Long-lived excitations in 2D electron fluids

Leonid Levitov
MIT

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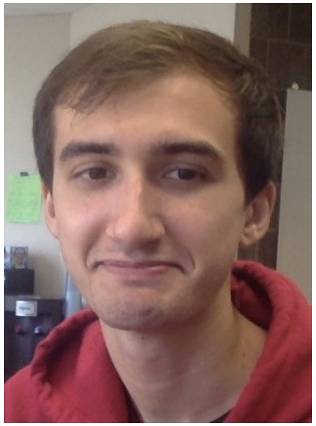
FTPI, University of Minnesota 06/12/2019

Team

Patrick Ledwith ‘19

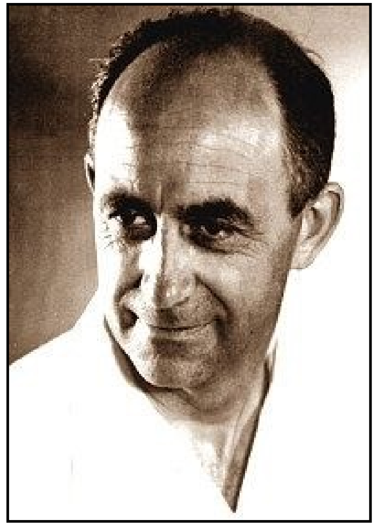


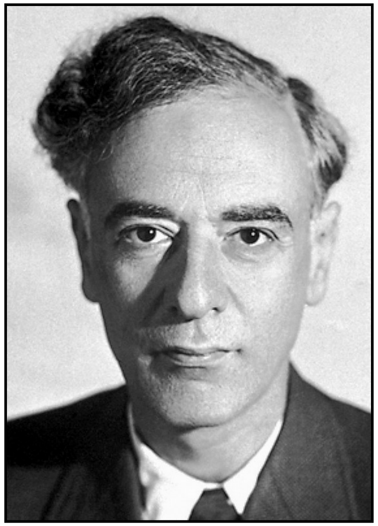
Lev Kendrick ‘19

Haoyu Guo ‘18

Andrey Shytov
(Exeter, UK)

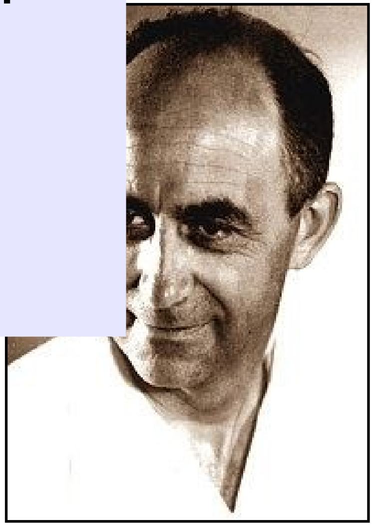
Landau Fermi-liquid theory:

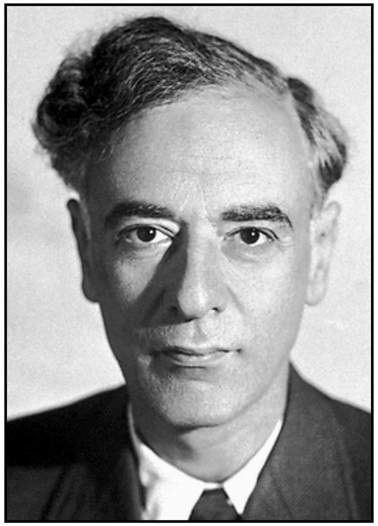




Landau Fermi-liquid theory:

- Strongly interacting fermions at a high density
- A challenging problem?
- Landau (1956): actually fermion exclusion makes it simple





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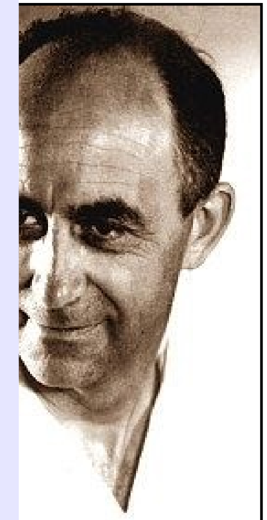
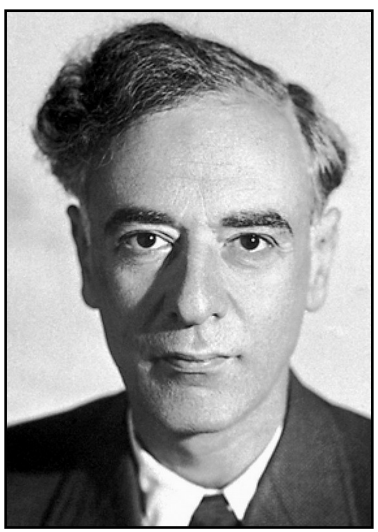
- All the action at the Fermi surface, $E \sim E_F$





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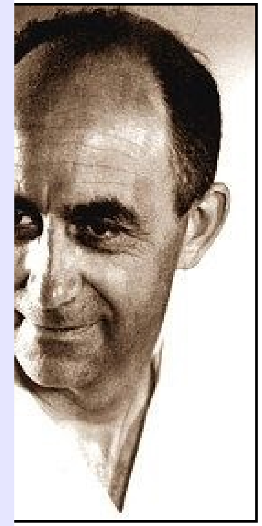

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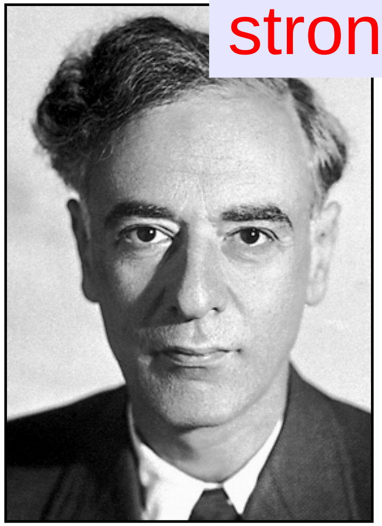

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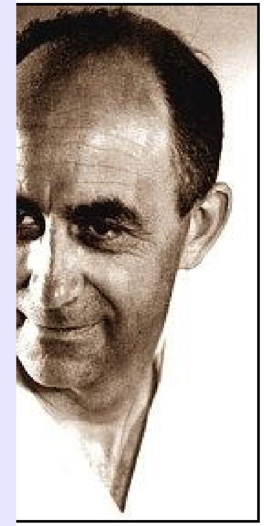

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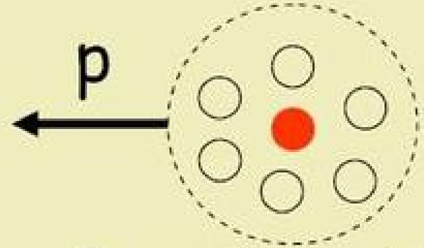
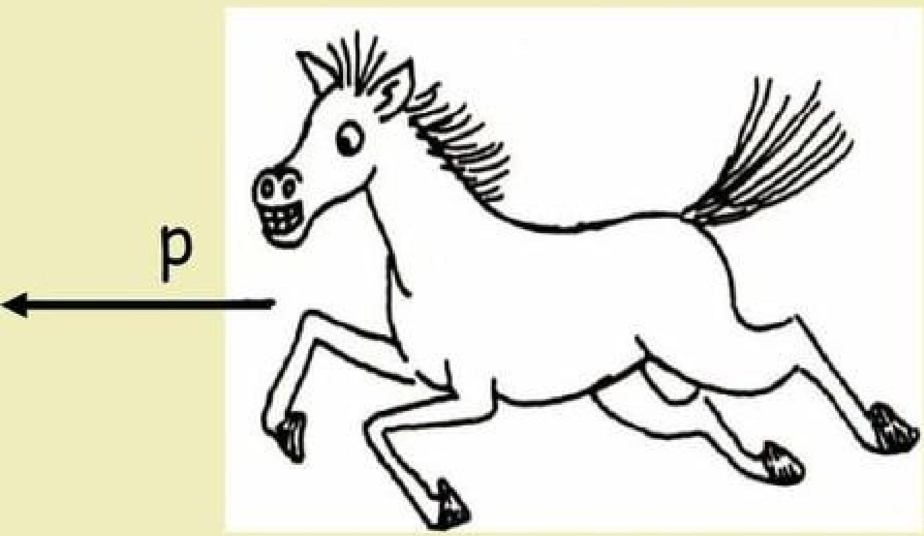
- Fermi sea (filled states with $E<E_F$)
- All the action at the Fermi surface, $E \sim E_F$
- Quasiparticles: quasi-free particles in a strongly interacting system
- Relatively long lifetimes: $\tau \sim 1/(E-E_F)^2$, $\tau \sim 1/(k_B T)^2$

Quasiparticle concept

Real particle



Quasi particle



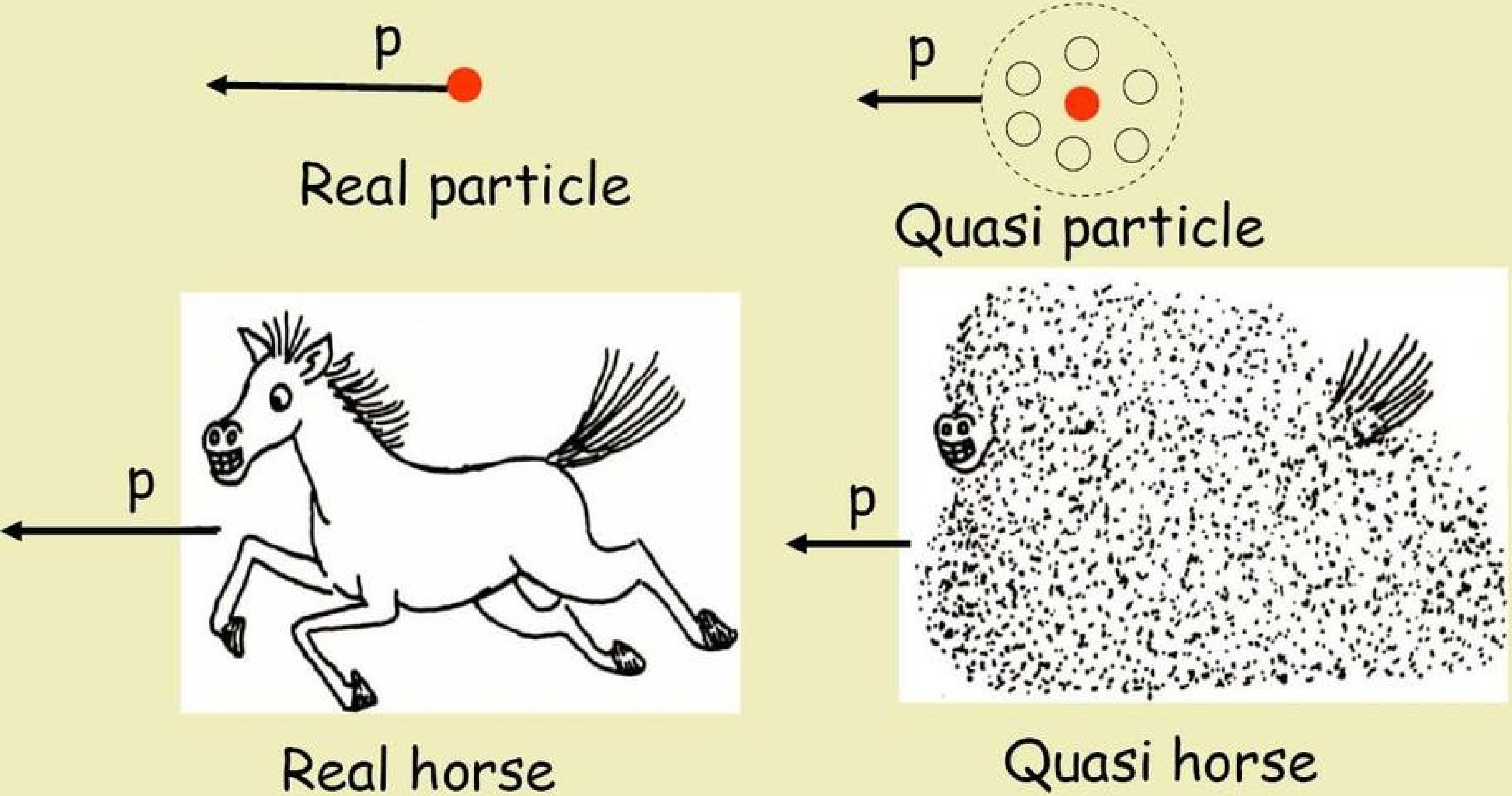
Real horse

Quasi horse

R. D. Mattuck, a guide to Feynman Diagrams in the MB problem, Dover, 1976

A quasiparticle has an effective mass, selfenergy (energy and lifetime).

