

Theory of a Planckian metal

Advances in strongly correlated electronic systems
ASCES 2019

Fine Theoretical Physics Institute, University of Minnesota
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Talk online: sachdev.physics.harvard.edu



Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \rightarrow 0,$$

where U is the strength of interactions, and E_F is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim U^2 T^2 \quad \text{with } \tau \sim \tau_{\text{eq}}$$



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- These times are much longer than the ‘Planckian time’ $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\text{eq}} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \rightarrow 0.$$



Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ , is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!



Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal T -linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142-147

A. Legros^{1,2}, S. Benhabib³, W. Tabis^{3,4}, F. Laliberté¹, M. Dion¹, M. Lizaire¹, B. Vignolle³, D. Vignolles³, H. Raffy⁵, Z. Z. Li⁵, P. Auban-Senzier⁵, N. Doiron-Leyraud¹, P. Fournier^{1,6}, D. Colson², L. Taillefer^{1,6*} and C. Proust^{3,6*}

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,^{1,2} Tristin Metz,² Christopher Eckberg,² Kevin Kirshenbaum,² Alex Hughes,² Renxiong Wang,² Limin Wang,² Shanta R. Saha,² I-Lin Liu,^{2,3,4} Nicholas P. Butch,^{2,4} Zhonghao Liu,^{5,6} Sergey V. Borisenko,⁵ Peter Y. Zavalij,⁷ and Johnpierre Paglione^{2,8}

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,^{1,*} Debanjan Chowdhury,^{1,*} Daniel Rodan-Legrain,¹ Oriol Rubies-Bigordà,¹ Kenji Watanabe,² Takashi Taniguchi,² T. Senthil,^{1,†} and Pablo Jarillo-Herrero^{1,†}

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown¹, Debayan Mitra¹, Elmer Guardado-Sanchez¹, Reza Nourafkan², Alexis Reymbaut², Charles-David Hébert², Simon Bergeron², A.-M. S. Tremblay^{2,3}, Jure Kokalj^{4,5}, David A. Huse¹, Peter Schauf^{1*}, Waseem S. Bakr^{1†}

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11 \text{ kbar}$	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of T -linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, *Nature Physics* **15**, 142 (2019)

1. Quantum matter with quasiparticles:
random matrix model
2. Quantum matter without quasiparticles:
the complex SYK model

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

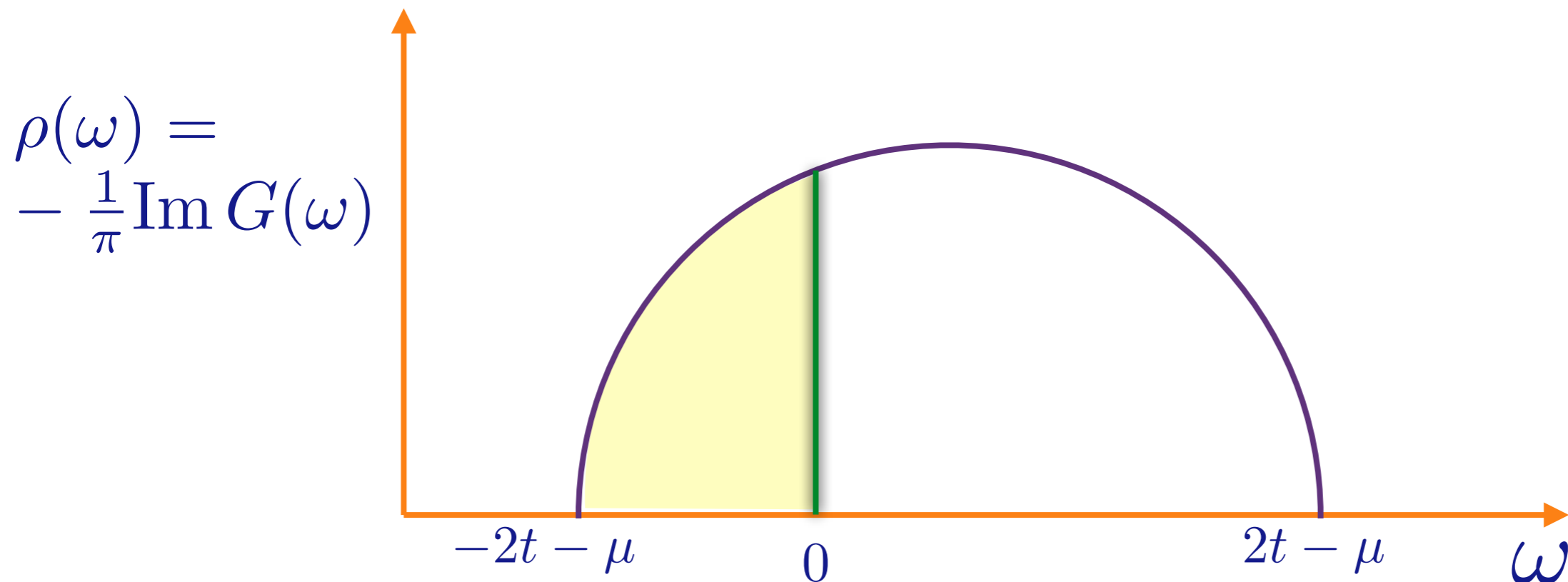
**Fermions occupying the eigenstates of a
 $N \times N$ random matrix**

A simple model of a metal with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

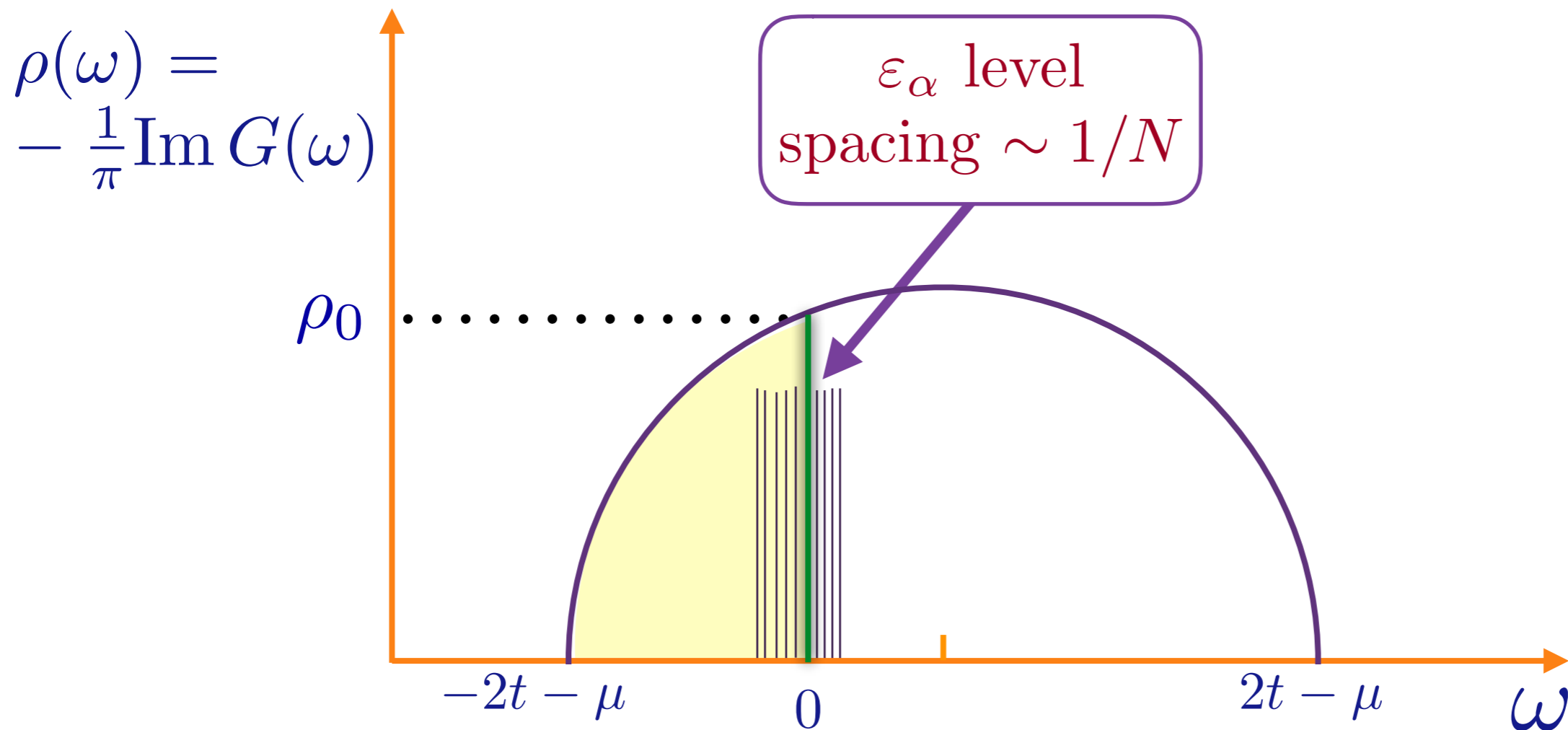
$G(\omega)$ can be determined by solving a quadratic equation.



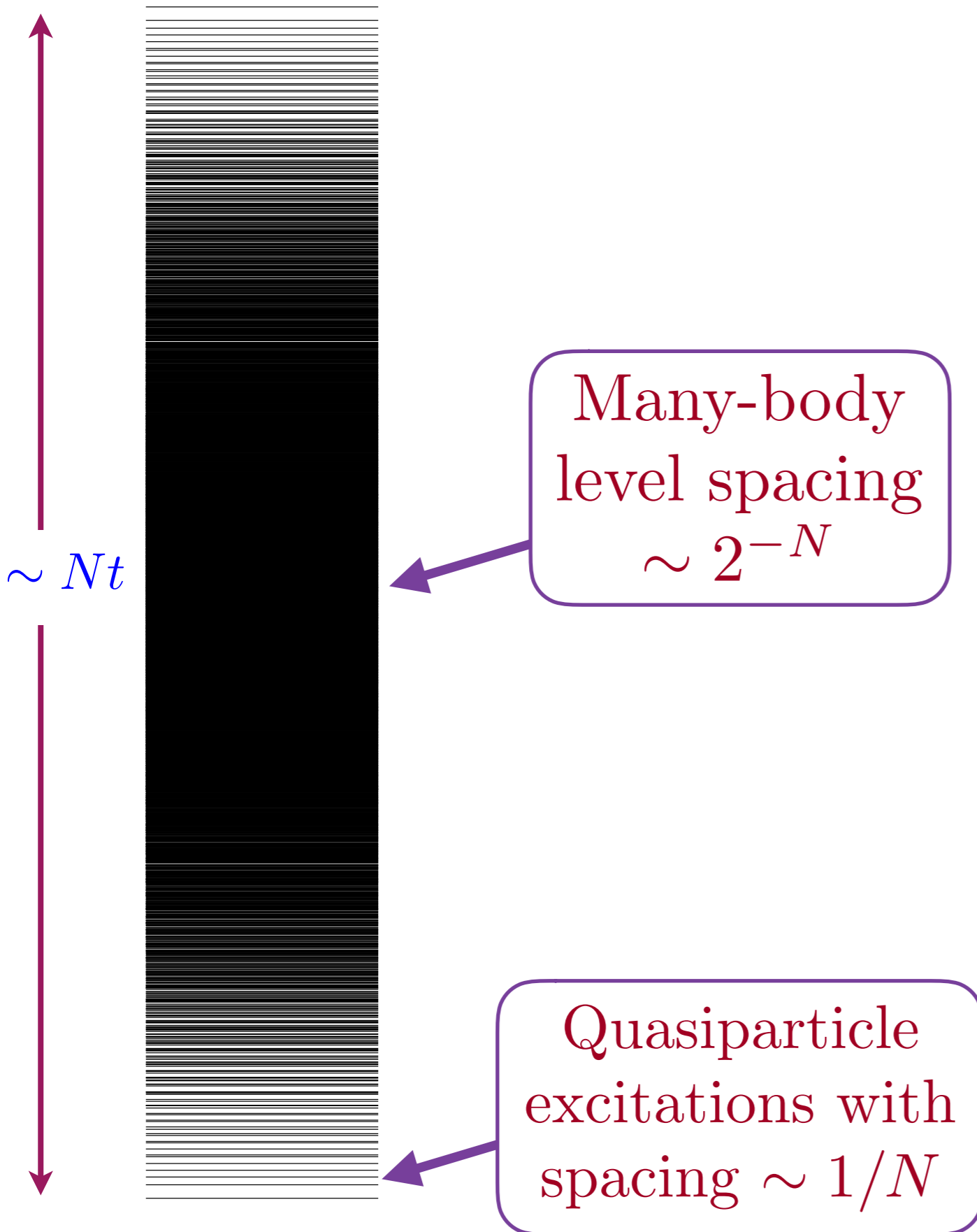
A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix t_{ij}/\sqrt{N} . The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F . The single-particle density of states is

$$\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha), \text{ and } \rho_0 \equiv \rho(\omega = 0).$$



A simple model of a metal with quasiparticles



There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^N n_{\alpha} \varepsilon_{\alpha},$$

where $n_{\alpha} = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_{α} have a level spacing $\sim 1/N$.

A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|\overline{U_{ij;kl}}|^2 = U^2$. We compute the lifetime of a quasiparticle, τ_α , in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamiltonian with energy ε_α . By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta)) \delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2 \end{aligned}$$

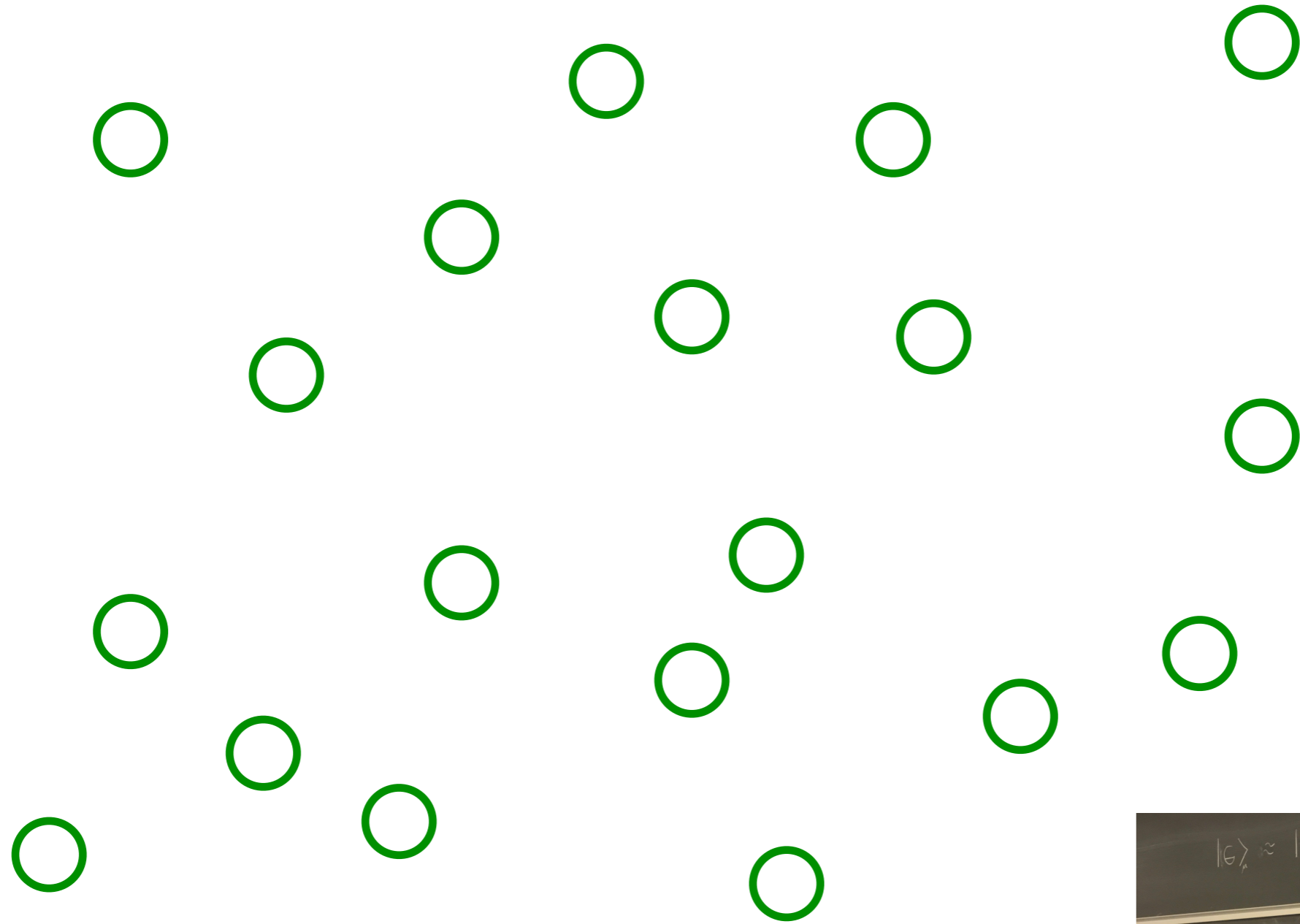
where ρ_0 is the density of states at the Fermi energy, and $f(\varepsilon) = 1/(e^{\varepsilon/T} + 1)$ is the Fermi function.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

