Essays on terms of trade

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Dedication

To my mother and grandmother. Thank you for always believing in me.

..no tenemos saca

Abstract

This dissertation consists of three chapters. The main topic I study through them is fluctuations in terms of trade and their effects in small open economies. In particular, I study their effects in the labor and financial markets.

In Chapter 1, I study the how the labor market and real GDP interact to fluctuations in the terms of trade. Conventional wisdom suggests that when terms of trade deteriorate in small open economies, real GDP decreases. In this paper, I document that the correlation between terms of trade and real GDP innovations varies widely between both positive and negative values across different countries. Furthermore, I show that this variation cannot be explained by income alone. This raises the question, why do countries' real GDPs react differently toward changes in their terms of trade? I show evidence that the way in which countries react toward changes in terms of trade is linked to the labor market. I build a real business cycle model in which a small open economy experiences terms of trade fluctuations in conjunction with real wage rigidity. I find that an economy with a real wage rigidity is able to produce either a positive or a negative correlation between terms of trade and real GDP innovations.

In Chapter 2, Sora Lee and I study the relationship between terms of trade fluctuations and changes in the sovereign government interest rate spreads in emerging economies. We propose a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. We incorporate two types of shocks to cover a foreign and a domestic uncertainty. We define as the domestic and the foreign uncertainty, GDP and terms of trade shock, respectively. The model is able to successfully increase the dispersion of sovereign interest rates when GDP

shocks are above the trend. This result seems to suggest that terms of trade is a good candidate to explain the volatility of interest rates in small open economies when they are not under recessions or crises.

In Chapter 3, I study the optimal choice of foreign issued debt incurred by a sovereign government in a small open economy. In particular, what are the shortcomings and benefits from issuing debt in a currency tightly linked to a trade partner country. I propose a two period model with uncertainty in the terms of trade and in a real exchange rate to show the benefits and costs. I find that when a government is in a deep recession, issuing debt in a currency that is not tightly linked to their trade becomes optimal.

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Chapter 1

Terms of trade and unemployment in the business cycle

1.1 Introduction

Conventional wisdom suggests that when terms of trade deteriorate, small open economies are negatively affected. This deterioration implies a contraction of available resources. Thus, the country becomes poorer, making macroeconomic indicators such as real GDP fall. This notion is summarized in Easterly, Islam and Stiglitz (2001) which states, "For small open economies, adverse terms of trade shocks can have much the same effect as negative technology shocks, and this is one of the important differences between macroeconomics in these economies and that which underlies some of the traditional closed economy models."

In this paper, I argue that the conventional narrative regarding terms of trade is not entirely consistent with the data. I document that the shock correlation between terms of trade and real GDP varies widely between positive and negative values across 122 countries. Furthermore, this variation is not explained by income levels alone. This raises the question, why do countries react differently toward terms of trade shocks? I show evidence that the way in which countries react toward terms of trade shocks is linked to their labor markets. Specifically, when unemployment reacts positively to increases in the terms of trade, real GDP reacts negatively.

Modeling the relationship between real GDP and terms of trade is not trivial. Kehoe and Ruhl (2008) show that standard models fail to generate a negative correlation between terms of trade and real GDP innovations. This is because an economy becomes poorer when facing an adverse terms of trade shock. With standard elasticities, this increases employment, and, as a result, real GDP increases. Because conventional wisdom suggests small open economies are negatively affected by deteriorating terms of trade, the literature has only focused on addressing what type of assumptions or frictions generate this negative relation. In this paper, I propose a mechanism that is able to account for not only a negative, but also a positive correlation between terms of trade and real GDP innovations. I build a real business cycle model in which a small open economy experiences terms of trade fluctuations in conjunction with real wage rigidity.

The model illustrates how the effect of an adverse terms of trade shock on labor and real GDP in a small open economy is determined by whether or not the real wage rigidity is binding. Given an adverse terms of trade shock, if the real wage rigidity does not bind, labor and real GDP increase in equilibrium. On the other hand, when the real wage rigidity does bind, firms hire less labor, increasing unemployment and decreasing labor and real GDP.

1.2 Kehoe-Ruhl Observation

It has been of particular interest to study how terms of trade influence small open economies. Mendoza (1995) finds that terms of trade shocks[1] tend to be large, persistent, and weakly procyclical accounting for nearly half of real GDP fluctuations. Moreover, in Mendoza (1997) states that for industrial and developing countries, volatility in the terms of trade has a large and adverse effect on economic growth. Expanding to these results, Kose (2002) finds that international prices fluctuations are the main driver of economic volatility in developing economies. One of the main sources of influence terms of trade affect an economy is via the exchange rates. In De Gregorio and Wolf (1994) show that the real exchange rate fluctuations is mainly driven by the volatility of the terms of trade. Expanding to this notion, Broda (2004) shows that countries with fixed exchange rate regimes experience larger contractions in real GDP.

The highlighted relationship between terms of trade changes and movements in real GDP is difficult to replicate theoretically though. Conventional wisdom suggests that real GDP should fall after a terms of trade deterioration. This was challenged in Kehoe and Ruhl (2008) by explaing why standard models with terms of trade shocks fail to generate changes in real GDP consistent with the data. There are ways to enrich the standard model to replicate falls in real GDP due to terms of trade deteriorations. Using Greenwood-Hercowitz-Huffman pref-

[1] In this paper, terms of trade is defined as the ratio of export to import unit values, measuring at constant import prices. In this sense, a deterioration of the terms of trade will be reflected as a fall in the terms of trade.

erences solve the issues because wealth effects are unexistant. In this way, when having negative terms of trade shocks, households are not willing to supply more labor even though they become poorer. Another way, de Soyres (2016) proposes a model with monopolistic competition and love for variety to generate falls in real GDP via contractions in producers profits when terms of trade deteriorate. Also, Costa (2017) proposes a model where natural resources are taken into consideration in the household preferences, generating falls in real GDP when terms of trade deteriorate due to idle resources. Finally, Benguria, Saffie and Urzúa (2018) uses a downward nominal rigidity and a fixed exchange rate regime to generate contractions in real GDP via falls in international prices when the rigidity binds.

In this paper I explore deeper this last mechanism. Specifically, I focus in the ability for wage rigidities to create opposite effects in real GDP for the same change in the terms of trade. To understand better this observation, let me illustrate it with a simple example.

1.2.1 Standard Model

Consider a production function $F(\cdot,\cdot,\cdot)$ owned by a representative firm that takes as inputs capital K, labor L, and foreign intermediate inputs M, satisfying INADA conditions. Let the technology be indexed by z, a productivity parameter. The firm rents capital at a price r, hires labor at a wage w, and buys foreign intermediate inputs at a price p. The price of p will represent the terms of trade.

Then, the representative firm's profit maximization problem is

$$\begin{aligned}
&\operatorname*{Max}_{Y,K,L,M} \{Y - rK - wL - pM\} \\
&\text{s.t. } Y \leq zF(K, L, M) \\
&Y, K, L, M \geq 0
\end{aligned}$$

This yields the following first order conditions,

$$\hat{r}(p) = z\frac{\partial F}{\partial K}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right), \qquad \hat{w}(p) = z\frac{\partial F}{\partial L}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right),$$

$$\text{and} \qquad p = z\frac{\partial F}{\partial M}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right).$$

Here, $\hat{K}(p)$ the optimal amount of capital stock rented, $\hat{L}(p)$ the optimal amount of labor hired, and $\hat{M}(p)$ the optimal amount of foreign intermediate inputs bought. Also, $\hat{r}(p)$ the equilibrium return to capital and $\hat{w}(p)$ the equilibrium real wage. Now, let us compute real GDP taking $p_0 > 0$ as the base year price,

$$GDP(p) \equiv \hat{Y}(p) - p_0\hat{M}(p) = zF(\hat{K}(p), \hat{L}(p), \hat{M}(p)) - p_0\hat{M}(p).$$

Notice that to compute real GDP, a base year price must be used. Computing the derivative with respect of terms of trade, we find that

$$\begin{aligned}
GDP'(p) = z\frac{\partial F}{\partial L}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right)\hat{K}'(p) + z\frac{\partial F}{\partial L}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right)\hat{L}'(p) \\
+ z\frac{\partial F}{\partial M}\left(\hat{K}(p), \hat{L}(p), \hat{M}(p)\right)\hat{M}'(p) - p_0\hat{M}'(p) \\
= \hat{r}(p)\hat{K}'(p) + \hat{w}(p)\hat{L}'(p) + (p - p_0)\,\hat{M}'(p).
\end{aligned}$$

Now, note that the capital stock is a decision settled in the previous period, $\hat{K}(p) = \overline{K}$. This is, the realization of current terms of trade p only has the influence in the next period's capital stock. Therefore, current equilibrium capital stock does not change with how the realization of terms of trade, $\hat{K}'(p) = 0$. Also, if changes in the terms of trade p are small and around the base year p_0, then we have that $p \approx p_0$. Therefore, the terms of trade first order effects in real GDP can be approximated as

$$GDP'(p) \approx \hat{w}(p)\hat{L}'(p). \tag{1.1}$$

With an inelastic supply of labor $\hat{L} = \overline{L}$, the optimal amount of labor is fixed regardless of the terms of trade, and thus $\hat{L}'(p) = 0$. Nevertheless, when relaxing this assumption, the optimal labor can change. Specifically, the optimal labor changes because the terms of trade changes the equilibrium real wage. For standard production functions, it is normal to find that real wages fall when there is a deterioration of the terms of trade, $\hat{w}'(p) < 0$. In addition, for standard elasticity of substitutions from the household preferences between consumption and leisure, it is common to find $\hat{L}'(p) > 0$. This is, labor increases in equilibrium for an increase in the terms of trade because households become poorer due to the fall in real wages. In other words, changes in the real GDP do not move in the opposite direction to the changes in the terms of trade because $GDP'(p) \geq 0$. This is the Kehoe-Ruhl observation, standard models are unable to replicate the negative relationship between changes in real GDP and changes in terms of trade that the data shows for some countries.

1.2.2 Wage Rigidity Mechanism

Now, let us assume there is a friction in the labor market, the real wage cannot fall lower than $\overline{w} > 0$. When the equilibrium real wage is such that $\hat{w}(p) \geq \overline{w}$, then real GDP and labor behaves the same way as the frictionless environment described in the previous subsection. Nonetheless, this relationship changes when the real wage rigidity binds. When the real wage rigidity binds, the equilibrium real wage is fixed, $\hat{w}(p) = \overline{w}$. Using the first order conditions for labor and foreign intermediate inputs and fixing the equilibrium current capital stock level to $\hat{K}(p) = \overline{K}$, the following relationship holds

$$\hat{L}'(p) = \left(\frac{p\frac{\partial^2 F}{\partial L \partial M}\left(\overline{K}, \hat{L}(p), \hat{M}(p)\right) - \overline{w}\frac{\partial^2 F}{\partial M^2}\left(\overline{K}, \hat{L}(p), \hat{M}(p)\right)}{\overline{w}\frac{\partial^2 F}{\partial M \partial L}\left(\overline{K}, \hat{L}(p), \hat{M}(p)\right) - p\frac{\partial^2 F}{\partial L^2}\left(\overline{K}, \hat{L}(p), \hat{M}(p)\right)} \right) \hat{M}'(p).$$

Because the production function $F(\cdot, \cdot, \cdot)$ satisfies the INADA conditions, we know that $\frac{\partial^2 F}{\partial L^2}$, $\frac{\partial^2 F}{\partial M^2} < 0$ and $\frac{\partial^2 F}{\partial L \partial M}$, $\frac{\partial^2 F}{\partial M \partial L} > 0$. Therefore, how the optimal demand of labor reacts towards terms of trade shares the same direction as how optimal foreign intermediate inputs react to terms of trade. In addition, with standard elasticities of substitution in the production function, the optimal foreign intermediate inputs react negatively towards terms of trade, $\hat{M}'(p) < 0$. When the real wage rigidity is binding, the equilibrium labor is demand determined. Therefore, because households are not in their labor supply curve, the equilibrium labor falls when terms of trade deteriorate. Furthermore, the household preferences will define a gap between the equilibrium labor and the desired labor supplied in the economy. This disparity describes the unvoluntary unemployment in equilibrium. Because equation (1.1) holds as well in this case, real GDP decreases when terms of trade deteriorate.

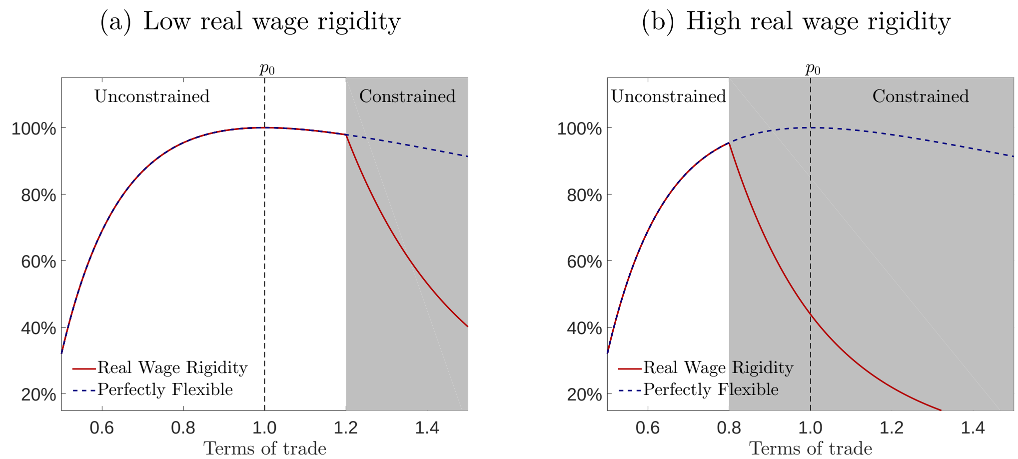

Figure 1.1: Closed form solution real GDP from deterministic example

Note: Deterministic example solution for real GDP using the following parameters: $\alpha_K = 0.10$, $\alpha_L = 0.45$, $\alpha_M = 0.45$, $\overline{K} = 1$, $\overline{L} = 1.00$, $z = 1.00$, $p_0 = 1.00$, $\overline{w}_{low} = 0.2017$, and $\overline{w}_{high} = 0.2812$. The two real wage rigidities were picked so the real wage constraint becomes binding at $\pm 0.20\%$ from the base terms of trade price p_0. The real GDP is deflated level is deflated by the real GDP obtained when there is no real wage rigidity and the terms of trade is at base $p_0 = 1$.

Figure 1.1 illustrates how the mechanism affect real GDP. I assume a constant returns to scale production function $F(K, L, M) = K^{\alpha_K} L^{\alpha_L} M^{\alpha_M}$ and an inelastic labor supply of labor $\overline{L}$. Both figures show with a dashed line how real GDP moves as terms of trade changes in a frictionless environment. The important thing to notice is that at the base price terms of trade, the slope of real GDP is zero. In other words, relatively small changes around the base year terms of trade yield approximately no changes in real GDP. Panel 1.1(a) shows what happens when there is a low real wage rigidity. In this case, small changes in the terms of trade around the base year give the same results as the frictionless case. In other words, the terms of trade has to deteriorate substantially in order for the real wage to fall below the real wage constraint. Thus, for small real wage rigidities, the economy behave identically to the frictionless economy. In the other hand, when the real wage rigidity is high enough, real GDP falls when terms of trade deteriorate. Panel 1.1(b) shows this case. When there is an increase in the terms of trade, under full-employment the amount of foreign intermediate inputs decreases. When this input decrease, the marginal productivity of labor falls, depressing real wages. If the real wage falls below the real wage rigidity, then labor will decrease in order to increase the marginal utility of labor and satisfy the real wage constraint. This effect guarantees falls in real GDP when terms of trade deteriorate.

1.3 Empirical Analysis

I use the World Bank's World Development Indicator (WDI) database for the empirical analysis. From it, I extract the series of GDP, exports, and imports in current and constant prices in local currency units. The analysis will be done yearly from 1980 to 2017. In order to consider a country inside my sample, there must be at least 20 consecutive years of observations. There are 122 countries that meet this criteria.

I classify my sample countries into four different groups: low income, mid-low income, mid-high income, and high income countries. Table 1.1 shows all the countries in the sample with its respective classification. I find 29 low income countries, 32 mid-low income countries, 31 mid-high income countries, and 30 high income countries. This classification is consistent and resembles significantly the one done by the World Bank.

Table 1.1: Sample classification of countries by income

Low Income		Mid-Low Income		Mid-High Income		High Income	
Bangladesh	Mozambique	Albania	Honduras	Argentina	Lebanon	Australia	Israel
Benin	Nicaragua	Algeria	Indonesia	Botswana	Lithuania	Austria	Italy
Burkina Faso	Pakistan	Armenia	Jordan	Brazil	Malaysia	The Bahamas	Japan
Cambodia	Rwanda	Azerbaijan	Macedonia	Bulgaria	Mauritius	Belgium	Luxembourg
Cameroon	Senegal	Belarus	Morocco	Chile	Mexico	Brunei	Macao
Congo Dem.	Sierra Leone	Belize	Namibia	Colombia	Panama	Canada	Netherlands
Gambia	Sudan	Bolivia	Nigeria	Costa Rica	Poland	Cyprus	New Zealand
Haiti	Tajikistan	Congo Rep.	Paraguay	Croatia	Portugal	Denmark	Norway
India	Tanzania	Cuba	Peru	Czech Rep.	Romania	Finland	Singapore
Kenya	Togo	Dominican Rep.	Philippines	Estonia	Russia	France	Slovenia
Kyrgyz Rep.	Uganda	Ecuador	Serbia	Gabon	Slovak Rep.	Germany	Spain
Madagascar	Uzbekistan	Egypt	Sri Lanka	Hungary	South Africa	Greece	Sweden
Mali	Vietnam	El Salvador	Thailand	Iran	Turkey	Hong Kong	Switzerland
Mauritania	Zimbawe	Eq. Guinea	Tunisia	Kazakhstan	Uruguay	Iceland	United Kingdom
Moldova		Eswatini	Ukraine	Korea	Venezuela	Ireland	United States
		Guatemala	West Bank Gaza	Latvia			

Note: To do the classification I use GDP-per-capita in constant 2010 US dollars from WDI database. Every year I compute the quartiles with respect of this series and establish these values as the limits between each one of the groups defined above. I then label each country depending which group dominated the most throughout the years of the sample.

The prevalent practice is to define terms of trade as the percentage ratio of unit value indexes of exports over imports. The WDI trade-weights these world price indexes specifically for every single country. I deviate from this and use another common characterization of terms of trade. I follow the definition of terms of trade as the ratio of price deflators between imports and exports. This is, for every year, I construct the series as

$$p_t \equiv \frac{M_t^{current}/M_t^{constant}}{X_t^{current}/X_t^{constant}},$$

where M_t^i stands for imports and X_t^i for exports in year t at local currency units where i represent current and constant prices.