Quasiparticle concept



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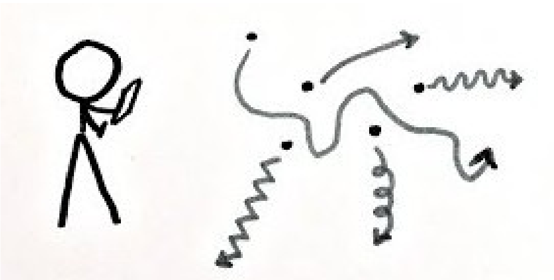
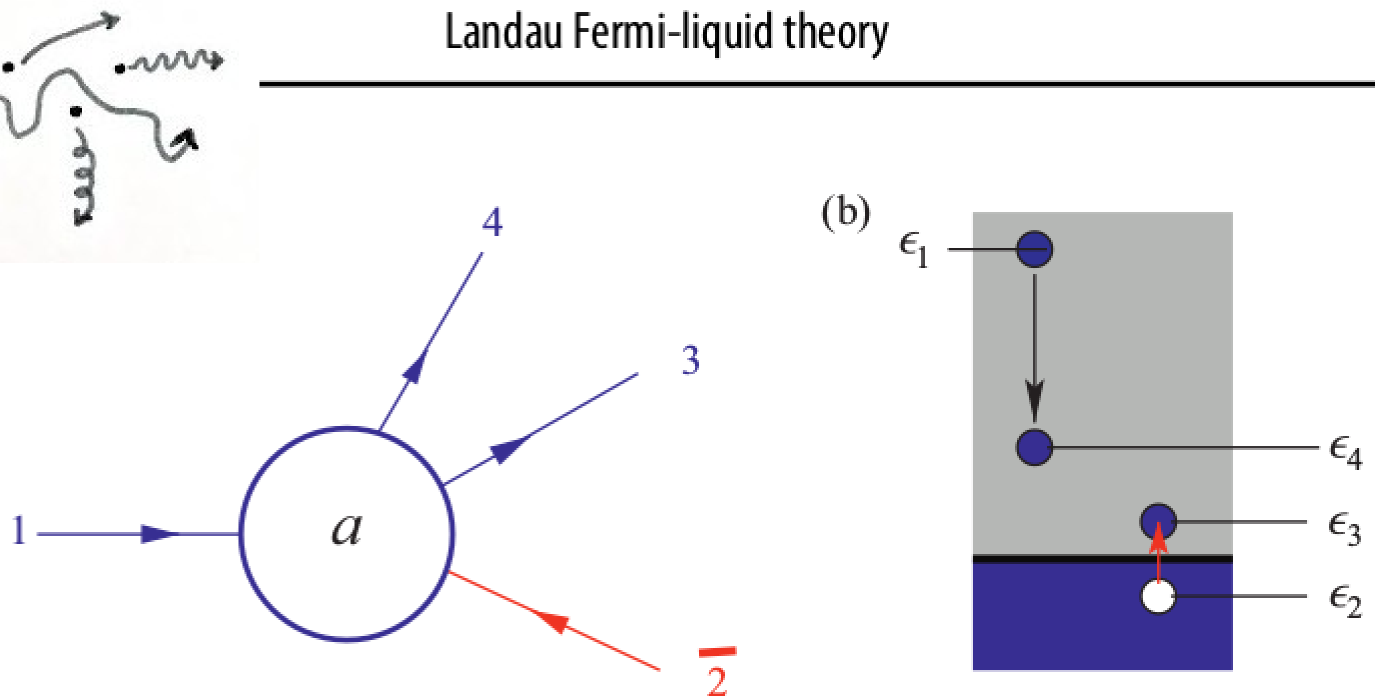
The phase space argument





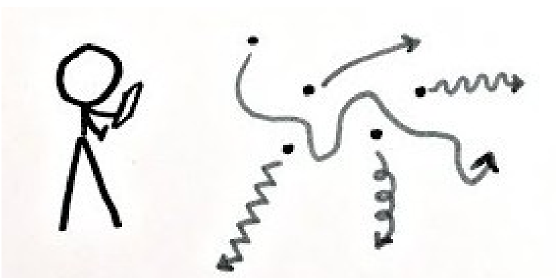
From: Coleman, Introduction to Many-body physics

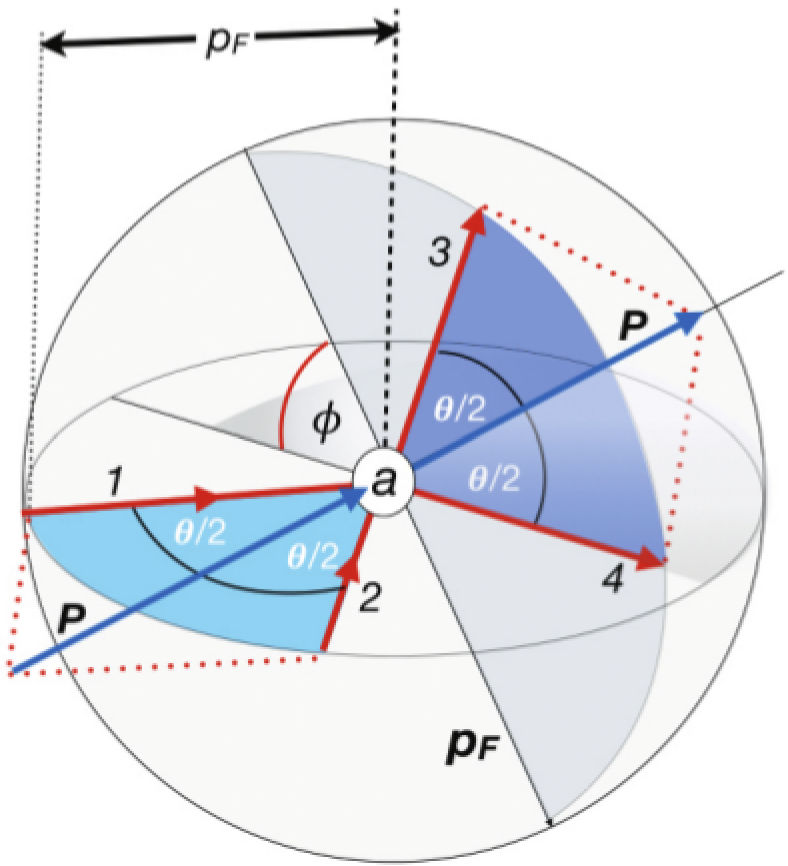
$$\gamma \sim \int \int \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) f(\epsilon_2)(1 - f(\epsilon_3))(1 - f(\epsilon_4)) \sim max[\epsilon_1^2, T^2]$$

Kinematics of ee scattering:

In 3D angular relaxation not a bottleneck (and thus doesn't matter)

Landau argument works





Kinematics of ee scattering:

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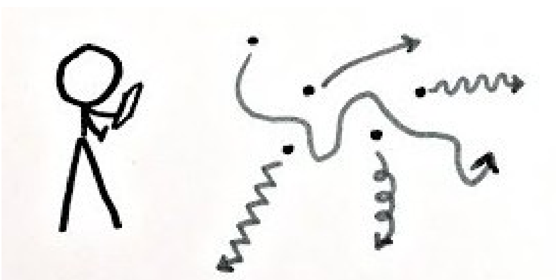
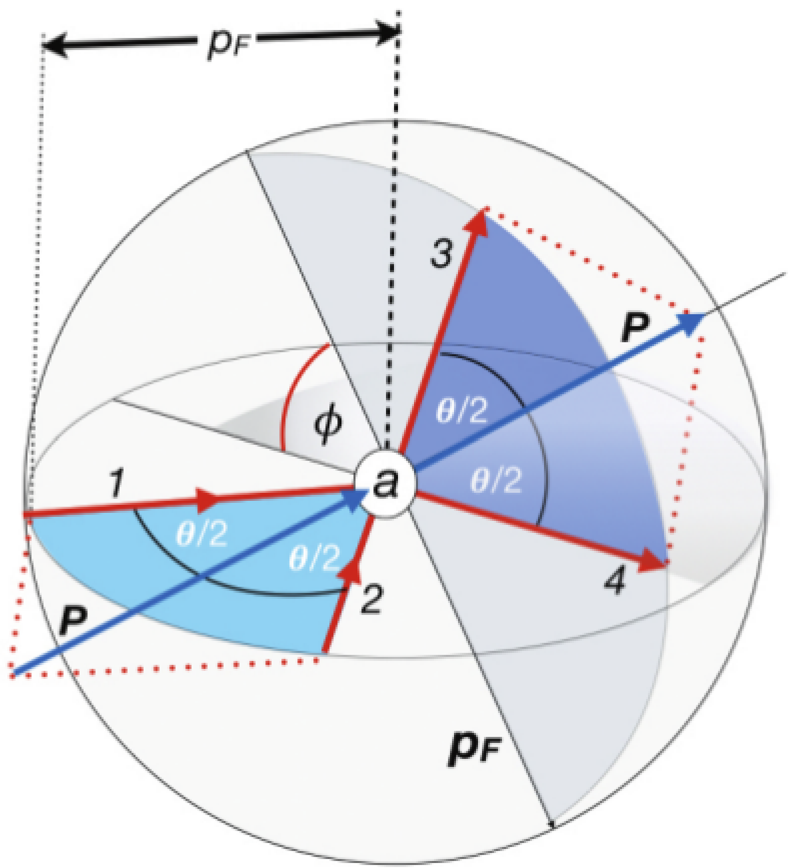
Landau argument works

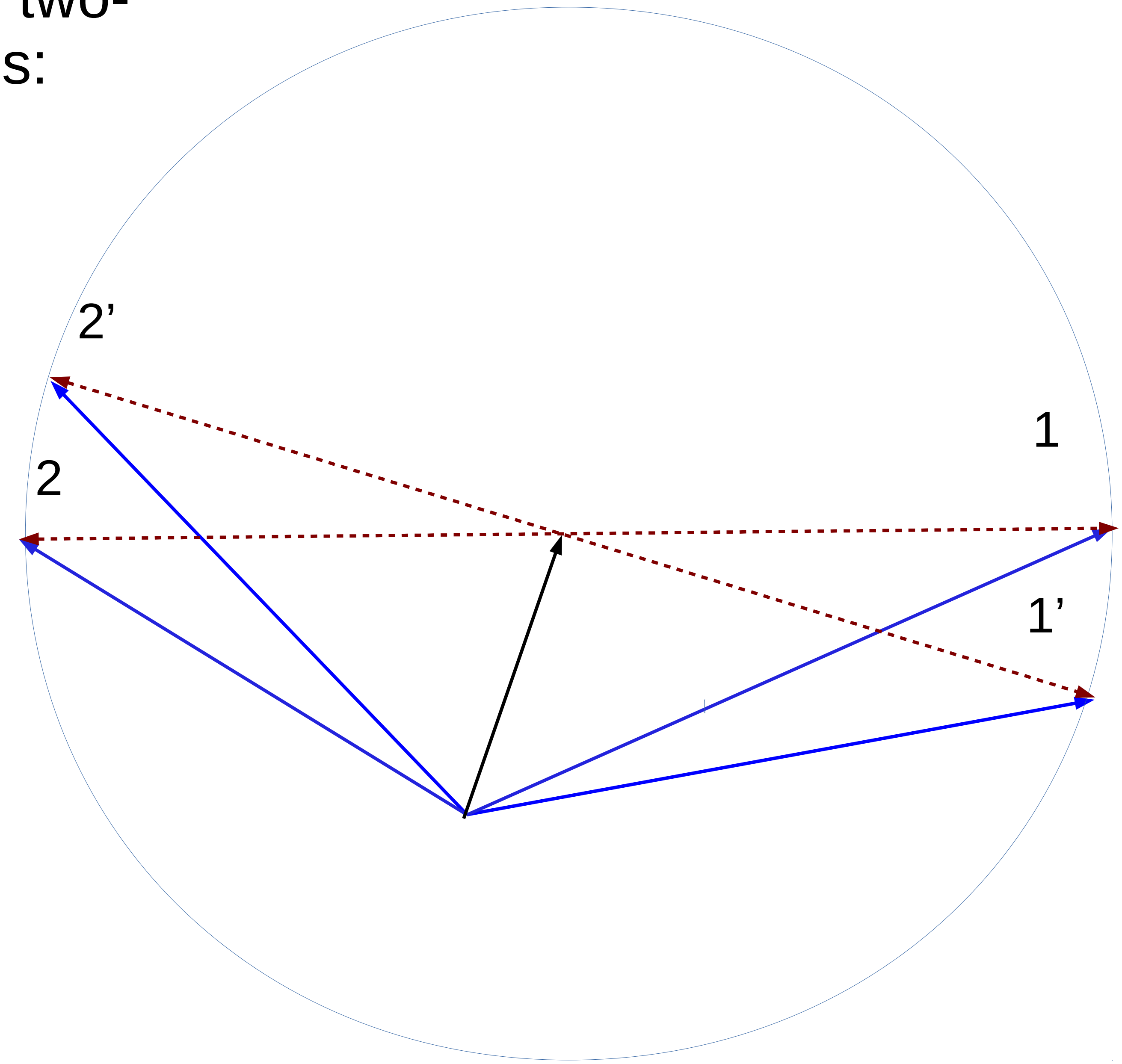
But in 2D it does matter!

