Essays on International Macroeconomics

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Dedication

To GOD

Abstract

This dissertation consists of three chapters. Chapter 1 is a critical survey of the literature on the real exchange rate and welfare. First paper researches on welfare associated with two different productivities and the real exchange rate. The second paper classifies countries by regions and analyzes the effects of the real exchange rate. The emerging Asian countries are more export-intensive, so the real depreciation stimulate their economic growth while other emerging countries grow faster with real appreciation. These results rationalize the different growth patterns in different regions.

In Chapter 2, I studied the role of foreign reserves. Some papers have argued that countries accumulate foreign reserves in order to deteriorate terms of trade to increase welfare. On the other hand, the optimal tariff theory argues that tariffs can increase the welfare of a country by improving its terms of trade. This paper provides a plausible explanation for the different foreign reserves policies regarding terms of trade. I build an endogenous growth model of a small open economy with technological spillovers generated from exports. Internalizing the growth effects from these externalities, the government decides whether to accumulate foreign reserves or to borrow from abroad. This paper finds that when the export externalities are large enough, it is optimal to hold positive foreign reserves to achieve faster growth through terms of trade deterioration. However, when the export externalities are small, the government holds negative foreign reserves.

In Chapter 3, Jorge Mondragon and I propose a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. We incorporate two types of shocks to cover foreign and domestic uncertainty. We define GDP and terms of trade shock as the domestic and the foreign uncertainty respectively. The model is able to successfully increase the dispersion of sovereign interest rates when GDP shocks are above the trend. This result seems to suggest that terms of trade is a good candidate to explain the volatility of interest rates in small open economies when they are not under recessions or crises.

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Chapter 1

The real exchange rate and the welfare

1.1 The Productivity and the real exchange rate

Coresetti, Martin, and Pesenti (2005) reveal the relationship between welfare and the real exchange rate and also terms of trade. More specifically, this paper research on welfare implications associated with productivity, market size, government spending, and trade liberalization. When those variables change, the prices such as real exchange rate, terms of trade, and CPI (the price level of the consumption basket) will be affected and this leads to change in the welfare of the Home country as well as trading partner countries.

This paper explains the movement of those variable by building a two-country model with monopolistic competition. Each firm in two countries produces different goods, which means this model has a feature of varieties. The consumer's preference depends on the varieties of goods. When these countries export, transaction cost will be induced. These components of the model generate the key results of this paper. The two-country and the transaction costs are adapted to analyze the international transmission between the two countries. The effects of the change in one country spill over into the other country so the welfare of two countries change simultaneously.[1] Also, the

[1]The spillover effects in this paper do not refer the technological spillovers from more advanced country to less developed country in externality literature. This illustrates how much of the changes in

imperfect competitions and love of varieties create a source of welfare gains. As more various goods are produced and consumed, the consumer's utility increases. In addition to varieties, the lower prices will also increase welfare since the lower prices allow the consumer to purchase at a cheaper price. Indeed, the welfare depends on both prices such as the real exchange rate and CPI and the consumer's preference over varieties.

There are two types of productivities. One is productivity in the production function and the other is productivity related to creating a new variety (or starting a new firm[2]). These two productivities are expressed in terms of costs. The high productivity related to the production reduces the cost of input, so more outputs will be produced with smaller inputs. Meanwhile, the second productivity is shown in the form of reduction of the entry cost. In other words, when productivity is high, more firms easily start producing a new variety since the entry cost becomes cheaper.

The key results

The main findings of the paper are documented as follows.

1. When productivity in Home country increases, the price of Home goods become cheaper which leads to a decrease in profits of firms in Home country. The decrease in profits decrease the incentive of starting a new business, so the number of varieties are lower.

2. The lower varieties mean the domestic goods supplied to the world will be smaller. However, the high productivity increases domestic production and deteriorate the real exchange rate and the terms of trade

3. When market size in Home country is bigger, the terms of trade improve. Furthermore, the output and consumption will be higher than other country and the real exchange rate depreciates.

4. The international spillover from domestic productivity gains to the Foreign country shows different patterns depends on the type of productivity. The rise in production productivity affects Foreign welfare ambiguously while the reduction of the entry costs

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one country transmit to the other country.

[2]Creating a new variety and starting a new firm are the same in this model since each firm produces different goods.

increases the welfare of the Foreign country by increasing the number of varieties.

5. The decrease in transaction costs has two effects: positive effects from lower cost and negative effects from the lower number of varieties. However, the numerical exercise shows the former effects are greater than the latter one.

The model

There are two countries in this world: Home and Foreign. All goods are tradable goods. There are three agents in this economy: Consumers, firms, and a government. This model is written in nominal terms, so there is a nominal exchange rate, price of domestic goods and Foreign goods denominated in the Home currency and the Foreign currency.

consumers There is a representative consumer in this economy consume the basket of differentiated goods produced by domestic firms and Foreign firms. The goods are produced by the firms in the two countries are all different. The utility function of the consumer is a function of the basket of consumption goods and labor.[3]

$$U_t = \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - \kappa l_t \tag{1.1}$$

The fact that the consumer cares about the number of varieties is the crucial factor to analyze international spillovers.

The consumers purchase the goods produced in domestic firms and also imported goods produced in the Foreign country. She pays taxes in a lump-sum fashion. In addition, she invests in firms in both countries but this investment is intra-temporal, meaning that the return will be generated in the same period as a form of profits. The sources of income are labor income and profits.

To obtain the main results, the utility-based consumer price index (CPI) is constructed. This utility-based CPI consists of the sum of prices of domestic varieties and

[3]CRRA for the consumption basket and constant marginal disutility over labor supply.

prices of Foreign varieties denominated in domestic currency. In addition, for the simplicity the elasticity of substitution across varieties is constant. From the first-order conditions, the following equation shows the relationship between the basket of consumption goods and the aggregate price level (CPI).

$$C_t = P_t^{-\psi} \tag{1.2}$$

where ψ is the intertemporal elasticity of substitution.

firms Firms in both countries produce the differentiated goods, denoted by n_t for the Home country and n_t^* for the Foreign country. Each variety is produced by using labor as an only input in a linear fashion.

$$Y_t(h) = \alpha_t l_t(h) \tag{1.3}$$

where h represents the variety at Home and α_t is labor productivity and the same for all firms in the Home.

At time period t, agents decide whether to open a new firm and produce a new variety. To set up a firm, the firm has to pay the fixed entry cost q_t. This entry cost depends on the labor productivity of ν_t, which is associated with creating a new variety. The investment from the consumer is used to finance this fixed entry cost. Furthermore, when firms export, they have to pay 'iceberg' transaction costs.

Mentioned above, there are two productivities in this paper: α in the production function and ν in the entry cost function. They play a different role in welfare analysis.

Government The government purchases the varieties produced only domestically and finances from the representative consumer. The government has a balanced budget constraint. The demand for each variety has the same price elasticity and the price index for the public goods is a function of prices of domestically produced varieties.

Main results and Mechanism

From the profit functions putting the optimality conditions in both countries, they show some main results related to international spillovers. First, when market size at Home becomes larger while other factors are constant, profits at Home increase more than those in Foreign. Second, when there are more firms at Home, profits at Home decrease more than those in Foreign. Third, the real depreciation of Home currency increase the profits at Home. Fourth, the increase in labor productivity in the production function decrease in Home profits. This is due to the intertemporal elasticity of substitution which is less than one. Even though the Home price drops, the demand does not increase sufficiently so the sales revenue decrease.

The effects of the increase in the number of varieties can be examined by the equilibrium profits functions. There are two opposite effects. The higher number of varieties decrease the price of the consumption bundle, so more consumption is available, which results in welfare gains. This increases the demand for goods and the firms' profits. On the other hand, due to the intra-tempral substitution, the demands decrease and the profits also drop. The net effects depend on the value of inter- and intra-temporal elasticity of substitution.

Consider the case where the trade costs drop, which is represented as trade liberalization in this paper. Lowering the transaction costs increase demand for goods and profits. This allows more firms to enter the market. However, more firms are engaged in a more competitive environment. In this case, the number of varieties will be dropped.

In terms of productivity, two different productivities are considered separately. If Home productivity in the production function increase, then it will lower the price of goods and profits for the Home firms also lower.[4] This makes the domestic firms exit the market and the number of varieties becomes smaller. In this situation, Home goods become relatively cheaper than Foreign goods, so the terms of trade deteriorate and also the real exchange rate depreciates. In this situation, the welfare gain for the Foreign country is ambiguous. Due to the lower prices of Home goods, the Foreign country can import at a cheaper price, which increases the welfare of the Foreign country. However, the number of varieties decreases so this negatively affects the welfare of the Foreign

[4]Since price elasticity is $\phi < 1$

country. As the second productivity, consider the higher productivity involved in the entry cost so now firms are able to enter the market at a lower cost. This directly increases the number of varieties at Home, and thus the terms of trade improve. However, the real exchange rate depreciates. Moreover, the increase in market size generates real exchange depreciation and the terms of trade improvement.

The last analysis is about welfare and international spillovers. The welfare at Home increases according to lower trade costs, higher productivities, and larger market size. These results are consistent with the mechanism mentioned above. However, the spillovers on the Foreign countries are undetermined. To investigate it, this paper presents the results obtained from the numerical exercise with some reasonable parameters. From the computation, the increase in Home productivity in the production function increase both domestic and Foreign welfare. Moreover, two countries experience welfare gains with large size of Home market.

Relationship with existing theories

This paper addresses several arguments insisted in previous literature in a different point of view. From the model in this paper, the authors show theoretically differentiated mechanism. The first argument that this paper address differently is that higher growth of a country induces the deterioration of the terms of trade. This paper insists that the opposite result is possible. Indeed, it can be a case that faster growth leads to the improvement of the terms of trade. This paper explains this possibility by investigating the relationship between the terms of trade and two types of productivities. When the firms at Home can produce more efficiently with higher productivity, this lowers the marginal costs of production and the terms of trade deteriorate. At the same time, if the productivity related to firm's entry costs is also high, then, this improves the terms of trade. Depending on the magnitude from two productivities, the terms of trade might deteriorate, but also might improve. In this regard, this paper can explain two cases.

The argument that faster growth can be achieved with any range of change of the terms of trade is, in fact, supported by other literature as well. Acemoglu and Ventura

(2002) support the idea that fast growth countries, which is featured by the higher accumulation of capital, experience the deterioration of the terms of trade. Meanwhile, Gagnon (2004) provides the theoretical model showing that the export-oriented countries can achieve faster growth without deteriorating the terms of trade. In addition, Bleaney and Greenaway (2001) study 14 sub-Saharan African countries and show that faster growth is associated with improvement of the terms of trade. This paper provides a wide range of explanations regarding the growth and terms of trade.

The second argument that this paper shows a different view from the previous literature is about trade liberalization. This paper provides the possibility that trade liberalization leads to a decrease in the number of varieties. This is against the argument that liberalization expands the number of varieties (Melitz (2003), Goldberg at el (2010)) and Debaere and Mostashari (2010)). This paper deal with two different effects of trade liberalization. One is a lower cost argument and the other one is higher competition argument. The former one starts with lowering the trade cost which increases the demand for domestic goods. This results in an increase in profits, so more firms enter the goods market and the number of varieties increases. On the other hand, higher liberalization makes higher competition with the Foreign firms, and thus more firms exist. By comparing these two effects, the number of varieties is determined.

I believe that more empirical work that supports the main point of this paper would make the theoretical model and mechanism more valuable. This paper addresses some debatable points mentioned above so it seems to be important that they provide empirical evidence.[5] In addition, choosing the two countries that fit the story can be helpful. For instance, to see the effects of liberalization, the U.S. and Canada might work for the story since they are big economies and members in NAFTA. Thus, analyzing the number of export varieties before and after NAFTA can give empirical evidence for their argument.

[5]Broda and Weinstin (2006), and Feenstra and Kee (2008) study the number of varieties in their paper. Especially Feensta and Kee (2008) uses the export variety so the measure of export variety can be used.

Introduction of Tariffs

The model in this paper includes the iceberg trade costs so when the firms export, they have to pay them. In this regard, the trade liberalization is characterized by the reduction of these trade costs. Meanwhile, trade liberalization is often expressed by the reduction of tariffs. Thus, it is expected that introducing the tariffs instead of iceberg trade costs also provide an interesting analysis.

As introducing tariff on imported goods, there are few things have to be changed. The budget constraint of the consumer at Home includes the tariffs while the iceberg trade costs do not affect the consumer's decision. Due to the tariffs, the consumer has to pay a higher price for Foreign goods. The budget constraint becomes

$$\int_0^{n_t} p_t(h)c_t(h)dh + \int_0^{n_t^*} (1+\tau)p_t(f)c_t(f)df + I_t \leq \omega_t l_t + \Pi_t \tag{1.4}$$

Where h and f refer to the Home and Foreign varieties respectively, and τ represents import tariffs that Home imposes. Due to the tariffs, the consumer's decision will be different from the iceberg trade cost scenario.

In addition to the consumer budget constraint, the government budget constraint is also affected by the tariffs. Since the government spends on only domestic varieties, the expenditure in the left-hand side consists of Home varieties. Meanwhile, the government finances the government expenditure with import tariffs imposed on Foreign varieties.

$$\int_0^{n_t} p_t(h)G_t(h)dh = \int_0^{n_t^*} \tau p_t(f)c_t(f)df \tag{1.5}$$

In this setting, we can analyze the effects of trade liberalization and government spending. Different from the iceberg trade costs, trade liberalization, which is lower tariffs, not only firms but also the consumer changes its decisions and the welfare will be changed as well. It is expected that with lower tariffs, the consumer's welfare increases since she can purchase imported goods at cheaper prices. However, firms might be hurt since they will be surrounded by more severe competition with Foreign firms. Moreover, when the government increases its spending, it must finance resources from the tariff revenue. In this situation, the import tariffs become an endogenous variable

so that the government can choose the optimal level of tariffs. This can provide policy implication regarding tariffs and the welfare analysis associated with the terms of trade and the real exchange rate is also available.

1.2 The different regions and the real exchange rate

Alfaro, Cunat, Fadinger, and Liu (2018) provide evidence that the benefits to changes in the real exchange rate vary depending on how much a country is focused on exports or imports. This paper embodies various factors: exports, imports, R&D investment, productivity, and financial constraint. More specifically, this paper analyzes the effects of the real exchange rate on firms' behavior such as investment in R&D, exports, and productivity growth. Also, the crucial analysis is conducted based on different regions: emerging Asian countries. other emerging countries, and advanced countries. This paper provides different implication depending on the regional classification.

The main argument of this paper is that in emerging Asian countries, the depreciation of the real exchange rate has a positive impact on the growth of TFP in terms of firm-level because they allow firms to invest more in R&D and more exports. On the contrary, the effects of real depreciations are shown opposite in emerging countries in Eastern Europe and Latin America. This phenomenon is derived by the fact that the emerging Asian countries are export-intensive while other emerging countries are import-intensive. Thus, the effects of the change in the real exchange rate show reversely in these two regions. Meanwhile, the advanced countries do not seem to be affected by the changes in the real exchange rate and this is because the import and export intensities are fairly similar in those advanced countries.

To investigate the mechanism of the model, consider the emerging Asian country case featuring an export-intensive country. The depreciation of the real exchange rate induces a higher demand for exports and results in higher profits of firms. These high profits allow the firms to borrow more and invest more in R&D. Hence, firms in emerging Asian countries can achieve higher productivity with real depreciations. The other emerging countries, in fact, have exactly the opposite mechanism. The real depreciations make imported goods more expensive so the firms in those countries suffer from

higher input prices. This occurs since those countries heavily rely on imports. This decrease the profits of the firms, which means that the firms cannot finance the money that is used in R&D. As a result, the growth of productivity will be lower with real depreciations. However, in the case of advanced countries, the positive effects of the real depreciation on exports are canceled out with its negative effects on imports so there is a negligible effect on productivity.

Stylized facts

There are several stylized facts regarding the real exchange rate and the firm's behavior.

First, the emerging Asian countries show the positive relationship between the real depreciation and growth of productivity, sale, and cash flow. Also, real depreciations allow more firms to engage in R&D and export.

Second, the real depreciation in other emerging economies are negatively associated with the growth of productivity, sale and cash flow. In addition, the probability of invest in R&D and export is low.

Third, the real depreciations have no impacts on those variables mentioned above in the firm-level.

Fourth, the real depreciations are positive for exporters but negative for importers who import intermediate goods.

Fifth, when comparing emerging countries with advanced countries, it can be concluded that the probability that firms export is relatively higher in emerging Asia while the probability that firms import is relatively higher in other emerging countries (Latin America, Eastern Europe). In the same regard, Asian countries have higher export-intensity; on the other hand, other emerging countries have higher import-intensity.

Sixth, Medium, and large firms consider two factors when choosing R&D investment. One is the level of internal cash flow and the other is the degree of development of the financial market. Cash flows are an important issue for other emerging economies and have also shown some impact on emerging Asian economies. However, it does not show much influence in advanced economies.

Empirical Evidence

To provide the empirical evidence, the authors use firm-level data and different data sets are used to supplement the deficiencies of one data source. There are two data sources to obtain information on sales, cash flow, R&D participation, export, and import in the firm-level. The analysis includes yearly based unbalance panel of manufacturing firms in 76 emerging economies and 23 advanced economies from 2001 to 2010. They construct the data for the change in the real exchange rate and define it in the following way.

$$\log(e_{c,t}) = \log(\frac{1}{P_{c,t}}) \tag{1.6}$$

The real exchange rate is defined as the relative price based on the U.S. and calculated with the PPP-based GDP price level. Hence, $P_{c,t}$ represents the GDP price level in PPP. In addition, they also construct the export- and import-weighted real exchange rate using bilateral sectoral export and import data. Thus, this data is country and sectoral specific. This allows the controlling country and time fixed effects.

They found that one percent of the real depreciation leads to increase in all of the growth of TFP, growth of sales, growth of cash flow, R&D probability, and export entry rate in emerging Asian countries. On the contrary, for the case of other emerging countries, the real depreciations significantly decrease only growth of TFP and growth of sales. In addition, there is no significant effect on any of these variables in advanced countries except cash flow, which shows a negative coefficient. They also ran other regressions to check the robustness. First, they conducted with the data excluding the financial crisis period. Second, they used IV instrument for the real exchange rate. In addition, they used export- and import-weighted sector-specific real exchange rate instead of the real exchange rate and analyzed the results.

These different regressions suggest consistent results for the emerging Asian countries and the developed countries, but the consistency of the results seem to be weaker for other emerging countries. When excluding crisis years, the decrease in TFP measured with gross output is not significant and the same result is shown when export- and import weighted real exchange rates are used.[6] Thus, it might be hasty to conclude

[6]The data period excluding the year of the global crisis is from 2000 to 2008.

that the results in other emerging countries are robust.

Furthermore, they divided firms into exporters and importers and investigated how the depreciation of the real exchange rate affect those firms. The higher TFP growth of exporters is associated with the real depreciations in all emerging countries while negative TFP growth of importers is shown. Moreover, the probability of engaging in R&D is significantly affected by the combination of the size of firms, cash flow, and the level of financial development.[7] The small firms do not show significant results, but medium and large firms show that the firms with higher growth of cash flow in the highly developed financial markets are more likely to invest in R&D.

The data analysis in this paper is based on the manufacturing firms. However, there are tradable services suggested by Jensen and Kletzer (2010). They classify tradable jobs based on offshorability, which means services that can be provided without using the internet or face-to-face. The firms providing these services can be included in the analysis and then the more detailed investigation can be possible.

Theoretical Framework

This paper builds a small open economy model with heterogeneous firms to investigate the mechanism supporting the empirical evidence. The important feature of the model is that the firms make a decision of investment in R&D and exporter and importer status. The firms differ by log-productivity, $\omega_{i,t} + \epsilon_{I,t}$ where $\omega_{i,t}$ follows a Markov process and $\epsilon_{I,t}$ is iid with mean 0 and standard deviation, σ_ϵ. In addition, the aggregate variables are taken as given to the firms so the real exchange rate is exogenous in this model. The real exchange rate follows an AR(1) process.

$$\log(e_t) = \gamma_0 + \gamma_1 \log(e_{t-1}) + \nu_t \tag{1.7}$$

The production function is a function of productivity, capital, labor, and materials. The material is made of domestic intermediate good and imported intermediate goods.

[7] The size of firms are classified based on quartiles and financial development is measured by private credit/GDP.

The price of the material depends on the price of domestic and imported intermediate goods and the real exchange rate. The real depreciation lowers the domestic factor price and relatively increase the imported price. This generates the fact that the real depreciations are good for exporters, bad for importers.

Furthermore, each firm faces a demand function for its good. Since firms are monopolists for their goods, they have a constant mark-up. If a firm is a non-importing firm, then revenue does not depend on the exchange rate but importing firm's revenue is negatively affected by the real exchange rate through imported intermediate pries. The exporting firms obtain higher revenue with real depreciation because the exports become cheaper. If the exporting firm also imports, then the revenue is lower since imported intermediates more expensive.

There are four types of firms: firms that export and import, exporting firm, importing firm, firm that sells only domestically. The firms trading internationally have to pay a fixed cost. By comparing profits from four cases, firms choose one of the types. Also, these firms have to choose whether they invest in R&D or not. When they invest in R&D, they have to pay a fixed cost, but it allows higher productivity as shown in Aw, Roberts, and Xu (2011). To support the idea that the small firms are not associated with R&D investment, this model includes the financial constraint. This illustrates the situation where the R&D fixed cost should be borrowed, but each firm can borrow as much as proportional to its profits. Thus, the small firms, characterized by less productivity firms, do not engage in R&D since their profits are more likely to be low.

Simulation and Mechanism

The results simulated with estimated parameters well support the empirical evidence. They showed three cases: 25% of the real depreciation, 25% of the real appreciations, and 12.5% of the real depreciation. In the case of emerging Asian countries, with 25% depreciation, the revenue TFP growth increases up to 6.5%. By dividing by components, we can see that the demand part dominates the import part. On the other hand, in the case of other emerging economies, because the effects of the import part are greater than those of the demand part, the revenue TFP growth shows negative with 25% of the real depreciation. Meanwhile, in the advanced countries, the change of the revenue

TFP growth is close to zero since two effects from the import part and the demand part are very similar. When unexpected the real appreciations hit the economy, all the results are opposite to the real depreciation cases, but the magnitude is much smaller. The revenue TFP in emerging Asia is -2% which is a quite similar magnitude that can be obtained by 12.5% of the real depreciation.[8]

The mechanism of these results are certainly clear. The differences among different regions come from export- and import- intensity. The emerging Asian countries are export-oriented countries, so the real depreciations lead to higher exports inducing higher profits. This higher profits enable the exporting firms to borrow more for paying the fixed cost of R&D investment. With higher R&D, the firms' productivity increases. On the other hand, other emerging countries are featured by the import-intensive countries. Thus, real appreciation actually reduces the cost of imports and increase profits. Since firms face financial constraint, the amount of borrowing depends on profits, so higher profits allow the firms to borrow more and invest more in R%D. Hence the productivity will be higher with the real appreciations.