The narrative of terms of trade is that small open economies are too small to offset the aggregate world supply and demand of the product they trade. In other words, small open economies take as given international prices. Under that narrative, it is common to assume that terms of trade shocks of every country follow an AR(1) process independent from its macroeconomic indicators. To obtain the terms of trade shocks, I take an HP-filter with a smoothing parameter of 100 to its log-series. Let $\tilde{p}_t$ represent the residuals between the original log-series and its trend. I consider the following process for terms of trade in every country

$$\tilde{p}_{t+1} = \rho_p \tilde{p}_t + \sigma_p \varepsilon_{t+1}, \qquad \text{where} \quad \varepsilon \sim N(0,1).$$

Table 1.2 shows the average of the autoregressive analysis of terms of trade residuals. The average autocorrelation for all the countries in our sample is 0.339. For high income countries, terms of trade shock tend to be a bit more persistent compared to mid-low and mid-high income countries. This value tells us that terms of trade shocks die quickly. In the literature, a standard value for this parameter is around 0.50. Using log-quadratic detrending recovers this value. Another feature of the analysis is that the volatility of the terms of trade shocks decreases as we move across groups with higher income. The volatility excluding high income countries is around 7.57%, a standard deviation consistent with the literature.

We now include other macroeconomic aggregate variables and see how terms of trade correlates with them. Table 1.3 shows the log-series HP-filter residuals using the same smoothing parameter as before. Let us start with how terms of trade correlates with real GDP. For low income, mid-low income, and high income countries, the correlation of real GDP is close to zero. Nevertheless, we

Table 1.2: Terms of trade residuals autorgressive analysis

Terms of Trade AR(1)	Low Income	Mid-Low Income	Mid-High Income	High Income	All Countries
ρ_p	0.291	0.322	0.339	0.404	0.339
σ_p	0.096	0.075	0.056	0.027	0.063
R_p^2	0.148	0.145	0.157	0.192	0.160

find a negative relationship of -0.164 for mid-high income countries. This value is consistent with the latest studies done for emerging economies. In the other hand, there is a small negative relationship of 0.016% for high income countries. Another feature we find is that for high income countries, investment does not correlate with investment series. Nevertheless, for the rest of the countries, there is a marked negative relationship between these two variables.

Table 1.3: Terms of trade and macroeconomic indicators correlations

Correlations with Terms of Trade	Low Income	Mid-Low Income	Mid-High Income	High Income	All Countries
GDP	-0.071	0.047	-0.164	-0.016	-0.050
C	-0.141	-0.044	-0.244	-0.147	-0.143
I	-0.261	-0.081	-0.218	-0.131	-0.170
NX	-0.279	-0.362	-0.123	-0.175	-0.236
L	0.070	-0.069	-0.098	-0.104	-0.052
U	-0.054	0.016	0.133	0.082	0.045

Figure 1.2 shows what in the data (1.1) represent for all the countries in the sample. Realize that the variation among countries in how terms of trade correlate real GDP and labor[2] related variables is diverse. Focus in Panel 1.2(a)

[2]The variables I consider are employment-popluation ratio and unemployment rate. I con-

and firstly realize that the variation between of terms of trade and real GDP. This correlation is not nearby the averages we found in the statistics table. This help us conclude that focusing in the averages among countries is misleading. This aggregation issue becomes more evident when we analyze the correlation between labor and terms of trade. Suggesting that this variation is close to zero does not reflect the different type of effect different countries have. Finally, we find that the relationship (1.1) is clear. How real GDP react to terms of trade is highly linked in how labor react to terms of trade. In Panel 1.2(b) we see the same how the correlation between terms of trade and real GDP is linked with how unemployment reacts to terms of trade. As we expected from the previous plot, the relationship flips. The correlation between terms of trade and real GDP becomes more positive as the correlation between terms of trade and unemployment increases.

1.3.1 Real Wage Rigidity

There are many ways to motivate a real wage rigidity. In Schmitt-Grohé and Uribe (2016) they propose that in the presence of nominal rigidities, having a PEGed currency or having rigidities in prices creates a real wage rigidity. I use this narrative to illustrate two country experiences, Greece and Ecuador. Figure 1.3 shows the correlation between terms of trade and real GDP for these two countries. Panel 1.3(a) shows the experience of Ecuador. In the year of 2000, Ecuador sacrificed their monetary autonomy adopting the dollar as its national currency. Before this experience, the correlation between terms of trade and

sider only variations over employment-population ratio and not hours worked due to data restrictions.

(a) Employment rate (b) Unemployment rate

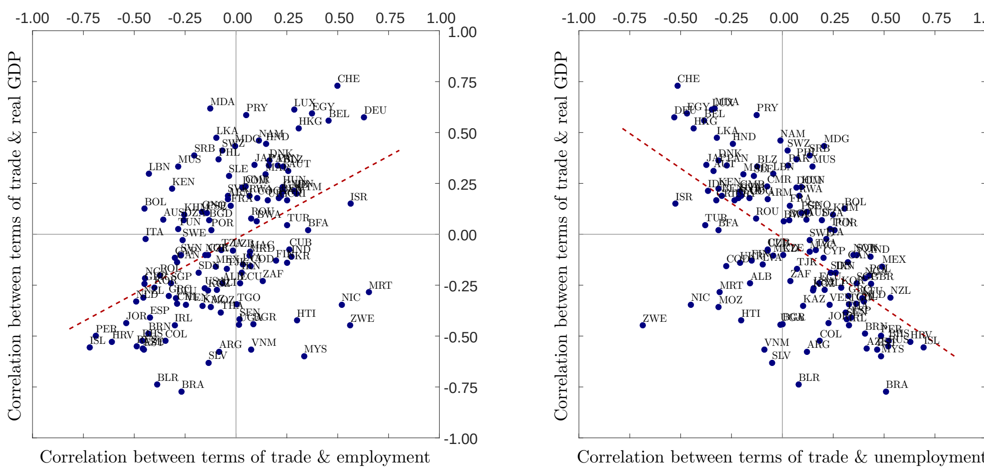

Figure 1.2: Relationship between terms of trade fluctuations, real GDP, and the labor market

Note: The correlations for each country are from 1991 to 2017. Real GDP comes from the WDI database in local currency at constant prices. The terms of trade series are constructed as the ratio between imports and exports price deflators using the WDI database also. Employment is represented as the employment to 15+ population ratio, meanwhile unemployment is represented by the unemployment rate modeled by the ILO database. All series are log HP-filtered using a smoothing parameter of 100.

real GDP was 0.112 while after it changed to -0.177. In the other hand, we see in Panel 1.3(b) what occurred to Greece. Greece the year 2000 entered the European monetary union. In this way they sacrificed their monetary autonomy by adopting the euro. Before this, the correlation between terms of trade and real GDP was 0.366, while after it moved to -0.408. Both examples experienced a deep change in the direction of how their economies reacted to terms of trade changes.

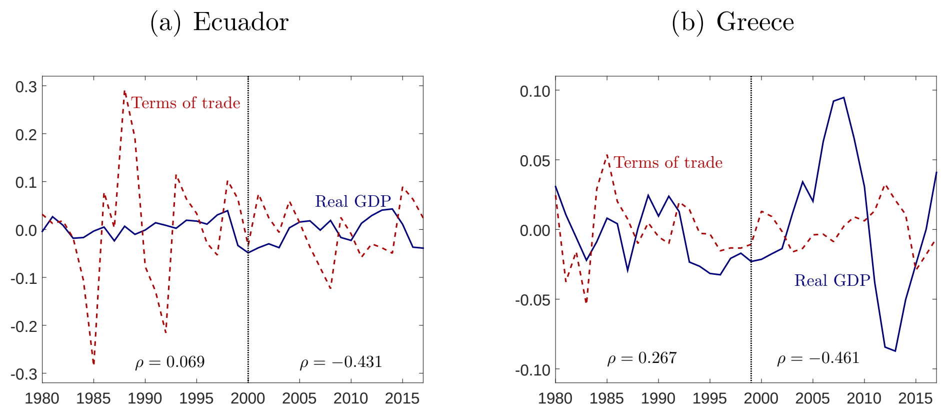

Figure 1.3: Ecuador and Greece correlation between terms of trade and real GDP

1.4 Model

I study a real business cycle model where a small open economy model is subject to exogenous terms of trade shocks. In the economy, there is only one firm producing final tradable goods. The final good firm imports foreign intermediate inputs paying a terms of trade price.

1.4.1 Households

Households' preferences over consumption are given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \; u(c_t, l_t) \right], \tag{1.2}$$

where $u(\cdot, \cdot)$ is the period utility function, c_t denotes private consumption in period t, l_t denotes the amount of hours supplied to work in period t, $\beta \in (0, 1)$ is the subjective discount factor, and $\mathbb{E}_0$ denotes the expectation operator conditional on the information set available at time 0. The period utility function $u(\cdot, \cdot)$ satisfy INADA conditons with respect of consumption, and with respect of labor it is only strictly decreasing.

Households have the option every period in investing in capital stock. The capital accumulation law of motion follows the rule

$$k_{t+1} = x_t + (1 - \delta)k_t, \tag{1.3}$$

where x_t represent the investment incurred in period t, k_t the capital stock accrued up to period t, and $\delta \in (0, 1)$ the depreciation rate of capital.

Each period, households spend c_t in final consumption and x_t in investment. Households' labor income is $w_t l_t$, where w_t is the real wage in period t. Also, households' capital income is $r_t k_t$ where r_t is the real return to capital payment. In addition, households' receive the profits from the representative firm π_t. Finally, $\Phi(\cdot)$ represent an adjustment cost function penalizing changes in capital. This capital adjustment cost function is a non-negative, increasing, and convex function, where $\Phi(0) = 0$. Considering all these, the households' budget constraint in terms of final goods is therefore given by

$$c_t + x_t + \Phi\left(k_{t+1} - k_t\right) = w_t l_t + r_t k_t + \pi_t. \tag{1.4}$$

The households' problem consists of choosing final consumption goods, investment, labor supply, and capital stock $\{c_t, x_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize (1.2) given the sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, profits $\{\pi_t\}_{t=0}^{\infty}$ and an initial capital stock $k_0 > 0$; subject to (1.3) and (1.4). In other words, the households' problem can be expressed as

$$\begin{aligned}
\underset{\{c_t, x_t, l_t, k_{t+1}\}_{t=0}^{\infty}}{\text{Max}} & \left\{\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)\right]\right\} \\
\text{s.t. } & c_t + x_t + \Phi\left(k_{t+1} - k_t\right) \leq w_t l_t + r_t k_t + \pi_t \\
& k_{t+1} \leq x_t + (1-\delta) k_t \\
& c_t, x_t, l_t, k_{t+1} \geq 0
\end{aligned}$$

The optimality conditions of this problem are denoted by the following intertemporal and intratemporal conditions,

$$\beta \mathbb{E}_t\left[\left(\frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)}\right)\left(\frac{(1-\delta) + r_{t+1} + \Phi'\left(k_{t+2} - k_{t+1}\right)}{1 + \Phi'\left(k_{t+1} - k_t\right)}\right)\right] = 1 \tag{1.5}$$

$$\frac{u_l\left(c_t, l_t\right)}{u_c\left(c_t, l_t\right)} = w_t. \tag{1.6}$$

1.4.2 Representative Final Good Firm

There is a representative firm that produces a final good in the economy every period t. To produce this good, the firm hires labor L_t at a real wage cost w_t, rents capital from the households K_t at a cost r_t, and buys foreign intermediate inputs at a terms of trade price p_t. The technology of the firm will be characterized by a production function $F(\cdot,\cdot,\cdot)$ that satisfies INADA conditions with a productivity z_t. Therefore, the period-by-period profit maximization problem can be described as

$$\begin{aligned}\pi_t = \underset{Y_t,K_t,L_t,M_t}{\text{Max}} & \left\{Y_t - w_t L_t - r_t K_t - p_t M_t\right\} \\ \text{s.t. } & Y_{F,t} \leq z_t F\left(K_t, L_t, M_t\right) \\ & Y_t, L_t, K_t, M_t \geq 0\end{aligned}$$

The representative firm's optimality conditions can be described as

$$z_t F_K\left(k_t, L_t, M_t\right) = r_t \tag{1.7}$$

$$z_t F_L\left(k_t, L_t, M_t\right) = w_t \tag{1.8}$$

$$z_t F_M\left(k_t, L_t, M_t\right) = p_t \tag{1.9}$$

1.4.3 Labor Market & Unemployment

The labor market will feature a real wage floor $\overline{w} \in \mathbb{R}_+$ that must be satisfied every period. This real wage floor will allow an excess supply of labor making involuntary unemployment exist in equilibrium. This real wage floor has been motivated in Schmitt-Grohé and Uribe (2016), where they explain that a presence of a downward nominal wage rigidity and a monetary policy aimed not to achieve full-employment can yield a real wage floor in the economy. Regarding the amount of labor exchanged in the economy, it must follow that the amount of labor demanded in the economy must not exceed its supply. In other words, the conditions $w_t \geq \overline{w}$ and $l_t \geq L_t$ must be satisfied in every period t. Using these, the labor market equilibrium implies that the following slackness condition must hold for all periods,

$$(w_t - \overline{w})\,(l_t - L_t) = 0. \tag{1.10}$$

This slackness condition joins both conditions assuring that the labor market will experience full-employment only if the real wage is above the real wage floor.

1.4.4 Unemployment Mechanism

In Figure 1.4 I show a graphical represenation of how a negative shock in the terms of trade can deliver different changes in the labor market. A negative shock in the terms of trade can be seen as an increase in p. Because $F(\cdot,\cdot,\cdot)$ satisfy INADA conditions, an increase in p imply a decrease in M by using (1.9). Now, suppose we start with a terms of trade p_0 and capital level K_0. In that sense, we start in the equilibrium A, with labor L_0 and real wages w_0. Now, imagine there is an increase in terms of trade $p_1 > p_0$, implying a decrease in intermediate foreign inputs $M_1 < M_0$ and in consumption for the households $c_1 < c_0$. This movement makes the supply of labor increase and the demand of labor decrease. Thus we reach the new equilibrium B with labor L_1 and real wage w_1. Nevertheless, if the new wage is below real wage floor $\overline{w} > 0$, equilibrium B will not be achieved because the real wage is too low. In this case, the economy will move to equilibrium C with labor L_2 and real wage w_2. Realise that in this case the real wage in equilibrium will be the floor $w_2 = \overline{w}$ and with this wage the households are willing to supply labor L_3. This discrepancy between the supply and demand of labor yields unemployment $L_3 - L_2$ in equilibrium. The important feauture of the model is that the presence of the real wage rigidity yields two different movements in labor when the real wage constraint is binding or not. If the real wage constraint is not binding, then an increase in the terms of trade will yield an increase in labor. Nonetheless, if the real wage constraint binds with the change of the terms of trade, labor will decrease in equilibrium. These opposite movements in labor affect the movements in real GDP producing the two different movements in real GDP when terms of trade changes.

1.4.5 Recursive Characterization

To solve the decentralized equilibrium, I consider the recursive form of the economy. To set the problem, it is necessary to divide the aggregate and individual states in the economy. The aggregate states are the total amount of capital stock accumulated in the economy K, the terms of trade shock p, and the productivity shock z. For tractability, define the state $s = (K, p, z)$ as the aggreate states in the economy.

The households are unable to see how their individual decisions affect the equilibrium prices or the aggregate states of the economy. Therefore, the households need to consider an individual state of capital stock in the economy k. Moreover, the optimal investment decision will imply a future individual capital stock k'. To find this stock, the households need to forecast the aggregate states in the economy. The terms of trade shock is assumed to follow an exogenous stochastic process. The aggregate capital is forecasted next period with the aggregate policy $\mathcal{K}(s)$ under rational expectations. In addition, the households need to also forecast what is the current level of aggregate labor available in the economy. The aggregate labor available is currently forecasted with the aggregate policy

$\mathcal{L}(s)$ under rational expectations. This aggregate level of labor in the economy will work as an upper boundary for what the household is able to supply of labor. With this, the maximization problem of the households can be expressed as

$$V(k,s) = \underset{c,x,l,k'}{\mathrm{Max}} \left\{ u\left(c,l\right) + \beta \mathbb{E}\left[V(k',s')\right] \right\} \tag{1.11}$$
$$\begin{aligned} \text{s.t. } & c + x + \Phi\left(k'-k\right) \leq w(s)l + r(s)k + \pi(s) \\ & k' \leq x + (1-\delta)k \\ & K' = \mathcal{K}(s) \\ & l \leq \mathcal{L}(s) \\ & c, x, l, k' \geq 0 \end{aligned}$$

The solution of the households' problem will yield a policy rule for consumption $\hat{c}(k,s)$, labor $\hat{l}(k,s)$, investment $\hat{x}(k,s)$, and future capital stock $\hat{k}(k,s)$.