revision of Fermi-liquid theory required





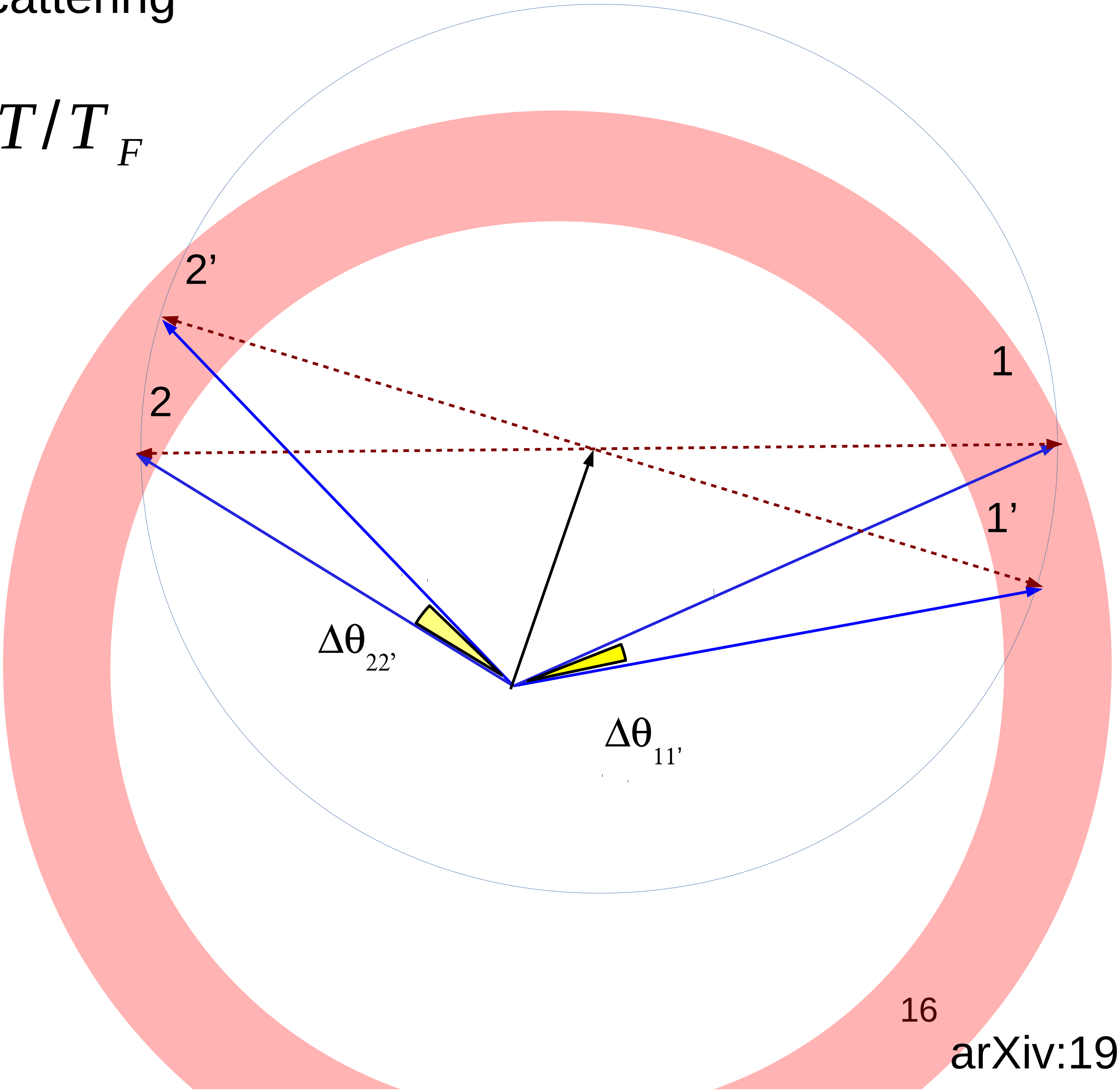
Kinematics of two-body collisions:



arXiv:1905.03751

Small scattering angles:

$$\Delta\theta \sim T/T_F$$



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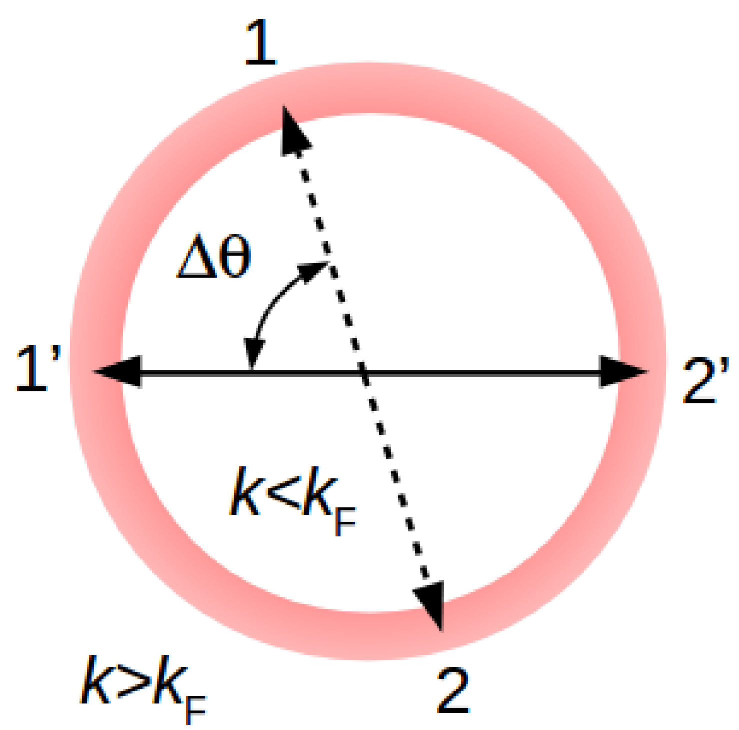
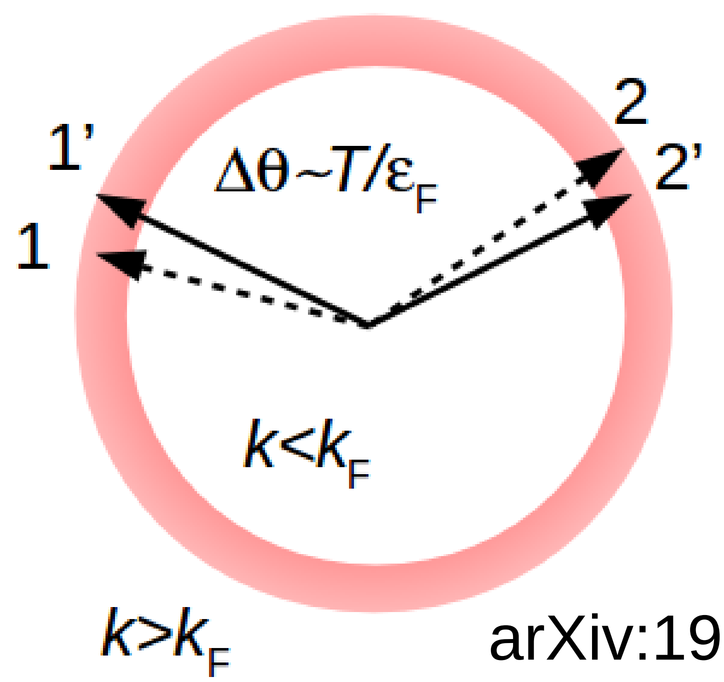
New behavior in 2D

- Momentum conservation and fermion exclusion single out two types of collisions: a) head-on, and b) small-angle
- Angular relaxation dominated by (near) head-on collisions.
- The **even-parity** and **odd-parity** parts of momentum distribution $\delta f(p)$ relax at different rates.
- Relaxation rates for the $\delta f(p)$ harmonics of the odd and even parity can differ by orders of magnitude: $\gamma'/\gamma \sim (T/T_F)^2$, $\gamma \sim T^2/T_F$





