The Impact of Add+VantageMR Professional Development: A study on teacher implementation following professional development

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Dedication

This thesis is dedicated to my family.

Special thank you to my parents who always encouraged me to continue my education and follow my dreams. Thank you to my husband, David, and three children, Nicholle, Hayden and Lydia, for their incredible patience through this whole process from start to finish.

Abstract

School districts in the United States spend a significant amount of human and financial resources on the design and facilitation of professional development experiences to impact teachers' understanding of how students think about and learn mathematics. In turn, these professional development opportunities hopefully lead to changes in instructional practices positively impacting students' feelings towards mathematics and their mathematics achievement. Empirical evidence to support components of effective professional development is mounting and has grown significantly in the past fifteen years (Desmoine, 2009).

This study provides a lens into three teachers' experiences implementing Math Recovery assessment and teaching practices after participating in Add+VantageMR® (AVMR) professional development. AVMR professional development focuses on student thinking, along with dynamic diagnostic assessment of students' known strategies to inform teaching. Teaching is complex, therefore several areas of opportunity exist to apply learning from AVMR. This study utilized a case study approach (Yin, 2014) and examined three first grade classroom by analyzing a series of video recorded mathematics lessons collected over four months. The study examined teachers' planning for, enacting, and reflecting upon mathematics lessons in the area of early number and operations. Three rounds of coding were applied to provide valuable insights into the ways Math Recovery Guiding Principles of Classroom Teaching and Dimensions of Mathematizing manifested in each classroom.

Findings from this study indicate teachers implement assessment and teaching practices promoted in Add+VantageMR® professional development when planning for, enacting and reflecting upon whole number mathematics lessons. Each participant's journey implementing AVMR course ideas was different, and evidence of impact was identified regardless of years of experience and teachers' participation in Course 1 or Course 2.

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Chapter One – Introduction

School districts in the United States spend a significant amount of human and financial resources on the design and facilitation of professional development experiences that potentially increase teachers' understanding of how students think about and learn mathematics. In turn, these professional development opportunities hopefully lead to improvement in instruction. This investment is done with the aim to make a difference in student achievement outcomes and thus meet district student achievement goals. Empirical evidence to support components of effective professional development is mounting and has grown significantly in the past fifteen years. This study adds to the literature by investigating the efficacy of a national professional development program Add+VantageMR® professional development (AVMR) among teachers of primary students.

Conceptual Framework

Advancements in the field make it possible to identify core features that have been repeatedly shown to relate to teacher improvement and tentatively to student achievement (Desimone, 2009). As a result of synthesizing this body of research, Desimone presents a core conceptual framework based on consensus from the field that can be used to study the effects of professional development on teachers and students. Within this framework, five core features of professional development are identified: content focus, active learning, coherence, duration, and collective participation. A content focus could be the most influential feature of professional development (Desimone, 2009). Studies over the past ten years indicate a link between activities executed in professional development that focus on how children learn content with an increase in teacher knowledge and skills, as well as improvements in practice. Active learning can take many forms in a professional development setting, such as active participation in discussions, observing expert teachers, or reviewing student work. Coherence is emphasized in two ways in the literature: coherence with teachers' current knowledge and beliefs, as well as a professional development focus that is coherent with school, district, and state reforms and policies.

Although there does not seem to be a convergence on the exact duration of quality professional development, studies do indicate that professional development consisting of 20 or more hours of contact time to be effective (Desimone, 2009; Supovitz & Turner, 2000). Supovitz and Turner (2000) found intense professional development over a longer period of time to be more effective than quick workshops. A final critical feature is collective participation. School teams or grade levels attend professional development together, and as a result, opportunities arise for interaction and discourse that can be powerful for teacher learning (Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001).

Although it is important to measure the effects of professional development experiences on teacher learning, there is also a need to evaluate the impact of teacher professional development on children's learning. Teachers' knowledge about student thinking strategies has shown to influence teacher decision making in the classroom as well as student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Carpenter et al. (1989) found that teachers who apply knowledge about children's thinking into their classrooms do so by assessing children's current areas of understanding and using this information as they choose lessons and question students during a lesson.

Add+VantageMR® professional development focuses on enhancing whole class and small group teaching and includes multiple components: (a) dynamic one-on-one interview assessments, (b) teaching tools, and (c) participant learning experiences. These three components are based on the Math Recovery® Intervention Specialist program (MRIS) Program (Wright, Martland, & Stafford, 2006; Wright, Martland, Stafford, & Stanger, 2006) and the Numeracy Intervention Research Project (NIRP) (Ellemor-Collins & Wright, 2009, 2011; Wright et al., 2007). Developed by Dr. Robert Wright between 1992-5 with funding from the Australian Research Council, the Math Recovery® Intervention Program is a one-on-one math intervention focused on intervention in the number learning of low-attaining first graders. Since that time, this initial work has been extended to the Numeracy Intervention Research Project (NIRP), which focused on one-on-one mathematics intervention for students in third and fourth grade.

Math Recovery® one-on-one intervention has a unique approach to intervention that includes intensive, highly interactive and targeted lessons at a students' zone of proximal development (Vygotsky, 1978). To become a Math Recovery® Intervention Specialist, participants engage in an intensive year-long study of mathematics one-on-one intervention, which includes video recording assessment and instructional sessions for teacher reflection and instructional planning.

The intensive study involved in becoming a Math Recovery® Intervention Specialist can be difficult to schedule into the already high demands of classroom teaching. There was a need to provide classroom and support educators with the same theory, assessment, and teaching tools. Add+VantageMR® professional development was created for this purpose. Therefore, the Add+VantageMR® professional development provides a less intense professional development experience that includes a very similar one-on-one student assessment and focuses on whole class teaching decisions based on individualized assessments. The proven impact of Math Recovery® Intervention provides reason to believe that transferring the same assessment and teaching practices into the classroom would yield a positive impact on student learning and teacher practice. Evaluation studies focused on elements of Math Recovery® Intervention professional development as a part of Add+VantageMR® professional development, such as the Learning Framework in Number, continue to impact the development of elementary mathematics curriculum and classroom practices around the world (Bobis et al., 2005).

Add+VantageMR® professional development (AVMR), released by the U.S. Math Recovery® Council in 2005, has a strong national presence and is beginning to reach international educational systems. Add+VantageMR® Courses consist of two four-day courses, where teachers spend 26 contact hours learning about content. Each course has a specific content focus in the area of whole number. Add+VantageMR® incorporates the aforementioned five core features of effective professional development practices and a model for children's thinking linked with each assessment. Robust content centers on children's thinking and incorporates research on children's thinking in a range of early number topics (Cobb, Wood, & Yackel, 1991; Mulligan & Mitchelmore, 1997; Steffe, Glasersfeld, Richards, & Cobb, 1983; R. Wright, 1994; Yackel, Cobb, & Wood, 1991). Assessments that are learned in the courses provide teachers with the avenues through which to apply knowledge about student learning to their own classroom students. Active learning is integral to teacher learning in each of the face-to-face instructional days. This practice of examining student thinking, followed by discussion with colleagues, is a core element throughout the courses. Video clips of students solving mathematical tasks are used to instruct teachers in the Learning Framework in Number (LFIN). The LFIN provides an overview of instruction in whole number mathematics and forms the foundation for Add+VantageMR® courses. Each course includes the opportunity for teachers to administer the assessments with children; many times students are from their own classroom. Each course concludes by involving teachers in activities wherein they participate in exemplary instructional activities and discuss ways to manageably differentiate these activities in the classroom setting and according to assessment results.

Collective participation is strongly encouraged by the U.S. Math Recovery Council®. When teachers participate in AVMR professional development as a grade level team and along with teachers from a cross-sectional of grade levels, the conversations about children's strategies and use of instruction are enriched and supported in the school building. Delivery models for AVMR promote collective participation in districts by offering leadership courses where Math Recovery® Intervention Specialists learn to facilitate AVMR courses within their own district. For districts who do not have an existing leader, U.S. Math Recovery® sends an instructor to the school district with a minimum of 12 participants to lead AVMR courses. Coherence is not as easily determined in the professional development itself, however, the use of video involving students of various elementary ages allows teachers to make connections between students in their own classrooms to the student strategies being discussed in the course.

Teachers who believe in the importance of conceptual understanding in mathematics and meeting children where they are find a strong coherence between their beliefs and AVMR. Add+VantageMR® has all the components of effective professional development. Teacher feedback indicates the courses themselves have a powerful impact on teacher learning (U.S. Math Recovery® Council). However, what remains a question is the ways in which the AVMR professional development creates change in the classroom.

There is a need for research on Add+VantageMR® professional development. Presently, there is no peer-reviewed research on Add+VantageMR® professional development and its impact in teacher learning. Additionally, research on high quality professional development is a growing field, and this study contributes to current research by offering a descriptive case study comparison of three classroom teachers as they implement assessment and instruction techniques after professional development.

Evaluation studies involving the effectiveness of Math Recovery® professional development, assessment, and instruction have shown favorable results, which provides reason to expect that Add+VantageMR® would have a positive influence on teacher learning and student achievement. Findings from this study hold implications for school districts investing in professional development for mathematics teachers with a goal to influence classroom practice and student achievement, as well as districts that have already invested in Add+VantageMR® programs. Additionally, this study will have implications for the U.S. Math Recovery® Council as they continue to develop and refine current program offerings.

Research Questions

As mentioned above, Add+VantageMR® features all components of effective professional development. There is robust content that centers on children's thinking, assessment, and instruction in early numbers and operations. Two courses are offered, each four days in length, for a total of 26 hours of instruction. During the courses, teachers are actively learning by reflecting on video of student thinking and practicing assessment protocols and instructional activities. Coherence with district initiatives is strongly encouraged, along with a systemic approach to professional development which includes training and support of in-district Add+VantageMR® leaders. Collective participation is also highly recommended. This not only offers a supportive environment to implement ideas from the courses into the classroom, but it offers the opportunity for coherence throughout a single building.

The intent of this study is to examine the ways, if any, in which classroom teachers incorporate ideas from Add+VantageMR® (AVMR) into their math classroom teaching practice. To examine teachers' implementation of ideas from AVMR professional development, the following research questions guide this study:

1. In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?
    a) In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?
    b) What are teachers using to determine children's current level of understanding?
    c) In what ways to teachers use course resources, such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 Year-Olds* (Wright,

Stanger, Stafford, & Martland, 2014) and/or *Developing Number Knowledge* (Wright, Ellemor-Collins, & Tabor, 2011) when planning a whole number lesson?

   d) In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number as well as skills that support learning about whole number operations?

2. In what ways does Add+VantageMR® professional development influence teachers instructional moves during a whole number lesson?

   a) In what ways are students engaged in activities from the course resources, such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 Year-olds* and/or *Developing Number Knowledge*?

   b) In what ways are students actively engaged in doing mathematics during the lesson?

   c) In what ways does the teacher use domains of progressive mathematization during a whole number lesson?

3. How does Add+VantageMR® professional development influence differentiation of a whole number lesson?

   a) In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?

   b) In what ways is the same activity modified during a lesson (modification may include offering different dice, number ranges, and/or instructional tools)?

4. How does Add+VantageMR® professional development influence teachers assessment of students learning during implementation of a whole number lesson?

a) In what ways do teachers make in-the-moment adjustments to the written lesson plan based on student responses, demonstrated understandings, and knowledge about children's thinking as it is related to the Learning Framework in Number?

b) In what ways do teachers describe children's strengths and areas of need for whole number understanding with specificity that is connected to observed student solution strategies and the Learning Framework in Number?

c) In what ways do teachers collect formative assessment data during the lesson that includes a student-thinking component?

Overview of the Chapters

This study is organized into five chapters. Chapter One offers a rationale for the study and presents the research questions. Chapter Two reviews the significant literature related to effective professional development and teacher noticing of mathematical student thinking. Chapter Three describes the research methodology used in the study. The results are presented in Chapter Four. Chapter Five concludes the study with a summary of the study, discussion of the results, and recommendations for future research.

Chapter Two - Literature Review

The overarching goal of this study is to investigate the ways, if any, in which classroom teachers incorporate ideas from Add+VantageMR® (AVMR) into their math classroom teaching practice. Teaching is complex, and therefore there are several areas of opportunity to apply learning from AVMR. AVMR professional development focuses on student thinking, along with formal and informal assessment of students' strategies to inform teaching. The research questions require a review of the literature on (a) effective professional development, effective classroom practice, (b) theoretical underpinnings of Add+VantageMR® professional development, (c) research on Add+VantageMR® professional development, (d) translating professional development into classroom practice, and (e) professional noticing of children's mathematical thinking. This literature review that follows is organized around these six topics.

Effective Professional Development

School districts in the United States spend a significant amount of human and financial resources on the design and facilitation of professional development experiences that potentially increase teachers' understanding of how students think about and learn mathematics. This investment is done with the aim to make a difference in student achievement outcomes, and thus meet district student achievement goals. Empirical evidence to support components of effective professional development has grown significantly in the past 15 years. Advancements in the field make it possible to identify core features that have been repeatedly shown to relate to teacher improvement, and tentatively, to student achievement. As a result of synthesizing this body of research, Desimone (2009) presents a core conceptual framework based on consensus from the field that can be used to study the effects of professional development on teachers and students. Within this framework, five core features of professional development are described: content focus, active learning, coherence, duration, and collective participation.

A content focus could be the most influential feature of professional development. Studies over the past 25 years indicate a link between activities executed in professional development that focus on how children learn content with an increase in teacher knowledge and skills, and improvements in practice. For example, research by Carpenter, Fennema, Peterson, Chiang, and Loef (1989) found that teachers who participated in Cognitively Guided Instruction professional development focused on children's thinking and development of addition and subtraction concepts spent significantly more instructional time talking about problems and discussing alternative solutions than did control classes. CGI teachers also allowed students to use multiple strategies more frequently. A follow-up longitudinal study by Fennema et al. (1996) observed teacher change over a four-year professional development experience focused on Cognitively Guided Instruction research on children's thinking and found participation in a content focused professional development centered on student thinking to have fundamental changes in the beliefs and instruction of 18 of the 21 participants.

Another study by Heck, Banilower, Weiss, and Rosenberg (2008) found a positive significant relationship between NSF's Local Systemic Change through Teacher Enhancement Initiative (LSC) professional development and teacher attitudes, perceptions of content and pedagogical preparedness, and classroom practices. The LSC professional development had a strong math and science content focus, among other key features of effective professional development.

Active learning, the second core feature, involves teachers being “active” participants in professional development, instead of passive recipients of information. Active participation can take many forms in a professional development setting, such as active participation in discussions, observing expert teachers, or reviewing student work. Garet, Porter, Desimone, Birman, and Yoon (2001) drew on data collected as part of a national evaluation of the Eisenhower Professional Development Program, a federal program supporting professional development in mathematics and science, to examine the relationship between features of professional development identified in literature and self-reported change in teachers’ knowledge, skills, and classroom teaching practices. Study participants were from a national probability sample of 1,027 mathematics and science teachers who participated in Eisenhower Professional Development Program activities. To determine a relationship between participation in professional development activities and teacher change in classroom practice, teacher self-report was used, and a model using ordinary least squares (OLS) regression revealed three core features having a significant, positive effect on change in knowledge and skills and classroom teaching practices: 1) focus on content knowledge, 2) opportunities for active learning, and 3) coherence with other learning activities. Desimone, Porter, Garet, Yoon, and Birman (2002) surveyed 207 teachers in 30 schools across five district to examine features of professional development and its effects on changing teaching practices in math and science classrooms from 1996-9. They found that features such as active learning opportunities increase the effect of the professional development on teacher's instruction. Findings also supported the link between professional development containing a content focus and teachers' use of those practices in the classroom. Additionally, Desimone et al. concluded that professional development is more effective in changing teachers' classroom practice when it has collective participation of teachers from the same school, department, or grade.

Coherence, the next core feature, is emphasized in two ways in the literature: coherence with teachers' current knowledge and beliefs, as well as a professional development focus that is coherent with school, district, and state reforms and policies. Penuel, Fishman, Yamaguchi, and Gallagher (2007) observed professional development strategies that foster curriculum implementation in the context of teacher participation in GLOBE professional development. The goal of professional development was to promote the implementation of the GLOBE curriculum into the classroom. Results indicated that meaningful, ongoing, and coherent professional development experiences consistent with local school and district goals had a positive impact on teacher implementation of the GLOBE curriculum. Teachers were more likely to feel prepared for student inquiry when professional development included links to standards and curriculum and classroom activities. Firestone, Mangin, Martinez, and Polovsky (2005) conducted a comparison of three urban New Jersey districts and found the district with the most coherent focus on helping teachers develop deeper knowledge about select subject areas had the greatest teacher-reported influence on teaching practice. Professional development in this district aligned with district and state educational goals and was delivered in a manner consistent with the pedagogy presented to teachers. Teachers observed each other's classrooms and took turns leading teacher professional development meetings.

Although there does not seem to be a convergence on the exact duration of quality professional development, studies do indicate that the duration of professional development matters; 20 or more hours of contact time is needed to be effective (Desimone, 2009). Supovitz and Turner (2000) surveyed 3,464 science teachers and 666 principals from 24 communities and found that dramatic changes in teaching practices occurred after 80 hours of intensive and sustained professional development activities.

A final critical feature is collective participation. School teams or grade levels attend professional development together, and as a result, opportunities arise for interaction and discourse that can be powerful for teacher learning. As stated above, Fennema et al. (1996) found that professional development is more effective in changing teachers' classroom practice when it has collective participation of teachers from the same school, department, or grade. Desimone et al. (2002) found both the collective participation of teachers and the duration of the professional development activity to significantly affect teacher learning when done in a professional development situation that focuses on content knowledge, has opportunities for active learning, and has coherence with other learning activities.

Effective Classroom Practice

Math Recovery® Guiding Principles for Classroom Teaching framework includes nine principles for classroom teaching (Wright, Stanger, Stafford, & Martland, 2006). These principles guide classroom instruction around the world and include the effective practices, such as initial and on-going assessment, cutting-edge teaching, and inquiry-based teaching (Wright et al. (2011). Formative assessment is considered to be an example of an effective practice for teaching. Principles to Action (NCTM, 2014), the latest in a series of landmark texts published by NCTM to move mathematics instruction in the future outlines eight mathematics practices; the eighth practice centers on using evidence of student thinking through formative assessment practices.

Literature commonly refers to assessment used for feedback to inform teaching as formative assessment (Black & William, 1998; Melmer, Burmaster, & James, 2008; Popham 2006, 2008). A seminal publication by Black and William (1998) stated research at the time "shows conclusively that formative assessment does improve learning," and that the gains in student achievement were "amongst the largest ever reported" (p. 61). This article prompted the publication of a number of subsequent articles, all of which support the finding that implementing formative assessment does indeed provide large return in increased student achievement. In fact, according to one scholarly search engine, Black and William (1998) is cited in nearly 6,000 publications. For example, Fennel, Swartz, Kobett & Wray use Black & William (1998, 2010) as support for their article *Classroom-Based Formative Assessments – Guiding Teaching and Learning* (NCTM, 2015).

Although there is this common acceptance that using data to inform instructional decisions is a proven practice to increase student achievement, not all are in agreement that the evidence is stately conclusive. Dunn and Mulvenon (2009) identify two cores issues with the limited research that has been used to make this claim. First, there is not a commonly accepted definition of formative assessment, and similarly, definitions that involve *how* assessment data are used as a part of the definition of formative assessment are quite confusing in conducting research, to say nothing of the difficulties arising for readers of the body of research. Secondly, the seminal piece by Black and William (1998) does not come without flaw; in fact, they raise the point that 80 percent of the effect sizes reported came from studies that were deemed to be poor to fair in quality. Additionally, they raise a concern in generalizing these results to the greater population. The studies were conducted within the realm of special education, with 83 percent of student participants in the studies identified as handicapped. More research is needed using sound practices in order to have conclusive evidence regarding the impact of assessment data in the form of formative assessment results showing substantial gains in student achievement.

It has been universally accepted within the mathematics education community that the more teachers know about children's thinking, the better the instructional decisions they will make. The body of research referred to Cognitive Guided Instruction (CGI) has consistently supported this tenet. In a series of CGI studies, Garet et al. (2001) found that learning to understand the development of children's mathematical thinking could lead to fundamental changes in teachers' beliefs and practices, and that these changes were reflected in students' learning (Carpenter et al., 1989; Fennema et al., 1996; Fennema, Franke, Carpenter, & Carey, 1993). The tenet that teachers should focus on student thinking is a core feature of Add+VantageMR.

Add+VantageMR assessment is unique from other studies using assessment to guide teaching in that AVMR assessment features one-on-one diagnostic assessment which is analyzed using a research based model of children's thinking. Similarly to other professional development offerings centered on children's thinking, AVMR integrates assessment of student thinking into instruction. Gearhart and Saxe (2004) support this notion in their work with the Integrating Mathematics Assessment (IMA) program. They found that as teachers implemented good lessons, they also needed to investigate children's thinking, and build activities and discussions upon children's understandings.

Theoretical underpinnings of Add+VantageMR®

AVMR is a professional development program that incorporates practices of successful professional development described by Desimone (2009). AVMR is a derivative program of Math Recovery® One-on-One Intervention and is designed for whole- and small-group classroom teaching. Utilizing the same theoretical principles about elementary mathematics assessment and instruction from Math Recovery® One-on-One intervention, AVMR provides classroom teachers with diagnostic interview assessments and teaching trajectories, which provide teachers with the tools necessary to identify and meet students' mathematical needs.

Math Recovery® is internationally recognized as a highly successful program of intervention in early number learning. Developed by Dr. Robert Wright from 1992-5 with funding from the Australian Research Council, the Math Recovery® Intervention Program is a one-on-one intervention targeted at low-performing first grade students. Math Recovery® One-on-One intervention is designed to support the one-on-one tutoring model and is not designed for whole class instruction. Math Recovery® is used extensively by school systems in the United States, United Kingdom, Ireland, and Australia, as well as several other countries, such as Canada and Vietnam. Two foundational books provide much of the detail of the one-on-one intervention program (Wright et al., 2000; Wright et al., 2002), and two additional books describe assessment and implications for classroom instruction with four – 11 year-olds (Wright et al., 2005; Wright et al., 2012).

The U.S. Math Recovery® Council is a 501(c)(3) non-profit organization dedicated since 2003 to overseeing Math Recovery® programs and its materials, methodology, and related intellectual property in the United States. It is the mission of U.S. Math Recovery® to transform numeracy education, to connect research with practice, and to empower educators to advance student mathematical thinking and success through Math Recovery® principles. The vision of U.S. Math Recovery® Council is to create and foster attitudes, cultures, and climates that lead to change in mathematics learning and teaching practices. U.S. Math Recovery® strives to do the following:

Ensure Math Recovery® programs and principles are widely attainable.

Deliver exceptional professional development.

Develop dynamic research-based materials.

Promote program fidelity and professional integrity within the Math Recovery® community.

Contribute to mathematics education research.

Serve as a catalyst for high quality systemic mathematics assessment and instruction.

Empower all students to reach their full mathematical potential.

Implementation of the program in the United States began in South Carolina in 1995. By 1999, the program had spread to several states in the western region. By 2004, implementation initiatives included school districts in 19 states including New England, California, and the Midwest, serving approximately 325 teachers, 60 leaders, and over 3,000 participating students. As of 2014, Math Recovery® has expanded to 36 states and grown to serve 130 leaders and well over 200,000 students. Additional information about the work of the U.S. Math Recovery® Council can be found at www.mathrecovery.org. Add+VantageMR® professional development (AVMR), released by the U.S. Math Recovery® Council in 2005, was developed in collaboration with Robert Wright. Its purpose is to make core principles of the Math Recovery® One-on-One Intervention Specialist program accessible and applicable to classroom teachers. Add+VantageMR® incorporates core features of effective professional development practices and a model for children's thinking linked with assessment. Robust content centers on children's thinking and incorporates research on children's thinking in a range of early number topics such as Number Word Sequences, Counting, Structuring Number, Base Ten, and Multiplication and Division (Cobb & Wheatley, 1988; Mulligan & Mitchelmore, 1997; Leslie P. Steffe, 1992; Leslie P Steffe, Cobb, von Glasersfeld, & Sinclair, 1988; Wright, 1991, 1994, 2000). Each course introduces an assessment for each topic and provides teachers with opportunities to apply knowledge about student learning to their own classroom of students through video recorded practice administering assessments. Active learning is integral to teacher learning in each of the face-to-face instructional days. This practice of examining student thinking, followed by discussion with colleagues, is a core element throughout the courses. Each course includes the opportunity for teachers to administer the assessments with children; many times students are from their own classroom. Each course concludes by involving teachers in activities wherein they participate in exemplary instructional activities and discuss ways to manageably differentiate these activities in the classroom setting according to assessment results. Add+VantageMR courses consist of two four-day courses; teachers spend 24 contact hours in each course, for a total of 48 hours devoted to learning about content. Each course has a specific content focus in the area of whole number.

Course One includes the following topics:

- Addition and subtraction to twenty.
- Structuring numbers (finger and spatial patterns, number combinations and partitions).
- Number words and numerals (forward and backward number word sequences and numeral identification).

Add+VantageMR Course Two includes:

- Multiplication and division.
- Place value concepts (multi-digit addition and subtraction).

Collective participation is strongly encouraged by the U.S. Math Recovery® Council. When teachers participate in AVMR professional development as a grade level team and along with teachers from a cross-sectional of grade levels, the conversations about children’s strategies and use of instructional is enriched and supported in the school building. Delivery models for AVMR promote collective participation in districts by offering leadership courses where Math Recovery® Intervention Specialists learn to facilitate AVMR courses within their own district. For districts who do not have an existing leader, U.S. Math Recovery® sends an instructor to the school district with a minimum of 12 participants. Coherence is not as easily determined in the professional development itself, however, the use of video involving students of various elementary ages allows teachers to make connections between students in their own classrooms to the student strategies being discussed in the course. Teachers who believe in the importance of conceptual understanding in mathematics and meeting children where they are find a strong coherence between their beliefs and AVMR. Add+VantageMR® has all the components of effective professional development. Teacher feedback indicates the courses themselves have a powerful impact on teacher learning. However, what remains in question are the ways in which the AVMR professional development creates change in the classroom.

Research on Add+VantageMR® Professional Development

Research on the impact of Add+VantageMR® on teacher learning is very limited. One doctoral thesis was found that examines Add+VantageMR® professional development. Briand (2013) studied the impact AVMR-type methods and strategies may have on teacher improvement and student learning. Using a case study design, Briand examined one first grade teacher's implementation level after AVMR Course One, along with the student achievement gains of three students in her class. Findings indicate that teachers would benefit from collegial conversation and follow-up experiences when implementing Add+VantageMR® assessment and instruction. While the research on AVMR is limited, studies on Math Recovery® Intervention do exist. Principles of assessment and instruction within AVMR are identical to those within Math Recovery® One-on-One intervention, therefore a review of the literature related to the impact of Math Recovery® Intervention is provided here.

A two-year experimental design research evaluation of Math Recovery® was conducted from 2007-9 in 20 elementary schools across five school districts in two states (Smith et al., 2013). In all, 343 children received one-on-one Math Recovery® intervention over the two years. During year one, 72.5 percent of the participants received free and reduced lunch, 46.3 percent were non-white, and 16.3 percent were limited English proficiency. In year two, 55.6 percent received free and reduced lunch, 46.5 percent were non-white, and 8.8 percent were limited English proficiency. Despite just 10 percent of participating students receiving the recommended number of 45-60 Math Recovery® lessons, there was a positive causal effect of Math Recovery® intervention at the end of first grade. As diagnostic program, effect sizes were found to be between +0.30 and +0.40 for students participating in Math Recovery® intervention.

The longitudinal effects of Mathematics Recovery substantiate the value of the program. Mathematics Recovery was fully implemented during the 1999-2000 school year at Roye-Williams Elementary School (RWES), a Title 1 school in Harford County, Maryland. One Math Recovery® teacher and three paraprofessionals were dedicated full-time to mathematics instruction and professional development. Prior to the implementation of Math Recovery®, only 30-40 percent of children scored proficient or better in mathematics on the Maryland state assessment program (MSPAP). In 2004, the fifth year of implementation, 76.6 percent of fifth grade children were proficient or better in mathematics on the Maryland State Assessment (MSA); 82.9 percent of fourth grade children scored proficient or better, with 22.9 percent scoring advanced, and 72.8 percent of third grade children scored proficient or better.

In 2006, the Kentucky Center for Mathematics was formed though an appropriation given to Northern Kentucky University with the goal of enhancing the mathematics within the state of Kentucky. This statewide project includes both Math Recovery® and AVMR and continues today. Math Recovery® is one of two intervention programs implemented as a part of the Primary Mathematics Intervention Program. Initial program results indicated that students participating in Math Recovery® Intervention gained slightly more than 2 grade levels and 70 percent of participants attained expected grade level proficiency after participating in Math Recovery® Intervention.

Results not only identified significant student achievement gains, but also documented change in teachers' beliefs, attitudes, and content knowledge towards mathematics. Math Intervention Teachers (MITs) in Math Recovery® Intervention reported an increase in mathematics content knowledge, perceived themselves to better identify best practices for classroom instruction that support teaching reasoning and problem solving skills to students, and moved away from the belief that mathematics mainly uses rules. MITs also reported greater knowledge of Kentucky core content standards (a 50 percent rate of agreement prior to participating increased to 100 percent at the end of year one). Results from a 2011-12 external evaluation of the Primary Mathematics Intervention Program at KCM indicated that the program enables sustained teacher growth, yields sustained gains in intervention students' achievement, encourages school-wide changes toward effective instructional practices, and enhances school-wide gains in student achievement. In 2014, Math Recovery® Intervention and

Add+VantageMR® programs are two of the five approved intervention programs by the Kentucky Department of Education that elementary schools can implement with Kentucky Mathematics Achievement Funds. More information about Kentucky's implementation can be found on the Kentucky Center for Math website, www.kentuckymathematics.org.

Translating Professional Development into Classroom Practice

Teachers seek out professional development and districts offer professional development to improve instructional practice for reasons outlined earlier within the student achievement section. This section focuses on the process of teacher change and the impact of professional development on the change process. Throughout the years, professional development as a whole has been under sharp criticism on its effectiveness on bringing about teacher change, and thus, a change in student learning outcomes. Guskey (1986) reported that nearly every major work on the topic of staff development up to that time emphasized the failings of professional development efforts.

Guskey (1986, 2002) presents a model to think about the change process differently, specifically in the area of teaching. The model focuses on three major outcomes of professional development: change in the classroom practices of teachers, change in their beliefs and attitudes, and change in the learning outcomes of students. Oftentimes, professional development seeks to alter the beliefs and attitudes of teachers with the thinking that a change in beliefs and attitudes will result in a change in classroom practice, and finally, a change in student learning outcomes. The model introduced by Guskey (1986, 2002) places these three outcomes to professional development in an alternative order, as shown in Figure 2. The model suggests that a change in teacher beliefs and attitudes follows a new practice in the classroom shown to impact student learning.

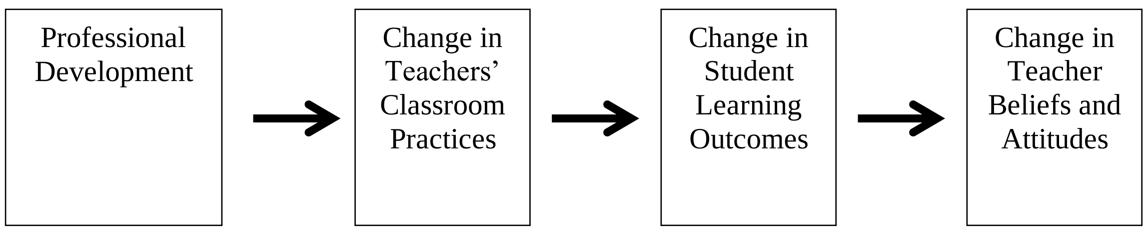


***Figure 2.1* A Model of Teacher Change. Guskey, T. (1986). Staff development and the process of teacher change. *Educational Researcher*, 15, p. 5. Copyright 1986 by the American Educational Research Association.**

This Model of Teacher Change is supported in literature. In a study of middle school science teachers' implementation of standards-based instructional practices, Johnson (2007) found that, in order to facilitate change in beliefs and practice, professional development providers need to include time for participants to engage in experiences with the instructional practices and offer opportunities for collegial dialogue. Stein and Wang (1988) observed 14 kindergarten through fourth grade classrooms in three pubic elementary schools during their first year of implementing the Adaptive Learning Environments Model. Findings revealed a statistically significant increase in individual teacher's self-efficacy following the implementation of instructional practices. Participants were followed from fall to spring during the first year of implementation. Interestingly, the greatest increase of program implementation occurred from fall to winter, whereas the greatest increase in self-efficacy occurred from winter to spring.

Furthermore, Fennema et al. (1996) and Franke, Carpenter, Levi, and Fennema (2001) examined changes in classroom practices and teachers beliefs for a total of seven years after implementation of professional development focused on children’s mathematical thinking. Fennema et al. (1996) conducted a three-year longitudinal study to explicitly examine the nature and pattern of change among 21 teachers and the relation between beliefs and instruction. During the time of the study, participants attended follow-up sessions and engaged in collegial conversation with an assigned mentor teacher as well as other colleagues, while at the same time implementing practices focused on children’s mathematical thinking. In year one, participants attended a follow-up session each week. By year two, follow-up support was about once every two weeks, and in year three, support was offered only occasionally. By the end of the study, 18 participants had grown at least one level in beliefs and practice and 12 had grown two levels.

Four years later, Franke et al. (2001) found that all teachers continued to implement principles from Cognitively Guided Instruction professional development on some level. Ten teachers demonstrated continued growth in beliefs and practice on the researcher-designed levels. Case studies (Carpenter et al., 2000; Fennema et al., 1993; Franke et al., 2001) confirm these findings and provide rich descriptions of teacher change and the ways teachers implemented CGI principles in the classroom. In summary, “developing an understanding of children’s thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own students” (Carpenter et al, 2000).

Professional Noticing of Children's Mathematical Thinking

Teacher noticing encompasses the processes through which teachers manage the "blooming, buzzing confusion of sensory data." Teacher noticing as a component of expertise is well documented (M. G. Sherin, V. R. Jacobs, & R. A. Philipp, 2011). M. G. Sherin et al. (2011) identifies two main processes to teacher noticing: 1) attending to particular events in an instructional setting, and 2) making sense of events in an instructional setting. The focus is on how, at a fine-grained level, the teacher interacts with the classroom work rather than solely on a teacher's reasoning. Decisions of research design and data analysis are made under the same assumptions as those outlined in the theoretical construct described as professional noticing (M. Sherin, V. Jacobs, & R. Philipp, 2011). Understanding a teacher's thinking and action while embedded in a classroom context is important to the purpose of this study, because it is in the classroom context where many of the Guiding Principles of Classroom Teaching will be most readily observed. Jacobs, Lamb, Philipp, and Schappelle (2011) collected data on the professional noticing of teachers engaged in sustained professional development focused on children's mathematical thinking. Three groups of practicing K-3 teachers and one group of prospective teachers totaling 131 individuals were asked to view the first of two videos featuring a kindergartener and react in writing to a provided prompt. Afterward, participants were shown the second video, which included the child's solution strategy to a word problem. Participants were then asked to provide a written account of what the child said and did. Analysis of responses revealed that if teachers decide how to respond on the basis of children's understandings, they are likely to also attend to children's strategies. However, if teachers attend to children's strategies, they may or may not decide how to respond on the basis of the understandings reflected in those strategies. With continued professional development support, teachers can become more skilled in deciding how to respond on the basis of children’s strategies. Being skilled in deciding how to respond based on children’s strategies is a significant component within the Guiding Principles for Classroom Teaching.

Summary

The purpose of this study is to contribute to research on the impact of professional development on classroom practice, specifically investigating what teachers are able to implement in the classroom after an Add+VantageMR® course. The literature review begins with core features of effective professional development followed by a description of effective classroom practice. The literature review then connects core features of effective professional development and effective classroom practices to Add+VantageMR®. An overview of research on the process of teacher change and the impact of professional development on the change process substantiates the reasonableness of claiming that teachers will have some kind of a change in classroom practice as a result of participating in Add+VantageMR®. To be able to identify changes in classroom practice at any point of the complex teacher role, two research questions draw heavily upon perspectives of professional noticing of children’s mathematical thinking and contribute to studies in this area.

This study contributes to the body of research on the effectiveness of Math Recovery® theory applied in AVMR professional development and serves as one of a few studies to specifically investigate the Add+VantageMR® program. This is one of the first studies to identify specific changes in classroom practice after an AVMR course.

Findings from this study hold implications for school districts investing in professional development for mathematics teachers with a goal to influence classroom practice and student achievement, as well as districts which have already invested in Add+VantageMR® programs.

Chapter Two provided a review of the literature around six topic areas related to the research questions. Chapter Three presents the methods used to answer the research questions. Chapter Four includes the results, while Chapter Five presents a summary of the study and a discussion of the results.

Chapter Three – Methods

This study was designed to examine which teaching practices, if any, teachers implement in the classroom from Add+VantageMR® professional development. This is determined by examining teachers' *planning for* and *enacting* mathematics lessons in the area of early number and operations. A case study approach (Yin, 2014) provides the opportunity to create a detailed description of each classroom in order to locate even the smallest change in classroom practice. Student achievement results in the content strands of number words and numerals and addition and subtraction to 20 are used to reference the potential impact on student achievement due to the changes made in classroom practice. Chapter One presented the rationale for the study and identified the research questions. Chapter Two reviewed the literature in five areas: (a) effective professional development and effective classroom practice, (b) theoretical underpinnings of Add+VantageMR® professional development, (c) research on Add+VantageMR® professional development, (d) translating professional development into classroom practice, and (e) professional noticing of children's mathematical thinking. This chapter presents a detailed look at the context for the study, a description of the study, and an explanation of how the data was collected and analyzed.

The study took place in a third ring Midwestern suburb during the first four months of the 2015-16 school year. Add+VantageMR® professional development has been a part of the district's goals and initiatives for mathematics since 2008. Over the past 10 years, district coaches created resources for teachers linking AVMR course content to state standards, district adopted curriculum, and district mathematics frameworks. A robust website houses these curriculum documents along, with a variety of additional links to materials that teachers can choose from when planning curriculum. The district's commitment to Add+VantageMR® made it an ideal site for this study. School and other district initiatives align with the same core values about teaching and learning as Add+VantageMR®. This is important to be able to greater connect changes in classroom practice to participation in AVMR courses. Study participants were determined through a search for volunteers. Individual school administrators were contacted by an email letter seeking study volunteers. Final study participants were determined based on the number of volunteers from a common grade level, administrator support for the initiative, and teacher enthusiasm for participating in professional development with the acumen to implement ideas after the course(s).

As previously mentioned, a case study approach provides the opportunity for an in-depth investigation of the classroom environment and how the teacher and students interact within that environment. A case study approach identifies nuances within teacher thinking, planning for lessons, and the moment-to-moment decisions teachers make during a lesson. Based on the core features of Add+VantageMR® professional development and the overarching research goal for this study, the following research questions and subtopics are addressed. Subtopics become guiding questions used during analysis and provide the structure for each case study description.

1. In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?
    a) In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?

b) What are teachers using to determine children's current level of understanding?

c) In what ways do teachers use course resources provided in course Teacher Handbooks or *Teaching Number in the Classroom with 4-8 Year-Olds* (Wright et al., 2014) and/or *Developing Number Knowledge* (Wright et al., 2011) when planning a whole number lesson?

d) In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number as well as skills that support learning about whole number operations?

2. In what ways does Add+VantageMR® professional development influence teachers' instructional moves during a whole number lesson?

   a) In what ways are students engaged in activities from the course resources, such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 Year-Olds* and/or *Developing Number Knowledge*?

   b) In what ways are students actively engaged in *doing* mathematics during the lesson?

   c) In what ways does the teacher use Guiding Principles for Classroom Teaching and domains of progressive mathematization during a whole number lesson?

3. How does Add+VantageMR® professional development influence differentiation of a whole number lesson?

   a) In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?

   b) In what ways do teachers modify an activity during a lesson (modification may include offering different dice, number ranges, and/or instructional tools)?

4. How does Add+VantageMR® professional development influence teachers' assessment of students learning during implementation of a whole number lesson?
   a) In what ways do teachers make in-the-moment adjustments to the written lesson plan based on student responses, demonstrated understandings, and knowledge about children's thinking as they are related to the Learning Framework in Number?
   b) In what ways do teachers describe children's strengths and areas of need for whole number understanding with specificity that is connected to observed student solution strategies and the Learning Framework in Number?
   c) In what ways do teachers collect formative assessment data during the lesson that includes a student-thinking component?

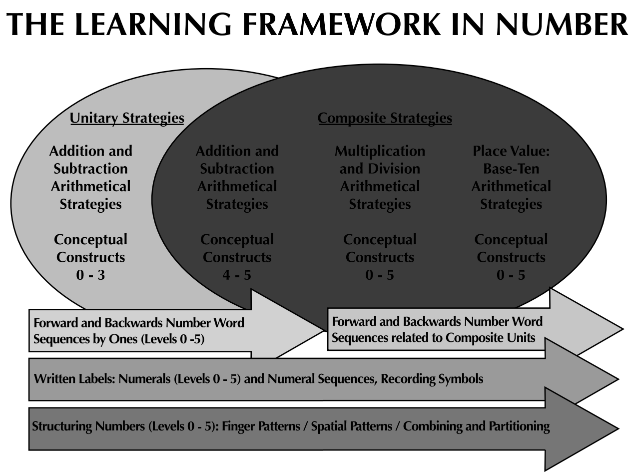
Add+VantageMR® Professional Development

This section provides greater detail on the structure and core features of Add+VantageMR® courses. Add+VantageMR® professional development includes a content focus on children's thinking and formative assessment, active learning opportunities during the course, coherence with standards and district initiatives, over twenty hours of contact time, collective participation with school colleagues. The core content focus features of Add+VantageMR® professional development follows a common structure for all courses. These core content features include teacher learning about research on children's thinking, formative assessment and pedagogical tools. These core features are organized in Three Grand Organizers.

Each course begins with a topic of number learning and engages participants in research about student thinking in that topic by introducing a model of student thinking and teaching using student video examples. Participants then study a diagnostic assessment to identify students' knowledge in that topic. Participants practice administering the assessment during the course by video-recording themselves administering course assessments to students. By reviewing the video recording, participants reflect on administration techniques and learn to analyze the assessment according to the model of student thinking. Finally, each course concludes by providing participants with pedagogical tools and engaging activities to support instructional decision making based on student assessment results in the classroom. Three Grand Organizers further describe the core theoretical assumptions of all Add+VantageMR® courses. The Three Grand Organizers are as follows: (a) six domains of arithmetic knowledge, (b) guiding principles for classroom teaching, and (c) dimensions of mathematizing. The following three paragraphs provide a short description of each of the three Grand Organizers.

Six Domains of Arithmetic Knowledge

The six domains of arithmetic knowledge are represented in the Learning Framework in Number. The Learning Framework in Number provides teachers with a way to form connections within and across content topics. **Error! Reference source not found.** shows the Learning Framework in Number graphic used throughout Add+VantageMR® course materials. The six domains or topics are shown in the ovals and arrows. Ovals represent conceptual topics requiring cognitive reorganization by the student (Steffe, 1988) and consist of the following topics: addition and subtraction, multiplication and division, and place value. Arrows represent topics considered to be skill-based and are necessary for the advancement of conceptual understandings represented in the ovals. Topics represented in the arrows are as follows: forward number



word sequences, backward number word

Figure 3.1. ***The Learning Framework In Number*****. (2005). Add+VantageMR® Course 1 Teacher Handbook.**

sequences, numeral identification, written numerals, and structuring numbers.

Add+VantageMR® Courses consist of two four-day courses organized by Domains of Arithmetic Knowledge. Add+VantageMR® Course One includes the following three domains: (a) addition and subtraction to 20; (b) structuring numbers, including finger and spatial patterns, number combinations, and partitions to 20; (c) number words and numerals, including forward and backward number word sequences, and numeral identification. Add+VantageMR® Course Two includes (a) multiplication and division and (b) place value concepts, including conceptual place value and multi-digit addition and subtraction.

Guiding Principles for Classroom Teaching

The Guiding Principles for Classroom Teaching are a result of researchers working with teachers and school systems during several Math Recovery® research and development projects in the 1990's (Wright, Stanger, Stafford, & Martland, 2006). The Guiding Principles provide a framework for teachers to think about instruction. Wright et al. (2006) outlines all nine principles and provides a short description for each. The Guiding Principles can be used to reflect upon individual practice. These Principles also offer a lens in which to engage in collegial conversations focused on instructional change. Rarely would all nine Guiding Principles be present in one lesson. Likewise, individual teachers may be well-versed in implementing some of the Principles and have other Principles yet to develop in practice.

Table 3.1. Guiding Principles for Classroom Teaching.

Guiding Principles for Classroom Teaching	Principle Description
Principle One The teaching approach is inquiry-based, that is, problem-based.	Students routinely are engaged in thinking hard to solve numerical problems which for them are quite challenging.
Principle Two Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.	This refers to the teacher's informed understanding of students' current knowledge and problem-solving strategies, and continual revision of this understanding.
Principle Three Teaching is focused just beyond the "cutting edge" of students' current knowledge.	This principle accords with Vygotsky's notion of zone of proximal development, that is, instruction should be focused just beyond the child's current levels of knowledge in the areas where the child is likely to learn successfully through sound teaching.

Principle Four Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involving particular instructional settings and tasks, and varying this selection on the basis of on-going observations.	This principle highlights the need to develop a bank of instructional procedures and to understand the role of each procedure, in terms of its potential to bring about advancements in children's current knowledge.
Principle Five The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.	This principle highlights the need for teachers to have a working model of children's knowledge of early number and the ways in which children's knowledge typically progresses. The belief is that teachers can develop an appropriate working model through reading, reflecting and observing, in conjunction with their teaching practice.
Principle Six Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.	This principle highlights the importance of observational assessment in determining children's specific learning needs, and the need for this assessment to be ongoing and to lead to action, that is, the fine0tuning of instruction on the basis of ongoing assessment.
Principle Seven Teaching supports and builds on students' intuitive, verbally-based strategies, and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.	This principle highlights that children's initial number knowledge is by and large verbally based rather than involving written forms. Thus we suppose that children's initial counting and calculating strategies mainly involve mentally computing with sound images of number words and number word sequences. The further development of number knowledge involves a gradual process of incorporating written symbols, including but not limited to numerals, and linking these symbols to already acquired verbally based number knowledge.
Principle Eight The teacher provides students with sufficient time to solve a given problem.	Students are frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking, and reflecting on the results of her or his thinking. This problem-solving and the mental processes of thinking hard and reflecting during problem-solving are a fundamental aspect of early number learning.
Principle Nine Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.	When young children work hard at problem-solving and their problem-solving is successful, this is typically a very positive experience for the learner. This kind of learning constitutes an intrinsic rewards beyond processes such as teacher affirmation and peer recognition.

Dimensions of Mathematizing

Mathematizing means to bring about greater sophistication to some mathematical activity (Wright et al. 2012). Dimensions of Mathematizing describe ways in which students advance their mathematical thinking and refers to the teacher moves that facilitate that advancement. Table 3.2 identifies all eleven dimensions related to mathematizing that are used within Add+VantageMR and provides a short description of each.

Table 3.2. Dimensions of Mathematizing.

Dimension of Mathematizing	Description of the Dimension
Complexifying Arithmetic	Develop more parts or more directions within an operation. Example: addition result unknown to missing addend.
Distancing the Setting	When a student begins by using an instructional setting or physical model and then is distanced from that setting or model using the following process: 1) students manipulate the materials, 2) students see the materials but do not manipulate them, 3) students see the materials only momentarily, and 4) students solve a task posed in verbal or written form without materials.
Extending the Range	A task is posed using higher numbers. Example 1-5, 0-10, 0-20, 0-100, 0-200, 0-1000, beyond 1000.
Formalizing Arithmetic	Placing more significance to form, especially in notating and language. Example: going from arrow notation to formal number sentences; developing more sophisticated language (from “take away” to “subtract”).
Organizing and Generalizing	Organizing involves forming categories. Example: separate ten-frame cards into five-wise (5+) and pair-wise (doubles) configurations; list partitions of 6 in order 0+6, 1+5, 2+4, 3+3. Organizing is closely aligned with generalizing.
Notating	A teacher can notate a students’ strategy or students can notate their own or others’ strategies. Notation can be a number sentence or recording a certain strategy, such as a jump strategy (34+45; 34 + 40 -> 74+5 -> 79)
Refining Computation Strategies	Students reflect on and discuss their computation strategies. During discussions, a teacher can draw attention to number relations and encourage efficiency, flexibility, and insight in computation. Additionally, the teacher can pose specific numbers in order to encourage particular strategies.

Structuring Numbers	Students notice and use number relations and develop a dense network of number relations. Example: describing an 8-dot ten-frame as 5-and-3, or as 10-less-2, or using “8 is close to 10” to solve 8 + 7.
Decimalising Numbers	Developing the practice of organizing numbers into ones, tens, hundreds, thousands, and so on.
Unitizing Numbers	Students come to regard numbers as units, a single whole object to be counted and acted upon. Example, the number of 3s in 12 is one 3, two 3s, three 3s, four 3s; each 3 is regarded as a unit.

AVMR provides classroom teachers with diagnostic interview assessments and teaching trajectories. Both provide classroom teachers with the pedagogical tools necessary to identify and meet students’ mathematical strengths and areas of instructional need.

Participants in AVMR courses learn about developmental models of student thinking and assessments by viewing authentic video of students engaged in mathematical problem solving. Participants then practice assessment administration and analysis as part of the course. Practice assessments are video-recorded by participants to allow for a fine-tuned look at student thinking and to provide teachers the opportunity for self-reflection. Each course concludes with a focus on using assessment results to inform instructional decisions. Course contact hours include time for participants to engage in activity and reflection. Participants engage in instructional tasks utilizing settings proven to be effective in advancing student mathematical thinking. It is the perspective of AVMR that settings take on an intentional role in facilitating the advancement of student mathematical thinking. Therefore, learning to fine-tune instructional activities based on student assessment results is an integral part of AVMR course goals.

Setting

The school district serves approximately 27,000 students from early childhood through grade 12. There are 18 elementary schools, of which three are magnet schools that focus on arts and science, international studies, and STEM. The school district first implemented Add+VantageMR® professional development in the 2006-7 school year when the program was first released in the United States. AVMR's focus on professional development, along with diagnostic interview-based assessment in early number aligned with district values and an existing goal to implement a district-wide assessment in kindergarten through grade two. Early implementation allowed for teacher and building choice. AVMR courses were financially supported at the district level; however, initial participation was voluntary at both the building and teacher level. In 2009, the school district started to require Add+VantageMR® Course One for all teachers in grades kindergarten through grade two. This requirement was still in place at the time of the study. The school in which this study took place has been classified as a STEM magnet school since 2006, as a response to identification as a racially identifiable elementary school by the State Department of Education. At the time of the study, the student population was identified as 2.0 percent American Indian/Alaskan Native, 15.3 percent Asian/Pacific Islander, 12.6 percent Hispanic, 25.7 percent Black, and 44.5 percent white. English Language Learners made up 11.7 percent of students, 11.2 percent receive special education services, and 46.8 percent utilize free/reduced priced school lunch. The elementary school has seen a rise in mathematics achievement scores from 2011-15, with more recent percentages of students proficient on the state mathematics assessments from

2013-15 being 64.4 percent, 68.3 percent, and 71.9 percent, respectively. Class sizes range from 22-24 students in each primary class.

Participants

Study participants included one male and two female teachers. Initially, a third grade class was included in the study, but was ultimately omitted because after the study was underway, the third grade team started to ability group for mathematics. As a result, only a portion of students in the class at the beginning of the study remained in the classroom for mathematics. The three teachers completing the study had varied experiences with Add+VantageMR® Courses One and Two. One first grade teacher, Julie, is a first-year teacher. She participated in Add+VantageMR® Course One in late September and early October and Course Two in late November and early December. Mark was in his 25th year of teaching, with all of those years in kindergarten or first grade. He first participated in Add+VantageMR® Course One in 2009 and Course Two as a two-day course in 2010. During the current study, Mark joined his colleagues to participate in Add+VantageMR® Course Two in the current four-day format. Carin has been teaching at this elementary school for five years, four of those years in first grade. She took Add+VantageMR® Course One during her first year in the district and Course Two during the study in late November and early December. The different cases provide different perspectives on what teachers take away from an Add+VantageMR® professional development course. Mark is well established as a district and school leader, serving on several school and district committees. He often finds himself trying to capture every minute possible for preparing lessons. Jessica's role is split between first grade math in the morning and literacy intervention and kindergarten social skills in the afternoon. Although she is pulled in many directions as a part of her role, she has the most time allocated for mathematics and has a desire to learn as much as possible; in her first year of teaching, she expressed feelings of not being adequately prepared to teach mathematics. It is common in teacher preparation programs in this area to have more courses in literacy than in mathematics instruction in Initial Licensure Programs.

