. Therefore this solution trades off a complete area monitoring in exchange for prolonging the network operational time. This paper contributes with the definition of disjoint dominating sets problem (DDS), hardness and a lower-bound results and proposes an efficient heuristic.

Recent literature addresses the topic of energy efficiency in wireless networks from different perspectives. An effective way to conserve energy is to schedule apriori the wireless node transmissions, allowing them to enter a low state energy while they are inactive. This idea is explored in [3], where authors study the communication from a base station to a large number of wireless nodes. Three access protocols are designed, considering two important factors: low delay and low energy requirements. These protocols propose a transmission scheduling strategy at the base station as well as a wake-up schedule at each node. In the grouped-tag TDMA protocols, the nodes are divided in groups and each group is assigned a TDMA slot for communication with the base station. In directory protocols, the base station broadcasts a directory which lists the destinations, permitting nodes to schedule their wake-up slots to coincide with the broadcast of their packets. In the pseudorandom protocols, the base station knows when every node is awake, based on sharing the seed for a random number generator, and thus knows when to send packets to specific destinations. In [10], the authors perform a comprehensive study of the problem of scheduling the communication between the central controller and other wireless nodes, with focus on energy conservation. The paper contributes three directory protocols that may be used by the central node to coordinate data transmissions considering multiple factors such as traffic-type (e.g. downlink, uplink, peer-to-peer) and the effects of packets errors.

In [7], the authors proposed a new multiaccess protocol, PAMAS, based on MACA [6], with the addition of a separate signaling channel. PAMAS achieves energy savings by powering off the nodes which are not actively transmitting or receiving packets. Their work is continued in [8] at the network layer, by introducing a power aware route selection strategy, using new power-aware metrics, such as energy consumed/packet and the time to network partition. A lowest cost routing algorithm may use one of these metrics when energy resources are critical, increasing the time to node failure.

In our previous work [1], we studied a method that increases wireless network lifetime by minimizing the length of the longest edge in the interconnecting tree through the deployment of additional relay nodes. We modeled this problem as the Bottleneck Steiner Tree problem, and designed a performance $\sqrt{3}+\epsilon$ approximation algorithm which constructs the solution as a minimum spanning tree over a weighted 3-hypergraph.

Energy efficiency is an important issue in wireless sensor networks design. In [12], the authors design a residual energy scan which approximately determines the remaining energy distribution within a sensor network. It uses localized algorithms which are then aggregated to build a composite scan. This tool may be used to inform users regarding the low power regions, who may decide to deploy new sensors in that area or to invoke some other power-aware strategies. This paper also points out the high energy cost of communication compared to computation. Therefore, if the application and infrastructure permit, energy savings are obtained by processing data locally, rather than transmitting it over long distances.

In [9], the authors propose an energy conservation technique for wireless sensor networks that works by selecting and successively activating mutually exclusive sets of sensor nodes, where every set completely covers the entire monitored area. Their method achieves energy savings by increasing the number of disjoint covers. The authors propose a heuristic solution to this problem.

Compared to the work in [9], the maximum number of disjoint dominating sets is greater or equal than the maximum number of covers. This is valid because the sensors in one cover also form a dominating set, which supplementary needs to satisfy the full area coverage constraint. Therefore, our approach potentially achieves better energy savings at the cost of small coverage lapses in the monitored area.

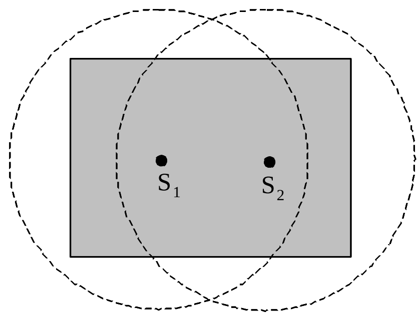


Figure 1: *Two sensors (S_1 and S_2) deployed in the target area*

Let us consider a simple example, as in *Figure 1*. In this case, there is only one set cover $\{S_1, S_2\}$, but there are two disjoint dominating sets $\{S_1\}$ and $\{S_2\}$. Considering disjoint dominating sets compared with set covers method, even if there are some uncovered parts of the target area, in this example the longevity of the network is doubled from the point of view of energy resources.

