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Dedication

To my beloved wife Gabriela, and my children Santiago, Benjamín, and Ignacio.

Abstract

The three chapters of this dissertation study topics on Public Economics. In the first chapter I analyze the feasibility of Universal Basic Income based on the Mirrlees model. In Chapter 2, together with my co-author, we study the role that demographics played in the great recession. Finally, in Chapter 3, I analyze the financing of public universities in Paraguay.

In Chapter 1, I analyze the convenience of a cash transfer program, or universal basic income (UBI), combined with a flat or linear marginal tax rate on income. Proponents of a policy of cash transfers argue that if combined with a simplification of welfare programs and the tax system, it could generate enough benefits through a reduction of administrative costs and a reduction of distortions, especially in the labor market. This idea is not new in economics, and numerical results in models as in Mirrlees (1971) have lump-sum components that can be interpreted as UBI. Also, Mirrlees has noted that the optimal marginal non-linear tax rate is close to a linear tax system. I start by double-checking Mirrlees' calculations and determining that those numbers hold for a wide range of parameters. For a utilitarian planner, if a simplification of the tax system results in a value of 0.18% - 0.5% of GDP, a linear tax system is desirable. Second, I analyze the effects of introducing uncertainty in the optimal non-linear tax system. I introduce uncertainty in the preference of the agent. I show that heterogeneity could be a factor in making the optimal non-linear tax system closer to a linear tax system. In the extreme case where heterogeneity increases to its maximum possible value, the optimal tax system tends toward a linear tax system.

In Chapter 2, jointly with Fausto Patiño Peña, we study the role of demographics in the Great Recession. Since the Great Recession, output and labor diverted from their pre-crisis long term trends. We show that demographics is

able to explain a significant portion of the gap between the long-term trend and the data, for both output and labor. An important reason why demographics play an important role during the crisis's recovery period is that the Great Recession coincides with the "baby boomers" entering the age cohorts associated with lower levels of labor force participation. Accounting for these demographic changes, we document that labor is converging to a different employment trend. Furthermore, we modify the standard growth model and calibrate it to capture the demographic features of the data for the period 1990 - 2015. Our results show that by 2015 the output and labor gap have been reduced by just 2.5% and 1.2% respectively.

In Chapter 3, I study the effects of tuition-free public universities in Paraguay. The funding of tuition-free public universities is highly regressive in Paraguay. Most of the students come from families that belong to the top of the income distribution. In this paper, I analyze the current free tuition system and study the effects of an alternative financing method. In the alternative system, the student does not pay anything while he is in school and returns the cost of his education in the future if he earns more than a certain amount. I estimate that this change in funding would reduce the cost for the government of financing tertiary education by 75 percent, an amount that would be covered by graduates who achieve high incomes in the future.

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1 Tax Simplicity and Universal Basic Income

1.1 Introduction

Is giving cash directly to people a radical idea? Although this idea is not new[1], he growing interest in universal basic income recently[2] has created a debate that should be informed by economic theory. Among other arguments for UBI[3], I focus on the argument that UBI generates benefits by simplifying the tax system and welfare programs. This argument states that a lump-sum transfer combined with a constant marginal tax rate on income [4] is preferable to a tax system that includes several tax rate brackets and welfare programs for those at the bottom of the distribution. .

To answer the question above, I'll use the model developed by Mirrlees (1971). In this model the tax system is generally described as a lump-sum transfer to the individual (as a UBI), and a non-linear tax rate on income. In that paper, Mirrlees concludes that *"an approximately linear income-tax schedule, with all the administrative advantages it would bring, is desirable"*. Other papers like Farhi and Werning (2013) and Heathcote, Tsujiyama (2015) find that a linear tax is close the optimal non-linear one. The first thing I do in this paper is to try to replicate the findings of these studies for several different parameters, making sure that the results are not specific to those authors' unique calibrations. I find that the closeness is robust for several parameters for a utilitarian planner.

Next, I show that adding heterogeneity to the preferences can also generate a flatter non-linear marginal tax rate. I follow Lockwood and Weinzierl (2015), where

[1] Friedman (1962) proposed something similar to UBI, a Negative Income Tax.

[2] Universal Basic Income can be defined as a program or policy that transfers income (normally in the form of cash) to all citizens or residents of a country or economy, regardless of their income or wealth.

[3] More notably in the last years: automation. But others include the possibility of reducing poverty and inequality, changing the welfare system that creates distortions in the labor market, simplifying the tax system, etc.

[4] The tax system is an affine system $T(y) = b + \tau y$, where b is the lump-sum component that can be positive (a lump-sum tax) or negative (a lump-sum transfer). For simplicity, I'll call this tax system a linear tax system.

they use a quasi-linear utility function, as in Diamond (1998), to simplify the Mirrlees model that allows for a simple characterization of the solution. With this model, I show that as the agents become more heterogeneous, the marginal tax rate becomes flatter.

Adding heterogeneity to the preferences make the problem harder to solve, since this adds more dimensions to the problem. As Judd, Ma, Saunders and Su (2006) argue, the multidimensional case doesn't have a principle like single-crossing that allows us to use the first order approach. Also, numerical solutions become much harder to calculate since constraints are not concave. Therefore simplifications are necessary to characterize the solution.

Related literature. The literature related to the Mirrlees model is long and a good summary can be found in Mankiw, Weinzeirl, Yagan (2009). Among the papers that try to answer a similar question than this paper there is Alari (2016), that compares universal transfers versus means tested in a partial equilibrium model, and finds that means tested are preferred. Saez (2002) shows that a negative income tax is optimal when behavioral responses are concentrated along the intensive margin. The paper by Damon, Marinescu (2017) analyzes the effects of the transfers of the Alaska Permanent Fund on the labor market. They do not find a negative effect on employment, and they do find an increase in part-time work. Marinescu (2018) analyzes the behavioral effects of cash transfer experiments realized in the past on labor, education, consumption, health and others. In general these experiments showed positive outcomes on the variables analyzed.

This paper is divided as follows. First, I solve numerically the standard Mirrlees model for different parameters and check if the linear system is “close” to the optimal non-linear marginal tax rate. Second, I describe the model with heterogeneity in preferences and show that heterogeneity can generate a flatter non-linear tax rate (closer to a constant marginal tax rate). Then I conclude.

1.2 Tax simplicity

In this section I'll confirm the conclusion found in Mirrlees (1971), that a linear tax system is close to the optimal non-linear tax system. The model is a standard Mirrlees model, which is described here briefly.

There is a continuum of individuals, each having the same preference: $u(c,l) = U(c) - V(l)$ defined over consumption x and labor l. Workers differ only in their productivity w, with density $f(w)$. A planner cannot observe productivity w, but can observe income $y = wl$. The agent maximizes its utility function $u(c,l)$ subject to the budget constraint:

$$c = y - T(y)$$

where $T(y)$ is the tax paid.

Government maximizes a social welfare function

$$W = \int_0^\infty G(u)f(w)dw$$

subject to the incentive compatibility constraints, and a resource constraint:

$$\int_0^\infty c(w)f(w)dw \leq \int_0^\infty y(w)f(w)dw - E$$

where E is the government expenditure (excluding transfers).

The parameters used are:

Table 1: Parameters of the model

Variable	Values	Source
Utility functions	Type 1: $u = log\left(c - \frac{l^{1+k}}{1+k}\right)$ Type 2: $log(c) - log\left(1 + \frac{l^{1+k}}{1+k}\right)$	Saez (2001)
Comp. elasticity$(1/k)$	$\{0.25,\ 0.5\}$	Saez (2001)
G (excluding transfers)	$\{0.1,\ 0.2,\ 0.3\}$	U.S. (2007) was 20.3%
People with ability $= 0$	5%	Social Security
Distribution $F(w)$	Log-normal up to 42$ per hour. Pareto (2) after that	CPS 2007, and Saez (2001)
Social welfare function	$G(u) = u,$	

To analyze the effects of moving from the full non-linear system to the linear system, I'll use two measures:

1. Welfare loss (%W), where $W(u(c(1-w\%), l); T, E) = W(u(\widetilde{c}, \widetilde{l}); \widetilde{T}, E)$. That is, moving to a linear system is equivalent to losing %W of consumption for every agent.

2. Saving needed (%S), where $W(u(c, l); T, E) = W(u(\widetilde{c}, \widetilde{l}); \widetilde{T}, E(1 - \%S))$.

The first is the standard measure of welfare loss from not doing the optimal policy. The second is the one that is more suitable for the goal of this exercise. The second measure finds the savings in the economy (through simplification of the tax system, and welfare programs) that are necessary to make the planner indifferent between the non-linear and the linear tax systems.

Table 2: Welfare loss and saving needed

Government spending (%GDP)	0.1		0.2		0.3	
	Compensated Elasticity					
	0.25	0.50	0.25	0.50	0.25	0.50
Type1 Utility						
Welfare loss	0.31%	0.37%	0.29%	0.43%	0.25%	0.42%
Saving needed (% GDP)	0.27%	0.33%	0.23%	0.34%	0.18%	0.34%
Type 2 Utility						
Welfare loss	0.50%	0.82%	0.50%	0.85%	0.51%	0.85%
Saving needed (% GDP)	0.38%	0.56%	0.49%	0.49%	0.52%	0.52%

Table 1 shows how close the two measures are for the linear tax compared to the non-linear one. In terms of welfare, using a linear tax system implies a reduction of equal to or less than 0.85% on average consumption. It should be noted that this comparison is with the optimal non-linear tax system, and not with the actual tax system. In Heathcote, Tsujiyama (2015)for example, they find that the linear tax system improves welfare compare to the actual system, and it achieves 71% of the welfare possible given by the non-linear system. So, these numbers should be interpreted as the distance in welfare between the linear system compared with the best the government can do, and not with the actual system of an economy.

The savings needed to make the planner indifferent is 0.52% of GDP or less, depending on the parameters. These numbers can be compared directly with administrative costs of welfare programs and tax systems. Also, there are other costs related to having a non-linear tax system.Many tax systems change their tax rates in a non-continuous fashion, and welfare programs depending on income of the participant can change drastically by an increase of a small amount of income. These facts create kinks and notches[5] in the choice set creating some behavioral responses by individuals. For example, East (2018) finds a decrease in labor supply by welfare recipients. Saez (2010) finds some accumulation of individuals (bunching) in kinks created by the Earned Income Tax Credit (EITC), but he argues that fiscal evasion can explain that bunching. More recently, with a richer database, Mortenson and Whitten (2016) find evidence of bunching in more kinks and this bunching is becoming more important over time. Also, Ruh and Staubli (2018) find strong evidence of bunching in Austria due to a notch created by the disability insurance program of that country.

In a recent document by the IDB[6] they find that 4.4% of GDP is lost because of inefficient public spending in Latin America, and 1.7% of that occurs in transfer programs. That 1.7% is greater than all the values estimated previously.

1.3 The role of Heterogeneous preferences on Optimal Income Taxation

In this section I'll show that heterogeneous preferences can be a sufficient condition to have a linear marginal tax rate on income. This result is in line with the results of Lockwood and Weinzierl (2015) and Judd, Ma, Saunders and Su (2006), where they find that heterogeneous preferences reduce redistribution.

Heterogeneous preferences present a challenge in solving the optimal problem, and even numerical solutions are hard to obtain, as explained in Judd, Ma, Saunders and Su (2006). The special case considered in Lockwood and Weinzierl (2015), with

[5] A kink is a non-smooth change in the choice set, as we see in changes in the marginal tax rates. A notch is a discontinuous jump in the choice set, a feature that is normally generated by losses of benefits in welfare programs due to small increases in income. A review of kinks and notches can be found in Slemrod (2019)

[6] See Izquierdo, Pessino, Vuletin (2018)

simplifying assumptions, allows for a simple characterization of the solution. They consider the Mirrlees model with a setup that is similar to Diamond (1998). By using a specific form of heterogeneous preferences, they find a solution that is similar to that in Diamond (1998), but with a different form of welfare weights. I'll use this formulation and show that the non-linear optimal marginal tax rate on income becomes flatter as heterogeneity increases. The extreme case, where heterogeneity is increased to its maximum possible amount, shows a linear marginal tax rate.

1.3.1 A model with homogeneous preferences

In this subsection and the following one I'll follow closely Lockwood and Weinzierl (2015) to describe the model with homogeneous preferences and heterogeneous preferences, and show how they compare to each other.

Individuals have a utility that is linear in consumption c and non-linear in labor effort l given by $u(c,l) = c - l^{1+1/\gamma}$where γ is the constant elasticity of labor supply. They have an unobservable ability $n \geq 0$ so that gross income y is equal to nl. Thus we can rewrite the utility function in the following form:

$$U(c,y,n) = c - (y/n)^{1+1/\gamma} \tag{1}$$

The distribution of ability is given by the cumulative density function $F(n)$with density $f(n)$.

The planner selects the allocation $\{c(n), y(n)\}$to maximize the social welfare function W, solving:

$$W = \max_{\{c(n),y(n)\}} \int_0^{\infty} g(n)\, U(c(n), y(n), n)\, f(n) dn \tag{2}$$

where $g(n) \geq 0$ is the welfare weight for type n assigned by the planner. If the planner is utilitarian, $g(n) = 1$ for all n.

The maximization problem in 2 is subject to the resource constraint:

$$\int_0^{\infty} (y(n) - c(n)) f(n) \, dn \geq E$$

where E is government spending,

The incentive compatibility (IC) constraints are:

$$U(c(n), y(n), n) \geq U(c(m), y(m), n), \ \forall \ , m, n$$

As done in the literature, we can define the income tax function as $T(y) = y - c$.

In this setup, as shown in Diamond (1998), the optimal tax function is characterized by the following first-order condition:

$$\frac{T'(y(n))}{1 - T'(y(n))} = \frac{1 + 1/\gamma}{nf(n)} (G(n) - F(n)), \ \forall n \tag{3}$$

where $G(n) = \frac{\int_0^n g(m)f(m)dm}{\int_0^{\infty} g(m)f(m)dm}$, normalized so that $G(0) = 0$ and $\lim_{n\to\infty} G(n) = 1$.

1.3.2 A model with heterogeneous preferences

A simple model that can add heterogeneity and keep the simplicity of the solution is developed in Lockwood and Weinzierl (2015), which I describe shortly below.