The real exchange rate and productivity

This paper does not address the relationship between the real exchange rate and the welfare directly but provides a constructive implication regarding productivity growth. First of all, this paper clearly illustrates the link of firms' behavior such as the real exchange rate, exports, imports, profits, borrowing, R&D, and productivity. The firm-level evidence regarding technological innovation and trade is often presented as a study limited to one country or one region(Aw et al(2011), Aghion at el(2012), and Bloom at el(2016)).[9] With the vast amount of data at the firm-level in various regions, this paper presents a new perspective that is the main point of this study.

The three regions are classified by geography but this paper also points out the export- and import- intensity of those regions. This is the critical factor that induces

[8] 12.5% of the real depreciation leads up to 2% of the growth of revenue TFP.

[9] Aw Roberts and Xu (2011) studies Taiwanese electronics industry. Aghion, Askenazy, Berman, Cette, and Eymard (2012) use French firm-level data. Bloom, Draca, and Van Reenen (2016) research technical improvement of twelve European countries respond to competition with China.

the main results. The emerging Asian countries are expressed by export-intensive countries, which is supported by various studies (De Melo(1985), Stubbs (1999), Kaplinsky and Morris (2008), Terjesen and Hessels (2009).) Asian four tigers (Taiwan, Hong Kong, Singapore, and South Korea) are well known as export-oriented countries. Also, China is a prominent example. Latin American and Eastern European countries are characterized by import-intensive countries. According to Seker (2010), the eastern European and central Asian countries display higher import intensity compared to export intensity. By comparing those three regions, this paper explicitly explains the effects of the real exchange rate.

In addition to the empirical evidence, the theoretical framework is also built in this paper. This helps quantify the impact of the real exchange rate and analyze the mechanism logically. Thus, the impact of the real exchange rate can be divided into the import part, demand part, and physical TFP part and it helps us to compare the magnitude clearly.

Inclusion of global crisis

To check the robustness the authors ran two regression: including and excluding global crisis period. The result is somewhat inconsistent for other emerging countries. The negative TFP growth is less significant (TFP computed from value-added) and no significant (TFP computed from the gross output) when excluding the years of global crisis. This can be rationalized by adding the financial crisis in the model. The financial constraint is expressed as follow.

$$\text{fixed cost for innovation} \leq \theta \Pi_{i,t} \tag{1.8}$$

Where $\theta \in [1, \bar{\theta}]$ is quality of the financial system. Then the smaller θ can be interpreted as a tighter constraint. We can think when a financial crisis hits the economy, θ becomes lower. Then, emerging countries suffer from real depreciations more. There are two channels: profits and θ. The real depreciations increase imported intermediate prices which are a large portion of the economy, so the profits will be smaller. Moreover, due to smaller θ, all firms can borrow much smaller amount during the financial crisis,

and this amplifies the negative growth of TFP.

This result is consistent with the two regressions. When including financial crisis, this amplification of negative TFP growth makes more significant and large effects of the real depreciation. In this regard, changing θ can explain some part of the empirical evidence.

Also, emerging Asian countries experienced a financial crisis twice: 1997 and 2008. The debt denomination can vary over time. Then, by comparing two crises, it can be possible to investigate the effects of debt denomination. In the benchmark analysis, debts are denominated in the local currency, but when debts are denominated in foreign currency, the debt burden is also affected by the change in the real exchange rate.

Externalities and export- and import- intensity

This model abstract from externalities often mentioned in the studies regarding technological innovation (Tylar(1981), Feder(1983), Grossman and Helpman (1991)). Their main argument is the there is a technological spillover from more advanced country to a less developed country when they trade. The import externalities can be larger when the country import more various imported capital goods and the export externalities can be derived by investing in R&D to compete with foreign firms or adopting higher technologies from trade partners.

These externalities may explain why some countries are export-intensive and some are import-intensive. This intensity is the crucial point of this paper but this paper does not deal with this issue. If we measure export externalities and import externalities in different regions, it can be concluded that emerging Asian countries are export-intensive since they have higher export externalities while Eastern European countries have higher import externalities. Then it can rationalize the export- and import-intensity in different regions.

When the externalities are introduced, a new channel is added to the effects of the change in the real exchange rate. Consider the real depreciation in emerging Asian countries and assume that these countries experience higher export externalities. Then the real depreciations stimulate exports and this increase profits which let the firms

invest in R&D more and the TFP grows faster. Also, more exports induce higher externalities which increase TFP. Combining two effects, the effects of the real depreciation can be larger.

Chapter 2

Growth, export externalities and accumulation of foreign reserves

2.1 Introduction

The accumulation of foreign reserves is a prevalent phenomenon in developing countries. Figure 2.1 shows the quartile of foreign reserves to GDP ratio. About half of the sample countries have foreign reserves that exceed 15% of GDP in the early 2000s. Considering that the average size of foreign reserves in 1980 was only 5% of GDP, this is a remarkable growth. A sizable literature offers different arguments for the accumulation of foreign reserves in developing countries, one of them being manipulation of terms of trade (real exchange rate).

Accumulation of foreign reserves can be used to make the real exchange rate depreciate, which is suggested by Benigno and Fornaro (2012). In their paper, economic growth is stimulated by importing intermediate inputs from abroad since they can take advantage of technological spillovers through imports. In this case, the government has an incentive to accumulate foreign reserves to induce real exchange rate depreciation. This leads to higher imports of intermediate goods and thus faster economic growth can be achieved. In this paper, I adopt their framework and incorporate export externalities instead of import externalities. Hence, the government considers faster growth through exports and thus has an incentive to hold foreign reserves to generate terms of trade deterioration to boost exports. As a result, the economy exports more and experiences

faster growth. This model can provide the justification of the behavior of the Asian Tigers (Hong Kong, South Korea, Taiwan, and Singapore) and China. They are well known for experiencing rapid economic growth using export expansion strategies and also have been holding a significant amount of foreign reserves.

However, there are counter-arguments regarding terms of trade policies: optimal tariff theory and consumption smoothing motivation. The optimal tariff theory has been developed in different ways (Kaldor (1940), Johnson (1953), Hamilton and Whalley (1983), Kennan and Riezman (1988), Helpman and Krugman (1989), and Syropoulos (2002)) since Bickerdike (1906). The main argument starts with governments facing monopoly/monopsony power. They have an incentive to impose tariffs which induces better terms of trade and increases their welfare at the cost of other countries' welfare. In these models, the optimal tariff level is strategically chosen to achieve the highest welfare of the home country. Also, consumption smoothing motivation in the growth model of a small open economy implies the improvement of terms of trade. Expecting higher income tomorrow, agents would borrow from abroad today to smooth their consumption over time. This is equivalent to the situation that households consume more imported goods with better terms of trade. As a result, the economy would benefit from consumption smoothing.

Considering these two forces related to the terms of trade, I build a model that can analyze which effect is dominant. I put the optimal tariff policy and consumption smoothing motivation together as a force of improvement of the terms of trade. Since price policy instruments such as tariffs or subsidies may be forbidden by the WTO or conflict with other trade agreements, I limit the policy tool for the management of the terms of trade to foreign reserves, which is not against these trade rules. In other words, when an economy accumulates foreign reserves, this means that the government buys foreign bonds, which are external savings. In this situation, the terms of trade are worse. On the contrary, the government can sell foreign bonds, which is external borrowing. This accumulating foreign reserves improves the terms of trade. This result is equivalent to the effects of the combination of the optimal tariff policy and the consumption smoothing motivation. Therefore, two forces can be expressed in terms of foreign reserves. By using them, I can build a model suggesting answers to the question why some countries accumulate foreign reserves while some do not.

To address this question, this paper proposes an endogenous growth model of a small open economy with technology spillovers through exports. There are two goods in this economy: domestic and foreign goods. The domestic goods are tradable, and thus can be exported or consumed domestically. The main feature of this model is the export externalities. The definition of export externalities in the paper is the spillover effects of exports on productivity growth. This can be achieved in two channels. First, to export domestic goods, the firms are exposed to more intense competition in the foreign market, which allows the firms to access a wide range of technology improvements through their activities in the foreign market.(De Melo (1985)) Furthermore, there is a case where knowledge can flow from international buyers and competitors. They normally advise how to improve efficiency and reduce costs.(Blalock and Gertler (2004)) These technological innovations are accumulated as a form of knowledge. Higher knowledge implies higher productivity when producing domestic goods. Due to the export externalities the economy can grow faster as it exports more.

This paper shows how governments in exporting countries use foreign reserves to increase their welfare. The optimal decisions depend on the magnitude of export externalities. If the export externalities are large enough, then the government accumulates foreign reserves to achieve faster economic growth. In the model, private agents fail to internalize the export externalities. Therefore, they export less than the socially optimal amount and the economy grows more slowly without the government intervention. On the other hand, the government takes externalities into account, and increases foreign reserves to encourage the private sectors to export more. The key assumption here is that the foreign reserves are denominated in foreign goods. Thus, the accumulation decision for the foreign reserves can change the terms of trade.[1] Increasing foreign reserves makes the domestic goods relatively cheaper so the domestic country can export more compared to the case where the government does not intervene. As a result, the economy experiences faster growth with technological spillovers through higher exports.

I compare the dynamics of several important variables in two different equilibria: the competitive equilibrium and the benevolent government equilibrium. In the competitive equilibrium (CE), consumers are not allowed to save and borrow. Also, since there is

[1]The terms of trade is defined as the price for imports over the price for exports. In this model, the terms of trade are equal to the real exchange rate since imported goods are foreign goods and exported goods are domestic goods.

no government, the foreign reserves are zero over time. However, in the benevolent government equilibrium (BGE), the government increases foreign reserves because it internalizes the faster growth achieved by export externalities. Therefore, the terms of trade in the BGE are higher than the one in the CE. This induces more exports, so the optimal consumption is less than the one in the CE for the first few periods. However, because the economic growth is faster, the consumption catches up to the one in the CE, and eventually goes beyond. As a result, the welfare under government intervention is higher than the one in the CE. This illustrates the inter-temporal trade-off from the foreign reserves. The increase in the foreign reserves would lead to less consumption today but more consumption in the future by speeding up growth. Once the economy reaches a steady-state, it no longer needs technological innovation through exports, so the government simply maintains enough foreign reserves to stay in the steady-state. This is the situation where the accumulation of foreign reserves improves welfare.

Nevertheless, it is not always optimal to choose to hold positive foreign reserves. To examine this, I vary the magnitude of export externalities and find that the government would borrow when externalities are positive but small. This is because the optimal foreign reserves are determined by comparing the benefits from faster growth through exports and the benefits from smoothing consumption and better terms of trade. If export externalities are large enough, the former effects outweigh the latter one, so the government sacrifices today's consumption to achieve faster growth by accumulating the foreign reserves. Consequently, the terms of trade deteriorate. However, when externalities are small, the latter force is stronger than the former one, so the government borrows to consume more today, which induces better terms of trade. This impedes the growth of the economy, but the economy gains welfare from the benefits of consumption smoothing and cheaper imported goods.

Literature review This paper is related to several branches of literature. First, there are papers about growth associated with knowledge spillovers through international trade. The theoretical study of cross-country knowledge spillovers was established by Grossman and Helpman (1991). They build an endogenous growth model of a small open economy and incorporate knowledge flow from abroad through international trade.

Since knowledge is non-rivalrous and non-excludable, it is hard to extract compensation from all agents who use it. This is how knowledge brings spillover benefits while trading. The authors argue that trade can affect growth in the long run through this mechanism. There is also a sizable literature studying knowledge flow across countries and they show knowledge transmission can be done through either importing foreign products or exporting to foreign markets. I focus on the latter channel in this paper as this channel is supported by other empirical papers.

The current literature provides empirical support for the export promotion policies achieving economic growth. Balassa (1978) and Balassa (1985) show that GDP growth is significantly affected by exports with eleven industrial countries. Tyler (1981) extends this argument to 55 middle income developing countries. He incorporates exports as an input of production function because export externalities may help more efficient production. Feder (1983) first develops a two-sector model with export and non-export sectors and estimate externalities based on the different marginal factor productivity between two sectors. This model indicates that the export sector has higher productivity that leads to the growth of the economy by reallocating resources from the non-export sector to the export sector. Bilginsoy and Khan (1994) improve Feder's specification by relaxing some assumptions and measuring the export externalities.[2] All these papers empirically show the positive correlation between economic growth and exports. The externalities are generally mentioned with efficiency from competitive management, the introduction of advanced techniques, training of higher quality labor, or economies of scale[3]. In this paper, I develop a theory of knowledge externalities through exports, which is consistent with the key results documented in the empirical literature.

There have been huge studies on foreign reserves. One explanation is that the accumulation of foreign reserves is a result of the precautionary motive. Developing economies are concerned about the situation of limited access to international financial markets. Therefore, they have the purpose of self-insurance. (Ranciere and Jeanne (2006), Aizenman and Lee (2007), Caballero and Panageas (2007), Alfro and Kanczuk

[2]Bilginsoy and Khan (1994) allow inter-sectoral externality and constant term in their regression and chose non-export sector growth as the dependent variable to avoid spurious correlation problem.

[3]chipman(1970), Helpman and Krugman(1985), and Romer(1986) use the increasing returns to express economies of scale.

(2009), Bianchi, Hatchondo, and Martinez (2016), and Hur and Kondo(2016)). Meanwhile, there is another factor that accounts for a significant amount of foreign reserves: acceleration of growth. Benigno and Fornaro (2012) build an endogenous growth model with knowledge spillovers through intermediate inputs and show how foreign reserves can increase imports of the inputs. They argue that by increasing foreign reserves, a real exchange rate is depreciated, and this leads to high intermediate inputs. In addition, it also makes labor reallocate to tradable sectors, so the economy grows faster. This paper focuses more on imports while Dooley, Folkerts-Landau and Garber (2005) are interested in exports. They mention that high foreign reserves are a part of strategies to boost the nation's growth by keeping the foreign exchange rate undervalued. It seems to be a rational claim when we see countries like China and South Korea since their export-oriented growth strategy is widely known as their growth engine. As opposed to these papers, I build an endogenous growth model of a small open economy with knowledge flow through exporting to foreign markets.

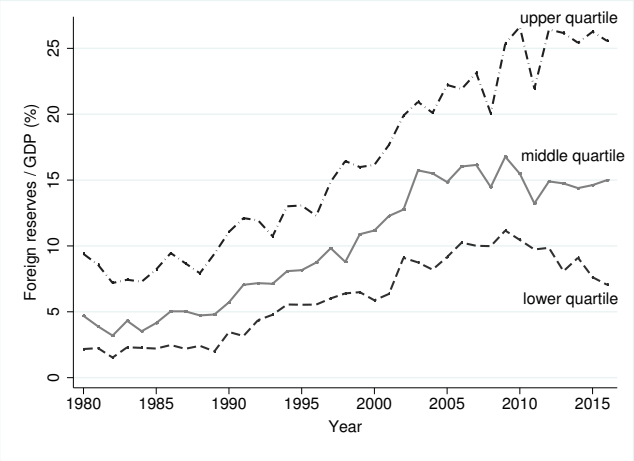
This paper is organized as follows: Section 2 describes empirical evidence on foreign reserves, exports, and growth in developing economies; Section 3 presents a baseline model and defines equilibria with and without a benevolent government to examine the role of the foreign reserve; Section 4 shows quantitative analysis to compare welfares; I then conclude in Section 5.

2.2 Data and empirical findings

This section describes definitions of main variables and documents a set of key findings. The data set contains annual data of 50 developing countries from 1980 to 2016. These countries were considered developing countries in 1995 according to the World Bank (1995) but are now classified as developed countries.[4] Since the paper analyzes the dynamics of growth, these countries, that used to be developing countries, are included

[4]World Bank classifies developing countries by low-income, middle-income, and transition countries. South Korea, Slovenia, Czech Republic, Slovak Republic, Latvia, and Lithuania are now listed as high-income countries in World Bank (Fantom and Serajuddin (2016)

Figure 2.1: Quartile of foreign reserves to GDP



Notes: The dot-dash line presents the 3rd quartile (75 percentile) of the 50 sample countries. The gray solid line and the black dot lines are the median quartile (50 percentile) and the 1st quartile (25 percentile) of the sample respectively.

in the data set. I divide the sample countries into two types of countries: countries following a policy of export growth and countries without this policy. Detailed definitions will be explained in the next section.

Using the data, I document the following facts.

1. Export-oriented countries have been accumulating foreign reserves more than non export-oriented countries.

2. Export-oriented countries show more of a significant positive correlation between growth rate and foreign reserves than others.

Before illustrating these facts, county growth policies are defined in the next section.

2.2.1 Export-oriented countries

Export-oriented countries are considered countries that have experienced a high growth rate of the exports-to-GDP ratio since 1980.[5] I assume that the countries who grow due

[5]Exports-to-GDP ratio can be used to define the export-oriented economy, but some developing countries are small in terms of size of GDP. Thus, even though the absolute value of exports is very

to the export externalities increase exports more compared to their economic size, so the exports-to-GDP ratio shows an increasing trend. To capture this feature, I take the average of the exports-to-GDP ratio from 1980 to 1990 and also one from 2006 to 2016 and calculate the growth rate using the two values. The reason for taking the average for 10 years is to obtain the general level of the ratio in the beginning and end of the sample period.[6] Finally, the export-oriented countries are classified by the following equation.

$$g\left(\frac{Exports}{GDP}\right)_{1980-2016} > 0.51$$

Countries that grew more than 51% in the exports-to-GDP ratio compared to 1980s are classified as an export-oriented country. By this criterion 0.51 is at the 60th percentile in the sample. In other words, the top 40% of countries in terms of their growth rate are classified in this group. According to this definition, 19 countries are included in the export-growth policy countries.[7]

I also define non export-oriented countries as a comparative group to the export-growth policy countries. I assume that these countries tend not to increase their exports-to-GDP ratio because exports are not critical to growth. As a result, the criteria value is set to be 0.26, which is at the 40th percentile in the sample. In other words, the countries whose growth of exports-to-GPD ratio is less than 26% during the sample period are classified as non export-growth policy countries. The list of countries in the two groups is shown in the Appendix(A.1).

Table 2.1 presents the mean values for main variables related to exports, GDP, and foreign reserves. The first three variables ($\frac{\text{Exports}}{\text{GDP}}$, $\frac{\text{Foreign Reserves}}{\text{GDP}}$, $g(\text{GDP})$) are the simple average over the sample period, from 1980 to 2016 and each column shows the

small, they could be included as a country with an export-growth policy. Also, China and Korea, which are well-known as the countries that follow a policy of export growth, are excluded with this definition. For this reason, I use the growth rate of exports-to-GDP ratio.

[6] when a specific year is picked the general exports-to-GDP ratio might be misled if the country is hit by shocks. For example, the exports-to-GDP ratio of Argentina in 2001 picked up to 26% from 10%, but this occurred because the GDP dropped dramatically due to the economic crisis in 2001, not because of increase in exports.

[7] The median of the growth of the export-to-GDP ratio is 0.44.

Table 2.1: Mean values for the main variables

	All countries	Export-oriented	Non export-oriented
Exports/GDP	29.5%	28.7%	26.3%
Foreign reserves/GDP	11.1%	11.6%	9.0%
g(GDP)	2.1%	3%	1.2%
g(Exports/GDP)	65%	140%	2%
g(Foreign reserves/GDP)	5.1%	6.0%	4.5%

Notes: $g(\cdot)$ means the growth rate of a variable. Exports/GDP, Foreign reserves/GDP, and g(GDP) are the mean of each variable from 1980 to 2016. To obtain the mean value of the growth rate of exports-to-GDP ratio for each country I calculated two values for the exports-to-GDP ratio: one for the first 10 years and one for the last 10 year of the sample period. Then I obtained the growth rate using the two values. The number shown in each column is the mean value in each group. The mean values of the growth rate of the foreign reserves-to-GDP ratio are also calculated in the same way.

mean of countries in each group : all countries, export-growth policy, and non export-growth policy countries. The growth of export- and foreign reserves-to-GDP ratios are calculated according to the steps described above.

First of all, it is clear from this Table 2.1 that mean values of the exports-to-GDP and the foreign reserves-to-GDP ratios do not show significant differences in the mean of all countries. The average level of the exports-to-GDP ratio is even higher for all countries. Thus, some countries that are not included in the export-oriented countries have higher exports. The export-oriented countries export more than non export-oriented countries. The foreign reserves-to-GDP ratio shows the same pattern. The foreign reserves-to-GDP ratio of all countries and export-oriented countries are quite similar while they have a bit higher level of foreign reserves than non export-oriented countries.

However, the average growth rates are different in the three groups. The growth rate of exports-to-GDP ratio is remarkably higher in the export-oriented countries. This is a natural result due to the definitions for the export-oriented countries. They grew more than 140 percent during the sample period while the non export-oriented countries only grew 2 percent. The mean for all countries is 65 percent. Also, countries following the export growth policy grew faster on average. This is consistent with the features of export-oriented countries. If the export-oriented countries grow through exports, high exports imply high growth of the economy. Furthermore, those countries show the high growth of foreign reserves as well. This supports the main argument of this paper. As

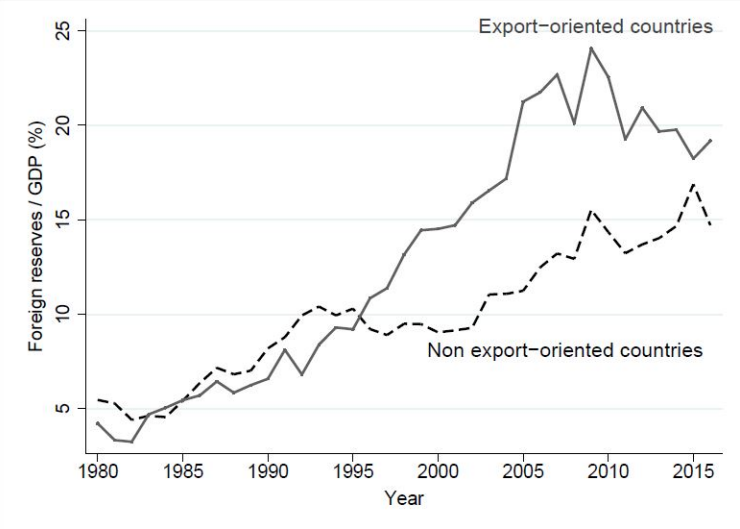
foreign reserves increase, countries export more, meaning higher growth of exports, and those countries grow faster. This is also related to the first empirical finding that I will document in the next section.

2.2.2 Foreign reserves and Exports

As shown in Table 2.1, the level of foreign reserves-to-GDP ratio in the export-oriented countries is higher than other countries. It is also worthwhile to see how foreign reserves change over time in two groups. If exports increase as foreign reserves are accumulated, then it can be concluded that the export-oriented countries accumulate foreign reserves faster than the non export-oriented countries. This fact is depicted in Figure 2.2. The gray solid line is the mean of the foreign reserves-to-GDP ratio of export-oriented countries and the black dash line is that of non export-oriented countries.