Research Design

This study used a holistic multiple-case case study design approach (Yin, 2014). A multiple-case study design allowed for an examination of a number of variables at play when examining the impact of AVMR professional development in real context. Figure 3.1 describes the participant background at the start of the study.

Data Collection

- Teacher Interview
- Classroom Observations
- Informal post-observation interview
- Student AVMR Assessment data

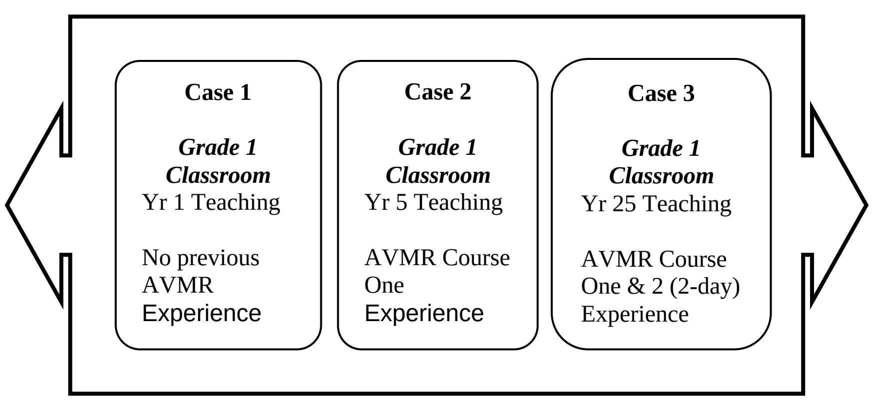


Figure 3.1 Overview of the Research Design

As shown in Figure 3.1, each classroom was treated as a separate and distinct bounded unit of study in order to look at patterns occurring within and across cases as related to the research questions. A three-case design allows for a stronger study by allowing for analytic conclusions and theoretical replication to be drawn across multiple cases, as opposed to results from a single case.

Methods

Data collection methods included teacher interviews, classroom observations, informal post-observation interviews, and AVMR pre- and post-student assessment data. Each data source was used to address research questions one through four pertaining to *planning for* instruction and *enacting* the planned lesson. Table 3.3 provides a timeline for data collection organized by research question.

Table 3.3. Data Collection Timeline.

Research Question	Aug/Sept	Oct	Oct	Nov/Dec	December
Q1: Planning for instruction	Teacher Interview Post Observation Interview Student Data	**Add+VantageMR® Course One**	Teacher Interview Post Observation Interview	**Add+VantageMR® Course Two**	Teacher Interview Post Observation Interview Student Data
Q2: Teaching moves during a Lesson	Class Observation Post Observation Interview		Class Observation Post Observation Interview		Class Observation Post Observation Interview
Q3: Differentiation	Teacher Interview Class Observation Post-Observation Interview Student Data		Teacher Interview Class Observation Post-Observation Interview		Teacher Interview Class Observation Post-Observation Interview Student Data
Q4: Assessment of student learning	Teacher Interview Class Observation Post-Observation Interview Student Data		Teacher Interview Class Observation Post-Observation Interview		Teacher Interview Class Observation Post-Observation Interview Student Data

Details pertaining to data collection are presented next.

Data Collection

Classroom Observations. Nine classroom observations for each teacher took place over the course of the study. Classroom observations, each consisting of three sequential lessons, took place over a four-month period. The first classroom observation occurred in September, a second in mid-November, and the final observation in December. Each lesson was video-recorded to allow for later analysis. Field notes and picture artifacts documented aspects of the lesson not visible to the video camera. Following each lesson, an informal post-lesson interview provided an opportunity for classroom teachers to share personal insights on the observed lesson. Often, personal insights included self-observations, reflections, and intended adjustments to the next day's lesson.

Semi-structured Interview. Semi-structured interview questions are open-ended in nature and determined in advance to the interview (Patton, 2002). Each individual interview focused on participant thinking on areas of quality math instruction, lesson design, and implementation. Teachers participated in a pre- and post- semi-structured interview; the first interview served as a baseline, while the second interview provided evidence of change in teachers' vision for high quality math instruction. The first interview took place at the beginning of the school year, in some cases during workshop week. The final interview took place for all participants at the conclusion of the study in December. One participant completed AVMR Course One during the study and participated in a semi-structured interview afterward, therefore having a total of three interviews.

For both the pre- and post-interview, *The Vision of High-Quality Mathematics Instruction Instrument* (Munter, 2014) was utilized to capture teacher vision for the role of the teacher, high quality math tasks, classroom discourse, and student engagement. The pre- and post-interview protocol started with one simple question: “If you were asked to observe another teacher’s math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?” The interviewer can choose from several pre-planned follow-up probes to glean more detail on the participant’s perspective of high quality math instruction. For example, in Mark’s opening response he stated: “Well, the kids would be engaged, [laughing] you would see students engaged and talking about whatever the activity was. You know. They would be using that vocabulary that applied to the activity.”

To follow-up the interviewer asked, “What might the activity be like?”

Follow-up probes are meant to add depth and specificity to responses in order to allow for a complete description of participant’s vision of high quality math instruction.

The post-interview included additional questions, such as, “What could be done to make it easier to use AVMR data in the classroom?” which provided further insight into phenomenon arising out of classroom observations. The complete interview protocols for both pre- and post-interview protocols, including the full VHQMI instrument and rubrics, can be found in the Appendix.

Student Data. The researcher conducted video-recorded pre- and post- student assessment interviews. Diagnostic interviews resulted in a numeric value representing a level or stage of students’ mathematical understanding in each of the domains comprising the Learning Framework in Number. Student assessment data was used to support classroom observation findings in research questions one, two, and three. Assessment data was shared with participating classroom teachers, providing an opportunity to investigate how teachers used AVMR data when planning for or enacting a whole number lesson.

One-on-one interview video recorded student assessment data provided each participating classroom teacher with a level or construct for the following five topics: addition and subtraction, forward number word sequence, backward number word sequence, numeral identification, and structuring numbers. The interview assessment took approximately ten to fifteen minutes to administer all assessment tasks. The assessment is dynamic in that the interviewer makes decisions during the assessment based on how a student responds to a set of tasks. For example, the assessment begins by saying, "Start counting from one until I tell you when to stop." The assessor then prompts the student to stop the sequence at 32. A successful attempt indicates the next task to be, "Start counting from 58 until I tell you when to stop." An unsuccessful attempt ends forward number word sequences and the interviewer moves to asking number word after tasks in the range of one-10. Assessment tasks include verbal sequencing tasks, numeral identification, writing numerals, finger patterns, dot patterns, covered addition and subtraction tasks using counters and screens, and bare number addition and subtraction tasks. Excerpts of the assessment can be found in Appendix A.

Assessments, administered and analyzed by the researcher, took place in September and December. Classroom teachers were provided completed assessment schedules along with a spreadsheet containing benchmark achievement levels and constructs for first graders during that time in the school year. To obtain a level or construct, student responses are placed upon the corresponding model of student thinking. For example, the complete set of student responses for the forward number word sequences and number word after tasks referenced previously are placed on the model for forward number word sequence as shown in Table 3.4. A level is chosen based on the description that best aligns with student responses on the assessment. Add+VantageMR® assessment seeks to determine the most advanced level in which a student demonstrates facility.

Table 3.4. Model for Forward Number Word Sequences.

Level	Description of student behaviors
Level Zero Emergent FNWS	The student cannot produce the FNWS from "one" to "ten."
Level One Initial FNWS to "ten"	The student can produce the FNWS from "one" to "ten." The student cannot produce the number word just after a given number word in the range "one" to "ten." Note: Students at Levels 1, 2, and 3 may be able to produce FNWS beyond "ten".
Level Two Intermediate FNWS to "ten"	The student can produce the FNWS from "one" to "ten." The student can produce the number word just after a given number word in the range "one" to "ten", but drops back to generate a running count when doing so.
Level Three Facile with FNWS to "ten"	The student can produce the FNWS from "one" to "ten." The student can produce the number word just after a given number word in the range "one" to "ten" without dropping back. The student has difficulty producing the number word just after a given number word for numbers beyond "ten."
Level Four Facile with FNWS to "thirty"	The student can produce the FNWS from "one" to "thirty." The student can produce the number word just after a given number word in the range "one" to "thirty" without dropping back. Note: Students at this level may be able to produce FNWS beyond "thirty."
Construct Five Facile with FNWS to "one hundred"	The student can produce FNWS in the range "one" to "one hundred". The student can produce the number word just after a given number word in the range "one" to "one hundred" without dropping back. Note: Students at this level may be able to produce FNWS beyond "one hundred."

Data Analysis

Teacher interview transcripts, classroom observation video analysis, post-observation interview field notes, and student AVMR assessment data results allowed for *data triangulation* (Patton, 2002) during analysis. Two of the three aforementioned core theoretical assumptions of AVMR professional development, Math Recovery® Guiding Principles for Classroom Instruction and a subset of the Dimension of Mathematizing, provided the framework for data analysis. Details pertaining to data analysis are presented next.

Classroom Observations. Two rounds of coding were applied to classroom observation video prior to formal analysis. The Guiding Principles for Classroom Teaching formed the basis for round one structural coding. Structural coding is defined as the application of a content-based or conceptual phrase representing a topic of inquiry to a segment of data that relates to specific research questions (Saldana, 2013). Math Recovery® Guiding Principles for Classroom Teaching were used as topics of inquiry and applied to classroom observation video. This ensured the use the Guiding Principles as labels and allowed for refinement of codes.

As a result of round one structural coding, five Dimensions of Mathematizing and three researcher created codes were added to initial codes. Four Dimensions of Mathematizing were added as descriptors under Guiding Principle Five: the teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies. Together, the four dimensions, a) distancing the setting, b) extending the range, c) complexifying arithmetic, and d) refining computational strategies describe teacher moves related to engendering more sophisticated mathematical strategies. Additionally, the dimension of notating was added to Guiding Principle 7: *teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic*.

In both cases, the Dimensions of Mathematizing provided specificity to each case, allowing for a rich description during data analysis. Three researcher created codes identified teacher behaviors considered to be in contrast to the intent of Math Recovery® Guiding Principles. Table 3.5 includes these new codes; a) providing a mathematically untrue statement, b) traditional telling to teach an important concept, and c) use of mathematical symbols detached from student thinking, along with a complete list of codes established through round one coding. Codes listed in Table 3.5 were applied during round two coding.

Table 3.5. Codes applied to Classroom Observation Video.

In Contrast of a Guiding Principle for Classroom Teaching
Providing a mathematically untrue statement
Traditional telling to teach a concept
Symbols detached from student thinking
Guiding Principles for Classroom Teaching
Principle One: Inquiry –based teaching *Dimension of Mathematizing* Problem centered task
Principle Two: Teaching Informed by assessment *Dimension of Mathematizing* Task based on Learning Framework in Number & aimed at student's current levels of understanding
Principle Three: "cutting edge" teaching *Dimension of Mathematizing* Teacher accounts for a range of student levels of understanding
Principle Four: Select from a bank of procedures
Principle Five: Engender more sophisticated strategies *Dimensions of Mathematizing* Distance the setting Grounded habituation Extending the range Complexifying arithmetic
Principle Six: Intense, ongoing observation *Dimensions of Mathematizing* Scaffolding a student through a task Micro-adjusting the task

Principle Seven: Teaching builds on verbal-based strategies *Dimensions of Mathematizing* Teachers' notation/symbolize student thinking Students verbalize mathematical strategies
Principle Eight: Sufficient time to solve problems
Principle Nine: Students experience intrinsic satisfaction from engaging in mathematical tasks

Second round coding included two passes through classroom observation video. One pass observed teacher moves and classroom structures, such as questioning; task presentation; teacher-student interactions in whole-group, small group or one-on-one situations; flexible student groupings; and available settings or manipulatives. A second pass observed student characteristics, such as engagement with the mathematics within the tasks, talk with other students and the teacher about mathematical solutions, and whether students were exhibiting intrinsic satisfaction from coming to mathematical solutions. In addition to applying codes displayed in Table 3.5, analytic memos provided a way to capture anecdotal notes during second round coding. Together, the application of codes and analytic memos provided a basis for identifying patterns related to each of the research question subtopics.

Teacher Interviews. As described in the data collection section, the pre- and post-interview protocol included the *Vision for High Quality Mathematics Instruction* interview protocol in the pre-interview, and the VHQMI protocol along with researcher created questions in the post-interview. VHQMI rubrics provided a level and descriptor for each of four dimensions: a) the role of the teacher, b) high quality math tasks, c) classroom discourse, and d) student engagement. Each of four levels indicates a relationship of alignment with instruction and research on high quality reformed based math classrooms. For example, a teachers' vision for the role of the teacher in the mathematics classroom that is closely aligned with reform teaching research on this same topic would be considered a level four on the VQHMI *role of the teacher* rubric. A teachers' vision for the *role of the teacher* in the mathematics classroom that is in opposition to research on high quality reformed based mathematics classrooms would be considered a level one on the rubric. Analysis procedures required alignment of each dimension to the Math Recovery® Guiding Principles for Classroom Teaching as shown in Table 3.6.

Teachers' vision for the types of mathematical tasks posed in a high-quality math classroom aligns with Guiding Principle One, Inquiry Based Teaching. VQHMI dimension Classroom Discourse consists of five sub-topics: 1) Patterns/structure of Classroom Talk, 2) Nature of Classroom Talk, 3) Student Questions, 4) Teacher Questions, and 5) Student Explanations. Sub-topics three and four, Teacher and Student Questions, align with Guiding Principle five, to engender more sophisticated strategies. Subtopics one, two, and five, Student Explanations and Patterns and Nature of Classroom Talk, align with Guiding Principle seven: teaching builds on verbal-based strategies. Finally, the VQHMI dimension Role of the Teacher aligns with Guiding Principle one, three, four, six, eight, and nine.

Table 3.6. Alignment of VHQMI Rubrics to Math Recovery® Guiding Principles for Classroom Teaching.

VHQMI Dimensions	Guiding Principles for Classroom Teaching
Mathematical Tasks	Principle One: Inquiry-based teaching

Classroom Discourse Teacher Questions Student Questions	Principle Five: Engender more sophisticated strategies
Classroom Discourse Patterns/structure of Classroom Talk Nature of Classroom Talk Student Explanations	Principle Seven: Teaching builds on verbal-based strategies
Role of the Teacher	Principle One: Inquiry-based teaching Principle Three: “cutting edge” teaching Principle Four: Select from a bank of procedures Principle Six: Intense, ongoing observation Principle Eight: Sufficient time to solve problems Principle Nine: Intrinsic satisfaction from solving problems that are determined to be quite challenging

To perform analysis, pre-and post-interviews were transcribed and transcriptions provided the basis for applying VHQMI level descriptors. A positive change in VHQMI level descriptor from pre- to post-interview supports a reported change in teachers’ vision of high quality math instruction after taking AVMR courses. For example, in the initial interview, Carole stated the role of the teacher is:

> …to have a framework for the lesson….like introducing the lesson as a whole group in front of the class with a SMART Board or whatever one desires or has access to. Teacher engages with students while walking around so that students do not get off track. A lot of informal assessment is important while walking around. The teacher is also a middle person when students are solving problems. Students can lead the lesson and the teacher intervenes when needed … when students are getting way off track.

Carole’s description fits the VQHMI rubric for level 2, Teacher as “Monitor.”

In December Julie’s description for the role of the teacher established a changed vision for the role of the teacher. For example, in the post-interview, Carole stated:

> I guess my lesson structure, I start with a mini-lesson, where I’m teaching all students, and then we break up into groups and do rotations. So, I am always with students, and the smaller groups … they’re leveled to where they’re at to certain assessments. As for me, I mean... I guess… I am always trying to meet them

> where they are at and try to push them further, but I am also wanting them to think for themselves too and try to, like, help them solve the problem together and not just give them. I think I am always formally and informally assessing them, too. Always checking where they are at and making sure …um… and that is my tricky part... is now making sure that the stuff I have for them in the groups *is* going to meet all their needs and so always trying to.... if I feel a student is getting more behind or moving up, like, always trying to pull them aside and do small little assessments to see if they can do what I have them [doing] or if it is too easy,"

The post-interview provides evidence for change in the role of the teacher. In September, Carole spoke of the classroom lesson consisting of the teacher at the front of the room leading a lesson, and then afterward circling the room to find students who are struggling so they do not "fall off the wagon." The December interview indicates a vision for the role of the teacher as a level four, more knowledgeable other. A small group rotation structure guides the lesson time. Julie is with students during that time and speaks of providing tasks for students to advance mathematical thinking.

The validated rubrics provide strength to the reported teacher change in vision from September to December. Classroom observations, teacher interview transcripts, informal post-observation interviews, and student assessment data provide the basis for each case description provided in Chapter 4. Guiding Principles for Classroom Teaching and Dimensions of Mathematizing set the framework to address the potential impact, if any, Add+VantageMR® professional development has upon the decisions teachers make when *planning for* and *enacting* mathematics lessons in the area of early number and operations.

Chapter Three presented the methods used to answer the research questions. Chapter Four includes the results, while Chapter Five presents a summary of the study and a discussion of the results.

Chapter 4 – Results

The purpose of this study is to examine the ways, if any, classroom teachers incorporate ideas from Add+VantageMR® (AVMR) into their math classroom teaching practice. Teacher interviews, classroom observations, informal post-observation interviews, and AVMR pre- and post- student assessment data were used to address four research questions for each of three teacher participants. Research question 1 seeks to understand the ways AVMR professional development influences teachers planning for a whole number lesson. Research question two investigates the ways AVMR professional development influences teachers' instructional moves during a whole number lesson. Research question three isolates differentiation methods to understand the ways AVMR professional development influences teachers' differentiation of a whole number lesson. Finally, research question four specifically addresses teachers' assessment of student learning during a whole number lesson. Teacher interviews and informal post-observation interviews provided a lens into the teachers' thinking prior to, during and after mathematics lessons. Classroom observations were used to identify changes in teacher instructional moves or responses during a whole number lesson as well as the types of assessment data teachers use in the moment of teaching a lesson and how they respond to their in the moment assessment of students' learning. Pre-and post AVMR assessment data identify changes in student achievement over a semester time to overlay onto changes in teacher practices before and during whole number lessons.

Each case represents a unique teacher perspective. Although each of the three participants teach first grade at the same school, significant differences exist in what they take away from Add+VantageMR® professional development and how they engage with that content in the classroom.

Overview of Teachers' Pedagogy

The three case descriptions that follow illustrate individual differences in teaching style and a classroom vision for teaching and learning. All three teachers appreciate and pursue opportunities to learn, to engage in committee work outside of regular teaching duties and to strive to actively engage students in mathematics lessons with the aim to advance students' mathematical knowledge and skills. All three teachers share a common goal to advance students' mathematical thinking in whole number and go about reaching that goal in three different ways. Carole begins each lesson with 15-20 minutes of whole group instruction, then continues the lesson by assigning students work to complete independently or in small table groups based on the whole group lesson. Each lesson concludes with a 5-minute whole group discussion wrapping up the activity and days' learning. Carole's lessons consistently follow this same structure throughout the study.

Mark begins each lesson with a whole group warm-up. The warm-up typically focuses on a specific skill necessary for advancing students' mathematical proficiency within whole number concepts and computation. Following the warm-up, Mark typically introduces an activity for students to complete individually or with a partner. Finally, most lessons conclude with a short whole class closure on the activity before dismissing students to go home.

At the initial interview, Jennifer stated it to be a goal to figure out a math workshop model. Beginning of the year observations revealed a classroom mathematics structure to be mainly a large group lesson followed by student work time. About one month into the study, Jennifer established a workshop model and continued that structure throughout the study. Students were placed into teacher selected small groups. Math time began with a whole group warm-up and then students rotated through 5 stations where one station was working with Jennifer.

Chapter Overview

The next section of this chapter is organized around these three cases in the following order: Carole, Mark and Jennifer. For each case, background information on the teacher is initially summarized. Then, each section is organized by research question and for each of the four research questions, sub-topics used during analysis are listed as guiding questions. Not all stated Guiding Principles are used for each case. Selection of Guiding Principles are dependent on ways individuals approach teaching mathematics. These sub-topic guiding questions provide the structure for each case study description. Following sub-questions Guiding Principles used to address each research question are listed. Then, a description from the perspective of before Add+VantageMR® professional development and after Add+VantageMR® professional development is provided by research question.

Data sources for analysis included teacher interviews, classroom observations and AVMR student data. Pre- and post-teacher interviews and classroom observations provide evidence for research question one regarding teachers planning for instruction of a whole number lesson. Classroom observations and Post-observation interviews are used to address research question two identifying changes in instructional choices during a whole number lesson. Pre- and post-teacher interviews, classroom observations and post-classroom observation interviews are used to address research question three regarding changes made when differentiating a whole number lesson. Pre- and post-teacher interviews, classroom observations and post-classroom observation interviews, and student data provide evidence for changes in teachers assessment of students' learning during a whole number lesson.

The Math Recovery Guiding Principles for Classroom Teaching framework is used as a lens for analyzing classroom observations for all research questions. For each research question, applicable Guiding Principles are identified and used to describe changes in classroom practice from before and after participation in AVMR professional development. Table 4.1 provides the complete list and short description of the nine Math Recovery® Guiding Principles for Classroom Teaching.

Table 4.1 Guiding Principles for Classroom Teaching

Principle 1 The teaching approach is inquiry-based, that is, problem-based.
Principle 2 Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.
Principle 3 Teaching is focused just beyond the 'cutting-edge' of students' current knowledge.
Principle 4 Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.
Principle 5 The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.

Principle 6 Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.
Principle 7 Teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.
Principle 8 The teacher provides students with sufficient time to solve a given problem.
Principle 9 Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.

Case 1: Carole

Background Information. At the start of the study, Carole is in her fifth year of teaching at Cedar Ridge Elementary school. Carole participated in AVMR Course 1 five years prior to the study as a new second grade teacher in the district. After one year teaching second grade, Carole moved to first grade and is teaching first grade during the study. Carole's previous experience with AVMR course 1 professional development five years before the study provided an opportunity to probe into Carole's perceived lasting influence of AVMR Course 1 professional development. Given district support of Add+VantageMR® professional development, it is expected that initial classroom observations and interviews would also reveal some influence from AVMR Course 1.

The first-grade team meets regularly throughout the year to discuss and plan each mathematics unit. First grade team members support each other by each completing smaller tasks to prepare lessons for the entire team. Carole is the team leader. Carole's long range instructional planning and reflection focused on appropriateness of content for students, rigor of math content and coherence across curricular units. During informal teacher/researcher conversations at the beginning of the study, Carole asked questions about the unit sequences and learning outcomes of the district adopted curriculum. Carole raised relevant questions focused on providing students appropriately challenging content. Carole's day-to-day planning goals aligned with long range unit planning decisions made by the team and outlined in district curriculum frameworks. District adopted curriculum frameworks provides an overview of state and district standards and district supported resources including relevant assessment and teaching references to Add+VantageMR professional development.

Carole participated in an initial interview during workshop week before students' first day of school. The first set of three classroom observations took place in mid to late September. Carole was provided a spreadsheet with individual AVMR assessment results for all students in the classroom. The second set of three classroom observations took place in October. Carole participated in AVMR Course 2 in late November/early December. The final set of three classroom observations took place in mid to late December before school closed for winter break. Carole participated in a final one-on-one interview in late December. Figure 4.1 below summarizes the data collection schedule for Carole. Following figure 4.1, results are organized by research question. For Carole, sub-question 2, 3 and 4 were useful in making sense of instructional practices before and after taking AVMR Course 2 professional development.

Figure 4.1. Data Collection Schedule: Carole

2010	**August 2015**	**September 2015**	**October 2015**	**Nov/Dec 2015**	**December 2015**	**December 2015**
AVMR Course 1	Initial one-on-one interview	Classroom observation #1	Classroom observation #2	AVMR Course 2	Classroom observation #3	Post one-on-one interview

Research question one. *In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?*

1. In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?
2. What are teachers using to determine children's current level of understanding?
3. In what ways do teachers use course resources provided in course Teacher Handbooks or Teaching Number in the Classroom with 4-8 year-olds and/or Developing Number Knowledge by Robert Wright, et al. when planning a whole number lesson?
4. In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number as well as skills that support learning about whole number operations?

Data Analysis.

Guiding Principles for Classroom Teaching applicable to Research Question one are as follows:

- Guiding Principle Two: *Teaching is informed by an initial comprehensive assessment and ongoing assessment through teaching. The latter refers to the teachers' informed understanding of children's current knowledge and problem-solving strategies and continual revision of this understanding.*
- Guiding Principle Three: *Teaching is focused just beyond the 'cutting-edge' of the child's current knowledge.*

- Guiding Principle Four: *Teachers exercise their professional judgement in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.*

For an explanation of each Guiding Principle for Classroom Teaching see Appendix A.

What are teachers using to determine children's current level of understanding?

Research Question One: sub-question 3

Carole's planning for instruction begins by using general knowledge of children's thinking and then refined by using detailed information regarding classroom students' current mathematical understandings. This is a specific application of Guiding Principle 2: Teaching is informed by an initial comprehensive assessment and ongoing assessment through teaching. Carole used the Bridges beginning of the year assessment to identify students' current math knowledge at the beginning of the year. Although the school district continued to focus on using AVMR assessment data in kindergarten through second grade since Carole's initial participation in the course, there was no evidence from the formal interview that Carole used AVMR assessments to plan for whole number lessons four years after participating in Add+VantageMR® Course 1. Initial classroom observations support this same finding. After receiving AVMR data on all students in her class between observations one and two, little to no evidence was found that Carole used the data to inform lesson planning. However, responses Carole provided in the initial questionnaire completed prior to the commencement of AVMR course 2 in late

November Carole indicated AVMR Course 1 assessments as being moderately useful in her current teaching assignment.

AVMR Course 1 professional development and the district focus on AVMR assessment and instruction within district created frameworks should have influenced Carole's selection of assessment resources to determine specific data regarding students' early numeracy skills. This is a missed opportunity to use detailed information regarding students' current understandings and skills to determine differentiation choices. This is done by determining the understandings required to be successful during upcoming lessons and how well students current understandings align with the designed lesson.

A variety of opportunities to attend to information about student's skills and understanding of beginning of the year math concepts were available to Carole before classroom observation one. Kindergarten teachers administered the assessment along with the addition and subtraction and structuring assessment at the end of the previous year. Some classroom teachers shared that data with first grade teachers. Therefore, first grade teachers have end of the year kindergarten data on some of their students at the start of first grade. The schools also have district-sponsored time before school resumes in the fall to administer one-on-one assessments. A set of literacy assessments are done at this time and typically time is available at the end of each time session for teachers to choose to administer a part or all of at least one of the AMVR assessment tasks.

Initial interviews and classroom observations provide a baseline for documenting changes throughout the study. During the initial interview Carole made no reference to administering assessments or considering AVMR assessment data in lesson decision making at the beginning of the year. Carole shared in the formal interview that her main source for data regarding students' current level of understanding came from a pencil-paper assessment administered whole group to students at the beginning of the year. The assessment is a part of the *Bridges* curriculum and addresses forward and backward number word sequences, more and less, ordering numbers on a number line, 10 more and 10 less, and crossing numbers over the decuple. Although Carole administered the beginning of the year assessment, there was no evidence that the assessment results were used to plan lessons. Initial classroom observations supported the finding that Carole did not use specific knowledge of children's thinking when planning for lessons prior to participation in AVMR Course 2.

The first two units taught at the start of the year followed the district-adopted curriculum, *Bridges* version one. Based on researcher collected AVMR assessment data that was shared with classroom teachers, there was opportunity to adjust the task and/or numbers used in the task with the existing lessons to reflect Guiding Principle 3: Teaching is focused just beyond the 'cutting-edge' of children's current knowledge. Tables 4.2, 4.3, 4.4 and 4.5 show the results of research collected AVMR data available to Carole after classroom observation set 1 and before classroom observation set 2. Each table identifies a topic in early numbers. Then provides a description of each level or construct followed by the number of students found to be at each level and construct in the fall and winter. Fall assessments were administered in September and Winter assessments were administered in late December. Each table identifies a progression of students' thinking for a topic. Forward Number Word Sequence, Backward Number Word Sequence and Numeral Identification are considered skills and therefore the model identifies a progression by describing levels of fluency. The Addition and Subtraction model documents a progression of cognitive shifts based on research by Steffe and colleagues (Steffe, Cobb & von Glasersfeld, 1988). These models along with their corresponding assessments form the content and focus of AVMR Course 1 professional development.

In the first lesson series observed, students were provided with a game page including a spinner with one 1, three 2s, one 3, and one 4. Below the spinner were four rows where each row had nine of the same numeral for students to trace. The object of the activity was for students to spin the spinner and then trace the numeral indicated by the spinner. Students then create a bar graph to show the number of times each numeral 1-4 was chosen by spinning the spinner. The following lesson followed this same format for numerals 5-8. The lesson activity as it was presented did not provide opportunity for Carole to make in-the-moment adjustments during the lesson and therefore did not allow for the presence of Guiding Principle 4: Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, varying this selection on the basis of ongoing observations.

Additionally, fall data revealed student knowledge to be well beyond the skills required to complete the beginning lessons. In the fall, seventeen of nineteen students can produce the FNWS and number word after beyond 10. Twelve of those students are fluent to 100. Carole received the data represented in Tables 4.2, 4.3, 4.4 and 4.5 in October between the first and second classroom observation.

Table 4.2. Forward Number Word Sequence (FNWS) Assessment Results

	Description	Fall	Winter
Level 6	Can produce FNWS from 1-1000. Can produce number word just after a number in this range without dropping back		2
Level 5	Can produce FNWS from 1-100. Can produce number word just after a number in this range without dropping back	12	14
Level 4	Can produce FNWS from 1-30. Can produce number word just after a number in this range without dropping back	5	3
Level 3	Can produce FNWS from 1-10. Can produce number word just after a number in this range without dropping back	2	0
Level 2	Can produce FNWS from 1-10. Can produce number word just after a number in this range, but drops back to generate a running count	0	0
Level 1	Can produce FNWS from 1-10. Cannot produce number word just after a number in this range	0	0
Level 0	Cannot produce FNWS from 1-10.	0	0

In the fall Numeral Identification data indicated 6 of 19 students can identify and write 1, 2, & 3 digit numerals. Fourteen of nineteen can identify 2-digit numerals. Four students are fluent identifying numerals to 20 and one student can identify numerals to ten and working on teen numbers. All students in the class are able to identify numbers to 10.

Table 4.3. Numeral Identification (NID) Assessment Results

	Description	Fall	Winter
Level 5	Can identify and write 1-, 2-, 3-, and 4-digit numerals.		1
Level 4	Can identify and write 1-, 2-, and 3-digit numerals.	6	9
Level 3	Can identify 1- and 2-digit numerals.	8	6
Level 2	Can identify numerals in the range 0 to 20.	4	2
Level 1	Can identify numerals in the range 0 to 10.	1	2
Level 0	Cannot identify some or all of the numerals in the range 0 to 10.	0	0

Fall Backward Number Word Sequence data shows that five students can say the backward number word sequence (BNWS) and provide the number word before a given number from 30. Of those five, three can produce the BNWS and number word before a given number in that range from 100. Thirteen of nineteen students can produce the BNWS and number word before from 10.

Table 4.4. Backward Number Word Sequence (BNWS) Assessment Results

	Description	Fall	Winter
Level 6	Can produce BNWS from 1-1000. Can produce number word just before a number in this range without dropping back		2
Level 5	Can produce BNWS from 1-100. Can produce number word just before a number in this range without dropping back	3	10
Level 4	Can produce BNWS from 1-30. Can produce number word just before a number in this range without dropping back	2	3
Level 3	Can produce BNWS from 1-10. Can produce number word just before a number in this range without dropping back	13	5
Level 2	Can produce BNWS from 1-10. Can produce number word just before a number in this range, but drops back to generate a running count	1	0
Level 1	Can produce BNWS from 1-10. Cannot produce number word just before a number in this range	1	0
Level 0	Cannot produce BNWS from 1-10.	0	0

The assessment for addition and subtraction used in the fall and winter identified students conceptual development up to construct 3; initial number sequence. At the beginning of the year, seven students used counting-on and counting-back strategies to solve addition and subtraction tasks. Five students were using figurative counting and seven students were using perceptual counting to solve addition and subtraction tasks. Only one student was not able to count visible collections accurately.

Table 4.5. Addition and Subtraction Assessment Results

Construct	Description	Fall	Winter
Construct 5: Facile number sequence - Non-count-by-ones strategies	The student uses a range of what are referred to as non-count-by-ones strategies. These strategies involve procedures, other than counting-by-ones but may also involve some county-by-ones. Thus, in additive and subtractive situations, the child uses strategies such as compensation, using a known result, adding to 10, commutativity, subtraction as the inverse of addition, awareness of the '10' in a teen number.	na	na
Construct 4: Intermediate number sequence - Counting-down-to	The student counts-down -to to solve missing subtrahend tasks (for example 17-14 as 16, 15, 14 - answer 3). The student can choose the more efficient of count-down-from and count-down-to strategies.	na	na
Construct 3: Initial number sequence - Counting-on and Counting-back	Child uses count-on rather than counting from "one", to solve addition or missing addend tasks (e.g.6+x=9). The child may use count-down-from strategy to solve removed items tasks (e.g.17-3 as 16, 15, 14-answer 14) but not count-down-to strategy to solve missing subtrahend tasks (e.g. 17-14 as 16, 15, 14 - answer 3).	7	12
Construct 2: Figurative counting	Can count the items in a screened collection but counting typically includes what adults might regard as a redundant activty. For example, when presented with two screened collections, told how many in each collection, and asked now many counters in all, the child will count from "one" instead of counting-on.	5	6
Construct 1: Perceptual counting	Can count perceived items but not those in screened (that is concealed) collections. This may involve seeing, hearing or feeling items.	7	4
Construct 0: Emergent counting	Cannot count visible items. The child either does not know the number words or cannot coordinate the number words with items.	1	0

After participating in AVMR Course 2, Carole's use of student data changed. The course provided practice assessing a student in Carole's class in early place value and multiplication and division. Classroom conversations focused on analyzing assessment data according to the model for each topic and differentiating tasks to meet students' at the cutting edge of their knowledge. After the course, Carole reached out to ask questions about the assessment data shared with her back in October that was before participating in AVMR Course 2. The data shared in October were from topics covered in AVMR Course 1.

Classroom observations following AVMR course 2 showed a different type of task incorporated for the lesson and students flexibly grouped during independent activities. Although tasks from the Math Recovery series texts are not the only source for tasks allowing for easy modification to meet students' zone of proximal development or cutting edge of learning, they are a source with several task options and a source that is discussed during AVMR professional development. Carole chose two tasks After AVMR course 2, Carole first asked questions about AVMR assessment data received in October.

After the course Carole used instructional resources from AMVR professional development to address place value skills discussed during the course. She chose lessons from *Developing Number Knowledge* by Robert Wright, et al., the main text in AMVR Course 2. One activity, *10 More,* targeted student skills of adding 10 to a single digit number. This lesson both targeted a grade level standard and addressed a skill appropriate for students' current mathematical knowledge. Another activity, *1 Less*, found in AVMR Course 1 resources and the online district website was planned for a substitute teacher to enact the day prior to *10 More*. Based on assessment data collected in December, this lesson was too easy for students. The lesson supported students learning to establish a composite unit leading to the development of counting-on and counting-back strategies. December assessment results indicated most students were already using counting-on and counting-back strategies or more advanced non-count-by-ones strategies as their main method of solving addition and subtraction problems. A non-challenging activity along with a poor report left by the substitute teacher prompted Carole to choose a task focused on the more difficult skill of adding *10 More* to a single digit number. The types of tasks, materials and rules of play observed during the final set of classroom observations were found to provide flexibility to the lessons and resulted in the observed presence of Guiding Principles for Classroom Teaching not present in the first two sets of classroom observations before participating in AVMR course 2.

In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number as well as skills that support learning about whole number operations?

AVMR assessment and instructional tools provide teachers with resources to work with students in small and large group figurations. Before and after AVMR Course 2, Carole begins each lesson with whole group instruction followed by students working in partners on a common activity. During observed lessons before participating in AVMR Course 2, Carole posed a series of questions to promote student thinking in place value followed by students determining different combinations to make 11 or the number of days in school each day. Planned questioning resulted in students thinking hard about tasks and sharing strategies used during number corner. This pre-planning allowed for the presence of several Guiding Principles present during a lesson as elaborated upon in research question 2.

The class lessons observed in September had students playing an activity where each student identifies a numeral 1 to 4 on a spinner to then trace the number on a worksheet. At the end of the time, students create a bar graph showing the quantity of each numeral spun. The following day's lesson continued the same activity with the numerals 5 to 8. The third curriculum lesson focused on making patterns with bugs. The lesson started with a praying mantis song involving rich science vocabulary followed by students completing a buggy pattern worksheet. To complete the worksheet, students drew the bug or wrote the word of the bug that came next in the pattern.

The Guiding principles are interrelated and this is particularly true for Guiding Principles 2, 3 and 4 addressing the first research question. It is difficult to implement one guiding principle without addressing each of the other two. Initial comprehensive assessment is required in order to choose appropriate task adjustments or modify numbers within lessons to be at students' cutting edge. Since Carole did not draw upon initial assessment of students' number words and numerals knowledge, it is not surprising that observed number lessons were found to not be at or near the cutting edge of learning for most students in the class according to researcher conducted AVMR assessments. Additionally, the curriculum provided pencil paper assessment either did not provide useful information about students' knowledge of number identification and sequences or Carole did not use the information when planning lessons at the beginning of the year.

Planning for instruction also includes the way math time is structured within the allotted time and how students are physically arranged for learning. It is reasonable to expect the structure for math time and the ways students are arranged for learning to be conducive to differentiating lessons and flexibly grouping students according to the task or learning outcome. Carole's math time is divided into two parts of the day. Twenty minutes before lunch students are engaged in Number Corner. Then the math lesson takes place for about 45 minutes at the end of the school day. The math lesson time is structured in a whole group, independent activity, whole group format. The initial whole group time provides instruction in order for students to complete the assigned activity independently. The final whole group time provides closure to the lesson. Classroom observations focused primarily on classroom lessons. Selected observations also included the Number Corner discussion. Number Corner followed the same structure throughout the study. Students begin by observing the pattern emerging on the calendar and making predictions about the color and symbol of the day's card. The Number Corner calendar cards come with the district adopted curriculum resource. This is followed by adding a stick to the number of days in school with a brief tens and one connection and moves right into students determining as many combinations as they can for the number of days in school. For instance, one observation took place on the twelfth day of school, so students determined and shared number combinations that make twelve.

Following AMVR Course 1, it would be expected that Carole's structure for math class and seating arrangements for students would be intentionally chosen prior to the lesson to establish an environment where Guiding Principles for Classroom Teaching are likely to be present and differentiation during math lessons is likely to occur. At the start of the year, students had assigned seating arrangements at tables located in the peripheral of the room. September and November observations seating assignments were establish based on literacy assessment data and behaviors. During lessons students chose a space on the carpet in front of the SMART board for whole group portions of the lesson and worked from these established seating arrangements during independent activities. The math lesson structure provides opportunity for differentiation, however the first two sets of classroom observations reveal this to not be utilized to its' full potential.

After participation in Add+VantageMR® Course 2, Carole intentionally determined a seating arrangement for students during math time based on her assessment of students' current knowledge and skills. Carole paired students during the partner activity portion of the lesson. Carole changed from having students complete an activity with the person they are regularly assigned to sit beside to intentionally pairing students while passing out the materials for the activities based on her knowledge of students' mathematics knowledge. This is significant step towards making instructional decisions with intentionality.

During the course teachers assessed one student from their own classroom in place value and multiplication and division. The course introduces developmental models for learning place value and multiplication and division. The beginning of Course 2 reviews content, specifically structuring numbers and addition and subtraction models, to clarify content from AVMR course one and connect to the AVMR Course 2 learning.

In the December interview, Carole enthusiastically shared how she was supporting the team in implementing tasks and activities from AVMR by purchasing additional supplies needed to readily adjust tasks chosen for mathematics lessons. Having appropriate materials available is important to planning lessons that will easily allow adjust to students cutting edge of learning. For example, after the course, Carole ordered spinners, a variety of types and number selections of dice, bead racks and other items to have available in the classroom. During the final interview Carole shared about the tools they were now using in the classroom after AVMR Course 2 and inquired about what numbers would be best to include on dice to ensure students are working on specific skills that would advance math knowledge. Here Carole is planning for differentiation to occur during a lesson. Not only is this planning for differentiation of the lesson ahead of time, but having dice available in the classroom with a variety of number combinations also allows for in-the-moment adjustments to occur as well.

In summary, changes occurred in Carole's planning for instruction following AVMR Course 2 professional development. Carole attended to students' individual mathematics data, (Guiding Principle 2), selected tasks more closely aligned with students' current understandings (Guiding Principle 4), and intentionally paired students according to knowledge of students' mathematics understanding (Guiding Principle 3), which in turn presented the opportunities to meet and extend students' current mathematical understanding during the lesson.

Research question 2. *In what ways does Add+VantageMR® professional development influence teachers' instructional moves during a whole number lesson?*

1. In what ways are students engaged in activities from the course resources such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 year-olds* and/or *Developing Number Knowledge* by Robert Wright, et al)?
2. In what ways are students actively engaged in ***doing*** mathematics during the lesson?
3. In what ways does the teacher use Guiding Principles and Domains of Progressive Mathematization during a whole number lesson?
    a. Do some Guiding Principles seem to be more readily implemented than others?

Data Analysis. Guiding Principles addressed

Guiding Principles for Classroom Teaching applicable to research question 2 are as follows:

- *Guiding Principle 1: The teaching approach is inquiry-based, that is, problem-based.*
- *Guiding Principle 3: Teaching is focused just beyond the 'cutting-edge' of students' current knowledge.*
- *Guiding Principle 4: Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*
- *Guiding Principle 5: The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.*
- *Guiding Principle 6: Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.*
- *Guiding Principle 7: Teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.*
- *Guiding Principle 8: The teacher provides students with sufficient time to solve a given problem.*

- *Guiding Principle 9: Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.*

Sub-question 3 is used to guide a reflection on the ways AVMR Course 2 professional development influenced Carole's instructional moves during a whole number lesson. To address research question 2, two rounds of coding were applied to classroom observation video prior to formal analysis. The Guiding Principles for Classroom Teaching formed the basis for round one structural coding. Structural coding is defined as the application of a content-based or conceptual phrase representing a topic of inquiry to a segment of data that relates to specific research questions (Saldana, 2013). Math Recovery® Guiding Principles for Classroom Teaching were used as topics of inquiry and applied to classroom observation video. This ensured the use of Guiding Principles as labels and allowed for refinement of codes.

As a result of round one structural coding, five Dimensions of Mathematizing and three researcher created codes were added to initial codes. Four Dimensions of Mathematizing were added as descriptors under Guiding Principle 5, the teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies. Together the four dimensions; a) distancing the setting, b) extending the range, c) complexifying arithmetic, and d) refining computational strategies describe teacher moves related to engendering more sophisticated mathematical strategies. Additionally, the dimension of notating was added to Guiding Principle 7, teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic. In both cases, the

Dimensions of Mathematizing provided specificity to the Guiding Principle allowing for detailed data analysis. Three researcher created codes identified classroom episodes considered to be in contrast of Math Recovery® Guiding Principles. Table 3.5 includes these new codes; a) providing a mathematically untrue statement b) traditional telling to teach an important concept and c) use of mathematical symbols detached from student thinking, along with a complete list of codes established through round one coding. Codes listed in Table 4.7 were applied during round two coding.

Table 4.6. Codes applied to Classroom Observation Video.

Contrasts Guiding Principles for Classroom Teaching
○ Providing a mathematically untrue statement
○ Traditional telling to teach a concept
○ Symbols detached from student thinking
Guiding Principles for Classroom Teaching
Principle One: Inquiry –based teaching *Dimension of Mathematizing* • Problem centered task
Principle Two: Teaching Informed by assessment *Dimension of Mathematizing* • Task based on Learning Framework in Number & aimed at student's current levels of understanding
Principle Three: Cutting edge teaching *Dimension of Mathematizing* • Teacher accounts for a range of student levels of understanding
Principle Four: Select from a bank of procedures
Principle Five: Engender more sophisticated strategies *Dimensions of Mathematizing* • Distance the setting • Grounded habituation • Extending the range • Complexifying arithmetic
Principle Six: Intense, ongoing observation *Dimensions of Mathematizing* • Scaffolding a student through a task • Micro-adjusting the task
Principle Seven: Teaching builds on verbal-based strategies *Dimensions of Mathematizing* • Teachers' notation/symbolize student thinking • Students verbalize mathematical strategies
Principle Eight: Sufficient time to solve problems
Principle Nine: Students experience intrinsic satisfaction from engaging in mathematical tasks

Data used to address research question 2 came predominantly from classroom observation data collection. The results section draws heavily from the coding analysis described above for classroom observations. It is important to recognize that Guiding Principles in their own right are not labels for moments in time, but overarching philosophies. Long and short periods of time within the lessons captured this this study are examples among many examples of how the Guiding Principles can manifest in the classroom. For this study, applying Guiding Principles as codes for examples that highlight one or more Guiding Principles provides a way to study impact of AVMR professional development after the four-day course, something that has not been done prior to this study. For analysis, each time a code was applied to a short or long section of the video within a lesson, it was calculated as an occurrence of that Guiding Principle for the purpose of identifying patterns across lessons. The codes were then tallied to determine the prevalence of manifestations of a single guiding principle or manifestations considered to be contradicting a Guiding Principles.

Manifestations of Guiding Principles and Domains of Progressive Mathematization during a whole number lesson.

The first two classroom observations took place before Carole participated in AVMR Course 2. Coding analysis of classroom observations revealed Carole most readily incorporated Guiding Principle 7; teaching supports and builds on students' intuitive, verbally-based strategies before participating in AVMR Course 2. The first two sets of classroom observations also revealed a high presence of classroom interactions determined to be in contrast of a Guiding Principle. The October series of lessons involved structuring numbers to 10 and within that lesson series there was a greater presence of events in contrast to the Guiding Principles than manifestations of the Guiding Principles.

After participating in AVMR Course 2, events in contrast to Guiding Principles were present less often than in each of the two sets of classroom observations before Carole participated in AMVR Course 2. The December lessons taking place after Carole participated in AVMR Course 2 included examples of Guiding Principle 3: Cutting edge teaching, Guiding Principle 4: Select from a bank of procedures, Guiding Principle 6: Intense, ongoing observation, and Guiding Principle 9: Students experience intrinsic satisfaction from engaging in mathematical tasks. Table 4.8 below shows the number of occurrences found of Guiding Principles and episodes considered to be in contrast to Guiding Principles for each set of classroom observations.   The table does not include the additional codes used to describe specific episodes contrasting the philosophy within Guiding Principles. Only two documented occurrences of mathematically untrue statements were identified in September.

Table 4.7. Observations and Guiding Principles.

	Number of occurrences in September		Number of Occurrences in October		Participated in Add+VantageMR Course 2	Number of Occurrences2 in December	
	Manifestation of GP	Contrasting the GP	Manifestation of GP	Contrasting the GP		Manifestatio n of GP	Contrastin g the GP
Guiding Principle 1	1			7		1	
Guiding Principle 3		3	1	1		3	1
Guiding Principle 4			1	2		2	
Guiding Principle 5	3	1		3			
Guiding	1	1	1	1		3	1

Principle 6						
Guiding Principle 7	11		6	2		2
Guiding Principle 8		3				
Guiding Principle 9	2	4		2	3	

Before AVMR Course 2: Excerpts from the Classroom

A single occurrence of Guiding Principles 1, 3, 4, 5 and 6 were identified along with 2 occurrences of Guiding Principle 9 before Carole participated in AVMR Course 2. The following paragraphs provide examples of classroom episodes and how codes were applied during data analysis. Excerpts are from classroom observations before Carole's participation in AVMR Course 2.

Particularly during the first two sets of observations Carole asked students to explain their thinking or to expand on the thinking of other students during a lesson or calendar time. These opportunities for students to verbalize math thinking are considered manifestations of Guiding Principle 7 within the classroom. Students refine strategies as they verbalize thinking. The teacher can then support the development of symbolic notation by connecting notation to student-shared strategies. Following is an example from the classroom. At the start of calendar time at the end of September, Carole asked students to identify patterns they see in the calendar pieces represented by a colored card with one of 3 creatures on it. The order of the creature cards and colors form an ABC pattern. One student comes to the board and adjusts the cards to make a new pattern. Carole accepts her response and asks students in the class if anyone would like to expand on the student's pattern. November observations confirm Carole's consistent use of asking students to explain or expand on their thinking as she continues to ask students what they notice and to identify patterns in situations. For example, lessons facilitated in November ask students to write equations for combinations of 6 and 7 with 2 or 3 addends. Students are provided with a picture of a set of *Unifix* cubes where each set has a different number to two or three colored cubes on the *SmartBoard*. One set has 3 red cubes and 4 yellow cubes. Students are to state that this set of cubes represents 3+4=7. Carole asks students to share what they notice about the sets of cubes allowing students to bring their own thinking to the mathematical situation. She also asks students to explain their thinking and strategy used to determine quantities that add up to 6 or 7. In the exchange below, Carole engages students in a conversation regarding three red cubes and four yellow cubes pictured on the *Smartboard*.

C: Alaina, what do you notice here? (pointing to the picture of 3 red and 4 yellow unifix cubes) It is called a train. Everyone say train.
Ss in chorus: Train
Alaina: 3 red and 4 yellow so you like 3 and add one except you add one is yellow.
Carole: Wait, wait – go back. What do you see?
Alaina: Three and 4 yellows and one is put together to make (pause) 8
Carole: Ok, What were you doing to get 8?
Alaina: I was counting the blocks
Carole: So you were counting each separate block?
A: Nods, yes.
Carole: What else can we do. OK, so that is one strategy. What else can we do? … Camilla?
Camilla: What I notice about the boxes ..
Carole: Yeah
Camilla: Is that .. There are actually 7.
Carole: 7 How did you know that?
Camilla: I counted
Carole: How did you count?
Camilla: I did the red 3 and then yellow 4 and
Carole: And you put them together
Another student: I counted too!!
Carole: That is that addition that we talk about in number corner. She put the 3 and the 4 together to make 7.

In order to engage students with each other's strategies, a hand signal system is in place for students to indicate agreement with another student or to communicate their thinking is the same as the student sharing. Students use this hand signal often during observations before and after Carole participated in AVMR Course 2. Carole asks students to expand on their thinking in appropriate places during the lesson. For example, to engender the advancement of strategies, Guiding Principle 5, during a Number Corner lesson in September Carole intentionally paused the lesson to elaborate on a range of students' strategies shared for equations involving 11. She connected students' shared informal strategy language to formal strategy labels such as non-count-ones, counting on, and using fingers. Carole then took additional student responses incorporating these strategy types shared before ending the lesson.

Observations before AVMR Course 2 revealed intentional use of mathematics vocabulary during lessons; also related to Principle 7. One September lesson focused on writing numerals 5-9. For this activity students used a spinner to determine a number to trace from 5-9. Students traced the numeral identified by each spin. Carole repeatedly brought in age appropriate math vocabulary during the lesson while asking questions and making statements such as, which one has the most (spins), (this) number has more than __, (this) number has less than___. Students adapted mathematical language into their own responses especially when making predictions within the lesson. Additionally, students demonstrated internal satisfaction, Guiding Principle 9, from making predictions and verbalizing thinking based on the design of the spinner.

While observations revealed positive examples of Guiding Principles in action as previously discussed regarding Guiding Principles 7 and 9, classroom observations also revealed occasions that are in contrast of Guiding Principles during the lesson. Guiding Principle one suggests that students should be actively engaged in learning with challenging tasks. Observations showed students were not involved in challenging based tasks.

In addition there were other instances where her instruction did not reflect a vision of teaching and learning provided by a Guiding Principle. Carole was shown to make connections to other math standards during a lesson; an instructional move that supports students making connections. Making connections is an important teaching strategy to develop conceptual knowledge. While this instructional move supports learning, at times during observations before AVMR Course 2, Carole made mathematical connections detached from students' current mathematically understandings and therefore created a situation where many students were confused or could not engage in thinking to respond. Following is an example of instruction that contradicts Guiding Principle 2, 3 and 6. Guiding Principles 2, 3, and 6 state that teaching is informed by assessment and aimed at the cutting edge of students' knowledge to allow for an appropriate challenge. Teachers engage in intense observation and adjust within the task according to in-the-moment assessment of students' understanding. The following description of a discussion about patterning is an example where Carole made a mathematical connection between the 3s skip-counting number sequence, but does not use information about students' current knowledge of skip-counting sequences and data from in-the-moment observations of students to adjust the task or provide supports for the mathematics required in the task. The lesson is a Number Corner lesson and one student recognized one skip count of 3 in the calendar pattern. Carole expanded on the connection with a discussion and series of questions related to skip counting sequences by asking the question, “What is between 3 and 9 if we are skip counting by 3s?” Students are silent and Carole answers 6 as a model of the response. The question is followed up with “What is between 9 and 15 if we are counting by 3s?” Again students are finding it very difficult to respond. Carole stated to the class, “we need to work on our 3s.” Then asks students, “Do you remember counting by 3s during lunch count this morning?” To Carole the questions during calendar were simply related to doing skip-counting sequences. However, the format of the questions required students to apply a known skip count by three sequence to a new situation by asking students to retrieve a number in the sequence between any two numbers in the skip count sequence. This is a very different type of cognitive demand than when students are asked to recite a skip counting sequence by 3s starting from 3 and continuing forward. This disconnect between students facility with the sequence and the elaboration of the pattern in calendar left students confused about the calendar pattern, in contrast to Guiding Principle Nine.