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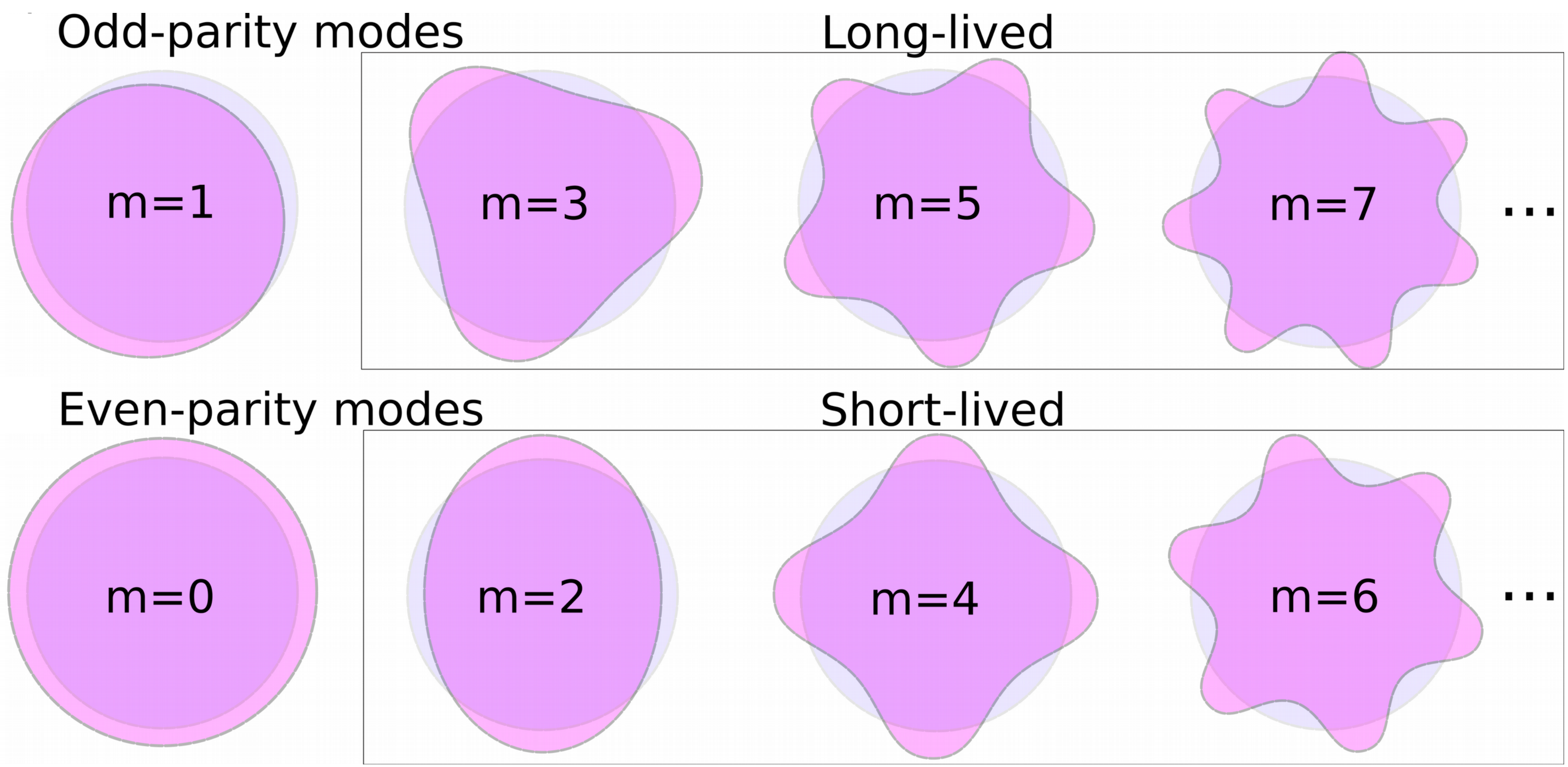
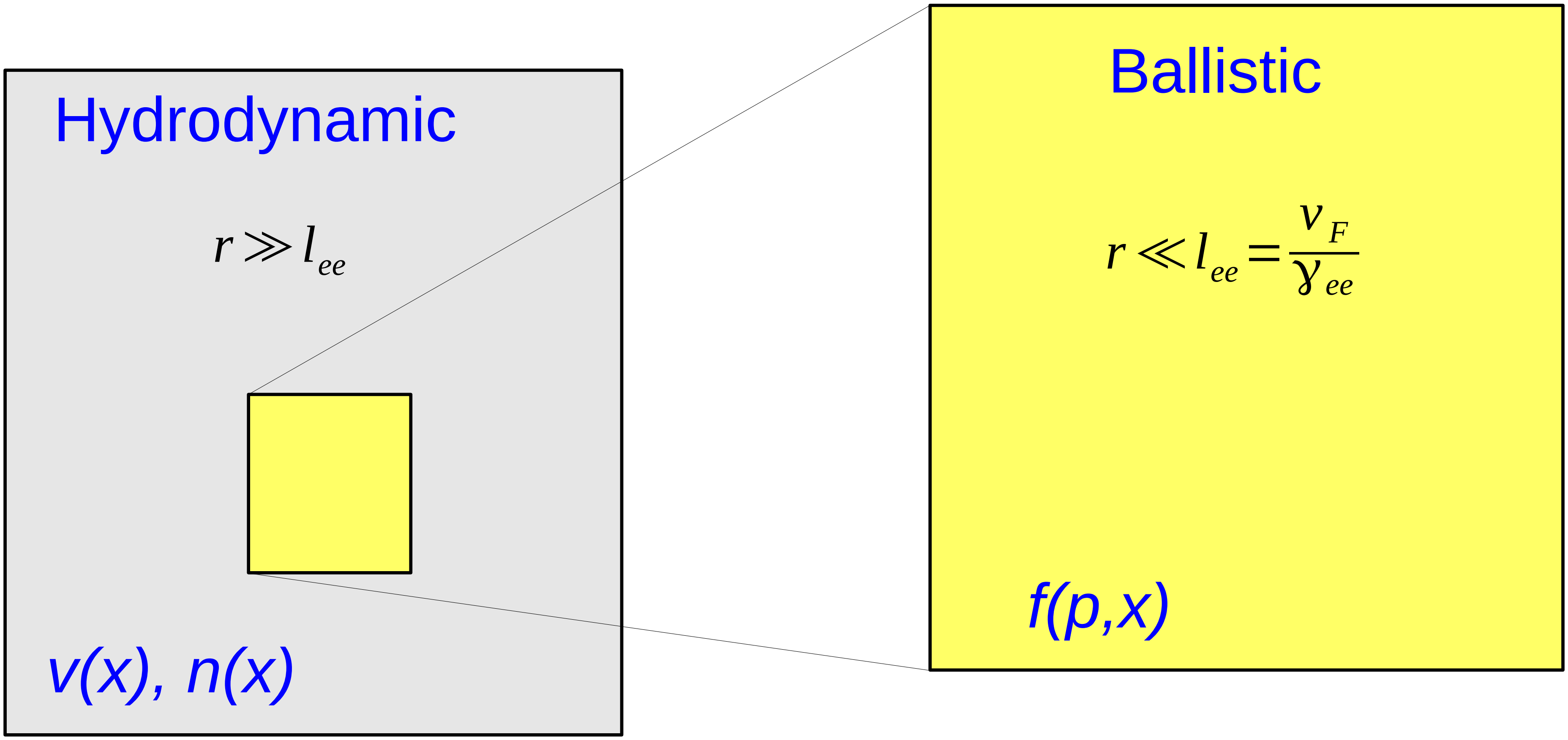
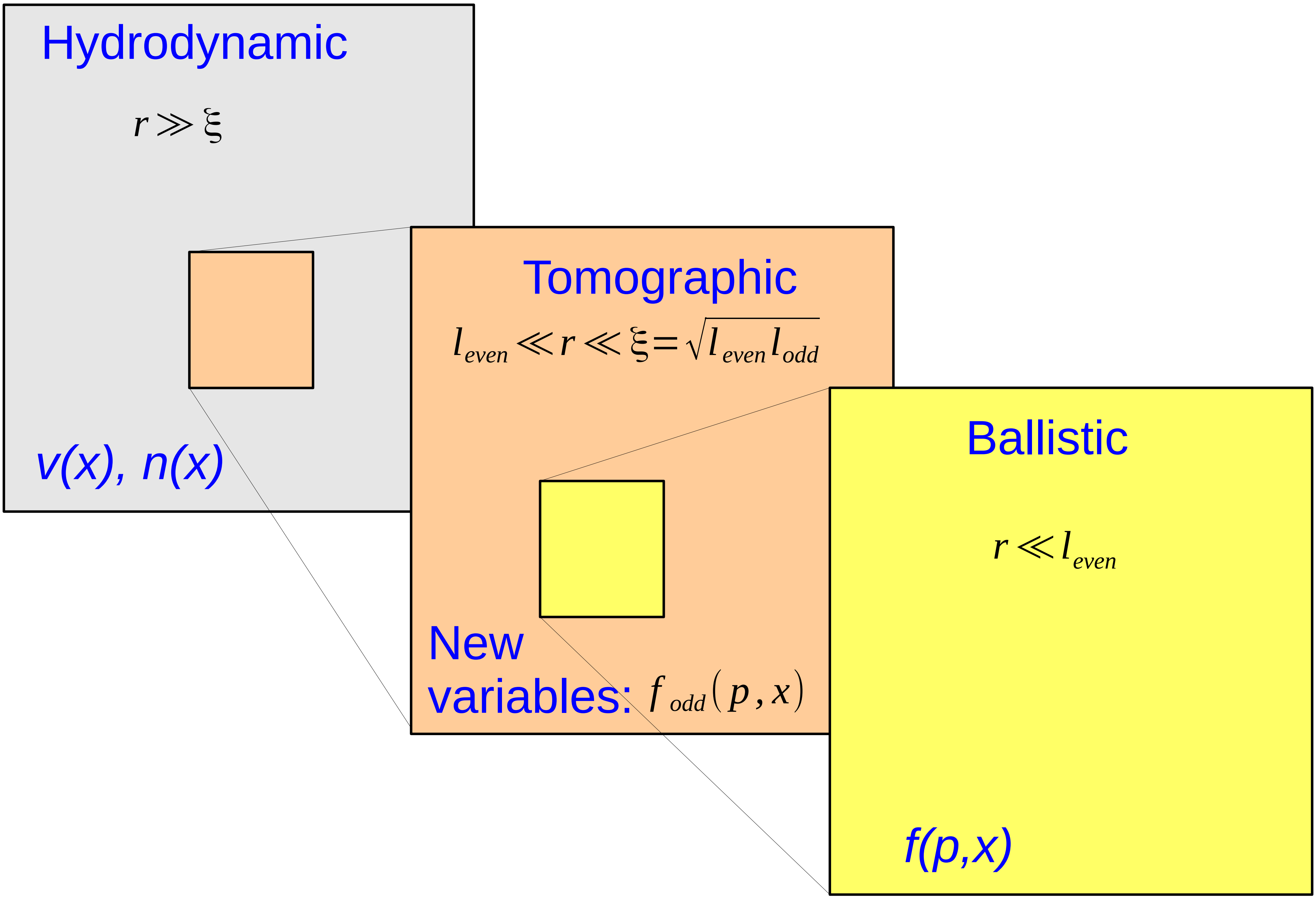


Conventional hierarchy

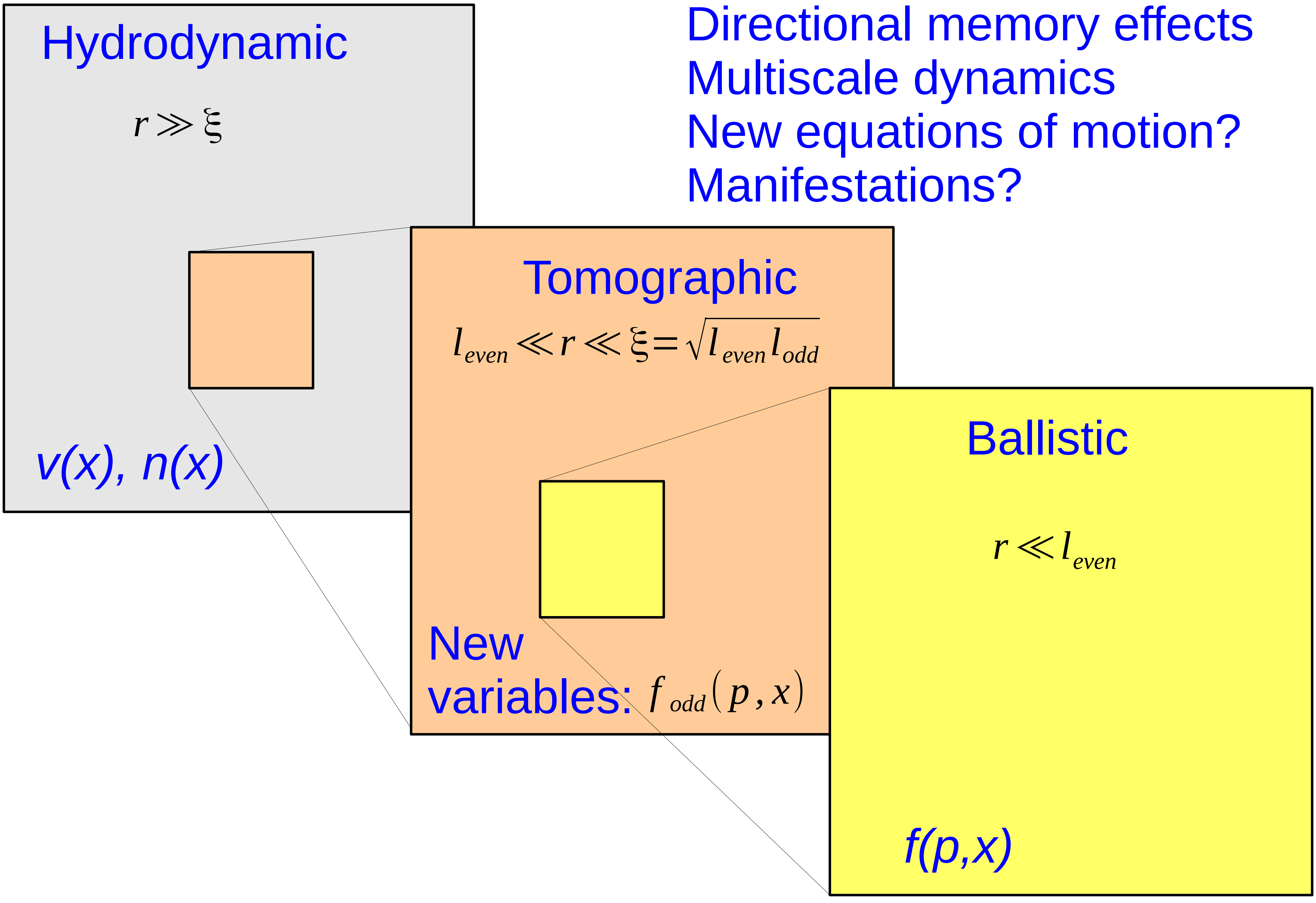


The new hierarchy



The new hierarchy

Directional memory effects
Multiscale dynamics
New equations of motion?
Manifestations?



Angular dynamics

Slow dynamics for odd-parity distributions $f(\theta+\pi)=-f(\theta)$

1) Long time scales

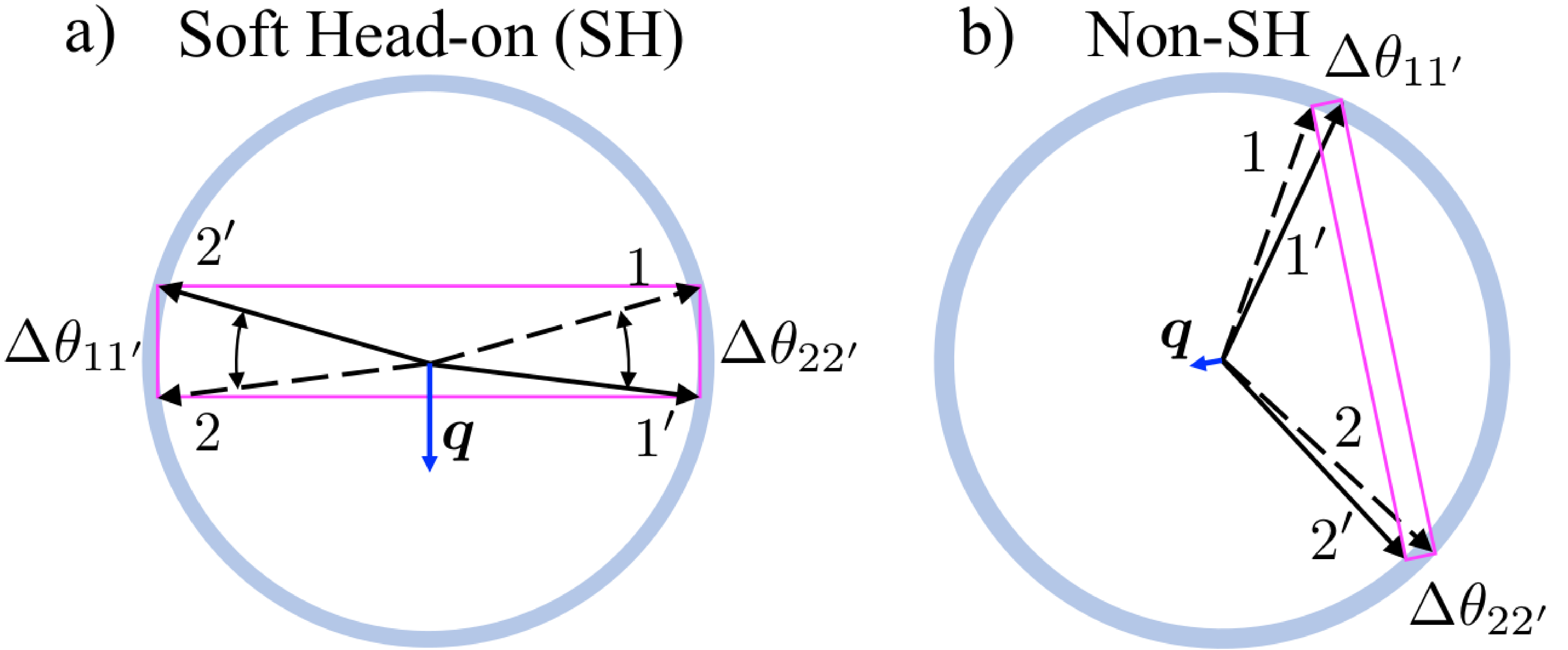
$$t \gg \tau_* \sim 1/T^2$$

2) Anomalous angular diffusion ("superdiffusion")

$$\left(\partial_t + D\,\partial_\theta^4\right) f\left(\theta\right) = 0$$

3) Directional memory ("tomographic" dynamics)