Definition 1 (Decentralized Recursive Competitive Equilibrium) *A decentralized recursive competitive equilibrium with real wage rigidity $\overline{w}$ is defined as a value function $\{V(k,s)\}$, a set of price policies $\{w(s), r(s)\}$, households' policies $\left\{\hat{k}\left(k,s\right), \hat{l}\left(k,s\right), \hat{x}\left(k,s\right), \hat{c}\left(k,s\right)\right\}$, households' desired labor supply policy $\{\hat{h}(s)\}$, firm policies $\left\{\hat{K}\left(s\right), \hat{L}\left(s\right), \hat{M}\left(s\right), \hat{Y}\left(s\right), \hat{\pi}(s)\right\}$, and the rational expectations policies $\{\mathcal{K}(s), \mathcal{L}(s)\}$; such that the following conditions are satisfied:*

1. *Taking the aggregate state $s = (K,p,z)$ and the price policies $\{w(s), r(s)\}$, the profits policy $\{\pi(s)\}$, and the aggregate forecast policies $\{\mathcal{K}(s), \mathcal{L}(s)\}$ as given; the value function $\{V(k,s)\}$ and the households policies $\left\{\hat{k}\left(k,s\right), \hat{l}\left(k,s\right), \hat{x}\left(k,s\right), \hat{c}\left(k,s\right)\right\}$ solve the problem (1.11).*

2. *Taking the aggregate state $s = (K,p,z)$, the wage policy $\{w(s)\}$, and the*

consumption policy $\{\hat{c}(k,s)\}$*; the households' desired labor supply policy* $\{\hat{h}(s)\}$ *satisfies*

$$w(s) = \frac{u_l\left(\hat{c}\left(K,s\right),\hat{h}(s)\right)}{u_c\left(\hat{c}\left(K,s\right),\hat{h}(s)\right)}$$

3. *Taking the aggregate state* $s = (K,p,z)$ *and the aggregate labor forecast policy* $\{\mathcal{L}(s)\}$*, the price policies* $\{w(s),r(s)\}$ *and the representative firms policies* $\left\{\hat{M}(s),\hat{Y}(s),\hat{\pi}(s)\right\}$ *satisfy*

$$\begin{aligned} w(s) &= zF_L\left(K,\mathcal{L}(s),\hat{M}(s)\right) \\ r(s) &= zF_K\left(K,\mathcal{L}(s),\hat{M}(s)\right) \\ p &= zF_M\left(K,\mathcal{L}(s),\hat{M}(s)\right) \\ \hat{Y}(s) &= zF\left(K,\mathcal{L}(s),\hat{M}(s)\right) \\ \hat{\pi}(s) &= \hat{Y}(s) - r(s)K - w(s)\mathcal{L}(s) - p\hat{M}(s) \end{aligned}$$

4. *The goods market clear*

$$\hat{Y}(s) = \hat{c}(K,s) + \hat{x}(K,s) + \Phi\left(\mathcal{K}(s) - K\right) + p\hat{M}(s)$$

5. *The labor market clears*

$$w(s) \geq \overline{w}, \qquad \hat{h}(s) \geq \mathcal{L}(s), \quad \text{and} \quad (w(s)-\overline{w})\left(\hat{h}(s)-\mathcal{L}(s)\right) = 0$$

6. *The rational expectations forecasts are consistent with private optimal deci-*

sion rules by the households

$$\mathcal{K}(s) = \hat{k}(K, s) \qquad \textit{and} \qquad \mathcal{L}(s) = \hat{l}(K, s)$$

1.5 Quantitative Analysis

In this section, I pick the functional forms and values to the parameters of the model. I solve the model numerically iterating over the optimal policy rules of capital and labor satisfying the equilibrium conditions of the households, firms, and market clearing. When solving for the optimal policy rules inside the grids, I use a standard linear interpolation. Then I perform a quantitative analysis to study how frictions in the labor market yield different reactions of macroeconomic aggregates to terms of trade shocks.

1.5.1 Baseline Calibration

I calibrate the model taking standard parameters from the literature and matching key moments in the data at an annual frequency for Greece from 1981 to 2017. I discretize the capital stock, the terms of trade, and the productivity space into grids of 45, 21, and 15 elements, respectively. For the capital stock grid, the elements are going to be equally separated with the steady state capital stock at its center element. For the terms of trade and productivity grids, I follow Tauchen and Hussey (1991) to obtain the probability transition matrix and the elements of the grids for both AR(1) processes.

Functional Forms. I use a separable utility function in consumption and labor proposed in King, Plosser and Rebelo (1988). Using this utility function consumption and leisure have different elasticities of substitution. The utility function is

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} - \chi \frac{l^{1+\nu}}{1+\nu},$$

where $\chi > 0$ measures the disutility of working, $\gamma > -1$ stands for the intertemporal elasticity of substitution for consumption, and $\nu > 0$ a parameter that determines the Frisch elasticity of labor supply.

The characterization of the capital adjustment cost function is broad in the literature. I follow the specification proposed in Mendoza (1991)

$$\Phi(k' - k) = \frac{\phi}{2} \left(k' - k\right)^2,$$

where $\phi > 0$ is a parameter that controls the penalty steepness of adjusting capital stock. This penalty allows the model to control and match better the investment fluctuations shown in the data.

I assume a Cobb-Douglas production function

$$F(K, L, M) = K^{\alpha_K} L^{\alpha_L} M^{\alpha_M},$$

where $\alpha_K, \alpha_L, \alpha_M \in (0,1)$. Furthermore, I assume that the production function has constant returns to scale, $\alpha_K + \alpha_L + \alpha_M = 1$.

Finally, I assume the log terms of trade and log productivity stochastic processes follow an AR(1) process each

$$\ln{(z')} = \rho_z \ln(z) + \sigma_z \varepsilon_z \qquad \text{and} \qquad \ln{(p')} = \rho_p \ln(p) + \sigma_p \varepsilon_p$$

where the auto-correlation parameter satisfy $|\rho_z|< 0$ and $|\rho_p|< 0$, and the shocks are $i.i.d.$ and normal distributed, $\varepsilon_p, \varepsilon_z \sim N(0,1)$.

Model Parameters. Table 3.1 shows all the baseline calibration values for the parameters of the model. I first specify some parameters using data directly and standard values found in the literature. After, I calibrate the rest of the parameters in two different ways. First, I calibrate the parameters such that the steady state solution of the model follows well-known trends in the literature. Then, I perform a Monte-Carlo simulation process and match common statistics with the data. I collect 10,000 simulations of 2,500 periods each, ignoring the first 500 periods to get rid of an initial state bias.

The first subset of parameters in Table 3.1 shows the characterization of the parameters using data directly and standard values found in the literature. I normalize the terms of trade base price index, the total labor supply of the households, and the production technology to 1. For the share of foreign intermediate inputs α_M, I use that the average of total imports as a share of GDP is 31%. For the capital share α_K, I use the standard value of 0.35 net of foreign intermediate inputs. Finally, using the constant returns to scale assumption, the labor share α_L will be the residual of the two previous shares. Using the perpetual inventory method explained in Conesa, Kehoe and Ruhl (2007), the depreciation rate of capital stock satisfies that on average the consumption of fixed capital as a share of GDP is of 10%. For the consumption intertemporal elasticity of substitution parameter, I set γ to the widely accepted value of 2. For the parameter of ν, I fix a Frisch elasticity of substitution of 3.5. This value falls in the range of Frisch elasticities used in macro models. For the terms of trade stochastic process, I use log-quadratic filter. I find that the log terms of trade shocks last around 2.4 years and have a standard deviation of 28%.

The second subset of parameters in Table 3.1 shows the parameters calibrated using the steady state and a simulation process. I set the discount parameter β so the average investment to GDP ratio is of 25% in the steady state. I fix χ to match that only a third of total available labor is used in the steady state. I calibrate the real wage rigidity such that the unemployment rate in the statistics of the simulation process is of 7%. Finally, the parameter that controls the capital stock adjustment penalty is set to match the ratio of volatilities between investment and GDP in the perfectly flexible wages environment. The volatility of investment is almost 3 times as big as the volatility of GDP.

Table 1.5 shows the long-run correlation statistics from the model using a Monte-Carlo simulation process. First, let us focus in the first two columns where I use the preferences I proposed above. I establish as the *Benchmark* the economy with the real wage rigidity. In the other hand, the *Perfectly Flexible* will describe the economy where there is no real wage constraint. The first row reports the correlation between real GDP and terms of trade innovations. The first takeaway is that in an economy where there is a wage rigidity the correlation is negative. Nevertheless, allowing the real wages to adjust freely, the correlation becomes positive. As explained before, how labor adjusts to the shocks in the terms of trade play a significant role to explain the different reaction of real GDP. I decompose the reaction of labor towards terms of trade into two components: how real wages react towards terms of trade, and how labor reacts towards real wages.

Table 1.4: Calibration table

Parameter	Value	Source
p_0	1.000	Base price index (Normalization)
$\overline{L}$	1.000	Total labor supply (Normalization)
z	1.000	Production technology (Normalization)
α_M	0.239	Average imports to GDP ratio ($M/GDP = 31.45$)
α_K	0.266	Capital share ($\alpha_K = 0.35(1 - \alpha_M)$)
α_L	0.495	Labor share ($\alpha_L = 1 - \alpha_K - \alpha_M$)
δ	0.076	Average cons. fixed capital to GDP ratio ($CFC/GDP = 10.11\%$)
γ	2.000	Standard consumption elasticity of substitution
ν	0.286	Frisch elasticity of substitution ($1/\nu = 3.5$)
ρ_p	0.585	Terms of trade shocks persistency ($1/(1 - \rho) = 2.41$)
σ_p	0.118	Terms of trade shocks standard deviation ($\sigma/(1 - \rho) = 28.46\%$)
ρ_z	0.921	Productivity shocks persistency (Data)
σ_z	0.021	Productivity shocks standard deviation (Data)
β	0.969	Steady state imports to GDP ratio ($x_{ss}/GDP_{ss} = 24.78\%$)
χ	22.790	Steady state labor ($l_{ss}/\overline{L} = 1/3$)
$\overline{w}$	0.389	Simulations unemployment rate ($h - l = 7.25\%$)
ϕ	9.092	Flexible wages volatility ratio ($\sigma(x)/\sigma(GDP) = 2.84$)

The former moves equally regardless of the wage rigidity. Nevertheless, the main difference resides in how labor adjusts towards movements in the real wages. When real wages are perfectly flexible, the wealth effect plays a significant role to make agents increase their labor supply. Hence, countering the substitution effect of making leisure more attractive. In the other hand, when the real wage constraint is present, this increase in the labor supply will only result in an increase in unemployment, not labor.

Table 1.5: Parameter Values

	KPR Preferences		GHH Preferences	
Statistic	Benchmark	Perfectly Flexible	Wage Rigidity	Perfectly Flexible
$\rho(G\hat{D}P, p)$	-0.386	0.098	-0.649	-0.253
$\rho(\hat{c}, p)$	-0.864	-0.906	-0.877	-0.943
$\rho(\hat{l}, p)$	-0.355	-0.061	-0.699	-0.991
$\rho(\hat{l}, \hat{w})$	0.107	-0.012	0.161	1.000
$\rho(\hat{w}, p)$	-0.958	-0.987	-0.973	-0.991

Table 1.5 last two columns show the same long-run correlation statistics using another type of preferences. The preferences proposed in Greenwood, Hercowitz and Huffman (1988) are widely used in the real business cycles literature because it simplifies the equilibrium by eliminating the wealth effect. Using this type of preferences make the correlation between terms of trade and real GDP negative regardless of the real wage rigidity. This is a direct consequence of eliminating the wealth effect in preferences. Even though this solves the Kehoe-Ruhl observation, it will not be able to account for a positive correlation between terms of trade and real GDP. This is a shortcoming by using these preferences because, as I documented before, the correlation between terms of trade and real GDP shocks

can take negative and positive values.

I also perform a short-term experiment to see how the correlations between terms of trade and real GDP and labor interact. Figure 1.5 shows this relationship with and without the real wage rigidity. The data shows that there is a positive relationship between these correlations from Figure 1.2(a). The model without a real wage rigidity shows almost no relationship whatsoever as shown in Figure 1.5(a). On the other hand, adding the friction in the labor market replicates what the data shows, as shown in Figure 1.5(b). As the correlation between terms of trade and labor increases, the correlation between terms of trade and real GDP increases also. Most importantly, it is able to replicate the heterogeneity in the responses from real GDP and employment due to fluctuations in the terms of trade.

In addition, Figure 1.6 analyzes the responses to the unemployment rate in the model. It is expected that in the flexible wages environment, there will be no possible action in this dimension because full-employment is always achieved. Recalling from Figure 1.2(a), there exists a negative relationship between the responses of real GDP and unemployment rates to fluctuations in the terms of trade in the data. Figure 1.6(a) shows that it is impossible to replicate the negative relationship shown in the data. On the other hand, Figure 1.6(b) is able to replicate the heterogeneity and relationship that the responses of unemployment and real GDP have in the data.

1.6 Conclusion

In this paper, I argue that a terms of trade deterioration can affect positively and negatively real GDP. I document that the business cycles correlation between terms of trade and real GDP vary widely between positive and negative values across 122 countries. Furthermore, this variation still persists after separating the countries into different income groups. I show evidence that how countries react towards terms of trade movements is tightly linked to their labor market. Specifically, when unemployment reacts positively to increases in the terms of trade, real GDP will react negatively. In addition, when employment reacts negatively to increases in the terms of trade, real GDP will also react negatively.

I construct a real business cycle model with terms of trade and productivity uncertainty and a friction in the labor market. I find that a friction in the labor market as a real wage floor can produce a different response in macroeconomic aggregates to fluctuations in the terms of trade. In particular, when the wage rigidity is not binding, a deterioration in the terms of trade produces an increase to real GDP and employment. On the other hand, when the wage constraint binds, unemployment must increase to stop the fall in real wages when there is a deterioration in the terms of trade. This is, employment and real GDP fall to keep real wages on the real wage floor level.

Finally, I due a simulation experiment of the model and analyze the results under a perfectly flexible wages and under a real wage floor. I find that under no frictions in the labor market, it is not possible to replicate the heterogeneity in the responses of real GDP, employment and unemployment rates to terms of trade fluctuations. On the other hand, when there are frictions in the labor market, the model is able to replicate the features in the data.

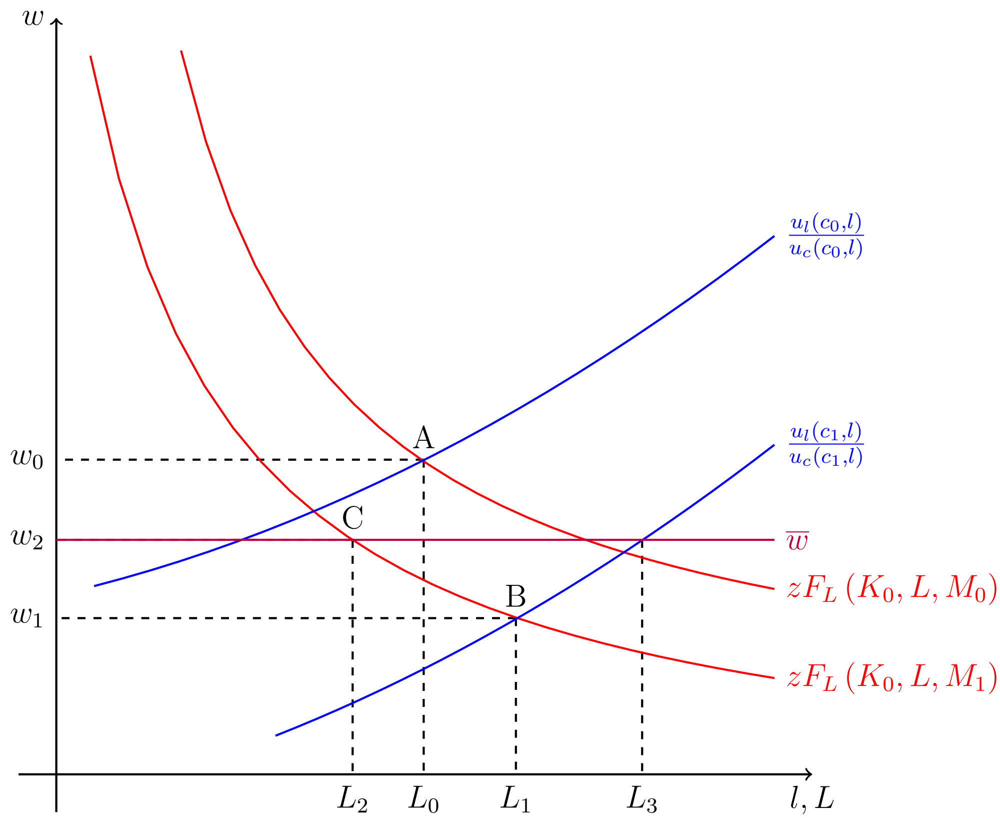

Figure 1.4: Representation of a negative terms of trade shock

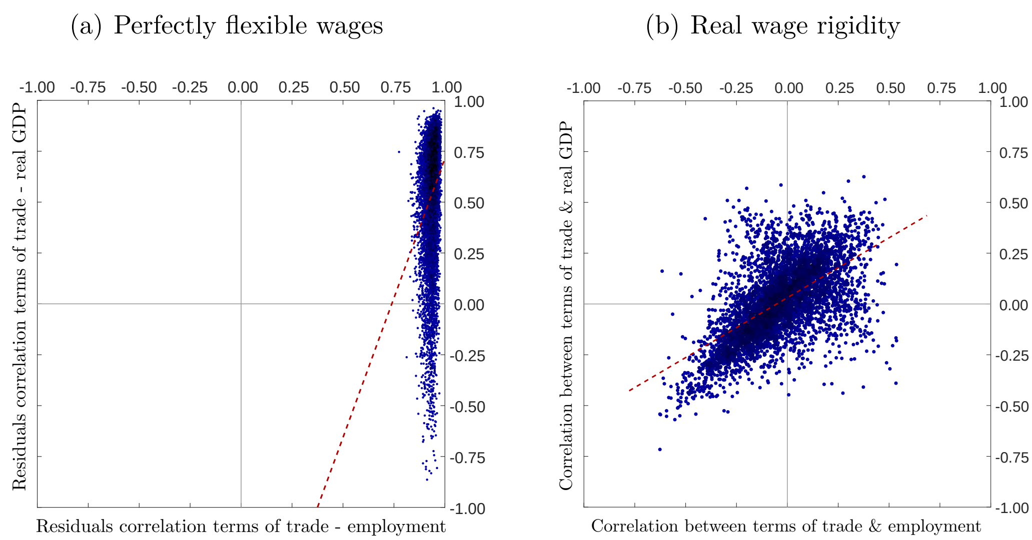

Figure 1.5: Relationship between correlation of real GDP and terms of trade residuals and correlation of employment and terms of trade

Note: The simulation process was done taking 5,000 collections of 40 periods each. Using an HP-filter with smoothing parameter of 100, the statistics are using a standard average across period collections from trend deviations.

(a) Perfectly flexible wages (b) Real wage rigidity

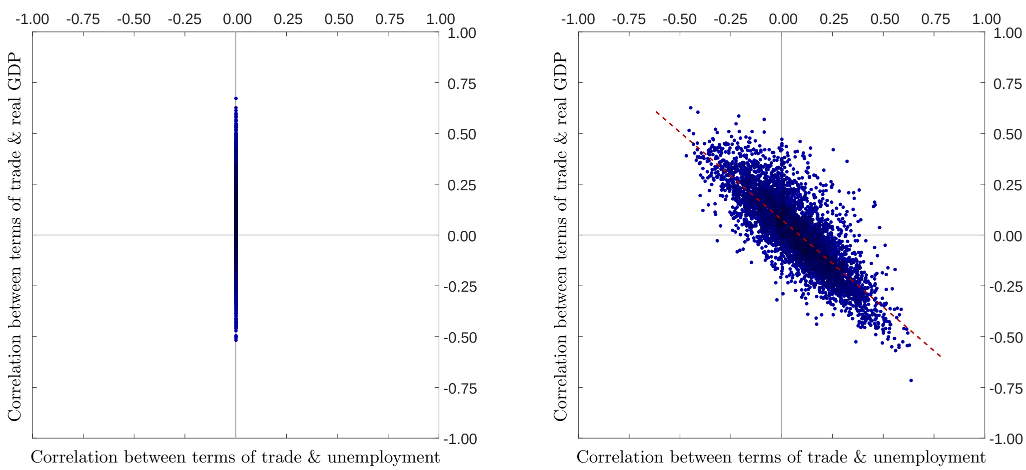

Figure 1.6: Relationship between correlation of real GDP and terms of trade residuals and correlation of unemployment rates and terms of trade

Note: The simulation process was done taking 5,000 collections of 40 periods each. Using an HP-filter with smoothing parameter of 100, the statistics are using a standard average across period collections from trend deviations.