The model is a modification of the previous subsection, where now an individual is defined by a two-dimensional type (w, θ), where $w \geq 0$ is now the unobservable ability so $y = wl$, and $\theta > 0$ is an unobservable preference parameter. This parameter is assumed to have a population average equal to one, and can be thought of as a taste parameter scaling the disutility of labor relatively to consumption. The utility function of the agent is now $u(c,l) = c - (l/\theta)^{1+1/\gamma}$. As in the previous section, the utility can be rewritten as:

$$U(c, y, w, \theta) = c - \left(\frac{y}{w\theta}\right)^{1+1/\gamma} \tag{4}$$

Here it is important to note that in (4) the two unknown parameters to the planner, w and θ, are entered in the utility function specifically to help solve the problem. If two individuals (w', θ'),(w'', θ'') are such that $w'\theta' = w''\theta''$, they will behave as if they are the same individual.

Thus, the product $w\theta$ is a sufficient statistic for this problem and the planner can consider individuals as a function of the product $w\theta$. The planner then will choose allocations $\{c(w\theta), y(w\theta)\}$.

As in the previous section, it is assumed that the planner seeks to maximize the welfare function W using welfare weights $b(w, \theta)$, depending now on both parameters. Let $H(w, \theta)$ denote the joint probability distribution of ability and preferences, with density $h(w, \theta)$. The planner's problem is:

$$W = \max_{\{c(w\theta), y(w\theta)\}} \int_0^\infty \int_0^\infty b(w, \theta) U(c(w\theta), y(w\theta), w, \theta) h(w, \theta) dw d\theta \tag{5}$$

The resource constraint in this case is:

$$\int_0^\infty \int_0^\infty (y(w\theta) - c(w\theta)) h(w, \theta) dw d\theta \geq E$$

and the IC constraints.

I'll use the same assumption used in Lockwood and Weinzierl (2015).

Assumption: $b(w, \theta) = b(w, \theta')$ for all θ and θ', or $b(w) \equiv b(w, \theta)$ for all θ.

Under this assumption, (5)becomes:

$$W = \max_{\{c(w\theta),y(w\theta)\}} \int_0^\infty \int_0^\infty b(w)\, U(c(w\theta), y(w\theta), w, \theta)\, h(w,\theta)\, dw d\theta \tag{6}$$

Now, to transform this model to an equivalent version of the homogeneous preferences, it is useful to make a change in variables denoting the unified type by n, so that $n = w\theta$.

The change of variables implies some changes in notation. Let $\hat{H}(\theta, n)$ now be the joint distribution of preferences and unified type, with density $\hat{h}(\theta, n) = h(n/\theta, n)$. Also, let $f(n) = \int_0^\infty \hat{h}(\theta, n)\, d\theta$. Then substituting the variables, the resource constraint and the IC constraint in both models are the same.

Further, the planner problem can be written as:

$$W = \max_{\{c(n),y(n)\}} \int_0^\infty \int_0^\infty b(n/\theta)\, U(c(n), y(n), n)\, \hat{h}(\theta, n)\, d\theta dn =$$

$$\max_{\{c(n),y(n)\}} \int_0^\infty \left(\int_0^\infty \frac{b(n/\theta)\, \hat{h}(\theta, n)\, d\theta}{f(n)} \right) U(c(n), y(n), n)\, f(n)\, dn$$

or

$$\max_{\{c(n),y(n)\}} \int_0^\infty \hat{b}(n)\, U(c(n), y(n), n)\, f(n)\, dn \tag{7}$$

where $\hat{b}(n) = \left(\int_0^\infty \frac{b(n/\theta)\hat{h}(\theta,n)d\theta}{f(n)} \right)$.

Note that (7)is equivalent to (2), with $\hat{b}(n)$ instead of $g(n)$.

Now, the optimal income tax will be given by equation (3), with

$$G(n) = \frac{\int_0^n \hat{b}(m)\, f(m)\, dm}{\int_0^\infty \hat{b}(m)\, f(m)\, dm} \tag{8}$$

So, the introduction of heterogeneity on preferences in this model creates another model that is equivalent to the original one, but with a modified welfare weight that will depend on the distribution of the preference parameter.

1.3.3 The role of heterogeneity on marginal income tax rates

I now proceed to show an analytical and a numerical result, showing the role of heterogeneity on the marginal income tax rate. The result applies for Pareto distributions, an important distribution for this literature since it is use to estimate the tail of the ability distribution as in Saez (2001).

I'll consider the simplified version of the model where the ability distribution is independent of the preference parameter. Assume the ability is distributed Pareto with parameter α, that is, w has support $[w_0, +\infty]$ and the pdf is $\frac{\alpha w_0^\alpha}{w^{\alpha+1}}$. The preference parameter$\theta$is distributed uniformly between $[\theta_L, \theta_H]$. Specifically, I'll consider the case where $\theta_L = 1 - \epsilon$ and $\theta_H = 1 + \epsilon$ for $\epsilon < 1$.

Let:

$$b(w) = \kappa w^{-\eta} \tag{9}$$

be the welfare weight assigned by the planner to the individual with ability w, for positive values of $\kappa$and η.

In the following proposition I state the main result, that the marginal tax rate goes to a constant for all n above some level of wages w.

Proposition: *Consider the income tax function (3) that solves the planner problem in (7), where $G(n)$ is given by (8). Further, let the welfare weight be given by (9) and the unified type distribution be given by the joint distribution of w and θ, where w is distributed Pareto and $\theta \sim U[1-\epsilon, 1+\epsilon]$ is uniformly distributed.*

Then, as $\epsilon \to 1$, $T' \to C$ for all $n \geq w_0\theta_H$, where C is a constant.

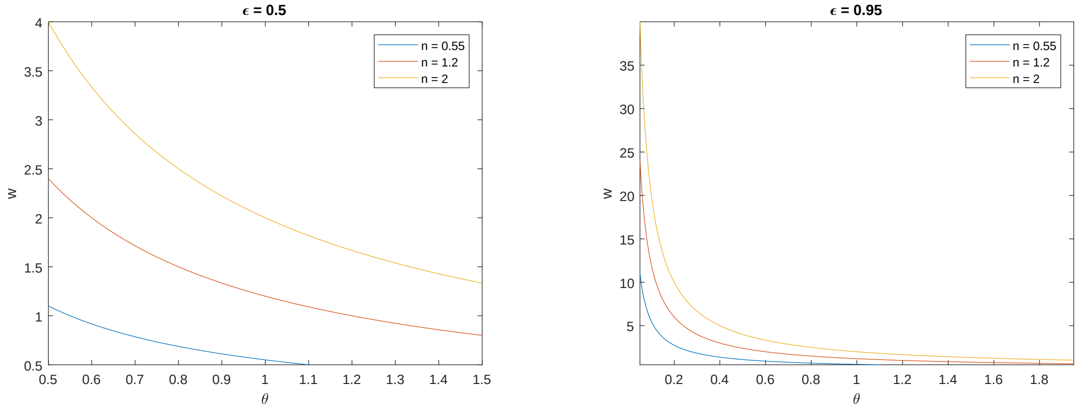
Proof: see appendix.

To understand this result we must notice that, as ϵ increases, every unified type n includes more and more w types (in the sense that for every n type, several w types are part of the unified n type). If $\epsilon = 0$, then $n = w$. But, if $\epsilon > 0$, for every n all the $w \epsilon [n/(1+\epsilon), n/(1-\epsilon)]$. Then, the welfare weight $G(n)$ includes several wages w and their corresponding values $b(w) = \kappa w^{-\eta}$. In fact, as $\epsilon \to 1$, for every $n \geq w_0\theta_H$ the unified type n includes all the w types from n/θ_H to ∞.

The economic intuition of this result rests on the fact that the parameter θ generates noise to the planner, where he is not able to distinguish what kind of $w-$type the agent is. The more variation (the bigger ϵ), the more noise there is. The only exception to the constant marginal tax rate is for those types $n \epsilon [w_0\theta_L, w_0\theta_H]$ .Those types are different because there is only a subset of θ values that can be combined with w to get n. For example, the unified type $n = w_0\theta_L$ has a unique value for θ that generates that value of n (which is θ_L). It can be seen in the figure below that for $n = 0.55$ there is no w^* that makes $n = w * \theta_H$.

The figures below show the w-types that every n-type implies for different values of θ. If ϵ is small, fewer w-types are included in each n-type. If ϵ is close to 1, more w-types are included making the modified welfare weight $\hat{b}(n)$ more alike for different n.

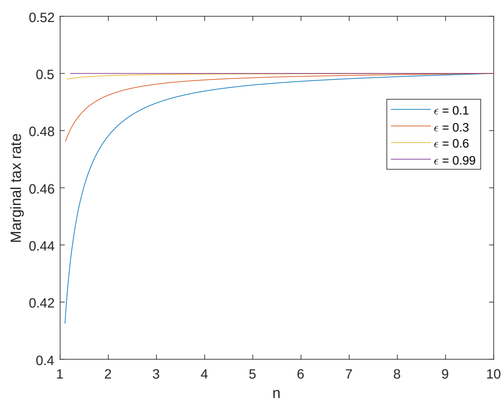
Figure 1: Individuals of w type included in each n type for different values of e



The exercise is done for $\theta \ \epsilon \ [1-\epsilon, 1+\epsilon]$ and $w \ \epsilon \ [0.5, \infty]$

To see how the marginal tax rate changes with ϵ. The next graph shows the evolution of the marginal tax rate for different values of ϵ.

Figure 2: Marginal tax rate for different values of ϵ



Values used in this graph: $w0 = 1$, $\alpha = 2$, $\kappa = 1.5$, $\gamma = 1$.

1.4 Conclusion

In this paper I have calculated the closeness of a linear tax system to the optimal non-linear tax system. I find that a linear tax system is close to the optimal non-linear tax system for a wide range of parameters. As others have noted, the linear system is better, in terms of welfare for the planner, than the actual tax system in the US. So, this closeness seems to hold not only for some specifications of the parameters model, but for a wide range of them. An obvious conclusion is that countries where administrative costs are high, and distortions created by welfare programs are important, have more to gain with a linear tax system.

Further, the characterization of the optimal non-linear tax system requires a lot of information that might not be available to the planner. In the second part of the paper I introduced heterogeneity into the preferences of the individuals, and I find that heterogeneity makes the non-linear tax system flatter. A key result of this paper is that heterogeneity could make the tax system completely linear.

Given these results, the answer to the question posed at the beginning is that it is not radical to give cash directly to people. A lump-sum transfer, that can be interpreted as UBI, is present on the Mirrlees model. Further, the non-linear tax system is already close to the linear tax system. If we consider that individuals have heterogeneous preferences, the non-linear tax system gets even closer to the linear tax system.

This paper should not be interpreted as a proof of the convenience of a policy like UBI; however, the paper does indicate that UBI should be considered seriously.

Further research should include dynamic aspects, as well as other uncertainties that the planner may face, such as not knowing exactly the distribution of abilities.

2 Demographics, Labor, and the Great Recession

2.1 Introduction

Recent literature has reached the consensus that after the Great Recession, output and labor in the United States diverted from their pre-crisis long term trends. Although many theories are trying to explain the sources of these patterns, one plausible candidate is demographics. The start of the crisis coincided with the "baby boomers" entering age cohorts associated with lower levels of labor force participation and retirement. Hence, this shift in the demographic composition has the potential of explaining the observed economic activity during the recovery.

The objective of this paper is to quantify the effects demographic changes had on the evolution of output and labor in the recovery period after the Great Recession. For this, we first carry out an in depth analysis of employment trends for the time period 1990 - 2015. We document that a significant portion of the literature is incorrect when comparing the evolution of labor after the crisis with its pre-crisis trend. More specifically, we construct a counterfactual trend in which we account for demographic effects on the intensive margin, number of hours worked, and the extensive margin, labor force participation. We find that our counter factual trend of employment, which accounts for demographics, reduced the gap in between the pre-crisis employment trend and the data by 83.7%.

Given this evidence of the potential effects of demographics on labor supply, we develop a growth model that incorporates demographics. More specifically, demographics affect the dynamics of the model through the growth rate of population and through changes in the age distribution of the population across time. We calibrate this model to match moments of the US pre-crisis economy.

Using this model, we first analyze how much demographics would have accounted for changes on output and labor in absence of the Great Recession. We document that 35% of the output gap between the pre-crisis trend and the data is explained by demographics. The channel through which demographics affect output is through a reduction in the hours worked by agents in the model. We then expand our analysis to also include fluctuations in total factor productivity. We find that this specification is able to reduce the gap in between output in the model and output in the data to 2.5%. Furthermore the gap in between labor in the model and in the data reduces to 1.2%.

This paper is organized as follows. Section 2 reviews literature related to demographics and the Great Recession. In Section 3, we carry out an in depth analysis of trends in employment, taking into account the effects that demographics has on labor supply. In Section 4 we develop a framework that modifies the standard Growth model to include demographic changes. In Section 5, we use this model to explain the patterns of macroeconomic variables that we perceive in the data. Section 6 concludes.

2.2 Literature Review

This paper is related to two main branches of the literature: the Great Recession and Demographics. Since 2008, many hypotheses have tried to understand the reasons behind the slow recovery in aggregate output and employment for the United States. UNA (2016) quantifies the contribution of different factors to explain their role in the slow recovery of aggregate variables. He documents that through 2013, output was 13 percentage points below its 1990 - 2007 trend, where the main contributors to this gap were the fall in business capital, productivity, and labor force participation.

From a more theoretical standpoint, the causes and mechanisms behind the Great Recession have been broad. For example, Schaal and Taschereau-Dumouchel (2015) set up standard neoclassical growth model with monopolistic competition and coordination failures to explain long recessions. They find that a big transitory shock, like the one in 2007, can force the economy into a steady state characterized by lower output and employment. On the other hand, Shimer (2012) sets up a search model with real wage rigidities to explain jobless recoveries. He documents that the interaction of rigid wages with search frictions are important for a persistent slow recovery in economic activity. Heathcote and Perri (2016), Mian and Sufi (2012), Mian, Rao and Sufi (2013) and Midrigan and Philippon (2016) study mechanisms by which a fall in housing prices, housing net worth, and tightening of credit standards caused declines in household debt, consumption and employment. Our paper is similar to these in the sense that it tries to understand the reduction in output and employment that occurred after the crisis of 2007. It differs from these as it tries to quantify the role of demographics in explaining the fall in output and labor.

The discussion on demographics and its effects on growth and employment has been increasing in the past few years in the literature. First, Hayashi and Prescott (2002), Chen, İmrohoroğlu, İmrohoroğlu (2016a) and DGEEC (2017), modify the standard growth model to account for dynamics in exogenous variables such as the growth in population. Among the caveats of only considering population growth in the standard neoclassical growth model is that demographics only affect the household by increasing its size across time. In other words, population growth does not take into account possible effects of changes in the population distribution across age groups as well as differences in agents' decision making at different age groups.