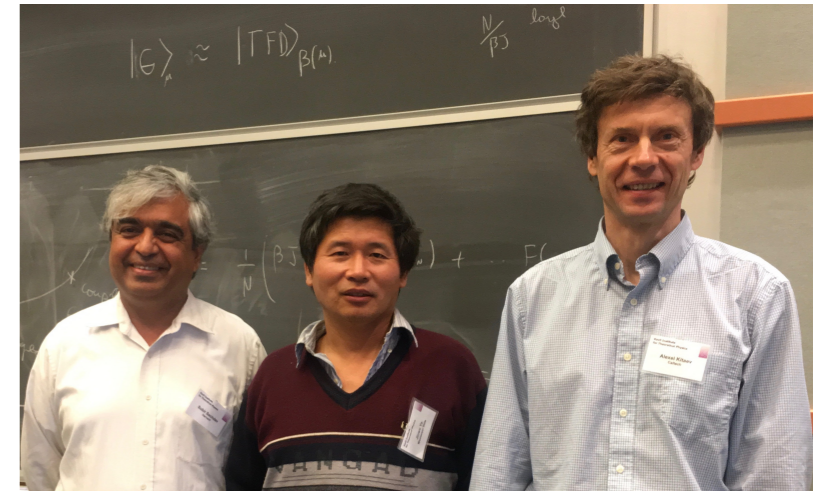
1. Quantum matter with quasiparticles:
random matrix model