According to the first graph (a), The foreign reserves accounted for only around 5% in GDP in 1980 and they showed a fairly similar pattern until 1995. During this period, the non export-oriented countries had even higher foreign reserves. Starting in 1995, the export-oriented countries began to overtake the group of other countries and kept increasing and reached over 20% right before 2007. This continuous growth of the foreign reserves suggests this importance as a policy in those countries. This is consistent with the idea that the countries that have export-led growth strategies can export more with higher foreign reserves. This argument is also supported by the graph (b). The increasing pattern of exports is quite similar to the foreign reserves in these countries. Both rose until the global crisis and decreased after 2010.

On the other hand, the average ratio in the non export-oriented countries is quite constant in the 1990s. It started increasing in the mid-2000s but failed to catch up to the foreign reserves-to-GDP in the countries with the export growth policy. Furthermore, there is no correlation between the foreign reserves-to-GDP ratio and exports-to-GDP ratio in these countries. While the foreign reserves exhibit a rising pattern, the exports are constant and the movement after 2005 even exhibits a decreasing pattern. From Figure 2.2, it can be concluded that the export-oriented countries have a higher level of foreign reserves, and the growth of the foreign reserves are faster.



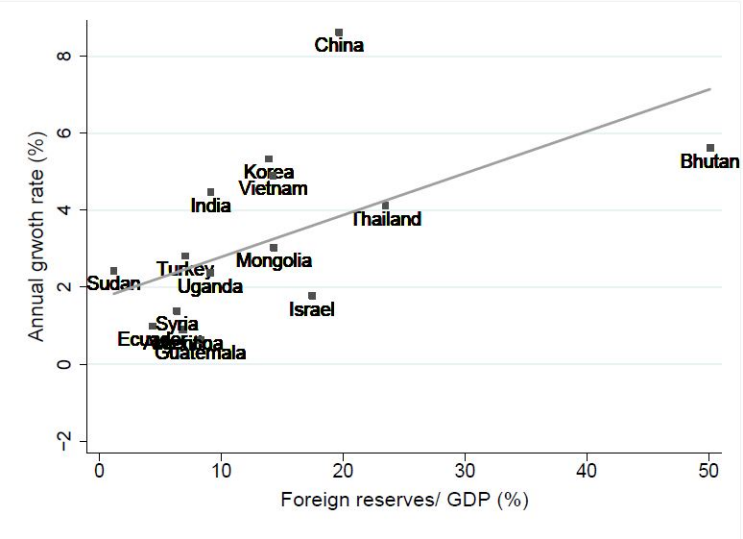
(a) Foreign reserves-to-GPD ratio

(b) Exports-to-GPD ratio

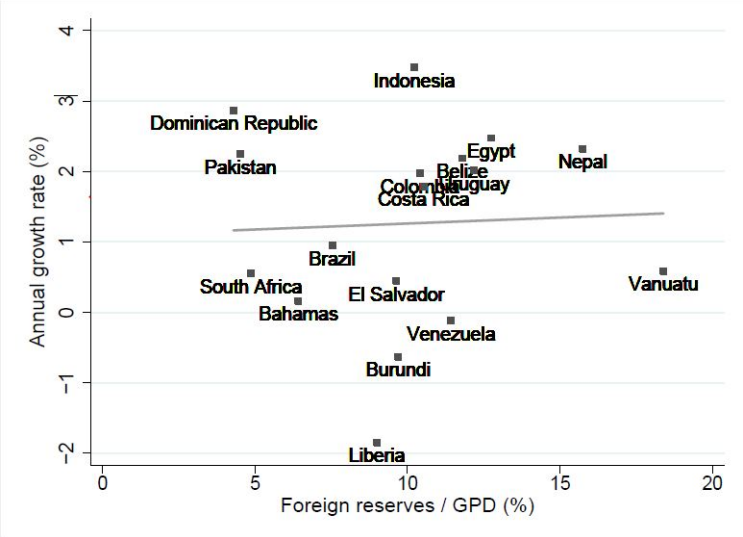
Notes: The gray solid line presents the mean of 19 countries following the export growth policy and the black dash line represents the mean of 18 countries without that policy.

Figure 2.2: Share of GDP from 1980 to 2016

2.2.3 Foreign reserves and Growth



(a) Countries with the export-growth policy



(b) Countries without the export-growth

Notes: Each plot shows the average value of annual growth rate and foreign reserves-to-GDP ratio of each country during the sample period.

Figure 2.3: Growth rate and Foreign reserves-to-GDP ratio

In addition to exports, economic growth also shows a positive relationship with foreign reserves in the export-oriented countries. Table 2.1 show that the export-oriented group has a higher growth rate of GDP per capita and also higher foreign reserves-to-GDP ratio compared to the other group of countries. Figure 2.3 provides a more detailed analysis of this fact using scatter plots. Plots (a) and (b) illustrate the relationship of annual growth rate and foreign reserves-to-GDP ratio in the export-oriented countries and non export-oriented countries respectively.

Countries having high foreign reserves experience faster growth if the countries are classified as an export-oriented country. The correlation coefficient is 0.56, indicating a significant positive correlation.[8] China displays the highest annual growth rate among those countries and is well-known as having export-led growth strategies. Furthermore, China has the most foreign reserves and it recorded 4 trillion dollars in 2014. This example supports the claim that high growth comes with high foreign reserves if the country follows the export growth policy. On the other hand, it is hard to say that there is a relationship between growth rate and foreign reserves-to-GDP ratio among the countries without the policy. Also, the correlation coefficient is 0.05, hardly different from 0.

Additionally, even though the export-oriented countries tend to hoard higher foreign reserves compared to the other country group, the ranges are quite similar, spanning from 0 to 25 if Bhutan is excluded. However, the range of annual growth rate is noticeably different. No country in Figure 2.3 (a) records a negative growth rate and some countries such as China, Korea, Vietnam, India, and Thailand experienced high growth– more than 4% – from 1980. Their foreign reserves-to-GDP ratios lie between 15% to 25%. Meanwhile, the growth rates of non export-oriented countries within this range are less than 2%. Given the fact that a 1% difference in annual growth rate can bring incredible differences over 37 years, the gap between the growth rates of the two groups is non-negligible. This indicates that foreign reserves are not correlated with the growth rate in the non export-oriented countries, but the export-oriented countries, in fact, cannot ignore the effects of foreign reserves. They reap the benefits from the high foreign reserves.

[8]Bhutan seems to be an extreme case but eliminating it will not change the positive correlation. Without Bhutan, the correlation coefficient is 0.62, which is higher than with it.

I have currently supported two claims regarding foreign reserves, exports, and growth. The following section discusses a model and assumptions in detail and describes the modeling elements required to describe the empirical results of the previous section.

2.3 Model

In this section, I develop an endogenous growth model of an almost small open economy (SOE) in the presence of export externalities. There are a home and a foreign country and produces domestic goods and foreign goods respectively. The domestic goods are either consumed in the home country or exported to the foreign country. Similar to Benigno and Fornaro (2012), the knowledge spillovers take place through exports.[9] In this way, the home country accumulates the knowledge stock that induces economic growth.

The domestic goods are exported goods and foreign goods are imported goods so the relative price between foreign goods and domestic goods is defined as the terms of trade. The export price is normalized to one so that terms of trade are effectively expressed in units of foreign goods.

This model is almost a small open economy in the sense that the interest rate of the foreign reserves and the foreign country's output are exogenously given. More precisely, the home country can affect the terms of trade, but not foreign income so this model is different from a small open economy. Also, the foreign country has a deep pocket, so the interest rate of the foreign reserves is the risk-free rate since it never defaults.

In this world, time is discrete and denoted by $t = 0, 1, \cdots, \infty$. In the home country, there are three agents: the representative consumer, the representative firm, and the government. The consumer chooses how many domestic and foreign goods to consume. The firm produces domestic goods given the knowledge stock and sells them to the domestic consumer or exports them to the foreign country. Lastly, the government chooses how much to save (or borrow) in the foreign country. I call this external saving

[9] Benigno and Fornaro (2012) explain the knowledge accumulation with imports. By importing foreign capital goods, firms in developing countries can benefit from the spillover effects of discovery from developed countries.

as *foreign reserves* and this is the only policy tool of the government in this economy. To see its role, I will propose two equilibria: the competitive equilibrium without the government and the equilibrium of the benevolent government problem.

2.3.1 Consumers

The consumers in this economy are infinitely lived and all identical. The representative consumer owns a domestic firm. She consumes a composite good of the domestic and the foreign goods. The preference is given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{2.1}$$

where β is the discount factor and the utility function $u(\cdot)$ is the Constant-Relative-Risk-Aversion (CRRA), which has the functional form of $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ where σ is a risk aversion parameter. The composite good C_t is an Armington-type of CES aggregator with elasticity of substitution $1/(1-\eta)$ between domestic goods c^D and imported foreign goods c^F

$$c_t = c(c_t^D, c_t^F) = \left[\omega (c_t^D)^{-\eta} + (1-\omega)(c_t^F)^{-\eta} \right]^{-\frac{1}{\eta}} \tag{2.2}$$

where $0 < \omega < 1$ denotes the share of expenditure on the domestic tradable goods. The income sources of the consumer are wages w_t, profits from the firm π_t, and transfers from the government T_t. The total income is used to consume domestic goods and foreign goods. Hence, the budget constraint is followed by:

$$c_t^D + p_t c_t^F = w_t L_t + \pi_t + p_t T_t \tag{2.3}$$

where p_t is terms of trade. I assume inelastic labor supply, so $L_t = 1$ for all t. The representative consumer receives transfers in terms of foreign goods since the government saves foreign reserves in the foreign country and they are denominated in foreign goods.

2.3.2 Firms

There is a large number of firms in this economy. Since all firms are identical I analyze the representative firm. The representative firm produces domestic goods that can be consumed domestically or exported. The firm produces using labor L_t, and the knowledge stock γ_t that shows the level of productivity in the economy. The production function has the following form:

$$Y_t = (\gamma_t L_t)^{\alpha} \tag{2.4}$$

The Y_t is the output of the domestic goods and α is labor share with $0 < \alpha < 1$.

The produced goods are sold in the domestic market and exported: $Y_t = q_t^d + x_t$ where q_t^d is domestic demand and x_t is foreign demand. The foreign demand function for the domestic goods in the almost small economy depends on the terms of trade and outputs in the foreign country $Y^{\star}$ which is exogenously given.[10]

$$x_t = \left(\frac{1}{p_t}\right)^{-\phi} Y^{\star} \tag{2.5}$$

where $\phi > 0$. $-\phi$ is the elasticity of demand for domestic tradable goods.

As a result, the firm's profit function is expressed as:

$$\pi_t = (\gamma_t L_t)^{\alpha} - w_t L_t \tag{2.6}$$

2.3.3 The Knowledge accumulation

The firm produces domestic goods given the available knowledge stock. I assume that the foreign country is more advanced, and the knowledge stock is non-rivalrous and non-excludable. These assumptions enable the knowledge spillovers from the foreign country to the home country. This knowledge spillover can be achieved by exports.

[10]This demand function is standard in small open economy models and the function depends on export price p^x and world price $P^{\star}$ as follow: $x_t = \left(\frac{p_t^x}{P_t^{\star}}\right)^{-\phi} Y^{\star}$ In this paper, I use import price as the world price so $\frac{p_t^x}{P_t^{\star}} = \frac{1}{p_t}$ and the demand function is obtained

By exporting, the home country adopts new technologies and ideas made in the foreign country and the knowledge can be accumulated in this process. This feature is captured in the law of motion of the knowledge stock:

$$\gamma_{t+1} = \psi\gamma_t + x_t^{\zeta_x}\gamma_t^{\zeta_\gamma} \tag{2.7}$$

where $\psi \geq 0$ and $\zeta_x + \zeta_\gamma \leq 1$. γ_t is the knowledge stock in period t and ψ shows the persistency of the knowledge stock. ζ_x presents the share of knowledge that exports contribute so $\zeta_x = 0$ means no export externalities.

I assume that the externality function in the knowledge accumulation equation presents decreasing returns to scale, i.e., $\zeta_x + \zeta_\gamma \leq 1$. This assumption ensures that the optimal level of foreign reserves is finite.[11]

2.3.4 The Government

The government trades foreign reserves and gives transfers to the consumer in this economy. The foreign reserves are one-period risk-free bonds issued by the foreign country. The interest rate of the foreign reserves is R_f. At time t, the government in the home country receives returns on the foreign reserves that were accumulated in the previous period $R_F A_t$, chooses the level of foreign reserves A_{t+1}, and gives transfers to the consumers in a lump sum fashion T_t. The government budget constraint is given as:

$$T_t = R_f A_t - A_{t+1} \tag{2.8}$$

[11] Suppose knowledge accumulation equation shows constant returns to scale, i.e., $\zeta_x + \zeta_\gamma = 1$. Consider the law of motion of the knowledge stock in the steady-state. Then $(1-\psi)\gamma^{*1-\zeta_\gamma} = x^{*\zeta_x}$

2.4 Equilibrium

2.4.1 Competitive equilibrium without the government

In this section, consider the situation where there is no government in this economy. Thus, the foreign reserves and transfers are set to be zero every period $t \geq 0$. Substituting $T_t = 0$ into (2.3), the consumer's budget constraint becomes

$$c_t^D + p_t c_t^F = w_t L_t + \pi_t \tag{2.9}$$

The consumer chooses consumption bundles $\{c_t^D, c_t^F\}_{t\geq 0}$ to maximize the expected value of utility (2.1) subject to (2.9), taking the terms of trade and wage, and profits $\{p_t, w_t, \pi_t\}_{t\geq 0}$ as given. The consumer's optimality condition is:

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{2.10}$$

$u_i(c_t^D, c_t^F)$ means a partial derivative of the utility function with respect to the ith argument at time t. The equation (2.10) indicates that the terms of trade (relative price) equals the marginal rate of substitution of the foreign and the domestic goods.

In the firm's problem, the firm chooses $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$ given the terms of trade $\{p_t\}_{t\geq 0}$ and the knowledge stock $\{\gamma_t\}_{t\geq 0}$. As mentioned in the previous section, there is a large number of firms, so each firm is too small to internalize the effects of export externalities. Therefore, the firm takes the knowledge stock as given every period.

Therefore, the firm's problem is not a maximization problem. Given the knowledge stock and inelastic labor supply, the output of the domestic goods is determined, and the level of exports is determined by the terms of trade in the foreign demand function for the domestic goods. Consequently, the amount of domestically consumed goods and the profits are automatically determined. The first-order condition of the firm is only for the labor demanded:

$$w_t = \alpha \gamma_t (\gamma_t L_t)^{(\alpha-1)} \tag{2.11}$$

The optimality condition (2.11) equates wage and the marginal product of labor.

Market clearing condition for domestic goods market should equate the output to

the amount of the domestic goods consumed in the home and the foreign country. Since consumers are identical, the market clearing condition is:

$$c_t^D + x_t = Y_t \tag{2.12}$$

Notice that the domestic goods that are consumed in the home are equal to those produced for the domestic market, so the market clearing condition (2.12) already reflected $c_t^T = q_t^d$. Combining (2.12) with the consumer's budget constraint (2.9) and the firm's profits function (2.6), the trade balance is obtained as :

$$x_t - p_t c_t^F = 0 \tag{2.13}$$

There is no external savings and borrowing in this equilibrium, so this economy always shows trade balance. Using the optimality conditions and the market-clearing conditions, the competitive equilibrium can be defined

Definition 1. ***Competitive Equilibrium***
The competitive equilibrium for the almost small open economy is defined by a set of the representative consumer's allocations $\{c_t^D, c_t^F\}_{t\geq 0}$*, the representative firm's allocations* $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$*, the terms of trade and the wages* $\{p_t, w_t\}_{t\geq 0}$*, and the stock of knowledge* $\{\gamma_{t+1}\}_{t\geq 0}$*, given initial knowledge stock and foreign output* $\{\gamma_0, Y^\star\}$ *such that the following conditions are satisfied:*

1. *Consumer: Given* $\{p_t, w_t, \pi_t\}_{t\geq 0}$*, the consumer's allocations* $\{c_t^D, c_t^F\}_{t\geq 0}$ *satisfy the optimality condition (2.10).*

2. *Firm: Given* $\{p_t, w_t, \gamma_t, Y^\star, z\}_{t\geq 0}$*, the firm chooses* $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$ *satisfying (2.4), (2.5), (2.6), and (2.11).*

3. *The market for domestic goods clears as (2.12) and trade balance satisfies (2.13).*

4. *The knowledge stock evolves according to (2.7).*

2.4.2 The Benevolent Government

In the previous section, the competitive equilibrium without the government was described. Now, consider the economy where the government can intervene in the market. The government in this economy is called as the benevolent government since it maximizes the consumer's utility knowing all the optimality decisions of the private agents. The government makes a foreign reserves decision which is not allowed to the private agents and pays (or collects) transfers (taxes) to the consumer in a lump-sum fashion. Then, the consumers choose their consumption bundles of domestic and foreign goods in a competitive way. Thus, the consumer's budget constraint in this benevolent government problem is the equation (2.3) including transfers T_t. Also, the trade balance should be different from that with no government case. Combining the government budget constraint (2.8) with the consumer's budget constraint (2.3), the trade balance is expressed as:

$$x_t - p_t c_t^F = p_t \left(A_{t+1} - R_f A_t \right) \tag{2.14}$$

One important feature of the government is that it can internalize the impact of growth through exports. This aspect is a significant difference from the private agents in this model. The government knows that choosing a higher level of foreign reserves deteriorates the terms of trade and it is advantageous for high exports. Since high exports contributes to the faster accumulation of knowledge, the home country can experience faster growth by hoarding foreign reserves.

Now consider the optimization problem of the benevolent government in a recursive form. The state variables are the current foreign reserves holding A, and the knowledge stocks γ. The crucial state variable is the later one. Since the representative private agents can not internalize the export externalities, the competitive equilibrium is solved without taking account of the dynamic process of knowledge accumulation involving the export externalities. However, the benevolent government takes into account it, so takes the knowledge stocks as a state variable. Combining the budget constraint (2.3) and (2.14), the resource constraint for the tradable good can be expressed as

$$c_t^D + x_t = (\gamma_t L_t)^\alpha \tag{2.15}$$

The benevolent government optimization problem consists of maximizing the value of consumer's life time utility (2.1) subject to the resource constraint for the tradable goods (2.15), the trade balance (2.14), the consumer's optimality condition (2.10), the foreign demand function for the domestic goods (2.5), and the law of motion of the knowledge stock (2.7). The government's recursive optimization problem can be expressed as following:

$$
\begin{aligned}
V(A,\gamma) = \underset{c^D,c^F,c,x,A'}{Max} & \{u(c(c^D,c^F)) + \beta V(A',\gamma')\} \qquad (2.16)\\
\text{s.t} \quad & c^D + x = (\gamma L)^{\alpha} \\
& x - pc^F = p\left(A' - R_f A\right) \\
& x = \left(\frac{1}{p}\right)^{-\phi} Y^{\star} \\
& \gamma' = \psi\gamma + (x)^{\zeta_x}(\gamma)^{\zeta_\gamma} \\
& p = \frac{1-\omega}{\omega}\left(\frac{c^D}{c^F}\right)^{1+\eta}
\end{aligned}
$$

Where $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and $c(c^D, c^F) = \left[\omega(c^D)^{-\eta} + (1-\omega)(c^F)^{-\eta}\right]^{-\frac{1}{\eta}}$. The notation for the variable with the prime superscript means variable in the next period. As shown in the problem the government knows the optimal actions of the private agents, and chooses an optimal allocation of consumption $\hat{c}^D(A,\gamma), \hat{c}^F(A,\gamma)$, exports $\hat{x}(A,\gamma)$, and foreign reserves $\hat{A}'(A,\gamma)$ to maximize the consumer's utility.

Definition 2. ***Recursive Equilibrium***

The recursive equilibrium for the benevolent government is defined by a pricing function $\hat{p}(A,\gamma)$, and a decision rule $\hat{A}'(A,\gamma)$ for the benevolent government with associated value function $V(A,\gamma)$, consumption and exports rules $\{\hat{c}^D(A,\gamma), \hat{c}^F(A,\gamma)\}$, $\hat{x}(A,\gamma)$, and a knowledge stock rule $ga\hat{m}ma(A,\gamma)$ such that the following conditions hold:

1. *Given $\hat{p}(A,\gamma)$, the decision rule $\hat{A}'(A,\gamma)$ solves the benevolent government recursive maximization problem (2.16) and the exports policy $\hat{x}(A,\gamma)$ satisfies (2.5).*

2. *Given $\hat{p}(A,\gamma)$, the tradable consumption plan $\hat{c}^D(A,\gamma)$ satisfies the resource constraint (2.15) of the economy and the imported goods consumption plan $\hat{c}^F(A,\gamma)$ satisfies the trade balance (2.14).*

3. *The knowledge stock $\hat{\gamma}(A,\gamma)$ evolves according to (2.7).*

2.4.3 Export Externalities and Efficiency

In the previous sections, I described the competitive equilibrium without the government and the recursive equilibrium of the benevolent government problem. Since the private agents cannot internalize the export externalities, the competitive equilibrium without the government cannot achieve the first best. However, the government can take into account the externalities, thus it will use its policy tool to make the private agents achieve the second best. I illustrate how externalities are considered in the government problem.

Consider the first order condition (A.8) for the benevolent government. Appendix (A.2) provides full parts of the characterization of the benevolent government allocation. This equation shows the crucial difference between the equilibrium solved by the benevolent government and that of the competitive equilibrium. The sequential notation is used to compare two equilibria.

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)\left(1 - \Psi_t\left(p_t^{-\phi(1-\zeta_x)}\right)\right)} \tag{2.17}$$

where $\Psi_t = \alpha\zeta_x\left(\gamma_t\right)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\left(\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right) > 0$.

To analyze this equation, consider two extreme cases: $\zeta_x = 0$ or $\zeta_x = 1$. $\zeta_x = 1$ cannot be realized in this economy due to the assumption of $\zeta_\gamma + \zeta_x \leq 1$. However, to see the meaning of the equation 2.17, relax the assumption for now. $\zeta_x = 0$ means there is no export externalities. In this case, the equation (2.17) equals the corresponding equation for the competitive equilibrium (2.10). The (2.10) equates the terms of trade to the marginal rate of substitution. On the contrary, when $\zeta_x = 1$, Ψ_t is always positive. This is what the additional term that the government considers when optimizing. The denominator on the right-hand side is divided into two terms: the marginal utility of

consuming one unit of domestic good, $u_1(c_t^D, c_t^F)$, and the marginal benefits of exporting one unit of tradable good, $\Psi_t \left(p_t^{-\phi(1-\zeta_x)}\right)$. The first term presents the increase in the marginal utility of the domestic goods today while the second term includes the marginal utility tomorrow. This term represents the benefits of increased knowledge stock implied by an additional unit of export. One more unit of export today induces higher knowledge tomorrow, and thus more output will be produced with a higher productivity tomorrow. As a result, the consumers will be able to consume more domestic goods tomorrow. In other words, this term refers to growth effects from the export externalities that are not internalized by the private agents. Furthermore, the benefits are realized in the next period. Then, due to the market-clearing condition, the government has to choose either to consume or to export. If the consumer decides to consume the domestic good today, it has to give up the benefits coming from growth so the marginal utility of domestic good is $u_1(c_t^D, c_t^F)\left(1 - \Psi_t \left(p_t^{-\phi(1-\zeta_x)}\right)\right)$. As the export externalities, ζ_x increase the benefits of exporting becomes larger so the marginal utility of domestic goods becomes smaller. Equivalently it is a decision of the increase in utility between today and tomorrow. Since the government internalizes the growth effects from the exports, it has an incentive to use foreign reserves as a policy tool to stimulate exports.