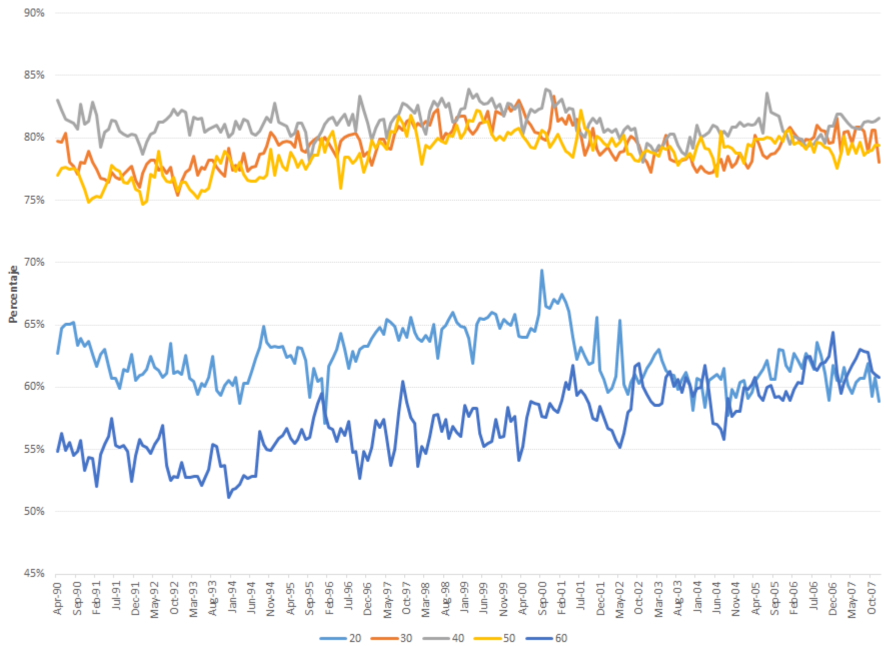
Using an empirical approach, Maestas, Mullen and Powell (2016) find that the effects of the population's age structure has an important impact on output per capita growth for the US. They document that a 10% increase in the population above 60 years causes a decrease in 5.5% in the growth rate of GDP per capita. Given these aspects, we consider a modification of the standard growth model, which accounts for differences in the population composition of age groups across time.

Since the Great Recession, there has been a bigger discussion on the relationship in between demographics and labor supply. In his quantitative approach, UNA (2016) estimates that of the 13 percentage point drop in output, 1.1 was explained by the effect of the aging of baby-boomers on labor force participation. Maestas, Mullen and Powell (2016) find that of the 5.5% reduction in the output growth rate caused by demographics, two-thirds is a result of slower growth in labor productivity of workers across the age distribution, while the rest is a result of slower growth in the labor force. To our knowledge, the closest work analyzing the effects of demographics on labor supply is by Henriksen and Cooley (2016). They set up a life-cycle model to examine how demographic induced changes in the intensive (hours worked) and extensive (employment) margins of labor supply affect the slowdown in output growth. Our paper differs to the aforementioned, as we analyze the specific effects that demographics have on macroeconomic aggregates through the lens of a modified growth model.

2.3 Employment Trends

We use monthly micro data from the Current Population Survey (CPS) obtained from the Integrated Public Use Microdata Series, IPUMS. To understand the effects of demographics on labor supply and output after the great recession, we start our analysis by documenting stable labor patterns before the crisis, for the period 1990 - 2007. We focus on this time period for the following reasons. We exclude the period before 1990 because women employment rate was raising as a consequence of the increase of their participation in the labor force. Additionally, after 2007, the Great Recession had a negative impact on labor.

Figure 3: Employment Ratio by Cohorts

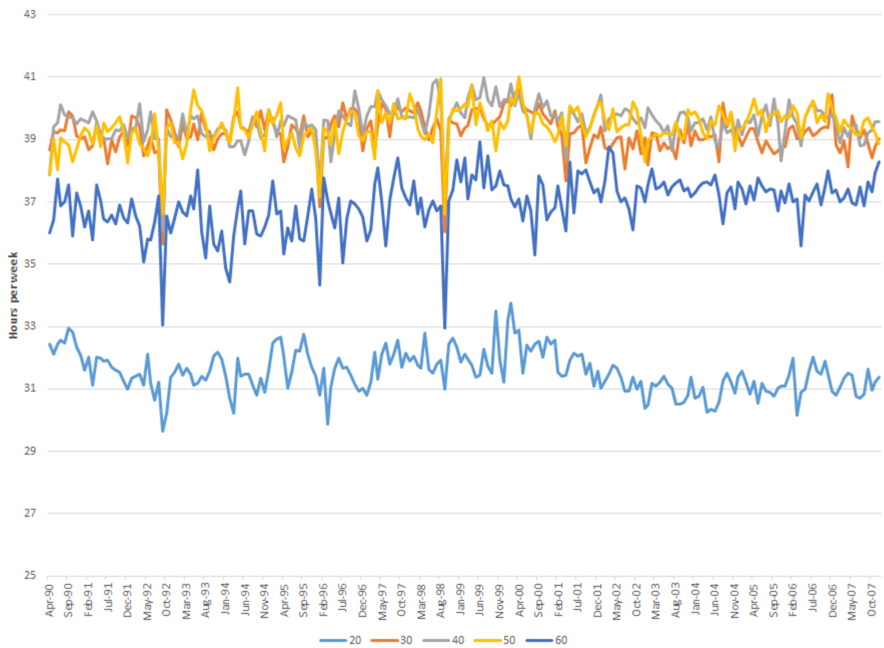


Source: Current Population Survey

We analyze employment and hours worked by age cohort in between 1900 and 2007. For each cohort, we observe the number of workers, and the total amount of hours worked. As Figure 1 shows, there is a stable evolution of the employment ratio (E_t^a), measured as the ratio of employment to population. For example, the monthly employment ratio for workers with age 40 fluctuated in between 78% and 83%; similar patterns are found across age cohorts. We estimate the average hours worked by those employed in each cohort (h_t^a), as the ratio of total hours to the total number of employed in each age group. Figure 2, shows that this statistic is also stable over time.

To statistically test for the stability in the employment ratio and hours worked, we fit a line through the time series of these statistics (linear regression). We find that the slope of the linear regression is statistically zero for most years between 25 and 50 years old. The slope is statistically negative for younger cohorts, and positive for older cohorts. However, in both cases the slope is relatively small[7].

Figure 4: Average Hours Worked by



Source: Current Population Survey

Using these statistics of the employment ratio and average hours worked, we construct a counterfactual of the total hours worked in the absence of the Great Recession. The motivation for this is to have an aggregate labor measure that allows us to compare the actual data to what would have happened without the crisis.

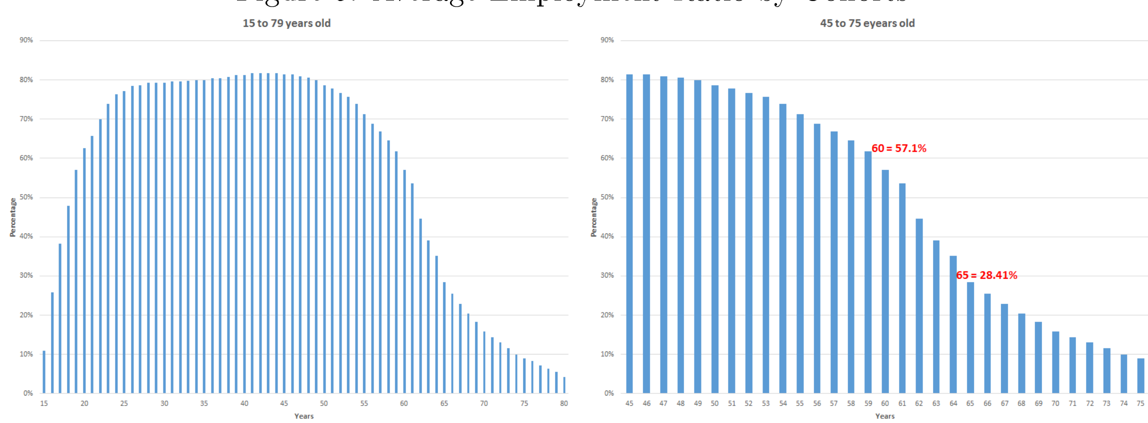
[7] The slope coefficient is statistically significant for ages 15 to 20 (negative) and above 55 (positive). On average the slope for younger cohorts implies a 2.7% decrease of employment ratio over 10 years, and for older cohorts implies an increase of 3.4% over 10 years.

We calculate the counterfactual in the following manner. Given the stability of the employment ratio and average hours worked for every cohort, we calculate the average of these measures across time as in equations (10) and (11):

$$\overline{E}^a = \frac{1}{T}\sum_{t=1990:2}^{2007:4} E_t^a, \tag{10}$$

$$\overline{h}^a = \frac{1}{T}\sum_{t=1990:2}^{2007:4} h_t^a. \tag{11}$$

Figure 5: Average Employment Ratio by Cohorts

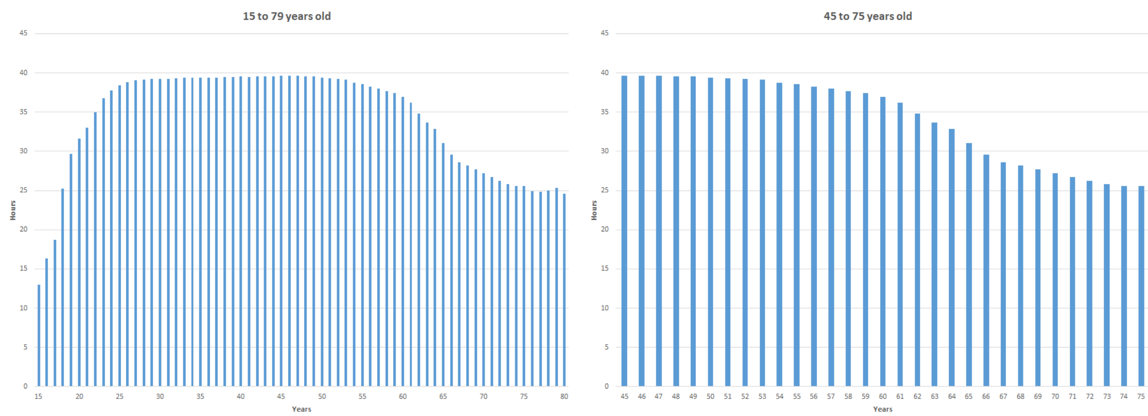


Source: Authors' calculation with data from CPS.

Figures 3 and 4 plot these statistics for every age group. The patterns portrayed in Figures 3 and 4 are similar. For young cohorts, the employment ratio is lower as most individuals in these cohorts are most likely with schooling responsibilities. For the case of average hours worked, young individuals also work a smaller number of hours, a result that is most likely due to their time being allocated to schooling. For age cohorts above 60, we can see that there is a fall in both employment ratio and average hours worked.

As expected, older individuals begin to retire at around the age of 60, which causes the employment ratio to fall. More specifically, in between the ages of 60 and 65 , the average employment ratio falls in about 30 percentage points. Also, the average hours worked falls for older individuals; in between the ages of 60 and 65, the average hours worked falls in more than 5 hours. Hence, older cohorts would affect aggregate labor supply through the extensive margin, by choosing to not supply labor, and the intensive margin, by choosing to work less hours.

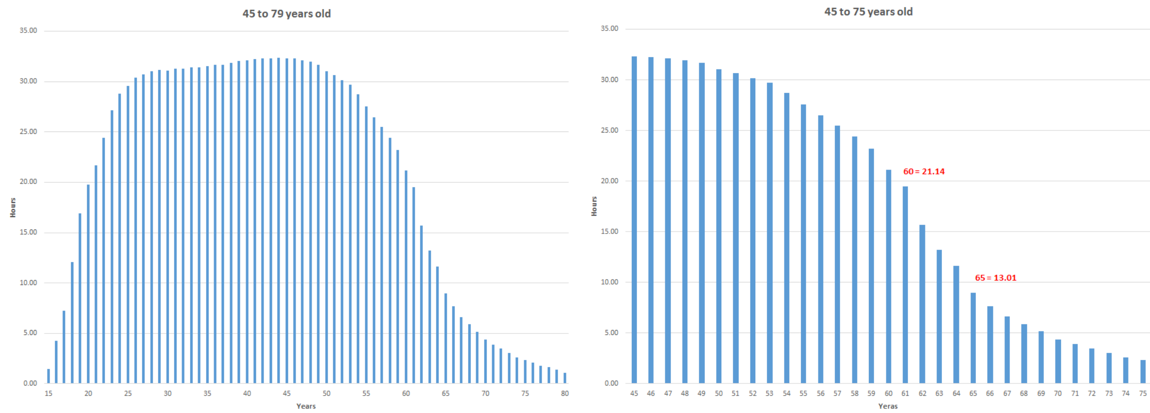
Figure 6: Average Hours Worked by Cohorts



Source: Authors' calculation with data from CPS.

The product in between $\bar{E}^a$ and $\bar{h}^a$ yields the number of hours worked per person in age cohort. Figure 5, shows this product. For older cohorts, there is a stronger decline in the number of hours worker per person in contrast to the average hours worked, as a consequence of labor supply falling through the extensive and intensive margins. Comparing the age cohort of 60 to that of 65, there is a decrease in the number of hours worked per person of more than 12 hours (58% drop).

Figure 7: Number of Hours Worked per Person by Cohorts



Source: Authors' calculation with data from CPS.

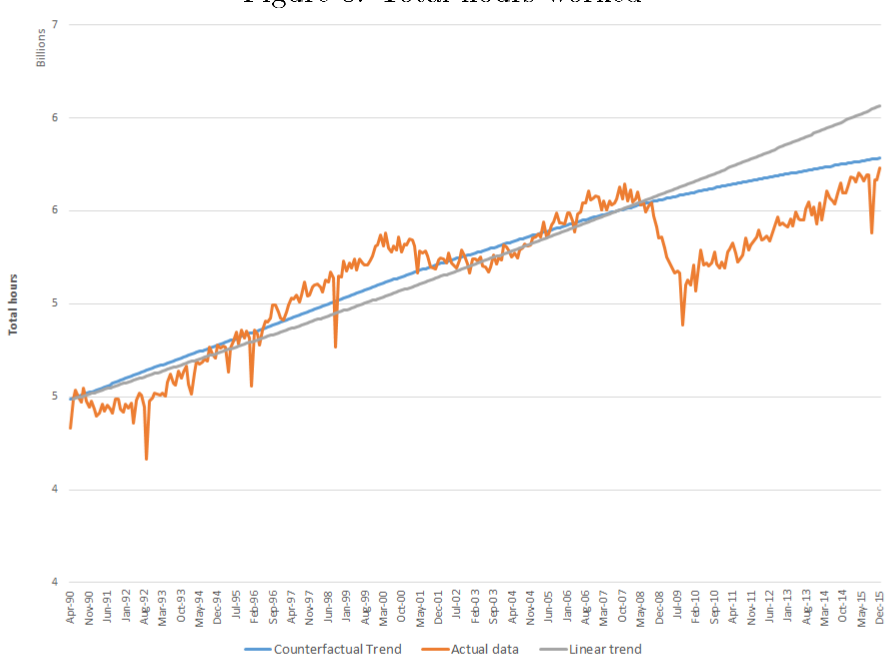
Focusing our analysis on the drop of labor supply of older cohorts is important, as it is a potential explanation of the apparent slow recovery of hours after the Great Recession. The years of the crisis coincide with the start of the baby boomer generation entering older cohorts and leaving the labor force. As a result, demographics played an important role during the years of the Great Recession, as they did in the 1980s and 1990s, when the baby boomers were at their most productive stage of their lives.