Let us consider a dominated sensor node S in a network with random sensor distribution, where the disjoint dominating sets method is used. According to the analysis from [11] applied in the context of the DDS problem, the percentage of the area sensed by S which could remain uncovered is in average 41% when S has only one dominator, 19% when S has two dominators and less than 5% when S has at least four dominators.

The rest of the paper is structured as follows. In section 2 we present the disjoint dominating sets problem and show that 3DDS is NP-complete. Section 3 continues with a heuristic for computing the maximum number of DDS. Section 4 presents performance evaluation results and section 5 concludes the paper.

2 Disjoint Dominating Sets (DDS) Problem

In this section we present the disjoint dominating sets problem and prove 3DDS problem is NP-complete. We also show that any polynomial-time approximation algorithm for the disjoint dominating sets problem has a lower bound of 1.5.

We assume that the coverage area of a sensor is a disc. We assume all sensors use the same sensing range. However our network model and the heuristic we propose do not require a circular sensing range. We model the sensor network with n sensors as an undirected graph with n vertices. An edge exists between vertices u and v if and only if u and v are each within other's sensing range. Next, we present some definitions used in our approach.

Given a graph $G = (V, E)$, an *independent set* (IS) S of G is a subset of V such that $\forall u, v \in S, uv \notin E$. S is *maximal* (denoted by MIS) if any vertex not in S has a neighbor in S [4]. A *dominating set* (DS) is a subset $V' \subseteq V$ such that all vertices not in V' have at least one neighbor in V. Two dominating sets V_i and V_j are disjoint if $V_i \cap V_j = \oslash$. We define the *disjoint dominating sets* (DDS) problem as the problem of finding a maximum number of disjoint dominating sets in G.

Lemma 1 *A MIS of G is also a DS of G.*

Proof: This results directly from the definition of MIS and DS. □

Theorem 1 *A graph $G = (V, E)$ contains two disjoint dominating sets if and only if G contains no isolated vertex.*

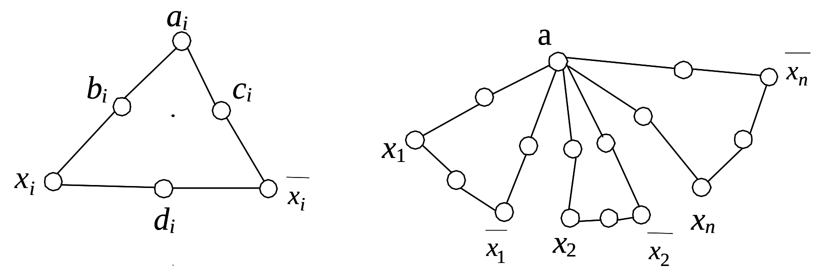
Proof: Let us first consider that G has two DDS, S_1 and S_2. We need to show that G has no isolated vertex. Suppose by contradiction that G has an isolated vertex v. S_1 is a DS and v is isolated, implies $v \in S_1$, therefore $v \notin S_2$. Results that S_2 does not dominate v, therefore S_2 is not a DS, which is a contradiction. Thus our assumption is false. Let us now consider that G has no isolated vertex. In this case, any maximal independent set and its complement form two DDS. □

The $3DDS$ problem asks to determine whether G contains three disjoint dominating sets or not.

Theorem 2 *$3DDS$ is NP-complete.*

Proof: It is easy to show that $3DDS \in NP$, since a nondeterministic algorithm needs only to guess three disjoint sets of vertices from V, and then verify in polynomial time whether each set is a dominating set.