2. Quantum matter without quasiparticles:
the complex SYK model

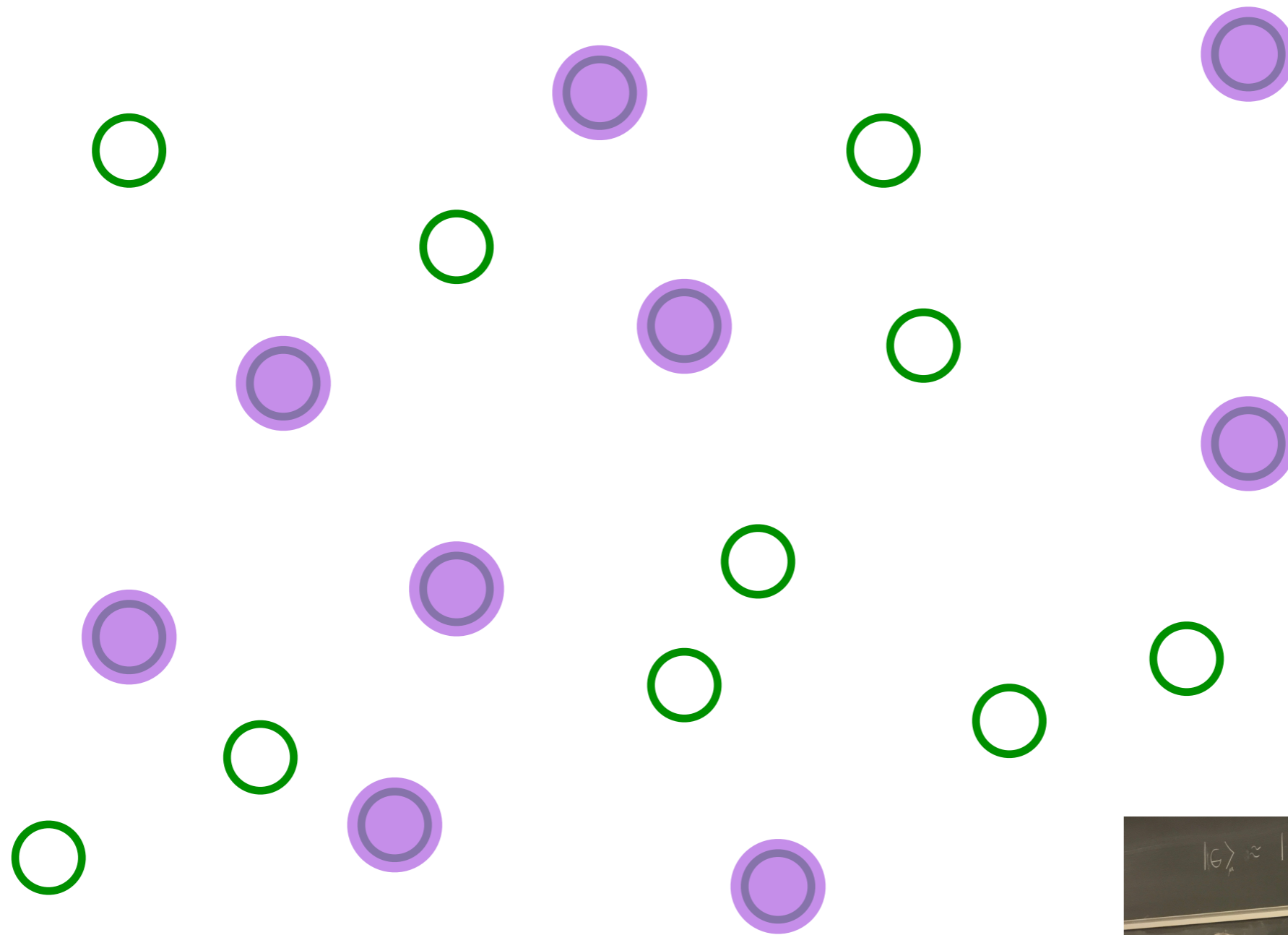
The Sachdev-Ye-Kitaev (SYK) model



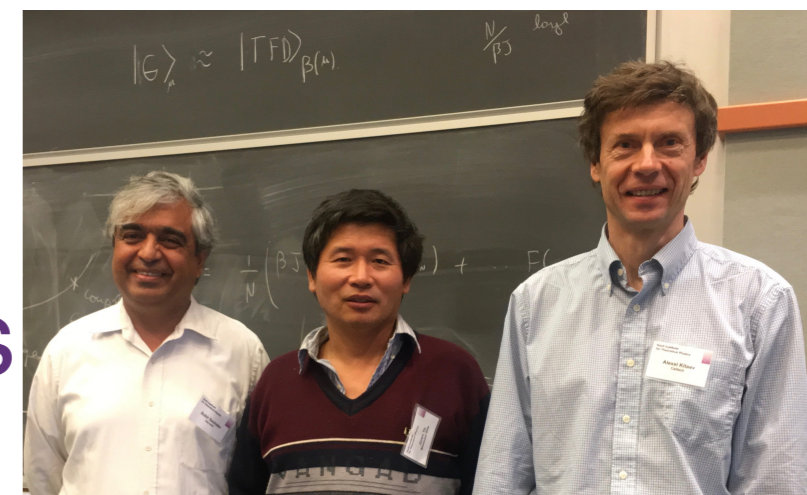
Pick a set of random positions



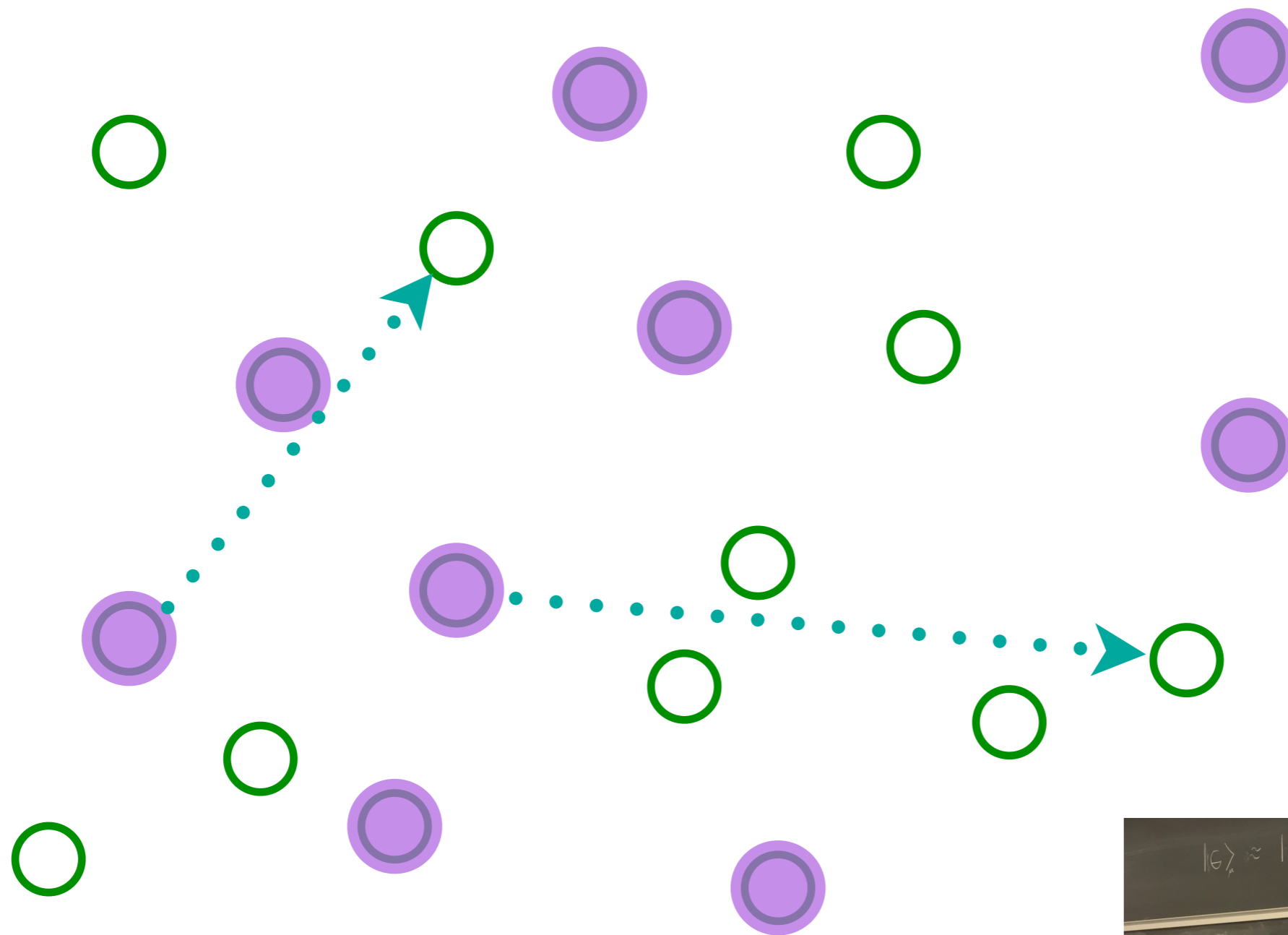
The SYK model



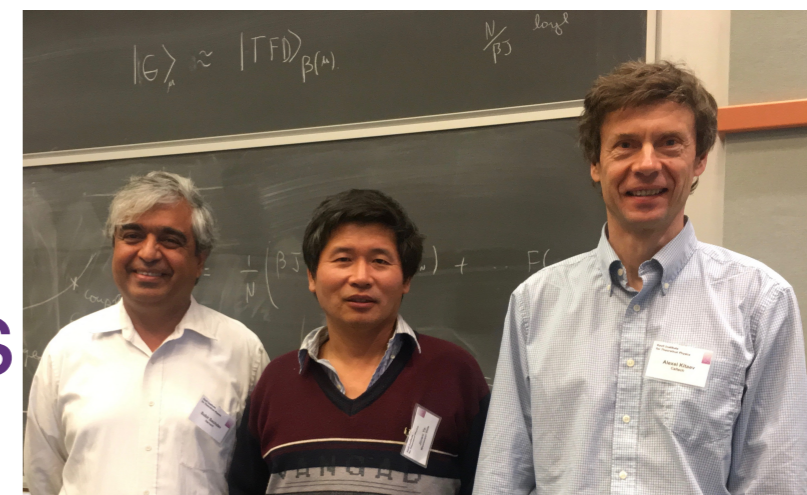
Place electrons randomly on some sites



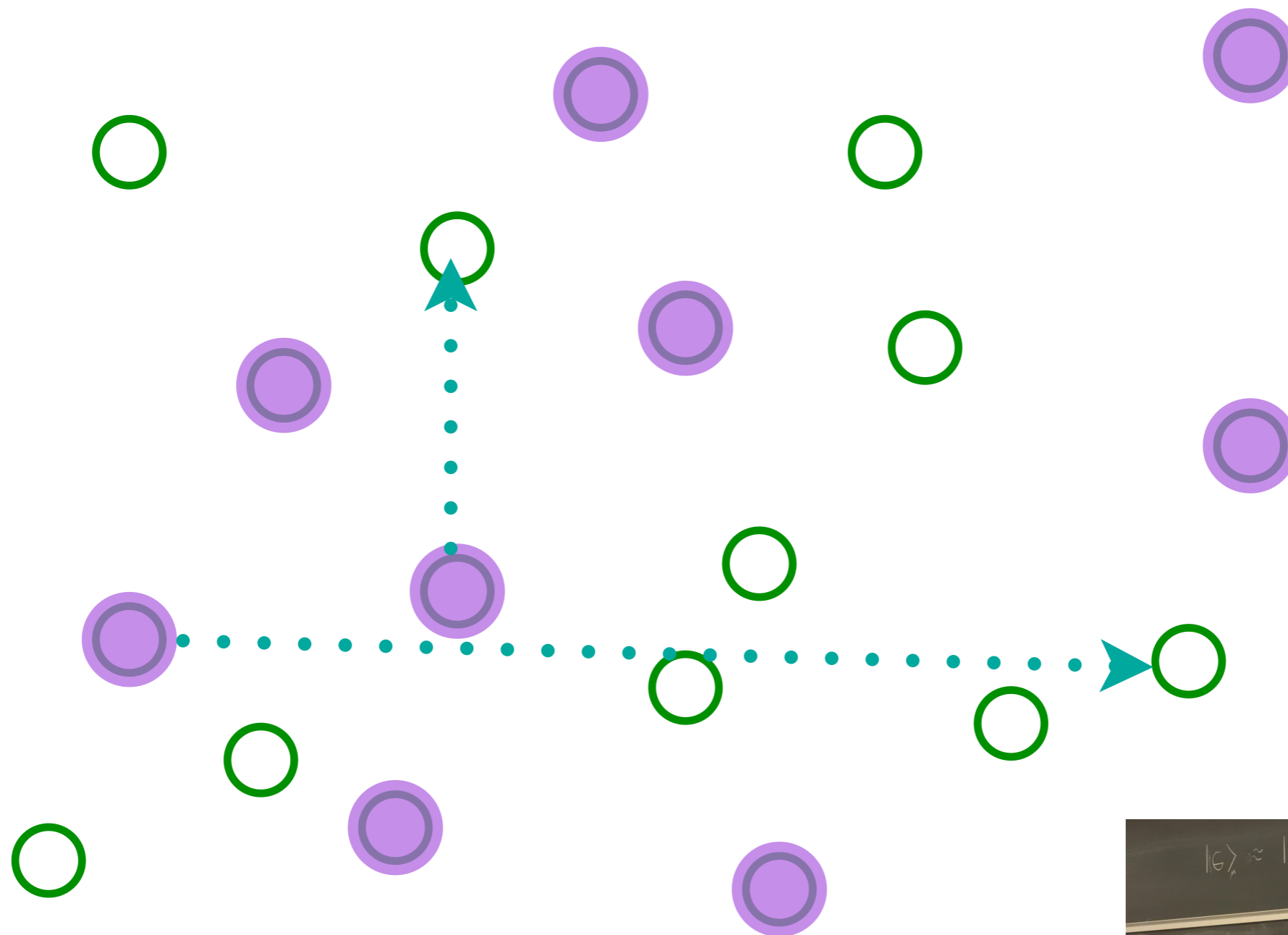
The SYK model



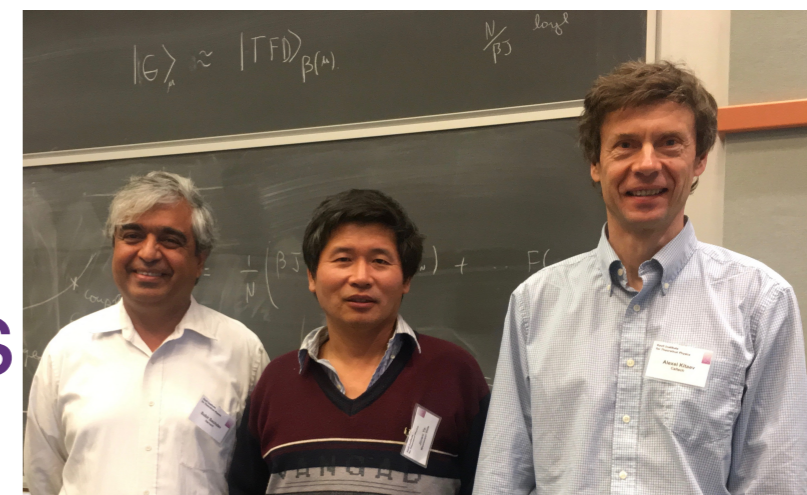
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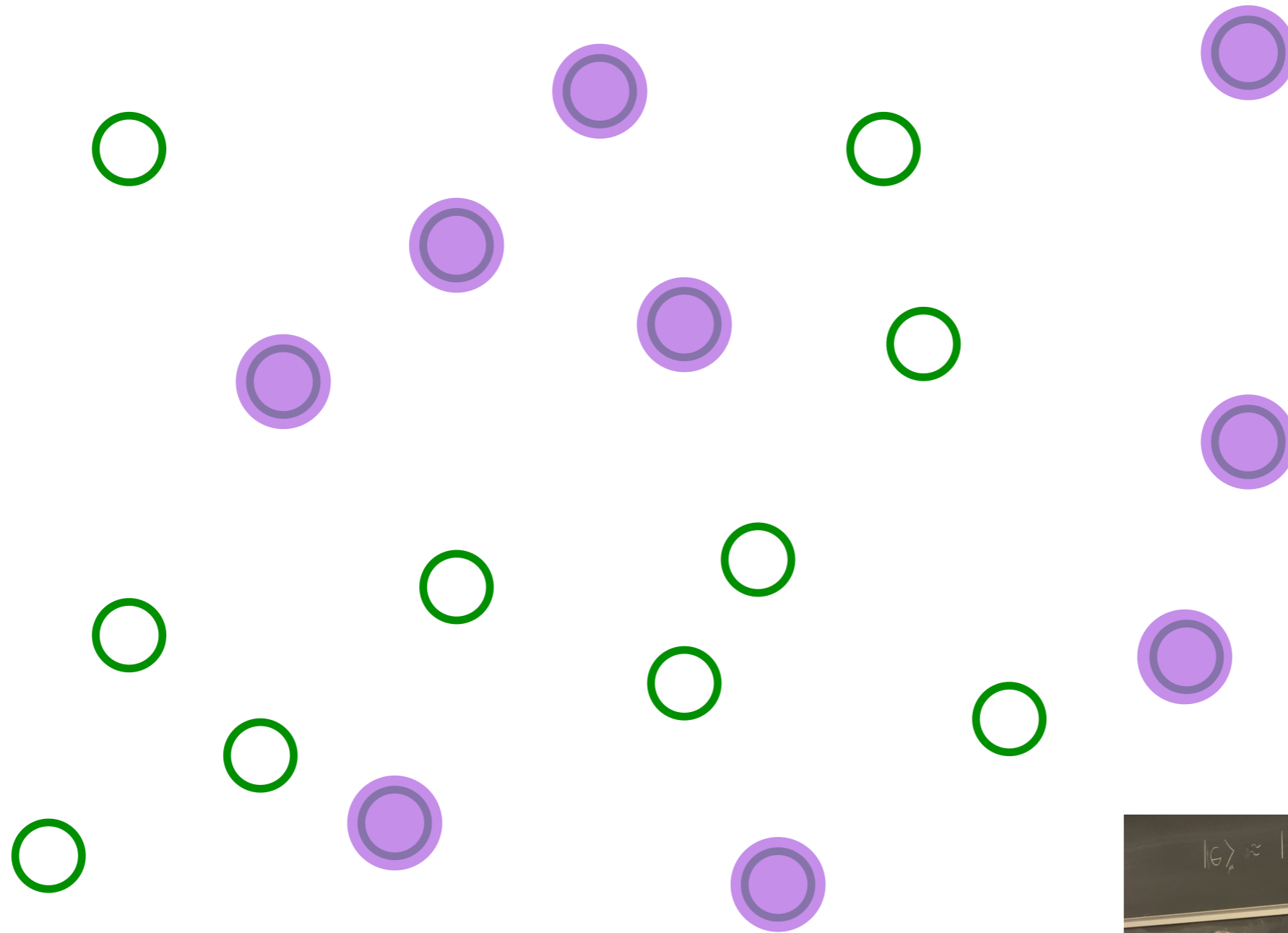
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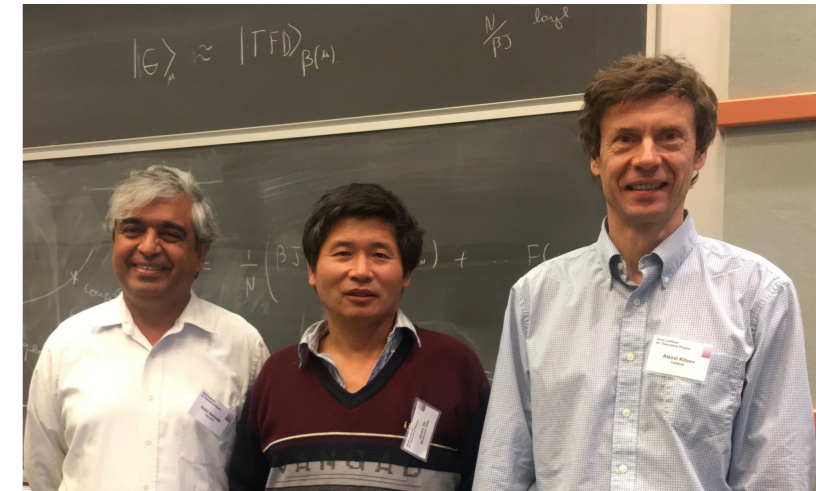
Place electrons randomly on some sites



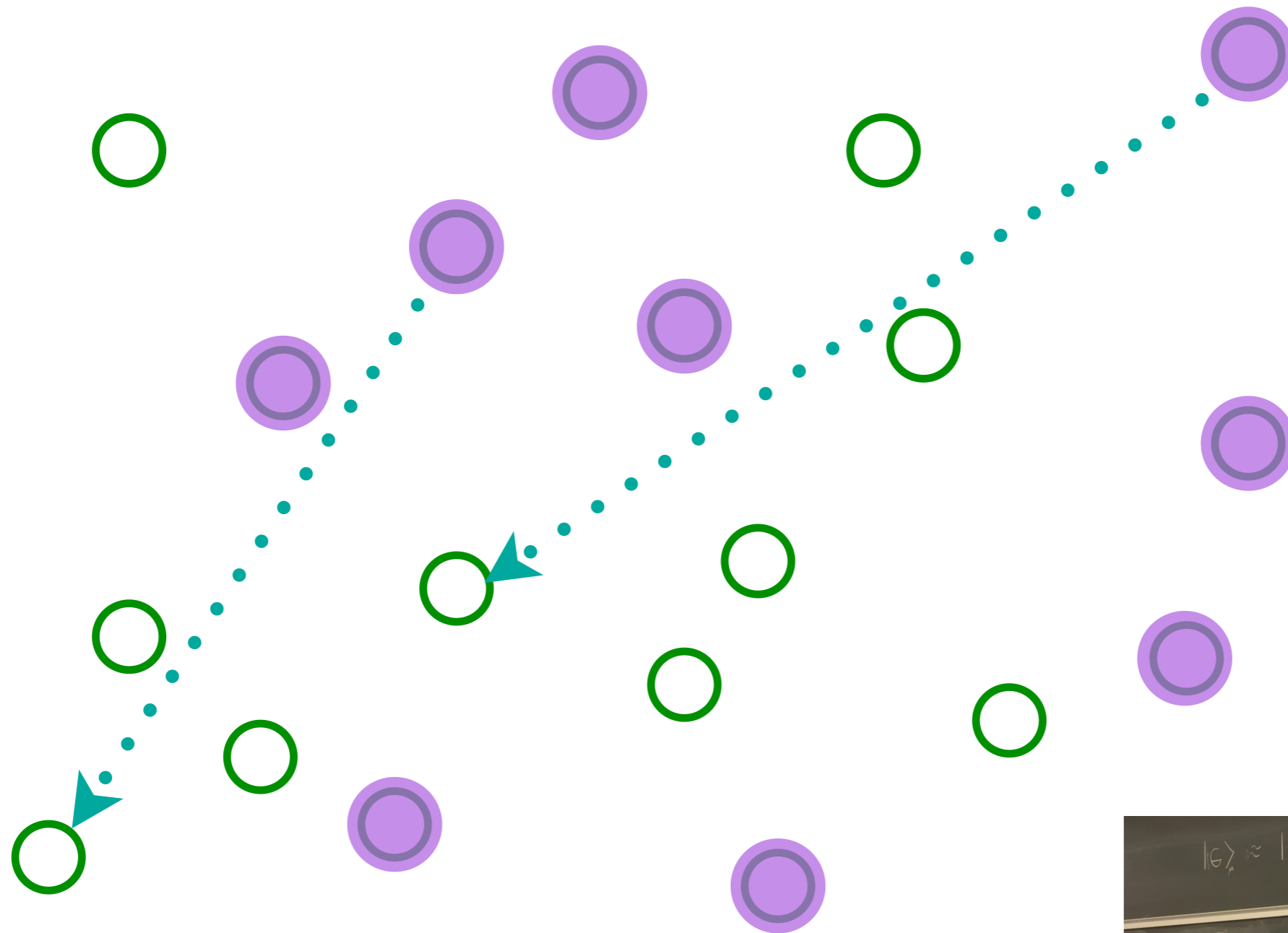
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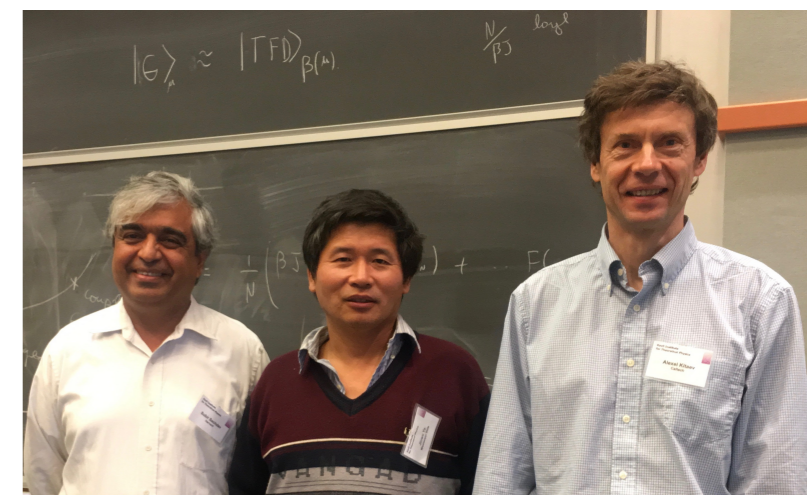
Entangle electrons pairwise randomly



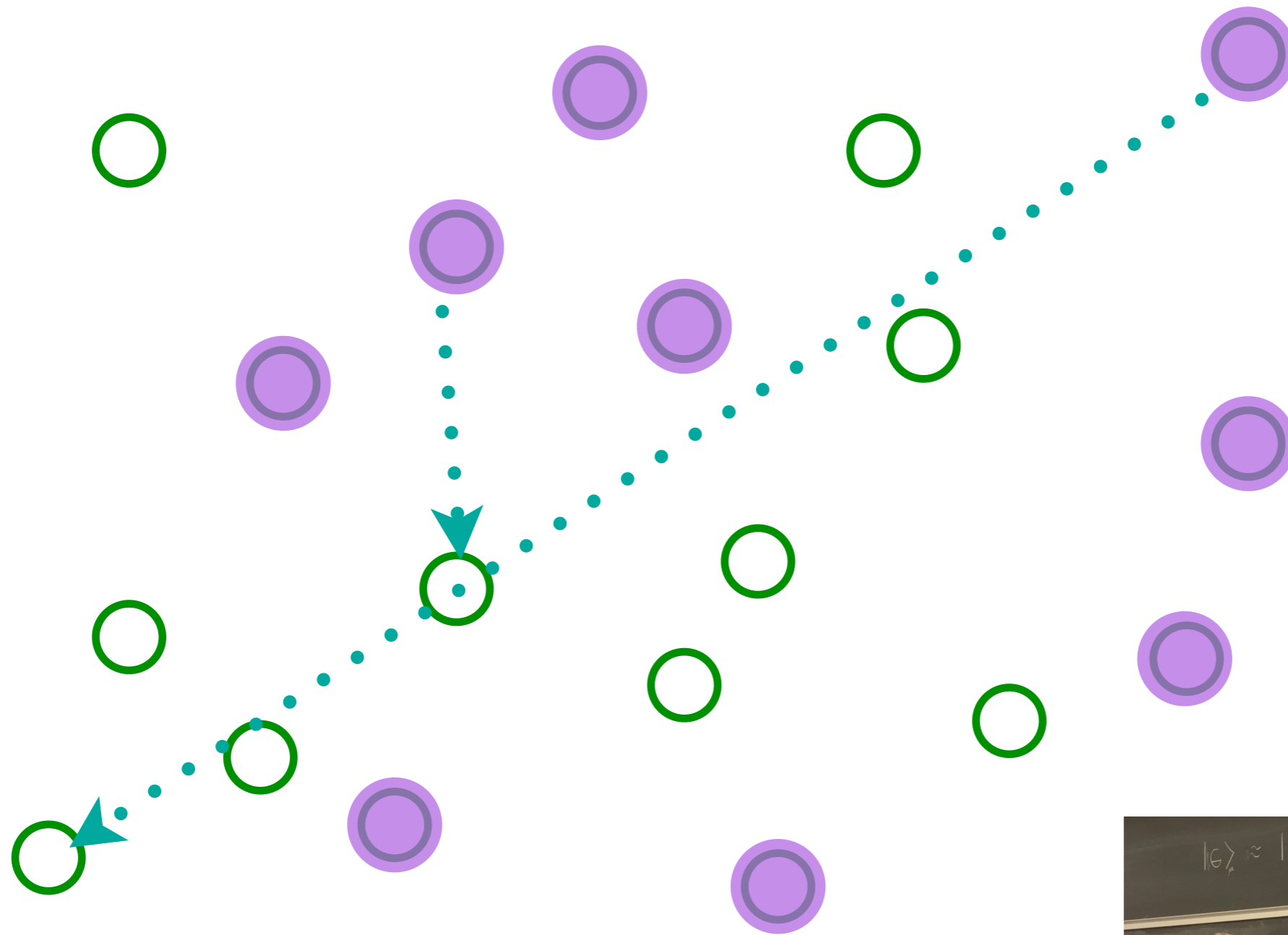
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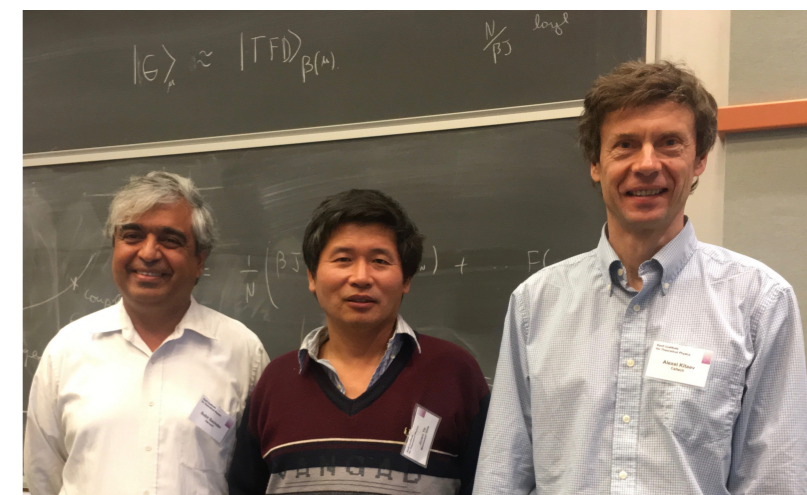
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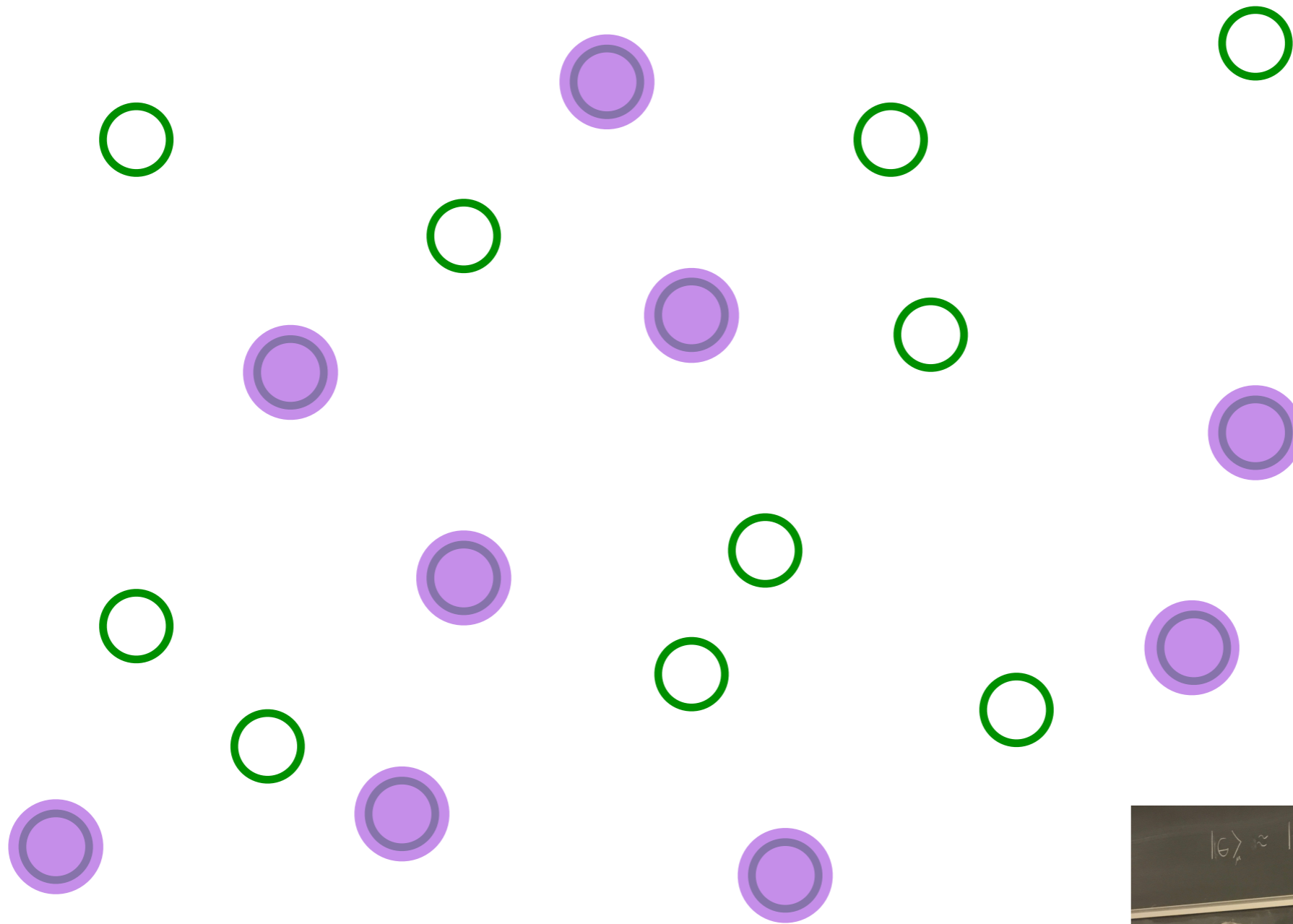
The SYK model