Next, we construct the total hours worked for the time period analyzed above 1990 - 2007, as well as the time period afterwards, 2008 - 2015. For each year t, we multiply the product in between $\bar{E}^a$ and $\bar{h}^a$, times the population in its corresponding cohort, (P_t^a). Then we sum across age cohorts:

$$H_t = \sum_{a=15}^{79} \overline{E}^a * \overline{h}^a * P_t^a.$$

This total hours worked after 2007 is our counterfactual measure of hours in absence of the Great Recession. Assuming that the employment ratio and average hours worked had not changed, which is a reasonable assumption given the stability of E_t^a and h_t^a, H_t is the level of hours we would have expected in the economy given only demographic changes, through P_t^a. Figure 6 plots three pieces of data: the linear trend of employment without taking into account demographics[8] (gray line), our employment counterfactual, H_t (blue line), and the actual data (orange line). It is evident from Figure 6 that the linear trend of total hours and the data move parallel to each other; in 2015 the gap in between these was -5.8% of the actual hours. On the other hand, the data and our counterfactual measure of employment are converging as the gap in between these is just -0.9% of the actual hours..

Figure 8: Total hours worked



Source: Authors' calculation with data from CPS and Census

[8]This is estimated as the fitted line for the data in between 1990 and 2007, and then using this fitted line to forecast the years after 2008.

We are conscious that the Great Recession constituted an important crisis, generating a big deviation of employment from its pre-crisis trend. However, it is also important to note that demographics play an important role in explaining the reason why employment did not recover to its trend before the crisis. The aging of the working age population implies a fall in the total hours worked, which is captured by our counterfactual employment trend. Comparing the data to this counterfactual employment trend shows that demographics is important in explaining the "slow" recovery in labor. Furthermore, it provides evidence that ignoring demographic changes from economic analysis can be detrimental not only for labor supply but for economic activity as a whole.

2.4 Growth Model and Demographics

In this section, we describe a variation of the growth model, in which the representative household is comprised of individuals with different ages. We will use this model to generate our quantitative results, similar to the approach by Hayashi and Prescott (2002) and DGEEC (2017). Below, aggregate variables are defined by capitalized letters, while per-capita variables are lower-cased.

2.4.1 Households

We assume there is a representative household with N_t members at time t. Population grows at rate $\gamma_{Nt} = \frac{N_t}{N_{t-1}}$. For each t, there is a number of P_t^a members of age a, so that $N_t = \sum_{a=s}^{S} P_t^s$, where s and S are the youngest and oldest ages in the household, respectively. Also, the household owns capital and rents it to firms. Further, the household solves the following problem:

$$\max_{\left\{\{c_t^a,h_t^a\}_{a=s}^S,K_{t+1},X_t\right\}_{t=0}^\infty} \sum_{t=0}^{\infty}\beta^t \sum_{a=s}^{S} P_t^a \left(\log\left(c_t^a\right)+\alpha^a \log\left(T-h_t^a\right)\right)$$

$$s.t.$$

$$\sum_{a=s}^{S} P_t^a c_t^a + X_t \leq w_t \sum_{a=s}^{S} P_t^a h_t^a + r_t K_t - \tau_t \left(r_t-\delta\right) K_t - \pi_t \quad for\ t=0,\ 1,...$$

$$K_{t+1} = X_t + (1-\delta) K_t \quad for\ t=0,\ 1,...$$

$$given\ K_0,$$

where aggregate consumption is $C_t = \sum_{a=s}^{S} P_t^a c_t^a$ and aggregate hours are $H_t = \sum_{a=s}^{S} P_t^a h_t^a$, T is the total time endowment per member, β is the discount factor, α^a is the share of leisure in the utility function for individuals with age a, w_t is the wage rate, r_t is the rental rate of capital, δ is the depreciation rate, τ_t is the tax rate con capital income, π_t is a lump-sum tax.

2.4.2 Firms

There is a representative firm, with the standard Cobb-Douglass Production Function, $Y_t = A_t K_t^\theta H_t^{1-\theta}$, where Y_t is aggregate output, A_t is total factor productivity, K_t is the capital stock rented by the firm, and H_t is the labor input of the firm measured in aggregate hours. We define θ as the share of capital in output. We assume that A_t grows at rate $\gamma_{At} = \left(\frac{A_t}{A_{t-1}}\right)^{\frac{1}{1-\theta}}$.

2.4.3 Government

The government taxes household's income on capital and lump-sum tax π_t, and uses these resources to finance government spending G_t so that the government budget balances every period:

$$G_t = \tau_t (r_t - \delta) K_t + \pi_t.$$

2.4.4 Competitive Equilibrium

The resource constraint of the economy is given by:

$$C_t + X_t + G_t = Y_t,$$

where C_t is aggregate consumption, X_t is aggregate investment and G_t is government purchases.

Given a government policy $\{G_t,\ \pi_t,\ \tau_t\}_{t=0}^{\infty}$, a competitive equilibrium for this economy is an allocation $\left\{\{c_t^a, h_t^a\}_{a=s}^{S},\ X_t,\ K_t,\ Y_t\right\}_{t=0}^{\infty}$ and a sequence of prices $\{w_t,\ r_t\}_{t=0}^{\infty}$, such that:

1. given the government policy and prices, the allocation solves the household's problem,

2. given the government policy and prices, the allocation maximizes firm's profits such that factor prices equal their marginal products, $w_t = (1-\theta) A_t \left(\frac{K_t}{H_t}\right)^{\theta}$ and $r_t = \theta A_t \left(\frac{K_t}{H_t}\right)^{\theta-1}$.

3. the government budget is satisfied,

4. and the market clearing condition holds: $\sum_{a=s}^{S} P_t^a c_t^a + K_{t+1} - (1-\delta) K_t + G_t = A_t K_t^{\theta} \left(\sum_{a=s}^{S} P_t^a h_t^a\right)^{1-\theta}$.

2.4.5 Numerical Solution

We solve the model in a similar manner to Hayashi and Prescott (2002) and DGEEC (2017). First, we compute the steady state of the U.S. economy in the sufficient distant future, using the calibrated parameters and exogenous variables. The steady state is obtained from the equilibrium conditions of the model. With this steady state, we apply a shooting algorithm toward this steady state from the given initial conditions, corresponding to the first trimester of 1990. The solution to this algorithm is an equilibrium transition path from the initial conditions to the final steady state.

The equilibrium conditions are characterized by the standard intratemporal condition, Euler equation, and resource constraint obtained from the household's and firm's optimality conditions:

$$\frac{\alpha^a c_t^a}{T - h_t^a} = (1-\theta) A_t \left(\frac{K_t}{\sum_{a=s}^{S} P_t^a h_t^a} \right)^{\theta} \quad \forall a,\ \forall t, \tag{12}$$

$$\frac{c_{t+1}^a}{c_t^a} = \beta \left[1 + (1-\tau_{t+1}) \left(\theta A_{t+1} \left(\frac{K_{t+1}}{\sum_{a=s}^{S} P_{t+1}^a h_{t+1}^a} \right)^{\theta-1} - \delta \right) \right] \quad \forall a,\ \forall t, \tag{13}$$

$$K_{t+1} = (1-\delta) K_t + A_t K_t^{\theta} \left(\sum_{a=s}^{S} P_t^a h_t^a \right)^{1-\theta} - \sum_{a=s}^{S} P_t^a c_t^a - G_t \quad \forall t. \tag{14}$$

To obtain the steady state, first we detrend all variables so that $\hat{x}_t = \frac{x_t}{A^{\frac{1}{1-\theta}}}$ for per capita variables and $\hat{x}_t = \frac{X_t}{A^{\frac{1}{1-\theta}} N_t}$ for aggregate variables. Equations (12) through (14) become:

$$\frac{\alpha^a \hat{c}_t^a}{T - h_t^a} = (1-\theta)\left(\frac{\hat{k}_t}{\sum_{a=s}^{S} \eta_t^a h_t^a}\right)^{\theta} \quad \forall a,\ \forall t, \tag{15}$$

$$\frac{\hat{c}_{t+1}^a}{\hat{c}_t^a} = \frac{\beta}{\gamma_{At+1}}\left[1 + (1-\tau_{t+1})\left(\theta\left(\frac{\hat{k}_{t+1}}{\sum_{a=s}^{S} \eta_{t+1}^a h_{t+1}^a}\right)^{\theta-1} - \delta\right)\right] \quad \forall a,\ \forall t, \tag{16}$$

$$\gamma_{At+1}\gamma_{Nt+1}\hat{k}_{t+1} = \hat{k}_t\left[\left(\frac{\hat{k}_{t+1}}{\sum_{a=s}^{S} \eta_{t+1}^a h_{t+1}^a}\right)^{\theta-1}(1-\psi_t) + (1-\delta)\right] - \sum_{a=s}^{S} \eta_t^a \hat{c}_t^a \quad \forall t, \tag{17}$$

where ψ_t is the ratio of government purchases to output, $\frac{G_t}{Y_t}$, and η_t^a is the ratio of the population of individuals of age a at time t to the total population at time t, $\frac{P_t^a}{N_t}$.

In steady state, detrended variables do not grow and the ratio of individuals of any age a with respect to total population remains constant. Hence the steady state equilibrium conditions are given by:

$$\frac{\alpha^a \hat{c}^a}{T - h^a} = (1-\theta)\left(\frac{\hat{k}}{\sum_{a=s}^{S} \eta^a h^a}\right)^{\theta} \quad \forall a, \tag{18}$$

$$1 = \frac{\beta}{\gamma_A}\left[1 + (1-\tau)\left(\theta\left(\frac{\hat{k}}{\sum_{a=s}^{S} \eta^a h^a}\right)^{\theta-1} - \delta\right)\right] \quad , \tag{19}$$

$$\gamma_A\gamma_N\hat{k} = \hat{k}\left[\left(\frac{\hat{k}}{\sum_{a=s}^{S} \eta^a h^a}\right)^{\theta-1}(1-\psi) + (1-\delta)\right] - \sum_{a=s}^{S} \eta^a \hat{c}^a. \tag{20}$$

2.5 Demographics and Macroeconomic Aggregates

2.5.1 Calibration

We calibrate the growth model described above to determine the effects of demographic changes on economic activity in the United States. The time period we use for calibration corresponds to 1990 - 2007. The model has four parameters that are the same for all the household: θ (capital share in production), δ (depreciation rate), β (discount factor), and T (total discretionary hours in a week). Also, there is an age specific parameter (α^a). For our analysis, we shut down the government, so that its revenue and expenditure is equal to zero. The values for the four common parameters are shown in Table 1. These are calculated in the standard way, as detailed in the Appendix.

Table 1: Parameters

θ	0.33
δ	0.058
β	0.948
T	100

The disutility of labor, α^a, is an age specific parameter that is chosen such that the average of hours worked in the model is the same as the hours worked in the data, for each age. Using the intratemporal condition of our model, (12), we obtain:

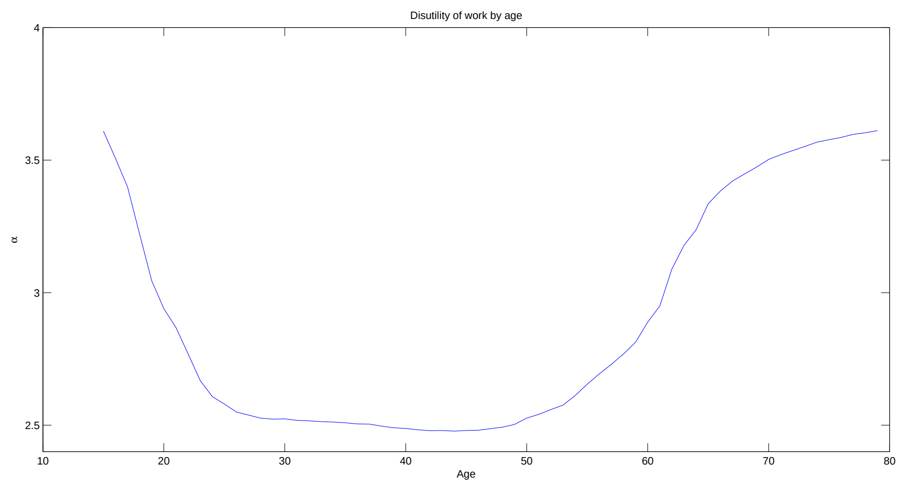
$$\frac{\alpha^i \hat{c}^i}{T - h^i} = \frac{\alpha^j \hat{c}^j}{T - h^j} \;\; \forall \; a = i, j.$$

The Euler condition, (13), implies that consumption level is the same for all ages. As a result, the above equation simplifies to:

$$\alpha^i = \frac{T - h^i}{T - h^j} \alpha^j \ \ \forall\, a = i, j.$$

Using our data counterparts, $\overline{h}^a$, calculated in equation (11), we calibrate α^a for all a. Figure 7 portrays the values of the disutility of labor by age. Our parameters vary between 2.45 and 3.56. These are higher than what is documented in the literature. The reason for this is that we consider hours per person by age as in figure 5, and not the average hours worked (as in figure 4).

Figure 9: Disutility of work by age: α^a



As mentioned before, we are interested in quantifying the effects of demographics on output and labor. We will carry out two experiments. The first only considers the effect of demographics on our model economy and sets up a counterfactual of how macroeconomic aggregates would have evolved in absence of the Great Recession. Demographics affect economic activity through the population growth rate, γ_{Nt}, and through the ratio of the population of individuals of age a at time t to the total population at time, η_t^a. Both γ_{Nt} and η_t^a are measured using data from the census. Using the solution method described in Section 4.5, we use these time series and a constant TFP growth rate $\bar{\gamma}_A = \frac{1}{T}\sum_{t=1990}^{2007}\gamma_{At}$ to obtain the equilibrium path for macroeconomic aggregates[9]. We compare the evolution of the aggregates in our model to those of the data to quantify the importance of demographics in explaining trends in economic activity. Our second experiment builds on the first one by considering demographic effects (γ_{Nt} and η_t^a) along with time-varying TFP growth rates, γ_{At}[10].