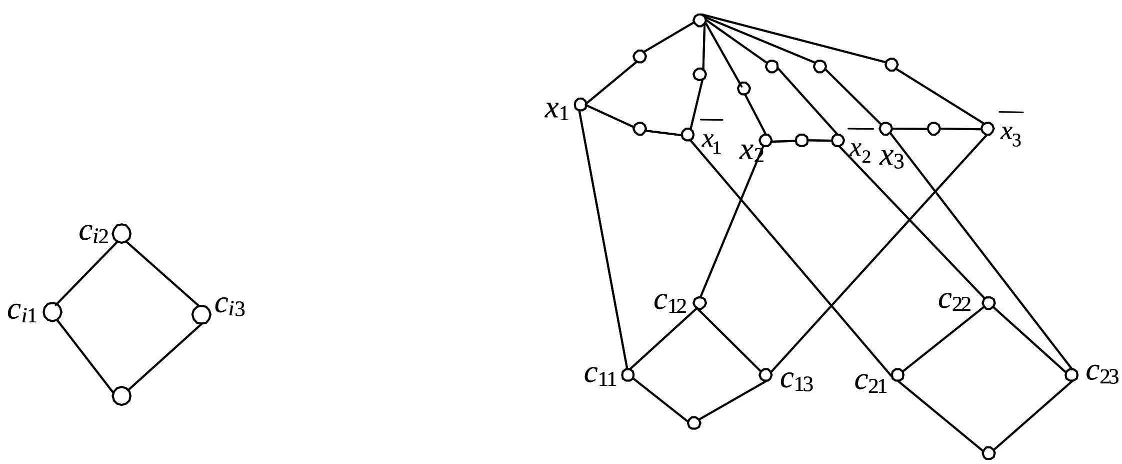
To show that $3DDS$ is NP-hard, we reduce the *Not-All-Equal 3SAT* problem [5] to it. A boolean formula is in conjunctive normal form (CNF) if it is expressed as an AND of clauses, each of which is the OR of one or more literals. A boolean formula is in *3-CNF* if each clause has exactly three distinct literals. *Not-All-Equal 3SAT* problem is defined as follows: given a *3-CNF* formula F, determine whether F has a satisfiable assignment such that every clause contains a literal with value 1 and a literal with value 0.



(a) Subgraph D_i (b) Graph D

Figure 2: *Construction of the graph D*

Let F be a 3-CNF formula with m clauses $C_1, C_2, ...C_m$, over n variables $x_1, x_2, ..., x_n$. We construct a graph G_F as follows. For each variables x_i, construct a subgraph D_i of six vertices as shown in *Figure 2(a)*. Connect the n subgraphs D_i , $1 \le i \le n$, into a graph D as shown in *Figure 2(b)*. For each clause C_i, $1 \le i \le m$, construct a subgraph C_i of four vertices as shown in *Figure 3(a)*. In addition, if $C_i = l_1 + l_2 + l_3$, then add into the graph G_F three edges connecting each c_{ik} to the vertex named l_k, $1 \le k \le 3$. *Figure 3(b)* shows the graph G_F for $F = (x_1 + x_2 + \bar{x_3})(\bar{x_1} + \bar{x_2} + x_3)$.



(a) Subgraph C_i (b) Graph G_F

Figure 3: *Construction of the graph G_F*

We now prove that F is satisfiable with *Not-All-Equal* assignment if and only if G_F has three disjoint dominating sets. First, we assume that F is satisfiable by a truth assignment on x_i such that every clause contains a literal with value 1 and a literal with value 0. We observe that for every subgraph D_i, the three disjoint dominating sets must be $\{a_i, d_i\}$, $\{x_i, c_i\}$, $\{\bar{x}_i, b_i\}$ respectively. Let us color $\{a_i, d_i\}$ red. If $x_i = 1$, then color $\{x_i, c_i\}$ blue and $\{\bar{x}_i, b_i\}$ yellow. If $x_i = 0$ then color $\{x_i, c_i\}$ yellow and $\{\bar{x}_i, b_i\}$ blue. Note that every clause contains a literal with value 1 and a literal with value 0. So for every subgraph C_i, there must be two vertices such that one is dominated by a blue vertex in D and another is dominated by a yellow vertex in D. Now, in the subgraph C_i, we color the vertex opposite to the vertex dominated by a blue vertex in blue, and the vertex opposite to the vertex dominated by a yellow vertex in yellow. The two vertices left are colored red. Clearly, all vertices in the same color form a dominating set. Therefore, G_F has three disjoint dominating sets.