Chapter 2

Sovereign spread movements in emerging economies: terms of trade matter

2.1 Introduction

This paper focuses on the high mean and volatility of interest rate spreads in emerging economies. We document that some emerging economies experience a decrease in the negative correlation between real GDP and interest rate spreads for the last two decades. As a matter of fact, it is observed in the data that interest rates oscillates regardless of a favorable domestic economic performance. Figure 2.1 shows the Mexican interest rate spread for the last twenty years and its relationship with real GDP and terms of trade. The interest rate spread in Mexico displays sharp rises in 1998 and 2014, even though the economy does not experience deep recessions during these years. Moreover, during these years, the interest rate spread in Mexico follow more closely the movements of the terms of trade series. This observation corresponds to the puzzle proposed in Tomz and Wright (2007). They found that the negative correlation between output and default of a country is remarkably week. Also, they show evidence of countries defaulting on their sovereign debts during good times while making repayments during bad times. This paper addresses this puzzle by arguing that foreign conditions can explain this issue.

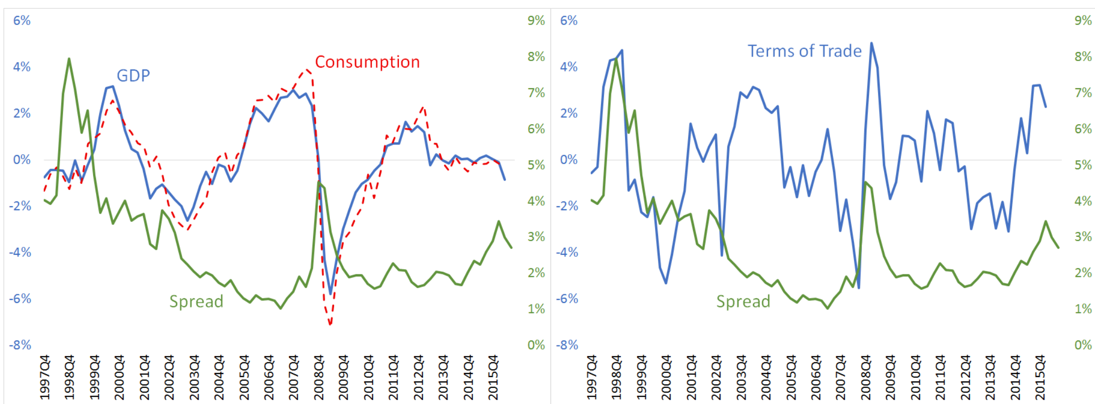

Figure 2.1: Spreads and Cyclical Components in Mexico

The framework presented in Eaton and Gersovitz (1981) is beneficial to analyze the spread behavior because this class of model is able to derive the interest rates endogenously [1]. However, it has been proven difficult to obtain three fea-

[1] Aguiar and Gopinath (2007) and Neumeyer and Perri (2005) assume exogenous interest rates since it is useful to explain business cycles of developing countries. High volatility and countercyclicality of interest rates are regarded as crucial parts to explain the cyclical movement of aggregate output and prices.

tures of the interest rates with sovereign default models. Arellano (2008) shows countercyclical interest rate spreads by introducing convex costs of default. This yields defaults to be more likely to occur during recessions. Nevertheless, the mean spread that the model provides is 3.58% which is relatively low compared to the mean spread of Argentina, which is 10.25%. Also, only few fluctuations of spread are observed with good economic conditions due to the structural features of the probability of default. In fact, the spread generated by the model is almost zero when the country is hit by good endowment shocks. Mendoza and Yue (2011) achieve large volatility of spread in their baseline model by introducing endogenous default costs, but fail also to capture the high mean spread shown in the data. This is because spread and default probabilities are linked to each other directly. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) are able to improve the spread behavior in the sovereign default models by incorporating long-duration bonds. This helps to increase mean and standard deviation of spread; however, those studies cannot explain why spreads can be high during good times in the economy. The high volatility of spreads is mainly accomplished by the large dispersion of spreads with low endowments while the standard deviation with high endowments is significantly small.

This paper proposes a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. The key feature of this paper is that the model incorporate an exogenous foreign shock called terms of trade. In the model, a negative terms of trade shock act in two ways. First, the country spends more in foreign products for consumption. Second, the terms of trade have direct impacts on the level of foreign currency debts that sovereigns owe to foreign lender. This model works with the assumption that sovereigns issue their bonds in foreign prices.

As shown in Eichengreen and Hausmann (1999) and Jeanne (2003), emerging economies tend to issue debt in foreign currency because their local currency present high fluctuations and lack of credibility [2] Since debt is issued in foreign currency, the countries are vulnerable to changes in world prices. In other words, when the adverse terms of trade shocks hit the economy, the sovereign immediately encounters an unexpected enlarged debt burden. Consequently, the probability of default is not only affected by countries GDP shocks but also the terms of trade shocks. This provides an explanation of why terms of trade shocks can lead to higher and more volatile spread movements, regardless of the GDP performance.

As mentioned above developing countries experience high volatility of terms of trade and output. More specifically, terms of trade shocks are more volatile than GDP shocks in emerging economies. This implies that the terms of trade shocks are an important factor to consider when studying them. Moreover, as shown in Kose (2002) an important source for the repayment of foreign debt is export revenue and this is largely affected by the terms of trade. Terms of trade are often studied in the sovereign default models. Na, Schmitt-Grohé, Uribe and Yue (2014), Gu (2015), and Asonuma (2016) endogenously induce the deterioration of the terms of trade and real exchange rate. This paper make distinctions from those papers by assuming terms of trade shocks as exogenous. Popov and Popov, Wiczer et al. (2014) assume an exogenous path of terms of trade but he examines the role of terms of trade penalties and focuses on changes in trade volumes. In contrast, we focus in analyzing the changes in debt burden contingent to terms of trade. Cuadra and Sapriza (2006) study also an exogenous

[2] Du and Schreger (2015) show that foreign currency debt composition has decreased since 2004. Nevertheless, they still have a significant level of foreign currency debt.

terms of trade shock when the production side buy intermediate imported goods. In their model, the terms of trade shocks are used as if they are productivity shocks so the terms of trade shocks generate real GDP movement. However, this mechanism violates the result in Kehoe and Ruhl (2008) that proves that terms of trade do not have first order effects in real GDP[3]. Moreover, they do not have the convex default costs so the frequency of default generated by the model is unusually small. In this paper we involve both endowment shocks and the terms of trade shocks, while also considering a convex default cost.

The rest of the paper is organized as follows. Section 2 empirical evidence, Section 3 the model, Section 4 quantitative analysis, Section 5 conclusion.

2.2 Empirical evidence

2.2.1 Currency composition of sovereign external debt

In this section, we construct the ratio of foreign currency sovereign external debt to total debt in developing countries. This helps to develop the idea that the terms of trade shocks are of importance to the fluctuations of the economy in emerging economies via foreign currency sovereign debt owed to foreign investors. The definition of external debt is adopted from Du and Schreger (2015). We deviate from their methodology because we are only interested in studying the government debt[4]. Hence, we use the definition of *sovereign external debt* as any

[3]They also show that the terms of trade do not act as a productivity shock in standard models while they do affect real income and consumption in a country.

[4]Their definition of *external debt* includes both public and private debt in order to analyze how default decisions are affected by debt denomination in public and private sectors. However, this paper considers only public debt. Thus, we define *sovereign external debt* instead of *external*

debt issued by the government in developing countries and owed to nonresidents, regardless of the market of issuance.

Debt is categorized by three dimensions: issue sector, issue currency, and issue market. Issuance sector is divided into the government and the corporate sector. The debt issued by the central or local governments is counted as government debt while all debt issued by the private sector is regarded as corporate debt. The classification of issue currency is determined by which currency debt is denominated when issued. Local currency (LC) debt refers to debt that is issued in the currency of issuance country while foreign currency (FC) debt is denominated in another country's currency. Lastly, issuance market is broken down into two markets. When debt is issued under the domestic law inside a country, it is called domestic debt; on the other hand, international debt follows foreign law and issued in international markets. Among these categories, this paper mainly address the combined category of government as issuer sector, foreign currency as issue currency, and both markets as issue market in order to study sovereign external debt.

Bank for International Settlements (BIS) provides amount of outstanding debt data by each classification. However, debt data by debt holder - nonresidents or residents -, which is the main part of definition of *external debt*, are not available. Hence, we follow Du and Schreger (2015) to construct the currency composition of *sovereign external debt*. They make two assumptions for debt holding of nonresidents. First, nonresidents hold all debts in international market, which implies that all international debts are regarded as external debt. Second, nonresidents do not hold any FC debt in domestic market[5]. Based on these two assumptions,

debt.

[5]They document that the amount of outstanding foreign currency debt in domestic market

the FC sovereign external debt is constructed as follows: amount of outstanding FC debt issued by the government in international market.

Table 2.1 provides the share of FC sovereign external debt in total sovereign debt. The total sovereign debt is defined as all debts issued by the government so it consists of both domestic debt and external debt. [6] The analysis of this paper is proceed based on sovereign external debt, and it is expected that countries with higher share of external debt denominated in FC are more likely to be exposed by terms of trade shocks. In Table 2.1, although the substantial heterogeneity for the ratio of FC external debt to total debt is observed, it is sensible that countries are considerably under the influence of it. Moreover, there are some countries that heavily rely on FC debt owed to foreign creditors such as Peru, Argentina, Lebanon, and Lithuania. In particular, the countries that experienced sovereign default events have a tendency to have higher percentage of FC external debt. Argentina had carried on more than 75% of external debt in FC until the default periods and reduced it to approximately 50% in the first half of the 2000's. Peru also has been maintained high share of FC external debt on average. In case of Russia, almost half of the total debt is FC debt owed to nonresidents in 2004 which is significantly large enough to be affected by exchange rate movements.

One of the results from Du and Schreger (2015) is that sovereigns have been using more LC when issuing external debt in government sector so there is a tendency of the decrease in the proportion of FC external debt in total external debt[7]. Nevertheless, analyzing FC external debt is worthy. Since they compare

is notably small so the second assumption is sensible.

[6] The definition of domestic debt in this context is any debt owed to residents within the country.

[7] They analyze $\frac{\text{FC external debt}}{\text{Total external debt}}$ while this paper analyze $\frac{\text{FC external debt}}{\text{External debt+Internal debt}}$. Thus the share of foreign currency external debt in this paper is affect by the amount of debt

Table 2.1: Foreign Currency Debt Composition

	Average	2004	2015
Argentina*	79.6	74.9	43.4
Brazil	7.3	4.9	4.4
Chile	12.9	20.8	16.3
Colombia	22.8	22.8	25.5
Croatia	45.7	59.3	46.1
Hungary	22.7	19.8	27.0
Indonesia	13.9	3.2	30.8
Lebanon	35.2	51.5	44.8
Lithuania	83.8	73.2	79.5
Malaysia	3.9	9.0	3.3
Mexico	16.2	27.3	15.1
Peru*	65.1	84.2	38.7
Philippines	24.9	33.9	23.4
Russia*	31.2	48.5	38.1
South Africa	7.2	10.6	9.7
Turkey	19.0	16.5	29.6

Notes: * indicates countries that experienced default events. 2005 data is used for Mexico and Malaysia for the 2004 column and 2007 data and 2008 data are used for South Africa and chile for the 2004 column respectively. They are first year of data availability.

the FC external debt with total external debt, the expansion or contraction of the amount of total external debt is not taken into consideration in their construction of currency composition. In other words, the importance of FC external debt could be underestimated if the countries issue more external debt than domestic debt. However, the measure of currency composition used in this paper reflects this issue since the definition of total sovereign debt include both domestic debt and external debt. If the amount of domestic debt gets smaller, then the share of FC external debt in total sovereign debt increases which means external debt in FC becomes a more essential part of the debt in the countries. Actually, all countries except Hungary and Russia in Table 2.1 display continuous increases in the amount of external debt denominated in FC.[8] Furthermore, it is not explicitly shown that the share of FC external debt has been decreasing in Figure 2.2 with our data construction. Indonesia, Hungary, Turkey, and Lithuania, for instance, have kept expanding the share of external debt in FC. The LC debt in domestic market rapidly rose in Croatia around 2004 so the share sharply decreased at that time but it started to issue more FC debt in international market in 2009 so the share has been following the growing trend since the time. In addition, other countries hold more or less a constant share of FC external debt. Those empirical evidence illustrates that external debt in FC are still a crucial part of debt in developing countries.

owe to residents in a country.

[8]Hungary and Russia has been reducing the amount of foreign currency external debt since 2014.

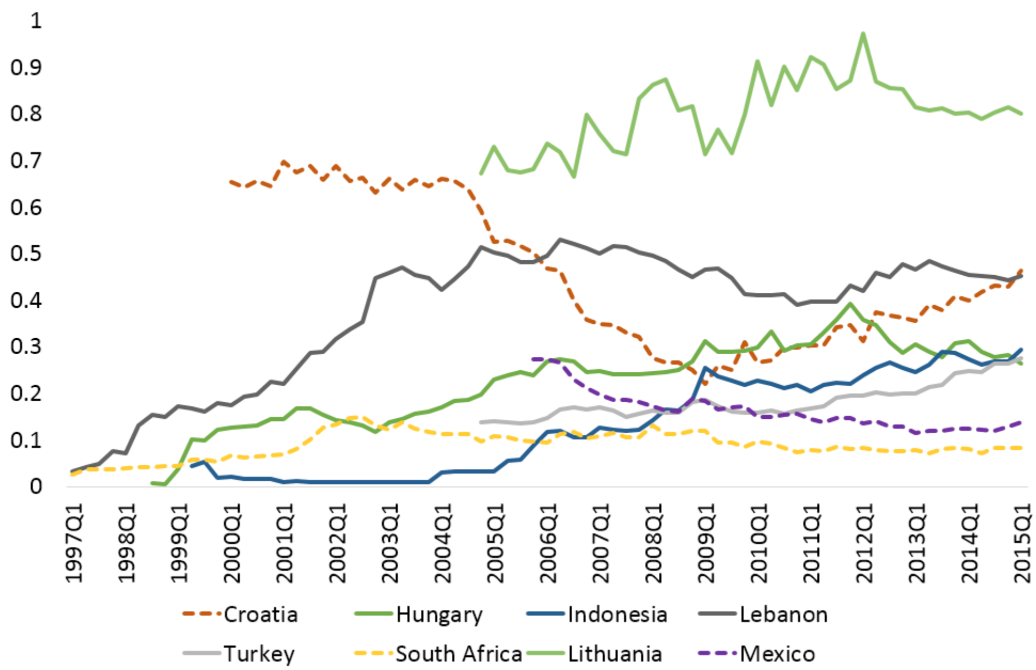

Figure 2.2: Foreign Debt Profile for Emerging Economies

2.3 Terms of trade, GDP and spread across countries

The data presented in Table 2.2 and 2.3 are statistics for the terms of trade, GDP, and spread across 24 developing countries. In this paper, the terms of trade (TOT) are defined as the price of imports relative to the price of exports.

$$TOT = \frac{P_M}{P_X}$$

In order to construct the terms of trade, quarterly merchandise customs imports and exports data 1991Q1 to 2015Q4 are obtained from World Bank Global Economic Monitor (GEM). [9] By using current and constant value of import and export each deflator is calculated. Afterwards the import and the export deflators are used for the price of imports and the price of exports respectively. Hence, the terms of trade are constructed by import deflator over export deflator. The quarterly GDP is also provided from GEM and the time period is the same as the one in imports and exports data. The interest rate spread data are taken from J.P. Morgan's EMBI + database.[10] The terms of trade and output are log and HP detrended.

Table 2.2 provides standard deviation of the terms of trade, GPD, and spread. It also provides the mean of spread in each country since this paper focuses on the behavior of spread. Although there is a cross-country heterogeneity, in almost all sample countries, the standard deviations of TOT is bigger than those of GDP. In other words, TOT is more fluctuate than GDP in most of countries. This fact is crucial for the analysis of the paper since this indicates that more fluctuations of the economy can be driven by the terms of trade shocks with high volatility. Also, defaulted countries, such as Argentina, Ecuador, and Russia, show much higher mean and volatile movement of spread. In particular, the volatility of TOT in Ecuador is approximately 14 times bigger than its GDP. Hence, it can be seen that Ecuador has been affected by volatile terms of trade shocks. However, it is

[9] World Development Indicators (WDI) provides imports and exports data across countries, but this is annual data. Thus, we interpolate annual data to transform to quarterly data and compare it with the quarterly merchandise customs imports and exports and we found that those two series are coincided through the sample period.

[10] The spread data are not a balanced data across countries, so we used series of spread available up to 2015Q4.

unclear that the terms of trade shocks is a main factor for movement of spread based on magnitude of standard deviation, but correlation with spread will help to improve this issue.

Table 2.3 shows different combinations of correlations among TOT, GDP and spread in the sample countries. First, the negative correlations between GDP and spread are achieved and also the positive correlations between TOT and spread are presented in most countries. This implies that the deterioration of the terms of trade coincide with the increase in spread in general. Moreover, even though it is hard to find a certain pattern between ρ(TOT,spread) and ρ(TOT,spread), there are nine countries having higher correlation of spread with TOT than with GDP. This can be direct evidence for the impacts of TOT on movement of interest rate spread. For example, Mexico have significantly high correlation of spread with TOT while there is almost no correlation with GDP: hence, it is reasonable to conclude that the the fluctuation of Mexican spread is mainly affected by the terms of trade shocks. This interpretation can be generalize to any countries showing higher correlation between spread and TOT.

2.4 Model

In this section, we propose a model of that incorporates two sources of uncertainty in a country, a domestic and a foreign. We work with a framework that extends the sovereign default models introduced in Eaton and Gersovitz (1981) and Arellano (2008). We use this last one to incorporate a source of external uncertainty called terms of trade.