The following lesson excerpt occurring prior to Carole’s participation in AVMR Course Two is an example where a series of events is in contrast to possible manifestations of the Guiding Principles. The lesson started whole group where students responded to what they noticed about pictures of groups of 6 or 7 cubes shown in 2 or 3 colors. During the discussion, students were explaining skip-count strategies to count cubes shown on the screen. Where this excerpt begins, students are to go back to their seats to create their own different combinations using 2 or 3 colors of cubes to total groups of 6 or 7 cubes.

Table 4.8. Classroom Excerpt.

T: You and your partner are going to make combinations. You partner with someone who has 6 or 7. Shares an example and says, hopefully your partner has something different. T is making partners. The teacher decides not to pass out the worksheet tool to accompany the lesson. T begins helping a partner group. Several students are off task….just sitting there with the materials.	
T comes up to me and knows this lesson is not going well. Teacher shares that she should have differentiated this lesson.	This would have been a very appropriate lesson; meeting Guiding Principle 1 & 4 had the opening discussion been connected to the thinking students needed to engage in to do this task independently. *In contrast of* Guiding Principle 4, 6 & 7.
Then teacher stops the class and showcases that Tanvi and her group have put their 7 and 6 together to make 13.	This changes the intent of the lesson. Now students are doing a join all problem instead of a structuring numbers problem.
Some students are just writing any true number sentences they know. They do not have cubes nearby.	*In contrast of* Guiding Principle 1
Another group is working with several colors together to make 13.	*In contrast of* Guiding Principle 1
A student comes up with 7+6+7 =20. Teacher draws attention to doubles and tells the student to go back to his seat and do 7+7+6 to keep the doubles together. Teacher asks the student to work that out. T is working on supporting him to explain his thinking, except that she has inserted a different strategy for him to use and explain instead of supporting his strategy. Putting materials away. No group debrief today.	*In contrast of* Guiding Principle 1, 5, 7 Student is confused. *In contrast of* Principle 9.

Carole communicated that the lesson was not going as planned. Lessons may often not go as planned in teaching and in this case Carole reflects upon this experience to plan for the following day's lesson. The following day's lesson is expanded upon within research question 3 since it was Carole's response to differentiate the lesson.

After AVMR Course 2: Excerpts from the Classroom

After participating in Add+VantageMR Course 2, manifestations of Guiding Principles changed. After AVMR Course 2, Carole noticeably attended to tasks and adjustments of tasks to make tasks meet the cutting edge of students' current knowledge as described in Guiding Principle 3. During the lesson in December, Carole observed students' strategies more intently than general behavior. She commented on *how* students were solving problems, and started engaging students in conversations about their solution strategies as they worked in the partner activity. This is an example of how Guiding Principle 6: Intense, ongoing observation leads to opportunities for students to engage in inquiry tasks with an appropriate amount of challenge. Although it is not necessary to use tasks from the Developing Number Knowledge text in Add+VantageMR Course 2, the tasks within this text are designed to be easily modified for students and implemented alongside any curriculum. Carole chose two tasks from this text for the December observations. The second lesson was at students' current level of understanding and thus provided an example of how Guiding Principle 1: Inquiry-based teaching manifests within a classroom lesson. Below is a transcript excerpt from this lesson after participation in AVMR Course 2 .

Table 4.9. Classroom Excerpt.

Lesson Description	Researcher Commentary
The focus for today's lesson is to playing the *10 more* activity	This activity comes from AVMR course 2 – Guiding Principle 4
The teacher introduces today's lesson as building flexibility by going back and forth from addition and subtraction.	
Teacher begins to demonstrate how to play. She rolls a 5. Multiple students in the class reply that 10 more would be 15. Teacher is writing the equation vertically.	Guiding Principle 3
Rolling again. "This is what you would do with a partner." 6	
Students are excited. There appears to be an appropriate amount of challenge in this activity.	Guiding Principle 9

Teacher calls upon a student to share their strategy for solving 6 and 10 more. The student replies, "I just know that when you have 6 then 10 more is 16." T: Sounds like you just know this – that you know these combinations. The teacher rolls a 1. Many students are very excited and yelling out 11. Students are excited and very engaged. It appears as though the students who are responding all know their 10plus. When called upon a student shares 10+1 is 11. A student shares how you could use the number line to figure out the answer. She said you could go to the 10 and go 1 more to figure out 1 + 10. The teacher highlights for the class that it is a great strategy to start at the bigger number. She started at the 10 and went 1 more.	
Using same partners that were assigned yesterday. Students go to their seats and find their partners to begin today's activity. Partners are engaged in playing and trying to block their partner. Group of students are counting on. In another group one student keeps blurting out the answers and finally his partner says that he wants to figure out the answer first.	Guiding Principle 3 (for a reasonable number of students) Guiding Principle 9
Students are playing and engaged in doing the activity correctly.	Guiding Principle 1, 3 & 9
The teacher stops the group and shares with the class another way to play that would make the game more challenging. Students can roll one die two times, add the numbers together and then add 10.	Guiding Principle 6 *This is the first that this Guiding Principle appears with this amount of teacher intentionality.
Another group is trying to add and then subtract. I cannot hear the students' adapted way of playing the game, but the teacher says that it is more challenging than what was just shared and the pair can decide which way to play.	
The teacher is going around the room to encourage students to play using rules that would provide an appropriate challenge. The additional challenge is having students roll a die more than 1 time.	Guiding Principle 6
The problem here is that the numbers represented on the board do not match the change in rolling of the dice. The teacher then suggests students identify a number to subtract to get to a number in the board. This is a missing subtrahend problem.	Opposite of Guiding Principle 6 *Although the mathematics is adjusted to provide a more cognitive challenge

The presence of Guiding Principles 3, 4, 6 and 9 is most notably after Carole's participation in AVMR Course 2. Each of these Principles is important for the advancement of student mathematical thinking. Guiding Principles 3, 4 and 6 focus on presenting tasks to students based on knowledge of students' current knowledge, the trajectory for student thinking and using assessment of students' current knowledge when making in-the-moment decisions during instruction. Guiding Principles 3, 4 and 6 did not have a notable presence in observations prior to Carole's participation in Add+VantageMR® Course 2.

As a result of providing students with tasks involving an appropriate level of challenge students are demonstrating behaviors of intrinsic satisfaction from thinking hard about mathematic- Guiding Principle 9. Prior to Add+VantageMR® Course 2, Guiding Principle 7 occurred most often. Students were spending time sharing their strategies with the whole class. In the lessons that followed Add+VantageMR® Course 2 students were still engaged in sharing computation strategies, however, they were sharing with their partner as they engaged in the mathematics. Carole utilized problem-based activities that were a part of the Add+VantageMR® resources, Principle 1. As a result, all students had much more time on task and were engaged in mathematics during the lesson instead of a select few having the opportunity to verbalize thinking.

Research question 3. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

1. In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?
2. In what ways do teachers modify an activity during a lesson. Modification may include offering different dice, number ranges, and/or instructional tools?

Data Analysis: Guiding Principles

Guiding Principles for Classroom Teaching applicable to research question 3 are as follows:

- *Guiding Principle Three: Teaching is focused just beyond the 'cutting-edge' of students' current knowledge.*
- *Guiding Principle Four: Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*

Research question one addressed the ways differentiation can happen as teachers plan for a lesson. Research question two focused on a variety of teacher moves related to Guiding Principles for Classroom Teaching during a lesson. Research question three specifically addresses differentiating for students through the lens of Guiding Principle Three, teaching is focused just beyond the 'cutting-edge' of students' current knowledge and Guiding Principle Four, teacher exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations. This includes short and long tern planning for lessons and in-the-moment decisions teachers make that result in students making mathematical connections to advance thinking or students practicing skills and refining strategies appropriate within a trajectory of student learning. Sub-questions 1 and 2 are used to guide a reflection on the ways AVMR Course 2 professional development influenced Carole's differentiation of a whole number lesson. The following paragraphs draw upon data collected during informal, pre- and post- interviews and provide example excerpts from Carole's classroom observations before and after her participation in AVMR Course 2.

In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration? In what ways do teachers modify an activity during a lesson?

Before AVMR Course 2: Excerpts from the Classroom

As stated within research question 1, before taking AVMR Course 2 Carole used the same assigned seating arrangement for mathematics that is used for all subject areas. Student seating arrangements were determined by beginning of the year reading assessment results. This is important because when students return to their desks from whole group instruction introducing the activity, students are then engaging in that activity with another student that may or may not be engaging in that activity in the same way. The tasks used during the first set of three lessons did not lend to students engaging at different entry points, a common way tasks are differentiated to meet the needs of more students. In that series of math lessons all students were spinning the same spinner and locating the identical number to trace on their worksheet followed by coloring a square for that number to create a bar graph. The tasks presented in the second set of classroom observations did lend to multiple entry points. In this series of tasks students are to create 2 or 3 addends that add up to 6 or 7. This series of tasks provided the opportunity for additional teacher decisions to be made in the moment of teaching to create examples of how Guiding Principle 3 and 4 can manifest in the classroom. In the excerpts that follow Carole knows from observation that the first lesson begins to not go as planned when students leave the whole group conversation and begin to work on the presented task in partners. Her evaluation of what is not going well and excerpts from the second day's lesson follow and highlight Carole's decisions and response to the events of day 1. As you will see in the examples, the results of these decisions create a different type of lesson that includes examples of episodes in contrast to the philosophies within Guiding Principles. The following paragraphs describe excerpts from observations during these lessons.

The following excerpt from the lesson observation transcript begins with the first lesson and followed by the second lesson.

Table 4.10. November Observation One.

Lesson Description	**Researcher Commentary**
Teacher (T): Begins by demonstrating equations written horizontally and then rewrites the same equation vertically to make sure students know the two written forms have the same meaning.	Guiding Principle Seven
The focus of this lesson is to have color unifix cubes in groupings that would support students in writing number equations to represent the different colors in the unifix train. They are doing these as a class and then they will build unifix trains on their own.	Guiding Principle Seven & Four
Students are to build a train of 6 or a train of 7 using two colors. The examples have 2 and 3 colors. Students are choosing whether they want to make a train of 6 or 7.	
T holds up two trains…a train of 6 with two colors and a train of 7 with 3 colors.	
T sets out a box of unifix cubes for the 6 group and then a tray of unifix cubes for the 7 people. Everyone has 2 minutes to make one train.	
Students are using colors, but the colors are not being placed next to each other to form groups. Students do not seem to be doing the activity as intended.	*In contrast of* Guiding Principle Four
	This lesson seems to be off focus from the intent of the lesson which I think would be to make combinations of 6 and 7 using 2 or 3 addends. It is hard to see the real focus of this lesson during the time…..Not sure what student learning is intended by watching the lesson as it is unfolding.
Students bring their trains to the rug. T: "So, some of you made 6. Some of you made 7. Some of you did not listen. I asked you to use either 2 or 3 colors. Ok. Holds up a multicolored train and says this is more and hands it back to the student it came from.	*In contrast of* Guiding Principle Nine. This publicly embarrasses the child.

T: You and your partner are going to make combinations. You partner with someone who has 6 or 7. Shares an example and says, hopefully your partner has something different. T is making partners. The teacher decides not to pass out the worksheet tool to accompany the lesson. T begins helping a partner group. Students are needing the practice with writing numerals the correct direction independently. Several students are off task….just sitting there with the materials.	
T comes up to me and knows this lesson is not going well. Teacher shares that she should have differentiated this lesson.	This would have been a very appropriate lesson; meeting Guiding Principle One & Four had the opening discussion been connected to the thinking students needed to engage in to do this task independently. *In contrast of* Guiding Principle Four, Six & Seven.
Then she stops the class and show cases that Tanvi and her group have put their 7 and 6 together to make 13. Some students are just writing any true number sentences they know. They do not have cubes nearby.	This is not a good choice.
Another group is working with several colors together to make 13.	*In contrast of* Guiding Principle One
A student comes up with 7+6+7 =20. Teacher draws attention to doubles and tells the student to go back to his seat and do 7+7+6 to keep the doubles together. Teacher asks the student to work that out. T is working on supporting him to explain his thinking, except that she has inserted a different strategy for him to use and explain instead of supporting his strategy.	*In contrast of* Guiding Principle One, Five, Seven Student is confused. *In contrast of* Principle Nine.

The following day, Carole facilitated a lesson with the same learning goal as the day before, but in her words, “differentiated”. The adjustment to the lesson was to have a new lesson that was much more teacher controlled resulting in teaching described to be in contrast to the guiding principles.

Table 4.11. November Observation Lesson Two.

Lesson Description	Researcher Commentary
Colored unifix cubes are on the board. There are several equations on the board as well. Students need to match the number sentence with the colored unifix train.	*In contrast of* Guiding Principle 1 & 3

T is calling on students and saying “Talk about this one.” the expressions are not identical to cubes. For example, there are 5 red and 1 yellow cubes connected and that train is supposed to match the expression 1+5. A student notices this.	
Talking about true and false. True means it **is the same as** and false means it is not the same as. 7=1+5 is shown on the Smartboard. Student shares that this equation is false and shares her thinking with the class.	Guiding Principle 7 Guiding Principle 7
T is unveiling a second number sentence and then calls on a student to share whether the equation is true or false. The teacher then asks that one person to share their thinking to support the student response of true or false.	
A student notices a pattern in the answers.	
Jun is back from taking a break for a behavior issue earlier. T makes an equation for him.	
This lesson is very prescribed to this point. It goes step by step and is controlled. Few students are participating in the mathematics and engaged in finding solutions.	*In contrast of* Guiding Principle 1. Students are not provided an opportunity to engage in hard thinking about the mathematics.
Now students are moving to a new seating position on the floor to be facing a chart that has sets of unifix cubes grouped in colors sitting in the pockets.	
Addresses that this is easier when all colors are together. T ties that into a learning from yesterday.	
Tanvi is called upon first and she shares about several of the trains and then moves the trains around to be a pattern instead of keeping combinations of 6 in one column and then combinations of 7 in another column.	
We want to think about what we did today. (referencing number sentences earlier) We want to make a number sentence.	*In contrast of* Guiding Principle 5. Unfortunately, the work earlier does not connect to the number sentences created here. This is matching a picture…the other was true false statements using only bare number.
T is writing a number sentence on a post-it to put into the pocket by a colored train. T picks up a train and asks a student to share what the number sentence would be. T writes that on the post it.	*In contrast of* Guiding Principle 1. Only 2-3 students are engaged in the mathematics during the lesson. The rest are watching what is happening and may or may not be thinking about the mathematics in their head.
T brings students focused over to smart board. This time there are pictures of unifix cubes that are outlined and s color in combinations to make 6 using 2 colors. Then at the bottom there are true and false statements for students to circle the answers. T ensures students understand how to complete the worksheet and then students go back to their seats to complete the worksheet.	

Students are getting started on the task right away. They are engaged in completing the worksheet. I do not think students are too cognitively challenged by the worksheet.	*In contrast of* Principle 1

After AVMR Course 2: Excerpts from the Classroom

After taking AVMR Course 2, changes were found to the way Carole differentiated for students. First, the classroom observation after Carole's participation in Course 2 revealed that students are arranged for math in partner and small groups according to informal assessment of students' mathematics knowledge. This is a positive change resulting in intentional decision making regarding which students are working together and for which purpose. The following classroom episode takes place after Carole's participation in AVMR Course 2. The activity chosen provided students with focused practice on 10 plus patterns. Students spin a number and then add 10 to that number. Once students left the whole group and starting working in partners, it was again transparent that adjustments needed to be made to the activity. Carole noticed through intense observation of students' strategies problems that several partner groups were finding the activity way too easy. This time, Carole made the decision, in the moment of teaching, to adjust the rules of the activity or to adjust the number size students were working with to adjust the activity to meet an appropriate level of challenge for students. In contrast to the lesson observed before Carole participated in AVMR Course 2, Carole was calmly observing student groups to identify what they found easy and to consider another way a partner set could play the game. Once Carole differentiated for one partner set, she quickly starting sharing that same modification with other partner groups as she noticed their facility with 10-plus problems. Following is the observation transcript from this moment in the lesson.

Table 4.12. December Lesson Observation One.

Lesson Description	**Researcher Commentary**
The focus for today's lesson is to playing the *10 more* activity The teacher introduces today's lesson as building flexibility by going back and forth from addition and subtraction.	This activity comes from AVMR course 2 – Guiding Principle 4
Teacher begins to demonstrate how to play. She rolls a 5. Multiple students in the class reply that 10 more would be 15. Teacher is writing the equation vertically.	Guiding Principle 3
Rolling again. "This is what you would do with a partner." 6	
Students are excited. There appears to be an appropriate amount of challenge in this activity. Teacher calls upon a student to share their strategy for solving 6 and 10 more. The student replies, "I just know that when you have 6 then 10 more is 16." T: Sounds like you just know this – that you know these combinations. The teacher rolls a 1. Many students are very excited and yelling out 11. Students are excited and very engaged. It appears as though the students who are responding all know their 10plus. When called upon a student shares 10+1 is 11. A student shares how you could use the number line to figure out the answer. She said you could go to the 10 and go 1 more to figure out 1 + 10. The teacher highlights for the class that it is a great strategy to start at the bigger number. She started at the 10 and went 1 more.	Guiding Principle 9
Using same partners that were assigned yesterday. Students go to their seats and find their partners to begin today's activity.	
Partners are engaged in playing and trying to block their partner. Group of students are counting on. In another group one student keeps blurting out the answers and finally his partner says that he wants to figure out the answer first.	Guiding Principle 3 (for a reasonable number of students) Guiding Principle 9
Students are playing and engaged in doing the activity correctly.	Guiding Principle 1, 3 & 9
The teacher stops the group and shares with the class another way to play that would make the game more challenging. Students can roll one die two times, add the numbers together and then add 10.	Guiding Principle 6 *This is the first that this Guiding Principle appears with this amount of teacher intentionality.
Another group is trying to add and then subtract. I cannot hear the students' adapted way of playing the game, but the teacher says that it is more challenging than what was just shared and the pair can decide which way to play.	
The teacher is going around the room to encourage students to play using rules that would provide an appropriate challenge. The additional challenge is having students roll a die more than 1 time.	Guiding Principle 6

The problem here is that the numbers represented on the board do not match the change in rolling of the dice. The teacher then suggests students identify a number to subtract to get to a number in the board. This is a missing subtrahend problem. [See video 12-18-15v2 for the following…..]	*In Contrast of* Guiding Principle 6 *Although the mathematics is adjusted to provide a more cognitive challenge
Students are using fingers to subtraction. Students are really struggling or thinking hard with subtraction. This is adding in a challenge that it too big of a leap from the original game.	*In Contrast of* Guiding Principle 3 The idea to differentiate is good as students were very successful with 10 plus combinations. However, the adjustments chosen changed the activity. Thus making the activity too difficult for students. Teacher is trying to make connections with addition and subtraction with students when the students are not yet at an understanding of subtraction to be able to work with the non-count-by-ones strategies she is trying to support students in using.
At the end of playing the game the Teacher came back to the large group. Talking about coming back to this game next week and having different ways of playing the game using different numbers.	Guiding Principle 6 – Intense observation
At closing the T asks for students to share anything else they learned from the day. One students shared a strategy that was just shared. The student is describing thinking on how to figure out the jump back from 19 to 16. The teacher records 19-16 = 4 on the board. That number sentence does not represent the solution being shared by the student.	*In Contrast of* Guiding Principle 7
The teacher begins to share a number line strategy where you could start at 19 and think about what gets you back to 10. T draws a jump back from 19 to 10 and then a jump forward to 11. The T draws 9 for going back but then adds the 1 to the 9 to say it was a jump of 10 to get from 19 to 11 instead of the correct answer of a jump of 8. The teacher says to the class that she got mixed up on that one and that she is learning right along with everyone. T then moves to say that kids can win by going vertically, horizontally, or diagonally on the board. Then she transitions to literacy.	*In Contrast of* Guiding Principle 7

After taking Add+VantageMR Course 2, Carole was able to respond to student need in the moment of teaching rather than continuing with a lesson she knew was not advancing students' mathematical knowledge. Carole also started to consider information about students' current knowledge and skills as well as the development of mathematics when enacting whole number math lessons. Carole also attended to student pairings while students are engaging in mathematics during the lesson.

Research question 4. *How does Add+VantageMR® professional development influence teachers' assessment of students learning during implementation of a whole number lesson?*

1. In what ways do teachers make in-the-moment adjustments to the written lesson plan based on student responses, demonstrated understandings and knowledge about children's thinking as it is related to the Learning Framework in Number.
2. In what ways do teachers describe children's strengths and areas of need for whole number understanding with specificity that is connected to observed student solution strategies and the Learning Framework in Number?
3. In what ways do teachers collect formative assessment data during the lesson that includes a student-thinking component?

Data Analysis: Guiding Principles

One Guiding Principle for Classroom Teaching is applicable to research question four.

- *Principle Two: Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.*

In the moment of enacting a lesson, teachers are required to attend to a variety of stimuli and quickly make a number of complex decisions. Research Question 4 focuses on teacher's noticing student solution strategies and the subsequent assessment of students' mathematical knowledge and shifts in that knowledge *during* a whole number lesson. Research Question 4 does not address action after making an informal assessment of students' understanding. The action a teacher takes in response to informal assessment information is better described as through differentiation decisions made during and after a whole number lesson, which is addressed within Research Question 3. Research

Question 4, sub-question 3 is used to guide the following reflection on the ways AVMR Course 2 professional development influenced Carole's noticing of student strategies and informal assessment during a whole number lesson.

Before and after taking AVMR Course 2, Carole provided numerous opportunities for students to share a range of strategies and expand upon their own thinking. Classroom observations before taking AVMR Course 2 reveal that although Carole observes students strategies, these observations are not intense nature and therefore indicate Carole does not place observations of student strategies within a framework which would provide her with informal assessment information during the lesson. The focus centers on the act of students collectively vocalizing a variety of accurate strategies. For example, student thinking is interrupted to place a teacher crafted solution with the students explanation or additional students from the class are asked to intercede with an explanation. In either case the explanations may or may not have been in line with the initial child's true solution strategy. Therefore, although students share strategies in the class, Carole does not act upon the strategies as a form of informal assessment. The follow excerpt from the first set of classroom observations provides an example.

> T: Now we say combinations that make 11.
> Now we want to think about our combinations to 11. (A little less than ½ the students are raising their hand.")
> Students are sharing equations that make 11 and T is recording the equations.
> Lium: 5+6 (First combination of 11 shared). T: I want to stop and talk about this one. What made you think of 5+6? What was your thinking?
>
> Lium uses his fingers to show 5 and then to show 5 on the other hand. He counts the first 5 and then counts from 1 to show the six would be the five there and one more to make 11.
>
> T: What number did you start with? Lium begins to explain. **T interrupts**: so you started at 1?
>
> Another student shares counting on from 5 to explain 5+6.

S: 7 plus, no 7 minus
T: **interrupts.** No I like the direction you were going…7 plus what equals 11? I like where you were going at first. Who can turn and help her?

Another girl student shares: I was going to say 7+11.

T: Ok….so let's think how we can get from 7 to 11. Who can help her?

S: 7+4….

T: 7+4 and how did you get there?
Tanvi: If you do 7+3 that makes 10. It's just like 1 more to make 11.
T: Did you guys hear that over there. What did she say? And how did she get there? What was her thinking.

Alaina: 8+3
T: What was your thinking?
S: Using my fingers

T took 3 more equations and two more combinations…taking student answers without sharing strategies.

T: Turn and talk…looking at responses…Is anything missing? Anything we could add.
Group is talking. Individual responses are inaudible.

T is taking some responses from the partner discussions. Students are showing a hand signal for having a similar idea showing they solved using a similar solution strategy.

After taking AVMR Course 2, Carole's attention to students' strategies and informal assessment remained unchanged. Carole's task selection changed after taking AVMR Course 2 and Carole's actions based on student observations during the lesson changed, however, what Carole attended to and placed within a framework of student thinking did not change. During the final set of classroom observations, Carole planned an activity where students worked in pairs to play an activity focused on adding 10 more to a single digit number. As students played Carole moved from group to group around the room attending to whether they were playing the game correctly and then listening or asking how a student solved a problem. After circling the room, Carole stopped the class to adjust the activity for the whole class based on students' overall quick responses and behaviors during play. After participating in AVMR Course 2, impact of course 2 would have been seen by the observance or Carole's indication during an interview that modifications in the lesson are based upon information gleaned from specific strategies students are using when solving whole number tasks. The following excerpt from the final interview reveals Carole's thoughts regarding assessment information. She is struggling with the idea that students may not have the same overall understanding of math and reading. For example, the held belief that a student high in reading would also be high in math and a student low in reading would also be low in math. Also, Carole's response to the impact AVMR Course 2 had on her use of assessment data indicates Carole's learning included a disequilibrium in her ideas regarding the nature of multiplication and division and place value toipcs more than the specific framework or trajectory embedded within the conceptual nature of each the topics themselves.

> *Interviewer: What did you do with the [assessment] information provided?*
>
> Carole: Sometimes it solidifies what you have already been seeing and the progression with where you started at the beginning of the year. There was a couple of surprises like oh they are not understanding structuring to this but they could do another area they were really strong…..it is when it is not consistent with the student and you knowing them.
>
> Carole: I think that is what threw me off with the Liam's. The same name but I know Liam W is this high reader and Liam G is at grade level, but it seemed like it was the data matching….Liam G is stronger in math. I don't know I just…. still understanding…. I feel like cross content they should be similar.
>
> Carole: Which isn't the case because I have my Blake by video tape is high in math but below in reading. Maybe because certain assessments he didn't do well with the curriculum on because of the reading ???
>
> *Interviewer: What role if any did participating in that 2nd course help in interpreting and understanding the data?*

> Carole: I think what I took away from it was just that I had never thought of multiplication that kids were maybe already doing it or understanding it. Or the understanding of place value in a different way. I thought I understood place value. That it was more than just .. I don't know, I think I am still trying to process a lot of it. There's just a lot that you can do with it. And I want to do it and it is just doing it. I want to understand the data. I want to get them to the next level. I want to take from the class the different things and then work with my team, alright here's this stuff it would help the get here, let's meet this next benchmark - you know the 120. Um the practical side you know…..

In summary, no change was found in the nature of Carole's noticing of student solution strategies and the subsequent assessment of students' mathematical knowledge and shifts in that knowledge *during* a whole number lesson. Carole's own conceptual knowledge of mathematics content and how math knowledge is related to reading academic content areas appears to provide a barrier to considering in detail a framework for children's thinking within a particular mathematics topic that is useful within the moment of teaching. Additionally, it is possible that additional experience is needed administering and analyzing one-on-one assessment data in order to make sense of the larger conceptual understandings where Carole is struggling to find a firm understanding.

Case 1 provided a rich description of Carole's planning for instruction and classroom teaching before and after participating in AVMR Course 2. Case 2 follows and provides a description of Mark's planning for instruction and classroom teaching before and after participating in AVMR Course 2. Case 3 provides a description of Jessica's planning for and classroom teaching before and after AVMR Course 1 and then after AVMR Course 2. All three teachers are members of a first grade team.

Case Two: Mark

Background Information. Mark has been a primary teacher for 23 years. He participated in the first district offered Add+VantageMR® Course One during the 2006-7 school year, nine years prior to the time of this study. During the study, Mark is in his third year of teaching first grade at Cedar Ridge Elementary School. Prior to teaching at Cedar Ridge, Mark taught kindergarten for 18 years in a different elementary school within the same district. Although Mark came to Cedar Ridge with years of experience teaching early number and other important primary grade topics, he shared during the initial interview that the STEM focus and differences in student population at Cedar Ridge Elementary require him to reflect upon and employ different teaching strategies in his current first grade classroom.

Figure 4.2. Data Collection Schedule: Mark

2006	August 2015	September 2015	October 2015	Nov/Dec 2015	December 2015	December 2015
AVMR Course One	Initial one-on-one interview	Classroom observation #1	Classroom observation #2	AVMR Course Two	Classroom observation #3	Post one-on-one interview

Research question one. *In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?*

1. In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?
2. What are teachers using to determine children's current level of understanding?
3. In what ways do teachers use course resources provided in course Teacher Handbooks or *Teaching Number in the Classroom with 4-8 Year-olds* (Wright et

al., 2014) and/or *Developing Number Knowledge* (Wright et al., 2011) when planning a whole number lesson?

4. In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number, as well as skills that support learning about whole number operations?

Data Analysis.

Guiding Principles for Classroom Teaching applicable to Research Question one are as follows:

- Guiding Principle Two: *Teaching is informed by an initial comprehensive assessment and ongoing assessment through teaching. The latter refers to the teachers' informed understanding of children's current knowledge and problem-solving strategies and continual revision of this understanding.*
- Guiding Principle Three: *Teaching is focused just beyond the "cutting edge" of the child's current knowledge.*
- Guiding Principle Four: *Teachers exercise their professional judgement in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.*

For an explanation of each Guiding Principle for Classroom Teaching, see Appendix A.

Background: Mark's Planning.

As a first grade teacher, Mark finds there to be a different set of joys and challenges in his current classroom than in his previous position. He is a leader in the building and serves on several committees, such as the teacher evaluation and curriculum committees. Mark is very knowledgeable about a range of teaching practices relevant to teaching elementary school. He is well respected by colleagues, as well as school and district leadership. As a result, Mark finds his planning time to be often interrupted or consumed by meeting with colleagues regarding building or district topics and attending committee meetings away from the classroom.

The first-grade team meets regularly throughout the year to discuss and plan each mathematics unit. First grade team members support each other by each completing smaller tasks to prepare lessons for the entire team. Mark contributes to team planning. During informal teacher/researcher conversations at the beginning of the study, Mark asked questions to better understand how to choose activities or tasks appropriate for students' current levels of understanding, as determined by the AVMR assessments. Mark also attended to links between AVMR level descriptors and district mathematics standards. He asked questions to better understand how to assess topics where level descriptors do not directly match district expectations. For example, district expectations for first graders are to say the number sequence forward to 120, whereas AVMR level five indicates a student can say the forward number word sequence to 100. Additionally, Mark expressed interest in utilizing small group instruction during math lessons, but indicated he was not sure how to organize small group instruction within the whole class lesson time.

Mark's day to day instructional goals align with district frameworks and objectives discussed as a first grade team. He drew heavily upon activities from a variety of resources, including texts used in Add+VantageMR® professional development, resources from the district Moodle webpage, and previous Cognitively Guided

Instruction courses. Mark did not use district adopted curriculum during classroom observations occurring before and after AVMR Course Two.

What are teachers using to determine children's current level of understanding?

The kindergarten team at Cedar Ridge Elementary school administers AVMR Course One in the fall, winter, and spring to monitor student progress according to district established benchmarks. Even though the kindergarten team has established a way to record and organize data, a building or district system to pass data to the next grade level has not yet been established. Therefore, some teachers will pass along AVMR data to the next grade levels and others will not, leaving first grade teachers with data on some students from spring of kindergarten. In the initial interview, Mark said he reviewed the data, but without having information on all of his students, he did not find it very useful for planning lessons. He would like to see data recorded at the district level, as it is for literacy, to ensure he has math assessment information on all of his students at the beginning of the year to use for planning lessons.

Initial interviews and classroom observations provide a baseline for documenting changes throughout the study. As expected, based on Mark's previous experience with Add+VantageMR® Course One, initial classroom observations in September reveal that Mark uses available AVMR assessment data to guide instruction which aligns with Guiding Principle Two. Once researcher-collected AVMR data was shared with Mark, he reviewed the data, and Classroom observations in November revealed that he started to use data more intentionally than before when planning lessons. During classroom observations in September, before Mark received researcher-collected student AVMR data, lessons were focused on a whole class activity. After receiving data, lessons started whole-group for the introduction of an activity, and transitioned to independent partner work. Lessons were based on AVMR Course One content and teaching practices. The September lesson involved playing Buzz, where students form a circle and take turns saying a sequence according to teacher directions. This activity is played during AVMR Course One professional development. The first game involved forward number word sequences from 19 to 25. The sequence was quite easy for many students in the class. The second round of the activity involved backward number word sequences from 22 to 18. This round proved to be very difficult for many students.

During November classroom observations, occurring after Mark received researcher-administered AVMR data, Mark chose an activity again played during AVMR Course One professional development for students to play in pairs. When it was time for students to engage in the activity, Mark identified student pairs according to AVMR data and provided each pair with a deck of cards marked with a sequence at or near students' current knowledge. Mark's use of assessment information not only is an example of Guiding Principle Two, using assessment to inform instruction, but it also allows for the presence of Guiding Principle Three, aiming instruction at students' cutting edge of knowledge. By using data on students' current understandings, Mark can then choose lessons involving tasks and number ranges to be on the cutting edge of students' current knowledge. Thus, initial interview and classroom observations identified the presence of Guiding Principles Two (*teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching*), Three (*teaching is focused just beyond the "cutting edge" of the child's current knowledge*) and Four (*professional judgment in* *selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks*) at the onset of the study.

Results of researcher-collected AVMR data available to Mark after classroom observation set one, before classroom observation set two, and then again at the conclusion of the study can be found in Appendix D. Each table identifies a topic in early numbers, then provides a description of each level or construct, followed by the number of students found to be at each level and construct in the fall and winter. Topics include Forward Number Word Sequence (FNWS), Backward Number Word Sequence (BNWS), Numeral Identification (NID), and Addition & Subtraction to 20. Fall class data results support initial findings in classroom observation set one, lesson one. During that lesson, the class was playing Buzz. Most students appeared to be secure in the content and not challenged by forward sequence from 19-25. Data from the fall administration of Forward Number Word Sequence assessment indicates that 13 of 17 students are fluent with sequences to at least 30. Ten of those students are fluent with sequences to 100. Similarly, data from the fall Backward Number Word Sequence assessment reveals only six of 17 students fluent with backward sequences from 30, supporting the finding that as a whole group, students struggled playing Buzz with the backward sequence from 22 to 18.

Between classroom observation set one in September and set two in November, fall Number Identification data indicated that three of 19 students can identify and write one-, two-, and three-digit numerals. Five of 19 can identify two-digit numerals. Six students can identify numerals to 20, two students can identify numerals to 10 and work on teen numbers, and three students are not able to identify numbers to 10.

Fall Backward Number Word Sequence data shows that six students can say the backward number word sequence (BNWS) and provide the number word before a given number from 30. Of those six, three can produce the BNWS and number word before from 100. Seven of 19 students can produce the BNWS and number word before from 10 and three more students can produce the BNWS from 10, but cannot state the number word that comes before a number in that range. Four of 19 students are not able to produce the BNWS from 10.

After participating in AVMR Course Two, Mark's use of student data became even more intentional. During classroom observations after participating in AVMR Course Two, Mark used and referenced the physical classroom data sheet, including all students' fall assessment scores for each of the number topics described above. He used the sheet to determine and refine partner groups engaging in a similar activity and to choose students to attend a small group with the teacher to provide challenging content.

In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number, as well as skills that support learning about whole number operations?

Planning for instruction also includes the way math time is structured within the allotted time, and how students are physically arranged for learning. It is reasonable to expect the structure for math time and the ways students are arranged for learning to be conducive to differentiating lessons and flexibly grouping students according to the task or learning outcome. During classroom observations before AMVR Course Two, Mark facilitated predominantly whole-group mathematics lessons. Each lesson included an activity for the day, where Mark posed a task and all students engaged in thinking about a response. Mark provided ample wait time to allow for all students to have a response, then proceeded to lead a discussion about student solutions to the task. Lessons typically modeled the following pattern: Mark posing a task, allowing students think time to solve the task, followed by a discussion about student strategies. The process repeated until math class ended. Table 4.14 is an excerpt from classroom observation set one, illustrating a typical whole class lesson format.

Table 4.13. Excerpt from Classroom Observation: Mark.

Classroom Observation Set One: Lesson Two
Mark has a vertical five frame with blue and red dots along the left to use to refill five frame presented on the SMART Board.

To start the lesson, Mark places three red dots in the five frame. Students are using a personal whiteboard to record responses. Once students record a response, they hold the whiteboard up in the air for Mark to see the responses.
Mark poses the questions in reference to the five frames displayed on the SMART Board: "How many dots do you see? How many are missing?"

Mark is providing students with ample wait time for students to figure out task. During that time he closely observes students engaging in finding a solution.

Mark takes student response for how many dots are seen, then records the numeral 3 on the board. He continues by asking, "How many are missing?" Students reply two. Mark records the addition sign and then writes the numeral 2 followed by the equal sign. Then he asks students, "How many altogether?"
Mark calls on one student, who replies five.

Erasing boards.

Now Mark places a blank five frame vertically and places one dot in the frame.

Mark: How many do you see? How many are missing?

Providing students wait time.

Students hold up their white board for Mark to see.

Mark: Emmanuel, how many do you see?
Emmanuel: One.
Mark: How many are missing?
Emmanuel: Four.
Mark: How did you figure that out? How did you figure out there was four missing?

Emmanuel replies, but is inaudible. Mark repeats while pointing to the four empty boxes, "So you counted these first and then one more." Mark points to the one dot provided in the frame.

Mark: Did anyone else look at it a different way?

Everett: I looked at two boxes and then another two boxes and put them together
Mark: And what did you get?
Everett: Four, and then put the one and mixed it altogether and it was 14.

Mark: 14? Oh. How many were missing, did you say?
Everett: Four.
Mark: And then how many did you see?
Everett: One.
Mark records a four on the board then an addition sign while saying, “So if I take the four and I add the one to it, how many do I have?”

A different student replies five.

Mark: So each time, what should the chips add up to?
Group of students: Five.
Mark: Five, because what is this group of ?
Students: Five frame.
Mark: That is a five frame.

Mark places a new vertical five frame and places four red.
Mark: How many do you see? How many are missing?

Wait time provided for students to figure and record responses.

Mark: Alexis, what did you come up with?
Alexis is responding, but inaudible.

Mark: Did you hear what Alexis said?
Mark then shares her strategy aloud, “I know if you have five, and then you take the one away, then you have four.”
Mark then writes 5-1=4 on the board to correspond with Alexis’ response. Then Mark records what Alexis has written on her white board, 4+1=5, and says, “What you explained to me looks like this,” while pointing to the subtraction problem.
Mark: Did anyone figure a different way? Kaden.
Kaden replies.
Mark: How many did what Kaden did? He saw that when I put the dots on I had 1+1+1+1 and that equals four and then I have one left.
Mark: Did anyone else do a different way? Maggie.

Maggie: I used my fingers.
Maggie put up one finger for each red dot, and then had one empty and one finger down, and that equaled five.

Mark: Anyone else have a different way?

No response.

M: OK, erase your boards.

Mark places up the next problem. “Who is up for a challenge?”

Half of students raise their hand.

M: We are going to go from a five frame to a 10 frame.
He instructs students to close their eyes while he sets up the frame. Mark sets up the 10 frame vertically with a pairwise five pattern, meaning the frame is placed as two columns of five squares. The dots are placed at the top of the frame by filling in the four.

M: How many do you see? And how many are missing?

Mark is providing wait time for students to solve and record on their white boards. Mark is walking around and supporting students individually for writing numerals the correct way and to solve the problem. One student is having a hard time creating a drawing on the white board that matches the 10 frame on the whiteboard.

Mark: Adrianne what did you get? How did you see the five? And then what did you do? How did you count these?
Mark is recording numerals to match Adrianne's explanation. Adrianne explains and Mark is having a hard time understanding her strategy, so he asks Adrianne to come to the front. Adrianne continues her explanation as Mark records her thinking.
Mark is still questioning her strategy and looks up at researcher as he works to understand and accurately record strategy.

Mark: What did someone else do?
Student shares that he counted them. He counted the five there by ones, and then counted the ones missing the same way. Mark records 1+1+1+1+1=5 to show the strategy of counting by ones for each the ones there and then ones missing.

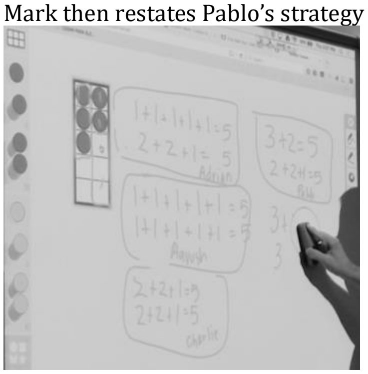
New student goes to the board to show how he counted by twos to get to five. Two, four, and then five.

M: Anybody do it a different way?

Pablo: I did three and two.
Mark: What do you know about three and two, Pablo?
Pablo: Three and two is five.
Mark: OK. Then what did you do?
Pablo: Then I did two, two, and one.

Mark points to the three red dots on the left, and then two red dots on the right to identify and share with the whole class where the three and two came from to make five. Then Mark asks Pablo to come up to the front and take the pointer to show the class where two, two, and one came from. Pablo takes the pointer and is looking at the white board and appears to be thinking hard about the task of pointing to the squares that he counted two, two, and one.
Mark continues to rephrase the question. The multiple questions stated so quickly is making it difficult for Pablo to respond.
Then Pablo points to the squares on the right and explains a strategy for coming up with five empty boxes that is different than the original two, two, and one. Mark addresses the researcher, who is observing behind the video camera, to clarify Pablo's strategy. With researcher assistance, the method for how Pablo got five empty squares by counting two, two, and one was determined.

Mark then restates Pablo's strategy and represents the verbal solutions by recording number sentences.



Lucy shared her strategy of looking at three dots and then saying "four, five" for the other two dots on the right. She then did the same for the empty boxes. Once finished recording her strategy, Mark says, "Actually, you counted on then."

M: So did you see what Lucy did? Do you know a good strategy for when you are doing counting? It is a strategy called counting on. And so she looked, and I saw this right away, and there is three, oh, three, four, five.

While saying this strategy out loud, Mark is pointing to the dots to link the counting on strategy to the verbal and the dots. M: And counting on is a great strategy to use. Look how many just out of this 10 frame, how many different strategies we came up with. Erase your boards.

After participating in AVMR Course Two, Mark incorporated a lesson with a small group of students while remaining students in the class continued the initial whole-group activity with a partner. The lesson opens by introducing a partner activity. Mark facilitates the introduction to the partner activity in the same manner as whole class lessons observed in classroom observation set one before forming partners using recorded AVMR assessment data for all students in the classroom. Once students are paired and setting up the partner activity, Mark joins the small group of students at the front table. An excerpt demonstrating this lesson format from classroom observation set three taking place in December after AVMR Course Two is provided in Table 4.15.

Table 4.14. Excerpt from Classroom Observation: Mark.

Introducing an activity called Nine-plus. Mark introduces the activity as a Bingo game. The gameboard has numbers 14-18. The spinner has numerals 6-9. Students spin to determine the number to add to nine. Students then cover the sum on the gameboard with a colored chip. Two players share one gameboard and play using a different color. To win, a player needs to get three counters in a row. Mark and student are demonstrating how to play the game. Mark spins a seven, then asks the class which strategy the student can use to solve the problem. Students try to respond with the answer and Mark keeps the focus on answering the question, “Which strategy can you use to solve this problem?” Another student responds that the student can use her fingers to solve the problem. Mark responds, “and if you use your fingers, but if she goes one, two, three…she runs out of fingers. She’s not going to use her toes.” Student replies that you can use your partners. Mark: We could, but is there another way where we do not need to use our fingers. A strategy that would be easier. Lucy replies, “Counting on.” Mark: Yes, counting on. Mark demonstrates the counting-on strategy and some students join by counting with Mark. Mark thinks aloud how to make a choice on which 16 to place the counter. Mark spins to have his turn. He spins an eight. Mark shares that he could use the counting on strategy. M: What other strategy is there? Caroline. Caroline shares a strategy that uses the previous problem, 9+7=16, to know 9+8=17.

Student player spins a five. Student player is thinking about how to solve 9+5. Mark provides her with time to think about her solution. Mark quiets the class to ensure she has time to think. She solves, getting the solution 14, and places a counter on the board.

Mark spins a seven. He solves out loud, saying, "I know 9+9 is 18, and so two less is 17." He then covers a spot on the board.

Student spins a nine. Mark quiets the class to provide wait time to provide student time to solve the problem. Student silently solves 9+9, responding 18. Mark then asks her to share her strategy (her strategy is inaudible in the video).

Mark discusses with the class that they will need to decide whether the winner of the game is the one who gets three in a row first, or if they play the game so that each player has the same number of turns.

Mark asks for questions.

Mark is clarifying how to win and what it means to have three in a row horizontally, vertically, and diagonally.

Mark shares that some students will be playing the Nine-plus game. Mark shares where to find materials to play the activity. Then Mark references his class assessment data sheet to record student names on the board that will work with Mark at the small group table.

Mark is choosing partner groups to play Nine-plus by AVMR assessment data.

Students are finding places to sit at the front table. Mark takes a moment to touch base with the support staff member working with a pair of students on Nine-plus before meeting the small group of students at the table.

Mark's lesson structure is conducive to the presence of Math Recovery® Guiding Principles for Classroom Teaching; in particular, Mark uses assessment data. This structure provides an environment for the presence of Guiding Principles Two, Three, and Four, which leads to the presence of remaining Guiding Principles, as addressed in research questions two through four.

Research question two. *In what ways does Add+VantageMR® professional development influence teachers' instructional moves during a whole number lesson?*

1. In what ways are students engaged in activities from the course resources such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 Year-olds* and/or *Developing Number Knowledge*?

2. In what ways are students actively engaged in *doing* mathematics during the lesson?
3. In what ways does the teacher use Guiding Principles and Domains of Progressive Mathematization during a whole number lesson?
4. Do some Guiding Principles seem to be more readily implemented than others?

Data Analysis. Guiding Principles addressed.

Guiding Principles for Classroom Teaching applicable to research question two are as follows:

- *Guiding Principle One: The teaching approach is inquiry-based, that is, problem-based.*
- *Guiding Principle Three: Teaching is focused just beyond the "cutting edge" of students' current knowledge.*
- *Guiding Principle Four: Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*
- *Guiding Principle Five: The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.*
- *Guiding Principle Six: Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.*

- *Guiding Principle Seven: Teaching supports and builds on students' intuitive, verbally-based strategies, and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.*
- *Guiding Principle Eight: The teacher provides students with sufficient time to solve a given problem.*
- *Guiding Principle Nine: Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.*

Sub-question three is used to guide a reflection on the ways AVMR Course Two professional development influenced Carole's instructional moves during a whole number lesson. The same coding procedure described for Carole was used for Mark.

Manifestations of Guiding Principles during a whole number lesson.

The first two classroom observations took place before Mark participated in AVMR Course Two. Based on Mark's prior experience with AVMR Course One, it is expected that coding analysis of classroom observations would reveal a presence of Guiding Principles during the initial classroom observations. Classroom observation analysis identified a greater than expected presence of Guiding Principles in lessons prior to Mark's participation in AVMR Course Two. Not only were all eight Guiding Principles identified during the two sets of classroom observations before AVMR Course

Two, there was a substantial representation from seven of eight Guiding Principles across all classroom observations. Table 4.16 shows the number of occurrences of Guiding Principles before and after Mark's participation in AVMR Course Two.

Table 4.15. Guiding Principles before and after AVMR Course Two: Mark.

	Number of occurrences in September		Number of Occurrences in October			Number of Occurrences2 in December	
	Manifestation of GP	Contrasting the GP	Manifestation of GP	Contrasting the GP		Manifestation of GP	Contrasting the GP
Guiding Principle One	3		1		Participated in Add+VantageMR® Course Two	1	1
Guiding Principle Three	4		2			7	1
Guiding Principle Four	1		4			9	
Guiding Principle Five	4		1			9	
Guiding Principle Six	3						
Guiding Principle Seven	8					3	
Guiding Principle Eight	5	1	1			2	
Guiding Principle Nine	1		1			1	

Classroom observations in October show fewer Guiding Principles than sets one and three. During informal conversations with the researcher during classroom observation set two, Mark recognized and shared that the lessons during this observation set were ones he could easily implement with little planning due to the additional work load he was experiencing at that time. One lesson did not have a number focus and was not included in the Table. Table 4.17 confirms the informal interview, showing a greater presence of Guiding Principle Four (*teachers exercise their professional judgment in* *selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations*) and a decrease in Guiding Principles Three, Five, Six, Seven, Eight, and Nine.

Before AVMR Course Two: Excerpts from the Classroom

Guiding Principle Seven, identified eight times, was the most prevalent in lessons prior to Mark's participation in AVMR Course Two. Guiding Principle Seven states that *teaching supports and builds on student's intuitive, verbally based strategies and these strategies are used as a basis for the development of written forms of arithmetic*. Mark naturally facilitated mathematical discussions where students think about a response to a problem and share the response with the class. As students share, Mark recorded student thinking using mathematical symbols, and connected or expanded strategies to engender the development of more sophisticated student strategies. To illustrate the way these Guiding Principles manifested in Mark's classroom, a sequence of two lessons is provided in Table 4.17 The lesson transcripts also illustrate the interconnectedness of Guiding Principles.

Table 4.16. Excerpt from Classroom Observation: Mark.

Classroom Observation Set One: Lesson One Mark offered a choice for the math activity today. Students chose to play Buzz from his selections for the short time remaining in math class. Starting Buzz for math class. Students are getting into circle to prepare for Buzz. Count forward from 19 to 25 (choosing to cross the 20 decuple). This is a fairly easy task for several students, however, some find it difficult to name the sequence 19, 20, 21 within the whole sequence played during the activity.	Guiding Principle One. Guiding Principle Two. Guiding Principle Three.
When students are not sure of the next number in the sequence, they guess numbers until they say the correct number coming next in the sequence. Towards the end of the game, Mark gives students five seconds to come up with the next number, and if they do not, then they sit down. Some students are not sure of next number and some students are not listening to the number said just before their turn.	Guiding Principle Five.

Kids showing behavior signs that they are having fun with this game. The girl that won learned the sequence during the game. She is doing a celebratory cheer.	Guiding Principle Nine.
Next game is counting backwards from 22 to 18. Mark says an example sequence. There are no visuals to accompany the sequence.	Guiding Principle One. Guiding Principle Two. *Attempts Guiding Principle Three.* Guiding Principle Five. *Including a numeral roll, numeral track, or number line would provide students with support to employ a different strategy than guessing for figuring out the next number in the sequence.*
Students are getting "squirrelly."	
Game commences again by continuing the backward sequence 22 to 18. From the start of this game, Mark employs the counts down from five seconds for response time. This is a much more difficult task for students.	*In contrast to Guiding Principle Eight.*
Mark says, "This is where you learn from your friends if you are not sure what the number is."	

Classroom Observation Set One: Lesson Two	
Mark has a vertical five frame with blue and red dots along the left to use to refill five frame presented on the SMART Board.	
To start the lesson, Mark places three red dots in the five frame. Students are using a personal whiteboard to record responses. Once students record a response, they hold the whiteboard up in the air for Mark to see the responses. Mark poses the questions in reference to the five frames displayed on the SMART Board: "How many dots do you see? How many are missing?"	Guiding Principle One. Guiding Principle Two. Guiding Principle Three.
Mark is providing students with ample wait time for students to figure out task.	Guiding Principle Eight.
Mark takes student response for how many dots are seen, then records the numeral 3 on the board. He continues by asking, "How many are missing?" Students reply two. Mark records the addition sign and then writes the numeral 2, followed by the equal sign. Then he asks students, "How many altogether?" Mark calls on one student, who replies five. Erasing boards.	Guiding Principle Seven. Guiding Principle Three.