Angular stepsizes for different processes: $\Delta\theta \sim T$ vs. $\Delta\theta \sim \sqrt{T}$



$$\Delta\theta_{11'} = \Delta\theta_{22'} \sim \sqrt{\frac{T}{T_F}}$$

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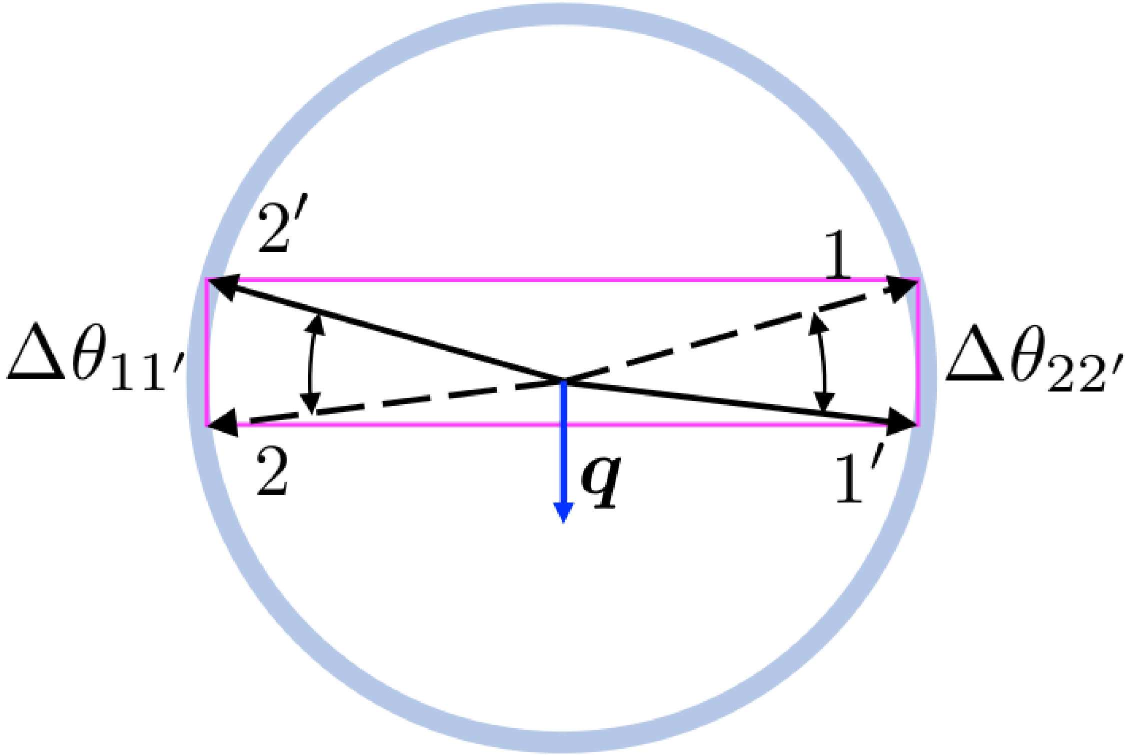
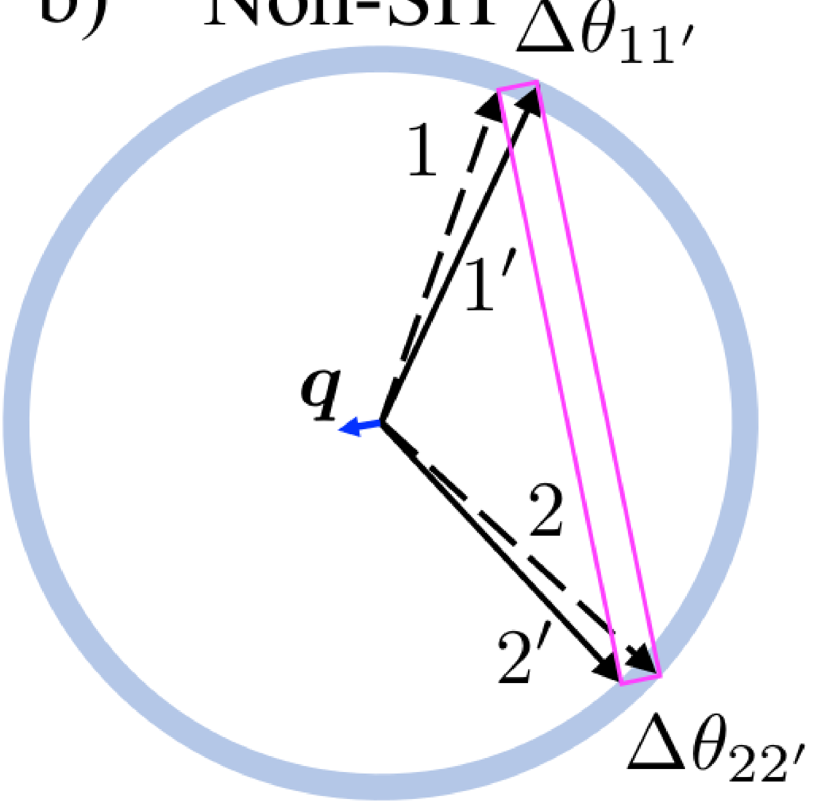
Angular stepsizes for different processes: $\Delta\theta \sim T$ vs. $\Delta\theta \sim \sqrt{T}$

a) Soft Head-on (SH)



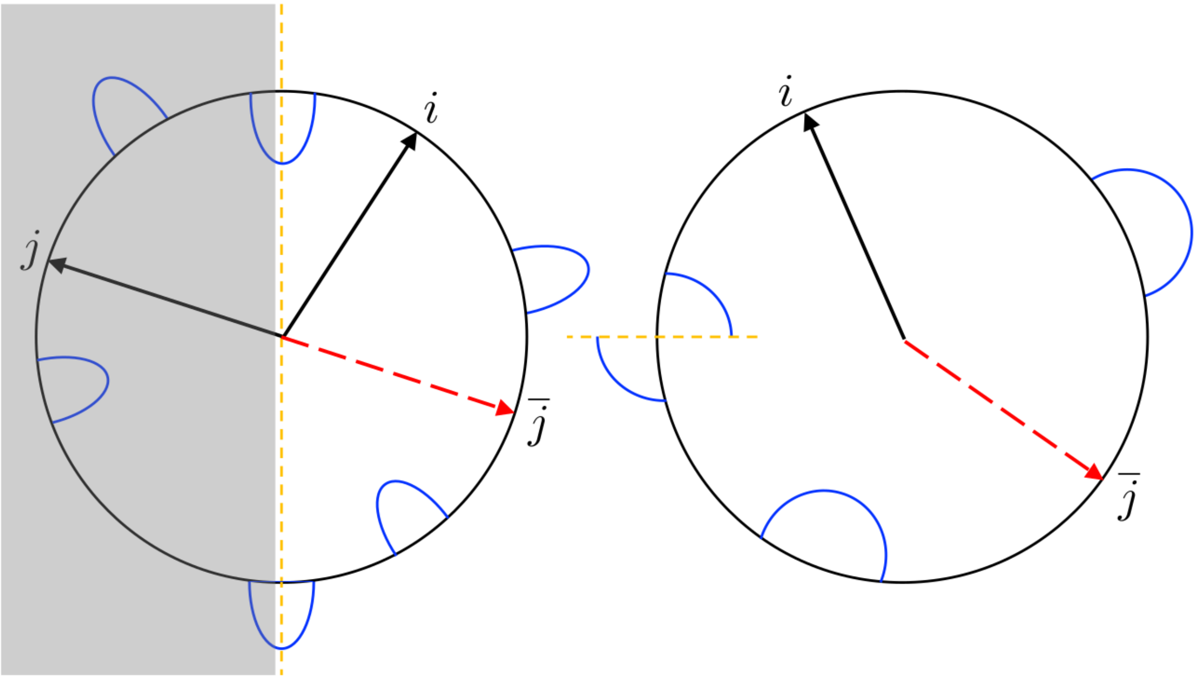
$$\Delta\theta_{11'} = \Delta\theta_{22'} \sim \sqrt{\frac{T}{T_F}}$$

b) Non-SH

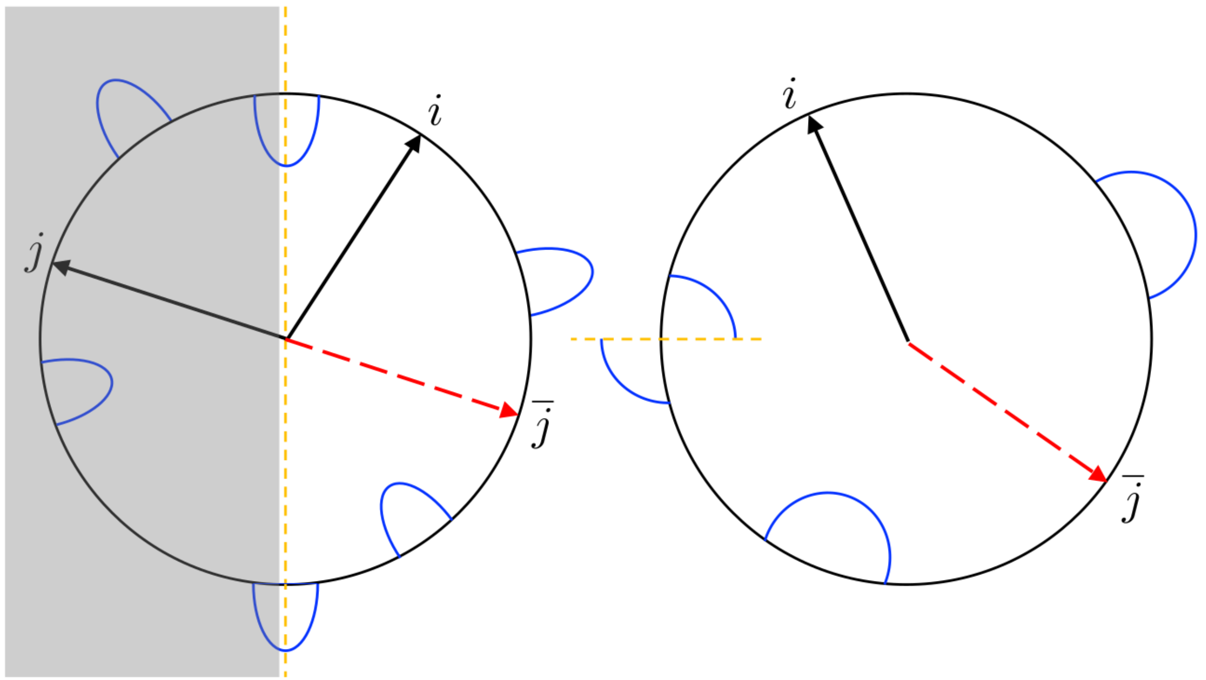


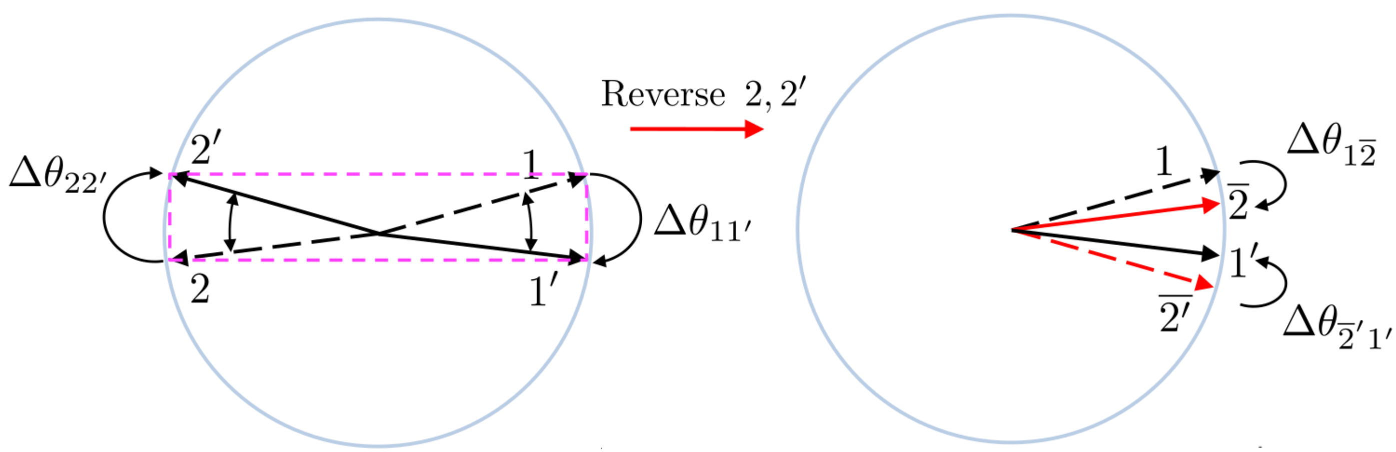
$$\Delta\theta_{11'} \sim \Delta\theta_{22'} \sim \frac{T}{T_F}$$