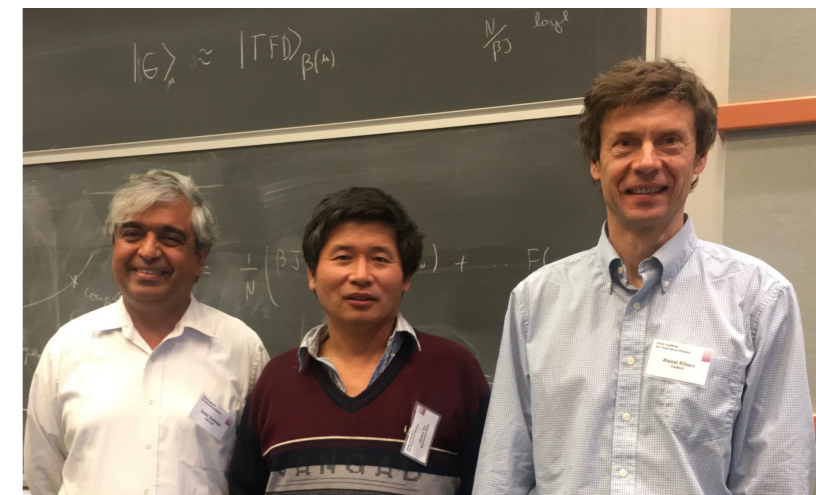
Entangle electrons pairwise randomly



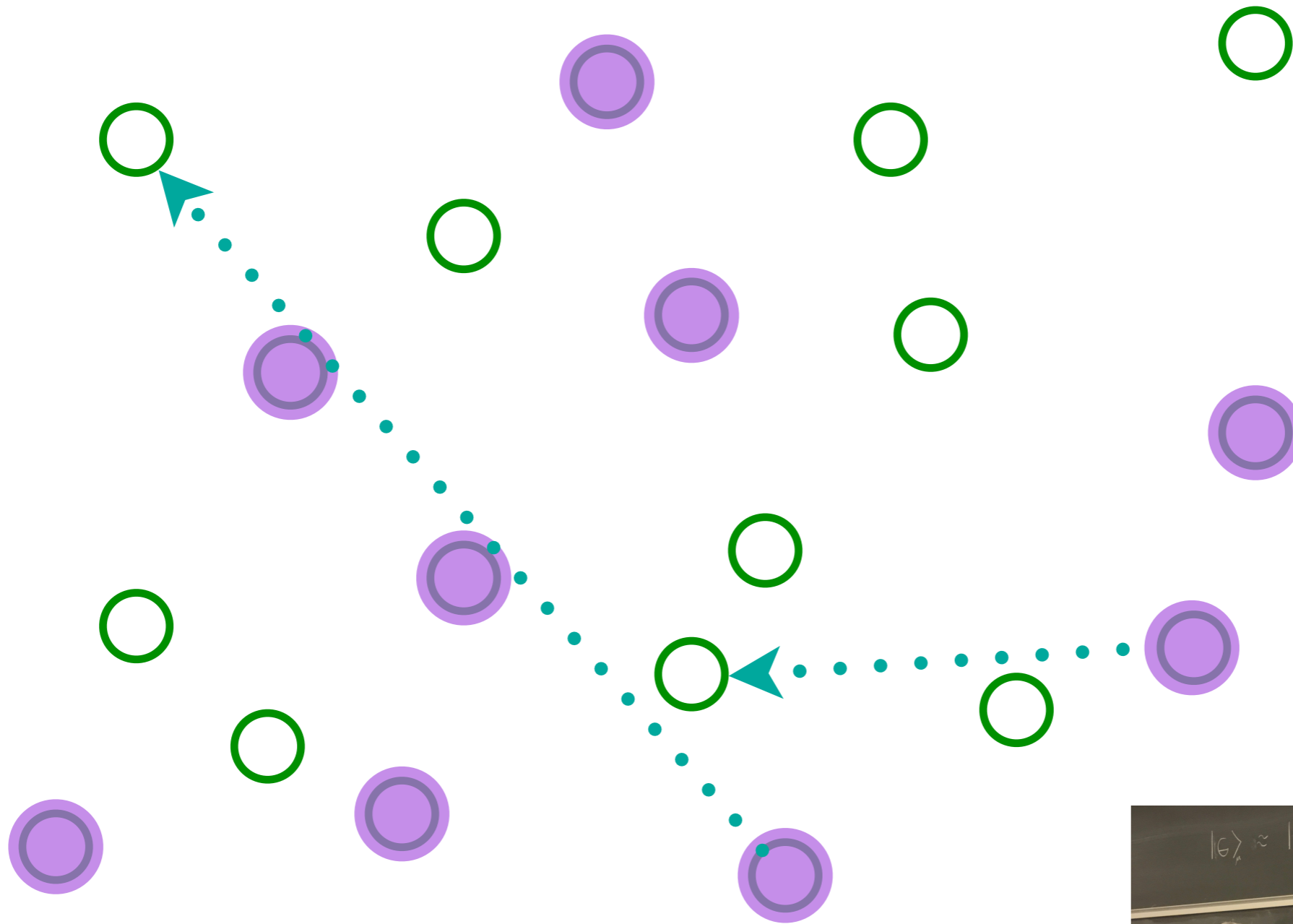
The SYK model



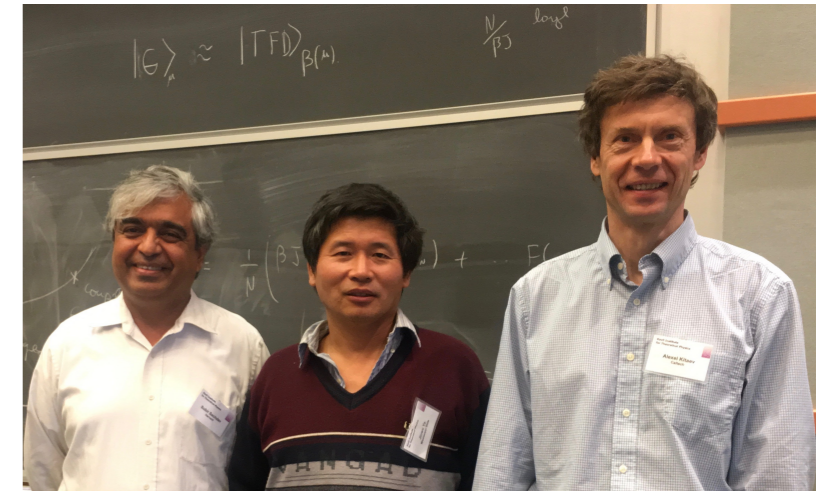
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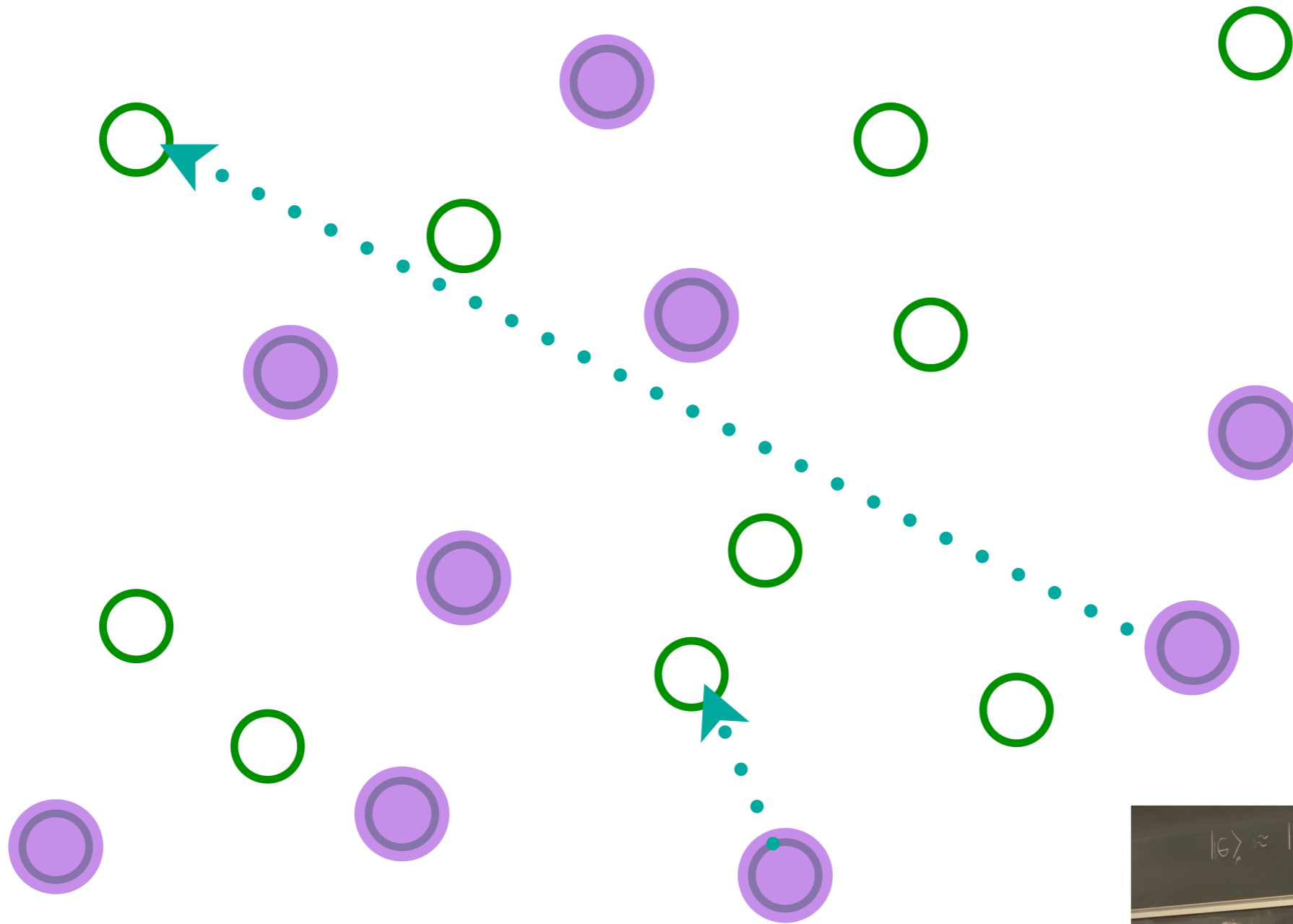
The SYK model