Now Mark places a blank five frame vertically and places one dot in the frame.	
Mark: How many do you see? How many are missing?	
Providing students wait time.	Guiding Principle Eight. Guiding Principle Six.
Students hold up their white board for Mark to see.	
Mark: Emmanuel, how many do you see? Emmanuel: One. Mark: How many are missing? Emmanuel: Four. Mark: How did you figure that out? How did you figure out there was four missing?	Guiding Principle Seven.
Emmanuel replies, but is inaudible. Mark repeats while pointing to the four empty boxes, "So you counted these first, and then one more." Mark points to the one dot provided in the frame.	Great example of connecting student responses to the visual frame.
Mark: Did anyone else look at it a different way?	
Everett: I looked at two boxes, and then another two boxes, and put them together. Mark: And what did you get? Everett: Four, and then put the one and mixed it altogether and it was 14.	Great example of eliciting multiple student responses.
Mark: 14? Oh. How many were missing, did you say? Everett: Four. Mark: And then how many did you see? Everett: One. Mark records a four on the board then an addition sign while saying, "So if I take the four and I add the one to it, how many do I have?" A different student replies five.	
Mark: So each time, what should the chips add up to? Group of students: Five. Mark: Five, because what is this group of? Students: Five frame. Mark: That is a five frame.	Guiding Principle Seven.
Mark places a new vertical five frame and places four red. Mark: How many do you see? How many are missing?	
Wait time provided for students to figure and record responses.	
Mark: Alexis, what did you come up with? Alexis is responding, but inaudible.	
Mark: Did you hear what Alexis said? Mark then shares her strategy aloud, "I know if you have five and then you take the one away, then you have 4." Mark then writes 5-1=4 on the board to correspond with Alexis' response. Then Mark records what Alexis has written on her white board, 4+1=5, and says, "What you	Guiding Principle Seven.

explained to me looks like this," while pointing to subtraction problem.	
Mark: Did anyone figure a different way? Kaden. Kaden replies. Mark: How many did what Kaden did? He saw that when I put the dots on, I had 1+1+1+1 and that equals four, and then I have one left. Mark: Did anyone else do a different way? Maggie.	Elicit multiple strategies.
Maggie: I used my fingers. Maggie put up one finger for each red dot, and then had one empty and one finger down, and that equaled five.	
Mark: Anyone else have a different way?	
No response.	
M: OK, erase your boards.	
Mark places up the next problem. "Who is up for a challenge?" Half of students raise their hand. M: We are going to go from a five frame to a 10 frame. He instructs students to close their eyes while he sets up the frame. Mark sets up the 10 frame vertically with a pairwise five pattern, meaning the frame is placed as two columns of five squares. The dots are placed at the top of the frame by filling in the four.	Extending the range.
M: How many do you see? And how many are missing?	Guiding Principle Eight.
Mark is providing wait time for students to solve and record on their white boards. Mark is walking around and supporting students individually for writing numerals the correct way and to solve the problem. One student is having a hard time creating a drawing on the white board that matches the 10 frame on the whiteboard.	Guiding Principle Six.
Mark: Adrianne, what did you get? How did you see the five? And then what did you do? How did you count these? Mark is recording numerals to match Adrianne's explanation. Adrianne explains and Mark is having a hard time understanding her strategy, so he asks Adrianne to come to the front. Adrianne continues her explanation as Mark records her thinking. Mark is still questioning her strategy and looks up at researcher as he works to understand and accurately record strategy.	Guiding Principle Seven.
Mark: What did someone else do? Student shares that he counted them. He counted the five there by ones and then counted the ones missing the same way. Mark records 1+1+1+1+1=5 to show the strategy of counting by ones for each the ones there and then ones missing	Elicit multiple strategies.
New student goes to the board to show how he counted	

by twos to get to five. Two, four, and then five.

M: Anybody do it a different way?
Pablo: I did three and two.
Mark: What do you know about three and two, Pablo?
Pablo: Three and two is five.
Mark: OK. Then what did you do?
Pablo: Then I did two, two, and one.

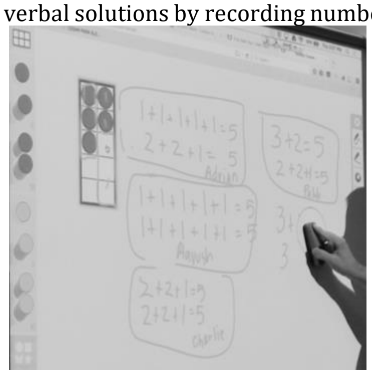
Mark points to the three red dots on the left and then two red dots on the right to identify and share with the whole class where the three and two came from to make five. Then Mark asks Pablo to come up to the front and take the pointer to show the class where two, two, and one came from. Pablo takes the pointer and is looking at the white board and appears to be thinking hard about the task of pointing to the squares that he counted two, two, and one. Mark continues to rephrase the question. The multiple questions stated so quickly is making it difficult for Pablo to respond. Then Pablo points to the squares on the right and explains a strategy for coming up with five empty boxes that is different than the original two, two, and one. Mark addresses the researcher, who is observing behind the video camera, to clarify Pablo's strategy. With researcher assistance, the method for how Pablo got five empty squares by counting two, two, and one was determined.

Guiding Principle Seven.

Guiding Principle Six.

Mark then restates Pablo's strategy and represents the verbal solutions by recording number sentences.

Guiding Principle Seven.



Guiding Principle Six.

Guiding Principle Five.

Lucy shared her strategy of looking at three dots and then saying "four, five" for the other two dots on the right. She then did the same for the empty boxes. Once finished recording her strategy, Mark says, "Actually, you counted on then."
M: So did you see what Lucy did? Do you know a good strategy for when you are doing counting? It is a strategy called counting on. And so she looked, and I saw this right away, and there is three, oh, three, four, five.
While saying this strategy out loud, Mark is pointing to the dots to link the counting on strategy to the verbal and the dots.
M: And counting on is a great strategy to use. Look how many just out of this 10 frame, how many different strategies we came up with. OK, close your eyes – Oh, erase your boards first.

At this time in the lesson students are significantly losing focus.	
Mark is making a new pattern on the 10 frame. This time, the 10 frame is horizontal. Four dots fill in the top row from left to right.	Guiding Principle Four.
Mark: How many do you see? How many are missing?	Guiding Principle Eight.
Students are working and thinking.	
Mark begins asking students about their strategies. Mark asks students to come up and use the pointer to share their strategy. Another student said he knows there are five across the top, and minus one is four. Then he counted up the bottom row by ones.	Elicits multiple student strategies.
Mark: Caroline, what did you do?	
Caroline: Counted by twos.	
Mark: OK, come on up here and show us how you counted by twos.	
Caroline pointed to the four red dots to do 2+2=4, then she counted the bottom squares on her fingers by one. Mark links that part of her strategy to another student's strategy.	Guiding Principle Five.
Everett: I knew you had two fours and then I knew two and two and two make six. Everett comes to front to show his strategy. He knows the top is five, and take one away is four. Then he points to two empty squares at a time to show the three twos. Then Everett changes midway through his explanation to say three and three empties make six.	
Mark represents verbal strategy with number sentences to match verbal explanations.	Guiding Principle Seven.
Emmanuel comes up to show his strategy. He determined four dots in the same way as others before. He had a new strategy for empties, where he looked at the five empties on the bottom and went one more to make six.	
Lesson ends.	
Game ends because students are not being kind to their friends. Students are talking and dancing in their spots. Focus is lost. Students stack chairs to prepare to go home.	

The lesson sequence illustrated how choosing a problem-based task (Guiding Principle One), and attending to AVMR assessment results (Guiding Principle Two), provides a basis for aiming instruction at a child's cutting edge of current knowledge (Guiding Principle Three). Mark engaged in careful observation of students' as the students engage in tasks during the lesson (Guiding Principle Six). Careful observation leads to making choices regarding number ranges within tasks. Understanding of children's numerical strategies (Guiding Principle Five) provides Mark with knowledge to build on student's intuitive, verbally based strategies as a basis for the development of written forms of arithmetic (Guiding Principle Seven). Providing students with adequate time to think hard about mathematical tasks (Guiding Principle Eight) often leads to intrinsic satisfaction from solving the task (Guiding Principle Nine) and students' overall positive feelings towards mathematics.

During classroom observations before participating in AVMR Course Two, four out of six activities selected for lessons were from AVMR Course One content or resources.

After AVMR Course Two: Excerpts from the Classroom

After participating in Add+VantageMR Course Two, classroom observations revealed that manifestations of Guiding Principles Three, Four, and Five to be the most prevalent. During the lessons in December, Mark drew upon a range of tasks (Guiding Principle Four) from Add+VantageMR Courses One and Two materials and Cognitively Guided Instruction to provide students with experiences adding one- and two-digit numbers based on students' current understandings (Guiding Principle Three). The excerpt below from December classroom observations demonstrates how Guiding Principles Three, Four, and Five are manifested in within Mark's classroom.

Table 4.17. Excerpt from December Classroom Observation.

Introducing an activity called Nine-plus. Mark introduces the activity as a Bingo game. The gameboard has numbers 14-18. The spinner has numerals 6-9. Students spin to determine the number to add to nine. Students then cover the sum on the gameboard with a colored chip. Two players share one gameboard and play using a different color. To win, a player needs to get three counters in a row.	Guiding Principle Four.
Mark and student are demonstrating how to play the game. Mark spins a seven, then asks the class which strategy the student can use to solve the problem. Students try to respond with the answer and Mark keeps the focus on answering the question, "Which strategy can you use to solve this problem?" Another student responds that the student can use her fingers to solve the problem. Mark responds, "And if you use your fingers, but if she goes one, two, three…she runs out of fingers. She's not going to use her toes." Student replies that you can use your partners. Mark: We could, but is there another way where we do not need to use our fingers. A strategy that would be easier. Lucy replies, "Counting on." Mark: Yes, counting on. Mark demonstrates the counting on strategy and some students join by counting with Mark. Mark thinks aloud how to make a choice on which 16 to place the counter	*Attempts Guiding Principle Five: Many students in the classroom are not yet ready to accommodate the counting-on strategy.*
Mark spins to have his turn. He spins an eight. Mark shares that he could use the counting-on strategy. M: What other strategy is there? Caroline. Caroline shares a strategy that uses the previous problem, 9+7=16, to know 9+8=17. Student player spins a five. Student player is thinking about how to solve 9+5. Mark provides her with time to think about her solution. Mark quiets the class to ensure she has time to think. She solves, getting the solution 14, and places a counter on the board. Mark spins a 7. He solves out loud saying, I know 9+9 is 18 and so 2 less is 17. He then covers a spot on the board. Student spins a nine. Mark quiets the class to provide wait time to provide student time to solve the problem. Student silently solves 9+9, responding 18. Mark then asks her to share her strategy (her strategy is inaudible in the video).	Eliciting multiple strategies. Guiding Principle Eight. Guiding Principle Eight.
Mark discusses with the class that they will need to decide whether the winner of the game is the one who gets three in a row first, or if they play the game so that each player has the same number of turns. Mark asks for questions. Mark is clarifying how to win and what it means to have three in a row horizontally, vertically, and diagonally.	
Mark then shares that some students will be playing the Nine-plus game. Mark shares where to find materials to play the activity. Then Mark references his class assessment data sheet to record student names on the board that will work with Mark at the small group table. Mark is choosing partner groups to play Nine-plus by AVMR	

assessment data. Students are finding places to sit at the front table. Mark takes a moment to touch base with the support staff member working with a pair of students on Nine-plus.	Guiding Principle Two.
Researcher moves around the room supporting students getting started on the Nine-plus game.	
Mark introduces a join change unknown word problem to the small group: Sparky has six rocks. Coco has some more rocks. They have 10 rocks altogether. Students are not sure how to solve. Mark then puts Sparky + Coco on the board. He asks students how many rocks Sparky has and records that number under Sparky, then asks if they know how many rocks Coco has. Students respond no, so Mark records a box under Coco followed by the = 10. A student responds that coco has 16 rocks. Mark then places 16 in the equation he wrote on the board to show that 16 would not make the equation true. Mark asks students what has to go into the blank to make 10. Mark asks students to erase the six and 10 in the problem and asks students to use the next set of numbers identified on the sheet. Mark is telling students to pick the set of numbers that is works for them. A set of numbers that is not too easy or not too hard. Numbers choices are in the two-digit and one-digit. He releases students to show how they would solve the problem on the paper. Students are recording the answer and Mark is asking that they also write how they solve the problem.	Guiding Principle One. Guiding Principle Two. Guiding Principle Three. Guiding Principle Four.
Students are loud, playing the Nine-plus game around the room.	
Some students in the small group are sitting and not sure how to solve the problem. Mark then begins to create a chart on the board to support students in how to set up and solve the problem. Mark is walking through the problem in a similar manner as the previous problem. Mark asks questions to walk through the problem and demonstrate how to solve. Mark is asking a student, "How many cookies does Coco have? How do you know?" Mark listens to the student explanation. Mark goes from student to student, asking what their solution is and how to solve the problem.	*In contrast to Guiding Principle One.*
Students playing Nine-plus are having a hard time staying focused with the game. Groups also need tools to support them in playing the game. Most students at the floor and not at the small group table are not yet able to use a counting on strategy fluently and need tools to count all.	*In contrast of Guiding Principle Three.*
Mark is sharing that his problem in the small group is designed to help students see how to get from a two-digit number to a decuple. Mark continues to listen to students share their strategies for getting up to the decuple. Students are using counting on for their strategies.	Guiding Principle Five.
Students are not engaged in the activity of listening to each other's strategies. Mark is connecting students' strategies to each other as he facilitates	

a small group strategy share. Mark now begins to record student strategies using written notation on the board once a strategy is shared using jumps of composite units. Previous strategies are involving counting on from starting number to total. Mark is clarifying and expanding upon using jumps of composite units to solve problems similar to what students solved today for students in the small group.	Eliciting multiple strategies. Guiding Principle Seven. Guiding Principle Five.
Lesson ends. Students are cleaning up to go home for the day.	
Mark is adding games to the work places with the Bridges curriculum. They are going to add the activities done last week, such as Nine-plus. Mark introduces today's activity as Guess my Number. The activity is based on the numeral track. Each partner set receives a line of five numerals to place in a folder with flaps that will cover all the numbers and allow numbers to be visible as either partner lifts a flap. Mark shares the materials required and demonstrates with a student partner. The activity begins with the student partner revealing a number by raising a flap. The number 19 is revealed. The student then points to another flap to ask Mark which number is under the flap. Mark guides students in determining a strategy to figure out which number is under the flap. Students are sharing to count on. However, counting on is counting forward, and Mark is supporting the class in vocabulary to indicate the strategy is to count back to determine the number under the flap. Students and Mark are playing a game together to demonstrate how to play. Now Mark and student switch roles. The student chooses a set of numbers to use for the activity. Mark shares with students about choosing a number set that is one that will challenge themselves. Mark is now asking the student helping to demonstrate the activity to determine which number is under the flap. Students in the class are observing or determining the answer themselves and then finding out if they are right after the student determines the number under the flap. Mark again focuses on choosing numbers that are a little more difficult that each student needs to think about to figure out. Mark has a range of number choices, teen numbers, one-30 sequence, one-10, count by 10s, counting by 100s from 600 to 1000. Students are going to play in partners. Mark is choosing partners and number ranges provided to each partner group. Mark is working with a small group of students while remaining students are working on the activity presented to the class. Mark sets up the table space to begin working with a group of seven students on place value and addition. Mark has bundles of popsicle sticks available for students. Each student has a plain piece of paper and pencil. Mark places a bundle under the screen and students say the quantity. This continues until there are three bundles and four sticks under a screen. Students then record 34 on their papers. Mark then repeats this process until five bundles and five sticks are under the second screen. Students are now to add the two numbers together: 34 + 55. Students are solving and recording their strategy on the paper. Mark	Guiding Principle Four. Guiding Principle Five. Guiding Principle Four. Guiding Principle Three. Guiding Principle Three. Guiding Principle Two. Guiding Principle Two. Guiding Principle Three. Guiding Principle Four.

is supporting one student with notation. The student shares his thinking as Mark records on paper. A student shares a strategy and Mark asks if the strategy is efficient.	Guiding Principle Seven.
All students are now done solving the problem, and all get the answer of 89. Mark is asking a student to share his strategy. His strategy is counting on from 34 by ones. Then Mark asks another student about her strategy. Mark is drawing out different strategies and talking about strategies that are efficient and why.	Guiding Principle Five.
Mark begins to demonstrate a strategy. He records 34+55= on the board. Mark is demonstrating the drop down strategy by adding the tens first and then adding the ones. He focuses on using tens language and maintaining the value of numbers.	
Researcher then ties that back into the bundles and sticks that are lying on the table. Now trying a different problem. Researcher and Mark are having a conversation about when to keep sticks covered or uncovered and the placement of a larger number on the right side when setting out bundles and sticks in two sets. Mark is setting up the second problem in the same way as the first. While this small group is going on, the rest of the class is engaged in the activity presented at the beginning of the hour.	*Attempting Guiding Principle Five.*
Students are working on 59+32. Sticks are on the table and left uncovered for students to look at when solving. Some students are trying the drop down strategy Mark shared previously.	Guiding Principle Five.
Mark is supporting one student who is using the sticks to count by ones even when they are bundled. Mark is asking her if there is a quicker way to count. He is appropriately presenting the sticks for her to add a bundle of 10 to a number involving tens and ones. Mark is supporting another student to record thinking and to work with two-digit numbers while maintaining the value of the tens instead of referring to tens numbers as a single digit, i.e., 30 instead of three. During explanations, Mark at times makes connections with the bundles and sticks and then at times begins to support student thinking without linking to the bundles and sticks. Time to clean up in preparation for end of the lesson. Students playing the activity on the floor begin to pick up supplies as Mark finishes working with students at the back table. Mark collects papers from group and has listened to each students strategy at the back table before releasing them back to their spots. Students begin to stack chairs and get ready to go home.	Guiding Principle Five.

Research question three. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

1. In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?

2. In what ways do teachers modify an activity during a lesson (modification may include offering different dice, number ranges, and/or instructional tools)?

Data Analysis: Guiding Principles

Guiding Principles for Classroom Teaching applicable to research question three are as follows:

- *Guiding Principle Three: Teaching is focused just beyond the "cutting edge" of students' current knowledge.*
- *Guiding Principle Four: Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*

In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration? In what ways do teachers modify an activity during a lesson?

Research question one addressed the ways differentiation can happen as teachers plan for a lesson. Research question two focused on a variety of teacher moves related to Guiding Principles for Classroom Teaching during a lesson. Research question three specifically addresses differentiating for students through the lens of Guiding Principle Three (*teaching is focused just beyond the "cutting edge" of students' current knowledge*) and Guiding Principle Four (*teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular* *instructional settings and tasks, and varying this selection on the basis of on-going observations*). This includes short and long term planning for lessons, in-the-moment decisions teachers make that result in students making mathematical connections to advance thinking, or students practicing skills and refining strategies appropriate within a trajectory of student learning. The following paragraphs draw upon data collected during informal, pre-, and post- interviews, and provide example excerpts from Mark's classroom observations before and after his participation in AVMR Course Two.

Before AVMR Course Two: Excerpts from the Classroom

During classroom observations before Mark participated in AVMR Course Two, there was very little presence of flexibly grouping or intentionally partnering students during a lesson. There was one occasion where Mark decided to partner students according to AVMR data within the final lesson in the second set of classroom observations. Within the lesson, Mark begins to suggest for students to choose a partner and then he reflects within the moment and decides to place students into partners to play the Treasure Hunt activity. Mark then modifies the activity according to partner set by providing partner sets with number sequences which are challenging for each group of students.

Table 4.18. Excerpt from Classroom Observation: Mark.

Mark begins by choosing a student for a partner to help demonstrate how to play Treasure Hunt. Each game will be played using two rows, each with their own color. Treasure Hunt is a number sequence activity where the teacher can choose any number work sequence for students to practice. The sequences can be forward or backward. To set up the game, the card with the color dot is used to show the start of each row. Then 10 cards are placed randomly upside down in a straight line to the right of each color dot. The remaining cards are used for a draw pile. To begin play, one student draws the first card from the draw pile and places it face up in the correct position in the sequence. The student then picks up the	Guiding Principle Four. Guiding Principle One.

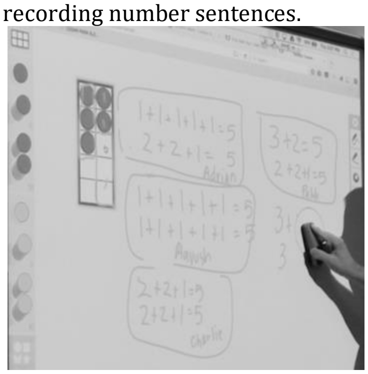
card already in that position face down and gives it to his partner to place in its correct place in the sequence. Students and Mark demonstrate using a strategy to place cards in the number sequence once a few cards have been correctly placed. Students are helping the student demonstrating how to play the game to correctly orientate the numeral 10. Mark and the student are showing how to play with the sequence one-10. Mark starts to tell a student to pick a friend and then he changes to assigning partners according to the current number sequence knowledge. Mark is passing out one-10 sequences, 11-20 sequences, counting by 10s, and 22-31 sequence.	Guiding Principle Two. Guiding Principle Three.

Mark also implemented modifications within a lesson in two additional lessons, thus meeting students' current understandings (Guiding Principle Three). During a lesson in September, Mark planned a lesson designed to support students' ability to visualize quantities promoting students advancement from perceptual to figurative counting. During the lesson, Mark begins with a five frame and then transitions to a 10 frame. The following lesson excerpt illustrates this modification.

Table 4.19. Excerpt from Classroom Observation.

Mark places a new vertical five frame and places four red. Mark: How many do you see? How many are missing? Wait time provided for students to figure and record responses. Mark: Alexis, what did you come up with? Alexis is responding, but inaudible. Mark: Did you hear what Alexis said? Mark then shares her strategy aloud, "I know if you have five and then you take the one away, then you have four." Mark then writes 5-1=4 on the board to correspond with Alexis' response. Then Mark records what Alexis has written on her white board, 4+1=5, and says, "What you explained to me looks like this," while pointing to the subtraction problem. Mark: Did anyone figure a different way? Kaden. Kaden replies. Mark: How many did what Kaden did? He saw that when I put the dots on I had 1+1+1+1, and that equals four, and then I have one left. Mark: Did anyone else do a different way? Maggie. Maggie: I used my fingers.	Guiding Principle Seven. Elicit multiple strategies.

Maggie put up one finger for each red dot and then had one empty and one finger down and that equaled five. Mark: Anyone else have a different way? No response. M: OK, erase your boards. Mark: places up the next problem. “Who is up for a challenge?” Half of students raise their hand. M: We are going to go from a five frame to a 10 frame. He instructs students to close their eyes while he sets up the frame. Mark sets up the 10 frame vertically with a pairwise five pattern, meaning the frame is placed as two columns of five squares. The dots are placed at the top of the frame by filling in the four. M: How many do you see? And how many are missing?	Extending the range.
Mark is providing wait time for students to solve and record on their white boards. Mark is walking around and supporting students individually for writing numerals the correct way and to solve the problem. One student is having a hard time creating a drawing on the white board that matches the 10 frame on the whiteboard.	Guiding Principle Eight. Guiding Principle Six.
Mark: Adrianne, what did you get? How did you see the five? And then what did you do? How did you count these? Mark is recording numerals to match Adrianne’s explanation. Adrianne explains and Mark is having a hard time understanding her strategy, so he asks Adrianne to come to the front. Adrianne continues her explanation as Mark records her thinking. Mark is still questioning her strategy and looks up at researcher as he works to understand and accurately record strategy.	Guiding Principle Seven.
Mark: What did someone else do? Student shares that he counted them. He counted the five there by ones and then counted the ones missing the same way. Mark records 1+1+1+1+1=5 to show the strategy of counting by ones for each the ones there and then ones missing New student goes to the board to show how he counted by twos to get to 5. Two, four, and then five.	Elicit multiple strategies.
M: Anybody do it a different way? Pablo: I did three and two. Mark: What do you know about three and two, Pablo? Pablo: Three and two is five. Mark: OK. Then what did you do? Pablo: Then I did two, two, and one. Mark points to the three red dots on the left and two red dots on the right to identify and share with the whole class where the three and two came from to make five. Then Mark asks Pablo to come up to the front and take the pointer to show the class where two, two, and one came from. Pablo takes the pointer and is looking at the white board and appears to be thinking hard about the task of pointing to the squares that he counted two, two, and one. Mark continues to rephrase the question. The multiple questions stated so quickly is making it difficult for Pablo to respond. Then Pablo points to the squares on the right and explains a strategy for coming up with five empty boxes that is different than the original two, two, and one. Mark addresses the researcher, who is observing	Guiding Principle

behind the video camera, to clarify Pablo’s strategy. With researcher assistance, the method for how Pablo got 5 empty squares by counting two, two, and one was determined.	Seven.
Mark then restates Pablo’s strategy and represents the verbal solutions by recording number sentences.	Guiding Principle Six.
	Guiding Principle Seven.
Lucy shared her strategy of looking at three dots and then saying “four, five” for the other two dots on the right. She then did the same for the empty boxes. Once finished recording her strategy, Mark says, “Actually, you counted on then.”	Guiding Principle Six.
M: So did you see what Lucy did? Do you know a good strategy for when you are doing counting? It is a strategy called counting on. And so she looked, and I saw this right away and there is three, oh, three, four, five.	
While saying this strategy out loud, Mark is pointing to the dots to link the counting on strategy to the verbal and the dots.	Guiding Principle Five.
M: And counting on is a great strategy to use. Look how many just out of this 10 frame, how many different strategies we came up with. OK, close your eyes – Oh, erase your boards first.	
At this time in the lesson, students are significantly losing focus.	

In another lesson, the task is to complete the missing number in a number word sequence of three numbers. The number range on the worksheet includes number sequences to 100. Mark provides students fluent with number sequences to 10 with a hundreds chart for support with sequences to 100. Additionally, every other decade is shaded gray for additional visual support distinguishing each decade.

Before taking AVMR Course Two, Mark shared that he wanted to implement small group instruction during math lesson time. Although lessons before participating in AVMR Course Two revealed manifestations of Guiding Principles, Guiding Principles Three and Four did not have a strong presence within classroom observations in September and October.

After AVMR Course Two: Excerpts from the Classroom

After taking AVMR Course Two, Mark determines a way to structure the math lesson to include working with a small group while remaining students in the class are actively engaged in a partner activity. Mark begins the December lessons similar to lessons observed before Mark's participation in AVMR Course Two. After students are familiar with the activity and possible strategies for solving problems, students continue the activity in partners, while Mark facilitates a small group lesson where challenging tasks are presented to students successful using composite strategies when solving addition and subtraction problems to 20. In this small group setting, Mark implements AVMR Course Two content. The following two lessons show how Mark modifies number ranges for students according to partners based on AVMR data. Mark then identifies students to work with in the small group using AVMR data.

Table 4.20. Excerpt from Classroom Observation.

Mark introduces today's activity as Guess my Number. The activity is based on the numeral track. Each partner set receives a line of five numerals to place in a folder with flaps that will cover all the numbers and allow numbers to be visible as either partner lifts a flap. Mark shares the materials required and demonstrates with a student partner. The activity begins with the student partner revealing a number by raising a flap. The number 19 is revealed. The student then points to another flap to ask Mark which number is under the flap.	Guiding Principle Four.
Mark guides students in determining a strategy to figure out which number is under the flap. Students are sharing to count on. However, counting on is counting forward, and Mark is supporting the class in vocabulary to indicate the strategy is to count back to determine the number under the flap. Students and Mark are playing a game together to demonstrate how to play.	Guiding Principle Five.
Now Mark and student switch roles. The student chooses a set of numbers to use for the activity. Mark shares with students about choosing a number set that is one that will challenge themselves.	Guiding Principle Four. Guiding Principle Three.

Mark is now asking the student helping to demonstrate the activity to determine which number is under the flap. Students in the class are observing or determining the answer themselves and then finding out if they are right after the student determines the number under the flap.	
Mark again focuses on choosing numbers that are a little more difficult that each student needs to think about to figure out.	Guiding Principle Three.
Mark has a range of number choices, teen numbers, one-30 sequence, one-10, count by 10s, counting by 100s from 600 to 1000.	
Students are going to play in partners. Mark is choosing partners and number ranges provided to each partner group.	Guiding Principle Two.
Mark is working with a small group of students while remaining students are working on the activity presented to the class.	
Mark sets up the table space to begin working with a group of seven students on place value and addition.	
Mark has bundles of popsicle sticks available for students. Each student has a plain piece of paper and pencil.	Guiding Principle Two. Guiding Principle Three.
Mark places a bundle under the screen and students say the quantity.	
This continues until there are three bundles and four sticks under a screen. Students then record 34 on their papers. Mark then repeats this process until five bundles and five sticks are under the second screen.	Guiding Principle Four.
Students are now to add the two numbers together: 34 + 55.	
Students are solving and recording their strategy on the paper. Mark is supporting one student with notation. The student shares his thinking as Mark records on paper.	
A student shares a strategy and Mark asks if the strategy is efficient.	Guiding Principle Seven.
All students are now done solving the problem, and all get the answer of 89. Mark is asking a student to share his strategy. His strategy is counting on from 34 by ones. Then Mark asks another student about her strategy. Mark is drawing out different strategies and talking about strategies that are efficient and why.	Guiding Principle Five.
Mark begins to demonstrate a strategy. He records 34+55= on the board.	
Mark is demonstrating the drop down strategy by adding the tens first and then adding the ones. He focuses on using tens language and maintaining the value of numbers.	
Researcher then ties that back into the bundles and sticks that are lying on the table.	*Attempting Guiding Principle Five.*
Now trying a different problem.	
Researcher and Mark are having a conversation about when to keep sticks covered or uncovered and the placement of a larger number on the right side when setting out bundles and sticks in two sets.	
Mark is setting up the second problem in the same way as the first.	
While this small group is going on, the rest of the class is engaged in the activity presented at the beginning of the hour.	
Students are working on 59+32.	Guiding Principle Five.
Sticks are on the table and left uncovered for students to look at when solving. Some students are trying the drop down strategy Mark shared previously.	
Mark is supporting one student who is using the sticks to count by ones even when they are bundled. Mark is asking her if there is a quicker way to count. He is appropriately presenting the sticks for her to add a bundle of 10 to a number involving tens and ones.	Guiding Principle Five.
Mark is supporting another student to record thinking and to work with two-digit numbers while maintaining the value of the tens instead of referring to tens numbers as a single digit, i.e., 30 instead of three.	
During explanations, Mark at times makes connections with the bundles and sticks and then at times begins to support student thinking without linking to the bundles and sticks.	

Time to clean up in preparation for end of the lesson. Students playing the activity on the floor begin to pick up supplies as Mark finishes working with students at the back table. Mark collects papers from group and has listened to each student's strategy at the back table before releasing them back to their spots. Students begin to stack chairs and get ready to go home.	

After participating in AVMR Course Two, Mark implemented small group instruction during math time as he hoped to do at the beginning of the study. Small group instruction provided a structure to engage students in tasks at the students' cutting edge of knowledge (Guiding Principle Three). Mark identified lessons for small group instruction from AVMR Course Two to challenge a group of students using non-count-by-ones strategies to solve addition facts to 20. Guiding Principle Four (*teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations*) is present when Mark adjusted the Nine-plus whole group activity and during in-the-moment decisions during small group instruction.

Research question four. *How does Add+VantageMR® professional development influence teachers' assessment of students learning during implementation of a whole number lesson?*

Data Analysis: Guiding Principles

One Guiding Principle for Classroom Teaching is applicable to research question four.

- *Guiding Principle Two: Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.*

Research question four focuses on teacher's noticing student solution strategies and the subsequent assessment of students' mathematical knowledge and shifts in that knowledge *during* a whole number lesson. Research Question four does not address action after making an informal assessment of students' understanding. The action a teacher takes in response to informal assessment information is better described as through differentiation decisions made during and after a whole number lesson, which is addressed within research question three. Research question four, sub-question three is used to guide the following reflection on the ways in which AVMR Course Two professional development influenced Mark's noticing of student strategies and informal assessment during a whole number lesson.

Before and after taking AVMR Course Two, Mark provided opportunities for students to share strategies and expand upon their own thinking. Classroom observations before taking AVMR Course Two reveal that Mark practices intense observation of students' strategies during a whole number lesson. For example, during one lesson in classroom observation set one, students were to determine the number of red dots placed within a five and then 10 frame. During this lesson, as in other lessons, Mark listened intently and posed questions to students to accurately record students' strategies and thinking as they truly occurred.

After AVMR Course Two, Mark listened intently and posed questions to students attending to specific strategies students are using to solve problems, as he did before taking AVMR Course Two. However, classroom observation data indicates a change in the way Mark rephrases, names, and connects strategies during a whole number lesson, providing evidence that Mark is generally placing student strategies within a framework or developmental model. Mark appropriately connects strategies in close sophistication. The excerpt below is part of the post interview occurring one month after AVMR Course Two. Mark shares that he would like more practice within an AVMR course administering and analyzing assessments to better be able to identify students' exact strategies and better place them within the framework during the lesson.

> M: In our class, something was mentioned that AVMR is more about giving you the data that you need, right? It's not that we're a program… You have the data and this is where your kids are at in these areas. Here's in the book. Here's things that you can do, but it's more prescriptive, but yet it's not. But [AVMR] doesn't go that next step, and I am not sure where you guys want to be…
>
> I: I would love to hear more about this part. What role does the class play? What are the take-aways from the class? From your perspective, I would love to hear more about that… how the class is helpful or not helpful.
>
> M: We went through and we picked a couple of students that we did the assessments on and got a chance, like when we went through AVMR One, I just remember we watched multiple videos and then you would code what they are doing and stuff to score it, to have the practice of going through that and stuff. For me, it is just some of the rote, and just doing that, like, in my head, I can think, "This is where they did this," and I can get a general idea, but I don't have the skill to hone in on that exactness. I can only get there generally, but not exactly… I think to have the practice when you get back, it is messy… because when you get a chance, it's like you are fumbling… Try some of these assessments, do them a few times, and bring them in, then really let us have the opportunity in a class to sit down and really analyze the data. We do our data analysis days here after literacy, how they did on writing about reading or whatever, but with the AVMR, you did those assessments and code and stuff and how to fill out the form from the forward number sequence on that sheet that has, you know, the number word after, or something like that, but with…
>
> I: The actual coding of it.
>
> M: The actual coding, so the sheet has the FNWS [Forward Number Word Sequence], BNWS [Backward Word Sequence]
>
> I: The student profile sheet.
>
> M: I could see directly, here's my fives, here's my fours, here's my threes, so to come back after doing the class, having done some of those assessments, you know,

> to code and stuff, and then to get some feedback, you said that they're this, but tell me how you arrived at that… because, you know, in our mind, of course you're right, but we may be not, because we are all going off of our perspective during the class.
>
> I: In what ways does that help in instruction?
>
> Mark: From when we first started doing AVMR, I don't know how many years ago, you know having been a kindergarten teacher, it's very prescriptive and it really gives me the range of my kids and where they are at mathematically. You know in whatever category who are the fives, and who are my zeroes and stuff. Then again, from the purple book, looking at activities in there that you can give those students to work on, to build those skills, and now, with having the red book, to have and pull activities out of that, and then starting to do some out of the blackline masters to go with the data. I find it valuable for math. It's the best. Do I get brownie points now? [laughing] From my usual teaching, it tells me where my kids are at, so I feel confident of what it tells me. You can definitely tell where the students' strengths are and where their weaknesses are and then build on those.

In summary, Mark used informal assessment before and after participating in AVMR Course Two. It is difficult to discern whether small modifications to Mark's language after participation in AVMR Course Two is a result from the course or occurs due to the different nature of the tasks from before to after AVMR Course Two. Excerpts from Mark's post interview reveal that Mark wants to have more experiences analyzing and talking about the results of analysis in order to increase his own understanding of the model. Mark shares that if he can increase his own understanding of the model, then he can accurately engage in informal assessment during a lesson with less of his own cognitive load.

Case Two provided a description of Mark's planning for instruction and classroom teaching before and after participating in AVMR Course Two. Case Three provides a description of Jessica's planning for and classroom teaching before and after AVMR Course One and then after AVMR Course Two.

Case Three: Jessica.

Background Information. At the start of the study, Jessica is starting her first year of teaching. She joins Cedar Ridge staff having a year-long co-teaching experience as a teacher candidate from a large university in the Midwest the year prior. Like many teachers new to a district, Jessica joined the team late in the summer, and is placed in a position that is created from more than one part-time position. She teaches first grade math and science for half the day each morning, and then teaches a small reading intervention group at Cedar Ridge Elementary in the afternoon. The last hour of her day is at a nearby elementary school within the district, where she teaches social skills in a kindergarten classroom.

Jennifer meets with the first grade team during full-team planning meetings as much as possible due to her complex schedule. During the study, Jessica contributed to the team by sharing activities and insights into working with flexible small groups during math time with her first grade colleagues. Jessica's lessons and activities align with team and district mathematics curriculum frameworks. She uses activities found on the district website, in district adopted curriculum, and teacher websites, such as teachers pay teachers when planning lessons.

Jessica participated in an initial interview during workshop week before students' first day of school. The first set of three classroom observations took place in mid to late September. Jessica was provided a spreadsheet with individual AVMR assessment results for all students in the classroom about the same time she participated in AVMR Course One. The second set of three classroom observations took place in October after participation in AVMR Course One. Jessica participated in AVMR Course Two in late

November/early December with two first grade colleagues also participating in this study. The final set of three classroom observations took place in mid to late December, before school closed for winter break, and after Jessica's participation in AVMR Course Two. Jessica participated in a final one-on-one interview in late December. Figure 4.3 below summarizes the data collection schedule for Jessica. Following figure 4.3, results are organized by research question. For Jessica, sub-questions two, three, and four were useful in making sense of instructional practices before and after taking AVMR Courses One and Two professional development.

Figure 4.3. Data Collection Schedule: Jessica.

August 2015	**September 2015**	**Sept/Oct 2015**	**October 2015**	**Nov/Dec 2015**	**December 2015**	**December 2015**
Initial one-on-one interview	Classroom observation #1	AVMR Course One	Classroom observation #2	AVMR Course Two	Classroom observation #3	Post one-on-one interview

Research question one. *In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?*

1. In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?
2. What are teachers using to determine children's current level of understanding?
3. In what ways do teachers use course resources provided in course *Teacher Handbooks or Teaching Number in the Classroom with 4-8 Year-olds* (Wright et al., 2014) and/or *Developing Number Knowledge* (Wright, et al., 2011) when planning a whole number lesson?

4. In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number, as well as skills that support learning about whole number operations?

Data Analysis.

Guiding Principles for Classroom Teaching applicable to Research Question One are as follows:

- Guiding Principle Two: *Teaching is informed by an initial comprehensive assessment and ongoing assessment through teaching. The latter refers to the teachers' informed understanding of children's current knowledge and problem-solving strategies and continual revision of this understanding.*
- Guiding Principle Three: *Teaching is focused just beyond the "cutting-edge" of the child's current knowledge.*
- Guiding Principle Four: *Teachers exercise their professional judgement in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.*

For an explanation of each Guiding Principle for Classroom Teaching, see Appendix A.

What are teachers using to determine children's current level of understanding?

Planning for instruction begins by using general knowledge of children's thinking and then refined by using detailed information regarding students' current mathematical understandings. This is a specific application of Guiding Principle Two: *Teaching is* *informed by an initial comprehensive assessment and ongoing assessment through teaching.*

Initial interviews and classroom observations provide a baseline for documenting changes throughout the study. During the initial interview, Jessica indicated it is important for a teacher to walk around and engage in informal assessment during a lesson. Before to taking AVMR Courses One and Two, Jessica did not reference specific sources for formal assessments or additional details regarding the types of information a teacher would look for while observing students and gaining informal assessment information. Initial classroom observations in September reveal that Jessica did not have a source for initial comprehensive assessment. As would be expected from a teacher in her first year, September lessons were followed from the beginning of the district adopted Bridges curriculum manual.

Changes were seen in the level of specificity Jessica used when talking about informal assessment. Researcher collected AVMR data was shared with Jessica about the same time she participated in AVMR Course One. During one informal conversation after her participation in AVMR Course One, Jessica shared she was thrilled to have the data. Additionally, she demonstrated genuine excitement after the course because now she knew how to implement flexible small group instruction during math lessons. Following AVMR Course 1, Jessica immediately started using AVMR Course One data when planning for mathematics lessons. Classroom observation set two demonstrates how she took the formal assessment information and knowledge of the models to determine an opening whole group lesson followed by math rotations. Jessica chose tasks for math rotations that were easily modified to meet students at their current understandings (Guiding Principles Two and Three). Additionally, the lessons planned for small group instruction as one of the math rotations are based on formal Course One assessment data and then continually refined in the moment through Jessica's intense observation of students' solution strategies (Guiding Principle Six). Table 4.23 is an excerpt from Classroom observation set two, showing Jessica's use of initial and ongoing assessment after participating in AVMR Course One.

Table 4.21. Excerpt from Classroom Observation: Jessica.

Transitioning to rotations as described in the beginning of the lesson.	
Jessica is getting students organized in their opening spots for rotations. She is demonstrating what to do for math facts on the iPad. She shares with students how to change the numbers for the game so students can switch numbers to challenge themselves. The counting/ordering game they are playing today is the same game they played yesterday, so they know how to play. At your desk: She sets up hundreds club and has the students starting at this station raise their hands. Then she names students at "Teacher's Choice" - this is the group working with Jessica. Hands-on group is playing work place games on the carpet. Jessica offers hundreds tables for students doing the number club. All students walk to their stations.	Guiding Principle Three.
Jessica walks around the room to see that students know where to go and how to get started. She stops at a student and adjusts her iPad game to order numbers in the 500's. Another student asks if they can go to 500s and she tells that student to be in the 100s. Jessica assigns other students to the 80s. Students are starting to engage in their activities at their stations.	Guiding Principles Two and Three.
Jessica is working with a small group of students needing support with numbers up to 10 and then 20. Each student is provided with a paper hundreds chart where every other column is shaded grey. Each paper is slightly different. Different numbers are missing on each hundreds chart.	Guiding Principles Two, Three, and Four.

As she starts the small group lesson, Jessica asks students to cover all rows except for the first row. Students are then unsure of the sequence to 10 and start to look at each other's paper. Jennifer then places folders on the table to isolate each student's paper. Students are to fill in the missing numbers on their number chart. Jessica asks if students notice a pattern on their hundreds charts. A student begins to notice that going down the column on the right is counting by 10s. Jessica asks the student to predict a number the next number that is covered in the pattern counting by 10s. Then Jessica transitions to counting by ones in a row with this same student. Jessica supports students and checks responses one by one. She then questions students drawing attention to patterns in the hundreds chart going down, as well as going across. In a one-on-one conversation with a student, Jessica is working on how to say and order teen numbers. She is posing questions for each student and providing isolated supports for identifying and saying numbers in and out of sequence in the range to 20. Jessica rings the bell for each group to clean up. Jessica comments that the last station was much longer than she thought it would be.	Guiding Principle Six. Guiding Principle One. Guiding Principle Three.
When groups are finished cleaning up their stations, students are to meet at the carpet in the front of the room. Jessica has a SMART Board page to show students where they go to next. She reminds students what station they are at next and what they do in the station. Students are going to their next stations. Jessica walks around the room to make sure everyone knows where they are going and what they are doing when they get there.	

Jessica continued to use AVMR assessment data after AVMR Course Two as well. Classroom observation data and post interview data after Jessica's participation in AVMR Course Two reveal that she continues to use AVMR data, collected formally and informally, to guide whole group, small group, and math rotation lesson planning (Guiding Principle Two). During the post interview in December, Jessica addresses assessment when asked about the role of a teacher in a high quality mathematics classroom. The following is an excerpt from this interview.

Jessica: Always with a student. I think that is why sometimes it's hard, because I feel like I am always on the go in the morning. I don't have time to sit. But…I guess my lesson structure. I start with a mini-lesson, where I'm teaching all students and then we break up into groups and do rotations. So, I am always with students and in the smaller groups they're leveled where [students are] in a certain assessment. As for me, I guess I am always trying to meet them where they are at and trying to push them further, but I am also wanting them to think for themselves, too. I try to like help them solve the problem together and not just give them answers…I don't know…I'm off my mark today.

Interviewer: This is good [laughter by both J and I].

Jessica: I think I am always formally and informally assessing them too.

Interviewer: Mm.

Jessica: I am always checking where they are at and making sure…that the stuff I have for them in the groups *is* going to meet all their needs. And so always trying to … if I feel a student is getting more behind or moving up, like always trying to pull them aside and do small little assessments to see if they can do what I have them or if it is too easy, just not doing … well nothing is going to harm them.

Interviewer: No.

Jessica: But I want to make sure that they are successful.

Interviewer: It sounds, like, successful in, like, advancing knowledge. Pushed in that way.

Jessica: Yes. That's where I am. It's struggles. That's where I struggle with it sometimes, cause part of it is - I want them to be able to work independently, which means they have to be working on things that they *do* know and it's just more repetition. Like getting it and also pushing them up. I do have a couple kiddos in my class that are just so uncomfortable doing anything on their own so they always need me, especially with math, so that is hard to get them. Because even my morning work, I've been doing more like more fun worksheets that's a little bit more engaging where they can color, but just in the morning, and I just don't want it to be busy work where they're just doing whatever, so now I am starting to do different worksheets and I've noticed that some of them are - they just don't want to do it unless I'm helping them or sitting right next to them. I'm trying to move them away from that to have them gain more independence, which means I feel that it's something where I have to give them more, not easy, but something to gain their confidence so they can. Which is tricky - there's just so many different components!

Jessica's responses regarding the role of the teacher in a high quality mathematics classroom reveal how integrated formal and informal assessment is to her approach to instruction. The initial interview before participating in AVMR Course One reveals that Jessica valued informal assessment before participating in AVMR Courses. Participation in AVMR Course One provided her with assessment tasks and a model or framework to use. Classroom observations and the post interview after Jessica participated in Courses One and Two suggest Jessica's participation in AVMR Course One had the greatest impact on Jessica's planning for instruction.

In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number, as well as skills that support learning about whole number operations?

Planning for instruction also includes the way math time is structured within the allotted time and how students are physically arranged for learning. It is reasonable to expect the structure for math time and the ways students are arranged for learning to be conducive to differentiating lessons and flexibly grouping students according to the task or learning outcome. During classroom observations before AMVR Course One, Jessica facilitated predominantly whole-group mathematics lessons from the district adopted curriculum resource. Each lesson from the Bridges curriculum includes Number Corner, a calendar based series of mathematics tasks; a whole group activity; and math work places, a set of independent activities students can do in small groups. Number Corner can be placed at any time during the day. Jessica opened each day with Number Corner and followed with the whole-group lesson and work places during classroom observations before participating in AVMR Course One. Table 4.24 is an excerpt from classroom observation set one, illustrating a typical lesson format.

Table 4.22. Excerpt from Classroom Observation: Jessica.

The lesson begins with Number Corner, a calendar exercise incorporating number concepts from the Bridges curriculum. Students share predictions for which picture and color card will come next in the calendar pattern cards.	
Then Jessica transitions the focus to another space on the board and poses the next question, "How many days have we been in school?" Students are confusing days in school with days in the month which is posted on another area of the board. Karl answers that they have been in school for seven days. Karl demonstrates how he counted the number of days by pointing to a hundreds grid with seven squares shaded in. Jessica: So you counted by ones. Is there another way to count to seven? Another student is called upon and demonstrates how he counted by threes to count to seven days.	Guiding Principle Seven.
Then Jennifer transitions the focus to another part of the board with 10 frames showing the number of days this month. Students add another tile to the 10 frame. On the board there is full 10 frame with another 10 frame positioned under the full frame. The bottom frame has six tiles on it in a pairwise pattern (three on top from left to right, and three on the bottom from left to right). Student shares there are 16 days altogether. Jessica: How did you get that so fast? Student: Because three and three is six. Jessica: He knows 3+3 equals 6 but how did he get 16? A different student: Because there is a 10 frame on top.	
Jessica: Boys and girls, raise your hands if you know what a 10 frame is. Some students raise hands and some do not Jessica: Do you see how there is five on top and five on the bottom? Do we all see that? Give me a thumbs up if you see the five red tiles and the five blue tiles. Jessica: We are going to be working with 10 frames today when we go back to the carpet so it is very important for you to listen. We know this is 10 because it is a 10 frame and then Garret knew that three on the top and three on the bottom equals six, so he added 10 plus six equals 16.	

Several students are saying out loud that six and 10 is 16. Wow you know you are adding, Garrett - adding is kind of your thing. Garrett: Math is my favorite thing in school.	
Transition to carpet spots at the front of the room from Number Corner spots. Jessica puts up a pocket chart. She has the Bridges curriculum manual in her hand and references it often during the lesson. Jessica is putting 10 frames representing teen numbers in the pocket chart. The 10 frames have bug pictures in patterns of five on them. Jessica: (Reading from the manual) I want you to look up here at these cards that show more than 10 bugs, and we are going to figure out ways to find out how many we have. Students vocalize quantities. Several different responses. J: How many bugs do I have? Students begin speaking out loud responding with different teen numbers. Jessica is pointing to a card. Jessica shares all the responses heard from students. J: Where are we getting all these? I want to hear how we are figuring these numbers out. Can someone raise their hand to tell me their math thinking?	Guiding Principle One.
Jessica: Maybe I shouldn't have all of these cards up here. She takes down all the cards except one down in order for students to focus on one frame to respond to. Jessica: We are going to listen to Evan. Evan: 10 plus one is eleven. Jessica: 11. That is great math thinking, Evan. Does anyone have another way to come up with 11? Student shares same thinking as Evan. Jessica: What are you thinking Lucas? Lucas: Two fives and another one is 11. Emma comes to the board and shares how she used twos to get to 16. Jessica asks her to point to the bugs and count out loud to share how she counted them. Emma points to two bugs as a time and says two, two, two, two, two, and one. Jessica asks if she can show her another way to count that. Student replies yes and Jessica proceeds to point to the bugs in the same way Emma did and says two, four, six, eight, 10 … Jessica pauses for Emma to say the next number in correspondence to one more bug. Emma says nine and another student shouts out 11, and then Emma says 11.	

Jessica: Takes a moment to share with the class that Emma's brain grew right there because she made a mistake. Ok – anyone else?	
A new card is shared. Jessica: Put it in your brain, don't say it out loud. I want all my friends a chance to count them in their own way. To keep students engaged and allow additional time for students to apply a strategy for figuring out the number of bugs on the card, Jessica calls a student's name and on asks, "Do you know how many are on this card?" Student will begin to reply and Jessica replies, "Oh, think about it and keep it in your brain." J: I see some of my friends looking at me smiling and they're saying 'Yep, I can tell just by looking.' I see some of my friends counting with their fingers. I assume they are counting by ones. Grace, what number did you get? *Lesson continues with discussion as before. Another card is shared.*	Guiding Principle Eight.
Jessica shares that next week the class will be playing a game using the bug ten frames and equation cards. Jessica: Raise your hand if you think you can do ten frames with bugs. Students raise their hands.	*In contrast to Guiding Principle Seven.*

After participating in AVMR Course One, Jessica changed the way she structured math lessons. Prior to AVMR Course One, Jessica shared during informal conversations that she wanted to incorporate guided math groups into her math structure. During one of those informal conversations before AVMR Course One, Jessica talked about a set of materials purchased from Teachers Pay Teachers. After taking AVMR Course One, Jessica communicated with excitement that now she knew how to incorporate guided math groups. The excerpt below from classroom observation set two, after AVMR Course One, shows the structure Jessica used for the remaining lessons in the study. Math time begins with Number Corner after a morning meeting. Following Number Corner is a whole group lesson, followed by math rotations. The overall structure is similar to classroom observations before Jessica's participation in AVMR Course One, however, the lessons changed after Jessica's participation in AVMR Course One.

Table 4.23. Excerpt from Classroom Observation: Jessica.

To begin the lesson, Jessica explains to students a new structure for math class. She has four stations, each with a different math activity for students to work on while assigned that station. Students will rotate through the stations once Jessica lets them know it is time to switch. One of the stations is meeting with Jessica. The other stations will include at your seat (hundreds club), hands-on (math work place games), and math facts (iPad). Jessica is going through what to do at each station and how to partner up when the activity requires it.	
Students will be at each station for 10-15 min. Jessica assures students that she will help students know what to do during rotations.	
Today is Tuesday Tally, so they transition to front of the room to practice how to make tally marks. Each student will use a whiteboard and marker to solve and share ways to make tally marks to represent a number. Jessica writes a number on the board and students record that number using tallies followed by holding the whiteboards in the air for Jessica to see responses. The numbers chosen are three and six. J: Let's see if my friends can remember the rule about making tallies. Jessica provides students with wait time and then has all students hold the whiteboard in the air for her to see. Jessica and students discuss why you place a diagonal tally to signify a group of five. Next number is 12. Jessica provides wait time and then has all students share whiteboards in the air as before. Jessica is correcting students as she reviews the white boards. She then calls on a student to come to the front and make his 12 tally marks on the SMART Board in front of the class. Jessica is turning this into a learning moment to provide space between lines when showing a number using tallies. Jessica thanks student for coming up to help us learn and she prompts class to give him a silent cheer.	
Next number is 32 Jessica: First before we start, who can read this number for me? Alberto says 32. Students begin to write and Jessica points out the correct way the previous student is leaving space between tally marks. Wait time is provided. Students are to put their hands on their head when done writing	Guiding

tallies. Students are employing different strategies to monitor when they have drawn 32 tallies. Jessica begins reviewing responses. She is calling on students to put their boards down once she has checked their work. Jessica asks a student if she can share his board. He had tallies drawn and then wrote an equation alongside, 30+2. Jessica elaborates more on different strategies for counting up 30 tally marks. Students are talking and sharing as well. One strategy highlighted is counting by fives to get to 30 and then two more. Another strategy shared is an equation indicating a count by 10 to get to 30 and then two more.	Principle Eight.
Transitioning to rotations as described in the beginning of the lesson.	
Jessica is getting students organized in their opening spots for rotations. She is demonstrating what to do for math facts on the iPad. She shares with students how to change the numbers for the game so students can switch numbers to challenge themselves. The counting/ordering game they are playing today is the same game they played yesterday, so they know how to play. At your desk: She sets up hundreds club and has the students starting at this station raise their hands. Then she names students at "Teacher's Choice" - this is the group working with Jessica. Hands-on group is playing work place games on the carpet. Jessica offers hundreds tables for students doing the number club. All students walk to their stations.	Guiding Principle Three.
Jessica walks around the room to see that students know where to go and how to get started. She stops at a student and adjusts her iPad game to order numbers in the 500's. Another student asks if they can go to 500s and she tells that student to be in the 100s. Jessica assigns other students to the 80s. Students are starting to engage in their activities at their stations.	Guiding Principles Two and Three.
Jessica is working with a small group of students needing support with numbers up to 10 and then 20. Each student is provided with a paper hundreds chart where every other column is shaded grey. Each paper is slightly different. Different numbers are missing on each hundreds chart. As she starts the small group lesson, Jessica asks students to cover all rows except for the first row. Students are then unsure of the sequence to ten and start to look at each other's paper. Jennifer then places folders on the table to isolate each student's paper. Students are to fill in the missing numbers on their number chart.	Guiding Principles Two, Three, and Four. Guiding Principle Six.