2.5.2 Results

The period 1990 - 2015 is a time frame which constitutes a transition from a fast growing population composed of middle-aged individuals to a slow growing population with older individuals. Incorporating this feature into our model, we analyze the transitional dynamics of several macroeconomic aggregates for the pre-crisis period (1990 - 2007) and the years after the crisis (2008 - 2015). As mentioned in Section 5.1, at first we only consider these demographic effects, and exclude any other exogenous time varying variables, such as TFP. This allows us to understand how the economy would have behaved in absence of the Great Recession.

The first aggregate we evaluate is the capital to output ratio. There are two effects at work in the demographic transition of this economy. First, the decrease in the population growth rate generates an increase in the consumption per capita over time. Second, the aging of population reduces the amount of labor offered to the market, while keeping constant the number of people consuming. This second effect

[9]For the first experiment's final steady state, we set the population growth to 0.4%, which is consistent with the census's estimation of the growth rate for the period 2050 - 2060. Also, we set the growth rate of TFP in steady state equal to the average growth rate between 1990 and 2007.

[10]For the second experiment's final steady state, we set the population growth rate equal to 0.4% and the TFP growth rate equal to the average TFP growth for the period 2008 - 2015.

reduces the consumption per capita over time. These two effects offset each other, as can be inferred from (16). As a consequence of this, our model predicts an almost time invariant trend for the capital to output ratio as can be observed in Figure 8.

Figure 10: Capital/Output ratio with only Demographic Changes

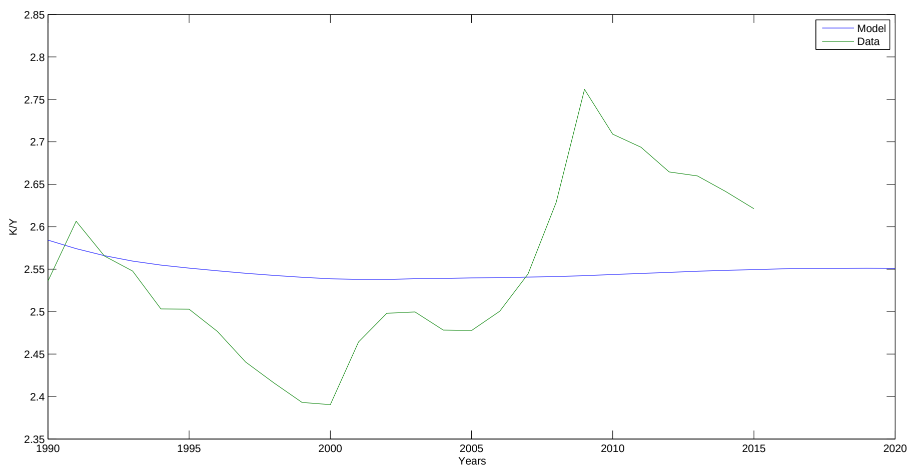
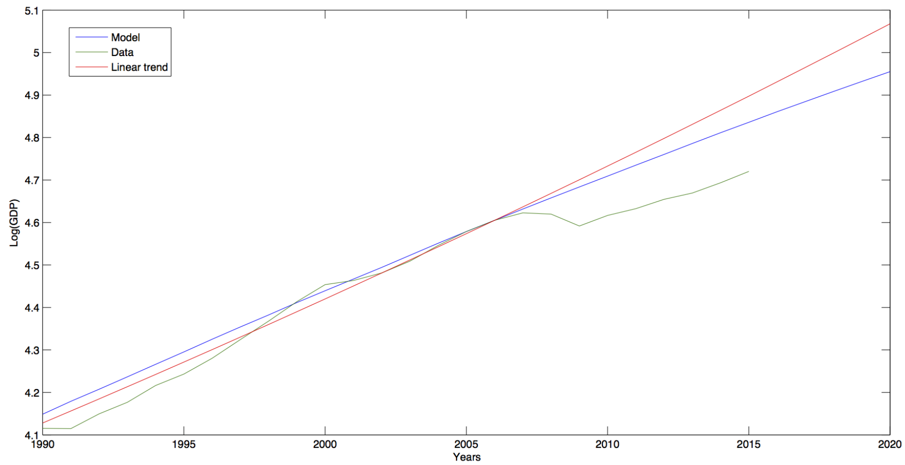


Figure 9 presents the evolution of output in the data since 1990 (green line). It is clear that after the Great Recession, output deviated from its previous trend (red line) and did not converge back to it[11]. A model which excludes the demographic changes we account for in our model would yield an output trend similar to the linear trend of Figure 9[12]. By considering demographic changes, through changes in the population growth rate and the population composition, our model is able explain at least part of the deviation of GDP from its trend previous trend.

[11] The linear trend is calculated as the trend of the data from 1990 to 2007.

[12] We solve our model with constant population growth rate and time invariant population composition, and obtain a output series similar to that of the linear trend.

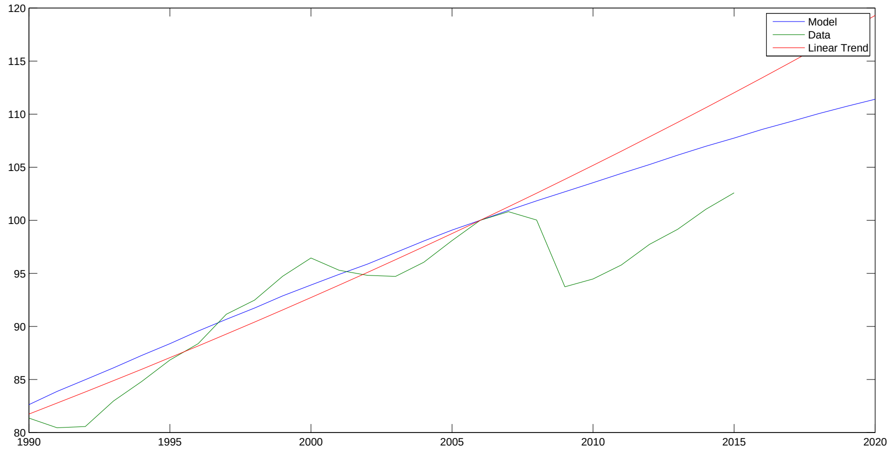
Figure 11: GDP with only Demographic Changes



The model with only demographic changes is not able to account of all of the drop in GDP, as the main contributor of the drop was the Great Recession, which is not modeled in our first experiment. Our exclusion of the negative TFP shock generated by the Great Recession is necessary for us to quantify the only effects of demographics in explaining the deviation of the data from its pre-crisis linear trend. We document that demographics accounts for 35% of the gap in between the pre-crisis linear trend and the data.

The channel through which changes in the population growth rate and the population composition affects output is by the reduction of labor supply. As the population ages, labor supply falls due to reduction in the hours supplied by the older cohorts. Figure 10 is the model counter part of Figure 6. When we compare the model's total hours to the counter factual employment trend of Figure 6, we can see that the fall of the latter is more pronounced. In the model, demographics only affects labor supply through the intensive margin, i.e. the number of hours supplied by each cohort. The counterfactual employment trend also takes into account the effects of demographics on the extensive margin. That is, older cohorts participate less in the labor force.

Figure 12: Labor with only Demographic Changes



Our second experiment builds on the previous one by incorporating TFP changes also into the analysis. Hence, this model will also account for TFP movements for the time period 1990 - 2015, where the most important was the negative TFP shock of the Great Recession. Figure 11 shows the evolution of the capital to output ratio. We can see that the model does a much better job of capturing the movements in the data. Comparing Figure 11 to Figure 8, we conclude that demographic changes which only affect the economy through labor supply are not able to account for the dynamics in the capital to output ratio. Also, the model predicts a higher level of capital-output due to the decrease in the TFP growth rate[13].

[13] The final steady state was calculated using the average TFP growth rate of the period 2008 - 2015, which is smaller to the growth rate of the period 1990 - 2007.

Figure 13: Capital-output ratio with Demographic and TFP Changes

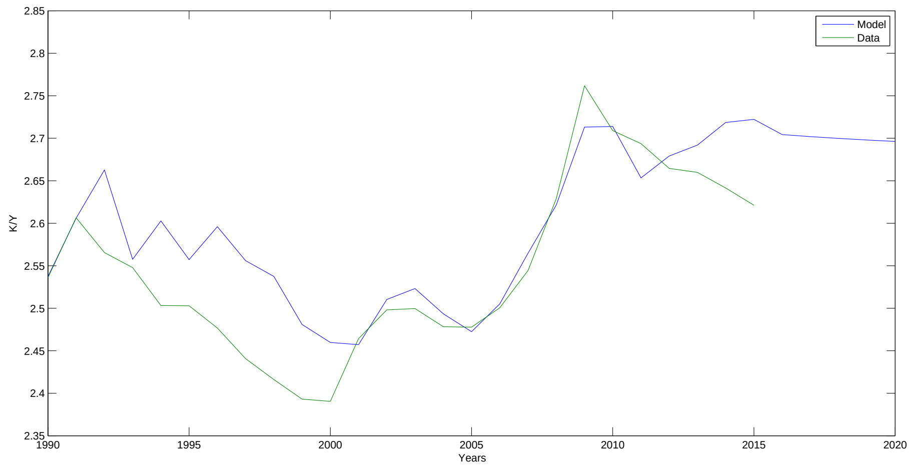
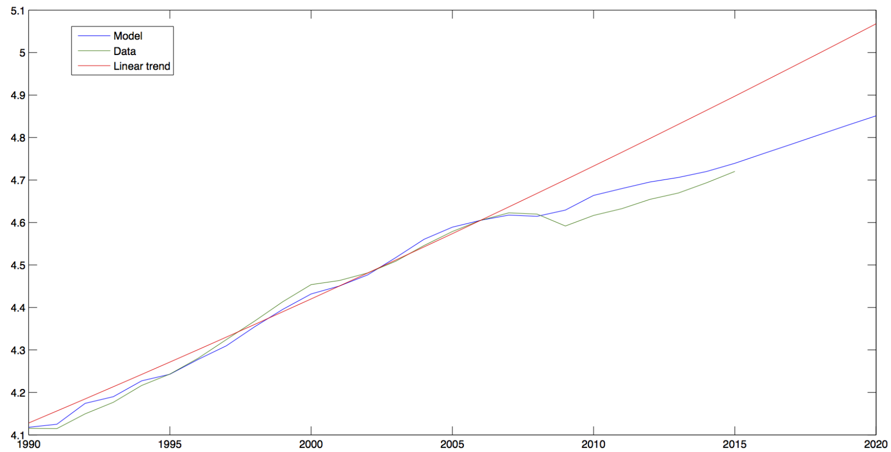


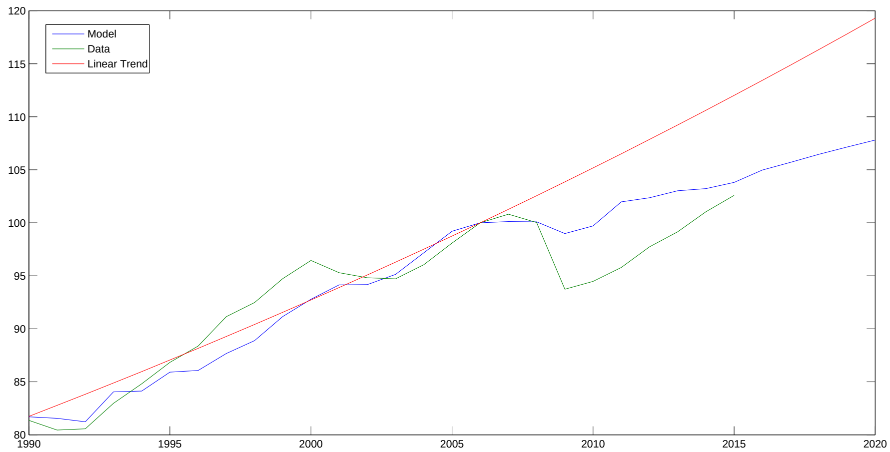
Figure 12 presents the evolution of GDP when we account for both demographic and TPF changes in our model. We can see that the model explains most of the drop in the GDP. Furthermore, by 2015 the gap between out model prediction and the data is only 2.5%. Thus the interaction of TFP and demographic changes do fairly well in capturing the evolution of output for the US economy.

Figure 14: GDP with Demographic and TFP Changes



Demographic and aggregate productivity changes generate a model employment counterpart that has similar movements to total employment hours seen in the data. For example, by adding TFP changes to the analysis, employment in the model falls in the year of the crisis, 2008. This feature was not captured in the previous exercise by construction. By 2015, the gap in between the model and data employment series was of about 1.2%. If our model was able to capture effects through the extensive margin, we suspect that this gap would be even smaller.

Figure 15: Labor with Demographic and TFP Changes



2.6 Conclusions

The effects of demographics may not be big or fast, but they have the potential of being important. The Great Recession coincides with a unique demographic period. At the start of the crisis, the generation of baby boomers started to enter retirement. Even though demographics is not what caused the Great Recession, it has the potential of explaining certain patterns for the slow recovery after the recession. For example, demographics play an important role in total hours worked and GDP not returning to their pre-crisis trend levels. In this paper, we quantify the effects demographics had on explaining the evolution of output and labor.

We develop a modified version of the standard growth model. This model incorporates demographics into the neoclassical framework through population growth rates and changes in population composition across time. We calibrate this model to test the implications of these demographic changes. First, we abolish the effects of the Great Recession on economic activity, so that we can evaluate how demographics affected output and labor for the time period 1990 - 2015. We find that the model explains 35% of the between output's pre-crisis trend and the data. We find that labor also drops but not substantially. This is a consequence of demographics only affecting labor through the intensive margin, amount of hours worked. If we considered a framework in which agents decided whether to participate in the labor force or not, the we expect that hours would drop further in the model. When we account for TFP changes for the time period analyzed, we find that the model does a better job of capturing movements in the data, for different macroeconomic aggregates.

It is important to note that our model only considers the effects of demographics on labor supply, which limits the effects of demographics on other variables. We are not considering potential interesting effects of savings decisions by different cohorts that can be important. Also, we are not considering effects that demographics can have on productivity, as suggested by Kuznets (1960). The first, can be carried out in a life cycle framework, while the second implies creating a theory of how productivity can be affected by aging and decreasing population growth rates. We consider both of these important in order to quantify the effects of demographics on economic activity. Our future research agenda will build upon the work of this paper and will seek to incorporate these two mechanisms into our analysis.