Consider a small open economy where there are two types of shocks, a domestic and a foreign. On one hand, the domestic shock is going to be represented by

Table 2.2: Descriptive Statistics in Emerging Economies

	σ(TOT)	σ(GDP)	σ(spread)	μ(spread)
Argentina	0.045	0.040	18.24	15.99
Brazil	0.041	0.015	3.92	5.66
Chile	0.090	0.018	0.59	1.46
China	0.036	0.010	0.55	1.18
Colombia	0.073	0.013	2.07	3.56
Dominican Rep	0.022	0.023	3.30	5.39
Ecuador	0.295	0.020	8.38	12.33
Egypt	0.034	0.015	1.74	2.55
Hungary	0.016	0.015	1.58	1.80
Indonesia	0.068	0.033	1.44	2.89
Kazakhstan	0.264	0.024	2.74	4.28
Korea	0.035	0.023	1.04	1.31
Malaysia	0.027	0.018	1.25	1.78
Mexico	0.040	0.023	1.51	2.75
Morocco	0.016	0.013	2.43	2.30
Peru	0.073	0.018	1.96	3.51
Philippines	0.046	0.013	1.52	3.46
Poland	0.096	0.014	0.90	1.71
Russia	0.098	0.029	11.26	7.28
South Africa	0.032	0.012	1.19	2.26
Sri Lanka	0.024	0.009	4.33	6.01
Tunisia	0.014	0.011	0.92	1.84
Turkey	0.030	0.036	2.20	4.02
Ukraine	0.023	0.043	6.23	7.46

Table 2.3: Correlations in Emerging Economies

	ρ(TOT,spread)	ρ(GDP,spread)	ρ(TOT,GDP)
Argentina	-0.026	-0.647	-0.066
Brazil	0.205	-0.198	-0.553
Chile	0.528	-0.335	-0.478
China	-0.080	-0.007	0.257
Colombia	0.267	-0.197	-0.485
Dominican Rep	0.020	-0.641	-0.006
Ecuador	0.517	-0.445	-0.687
Egypt	0.082	-0.189	0.084
Hungary	0.141	-0.290	0.222
Indonesia	0.574	-0.284	-0.507
Kazakhstan	-0.440	-0.607	0.541
Korea	-0.209	-0.633	0.524
Malaysia	0.061	-0.485	0.116
Mexico	0.443	0.005	-0.466
Morocco	-0.126	0.135	-0.099
Peru	0.332	-0.156	-0.263
Philippines	0.029	-0.326	0.262
Poland	0.007	0.159	0.137
Russia	0.241	-0.487	-0.681
South Africa	0.223	-0.250	-0.021
Sri Lanka	0.433	-0.430	0.229
Tunisia	0.556	-0.162	0.017
Turkey	0.169	-0.449	0.079
Ukraine	-0.194	-0.423	-0.157

real GDP movements. On the other, the foreign shock is going to be represented by terms of trade movements. Let y_t and p_t represent output and terms of trade in period t, respectively. As discussed previously, it is standard to assume terms of trade as an exogenous variable for small open economies. We consider this in order to construct the following stochastic system

$$\begin{bmatrix} \ln y_{t+1} \\ \ln p_{t+1} \end{bmatrix} = A \begin{bmatrix} \ln y_t \\ \ln p_t \end{bmatrix} + \begin{bmatrix} \varepsilon^y_{t+1} \\ \varepsilon^p_{t+1} \end{bmatrix}, \tag{2.1}$$

where the roots of the second order polynomial $det\,(I_2 - Ax) = 0$ lie outside the complex unit circle and the errors vector is a binormal distribution with mean $\overline{0}$ and variance-covariance matrix Σ. With this formulation we are able to tailor a correlation between contemporanous real GDP and terms of trade.[11]

There are two types of tradable goods in this economy, a domestic and a foreign. In every period, a representative household purchases these two types of goods and transfom them into a final consumption good using the following Armington aggregator technology,

$$C = \left(\lambda c_d^{-\eta} + (1-\lambda) c_f^{-\eta}\right)^{-\frac{1}{\eta}},$$

where C represents the final aggregated good consumption, c_d the domestic good consumption, c_f the foreign good consumption, $\lambda \in [0, 1]$ a parameter that captures home bias, and $\eta \in [-1, \infty)$ a parameter that controls the elasticity of

[11] As shown in Kehoe and Ruhl (2008), it is a common mistake to misrepresent what real GDP is in a model. Moreover, standard models with a production side that buy imported goods at the price of terms of trade capture a spurious correlation between real GDP and terms of trade. If real GDP is measured correctly in them, the correlation between real GDP and terms of trade should be close to zero.

substitution between domestic and foreign goods. The representative household is able to purchase these goods taking the output available in the period and an amount of taxes that the government issues in a lump-sum fashion as given. The representative household is a risk averse agent that obtains utility from the stream of final consumption goods obtained in every period as

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(C_t\right) \right],$$

where $\beta \in (0,1)$ is a parameter that captures a discount factor across periods and $u(\cdot)$ is an increasing and strictly concave function.

In this economy there is also a benevolent government whose objective is to maximize the utility of the representative household. In order to achieve this, it has two main decisions to take. The first decision is either to honor or default in its sovereign bonds obligations. If the government takes the decision of honoring them, it has access to international financial markets where it can buy or sell one period maturity sovereign bonds. Then, the second decision is how much of these bonds to purchase given a price schedule contingent to the amount of these new bonds and the shocks the economy experiences. Let B, B', and $q(B', y, p)$ represent the amount of sovereign bonds due, the amount of new sovereign bonds purchases, and the price of these at any given period.

We introduce the assumption that the international financial markets are managed only in foreign goods. This is, the sovereign bonds returns and purchases have to be done in foreign terms. Because of this, the value of sovereign bonds in each period will be subject to the terms of trade shock.

We model two types of penalties for defaulting in sovereign debt, exclusion of financial markets and output losses. With these penalties we want to capture the fact that countries that default in their debt experience a temporary exclusion to international borrowing and periods of poor output performance. We assume that if a government defaults in their debt, this will be erase entirely and it will enter a financial autarkic environment. In addition, the country will experience an output cost that limit its endowment. The economy will remain in this financial autarky for a stochastic number of periods and will re-enter the financial markets with an exogenous probability.

Let x represent exports, the resources exiting the economy. The balance trade condition for the repayment state can be expressed as

$$x - pc_f = pq(B', y, p)B' - pB.$$

The left-hand side is the current account while the right-hand side is the negative of the capital account of the economy. This is, any surplus(deficit) that the capital account experiences due to the government's sovereign debt position will imply a deficit(surplus) of the current account. The balance trade condition for the default state can be expressed as

$$x - pc_f = 0.$$

Considering the exclusion of financial markets penalty of default, the balance trade condition will imply that the current account cannot experience any kind of suplus or deficit.

The resource constraint of the government will show how the domestic good endowment can be consumed in every period. On one hand, if the government chooses to repay its debt, the resource constraint will be

$$c_d + x = y.$$

On the other hand, if the government chooses to default,

$$c_d + x = h(y),$$

where $h(\cdot)$ is an increasing function such that $h(y) \leq y$[12].

There is a representative foreign lender who is able to borrow and lend risk-free bonds at a constant international interest rate $r^* > 0$. We assume it has perfect information about the small open economy. This is, it can observe the level of output and terms of trade that the small open economy experiences every period. The foreign lender is risk neutral[13] and maximizes expected profits over risky sovereign bonds from the small open economy. We assume that the foreign lender maximizes profit only in foreign goods terms. Let δ represent the probability of the government to default in its sovereign debt position. Taking as given the default probability and the bond price, the foreign creditor chooses $\tilde{B}'$ to maximizes

$$\operatorname*{Max}_{\tilde{B}'} \left\{ q\tilde{B}' - \left(\frac{1-\delta}{1+r^*} \right) \tilde{B}' \right\}.$$

By the risk neutral nature of the foreign lender, the bond price schedule solu-

[12]Realise that the output cost is defined as $y - h(y)$ and is non-negative by definition.

[13]Cole and Kehoe (1996) explains that the risk neutrality of the foreign lenders reflect the fact that the size of an individual sovereign transaction is relatively small compared to the total international credit market.

tion in equilibrium must satisfy the first order condition of the previous problem. The bond price schedule will be a result of the following break-even condition,

$$q = \frac{1-\delta}{1+r^*}. \tag{2.2}$$

Realising that $\delta \in [0,1]$, we can infer that $q \in \left[0, \frac{1}{1+r^*}\right]$. Defining the sovereign interest rate as $r \equiv \frac{1}{q} - 1$, we obtain that $r = [r^*, \infty)$. Finally, we define the sovereign bond spread as $S \equiv r - r^*$.

The timing of the government problem is the following. In the beginning of the period the government realises the amount of sovereign bonds due B, the domestic goods endowent shock y, and the terms of trade shock p. The government asses the optimal relationship between domestic and foreign goods using p and the preferences of the representative household. Then the government decides whether to honor or default in its debt obigations. If it decides to honor its debt, the government updates its sovereign bond holdeings B' taking as given the sovereign bond price schedule $q(B', y, p)$ and constrained to its resource constraint and the balance trade condition. The foreign lender takes the bond price q as given and supplies $\tilde{B}'$ matching B'. Purchases of foreign and domestic take place. Finally, the representative household consumes the final good by aggregating the domestic and foreign goods.

2.4.1 Recursive Equilibrium

There are three variables that define the state of the government in every period: the sovereign bonds due, the output shock, and the terms of trade shock. Define $V(B, y, p)$ as the value function of the government at the beginning of every period. Let us model the first decision of the government as

$$V(B,y,p) = \text{Max}\left\{V^R(B,y,p), V^D(y,p)\right\}, \tag{2.3}$$

where $V^R(B,y,p)$ and $V^D(y,p)$ represent the value of the government if it repays and defaults in its debt obligations, respectively. Here, the government chooses which environment will yield the highest welfare for the representative household captured by $V^R(B,y,p)$ and $V^D(y,p)$.

When the government chooses to repay, it chooses the amount of sovereign bonds to sell or purchase as well as the household allocations that will maximize its welfare subject to the resource constraint, the balance trade condition, the aggregation technology, and a no-Ponzi condition. In order to do this, it takes the price schedule for the bonds as given and a lower boundary for the sovereign bonds issuance $\mathbb{B} > 0$. Thus,

$$V^R(B,y,p) = \underset{x,c_d,c_f,C,B'}{\text{Max}} \left\{u(C) + \beta\mathbb{E}_{(y',p')}\left[V(B',y',p')|(y,p)\right]\right\} \tag{2.4}$$

$$\begin{aligned}
\text{s.t. } & c_d + x = y && \text{(Resource Constraint)}\\
& x - pc_f = pq(B',y,p)B' - pB && \\
& && \text{(Balance Trade Condition)}\\
& C = \left(\lambda c_d^{-\eta} + (1-\lambda)c_f^{-\eta}\right)^{-\frac{1}{\eta}} && \text{(Aggregation Technology)}\\
& B' \geq -\mathbb{B}. && \text{(No-Ponzi Condition)}
\end{aligned}$$

When the government chooses to default, it chooses the household allocations that will maximize its welfare subject to the resource constraint, the balance trade condition, and the aggregation technology. In order to do this, it takes the default penaltiy and the probability of returning to the financial markets $\phi \in [0,1]$ as given. Recall that the default penalties are the output costs and the zero current account restriction due to the financial market exclusion. Thus,

$$V^D(y,p) = \underset{x,c_d,c_f,C}{\text{Max}} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[\phi V(0,y',p') + (1-\phi) V^D(y',p') | (y,p) \right] \right\} \tag{2.5}$$

$$\begin{aligned} \text{s.t. } & c_d + x = h(y) && \text{(Resource Constraint)} \\ & x - pc_f = 0 && \text{(Balance Trade Condition)} \\ & C = \left(\lambda c_d^{-\eta} + (1-\lambda) c_f^{-\eta} \right)^{-\frac{1}{\eta}}. && \text{(Aggregation Technology)} \end{aligned}$$

In order to define what is the probability of default for a government, it is useful to characterize the set of output and terms of trade states in which a government finds optimal to default contingent to a level of sovereign bond holdings. Specifically, define the default set as the

$$\mathcal{D}(B) = \left\{ (y,p) \in \mathbb{R}^2_{++} : \quad V^D(y,p) > V^R(B,y,p) \right\}. \tag{2.6}$$

This set expresses that if the government sells B' and the shocks of next period are $(y',p') \in \mathcal{D}(B')$, then the government will find it optimal to default on B' next period. Because of this, we can define the probability for a government to default on B' by measuring how likely is to end up in the states that live in $\mathcal{D}(B')$. Given the stochastic process that govern the movements of output and terms of trade shocks, call $f(\cdot)$ the probability density function between shock states. Thus, the probability of default can be expressed as

$$\delta(B', y, p) = \int_{(y',p')\in\mathcal{D}(B')} f\left((y', p')|(y, p)\right) d(y', p'). \tag{2.7}$$

Consider the case where the government chooses a level of sovereign bonds B' such that there are no possible states in the next period in which it will default on them, $\mathcal{D}(B') = \emptyset$. Then, with that amount of sovereign bonds the probability of default will be zero, $\delta(B', y, p) = 0$. Also, consider the case when the government chooses a level of sovereign bonds B' such that default is for sure regardless of the shock realisations in the next period, $\mathcal{D}(B') = \mathbb{R}^2_{++}$. Then, with that amount of sovereign bonds the probability of default will be one, $\delta(B', y, p) = 1$. Finally, realise that if the output and terms of trade shocks have no persistency whatsoever, then the probability of default will only be contingent in the sovereign bond issues or purchases.

Now, the bond price schedule must satisfy the break-even condition (2.2) from the representative foreign lender problem. Considering the probability of default, the representative foreign lender must be consistent to (2.7). Thus, the bond price schedule will be

$$q(B', y, p) = \frac{1 - \delta(B', y, p)}{1 + r^*}. \tag{2.8}$$

Realise that this break-even condition will yield zero profit in expectation to the representative foreign lender regardless of the quantity of sovereign bonds it purchases or sells $\tilde{B}'$. Therefore, it will be willing to cover any amount of sovereign bonds the government finds optimal to choose during the repayment state. In other words, the sovereign bond market always clear in equilibrium

$\tilde{B}' = B'$.

Definition 2 (Recursive Equilibrium) *The recursive equilibrium of this small open economy will be a set of government value functions $V(B, y, p)$, $V^R(B, y, p)$, $V^D(y, p)$ and a sovereign bonds policy rule $\hat{B}'(B, y, p)$, a set of household consumption policy rules $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, and $\hat{C}(B, y, p)$, an exports policy rule $\hat{x}(B, y, p)$, a default set $\mathcal{D}(B, y, p)$, a default probability schedule $\delta(B, y, p)$, and bond price schedule $q(B, y, p)$ such that the following conditions are satisfied:*

- *Benevolent Government (Initial)*
  The default set $\mathcal{D}(B, y, p)$ is consistent with the set of government value functions $V(B, y, p)$, $V^R(B, y, p)$, and $V^D(y, p)$ and solves (2.3).

- *Benevolent Government (Repayment)*
  If $(y, p) \notin \mathcal{D}(B)$ and taking as given the bond price schedule $q(B, y, p)$, the government chooses the sovereign bonds policy rule $\hat{B}'(B, y, p)$, the set of household consumption policy rules $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, and $\hat{C}(B, y, p)$, and the exports policy rule $\hat{x}(B, y, p)$ in order to solve (2.4) and its solution is consistent with $V^R(B, y, p)$.

- *Benevolent Government (Default)*
  If $(y, p) \in \mathcal{D}(B)$, the government chooses the set of household consumption policy rules $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, and $\hat{C}(B, y, p)$, and the exports policy rule $\hat{x}(B, y, p)$ in order to solve (2.5) and its solution is consistent with $V^D(y, p)$.

- *Default probability*
  The default probability schedule $\delta(B, y, p)$ is consistent with the default set $\mathcal{D}(B, y, p)$ and (2.7).

► *Bond Pricing*
The bond pricing schedule $q(B, y, p)$ is consistent with the probability of default schedule $\delta(B, y, p)$ and (2.8).

2.4.2 Aggregate Recursive Equilibrium

The problem described in the previous section has an intratemporal condition between consumption of foreign and domestic goods by the representative household. Specifically, this intratemporal condition balances the terms of trade with the marginal rate of substitution between foreign and domestic consumption goods,

$$p = \frac{u'(C) \cdot \frac{\partial C}{\partial c_f}}{u'(C) \cdot \frac{\partial C}{\partial c_d}} = \left(\frac{1-\lambda}{\lambda}\right) \left(\frac{c_d}{c_f}\right)^{1+\eta}. \tag{2.9}$$

Using the final consumption aggregator and (2.9), let us define the final consumption price index as

$$\mathcal{P}(p) \equiv \left(\lambda^{\frac{1}{1+\eta}} + (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)^{\frac{1+\eta}{\eta}}. \tag{2.10}$$

This price lets us weight the price of the final consumption good in the economy considering how important are domestic and foreign goods in its aggregation. Notice that the limit expression of home bias follow,

$$\lim_{\lambda \to 0} \{\mathcal{P}(p)\} = 1 \qquad \text{and} \qquad \lim_{\lambda \to 1} \{\mathcal{P}(p)\} = p.$$

On one hand, if there is complete home bias, the price index of the final consumption good is not affected at all by the terms of trade. This result is intuitive because it tells us that the representative household does not derive any utility from foreign consumption goods. On the other hand, if there is complete foreign bias, the price index of the final consumption good is the complete terms of trade price.

Consider an aggregate version of the government's problem when it chooses to repay,

$$V^R(B,y,p) = \underset{C,B'}{\text{Max}} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[V(B',y',p') | (y,p) \right] \right\} \tag{2.11}$$

$$\text{s.t. } \mathcal{P}(p)C + pq(B',y,p) = y + pB \qquad \text{(Resource Constraint)}$$

$$B' \geq -\mathbb{B}. \qquad \text{(No-Ponzi Condition)}$$

Also, consider the an aggregate version of the government's problem when it chooses to default,

$$V^D(y,p) = \underset{c_d,c_f,C}{\text{Max}} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[\phi V(0,y',p') + (1-\phi) V^D(y',p') | (y,p) \right] \right\} \tag{2.12}$$

$$\text{s.t. } P(p)C = h(y) \qquad \text{(Resource Constraint)}$$

Definition 3 (Aggregate Recursive Equilibrium) *The aggregate recursive equilibrium of this small open economy will be a set of government value functions* $V(B,y,p)$, $V^R(B,y,p)$, $V^D(y,p)$ *and a sovereign bonds policy rule* $\hat{B}'(B,y,p)$, *a household final consumption policy rule* $\hat{C}(B,y,p)$, *a final consumption good price index* $\mathcal{P}(p)$, *a default set* $\mathcal{D}(B,y,p)$, *a default probability schedule* $\delta(B,y,p)$, *and bond price schedule* $q(B,y,p)$ *such that the following conditions are satisfied:*

- *Benevolent Government (Initial)*
  The default set $\mathcal{D}(B, y, p)$ is consistent with the set of government value functions $V(B, y, p)$, $V^R(B, y, p)$, and $V^D(y, p)$ and solves (2.3).