Jessica asks if students notice a pattern on their hundreds charts. A student begins to notice that going down the column on the right is counting by 10s. Jessica asks the student to predict a number the next number that is covered in the pattern counting by 10s. Then Jessica transitions to counting by ones in a row with this same student. Jessica supports students and checks responses one by one. She then questions students drawing attention to patterns in the hundreds chart going down as well as going across. In a one-on-one conversation with a student, Jessica is working on how to say and order teen numbers. She is posing questions for each student and providing isolated supports for identifying and saying numbers in and out of sequence in the range to 20. Jessica rings the bell for each group to clean up. Jessica comments that the last station was much longer than she thought it would be.	Guiding Principle One. Guiding Principle Three.
When groups are finished cleaning up their stations, students are to meet at the carpet in the front of the room. Jessica has a smartboard page to show students where they go to next. She reminds students what station they are at next and what they do in the station. Students are going to their next stations. Jessica walks around the room to make sure everyone knows where they are going and what they are doing when they get there.	
Jessica stops to support a student with hundreds club. Jessica stops again to provide a student with a place value flip chart to support writing three digit numbers between 110-20.	Guiding Principle Four.
Jessica then begins to work with the small group assigned to her group for the rotation. She is taking out an activity from an AVMR binder of activities provided by the district math coaches. The master sheet she was planning to use is missing and so Jessica comes up with another activity in the moment. She provides each student with a completed hundreds chart and puts up folders in order for each student to have a private work space. She starts off by asking each student to point to the number Four.	Guiding Principle Four.
Jessica asks students to circle the number four. Now she asks students to point to the number that comes after four. A student asks to circle that number too. One students says the number five. Jessica asks him if that number is even or odd. Student is not sure, so Jessica moves on with the small group lesson.	
The next number to circle is 10. Then Jessica asks what the number is after 10. Jessica asks students to point to the number 20. Jessica asks, "Where is the number that comes before 20?" She has a student share who pointed to 19 and then asks other students in the group to also point to 19.	Guiding Principles Two and Three.

Jessica folds each of the hundreds chart papers in order to have less visual clutter for students when answering questions. J: Hunter, how did you know that this number is the number before 20? Now Jessica brings out a numeral roll as a support and asks Hunter to identify 20, and then point to the number that comes before 20, followed by the number that comes after.	Guiding Principles Four and Six.
Jessica asks the next student to point to 15. Jessica needs to support the student to identify 15 and not 50. Then she asks this student to show the number after 15. Student does not know what after means. Jessica clarifies by using hand gestures and the numeral roll. Then Jessica scaffolds down to a smaller number. Then asks the student to identify the answer after six, followed by the number that comes before six. She poses another question to this student. This time she identifies four, and asks which number comes after four, and which number comes before four. The student is timid, but correct for bot responses.	Guiding Principle Six.
Jessica is again looking for the worksheet that breaks down number work before and after.	
She then poses a question to Austin. She asks him to identify 20 and point to the number that comes before 20 and after 20. Jessica rings the bell to signal to clean up and join her on the carpet. They are finishing with a conversation about this week's focus is to practice rotations and get into the routine. Students in the hands-on group are reflecting on the importance of putting cards back in an organized fashion for work place games. Cleaning up. End of math time.	

After Jessica participated in AVMR Course One, there was a change in Guiding Principles identified during the lesson. These changes begin by Jessica's use of AVMR assessment data when planning lessons. Before taking AVMR Course One, Guiding Principles Two, Three, and Four were not identified in classroom observations. After participating in AVMR Course One, a strong presence of Guiding Principles Two, Three, and Four is identified in classroom observations. The presence of Guiding Principles Two, Three, and Four leads to the presence of remaining Guiding Principles, as addressed in research questions two through four.

Research question two. *In what ways does Add+VantageMR® professional development influence teachers' instructional moves during a whole number lesson?*

1. In what ways are students engaged in activities from the course resources such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 Year-olds* (Wright et al., 2014) and/or *Developing Number Knowledge* (Wright et al., 2011)?
2. In what ways are students actively engaged in *doing* mathematics during the lesson?
3. In what ways does the teacher use Guiding Principles and Domains of Progressive Mathematization during a whole number lesson?
4. Do some Guiding Principles seem to be more readily implemented than others?

Data Analysis. Guiding Principles addressed.

Guiding Principles for Classroom Teaching applicable to research question two are as follows:

- *Guiding Principle One: The teaching approach is inquiry-based, that is, problem-based.*
- *Guiding Principle Three: Teaching is focused just beyond the "cutting edge" of students' current knowledge.*
- *Guiding Principle Four: Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*

- *Guiding Principle Five: The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.*
- *Guiding Principle Six: Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.*
- *Guiding Principle Seven: Teaching supports and builds on students' intuitive, verbally-based strategies, and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.*
- *Guiding Principle Eight: The teacher provides students with sufficient time to solve a given problem.*
- *Guiding Principle Nine: Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.*

Sub-questions one, two, and three are used to guide a reflection on the ways AVMR Courses One and Two professional development influenced Jessica's instructional moves during a whole number lesson.

Manifestations of Guiding Principles during a whole number lesson.
Sub-question 3

The first classroom observation set took place before Jessica participated in AVMR Course One. The Second classroom observation set took place after Jessica participated in AVMR Course One and before she participated in AVMR Course Two. The final classroom observation set took place after Jessica participated in AVMR

Course Two. During classroom observation set one, three Guiding Principles were identified before Jessica's participation in AVMR Course One: Guiding Principles One, Seven, and Eight. Lessons were problem-centered and students were provided opportunities to verbalize mental strategies with their peers. Jessica provided wait time for students to solve and communicate solution strategies and mathematical thinking.

After participating in AVMR Course One, a strong presence of Guiding Principles Three, Four, and Six were identified throughout the lessons. This continued to be true after Jessica's participation in AVMR Course Two, with the addition of Guiding Principle Five: *the teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies*. Table 4.26 shows the number of occurrences of Guiding Principles before and after Jessica's participation in AVMR Courses One and Two.

Table 4.24. Occurrences of Guiding Principles before and after AVMR: Jessica.

	Number of occurrences in September		Participated in Add+VantageMR® Course Two	Number of Occurrences in October		Participated in Add+VantageMR® Course Two	Number of Occurrences in December	
	Manifestation of GP	Contrasting the GP		Manifestati on of GP	Contrasting the GP		Manifestati on of GP	Contrast ing the GP
Guiding Principle One	1			2				
Guiding Principle Three				7			4	
Guiding Principle Four				7			4	
Guiding Principle Five							1	
Guiding Principle Six				5			3	
Guiding Principle	2	1					2	

Seven				
Guiding Principle Eight	1	1	1	1
Guiding Principle Nine			1	1

Table 4.26 confirms the informal interview showing a greater presence of Guiding Principle Four: *teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.* The table also shows a decrease in Guiding Principles Three, Five, Six, Seven, Eight, and Nine

.

Before AVMR Course One: Excerpts from the Classroom.

A single code was applied for Guiding Principles One and Eight during classroom observations before Jessica participated in AVMR Course One. Two occurrences were coded as a manifestation of Guiding Principle Seven. During this first set of observations, Jessica followed the Bridges instructional manual, referring to the manual to pose questions during the lesson. The task identified and enacted from the district adopted curriculum set the stage for problem based tasks, providing the opportunity for students to think deeply about mathematics and affording the opportunity for students to share solution strategies. Students were engaged in mathematical thinking. The following excerpt from classroom observation set one, lesson two shows the lesson. There were no episodes in the lesson in contrast to Guiding Principles. However, Table 4.27 does show the limited presence of Guiding Principles before Jessica’s participation in AVMR Course One.

Table 4.25. Excerpt from Classroom Observation.

Lesson Two

Lesson today is from Bridges curriculum. J: What does this look like to you? Alexis: A calendar. Jessica: I don't think so. Then another student points to the number calendar at the back of the room and says, "It's kind of like this, a pattern of bugs - you can put on beetles, butterflies, mantis." Jessica: Ahhh, Why doesn't this look like a calendar to me? Kaden says something about matching. Jessica: You think I am going to do something about matching - you are exactly right, Kaden, that is what I am going to do. Boys and girls, I want to let you know… Jessica puts a purple square matching the grid on the screen in front of students to the calendar the class is building in the back of the room for Number Corner. One of the students generalizes that the purple squares follows a diagonal in the back and the teacher can fill in purple squares in diagonal pattern on the table on the Smartboard. Jessica is starting to touch each square and recite an a, b, c, pattern to check all of her purple squares in already and fill in the remaining purple squares.	
Jessica: Now these are butterflies. Let's count how many butterflies there are in our pattern. She calls on a student to come to the front to count the butterflies. Student notices that purple should be beetles, not butterflies. Jessica: Oops, I just confused you! Should I add in the butterflies now? Student nods yes. Jessica quickly fills in the butterfly pattern with a green letter B. Jessica: I am confusing Charlotte so much. Can I explain to you what I am doing right now so I am not confusing anyone? Group of students nod and say yes. Jessica is explaining what she is doing to complete the chart to match the pattern from Number Corner. She continues to ask students to give a thumbs up if they understand. J: So now can you understand what is on our calendar. Give me a thumbs up if you understand.	
J: Our job today, boys and girls, is to count up how many Praying Mantis we have, how many beetles we have, and how many butterflies we have, and put them in our ten frames. There are pictures of ten frames to the right of the calendar grid on the SMART Board in the front of the class.	

J: Kylie, remember how you said you were going to the apple orchard and I said maybe you should make tallies for how many apples you pick? Kylie shakes her head. J: And then I was telling you that maybe we should make 10 frames when you get back on Monday and we can show everybody how many apples you picked? That is kind of what we are doing right now. Hunter shares that Jessica can move the colored squares right into the 10 frames. Jessica confirms his comments and shows how she will move a colored square into the frame. Gabby comes to the front to count all of the butterflies in the pattern on the smartboard. She counts by ones to get seven. Jessica then leads the class in counting in order to check Gabby's number. The class counts to eight altogether. Jessica: Oh, Gabby, we must have missed one. Who thinks they can make eight in their 10 frame? Wait time.	Guiding Principle Eight.
Jessica calls on Gabby to choose someone. Gabby chooses Charlotte to come to the board to show eight on the 10 frame. Charlotte proceeds to make eight in the frame by dragging red squares and placing five on top and three on the bottom. Jessica says to the class, "Are you watching? Let's see how she is doing it and maybe you do it differently." J: Is she right? Give me a thumbs up on your knee if you think she is right. Put your hand in the air if you think she made a mistake and you want to help her. Jessica calls on a student. The student begins to offer a suggestion, and through conversation they determine that Charlotte was right. The student saw five on top and three on the bottom. Emma: Charlotte is right, because if you have 10 and then take two out, that is eight. Jessica: Oh - we like subtraction in here, huh? Remember how we already made a subtraction problem over there? Group of students: Yeah. J: Same thing. Emma. Her math thinking right now is subtraction. Jessica repeats what Emma said, and tells the students that she is going to take what Emma said and show the class using numbers. Jessica proceeds to record 10 - 2 = 8 on the board. Then Jessica records 5+3=8 to record an equation for the first student's strategy.	Guiding Principle Seven.
Jessica calls on Karl. Karl shares that we can put four on the top and four on the bottom in the frame to have eight that way. Jessica moves the squares from a five and three pattern to a four and four pattern. Jessica records using a number sentence. Student: I have another way.	

Jessica: How? The student comes to the board to count the eight squares by twos. Jessica: Can I record that in a math equation? Jessica then matches the equation numerals to the squares in the picture. She shows the student another way to see his strategy by keeping the four and four pattern. Jessica: We need to wrap this up. This is really cool. We were supposed to do so much more with this. Do you see all the different ways my friends came up with seeing the number eight? Give me a thumbs up if you see. Give me two thumbs up if you see yet another way. We could probably talk about this all day, because there are other ways to make eight, aren't there? Students: Maybe we can do it later. Jessica: Yeah - maybe we can talk about this more over lunch. Students are excited about the lesson.	
Jessica leads the class in student yoga before transitioning to part two.	

Part 2

Transitions to next part of Bridges lesson. This part begins with rows of sorting cards shown on the smartboard. The cards each have a different water animal on them. Jessica: Boys and girls, what do you see? Raise your hand. No shout outs right now. Kaden, What do you see? Kaden responds. J: What did we decide yesterday when we were sorting them into cups? One student remembers they called them sea creatures.	
Jessica has cards in a random card generator app on the SMART Board. The cards with sea creatures on them are entered into the random generator. “Long snouts” comes up in the random card generator. Jessica: Students, when you are sorting your creatures today, you are going to look for creatures that have long snouts. Then next card randomly selected is “Walrus.” Jessica is sharing directions for students to return to their table spots and sort sea creatures into piles that are walruses and another pile with creatures having long snouts.	
Students are going back to their seats. Jessica is bringing around sea creatures for each table to begin the activity. Students begin sorting sea creatures into ones with long snouts and walruses. Jessica is coming around to collect all the sea creatures that do not have a long snout or are a walrus.	

Jessica is now calling table groups to come to the carpet at the front of the room to share the number of creatures with snouts and walruses they had in their table groups. Jessica has a T-chart on chart paper with Walrus on the left and Long snout on the right. Jessica is explaining that they will use tallies to keep track of how many walruses the class found.	
Students are now bringing up their Walrus creatures and setting in a pile at the front of the chart. Then Jessica asks students to bring up creatures with long snouts to set in another pile at the front of chart. Jessica begins to count walruses and ten decides to put up a pocket chart where she can set the walrus animals inside the pocket chart for all students to see as they count. Jessica begins to draw tallies and students are to monitor the count to tell her when to stop drawing tallies. Students are having a very difficult time staying focused. Students point out that the ten walruses are written with two sets of five tallies.	
Now Jessica is setting the long snout sea creatures into the pocket chart and counting the total number of sea creatures with snouts. There are crabs that groups included with sea creatures with long snouts. Jessica asks students to explain their thinking when they chose those as creatures with long snouts. The class agrees that the crabs can be taken out. The class counts the remaining creatures together. Jessica then draws tallies as the class counts out loud	
Jessica leads the class in movement. She has students pretend to jump rope while counting to 10 and then 20. Students are going to engage in work places next. Each table group as a different work place. One group is copying and creating patterns with polygons. Another is counting sea creatures by placing them on empty 10 frames, then determining a total. A third group is creating shapes on geoboards, and the fourth group is creating three dimensional figures using Unifix cubes. Students are engaged in thinking and working within their work places. Students placing sea creatures into empty 10 frames are using count by ones methods and then recounting using counting by 10s. Jessica is walking from table to table supporting students and asking questions about their math work.	

After AVMR Course One: Excerpts from the Classroom

After participating in Add+VantageMR Course One, classroom observations revealed that manifestations of Guiding Principles Three, Four, and Six to be the most prevalent. As discussed in research question one, Jessica's first change was to implement math rotations, which is Jessica's form of implementing guided math groups. Research question two examines Jessica's decisions and teacher moves during whole-group lessons and math rotations. Classroom observation set two, lesson one demonstrates the changes in the ways Jennifer enacts a whole number lesson after participating in AVMR Course One. Jennifer often brings in a new setting to support student's thinking while engaging in a problem based task. For example, in the Teacher's Choice rotation, students are working on number words and numeral skills, where each group is working on the same skill. However, the task and questions Jennifer poses during the small group change based on the students current number words and numerals skills and understandings identified from AVMR Course One assessments. Additionally, Jessica brings in settings, such as a numeral roll and place value flip chart, to support students in their thinking. The excerpt below from classroom observations after Jessica's participation in AVMR Course One shows this example among additional examples of Jessica's changes.

Table 4.26. Excerpt from Classroom Observation After AVMR Course One: Jessica.

To begin the lesson, Jessica explains to students a new structure for math class. She has four stations each with a different math activity for students to work on while assigned that station. Students will rotate through the stations once Jessica lets them know it is time to switch. One of the stations is meeting with Jessica. The other stations will include at your seat (hundreds club), hands-on (math work place games), and math facts (iPad). Jessica is going through what to do at each station and how to partner up when the activity requires it.	
Students will be at each station for 10-15 min. Jessica assures students that she will help students know what to do during rotations.	
Today is Tuesday Tally, so they transition to front of the room to practice how to make tally marks. Each student will use a whiteboard and marker to solve and share ways to make tally marks to represent a number. Jessica writes a number on the board and students record that number using tallies	

followed by holding the whiteboards in the air for Jessica to see responses. The numbers chosen are three and six. J: Let's see if my friends can remember the rule about making tallies. Jessica provides students with wait time and then has all students hold the whiteboard in the air for her to see. Jessica and students discuss why you place a diagonal tally to signify a group of five. Next number is 12. Jessica provides wait time and then has all students share whiteboards in the air as before. Jessica is correcting students as she reviews the white boards. She then calls on a student to come to the front and make his 12 tally marks on the smartboard in front of the class. Jessica is turning this into a learning moment to provide space between lines when showing a number using tallies. Jessica thanks student for coming up to help us learn and she prompts class to give him a silent cheer.	
Next number is 32 Jessica: First before we start, who can read this number for me? Alberto says 32. Students begin to write and Jessica points out the correct way the previous student is leaving space between tally marks. Wait time is provided. Students are to put their hands on their head when done writing tallies. Students are employing different strategies to monitor when they have drawn 32 tallies. Jessica begins reviewing responses. She is calling on students to put their boards down once she has checked their work. Jessica asks a student if she can share his board. He had tallies drawn and then wrote an equation alongside, 30+2 Jessica elaborates more on different strategies for counting up 30 tally marks. Students are talking and sharing as well. One strategy highlighted is counting by fives to get to 30 and then two more. Another strategy shared is an equation indicating a count by 10 to get to 30 and then two more.	Guiding Principle Eight.
Transitioning to rotations as described in the beginning of the lesson.	
Jessica is getting students organized in their opening spots for rotations. She is demonstrating what to do for math facts on the iPad. She shares with students how to change the numbers for the game so students can switch numbers to challenge themselves. The counting/ordering game they are playing today is the same game they played yesterday, so they know how to play. At your desk: She sets up hundreds club and has the students starting at this station raise their hands.	Guiding Principle Three.

Then she names students at “Teacher’s Choice” - this is the group working with Jessica. Hands-on group is playing work place games on the carpet. Jessica offers hundreds tables for students doing the number club. All students walk to their stations.	
Jessica walks around the room to see that students know where to go and how to get started. She stops at a student and adjusts her iPad game to order numbers in the 500’s. Another student asks if they can go to 500s and she tells that student to be in the 100s. Jessica assigns other students to the 80s. Students are starting to engage in their activities at their stations.	Guiding Principles Two and Three.
Jessica is working with a small group of students needing support with numbers up to 10 and then 20. Each student is provided with a paper hundreds chart where every other column is shaded grey. Each paper is slightly different. Different numbers are missing on each hundreds chart. As she starts the small group lesson, Jessica asks students to cover all rows except for the first row. Students are then unsure of the sequence to 10 and start to look at each other’s paper. Jennifer then places folders on the table to isolate each student’s paper. Students are to fill in the missing numbers on their number chart. Jessica asks if students notice a pattern on their hundreds charts. A student begins to notice that going down the column on the right is counting by 10s. Jessica asks the student to predict a number the next number that is covered in the pattern counting by 10s. Then Jessica transitions to counting by ones in a row with this same student. Jessica supports students and checks responses one by one. She then questions students drawing attention to patterns in the hundreds chart going down, as well as going across. In a one-on-one conversation with a student, Jessica is working on how to say and order teen numbers. She is posing questions for each student and providing isolated supports for identifying and saying numbers in and out of sequence in the range to 20. Jessica rings the bell for each group to clean up. Jessica comments that the last station was much longer than she thought it would be.	Guiding Principles Two, Three, and Four. Guiding Principle Six. Guiding Principle One. Guiding Principle Three.
When groups are finished cleaning up their stations, students are to meet at the carpet in the front of the room. Jessica has a smartboard page to show students where they go to next. She reminds students what station they are at next and what they do in the station. Students are going to their next stations.	

Jessica walks around the room to make sure everyone knows where they are going and what they are doing when they get there.	
Jessica stops to support a student with hundreds club. Jessica stops again to provide a student with a place value flip chart to support writing three digit numbers between 110-20.	Guiding Principle Four
Jessica then begins to work with the small group assigned to her group for the rotation. She is taking out an activity from an AVMR binder of activities provided by the district math coaches. The master sheet she was planning to use is missing, and so Jessica comes up with another activity in the moment. She provides each student with a completed hundreds chart and puts up folders in order for each student to have a private work space. She starts off by asking each student to point to the number four.	Guiding Principle Four.
Jessica asks students to circle the number four. Now she asks students to point to the number that comes after four. A student asks to circle that number too. One students says the number five. Jessica asks him if that number is even or odd. Student is not sure, so Jessica moves on with the small group lesson.	
The next number to circle is 10. Then Jessica asks what the number is after 10. Jessica asks students to point to the number 20. Jessica asks, "Where is the number that comes before 20?" She has a student share who pointed to 19 and then asks other students in the group to also point to 19. Jessica folds each of the hundreds chart papers in order to have less visual clutter for students when answering questions. J: Hunter, how did you know that this number is the number before 20? Now Jessica brings out a numeral roll as a support and asks Hunter to identify 20, and then point to the number that comes before 20, followed by the number that comes after.	Guiding Principles Two and Three. Guiding Principles Four and Six.
Next student Jessica asks to point to 15. Jessica needs to support the student to identify 15 and not 50. Then she asks this student to show the number after 15. Student does not know what after means. Jessica clarifies by using hand gestures and the numeral roll. Then Jessica scaffolds down to a smaller number. Then asks the student to identify the answer after six, followed by the number that comes before six. She poses another question to this student. This time she identifies four and asks which number comes after four, and which number comes before four. The student is timid, but correct for bot responses.	Guiding Principle Six.
Jessica is again looking for the worksheet that breaks down number work before and after.	
She then poses a question to Austin. She asks him to identify 20 and point to the number that comes before 20 and after 20.	

Jessica rings the bell to signal to clean up and join her on the carpet. They are finishing with a conversation about this week's focus is to practice rotations and get into the routine. Students in the hands-on group are reflecting on the importance of putting cards back in an organized fashion for work place games. Cleaning up. End of math time.	

The third and final set of classroom observations taking place after Jennifer's participation in AVMR Course Two provide additional examples of Guiding Principles Three through Six. The classroom excerpt below provide examples of manifestations of these Guiding Principles in whole number lessons in addition and subtraction. There is no change in the presence of Guiding Principles after Jessica's participation in AVMR Course Two. However, the consistently strong presence of Guiding Principles Three, Four, and Six provide evidence for the lasting impact AVMR Course One had on the ways Jessica was able to focus instruction on students' cutting edge; exercise professional judgement in selecting from a bank of teaching procedures, including instructional settings and tasks; and intense, ongoing observation, resulting in adjustments or fine-tuning within a lesson based on these observations.

Table 4.27. Excerpt from Classroom Observation.

Lesson starts by playing Buzz, counting by fives from five to 70. When a student makes an error, Jessica goes back two people in the sequence to provide a running start again for that student. When a student does not know the next five in the forward sequence, Jessica asks her what five more than 60 is. Then when she is not sure, she supports her in getting the correct number in the sequence so she can continue in the game. The game is moving very quickly. Students are staying on task and engaged. After a winner is determined, all students stand to count by fives together to 100.	Guiding Principles Four and Six.
Jessica is introducing the activities for today for each of the rotation topics.	

She is introducing a new activity called Flip Cards. The Flip Cards are ones she printed from a teacher website. Students receive a packet of cards at their rotation. Each card has a sequence of numbers and students either determine the missing number or choose from a list of possible numbers provided to complete a sequence. Students record solutions in their math notebook. Jessica communicates that a hundreds board can be used to help you solve some of the problems if students need it.	Guiding Principle Four.
Groups are switched today. Students are going to their rotation spots.	
Teacher's Choice table is playing a dice activity. Students are recording the date and activity to be played in their math notebooks. Students roll two dice and add the quantities together. She leaves students to pay the activity at the table while Jessica walks around the room observing students in rotation spots. Students at the table are recording addition equations in their math notebooks and finding solutions to the addition problems rolled with the dice.	Guiding Principles Two and Three. Guiding Principle Seven.
Jessica asks students if they are ready for a challenge. Students respond yes. This time, students are instructed to roll both dice and add them together. Then students write this number in their notebook. Students then roll one die and subtract that number from the number they just recorded in their notebook. Jessica rolls the dice to show the following example: Her first roll was six and four, so she recorded 10 in her notebook. Then she rolled a three, and so she recorded 10 – 3 = in her notebook. One student solves the problem and shares with Jessica how she determined the difference to be seven. Students quickly begin playing again independently at the teacher table. Students are employing a variety of strategies, including count down from and finger patterns. One student is have a difficult time with subtraction. Jessica brings out the numeral roll to identify the direction to go when subtracting. The student is still having difficulty with subtraction. He begins to use the numeral roll to support determining a solution.	Guiding Principles Two, Three, and Six.
Bell rings. Time to clean up. Bell rings to rotate to next station.	
Next group is coming to the teacher table. Students are labeling their notebook. This group is playing the game by rolling one die and then recording the number, then rolling the second die and adding it to the first roll. Students are using pips in the dice to solve addition problems.	
Jessica is supporting a student at the table in writing numerals correctly. She brings out the numeral roll for the student to match his numerals to the numerals on the numeral roll. Jessica stops the group to bring everyone back to focus and be on task. Jessica then turns to students at the group and says, "Can I challenge you?"	

Students do not respond. Now students are to roll both dice and point to the number that is the largest. Students are to write that number down on the paper and then subtract the smaller number. Jessica demonstrates subtraction by covering dots on the larger die. Students roll a six and two. Jessica takes the six die and covers two dots with her hand. Students are using their fingers and looking at the pips on the die to determine the difference. Jessica continues to have conversations with students one-on-one regarding subtraction. She continues to cover the number of dots on the larger die to support students in determining the difference. Students are having a difficult time playing this modification on their own. Clean up and rotate to the next rotation. The next group comes to the teacher table. Their bucket is messy and so Jessica is sorting it quickly. She is also talking to the group about following the rules during rotations. Several assignments are found in the bucket without a name. Jessica takes time for each student to go through their folder to identify which student should get the unnamed assignments. Bell rings and math time is over. The last group cleaned out their folders to Teacher's Choice time.	Guiding Principles Two, Three, and Four.

In summary, the most profound changes during a whole number lesson occurred after Jessica participated in AVMR Course One. Classroom observations and formal interviews revealed that after participating in AVMR Course One, Jessica's teaching became fore focused on the cutting edge of students understanding. She started to select from a bank of teacher procedures based on her on-going observations of students' solution strategies. Jessica engaged in intense and ongoing observation of student solution strategies within small groups, as well as in whole-group instruction, and in both situations, made fine-tuned adjustments to the tasks or settings. There was not a noticeable change in the presence of Guiding Principles after Jessica's participation in AVMR Course Two; however, the identified changes in the presence of Guiding

Principles found after her participation in AVMR Course One remained just as strong as before taking AVMR Course Two.

Research question three. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

1. In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?
2. In what ways do teachers modify an activity during a lesson (modification may include offering different dice, number ranges, and/or instructional tools)?

Data Analysis: Guiding Principles

Guiding Principles for Classroom Teaching applicable to research question three are as follows:

- *Guiding Principle Three: Teaching is focused just beyond the "cutting edge" of students' current knowledge.*
- *Guiding Principle Four: Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations.*

In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration? In what ways do teachers modify an activity during a lesson?

Research question one addressed the ways differentiation can happen as teachers plan for a lesson. Research question two focused on a variety of teacher moves related to

Guiding Principles for Classroom Teaching during a lesson. Research question three specifically addresses differentiating for students through the lens of Guiding Principle Three: *teaching is focused just beyond the "cutting edge" of students' current knowledge,* and Guiding Principle Four: *teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations*. This includes short and long term planning for lessons, in-the-moment decisions teachers make that result in students making mathematical connections to advance thinking, or students practicing skills and refining strategies appropriate within a trajectory of student learning. The following paragraphs draw upon data collected during informal, pre- and post- interviews and provide example excerpts from Jessica's classroom observations before and after her participation in AVMR Course Two.

Before AVMR Course One: Excerpts from the Classroom

During classroom observations before Jennifer participated in AVMR Course One, Jessica made in-the-moment decisions to reduce students' confusion during a task. The following short excerpts provide examples in bold. Much of the lesson time was facilitated whole group, except when students were randomly placed into small groups to participate in work places.

Table 4.28. Excerpt from Classroom Observation.

Then Jennifer transitions the focus to another part of the board with 10 frames showing the number of days this month. Students add another tile to the 10 frame. On the board there is a full 10 frame, with another 10 frame positioned under the full frame. The bottom frame has six tiles on it in a pairwise pattern (three on top from left to right, and three on the	

bottom from left to right). Student shares there are 16 days altogether. Jessica: How did you get that so fast? Student: Because three and three is six. Jessica: He knows 3+3 equals six, but how did he get 16? A different student: Because there is a 10 frame on top.	
Jessica: Boys and girls, raise your hands if you know what a 10 frame is. Some students raise hands and some do not. Jessica: Do you see how there is five on top and five on the bottom? Do we all see that? Give me a thumbs up if you see the five red tiles and the five blue tiles. Jessica: We are going to be working with 10 frames today when we go back to the carpet, so it is very important for you to listen. We know this is 10 because it is a 10 frame, and then Garret knew that three on the top and three on the bottom equals six, so he added 10 plus six equals 16. Several students are saying out loud that six and 10 is 16. J: Wow you know you are adding, Garrett - adding is kind of your thing. Garrett: Math is my favorite thing in school.	

Excerpt example 2

Transition to carpet spots at the front of the room from Number Corner spots. Jessica puts up a pocket chart. She has the Bridges curriculum manual in her hand and references it often during the lesson. Jessica is putting 10 frames representing teen numbers in the pocket chart. The 10 frames have bug pictures in patterns of five on them. Jessica: (Reading from the manual) I want you to look up here at these cards that show more than 10 bugs, and we are going to figure out ways to find out how many we have. Students vocalize quantities. Several different responses. J: How many bugs do I have? Students begin speaking out loud responding with different teen numbers. Jessica is pointing to a card. Jessica shares all the responses heard from students. J: Where are we getting all these? I want to hear how we are figuring these numbers out. Can someone raise their hand to tell me their math thinking?
Jessica: Maybe I shouldn't have all of these cards up here. She takes down all the cards except one down in order for students to focus on one

frame to respond to.

Jessica: We are going to listen to Evan.

Evan: 10 plus one is eleven

Jessica: 11. That is great math thinking, Evan. Does anyone have another way to come up with 11?

Before taking AVMR Course One, Jessica did not group students based on students' current knowledge. Additionally, she did not modify lessons from how they were written in the Bridges curriculum manual.

After AVMR Course One: Excerpts from the Classroom

After taking AVMR Course One, the ways Jessica grouped students for short periods of time and modified an activity during a lesson changed considerably. Almost immediately, Jessica implemented a math rotation system, where she chose four types of stations, including one station where she works with a small group of students on a skill in whole number. Activities chosen for the teacher-led small group and independent stations were of the type where they were easily modified by changing a number range or bringing in a setting or manipulative to support students' thinking during the activity. Jessica used activities from AVMR Course resources, general teacher resource websites, the district website with activities matched to standards within the district adopted frameworks, and a district AVMR binder assembled by district math coaches and provided to each school.

During one observation, the page Jessica planned for the small group at Teacher's Choice was missing. This was the third group working on number sequences and the first groups whose focus was on the number work after and the number work before in a number sequence. Once Jennifer realized the page was missing, she quickly passed out a hundreds chart with only a few numbers filled in and determined on the spot a task to pose and follow-up questions to push students' thinking. This was in stark contrast to having the manual in hand and reading tasks and questions from the lesson book, as observed before Jessica's participation in AVMR Course One. The following excerpt is from this small group at Teacher's Choice.

Table 4.29. Excerpt from Teacher's Choice.

Jessica stops to support a student with hundreds club. Jessica stops again to provide a student with a place value flip chart to support writing three digit numbers between 110-120.	Guiding Principle Four.
Jessica then begins to work with the small group assigned to her group for the rotation. She is taking out an activity from an AVMR binder of activities provided by the district math coaches. The master sheet she was planning to use is missing, and so Jessica comes up with another activity in the moment. She provides each student with a completed hundreds chart and puts up folders in order for each student to have a private work space. She starts off by asking each student to point to the number four.	Guiding Principle Four.
Jessica asks students to circle the number four. Now she asks students to point to the number that comes after four. A student asks to circle that number too. One students says the number five. Jessica asks him if that number is even or odd. Student is not sure, so Jessica moves on with the small group lesson.	
The next number to circle is 10. Then Jessica asks what the number is after 10. Jessica asks students to point to the number 20. Jessica asks, "Where is the number that comes before 20?" She has a student share who pointed to 19 and then asks other students in the group to also point to 19. Jessica folds each of the hundreds chart papers in order to have less visual clutter for students when answering questions. J: Hunter, how did you know that this number is the number before 20? Now Jessica brings out a numeral roll as a support and asks Hunter to identify 20, and then	Guiding Principles Two and Three.

point to the number that comes before 20, followed by the number that comes after.	Guiding Principles Four and Six.
Next student Jessica asks to point to 15. Jessica needs to support the student to identify 15 and not 50. Then she asks this student to show the number after 15. Student does not know what after means. Jessica clarifies by using hand gestures and the numeral roll. Then Jessica scaffolds down to a smaller number. Then asks the student to identify the answer after six, followed by the number that comes before six. She poses another question to this student. This time she identifies four, and asks which number comes after four, and which number comes before four. The student is timid, but correct for bot responses.	Guiding Principle Six.
Jessica is again looking for the worksheet that breaks down number work before and after.	
She then poses a question to Austin. She asks him to identify 20 and point to the number that comes before 20 and after 20. Jessica rings the bell to signal to clean up and join her on the carpet. They are finishing with a conversation about this week's focus is to practice rotations and get into the routine. Students in the hands-on group are reflecting on the importance of putting cards back in an organized fashion for work place games. Cleaning up. End of math time.	

Again, the considerable changes Jessica implemented after taking AVMR Course One remained in place after Jessica's participation in AVMR Course Two. During whole-group and small group lessons, Jessica provided students opportunities to choose their own numbers. Students tended to appropriately challenge themselves as shown in the excerpt below.

Table 4.30. Excerpt from Classroom Observation.

Lesson starts with the class preparing to do an activity called Number Rings. Instructions are that each student puts +1 in the center of the ring. Then when they get the thumbs up from Jessica, they can put any number they would like in the ring. Students are getting supplies to do the ring activity and going to their seats to begin working.	

The rings have an open center, where students record a number they are adding to each of the numbers written on the ring. The numbers 0-9 are written on the ring. Students record the sum around the outside of the ring.	
Jessica is supporting a student with this activity by providing her Unifix cubes to build the number written on the ring and then add one cube to that number to be able to determine the total. Once Jessica checks a student's board, she allows the student to choose a number for the center. One student has written +100. As Jessica goes around the room she provides Unifix cubes to additional students who need to have the cubes in order to determine the sum of two numbers. For students requiring Unifix cubes to solve, Jessica is telling them to now place two Unifix cubes in the center to do +2 to each number written on the ring after completing +1.	Guiding Principles Four and Six. Guiding Principle Five.
Another student has placed +10 inside the ring. Another is +5. Students are choosing numbers that challenge their thinking. A student has chosen +100. This is a bit out of reach for this student, but he is motivated to figure out the correct sums. Students are thinking and working. Jessica continues to rotate around the room asking students about their strategies and checking their answers.	
Students are being supported in correctly writing numerals as well as addition. Chime rings to end the activity. Students begin to clean up.	
Students are at the front carpet. Students are released by groups to their rotation spots. There is only time to do one rotation today. The hand-on group can do beach ball addition or dice addition that they were doing yesterday in small groups with Jessica. Teacher's Choice group is cutting out a clock to glue in their math notebook. Students are cutting out an "I can" statement, and then pasting the minute and hour hand to represent 10:00. Some students are having a difficult time focusing on the activity assigned for their groups during rotations today. Jessica is talking to students off task. Bell rings. Math time ends.	

The class is completing a board with the number 13 altogether so all students know how to fill out the page. The page has a number in the center. In the top left quadrant, students record number sentences for the number 13. In the top right quadrant, there are empty 10 frames to show the number recorded in the middle of the document. In the bottom right, students record the number using tally marks, and in the bottom left there is a hundred grid to shade in the number of boxes indicated in the center. Jessica is calling on students to come up one at a time to the board to record the representation of the numeral in one of the quadrants until all quadrants have completed representations of the number 13.	Guiding Principle Seven.

Jessica asked different students to share their equations. Then she tells students to erase their boards and choose their own number to complete each quadrant on the board. She is supporting students who have incorrect responses and checking students' boards for accuracy.	
Jessica stops to support a student who chose the number 36. She has 26 dots filled in the 10 frames and has an equation written as well. After listening to the student's strategy for writing the equation, Jessica notices that she used the 10 frames to record the equation. She only had 26 dots completed, and so Jessica communicated to the student that her equation was right for the number 26. The student is then going back to the 10 frames to accurately complete frames to show 36. Jessica questions the student by asking how many frames she filled in completely. The student replies three. Jessica asks, "How many dots is that?" Student replies 30. Emma then is asked to share her equation with the class at the board. Jessica shares with the class how the student used the dots in the 10 frame to create the equation. Then Jessica asks the student to share how she ended up adding 10 more to have an accurate equation. Student is not sure, so Jessica explains. Jessica communicated that her math thinking was cool and so she wanted to share it with the class. Student then starts to share her math thinking. Jessica expands on the types of strategies that the student used. She then provides students more time to work on their number worksheet/boards.	Guiding Principle Three.
Students are working independently. Jessica is walking around the room having one-on-one conversations with students.	

Additionally, the settings Jennifer brought in for support, coupled with questions posed to students as illustrated in the excerpts above, provide exemplary examples of Guiding Principle Three (*teaching is focused just beyond the "cutting edge" of students' current knowledge*) and Guiding Principle Four (*teachers exercise their professional judgement in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations*). Guiding Principle Five (*the teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated* *strategies*) and Guiding Principle Six (*teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of observation*) were not originally identified for research question three. However, the way both Guiding Principles Five and Six intertwined with Jessica's teaching during times of cutting-edge instruction and varying tasks selection is work noting in the previous example. Jessica's use of intense observation and on-going assessment during the lesson is expanded upon in research question four.

Research question four. *How does Add+VantageMR® professional development influence teachers' assessment of students learning during implementation of a whole number lesson?*

1. In what ways do teachers make in-the-moment adjustments to the written lesson plan based on student responses, demonstrated understandings, and knowledge about children's thinking as it is related to the Learning Framework in Number?
2. In what ways do teachers describe children's strengths and areas of need for whole number understanding with specificity that is connected to observed student solution strategies and the Learning Framework in Number?
3. In what ways do teachers collect formative assessment data during the lesson that includes a student-thinking component?

Data Analysis: Guiding Principles

One Guiding Principle for Classroom Teaching is applicable to research question four.

- *Guiding Principle Two: Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.*

In the moment of enacting a lesson, teachers are required to attend to a variety of stimuli and quickly make a number of complex decisions. Research question four focuses on the teacher's noticing of student solution strategies and the subsequent assessment of students' mathematical knowledge and shifts in that knowledge *during* a whole number lesson. Research Question four, sub-question three is used to guide the following reflection on the ways AVMR Courses One and Two professional development influenced Jessica's noticing of student strategies and informal assessment during a whole number lesson.

Classroom observations after AVMR Course One and then again after AVMR Course Two reveal the intense nature of Jessica's observations of student thinking during a lesson. Jessica clearly places observations within a model of children's thinking. When students share thinking or engage in solving a task, Jessica makes decisions regarding a setting to provide or a question where the settings and questions are appropriate moves for the advancement of students' thinking. For example, when students were engaging in ring math, two students needed Unifix cubes in order to build both addends. Additionally, Jessica chose one as the second addend. Once students solved with an addend of one, she told students to choose an addend of two for the second round of the activity. The intense observations and knowledge of the model to place the observations of students' strategies indicated that students needed to continue using Unifix cubes in additive situations where the second addend is three or less.

In summary, the way Jessica was able to plan and enact tasks at the cutting edge of students' current understandings after participating in AVMR Course One was remarkable. She consistently engaged in on-going, intense observation when students were in small group and large group. She then responded by modifying the task through changes in questioning or presentation of the task. Jessica's use of differentiation changed after participating in AVMR Course One in part by her change to implementing small group instruction, which allowed the opportunity for her to take the same lesson objective and similar tasks to make fine-tuned adjustments at students' cutting edge of current knowledge in the supportive environment of a small group. Again, these changes continued to be present in classroom observations after Jessica's participation in AVMR Course Two.

Case One provided a description of Carole's planning for instruction and classroom teaching after participating in AVMR Course Two. Case Two provided a description of Mark's planning for instruction and classroom teaching before and after participating in AVMR Course Two. Case Three provides a description of Jennifer's planning for and classroom teaching before and after AVMR Course One and then after AVMR Course Two.

Conclusion

The purpose of this study is to examine the ways, if any, classroom teachers incorporate ideas from Add+VantageMR® (AVMR) into their math classroom teaching practice. Guiding Principles for Classroom teaching were used as a lens to describe changes in teaching after participation in AVMR Courses One and Two. The three cases presented and described in Chapter Four are each unique teacher experiences from within a first grade team.

Carole had taken AVMR Course One prior to the study. She did not use AVMR assessment in her teaching practice before or after taking AVMR Course Two during the study. Many guiding principles for classroom teaching emerge from initial and ongoing assessment, therefore, Carole had the least identified manifestations of Guiding Principles during the study. However, changes were identified after Carole's participation in AVMR Course Two. In particular, changes were identified when addressing research questions two and three. After AVMR Course Two, Carole selected tasks for classroom lessons which allowed opportunities to differentiate the lesson. Students were spending more time doing mathematics at their cutting edge of learning. After taking AVMR, Carole was considering and trying to make sense of events involving assessing, teaching and learning mathematics. One of her highest readers performed lower than one of the students in her class performing lower in reading concepts. Carole was still thinking about how that could be at the conclusion of the study. Her own thoughts about students learning of mathematics was that if you were a good reader, you were also high performing in math, and if a student struggled with reading concepts, the student struggles with learning mathematics because you need to read problems and directions when doing math.

Carole also expressed disequilibrium in her feelings about flexibly grouping students within the mathematics lesson. She was thinking where that belongs. In her final interview, Carole talked about a teacher that does intervention with students for 10 minutes a day in the grade level. She was thinking about and trying to make sense of the role of the interventionist teacher and her role as the classroom teacher. The following quote from the final interview is a good summary of Carole's position towards assessment, teaching and learning of mathematics at the end of the study.

> There's just a lot that you can do with it, and I want to do it, and it is just doing it, I think. I want to understand the data. I want to get them to the next level. I want to take from the class the different things and then work with my team – "Alright, here's this stuff, it would help them get here, let's meet this next benchmark," the practical side, you know…

There is an overwhelming nature to the amount of new information gained from AVMR Course Two. Carole wants to use AVMR assessment and use information gleaned from the assessments to inform instruction, but details as to how to do that is unclear after the course.

Mark entered the study with a desire to implement flexible small grouping into his regular classroom practice. He also reviewed assessment data provided by the kindergarten team or researcher during the study. At the start of the study, Mark asked questions regarding which tasks to pose in order to advance student mathematical thinking. After taking AVMR Course Two, Mark was able to implement flexible small groups. He was also able to choose tasks for students. During classroom observations, Mark's next questions included enacting tasks and questioning and responding to informal assessment during the lesson. At the end of the study, Mark shares that he is limited by time - time to put together activities and materials after lesson planning in order to implement small group and instruction responsive to what students know regularly. In the final interview, Mark shared:

> From when we first started doing AVMR, I don't know how many years ago, you know, having been a kindergarten teacher, it's very prescriptive and it really gives me the range of my kids and where they are at mathematically. You know, in whatever category, who are the fives, and who are my zeroes and stuff. Then again from the purple book, looking at activities in there that you can give those students to work on, to build those skills, and now with having the red book to have and pull activities out of that, and then starting to do some of out of the blackline masters to go with the data. I find it valuable for math. It's the best.

Mark and Carole are at different places with what they take away from AVMR Courses One and Two and what they are able to implement after AVMR Courses. Carole is thinking hard about where to get started and firming her own beliefs about assessing, teaching, and learning of mathematics. Mark knows how to get started and he needs to be able to more readily access activities in order to maximize his limited amount of time available for planning and preparing for math lessons among teaching all subjects and being involved in district committees. Both can benefit from support after the course. However, the type of support is different.

Jennifer is a first year teacher and took both AVMR Course One and Course Two during the study. Similar to Mark, Jennifer knew from the start of the year that she wanted to implement flexible small group instruction within the mathematics lesson time. She used student assessment data from the start of the study, asking questions to understand what the level and construct numbers meant for students' current understandings and where to target instruction. During the study, Jessica asked questions about what the data means and where to find resources among the many resources available from the district. She enthusiastically awaited assessment data at the conclusion of the study to guide her flexible grouping decisions and lessons after winter break.

Jessica's use of formal and informal assessment data provided a basis for the presence of many manifestations of Guiding Principles for Classroom teaching immediately after taking AVMR Course One and continuing after AVMR Course Two. Jessica was able to implement a flexible small group instruction model using assessment and teaching tools after taking an AVMR Course. She may benefit from support in the future, but areas where she could benefit from additional support was not readily apparent as a result of the study. She was excited about the new learning and the ways in which she was immediately able to implement the new learning were impressive.

Chapter Four provided a rich description of each of three cases, followed by a summary of what each teacher was able to take away and implement after participating in an AVMR Course. Further conclusions and implications based on the results of this study will be discussed in Chapter Five.

Chapter 5 – Summary, Conclusions, and Implications

School districts in the United States spend a significant amount of human and financial resources on the design and facilitation of professional development experiences that potentially increase teachers' understanding of how students think about and learn mathematics. In turn, these professional development opportunities hopefully lead to change in classroom instruction. This investment is done with the aim to make a difference in student achievement outcomes and thus meet district student achievement goals. Empirical evidence to support components of effective professional development is mounting and has grown significantly in the past fifteen years. This study adds to the literature by investigating the efficacy of a national professional development program Add+VantageMR® professional development (AVMR) among teachers of primary students.

Summary

The overarching goal of this study is to investigate the ways, if any, in which classroom teachers incorporate ideas from Add+VantageMR® (AVMR) into their math classroom teaching practice. Teaching is complex and therefore there are several areas of opportunity to apply learning from AVMR. AVMR professional development focuses on student mathematical thinking along with formal and informal assessment of students' strategies to inform teachers decision-making when designing instruction to advance student mathematical thinking.

This study was designed to examine what teaching practices, if any, teachers implement in the classroom from Add+VantageMR® professional development. This was determined by examining teachers' *planning for* and *enacting* mathematics lessons in the area of early number and operations. A case study approach, (Yin, 2014) provided the opportunity to create a detailed description of each classroom in order to locate even the smallest change in classroom practice. Student achievement results in the content strands of number words and numerals and addition and subtraction to 20 are used support classroom observation analysis by research question.

Based on the core features of Add+VantageMR® professional development and the overarching research goal for this study the following research questions and subtopics were addressed. Subtopics became guiding questions used during analysis and provided the structure for each case study description.

1. In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?
    a. In what ways do teachers use knowledge of children's thinking when choosing instructional tasks and assigning number ranges for mathematics problems?
    b. What are teachers using to determine children's current level of understanding?
    c. In what ways do teachers use course resources provided in course Teacher Handbooks or *Teaching Number in the Classroom with 4-8 year-olds* and/or *Developing Number Knowledge* by Robert Wright, et al. when planning a whole number lesson?

d. In what ways do teachers structure the mathematics class time to allow for development of conceptual understandings in whole number as well as skills that support learning about whole number operations?

2. In what ways does Add+VantageMR® professional development influence teachers instructional moves during a whole number lesson?

   a. In what ways are students engaged in activities from the course resources such as the provided Teacher Handbook or *Teaching Number in the Classroom with 4-8 year-olds* and/or *Developing Number Knowledge* by Robert Wright, et al)?

   b. In what ways are students actively engaged in ***doing*** mathematics during the lesson?

   c. In what ways does the teacher use Guiding Principles for Classroom Teaching and domains of progressive mathematization during a whole number lesson?

3. How does Add+VantageMR® professional development influence differentiation of a whole number lesson?

   a. In what ways do teachers group students based on students' current knowledge and for purposes that are specific and short in duration?

   b. In what ways do teachers modify an activity during a lesson. Modification may include offering different dice, number ranges, and/or instructional tools?

4. How does Add+VantageMR® professional development influence teachers assessment of students learning during implementation of a whole number lesson?

a. In what ways do teachers make in-the-moment adjustments to the written lesson plan based on student responses, demonstrated understandings and knowledge about children's thinking as it is related to the Learning Framework in Number.

b. In what ways do teachers describe children's strengths and areas of need for whole number understanding with specificity that is connected to observed student solution strategies and the Learning Framework in Number?

c. In what ways do teachers collect formative assessment data during the lesson that includes a student-thinking component?

This study used a holistic multiple-case case study design approach (Yin, 2014). A multiple-case case study design allowed for an examination of a number of variables at play when examining the impact of AVMR professional development in real context.

Figure 5.1 describes the participant background at the start of the study.

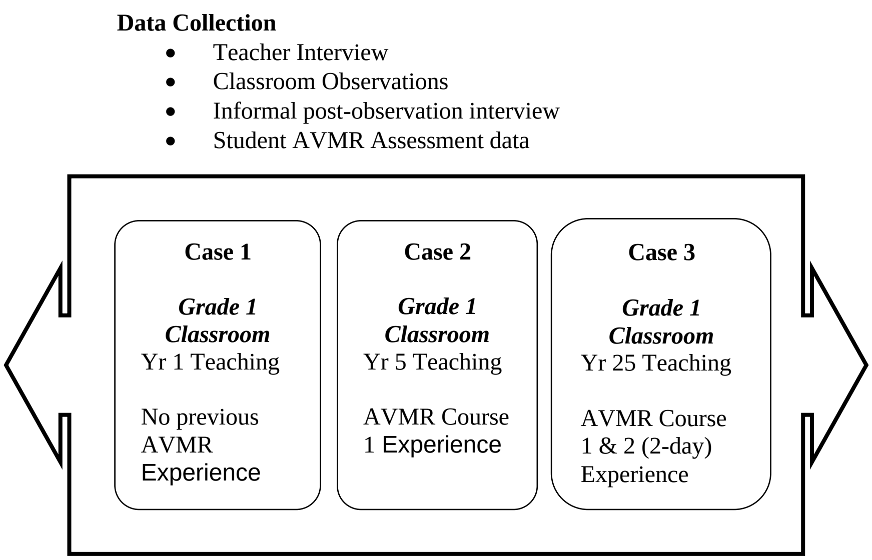


Figure 5.1 Overview of the Research Design

As shown in Figure 5.1, each classroom was treated as a separate and distinct bounded unit of study in order to look at patterns occurring within and across cases as related to the research questions. A three case design allows for a stronger study by allowing for analytic conclusions and theoretical replication to be drawn across multiple cases as opposed to results from a single case.

The study took place in a third ring midwestern suburb during the first 4 months of the 2015-2016 school year. Add+VantageMR® professional development has been a part of the district's goals and initiatives for mathematics since 2008. Over the past ten years, district coaches created resources for teachers linking AVMR course content to state standards, district adopted curriculum and district mathematics frameworks. A robust website houses these curriculum documents along with a variety of additional links to materials that teachers can choose from when planning curriculum. The district's commitment to Add+VantageMR® made it an ideal site for this study. School and other district initiatives align with the same core values about teaching and learning as Add+VantageMR®.

Data collection methods included teacher interviews, classroom observations, informal post-observation interviews, and AVMR pre- and post- student assessment data. Each data source was used to address research questions one through four pertaining to *planning for* instruction and *enacting* the planned lesson. Figure 5.2 provides a timeline for data collection organized by research question.