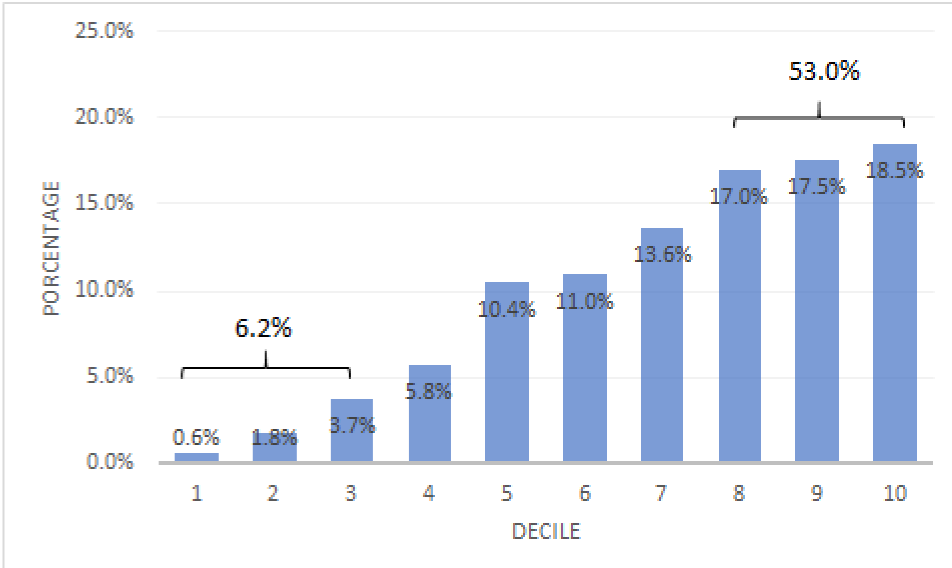
3 Financing of Public Universities in Paraguay: A solution to its regressivity

3.1 Introduction

The financing of university studies by the national state in Paraguay dates from 1889 when the National University of Asunción was created. In 2016 there were eight national universities. The countries of Latin America, as well as much of the world, finance tertiary education. In Paraguay, the State provided about 89% of the budget of all national universities in 2016. Other income, such as fees paid by students, covered the remaining 11%. There is an aspect of its financing by the state that affects the redistribution of income. This state expenditure, which is a subsidy in kind, go mostly to people who belong to high-income households. In 2016 only 6.2% of the students belonged to the poorest 30% of the population, while 53% came from the richest 30%. This expense increases income inequality in the country. In addition, the investment required by these universities is increasing. Compared to 0.55% of the GDP that they required in 2003, between 2012 and 2016 they required almost double, an average of 1% of the GDP. I propose to solve this problem of regressivity with two changes to the current system. First, eliminate all fees that, even if low, can be a real impediment for students with limited resources. Second, create an additional income tax for all those people who have studied at a national university and who earn more than a certain amount to be established. In no case, throughout the life of this person, the collection would exceed in real terms the cost of education. A similar reform was done in England starting in 1998. Murphy, Scott-Clayton and Wyness (2018) describe the effects of this reform and they find that charging tuition fees increased quality, quantity, and equity in higher education.

The simulations show that with a tax that does not exceed 8% gross for those with higher incomes, the recovery per student can reach 75%. This paper is divided into four parts. The first presents the data that show the regressivity of the financing

Figure 16: Students of undergraduate programs by economic decile



Source: Permanent Household Survey of Paraguay, 2016

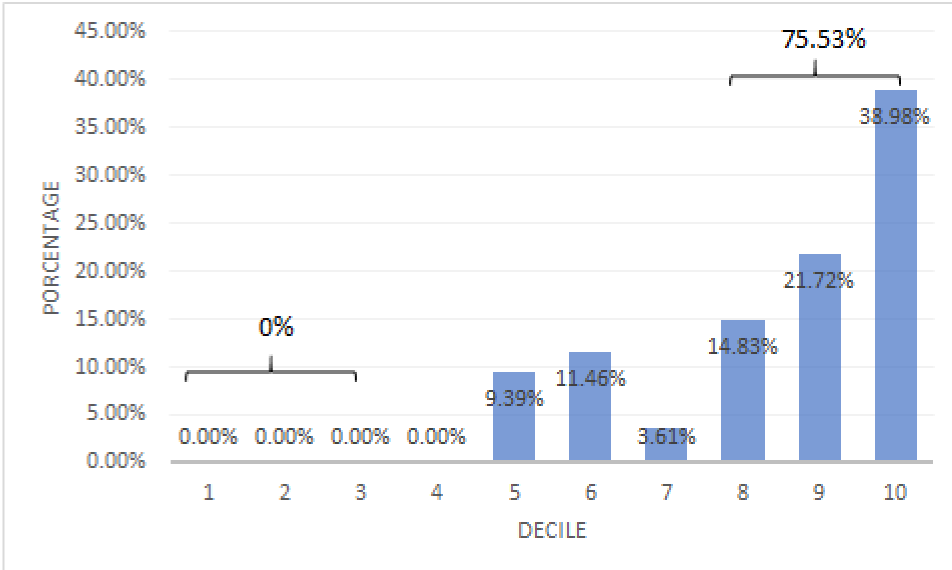
of public universities. The second part talks about the costs of public universities. The third proposes a change in the financing system and simulates collection. The fourth concludes the paper.

3.2 The regressivity of the funding of public universities

The Permanent Household Survey of Paraguay (EPH, *Encuesta Permanente de Hogares* in Spanish) provides the economic level of those who attend public universities (PUs). Students of PU undergraduate programs come mainly from households that come from the highest deciles of the population, as shown in Figure 1.

In 2016, 53% of students came from households that are in the top three deciles, while the lowest three deciles provided only 6.20% of students. The PUs also provide postgraduate university programs. In these programs, the representation of households with greater resources is even greater, and the representation of lower income households is even lower. Figure 2 shows the income distribution of students in post-

Figure 17: Students of graduate programs by economic decile



Source: Permanent Household Survey of Paraguay, 2016

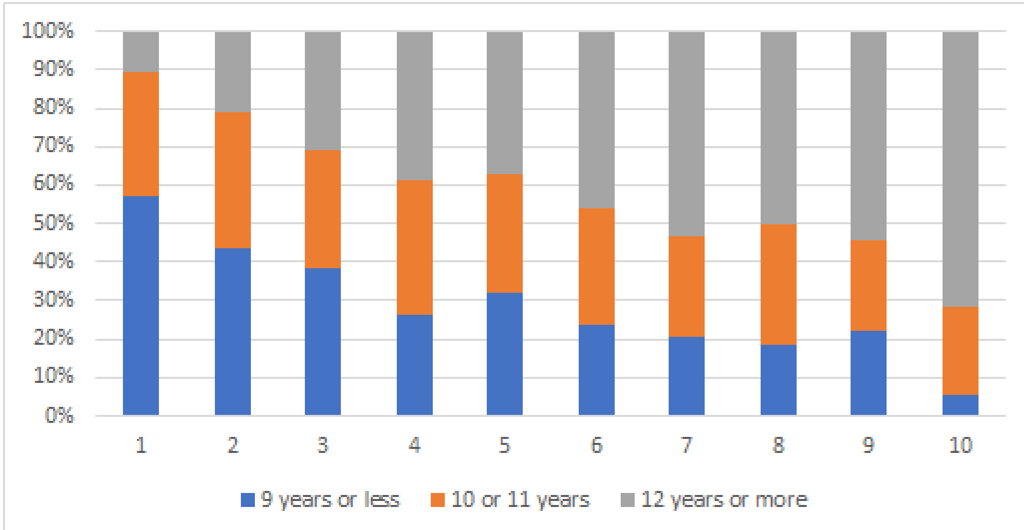
graduate programs.

From 2001 to 2016 there was not a single student from the three lowest deciles. In these years, the three highest deciles contributed between 75% and 93.69% of the students.

The fact that mostly high-income people attend PUs makes the state's spending on PUs regressive.

The paper by Gimenez, Lugo, Colman, Galeano and Farfan (2017) calculates that all state expenditures are progressive, except for spending on university education, which is regressive, although the value is close to zero. This same document calculates the concentration index for different taxes and expenses of the state. The authors find that all taxes are absolutely progressive. Also, almost all expenses are absolutely progressive, with the exception of subsidies for public transport and tertiary education. Tertiary education is relatively progressive, that is, the concentration of tertiary education spending goes mainly to the richest, but its concentration is not as unequal

Figure 18: Years of study completed by economic decile, people 18 and 19 years old



Source: Permanent Household Survey of Paraguay, 2016

as that of income.

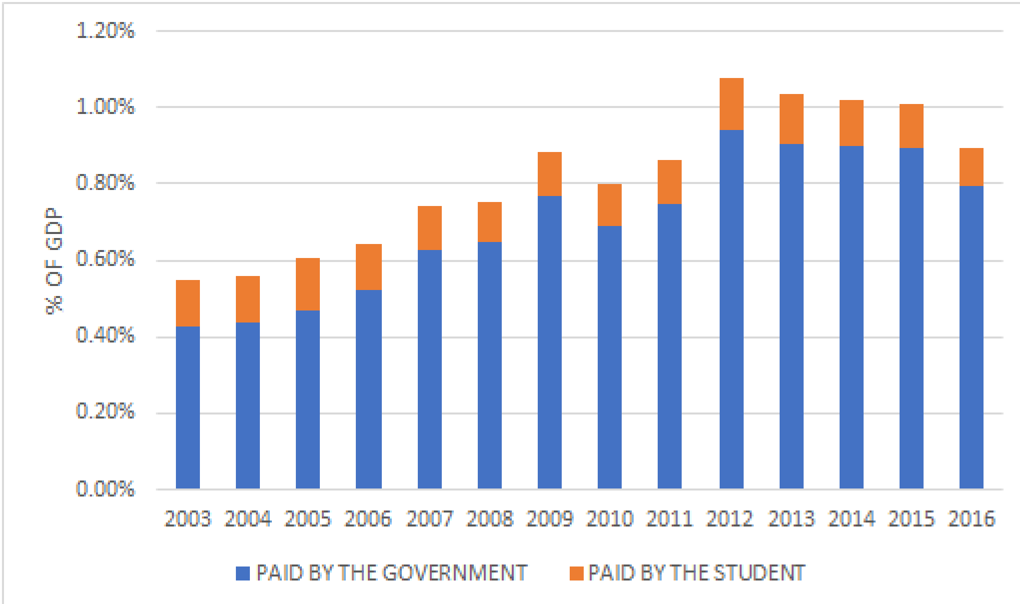
In the article by Sanguinetti, Berniell, Alvarez, Ortega, Arreaza and Penfold (2012), the concentration index of tertiary education is calculated for other Latin American countries. This document finds that, for Argentina, Bolivia, Brazil, Mexico, and Peru, the index is positive (as in the Paraguayan case). That is, this spending goes mostly to high-income people. Although the reasons why PU students are mostly high income are several, the EPH very concretely shows that the economic situation is in itself a very important factor. The poorest tend to abandon primary and secondary education at much higher rates than the rich. Figure 3 shows how many years of formal education were completed by 18- and 19-year-olds, where 12 years is expected if they have finished high school.

To access the university, one should have completed secondary education. Only 10.6% of the poorest have done so, compared with 71.5% of the richest. These data are very illustrative in pointing out that the usufruct of the PUs is concentrated in households with higher incomes. Finally, it should be noted that the National University of Asunción (UNA, *Universidad Nacional de Asunción* in Spanish) generates some reports on the socioeconomic level of new entrants. These reports, contrary to the above, report that students are mostly low income. In the appendix, I analyze the causes of why this report classifies students as low income when they are not necessarily so. The quality of these reports is suspect, partly based on the fact that it reports that in 2010 there were only three high-income students in the entire UNA of the more than 25,000 of students. According to incoming reports from 2011 to 2016, only ten more high-income students enrolled.

3.3 The costs of financing public universities

In 2003, Paraguay was financing four public universities: the National University of Asunción, the National University of the East, the National University of Pilar, and the National University of Itapúa. In 2007, the National University of Concepción received its first budget item, and in 2008, so did the National University of Villarrica del Espíritu Santo. In 2009 the National University of Caaguazú was added to the list and finally, in 2011, the National University of Canindeyú was included. By 2016, there were eight universities funded at least in part by the state The budget of these universities from 2011 to 2016 was 0.98% of GDP on average, approximately US$250 million for 2016. UNA receives 75% of this budget, and the remaining 25% is shared among the other seven universities. Not all the funding of national universities falls to the state. Students finance part of this expense through tuition, exam fees, and other fees. There is also other income from the production of goods, services, and donations. All of the income that comes from students is budgeted as "Source 30". The portion of the budget financed by Source 30 reached 21.99% in 2003, but it

Figure 19: Public Universities' Budget as a percent of GDP



Source: Ministry of Finance, Paraguay.

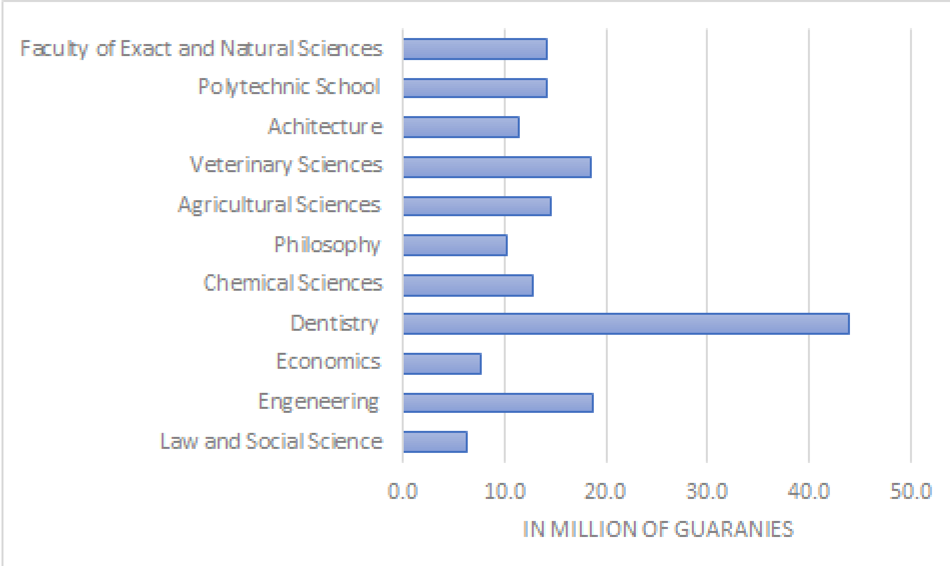
decreased overtime and by 2016 it reduced by almost half, to 11.59%. The rest is financed by the State, whose budgetary name is "Source 10".