- *Benevolent Government (Repayment)*
  If $(y, p) \notin \mathcal{D}(B)$ and taking as given the bond price schedule $q(B, y, p)$ and the final consumption good price index $\mathcal{P}(p)$, the government chooses the sovereign bonds policy rule $\hat{B}'(B, y, p)$ and the household final consumption policy rule $\hat{C}(B, y, p)$ in order to solve (2.4) and its solution is consistent with $V^R(B, y, p)$.

- *Benevolent Government (Default)*
  If $(y, p) \in \mathcal{D}(B)$ and taking as given the final consumption good price index $\mathcal{P}(p)$, the government chooses household final consumption policy rules $\hat{C}(B, y, p)$ in order to solve (2.5) and its solution is consistent with $V^D(y, p)$.

- *Price Index*
  The final consumption good price index follows (2.10).

- *Default probability*
  The default probability schedule $\delta(B, y, p)$ is consistent with the default set $\mathcal{D}(B, y, p)$ and (2.7).

- *Bond Pricing*
  The bond pricing schedule $q(B, y, p)$ is consistent with the probability of default schedule $\delta(B, y, p)$ and (2.8).

Proposition 4 (Recursive Equilibrium Isomorphism) *The equilibriums defined in Definition 2 and Definition 3 are isomorphic.*

Proof See Appendix 4.1.

This transformation of the original problem is very useful for solving the model and to understand how terms of trade shocks work in our environment. Firstly, realise that the final consumption good price index is non-decreasing[14] in terms of trade regardless of the parameters λ and η,

$$P'(p) = \left(\frac{(1-\lambda)P(p)}{p} \right)^{\frac{1}{1+\eta}} \geq 0.$$

This implies that, regardless of the complementarity or substitutaility of the domestic and foreign goods, the final consumption good price index will keep a monotonic behaviour throughout all the domain of the terms of trade. Moreover, the final consumption good price index will work as a shock absorber of the terms of trade shock. This is, the households will only experience a fraction of the terms of trade shock in terms of final good expenditure. Therefore, terms of trade will have two main effects in the model. The first effect is adjsuting the price of final consumption goods. The second effect is expanding or contracting the debt burden of soverign bonds. This last effect is present under the assumption that the sovereign government issues debt in foreign currency.

Proposition 5 (Default Sets Monotonicity) *Pick an arbitrary level of sovereign bonds B_1 such that $\mathcal{D}(B_1) \neq \emptyset$, if $B_2 \leq B_1$ then $\mathcal{D}(B_1) \subseteq \mathcal{D}(B_2)$.*

Proof See Appendix 4.1.

[14] Furthermore, it is strictly increasing as long as we assume there is no full home-bias in the model $\lambda \in [0, 1)$

This result is originally taken from Eaton and Gersovitz (1981), Arellano (2008), and Chatterjee, Corbae, Nakajima and Rios-Rull (2007) and it is common in sovereign default models. This result tells us that incentives are monotonic with respect of sovereign bonds. Hence, it is mainly followed because the bond pricing $q(B', y, p)$ is independent from the sovereign bonds due in a period. In our model, this relationship does not exist because of the assumption of risk neutrality from the representative foreign lender. Thus, given a level of output and terms of trade, the sovereign bonds due in the period acts only as a shifter in the available amount of resources in the economy. Therefore, if for a level of sovereign bonds a government finds optimal to default, then for a lower level of soverign bonds the default decision will still be optimal because the government will have less resources overall.

Using Proposition 5 we can conclude that the bond pricing $q(B, y, p)$ is non-decreasing in sovereign bonds. Let us focus in the case where the government borrows resources from foreign lenders. As the government increases the amount of borrowing, the actual amount of resources received will decrease because the bond pricing contracts. This resources reduction of borrowing compensates the default probability that the government can incur in the following period.

2.4.3 No Persistency Case

Let us study the case in which the stochastic system proposed in (2.1) has no persistency, therefore it is i.i.d. binormal distribution with mean $\overline{0}$ and variance-covariance matrix Σ. In this case, the probability of default and the sovereign bond price schedule lose their contingency with respect of GDP and terms of trade. This is because current levels of GDP and TOT do not provide any information about the future realizations of them. We assume also that there are no penalty costs, $h(y) = y$; and default is permanent, $\phi = 0$.

Proposition 6 (No Resources Inflows) *For every sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, every feasible B' will yield no resources inflows, $q(B')B' - B \geq 0$.*

Proof See Appendix 4.1.

Default episodes happen when governments are unable to roll-over their debt. This idea is captured with Proposition 6. If the government is in a state where he chooses to default, all the feasible issuances of sovereign bonds must have not been enough to cover the sovereign bonds due in the period. Specifically, there was no feasible issuances of sovereign bonds that could have given a positive flux of resources from outside, $B - q(B')B' \ngtr 0$.

Proposition 7 (GDP Default Incentives) *Pick an arbitrary level of terms of trade p and sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, if $y_2 \leq y_1$ and $(y_1, p) \in \mathcal{D}(B)$ then $(y_2, p) \in \mathcal{D}(B)$.*

Proof See Appendix 4.1.

Default episodes also happen when economies experience recessions, periods where GDP levels are below the trend. Proposition 7 is able to capture this idea. This shows that there if a government finds optimal to default on a level of sovereign bonds for a given recession, any deeper recession would make it default as well. This result is driven mainly because the country will be poorer and there are not contracts available that provide an influx of resources from abroad.

Proposition 8 (Terms of Trade Default Incentives) *Pick an arbitrary level of GDP y and sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, if $p_2 \geq p_1$ and $(y, p_1) \in \mathcal{D}(B)$ then $(y, p_2) \in \mathcal{D}(B)$.*

Proof See Appendix 4.1.

We have a similar result for the case of terms of trade rising higher than the trend. Proposition 8 is able to capture this idea. This shows that there if a government finds optimal to default on a level of sovereign bonds for a level of terms of trade, any higher level of terms of trade would make it default as well. This result is driven mainly because the country will experience more expensive goods from the exterior and the resources that flow outside the country will grow because of the foreign good coversion.

2.5 Quantitative analysis

In this section, we study Mexico and its business cycles statistics. Also, we describe the calibration process of the model fitting the parameters to the Mexican economy. We solve the aggregate recursive equilibrium described in Definition 3 applying a value function iteration process using a grid search method. The complete algorithm can be found in Appendix 4.2.

2.5.1 Data

Let us study the Mexican business cycle behavior as an emerging economy. Using OECD Statistics, we obtain quarterly data seasonally adjusted at quarterly levels, and at current and constant prices from 1993Q1 to 2016Q2 for the series of *gross domestic product*, *private final consumption expenditure*, *exports of goods and services*, and *imports of goods and services.* Also, using Global Financial Database, we obtain quarterly data from 1997Q4 to 2016Q3 for the series of *Emerging Markets Bond Index (EMBI+)*. The series of EMBI+ is provided by J.P. Morgan and portrays the long-term spread between yields from sovereign bond and the U.S. Treasuries. We construct the terms of trade series as the ratio of imports price delfator and exports price deflator following Kehoe and Ruhl (2008) methodology[15]. We apply the *HP-filter* with a smoothing parameter of 1600 to the real consumption, real output and terms of trade log-series in order to obtain the cyclical components of them. In addition, we compute the ratio of the difference between exports and imports, over GDP to construct the series of trade balance.

Table 2.4: Business Cycles Statistics for Mexico

Variable	μ	σ	$\rho(\cdot,\text{Spread})$	$\rho(\cdot,\text{GDP})$	$\rho(\cdot,\text{TOT})$
Spread	2.71%	1.47%	-	0.0352	0.2178
GDP	-	2.22%	0.0352	-	-0.4480
Terms of Trade	-	2.56%	0.2178	-0.4480	-
Consumption	-	2.65%	-0.0367	0.9518	-0.4673
Trade Balance	-0.84%	1.62%	0.3335	-0.4819	0.1501

Table 2.4 shows the business cycles statistics for the Mexican economy. The table shows regular characteristics of emerging economies shown in Neumeyer and

[15]There are other ways to compute terms of trade. For example, Mendoza (1995) uses the ratio of exports and imports volumes. We choose not to use this methodology for convenience. Establishing terms of trade as the ratio of imports and exports deflator matches closely the movements of real exchange rates.

Perri (2005). The volatility of consumption is higher compared to the volatility of GDP. Another important characteristic is that trade balance is countercyclical. Nevertheless, we are not able to find that interest rates (captured by the sovereign spread series) are countercyclical after 1997Q4[16].

Terms of trade also play an important feature. As documented in Mendoza (1995), Kose (2002), and Broda (2004), there is a strong negative relationship between terms of trade and real GDP in emerging countries. Also, the volatility of terms of trade is higher compared to the volatility of GDP. An important feature we are able to provide is that the correlation between terms of trade and the spreads is significantly higher than the correlation between GDP and spreads. This seems to suggest that the movements the Mexican experienced after 1997 may be better explained by movements in terms of trade rather than movements in real GDP.

2.5.2 Calibration

We use the real GDP and terms of trade *HP-filter* log-series in order to estimate (2.1). We assume that the errors vector components are independent from each other[17] and distributed as $\varepsilon^y_{t+1} \sim N(0, \sigma^2_y)$ and $\varepsilon^y_{t+1} \sim N(0, \sigma^2_y)$. We estimate

[16] We analyzed this issue further. We use the database provided by Neumeyer and Perri (2005) and construct an implied EMBI+ series from 1994Q1 to 1997Q3. In the database the authors provide the interest rate of several emerging economies from 1994Q1 to 2002Q2. We take the series of Mexican interest rates and substract the yield of US 10-year Treasury constant maturity. We find that the constucted EMBI+ series resembles closely the original EMBI+ series in the quarters they overlap with the exception of the period 2001Q2-2002Q2. Filling the missing quarters for the spread series we are able to find that the interest rates (captured by the sovereign spread series) is countercyclical, with a correlation of -0.4275.

[17] We make this assumption for simplicity. Nevertheless, we are expecting in relaxing this. The error of the real GDP regression surely is correlated with contemporanous terms of trade.

the matrix A using standard OLS regressions[18],

$$A = \begin{bmatrix} 0.8399 & -0.0291 \\ -0.1466 & 0.5367 \end{bmatrix}.$$

Once estimated the matrix A, we recover the observed errors and estimate the standard deviations of the errors. The standard deviations of the GDP and terms of trade errors are $\sigma_y = 0.0105$ and $\sigma_p = 0.0203$, respectively. Finally, we discretize the 2-dimensional VAR(1) process into a 289 Markov chain (17 GDP and 17 terms of trade shock levels) using a quadrature method algorithm following Tauchen and Hussey (1991) with a 3 standard deviations mean centered bandwidth.

We use a standard CRRA utility function to convey the representative household preferences,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

where σ represents the constant relative risk aversion parameter. We settle this risk aversion parameter to the value of 2. This is a common value in the international real business cycles literature.

During default episodes we impose GDP penalty via the increasing function $h(\cdot)$. An important issue to address when modeling this is its sensitivity contingent to the state of the economy. We use Arellano (2008) convex GDP cost

In this way, the assumption of the errors being independent should be worked upon.

[18]We make this assumption to capture a higher correlation between contemporanous real GDP and terms of trade. Terms of trade can be considered as exogenous for small open economies as motivated in Broda (2004). Therefore, previous real GDP affecting contemporaneous terms of trade should be restricted.

formulation,

$$h(y) = \begin{cases} y & \text{if } y \leq \kappa\mathbb{E}[y] \\ \kappa\mathbb{E}[y] & \text{if } y > \kappa\mathbb{E}[y] \end{cases},$$

where $\kappa > 0$ is a contraction of the long-run mean of GDP. This formulation has the advantage of making default less sensitive to GDP shocks. In particular, defaulting with a GDP level below the threshold of $\kappa\mathbb{E}[y]$ there is no GDP penalty. Nevertheless, above this threshold the GDP penalty increases the higher GDP is.

We use the Global Financial Database in order to obtain the series of *US 10-year Treasury constant maturity yield*. We pick a 10-year Treasury maturity bond because the EMBI+ series relies on long-term maturity bonds. We fix the risk-free interest rate as the average yield from 1997Q1 to 2016Q3, which is 1.62%.

The literature show a wide variety of possible elasticity of substitution between domestic and foreign goods. As noted by Ruhl (2008), this elasticity can be small to account the quarterly fluctuations in trade balances and terms of trade, or high to account the growth in trade due to trade liberalization. In Kose, Towe and Meredith (2004) they propose this elasticity of substitution to be 1.05 for Mexico when analyzing the NAFTA effect on trade. We choose not to use this level due to the different time span of study we are interested. Nevertheless, we find that other updated papers for the Mexican economy have a similar elasticity of substitution that considers the trade liberalization that Mexico has experienced in the past decades. We use the elasticity of substitution between domestic and foreign goods presented in Cuadra and Nuguer (2016) to calibrate η. They use an elasticity of subtitution of 1.5556, which implies a parameter η of -0.3571.

We use the intratemporal condition (2.9) in order to calibrate the home bias parameter. Realise that the intratemporal condition can be rewritten as

$$\left(\frac{c_f}{C}\right)^{1+\eta} = \frac{(1-\lambda)\mathcal{P}(p;\lambda,\eta)}{p} \tag{2.13}$$

In order to construct the series of consumption of foreign goods, we obtain from the Mexican central bank the annual share of imported consumption goods from total imports from 1997 to 2015. We find that the average share is of 13.18% during this period[19]. Using *total imports of goods and services*, *private final consumption expenditure*, and terms of trade quarterly series, the parameter value of $\eta = -0.3571$, the fixed share of imported consumption goods, and the final consumption good price index (2.10) formula; we find a series of the home bias λ_t that solves in every quarter the intratemporal condition (2.13). We find that the average home bias is of 0.8748 from 1997Q4 to 2016Q2.

We calibrate the default GDP penalty and the discount factor in order to match two moments of the Mexican economy. Mexico has defaulted in its sovereign debt twice (1928 and 1982) in the last hundred years[20]. This gives a rough estimate of a 2% default probability. We then focus the targets to be this default probability and the standard deviation of trade balance over GDP ratio shown in Table 2.4. Finally, we keep the probability of re-entry to financial markets proposed by Arellano (2008)[21]. Table 2.5 presents the parameters specification

[19] We also find that from 2007 to 2015 this share almost doubled from 8.5% to 14.24%. We also find that most of this increase happened in the first six years of the sample. For this reason, we consider this share as a constant for the calibration process.

[20] As noted in Reinhart and Rogoff (2009), the default years were when two important international crises happened, the *Great Depression* and the *Emerging Markets Crises*. During the *Tequila Crisis* in 1994, Mexico was close to default with its international lenders. Thanks to the internatinal help from the USA, Mexico was able to have dodge this.

[21] In Gelos, Sahay and Sandleris (2011), it is shown that the average waiting period for re-

from the calibration and estimation strategy.

Table 2.5: Parameters Specification

Parameter	Value	Source
A	$\begin{bmatrix} 0.8399 & -0.0291 \\ -0.1466 & 0.5367 \end{bmatrix}$	OLS estimators
σ_y	0.0105	Observed errors
σ_p	0.0203	Observed errors
σ	2.00	IRBC Literature
r^*	1.62%	US 10-year Treasury
η	-0.3571	Cuadra and Nuguer (2016)
λ	0.8748	Intratemporal Condition
β	0.9530	Default probability
κ	0.9690	Trade balance volatility
ϕ	0.2820	Arellano (2008)

entry after a default has decreased significantly. They show that this average fell from 5 years in the 1980's to 1.6 years in the 1990's. Nevertheless, in Alessandro et al. (2011), they conclude that this comparison between decades is not fair. In particular, that the decrease in average waiting period for re-entry has not decreased throughout time. They provide evidence that, if re-entry to to financial markets do not happen after three years of the default, it is significantly harder to achieve this re-entry. This seems to suggest that the probability of re-entry has not experienced important movements throughout the years.

2.5.3 Results

We use a value function iteration process using a grid search method[22] to solve the model. To study the policy rules we establish a high and low level of GDP and terms of trade as $\pm 0.0489\%$ and $\pm 0.0467\%$ deviations from their means, respectively. We then do a 100 simulation processes for the economy for 10,000 periods, starting with zero sovereign bond holdings and in the long-run level of GDP and terms of trade. We burn the first 500 periods of each simulation and compute the business cycles statistics of the model as the averages of the simulations.

Figure 2.3 shows the boundary limits of the default set (2.6). Because of Proposition 5 we know that these sets are monotonic with respect of the level of sovereign bond holdings. The left panel shows the relationship with respect of GDP fixing the terms of trade level. The default set is the area south west of the boundaries. We find that higher levels of sovereign debt increases the levels of GDP inside the the default set. In addition, having a higher terms of trade level increases slightly the levels of GDP inside the default set. The right panel shows the relationship with respect of terms of trade fixing the GDP level. The default set is the area north west of the boundaries. We find that higher levels of sovereign debt increases the levels of terms of trad inside the default set. In addition, having a higher GDP levles decreases significantly the levels of terms of trade inside the default set.

Figure 2.4 shows the pricing of the sovereign bonds. The left panel shows us how the sovereign bond price schedule decreases in value as the sovereign bond

[22]We use this method as a first approach to solving the equilibrium of the model. As Hatchondo, Martinez and Sapriza (2010) shows, grid search methods can give spurious results in the business cycle analysis.