The odd-parity configuration phase space: a half-circle $S^1/Z_2 = RP^1 \equiv S^1$



Equal and opposite angular steps

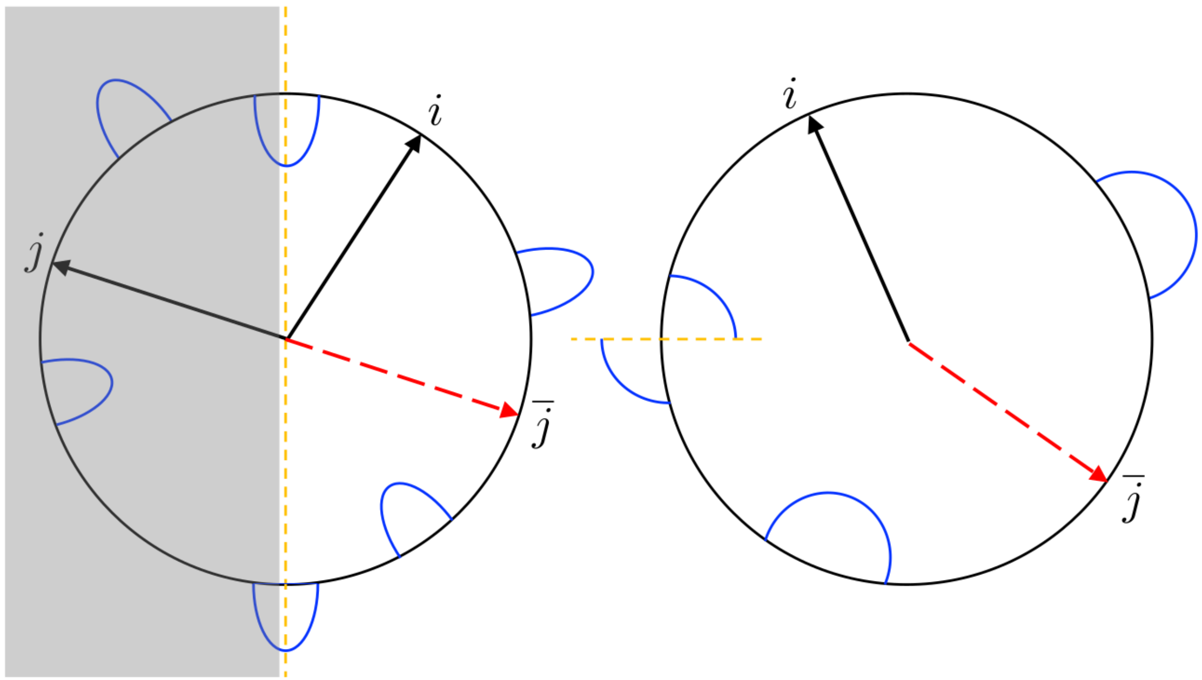




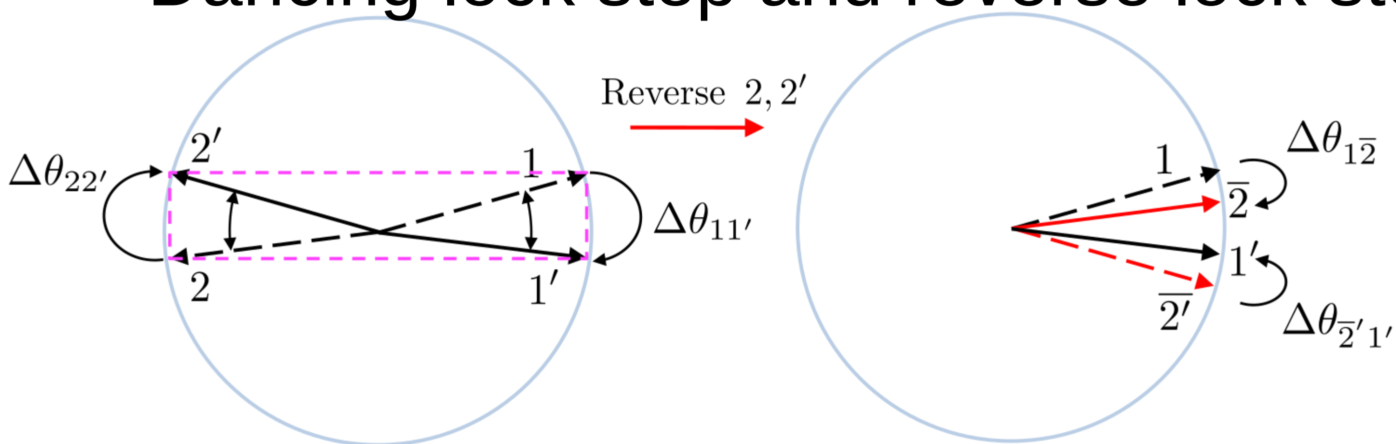
$$\Delta\theta_{11'} = \Delta\theta_{22'} \sim \sqrt{T}$$

$$\Delta\theta_{1\bar{2}} = -\Delta\theta_{\bar{2}'1'} \sim \sqrt{T}$$

Equal and opposite angular steps



Dancing lock step and reverse lock step:



$$\Delta\theta_{11'} = \Delta\theta_{22'} \sim \sqrt{T}$$

$$\Delta\theta_{1\bar{2}} = -\Delta\theta_{\bar{2}'1'} \sim \sqrt{T}$$

Angular superdiffusion

Naively, diffusivity = collision rate x (stepsize)^2

$$D = c\,\frac{T^2}{T_F}\,\Delta\theta^2 \sim T^3, \quad \Delta\theta \sim \sqrt{\frac{T}{T_F}}$$

However, the actual diffusivity vanishes due to strongly-correlated pairwise equal stepsizes:

$$D \sim \frac{T^2}{T_F}\left(\Delta\theta_{12} + \Delta\theta_{2'1'}\right)^2 = 0$$

Gives superdiffusion at next order:

$$D \sim \frac{T^2}{T_F}\,\Delta\theta^4 \sim T^4$$

Memory effects in time evolution

$$f(\theta, t) = \sum_m f_m e^{im\theta} e^{-\gamma_m t}$$

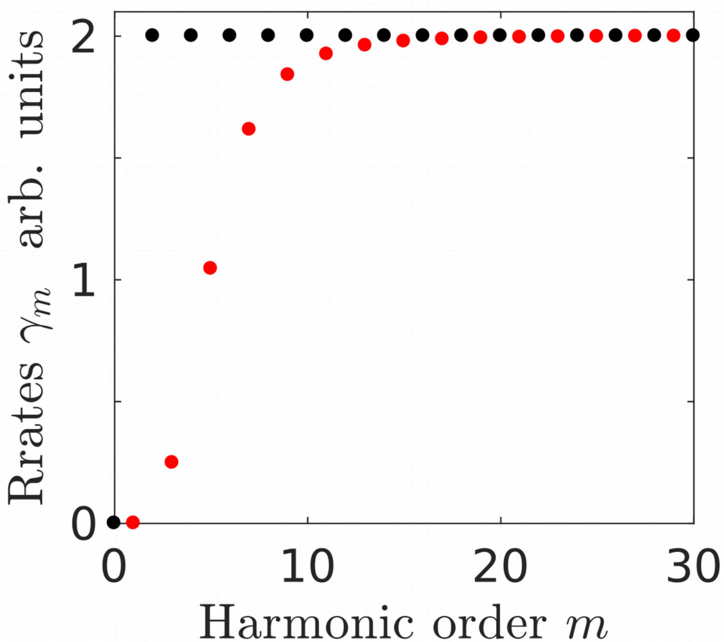
$$\gamma_{m\,even} = \gamma_0 \sim T^2$$

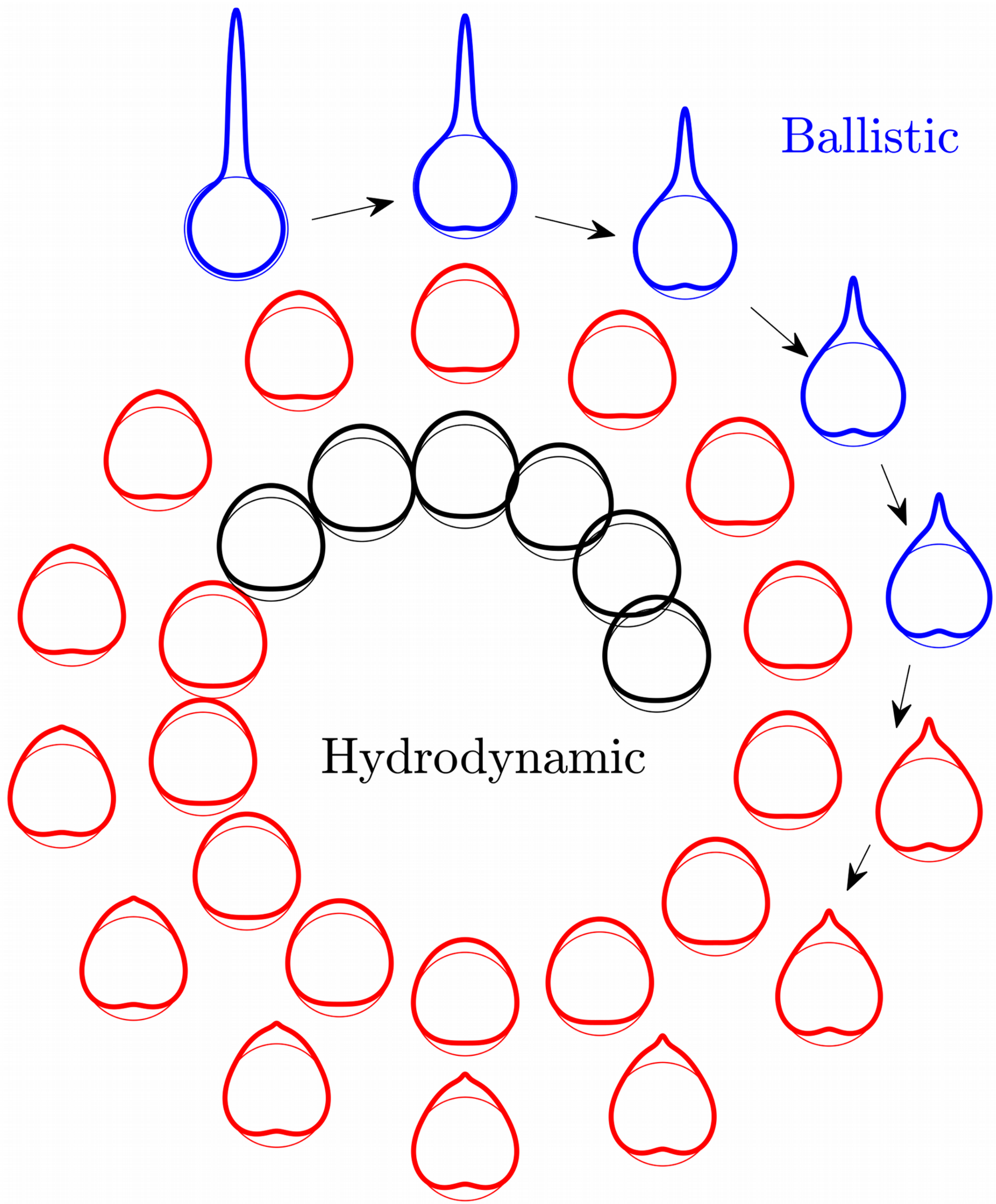
$$\gamma_{m\,odd} = Dm^4 \sim T^4 m^4$$

$$m < m_* = (\gamma_0 / D)^{1/4} \sim T^{-1/2}$$

Time evolution phases

Tomographic
(odd-parity
angular
diffusion)