To see how the government uses the foreign reserves, compare the terms of trade in the two equilibria. Define p_t^c and p_t^g the terms of trade under the competitive equilibrium and benevolent government problem respectively.

$$p_t^c = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \quad , \qquad p_t^g - \Psi_t (p_t^g)^{1-\phi(1-\zeta_x)} = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{2.18}$$

Since $\Psi_t > 0$ and $p_t^g > 0$, with positive export externalities the terms of trade under the government problem are higher than that under the competitive equilibrium, $p_t^c < p_t^g$ if both has the same consumption bundle. The government takes into account the growth effects through exports, so the opportunity cost of the domestic goods becomes higher under the government problem. In other words, consumption of one unit of the domestic goods today means that the consumer has to give up the utility from the consumption on the foreign goods today and also the higher consumption on the domestic good tomorrow. Hence, the cost of consumption on the today's domestic goods is relatively expensive. As a result, the government wants the market to achieve p_t^g. The

lower terms of trade in the competitive environment mean that too many foreign goods are consumed. Thus, by accumulating foreign reserves in terms of foreign goods, the government makes the consumer consume fewer foreign goods, and thus the higher terms of trade can be obtained. This is how the government manages foreign reserves.

2.5 Quantitative Analysis

Here I study the quantitative implication of the model using a baseline calibration based on data from Taiwan, well known as an export-oriented country hoarding sizeable foreign reserves. I investigate the mechanism of the model and provide a welfare analysis over the sample period.

2.5.1 Calibration

The parameters in the model are calibrated with yearly frequency. Table 2.2 shows the calibrated parameter values. The weight on domestic goods in utility function ω is set by the ratio of consumption on tradable goods to total consumption on tradable goods averaged over the sample period. Tradable goods are defined as non-service goods and the domestic tradable consumption is obtained by subtracting import of consumption goods from the total consumption of tradable goods.

The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate R_f is set to 1%, which are standard values in international macro studies. The curvature of Armington aggregate of domestic tradable goods and imported goods η is set to be 0.11, following the calibration of Heathcote and Perri (2002). This implies the elasticity of substitution between domestic goods and foreign goods is 0.9. According to Tallman and Wang (1994), Taiwan's labor share in output α was 0.66 during the period from 1965 to 1990. The value of elasticity of demand for exports ϕ is obtained from Imbs and Mejean (2010). ω is calculated with weight on domestic tradable goods.

Finally, the constant term in the knowledge process ψ knowledge and export contribution parameters, ζ_γ and ζ_x, are calibrated. First, I could obtain elasticity of knowledge

Table 2.2: Parameters

Parameter	Description	Value	Source/Data
β	Discount factor	0.96	4% interest rate
σ	Risk aversion	2.00	Standard value
ω	Weight on domestic tradable goods	0.64	$\frac{1}{T}\sum \frac{Non-importcons.}{Trad.cons.}$
η	Amington curvature	0.11	Heathcote and Perri(2002)
α	Labor share in output	0.66	Tallman and Wang (1994)
ϕ	Elasticity of export demand	1.50	Imbs and Mejean(2010)
R_f	Gross risk-free interest rate	1.01	standard value
ψ	Constant in knowledge process	0.10	2.1% growth rate of non-exporters
ζ_x	export contribution	0.61	8.1% higher productivity of exporters
ζ_γ	knowledge contribution	0.18	Elasticity of knowledge on productivity

spillovers on productivity which is 0.18[12] and set it to ζ_γ. Taiwan has an exporting premium which means that after experiencing exporting, the growth of productivity of the firms become faster. Controlling self-selection effects, Aw, Chung, and Roberts (2000) suggest that the productivity differential between exporters and non-exporters is 6 percent from 1981 to 1991. ψ and ζ_x are calibrated to match 1.77% of productivity growth of non-exporters and 6% higher productivity in the exporting sector.

2.5.2 Results

Based on this calibration, I analyze the solutions of two equilibria: the competitive equilibrium and the equilibrium of the benevolent government. I simulate the models to analyze the growth of knowledge, output, and consumption in the long run and Figure 2.4 shows the two equilibria. The red solid line represents the competitive equilibrium without the government and the blue dash line displays the equilibrium of the benevolent government's problem. The initial foreign reserves and the initial knowledge stock is 0

[12]followed by Tsai and Wang (2004).

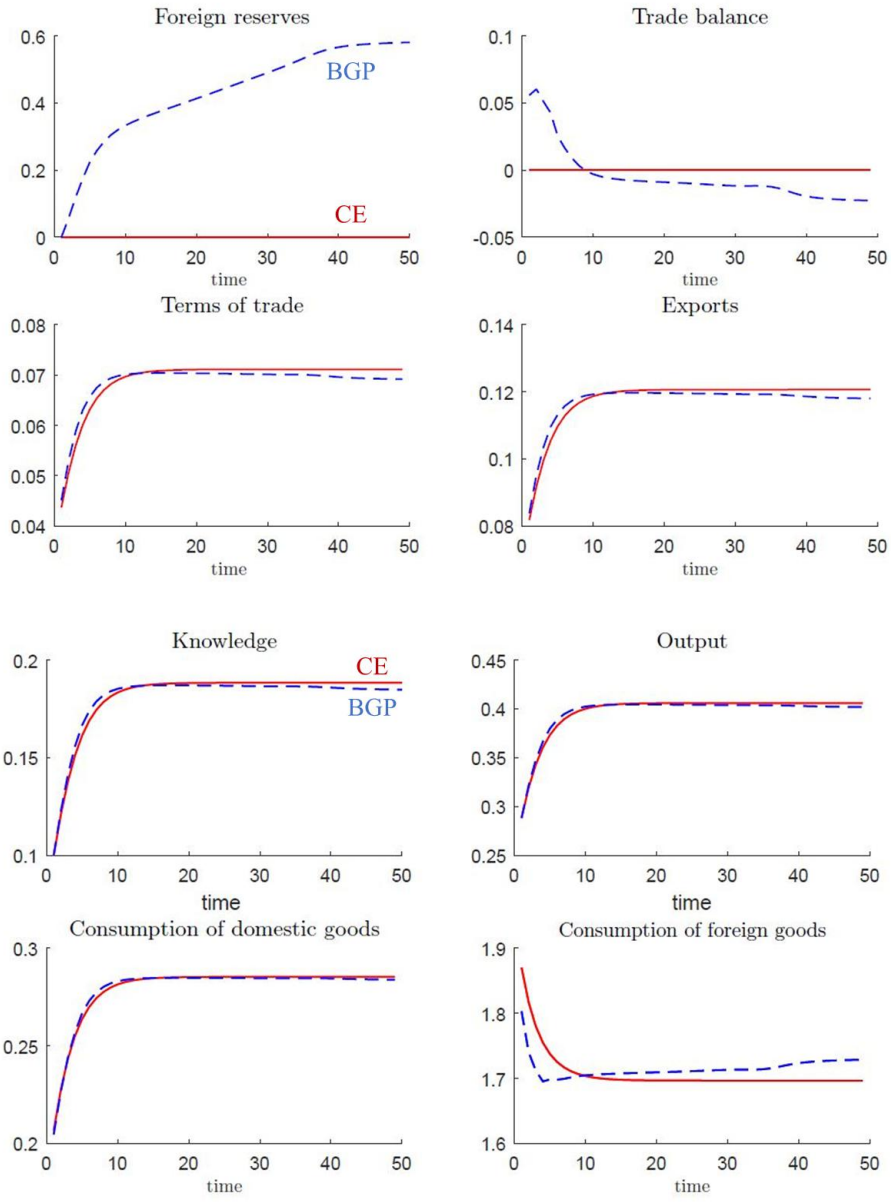
and 0.1 respectively and I simulate for 50 periods.

I first show the simulation results from the competitive equilibrium without the government. The private agents are not allowed to save in the foreign country and there is no government in this equilibrium, the foreign reserves are zero all the time. This also implies a zero trade balance. The economy exports as much as imports times terms of trade. Given the initial knowledge stock, the economy accumulates knowledge by exporting domestic goods. Hence, knowledge stock grows over time, and thus the output also grows since the knowledge stock is an input factor of the production. As output increases, the consumption of domestic and foreign goods increase. The terms of trade are less than 1. This means domestic goods are more expensive than foreign goods. For this reason, the consumption of domestic goods is less than the consumption of foreign goods. Eventually, this economy converges.

On the other hand, internalizing the effects of knowledge externalities, the benevolent government accumulates foreign reserves over time. During the first few periods, the government saves in the foreign country very quickly, so the foreign reserves sharply increase. Meanwhile, the government policy induces the higher terms of trade and this enables the economy to stimulate exports. Thus, the knowledge stock increases as the exports rise. When the knowledge converges to the steady-state value, the government does not need to boost exports anymore. This is a consequence of decreasing returns to scale of the externality function in the law of motion of knowledge. Larger exports give lower marginal benefits of exporting, and thus the knowledge does not significantly grow at a certain level of exports. Thus, after the knowledge converges, the government attempts to keep exports as much as maintaining the steady-state level of the knowledge stock. Therefore, the foreign reserves show overshooting during the first few periods to boost exports but decrease to the point where can keep the steady-state value of the knowledge stock.

The accumulation of the foreign reserves results in the lower consumption of both goods compared to the competitive equilibrium. The consumption on foreign goods is lower because the government collects the lump-sum taxes in terms of foreign goods from the consumer to finance the foreign reserves. In addition, the deterioration of terms of trade caused by accumulating of the foreign reserves makes the foreign goods

Figure 2.4: Simulation results

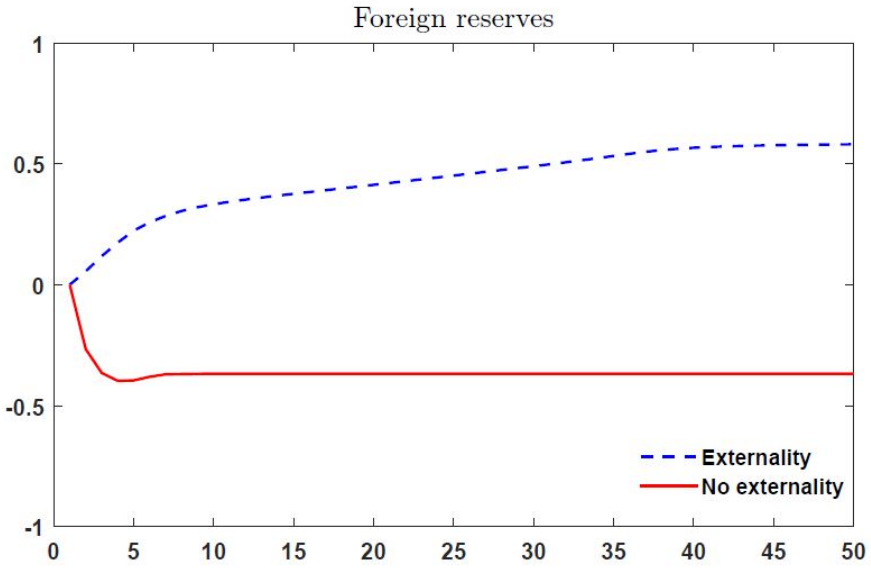


Notes: red solid lines are for the competitive equilibrium without the government intervention and blue dash lines are for the benevolent government.

more expensive. Thus, the consumer consumes less foreign goods. Furthermore, the consumption on domestic goods is also lower. This is because more domestic goods are exported due to the accumulation of foreign reserves. However, higher foreign reserves generate higher knowledge stocks and outputs in the future, so the economy grows faster. Since the economy grows faster consumers consume more with higher output. Once the economy reaches the steady-state, the government decreases the speed of accumulation of the foreign reserves and terms of trade starts being improved. Thus, when the steady-state is achieved, the consumption of domestic goods is lower than the CE while the consumption of foreign goods is higher than the CE. This shows the government takes into account the growth effects from the export externalities.

It is more clear to show the role of the government decision by comparing the level of the foreign reserves corresponding to no externality case. Figure 2.5 shows that the different optimal level of foreign reserves when there is no externality.

Figure 2.5: The foreign reserves with externality and no externality



Notes: The blue dash line is the level of the foreign reserves with externalities, $\zeta_x = 0.61$ and the red solid line is the foreign reserves with no externality, $\zeta_x = 0.00$.

With the benchmark parameter, $\zeta_x = 0.61$, the government accumulates the foreign reserves over time and the level of foreign reserves is 0.5 at the steady-state level. On the other hand, the optimal foreign reserves are negative when there is no externality, $\zeta_x = 0.00$. In other words, the government does not save in the foreign country, rather borrows from the foreign country. This result follows the argument that the benefits of improving the terms of trade and consumption smoothing are greater than the benefits of faster growth through export externalities.

To describe the impact on the welfare of the government policy, I compute the welfare gains from accumulating foreign reserves. Define CE as the competitive equilibrium with no government and BG as the equilibrium solved by the benevolent government. The welfare gain is measured by the effect of considering the externalities that make the CE equal to the BG in proportion to the consumption in the CE. This calculation explicitly takes into account the cost of reducing consumption when switching to an economy without foreign reserves. Welfare gains from accumulating foreign reserves, θ defined as

$$\sum_{t=0}^{\infty} \beta^t u((1+\theta^{CE})c_t^{CE}) = \sum_{t=0}^{\infty} \beta^t u(c_t^{BG}) \tag{2.19}$$

With the calibrated values, the welfare gains from accumulating foreign reserves are 0.05% of permanent consumption when the government intervenes. This is less than the average foreign reserves to GDP ratio since 1980. However, this number is fairly close to the average ratio prior to 2000.

2.5.3 Export externalities and foreign reserves decisions

In the previous section, I show the case where the government accumulates foreign reserves with the calibrated parameters. However, it is not always optimal for the government to accumulate foreign reserves. The optimal level of the foreign reserves is determined by comparing the growth effects through exports and the benefits of the consumption smoothing and the tariff policy. The magnitude of the two effects depends on the size of the export externalities. Suppose the export externalities are large, then exporting the domestic goods significantly increases the level of knowledge stock in the

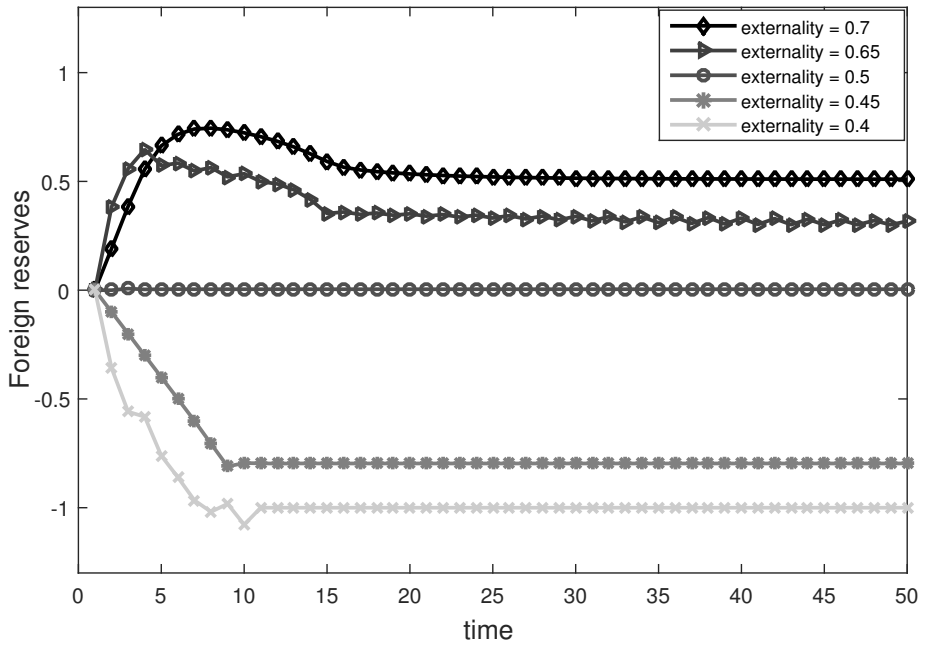
next period. With the high knowledge stock, the economy can produce more domestic goods tomorrow. Thus, the economy experiences faster growth by sacrificing today's consumption of domestic goods. In this regard, the government has an incentive to hold positive foreign reserves to induce high terms of trade, which can lead to high exports for fast growth.

On the other hand, when export externalities are small, the benefits from the consumption smoothing motivation and improvement of terms of trade is stronger than the growth effects. Therefore, the government holds negative foreign reserves, which is borrowing from abroad at the risk-free interest rate. By borrowing in terms of foreign goods the terms of trade become lower, meaning that imported foreign goods become cheaper so the consumers consume more foreign goods today. Also, lower terms of trade imply that the government decides not to stimulate exports and consume more domestic goods today as well as imported goods. This is related to the consumption smoothing motivation because the agents know that the economy will have higher income tomorrow, so it borrows foreign goods today. In addition, similar to the optimal tariff policy, the government induces better terms of trade to increase the consumer's welfare by borrowing foreign goods.

This mechanism is shown in Figure 2.6. This graph illustrates the optimal foreign reserves level depending on the size of the export externalities. I vary the externality parameter ζ_x and see the government decisions on foreign reserves. The baseline model has externality parameter 0.65 and the foreign reserves reach to 0.74 in period 10 and decrease to 0.51 which is the steady-state level. As the size of foreign reserves become smaller, the optimal level of foreign reserves also decreases. When the externality parameter is 0.5, the government is almost indifferent between intervention and no intervention. Once the externality parameter is less than 0.5, consumption today gives higher welfare than exporting, so the government borrows to improve terms of trade.

This exercise provides different foreign reserves policy implications in developing countries. The countries having high export externalities would hold high foreign reserves to enhance exports through the deterioration of the terms of trade. On the other hand, the countries having low export externalities would receive capital inflow from foreign countries to gain welfare improvement through consumption smoothing and better terms of trade.

Figure 2.6: Export externalities and Foreign reserves decisions



Notes: Externality is the export externality parameter ζ_x. Each line represents the optimal foreign reserves obtained by solving the benevolent government problem with different externality parameter.

2.5.4 Competitive equilibrium with private savings

I showed the optimal actions of the private agents are less efficient than the benevolent government. To achieve higher welfare the government uses foreign reserves. Then it is natural to consider that the existing government's external saving technology might be the source of efficiency because the private agents are not allowed to save. Hence, in this section, I discuss the welfare gains just from internalizing the externalities by comparing the competitive equilibrium with private savings and no government to the equilibrium of the benevolent government.

All environments are the same except the consumer's problem. The consumer now

can save foreign goods in the foreign country by buying foreign bonds with the risk-free interest rate. This will change two equations compared to competitive equilibrium without savings: the consumer's budget constraint and the trade balance. The budget constraint of the consumer includes savings of s. These are the external savings, so it must be saved in terms of foreign goods. Since the government does not exist, transfers are dropped from the budget constraint. The following equation is the consumer's budget constraint with savings.

$$c_t^D + p_t(c_t^F + s_{t+1}) = w_t L_t + \pi_t + p_t R_f s_t \tag{2.20}$$

Also, substituting (2.20) and (2.6) into (2.12), the trade balance becomes

$$x_t - p_t c_t^F = p_t \left(s_{t+1} - R_f s_t\right) \tag{2.21}$$

which is similar to the benevolent government trade balance. The definition and the characterization of the competitive equilibrium for this economy with private saving is defined in appendix (A.3).

In this setup, the private agents sell the foreign bonds i.e., borrowing rather than buying them i.e., lend. Figure 2.7 illustrates the simulation results when private borrowing is allowed. The red solid lines are the competitive equilibrium while the blue dash lines show the government decision. The consumers know that they will have a higher output tomorrow due to the accumulation of knowledge. Hence, the consumers borrow to smooth their consumption. In fact, this action badly affects the growth of the economy. The borrowing in the private sector deteriorates the terms of trade in the first 10 periods, so the economy exports less. Thus, during these periods, the imports are much higher with better terms of trade. This hinders knowledge accumulation, and thus the economy grows slowly. This situation shows the opposite of the action of the benevolent government. Because the future growth benefits from the export externalities outweigh the utility from consuming foreign goods today, the government would save. On the contrary, not internalizing the externalities, the private agents would borrow. Therefore, opening a capital account is not favorable to the government and the accumulation of the foreign reserves generates higher welfare gain than the competitive

equilibrium with no private saving.

The welfare gains from correcting externalities are higher when private savings are allowed. I compute the welfare gains with the following way.

$$\sum_{t=0}^{\infty} \beta^t u((1+\theta^{CE_s})c_t^{CE_s}) = \sum_{t=0}^{\infty} \beta^t u(c_t^{BG}) \tag{2.22}$$

θ^{CE_s} represents the welfare gains and $c_t^{CE_s}$ is the consumption level of the competitive equilibrium with private saving at each period t. The welfare gains in this environment are 0.60%.

2.5.5 Subsidy on exports

I examined the role of foreign reserves as a policy tool, but the government could have used the export subsidy (or tariff) if the WTO allowed them. Thus, I investigate the welfare implication of the export subsidy as a government policy tool in this section. This means that the government will choose the optimal level of subsidies on the export. The results from this environment will help to compare with the argument of the optimal tariff theory.[13]

In terms of model, the domestic consumer still receives subsidies from the government so there is no change in the consumer problem. However, the government budget constraint includes expenditure associated with the export subsidies. In other words, the government finances the export subsidies by tax revenues from the consumer and the tax is a lump-sum fashion. As same as the model with the foreign reserves the subsidy is denominated in the foreign goods. In this environment the government budget constraint is :

$$T_t = x_t \tau_t \tag{2.23}$$

where τ_t is the export subsidies.

[13] I use subsidies on the exports, not the tariffs. But I can interpret the negative subsidies as tariffs.

Not only the government budget constraint, but the domestic goods market-clearing condition is also affected by the export subsidies. Domestic output will be distributed to domestic consumption and exports. Moreover, $\tau_t x_t$ are subsidized by the government.[14]

$$c_t^D + (1-\tau_t)x_t = Y_t \tag{2.24}$$

The government pays $\tau_t x_t$ with the lump-sum tax collected from the consumer in terms of foreign goods. Combining the consumer's budget constraint (2.2), the government budget constraint (2.23), and domestic goods market clearing condition (2.24), the trade balance can be obtained.

$$p_t c_t^F - (1-\tau_t)x_t = p_t \tau_t x_t \tag{2.25}$$

Furthermore, the foreign demand function for the export is changed because the export goods become cheaper in the foreign country due to the export subsidy. Hence, the demand for the export with an export subsidy is as follow.

$$x_t = \left((1-\tau)\frac{1}{p_t}\right)^{-\phi} Y^\star \tag{2.26}$$

This part is crucial for analyzing the export subsidy. Figure 2.8 shows the levels of subsidies and volume of the exports according to the different externality parameters, ζ_x. When the externalities are high enough, the subsidies on the exporting goods are big. Since the economy has high export externalities, it prefers exporting today so provides subsidies to the exporting sector. Due to the subsidies, the exporting goods become cheaper, meaning that the terms of trade deteriorate. This stimulates exports and the economy will grow faster. This result is also shown in the right-hand side of Figure 2.8. With the high externality parameter, the economy exports more. As the externalities become smaller, the subsidies become smaller and subsidies turn into taxes

[14]It might be easy for the negative sign of τ. If τ is negative, then it will be a tax on exports. Thus, the firm has to pay export tax to the government and the government gives subsidies to the consumer by using the export tax revenue.