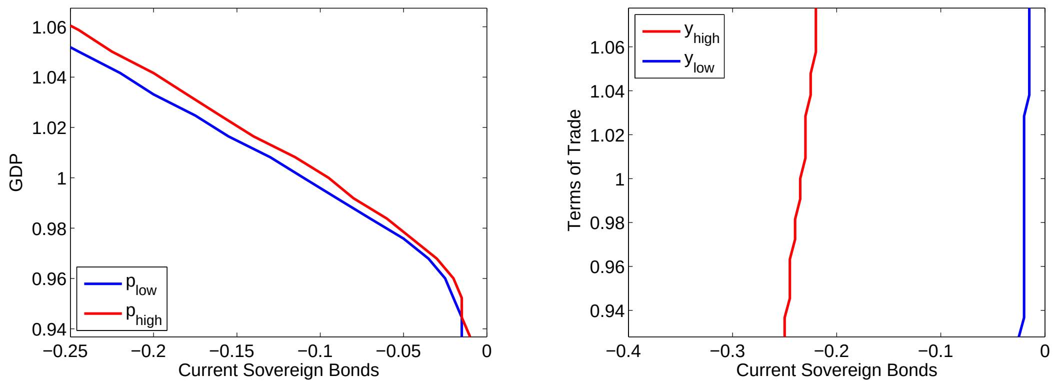

Figure 2.3: Default Set Boundaries

becomes more negative. The right panel shows the implied interest rate using a the domain of sovereign bonds in which the sovereign bond pricing is strictly positive. These graphs deliver two important features. Firstly, this model is able to create countercyclical interest rates due to the link between probability of default and GDP levels. Secondly, the movements of terms of trade increase in importance for sovereign bond price schedule as GDP increases. Moreover, this rise in the price schedule dispersion increases the sovereign intrest rates possibilities. This result helps us explain the movements of spreads during periods where GDP is not below trend. Furthermore, it suggests that terms of trade matter when analyzing interest rates and default likelihoods of emerging economies. This is a step towards explaining the puzzle shown in Tomz and Wright (2007).

Table 2.6 shows the business cycle statistics of the model. The business cycles is able to recover a couple of the statistics shown in Table 2.4. But in general, the experiment fails because it is not able to match most of the moments in the

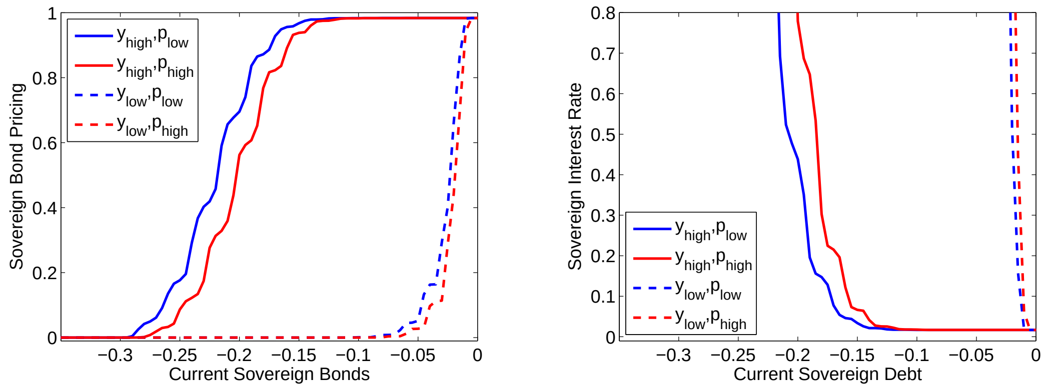

Figure 2.4: Sovereign Bond Pricing and Sovereign Interest Rate

data. Despite of this, it is an important step towards understanding the flaws and improving the model.

Table 2.6: Model Business Cycles Statistics

Variable	μ	σ	$\rho(\cdot,\text{Spread})$	$\rho(\cdot,\text{GDP})$	$\rho(\cdot,\text{TOT})$
Spread	0.65%	0.74%	-	-0.2009	0.2567
GDP	-	2.24%	-0.2009	-	-0.2388
Terms of Trade	-	2.52%	0.2567	-0.2388	-
Consumption	-	2.01%	-0.3073	0.9386	-0.2376
Trade Balance	0.00%	0.09%	0.3788	-0.2628	-0.0385
Default probability	2.46%	-	-	-	-
Debt-GDP ratio	3.05%	-	-	-	-

2.6 Conclusion

We propose a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. We incorporate two types of shocks to cover a foreign and a domestic uncertainty. We define as the domestic and the foreign uncertainty, GDP and terms of trade shock, respectively. The model is able to succesfully increase the dispersion of sovereign interest rates when GDP shocks are above the trend. This result seems to suggest that terms of trade is a good candidate to explain the volatility of interest rates in small open economies when they are not under recessions or crises.

Unfortunately, our business cycles exercise have room for improvements. Nevertheless, the results presented is a great step to explain the behavior of interest rates in emerging economies. Below, we present three issues we are currently working in order to improve our line of research.

Firstly, the VAR(1) process does not capture correctly the dynamics between GDP and terms of trade shown in the data. This can be shown by the small correlation between their contemporaneous realizations in the simulation process. In particular, we are confident that $\mathbb{E}\left[\varepsilon_t^y | \ln p_t\right] \neq 0$, making our estimators in A biased. Moreover, we are not implementing completely the exogeneity assumption of terms of trade. Specifically, we let the future realization of terms of trade be affected by the current level of GDP. Furhter work of the model will consider improvements in the VAR(1) process presented in (2.1), for example

$$\begin{bmatrix} \ln y_{t+1} \\ \ln p_{t+1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ 0 & a_3 \end{bmatrix} \begin{bmatrix} \ln y_t \\ \ln p_t \end{bmatrix} + \begin{bmatrix} 1 & b_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^y \\ \varepsilon_{t=1}^p \end{bmatrix}.$$

Another weakness of our results is our calibration strategy. We let the tolerance in the algorithm to be high for a faster convergence. The tolerance provided is of $1.0e^{-}1$, which yields really loose results. Moreover, we only do one simulation process in order to compute the target statistics of default probability and standard deviation of trade balance over GDP. Future improvements will encompass a more serious calibration process with a higher tolerance and number of simulation processes. Moreover, we will include as part of it the probability of re-entry to financial markets by targeting the ratio of debt over GDP.

Finally, our computation of the equilibrium can improve greatly. Unfortunately, having three state variables increases greatly the computational cost in terms of time. Because of this we have coarse grids that might miss important movements. Furthermore, as noted by Hatchondo, Martinez and Sapriza (2010), grid search methods can yield spurious results in the business cycles statitics of the model. Future improvements will work on this by imposing finer grids in the state variables. In addition, we will move from the grid search method to methods that are able to capture movements between the grid elements[23].

[23]There are a great variety of methods that allow movements inside the grid elements. In particular, we are currently working on linear and quadratic interpolation methods. We are also interested in implementing innovative methods that have shown effiency in solving these types of models. Specifically, we are interested in the alogorithms provided in McGrattan (1996) and Gordon and Qiu (2015).

Chapter 3

Optimal foreign currency debt denomination

3.1 Introduction

The popular question regarding debt currency denomination is whether it should be incurred in domestic or foreign currency. This question has been studied extensively regarding private and sovereign governments, focusing in the benefits and shortcomings from issuing debt in a currency different to the domestic currency. For example, Du and Schreger (2015) study the patters of sovereign debt in domestic and foreign currency across time. However, not all foreign currencies are the same, and this dimension is missing in the literature. The question I address in this paper is, what type of foreign currency should a country issue debt? In particular, should a country issue debt in its trade partner currency? I find that as an economy falls into a recession and more debt issuance is desired, issuing debt in a currency not linked to its trading partner currencies becomes attractive.

3.2 Two Period Model

Consider a two period small open economy inhabited with a representative firm, a representative household, and a benevolent government. There are two different type of foreign goods. One of the foreign goods is linked with the trade in the small open economy has. The real exchange rate between this foreign good and the domestic produced good will be the terms of trade p. On the other hand, the other foreign good is not linked to the trade in the small open economy. The real exchange rate between this foreign and the domestic produced goods can be described by e. In the first period there is no uncertainty and the terms of trade and the real exchange rate start at their long-run level $\overline{p}$ and $\overline{e}$, respectively. For simplicity, I will assume these long-run levels are the same and normalized to one. Nevertheless, in the second period there is uncertainty regarding both, the terms of trade p and the real exchange rate e.

3.2.1 Representative Household

The representative household maximizes its two period utility by consuming final goods. Every period, the total disposable income of the household comes from the profits of representative firm and a lump-sum tax/transfer by the government. The utility maximization problem of the household can be expressed as

$$\begin{aligned}
&\underset{c}{\text{Max}} \left\{ u\left(c_1\right) + \beta \mathbb{E}\left[u\left(c_2\right)\right] \right\} \\
&\text{s.t. } \; c_1 = \pi_1 + T_1 \\
&\qquad\; c_2 = \pi_2 + T_2
\end{aligned}$$

where the utility function $u(\cdot)$ is a strictly increasing and concave function and $\beta \in (0,1)$ the discount factor.

3.2.2 Representative Firm

The representative firm every period observes the terms of trade realization and maximize its profits by purchasing foreign intermediate inputs. The profit maximization problem can be expressed as

$$\begin{aligned}
\pi(p) = &\underset{Y,M}{\text{Max}} \left\{ Y - pM \right\} \\
&\text{s.t. } \; Y = zM^{\alpha}
\end{aligned}$$

The first order condition of the previous problem yields the following optimal allocations

$$\begin{aligned}
&\hat{M}(p) = \left(\frac{\alpha z}{p}\right)^{\frac{1}{1-\alpha}}, \quad \hat{Y}(p) = \left(z\left(\frac{\alpha}{p}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}, \quad \text{and} \\
&\pi(p) = (1-\alpha)\left(z\left(\frac{\alpha}{p}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

3.2.3 Foreign International Lenders

The foreign international lenders can invest in a world risk-free asset and in two different risky government bonds in the first period. For simplicity, I assume that the risk-free asset and one of the risky government bonds are denominated in the same foreign good. The other risky government bond will be denominated in the foreign good the small open economy has a trade relationship with. The international lenders have deep pockets in the first period and are assumed to be risk-neutral. The maximization problem of the international lenders can be described as

$$\begin{aligned} &\text{Max} \left\{ x_1 + \frac{1}{1+r^e} x_2 \right\} \\ &\text{s.t. } \; x_1 = I - q_e b_e - q_p b_p \\ &\qquad\; x_2 = \mathbb{E}_{p,e} \left[(1-d_e) b_e + (1-d_p) \frac{p}{e} b_p \right], \end{aligned}$$

where the d_e and d_p represent if the government default in the risky bond in the second period. The first order conditions of this problem yield the following debt pricing rules

$$q_e = \frac{1}{1+r_e} \mathbb{E}\left[(1-d_e)\right] \qquad \text{and} \qquad q_p = \frac{1}{1+r^e} \mathbb{E}\left[(1-d_p)\frac{p}{e}\right].$$

3.2.4 Benevolent Government

The government is benevolent with respect of the representative household's welfare and issues debt b in the first period and in the second period chooses either to default or repay it. The government issues new debt in the first period and it can choose whether to issue debt in the currency of its trade partner (real exchange rate of p) or in a different independent currency (real exchange rate of e). If it defaults in the second period, it will receive a penalty of $\kappa \in (0,1)$ tradabale goods over the total profits of the representative firm. Finally, the government starts with an initial amount of debt to be paid in final goods b_0. The maximization problem in the first period of the benevolent government can be summarized in

$$V(b_0) = \text{Max}\left\{V_p(b_0), V_e(b_0)\right\}, \tag{3.1}$$

where $V_p(b_0)$ and $V_e(b_0)$ represent the environment of choosing foreign denominated debt in the trade partner currency or the independent currency, respectively. The maximization problem when debt is in the trade partner currency can be expressed as

$$\begin{aligned} V_p(b_0) = \underset{c_1,c_2,b}{\text{Max}} & \left\{u(c_1) + \beta \mathbb{E}_p\left[u(c_2)\right]\right\} \\ \text{s.t. } & c_1 + b_0 = \pi(\overline{p}) + q_p(b)\, b \\ & c_2 = \begin{cases} \pi(p) - pb & \text{if} \quad repays \\ (1-\kappa)\,\pi(p) & \text{if} \quad defaults \end{cases} \end{aligned}$$

On the other hand, the maximization problem when debt is in the an independent foreign currency can be expressed as

$$
\begin{aligned}
V_e(b_0) = \underset{c_1,c_2,b}{\text{Max}} & \left\{u\left(c_1\right) + \beta \mathbb{E}_{p,e}\left[u\left(c_2\right)\right]\right\} \\
\text{s.t. } & c_1 + b_0 = \pi\left(\overline{p}\right) + q_e\left(b\right) b \\
& c_2 = \begin{cases} \pi\left(p\right) - eb & \text{if} \quad repays \\ \left(1-\kappa\right)\pi\left(p\right) & \text{if} \quad defaults \end{cases}
\end{aligned}
$$

3.2.5 Equilibrium

Let us describe the default sets for both problems as

$$
\mathcal{D}_p(b) \equiv \{p \in \mathbb{R}_+ : \quad u\left((1-\kappa)\pi(p)\right) > u\left(\pi(p) - pb\right)\} \qquad \text{and} \tag{3.2}
$$

$$
\mathcal{D}_e(b) \equiv \left\{(p,e) \in \mathbb{R}^2_+ : \quad u\left((1-\kappa)\pi(p)\right) > u\left(\pi(p) - eb\right)\right\}. \tag{3.3}
$$

Using these sets, we can rewrite the debt pricing as

$$
q_p(b) = \frac{1}{1+r_e} \int \int_{p \in \mathcal{D}_p(b)} \frac{p}{e} dF(p) dF(e) \qquad \text{and} \tag{3.4}
$$

$$
q_e(b) = \frac{1}{1+r_e} \int_{(p,e) \in \mathcal{D}_e(b)} dF(p,e). \tag{3.5}
$$

Moreover, we can define the second period discounted expected utility as

$$\begin{aligned}
\mathcal{V}_p(b) &\equiv \beta \mathbb{E}_p\left[u(c_2)\right] \\
&= \beta\left[\int_{p\in\mathcal{D}_p(b)} u(\pi(p)-pb)dF(p) + \int_{p\notin\mathcal{D}_p(b)} u((1-\kappa)\pi(p))dF(p)\right] \qquad \text{and} \\
\mathcal{V}_e(b) &\equiv \beta \mathbb{E}_{p,e}\left[u(c_2)\right] \\
&= \beta\left[\int_{(p,e)\in\mathcal{D}_e(b)} u(\pi(p)-eb)dF(p,e) + \int_{(p,e)\notin\mathcal{D}_e(b)} u((1-\kappa)\pi(p))dF(p,e)\right].
\end{aligned}$$

Figure 3.1 shows these relationships and how they change when higher debt is issued in the first period. Figure 3.1(a) shows us that for low levels of debt, there is a higher price for debt issued in the trading partner currency. Nevertheless, this changes as higher debt starts to be issued. This can be explained as the more likely the government is to default, a negative terms of trade shock makes profits fall and debt to rise, making default attractive when debt is issued in the trade partner currency. Nevertheless, when debt is issued in a different foreign exchange rate, when terms of trade deteriorate, profits fall but debt does not necessarily increases. As a matter of fact, the exchange rate may appreciate, making the debt burden fall and making default not as attractive. Figure 3.1(b) shows the discounted expected utility from the second period. It also shows that as debt increases and comes closer to default, issuing debt in a foreign currency not linked to the trade partner is more attractive due to the possibility of an appreciation of the currency, making the debt burden in the second period to fall.

Using these definitions, I state the following definition for a recursive equilibrium of the two period model.

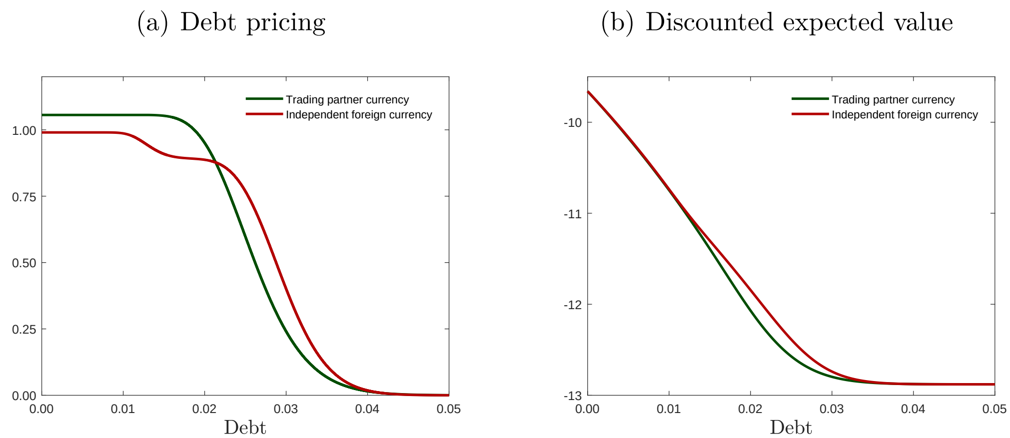

Figure 3.1: Debt pricing and second period discounted expected value contingent in debt

Note: To construct these functions contingent to debt I use the parameters established in the following section. The debt axis is regarding the debt incurred in the first period.

Definition 9 (Recursive Equilibrium) *A recursive equilibrium with initial debt b_0 is defined as value functions $\{V(b_0), V_p(b_0), V_e(b_0)\}$, a set of debt pricing schedules $\{q_p(b), q_e(b)\}$, and default sets $\{\mathcal{D}_p(b), \mathcal{D}_e(b)\}$; such that the following conditions are satisfied:*

1. *The decision of whether to issue debt in the trade partner currency or a different foreign currency satisfies* (3.1).

2. *Taking the debt pricing schedule $q_p(b)$, the following problem is solved*

$$V_p(b_0) = \underset{b}{\mathrm{Max}} \left\{ u\left(\pi\left(\overline{p}\right) + q_p\left(b\right) b - b_0\right) + \mathcal{V}_p(b) \right\}.$$

3. *Taking the debt pricing schedule $q_e(b)$, the following problem is solved*

$$V_e(b_0) = \underset{b}{\mathrm{Max}} \left\{ u\left(\pi\left(\overline{p}\right) + q_e\left(b\right) b - b_0\right) + \mathcal{V}_e(b) \right\}.$$

4. *The default sets satisfy* (3.2) *and* (3.3).

5. *The debt pricing schedules satisfy* (3.4) *and* (3.5).

3.3 Quantitative Analysis

In this section, I pick the functional forms and values to the parameters of the model. I solve for both currencies a grid search method to find the optimal debt. Then I choose the the currency that yields the highest utility in the first period given an initial debt paid in the first period. I then study how the currency choice change as the initial debt increases.

3.3.1 Baseline Calibration

I calibrate the model taking parameters values widely accepted in the literature. I discretize the initial debt, the period 1 debt, and the terms of trade space into grids of 5001 elements each. For the grids regarding debt, the elements are going to be equally separated from 0.0 to 0.1. For the terms of trade, I follow Tauchen and Hussey (1991) to obtain the probability transition matrix and the elements of the grid for an AR(1) process. For the stochastic process of the different exchange rate, I take set it to take only two values in such a way that the average of the exchange rate shock is the long-run level of the terms of trade shock.