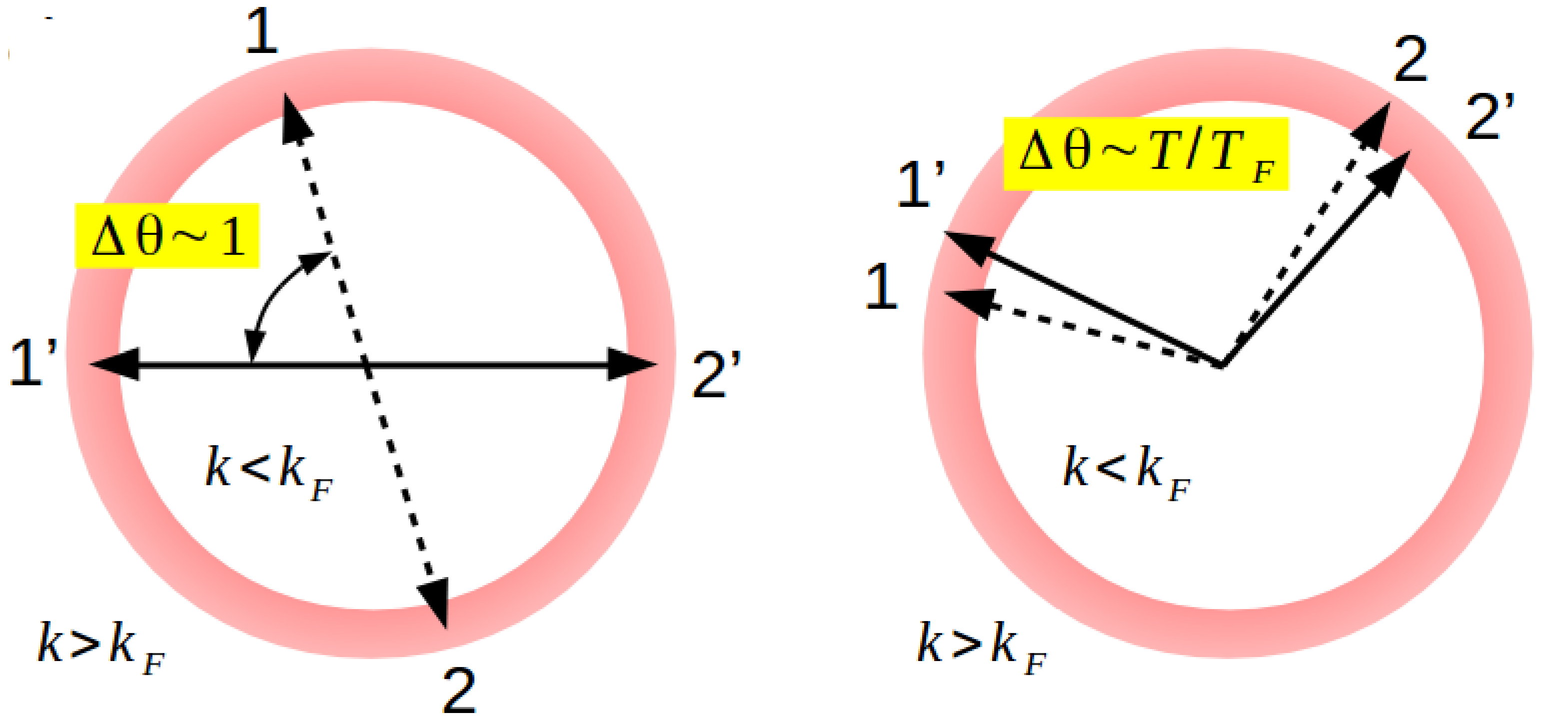


The rates γ_m

Estimating the rates

Angular diffusion

$$\gamma_{m\,even} \sim R_* \frac{T^2}{T_F^2} \qquad \gamma_{m\,odd} \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$



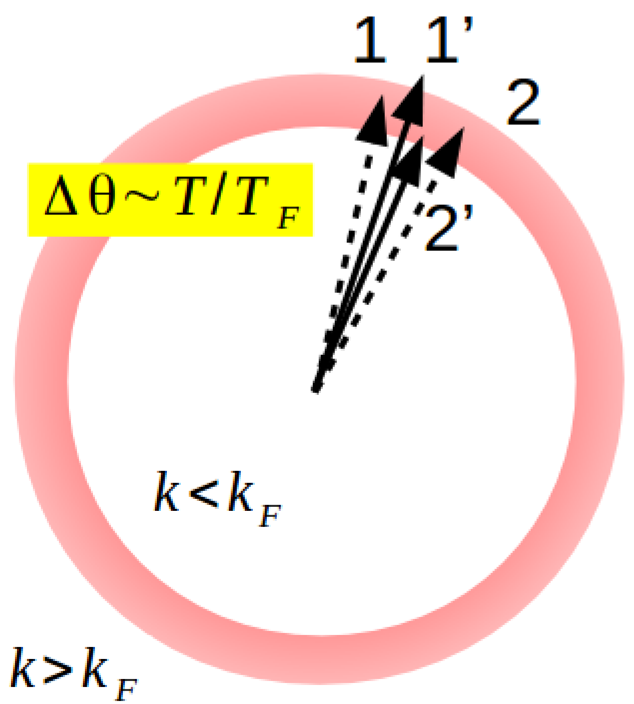
arXiv:1905.03751

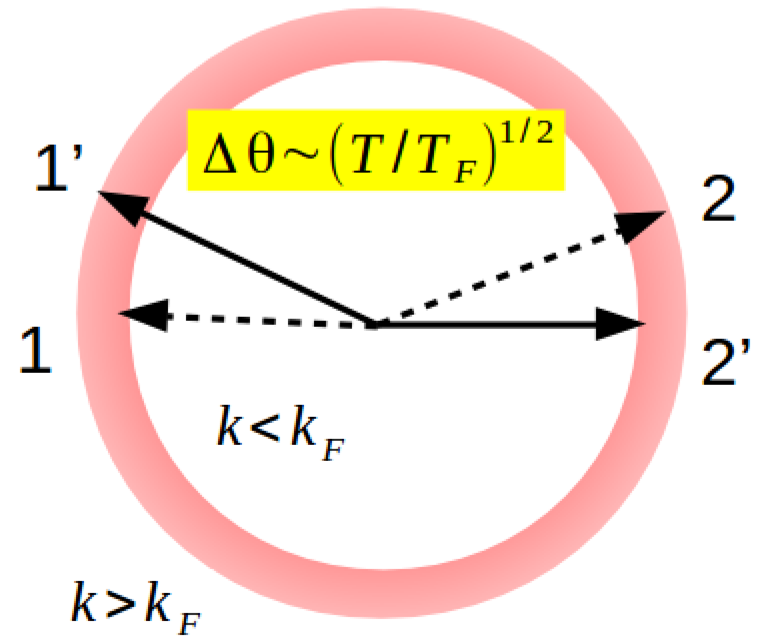
Estimating the rates: odd *m*

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

Naively:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 = R_* \frac{T^3}{T_F^3} m^2$$





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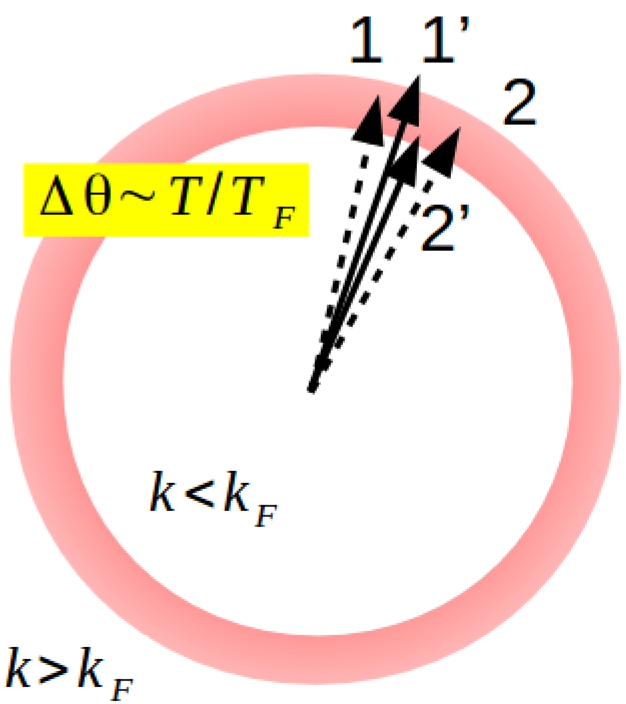

Naively:

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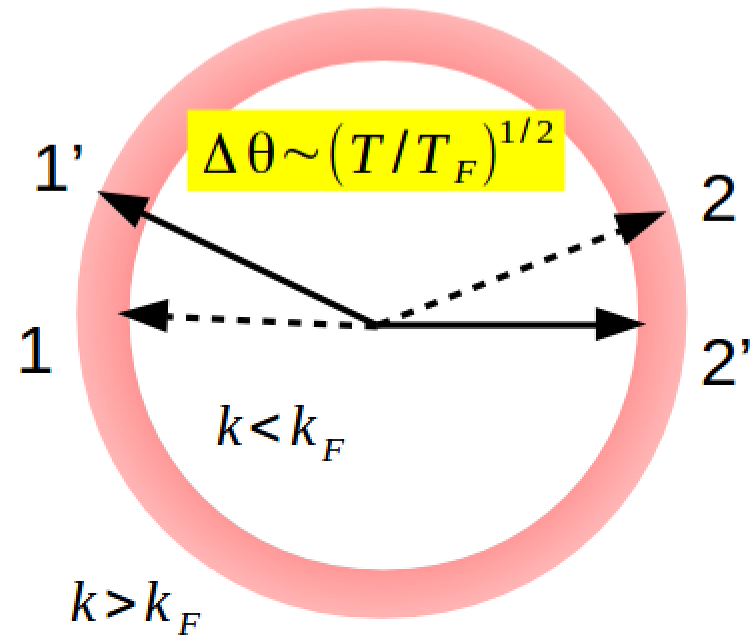
Is this true? Not quite



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Estimating the rates: odd *m*

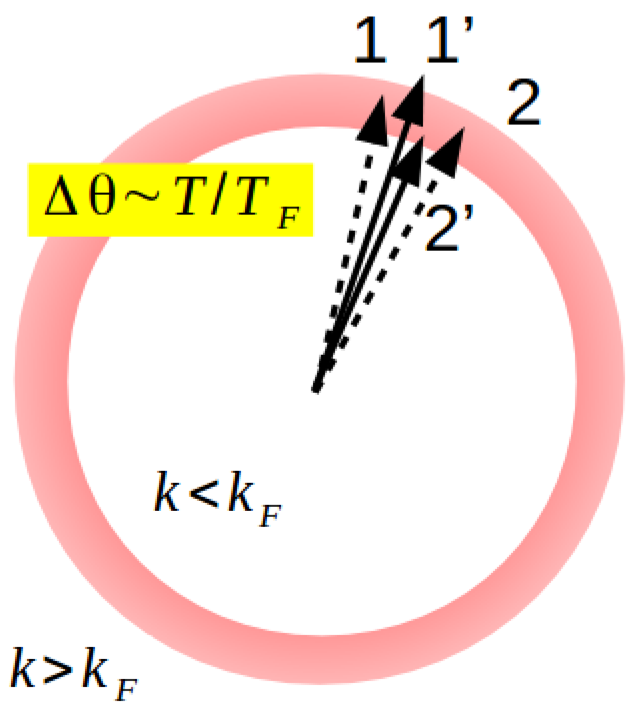
Actually:

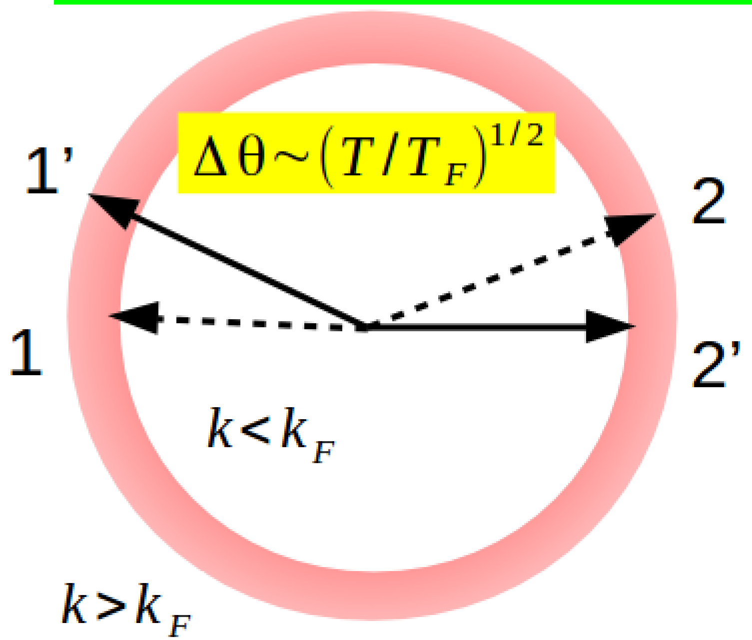
Angular superdiffusion

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} (\Delta\theta^2 m^2)^2 = R_* \frac{T^4}{T_F^4} m^4$$