Conversely, assume that graph G_F has three disjoint dominating sets. Let us color them red, blue and yellow respectively. We observe that each subgraph D_i must contain $\{a_i, d_i\}$, $\{x_i, c_i\}$ and $\{\bar{x}_i, b_i\}$ respectively. Assume $\{a_i, d_i\}$ are red. For each subgraph C_i, there must be two vertices red, because no vertex in C_i can be dominated by a red vertex. The other two vertices must be blue and yellow respectively. The vertex opposite to the blue vertex must be dominated by a blue vertex in D. The vertex opposite to the yellow vertex must be dominated by a yellow vertex in D. Set $x_i = 1$ if x_i is blue and $x_i = 0$ if x_i is yellow. Then every clause must contain a literal with value 1 and a literal with value 0. Therefore, F is satisfiable with *Not-All-Equal* assignment. □

Corollary *If $P \neq NP$, then the maximum disjoint dominating sets problem has no polynomial time α-approximation for any $1 \leq \alpha < 3/2$.*

Proof: Suppose that such an approximation algorithm A exists. We will show that A could be used to obtain a polynomial time algorithm for the $3DDS$ problem, contradicting the assumption that $P \neq NP$. For any input graph G, if G has $3DDS$, then A produces more than $3/1.5$ disjoint dominating sets, i.e., at least $3DDS$. If G does not have $3DDS$, then A produces at most $2DDS$. Therefore the polynomial-time approximation algorithm A can tell whether G has $3DDS$ or not, contradicting the assumption that $NP \neq P$. □

3 A Heuristic To Compute Maximum Disjoint Dominating Sets

In this section we present a heuristic to compute maximum number of disjoint dominating sets in a graph $G = (V, E)$. This algorithm consists of two phases. First, all vertices in G are colored using the sequential coloring algorithm. Then, based on this coloring, a heuristic is employed to construct the disjoint dominating sets. The analysis of this heuristic indicates it has the running time of $O(n^3)$, where n is the number of vertices in V.

Let us now consider the sequential coloring algorithm as follows:

Algorithm 1:

1. Give an ordering of all vertices.

2. Color the vertices one by one according to the ordering, with the least possible color (starting with 1) not appearing in its neighbors.

With the sequential coloring algorithm, each graph G is colored with at most $\triangle(G)+1$ colors, regardless of the chosen vertex ordering, where $\triangle(G)$ represents the maximum vertex degree of G.

Theorem 3 *For the coloring produced by Algorithm 1, each vertex with color i must have a neighbor with color j, for any $j < i$.*

Proof: Assume by contradiction that k is the smallest color such that i does not have a neighbor with color k, where $k < i$. Then, during the coloring procedure, in accordance with *step* 2, i is colored with k, contradicting our assumption. □

Corollary *All vertices with color 1 form a dominating set.*

We design a heuristic for computing maximum disjoint dominating sets as follows:

Algorithm 2:

1. Given $G = (V, E)$, order its vertices into a list, in the decreasing order of the vertex degree.

2. Compute a vertex coloring using the *Algorithm* 1.

3. Set D_1 = all vertices with color 1. Set $k = 1$. Set $max = minimum(d+1, colors)$, where d is the minimum vertex degree of G and $colors$ is the number of colors used by *Algorithm* 1.

4. while $(k < max)$

5. D_{k+1} = all vertices with color k+1;

6. for each color $i = 1..k$

7. Check every vertex v with color i. If v is not dominated by a vertex in color $k+1$, then we distinguish two cases. If v has a neighbor with color greater than $k+1$, then choose a neighbor with largest color and color it with $k+1$. If v does not have a neighbor with color greater than $k+1$ then go to line 11.

8. end for

9. $k++$

10. end while

11. Return k disjoint dominating sets: D_1, D_2, ..., D_k.

The complexity of this heuristic is $O(n^3)$. The maximum number of disjoint dominating sets is upper bounded by $d + 1$, where d is the minimum vertex degree of G. The heuristic first colors all vertices using the *Algorithm* 1. Then, starting with the smallest color, the set of vertices with same color is checked whether it forms a dominating set or not. In conformity with *Theorem* 3, only vertices with smaller color need to be checked whether they are dominated by the current set D_{k+1}(step 7). If a vertex v is not currently dominated by D_{k+1}, we look for a neighbor u which is not part of $D_1, ..., D_k$ and include it into D_{k+1}, by recoloring it with color $k + 1$. If we cannot find such a neighbor u, then the construction of D_{k+1} fails, the algorithm returning k dominating sets. Note that if a vertex v cannot be dominated by D_{k+1}, then it cannot be dominated by any D_i for $i > k + 1$.