Functional Forms. The utility function is a standard CRRA

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ represents the risk aversion parameter of the households.

I assume a decreasing returns to scale production function

$$F(M) = zM^{\alpha},$$

where $\alpha < 1$.

I assume the log terms of trade process follow an AR(1) process

$$\ln(p') = \rho \ln(p) + \sigma\varepsilon$$

where the auto-correlation parameter satisfy $|\rho| < 0$, and the shocks are *i.i.d.* and normal distributed, $\varepsilon \sim N(0,1)$.

Finally, I assume that the real exchange rate follow a Bernoulli distribution with parameter λ. I also set that the mean of the exchange rate coincides with the long-run level of the terms of trade shock.

Model Parameters. Table 3.1 shows all the baseline calibration values for the parameters of the model. I set values of the parameters widely used in the literature. I normalize the productivity of the final good production as well as the long-run level of the log terms of trade stochastic process. Its auto-correlation and standard deviation are consistent to the values found in the literature.

Table 3.1: Calibration table

Parameter	Value
$\overline{p}$	1.00
$\overline{e}$	1.00
z	1.00
α_M	0.75
γ	2.00
λ	0.80
ρ	0.50
σ	0.08
r	1.01
β	0.99

Figure 3.2 show the optimal debt of the sovereign government contingent to an initial debt the government needs to pay in the first period. This can also be seen as a contraction in the economy measuring an initial crisis in the first period. This makes the government incur in more debt and therefore more close

to a default scenario in the second period. Figure 3.2(a) show the optimal debt incurred in each type of foreign currency denomination. Figure 3.2(b) portrays the same curves but considering considering when it is optimal to incur the debt in the trading partner currency or not. The main message of the model is that the higher debt is needed in the first period, issuing debt in a different currency becomes more attractive. The figure shows that for initial debt levels of 0.72, debt in the trading partner currency is optimal. However, for higher levels of initial debt, the government will find more attractive to issue debt in a currency not linked to its terms of trade.

(a) Optimal debt (b) Optimal currency

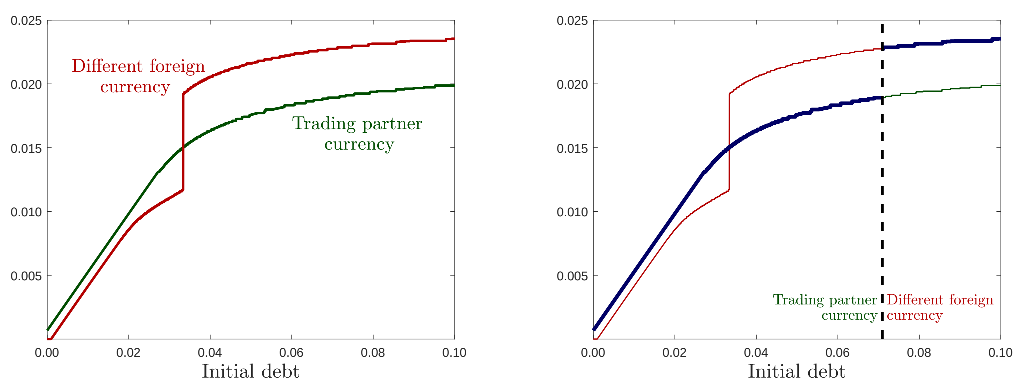

Figure 3.2: Optimal debt issuance contingent to initial debt

3.4 Conclusion

In this paper, I deviate from the question of whether a country should issue debt in domestic or foreign currency. Instead I focus in the question of what type of foreign currency should a country issue its debt. To answer this question I create a two period model where a sovereign government chooses whether to issue debt in a currency tied to their terms of trade or not. I find that when a government is not in a recession, a government is more prone to choose a foreign currency linked to their trading patterns. Nevertheless, as soon as a government is in a recession and needs to incur in higher levels of debt, changing to a currency not linked to its trading patterns becomes more attractive. This is due to the probability of the domestic currency to appreciate and contract the debt burden when there is a deterioration in the terms of trade.

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Chapter 4

Appendix

4.1 Propositions proofs

4.1.1 Proposition 4 (Recursive Equilibrium Isomorphism)

Proof Let us use this intratemporal condition (2.9) and the household's final consumption aggregator,

$$\begin{aligned}
C &= \left(\lambda c_d^{-\eta} + (1-\lambda) c_f^{-\eta}\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right) \left(\frac{c_d}{c_f}\right)^{\eta}\right)\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right) \left(\left(\frac{\lambda}{1-\lambda}\right) p\right)^{\frac{\eta}{1+\eta}}\right)\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)\right)^{-\frac{1}{\eta}} \\
&= \left(\frac{c_d}{\lambda^{\frac{1}{1+\eta}}}\right) \left(\lambda^{\frac{1}{1+\eta}} + (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)^{-\frac{1}{\eta}} .
\end{aligned}$$

Using the final consumption good price index (2.10), we can rewrite the previous expression as

$$\frac{c_d^{1+\eta}}{\lambda} = \mathcal{P}(p)C^{1+\eta}. \tag{4.1}$$

Using the the intratemporal condition (2.9) and mixing the budget and the balanced trade constraints of (2.4) and (2.5), we can realise that the consumption of domestic and foreign expenditure can be expressed as

$$\begin{aligned}
c_d + pc_f &= c_d + \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{c_d}{c_f}\right)^{1+\eta} c_f \\
&= c_d + \left(\frac{1-\lambda}{\lambda}\right) c_d^{1+\eta} c_f^{-\eta} \\
&= \left(\frac{c_d^{1+\eta}}{\lambda}\right)\left(\lambda c_d^{-\eta} + (1-\lambda)c_f^{-\eta}\right) \\
&= \frac{c_d^{1+\eta}}{\lambda C^{\eta}}.
\end{aligned}$$

Thus, using (4.1) we reach the expression that the household's consumption expenditure can be expressed in terms of the final consumption good and the final consumption good price index (2.10),

$$c_d + pc_f = \mathcal{P}(p)C.$$

Furthermore, using (2.9) and (4.1), we can construct how the consumption of domestic and foreign goods can be decomposed from the aggregate final consumption good as

$$c_d = (\lambda \mathcal{P}(p))^{\frac{1}{1+\eta}} C \qquad \text{and} \qquad c_f = \left(\frac{(1-\lambda)\mathcal{P}(p)}{p} \right)^{\frac{1}{1+\eta}} C.$$

Finally, realise that the restrictions of the maximization problems (2.4) and (2.5) are the same as the ones described in (2.11) and (2.12). $\blacksquare$

4.1.2 Proposition 5 (Default Sets Monotonicity)

Proof Set a level of sovereign debt B_1 and levels of output and terms of trade such that $(y,p) \in \mathcal{D}(B_1)$. Then, it follows that $V^D(y,p) > V^R(B_1,y,p)$. Pick an arbitrary level of sovereign debt B_2 such that $B_2 \leq B_1$. Let us study the resource constraint in (2.11). Define the budget set of the government contingent to the amount of sovereign bonds due as

$$\mathcal{B}(B) = \left\{ (C, B') \in \mathbb{R}_+ \times [\mathbb{B}, \infty) : \quad P(p)C + pq(B', y, p) \leq y + pB \right\}.$$

By construction, acknowledge that $\mathcal{B}(B_2) \subseteq \mathcal{B}(B_1)$. Because the government is maximizing over a subset of a set, it follows that $V^R(B_1,y,p) \geq V^R(B_2,y,p)$. Joining the inequalities, we conclude that

$$V^D(y,p) > V^R(B_1,y,p) \geq V^R(B_2,y,p).$$

In other words, $(y,p) \in \mathcal{D}(B_2)$. Finally, because the levels of sovereign debt B_2 was taken arbitrarily, we can conclude that $\mathcal{D}(B_1) \subseteq \mathcal{D}(B_2)$. $\blacksquare$

4.1.3 Proposition 6 (No Resources Inflows)

Proof Pick an arbitrary $(y,p) \in \mathcal{D}(B)$ and realise this implies $V^D(y,p) > V^R(B,y,p)$. Acknowledge that the resource constraints found in (2.11) and in (2.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

Therefore,

$$\begin{aligned}
u\left(\frac{y}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y',p')\right] > \\
&> \operatorname*{Max}_{B'}\left\{u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right\} \\
&\geq \operatorname*{Max}_{B'}\left\{u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y',p')\right]\right\} \\
&\geq u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y',p')\right],
\end{aligned}$$

for all feasible B'. Thus,

$$u\left(\frac{y}{\mathcal{P}(p)}\right) > u\left(\frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right)\right).$$

Because $u(\cdot)$ is an increasing function,

$$\frac{y}{\mathcal{P}(p)} \geq \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right).$$

Then, we arrive to

$$q(B')B' - B \geq 0.$$

Finally, because we picked an arbitrary $(y,p) \in \mathcal{D}(B)$, we can conclude that for all feasible B' there are no resources inflows, $q(B')B' - B \geq 0$. ∎

4.1.4 Proposition 7 (GDP Default Incentives)

Proof Set a level of GDP, terms of trade and sovereign bonds such that $(y_1, p) \in \mathcal{D}(B) \neq \emptyset$. Then, it follows that $V^D(y_1, p) > V^R(B, y_1, p)$. Pick an arbitrary level of GDP y_2 such that $y_2 \leq y_1$. Acknowledge that the resource constraints found in (2.11) and in (2.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

To make the proof easier, call

$$B_1' = \arg\underset{B'}{\text{Max}}\left\{u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_1')B_1' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_1', y', p')\right]\right\} \quad \&$$
$$B_2' = \arg\underset{B'}{\text{Max}}\left\{u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2', y', p')\right]\right\}.$$

Realise that, in particular, these expressions imply,

$$u\left(\frac{y_1}{\mathcal{P}(p)}-\frac{p}{\mathcal{P}(p)}\left(q(B_1')B_1'-B\right)\right)+\beta\mathbb{E}_{(y',p')}\left[V\left(B_1',y',p'\right)\right]\geq$$
$$u\left(\frac{y_1}{\mathcal{P}(p)}-\frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2'-B\right)\right)+\beta\mathbb{E}_{(y',p')}\left[V\left(B_2',y',p'\right)\right].$$

Using Proposition 6 and because $\mathcal{D}(B)\neq\emptyset$ and $u(\cdot)$ is increasing, it follows that $q(B')B'-B\geq 0$ for every feasible B'. In particular, for the optimal level of sovereign bonds using y_2 level of GDP, $q(B_2')B_2'-B\geq 0$.

Because $u(\cdot)$ is strictly concave and $q(B_2')B_2'-B\geq 0$, we know that

$$u\left(\frac{y_1}{\mathcal{P}(p)}-\frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2'-B\right)\right)-u\left(\frac{y_2}{\mathcal{P}(p)}-\frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2'-B\right)\right)\geq$$
$$\geq u\left(\frac{y_1}{\mathcal{P}(p)}\right)-u\left(\frac{y_1}{\mathcal{P}(p)}\right).$$

Moreover, the right hand side can be expressed as

$$\begin{aligned}u\left(\frac{y_1}{\mathcal{P}(p)}\right)-u\left(\frac{y_2}{\mathcal{P}(p)}\right)&=\left(u\left(\frac{y_1}{\mathcal{P}(p)}\right)+\beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right)\\&\qquad-\left(u\left(\frac{y_2}{\mathcal{P}(p)}\right)+\beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right)\\&=V^D(y_1,p)-V^D(y_2,p).\end{aligned}$$

Therefore,

$$
\begin{aligned}
V^D(y_1,p) - V^D(y_2,p) &\leq u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) \\
&\quad - u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) \\
&= \left(u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\quad - \left(u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\leq \left(u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_1')B_1' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_1',y',p')\right]\right) \\
&\quad - \left(u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&= V^R(B,y_1,p) - V^R(B,y_2,p) \\
&< V^D(y_1,p) - V^R(B,y_2,p).
\end{aligned}
$$

Then we arrive to following expression $V^D(y_2,p) > V^R(B,y_2,p)$. Finally we can conclude, because the level of GDP y_2 was taken arbitrarily, we can conclude that $(y_2,p) \in \mathcal{D}(B)$.

∎

4.1.5 Proposition 8 (Terms of Trade Default Incentives)

Proof Set a level of GDP, terms of trade and sovereign bonds such that $(y,p_1) \in \mathcal{D}(B) \neq \emptyset$. Then, it follows that $V^D(y,p_1) > V^R(B,y,p_1)$. Pick an arbitrary level of terms of trade p_2 such that $p_2 \geq p_1$. Acknowledge that the resource constraints found in (2.11) and in (2.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

To make the proof easier, call

$$B_1' = \arg\underset{B'}{\text{Max}} \left\{ u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)} \left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_1', y', p')\right] \right\} \quad \&$$
$$B_2' = \arg\underset{B'}{\text{Max}} \left\{ u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)} \left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_2', y', p')\right] \right\}.$$

Realise that, in particular, these expressions imply,

$$u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)} \left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_1', y', p')\right] \geq$$
$$u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)} \left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_2', y', p')\right].$$

Using Proposition 6 and because $\mathcal{D}(B) \neq \emptyset$ and $u(\cdot)$ is increasing, it follows that $q(B')B' - B \geq 0$ for every feasible B'. In particular, for the optimal level of sovereign bonds using y_2 level of GDP, $q(B_2')B_2' - B \geq 0$.

Acknowledge that the final consumption good price index is increasing,

$$\mathcal{P}'(p) = \left(\frac{(1-\lambda)\mathcal{P}(p)}{p}\right)^{\frac{1}{1+\eta}} \geq 0.$$

Therefore, the ratio $\frac{p}{\mathcal{P}(p)}$ is increasing too,

$$\begin{aligned}
\frac{d}{dp}\left(\frac{p}{\mathcal{P}(p)}\right) &= \frac{\mathcal{P}(p) - p\mathcal{P}'(p)}{\left(\mathcal{P}(p)\right)^2} \\
&= \frac{1 - (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}} \left(\mathcal{P}(p)\right)^{-\frac{\eta}{1+\eta}}}{\mathcal{P}(p)} \\
&= \frac{\left(\mathcal{P}(p)\right)^{\frac{\eta}{1+\eta}} - (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}}{\left(\mathcal{P}(p)\right)^{\frac{1+2\eta}{1+\eta}}} \\
&= \left(\frac{\lambda}{\left(\mathcal{P}(p)\right)^{1+2\eta}}\right)^{\frac{1}{1+\eta}} \\
&\geq 0.
\end{aligned}$$

In other words, it follows that $\frac{p_2}{\mathcal{P}(p_2)} \geq \frac{p_1}{\mathcal{P}(p_1)}$ because $p_2 \geq p_1$.

Using the previous result and because $u(\cdot)$ is strictly concave and $q(B_2')B_2' - B \geq 0$, we know that

$$\begin{aligned}
u\left(\frac{y}{\mathcal{P}(p_1)}\right) - u\left(\frac{y}{\mathcal{P}(p_2)}\right) &\leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) \\
&\quad - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) \\
&\leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) \\
&\quad - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right)
\end{aligned}$$

Moreover, the left hand side can be expressed as

$$\begin{aligned}
u\left(\frac{y}{\mathcal{P}(p_1)}\right) - u\left(\frac{y}{\mathcal{P}(p_2)}\right) &= \left(u\left(\frac{y}{\mathcal{P}(p_1)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) \\
&\qquad - \left(u\left(\frac{y}{\mathcal{P}(p_2)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) \\
&= V^D(y,p_1) - V^D(y,p_2).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&V^D(y,p_1) - V^D(y,p_2) \leq \\
&\quad \leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) \\
&\quad = \left(u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\qquad - \left(u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\quad \leq \left(u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_1')B_1' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_1',y',p')\right]\right) \\
&\qquad - \left(u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\quad = V^R(B,y,p_1) - V^R(B,y,p_2) \\
&\quad < V^D(y,p_1) - V^R(B,y,p_2).
\end{aligned}$$

Then we arrive to following expression $V^D(y,p_2) > V^R(B,y,p_2)$. Finally we can conclude, because the level of terms of trade p_2 was taken arbitrarily, we can conclude that $(y,p_2) \in \mathcal{D}(B)$. ∎

4.2 Computational algorithm

We extend the algoritm described in Arellano (2008) and incorporate the one loop enhancement proposed in Hatchondo, Martinez and Sapriza (2010) using a grid search method. The following is the algorithm we follow to solve the model proposed in Definition 3 and the calibration strategy for the discount factor β and the GDP default penalty parameter κ:

1) Fix the calibration targets of default probability $\bar{d}$ and standard deviation of trade balance over GDP $\bar{s}$.

2) Discretize B space and dicretize (y,p) space using Tauchen and Hussey (1991)

3) Propose a guess for the discount factor β and the GDP default penalty parameter κ,

$$\beta = 0.95 \qquad \text{and} \kappa = 1.00$$

4) Propose a guess for the set of value functions V, V^R, V^D and the sovereign bond price schedule q,

$$V = [0], \qquad V^R = [0], \qquad V^D = [0], \qquad \text{and} \qquad q = \left[\frac{1}{1 + r^*}\right].$$

5) For every state (B, y, p), solve the repayment state maximization problem (2.11) and compute an implied repayment state value function $\hat{V}^R$.

6) For every state (y, p), solve the repayment state maximization problem (2.12) and compute an implied default state value function $\hat{V}^D$.

7) For every state (B, y, p), solve the maximization problem and compute an implied default state value function $\hat{V}$.

8) Using the implied value functions $\hat{V}^R$ and $\hat{V}^D$, construct default set $\mathcal{D}$

9) Using the default set $\mathcal{D}$, construct default probability schedule δ

10) Using the default probability schedule δ, construct an implied sovereign bond price schedule $\hat{q}$

11) Compute the error term as

$$z_1 = ||V - \hat{V}||_\infty + ||V^R - \hat{V}^R||_\infty + ||V^D - \hat{V}^D||_\infty + ||q - \hat{q}||_\infty.$$

12) Update guesses $V = \hat{V}$, $V^R = \hat{V}^R$, $V^D = \hat{V}^D$, and $q = \hat{q}$.

13) If $z_1 \geq 1.0e^{-6}$, return to 5).

14) Construct one simulation of 10,000 periods with 500 burn-ins starting with the zero sovereign bonds and the long-run levels of GDP and terms of trade

15) Construct the implied default probability α and the implied standard deviation of trade balance over GDP s

16) Compute the error term as

$$z_2 = \frac{\left|d - \bar{d}\right| + |s - \bar{s}|}{2}.$$

17) If $z_2 \geq 1.0e^{-1}$, update guesses β and κ, then return to 4)