(negative subsidies) with even the positive externalities if it is small. This implies that the economy can achieve higher utility by consuming goods more today rather than exporting goods if the export externalities are small. This result is consistent with Figure 2.6.

I also calculate the welfare gain using subsidies in the situation where the private savings are allowed, and it is 17%. This is relatively higher than the welfare gain using foreign reserves so it can be concluded that subsidies (taxes) are more efficient than foreign reserves. Nevertheless, the direct export subsidies (taxes) are forbidden by the WTO rules, the foreign reserves can be used as an alternative policy tool.

2.6 Conclusion

In this paper, I propose an endogenous growth model of a small open economy with export externalities that provide a plausible explanation for different policies regarding the foreign reserves in various countries. I argue that some countries with high export externalities can accelerate their economic growth by accumulating foreign reserves and this can be achieved through export spillovers from the trade relationships with foreign countries.

The key factor in the model is knowledge spillovers through exports, and it can be internalized by the government, but not by private agents. The consumers and the firms do not consider the growth effects from the exports so the exports would be less than the socially optimal level if there is no government intervention. However, the government takes into account the export externalities that induce faster growth of the economy, and thus it uses the foreign reserves as a policy tool to manipulate the terms of trade for stimulating exports. By increasing the foreign reserves, the terms of trade are deteriorated, so the price of domestic goods become cheaper. This leads to higher foreign demand for domestic goods, so the home country exports more and grow faster.

However, this mechanism works only if the export externalities are large enough. When the export externalities are positive but small, the government would hold a negative level of foreign reserves, implying borrowing from abroad at the risk-free interest

rate. This is because the benefits of the consumption smoothing motivation and improvement of terms of trade by borrowing foreign goods outweigh the growth effects from higher exports. Therefore, the government has an incentive to improve the terms of trade to increases its welfare.

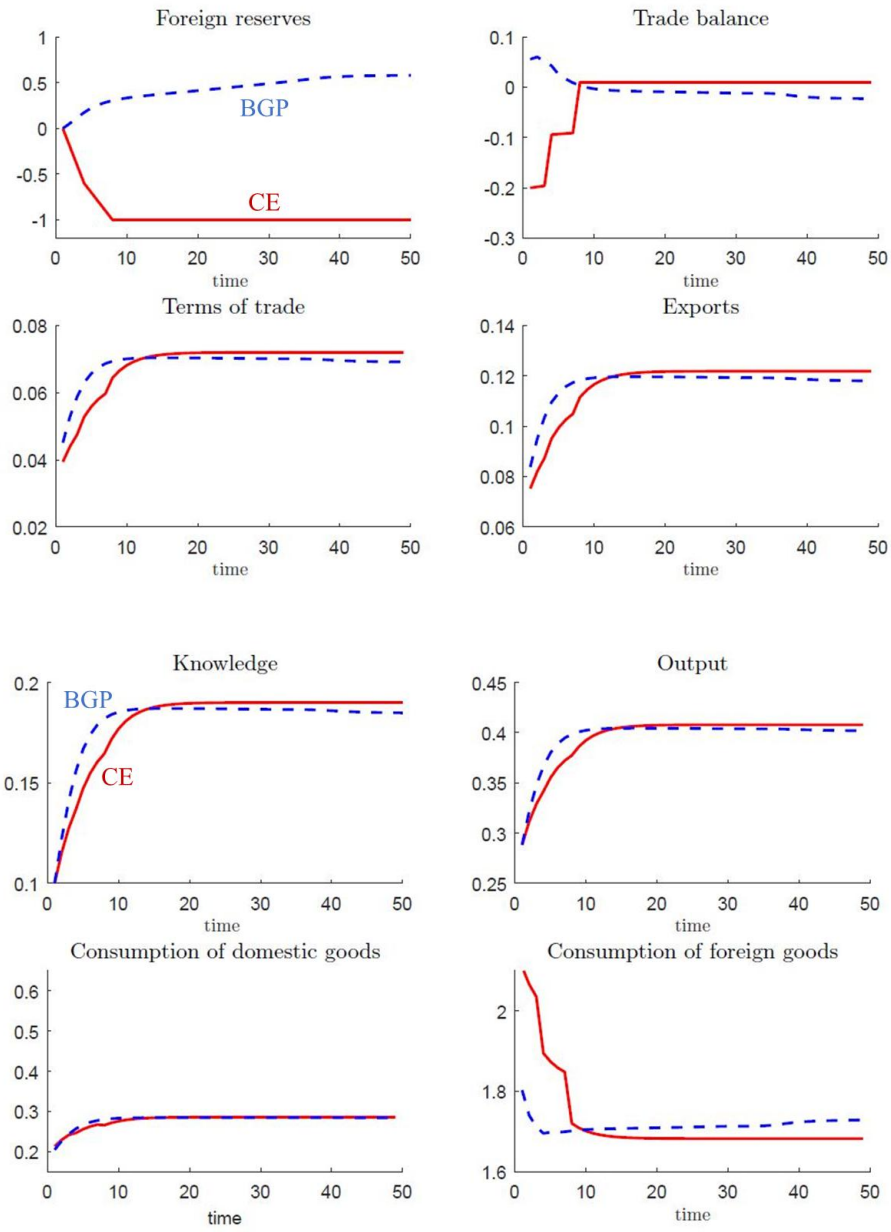
I calibrated the model with data from Taiwan and showed the optimal choice for the foreign reserves. I found that the government accumulates foreign reserves faster until the economy reaches the steady-state level. During this period, the terms of trade are high and show faster growth of the economy with higher exports. Once it converges to the steady-state, it is not necessary to enhance the exports anymore, so the optimal foreign reserves are determined at the level that maintains the steady-state level of the economy. The welfare gain from accumulating foreign reserves is 0.05% of permanent consumption in the economy with no government policy when the consumers cannot save internationally.

Also, I vary the externality parameters to examine the different optimal level of foreign reserves. The results show that as the export externalities become smaller, the growth effects through exports become smaller too, and thus the optimal level of foreign reserves decreases. Therefore, even though the export externalities are positive, the government chooses to hold negative foreign reserves.

When I allow private saving, the welfare gain becomes larger, 0.60% since the private agents who do not internalize the externalities would borrow by expecting higher income in the future. Thus, the welfare gain from correcting externalities is bigger in this situation.

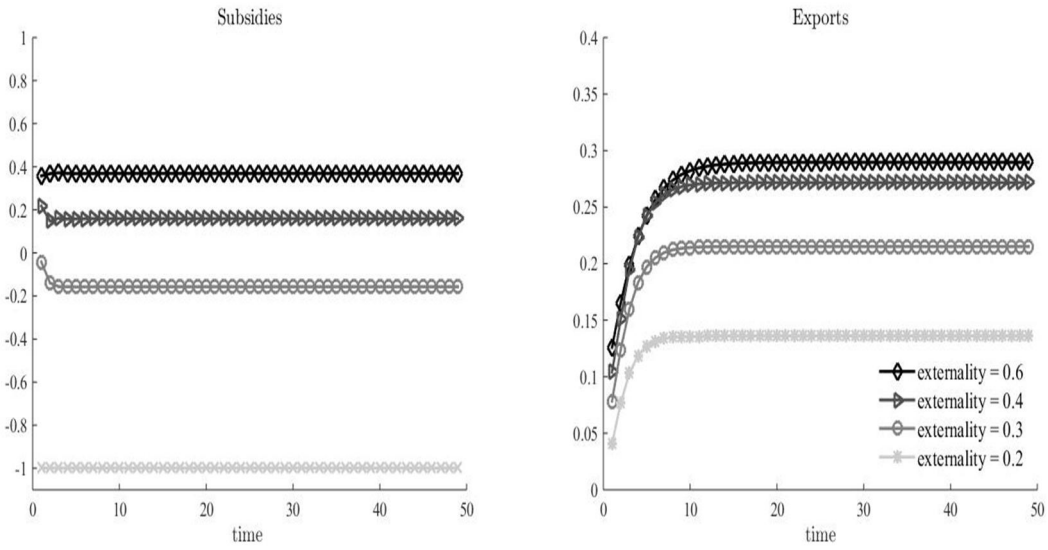
Comparing the welfare gain using subsidies and foreign reserves, the export subsidies are more efficient than foreign reserves. However, the international trade rules do not allow the export subsidies so foreign reserves can be used as the government policy tool in many countries.

Figure 2.7: Simulation results



Notes: red solid lines are for the competitive equilibrium allowing private saving without the government intervention and blue dash lines are for the benevolent government.

Figure 2.8: Subsidies and export according to different externalities



Notes: Externality is the export externality parameter ζ_x. Each line represents the optimal subsidies and exports obtained by solving the benevolent government problem with different externality parameter.

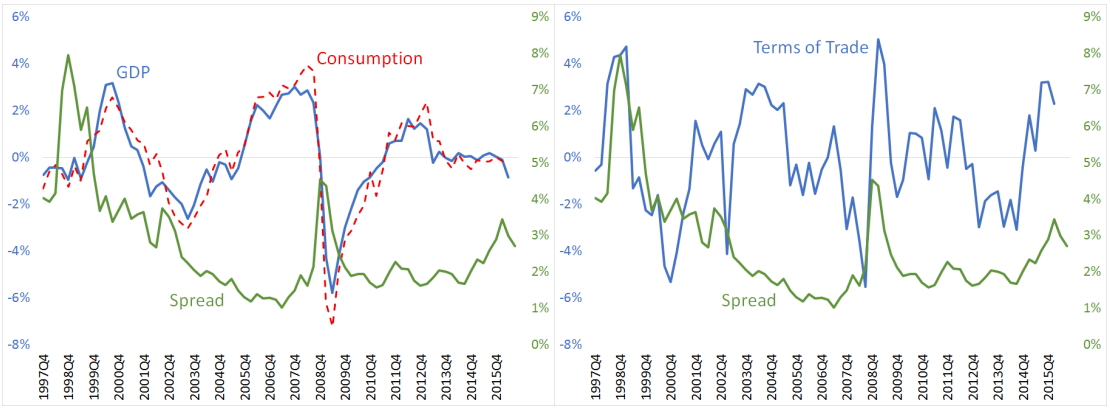
Chapter 3

Sovereign spread movements in emerging economics: terms of trade matter

3.1 Introduction

This paper focuses on the high mean and volatility of interest rate spreads in emerging economies. We document that some emerging economies experience a decrease in the negative correlation between real GDP and interest rate spreads for the last two decades. As a matter of fact, it is observed in the data that interest rates oscillate regardless of a favorable domestic economic performance. Figure 3.1 shows the Mexican interest rate spread for the last twenty years and its relationship with real GDP and terms of trade. The interest rate spread in Mexico displays sharp rises in 1998 and 2014, even though the economy does not experience deep recessions during these years. Moreover, during these years, the interest rate spread in Mexico follow more closely the movements of the terms of trade series. This observation corresponds to the puzzle proposed in Tomz and Wright (2007). They found that the negative correlation between output and default of a country is remarkably week. Also, they show evidence of countries defaulting on their sovereign debts during good times while making repayments during bad times. This paper addresses this puzzle by arguing that foreign conditions can explain this issue.

Figure 3.1: Spreads and Cyclical Components in Mexico



The framework presented in Eaton and Gersovitz (1981) is beneficial to analyze the spread behavior because this class of model is able to derive the interest rates endogenously.[1] However, it has been proven difficult to obtain three features of the interest rates with sovereign default models. Arellano (2008) shows countercyclical interest rate spreads by introducing convex costs of default. This yields defaults to be more likely to occur during recessions. Nevertheless, the mean spread that the model provides is 3.58% which is relatively low compared to the mean spread of Argentina, which is 10.25%. Also, only few fluctuations of spread are observed with good economic conditions due to the structural features of the probability of default. In fact, the spread generated by the model is almost zero when the country is hit by good endowment shocks. Mendoza and Yue (2012) achieve large volatility of spread in their baseline model by introducing endogenous default costs, but fail also to capture the high mean spread shown in the data. This is because spread and default probabilities are linked to each other directly. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) are able to improve the spread behavior in the sovereign default models by incorporating long-duration bonds. This helps to increase mean and standard deviation of spread; however, those studies cannot explain why spreads can be high during good

[1] Aguiar and Gopinath (2007) and Neumeyer and Perri (2005) assume exogenous interest rates since it is useful to explain business cycles of developing countries. High volatility and countercyclicality of interest rates are regarded as crucial parts to explain the cyclical movement of aggregate output and prices.

times in the economy. The high volatility of spreads is mainly accomplished by the large dispersion of spreads with low endowments while the standard deviation with high endowments is significantly small.

This paper proposes a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. The key feature of this paper is that the model incorporates an exogenous foreign shock called terms of trade. In the model, a negative terms of trade shock act in two ways. First, the country spends more in foreign products for consumption. Second, the terms of trade have direct impacts on the level of foreign currency debts that sovereigns owe to foreign lender. This model works with the assumption that sovereigns issue their bonds in foreign prices. As shown in Eichengreen and Hausmann (1999) and Jeanne (2003), emerging economies tend to issue debt in foreign currency because their local currency present high fluctuations and lack of credibility[2] Since debt is issued in foreign currency, the countries are vulnerable to changes in world prices. In other words, when the adverse terms of trade shocks hit the economy, the sovereign immediately encounters an unexpected enlarged debt burden. Consequently, the probability of default is not only affected by countries GDP shocks but also the terms of trade shocks. This provides an explanation of why terms of trade shocks can lead to higher and more volatile spread movements, regardless of the GDP performance.

As mentioned above developing countries experience high volatility of terms of trade and output. More specifically, terms of trade shocks are more volatile than GDP shocks in emerging economies. This implies that the terms of trade shocks are an important factor to consider when studying them. Moreover, as shown in Kose (2002) an important source for the repayment of foreign debt is export revenue and this is largely affected by the terms of trade. Terms of trade are often studied in the sovereign default models. Na, Schmitt-Grohé, Uribe, and Yue (2018), Gu (2015), and Asonuma (2016) endogenously induce the deterioration of the terms of trade and real exchange rate. This paper makes distinctions from those papers by assuming terms of trade shocks as exogenous. Popov and Wiczer (2014) assume an exogenous path of terms of trade but he examines the role of terms of trade penalties and focuses on changes in trade volumes. In contrast,

[2]Du and Schreger (2016) show that foreign currency debt composition has decreased since 2004. Nevertheless, they still have a significant level of foreign currency debt.

we focus in analyzing the changes in debt burden contingent to terms of trade. Cuadra and Sapriza (2006) study also an exogenous terms of trade shock when the production side buy intermediate imported goods. In their model, the terms of trade shocks are used as if they are productivity shocks so the terms of trade shocks generate real GDP movement. However, this mechanism violates the result in Kehoe and Ruhl (2008) that proves that terms of trade do not have first order effects in real GDP[3] Moreover, they do not have the convex default costs so the frequency of default generated by the model is unusually small. In this paper we involve both endowment shocks and the terms of trade shocks, while also considering a convex default cost.

The rest of the paper is organized as follows. Section 2 empirical evidence, Section 3 the model, Section 4 quantitative analysis, Section 5 conclusion.

3.2 Empirical Evidence

3.2.1 Currency composition of sovereign external debt

In this section, we construct the ratio of foreign currency sovereign external debt to total debt in developing countries. This helps to develop the idea that the terms of trade shocks are of importance to the fluctuations of the economy in emerging economies via foreign currency sovereign debt owed to foreign investors. The definition of external debt is adopted from Du and Schreger (2016). We deviate from their methodology because we are only interested in studying the government debt.[4] Hence, we use the definition of *sovereign external debt* as any debt issued by the government in developing countries and owed to nonresidents, regardless of the market of issuance.

Debt is categorized by three dimensions: issue sector, issue currency, and issue market. Issuance sector is divided into the government and the corporate sector. The debt issued by the central or local governments is counted as government debt while all debt issued by the private sector is regarded as corporate debt. The classification of issue currency is determined by which currency debt is denominated when issued. Local

[3]They also show that the terms of trade do not act as a productivity shock in standard models while they do affect real income and consumption in a country.

[4]Their definition of *external debt* includes both public and private debt in order to analyze how default decisions are affected by debt denomination in public and private sectors. However, this paper considers only public debt. Thus, we define *sovereign external debt* instead of *external debt*.

currency (LC) debt refers to debt that is issued in the currency of issuance country while foreign currency (FC) debt is denominated in another country's currency. Lastly, issuance market is broken down into two markets. When debt is issued under the domestic law inside a country, it is called domestic debt; on the other hand, international debt follows foreign law and issued in international markets. Among these categories, this paper mainly addresses the combined category of government as issuer sector, foreign currency as issue currency, and both markets as issue market in order to study sovereign external debt.

Bank for International Settlements (BIS) provides amount of outstanding debt data by each classification. However, debt data by debt holder - nonresidents or residents -, which is the main part of definition of *external debt*, are not available. Hence, we follow Du and Schreger (2016) to construct the currency composition of *sovereign external debt*. They make two assumptions for debt holding of nonresidents. First, nonresidents hold all debts in international market, which implies that all international debts are regarded as external debt. Second, nonresidents do not hold any FC debt in domestic market.[5] Based on these two assumptions, the FC sovereign external debt is constructed as follows: amount of outstanding FC debt issued by the government in international market.

Table 3.1 provides the share of FC sovereign external debt in total sovereign debt. The total sovereign debt is defined as all debts issued by the government so it consists of both domestic debt and external debt.[6] The analysis of this paper is proceed based on sovereign external debt, and it is expected that countries with higher share of external debt denominated in FC are more likely to be exposed by terms of trade shocks. In Table 3.1, although the substantial heterogeneity for the ratio of FC external debt to total debt is observed, it is sensible that countries are considerably under the influence of it. Moreover, there are some countries that heavily rely on FC debt owed to foreign creditors such as Peru, Argentina, Lebanon, and Lithuania. In particular, the countries that experienced sovereign default events tend to have higher percentage of FC external debt. Argentina had carried on more than 75% of external debt in FC until the default periods and reduced it to approximately 50% in the first half of the 2000's. Peru also has

[5]They document that the amount of outstanding foreign currency debt in domestic market is notably small so the second assumption is sensible.

[6]The definition of domestic debt in this context is any debt owed to residents within the country.

Table 3.1: Foreign Currency Debt Composition

	Average	2004	2015
Argentina*	79.6	74.9	43.4
Brazil	7.3	4.9	4.4
Chile	12.9	20.8	16.3
Colombia	22.8	22.8	25.5
Croatia	45.7	59.3	46.1
Hungary	22.7	19.8	27.0
Indonesia	13.9	3.2	30.8
Lebanon	35.2	51.5	44.8
Lithuania	83.8	73.2	79.5
Malaysia	3.9	9.0	3.3
Mexico	16.2	27.3	15.1
Peru*	65.1	84.2	38.7
Philippines	24.9	33.9	23.4
Russia*	31.2	48.5	38.1
South Africa	7.2	10.6	9.7
Turkey	19.0	16.5	29.6

Notes: * indicates countries that experienced default events. 2005 data is used for Mexico and Malaysia for the 2004 column and 2007 data and 2008 data are used for South Africa and Chile for the 2004 column respectively. They are first year of data availability.

been maintained high share of FC external debt on average. In case of Russia, almost half of the total debt is FC debt owed to nonresidents in 2004 which is significantly large enough to be affected by exchange rate movements.

One of the results from Du and Schreger (2016) is that sovereigns have been using more LC when issuing external debt in government sector so there is a tendency of the decrease in the proportion of FC external debt in total external debt.[7] Nevertheless, analyzing FC external debt is worthy. Since they compare the FC external debt with total external debt, the expansion or contraction of the amount of total external debt is not taken into consideration in their construction of currency composition. In other words, the importance of FC external debt could be underestimated if the countries issue

[7]They analyze $\frac{\text{FC external debt}}{\text{Total external debt}}$ while this paper analyze $\frac{\text{FC external debt}}{\text{External debt+ Internal debt}}$. Thus, the share of foreign currency external debt in this paper is affect by the amount of debt owe to residents in a country.

more external debt than domestic debt. However, the measure of currency composition used in this paper reflects this issue since the definition of total sovereign debt includes both domestic debt and external debt. If the amount of domestic debt gets smaller, then the share of FC external debt in total sovereign debt increases which means external debt in FC becomes a more essential part of the debt in the countries. Actually, all countries except Hungary and Russia in Table 3.1 display continuous increases in the amount of external debt denominated in FC.[8] Furthermore, it is not explicitly shown that the share of FC external debt has been decreasing in Figure 3.2 with our data construction. Indonesia, Hungary, Turkey, and Lithuania, for instance, have kept expanding the share of external debt in FC. The LC debt in domestic market rapidly rose in Croatia around 2004 so the share sharply decreased at that time but it started to issue more FC debt in international market in 2009 so the share has been following the growing trend since the time. In addition, other countries hold more or less a constant share of FC external debt. Those empirical evidence illustrates that external debt in FC are still a crucial part of debt in developing countries.

3.3 Terms of trade, GDP and spread across countries

The data presented in Table 3.2 and 3.3 are statistics for the terms of trade, GDP, and spread across 24 developing countries. In this paper, the terms of trade (TOT) are defined as the price of imports relative to the price of exports.

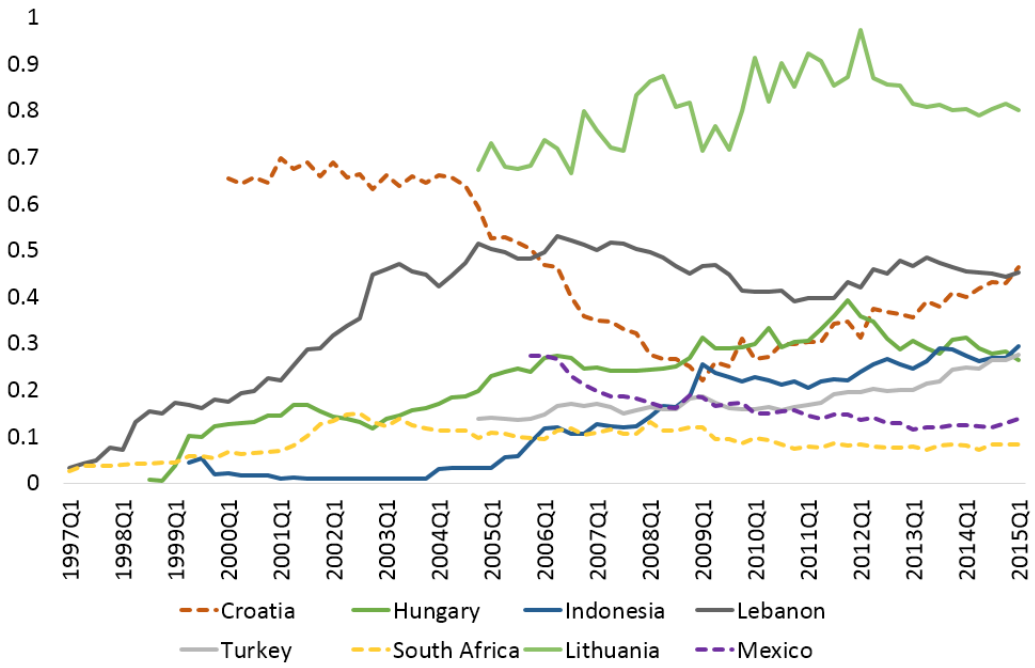
$$TOT = \frac{P_M}{P_X}$$

In order to construct the terms of trade, quarterly merchandise customs imports and exports data 1991Q1 to 2015Q4 are obtained from World Bank Global Economic Monitor (GEM).[9] By using current and constant value of import and export each deflator is calculated. Afterwards the import and the export deflators are used for the price of imports and the price of exports respectively. Hence, the terms of trade are constructed

[8] Hungary and Russia has been reducing the amount of foreign currency external debt since 2014.