Figure 5.2 Data Collection Timeline

Research Question	Aug/Sept	Oct	Oct	Nov/Dec	December
Q1: Planning for instruction	Teacher Interview Post Observation Interview Student Data	**Add+VantageMR® Course 1**	Teacher Interview Post Observation Interview	**Add+VantageMR® Course 2**	Teacher Interview Post Observation Interview Student Data
Q2: Teaching moves during a Lesson	Class Observation Post Observation Interview		Class Observation Post Observation Interview		Class Observation Post Observation Interview
Q3: Differentiation	Teacher Interview Class Observation Post-Observation Interview Student Data		Teacher Interview Class Observation Post-Observation Interview		Teacher Interview Class Observation Post-Observation Interview Student Data
Q4: Assessment of student learning	Teacher Interview Class Observation Post-Observation Interview Student Data		Teacher Interview Class Observation Post-Observation Interview		Teacher Interview Class Observation Post-Observation Interview Student Data

The Math Recovery Guiding Principles for Classroom Teaching framework is used as a lens for analyzing classroom observations for all research questions. For each research question, applicable Guiding Principles were identified and used to describe changes in classroom practice from before and after participation in AVMR professional development. Table 5.1 provides the complete list and short description of the nine Math Recovery® Guiding Principles for Classroom Teaching.

Table 5.1 Guiding Principles for Classroom Teaching

Principle 1 The teaching approach is inquiry-based, that is, problem-based.
Principle 2 Teaching is informed by an initial, comprehensive assessment and on-going assessment through teaching.
Principle 3 Teaching is focused just beyond the 'cutting-edge' of students' current knowledge.
Principle 4 Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going

observations.

Principle 5 The teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies.
Principle 6 Teaching involves intensive, on-going observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.
Principle 7 Teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic which accord with students' verbally-based strategies.
Principle 8 The teacher provides students with sufficient time to solve a given problem.
Principle 9 Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.

Classroom Observations. Nine classroom observations for each teacher took place over the course of the study. Classroom observations, each consisting of three sequential lessons, took place over a four-month period. The first classroom observation occurred in September, a second in mid-November and the final observation in December. Each lesson was video recorded to allow for later analysis. Field notes and picture artifacts documented aspects of the lesson not visible to the video camera. Following each lesson, an informal post-lesson interview provided an opportunity for classroom teachers to share personal insights on the observed lesson. Often, personal insights included self-observations, reflections and intended adjustments to the next day's lesson.

Two rounds of coding were applied to classroom observation video prior to formal analysis. The Guiding Principles for Classroom Teaching formed the basis for round one structural coding. As a result of round one structural coding, five Dimensions of Mathematizing and three researcher created codes were added to initial codes. Table 5.2 shows researcher creator codes; a) providing a mathematically untrue statement b) traditional telling to teach an important concept and c) use of mathematical symbols detached from student thinking indicating behaviors identified to be in contrast to the Guiding Principles of Classroom Teaching. Researcher created codes are followed by Dimensions of Mathematizing added as additional features within the appropriate Guiding Principle.

Table 5.2 Codes applied to Classroom Observation Video

Contrasts Guiding Principles for Classroom Teaching
○ Providing a mathematically untrue statement
○ Traditional telling to teach a concept
○ Symbols detached from student thinking
Guiding Principles for Classroom Teaching
Principle One: Inquiry –based teaching *Dimension of Mathematizing* • Problem centered task
Principle Two: Teaching Informed by assessment *Dimension of Mathematizing* • Task based on Learning Framework in Number & aimed at student's current levels of understanding
Principle Three: Cutting edge teaching *Dimension of Mathematizing* • Teacher accounts for a range of student levels of understanding
Principle Four: Select from a bank of procedures
Principle Five: Engender more sophisticated strategies *Dimensions of Mathematizing* • Distance the setting • Grounded habituation • Extending the range • Complexifying arithmetic
Principle Six: Intense, ongoing observation *Dimensions of Mathematizing* • Scaffolding a student through a task • Micro-adjusting the task
Principle Seven: Teaching builds on verbal-based strategies *Dimensions of Mathematizing* • Teachers' notation/symbolize student thinking • Students verbalize mathematical strategies
Principle Eight: Sufficient time to solve problems
Principle Nine: Students experience intrinsic satisfaction from engaging in mathematical tasks

Four Dimensions of Mathematizing were added as descriptors under Guiding Principle 5, the teacher understands students' numerical strategies and deliberately engenders the development of more sophisticated strategies. Together the four dimensions; a) distancing the setting, b) extending the range, c) complexifying arithmetic, and d) refining computational strategies describe teacher moves related to engendering more sophisticated mathematical strategies. Additionally, the dimension of notating was added to Guiding Principle 7, teaching supports and builds on students' intuitive, verbally-based strategies and these are used as a basis for the development of written forms of arithmetic. In both cases, the Dimensions of Mathematizing provided specificity to each case allowing for a rich description during data analysis. When discussed within each case, the overarching Guiding Principle is discussed and in inclusive of the identified Dimensions of Mathematizing.

Second round coding included two passes through classroom observation video. One pass observed teacher moves and classroom structures such as questioning, task presentation, teacher-student interactions in whole group, small group or one-on-one situations, flexible student groupings, and available settings or manipulatives. A second pass observed student characteristics such as engagement with the mathematics within the tasks, talk with other students and the teacher about mathematical solutions, and whether students were exhibiting intrinsic satisfaction from coming to mathematical solutions. In addition to applying codes displayed in Table X, analytic memos provided a way to capture anecdotal notes during second round coding. Together, the application of codes and analytic memos provided a basis for identifying patterns related to each of the research question subtopics.

Semi-structured Interview. Each individual interview focused on participant thinking on areas of quality math instruction, lesson design and implementation. Teachers participated in a pre- and post- semi-structured interview where the first interview served as a baseline while the second interview provided evidence of change in teacher's vision for high quality math instruction. The first interview took place at the beginning of the school year, in some cases during workshop week. The final interview took place for all participants at the conclusion of the study in December. One participant completed AVMR Course 1 during the study and participated in a semi-structured interview afterward, therefore having a total of three interviews.

For both the pre- and post-interview *The Vision of High-Quality Mathematics Instruction Instrument (Munter, 2014)* was utilized to capture teacher vision for the role of the teacher, high quality math tasks, classroom discourse, and student engagement. The pre- and post-interview protocol started with one simple question, "If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?" The interviewer can choose from several pre-planned follow-up probes to glean more detail on the participant's perspective of high quality math instruction.

The post-interview included additional questions such as, *What could be done to make it easier to use AVMR data in the classroom?*, which provided further insight into phenomenon arising out of classroom observations.

To perform analysis, pre-and post-interviews were transcribed and transcriptions provided evidence and support for classroom observations when coding for manifestations of the Math Recovery Guiding Principles for Classroom Teaching.

Student Data. The researcher conducted video-recorded pre- and post- student assessment interviews. Diagnostic interviews resulted in a numeric value representing a level or stage of students' mathematical understanding in each of the domains comprising the Learning Framework in Number. Student assessment data was used to support classroom observation findings in research question one, two and three. Assessment data was shared with participating classroom teachers providing an opportunity to investigate how teachers used AVMR data when planning for or enacting a whole number lesson.

Assessments, administered and analyzed by the researcher, took place in September and December. Classroom teachers were provided completed assessment schedules along with a spreadsheet containing benchmark achievement levels and constructs for first graders during that time in the school year.

Overview of Add+VantageMR Professional Development.

Key characteristics of Add+VantageMR® professional development.

Empirical evidence to support components of effective professional development has grown significantly in the past 15 years. Advancements in the field make it possible to identify core features that have been repeatedly shown to relate to teacher improvement, and tentatively, to student achievement. As a result of synthesizing this body of research, Desimone (2009) presented a core conceptual framework based on consensus from the field that can be used to study the effects of professional development on teachers and students. Within this framework, five core features of professional development are described: content focus, active learning, coherence, duration, and collective participation. Add+VantageMR® incorporates all five core features of effective professional development practices.

Robust content centers on children's thinking and incorporates research on children's thinking in a range of early number topics such as Number Word Sequences, Counting, Structuring Number, Base Ten, and Multiplication and Division (Cobb & Wheatley, 1988; Mulligan & Mitchelmore, 1997; Leslie P. Steffe, 1992; Leslie P Steffe, Cobb, von Glasersfeld, & Sinclair, 1988; Wright, 1991, 1994, 2000). Each course introduces an assessment for each topic and provides teachers with opportunities to apply knowledge about student learning to their own classroom of students through video recorded practice administering assessments.

Active learning is integral to teacher learning in each of the face-to-face instructional days. This practice of examining student thinking, followed by discussion with colleagues, is a core element throughout the courses. Each course includes the opportunity for teachers to administer the assessments with children; many times students are from their own classroom. Each course concludes by involving teachers in activities wherein they participate in exemplary instructional activities and discuss ways to manageably differentiate these activities in the classroom setting according to assessment results.

Coherence with district initiatives, existing curriculum and across grade levels is encouraged through US Math Recovery Council leadership courses. Instructors are encouraged to develop support materials and networks for teachers in order to make transparent coherence between Add+VamtageMR professional development and district initiatives.

Add+VantageMR is an intensive course in content and duration. Each course consists of two four-day courses; teachers spend 24 contact hours in each course, for a total of 48 hours devoted to learning about content. Each course has a specific content focus in the area of whole number.

Collective participation is strongly encouraged by the U.S. Math Recovery Council®. When teachers participate in AVMR professional development as a grade level team and along with teachers from a cross-sectional of grade levels, the conversations about children's strategies and use of instructional is enriched and supported in the school building. Delivery models for AVMR promote collective participation in districts by offering leadership courses where Math Recovery® Intervention Specialists learn to facilitate AVMR courses within their own district.

During AVMR Course 1 and 2, teachers learn about the Learning Framework in Number and how to assess and pose teaching tasks within each domain within the Learning Framework in Number. Additionally, teachers learn about the Math Recovery Guiding Principles for Classroom Teaching and a few of the nine identified Dimensions of Mathematizing. Each AVMR Course is 24 hours of instruction within a four-day time frame. Following the face-to-face days, teachers, schools and districts have a great amount of flexibility in how they implement AVMR assessment and teaching tools. Teaching is complex. AVMR Course 1 and 2 content can potentially impact teacher's planning for instruction, task selection, ways they structure math time or students to engage in math content, and teachers' noticing during a lesson. Adding to the complexity of studying professional development such as AVMR is the limited availability of high quality instruments to investigate fine changes in classroom practice.

A summary of findings for each case elaborated upon in chapter four is found below. Changes in classroom practice occurring after participation in AVMR professional development are shared and linked to AVMR professional development.

Case 1: Carole

Research Question 1. *In what ways does Add+VantageMR professional development influence teachers planning for instruction of a whole number lesson?*

Although student data was available for Carole at the beginning of the school year, Carole's use of student data changed after participating in AVMR course two. For Carole, the focus placed on analyzing student assessment data and planning instruction based on AVMR assessment results during AVMR course two appeared to impact her planning for a whole number lesson by providing detailed information about her students current knowledge. With detailed information, Carole was able to make decisions regarding task choice and differentiation, seating placement and partner selection. After participating in AVMR course two, Carole reached out to ask questions about the assessment data shared early in the school year. Although the data was now two months old, Carole was able to attend to interpreting the data differently than she could prior to participating in AVMR course two. It is interesting to note that the mathematics topics assessed earlier in the year were not the same topics discussed in course two. It seems that professional learning experiences relating to interpreting AVMR in any assessment topic supported Carole in interpreting data from all math topics.

After AVMR course two Carole chose instructional activities or tasks more closely aligned with students current mathematical understandings than task sections prior to AVMR course two. The instructional activities after course two provided the opportunity to differentiate for students in order to align more closely to students cutting edge of learning, and Carole made decisions in the moment of teaching to adjust the task during the lesson. This is in contrast to classroom observations before AVMR course two where students were engaged in tasks that were way too easy and although the activity could have been adjusted for students, all students were asked to engage in the activity in the same way. AVMR Course two provided practice adjusting tasks for students at levels of understanding described within the model for children's thinking. For example, in the area of place value, six levels called constructs are described in increasing level of mathematical sophistication. During AVMR professional development participants work in small groups to read through an instructional activity and share with the whole group how that learning activity can be adjusted using elements of the dimensions of mathematizing such as materials, number choices, and screening. Practicing with peers within the professional development experience seems to translate into the classroom.

Following AVMR course two Carole started to flexibly group students during independent work based on mathematics data. After participation in Add+VantageMR® Course 2, Carole intentionally created a seating arrangement for students during math time based on her assessment of students' current knowledge and skills in mathematics. Carole paired students during the partner activity portion of the lesson. Carole changed from having students complete an activity with the person they are regularly assigned to sit beside to intentionally pairing students while passing out the materials for the activities based on her knowledge of students' mathematics knowledge. Before taking AVMR course two students collaborated in mathematics within their assigned seating arrangements based on before school reading assessment data. The type of assessment data teachers receive from AVMR assessments provides specific information about students math knowledge that teachers can use to base decisions in the classroom. During AVMR professional development the course addresses ways to use assessment data from AVMR assessments to inform decisions in the classroom. Specifically, the course does address seating arrangements and how to partner students for particular activities. During the post-interview Carole was thinking hard about the relationship between students reading and math skills and understandings. Before taking AVMR course 2, she believed students' knowledge and skills in reading would be similar to knowledge and skills in mathematics. For example, if a student was performing below grade level in reading then the same child would be performing below grade level in mathematics. Two students, in particular, did not align with that assumption. At the end of the study, Carole was very surprised to learn that students' knowledge and skills in reading can be very different than students' knowledge and skills in mathematics.

Content addressed in AVMR Course 2 included place value and multiplication and division. In particular, Carole made connections between teacher learning in regards to learning progressions for place value. During the course, Carole assessed a student in her classroom and gained insight into student thinking in place value of students similar to the student she assessed. The course addressed how to choose instructional tasks providing students opportunities to engage with place value concepts meaningfully and in such a way to promote advancements in students' thinking. During the course, tasks provided in the course text, *Developing Number Knowledge* by Robert Wright et al, were used to discuss how to differentiate a task for students thinking about place value at a variety of places along the trajectory. Although the learning can be applied to tasks found in any commercial curriculum, using tasks from course texts during an AVMR course provides a common context for all participants. After AVMR course two Carole used instructional resources from AMVR professional development to address place value skills discussed during the course. She chose lessons from *Developing Number Knowledge* by Robert Wright, et al., the main text in AMVR Course two. During AVMR professional development participants practice planning a series of activities according to the model for each math topic. Ways to incorporate assessment and the results from assessment in the classroom are discussed in such a way that teachers are left to apply to their own classroom practice in a way that works best with their own practice and style of teaching. Observed changes in planning for instruction align with Guiding Principles of Classroom teaching and practice activities done in the course.

In summary, changes occurred in Carole's planning for instruction following AVMR Course 2 professional development. Carole attended to students' individual mathematics data, (Guiding Principle 2), selected tasks more closely aligned with students' current understandings (Guiding Principle 4), and intentionally paired students according to knowledge of students' mathematics understanding (Guiding Principle 3), which in turn presented the opportunities to meet and extend students' current mathematical understanding during the lesson.

Research Question Two. *In what ways does Add+VantageMR professional development influence teachers' instructional moves during a whole number lesson?*

Math Recovery Guiding Principles for Classroom teaching were used to identify changes in classroom practice during a lesson associated with AVMR professional development. During AVMR course two, the Guiding Principles are specifically addressed when transitioning from learning about assessment to learning about instruction. Additionally, Guiding Principles are foundational to the participant learning goals throughout the course. During coding procedures, three identified behaviors in contrast to the Guiding Principles were identified. Classroom observations after Carole's participation in AVMR course two showed a decrease in behaviors in contrast from twelve and eighteen occurrences before AVMR course two to four occurrences after AVMR course two. Additionally, before taking AVMR course two the most prevalent Guiding principle found in classroom observations was Guiding Principle 7, teaching builds on verbal-based strategies where observers would witness teachers' providing notation/symbolize student thinking and students verbalize mathematical strategies. Most notably after Carole's participation in AVMR Course 2 is the presence of Guiding Principles 3, 4, 6 and 9. Each of these Principles is important for the advancement of student mathematical thinking. Guiding Principles 3, 4 and 6 focus on presenting tasks to students based on knowledge of students' current knowledge, the trajectory for student thinking and using assessment of students' current knowledge when making in-the-moment decisions during instruction. Guiding Principles 3, 4 and 6 did not have a notable presence in observations prior to Carole's participation in Add+VantageMR® Course 2.

As a result of providing students with tasks involving an appropriate level of challenge students are demonstrating behaviors of intrinsic satisfaction from thinking hard about mathematic- Guiding Principle 9. In the lessons that followed Add+VantageMR® Course two students were still engaged in sharing computation strategies, however, they were sharing with their partner as they engaged in the mathematics. Carole utilized problem-based activities that were a part of the Add+VantageMR® resources, Principle 1. As a result, all students had much more time on task and were engaged in mathematics during the lesson instead of a select few having the opportunity to verbalize thinking.

A second way AVMR professional development can influence a teacher's instructional moves during the lesson is through the ways students are engaged in activities from the course resources such as the course teacher handbook or the course text *Developing Number Knowledge* by Robert Wright, et al. Prior to AVMR course two, Carole followed each lesson within the district adopted curriculum without adjusting for students' current mathematics levels and skills of students in the class. Lessons were too easy for most of the class. For example, the opening lesson objective was to write numerals to 5 and then numerals to 9. Beginning of the year assessment data revealed that fourteen of nineteen students in the class can read and write two digit numbers. As a result, students did not engage in the tasks or math content.

Research Question Three. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

Before taking AVMR course two, Carole did not differientiate a lesson during planning nor during the lesson. Notable in the classroom observations after AVMR course two is Carole's use of differentiating a lesson both in the planning stages and during the math lesson. Carole was responsive to students during the lesson. She was observing and listening to students while they were working in pairs. Before AVMR course two Carole would go around the room to each of the groups, but her attention and interactions focused on whether students were following directions and doing the lesson activities rather than attending to solution strategies and keeping students engaged at the cutting edge of their own learning.

After taking AVMR Course two, changes were found to the way Carole differentiated for student learning before and during the mathematics lesson. Carole was able to respond to student need in the moment of teaching rather than continuing with a lesson she knew was not advancing students' mathematical knowledge. Carole also started to consider information about students' current knowledge and skills as well as the development of mathematics when enacting whole number math lessons. She intentionally paired students for the day's activity and then adjusted the activity during the lesson and while students were engaging in mathematics. This is a change from the first set of observations in which Carole paired students according to information about students' knowledge and skills in reading.

Research Question Four. *How does Add+VantageMR professional development influence teachers' assessment of students learning during implementation of a whole number lesson?*

No change was found in the nature of Carole's noticing of student solution strategies and the subsequent assessment of students' mathematical knowledge and shifts in that knowledge *during* a whole number lesson. Carole's own conceptual knowledge of mathematics content and how math knowledge is related to other academic content areas appears to provide a barrier to considering in detail a framework for children's thinking within a particular mathematics topic that is useful within the moment of teaching. Additionally, it is possible that additional experience is needed administering and analyzing one-on-one assessment data in order to make sense of the larger conceptual understandings of which Carole is struggling to find a firm understanding.

Case 2: Mark

Mark had the most professional development and teaching experience of each of the study participants. As expected classroom observations and initial interviews identified evidence of implementing AVMR course one formative assessment practices and revealed manifestations of Math Recovery Guiding Principles within his classroom practice. After participating in AVMR course two, Mark finetuned his use of assessment to incorporate the diagnostic nature of the AVMR course assessments leading to evidence of classroom tasks and practice focused just beyond the "cutting-edge" of students' current knowledge.

Research Question One. *In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?*

In the initial interview, Mark said he reviewed AVMR data shared from some kindergarten classrooms, but without having information on all of his students, he did not find it very useful for planning lessons. In the future, he hoped to see data recorded at the district level, as it is for literacy, to ensure he has math assessment information on all of his students at the beginning of the year to use for planning lessons. Between observation one and two, Mark received researcher collected AVMR data on all of his students. He reviewed the data, linked level descriptors to state and district grade level standards and asked questions to better understand how to choose activities or tasks appropriate for students' current levels of understanding as revealed by AVMR assessments. Classroom observations found lessons to be differentiated more closely aligned to classroom assessment results. Additionally, Mark expressed interest in utilizing small group instruction during math lessons, but indicated he was not sure how to organize small group instruction within the whole class lesson time.

AVMR Course Two professional development included a review of critical Course one content forming a basis for AVMR Course two content. AVMR Course Two addresses two core content strands; place value and multiplication and division. Each strand is broken down into learning to administer and analyze a diagnostic assessment in that strand. Dimensions of Mathematizing is introduced with a focus on a subset of dimensions and Math Recovery Guiding Principles for Classroom teaching are reviewed. When learning about instructional progressions, course participants apply how to choose and differentiate tasks based on where students are found to be using the diagnostic assessment for that strand.

After participating in AVMR Course Two, Mark's use of data on his own students became more intentional than before taking AVMR Course Two. During classroom observations, Mark used and referenced the physical classroom data sheet with all students fall assessment scores for each of the number topics covered in AVMR course 1; number words and numerals, structuring numbers, and addition and subtraction. He used the data sheet to determine and refine partners engaging in a similar activity. He also determined a way to implement small group instruction during the allotted math time.

During the observed lesson, Mark referred to the data sheet to identify students to attend a small group lesson providing students with more challenging content than the whole class activity. He also incorporated his knowledge from Cognitively Guided Instruction problem types and Math Recovery teacher moves to link visible materials and symbolic notation to create a robust task with multiple entry points.

Research Question Two. *In what ways does Add+VantageMR® professional development influence teachers instructional moves during a whole number lesson?*

As expected, manifestations of guiding principles were observed in Mark's classroom during the first set of baseline classroom observations. In fact, all eight Guiding Principles were identified across observed lessons before Mark's participation in AVMR Course Two. The most prevalent Guiding Principle was principle seven, *teaching supports and builds on students' intuitive, verbally based strategies, forming the basis for the development of written forms of arithmetic.*

After Mark's participation in Add+VantageMR Course 2, classroom observations revealed a strong representation of all nine guiding principles, however, the greatest representation after his participation in AVMR course two was found in Guiding Principles three, four and five; *GP3: teaching is focused just beyond the "cutting-edge" of the child's current knowledge, GP4: teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations GP5: teachers understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies.* The increased presence of tasks positioned at the cutting edge of students' current knowledge and instruction aimed to engender more sophisticated strategies alongside the use of verbally based strategies as the basis for developing written forms of arithmetic creates a math learning environment conducive to advancing students' knowledge, skills and understanding of math concepts. Classroom observations taking place after Mark's participation in AVMR Course Two illustrated how choosing a problem-based task (Guiding Principle One), and attending to AVMR assessment results (Guiding Principle Two), provides a basis for aiming instruction at a child's cutting edge of current knowledge (Guiding Principle Three). Additionally, Mark engaged in careful observation of students' they engaged in tasks during the lesson (Guiding Principle Six). Careful observation led to making choices regarding number ranges within tasks. The understanding of children's numerical strategies (Guiding Principle Five) provided Mark with knowledge to build on student's intuitive, verbally based strategies as a basis for the development of written forms of arithmetic (Guiding Principle Seven). Providing students with adequate time to think hard about mathematical tasks (Guiding Principle Eight) led to intrinsic satisfaction from solving the task (Guiding Principle Nine) ultimately leading to students' overall positive feelings towards mathematics.

Research Question Three. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

The most notable change from before to after Mark's participation in AVMR Course Two is Mark's implementation of small group instruction during the allotted mathematics time. Before participating in AVMR Course Two Mark shared his desire to implement small group instruction during math time, however, he was unsure how to make it happen. After participating in AVMR Course Two, Mark was implementing small group instruction during the observed lessons.

In the final interview, when asked, "*What would be helpful as a classroom teacher to be able to more easily use data?",* Mark shared that AVMR provides the best information about his students' strengths, holes and next steps instructionally. However, materials management and the time to prepare materials for lessons by creating appropriate activities from the Math Recovery series texts or blackline masters included in the course are barriers to differentiating whole number lessons.

Research Question 4. *How does Add+VantageMR® professional development influence teachers assessment of students learning during implementation of a whole number lesson?*

Before and after taking AVMR Course Two, Mark provided opportunities for students to share strategies and expand upon their own thinking. Classroom observations before taking AVMR Course Two reveal that Mark practices intense observation of students' strategies during a whole number lesson. Across all observed lessons, Mark listened intently and posed questions to students to accurately record students' strategies and thinking as they truly occurred.

After AVMR Course Two, Mark listened intently and posed questions to students attending to specific strategies students are using to solve problems, as he did before taking AVMR Course Two. However, classroom observation data indicates a change in the way Mark rephrases, names, and connects strategies during a whole number lesson, providing evidence that Mark is generally placing student strategies within a framework or developmental model. Mark appropriately connects strategies in close sophistication.

During the final interview, Mark shares that he would like more practice within an AVMR course administering and analyzing assessments to better be able to identify students' exact strategies and better place them within the framework during a lesson.

Case 3: Jessica

Jessica is a first year teacher during the study. She participated in a year-long co-teaching student teacher placement before coming to Cedar Ridge. She teaches half the day in first grade, then completes the remainder of her day teaching a kindergarten skills group and reading intervention groups. Jessica is the only teacher in the study participating in AVMR Course One and Course 2 during the study. Both colleagues participating in the study took AVMR Course 1 prior to the study and participated in AVMR Course Two during the study. Across all four research questions, the greatest change occurred after Jessica participated in AVMR Course One.

Research Question 1. *In what ways does Add+VantageMR® professional development influence teachers planning for instruction of a whole number lesson?*

During the initial interview, occurring before Jessica participated in AVMR Course One, Jessica describes the importance of engaging with students during the lesson and meeting students at their current knowledge and skills. She also shares her desire to determine a way to incorporate small group instruction into mathematics class time. Prior to participating in AVMR Course One, Jessica purchased a Daily 5 for math resource on *Teachers Pay Teachers* website. The planned lessons at the time of initial classroom observations are from the district adopted Bridges curriculum. During the lessons, Jessica has the teaching manual open and reads the lesson out of the manual for the duration of the lesson. Some students are participating in the lesson and many students are not able to access lesson content.

At the time of AVMR Course One, researcher collected AVMR data was shared with Jessica. After Jessica participated in AVMR Course One, she requested an informal meeting to share and confirm her excitement to apply learning from AVMR Course One to setting up small group instruction. After AVMR Course One Jessica's planning for instruction changed to include small group instruction and a series of math rotations using AVMR assessment data results.   Small group and math rotation tasks are selected from a bank of activities coming from Bridges curriculum, district AVMR resources and Math Recovery texts (*GP4*). Tasks and questioning are selected to focus just beyond the "cutting edge" of students current knowledge and skills (*GP3*).

Research Question 2: *In what ways does Add+VantageMR® professional development influence teachers instructional moves during a whole number lesson?*

Changes in Jessica's teaching was also identified during a whole number lesson. As expected, before participating in AVMR Course One, very few manifestations of Math Recovery Guiding Principles for Classroom Instruction were identified during classroom lessons. The few identified fall primarily in the areas of Guiding Principal seven and eight. (*Teaching supports and builds on students' intuitive, verbally based strategies for the development of written forms of arithmetic and providing sufficient wait time for students to solve problems)* After participating in AVMR Course One, classroom observations revealed over twenty occurrences demonstrating manifestations of Guiding Principles three, four and six to be the most prevalent. (*Teaching is focused just beyond the "cutting-edge" of students' current knowledge, teachers select tasks from* *a bank of activities varying this selection on the basis of ongoing observations and engaging in intense, ongoing observation by the teacher and continual micro-adjusting teaching on the basis of observation*) Jennifer often brought in a new manipulative or visual referred to as a *setting* in Math Recovery to facilitate student connections during problem solving. During small group she also had notable success adjusting a task slightly and changing questions to keep a task at students' cutting-edge of knowledge.

The most profound change occurred after Jessica's participation in AVMR Course One. Following AVMR Course 2, no change was found in the presence of Guiding Principles during a whole number lesson. Guiding Principles three, four, and six provide evidence for the lasting impact AVMR Course One had on the ways Jessica was able to focus instruction on students' cutting edge, exercise professional judgement in selecting from a bank of teaching procedures including instructional settings and tasks, and intense, ongoing observation resulting in adjustments or fine-tuning within a lesson based on these observations.

Research Question 3. *How does Add+VantageMR® professional development influence differentiation of a whole number lesson?*

After taking AVMR Course 1, the ways Jessica grouped students for short periods of time and modified an activity during a lesson changed considerably. Almost immediately Jessica implemented a math rotation system where she chose four types of stations including one station where she works with a small group of students on a skill in whole number. Activities chosen for the teacher-led small group and independent stations were of the type where they were easily modified by changing a number range or bringing in a setting or manipulative to support students' thinking during the activity.

Jessica used activities from AVMR Course resources, general teacher resource websites, the district website with activities matched to standards within the district adopted frameworks, and a district AVMR binder assembled by district math coaches and provided to each school. Again, the considerable changes Jessica implemented after taking AVMR Course 1 remained in place after Jessica's participation in AVMR Course 2.

Additionally, the settings Jennifer brought in for support coupled with questions posed to students as illustrated in the excerpts above provide exemplary examples of the way Guiding Principles 3, teaching is focused just beyond the 'cutting-edge' of students' current knowledge and Guiding Principle 4, teachers exercise their professional judgement in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of on-going observations. Guiding Principle 5, the teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies and Guiding Principle 6, teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of observation were not originally identified for Research Question 3.

Research Question 4. *How does Add+VantageMR® professional development influence teachers assessment of students learning during implementation of a whole number lesson?*

Additionally, the way Jessica was able to plan and enact tasks at the cutting edge of students' current understandings after participating in AVMR Course 1 was remarkable. She consistently engaged in on-going, intense observation when students were in small group and large group. She then responded by modifying the task through changes in questioning or presentation of the task. Jessica's use of differentiation changed after participating in AVMR Course 1 in part by her change to implementing small group instruction which allowed the opportunity for her to take the same lesson objective and similar tasks to make fine-tuned adjustments at students' cutting edge of current knowledge in the supportive environment of a small group. Again, these changes continued to be present in classroom observations after Jessica's participation in AVMR Course 2.

Each of the cases presented unique insight into what teachers can take away and implement after participating in AVMR professional development. While each teacher revealed noticeable change after participating in AVMR Course 1 or 2, the area of change differed in each case. Carole implemented a new task from AVMR Course materials followed by making adjustments to the task during the lesson which increased the challenge for students who needed the challenge. Mark implemented small group instruction and explored teaching two-digit addition with materials in ways he did not do before participating in AVMR Course 2. He was also able to meet students with appropriate challenge and choose tasks based on student assessment data. Jessica implemented small group instruction and was able to put into practice her vision for math instruction as a first year teacher. The strong presence of a variety of Guiding Principles revealed a remarkable impact by Jessica's participation in AVMR Course 1 and 2.

Implications for Action and Future Research

Very few studies have been conducted evaluating the impact of AVMR professional development. AVMR professional development is grounded in teachers learning about student thinking and diagnostic assessment in identified domains of whole number arithmetic knowledge. These domains comprise the Learning Framework in Number. The Learning Framework in Number highlights unique progressions within each domain and the interconnectedness of the domains when considering student learning across the domains. One possible explanation for the limited number of studies evaluating AVMR is the complexities involved when considering teachers' implementation of course content.

This study provides an example of how overarching theoretical principles within AVMR professional development such as the Guiding Principles for Classroom Teaching can be used as a lens to investigate fine-tuned changes in teacher practice leading up to and during a whole number lesson. The Guiding Principles were also useful to investigate teacher moves regarding planning for differentiating instruction and making differentiation decisions during a lesson. Finally, this study provided a look into what teachers attend to prior to acting on what they see when making informal assessment decisions. By examining one way to investigate changes in teacher practice after participating in an AVMR Course, the hope is this will lead to additional studies involving more teachers at a variety of grade levels. Additionally, Guiding Principles proved to be a useful lens to investigate teacher change after an AVMR Course. Perhaps additional studies could narrow the scope to concentrate on a set of interconnected Guiding Principles.

Concluding remarks

Teachers take away a great deal of information following an AVMR Course. Testimonials and course reviews collected by the US Math Recovery Council show a great amount of enthusiasm immediately following a course. However, putting course content into practice after the course can be overwhelming. As this study reveals, implementing ideas from an AVMR Course is indeed a journey over years of teaching. Continued learning experiences with someone having a great depth of knowledge regarding Math Recovery theory, assessment and teaching is required to support teachers in a journey of implementing assessment and teaching practices.

Study results have implications for US Math Recovery Council course development as well as suggestions for future development. This study reveals three unique experiences by three first grade teachers following AVMR Course 1 and 2. Two of three cases highlighted the importance of support experiences short in nature and targeted to a particular nuance of planning, enacting and reflecting on classroom practice. Results suggest it would be beneficial for implementation of Math Recovery teaching for US Math Recovery Council is to consider creating a variety of support options after a course.

AVMR Course content can have a profound impact on classroom practice. Like Jessica, teachers may readily accommodate much of the learning about assessment and teaching into classroom practice without a lot of additional outside support. This may be contributed to Jessica's time in her teaching career. As a new teacher, Jessica has the most to learn and potentially could be more open to incorporating practices from professional development. However, this could also be attributed to the fact that Jessica was the only teacher in the study without previous AVMR course one experience. For others like Carole, AVMR courses may create disequilibrium on held beliefs regarding the foundation of math concepts and the assessment and teaching of a conceptual understanding of mathematics from a constructivist perspective. In this case, some classroom change occurs immediately after a course, but in order for there to be lasting impact on classroom instruction, additional professional development experiences are needed to support teachers in implementing small changes in the classroom with the aim to impact beliefs about teaching and learning of mathematics. Still other teachers may be similar to Mark and need additional learning experiences to make changes to a particular element of classroom practice such as implementing small group instruction or implementing a warm-up routine.

Professional development is a long-term investment into the knowledge of a teacher and an investment that makes a difference in classrooms. The three cases presented in this study reveal three unique ways first grade teachers implemented learning from AVMR professional development. Through the stories of these three teachers, school leaders and future researchers gain insight into the level of impact of AVMR professional development. The next section includes limitations of this study and implications for future research.

Limitations and Future research.

This study provided a lens into three teachers' experiences implementing Math Recovery assessment and teaching practices after participating in Add+VantageMR Courses through a case-study approach. Analysis of results provide valuable insight into implementing course ideas, however, it is important to include limitations of the study. For each case, the classroom environment was described and considered holistically. However, school factors such as principal support and engagement, were not considered in detail. Additionally, three of five classroom teachers making up the first grade team participated in the study. Having the full first grade team participate in the study could have afforded the opportunity to examine the role of collegial support for individual teachers implementing AVMR course assessments and teaching practices.

This study is an initial study of teacher change following AVMR professional development. This study identifies areas for further investigation such as extending a study to upper elementary grade levels. A study involving upper elementary grade levels would allow for further investigation of teachers' application of AVMR course two content, place value and multiplication and division concepts. Another consideration for future research is to have classroom teacher participants administer assessments to students in their classroom. It is possible that administering assessments to students would impact teachers' understanding of the models within each topic and the ways teachers apply learning from AVMR professional development into their classroom. Finally, further research can focus on one or two elements of a teacher's practice based on findings in this broad view of classroom. For example, this study investigated the impact teachers' participation in AVMR courses has on mathematics planning for a whole number lesson, enacting a lesson, differentiating a lesson and teachers' in-the-moment assessment of student math knowledge. A study narrowed in focus on one of those areas would provide the opportunity for in-depth analysis.

Appendix A – Guiding Principles

Guiding Principles for Classroom Teaching

Principle One

The teaching approach is inquiry based, that is, problem based. Children routinely are engaged in thinking hard to solve numerical problems which for them are quite challenging.

The inquiry-based approach to teaching number is sometimes referred to as learning through problem-solving, or problem-based learning. In this approach, the central learning activity for children is to solve tasks that constitute genuine problems, that is, problems for which the children do not have a ready-made solution. What follows is that the issue of whether a particular task is appropriate as a genuine problem largely depends on the extent of the children's current knowledge.

Principle Two

Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching. The latter refers to the teacher's informed understanding of children's current knowledge and problem-solving strategies, and continual revision of this understanding.

Assessment for providing specific and detailed information to inform instruction is the critical ingredient in our approach to teaching early number. It is essential to conduct a detailed assessment of children's current **number**

knowledge, and to use the results of assessment in designing instruction. In each of Chapters three to 10, the second section of the chapter contains detailed descriptions of assessment tasks and notes on their use. These have the explicit purpose of informing the design of instruction. The second aspect of this principle, ongoing assessment through observation and reflection, is equally as important as initial assessment.

Principle Three

Teaching is focused just beyond the "cutting-edge" of the child's current knowledge.

This principle accords with Vygotsky's notion of zone of proximal development, that is, instruction should be focused just beyond the child's current levels of knowledge in the areas where the child is likely to learn successfully through sound teaching. This principle is very important in our focus on the teaching of early number. The principle highlights the importance of assessment to inform teaching. Assessment provides the teacher with a profile of children's knowledge and the teacher focuses instruction so that children will be moved beyond their current levels of knowledge.

Principle Four

Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.

This principle highlights the need to develop a bank of instructional procedures and to understand the role of each procedure, in terms of its potential to bring about advancements in students' current knowledge. In each of Chapters three to 10, the third section of the chapter includes up to ten examples of learning activities which can be used to develop an appropriate bank of teaching procedures. Also, the second section of each chapter contains an extensive set of assessment tasks. These tasks constitute an additional source of instructional procedures because the tasks are easily adapted for instruction.

Principle Five

The teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies.

This principle highlights the need for teachers to have a working model of children's knowledge of early number and the ways in which children's knowledge typically progresses. In each of Chapters three to 10, the first section of the chapter provides a detailed overview of the development of an aspect of **early number** knowledge. Our belief is that teachers can develop an appropriate working model through reading, reflecting and observing, in conjunction with their teaching practice.

Principle Six

Teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.

This principle highlights the importance of observational assessment in determining children's specific learning needs, and the need for this assessment to be ongoing and to lead to action, that is, the fine-tuning of instruction on the basis of ongoing assessment.

Principle Seven

Teaching supports and builds on the child's intuitive, verbally based strategies, and these are used as a basis for the development of written forms of arithmetic which accord with the child's verbally based strategies.

This principle highlights that children's initial number knowledge is, by and large, verbally based, rather than involving written forms. Thus, we suppose that children's initial counting and calculating strategies mainly involve mentally computing with sound images of number words and number word sequences. The further development of number knowledge involves a gradual process of incorporating written symbols, including, but not limited to **numerals**, and linking these symbols to already acquired, verbally based number knowledge.

Principle Eight

The teacher provides the child with sufficient time to solve a given problem. Consequently the child is frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking and reflecting on the results of her or his thinking.

In our research and development work in early number learning over the last 20 years and longer, we have always emphasized the importance of sustained thinking and reflection for the learning of mathematics. The topic of early number learning is well suited to significant problem-solving by children. This problem-solving and the mental processes of thinking hard and reflecting during problem-solving are, we believe, a fundamental aspect of early number learning.

Principle Nine

Children gain intrinsic satisfaction from their problem-solving, their realization that they are making progress, and from the verification methods they develop.

This principle relates to Principle Eight. Our experience in working closely with teachers and students for many years on the topic of early number learning is that when young children work hard at problem-solving and their problem-solving is successful, this is typically a very positive experience for the learner. To go further, we argue that this kind of learning is a kind of cognitive therapy, having intrinsic rewards beyond such processes as teacher affirmation and peer recognition.

Appendix B – Interview Questions

Pre- and Post-Interview Questions

Vision of High-Quality Mathematics Instruction Instrument Interview Questions

Interview Questions:
I'd like to ask you a few questions about your view of high quality mathematics instruction.

Taken from:
Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education, 45*(5), 584-635.

1. If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?

*Notes to interviewer:

- Probe on depth/specificity of response until you understand what the participant describes (e.g., If a teacher says "student engagement," ask "Engaged in what?").
- Keep the form/function distinction in mind. Ask participants why they think ____ is important (e.g., Why do you think it's important for kids to work in groups? Why do you think it's important to hold a whole-class discussion?).
- If the interviewee talks about the structure of discourse (who's talking to whom and when), probe on content (and vice versa). If the interviewee says, "Teachers (or students) should be asking questions," probe to find out the kinds of questions the teacher (or students) should ask and for what purpose, as well as whether they conceive of discussion as happening in whole-class settings and/or in small groups alone.

   a. Is there anything else you would look for? (Ask BEFORE probing on the following issues.)
   b. What are some of the things you would expect to find the teacher actually doing in the classroom for instruction to be of high quality?
   c. What kinds of problems or mathematical tasks would you expect to see the students working on for instruction to be of high quality?
      i. Can you please describe a _____[use the word or phrase—e.g., "task" or "problem"— that the participant used for "task"] that you would consider to be of high quality?
   d. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
      i. Would you expect to see the entire class participating in a

single discussion, or would students be talking primarily in small groups?

Additional Post-Interview Questions

When did you take AVMR 1?
When did you take AVMR 2?
How long have you been teaching in district 196?
Which grade levels?
How long have you been teaching?

1. Prior to receiving the Add+VantageMR data, what were you using to assess your students' math understanding?
    a. How did you decide which math topics to assess?
2. What are the strengths of those assessment methods?
3. What are the weaknesses of those assessment methods?
4. Once you received the AVMR data, what did you do with the information on the spreadsheet?
    a. What did you think?
    b. What questions did you have?
    c. What do you need?
    d. What role does the participation in the course play at this moment in time?
5. How have you been using the AVMR assessment data?
    a. How would you like to be using the assessment data?
6. How does your knowledge of children's thinking impact decision making when planning for instruction?
    a. How does your knowledge of children's thinking impact decision making during a classroom lesson?
7. How often do you reference AVMR course resources such as the textbook, teacher handbook, or Math Recovery® website?
    a. If teacher references course resources, follow up with: How do you use those materials?

Appendix C – Classroom Observations and Interview: Carole

Carole: Classroom Observations and Interview

Classroom Observations Descriptions

September Observation One Students are getting settled. Carole uses classroom management techniques to get students seated and ready to learn. **Number Corner Time** Number Corner occurs for 20 minutes before going to lunch. A calendar is up and shows a pattern involving three colors and three bug pictures. Question: Tell me about Tuesday. What should Tuesday be? Teacher is pointing to today's card, which is flipped backwards to hide the color and bug. Student responds, "butterfly."	
T (Carole): A butterfly. Tell me your thinking? Student shares the existing pattern and thus the prediction for today's pattern. T shows the card and then asks students to show a sign if they also were predicting a butterfly.	Evidence of drawing out student thinking.
Pointing to popsicle sticks in a chart pocket - C asks, "What do I need to do to make this 11?" S: Add one.	Principle One.
T asks the class if they agree. Many students are showing a hand signal that they agree with the strategy of adding one. T is asking for other responses. T: How many do I have in the ones place? How many days	Showing agreement allows more students than the one student sharing to express their verbally based strategy. Links to Principle Seven.

in school? S: 11 T: Now we say combinations that make 11. Now we want to think about our combinations to 11 (a little fewer than ½ the students are raising their hand). Students are sharing equations that make 11 and T is recording the equations.	Principle Seven. The teacher is accepting student responses as combinations when they are expressions for 11. This is significant because knowing and working with combinations and partitions of numbers up to 20 is a grade level standard. There is potential here to facilitate this discussion in such a way that it would intentionally meet that learning outcome.
Lium: 5+6 (First combination of 11 shared). T: I want to stop and talk about this one. What made you think of 5+6? What was your thinking? Lium uses his fingers to show five and then to show five on the other hand. He counts the first five and then counts from 1 to show the six would be the five there and one more to make 11.	Principle Seven.
T: What number did you start with? Lium begins to explain. **T interrupts**: so you started at one? Another student shares counting on from five to explain 5+6.	*In contrast to* Principle Eight.
S: Seven plus, no, seven minus. T: **interrupts.** No I like the direction you were going… seven plus what equals 11? I like where you were going at first. Who can turn and help her? Another girl student shares: I was going to say 7+11.	*In contrast to* Principle Eight.

T: Ok… so let's think, how we can get from seven to 11. Who can help her? S: 7+4… T: 7+4, and how did you get there? Tanvi: If you do 7+3 that makes 10. It's just like one more to make 11. T: Did you guys hear that over there? What did she say? And how did she get there? What was her thinking?	Principle Seven.
Alaina: 8+3. T: What was your thinking? S: Using my fingers.	Principle Seven.
T took three more equations and two more combinations, taking student answers without sharing strategies. T: Turn and talk…looking at responses…Is anything missing? Anything we could add? Group is talking. Individual responses are inaudible.	
T is taking some responses from the partner discussions. Students are showing a hand signal for having a similar idea, showing they solved using a similar solution strategy. *** End of Number corner***	Link to Principle Seven.
Lesson – The lesson time is the last 45 minutes of the day. The activity is on from the District Curriculum (Bridges, first edition). Students are to spin a spinner to identify the dotted number on the page to trace. Numbers 1-4 are on a today's spinner. The number two is on the spinner twice.	*In contrast to* Principles Two and Three. This task is too easy for all students in the class.
T focused conversations on using the vocabulary **more** and **less**. Most students had more twos and then less threes. They graphed as a class to show which number was spun more or less often.	Guiding Principle Seven.
Students participated in the activity and then graphed the results as a group. Most students spend the math time watching the teacher and two-three students engage in the math. **September Observation Two (Picture day)**	*In contrast to* Principles One & Nine.

Number Corner Going over the pattern. Predicting the next day's card. Agreeing with each other's predications.	Link to Principle Seven.
T adds a stick to make 12. T: To make 12, we add one more. If we want to go back to 10, we take one away. One less is 10. Now we are at 11. One more is 12. So now we have a bundle of 10 and two ones. So 10+2 is 12.	Principle Five.
Getting responses for ways to come up with 12. S: 5+6. T: 5+6 is 11, we want 12. S: 6+6. Taking responses of combinations from students without asking for students to share thinking. S: Shares 12-6. T: we want to get to 12, so how about 12 – (makes a zero with fingers). Class says 12-0.	
T: We can never start with a lower number to subtract.	Mathematically untrue statement.
Classroom Lesson This is the same activity as yesterday, except with the numerals 5-8 instead of 1-4. The lesson for today is the numeral writing worksheet. There is a spinner with numbers 5-8, and when the number is rolled, the player traces a number on the board. The activity is from the District Curriculum (Bridges, first edition).	*In contrast to* Principles Two and Three. This task is too easy for all students in the class.
Talking about predicting what will be spun before spinning. Student notices there are two fives on the spinner.	Guiding Principle Five.
Doing this activity for eight or nine minutes. Then will do another activity.	
T is supporting students to be predicting and thinking whether or not they will have more of a certain number than T.	Guiding Principle Five.
When they come back, they will talk about graphing and share which numbers they have the **most** of.	
T: Is that a word we are familiar with?	

A student asks a friend to share what **most** means. Alana says a number that is more than the others. Another student notices there are two fives and two sevens on the spinner, and so thinks there will be more. Another student shares a pattern he sees in the spinner. T: Thank you for carrying over that pattern thinking from our number corner work. T: If we were to continue this pattern, how can we add to it? S: [Explains an idea]. T: How can we continue this one? What would come after eight? Student is not tracking her meaning….Then teacher puts the numbers of the spinner in a straight line: 5, 5, 6, 7, 7, 8. One student says five. T asks if you could continue to nine. Student says no. Students are setting up their spaces to do the spinning and tracing numerals activity. Students begin doing the activity. Students are working. T: Using vocabulary….Who is getting more fives? Who is getting more sevens? Who is getting more eights? OK, I will check in again in a couple of minutes. Students go back to working. Students are spinning and talking. There is little cognitive load present in this activity. The numbers are all tracing lines that are there. This is a very social math activity. Little cognitive demand. Students are doing the activity. T: [Ends activity]. What do you have the most of? Calls on individual students to share the number the student has the most of. T: This vocabulary is good for many… T: We are going to take another minute… so finish up your spins.	 Guiding Principle Nine. This is not directly related to a Guiding Principle, but would be a mathematical habit of mind to identify and extend patterns. *In contrast to* Guiding Principles One & Nine.

T: Stop and bring sheets to the rug.	
Graphing an example altogether. She calls on a student to share the number of fives, and then T colors in that number. Going from student to student, asking the number of that number and then shading it in.	
Then asking students questions, such as which number has the most. This number is less than. This number is more than.	Guiding Principle Seven.
Students go back to their seats and graph their own numbers.	
Students are talking and coloring in their graphs. Again, a social activity. Students are somewhat engaged. Students are not academically challenged.	*In contrast to* Guiding Principles One & Nine.
Some students have colored all the bars the same height, linking back to students who are spinning and spinning in order to get many of all or certain numbers.	*In contrast to* Guiding Principles One & Nine.
Students place papers in backpack to share with their families.	
END OF LESSON	
September Lesson Three. Tanvi discusses changing the pattern by moving the cards. T: I like that you are thinking flexibly. Tanvi moves the first row as an example.	This is not directly related to a Guiding Principle, but would be a mathematical habit of mind to identify and extend patterns.
T: Does anyone want to add onto Tanvi's thinking?	
Next student shows a pattern of numbers counting backwards.	
T: Who want to show an addition to Cole's thinking?	Again, not directly related to a Guiding Principle, however, worth noting as an effective teacher move.

Students are sharing patterns they see. **Teacher is focusing on the words students use** …. T: I like that word…I notice… Student shares a skip counting pattern.	Guiding Principle Seven.
T: What is between three and nine if we are skip counting by threes? Students are silent and Teacher demonstrates counting by threes as an example. T: What is between nine and 15 if we are counting by threes? Most students are finding this conversation very difficult to engage in.	*In contrast to* Guiding Principles Three & Six. The teacher is adjusting and fine tuning instruction, however, the adjustments are making the mathematics out of reach for many in the class.
T: We are going to work on our threes again.	
T: Remember this morning, when we were doing lunch count and we were counting by threes? T: We need to move on to something else.	*In contrast to* Guiding Principle Five.
T is giving high fives for all that work and thinking happening.	Link to Guiding Principle Nine.
Now they are working with 13 days of school. A student is called out for not paying attention. A different student speaks out on the behalf of that student to share with the teacher that she was counting what was next. Teacher asks the first student to share what she was doing, and she was figuring out with her fingers that 10+3 is 13. T: Now that is some good thinking. T apologizes for not realizing she was on task.	*In contrast to* Guiding Principle Eight.
As T is putting in another stick, students begin sharing equations for 13. Today, students are sharing combinations, and for first several responses, T is only taking equations. One subtraction and T applauds getting a subtraction equation.	Although this is not the opposite of a specific Guiding Principle, the teacher is providing positive feedback that is not mathematically sound for accurately identified student good thinking.

LESSON Doing a science and patterning lesson for math today. Lesson starts by playing a praying mantis song. Then they play again and sing along. This lesson is from the district adopted curriculum, (Bridges, first edition).	The learning here is focused mainly on science topics.
They discuss the meaning of the song…science vocabulary rich. Discussing the praying mantis life cycle.	Science content.
Students then complete the buggy pattern on their worksheet. Students draw the bug or write the word of the bug that comes next in the pattern. This is a part of the math unit theme. The number corner calendar is a bug pattern.	*In contrast to* several Guiding Principles. This worksheet is a very loose mathematics content connection.