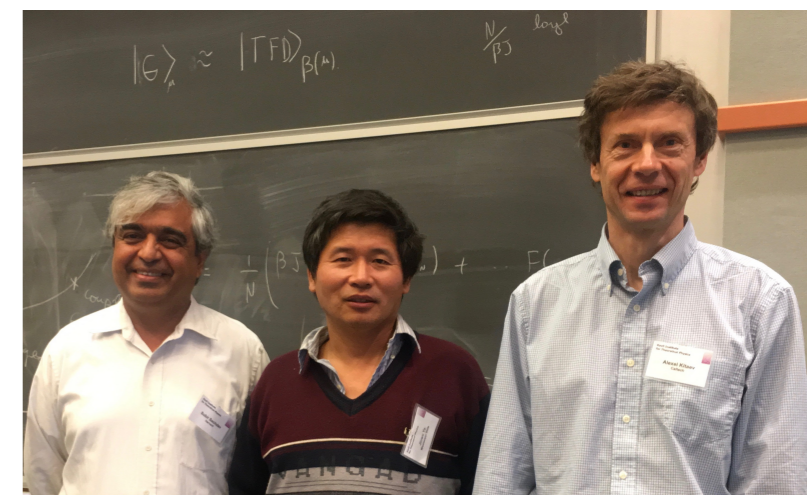
Entangle electrons pairwise randomly



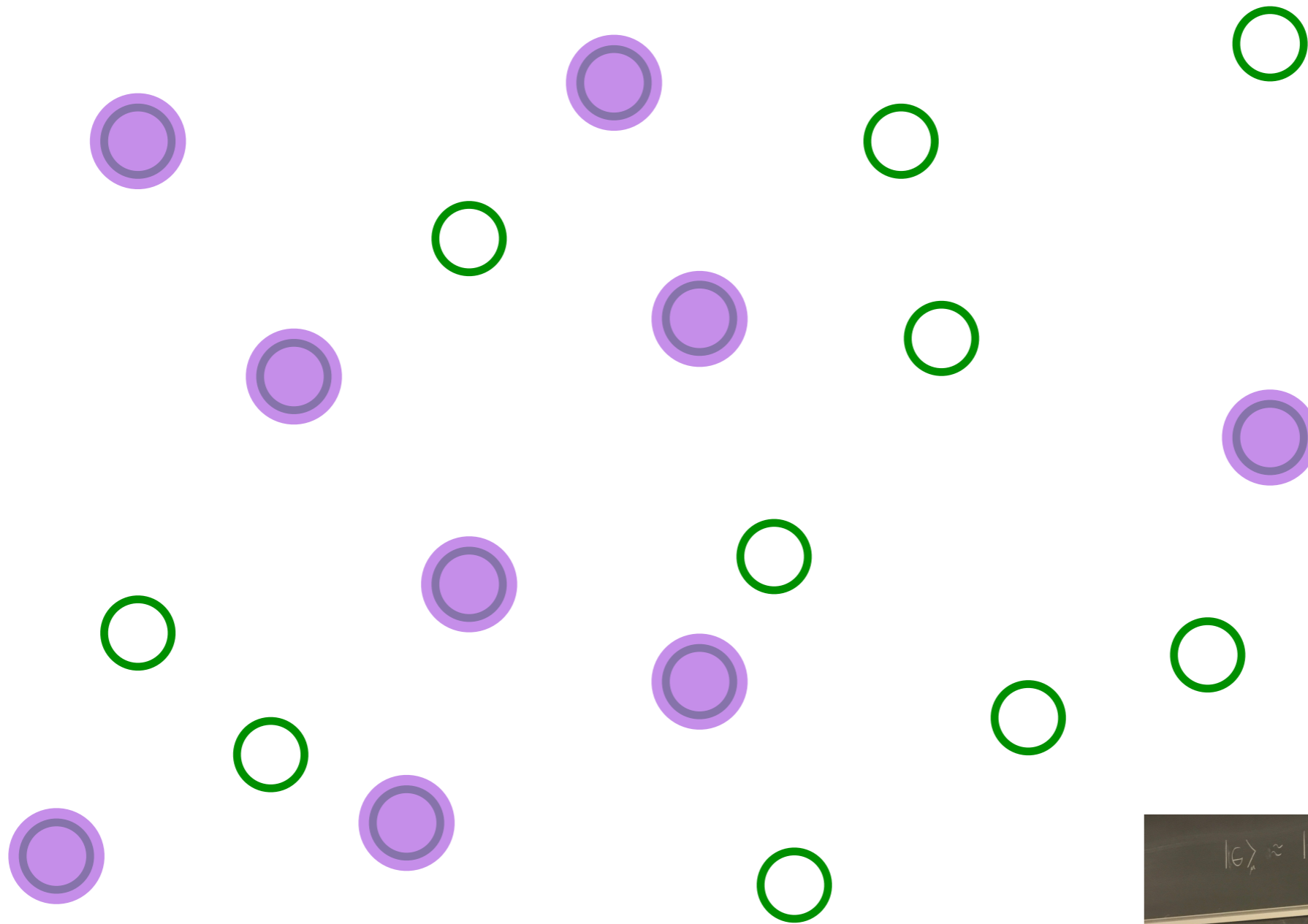
The SYK model



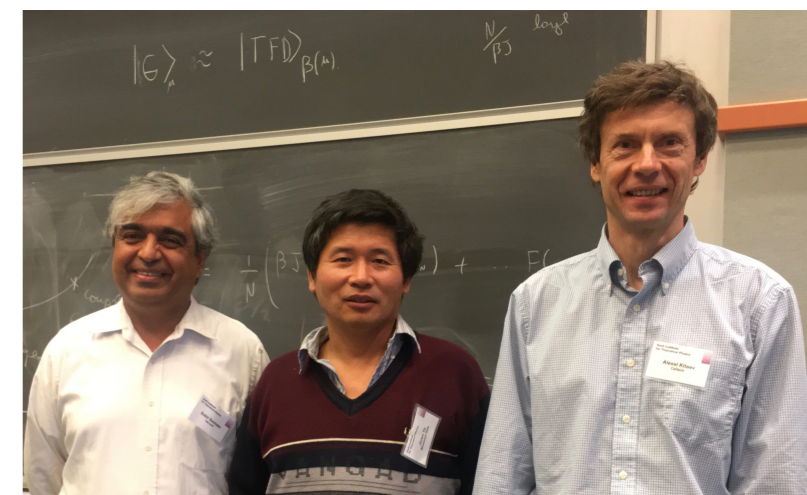
Entangle electrons pairwise randomly



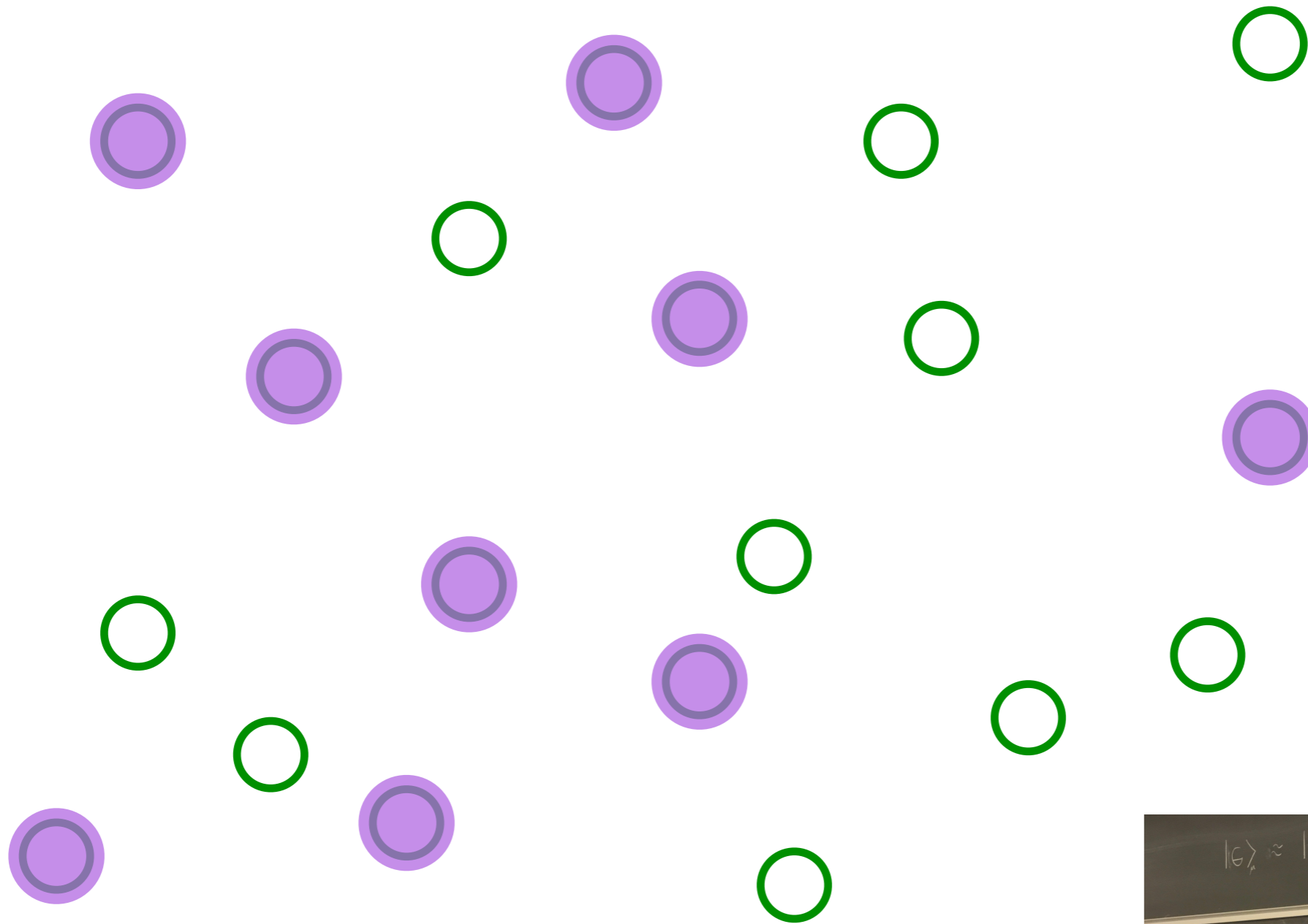
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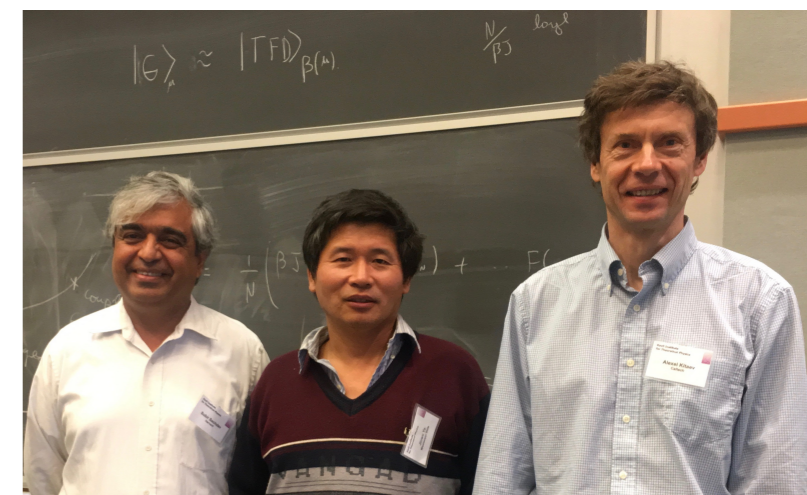
Entangle electrons pairwise randomly



The SYK model



This describes both a strange metal
and a black hole!



The complex SYK model

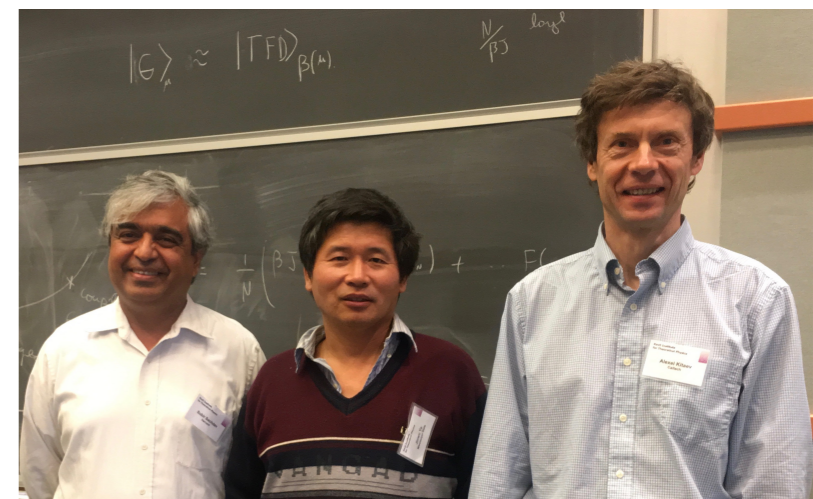
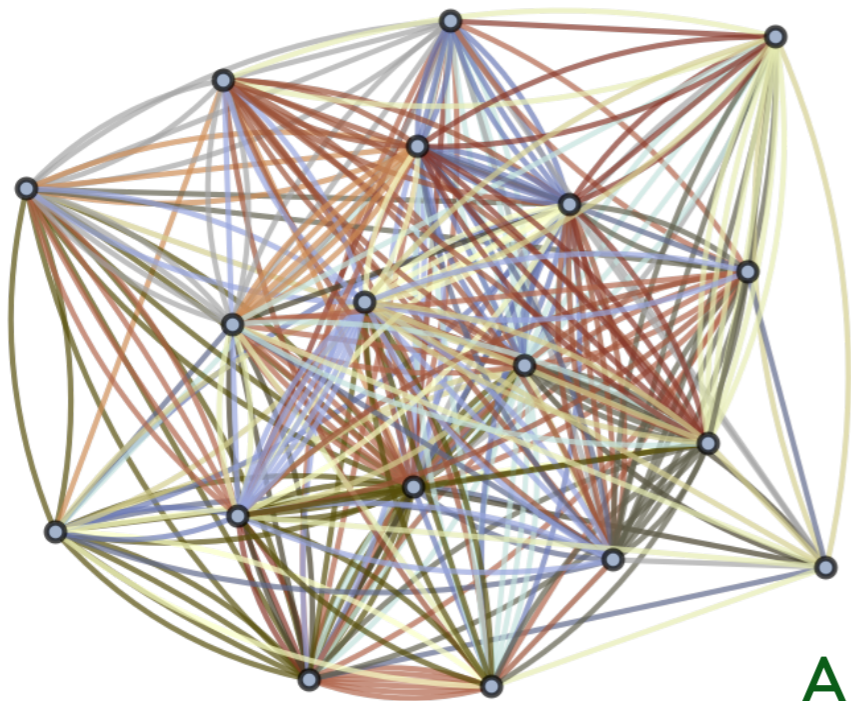
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta; \gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta; \gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta; \gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.



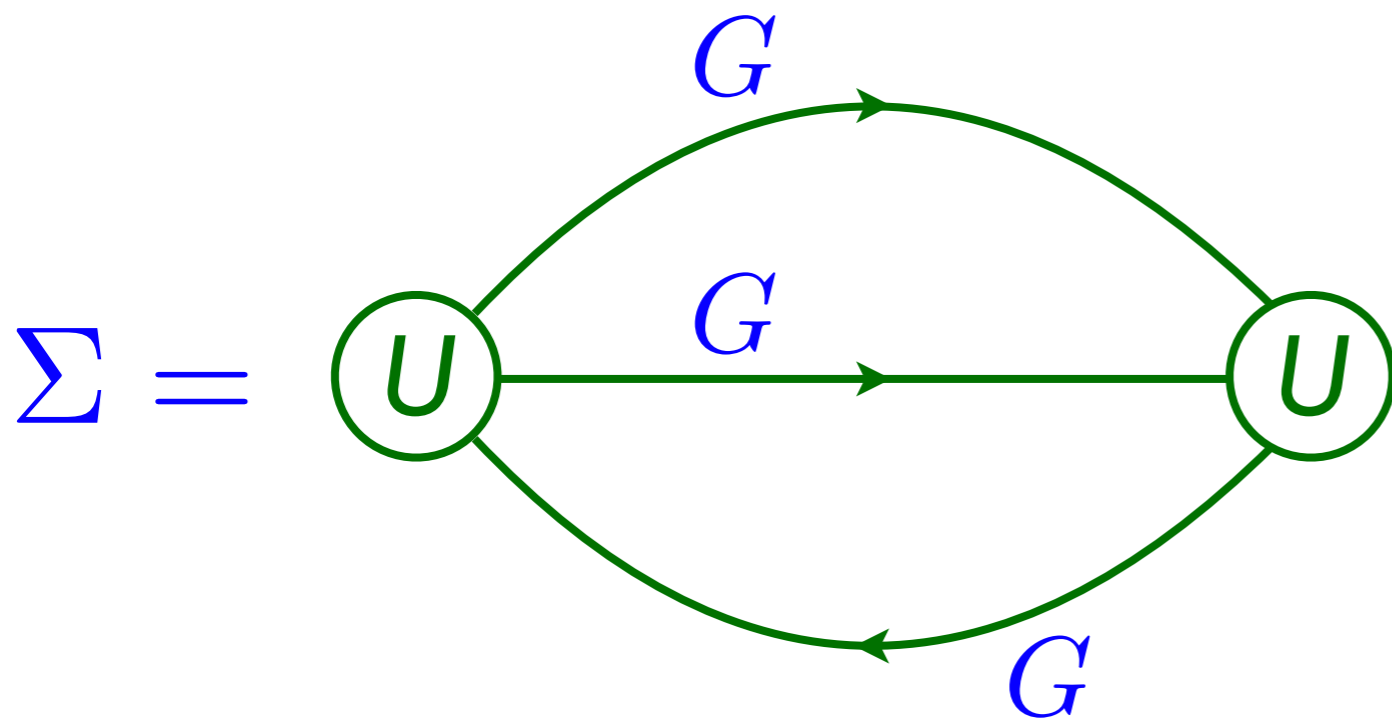
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)



The complex SYK model

The large N limit is given by the sum of “melon” Feynman graphs

For long times $\tau > 0$

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = e^{\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

$$\langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = e^{-\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E} = \mathbb{C}(\epsilon/U)$ determines the particle-hole asymmetry,

and has a universal “Luttinger” relation to \mathcal{Q} .

In a Fermi liquid,

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = \langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = \tilde{A}/\tau$$

The SYK model

There are 2^N many body levels with energy E . Shown are all values of E for a single cluster of size $N = 12$. The $T \rightarrow 0$ state has an entropy $S_{GPS} = Ns_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi\mathcal{E}.$$

At $Q = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\dots$$

where G is Catalan's constant.

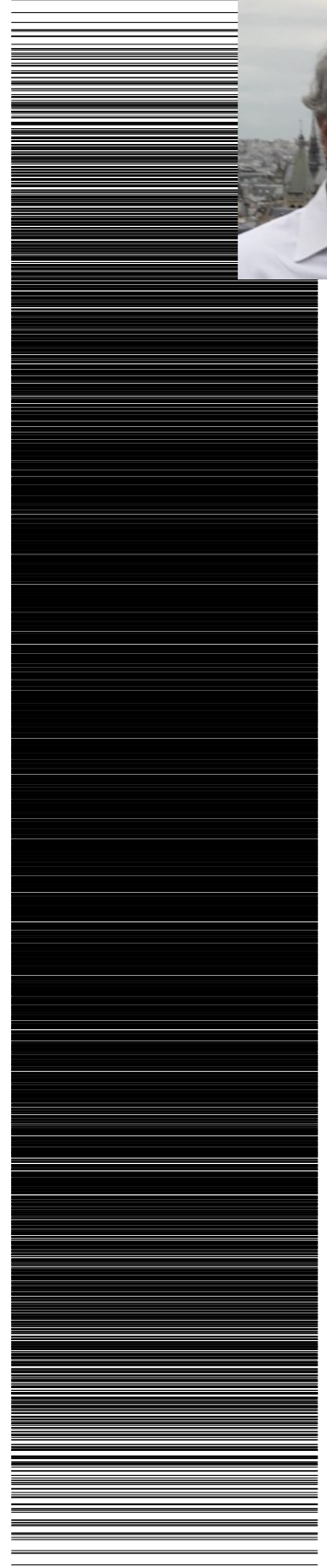
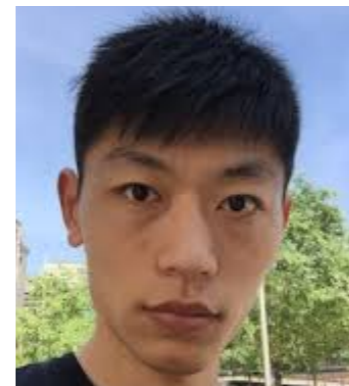


GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)



The complex SYK model

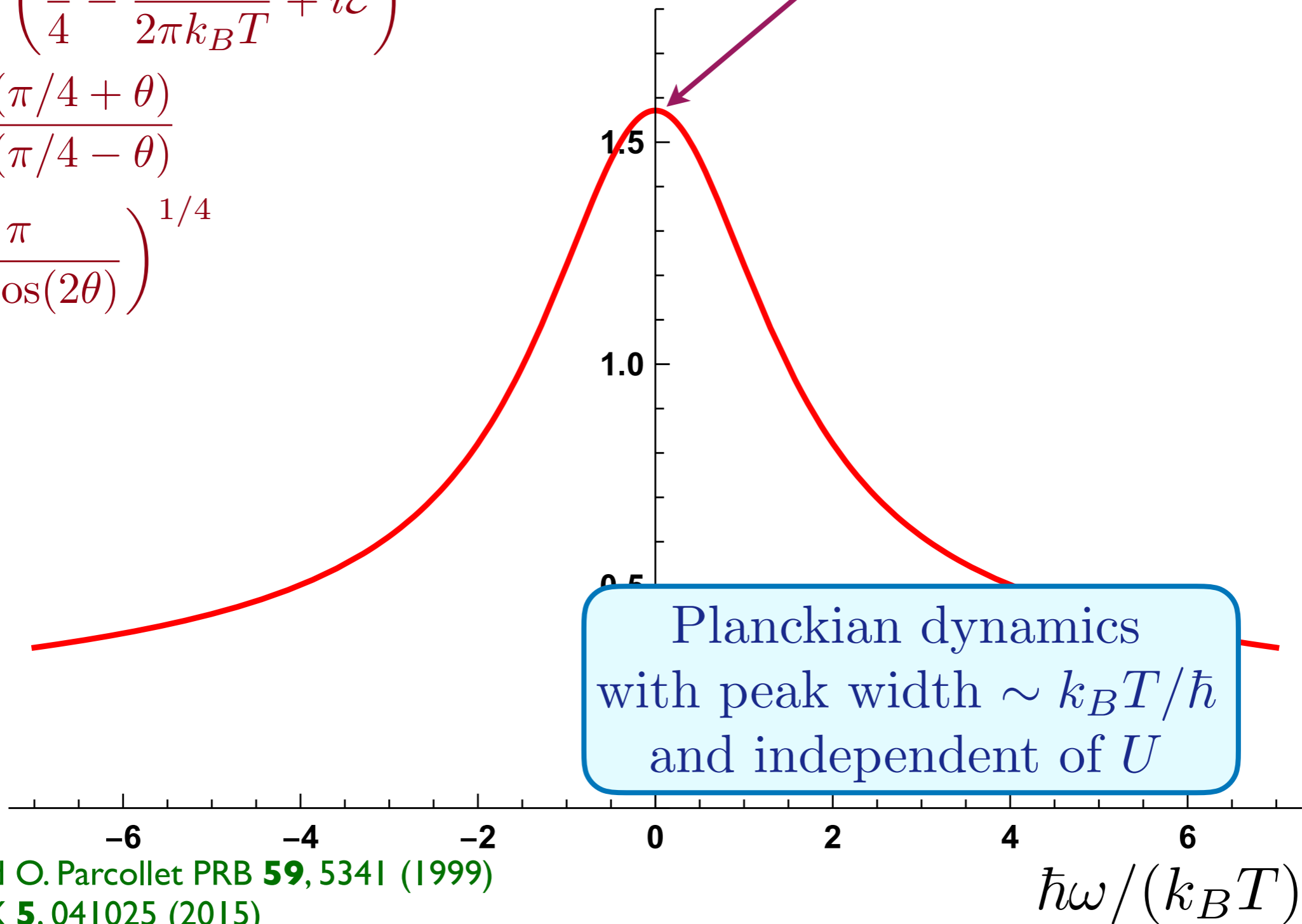
$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) = \frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$