[9] World Development Indicators (WDI) provides imports and exports data across countries, but this is annual data. Thus, we interpolate annual data to transform to quarterly data and compare it with the quarterly merchandise customs imports and exports and we found that those two series are coincided through the sample period.

Figure 3.2: Foreign Debt Profile for Emerging Economies



by import deflator over export deflator. The quarterly GDP is also provided from GEM and the time period is the same as the one in imports and exports data. The interest rate spread data are taken from J.P. Morgan's EMBI + database.[10] The terms of trade and output are log and HP detrended.

Table 3.2 provides standard deviation of the terms of trade, GPD, and spread. It also provides the mean of spread in each country since this paper focuses on the behavior of spread. Although there is a cross-country heterogeneity, in almost all sample countries, the standard deviations of TOT are bigger than those of GDP. In other words, TOT is more fluctuate than GDP in most of countries. This fact is crucial for the analysis of the paper since this indicates that more fluctuations of the economy can be driven by the terms of trade shocks with high volatility. Also, defaulted countries, such as Argentina, Ecuador, and Russia, show much higher mean and volatile movement of spread. In

[10]The spread data are not a balanced data across countries, so we used series of spread available up to 2015Q4.

Table 3.2: Descriptive Statistics in Emerging Economies

	σ(TOT)	σ(GDP)	σ(spread)	μ(spread)
Argentina	0.045	0.040	18.24	15.99
Brazil	0.041	0.015	3.92	5.66
Chile	0.090	0.018	0.59	1.46
China	0.036	0.010	0.55	1.18
Colombia	0.073	0.013	2.07	3.56
Dominican Rep	0.022	0.023	3.30	5.39
Ecuador	0.295	0.020	8.38	12.33
Egypt	0.034	0.015	1.74	2.55
Hungary	0.016	0.015	1.58	1.80
Indonesia	0.068	0.033	1.44	2.89
Kazakhstan	0.264	0.024	2.74	4.28
Korea	0.035	0.023	1.04	1.31
Malaysia	0.027	0.018	1.25	1.78
Mexico	0.040	0.023	1.51	2.75
Morocco	0.016	0.013	2.43	2.30
Peru	0.073	0.018	1.96	3.51
Philippines	0.046	0.013	1.52	3.46
Poland	0.096	0.014	0.90	1.71
Russia	0.098	0.029	11.26	7.28
South Africa	0.032	0.012	1.19	2.26
Sri Lanka	0.024	0.009	4.33	6.01
Tunisia	0.014	0.011	0.92	1.84
Turkey	0.030	0.036	2.20	4.02
Ukraine	0.023	0.043	6.23	7.46

particular, the volatility of TOT in Ecuador is approximately 14 times bigger than its GDP. Hence, it can be seen that Ecuador has been affected by volatile terms of trade shocks. However, it is unclear that the terms of trade shocks is a main factor for movement of spread based on magnitude of standard deviation, but correlation with spread will help to improve this issue.

Table 3.3 shows different combinations of correlations among TOT, GDP and spread in the sample countries. First, the negative correlations between GDP and spread are achieved and also the positive correlations between TOT and spread are presented in most countries. This implies that the deterioration of the terms of trade coincide with

Table 3.3: Correlations in Emerging Economies

	ρ(TOT, spread)	ρ(GDP, spread)	ρ(TOT,GDP)
Argentina	-0.026	-0.647	-0.066
Brazil	0.205	-0.198	-0.553
Chile	0.528	-0.335	-0.478
China	-0.080	-0.007	0.257
Colombia	0.267	-0.197	-0.485
Dominican Rep	0.020	-0.641	-0.006
Ecuador	0.517	-0.445	-0.687
Egypt	0.082	-0.189	0.084
Hungary	0.141	-0.290	0.222
Indonesia	0.574	-0.284	-0.507
Kazakhstan	-0.440	-0.607	0.541
Korea	-0.209	-0.633	0.524
Malaysia	0.061	-0.485	0.116
Mexico	0.443	0.005	-0.466
Morocco	-0.126	0.135	-0.099
Peru	0.332	-0.156	-0.263
Philippines	0.029	-0.326	0.262
Poland	0.007	0.159	0.137
Russia	0.241	-0.487	-0.681
South Africa	0.223	-0.250	-0.021
Sri Lanka	0.433	-0.430	0.229
Tunisia	0.556	-0.162	0.017
Turkey	0.169	-0.449	0.079
Ukraine	-0.194	-0.423	-0.157

the increase in spread in general. Moreover, even though it is hard to find a certain pattern between ρ(TOT, spread) and ρ(TOT, spread), there are nine countries having higher correlation of spread with TOT than with GDP. This can be direct evidence for the impacts of TOT on movement of interest rate spread. For example, Mexico have significantly high correlation of spread with TOT while there is almost no correlation with GDP: hence, it is reasonable to conclude that the fluctuation of Mexican spread is mainly affected by the terms of trade shocks. This interpretation can be generalized to any countries showing higher correlation between spread and TOT.

3.4 Model

In this section, we propose a model of that incorporates two sources of uncertainty in a country, a domestic and a foreign. We work with a framework that extends the sovereign default models introduced in Eaton and Gersovitz (1981) and Arellano (2008). We use this last one to incorporate a source of external uncertainty called terms of trade.

Consider a small open economy where there are two types of shocks, a domestic and a foreign. On one hand, the domestic shock is going to be represented by real GDP movements. On the other, the foreign shock is going to be represented by terms of trade movements. Let y_t and p_t represent output and terms of trade in period t, respectively. As discussed previously, it is standard to assume terms of trade as an exogenous variable for small open economies. We consider this in order to construct the following stochastic system

$$\begin{bmatrix} \ln y_{t+1} \\ \ln p_{t+1} \end{bmatrix} = A \begin{bmatrix} \ln y_t \\ \ln p_t \end{bmatrix} + \begin{bmatrix} \varepsilon^y_{t+1} \\ \varepsilon^p_{t+1} \end{bmatrix}, \tag{3.1}$$

where the roots of the second order polynomial $det\,(I_2 - Ax) = 0$ lie outside the complex unit circle and the errors vector is a binormal distribution with mean $\overline{0}$ and variance-covariance matrix Σ. With this formulation we are able to tailor a correlation between contemporaneous real GDP and terms of trade.[11]

There are two types of tradable goods in this economy, a domestic and a foreign. In every period, a representative household purchases these two types of goods and transform them into a final consumption good using the following Armington aggregator technology,

$$C = \left(\lambda c_d^{-\eta} + (1-\lambda) c_f^{-\eta}\right)^{-\frac{1}{\eta}},$$

where C represents the final aggregated good consumption, c_d the domestic good consumption, c_f the foreign good consumption, $\lambda \in [0,1]$ a parameter that captures home

[11] As shown in Kehoe and Ruhl (2008), it is a common mistake to misrepresent what real GDP is in a model. Moreover, standard models with a production side that buy imported goods at the price of terms of trade capture a spurious correlation between real GDP and terms of trade. If real GDP is measured correctly in them, the correlation between real GDP and terms of trade should be close to zero.

bias, and $\eta \in [-1, \infty)$ a parameter that controls the elasticity of substitution between domestic and foreign goods. The representative household is able to purchase these goods taking the output available in the period and an amount of taxes that the government issues in a lump-sum fashion as given. The representative household is a risk averse agent that obtains utility from the stream of final consumption goods obtained in every period as

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(C_t\right) \right],$$

where $\beta \in (0, 1)$ is a parameter that captures a discount factor across periods and $u(\cdot)$ is an increasing and strictly concave function.

In this economy there is also a benevolent government whose objective is to maximize the utility of the representative household. In order to achieve this, it has two main decisions to take. The first decision is either to honor or default in its sovereign bonds obligations. If the government takes the decision of honoring them, it has access to international financial markets where it can buy or sell one period maturity sovereign bonds. Then, the second decision is how much of these bonds to purchase given a price schedule contingent to the amount of these new bonds and the shocks the economy experiences. Let B, B', and $q(B', y, p)$ represent the amount of sovereign bonds due, the amount of new sovereign bonds purchases, and the price of these at any given period.

We introduce the assumption that the international financial markets are managed only in foreign goods. This is, the sovereign bonds return and purchases have to be done in foreign terms. Because of this, the value of sovereign bonds in each period will be subject to the terms of trade shock.

We model two types of penalties for defaulting in sovereign debt, exclusion of financial markets and output losses. With these penalties we want to capture the fact that countries that default in their debt experience a temporary exclusion to international borrowing and periods of poor output performance. We assume that if a government defaults in their debt, this will be erased entirely and it will enter a financial autarkic environment. In addition, the country will experience an output cost that limit its endowment. The economy will remain in this financial autarky for a stochastic number of periods and will re-enter the financial markets with an exogenous probability.

Let x represent exports, the resources exiting the economy. The balance trade condition for the repayment state can be expressed as

$$x - pc_f = pq(B', y, p)B' - pB.$$

The left-hand side is the current account while the right-hand side is the negative of the capital account of the economy. This is, any surplus(deficit) that the capital account experiences due to the government's sovereign debt position will imply a deficit(surplus) of the current account. The balance trade condition for the default state can be expressed as

$$x - pc_f = 0.$$

Considering the exclusion of financial markets penalty of default, the balance trade condition will imply that the current account cannot experience any kind of surplus or deficit.

The resource constraint of the government will show how the domestic good endowment can be consumed in every period. On one hand, if the government chooses to repay its debt, the resource constraint will be

$$c_d + x = y.$$

On the other hand, if the government chooses to default,

$$c_d + x = h(y),$$

where $h(\cdot)$ is an increasing function such that $h(y) \leq y$.[12]

There is a representative foreign lender who is able to borrow and lend risk-free bonds at a constant international interest rate $r^* > 0$. We assume it has perfect information about the small open economy. This is, it can observe the level of output and terms of trade that the small open economy experiences every period. The foreign lender is risk neutral[13] and maximizes expected profits over risky sovereign bonds from the small

[12]Realize that the output cost is defined as $y - h(y)$ and is non-negative by definition.

[13]Cole and Kehoe (1996) explains that the risk neutrality of the foreign lenders reflect the fact that

open economy. We assume that the foreign lender maximizes profit only in foreign goods terms. Let δ represent the probability of the government to default in its sovereign debt position. Taking as given the default probability and the bond price, the foreign creditor chooses $\tilde{B}'$ to maximizes

$$\max_{\tilde{B}'} \left\{ q\tilde{B}' - \left(\frac{1-\delta}{1+r^*} \right) \tilde{B}' \right\}.$$

By the risk neutral nature of the foreign lender, the bond price schedule solution in equilibrium must satisfy the first order condition of the previous problem. The bond price schedule will be a result of the following break-even condition,

$$q = \frac{1-\delta}{1+r^*}. \tag{3.2}$$

Realizing that $\delta \in [0,1]$, we can infer that $q \in \left[0, \frac{1}{1+r^*}\right]$. Defining the sovereign interest rate as $r \equiv \frac{1}{q} - 1$, we obtain that $r = [r^*, \infty)$. Finally, we define the sovereign bond spread as $S \equiv r - r^*$.

The timing of the government problem is the following. In the beginning of the period the government realizes the amount of sovereign bonds due B, the domestic goods endowment shock y, and the terms of trade shock p. The government asses the optimal relationship between domestic and foreign goods using p and the preferences of the representative household. Then the government decides whether to honor or default in its debt obligations. If it decides to honor its debt, the government updates its sovereign bond holdings B' taking as given the sovereign bond price schedule $q(B', y, p)$ and constrained to its resource constraint and the balance trade condition. The foreign lender takes the bond price q as given and supplies $\tilde{B}'$ matching B'. Purchases of foreign and domestic take place. Finally, the representative household consumes the final good by aggregating the domestic and foreign goods.

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the size of an individual sovereign transaction is relatively small compared to the total international credit market.

3.4.1 Recursive Equilibrium

There are three variables that define the state of the government in every period: the sovereign bonds due, the output shock, and the terms of trade shock. Define $V(B, y, p)$ as the value function of the government at the beginning of every period. Let us model the first decision of the government as

$$V(B, y, p) = \max \left\{ V^R(B, y, p), V^D(y, p) \right\} \tag{3.3}$$

where $V^R(B, y, p)$ and $V^D(y, p)$ represent the value of the government if it repays and defaults in its debt obligations, respectively. Here, the government chooses which environment will yield the highest welfare for the representative household captured by $V^R(B, y, p)$ and $V^D(y, p)$.

When the government chooses to repay, it chooses the amount of sovereign bonds to sell or purchase as well as the household allocations that will maximize its welfare subject to the resource constraint, the balance trade condition, the aggregation technology, and a no-Ponzi condition. In order to do this, it takes the price schedule for the bonds as given and a lower boundary for the sovereign bonds issuance $\mathbb{B} > 0$. Thus,

$$V^R(B, y, p) = \max_{x, c_d, c_f, C, B'} \left\{ u(C) + \beta \mathbb{E}_{(y', p')} \left[V(B', y', p') | (y, p) \right] \right\} \tag{3.4}$$

$$\begin{aligned}
\text{s.t. } & c_d + x = y && \text{(Resource Constraint)} \\
& x - pc_f = pq(B', y, p)B' - pB && \text{(Balance Trade Condition)} \\
& C = \left(\lambda c_d^{-\eta} + (1 - \lambda) c_f^{-\eta} \right)^{-\frac{1}{\eta}} && \text{(Aggregation Technology)} \\
& B' \geq -\mathbb{B}. && \text{(No-Ponzi Condition)}
\end{aligned}$$

When the government chooses to default, it chooses the household allocations that will maximize its welfare subject to the resource constraint, the balance trade condition, and the aggregation technology. In order to do this, it takes the default penalty and the probability of returning to the financial markets $\phi \in [0, 1]$ as given. Recall that the default penalties are the output costs and the zero current account restriction due to

the financial market exclusion. Thus,

$$V^D(y,p) = \max_{x,c_d,c_f,C} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[\phi V(0,y',p') + (1-\phi) V^D(y',p') | (y,p) \right] \right\} \quad (3.5)$$

$$\text{s.t. } c_d + x = h(y) \qquad \text{(Resource Constraint)}$$

$$x - pc_f = 0 \qquad \text{(Balance Trade Condition)}$$

$$C = \left(\lambda c_d^{-\eta} + (1-\lambda) c_f^{-\eta} \right)^{-\frac{1}{\eta}}. \qquad \text{(Aggregation Technology)}$$

In order to define what is the probability of default for a government, it is useful to characterize the set of output and terms of trade states in which a government finds optimal to default contingent to a level of sovereign bond holdings. Specifically, define the default set as the

$$\mathcal{D}(B) = \left\{ (y,p) \in \mathbb{R}^2_{++} : \quad V^D(y,p) > V^R(B,y,p) \right\}. \quad (3.6)$$

This set expresses that if the government sells B' and the shocks of next period are $(y',p') \in \mathcal{D}(B')$, then the government will find it optimal to default on B' next period. Because of this, we can define the probability for a government to default on B' by measuring how likely is to end up in the states that live in $\mathcal{D}(B')$. Given the stochastic process that govern the movements of output and terms of trade shocks, call $f(\cdot)$ the probability density function between shock states. Thus, the probability of default can be expressed as

$$\delta(B',y,p) = \int_{(y',p') \in \mathcal{D}(B')} f\left((y',p') | (y,p) \right) d(y',p'). \quad (3.7)$$

Consider the case where the government chooses a level of sovereign bonds B' such that there are no possible states in the next period in which it will default on them, $\mathcal{D}(B') = \emptyset$. Then, with that amount of sovereign bonds the probability of default will be zero, $\delta(B',y,p) = 0$. Also, consider the case when the government chooses a level of sovereign bonds B' such that default is for sure regardless of the shock realizations in the next period, $\mathcal{D}(B') = \mathbb{R}^2_{++}$. Then, with that amount of sovereign bonds the probability of default will be one, $\delta(B',y,p) = 1$. Finally, realize that if the output and terms of trade shocks have no persistency whatsoever, then the probability of default

will only be contingent in the sovereign bond issues or purchases.

Now, the bond price schedule must satisfy the break-even condition (3.2) from the representative foreign lender problem. Considering the probability of default, the representative foreign lender must be consistent to (3.7). Thus, the bond price schedule will be

$$q(B', y, p) = \frac{1 - \delta(B', y, p)}{1 + r^*}. \tag{3.8}$$

Realize that this break-even condition will yield zero profit in expectation to the representative foreign lender regardless of the quantity of sovereign bonds it purchases or sells $\tilde{B}'$. Therefore, it will be willing to cover any amount of sovereign bonds the government finds optimal to choose during the repayment state. In other words, the sovereign bond market always clear in equilibrium $\tilde{B}' = B'$.

Definition 3 (Recursive Equilibrium)**.** *The recursive equilibrium of this small open economy will be a set of government value functions* $V(B, y, p)$, $V^R(B, y, p)$, $V^D(y, p)$ *and a sovereign bonds policy rule* $\hat{B}'(B, y, p)$, *a set of household consumption policy rules* $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, *and* $\hat{C}(B, y, p)$, *an exports policy rule* $\hat{x}(B, y, p)$, *a default set* $\mathcal{D}(B, y, p)$, *a default probability schedule* $\delta(B, y, p)$, *and bond price schedule* $q(B, y, p)$ *such that the following conditions are satisfied:*

- *Benevolent Government (Initial)*
  The default set $\mathcal{D}(B, y, p)$ *is consistent with the set of government value functions* $V(B, y, p)$, $V^R(B, y, p)$, *and* $V^D(y, p)$ *and solves* (3.3).

- *Benevolent Government (Repayment)*
  If $(y, p) \notin \mathcal{D}(B)$ *and taking as given the bond price schedule* $q(B, y, p)$, *the government chooses the sovereign bonds policy rule* $\hat{B}'(B, y, p)$, *the set of household consumption policy rules* $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, *and* $\hat{C}(B, y, p)$, *and the exports policy rule* $\hat{x}(B, y, p)$ *in order to solve* (3.4) *and its solution is consistent with* $V^R(B, y, p)$.

- *Benevolent Government (Default)*
  If $(y, p) \in \mathcal{D}(B)$, *the government chooses the set of household consumption policy*

rules $\hat{c}_d(B, y, p)$, $\hat{c}_f(B, y, p)$, *and* $\hat{C}(B, y, p)$, *and the exports policy rule* $\hat{x}(B, y, p)$ *in order to solve* (3.5) *and its solution is consistent with* $V^D(y, p)$.

- *Default probability*
  The default probability schedule $\delta(B, y, p)$ *is consistent with the default set* $\mathcal{D}(B, y, p)$ *and* (3.7).

- *Bond Pricing*
  The bond pricing schedule $q(B, y, p)$ *is consistent with the probability of default schedule* $\delta(B, y, p)$ *and* (3.8).

3.4.2 Aggregate Recursive Equilibrium

The problem described in the previous section has an intratemporal condition between consumption of foreign and domestic goods by the representative household. Specifically, this intratemporal condition balances the terms of trade with the marginal rate of substitution between foreign and domestic consumption goods,

$$p = \frac{u'(C) \cdot \frac{\partial C}{\partial c_f}}{u'(C) \cdot \frac{\partial C}{\partial c_d}} = \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{c_d}{c_f}\right)^{1+\eta}. \tag{3.9}$$

Using the final consumption aggregator and (3.9), let us define the final consumption price index as

$$\mathcal{P}(p) \equiv \left(\lambda^{\frac{1}{1+\eta}} + (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)^{\frac{1+\eta}{\eta}}. \tag{3.10}$$

This price lets us weight the price of the final consumption good in the economy considering how important are domestic and foreign goods in its aggregation. Notice that the limit expression of home bias follow,

$$\lim_{\lambda \to 0} \{\mathcal{P}(p)\} = 1 \qquad \text{and} \qquad \lim_{\lambda \to 1} \{\mathcal{P}(p)\} = p.$$

On one hand, if there is complete home bias, the price index of the final consumption good is not affected at all by the terms of trade. This result is intuitive because it tells us that the representative household does not derive any utility from foreign consumption

goods. On the other hand, if there is complete foreign bias, the price index of the final consumption good is the complete terms of trade price.

Consider an aggregate version of the government's problem when it chooses to repay,

$$V^R(B,y,p) = \max_{C,B'} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[V(B',y',p') | (y,p) \right] \right\} \tag{3.11}$$

$$\text{s.t. } \mathcal{P}(p)C + pq(B',y,p) = y + pB \qquad \text{(Resource Constraint)}$$

$$B' \geq -\mathbb{B}. \qquad \text{(No-Ponzi Condition)}$$

Also, consider the aggregate version of the government's problem when it chooses to default,

$$V^D(y,p) = \max_{c_d,c_f,C} \left\{ u(C) + \beta \mathbb{E}_{(y',p')} \left[\phi V(0,y',p') + (1-\phi) V^D(y',p') | (y,p) \right] \right\} \tag{3.12}$$

$$\text{s.t. } P(p)C = h(y) \qquad \text{(Resource Constraint)}$$

Definition 4 (Aggregate Recursive Equilibrium)**.** *The aggregate recursive equilibrium of this small open economy will be a set of government value functions $V(B,y,p)$, $V^R(B,y,p)$, $V^D(y,p)$ and a sovereign bonds policy rule $\hat{B}'(B,y,p)$, a household final consumption policy rule $\hat{C}(B,y,p)$, a final consumption good price index $\mathcal{P}(p)$, a default set $\mathcal{D}(B,y,p)$, a default probability schedule $\delta(B,y,p)$, and bond price schedule $q(B,y,p)$ such that the following conditions are satisfied:*

- *Benevolent Government (Initial)*
  The default set $\mathcal{D}(B,y,p)$ is consistent with the set of government value functions $V(B,y,p)$, $V^R(B,y,p)$, and $V^D(y,p)$ and solves (3.3)*.*

- *Benevolent Government (Repayment)*
  If $(y,p) \notin \mathcal{D}(B)$ and taking as given the bond price schedule $q(B,y,p)$ and the final consumption good price index $\mathcal{P}(p)$, the government chooses the sovereign bonds policy rule $\hat{B}'(B,y,p)$ and the household final consumption policy rule $\hat{C}(B,y,p)$ in order to solve (3.4) *and its solution is consistent with $V^R(B,y,p)$.*

- *Benevolent Government (Default)*
  If $(y,p) \in \mathcal{D}(B)$ and taking as given the final consumption good price index $\mathcal{P}(p)$,

the government chooses household final consumption policy rules $\hat{C}(B, y, p)$ *in order to solve* (3.5) *and its solution is consistent with* $V^D(y, p)$.