The increase over time in the PU budget is due not only to the increase in the number of national universities in that period but also to an increase in the budget for the four PUs already existing in 2003. These four original universities had a budget of 0.55% of the GDP in 2003 and increased to 0.80% of GDP in 2016.

The budget of the PUs also shows an increase in their participation in the total budget of the state. In 2003, the PU budget represented 4.48% of the total budget of the state. In 2013, it reached as high as 6.28%, while it fell to 5.33% in 2016 (which is still higher than in 2003).

Not all PU expenses can be attributed to tertiary education itself. The Paraguay-Brazil Experimental College or CEPB (a primary and secondary school), for example, is part of UNA. This school represents approximately 1% of the entire UNA budget.

Figure 20: Average cost per student per college



Note: the College of Medical Sciences is excluded since it is not possible to distinguish the costs of tertiary education from the costs of the Hospital de Clínicas (an important and big hospital). The result of doing the same calculation for this faculty with all expenses gives 185 million of guaranies (national currency of Paraguay) per student. Source: 2016 Statistical Yearbook, National University of Asunción.

Much more important is the Hospital de Clínicas. This hospital is part of the budget of the UNA in the College of Medical Sciences. Although it is a necessary aspect of medical education because it is a hospital-school, only a part of the costs of this hospital can be attributed to medical training. This point is important since in 2016 40% of everything spent by the UNA was in the College of Medical Sciences.

The costs per student of the different faculties (departments) can be obtained for the UNA thanks to the statistical yearbook of the UNA. In figure 5, the expenditures of each faculty are divided by the number of students (of degree and postgraduate) in each faculty.

As stated previously, the College of Medical Sciences' numbers include all the operating costs of the Hospital de Clínicas, so they cannot be interpreted as a good approximation of the average cost of medical education.

Leaving aside the College of Medical Sciences, the rest of the faculties have an annual cost of between Gs. 6.33 million up to Gs. 43.8 million. The School of Law and Social Sciences and the College of Economic Sciences produce a professional at the lowest cost. The College of Dentistry, the most expensive, invests an annual average of Gs. 43.8 million per student.

These costs per student have been growing in real terms in recent years in the UNA. The following graph shows the average costs per student in terms of minimum wages (MW) for periods of five years, the average length of an undergraduate degree program in Paraguay. The reason for presenting the cost in minimum wages is that in Paraguay the minimum wages are adjusted once a year to preserve the value against inflation.

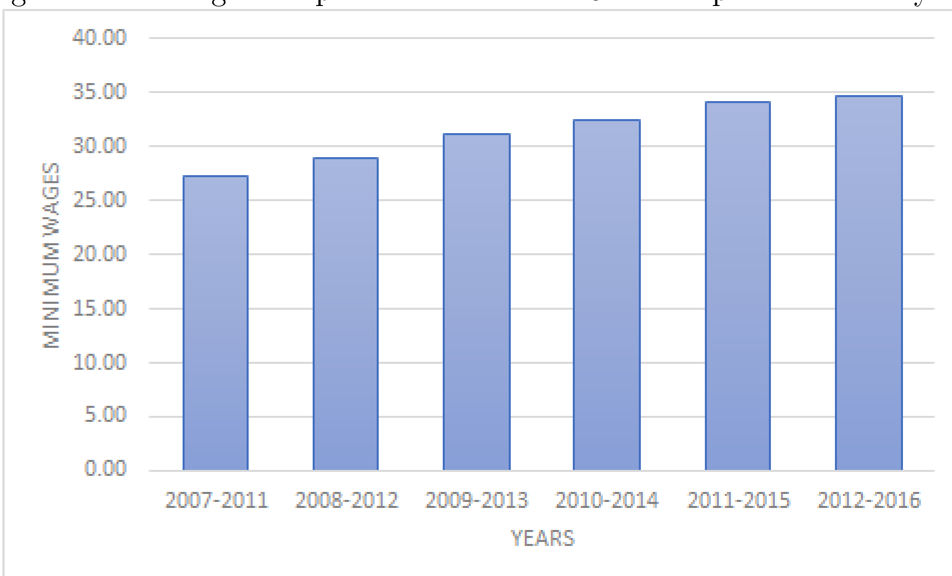
The PUs went from having a budget that represents approximately 0.5% to 1% of GDP in 13 years. This growth is due not only to a growth in the number of universities and students but also to the increase in the average real cost per student.

3.4 Proposed policy change and analysis of amount of recovery

The high concentration of students in the highest deciles of the population and the increase (in absolute terms and relative to GDP or the total state budget) of expenditures in the PUs imply that this state expenditure is regressive and increasingly important.

Therefore, below, I propose a change in the financing of public universities that allows the regression of this state expenditure to be eliminated without affecting other objectives that the PUs have. A proposal is then sought that does not affect the public nature of the PUs and their mission to be social mobilizers and creator of

Figure 21: Average cost per student for the UNA for periods of five years



The entire budget of the UNA is counted, excluding the Faculty of Medical Sciences. The bars are the result of dividing all these costs by the number of students of the UNA (excluding the Faculty of Medical Sciences), dividing them by the current minimum wage in each year, and adding them for the corresponding periods. It can be interpreted that a student who started the undergraduate program in 2007 and finished it in 2011 cost on average 27.21 MW. Source: 2016 Statistical Yearbook, National University of Asunción.

science, to give those with limited resources opportunities to earn a university degree, to promote meritocracy, etc.

The proposal has two parts:

1. Eliminate all fees that students must pay to complete the degree Although this represents only 11% or less of the entire PU budget, for low-income students it can represent a barrier to start or continue their university studies.

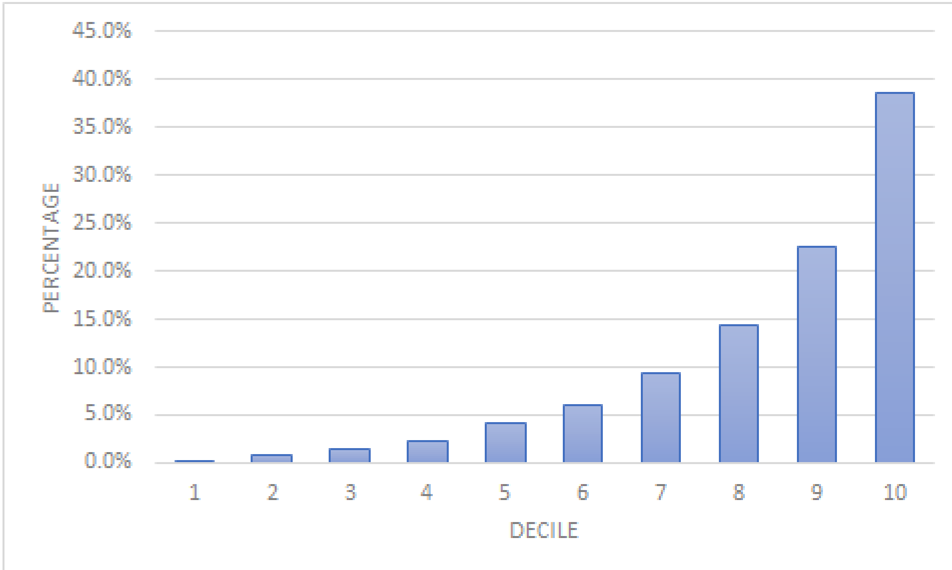
2. Establish a tax on the additional income that the people who have studied in a PU will pay in the future; these students will accumulate a debt (equal to the cost of their studies) with the state. These debts will be updated with the inflation rate every year and must be paid if the people achieve income above a certain amount to be established. If the income exceeds this amount, an additional income tax rate will be added to that established by the Personal Income Tax (IRP, *Impuesto a la Renta Personal* in Spanish)

This proposal is similar to systems established in England, Australia, and Uruguay, among other countries. Murphy, Scott-Clayton, and Wyness (2018) explain in detail the effects of the reform in England. In England, students don't pay while studying, but they repay later in the future is their annual income is above 21,000 pounds.

This system reduces the PU main problem, which is regressivity, since the beneficiaries of this expense pay for the cost of their education if their income is above a certain amount.

This system does not eliminate social mobility (in fact, it improves it by eliminating tariffs) and it still continues to reward meritocracy. Although it may not be free, you will never pay more than what was invested, and if you do not reach an income high enough in the future, you will pay less than what was received.

Figure 22: People 25 years old or older who attended the university



Source: Prepared by the author based on data from the EPH 2016

One of the main facts that motivate this proposal is that those who have attended university tend to have high incomes in the future. The following graph shows what deciles belong to those who have attended at least one year of university and today are over 25 years old.

With this proposal, the collection generated by each student will be between Gs 0 and the entire investment in the student. For each individual, the expected collection is equal to:

$$R_i = min\left\{\tau\left(I_j\right)\sum_{j=t_0}^{T}(1-P_j)\int I_j\left(\theta_j\right)d\left(\theta_j\right), Investment\ per\ student\right\}$$

Where the collection R_i is the sum of all future income I_j (which will depend randomly on the shock \theta_j) discounted the mortality rate P_j, multiplied by the tax rate \tau_j. As the collection cannot exceed what has been invested in the

Table 3: Parameters

Parameter	Value
Investment per student	34.59 MW
t_0	25
T	65

student, the final collection will be the minimum between the two objects.

3.4.1 Collection estimate

To estimate the revenue generated by this financing system, data from the 2016 EPH will be used. In the survey, we can obtain the income of people who have completed or partially attended university programs. Using this survey, one cannot distinguish between public and private university attendance, so I will use the income of all those with university degrees or who have attended, but not completed a university degree.

The income data will be sorted by the age of the person. For the estimation, it will be assumed that this income is equal to the expected income of the person with a university education.

From the population projections of the Directorate of Surveys, Statistics, and Censuses, mortality by age will be calculated.

With the mortality and income data, 50,000 people who passed through a PU will be simulated. Income will be treated as random shocks, where the probability of obtaining income $\underline{I}_j$ will be equal to the proportion of people with age j and income equal to $\underline{I}$ observed in the EPH 2016.

The parameters used in the estimation are found in the following table:

The investment per student is calculated as the average expenditure per student for the UNA, taking all the faculties into account with the exception of the Faculty of Medical Sciences. That investment is equal to Gs. 60,680,000 for the five years

Table 4: Tax rate bracket

Bracket (τ_I)	Rate
0 - 3 MW	0%
3 - 4 MW	1%
4 - 5 MW	2%
5 - 6 MW	3%
6 - 7 MW	4%
7 - 8 MW	5%
8 - 9 MW	6%
10 - 11 MW	7%
11 or more	8%

Table 5: Recovery for other tax rates and costs

		Cost per student in monthly MWs		
		34.59	40	45
Tax rates	1% less in each bracket	66.3%	61.6%	57.3%
	Same bracket	74.1%	69.8%	66.4%
	1% more in each bracket	79.78%	76.4%	73.1%

between 2012 and 2016. To discount the effects of inflation, all values will be expressed in the number of minimum wages (MW) of the year. This investment corresponds to 34.59 MW per student, for a period of five years.

The ages from 25 to 65 years will be used.

The tax rates for the fiscal year are established in the following table:

Results: the results of the simulation show that 52.94% of students would pay the cost of their university study in its entirety. The rest is partially paid, putting the total recovery of the investment at 74.1%.

We can do some robustness analysis by adjusting the investment per student and the interest rates, shown in the table below:

3.5 Conclusion

PUs in Paraguay attract mostly students who belong to households in the highest income deciles. The reasons why this happens are beyond the scope of this paper. One of them can be seen in the EPH since the family income itself is important for explaining the difference in dropout rates of primary and secondary education. Only 10% of 18- and 19-year-olds of the lowest decile have finished secondary education, compared with 71% of the highest decile.

The cost of PUs is not negligible. It represents approximately 1% of GDP or just over 5% of all state spending and this number is steadily increasing.

We propose a tax that corrects the problem of the regressivity of this expense, without affecting the main objectives of the PU. This tax, similar to that implemented in other countries, is easy to apply. Because of its design, it only affects those who earn enough to pay it.

Our simulation shows that with maximum rates of 8%, the system can generate a recovery of at least 75% of everything invested per student.

References

Better spending for better lives: how Latin America and the Caribbean can do more with less / edited by Alejandro Izquierdo, Carola Pessino and Guillermo Vuletin.

Chen, K., , A., & İmrohoroğlu, S. (2006a). “The Japanese Saving Rate”. The American Economic Review, 96(5), 1850-1858. Retrieved from http://www.jstor.org/stable/30034999

Chen, K., Imrohoroglu, A. & Imrohoroglu, S. (2006b). "Secular Movements in U.S. Saving and Consumption," 2006 Meeting Papers 154, Society for Economic Dynamics.

Diamond, P. (1998). "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." The American Economic Review 88, no. 1: 83-95.

Dirección General de Encuestas Estadísticas y Censos. (2017). “Encuesta Permanente de Hogares”.

East, C. (2018). "Immigrants’ labor supply response to Food Stamp access," Labour Economics, Elsevier, vol. 51(C), pages 202-226.

Farhi, E., and Werning, I. (2012) "Insurance and Taxation over the Life Cycle." The Review of Economic Studies 80, no. 2 (283): 596-635.

Flood, Sarah, Miriam King, Steven Ruggles, and J. Robert Warren. Integrated Public Use Microdata Series, Current Population Survey: Version 4.0. [Machine-readable database]. Minneapolis: University of Minnesota, 2015.

Friedman, M. and Friedman, R. (1962). Capitalism and freedom.

Gimenez, L., Lugo, M., Martinez, S., Colman, H., Galeano, J., Farfan, G. (2017) “Paraguay: Análisis del sistema fiscal y su impacto en la pobreza y la equidad”. Commitment to Equity (CEQ) Working Paper Series 74, Tulane University, Department of Economics.

Hall, R. E. (2014). "Quantifying the Lasting Harm to the U.S. Economy from the Financial Crisis," NBER Macroeconomics Annual 2014, Volume 29, pages 71-128 National Bureau of Economic Research, Inc.

Hayashi, F. and Prescott, E. (2002). "The 1990s in Japan: A Lost Decade," Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 5(1), pages 206-235, January.

Heathcote, J. and Perri, F. (2016). “Wealth and Volatility”. Working Paper. Federal Reserve Bank of Minneapolis and CEPR.

Heathcote, J. and Tsujiyama, H. (2015). “Optimal Income Taxation: Mirrlees Meets Ramsey”. CEPR Discussion Paper No. DP10380.

Henriksen, E. and Cooley, T. (2016). “The Demographic Deficit”. Working paper.