The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

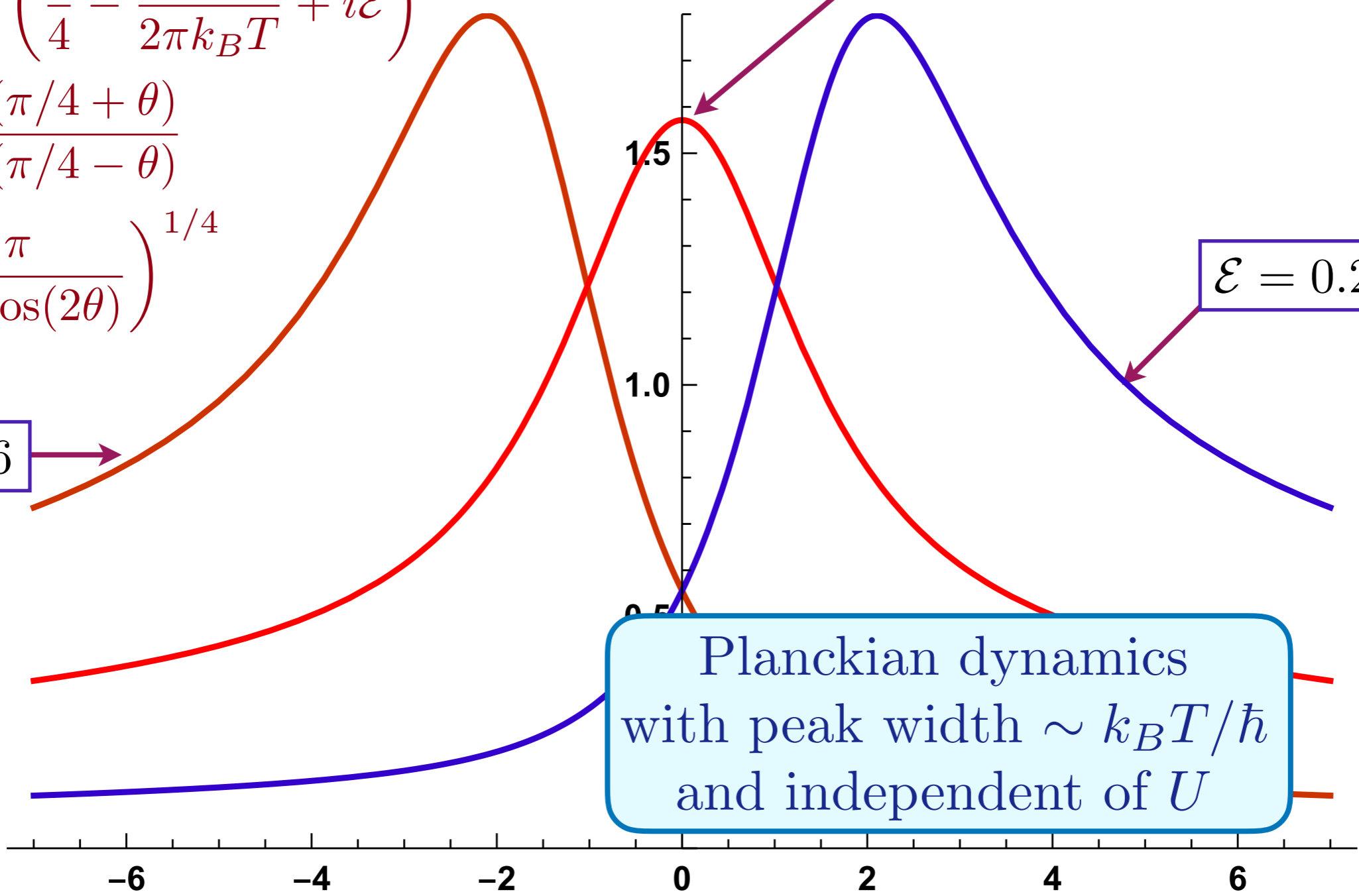
$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$

$$\mathcal{E} = 0.26$$

$$\mathcal{E} = -0.26$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

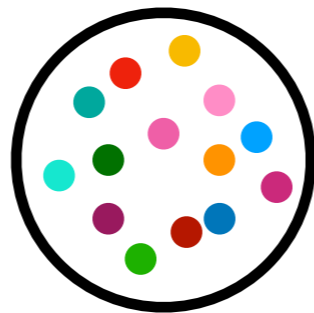
$\hbar\omega/(k_B T)$

The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

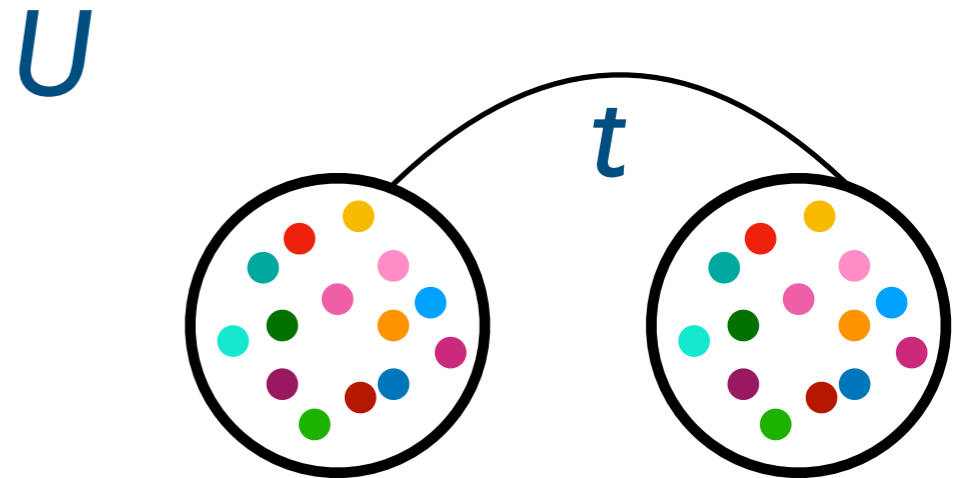
$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Equivalent to an
 “eternal traversable wormhole”
 between two black holes with
 AdS₂ horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

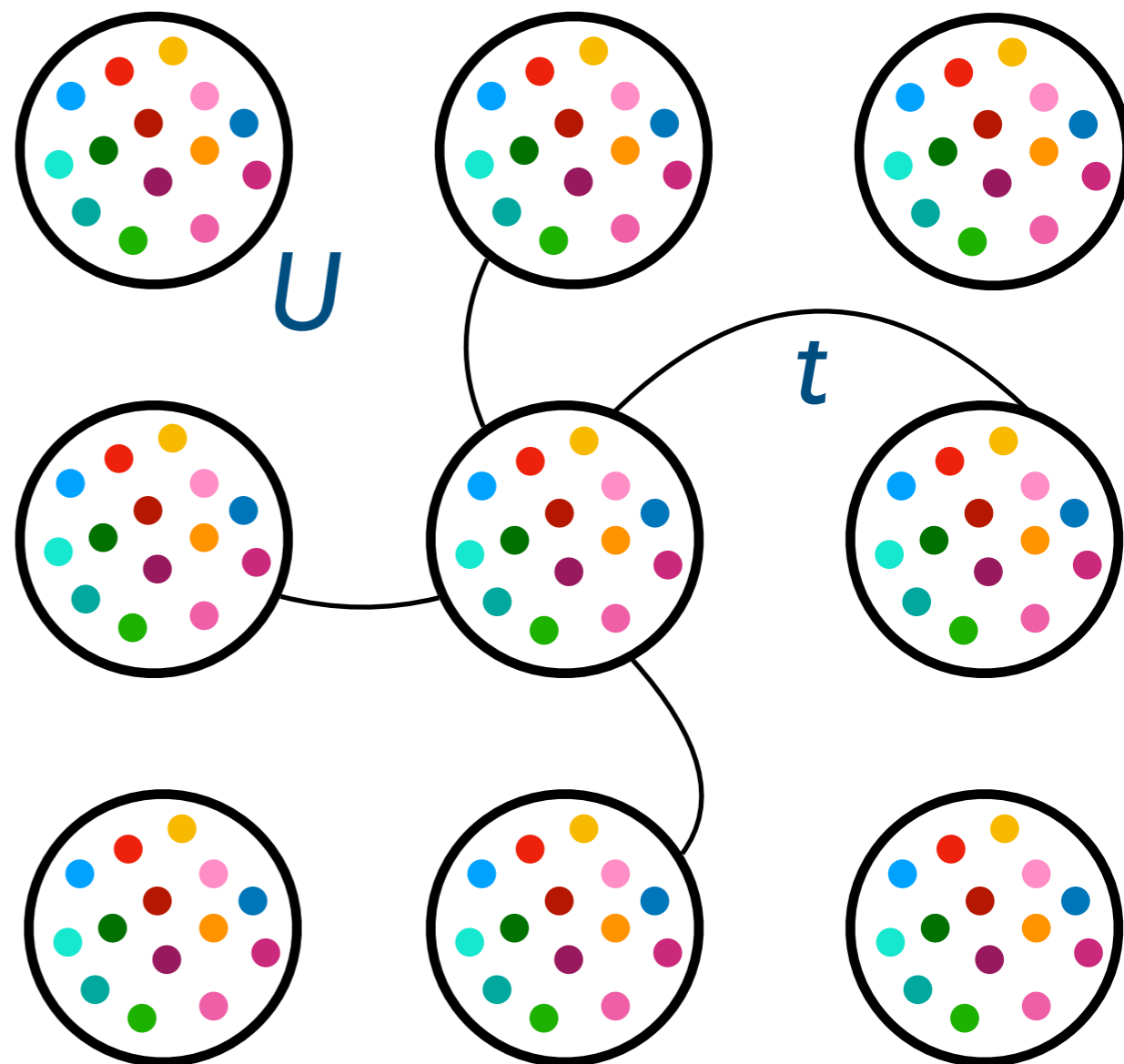
Choose U on-site,
and the same on all sites;
yields ‘incoherent metal’
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega / (k_B T))$$

independent of \mathbf{k} .

There is linear-in- T resistivity
but only with bad metal
behavior with $\rho > h/e^2$, and
co-efficient dependent upon U :

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);
Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman,
Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy,
Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{i, \alpha \beta; \gamma \delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

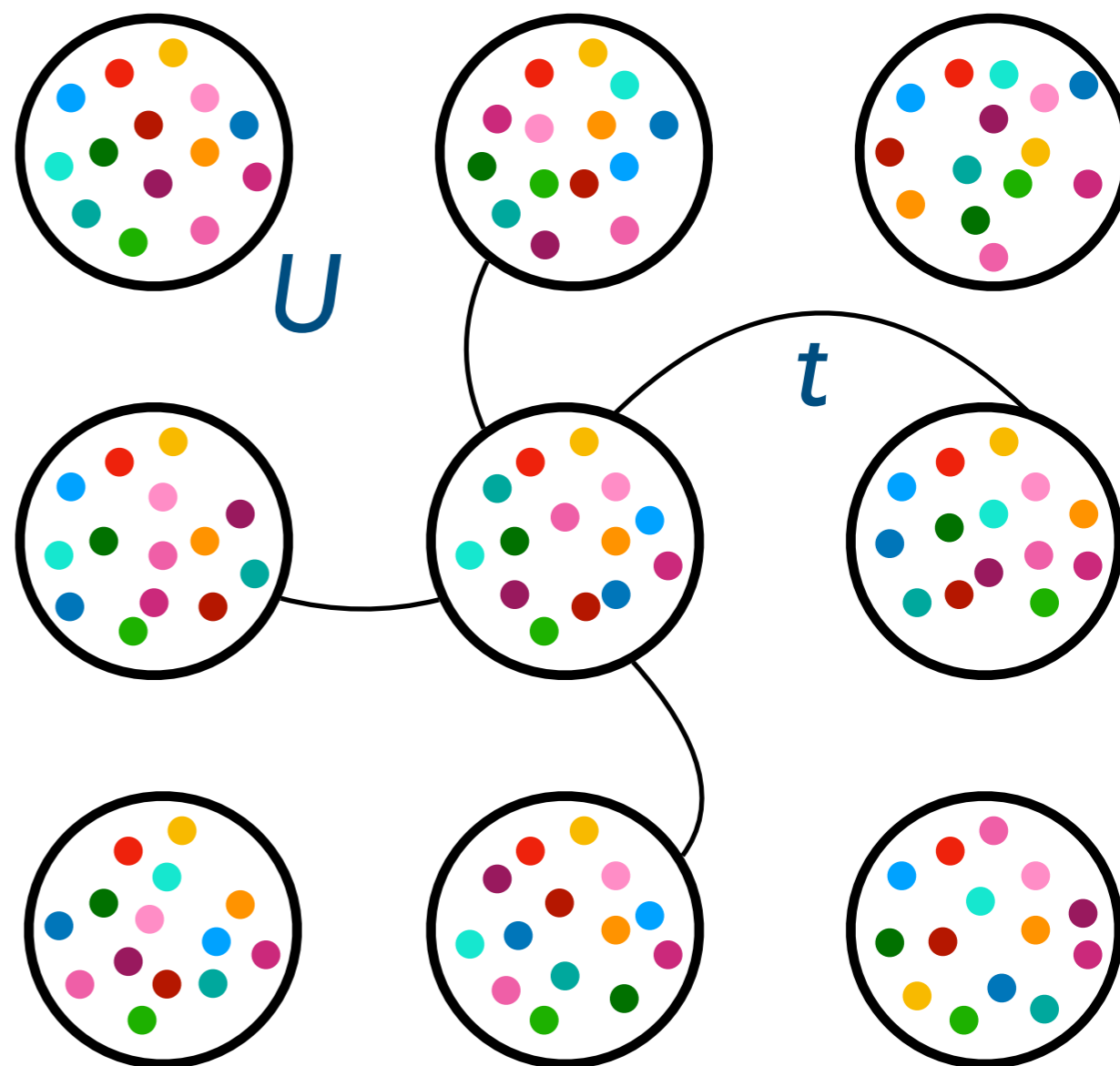
Choose U on-site,
and independent between sites;
yields ‘incoherent metal’
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega / (k_B T))$$

independent of \mathbf{k} .

There is linear-in- T resistivity
but only with bad metal
behavior with $\rho > h/e^2$, and
co-efficient dependent upon U :