- *Price Index*
  The final consumption good price index follows (3.10).

- *Default probability*
  The default probability schedule $\delta(B, y, p)$ *is consistent with the default set* $\mathcal{D}(B, y, p)$ *and* (3.7).

- *Bond Pricing*
  The bond pricing schedule $q(B, y, p)$ *is consistent with the probability of default schedule* $\delta(B, y, p)$ *and* (3.8).

Proposition 3.4.1 (Recursive Equilibrium Isomorphism)**.** *The equilibriums defined in Definition 3 and Definition 4 are isomorphic.*

Proof. See Appendix B.1. ■

This transformation of the original problem is very useful for solving the model and to understand how terms of trade shocks work in our environment. Firstly, realize that the final consumption good price index is non-decreasing[14] in terms of trade regardless of the parameters λ and η,

$$P'(p) = \left(\frac{(1-\lambda)P(p)}{p} \right)^{\frac{1}{1+\eta}} \geq 0.$$

This implies that, regardless of the complementarity or substitutability of the domestic and foreign goods, the final consumption good price index will keep a monotonic behavior throughout all the domain of the terms of trade. Moreover, the final consumption good price index will work as a shock absorber of the terms of trade shock. This is, the households will only experience a fraction of the terms of trade shock in terms of final good expenditure. Therefore, terms of trade will have two main effects in the model. The first effect is adjusting the price of final consumption goods. The second effect is

[14] Furthermore, it is strictly increasing as long as we assume there is no full home-bias in the model $\lambda \in [0, 1)$

expanding or contracting the debt burden of sovereign bonds. This last effect is present under the assumption that the sovereign government issues debt in foreign currency.

Proposition 3.4.2 (Default Sets Monotonicity)**.** *Pick an arbitrary level of sovereign bonds* B_1 *such that* $\mathcal{D}(B_1) \neq \emptyset$*, if* $B_2 \leq B_1$ *then* $\mathcal{D}(B_1) \subseteq \mathcal{D}(B_2)$*.*

Proof. See Appendix B.1. ■

This result is originally taken from Eaton and Gersovitz (1981), Arellano (2008), and Chatterjee, Corbae, Nakajima and Ríos-Rull (2007) and it is common in sovereign default models. This result tells us that incentives are monotonic with respect of sovereign bonds. Hence, it is mainly followed because the bond pricing $q(B', y, p)$ is independent from the sovereign bonds due in a period. In our model, this relationship does not exist because of the assumption of risk neutrality from the representative foreign lender. Thus, given a level of output and terms of trade, the sovereign bonds due in the period acts only as a shifter in the available amount of resources in the economy. Therefore, if for a level of sovereign bonds a government finds optimal to default, then for a lower level of sovereign bonds the default decision will still be optimal because the government will have less resources overall.

Using Proposition 3.4.2 we can conclude that the bond pricing $q(B, y, p)$ is non-decreasing in sovereign bonds. Let us focus in the case where the government borrows resources from foreign lenders. As the government increases the amount of borrowing, the actual amount of resources received will decrease because the bond pricing contracts. This resources reduction of borrowing compensates the default probability that the government can incur in the following period.

3.4.3 No Persistency Case

Let us study the case in which the stochastic system proposed in (3.1) has no persistency, therefore it is i.i.d. binormal distribution with mean $\overline{0}$ and variance-covariance matrix Σ. In this case, the probability of default and the sovereign bond price schedule lose their contingency with respect of GDP and terms of trade. This is because current levels of GDP and TOT do not provide any information about the future realizations of them. We assume also that there are no penalty costs, $h(y) = y$; and default is permanent, $\phi = 0$.

Proposition 3.4.3 (No Resources Inflows)**.** *For every sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, every feasible B' will yield no resources inflows, $q(B')B' - B \geq 0$.*

Proof. See Appendix B.1. ■

Default episodes happen when governments are unable to roll-over their debt. This idea is captured with Proposition 3.4.3. If the government is in a state where he chooses to default, all the feasible issuances of sovereign bonds must have not been enough to cover the sovereign bonds due in the period. Specifically, there was no feasible issuances of sovereign bonds that could have given a positive flux of resources from outside, $B - q(B')B' \not> 0$.

Proposition 3.4.4 (GDP Default Incentives)**.** *Pick an arbitrary level of terms of trade p and sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, if $y_2 \leq y_1$ and $(y_1, p) \in \mathcal{D}(B)$ then $(y_2, p) \in \mathcal{D}(B)$.*

Proof. See Appendix B.1. ■

Default episodes also happen when economies experience recessions, periods where GDP levels are below the trend. Proposition 3.4.4 is able to capture this idea. This shows that there if a government finds optimal to default on a level of sovereign bonds for a given recession, any deeper recession would make it default as well. This result is driven mainly because the country will be poorer and there are not contracts available that provide an influx of resources from abroad.

Proposition 3.4.5 (Terms of Trade Default Incentives)**.** *Pick an arbitrary level of GDP y and sovereign bonds B such that $\mathcal{D}(B) \neq \emptyset$, if $p_2 \geq p_1$ and $(y, p_1) \in \mathcal{D}(B)$ then $(y, p_2) \in \mathcal{D}(B)$.*

Proof. See Appendix B.1. ■

We have a similar result for the case of terms of trade rising higher than the trend. Proposition 3.4.5 is able to capture this idea. This shows that there if a government finds optimal to default on a level of sovereign bonds for a level of terms of trade, any higher level of terms of trade would make it default as well. This result is driven mainly because the country will experience more expensive goods from the exterior and the resources that flow outside the country will grow because of the foreign good conversion.

3.5 Quantitative Analysis

In this section, we study Mexico and its business cycles statistics. Also, we describe the calibration process of the model fitting the parameters to the Mexican economy. We solve the aggregate recursive equilibrium described in Definition 4 applying a value function iteration process using a grid search method. The complete algorithm can be found in Appendix B.2.

3.5.1 Data

Let us study the Mexican business cycle behavior as an emerging economy. Using OECD Statistics, we obtain quarterly data seasonally adjusted at quarterly levels, and at current and constant prices from 1993Q1 to 2016Q2 for the series of *gross domestic product*, *private final consumption expenditure*, *exports of goods and services*, and *imports of goods and services*. Also, using Global Financial Database, we obtain quarterly data from 1997Q4 to 2016Q3 for the series of *Emerging Markets Bond Index (EMBI+)*. The series of EMBI+ is provided by J.P. Morgan and portrays the long-term spread between yields from sovereign bond and the U.S. Treasuries. We construct the terms of trade series as the ratio of imports price deflator and exports price deflator following Kehoe and Ruhl (2008) methodology.[15] We apply the *HP-filter* with a smoothing parameter of 1600 to the real consumption, real output and terms of trade log-series in order to obtain the cyclical components of them. In addition, we compute the ratio of the difference between exports and imports, over GDP to construct the series of trade balance.

Table 3.4 shows the business cycles statistics for the Mexican economy. The table shows regular characteristics of emerging economies shown in Neumeyer and Perri (2005). The volatility of consumption is higher compared to the volatility of GDP. Another important characteristic is that trade balance is countercyclical. Nevertheless, we are not able to find that interest rates (captured by the sovereign spread series) are countercyclical after 1997Q4.[16]

[15]There are other ways to compute terms of trade. For example, e, Mendoza (1995) uses the ratio of exports and imports volumes. We choose not to use this methodology for convenience. Establishing terms of trade as the ratio of imports and exports deflator matches closely the movements of real exchange rates.

[16]We analyzed this issue further. We use the database provided by Neumeyer and Perri (2005) and

Table 3.4: Business Cycles Statistics for Mexico

Variable	μ	σ	$\rho(\cdot$,Spread)	$\rho(\cdot$,GDP)	$\rho(\cdot$,TOT)
Spread	2.71%	1.47%	-	0.0352	0.2178
GDP	-	2.22%	0.0352	-	-0.4480
Terms of Trade	-	2.56%	0.2178	-0.4480	-
Consumption	-	2.65%	-0.0367	0.9518	-0.4673
Trade Balance	-0.84%	1.62%	0.3335	-0.4819	0.1501

Terms of trade also play an important feature. As documented in Mendoza (1995), Kose (2002), and Broda (2004), there is a strong negative relationship between terms of trade and real GDP in emerging countries. Also, the volatility of terms of trade is higher compared to the volatility of GDP. An important feature we are able to provide is that the correlation between terms of trade and the spreads is significantly higher than the correlation between GDP and spreads. This seems to suggest that the movements the Mexican experienced after 1997 may be better explained by movements in terms of trade rather than movements in real GDP.

3.5.2 Calibration

We use the real GDP and terms of trade *HP-filter* log-series in order to estimate (3.1). We assume that the errors vector components are independent from each other[17] and distributed as $\varepsilon^y_{t+1} \sim N(0, \sigma^2_y)$ and $\varepsilon^y_{t+1} \sim N(0, \sigma^2_y)$. We estimate the matrix A using

construct an implied EMBI+ series from 1994Q1 to 1997Q3. In the database the authors provide the interest rate of several emerging economies from 1994Q1 to 2002Q2. We take the series of Mexican interest rates and subtract the yield of US 10-year Treasury constant maturity. We find that the constructed EMBI+ series resembles closely the original EMBI+ series in the quarters they overlap with the exception of the period 2001Q2-2002Q2. Filling the missing quarters for the spread series we are able to find that the interest rates (captured by the sovereign spread series) is countercyclical, with a correlation of -0.4275.

[17] We make this assumption for simplicity. Nevertheless, we are expecting in relaxing this. The error of the real GDP regression surely is correlated with contemporaneous terms of trade. In this way, the assumption of the errors being independent should be worked upon.

standard OLS regressions[18],

$$A = \begin{bmatrix} 0.8399 & -0.0291 \\ -0.1466 & 0.5367 \end{bmatrix}.$$

Once estimated the matrix A, we recover the observed errors and estimate the standard deviations of the errors. The standard deviations of the GDP and terms of trade errors are $\sigma_y = 0.0105$ and $\sigma_p = 0.0203$, respectively. Finally, we discretize the 2-dimensional VAR(1) process into a 289 Markov chain (17 GDP and 17 terms of trade shock levels) using a quadrature method algorithm following Tauchen and Hussey (1991) with a 3 standard deviations mean centered bandwidth.

We use a standard CRRA utility function to convey the representative household preferences,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

where σ represents the constant relative risk aversion parameter. We settle this risk aversion parameter to the value of 2. This is a common value in the international real business cycles literature.

During default episodes we impose GDP penalty via the increasing function $h(\cdot)$. An important issue to address when modeling this is its sensitivity contingent to the state of the economy. We use Arellano (2008) convex GDP cost formulation,

$$h(y) = \begin{cases} y & \text{if } y \leq \kappa \mathbb{E}[y] \\ \kappa \mathbb{E}[y] & \text{if } y > \kappa \mathbb{E}[y] \end{cases},$$

where $\kappa > 0$ is a contraction of the long-run mean of GDP. This formulation has the advantage of making default less sensitive to GDP shocks. In particular, defaulting with a GDP level below the threshold of $\kappa \mathbb{E}[y]$ there is no GDP penalty. Nevertheless, above this threshold the GDP penalty increases the higher GDP is.

[18] We make this assumption to capture a higher correlation between contemporaneous real GDP and terms of trade. Terms of trade can be considered as exogenous for small open economies as motivated in Broda (2004). Therefore, previous real GDP affecting contemporaneous terms of trade should be restricted.

We use the Global Financial Database in order to obtain the series of *US 10-year Treasury constant maturity yield*. We pick a 10-year Treasury maturity bond because the EMBI+ series relies on long-term maturity bonds. We fix the risk-free interest rate as the average yield from 1997Q1 to 2016Q3, which is 1.62%.

The literature show a wide variety of possible elasticity of substitution between domestic and foreign goods. As noted by Ruhl (2008), this elasticity can be small to account the quarterly fluctuations in trade balances and terms of trade, or high to account the growth in trade due to trade liberalization. In Kose, Towe and Meredith (2004) they propose this elasticity of substitution to be 1.05 for Mexico when analyzing the NAFTA effect on trade. We choose not to use this level due to the different time span of study we are interested. Nevertheless, we find that other updated papers for the Mexican economy have a similar elasticity of substitution that considers the trade liberalization that Mexico has experienced in the past decades. We use the elasticity of substitution between domestic and foreign goods presented in Cuadra and Nuguer (2018) to calibrate η. They use an elasticity of substitution of 1.5556, which implies a parameter η of -0.3571.

We use the intratemporal condition (3.9) in order to calibrate the home bias parameter. Realize that the intratemporal condition can be rewritten as

$$\left(\frac{c_f}{C}\right)^{1+\eta} = \frac{(1-\lambda)\mathcal{P}(p;\lambda,\eta)}{p} \tag{3.13}$$

In order to construct the series of consumption of foreign goods, we obtain from the Mexican central bank the annual share of imported consumption goods from total imports from 1997 to 2015. We find that the average share is of 13.18% during this period.[19] Using *total imports of goods and services*, *private final consumption expenditure*, and terms of trade quarterly series, the parameter value of $\eta = -0.3571$, the fixed share of imported consumption goods, and the final consumption good price index (3.10) formula; we find a series of the home bias λ_t that solves in every quarter the intratemporal condition (3.13). We find that the average home bias is of 0.8748 from 1997Q4 to 2016Q2.

[19]We also find that from 2007 to 2015 this share almost doubled from 8.5% to 14.24%. We also find that most of this increase happened in the first six years of the sample. For this reason, we consider this share as a constant for the calibration process.

We calibrate the default GDP penalty and the discount factor in order to match two moments of the Mexican economy. Mexico has defaulted in its sovereign debt twice (1928 and 1982) in the last hundred years.[20] This gives a rough estimate of a 2% default probability. We then focus the targets to be this default probability and the standard deviation of trade balance over GDP ratio shown in Table 3.4. Finally, we keep the probability of re-entry to financial markets proposed by y Arellano (2008).[21] Table 3.5 presents the parameters specification from the calibration and estimation strategy.

Table 3.5: Parameters Specification

Parameter	Value	Source
A	$\begin{bmatrix} 0.8399 & -0.0291 \\ -0.1466 & 0.5367 \end{bmatrix}$	OLS estimators
σ_y	0.0105	Observed errors
σ_p	0.0203	Observed errors
σ	2.00	IRBC Literature
r^*	1.62%	US 10-year Treasury
η	-0.3571	Cuadra and Nuguer (2016)
λ	0.8748	Intratemporal Condition
β	0.9530	Default probability
κ	0.9690	Trade balance volatility
ϕ	0.2820	Arellano (2008)

[20] As noted in Reinhart and Rogoff (2009), the default years were when two important international crises happened, the *Great Depression* and the *Emerging Markets Crises*. During the *Tequila Crisis* in 1994, Mexico was close to default with its international lenders. Thanks to the international help from the USA, Mexico was able to have dodge this.

[21] In Gelos, Sahay and Sandleris (2011), it is shown that the average waiting period for re-entry after a default has decreased significantly. They show that this average fell from 5 years in the 1980's to 1.6 years in the 1990's. Nevertheless, in Alessandro et al. (2011), they conclude that this comparison between decades is not fair. In particular, that the decrease in average waiting period for re-entry has not decreased throughout time. They provide evidence that, if re-entry to to financial markets do not happen after three years of the default, it is significantly harder to achieve this re-entry. This seems to suggest that the probability of re-entry has not experienced important movements throughout the years.

3.5.3 Results

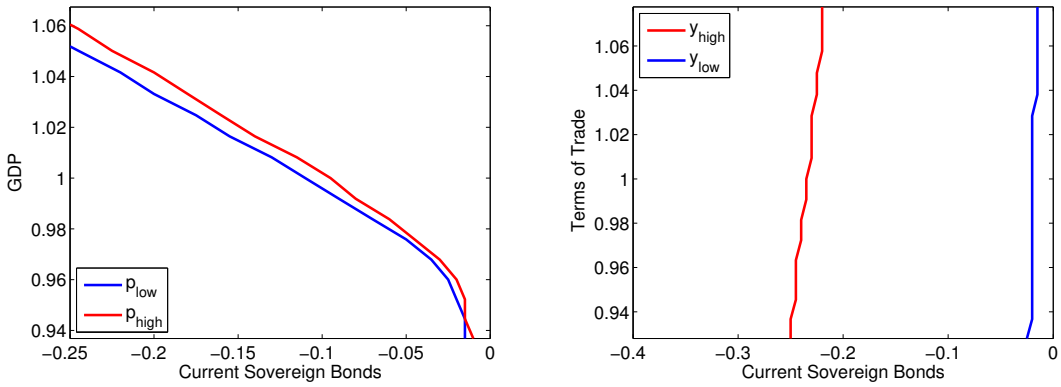
We use a value function iteration process using a grid search method[22] to solve the model. To study the policy rules we establish a high and low level of GDP and terms of trade as $\pm 0.0489\%$ and $\pm 0.0467\%$ deviations from their means, respectively. We then do a 100 simulation processes for the economy for 10,000 periods, starting with zero sovereign bond holdings and in the long-run level of GDP and terms of trade. We burn the first 500 periods of each simulation and compute the business cycles statistics of the model as the averages of the simulations.

Figure 3.3 shows the boundary limits of the default set (3.6). Because of Proposition 3.4.2 we know that these sets are monotonic with respect of the level of sovereign bond holdings. The left panel shows the relationship with respect of GDP fixing the terms of trade level. The default set is the area south west of the boundaries. We find that higher levels of sovereign debt increase the levels of GDP inside the the default set. In addition, having a higher terms of trade level increases slightly the levels of GDP inside the default set. The right panel shows the relationship with respect of terms of trade fixing the GDP level. The default set is the area north west of the boundaries. We find that higher levels of sovereign debt increase the levels of terms of trad inside the default set. In addition, having a higher GDP levles decreases significantly the levels of terms of trade inside the default set.

Figure 3.4 shows the pricing of the sovereign bonds. The left panel shows us how the sovereign bond price schedule decreases in value as the sovereign bond becomes more negative. The right panel shows the implied interest rate using the domain of sovereign bonds in which the sovereign bond pricing is strictly positive. These graphs deliver two important features. Firstly, this model is able to create countercyclical interest rates due to the link between probability of default and GDP levels. Secondly, the movements of terms of trade increase in importance for sovereign bond price schedule as GDP increases. Moreover, this rise in the price schedule dispersion increases the sovereign interest rates possibilities. This result helps us explain the movements of spreads during periods where GDP is not below trend. Furthermore, it suggests that terms of trade

[22] We use this method as a first approach to solving the equilibrium of the model. As Hatchondo, Martinez and Sapriza (2010) shows, grid search methods can give spurious results in the business cycle analysis.

Figure 3.3: Default Set Boundaries



matter when analyzing interest rates and default likelihoods of emerging economies. This is a step towards explaining the puzzle shown in Tomz and Wright (2007).

Figure 3.4: Sovereign Bond Pricing and Sovereign Interest Rate

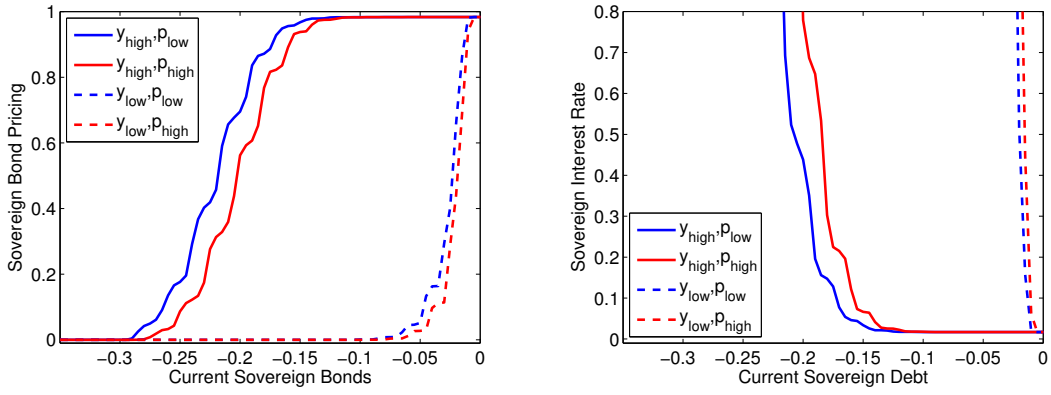


Table 3.6 shows the business cycle statistics of the model. The business cycles is able to recover a couple of the statistics shown in Table 3.4. But in general, the experiment fails because it is not able to match most of the moments in the data. Despite of this, it is an important step towards understanding the flaws and improving the model.

Table 3.6: Model Business Cycles Statistics

Variable	μ	σ	$\rho(\cdot$,Spread)	$\rho(\cdot$,GDP)	$\rho(\cdot$,TOT)
Spread	0.65%	0.74%	-	-0.2009	0.2567
GDP	-	2.24%	-0.2009	-	-0.2388
Terms of Trade	-	2.52%	0.2567	-0.2388	-
Consumption	-	2.01%	-0.3073	0.9386	-0.2376
Trade Balance	0.00%	0.09%	0.3788	-0.2628	-0.0385
Default probability	2.46%	-	-	-	-
Debt-GDP ratio	3.05%	-	-	-	-

3.6 Conclusion

We propose a stochastic general equilibrium model of sovereign default with endogenous default risk in order to explain the interest rate behavior in emerging economies. We incorporate two types of shocks to cover a foreign and a domestic uncertainty. We define as the domestic and the foreign uncertainty, GDP and terms of trade shock, respectively. The model is able to successfully increase the dispersion of sovereign interest rates when GDP shocks are above the trend. This result seems to suggest that terms of trade is a good candidate to explain the volatility of interest rates in small open economies when they are not under recessions or crises.

Unfortunately, our business cycles exercise has room for improvements. Nevertheless, the results presented is a great step to explain the behavior of interest rates in emerging economies. Below, we present three issues we are currently working in order to improve our line of research.

Firstly, the VAR(1) process does not capture correctly the dynamics between GDP and terms of trade shown in the data. This can be shown by the small correlation between their contemporaneous realizations in the simulation process. In particular, we are confident that $\mathbb{E}\left[\varepsilon_t^y | \ln p_t\right] \neq 0$, making our estimators in A biased. Moreover, we are not implementing completely the exogeneity assumption of terms of trade. Specifically, we let the future realization of terms of trade be affected by the current level of GDP. Furhter work of the model will consider improvements in the VAR(1) process presented

in (3.1), for example

$$\begin{bmatrix} \ln y_{t+1} \\ \ln p_{t+1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ 0 & a_3 \end{bmatrix} \begin{bmatrix} \ln y_t \\ \ln p_t \end{bmatrix} + \begin{bmatrix} 1 & b_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^y_{t-1} \\ \varepsilon^p_{t=1} \end{bmatrix}.$$

Another weakness of our results is our calibration strategy. We let the tolerance in the algorithm to be high for a faster convergence. The tolerance provided is of $1.0e^{-}1$, which yields really loose results. Moreover, we only do one simulation process in order to compute the target statistics of default probability and standard deviation of trade balance over GDP. Future improvements will encompass a more serious calibration process with a higher tolerance and number of simulation processes. Moreover, we will include as part of it the probability of re-entry to financial markets by targeting the ratio of debt over GDP.