From the point of view of our environment monitoring application model, all vertices should be part of a dominating set. By executing the *Algorithm 2*, there may be some vertices left, which cannot form another dominating set. Such a vertex is added to a dominating set based on the following criteria. We determine the set where this vertex has greatest contribution in covering parts of the uncovered target area. Ties are broken by giving priority to the set with fewest number of vertices.

4 Performance Evaluation

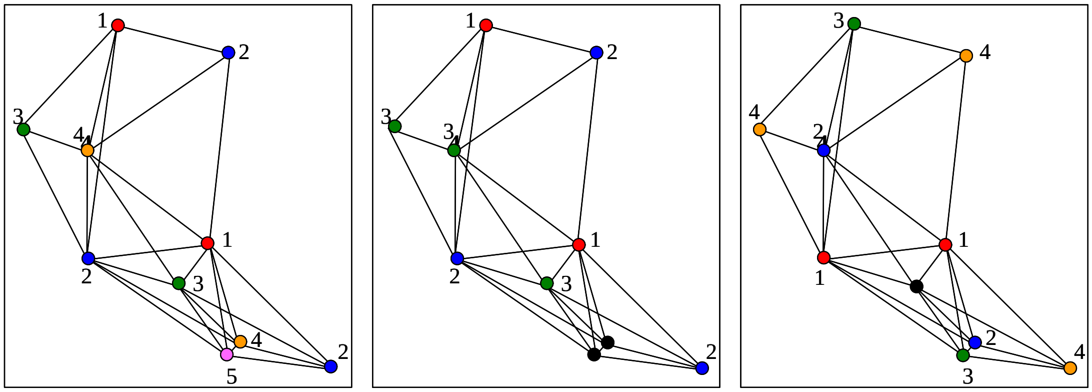
In this section we evaluate the performance of the heuristic proposed for computing the disjoint dominating sets.We simulate a stationary network with sensor nodes randomly located in a $1000m \times 1000m$ area. We assume the transmission range is equal for all the sensors in the network.

For small network sizes we compare the result produced by *Algorithm 2* with an optimal solution obtained through exhaustive search.

Transmission Range	Computed DDS	Optimal DDS
400	2	2
450	4	4
500	4	4
550	3	4
600	5	5
650	5	5
700	6	6
750	6	6
800	7	7

Table 1: *Number of DDS in a 10 node network with random locations, with variable transmission ranges.*

In *Table 1* we perform measurements by varying the transmission range of the sensor



(a) Graph coloring (b) Computed DDS (c) Optimal DDS

Figure 4: *Graph coloring and number of DDS in a 10 node network with random location, when transmission range is 550m.*

nodes between *400m* and *800m*. In determining the maximum number of DDS in a graph, the *minimum graph degree + 1* constitutes an upper bound value, because every node v can be dominated by at most *degree(v) + 1* disjoint dominating sets. We observe that as the graph connectivity increases, due to a higher propagation range, the number of disjoint dominating sets increases, too. This is because the graph minimum degree increases and fewer nodes are necessary for each dominating set. For this small number of sensor nodes, the differences between the optimal solution and the result produced by our heuristic are minimal.

In *Figure 4* we present the experiment for 10 nodes, when transmission range is *550m*. *Figure 4(a)* shows the graph coloring that resulted from performing step 2 in *Algorithm 2*, using five colors, labeled from 1 to 5. *Figure 4(b)* shows the three DDS output by *Algorithm 2* whereas *Figure 4(c)* shows four DDS obtained through exhaustive search. In *Figure 4(b),(c)* the nodes labeled with the same number belong to the same dominating set.

In the next experiment we test our heuristic for a 100 node network, with nodes randomly distributed. In *Figure 5* we compare the number of DDS obtained when nodes are initially arranged in a *decreasing* versus *increasing* order of the vertex degrees in step 1 of the *Algorithm 2*. We conclude that ordering node by decreasing degree produces better results in general. This is because the first disjoint sets contain fewer nodes, therefore the available node pool for building the other sets is larger. The number of DDS computed by our heuristic is compared with *minimum graph degree + 1*, which is the maximum possible value. Considering the nodes ordered by decreasing degree in the coloring step, we repeated the experiment illustrated in *Figure 5*, for 50 times, with different random node distributions. For these measurements, the number of disjoint dominating sets determined by our heuristic is on average within 5% from the corresponding value of *minimum graph degree + 1*.