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$

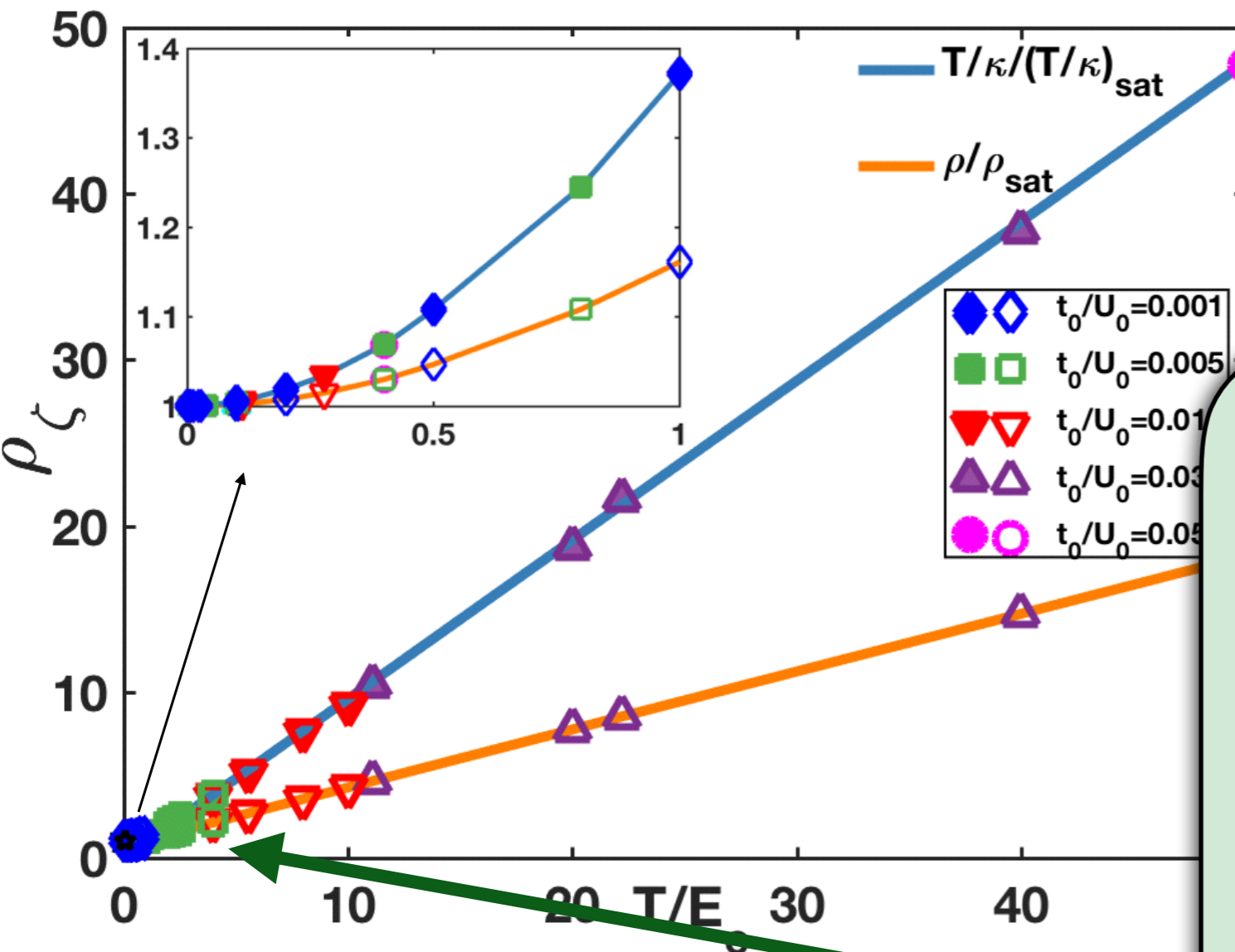


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Coupled SYK Islands

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

For $T < E_c$, the resistivity, ρ , and entropy density, s , are

$$\rho = \frac{h}{e^2} \left[c_1 + c_2 \left(\frac{T}{E_c} \right)^2 \right]$$

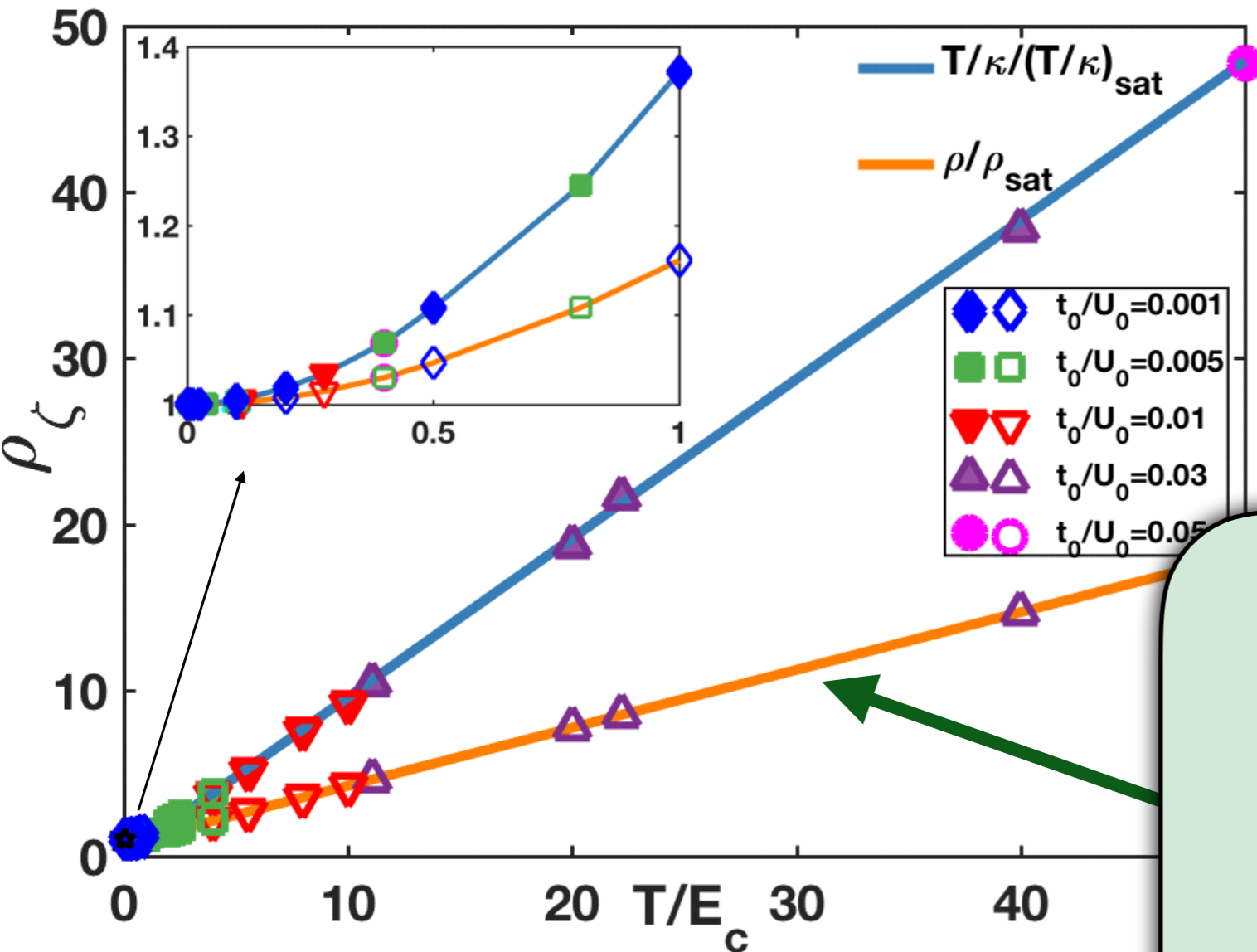
$$s \sim s_0 \left(\frac{T}{E_c} \right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

For $E_c < T < U$, the resistivity, ρ , and entropy density, s , are

$$\rho \sim \frac{h}{e^2} \left(\frac{T}{E_c} \right), \quad s = s_0$$

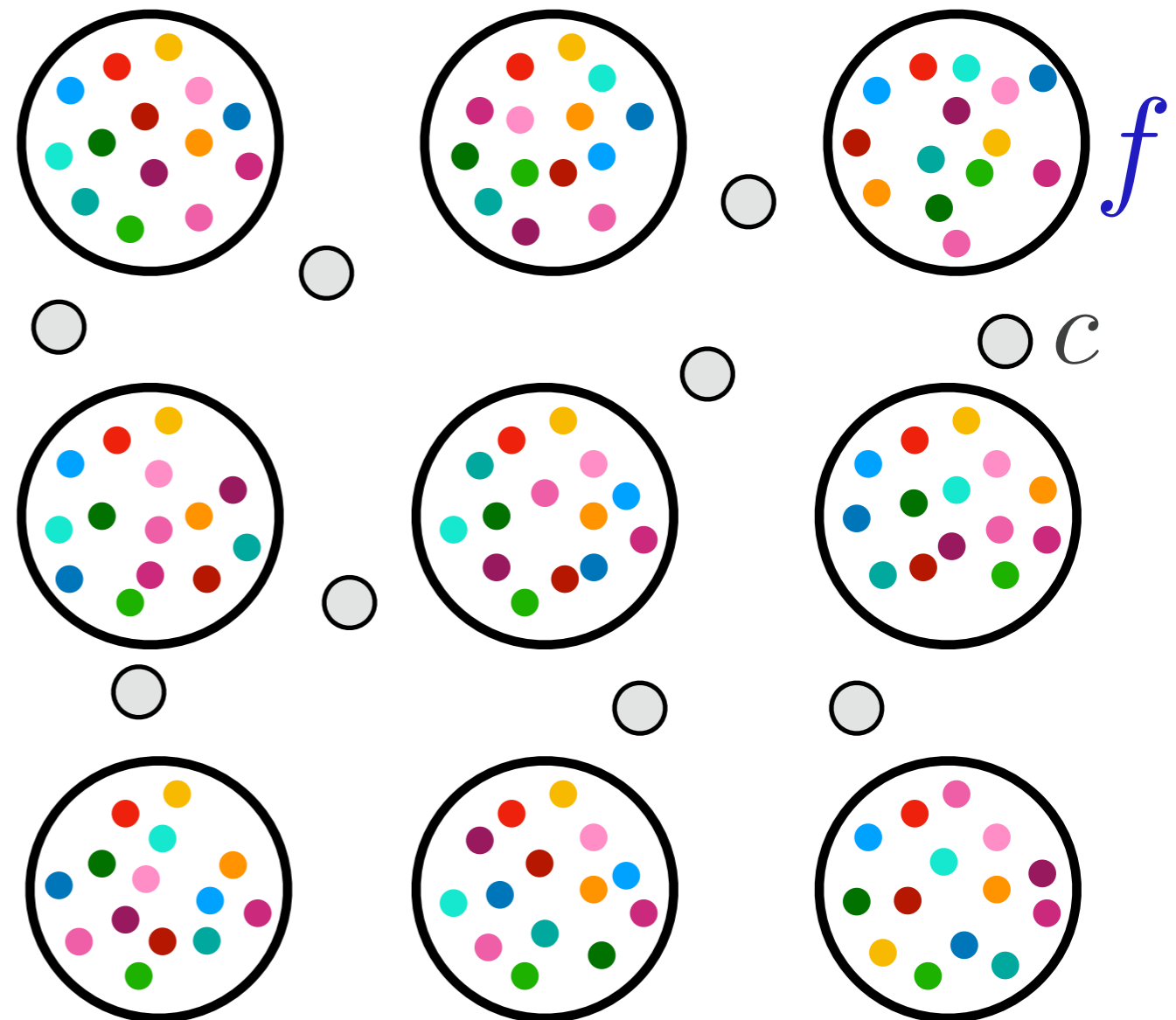
Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

A Kondo-SYK model

Mobile electrons (c) coupled to SYK quantum islands (f) with exchange interactions.

Has a regime where the c electrons form a marginal Fermi liquid with a linear-in- T resistivity dependent upon interaction strength, and a small Fermi surface which does not count the f electrons.



Similar results for many earlier 'marginal Fermi liquid' and holographic models

Generalized SYK models

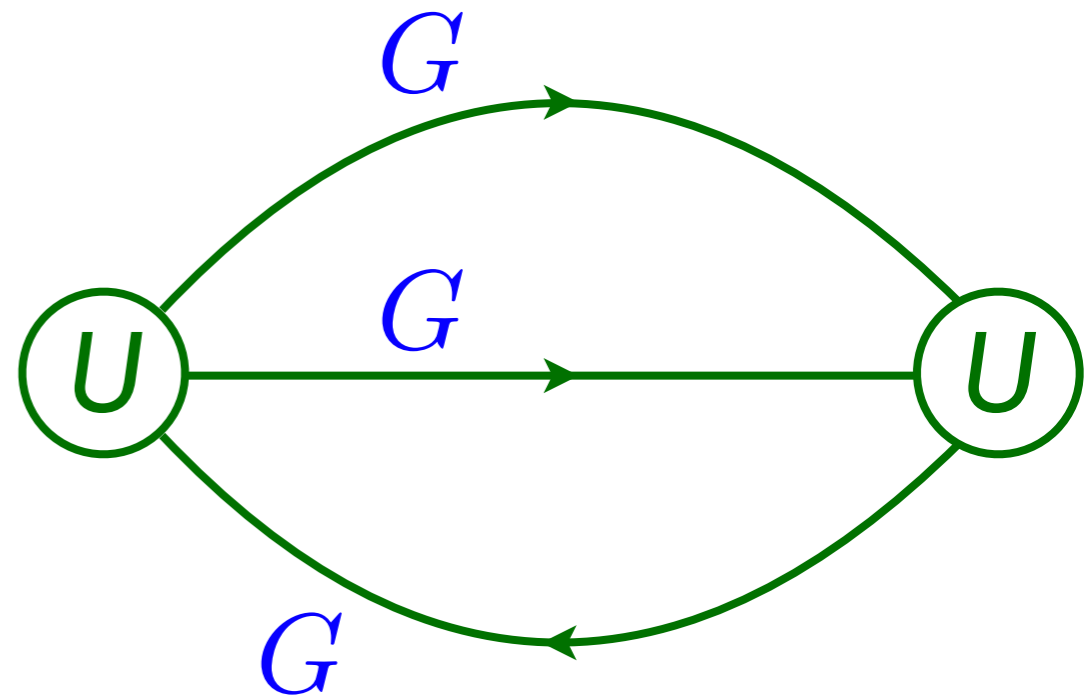
$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)
 ϵ_k has a range of values of width W .

The large N limit is still given by the sum of “melon” diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$$\Sigma =$$



Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

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 ϵ_k has a range of values of width W .

The large N limit is still given by the sum of “melon” diagrams.

For many generic models in this class, $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$, and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Pengfei Zhang, PRB **96**, 205138 (2017)

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

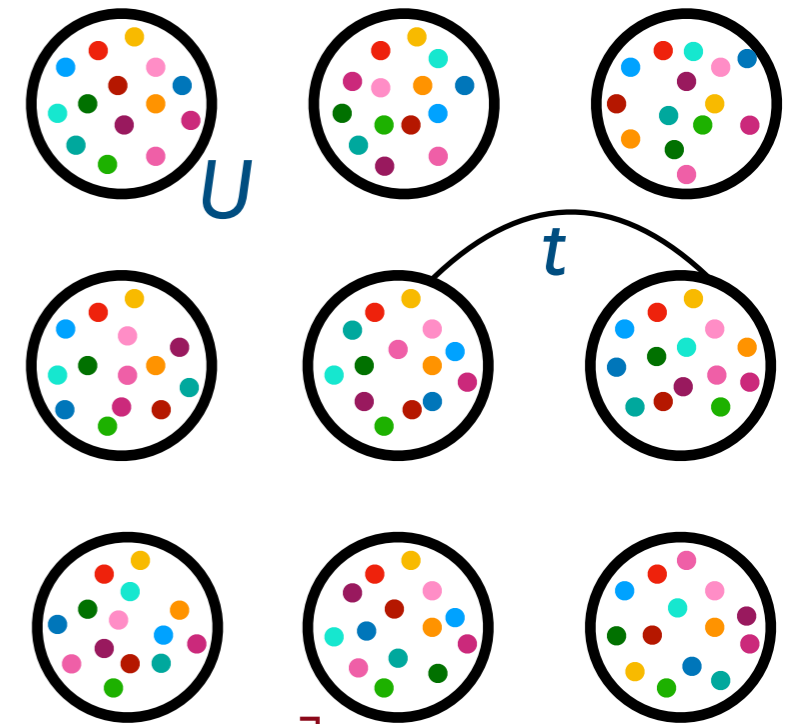
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

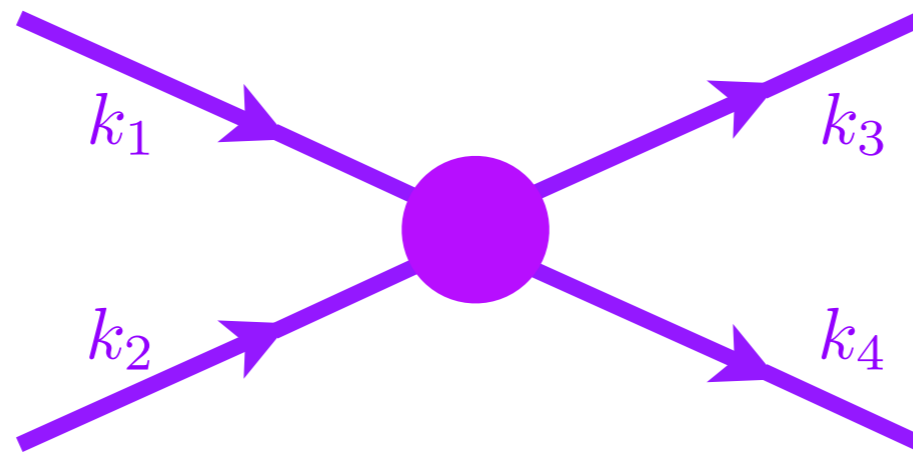
$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

Rewriting of lattice model of incoherent and bad metal in momentum space



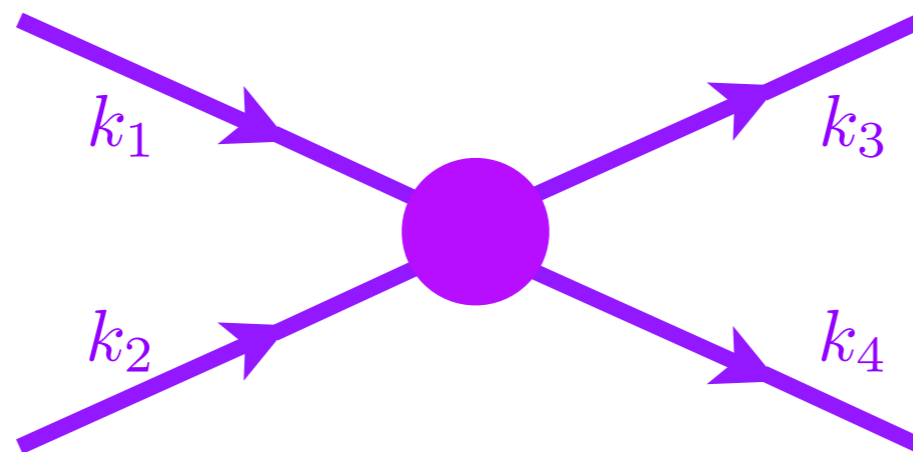
$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k , which we have already accounted for.

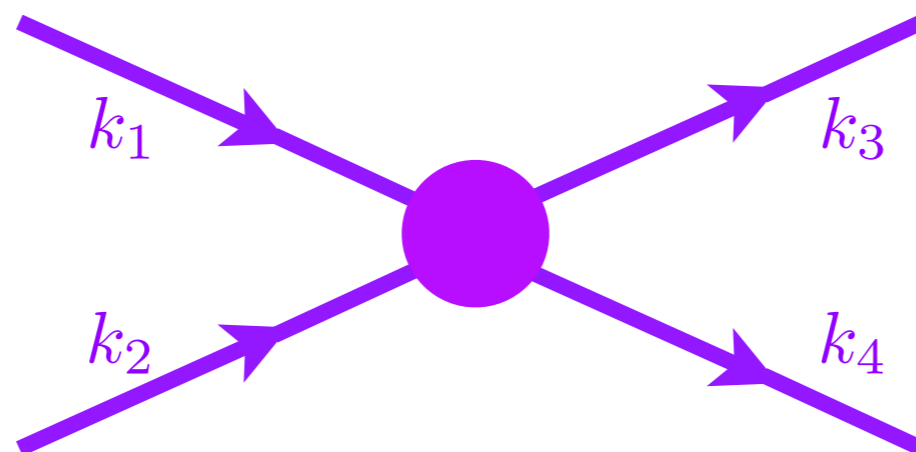
Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k , which we have already accounted for.