November

Classroom Observation One. At the start of the lesson, the purpose appears to be using Unifix cubes to make a grouping. T: What is a grouping? Where have we made a group before? Alaina: Where they go together. T: And how do we make those groups? S: By tens. Tanvi: We can group by fives. Students and Teacher mention we can make groups of threes, fours, etc.	Guiding Principle Seven.
Students try to demonstrate groups of three by skip counting. The group is having a difficult time. T: We are at nine. How do we find the next group of three? S begins to share about counting. T picks up and supports counting on from six. T: We are at 12, where do we go now?	Guiding Principle Three. Guiding Principle Six.
T: Now we are going to change and work with six and seven. Not skip counting, but using strategies with skip counting. What do you notice here with this Unifix train? On the board there is a picture of colored Unifix cubes. The first one is three red cubes and four yellow cubes. T	*In contrast to* Guiding Principle Five. This is quite confusing mathematically. Students begin to share how to skip

engages students in a conversation about this one first.	count to determine combinations of six & seven. The teacher honors the student response and tries to build off of it. In this case, skip counting would not be the type of counting that would aid in this mathematical problem-solving.
Student shares three and four and then shares that there are eight blocks because she counted them. T: OK that is one strategy. What else can we do? S: There are actually 7. T: How did you count them? S: I put three and four together to make seven.	Guiding Principle Seven.
Going to a new grouping. T: Abdi, what do you notice about this grouping? Two red, two yellow, two green. Abdi shares that he added the twos. Another student chimes in that it is skip counting. T: OK.	
Students begin sharing about skip counting by twos for each of the groupings up on the board. T stops the class to draw attention to language of one student who said he was thinking in his head to skip count.	
T asks another student to share. This student is having a hard time sharing a specific counting and keeps telling T that he is thinking in his head.	
S: I counted up with the colors. Five blues plus one red to get six. T begins by demonstrating equations written horizontally and then rewrites the same equation vertically to make sure students know the two written forms have the same meaning.	Guiding Principle Seven.
The focus of this lesson is to have color Unifix cubes in groupings that would support students in writing number equations to represent the different colors in the Unifix train. They are doing these as a class, and then they will	Guiding Principles Four & Seven.

build Unifix trains on their own. Students are to build a train of six or a train of seven using two colors. The examples have two and three colors. Students are choosing whether they want to make a train of six or seven. T holds up two trains: a train of six with two colors and a train of seven with three colors. T sets out a box of Unifix cubes for the six group and then a tray of Unifix cubes for the seven people. Everyone has two minutes to make one train.	
Students are using colors, but the colors are not being placed next to each other to form groups. Students do not seem to be doing the activity as intended.	*In contrast to* Guiding Principle Four. This lesson seems to be off focus from the intent of the lesson, which I think would be to make combinations of six and seven using two or three addends. It is hard to see the real focus of this lesson during the time…..Not sure what student learning is intended by watching the lesson as it is unfolding.
Students bring their trains to the rug. T: So, some of you made six. Some of you made seven. Some of you did not listen. I asked you to use either two or three colors. Ok. T holds up a multicolored train and says, "This is more," and hands it back to the student it came from. T: You and your partner are going to make combinations. You partner with someone who has six or seven. T shares an example and says, "Hopefully, your partner has something different." T is making partners. The teacher decides not to pass out	*In contrast to* Guiding Principle Nine. This publically embarrasses the child.

the worksheet tool to accompany the lesson. T begins helping a partner group. Students are needing the practice with writing numerals the correct direction independently. Several students are off task, just sitting with the materials.	
T comes up to me and knows this lesson is not going well. She shares that she should have differentiated this lesson.	This would have been a very appropriate lesson, meeting Guiding Principles One & Four, had the opening discussion been connected to the thinking students needed to engage in to do this task independently. *In contrast to* Guiding Principles Four, Six, & Seven.
T stops the class and showcases that Tanvi and her group have put their seven and six together to make 13.	This changes the intent of the lesson.
Some students are just writing any true number sentences they know. They do not have cubes nearby.	*In contrast to* Guiding Principle One.
Another group is working with several colors together to make 13.	*In contrast to* Guiding Principle One.
A student comes up with 7+6+7 =20. T draws attention to doubles and tells the student to go back to his seat and do 7+7+6 to keep the doubles together. T asks the student to work that out.	*In contrast to* Guiding Principles One, Five, & Seven.
T is working on supporting him to explain his thinking, except that she has inserted a different strategy for him to use and explain instead of supporting his strategy. Putting materials away. No group debrief today.	Student is confused. *In contrast to* Principle Nine.
November Observation Two. There is an outside observer in the room today. Colored Unifix cubes are on the board. There are several	*In contrast to* Guiding

equations on the board as well. Students need to match the number sentence with the colored Unifix train. T is calling on students and saying, "Talk about this one." The expressions are not identical to cubes. For example, there are five red and one yellow cubes connected, and that train is supposed to match the expression 1+5. A student notices this.	Principles One & Three.
Talking about true and false. True means it **is the same as** and false means it is not the same as. 7=1+5 is shown on the Smartboard. Student shares that this equation is false and shares her thinking with the class. T is unveiling a second number sentence and then calls on a student to share whether the equation is true or false. The teacher then asks that one person to share their thinking to support the student response of true or false. A student notices a pattern in the answers. Jun is back from taking a break for a behavior issue earlier. T makes an equation for him.	Guiding Principle Seven. Guiding Principle Seven.
This lesson is very prescribed to this point. It goes step by step and is controlled. Few students are participating in the mathematics and engaged in finding solutions.	*In contrast to* Guiding Principle One. Students are not provided an opportunity to engage in hard thinking about the mathematics.
Now students are moving to a new seating position on the floor to be facing a chart that has sets of Unifix cubes grouped in colors sitting in the pockets. T addresses that this is easier when all colors are together. T ties that into a learning from yesterday. Tanvi is called upon first and she shares about several of the trains and then moves the trains around to be a pattern, instead of keeping combinations of six in one column and then combinations of seven in another column.	
T: We want to think about what we did today [referencing	*In contrast to* Guiding

number sentences earlier]. We want to make a number sentence.	Principle Five. Unfortunately, the work earlier does not connect to the number sentences created here. This is matching a picture; the other was true false statements using only bare number.
T is writing a number sentence on a Post-it note to put into the pocket by a colored train. T picks up a train and asks a student to share what the number sentence would be. T writes that on the post it.	*In contrast to* Guiding Principle One. Only two or three students are engaged in the mathematics during the lesson. The rest are watching what is happening and may or may not be thinking about the mathematics in their head.
T brings students focused over to SMART Board. This time there are pictures of Unifix cubes that are outlined and colored in combinations to make six using two colors. Then at the bottom there are true and false statements for students to circle the answers. T ensures students understand how to complete the worksheet and then students go back to their seats to complete the worksheet.	
Students are getting started on the task right away. They are engaged in completing the worksheet. I do not think students are too cognitively challenged by the worksheet.	*In contrast to* Principle One.

December

Lesson preceding observation one. Students played the activity *one less* with a substitute teacher.	This activity comes from AVMR Course One.
Observation one.	
The focus for today's lesson is to playing the *10 more* activity	This activity comes from AVMR Course Two.

The teacher introduces today's lesson as building flexibility by going back and forth from addition and subtraction. Teacher begins to demonstrate how to play. She rolls a five. Multiple students in the class reply that 10 more would be 15. Teacher is writing the equation vertically. Rolling again. T: This is what you would do with a partner. Students are excited. There appears to be an appropriate amount of challenge in this activity. Teacher calls upon a student to share their strategy for solving six and 10 more. S: I just know that when you have six, then 10 more is 16. T: Sounds like you just know this – that you know these combinations. The teacher rolls a one. Many students are very excited and yelling out 11. Students are excited and very engaged. It appears as though the students who are responding all know their 10plus. When called upon, a student shares 10+1 is 11. A student shares how you could use the number line to figure out the answer. She says you could go to the 10 and go one more to figure out 1 + 10. The teacher highlights for the class that it is a great strategy to start at the bigger number. She started at the 10 and went one more. Using same partners that were assigned yesterday, students go to their seats and find their partners to begin today's activity. Partners are engaged in playing and trying to block their partner. Group of students are counting on. In another group, one student keeps blurting out the answers, and finally, his partner says that he wants to figure out the answer first.	Guiding Principle Three. Guiding Principle Nine. Guiding Principle Three (for a reasonable number of students). Guiding Principle Nine.

Students are playing and engaged in doing the activity correctly.	Guiding Principles One, Three, & Nine.
The teacher stops the group and says, “I have been around to every group, and it sounds like you know this game pretty well. This is your challenge; you can do it two ways. I am working with Cole and Hudson. They rolled the die twice, instead of just once, and added those together, and then added ten, or you can do what Hudson did. He rolled two dice and added them together and then got nine, so 19, and there is not a number 19 on the board, so he subtracted a number to get to one of the numbers to cover available on the board.	Guiding Principle Six. *This is the first that this Guiding Principle appears with this amount of teacher intentionality.
Another group is trying to add and then subtract. I cannot hear the students’ adapted way of playing the game, but the teacher says that it is more challenging than what was just shared and the pair can decide which way to play.	
The teacher is going around the room to encourage students to play using rules that would provide an appropriate challenge. The additional challenge is having students roll a die more than one time.	Guiding Principle Six.
The problem here is that the numbers represented on the board do not match the change in rolling of the dice. The teacher then suggests students identify a number to subtract to get to a number in the board. This is a missing subtrahend problem.	*In contrast to* Guiding Principle Six. *Although, the mathematics is adjusted to provide a more cognitive challenge
[See video 12-18-15v2 for the following] Students are using fingers to subtract. Students are really struggling or thinking hard with subtraction. This is adding in a challenge that it too big of a leap from the original game.	*In contrast to* Guiding Principle Three.
	The idea to differentiate is good as students were very successful with 10 plus combinations. However, the adjustments chosen changed the activity, thus making the activity too difficult for students. Teacher is trying to make connections with addition and subtraction with

	students when the students are not yet at an understanding of subtraction to be able to work with the non-count-by-ones strategies she is trying to support students in using.
At the end of playing the game the teacher comes back to the large group and talks about coming back to this game next week and having different ways of playing the game using different numbers.	Guiding Principle Six – Intense observation.
At closing, the T asks for students to share anything else they learned from the day. One student shared a strategy that was just shared. The student is describing thinking regarding how to figure out the jump back from 19 to 16. The teacher records 19-16 = 4 on the board. That number sentence does not represent the solution being shared by the student.	*In contrast to* Guiding Principle Seven.
The teacher begins to share a number line strategy where you could start at 19 and think about what gets you back to 10. T draws a jump back from 19 to 10 and then a jump forward to 11. The T draws nine for going back, but then adds the one to the nine to say it was a jump of 10 to get from 19 to 11, instead of the correct answer of a jump of eight. The teacher says to the class that she got mixed up on that one and that she is learning right along with everyone. T then moves to say that kids can win by going vertically, horizontally, or diagonally on the board. Then she transitions to literacy.	*In contrast to* Guiding Principle Seven.

Interview Transcript: Carole
Interview 1 – September 3, 2015

I: If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?

C: Student engagement and their response. Where the teacher then leads those conversations, so they're having a lesson on addition, what are they setting it up, where are the kids coming from - that prior knowledge and where's that lesson going? Are they having strong conversations? You know, or are all the kids kind of sharing? Are they pair sharing and adding to the conversation? It doesn't just stop with "that's the right answer." You're continuing. You know, where's the thinking behind where they're going?

I: What would children be engaged in?

C: The conversation and then the activity. So then it would lead into an activity. You know, if it was manipulatives or a story problem that they're…they're moving their pencil, they're moving their things, showing their neighbor, they're showing they're excited about it.

I: What are some of the things - you mentioned discussions and talking and moving those discussions along.

C: Sure.

I: What might those discussions look like? What might they sound like? What might they be about?

C: I know in the past I've had them even during the lunch count, so adding up, you know, who's having hot lunch, who's having, you know. And then is there a better way to get there? You know, if you seeing them doing this, is there a better strategy? So talking about those, charting those strategies. Do you have a different way to arrive at the same number? But what's the best way to get there? So, I always try to talk about that best strategy that you're using but never saying that it was wrong, you know, so you don't want to discourage. You know you don't want your EL student to say, "but I had to use my fingers," you know, I had to do this because they are not all going to be yet at that same place, but making sure that they're seeing there could be a better way.

I: Why is that important?

C: Um – well, you don't want them already giving up when it's just beginning. And even at the end of the year, you want it to continue their passion for math for the language with math. And just how they view that math is all around them, you know, like [laughing]. That's why we start with different things, like when we're walking down the hallway looking at the different tiles, or how many are in groups, the grouping part. Um, so just identifying and making the most out of every opportunity, I guess.

I: What are some of the things you would expect to find the teacher doing in the classroom for instruction to be of high quality?

C: That there would be the vocab maybe that they are experiencing, the language available, the words, modeling. Like, in our number corner area, counting the days. So, having those things they can see visually. Making sure you have the right tools for that, especially in first grade, with that place value, such a hard concept, so moving the ones to the tens, and maybe having them be able to even have them touch that. Um, definitely having the right tools, the visual and they are able to interact with whatever, you know, you're trying to explain. [laughing] Their attention span is a limited, right. [laughing]

I: You mentioned problems or activities that kids would be engaged in. What kinds of activities or what kinds of problems?

C: I know we begin with a lot of sorting, a lot of patterns, shapes - we deal with that in geometry. A lot of real life situations. There's a money component, telling time to that hour and half hour. That is all I can think of.

I: That is good… [laughter from both C and I] Follow-up a little bit for discussion. Um, would you see the entire class participating in a single discussion, or would students be taking their discussions into small groups? I think you mentioned a little bit about, that but just to touch on that a little bit more.

C: Ideally, [discussion] would probably begin whole-group, but then it would ideally move out into small groups, but at this age, um, you know, you are hoping you are still hearing those conversations in their small group. And a lot of parent volunteers, usually they can take a group that seemed to be - even if it was a high achiever or low, you know, they were able to kind of continue that work then. Yeah, I kind of like to move around and make sure that it is not lost in that whole group having those questioning [laughing] asking the right questions.

I: Who's asking questions?

C: Well, we are hoping they are. Yeah. And then…

I: Kinds of questions?

C: That's a good question! [laughing] How did you arrive at that answer? Is there another way to get to that answer? Is there a quicker way to get to that answer? Why should we get to this answer? How did you work as a group to get to this answer? Um, is there an answer? I don't know. [Laughing]

I: This is last….Conversation and talking. We have talked about who's talking and a little bit about when, but if there is anything else about when students would be talking and who they are talking to, but anything else you want to add about the content they may be talking about, or not necessarily.

C: At this level, it's grouped around topics that interest them so, you know, [bell rang] the farm, animals, oh penguins. We spend a lot of time on that. So it's integrated throughout our reading unit too. So whatever we are doing in math, you know, we are trying to tie it all in together. So the conversations are not just that math they should be throughout the day. Ideally, and some kids like one more than the other, but it seems they all gravitate towards at least some part. And maybe the reading does, and they see it in the math conversations, those story problems, "Oh, we are studying?" "Oh yeah," I can identify with… You know, the math is because we are studying that in reading, you know, so it's, like, yeah, I do know what that is. You don't have to spend so much time explaining the story problem.

I: Thank you very much!

Interview Two

C: So, I was trying to start, and these were the spinners we were trying to use, so I don't know if that…

I: And then these are really nice especially for subtraction too because it adds the structure.

C: A few kids actually ask for them.

I: Awesome.

C: Because, you know, I introduced them…

I: I need to run over….if you don't necessarily….

C: And then we ordered these from Lakeshore…

I: And then we ordered the dice…

C: You know, I have plenty because my mom has ordered over the years, but always confuse you need a ten-sided dice. Is this where our maker space guy should be making us dice on the 3-D printer?

I: Oh yeah.

C: Because there are ….like, 10 sides are cheap enough where you might want to buy just enough, but other kinds, because I know we are talking about making a before and after game, so it pinned kids into having to work over crossing the decuple, so then it's nice to have a dice that has a nine, zero, and a one. Two zeroes and two ones, because then you would pair it with, like, a tens dice. The tens dice would have 10 sides, just like the zero to nine, except that one has 10, 20, 30, 40, all the way up to 90. These, and one of these, and rolling them, and then you are always getting a two-digit number that is 50, and 51, nine, so either way, so if you got 50, then you would have to do 49, 50, 51.

C: OK.

I: And, like, if you ended up with 61, then you would have kids still need to do 60 and 62. There's still quite a few kids forgetting the 60…you know they go 61, and then 59. Then that would be…

C: And that was that before and after?

I: Yeah. There's more that are kind of like that too, or if you had another dice that is maybe just six, seven, eight, nine, maybe you want two sevens or two eights…

C: And then here. What was it again? Was it was zero to nine?

I: Oh no, zero, one, and nine.

C: OK.

I: It made me think about that we should have a list, so that as we develop new program, a list, here's some great dice just to have on hand, you know…

C: Exactly.

I: It could be really useful.

C: Because a lot of teachers like to be looking for things, "Oh I'm going to run to the Good Will, oh, over the summer, I could be looking at the Dollar Store…"

I: Yes, yes.

C: Buying something or creating something.

I: Yes – yeah, so it would be nice to start a list. Alright, we can start with ….one part is that same question from the fall, and then the other one, I have specific questions just related to the whole thing in general.

C: [laughing]

I: No, no, and I want to reiterate, it's nothing that you have to….and really, really feel free to be honest. The good, the bad, the ugly, whatever. Because that is the most helpful actually.

C: I don't know [laughing]

I: It really is. And that's OK.

C: I haven't had enough coffee. I might not ever interview people in the morning again.

I: No, no.

I: If you walked into any classroom how would you know that good math instruction was happening? What would you see?

C: Engagement for sure of students. Materials available, if it was applicable to the activity. Teacher interacting, moving around, making sure, maybe posting of learning that has been happening.

I: What about what would the kids be doing?

C: They would be interacting with numbers, maybe there's a story problem that is happening, um, working maybe with a group, you know, or a partner. They could be working with a teacher. Maybe there's a small group happening based on needs. Pulling aside a few or pulling aside one to show this is how they solved, can you share with Johnny how you got it. Kind of like a before, like an intro, activity, and then kind of wrapping it up and sharing out how did you arrive at what you were doing? And making sure there is solidifying that learning with what are you taking away, what is something new that you have learned? How could you apply this in another situation? There should be excitement, too.

I: By who?

C: By both? [laughing] By both. Depending on time of day. Just kidding. There could be a limit. They could know too, which materials they can use or not use.

I: The students?

C: Yeah and there should be observation. Oh good, they are using this strategy, or they have moved away from non-counting by the ones. They know how to look at a number together, and is it best for me to do it this way, or that way, or show my several different ways to arrive if it's addition or subtraction.

I: How about questions that might be present. Or questioning in general….the kinds of questions?

C: That they would be using?

I: Either – either.

C: What was your strategy, or what was your thinking, um, could you tell somebody else? Could they solve it the way you solved it? Would they arrive at the same way…..I keep calling Johnny - I don't have a Johnny - but how did Johnny get to that? Is there a more efficient way you could get there? Is there something that, um, you could share with your neighbor?

I: Alright - sounds good. No, I think … You talked about how kids might work together and the types of activities, you mentioned problems…maybe anything more about the kinds of activities. I know you said engaged in them - like the kinds of activities that you might see.

C: A lot of…. now lately, we have been doing more of these activities that are more of a game style or a partner style. It could be where they need to be solving on their own, so independent work, so really kind of a mixture, depending on where students are at.

I: Thank you. First ones are really easy. They are just to make sure that I know I have asked them

I: Before receiving any AVMR data, or just at the beginning of the year before the study started, what were you using to assess the students' math understanding? What kinds of pieces were available and that you would use?

C: From our curriculum there was a general piece based on forward number, backward sequencing, more and less as well, so it was a paper thing that students did individually, then there is a few students, based on how their reading assessments went, I had because there was a lot of reading by the directions that, I just sat them by me, so I kind of worked with them individually. So it was a lot of, do they know their numbers, do they understand forward and backward? Could they order numbers anywhere on the number line, and then the more or less, what's 10 more 10 less than a certain number? What's the decuple? And that was something from Bridges.

I: How did you decide which math topics to assess? I think I hear the curriculum had this resource, and the topics assessed really was guided by the curriculum resource.

C: Pretty much….kind of as a team deciding about what we know coming from kindergarten what we want to double check.

I: OK, so in conjunction.

C: Are they coming in even knowing their numbers, and can they write them, and where should we go from …you know, our first is that we need to be able to count to 50 in that first trimester, forward and backward, so understanding can they do any of that in relation to anywhere in the number line too, up to 50.

I: So district guides your frameworks, also in conjunction with the curriculum resources, in conjunction with the team, is that correct?

C: Correct.

I: And topics just mean as right…

C: So pretty basic right at the beginning.

I: What are the strengths in those assessments that you use?

C: It is helpful to see quickly. It's quick assessments that you just see quickly - Johnny can't, or he has lots of referrals - seven & nine - so we have to get that turned around…..he actually in literacy, has the b and d….so you see some connections, you know, the reversals is pretty huge that first…I comment in the report cards too, working on four, they are just writing, but it seems to just work its way out at the end of the year.

I: It does, it does.

C: Parents don't, don't get too worried, and then just knowing what to talk about in number corner quickly there too, as we are talking through number of the day and things that the students, so those conversations to have, it's those…

I: What are the weaknesses of those assessments?

C: Again, sometimes I think there is a lot of reading to it so it is driven on some of that ability for the students. A lot of times, we are not seeing the strategy they are using if they are not right in front of me, when it is not isolated. I am being more with that couple, because they cannot even read what they're supposed to do, and I cannot see the person, you know, using her fingers, or yeah……a lot of the teacher observation is gone. Truly, like, we just do it like reading, where it is a one-on-one thing, but then that would be back to the AVMR, which you would do.

I: So then, the next questions relate to that a little bit. Once you finally received the spreadsheet of data [laughing] What were you…What did you do with the information? How was that information helpful? This question is imbedded in a larger conversation about just getting the data being useful if you haven't done the assessments? Is getting the numbers useful at all? Some of these questions are, was it useful just getting the numbers, and there is this whole other piece about having the numbers is one thing, but then knowing what to do with them is a whole other thing. What did you do with the information? Like, honestly, what did you think?

C: Sometimes it solidifies what you have already been seeing and the progression with where you started at the beginning of the year. There was a couple of surprises, like, oh, they are not understanding structuring to this, but they could do another area, they were really strong…..it is when it is not consistent with the student and you knowing them.

I:

C: I think that is what threw me off with the Liams…the same name, but I know Liam W. is this high reader and all this, and the other Liam is at grade level, but it seemed like it was the data matching…Liam G. is stronger in math. I don't know, I just…still understanding…I feel like, cross content, they should be similar…

I: [laughing]

C: Which isn't the case, because I have my Blake by video tape, is high in math but below in reading. Maybe because [in] certain assessments, he didn't do well with the curriculum on, because of the reading…

I: Ah, yes…

C: I don't know where I am going with that. The data was helpful, it's the next step, it would have been nice to talk about where to go with that.

I: And that leads into the next. What questions did you have? What did you need? In that in-between time…

C: It's a tricky conversations of grouping, and knowing do I pull the kids, and I am just doing that and just let the other ones go, and having but then wanting those other ones knowing and they still have stuff too going. So where's the critical right now this time of year with those red students.

I: Right, right. And how to meet them?

C: Yeah, grouping and meeting their needs. Like, do I just pull some out…..Because we have done this 10 for 10 program … 10 kids for 10 minutes.

I: Oh - I don't know that I have heard anyone say anything about that.

C: It's based on the AVMR just number and numeral assessment in first grade. Somebody last year, it has been very hit or miss, but based on that data, someone would work with them for 10 minutes. Pick your two from each class and they would go and

work with someone on different activities to get them where they needed to be. Someone trained to understand that data it hasn't been consistent though.
I don't know where I was going with that. But back to the data stuff, and what to do with it
Or do you just meet the whole needs with a whole group activity, like sensing…..they are all here, with those ones I just showed you to be working on this to be stronger, and then come back and see if by doing these activities they have been pushed as a whole group. Because they are working with someone that is a little bit higher, even though I do not purposely sometimes they seek that out. Or just…I don't know, they seek it out.

I: They do that a lot with their peers. Where there any other pieces that you needed. There were questions that you had… that opportunity to ask questions, look at it, what does this mean?… maybe seeing the actual assessment or you gave it to us… or to look within the assessment, why did they miss this little part? They got all of this… kind of how we analyzed during the class?
Got it all the way through here, but they didn't get it here, and this section, and so just kind of understanding truly the levels, maybe…

I: What do you need to really understand those levels? What role, if any, did participating in that 2nd course… help in interpreting, understanding the data? Was there anything in particular from coming to that course that helped you understand what to do with that data? Or not necessarily, because I know the topics are different, but there are some processes that may or may not be helpful.

C: I think what I took away form it was just that we, or I, had never thought of multiplication, that kids were maybe already doing it or understanding it, like, or the understanding of place value in a different way - I thought I understood place value, that it was more than just, I don't know, I think I am still trying to process a lot of it, like, it's more than… [laughing] There's just a lot that you can do with it, and I want to do it, and it is just doing it, I think I want to understand the data. I want to get them to the next level. I want to take from the class the different things and then work with my team, alright here's this stuff it would help them get here, let's meet this next benchmark you the 120. Um, the practical side you know…

I: I want the good, the bad and the ugly.

C: You are right, I have to have this stuff out in front of me….I have to take everything out of the bag and have it in front of me, because if I leave it in the bag…
I got to get the tools out. That is why I am starting here, ok, I wanted to do that empty number line. I want to get that up there. I want to talk about that stuff, you know I want to.

I: So it may not have necessarily been helpful in because the topics were different.

C: Yes.

I: So, the data you received at the beginning of the year were different topics covered into the course. So it didn't help in any way connect?

C: It helped me understand where they need to get. You know, when I talked to Katie, you know, the third grade, or the strategies not happening in second grade, you know, it helped me think about where they need to be going, or when they arrive, what would be ones that would be helpful to them, um, so I put those couple of…

I: At the very end, it helped to know where they are headed, but not tying into the data piece from the beginning of the year that was first grade, just the forward, backward, numeral ID, addition, subtraction was there.
OK, that makes sense.

C: But it was a critical thing to talk about the structuring, and that needs to be solidified, and that needs to be the foundation, to go back and really revisit that, and making sure that is strong to support the multiplication, you know, what's coming up for the addition and subtraction with two digit. You know, that's important to know that needs to be solid, so some activities, be doing, you know, quickly with them already, or…

I:

C: Or just the language.

Appendix D – Classroom Observations and Interview: Mark

Mark
Classroom Observations Descriptions.

September

Classroom Observation Set One: Lesson One Finishing Science observations prior to commencing mathematics class. They are preparing for a research station. They will be getting butterflies and observing behavior as they form a chrysalis and then emerge as butterflies. This is during the scheduled start time communicated for math class. Student Yoga to prepare for math class. Mark offered a choice for the math activity today. Students chose to play Buzz from his selections for the short time remaining for math class. Starting Buzz for math class. Students are getting into a circle to prepare for Buzz.	10 minutes.
Count forward from 19 to 25 (choosing to cross the 20 decuple). This is a fairly easy task for several students; however, some find it difficult to name the sequence 19, 20, 21 within the whole sequence played during the activity. When students are not sure of the next number in the sequence, they guess numbers until they say the correct number coming next in the sequence. Then towards the end of the game, Mark gave students five seconds to come up with the next number, and if they did not, then they sat down. Some students are not sure of next number, and some students are not listening to the number said just before their turn. Kids showing behavior signs that they are having fun with this game.	Guiding Principle One. Guiding Principle Two. Guiding Principle Three. Guiding Principle Five.
The girl who won learned the sequence during the game. She is doing a celebratory cheer.	Guiding Principle Nine.
Next game is counting backwards from 22 to 18. Mark says an example sequence.	Guiding Principle One. Guiding Principle Two.

There are no visuals to accompany the sequence. Students are getting “squirrelly.” Game commences again by continuing the backward sequence 22 to 18. From the start of this game, Mark employs the countdowns from five seconds for response time. This is a much more difficult task for students. Mark says, “This is where you learn from your friends if you are not sure what the number is.” Game ends because students are not being kind to their friends. Students are talking and dancing in their spots. Focus is lost. Students stack chairs to prepare to go home.	*Attempts Guiding Principle Three.* Guiding Principle Five. *Including a numeral roll, numeral track, or number line would provide students with support to employ a different strategy than guessing for figuring out the next number in the sequence.* *In contrast to Guiding Principle Eight.*

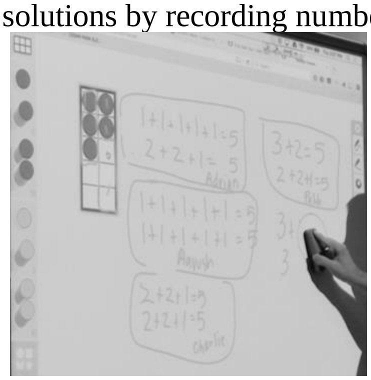
September Set One.
Lesson Two.

Classroom Observation Set One: Lesson Two.	
Mark has a vertical five frame with blue and red dots along the left to use to refill five frame presented on the SMART Board.	
To start the lesson, Mark places three red dots in the five frame. Students are using a personal whiteboard to record responses. Once students record a response, they hold the whiteboard up in the air for Mark to see the responses. Mark poses the questions in reference to the five frames displayed on the SMART Board: How many dots do you see? How many are missing?	Guiding Principle One. Guiding Principle Two. Guiding Principle Three.

Mark is providing students with ample wait time for students to figure out task.	Guiding Principle Eight.
Mark takes student responses for how many dots are seen, then records the numeral 3 on the board. He continues by asking, How many are missing? Students reply two. Mark records the addition sign and then writes the numeral 2 followed by the equal sign. Then he asks students, "How many altogether?" Mark calls on one student, who replies five.	Guiding Principle Seven Guiding Principle Three.
Erases boards. Now Mark places a blank five-frame vertically and places one dot in the frame. Mark: How many do you see? How many are missing? Provides students wait time. Students hold up their white board for Mark to see Mark: Emmanuel, how many do you see? Emmanuel: One. Mark: How many are missing? Emmanuel: Four. Mark: How did you figure that out? How did you figure out there were four missing? Emmanuel replies but is inaudible. Mark repeats while pointing to the four empty boxes, "So you counted these first, and then one more." Mark points to the one dot provided in the frame. Mark: Did anyone else look at it a different way? Everett: I looked at two boxes and then another two boxes and put them together. Mark: And what did you get? Everett: Four, and then put the one and mixed it altogether and it was 14. Mark: 14? Oh. How many were missing did you say?	Guiding Principle Eight. Guiding Principle Six. Guiding Principle Seven. Great example of connecting student responses to the visual frame. Great example of eliciting multiple student responses.

Everett: Four. Mark: And then how many did you see? Everett: One. Mark records a four on the board then an addition sign while saying, "So if I take the four and I add the one to it, how many do I have?" A different student replies five. Mark: So each time, what should the chips add up to? Group of students: Five. Mark: Five, because what is this group of? Students: Five frame. Mark: That is a five frame.	Guiding Principle Seven.
Mark places a new vertical five frame and places four red. Mark: How many do you see? How many are missing? Wait time is provided for students to figure and record responses. Mark: Alexis, what did you come up with? Alexis is responding, but inaudible. Mark: Did you hear what Alexis said? Mark then shares her strategy aloud. "I know if you have five and then you take the one away, then you have four." Mark then writes 5-1=4 on the board to correspond with Alexis' response. Then Mark records what Alexis has written on her white board, 4+1=5, and says, "What you explained to me looks like this [pointing to subtraction problem]. Mark: Did anyone figure a different way? Kaden. Kaden replies. Mark: How many did what Kaden did? He saw that when I put the dots on, I had 1+1+1+1 and that equals four, and then I have one left. Mark: Did anyone else do a different way? Maggie. Maggie: I used my fingers. Maggie puts up one finger for each red dot, and then has one empty and one finger down, and that equals five. Mark: Anyone else have a different way? [No response.] OK, erase your boards.	Guiding Principle Seven. Elicit multiple strategies.
Mark places up the next problem. "Who is up for a	

challenge?" Half of students raise their hand. M: We are going to go from a 5 frame to a 10 frame. He instructs students to close their eyes while he sets up the frame. Mark sets up the ten-frame vertically with a pairwise five pattern, meaning the frame is placed as two columns of five squares. The dots are placed at the top of the frame by filling in the four.	Extending the range.
M: How many do you see? And how many are missing?	Guiding Principle Eight.
Mark is providing wait time for students to solve and record on their white boards. Mark is walking around and supporting students individually for writing numerals the correct way and to solve the problem. One student is having a hard time creating a drawing on the white board that matches the ten frame on the whiteboard.	Guiding Principle Six.
Mark: Adrianne, what did you get? How did you see the five? And then what did you do? How did you count these? Mark is recording numerals to match Adrianne's explanation. Adrianne explains and Mark is having a hard time understanding her strategy, so he asks Adrianne to come to the front. Adrianne continues her explanation as Mark records her thinking. Mark is still questioning her strategy and looks up at researcher as he works to understand and accurately record strategy.	Guiding Principle Seven.
Mark: What did someone else do? Student shares that he counted them. He counted the five there by ones, and then counted the ones missing the same way. Mark records 1+1+1+1+1=5 to show the strategy of counting by ones for each of the ones there and then the ones missing. New student goes to the board to show how he counted by twos to get to five. Two, four, and then five. M: Anybody do it a different way? Pablo: I did three and two. Mark: What do you know about three and two, Pablo? Pablo: Three and two is five.	Elicit multiple strategies.

Mark: OK. Then what did you do? Pablo: Then I did two, two, and one.	
Mark points to the three red dots on the left and then two red dots on the right to identify and share with the whole class where the three and two came from to make five. Then Mark asks Pablo to come up to the front and take the pointer to show the class where two, two, and one came from. Pablo takes the pointer and is looking at the white board and appears to be thinking hard about the task of pointing to the	Guiding Principle Seven.
squares that he counted two, two, and one. Mark continues to rephrase the question. The multiple questions stated so quickly are making it difficult for Pablo to respond. Then Pablo points to the squares on the right and explains a strategy for coming up with five empty boxes that is different than the original two, two, and one. Mark addresses the researcher who is observing behind the video camera to clarify Pablo's strategy. With researcher assistance, the method for how Pablo got five empty squares by counting	Guiding Principle Six.
two, two, and one is determined. Mark then restates Pablo's strategy and represents the verbal solutions by recording number sentences.	Guiding Principle Seven.
 **Figure D. Pablo's Strategy.**	Guiding Principle Six. Guiding Principle Five.
Lucy shared her strategy of looking at three dots and then saying four, five for the other two dots on the right. She then did the same for the empty boxes. Once finished recording her strategy, Mark says, "Actually you counted on then." Mark: So did you see what Lucy did? Do you know a good strategy for when you are doing counting? It is a strategy called counting on. While describing the strategy, Mark is pointing to the dots to link the counting on strategy to the verbal and the dots. M: And counting on is a great strategy to use. Look how many just out of this ten frame, how many different	

strategies we came up with. OK, close your eyes – Oh, erase your boards first. At this time in the lesson, students are significantly losing focus.	
Mark is making a new pattern on the ten frame. This time, the ten frame is horizontal. Four dots fill in the top row from left to right. Mark: How many do you see? How many are missing? Students are working and thinking. Mark begins asking students about their strategies. Mark asks students to come up and use the point to share his strategy. Another student said he knows there are five across the top and minus one is four. Then he counted up the bottom row by ones. Mark: Caroline, what did you do? Caroline: Counted by twos. Mark: OK, come on up here and show us how you counted by twos. Caroline points to the four red dots to do 2+2=4, then she counts the bottom squares on her fingers by one. Mark links that part of her strategy to another student's strategy. Everett: I knew you had two fours, and then I knew two and two and two make six. Everett comes to front to show his strategy. He knows the top is five and take one away is four. Then he points to two empty squares at a time to show the twos. Then Everett changes midway through his explanation to say three and three empties make six. Mark represents verbal strategy with number sentences to match verbal explanations. Emmanuel comes up to show his strategy. He determined four dots in the same way as others before. He had a new strategy for empties, wherein he looked at the five empties	Guiding Principle Four. Guiding Principle Eight. Elicits multiple student strategies. Guiding Principle Five. Guiding Principle Six.

on the bottom and went one more to make six.	
Lesson ends.	

November Classroom Observations.
Lesson One.

Mark is going over a worksheet for students to work on during math time. The worksheet has three sequential numerals in the range of teens to over 60, where each sequence has one missing numeral placed randomly on a page. There are a total of about 10 sequences in all.	Guiding Principle Four. Guiding Principle Three.
Mark begins to go over the instructions for the back side of the worksheet. On this side is a color by number picture, where students solve the equation within an area of space to determine a sum or difference. Students then match the sum to a chart showing the color to color in the space of that sum or difference.	
Mark and students are having a conversation regarding symbols for addition and symbols for subtraction, along with vocabulary used with each of the symbols. Mark and students are solving two problems together as a group, one addition and one subtraction.	
Mark is going over rules for working independently while working with students at the table. Mark is also guiding students in roleplay for how they can help their neighbor with a math problem. He is guiding students in how they can avoid stealing their neighbor's math thinking. Mark is passing out worksheets for students to go back to their table spots to continue working.	Guiding Principle Eight.
Mark talked with the researcher at this time to explain that this week is the end of the first trimester and conferences are coming up. There are also expectations to complete a certain amount of reading assessments one-on-one with students. He feels badly, but the reality is that the lesson needed to be something he could quickly put together. He chose activities working on before and after, as well as addition and subtraction indicating that he made the best	Guiding Principle Four. Guiding Principle Two.

choices he could even though he was not proud of pulling a worksheet.	
For the remainder of the class time, students are working on the worksheet. Students are able to employ their different strategies to each of the worksheet pages. Mark provides numeral sequence support for two students whose AVMR assessment data indicates their current knowledge is number sequences from one-10. The numeral support for students is a hundreds chart with every other row shaded gray. Mark sits beside students to do individual instruction on the number system in the ranges within the worksheet.	Guiding Principle Four. Guiding Principle Five.
Lesson ends.	

November Classroom Observations.
Lesson Two.

Today's lesson is an activity shared by the Talented and Gifted teacher in the building. The activity is called Q-Bitz, and consists of 16 tiles, each with a design on it. The object of the activity is to choose a card with a specific pattern and then to use the tiles to replicate the design shown on the card.	This lesson does not have a number focus.
Mark is sharing his own thinking out loud with the class to show how to go about replicating the design on the card.	
Mark asks students to share whether or not they want to work individually or with a partner.	
Mark shares how to work jointly with a partner so that both students are able to think hard about creating the design on the card. Students and Mark talk through specific scenarios involving working with a partner and how to problem solve in the partnership. Students receive their own puzzle and choose to work with a partner or alone. Students work for the remainder of the class period.	

November Classroom Observations.
Lesson Three.

For today's lesson, students are going to play Treasure Hunt. Several students remember how to play Treasure Hunt from before.	Guiding Principle Four. Guiding Principle One.
Mark begins by choosing a student for a partner to help demonstrate how to play Treasure Hunt. Each game will be played using two rows, each with their own color. Treasure Hunt is a number sequence activity where the teacher can choose any number work sequence for students to practice. The sequences can be forward or backward. To set up the game, the card with the color dot is used to show the start of each row. Then 10 cards are placed randomly upside down in a straight line to the right of each color dot. The remaining cards are used for a draw pile. To begin play, one student draws the first card from the draw pile and places it face up in the correct position in the sequence. The student then picks up the card already in that position face down and gives it to his partner to place in its correct place in the sequence.	
Students and Mark demonstrate using a strategy to place cards in the number sequence once a few cards have been correctly placed. Students are helping the student demonstrating how to play the game to correctly orientate the numeral 10. Mark and the student are showing how to play with the sequence one-10. Mark starts to tell a student to pick a friend, and then he changes to assigning partners according to the current number sequence knowledge. Mark is passing out one-10 sequences, 11-20 sequences, counting by 10s, and 22-31 sequence.	Guiding Principle Two. Guiding Principle Three.
Students are playing the activity. Mark is playing the activity with a student who does not have a partner and needs support in number sequences from 22-31. Students are actively engaged in the activity.	Guiding Principle Nine.
Mark stops the class to have all students bring their voices down. Students continue to play. Mark continues to work with students in the class as they need help.	

Lesson ends.	

December Classroom Observations.
Lesson One.

Introducing an activity called Nine-plus. Mark introduces the activity as a Bingo game. The gameboard has numbers 14-18. The spinner has numerals 6-9. Students spin to determine the number to add to nine. Students then cover the sum on the gameboard with a colored chip. Two players share one gameboard and play using a different color. To win, a player needs to get three counters in a row.	Guiding Principle Four.
Mark and student are demonstrating how to play the game. Mark spins a seven, then asks the class which strategy the student can use to solve the problem. Students try to respond with the answer and Mark keeps the focus on answering the question, "Which strategy can you use to solve this problem?" Another student responds that the student can use her fingers to solve the problem. Mark responds, "And if you use your fingers, but if she goes one, two, three, she runs out of fingers. She's not going to use her toes." Student replies that you can use your partners. Mark: We could, but is there another way where we do not need to use our fingers. A strategy that would be easier. Lucy: Counting on. Mark: Yes, counting on. Mark demonstrates the counting-on strategy and some students join by counting with Mark. Mark thinks aloud how to make a choice on which 16 to place the counter	*Attempts Guiding Principle Five: Many students in the classroom are not yet ready to accommodate the counting-on strategy.*
Mark spins to have his turn. He spins an eight. Mark shares that he could use the counting-on strategy. M: What other strategy is there? Caroline. Caroline shares a strategy that uses the previous problem, 9+7=16 to know 9+8=17. Student player spins a five. Student player is thinking about how to solve 9+5. Mark provides her with time to think about her solution. Mark quiets the class to ensure she has time to think. She solves, getting the solution 14	Eliciting multiple strategies. Guiding Principle Eight.

and places a counter on the board. Mark spins a seven. He solves out loud, saying, "I know 9+9 is 18, and so two less is 17." He then covers a spot on the board. Student spins a nine. Mark quiets the class to provide wait time to provide student time to solve the problem. Student silently solves 9+9, responding 18. Mark then asks her to share her strategy (her strategy is inaudible in the video).	Guiding Principle Eight.
Mark discusses with the class that they will need to decide whether the winner of the game is the one who gets three in a row first, or if they play the game so that each player has the same number of turns. Mark asks for questions. Mark is clarifying how to win and what it means to have three in a row horizontally, vertically, and diagonally.	
Mark then shares that some students will be playing the Nine-plus game. Mark shares where to find materials to play the activity. Then Mark references his class assessment data sheet to record student names on the board that will work with Mark at the small group table. Mark is choosing partner groups to play Nine-plus by AVMR assessment data. Students are finding places to sit at the front table. Mark takes a moment to touch base with the support staff member working with a pair of students on Nine-plus.	Guiding Principle Two.
Researcher moves around the room, supporting students getting started on the Nine-plus game.	
Mark introduces a join change unknown word problem to the Small group: Sparky has six rocks. Coco has some more rocks. They have 10 rocks altogether. Students are not sure how to solve. Mark then puts Sparky + Coco on the board. He asks students how many rocks Sparky has and records that number under Sparky. Then asks if they know how many rocks Coco has, and students respond "No," so Mark records a box under Coco followed by the = 10.	Guiding Principle One. Guiding Principle Two. Guiding Principle Three. Guiding Principle Four.

A student responds that coco has 16 rocks. Mark then places 16 in the equation he wrote on the board to show that 16 would not make the equation true. Mark asks students what has to go into the blank to make 10. Mark asks students to erase the six and 10 in the problem and asks students to use the next set of numbers identified on the sheet. Mark is telling students to pick the set of numbers that is works for them. A set of numbers that is not too easy or not too hard. Numbers choices are in the two-digit + one-digit. He releases students to show how they would solve the problem on the paper. Students are recording the answer and Mark is asking that they also write how they solve the problem.	
Students are loud playing the Nine-plus game around the room.	
Some students in the small group are sitting and not sure how to solve the problem. Mark then begins to create a chart on the board to support students in how to set up and solve the problem. Mark is walking through the problem in a similar manner as the previous problem. Mark asks questions to walk through the problem and demonstrate how to solve. Mark is asking a student, “How many cookies does Coco have? How do you know?” Mark listens to the student explanation. Mark goes from student to student, asking what their solution is and how to solve the problem.	In contrast to Guiding Principle One.
Students playing Nine-plus are having a hard time staying focused with the game. Groups also need tools to support them in playing the game. Most students at the floor and not at the small group table are not yet able to use a counting-on strategy fluently and need tools to count all.	In contrast to Guiding Principle Three.
Mark is sharing that his problem in the small group is designed to help students see how to get from a two-digit number to a decuple. Mark continues to listen to students share their strategies for getting up to the decuple. Students are using counting-on for their strategies.	Guiding Principle Five.
Students are not engaged in the activity of listening to each	

other's strategies. Mark is connecting students' strategies to each other as he facilitates a small group strategy share. Mark now begins to record student strategies using written notation on the board once a strategy is shared using jumps of composite units. Previous strategies are involving counting-on from starting number to total. Mark is clarifying and expanding upon using jumps of composite units to solve problems similar to what students solved today for students in the small group.	Eliciting multiple strategies. Guiding Principle Seven. Guiding Principle Five.
Lesson ends. Students cleaning up to go home for the day.	

December Classroom Observations.
Lesson Two.

Opening of lesson: Mark flashes a dot patterns on cards showing a red pattern and a blue pattern side by side for student to determine the total of red, total of blue, and a composite total. He asks students if they want an easy or hard one. Many students say hard. Some say easy. He shows a card with one dot on the left and blank on the right. Then he shows a card with one dot on each side. Students share. Mark: How did you know? He flashes another pattern. Mark: How did you know, Caroline? He flashes a three on the left and three on the right. Mark: What did you see, Antonio? Did anyone see it a different way? Abby shares that she saw five and one. Mark has Abby come to the front to show where she saw the patterns. Mark: What did you see, Alexis? Alexis: I saw two, four, six. Mark asks students to raise their hands if they saw it like Antonio, like Abby, or like Alexis. Students raise their hand if they saw it like their classmate.	Guiding Principle Four. Guiding Principle Two. Guiding Principle Three. Eliciting multiple ways to see and group the dots. Guiding Principle Four. Guiding Principle Five.
Mark flashes a new pattern. First student says that there are a total of seven dots and explains strategy. Mark: Does anyone have a different way? Mohomad says eight. Mark places the card up for all students to see and share how they found eight. Caroline counted by twos. Another student saw six,	

and then said seven, eight. Mark asks how many saw as the students shared already. Then asks if anyone did it a different way. A students shares that he saw five and three. He demonstrates that he saw five in the red dots, and then Mark questions the class to see if anyone saw similarly, and to highlight the possibility of seeing five by looking at three red dots and two blue dots, and then adding on three more red. Another student shares seeing a four and four and comes up to show the class where she saw the two patterns. Mark flashes a five and four pattern. Students enthusiastically raise their hands. A student says they saw five and four to get eight. Mark stops to group solve five and four using finger pattern.	 Guiding Principle Five. Guiding Principle Four.
Now Mark is flashing an easy pattern on one side and a hard problem on the right side. He is giving students a choice at which card they choose to look. He then asks students to share whether they chose the easy or hard one. Mark: What did you see? How did you see it? Mark pauses to provide students thinking time before asking students to share strategies. The lesson continues in the same way. Students come to the board to share how they saw, grouped the dots visually, and found the total number of dots. Mark continues to ask for a variety of strategies and connects to each other's strategies. Mark's language and the language of students facilitates understanding addition of two addends.	Guiding Principle Two. Guiding Principle Three. Guiding Principle Eight. Guiding Principle Seven.
Mark transitions to a discussion about Nine-plus. Students are going to play Nine-plus again today. Mark is having a discussion about the importance of being accurate as they play, and not just trying to win	

the game. He then begins to demonstrate and have the class join in how to use two color counters as tools to solve the Nine-plus problems. Mark spins to get a six. Then Mark asks for students to share their strategy for solving. He calls on Sebastian, who shares a make-10 strategy for solving 9+6. M: So when you are playing the game, I can count in my head, or I can use my fingers, but if that is too hard, I can use these [points to 2 color counters]. Mark shares with students to place out six red counters to use to count on with. The group is talking about how to use the counters to determine a solution.	Guiding Principle Five. Guiding Principle Four. Guiding Principle Three.
Researcher intervenes to share placing out the nine white in a five-wise pattern first, and then placing the red chips in a six dot pattern to show the two addends in a pattern.	
It is close to time to go home. Even though there is just a short time to play, Mark begins to pass out materials and create partners.	Guiding Principle Two.
Students set up quickly and begin playing Nine-plus. They are focused on the mathematics today. Mark brings three students to the small group table needing to have the nine and number spun placed out with white and red counters.	
Mark is working with students at the small group pattern and leading discussions with students about the nine and six pattern. Students are solving. Groups are self-differentiating. Some have chosen to use counters, others using fingers, and others using composite strategies. One more spin before class ends. Class ends.	Guiding Principle Three. Guiding Principle Nine.

December Classroom Observations.
Lesson Three.

Mark is adding games to the work places with the Bridges curriculum. They are going to add the activities done last week, such as Nine-plus.	
Mark introduces today's activity as Guess My Number. The activity is based on the numeral	Guiding Principle Four.

track. Each partner set receives a line of five numerals to place in a folder with flaps that will cover all the numbers and allow numbers to be visible as either partner lifts a flap. Mark shares the materials required and demonstrates with a student partner.	
The activity begins with the student partner revealing a number by raising a flap. The number 19 is revealed. The student then points to another flap to ask Mark which number is under the flap. Mark guides students in determining a strategy to figure out which number is under the flap. Students are sharing to count-on. However, counting on is counting forward and Mark is supporting the class in vocabulary to indicate the strategy is to count back to determine the number under the flap. Students and Mark are playing a game together to demonstrate how to play.	Guiding Principle Five.
Now Mark and student switch roles. The student chooses a set of numbers to use for the activity. Mark shares with students about choosing a number set that is one that will challenge themselves. Mark is now asking the student helping to demonstrate the activity to determine which number is under the flap. Students in the class are observing or determining the answer themselves, and then finding out if they are right after the student determines the number under the flap.	Guiding Principle Four. Guiding Principle Three.
Mark again focuses on choosing numbers that are a little more difficult - that each student needs to think about to figure out. Mark has a range of number choices, teen numbers, one-30 sequence, one-10, count by 10s, counting by 100s from 600 to 1000, etc.	Guiding Principle Three.
Students are going to play in partners. Mark is choosing partners and number ranges are provided to each partner group. Mark is working with a small group of students while remaining students are working on the activity presented to the class.	Guiding Principle Two.
Mark sets up the table space to begin working with a group of seven students on place value and addition.	Guiding Principle Two.

Mark has bundles of popsicles available for students. Each student has a plain piece of paper and pencil. Mark places a bundle under the screen and students say the quantity. This continues until there are three bundles and four sticks under a screen. Students then record 34 on their papers. Mark then repeats this process until five bundles and five sticks are under the second screen. Students are now to add the two numbers together: 34 + 55. Students are solving and recording their strategy on the paper. Mark is supporting one student with notation. The student shares his thinking as Mark records on paper. A student shares a strategy and Mark asks if the strategy is efficient.	Guiding Principle Three. Guiding Principle Four. Guiding Principle Seven.
All students are now done solving the problem and all get the answer of 89. Mark is asking a student to share his strategy. His strategy is counting on from 34 by ones. Then Mark asks another student about her strategy. Mark is drawing out different strategies and talking about strategies that are efficient and why.	Guiding Principle Five.
Mark begins to demonstrate a strategy. He records 34+55= on the board. Mark is demonstrating the drop down strategy by adding the tens first and then adding the ones. He focuses on using tens language and maintaining the value of numbers. Researcher then ties that back into the bundles and sticks that are lying on the table.	*Attempting Guiding Principle Five.*
Now trying a different problem. Researcher and Mark are having a conversation about when to keep sticks covered or uncovered and the placement of a larger number on the right side when setting out bundles and sticks in two sets. Mark is setting up the second problem in the same way as the first.	
While this small group is going on, the rest of the class is engaged in the activity presented at the beginning of the hour.	
Students are working on 59+32. Sticks are on the table and left uncovered for	Guiding Principle Five.

students to look at when solving. Some students are trying the drop down strategy Mark shared previously.	
Mark is supporting one student who is using the sticks to count by ones even when they are bundled. Mark is asking her if there is a quicker way to count. He is appropriately presenting the sticks for her to add a bundle of 10 to a number involving tens and ones.	Guiding Principle Five.
Mark is supporting another student to record thinking and to work with two digit numbers while maintaining the value of the tens, instead of referring to tens numbers as a single digit, i.e. 30 instead of three.	
During explanations, Mark at times makes connections with the bundles and sticks, and then at times begins to support student thinking without linking to the bundles and sticks.	
Time to clean up in preparation for end of the lesson. Students playing the activity on the floor begin to pick up supplies as Mark finishes working with students at the back table.	
Mark collects papers from group and has listened to each student's strategy at the back table before releasing them back to their spots. Students begin to stack chairs and get ready to go home.	

Interview Transcript
Mark
Interview 1 – September 3, 2015

I: If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?

M: As far as like the mini-lesson, the questions that are being posed to the students and the skill or whatever is being addressed and that how was their approach to it as far as *getting at the prior knowledge with the students*, and then going from there and then also *guiding those activities with the students and meeting those needs.*

I: What kinds of activities might you see?

M: Well, you know, with first graders - *hands-on*. Again, just *engaging activities*, the kids are *doing some problem solving* and coming at it and being able to come at it, whatever it may be and *from their level* and where it is *tiered to meet the students' abilities*.

I: Why would that be important? You mentioned having activities that are tiered and having activities that they are engaged in, hands-on. Why would those pieces be important?

M: Well, it *gets to how you look at your students, and knowing that every student's abilities are different* and whether you address it through hands-on activities. Also, *they are able to come into the activity where they are going to feel successful*, where they can still be challenged. And it's going to take some work – it's not busy work or anything. It is going to *build capacity for them* and then I can throw in there too during a lesson or throughout the time of math too. Then they're *pulling students to meet particular needs* as well, here's a lesson, and go at it looking at students where they see certain needs and stuff, and where they can go into those students, you know, hone whatever the skill might be, or whatever.

I: What might you find the teacher *doing* in that classroom?

M: Like during the lesson?

I: Yeah.