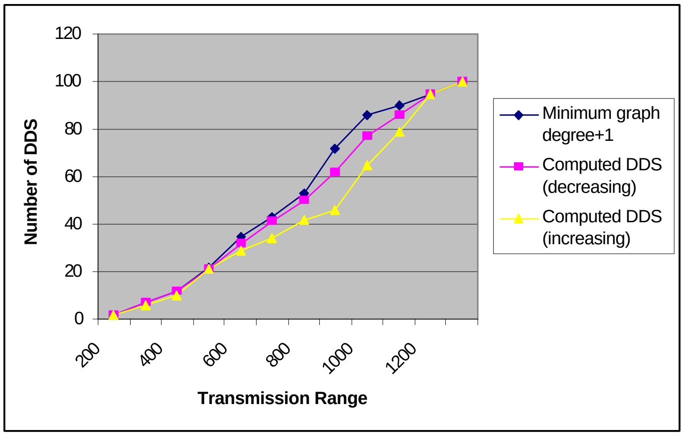


Figure 5: *Number of DDS in a 100 node network with random location, for different transmission ranges.*

5 Conclusion

Limited battery power is a critical constraint when designing wireless sensor network applications. Therefore, power aware strategies should be designed to prolong network lifetime and to decrease the deployment cost. In this paper we propose organizing the sensor nodes into a maximum number of disjoint dominating sets as a means to reduce energy consumption. The transmitting sets are alternated in a round-robin fashion, such that at a specific time only one set is responsible for sensing the target area, permitting all other nodes to be in a low energy, sleeping state. We presented a theoretical analysis for this problem and proposed an efficient heuristic. We evaluated its performance against the optimal solution with simulations and analyzed the algorithm behavior for large networks.

References

[1] I. Cardei, M. Cardei, L. Wang, B. Xu. D.-Z. Du, *"Optimal Relay Location for Resource-Limited Energy-Efficient Wireless Communication"*, submitted to IEEE Transactions on Wireless Communications.

[2] L. Benini, G. Castelli, A. Macii, E. Macii, et al.,*"A discrete-time battery model for high-level power estimation"*, Design, Automation and Test in Europe Conference, pp. 35-39, 2000.

[3] I. Chlamtac, C. Petrioli, J. Redi, *"Energy-Conserving Access Protocols for Identification Networks"*, IEEE/ACM Transactions on Networking, Vol. 7, No. 1, pp 51-59, 1999.

[4] T. S. Cormen, C. E. Leiserson, R. L. Rivest, *"Introduction to Algorithms"* edited by MIT Press and McGraw-Hill Book Company.

[5] M. R. Garey, D. S. Johnson, *"Computers and Intractability. A guide to the Theory of NP-Completeness"*, Freeman, New York, 1979.

[6] P. Karn, *"MACA - a New Channel Access Method for Packet Radio"*, ARRL/CRRL Amateur Radio 9th Computer Networking Conference, pp 134-140, 1990.

[7] C.S. Raghavendra, S. Singh, *"PAMAS-Power Aware Multi-Access protocol with Signalling for Ad Hoc Networks"*, Computer Communications Review, July 1998.

[8] M. Woo, S. Singh, C.S. Raghavendra *"Power-Aware Routing in Mobile Ad Hoc Networks"*, MobiCom 98.

[9] S. Slijepcevic, M. Potkonjak *"Power Efficient Organization of Wireless sensor Networks"*, IEEE International Conference on Communications, June 2001, Helsinki, Finland.

[10] J. Stine, G. de Veciana *"Improving Energy Efficiency of Centrally Controlled Wireless Data Networks"*, Wireless Networks, Vol. 8, No. 6, pp 681 - 700, 2002.

[11] Y.-C. Tseng, S.-Y. Ni, Y.-S. Chen, J.-P. Sheu *"The Broadcast Storm Problem in a Mobile Ad Hoc Network"*, Wireless Networks, Vol. 8, pp 153-167, 2002.

[12] Y. J. Zhao, R. Govidan, D. Estrin, *"Residual Energy Scans for Monitoring Wireless Sensor networks"*, IEEE Wireless Communications and Networking Conference, Orlando, FL, USA, 2002.