Keep only the interactions resonant in the bare quasiparticle energy with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

Resonant SYK model



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Keep only the interactions resonant in the bare quasiparticle energy with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only interactions resonant in the bare quasiparticle energies
with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

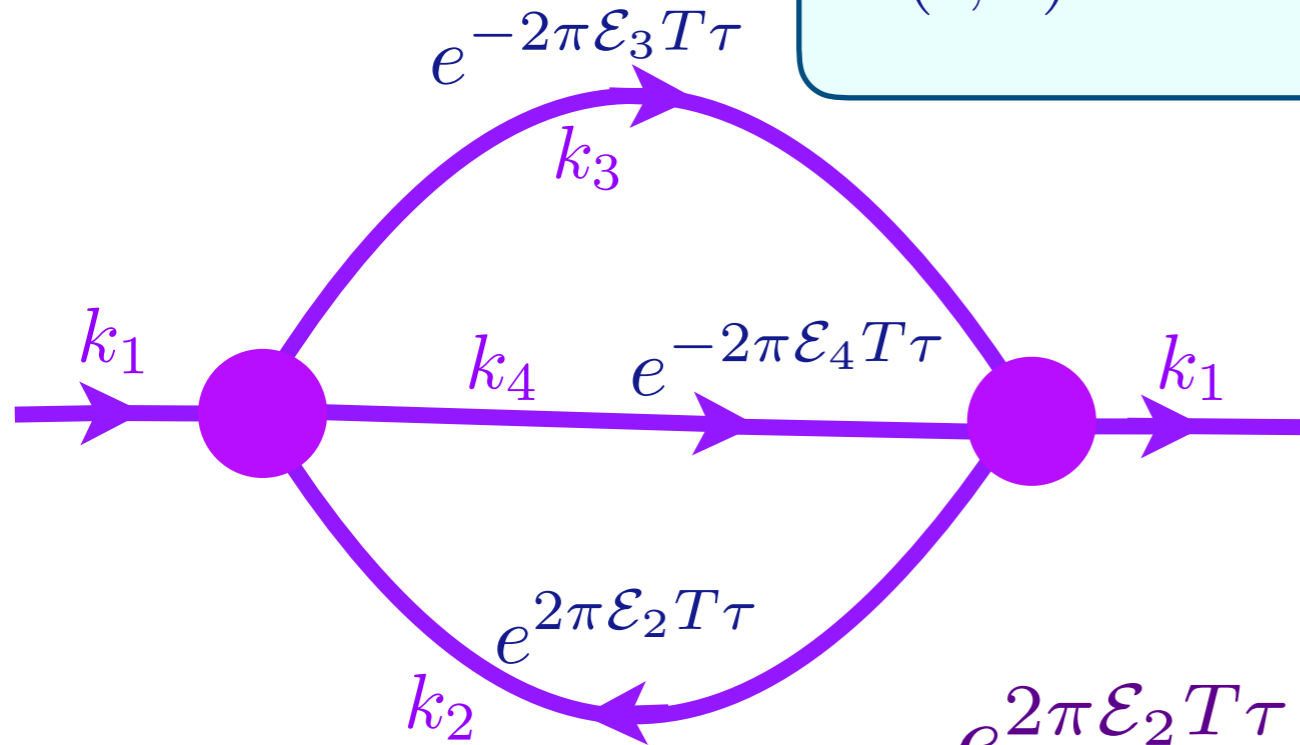
This implies off-site interactions with correlations which decay with a power-law in space.



Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi\mathcal{E}T\tau} \left(\frac{T}{\sin(\pi T\tau)} \right)^{1/2}, \quad 0 < \tau < 1/T.$$



$$e^{2\pi\mathcal{E}_2 T\tau} e^{-2\pi\mathcal{E}_3 T\tau} e^{-2\pi\mathcal{E}_4 T\tau} = e^{-2\pi\mathcal{E}_1 T\tau}$$

if

$$\mathcal{E}_a = \mathbb{C}\epsilon_a/U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a Planckian metal as $T \rightarrow 0$ with a remnant Fermi surface:
 $G(k, \omega) = G_{\text{SYK}}(\epsilon_k, \hbar\omega/(k_B T))$,
 with $\mathcal{E}_k = \mathbb{C}\epsilon_k/U$

Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$
$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter \mathcal{E} is independent of k ,
and determined by the total density

Planckian metal with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$

The particle-hole asymmetry changes as
we cross the Fermi surface



The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$

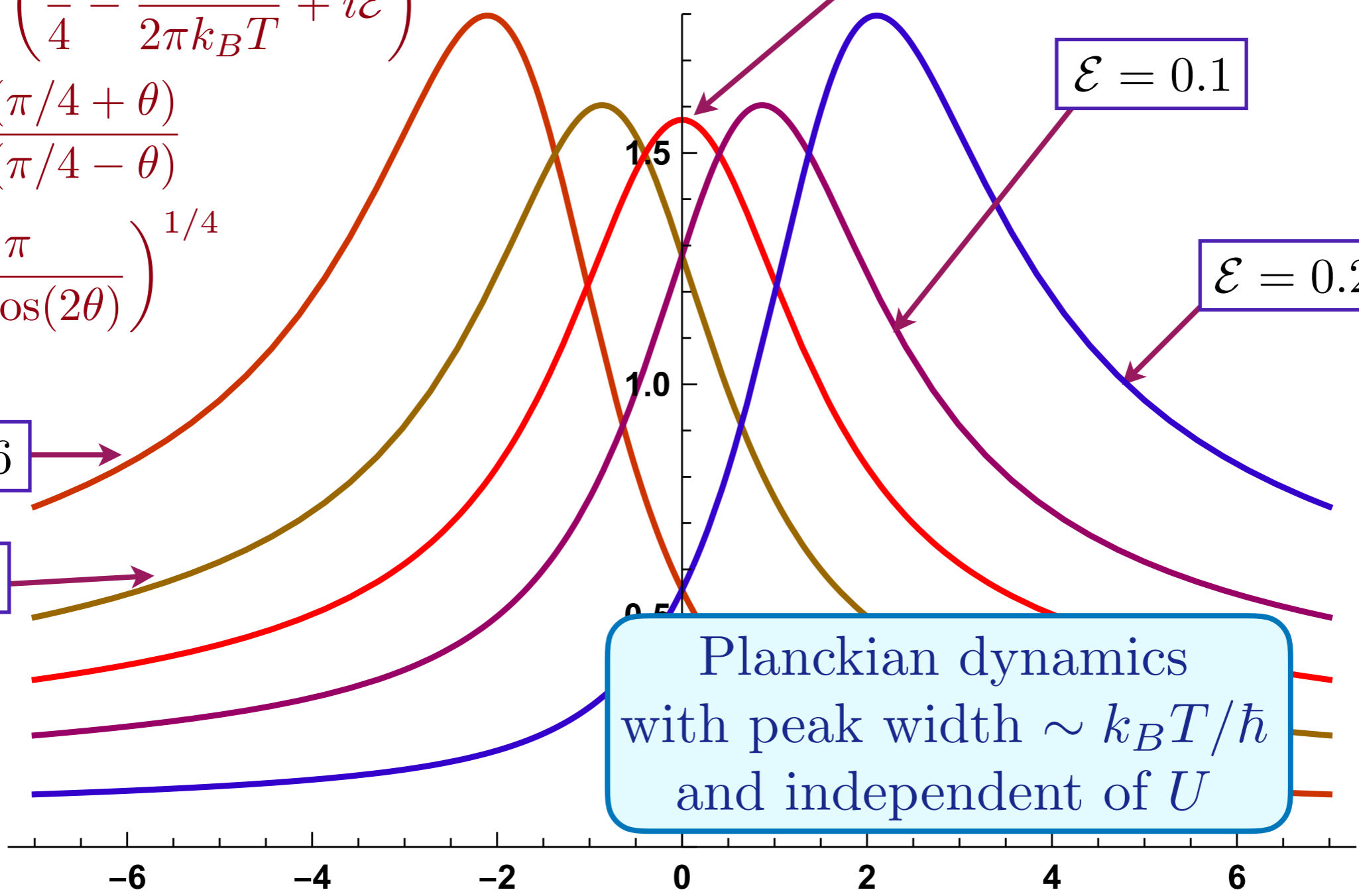
$$\mathcal{E} = 0.1$$

$$\mathcal{E} = 0.26$$

$$\mathcal{E} = -0.26$$

$$\mathcal{E} = -0.1$$

Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U



$\hbar\omega/(k_B T)$

The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) =$$

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$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

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$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$

$$\mathcal{E} = 0.1$$

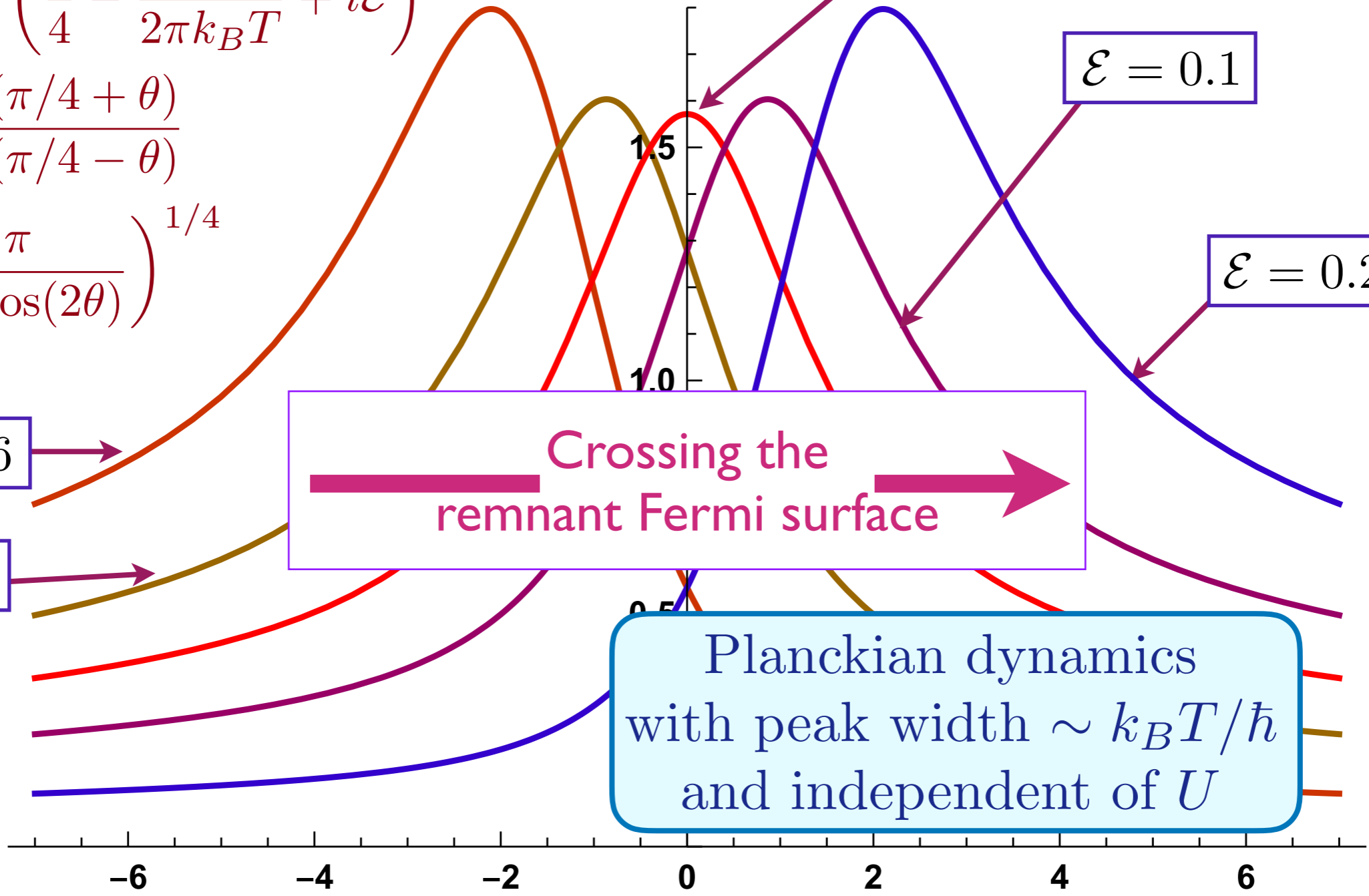
$$\mathcal{E} = 0.26$$

$$\mathcal{E} = -0.26$$

$$\mathcal{E} = -0.1$$

Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U



Resonant SYK model

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only
interactions resonant in the bare quasiparticle energies
with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

Resistivity of a [Planckian metal](#) as $T \rightarrow 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{Ne^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im} G_{\text{SYK}}^R \left(\epsilon, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left(\frac{\omega}{2T} \right)$$

Resonant SYK model

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

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Resistivity of a Planckian metal as $T \rightarrow 0$

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$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}, \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U,$$

where

$$m^* = \frac{d V_{FS}}{\oint_{FS} |\mathbf{v}_F|},$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^* .

Resonant SYK model

Resistivity of a Planckian metal as $T \rightarrow 0$

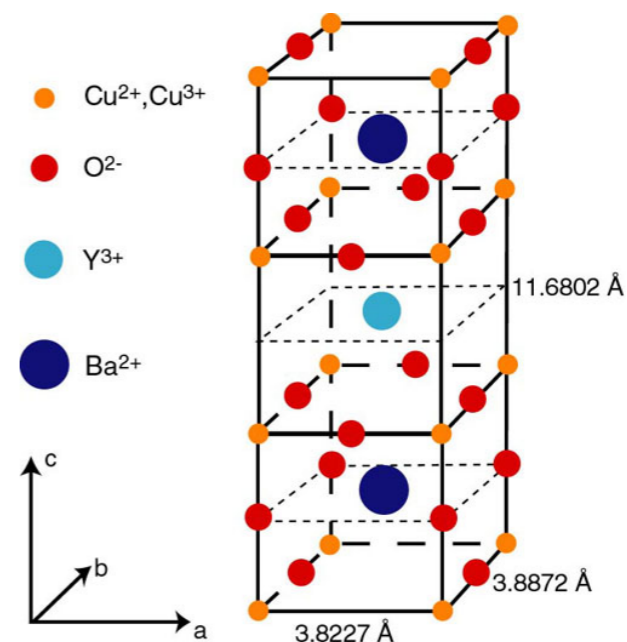
$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

The number \mathbb{C} is defined by $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$ as $|\epsilon_k| \rightarrow 0$. This is determined by UV physics, and is very weakly dependent upon the ratio of the energy width of the interactions, W_U , to U .



Aavishkar Patel



A. Patel and S. Sachdev, arXiv:1906.03265

Resonant SYK model

Take the independent momentum shell limit, $W_U/U \rightarrow 0$,

$$\overline{U(k_1, k_2, k_3, k_4)U^*(k_5, k_6, k_7, k_8)} =$$

$$U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

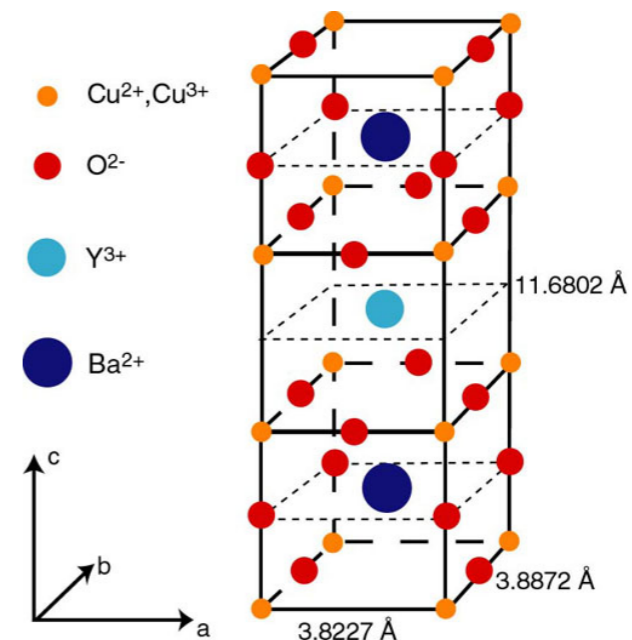
$$\times \left[\delta(\epsilon_{k_1} - \epsilon_{k_2})\delta(\epsilon_{k_2} - \epsilon_{k_3})\delta(\epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} - \epsilon_{k_6})\delta(\epsilon_{k_6} - \epsilon_{k_7})\delta(\epsilon_{k_7} - \epsilon_{k_8}) \right]$$

$\mathbb{C} = 0.41$ as in a single SYK model,
and we obtain a Planckian metal with

$$\rho = \frac{m^*}{ne^2} 1.11 \frac{k_B T}{\hbar}$$



Aavishkar Patel



A. Patel and S. Sachdev, arXiv:1906.03265

Planckian metals with a remnant Fermi surface

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \rightarrow 0$.
- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasi-particle dispersion ϵ_k , while resonant interactions have to be treated self-consistently.
- The resonance is a single ‘fine-tuning’ condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \rightarrow 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $\epsilon_k = 0$, and an effective mass m^* defined by the dispersion of ϵ_k , with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.



Aavishkar Patel (Harvard \rightarrow Miller Fellow at Berkeley)