Finally, our computation of the equilibrium can improve greatly. Unfortunately, having three state variables increases greatly the computational cost in terms of time. Because of this we have coarse grids that might miss important movements. Furthermore, as noted by y Hatchondo, Martinez and Sapriza (2010), grid search methods can yield spurious results in the business cycles statistics of the model. Future improvements will work on this by imposing finer grids in the state variables. In addition, we will move from the grid search method to methods that are able to capture movements between the grid elements.[23]

[23]There are a great variety of methods that allow movements inside the grid elements. In particular, we are currently working on linear and quadratic interpolation methods. We are also interested in implementing innovative methods that have shown efficiency in solving these types of models. Specifically, we are interested in the algorithms provided in McGrattan (1996) and Gordon and Qiu (2018).

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Appendix A

Appendix to Chapter 2

A.1 Data

All data is annual data from 1980 to 2016. The data availability varies by countries so countries with missing data on their exports, real GDP are dropped from the sample. Consequently, the final sample consists of 50 countries. All data is obtained from the World Development Indicators (WDI).

- Exports : The dollar value of exporting goods and services.
- Growth rate of GDP: Growth rate is calculated with Growth rate of real GDP per capita. The real GDP is expressed in local currency.
- Foreign reserves: Gross foreign exchange reserves minus gold.

A.1.1 Export-orientation

The following table (A.1) and table (A.2) list export-oriented countries and non export-oriented countries in the sample. It presents the growth of exports to GDP ratio.

Table A.1: countries with a export growth policy

Country	$g\left(\frac{Exports}{GDP}\right)$	$\left(\frac{Exports}{GDP}\right)_{1980}$	$\left(\frac{Exports}{GDP}\right)_{2016}$
Argentina	0.99	0.09	0.18
Bhutan	1.34	0.18	0.42
Bolivia	0.73	0.23	0.40
Burkina	1.11	0.10	0.20
China	1.71	0.10	0.27
Ecuador	0.66	0.17	0.28
Guatemala	0.52	0.16	0.24
Hong Kong	0.95	1.02	2.00
India	2.74	0.06	0.22
Israel	1.31	0.11	0.25
Korea	0.65	0.29	0.48
Mexico	0.87	0.16	0.31
Mongolia	0.85	0.27	0.50
Sudan	0.98	0.07	0.15
Syria	1.48	0.16	0.39
Thailand	1.65	0.26	0.69
Turkey	0.68	0.13	0.22
Uganda	0.62	0.12	0.19
Vietnam	6.70	0.10	0.78

Table A.2: countries without a export growth policy

Country	$g\left(\frac{Exports}{GDP}\right)$	$\left(\frac{Exports}{GDP}\right)_{1980}$	$\left(\frac{Exports}{GDP}\right)_{2016}$
Bahamas	-0.42	0.64	0.37
Belize	0.06	0.55	0.59
Brazil	0.20	0.10	0.12
Burundi	-0.25	0.10	0.08
Colombia	0.15	0.14	0.17
Costa Rica	0.11	0.31	0.35
Dominican Republic	0.02	0.24	0.25
Egypt	-0.05	0.22	0.21
El Salvador	0.22	0.22	0.27
Indonesia	-0.01	0.26	0.25
Liberia	-0.43	0.62	0.35
Nepal	-0.02	0.11	0.11
Pakistan	0.03	0.12	0.12
Senegal	-0.02	0.27	0.27
South Africa	0.09	0.28	0.31
Uruguay	0.20	0.21	0.26
Vanuatu	0.04	0.42	0.43
Venezula	0.10	0.24	0.27

A.2 Benevolent government allocation

In this appendix, the benevolent government allocations are characterized. The benevolent government chooses $\{c_t^D, c_t^F, c_t, x_t, A_{t+1}\}_{t\geq 0}$ to solve the benevolent government problem (2.16) The first order conditions of the government's problem are

$$\beta^t u_1(c_t^D, c_t^F) = \lambda_t \tag{A.1}$$

$$\beta^t u_2(c_t^D, c_t^F) = \mu_t \tag{A.2}$$

$$\beta R \left(\frac{p_{t+1}}{p_t}\right) \mu_{t+1} = \mu_t \tag{A.3}$$

$$\frac{\mu_t}{\lambda_t} = p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{A.4}$$

λ_t and μ_t are the Lagrange multiplier associated with (2.12) and (2.14) respectively. Substituting (A.1) and (A.2) into (A.4) and using the functional forms for the CRRA utility and the CES aggregate consumption, I can obtain

$$\frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} = p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{A.5}$$

$$= p_t - p_t \left[\alpha\zeta(\gamma_t)^{\zeta_\gamma}(x_t)^{1-\zeta_x}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{A.6}$$

$$= p_t - p_t \left[(p_t)^{-\phi(1-\zeta_x)}\alpha\zeta(\gamma_t)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)\right] \tag{A.7}$$

$$= p_t \left(1 - p_t^{-\phi(1-\zeta_x)}\Psi_t\right) \tag{A.8}$$

where $\Psi_t = \alpha\zeta(\gamma_t)^{\zeta_\gamma}\left(\frac{u_1(c_{t+1}^D, c_{t+1}^F)}{u_1(c_t^D, c_t^F)}\frac{Y_{t+1}}{\gamma_{t+1}}\right)$ The (A.8) is equivalent to the equation (2.17) and the management of foreign reserves are implied by this expression.

A.3 Competitive Equilibrium allocation for the economy with private savings

This appendix provides the characterization of the competitive equilibrium for the economy with private savings. The consumer chooses $\{c_t^D, c_t^F, c_t, s_{t+1}\}_{t\geq 0}$ and the first order conditions are

$$p_t = \frac{u_2(c_t^D, c_t^F)}{u_1(c_t^D, c_t^F)} \tag{A.9}$$

$$\beta R \left(\frac{p_{t+1}}{p_t}\right) \lambda_{t+1}^s = \lambda_t^s \tag{A.10}$$

$$\tag{A.11}$$

where λ_t^s is the Lagrange multiplier associated with (2.20). The equation (A.9) equates the terms of trade and marginal rate of substitution and the equation (A.10) is the Euler equation. The definition of competitive equilibrium for this economy is shown below.

Definition 5. ***Competitive Equilibrium***
The competitive equilibrium for the small open economy with private saving is defined by a set of the representative consumer's allocations $\{c_t^D, c_t^F, s_{t+1}\}_{t\geq 0}$, the representative firm's allocations $\{Y_t, L_t, q_t^d, x_t\}_{t\geq 0}$, the terms of trade and the wages $\{p_t, w_t\}$, and the stock of knowledge $\{\gamma_{t+1}\}_{t\geq 0}$, given initial savings, initial knowledge stock, foreign output, and productivity, $\{s_0\gamma_0, Y^\star, z\}$ such that the following conditions are satisfied:

1. *Consumer: Given $\{p_t, w_t, \pi_t\}_{t\geq 0}$, the consumer's allocations $\{c_t^D, c_t^F, s_{t+1}\}_{t\geq 0}$ stratifies optimality condition (A.9).*

2. *Firm: Given $\{p_t, \gamma_t, Y^\star, z\}_{t\geq 0}$, the firm chooses $\{Y_t, L_t, q_t^d, x_t, \pi_t\}_{t\geq 0}$ satisfying (2.4), (2.5), (2.6), and (2.11).*

3. *The market for tradable goods clears as (2.12) and trade balance satisfies (2.25).*

4. *The knowledge stock evolves according to (2.7).*

Appendix B

Appendix to Chapter 3

B.1 Proposition Proof

B.1.1 Proposition 3.4.1 (Recursive Equilibrium Isomorphism)

Proof. Let us use this intratemporal condition (3.9) and the household's final consumption aggregator,

$$\begin{aligned}
C &= \left(\lambda c_d^{-\eta} + (1-\lambda)c_f^{-\eta}\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{c_d}{c_f}\right)^{\eta}\right)\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right)\left(\left(\frac{\lambda}{1-\lambda}\right)p\right)^{\frac{\eta}{1+\eta}}\right)\right)^{-\frac{1}{\eta}} \\
&= c_d \left(\lambda \left(1 + \left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)\right)^{-\frac{1}{\eta}} \\
&= \left(\frac{c_d}{\lambda^{\frac{1}{1+\eta}}}\right)\left(\lambda^{\frac{1}{1+\eta}} + (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}\right)^{-\frac{1}{\eta}}.
\end{aligned}$$

Using the final consumption good price index (3.10), we can rewrite the previous expression as

$$\frac{c_d^{1+\eta}}{\lambda} = \mathcal{P}(p)C^{1+\eta}. \tag{B.1}$$

Using the intratemporal condition (3.9) and mixing the budget and the balanced trade constraints of (3.4) and (3.5), we can realize that the consumption of domestic and foreign expenditure can be expressed as

$$\begin{aligned} c_d + pc_f &= c_d + \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{c_d}{c_f}\right)^{1+\eta} c_f \\ &= c_d + \left(\frac{1-\lambda}{\lambda}\right) c_d^{1+\eta} c_f^{-\eta} \\ &= \left(\frac{c_d^{1+\eta}}{\lambda}\right)\left(\lambda c_d^{-\eta} + (1-\lambda)c_f^{-\eta}\right) \\ &= \frac{c_d^{1+\eta}}{\lambda C^{\eta}}. \end{aligned}$$

Thus, using (B.1) we reach the expression that the household's consumption expenditure can be expressed in terms of the final consumption good and the final consumption good price index (3.10),

$$c_d + pc_f = \mathcal{P}(p)C.$$

Furthermore, using (3.9) and (B.1), we can construct how the consumption of domestic and foreign goods can be decomposed from the aggregate final consumption good as

$$c_d = (\lambda \mathcal{P}(p))^{\frac{1}{1+\eta}} C \qquad \text{and} \qquad c_f = \left(\frac{(1-\lambda)\mathcal{P}(p)}{p}\right)^{\frac{1}{1+\eta}} C.$$

Finally, realize that the restrictions of the maximization problems (3.4) and (3.5) are the same as the ones described in (3.11) and (3.12). ■ ■

B.1.2 Proposition 3.4.2 (Default Sets Monotonicity)

Proof. Set a level of sovereign debt B_1 and levels of output and terms of trade such that $(y, p) \in \mathcal{D}(B_1)$. Then, it follows that $V^D(y, p) > V^R(B_1, y, p)$. Pick an arbitrary level of sovereign debt B_2 such that $B_2 \leq B_1$. Let us study the resource constraint in (3.11). Define the budget set of the government contingent to the amount of sovereign bonds

due as

$$\mathcal{B}(B) = \left\{(C, B') \in \mathbb{R}_+ \times [\mathbb{B}, \infty) : \quad P(p)C + pq(B', y, p) \leq y + pB\right\}.$$

By construction, acknowledge that $\mathcal{B}(B_2) \subseteq \mathcal{B}(B_1)$. Because the government is maximizing over a subset of a set, it follows that $V^R(B_1, y, p) \geq V^R(B_2, y, p)$. Joining the inequalities, we conclude that

$$V^D(y, p) > V^R(B_1, y, p) \geq V^R(B_2, y, p).$$

In other words, $(y, p) \in \mathcal{D}(B_2)$. Finally, because the levels of sovereign debt B_2 was taken arbitrarily, we can conclude that $\mathcal{D}(B_1) \subseteq \mathcal{D}(B_2)$. ■ ■

B.1.3 Proposition 3.4.3 (No Resources Inflows)

Proof. Pick an arbitrary $(y, p) \in \mathcal{D}(B)$ and realize this implies $V^D(y, p) > V^R(B, y, p)$. Acknowledge that the resource constraints found in (3.11) and in (3.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

Therefore,

$$\begin{aligned}
&u\left(\frac{y}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y', p')\right] \\
&\qquad > \max_{B'}\left\{u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y', p')\right]\right\} \\
&\qquad \geq \max_{B'}\left\{u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y', p')\right]\right\} \\
&\qquad \geq u\left(\frac{y - p(q(B')B' - B)}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V^D(y', p')\right],
\end{aligned}$$

for all feasible B'. Thus,

$$u\left(\frac{y}{\mathcal{P}(p)}\right) > u\left(\frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right)\right).$$

Because $u(\cdot)$ is an increasing function,

$$\frac{y}{\mathcal{P}(p)} \geq \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B')B' - B\right).$$

Then, we arrive to

$$q(B')B' - B \geq 0.$$

Finally, because we picked an arbitrary $(y, p) \in \mathcal{D}(B)$, we can conclude that for all feasible B' there are no resources inflows, $q(B')B' - B \geq 0$. ■ ■

B.1.4 Proposition 3.4.4 (GDP Default Incentives)

Proof. Set a level of GDP, terms of trade and sovereign bonds such that $(y_1, p) \in \mathcal{D}(B) \neq \emptyset$. Then, it follows that $V^D(y_1, p) > V^R(B, y_1, p)$. Pick an arbitrary level of GDP y_2 such that $y_2 \leq y_1$. Acknowledge that the resource constraints found in (3.11) and in (3.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

To make the proof easier, call

$$B_1' = \arg\max_{B'} \left\{ u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')} \left[V(B_1', y', p')\right] \right\} \qquad \text{and}$$
$$B_2' = \arg\max_{B'} \left\{ u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')} \left[V(B_2', y', p')\right] \right\}.$$

Realize that, in particular, these expressions imply,

$$u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')} \left[V(B_1', y', p')\right] \geq$$
$$u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)} \left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')} \left[V(B_2', y', p')\right].$$

Using Proposition 3.4.3 and because $\mathcal{D}(B) \neq \emptyset$ and $u(\cdot)$ is increasing, it follows that $q(B')B' - B \geq 0$ for every feasible B'. In particular, for the optimal level of sovereign bonds using y_2 level of GDP, $q(B_2')B_2' - B \geq 0$.

Because $u(\cdot)$ is strictly concave and $q(B_2')B_2' - B \geq 0$, we know that

$$u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) \geq u\left(\frac{y_1}{\mathcal{P}(p)}\right) - u\left(\frac{y_1}{\mathcal{P}(p)}\right).$$

Moreover, the right-hand side can be expressed as

$$\begin{aligned} & u\left(\frac{y_1}{\mathcal{P}(p)}\right) - u\left(\frac{y_2}{\mathcal{P}(p)}\right) \\ &= \left(u\left(\frac{y_1}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) - \left(u\left(\frac{y_2}{\mathcal{P}(p)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) \\ &= V^D(y_1,p) - V^D(y_2,p). \end{aligned}$$

Therefore,

$$\begin{aligned} V^D(y_1,p) - V^D(y_2,p) & \\ &\leq u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) \\ &= \left(u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\ &\quad - \left(u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\ &\leq \left(u\left(\frac{y_1}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_1')B_1' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_1',y',p')\right]\right) \\ &\quad - \left(u\left(\frac{y_2}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\ &= V^R(B,y_1,p) - V^R(B,y_2,p) < V^D(y_1,p) - V^R(B,y_2,p). \end{aligned}$$

Then we arrive to following expression $V^D(y_2,p) > V^R(B,y_2,p)$. Finally we can conclude, because the level of GDP y_2 was taken arbitrarily, we can conclude that $(y_2,p) \in \mathcal{D}(B)$. ■ ■

B.1.5 Proposition 3.4.5 (Terms of Trade Default Incentives)

Proof. Set a level of GDP, terms of trade and sovereign bonds such that $(y, p_1) \in \mathcal{D}(B) \neq \emptyset$. Then, it follows that $V^D(y, p_1) > V^R(B, y, p_1)$. Pick an arbitrary level of terms of trade p_2 such that $p_2 \geq p_1$. Acknowledge that the resource constraints found in (3.11) and in (3.12) can be rewritten respectively as

$$C = \frac{y}{\mathcal{P}(p)} - \frac{p}{\mathcal{P}(p)}\left(q(B')B' - B\right) \qquad \text{and} \qquad C = \frac{y}{\mathcal{P}(p)}.$$

To make the proof easier, call

$$B_1' = \arg\max_{B'} \left\{ u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_1', y', p')\right] \right\} \qquad \text{and}$$
$$B_2' = \arg\max_{B'} \left\{ u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_2', y', p')\right] \right\}.$$

Realize that, in particular, these expressions imply,

$$u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_1')B_1' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_1', y', p')\right] \geq$$
$$u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) + \beta \mathbb{E}_{(y',p')}\left[V(B_2', y', p')\right].$$

Using Proposition 3.4.3 and because $\mathcal{D}(B) \neq \emptyset$ and $u(\cdot)$ is increasing, it follows that $q(B')B' - B \geq 0$ for every feasible B'. In particular, for the optimal level of sovereign bonds using y_2 level of GDP, $q(B_2')B_2' - B \geq 0$.

Acknowledge that the final consumption good price index is increasing,

$$\mathcal{P}'(p) = \left(\frac{(1-\lambda)\mathcal{P}(p)}{p}\right)^{\frac{1}{1+\eta}} >\geq 0.$$

Therefore, the ratio $\frac{p}{\mathcal{P}(p)}$ is increasing too,

$$\begin{aligned}
\frac{d}{dp}\left(\frac{p}{\mathcal{P}(p)}\right) &= \frac{\mathcal{P}(p) - p\mathcal{P}'(p)}{\left(\mathcal{P}(p)\right)^2} \\
&= \frac{1-(1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}} \left(\mathcal{P}(p)\right)^{-\frac{\eta}{1+\eta}}}{\mathcal{P}(p)} \\
&= \frac{\left(\mathcal{P}(p)\right)^{\frac{\eta}{1+\eta}} - (1-\lambda)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}}{\left(\mathcal{P}(p)\right)^{\frac{1+2\eta}{1+\eta}}} \\
&= \left(\frac{\lambda}{\left(\mathcal{P}(p)\right)^{1+2\eta}}\right)^{\frac{1}{1+\eta}} \\
&\geq 0.
\end{aligned}$$

In other words, it follows that $\frac{p_2}{\mathcal{P}(p_2)} \geq \frac{p_1}{\mathcal{P}(p_1)}$ because $p_2 \geq p_1$.

Using the previous result and because $u(\cdot)$ is strictly concave and $q(B_2')B_2' - B \geq 0$, we know that

$$\begin{aligned}
&u\left(\frac{y}{\mathcal{P}(p_1)}\right) - u\left(\frac{y}{\mathcal{P}(p_2)}\right) \\
&\qquad \leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) \\
&\qquad \leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right)
\end{aligned}$$

Moreover, the left-hand side can be expressed as

$$\begin{aligned}
&u\left(\frac{y}{\mathcal{P}(p_1)}\right) - u\left(\frac{y}{\mathcal{P}(p_2)}\right) \\
&= \left(u\left(\frac{y}{\mathcal{P}(p_1)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) - \left(u\left(\frac{y}{\mathcal{P}(p_2)}\right) + \beta\mathbb{E}_{(y',p')}\left[V(y',p')\right]\right) \\
&= V^D(y,p_1) - V^D(y,p_2).
\end{aligned}$$

Therefore,

$$
\begin{aligned}
&V^D(y,p_1) - V^D(y,p_2) \\
&\qquad \leq u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) - u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) \\
&\qquad = \left(u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\qquad\quad - \left(u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p_2)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\qquad \leq \left(u\left(\frac{y}{\mathcal{P}(p_1)} - \frac{p_1}{\mathcal{P}(p_1)}\left(q(B_1')B_1' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_1',y',p')\right]\right) \\
&\qquad\quad - \left(u\left(\frac{y}{\mathcal{P}(p_2)} - \frac{p_2}{\mathcal{P}(p)}\left(q(B_2')B_2' - B\right)\right) + \beta\mathbb{E}_{(y',p')}\left[V(B_2',y',p')\right]\right) \\
&\qquad = V^R(B,y,p_1) - V^R(B,y,p_2) \\
&\qquad < V^D(y,p_1) - V^R(B,y,p_2).
\end{aligned}
$$

Then we arrive to following expression $V^D(y,p_2) > V^R(B,y,p_2)$. Finally we can conclude, because the level of terms of trade p_2 was taken arbitrarily, we can conclude that $(y,p_2) \in \mathcal{D}(B)$. ■ ■

B.2 Computational Algorithm

We extend the algorithm described in Arellano (2008) and incorporate the one loop enhancement proposed in Hatchondo, Martinez, and Sapriza (2010) using a grid search method. The following is the algorithm we follow to solve the model proposed in Definition 4 and the calibration strategy for the discount factor β and the GDP default penalty parameter κ:

1) Fix the calibration targets of default probability $\bar{d}$ and standard deviation of trade balance over GDP $\bar{s}$.

2) Discretize B space and discretize (y,p) space using Tauchen and Hussey (1991).

3) Propose a guess for the discount factor β and the GDP default penalty parameter

κ,

$$\beta = 0.95 \qquad \text{and} \kappa = 1.00$$

4) Propose a guess for the set of value functions V, V^R, V^D and the sovereign bond price schedule q,

$$V = [0], \qquad V^R = [0], \qquad V^D = [0], \qquad \text{and} \qquad q = \left[\frac{1}{1+r^*}\right].$$

5) For every state (B, y, p), solve the repayment state maximization problem (3.11) and compute an implied repayment state value function $\hat{V}^R$.

6) For every state (y, p), solve the repayment state maximization problem (3.12) and compute an implied default state value function $\hat{V}^D$.

7) For every state (B, y, p), solve the maximization problem (3.3) and compute an implied default state value function $\hat{V}$.

8) Using the implied value functions $\hat{V}^R$ and $\hat{V}^D$, construct default set $\mathcal{D}$

9) Using the default set $\mathcal{D}$, construct default probability schedule δ

10) Using the default probability schedule δ, construct an implied sovereign bond price schedule $\hat{q}$

11) Compute the error term as

$$z_1 = ||V - \hat{V}||_\infty + ||V^R - \hat{V}^R||_\infty + ||V^D - \hat{V}^D||_\infty + ||q - \hat{q}||_\infty.$$

12) Update guesses $V = \hat{V}$, $V^R = \hat{V}^R$, $V^D = \hat{V}^D$, and $q = \hat{q}$.

13) If $z_1 \geq 1.0e^{-6}$, return to 5).

14) Construct one simulation of 10,000 periods with 500 burn-ins starting with the zero sovereign bonds and the long-run levels of GDP and terms of trade

15) Construct the implied default probability α and the implied standard deviation of trade balance over GDP s

16) Compute the error term as

$$z_2 = \frac{\left|d - \bar{d}\right| + |s - \bar{s}|}{2}.$$

17) If $z_2 \geq 1.0e^{-1}$, update guesses β and κ, then return to 4)