Jones, D. and Marinescu, I. (2017). “The Labor Market Impacts of Universal and Permanent Cash Transfers: Evidence from the Alaska Permanent Fund.” Working paper.

Judd, K. L., Ma, D., Saunders, M. and Su, C. (2006). “Optimal Income Taxation with Multidimensional Taxpayer Types.” .

Kuznets, S. (1960). “Population Change and Aggregate Output”. Chapter in “Demographic and Economic Change in Developed Countries”. Columbia University Press. Pages 324 - 351.

Lockwood, B. and Weinzierl, M. (2015). "De Gustibus non est Taxandum: Heterogeneity in preferences and optimal redistribution," Journal of Public Economics, Elsevier, vol. 124(C), pages 74-80.

Mankiw, N. G., Weinzierl, M. and Yagan, D. (2009). "Optimal Taxation in Theory and Practice." Journal of Economic Perspectives 23, no. 4: 147–174.

Marinescu, I. (2018). "No Strings Attached: The Behavioral Effects of U.S. Unconditional Cash Transfer Programs," NBER Working Papers 24337, National Bureau of Economic Research, Inc.

Maestas, N., Mullen, K. and Powell, D. (2016). "The Effect of Population Aging on Economic Growth, the Labor Force and Productivity," Working Papers 1063-1, RAND Corporation.

Mian, A. and Sufi, A. (2012). "What explains high unemployment? The aggregate demand channel," Chicago Booth Working Paper.

Mian A., Rao, K. and Sufi,A. (2013), “Household Balance Sheets, Consumption, and the Economic Slump”,Quarterly Journal of Economics, 128, 1687–1726.

Midrigan, V. and Philippon, T. (2016). "Household Leverage and the Recession," CEPR Discussion Papers 11407, C.E.P.R. Discussion Papers.

Mirrlees, J. A. "An Exploration in the Theory of Optimum Income Taxation." The Review of Economic Studies 38, no. 2 (1971): 175-208.

Mortenson, J.. and Whitten, A. (2016). “How Sensitive Are Taxpayers to Marginal Tax Rates ? Evidence from Income Bunching in the United States.”

Murphy, R., Scott-Clayton, J. and Wyness, G. (2018). "The end of free college in England: Implications for enrolments, equity, and quality," Economics of Education Review.

Paulus, A. (2016), The antipoverty performance of universal and means-tested benefits with costly take-up, No 16/12, ImPRovE Working Papers, Herman Deleeck Centre for Social Policy, University of Antwerp

Ruh, P. & Staubli, S. (2018). "Financial Incentives and Earnings of Disability Insurance Recipients: Evidence from a Notch Design," CEPR Discussion Papers 12979, C.E.P.R. Discussion Papers.

Saez, E.. (2001). “Using Elasticities to Derive Optimal Income Tax Rates”. Review of Economic Studies. 68. 205-29.

Saez, E.. (2002). “Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses”. The Quarterly Journal of Economics. 117. 1039-1073.

Saez, E.l. (2010). "Do Taxpayers Bunch at Kink Points?" American Economic Journal: Economic Policy 2, no. 3 : 180-212.

Sanguinetti, P., Berniell, L., Álvarez, F., Ortega, D., Arreaza, A., and Penfold, M. (2012) “Finanzas públicas para el desarrollo. Fortaleciendo la conexión entre ingresos y gastos” (Reporte de Economía y Desarrollo (RED)). Caracas: CAF. Retrieved from http://scioteca.caf.com/handle/123456789/169

Schaal, E. and Taschereau-Dumouchel, M. (2015). "Coordinating Business Cycles," 2015 Meeting Papers 178, Society for Economic Dynamics.

Shimer, R. (2012). "Wage rigidities and jobless recoveries," Journal of Monetary Economics, Elsevier, vol. 59(S), pages S65-S77.

Slemrod, J. (2013). Buenas notches: Lines and notches in tax system design. eJournal of Tax Research. 11. 259-283.

The National University of Asunción. “Anuario Estadístico 2016”.

The National University of Asunción. “Informe Socioeconómico de Ingresantes”, years 2010, 2011, 2012, 2013, 2014, 2015, 2106.

Appendix A. Appendix to chapter 1

Proof of proposition.

Given the distribution for w and θ the distribution of the unified-type n is given by:

$$f(n) = \begin{cases} \frac{\alpha w_0^\alpha}{n^{\alpha+1}} \frac{1}{\theta_H - \theta_L} \left[\frac{(n/w_0)^{\alpha+2} - \theta_L^{\alpha+2}}{\alpha+2} \right] & \text{if } n/w_0 < \theta_H \\ \frac{\alpha w_0^\alpha}{n^{\alpha+1}} \frac{1}{\theta_H - \theta_L} \left[\frac{\theta_H^{\alpha+2} - \theta_L^{\alpha+2}}{\alpha+2} \right] & \text{if } n/w_0 \geq \theta_H \end{cases}$$

The modified welfare weight $\hat{b}(n) = \left(\int_0^\infty \frac{b(n/\theta)\hat{h}(\theta,n)d\theta}{f(n)} \right)$ is then:

$$\hat{b}(n) = \begin{cases} \frac{\kappa(\alpha+2)}{n^\eta(\eta+1)} \left[\frac{\theta_H^{\eta+1} - \theta_L^{\eta+1}}{(n/w_0)^{\alpha+2} - \theta_L^{\alpha+2}} \right] & \text{if } n/w_0 < \theta_H \\ \frac{\kappa(\alpha+2)}{n^\eta(\eta+1)} \left[\frac{\theta_H^{\eta+1} - \theta_L^{\eta+1}}{\theta_H^{\alpha+2} - \theta_L^{\alpha+2}} \right] & \text{if } n/w_0 \geq \theta_H \end{cases}$$

Then $\hat{b}(n) f(n) = \frac{\kappa\alpha w_0^\alpha}{n^{\alpha+\eta+1}(\eta+1)} \left[\theta_H^{\eta+1} - \theta_L^{\eta+1} \right]$.

From (8)we have:

$$G(n) = \frac{\int_0^n \hat{b}(m) f(m)\, dm}{\int_0^\infty \hat{b}(m) f(m)\, dm} = \frac{\int_0^n \frac{\kappa\alpha w_0^\alpha}{m^{\alpha+\eta+1}(\eta+1)} \left[\theta_H^{\eta+1} - \theta_L^{\eta+1} \right] dm}{\int_0^\infty \frac{\kappa\alpha w_0^\alpha}{m^{\alpha+\eta+1}(\eta+1)} \left[\theta_H^{\eta+1} - \theta_L^{\eta+1} \right] dm}$$

$$= \frac{\frac{\kappa\alpha w_0^\alpha}{(\eta+1)} \left[\theta_H^{\eta+1} - \theta_L^{\eta+1} \right] \int_{w_0\theta_L}^n \frac{1}{m^{\alpha+\eta+1}} dm}{\frac{\kappa\alpha w_0^\alpha}{(\eta+1)} \left[\theta_H^{\eta+1} - \theta_L^{\eta+1} \right] \int_{w_0\theta_L}^{+\infty} \frac{1}{m^{\alpha+\eta+1}} dm} = \frac{\int_{w_0\theta_L}^n \frac{1}{m^{\alpha+\eta+1}} dm}{\int_{w_0\theta_L}^{+\infty} \frac{1}{m^{\alpha+\eta+1}} dm}$$

$$= \frac{\frac{1}{w_0\theta_L} - \frac{1}{n^{\alpha+\eta}}}{\frac{1}{w_0\theta_L}} = \frac{n^{\alpha+\eta} - w_0\theta_L}{n^{\alpha+\eta}}$$

From (3) we have that:

$$\frac{T'(y(n))}{1 - T'(y(n))} = \frac{1 + 1/\gamma}{n f(n)} (G(n) - F(n))$$

$$= \frac{1+1/\gamma}{n\frac{\alpha w_0^\alpha}{n^{\alpha+1}}\frac{1}{\theta_H-\theta_L}\left[\frac{\theta_H^{\alpha+2}-\theta_L^{\alpha+2}}{\alpha+2}\right]}\left(\frac{n^{\alpha+\eta}-w_0\theta_L}{n^{\alpha+\eta}}-\left(1-\frac{\theta_H^{\alpha+2}-\theta_L^{\alpha+2}}{(\alpha+2)(\theta_H-\theta_L)}\left(\frac{w_o}{n}\right)^\alpha\right)\right)$$

$$= \frac{(1+1/\gamma)(\theta_H-\theta_L)(\alpha+2)}{\alpha\left[\theta_H^{\alpha+2}-\theta_L^{\alpha+2}\right]}\left(\frac{n}{w_0}\right)^\alpha\left(\frac{n^{\alpha+\eta}-w_0\theta_L}{n^{\alpha+\eta}}-1+\frac{\theta_H^{\alpha+2}-\theta_L^{\alpha+2}}{(\alpha+2)(\theta_H-\theta_L)}\left(\frac{w_o}{n}\right)^\alpha\right)$$

$$= \frac{(1+1/\gamma)(\theta_H-\theta_L)(\alpha+2)n^\alpha}{\alpha w_0^\alpha\left[\theta_H^{\alpha+2}-\theta_L^{\alpha+2}\right]}\left(\frac{n^{\alpha+\eta}-w_0\theta_L}{n^{\alpha+\eta}}-1\right)+\frac{(1+1/\gamma)}{\alpha}$$

The first term of the equation goes to zero as $\epsilon \to 1$, since the parentheses term goes to zero (while the term not in parenthesis goes to a constant).

So:

$$\frac{T'(y(n))}{1-T'(y(n))} \to \frac{(1+1/\gamma)}{\alpha},\ \forall n \geq w_0\theta_H$$

Appendix B. Appendix to chapter 2

To calibrate the model, we use standard methods of Growth Accounting. Parameters δ and K_0, the depreciation rate and the initial capital are calculated together, so that two conditions are satisfied. We use 1964 as the first data of capital, so any miscalculation of the initial capital is reduced by depreciation over time. First, the initial capital output ratio of the data is equal to the average of the capital output ratio for the first ten years,

$$\frac{K_{1964}}{Y_{1964}} = \frac{1}{10}\sum_{1964}^{1974}\frac{K_t}{Y_t}.$$

Second, the depreciation rate times the average of the capital-output ratio of the model is equal to the average of the ratio of depreciation over GDP in the data.

$$\frac{1}{44}\sum_{1964}^{2007}\frac{\delta K_t}{Y_t} = \frac{1}{44}\frac{depreciation}{Y_t}$$

θ is calculated as the sample average over 1990 to 2007 of the compensation of capital. α^a is calculated for every age with the formula:

$$\alpha^a = \frac{T - h^a}{T - h^b}\alpha^b,$$

such that the hours decided by each cohort matches the sample average of the data. β is calculated using the intertemporal equation 16. T is the total discretionary hours in a week.

Appendix C. Appendix to chapter 3

The National University of Asunción, through its "Socioeconomic Report of Entrants," presents numbers that contradict those presented by the EPH. For example, it says that in 2016 more than 97% of entrants were from "Low," "Medium Low," or "Medium" income and the remaining less than 3% were from "Medium-high" or "High" income. The UNA Socioeconomic Report of Entrants presents enormous problems to determine the actual social stratum of the members. Before going to the methodological and incentive reasons why the report should be considered inaccurate, I will first mention a very telling inaccuracy. The following table shows the number of "High" income students that were enrolled in 2010 (especially the students, not just incoming students) and then the number of "High" income students who were enrolled from 2011 to 2016.

Table 6: Students of “High” income level at the UNA

Surveyed	Year	Number of respondents	“High” income people	Percentage
All the students	2010	25,227	3	0.01%
Incoming students	2011	5,474	2	0.04%
Incoming students	2012	5,379	0	0.00%
Incoming students	2013	5,085	5	0.10%
Incoming students	2014	6,011	1	0.02%
Incoming students	2015	5,441	1	0.02%
Incoming students	2016	4,870	1	0.02%

There were three students of "High" income in the UNA in 2010 and then they enrolled only ten more from 2011 to 2016. In the last decade, a total of 13 high-income students went through the entire UNA. Only the Faculties of Law, Philosophy, Agronomy, Exact Sciences, and Economics ever saw "High" students, the rest of the faculties none. This data is very striking. Below are some of the reasons why so few students enter the "High" income classification based on the 2016 report: 1. The methodology does not really measure social stratum. One may imagine that if five categories are in use, category 1 should represent the poorest 20% of the country, 2 should be the next 20-40%, and so on. However, this is not the case here. Rather, it is based on a system of points where family income only represents 15% (15 points out of 100). That the student works, for example, is worth 12 (16 maximum, 4 minimum) points, the academic level and the occupation of the parents are worth 34 points. Then, to be of "medium-high" income, you must have at least 71 points, and for "high" income, 87 points. It could well be that, if we measure the whole country, only 1% enter into what this report defines as high income. Some characteristics of this report: a. If the entrant does not work, he automatically loses 12 points. For only that reason he can no longer be considered high income, regardless of family income or any wealth that he may have. But does this really measure if one is high or low income? It is likely that those with high incomes are the ones who decide not to work since they must dedicate time to income. In addition, there are programs that, for the time they demand, make it impossible to work. What is found is quite the opposite: in this case not working is probably an indicator of having a high income and not a low one. The same report shows that 70% of all students do not work. b. If both parents have "only" college degrees, two points are lost. They have to have at least Masters degrees or Ph.D.s to earn all the points in this category. c. If there are five or more people living in one household, points are lost. Income is not taken into consideration. d. If only one parent works, 6 points are lost. e. If one does not own a car, 5 points are lost. f. Having landline telephones is worth 1 point. But one must own two telephones to earn the whole point. Having only one telephone costs half a point. g. All this makes it very difficult to be categorized as "high" or "medium-high" income. By way of illustration, let us consider a non-working student whose father has gone to college, who has 3 siblings, a mother who does not work, only one telephone, and the father earns the equivalent of 30 monthly minimum wages. This report would classify the student as “middle” income, while the EPH shows that this family belongs to the highest decile and is among the richest families in the country.

2. This report engages in what is known as "adverse selection". This survey is not only used for statistical purposes, but it is also used to apply for scholarships. Therefore, those who really have low incomes will be sure to complete the survey, while those who really have high income may do so, but may not. Only 64% of university entrants answered this survey.

3. There is also a "hidden Information" problem. If pretending to be a family income situation is more precarious than in reality increases the probability of obtaining a scholarship, many will do so. In other words, there is a strong incentive for people to lie on this survey in order to be eligible for a scholarship.

There are potentially many other problems such as, for example, that students do not know the exact income of parents and others. But I think these points are enough to explain the difference with the EPH.