M: Initially introducing the lesson activity, you know, going through some of the objectives. Going through what I am introducing as they are getting into the activity part of it or whatever, then the teacher's kind of watching what's going on, you know, if they're active or not. Here's the lesson, and I'm just going to let go and engage with you. Engaging with the students, and addressing whatever needs are arising for the students, pulling them maybe ahead of time where the lesson falls in a unit. Maybe you need to be pulling students in a group and that…

I: Could you please describe what classroom discussion would look like and sound like when instruction is high quality?

M: Well, again, it gets back to having good questions and, you know, having the good questions to address the students, but then giving them the opportunity to discuss, whether it be as a whole group or doing the turn and talks, or to do, you know, where they begin sharing with a partner and stuff. Maybe, you know, "This is how I do it, how would you do it?"

I: Yes, yes. Describe what classroom discussion would look like and sound like if instruction were of high quality.

M: Once students have had the opportunity to engage, then pulling back, you know, as a group, and then also have an opportunity to share where I am coming from where they can kinda see, "This is how Johnny did it, this is how Mathaz did it…"

I: You talked a bit about who's talking, when they were talking, whom they're talking to… Would there be anything additional you would like to add about content, what they are talking about, or not necessarily?

M: I am looking more at the general lesson format, but, whatever….obviously, whatever the specific content might be, you know, you're introducing… whether it be vocabulary, or if it's a concept or whatever, that you're explaining that, and giving, you know, some of that introduction to it, and then giving the students practice, whether they're doing practice on their own, in groups, or if you are doing guided practice with a small group or…
I: All right - that's it.

Post Interview

I: If you walk into any math classroom, how would you know that great math instruction is happening? What would you see and hear?

M: Well, the kids would be engaged… you would see students engaged and talking about whatever the activity was, you know, they would be using that vocabulary that applied to the activity.

I: What might the activity be like? How might you describe a great activity?

M: Well, again, something where they're engaged, whether it be something that is either manipulatives, or whether it be dice or cards or some kind of a counting game, or some kind of problem solving.

I: What might the teaching be doing?

M: Well, I guess it depends what the focus was for the teacher or whatever, but I guess it could be walking around… touching base with struggling students, and see how they are doing, and if they're off to a good start on whatever it is they are doing, then they can kinda understand what is happening, and just seeing where the students are at, and they are applying the game. I found through an observation for our q-comp… we were playing a game, Plus Nine, and the students, it seemed to be more of a competition than it was a game, and so the math part was maybe getting lost. When they pulled back to doing the counting and using - because some of the students with counting on from nine had some difficulty - and so then, with doing the counters and stuff, it kinda pulled them back to being engaged in the activity…The question again was…?

I: Of the activities themselves…and the teacher…

M: And what the teacher is doing?

I: Yes.

M: So that would be one aspect, getting the students doing an activity, and then either pulling individual students, or pulling small group of students, to do work with him or her on, you know, an appropriate activity for that student, or a group of students.

I: You mentioned students being engaged, how do you know the students are engaged? What will you see them doing, or how will you know?

M: Back to the Plus Nine game and stuff, where you can see them doing the activity, you know, they are purposeful in, like, say if it's like a dice game, they're doing the counting that they should be doing, or if they are playing compare, or if it's double compare, they're adding numbers, they're doing the counting and maybe saying 10 plus whatever and the number. Saying the answer, using the right vocabulary, asking some of those questions that were posed, and if they're with a partner, they're saying, "how did you get that?"… Using the language of what the activity is involved or what is involved in the activity.

I: Thinking about the teacher again, what kinds of questions might the teacher be using or asking?

M: "How did you come up with that?" We have talked about strategies and stuff, the vocabulary we've used, like, "What strategy did you use?" You know, to hopefully get them to engage, be able to verbalize what they did, and continually kind of process through their thought process and stuff, and ask them "How did you do that?" or "What did you do next?"

I: I think that covers….let me look at my one sheet. students with each other, …… when the students are talking with each other….you have talked about that with the vocabulary …..(I am)

M: It gets back to doing the activity so they are able to do it independently, and part of it becomes to practice, and modeling and stuff. I've found… sometimes less is more, and I want to do more than one game and do something else, but then they really have not had a chance to practice the game. Especially last week, with the Nine Plus game, when we did it the second day, with getting past that it's not a competition, it is a game, and also helping bridge students having difficulty holding onto the number to actually visually see the nine and then whatever they spun, the number that they spun, and counting on the chips and the counters from nine, and seeing that being a bridge from some of the students that could not hold onto the nine. Am I answering your question?

I: Yep! What would be helpful as a classroom teacher to be able to more easily use data?

M: Using the data?

I: Yeah. What could make that easier to do? And it could be anything. As a Math Recovery® organization, what might they do or what could districts do to be helpful to teachers?

M: The one thing I would say is with the math recourse we have. Right now, Bridges has the work places, a lot of it was already ready-made and stuff like that. To have something like a box I could go to, and it had the stuff there, rather than like right now - it's on our Moodle page. We have, like, right now from the class, we have the blackline masters, but it is a matter of *who prepares it.* So it's all that preparation stuff, and it gets down to time for teachers - that is the most valuable thing that we have, and don't have, because of everything that is on our plate. So, like, today, if people are watching this video, they are going to be like, "What is this guy?"

M: So, I go have a seat at the table, but I've got my stuff from this morning there. And it's, like, having to do everything. I have to transition from here to here. I was talking with you during our prep time, when I should have probably been doing that, but it's all the demands that happen during the day. We have each of the constructs, you know, for the levels, and here are games that go with each of those. We were looking in the book to have - for addition and subtraction or conceptual place value and stuff - games that work at that construct and stuff, or kits, you know what I mean, it's just like the AVMR binder we've been talking about, and stuff that have seemed to disappeared [laughing].

I: Only finished…not just a master copy it's actually finished the games you could just pull out and blop in front of kids ….

M: It's one thing that Jenny has been trying to do, you know, her focus is math and that is what she gets to focus on. So she has been able to do that and she's been able to help for Carole and I, and she's pulling things off the Moodle page and printing things up to share. You know, it almost seems like, "Well, why aren't you doing that?" For teachers, we are supposed to individualize things for all these students, and all these different levels, and how do you meet the needs of those kids? That's the demand for us… but to have something made that I can pick and choose, and that is going to work for him or her and stuff, that's going to help more. I need this activity for this student and for this student, and that's what I appreciate with Bridges. We had a workshop day where we had the box, so we sat down and for how many hours – "OK, here's unit one, here's the file folders, here's all the games that go with these work places, here's unit two, here's the file folders," all the stuff is in there, so when we left, it is like we had it all from unit one to six and we had the work place games. We had investigations before, they offered a lot of resources too, but with Bridges, that is the part that I really appreciate. Again, like I said, the demands for time…

I: I went back and I already went back and there is some things to think hard about and there's some things that like they do not need to be thought hard about and ok which dice could be good. That is something that could just be provided. Here is a list of things adn this is helpful too and it is actually in a box and actually someone at the office is not in teaching said it would be great to have in a box.

M: In our class, something was mentioned that AVMR is more about giving you the data that you need, right? It's not that *we're a program*… You have the data and *this is where your kids are at in these areas*. Here's in the book. Here's things that you can do, but it's more prescriptive, but yet it's not. But [AVMR] doesn't go that next step, and I am not sure where you guys want to be…

I: I would love to hear more about this part. What role does the class play? What are the take-aways from the class? From your perspective, I would love to hear more about that… how the class is helpful or not helpful.

M: We went through and we picked a couple of students that we did the assessments on and got a chance, like when we went through AVMR One, I just remember we watched multiple videos and then you would code what they are doing and stuff to score it, to have the practice of going through that and stuff. For me, it is just some of the rote, and just doing that, like, in my head, I can think, "This is where they did this," and I can get a general idea, but I don't have the skill to hone in on that exactness. I can only get there generally, but not exactly… I think to have the practice when you get back, it is messy… because when you get a chance, it's like you are fumbling… Try some of these assessments, do them a few times, and bring them in, then really let us have the opportunity in a class to sit down and really analyze the data. We do our data analysis days here after literacy, how they did on writing about reading or whatever, but with the AVMR, you did those assessments and code and stuff and how to fill out the form from the forward number sequence on that sheet that has, you know, the number word after, or something like that, but with…

I: The actual coding of it.

M: The actual coding, so the sheet has the FNWS, BNWS

I: The student profile sheet.

M: I could see directly, here's my fives, here's my fours, here's my threes, so to come back after doing the class, having done some of those assessments, you know, to code and stuff, and then to get some feedback, you said that they're this, but tell me how you arrived at that… because, you know, in our mind, of course you're right, but we may be not, because we are all going off of our perspective during the class.

I: In what ways does that help in instruction?

Mark: From when we first started doing AVMR, I don't know how many years ago, you know having been a kindergarten teacher, it's very prescriptive and it really gives me the range of my kids and where they are at mathematically. You know in whatever category who are the fives, and who are my zeroes and stuff. Then again, from the purple book, looking at activities in there that you can give those students to work on, to build those skills, and now, with having the red book, to have and pull activities out of that, and then starting to do some out of the blackline masters to go with the data. I find it valuable for math. It's the best. Do I get brownie points now? [laughing] From my usual teaching, it tells me where my kids are at, so I feel confident of what it tells me. You can definitely tell where the students' strengths are and where their weaknesses are and then build on those.

Appendix E – Classroom Observations and Interview: Jessica

Jessica
Classroom Observation Set One
September Lesson One

The lesson begins with Number Corner, a calendar exercise incorporating number concepts from the Bridges Curriculum. Students share predictions for which picture and color card will come next in the calendar pattern cards.	
Then Jessica transitions the focus to another space on the board and poses the next question, "How many days have we been in school?" Students are confusing days in school with days in the month, which is posted on another area of the board. Karl answers that they have been in school for seven days. Karl demonstrates how he counted the number of days by pointing to a hundreds grid with seven squares shaded in. Jessica: So you counted by ones. Is there another way to count to seven? Another student is called upon and demonstrates how he counted by threes to count to seven days.	Guiding Principle Seven.
Then Jessica transitions the focus to another part of the board, with 10 frames showing the number of days this month. Students add another tile to the 10 frame. On the board, there is full 10 frame with another 10 frame positioned under the full frame. The bottom frame has six tiles on it in a pairwise pattern (three on top from left to right, and three on the bottom from left to right). Student shares there are 16 days altogether. Jessica: How did you get that so fast? Student: Because three and three is six.	

Jessica: He knows 3+3 equals six, but how did he get 16? A different student: Because there is a 10 frame on top.	
Jessica: Boys and girls, raise your hands if you know what a 10 frame is. Some students raise hands and some do not. Jessica: Do you see how there is five on top and five on the bottom? Do we all see that? Give me a thumbs up if you see the five red tiles and the five blue tiles. Jessica: We are going to be working with 10 frames today when we go back to the carpet so it is very important for you to listen. We know this is 10, because it is a 10 frame, and then Garret knew that three on the top and three on the bottom equals six, so he added 10 plus six equals 16… Several students are saying out loud that six and 10 is 16. J: Wow, you know you are adding, Garrett - adding is kind of your thing. Garrett: Math is my favorite thing in school.	
Transition to carpet spots at the front of the room from Number Corner spots Jessica puts up a pocket chart. She has the Bridges curriculum manual in her hand and references it often during the lesson. Jessica is putting 10 frames representing teen numbers in the pocket chart. The 10 frames have bug pictures in patterns of five on them. Jessica: [Reading from the manual] I want you to look up here at these cards that show more than 10 bugs, and we are going to figure out ways to find out how many we have. Students vocalize quantities. Several different responses. J:How many bugs do I have? Students begin speaking out loud responding with different teen	Guiding Principle One.

numbers. Jessica is pointing to a card. Jessica shares all the responses heard from students. J: Where are we getting all these? I want to hear how we are figuring these numbers out. Can someone raise their hand to tell me their math thinking? Can someone share with me their math thinking?	
Maybe I shouldn't have all of these cards up here. She takes down all the cards except one down in order for students to focus on one frame to respond to. Jessica: We are going to listen to Evan. Evan: 10 plus one is eleven. Jessica: 11. That is great math thinking, Evan. Does anyone have another way to come up with 11? Student shares same thinking as Evan. Jessica: What are you thinking, Lucas? Lucas: Two fives and another one is 11. Emma comes to the board and shares how she used twos to get to 16. Jessica asks her to point to the bugs and count out loud to share how she counted them. Emma points to two bugs as a time and says, "Two, two, two, two, two, and one." Jessica asks if she can show her another way to count that. Student replies yes and Jessica proceeds to point to the bugs in the same way Emma did and says, "Two, four, six, eight, 10…" Jessica pauses for Emma to say the next number in correspondence to one more bug. Emma says nine, and another student shouts out 11, and then Emma says 11. Jessica takes a moment to share with the class that Emma's brain grew right there because she made a mistake. J: OK, anyone else?	
A new card is shared.	

Jessica: Put it in your brain, don't say it out loud. I want all my friends to have a chance to count them in their own way. To keep students engaged and allow additional time for students to apply a strategy for figuring out the number of bugs on the card, Jessica calls a student's name and asks, "Do you know how many are on this card?" Student begins to reply and Jessica replies, "Oh, think about it and keep it in your brain." J: I see some of my friends looking at me smiling and they're saying yep, I can tell just by looking. I see some of my friends counting with their fingers. I assume they are counting by ones. Grace, what number did you get? G: 15. J: How did you get 15? Grace comes to the board to share how she determined 15. She says she counted going down [she points to the three dots in columns]. Jessica: Were you counting by threes like this, or were you counting by ones? The student begins to show how she counted by pointing at the card. Jessica: The card has two frames, one above the other. The frames show five dots, then five dots, and then another five dots. The student counted to nine by ones, pointing and counting in columns of three. Jessica: Thank you, Grace. Karl, what did you do? Karl comes up to show how he knew 15 bugs. He said he saw one 10 and then another 10 and counted by ones. Jessica: I like what you are saying. You said you saw one 10 and another 10. She points and provides language for the strategy shared by Karl. Charlotte says she counted by fives.	Guiding Principle Eight.

Jessica now brings out a 10+5 equation card written vertically and asks students to raise their hand if they have seen something like this before. A few students raise their hands. Jessica: Is this new to you? Have you seen something like this before? Jessica calls on individual students to ask the question and listens to responses. Jessica: This is a number sentence, boys and girls. I am going to show you. We are going to play a game using these next week. Jessica begins to explain the symbols used on the card, showing an equation in vertical form. Jessica is matching the number sentence symbolic notation with the picture of bugs on 10 frames. Now Jessica is putting up 10 frame cards on the left and equation cards on the right in random order. Jessica: Who thinks they can match a number sentence card with one of these 10 frames? Evan pulls a 10+1 card and matches with the frame picture for 11. Jessica is asks the class if Evan is correct. Jessica calls on a student comes up to match the 10+10 equation with a 10 frame card. Lesson continues with Jessica drawing sticks to choose a student to come to the front to match an equation card with the 10 frame card.	*In contrast toGuiding Principle Seven.*
Jessica shares that next week the class will be playing a game using the bug 10 frames and equation cards. Jessica: Raise your hand if you think you can do 10 frames with bugs. Students raise their hands.	

Lesson Two.

Lesson today is from Bridges curriculum. Jessica: What does this look like to you? Alexis: A calendar.	

Jessica: I don’t think so. Then another student points to the number calendar at the back of the room and says, “It’s kind of like this, a pattern of bugs - you can put on beetles, butterflies, mantis.” Jessica: Ahhh. Why doesn’t this look like a calendar to me? Kaden says, “matching.” Jessica: You think I am going to do something about matching - you are exactly right, Kaden. That is what I am going to do. Boys and girls, I want to let you know… Jessica puts a purple square matching the grid on the screen in front of students to the calendar the class is building in the back of the room for Number Corner. One of the students generalizes that the purple squares follows a diagonal in the back and the teacher can fill in purple squares in diagonal pattern on the table on the SMART Board. Jessica is starting to touch each square and recite an a, b, c, pattern to check all of her purple squares in already and fill in the remaining purple squares.	
Jessica: Now these are butterflies. Let’s count how many butterflies there are in our pattern. She calls on a student to come to the front to count the butterflies. Student notices that purple should be beetles, not butterflies. Jessica: Oops, I just confused you! Should I add in the butterflies now? Student nods. Jessica quickly fills in the butterfly pattern with a green letter B. Jessica: I am confusing Charlotte so much. Can I explain to you what I am doing right now so I am not confusing anyone? Group of students nod and say yes.	

Jessica is explaining what she is doing to complete the chart to match the pattern from number corner. She continues to ask students to give a thumbs up if they understand. J: So now can you understand what is on our calendar. Give me a thumbs up if you understand.	
Our job today, boys and girls, is to count up how many Praying Mantis we have, how many beetles we have, and how many butterflies we have, and put them in our 10 frames. There are pictures of 10 frames to the right of the calendar grid on the SMART Board in the front of the class. J: Kylie, remember how you said you were going to the apple orchard and I said that maybe you should make tallies for how many apples you pick? Kylie shakes her head. J: And then I was telling you that maybe we should make 10 frames when you get back on Monday and we can show everybody how many apples you picked? That is kind of what we are doing right now. Hunter shares that Jessica can move the colored squares right into the 10 frames. Jessica confirms his comments and shows how she will move a colored square into the frame. Gabby comes to the front to count all of the butterflies in the pattern on the SMART Board. She counts by ones to get seven. Jessica then leads the class in counting in order to check Gabby's number. The class counts to eight altogether. Jessica: Oh, Gabby, we must have missed one. Who thinks they can make eight in their 10 frame? Wait time.	Guiding Principle Eight.
Jessica calls on Gabby to choose someone. Gabby chooses Charlotte to come to the board to show eight on the 10 frame. Charlotte proceeds to make eight in the frame by dragging red	

squares and placing five on top and three on the bottom. Jessica says to the class, “Are you watching? Let’s see how she is doing it and maybe you do it differently.” J: Is she right? Give me a thumbs up on your knee if you think she is right. Put your hand in the air if you think she made a mistake and you want to help her. Jessica calls on a student. The student begins to offer a suggestion and through conversation they determine that Charlotte was right. The student saw five on top and three on the bottom. Emma: Charlotte is right because if you have 10 and then take two out that is eight. Jessica: Oh - we like subtraction in here, huh? Remember how we already made a subtraction problem over there? Group of students: Yeah. J: Same thing. Emma. Her math thinking right now is subtraction. Jessica repeats what Emma said, and tells the students that she is going to take what Emma said and show the class using numbers. Jessica proceeds to record 10 - 2 = 8 on the board. Then Jessica records 5+3=8 to record an equation for the first student’s strategy.	Guiding Principle Seven.
Jessica calls on Karl. Karl shares that they could put four on the top and four on the bottom in the frame to have eight that way. Jessica moves the squares from a five and three pattern to a four and four pattern. Jessica records using a number sentence. Student: I have another way. Jessica: How? The student comes to the board to count the eight squares by twos. Jessica: Can I record that in a math equation? Jessica then matches the equation numerals to the squares in the picture. She shows the student another way to see his strategy by keeping the four and four pattern.	

Jessica: We need to wrap this up. This is really cool. We were supposed to do so much more with this. Do you see all the different ways my friends came up with seeing the number eight? Give me a thumbs up if you see. Give me two thumbs up if you see yet another way. We could probably talk about this all day because there are other ways to make eight, aren't there? Students: Maybe we can do it later. Jessica: Yeah - maybe we can talk about this more over lunch. Students are excited about the lesson.	
Jessica leads the class in student yoga before transitioning to part two.	

Part Two.

Transitions to next part of Bridges lesson. This part begins with rows of sorting cards shown on the SMART Board. The cards each have a different water animal on them. Jessica: Boys and girls, what do you see? Raise your hand. No shout outs right now. Kaden, what do you see? Kaden responds. J: What did we decide yesterday when we were sorting them into cups? One student remembers they called them sea creatures.	
Jessica has cards in a random card generator app on the SMART Board. The cards with sea creatures on them are entered into the random generator. "Long snouts" comes up in the random card generator. Jessica: Students, when you are sorting your creatures today, you are going to look for creatures that have long snouts. Then next card randomly selected is "walrus."	

Jessica is sharing directions for students to return to their table spots and sort sea creatures into piles that are walruses and another pile with creatures having long snouts.	
Students are going back to their seats. Jessica is bringing around sea creatures for each table to begin the activity. Students begin sorting sea creatures into those with long snouts and walruses. Jessica is coming around to collect all the sea creatures that do not have a long snout or are a walrus.	
Jessica is now calling table groups to come to the carpet at the front of the room to share the number of creatures with snouts and walruses they had in their table groups. Jessica has a T-chart on chart paper with Walrus on the left and Long snout on the right. Jessica is explaining that they will use tallies to keep track of how many walruses the class found.	
Students are now bringing up their walrus creatures and setting in a pile at the front of the chart. Then Jessica asks students to bring up creatures with long snouts to set in another pile at the front of chart. Jessica begins to count walruses and decides to put up a pocket chart where she can set the walrus animals inside the pocket chart for all students to see as they count. Jessica begins to draw tallies and students are to monitor the count to tell her when to stop drawing tallies. Students are having a very difficult time staying focused. Students point out that the 10 walruses are written with two sets of five tallies.	
Now Jessica is setting the long snout sea creatures into the pocket chart and counting the total number of sea creatures with snouts. There are crabs that groups included with sea creatures with	

long snouts. Jessica asks students to explain their thinking when they chose those as creatures with long snouts. The class agrees that the crabs can be taken out. The class counts the remaining creatures together. Jessica then draws tallies as the class counts out loud.	
Jessica leads the class in movement. She has students pretend to jump rope while counting to 10, and then 20. Students are going to engage in work places next. Each table group as a different work place. One group is copying and creating patterns with polygons. Another is counting sea creatures by placing them on empty 10 frames, then determining a total. A third group is creating shapes on geoboards, and the fourth group is creating three dimensional figures using Unifix cubes. Students are engaged in thinking and working within their work places. Students placing sea creatures into empty 10 frames are using count by ones methods, and then recounting using counting by 10s. Jessica is walking from table to table, supporting students and asking questions about their math work.	

Classroom Observation Set Two.

November Lesson One.

To begin the lesson, Jessica explains to students a new structure for math class. She has four stations, each with a different math activity for students to work on while assigned that station. Students will rotate through the stations once Jessica lets them know it is time to switch. One of the stations is meeting with Jessica. The other stations will include at your seat (hundreds club), hands-on (math work place games), and math facts (iPad). Jessica is going through what to do at each station and how to partner up when the activity requires it.	
 Students will be at each station for 10-15 min.	

Jessica assures students that she will help students know what to do during rotations.	
Today is Tuesday Tally, so they transition to front of the room to practice how to make tally marks. Each student will use a whiteboard and marker to solve and share ways to make tally marks to represent a number. Jessica writes a number on the board and students record that number using tallies, followed by holding the whiteboards in the air for Jessica to see responses. The numbers chosen are three and six. J: Let's see if my friends can remember the rule about making tallies. Jessica provides students with wait time and then has all students hold the whiteboard in the air for her to see. Jessica and students discuss why you place a diagonal tally to signify a group of five. The next number is 12. Jessica provides wait time and then has all students share whiteboards in the air as before. Jessica is correcting students as she reviews the white boards. She then calls on a student to come to the front and make his 12 tally marks on the SMART Board in front of the class. Jessica is turning this into a learning moment to provide space between lines when showing a number using tallies. Jessica thanks the student for coming up to help learn and she prompts class to give him a silent cheer.	
The next number is 32. Jessica: First, before we start, who can read this number for me? Alberto says thirty-two. Students begin to write and Jessica points out the correct way the previous student is leaving space between tally marks.	Guiding Principle

Wait time is provided. Students are to put their hands on their head when done writing tallies. Students are employing different strategies to monitor when they have drawn 32 tallies. Jessica begins reviewing responses. She is calling on students to put their boards down once she has checked their work. Jessica asks a student if she can share his board. He had tallies drawn and then wrote an equation alongside (30+2). Jessica elaborates more on different strategies for counting up 30 tally marks. Students are talking and sharing as well. One strategy highlighted is counting by fives to get to 30, and then two more. Another strategy shared is an equation indicating a count by 10 to get to 30, and then two more.	Eight.
Transitioning to rotations as described in the beginning of the lesson.	
Jessica is getting students organized in their opening spots for rotations. She is demonstrating what to do for math facts on the iPad. She shares with students how to change the numbers for the game so students can switch numbers to challenge themselves. The counting/ordering game they are playing today is the same game they played yesterday, so they know how to play. At your desk: She sets up hundreds club and has the students starting at this station raise their hands. Then she names students at "Teacher's Choice" - this is the group working with Jessica. Hands-on group is playing work place games on the carpet. Jessica offers hundreds tables for students doing the number club. All students walk to their stations.	Guiding Principle Three.
Jessica walks around the room to see that students know where	

to go and how to get started. She stops at a student and adjusts her iPad game to order numbers in the 500's. Another student asks if they can go to 500s and she tells that student to be in the 100s. Jessica assigns other students to the 80s. Students are starting to engage in their activities at their stations.	Guiding Principles Two & Three.
Jessica is working with a small group of students needing support with numbers up to 10 and then 20. Each student is provided with a paper hundreds chart where every other column is shaded grey. Each paper is slightly different. Different numbers are missing on each hundreds chart. As she starts the small group lesson, Jessica asks students to cover all rows except for the first row. Students are then unsure of the sequence to 10, and start to look at each other's paper. Jennifer then places folders on the table to isolate each student's paper. Students are to fill in the missing numbers on their number chart. Jessica asks if students notice a pattern on their hundreds charts. A student begins to notice that going down the column on the right is counting by 10s. Jessica asks the student to predict a number the next number that is covered in the pattern counting by 10s. Then Jessica transitions to counting by ones in a row with this same student. Jessica supports students and checks responses one by one. She then questions students drawing attention to patterns in the hundreds chart going down, as well as going across. In a one-on-one conversation with a student, Jessica is working on how to say and order teen numbers. She is posing questions for each student and providing isolated	Guiding Principles Two, Three, & Four. Guiding Principle Six. Guiding Principle One. Guiding Principle Three.

supports for identifying and saying numbers in and out of sequence in the range to 20. Jessica rings the bell for each group to clean up. Jessica comments that the last station was much longer than she thought it would be.	
When groups are finished cleaning up their stations, students are to meet at the carpet in the front of the room. Jessica has a SMART Board page to show students where they go to next. She reminds students what station they are at next and what they do in the station. Students are going to their next stations. Jessica walks around the room to make sure everyone knows where they are going and what they are doing when they get there.	
Jessica stops to support a student with hundreds club. Jessica stops again to provide a student with a place value flip chart to support writing three digit numbers between 110-120.	Guiding Principle Four.
Jessica then begins to work with the small group assigned to her group for the rotation. She is taking out an activity from an AVMR binder of activities provided by the district math coaches. The master sheet she was planning to use is missing, and so Jessica comes up with another activity in the moment. She provides each student with a completed hundreds chart and puts up folders in order for each student to have a private work space. She starts off by asking each student to point to the number four.	Guiding Principle Four.
J: Circle the number four. Now she asks students to point to the number that comes after four. A student asks to circle that number too. One students says the	

number five. Jessica asks him if that number is even or odd. Student is not sure, so Jessica moves on with the small group lesson.	
The next number to circle is 10. Then Jessica asks what the number is after 10. Jessica asks students to point to the number 20. Jessica asks, "Where is the number that comes before 20?" She has a student share who pointed to 19 and then asks other students in the group to also point to 19. Jessica folds each of the hundreds chart papers in order to have less visual clutter for students when answering questions. J: Hunter, how did you know that this number is the number before 20? Now Jessica brings out a numeral roll as a support and asks Hunter to identify 20, and then point to the number that comes before 20, followed by the number that comes after.	Guiding Principles Two & Three. Guiding Principles Six & Four.
The next student Jessica asks to point to 15. Jessica needs to support the student to identify 15 and not 50. Then she asks this student to show the number after 15. Student does not know what after means. Jessica clarifies by using hand gestures and the numeral roll. Then Jessica scaffolds down to a smaller number. She asks the student to identify the answer after six, followed by the number that comes before six. She poses another question to this student. This time, she identifies four and asks which number comes after four, and which number comes before four. The student is timid, but correct for both responses.	Guiding Principle Six.
Jessica is again looking for the worksheet that breaks down number work before and after.	
She then poses a question to Austin. She asks him to identify 20 and point to the number that comes before 20 and after 20. Jessica rings the bell to signal to clean up and join her on the	

carpet. They are finishing with a conversation about this week's focus to practice rotations and get into the routine. Students in the hands-on group are reflecting on the importance of putting cards back in an organized fashion for work place games. Cleaning up. End of math time.	

Lesson Two.

Jessica begins the lesson by posting a hundreds chart with numbers up to 120. She asks students to record the number 41 on their boards and hold up the whiteboard to show her. She asks students to erase and then write the number 14. A student is talking about how this number is backwards to the one they just wrote.	Guiding Principle Six.
Jessica shares with the whole class student thinking that she saw students do for the previous problem. She saw some students write down a four and then pause to think about which side the one goes on. Jessica continues to think aloud about writing teen numbers. She brings to students' attention that in number club, some students were writing teen numbers backwards. Jessica asks student to write 18 on the board. Then she asks students to write 81 on their boards.	
J: Raise your hand if you see the number 22 on the SMART Board. Next she asks students to write the number that comes after 22. Wait time for students to think about responses. Jessica is showing students where they can get help	Guiding Principle Four.

identifying a number that comes after another number. She points out the number line up on the wall and a numeral roll. Jessica is demonstrating the direction for after by pointing to a numeral roll and using hand actions for after. What comes after 23? A student uses the sequence one, two, three, four to determine after 23 is 24. Jessica tells students to erase boards.	
Jessica: I see the number 16. What is the number that comes before 16? If this is too easy, think about what number comes before 116. Thinking time. Students are recording responses on their whiteboards. Jessica asks a student to come to the board to draw a circle around the number that comes before 16. Then she asks another student to come up and circle the number that comes before 116. J: Boys and girls - what number comes after 16? Class replies 17. J: What number comes after 116? Class replies 117. Students are putting away whiteboards and markers to transition to rotations.	Guiding Principle Six.
Students go to spots for rotations. They begin working.	
Students at the table with Jennifer have a hundreds chart with a small number of numbers filled into the chart. Students are writing in the missing numbers on the chart going to 120. Jessica asks students to fill in the number 51, then the whole row of 50s. Jessica supports a student in correctly forming a seven. She is providing students time to work. Then she supports a student	Guiding Principle Three.

with the 50s row. She instructs the remaining student to fill in the top row. With this same student Jessica is supporting the student as the student learns that the counting sequence on a hundreds chart goes from left to right. A different student identifies a pattern going down from one. Jessica asks the student if she can fill in the rest of the column. The student begins to fill in the numbers down the 1s column. Jessica brings this pattern to the attention of the whole group. Jessica begins by asking whether the student who is filling in the numbers down the one column is counting by ones. Many students say yes and a student hesitantly says no. Jessica shares that numbers going down the ones column are counting by 10. Most students are confused by this. Jessica brings in the place value flip chart to show how numbers are increasing by 10 going down the column. She demonstrates how we can count by 10s starting with one, i.e. one, 11, 21, 31, 41, etc.	Guiding Principle One. *Approaching Guiding Principle Four.*
Jessica tells students to start counting by 10s starting with the twos column. Students begin filling in numbers down the twos column. Then Jessica has students complete the remaining columns. Jessica observes students and corrects students as she sees numbers recorded incorrectly. Then helps a student unsure of how to write numbers past 100. Bell rings for students to clean up at their rotation spots. Jessica: supports students in knowing where to go next.	
Students are moving to the next rotation spot. Students begin working. The group with Jessica has a hundreds chart that is mostly completed, with some numbers missing. She is asking students to fill in the number five and then county by fives while completing the chart. Jessica brings out a numeral roll to support students A student notices that when they count by fives, they will	Guiding Principle Four.

always land in one of two columns. The students count by fives to notice that some numbers end with five and some end with zero. Students are counting by fives to themselves and then completing the chart. Jessica is supporting a student who got off in labeling numbers by skip counting by fives. Jessica is highlighting a students' thinking. Lucas said that all of the numbers in the five column are odd and all the numbers in the 10s column are even. Jessica: Do you know why that is? A student explains why that is and crosses her fingers to show that each number has a partner. Now Jessica is sharing that you tell whether a number is odd or even by looking at the ones place. She goes through numbers, counting by 10 and starting at five to identify each number as odd or even. Then they move to the 10s column and do the same. Jessica then directs their attention to the twos column. Jessica asks students to complete the twos column. Jessica: If we are going down the twos column, what are we counting by? Student is confused. Jessica shares with him the pattern and the skip count by 10 sequence starting with two.	
A different student is now counting by ones in the twenties and is finding it difficult to change from counting by 10s to counting by ones. Jessica uses the place value flip chart to demonstrate counting by 10s starting with two. Another student shares that he can use numbers that are there to figure out missing numbers. Jessica is supporting another student completing the chart past 100 in the twos column. Jessica rings the bell. End of math time.	Guiding Principle Three. Extending the range.

Classroom Observation Set Three.

December Lesson One.

Lesson starts with the class preparing to do an activity called Number Rings. Instructions are that each student puts +1 in the center of the ring. Then when they get the thumbs up from Jessica, they can put any number they would like in the ring. Students are getting supplies to do the ring activity and going to their seats to begin working. The rings have an open center where students record a number they are adding to each of the numbers written on the ring. The numbers 0-9 are written on the ring. Students record the sum around the outside of the ring.	
Jessica is supporting a student with this activity by providing her Unifix cubes to build the number written on the ring and then add one cube to that number to be able to determine the total. Once Jessica checks a student's board, she allows the student to choose a number for the center. One student has written +100. As Jessica goes around the room, she provides Unifix cubes to additional students who need to have the cubes in order to determine the sum of two numbers. For students requiring Unifix cubes to solve, Jessica is telling them to now place two Unifix cubes in the center to do +2 to each number written on the ring after completing +1.	Guiding Principles Six & Four. Guiding Principle Five.
Another student has placed +10 inside the ring. Another is +5. Students are choosing numbers that challenge their thinking. A student has chosen +100. This is a bit out of reach for this student, but he is motivated to figure out the correct sums. Students are thinking and working. Jessica continues to rotate around the room, asking students about their strategies and checking their answers.	
Students are being supported in correctly writing numerals, as	

well as addition. Chime rings to end the activity. Students begin to clean up.	
Students are at the front carpet. Students are released by groups to their rotation spots. There is only time to do one rotation today. The hand-on group can do beach ball addition or dice addition that they were doing yesterday in small groups with Jessica. Teacher's Choice group is cutting out a clock to glue in their math notebook. Students are cutting out an "I can" statement and then pasting the minute and hour hand to represent 10:00. Some students are having a difficult time focusing on the activity assigned for their groups during rotations today. Jessica is talking to students off task. Bell rings. Math time ends.	

December Classroom Observation Set Three.

Lesson Two.

The class is completing a board with the number 13 altogether so all students know how to fill out the page. The page has a number in the center. In the top left quadrant, students record number sentences for the number 13. In the top right quadrant, there are empty 10 frames to show the number recorded in the middle of the document. In the bottom right, students record the number using tally marks, and in the bottom left, there is a hundred grid to shade in the number of boxes indicated in the center. Jessica is calling on students to come up one at a time to the board to record the representation of the numeral in one of the quadrants until all quadrants have completed representations of the number 13. Jessica asked different students to share their equations. Then she tells students to erase their boards and choose their own number to complete each quadrant on the board.	Guiding Principle Seven.

She is supporting students who have incorrect responses and checking students' boards for accuracy.	
Jessica stops to support a student who chose the number 36. She has 26 dots filled in the 10 frames and has an equation written as well. After listening to the student's strategy for writing the equation, Jessica notices that she used the 10 frames to record the equation. She only had 26 dots completed, and so Jessica communicates to the student that her equation was right for the number 26. The student goes back to the 10 frames to accurately complete frames to show 36. Jessica questions the student by asking how many frames she filled in completely. The student replies three. J: How many dots is that? Student replies 30. Emma then is asked to share her equation with the class at the board. Jessica shares with the class how the student used the dots in the 10 frame to create the equation. Then Jessica asks the student to share how she ended up adding 10 more to have an accurate equation. Student is not sure, so Jessica explains. Jessica communicated that her math thinking was cool and so she wanted to share it with the class. Student then starts to share her math thinking. Jessica expands on the types of strategies that the student used. She then provides students more time to work on their number worksheet/boards.	Guiding Principle Three.
Students are working independently. Jessica is walking around the room, having one-on-one conversations with students.	
Students are transitioning to rotation spots. Rotations have the same categories as they did in November. Students are working on iPads for number facts. They are playing card games for hands on. Students are playing an addition dice activity for hands-on. Jessica provides students with different dice in order to adjust the addition problems.	

The activity at the teacher table is cutting and assembling a clock picture with hour and minute hands into their math notebooks.	
Students are having a hard time staying focused on their tasks in rotations.	
Jessica asks students to begin cutting out the "I can" statement first to paste in their notebook, then the clock and clock components. While students are cutting and pasting, Jessica is walking around the room and supporting students in their rotation activities. She then comes back to the group working at teachers' choice. She asks students to past the minute and hour hand to say 10:00. Jessica shows students where to place the hour and minute hand. Students are then instructed to copy how to write 10:00 from Jessica's teacher notebook.	
Change rotation activities. Each student in the Teacher's Choice group is cutting out the "I can" statement and the clock, then pasting the hands at 10:00. Each group is doing the exact same task at teacher choice.	
Lesson ends.	

December Classroom Observation Set Three.

Lesson Three.

Lesson starts by playing Buzz, counting by fives from five to 70. When a student makes an error, Jessica goes back two people in the sequence to provide a running start again for that student. When a student does not know the next five in the forward sequence, Jessica asks her what five more than 60 is. Then when she is not sure, Jessica supports her in getting the correct number in the sequence so she can continue in the game. The game is moving very quickly. Students are staying on task and engaged. After a winner is determined, all students stand to count by fives together to 100.	Guiding Principles Six & Four.

Jessica is introducing the activities for today for each of the rotation topics. She is introducing a new activity called Flip Cards. The Flip Cards are ones she printed from a teacher website. Students receive a packet of cards at their rotation. Each card has a sequence of numbers and students either determine the missing number, or choose from a list of possible numbers provided to complete a sequence. Students record solutions in their math notebook. Jessica communicates that a hundreds board can be used to help solve some of the problems if students need it.	Guiding Principle Four.
Groups are switched today. Students are going to their rotation spots.	
Teacher's Choice table is playing a dice activity. Students are recording the date and activity to be played in their math notebooks. Students roll two dice and add the quantities together. She leaves students to pay the activity at the table while Jessica walks around the room observing students in rotation spots. Students at the table are recording addition equations in their math notebooks and finding solutions to the addition problems rolled with the dice.	Guiding Principles Two & Three. Guiding Principle Seven.
Jessica asks students if they are ready for a challenge. Students respond yes. This time, students are instructed to roll both dice and add them together. Then students write this number in their notebook. Students then roll one die, and subtract that number from the number they just recorded in their notebook. Jessica rolls the dice to show the following example: Her first roll was six and four, so she recorded 10 in her notebook. Then she rolled a three, so she recorded 10 – 3 = in her notebook. One student solves the problem and shares with Jessica how she determined the difference to be seven.	Guiding Principles Two, Three, & Six.

Students quickly begin playing again independently at the teacher table. Students are employing a variety of strategies, including count down from and finger patterns. One student is having a difficult time with subtraction. Jessica brings out the numeral roll to identify the direction to go when subtracting. The student is still having difficulty with subtraction. He begins to use the numeral roll to support determining a solution.	
Bell rings. Time to clean up. Bell rings to rotate to next station.	
Next group is coming to the teacher table. Students are labeling their notebooks. This group is playing the game by rolling one die and recording the number, then rolling the second die and adding it to the first roll. Students are using pips in the dice to solve addition problems.	
Jessica is supporting a student at the table in writing numerals correctly. She brings out the numeral roll for the student to match his numerals to the numerals on the numeral roll. Jessica stops the group to bring everyone back to focus and be on task Jessica then turns to students at the group and says, "Can I challenge you?" Students do not respond. Now students are to roll both dice and point to the number that is the largest. Students are to write that number down on the paper and then subtract the smaller number. Jessica demonstrates subtraction by covering dots on the larger die, i.e. students roll a six and two. Jessica takes the six die and covers two dots with her hand. Students are using their fingers and looking at the pips on the die to determine the difference. Jessica continues to have conversations with students one-on-one regarding subtraction. She continues to cover the number of dots on the larger die to support students in determining the difference. Students are having a difficult time playing this modification on their own.	Guiding Principles Two, Three, & Four.

Clean up and rotate to the next rotation. The next group comes to the teacher table. Their bucket is messy, and so Jessica is sorting it quickly. She is also talking to the group about following the rules during rotations. Several assignments are found in the bucket without a name. Jessica takes time for each student to go through their folder to identify which student should get the unnamed assignments. Bell rings and math time is over. The last group cleaned out their folders to Teacher's Choice time.	

Interview Transcript
Jessica
Interview 1 – September 3, 2015

I: If you observed another teachers math classroom, how would you know great math instruction is happening?

J: Hands on math - since kids learn in different since they learn in different ways
I think there would be some adaptations for certain kids. you'll see some kids with manipulatives counting and you will see some kids with number lines, counting, so I think its all ….um….manipulatives
different for each kid — I have noticed that some will need extra visual help and hands on help -
and some kids can just pick up from the board and maintain it

I: What are something you might see teacher doing?

J: Ummm in math where the teacher…..Framework of lesson - introducing the lesson as a whole to the group - being up in front of classroom maybe with a smartboard or what have you
and then engaging with each student - walking around making sure that, especially with math the moment they start getting off track its really easy for them to fall off the wagon and not get back on again. so Making sure you know like where your low students are in the classroom… um and like do a lot of informal assessment I guess walking around so like during math I guess I would definitely, not that you would do it is reading or in writing, but um a lot of engagement from the teacher walking around and making sure um

but then also Having the students participating and helping — I feel like math that's one way there's so many different ways that the kids can solve a certain problem so being like the middle person sometimes as a teacher, letting the students kind of lead the lesson but intervening when needed…their getting way off track. They're not going about it the right way..
I: What might you see the students **doing**?
J: Hopefully listening (laughing) um I….like physically?

I: Yeah yeah kind of like that Looks like, sounds like
J:OK um I guess their task at hand that the teacher wants them to do….any way that they know how to solve a certain problem. I guess that is one thing I have realized being a teacher a short year - student teaching - is that Every kid is learning so differently and that like 1 worksheet for all the kids can mean every kid doing same problem, kids are doing half worksheet so at least their all engaged in some type of activity whether its the same as one of the other kids is doing or not
I: What kinds of tasks might kids be working on?
Ummmm …….like, counting? like that kind of task or???
I: It could be. you mentioned worksheets before and I think you said solve problems before but even In general - the kinds of math tasks.
J: I think there should be independent work, back to every kid learns different so their definitely should be independent work at their comfort level or instruction level …..math work places like discover and learn new things with their work places, they should….maybe guided math. um going on at the tables….but i think Math is one of those subjects where you can use all forms of teaching in 1 lesson…so many things??? (laugh)
I: What kinds of discussions or talking? or like whose talking and how much in one lesson
J: I think from what I've noticed is Math is a very is one of those subject kids love leading almost where like we can bring up like the subject were talking about and start and they always like to oh I know how to do that …its 5+5 ….but i know how to do it this way and I'm like no you do it this way I think is more like an engagement with all the students, more of a conversation…..like this is how we're going to do it you know that's how I would like to lead a lesson is having the kids or the students um help me find that answer instead of this is how you do it this is why …. eventually you explain the ways you would do it…it's kind of like a shared job in math -teacher will mentor you know making sure - mediator making sure everything's going the right way but at the same time students drive that, depending on i guess the lesson….many different ways to solve a word problem. its fund to hear how the kids would go about solving the problem instead of just giving them the answer.

Post Interview
December, 2015

A: How do you know great math instruction is happening
Well now, … being in the classroom for a little while…I think before I made a comment that everyone is busy and everyone is like I mean I guess they ***look*** busy, but now I realize um when they are doing math
like the other day a lot of my stations were really solid sometimes I feel liek they are not where i am not sure if they are going to work out like they are it was so quite in my room - when i looked around everyone was very on task and you could tell they were in to what they were doing compared to being "busy". I feel, I don't know just engagement and I also love it when kids can teach each other about math. That is really what I am REALLY focusing on now that I am not just one teacher in the classroom there are many teachers in teh classroom. so if there is more than one person knows what to do they can go to them instead of just me….I am REALLY focusing on kiddos teach each other.
I guess its hard to actually just come in its really just come in and look at a classroom too see if their I feel like it is more than just looking at a classroom. I feel like it is diving into what they are actually doing.

Interviewer: Yeah - *Yeah*
Say more about teacher talk and the type of talk happening.

Jessica: I feel math is more of a conversation than me really teaching at them. There's a couple different ways. like me with a small group of kiddos or one-on-one conferencing where I am walking around talking with them or also I am really trying to get to um my mini-lesson with all the kids, my big lesson is more of a conversation going back and forth on how we got that number. What are other ways that we can um solve a problem or have more math thinking coming from the first grade language instead of just my language. um I don't know

A: Yep um So um you mentioned activities and what kids would be doing and when they are really solid. Please describe a little bit more on what kinds of activities are the types that are ideal

J: get them focused

A: Yeah

J: like the games really obviously any time you can play a game with a kid they are super excited but the fact that they - some of them don't even understand that when they are playing a game they are actually doing math.
They just think they are rolling a dice and its fun and trying to beat each other and have competition. But I think a lot of the hands on games now we are really branching away from …like before we had buckets like work place buckets where it's just like getting

used to materials it would get super noisy and they would get to do whatever they want. now I am noticing that if we do have to bring out the Unifix cubes they are not always just building stuff they are actually trying to make patterns and taking and I don't tell them necessarily to do that so they are kind of taking upon themselves. um anything that involves partner work and working with another student in math I think is ideal. They really get a kick out of that and I feel like they are more i mean its more they are not socializing but it gives them a time because the afternoon is very sit and listen to books so math is so much more hands on and I want them to understand it is not just paper and pencils and I take out the whiteboards and I do a lot of things on the whiteboards like every tool possible something that's different even when they are counting and we are doing addition on the whiteboard with like the number line they all have cubes in their hand they are trying to figure it out on their own um cause they are all at different places in the math I am really trying to differentiate whose where we had these math packets for the penguin units. There are 3 different sets of numbers so the kiddos will pick numbers they think they can work best in their equation so like having them direct their own learning.
um of course the technology piece we have the math apps that we have been doing.
I just have them choose a couple…I've noticed watching some of them that are very squirrely and not on task some of time then when they get their iPads, there calming machine they will sit and they really are doing what they are supposed to be doing. just a different aspect of the learning bringing that in there

Interviewer: How about the teacher? What would the teacher be doing?

Jessica: Always with a student. I think that is why sometimes it's hard, because I feel like I am always on the go in the morning. I don't have time to sit. But ..**it is**, um, I guess my lesson structure. I start with a mini-lesson where I'm teaching all students and then we break up into groups and do rotations. So, I am always with students and in the smaller groups they're leveled where they're [students] in a certain assessment. As for me, I guess I am always trying to meet them where they are at and trying to push them further, but I am also wanting them to think for themselves, too. I try to like help them solve the problem together and not just give them answers. …. I don't know….I'm off my mark today.

Interviewer: This is good (laughter by both J and I)
this is just great and this is kind of an odd question And it really is it really fits and the categories that the questions are really meant to cover after that is the classroom environment, questioning, talk - student talk and teacher talk , talk and discussion, equality a little bit which comes through in differentiation um yeah and the role of the teacher is the last one.

Jessica: I think I am always formally and informally assessing them too.
Interviewer: mm

Jessica: I am always checking where they are at and making sure - um and that is my tricky part -is now making sure that the stuff I have for them in the groups ***is*** going to meet all their needs. And so always trying to … if I feel a student is getting more behind or moving up, like always trying to pull them aside and do small little assessments to see if they can do what I have them or if it is too easy, just not doing … well nothing is going to harm them.

Interviewer: No
Jessica: But I want to make sure that they are successful.
Interviewer: It sounds like successful in like advancing knowledge. Pushed in that way.

Jessica: Yes. That's where I am. It's struggles. That's where I struggle with it sometimes, cause part of it is - I want them to be able to work independently, which means they have to be working on things that they ***do*** know and it's just more repetition. Like getting it and also pushing them up. I do have a couple kiddos in my class that is they are just so uncomfortable doing anything on their own so they always need me, especially with math, so that is hard to get them. Because even my morning work, I've been doing more like more fun worksheets that's a little bit more engaging where they can color, but just in the morning, and I just don't want it to be **busy** work where they're just doing whatever so now I am starting to do different worksheets and I've noticed that some of them are - they just don't want to do it unless I'm helping them or sitting right next to them. I'm trying to move them away from that to have them gain more independence, which means I feel that it's something where I have to give them more, not easy, but something to gain their confidence so they can. Which is tricky there's just so many different components!

Interviewer: There are (Jessica and Interviewer laugh) That works great for the last question. Did you have questions for me? You had mentioned data or anything else?

Jessica: Do you have post data on Evan and Grant?

Interviewer: Yes, they were in that last email, but you probably need the sheets.

Jessica: [Pointing to the Addition and Subtraction column on the data spreadsheet showing student results from fall and winter] So this is SEAL - addition and subtraction? And this is structuring? (pointing)

Interviewer: Structuring is the topic we haven't really talked about before. Once students are a 3 [in the Addition & Subtraction assessment], they know how to count on and they know how to count back, and have initial composite understanding, then structuring is the next instructional place for them. But it's not that if they are not a 3, they can't do those structuring games. Absolutely they can, but students thinking at a construct 3 is at a place where it is all clicking and they're understanding strategies. It is really important that if they get too much structuring jumping to strategies before getting the *counting experiences then sometimes they can get holes that come back again in multiplication and division and struggles with composite thinking.*

Jessica: I am excited….

Interviewer: I know, look at that! — Yeah.

Jessica: Kaden is moving up too….I am excited …no more 0…color wise I still feel there is a big
Interviewer: Now with students at a Construct 2, it will be good to look at their assessment because many are close to construct 3. They are counting on and it really is just subtraction.

J: Subtraction. We really are working on the subtraction unit. Ao if I want to regroup my kiddos should I use this data?

Interviewer: Yes - Yes
so to regroup use this one….
going over specific students and their fantastic growth and improvement
Even moved up — (14:33)
I: shares some strategies for distancing the setting during a game …..
J: shares how she is using distancing the setting currently 15:12
Jenally is still struggling
I: all of these guys especially - all the number talks and different strategies are for sure making sense
sharing data for even and grant
J: ____ is amazing….it is almost in number corner that “oh buddy…you can’t be doing this to some of my kids right now not even fair
I yeah
going over quickly what the numbers mean…..
BNWS - being in the yellow if a 4…..district has need to get them to be 5s
going over NID — reading and writing numbers
I: And even if you are not in the small group mode with the rotations with um in these groupings you can still be pretty …with the number choices and whether they are or they are not….picking games
J: It these kiddos are the ones in the green that are picking the high numbers because they want to challenge themselves with more challenging numbers…the ones in the yellow…its funny I’m doing that Jan Jo Boaler
I: oh yeah - Jo Boaler
and she had that comment about the students that you give positive affirmation to they will choose the number…how did that go…they will choose numbers that are higher to challenge themselves….but the ones that you say oh you are so smart your so intelligent they want to maintain that mindset and so they choose numbers that they know they can do. so that they are always getting them right. That they are continuing to be smart. which is really interesting….I have 2 kiddos in particular that huh that makes sense now so i

have been able to push them a little more than i normally would but structuring — which test is that again
discussing the structuring….

structuring as a assessment Jessica can do structuring when
discussing what the different assessments are……

J: Is this something that would be really beneficial for me to do the beginning of the year next year
I: Yes
J: cause I feel like that's the one thing i am stressing — holy moly - it is already 9:00
I: sorry
J no its fine
I was like this has been so nice being a new teacher having all this data and like I am super pumped because I will be able to my goal is to assess all of them again in these so that I can pass them along to the next teachers and but I was like at the beginning of next year it was going to be tough doing all that if I can do this and I can get all of these by just doing this test that would be huge right?
I: Yes - absolutely
J: Then to create my groups I would want to be going off my addition/subtraction or my structuring
I yes - because this one takes precedence…..let's say they cannot subtract

Discussing how FNWS, BNWS is related to Add/Subr

I think with some of my kiddos I think i would have benefited from doing some NID stuff right away like with all of them like as a refresher…
I: Yes
So when we do intervention…..an intervention cycle will include more FNWS, BNWS, NID right away …..
J: Is there an actual intervention - like protocol….how do you play that game
    discussion about how to get MRIS training — what it's called and how to get connected to training\
Discussion about how to use green book for small groups in the classroom.
J: so this you want to push them up…..

independent vs instructional level …

Appendix F – Works Cited

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