Origin: correlated angular shifts in ee scattering





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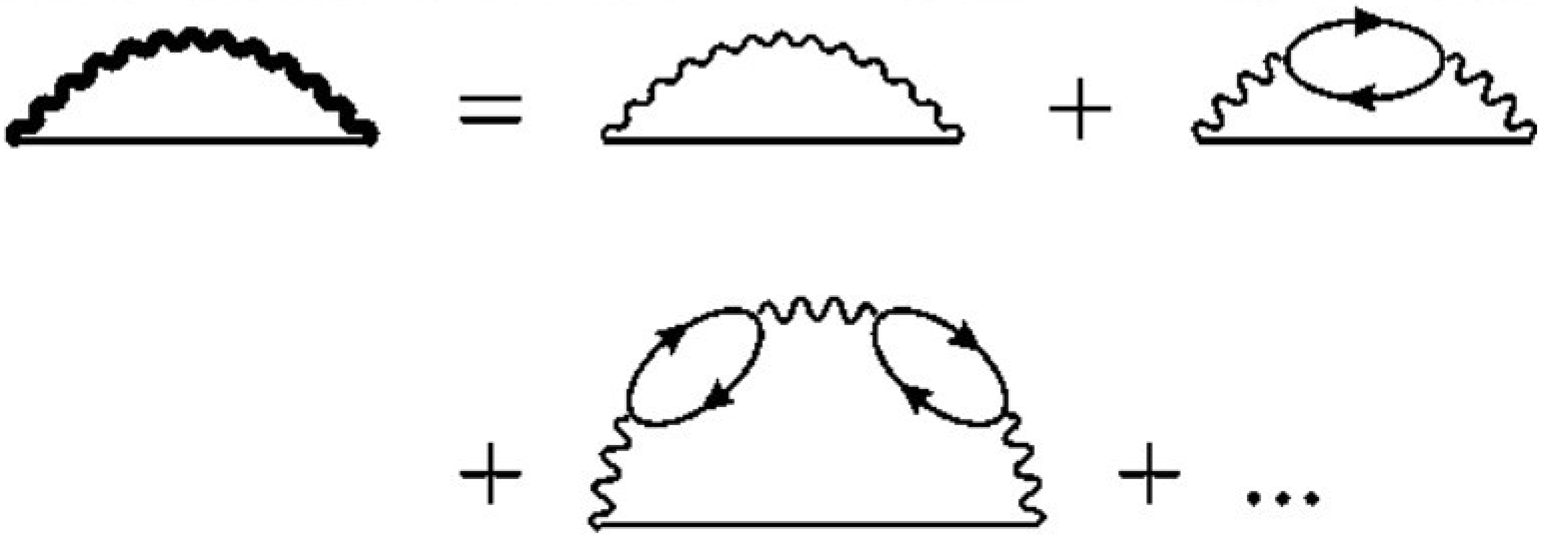
Cf. lifetimes from selfenergy in 2D

$$\gamma = -2\Sigma''(\epsilon, p) \sim T^2 \ln(1/T)$$

Chaplik 1971
Hodges, Smith, Wilkins 1971
Bloom 1975
Giuliani, Quinn 1982
Menashe, Laikhtman 1996
Zheng DasSarma 1996
Chubukov, Maslov 2003



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Hodges, Smith, Wilkins 1971
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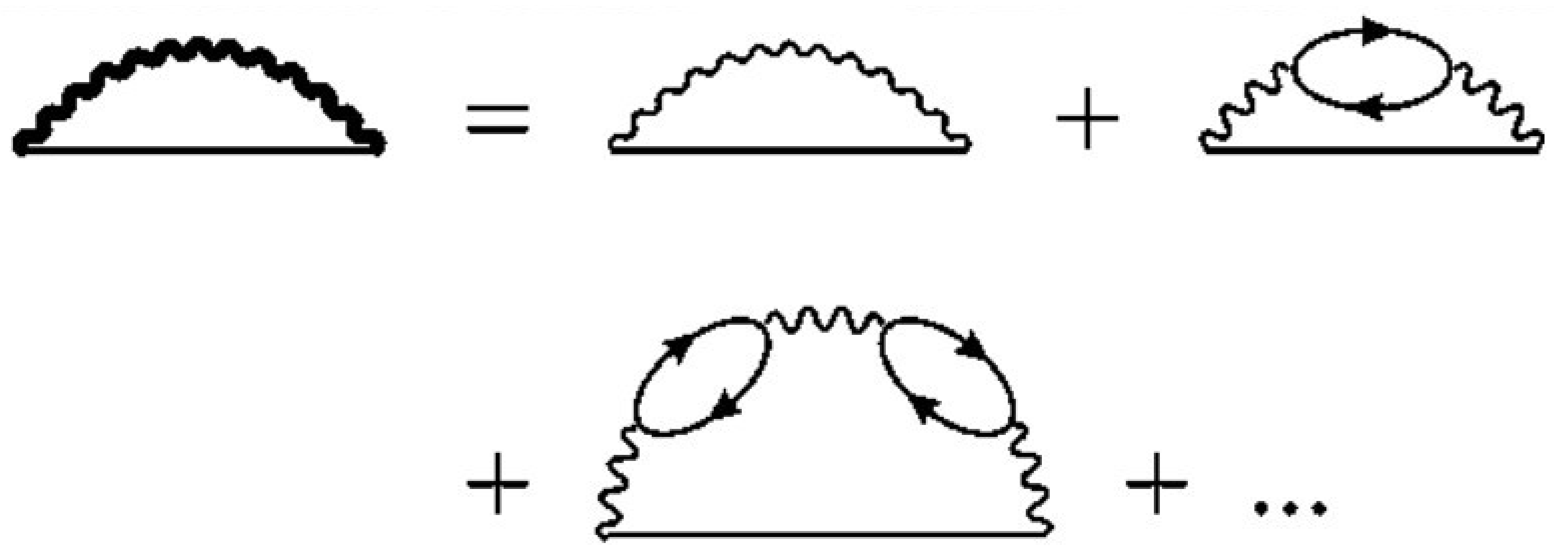
Dominated by the fast pathways (rapid decays),
Insensitive to slowly decaying modes

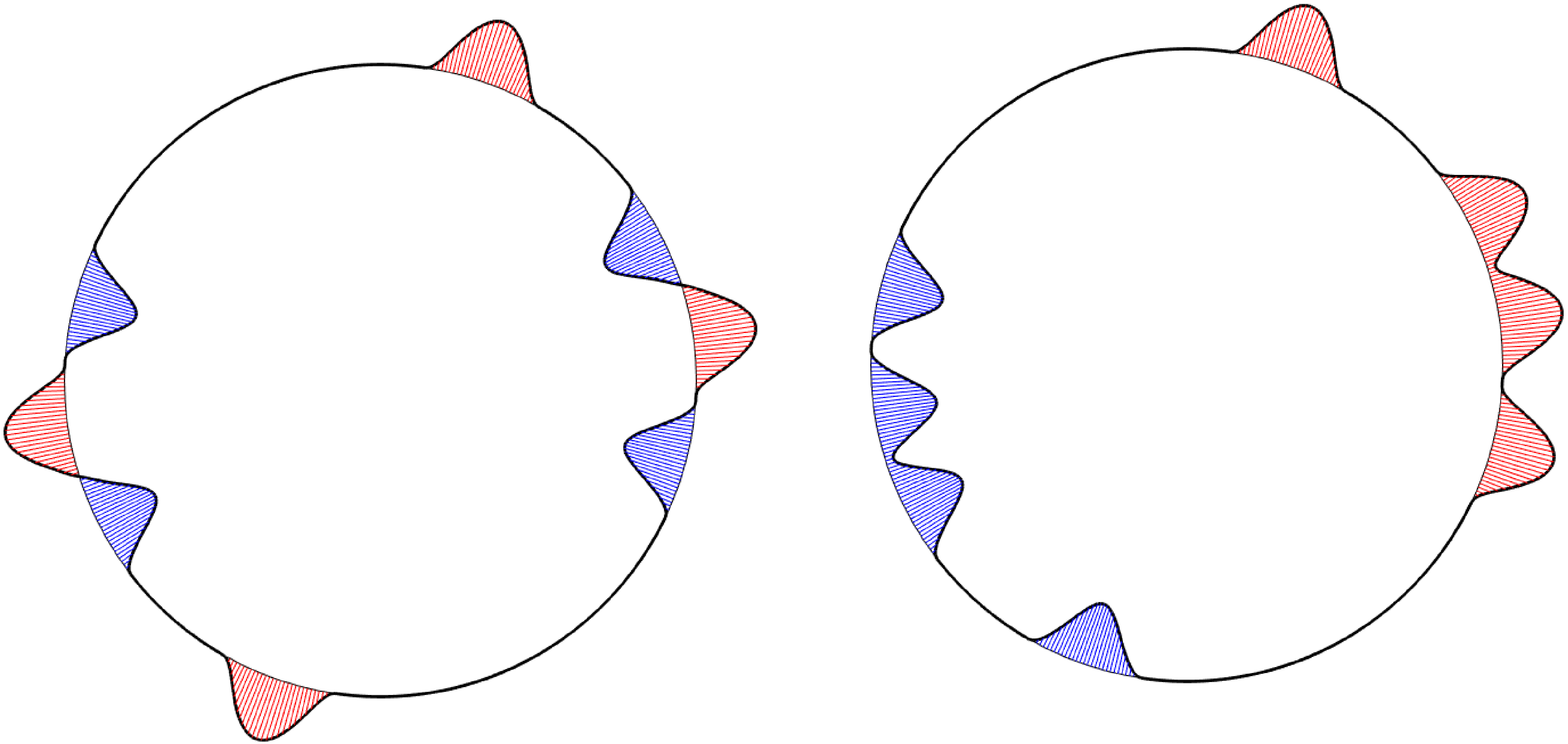


Macroscopic manifestations: Nonlocal conductivity and Scale-dependent viscosity

Active degrees of freedom: loops in *p*-space

Even parity

Odd parity



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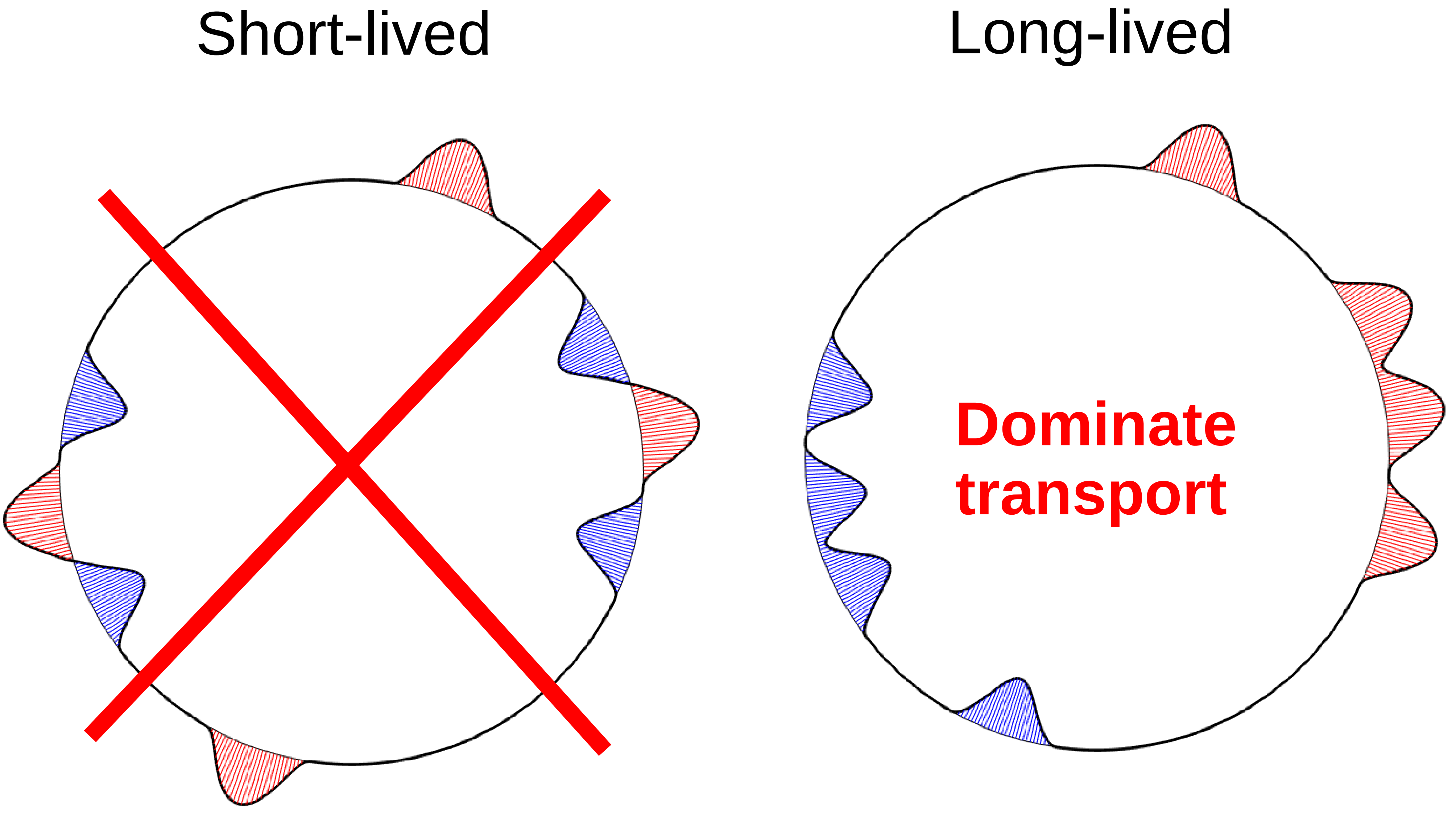

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Tomographic dynamics

Integrate out the even-*m* modes, derive a closed-form master equation for the odd-*m* modes, valid at the length scales $r > l_{ee} = v/\gamma$

$$\left(\partial_t - I_+\right)\delta f_p^+(\vec{x},t) + \vec{v}\,\vec{\nabla}\,\delta f_p^-(\vec{x},t) = 0,$$
$$\left(\partial_t - I_-\right)\delta f_p^-(\vec{x},t) + \vec{v}\,\vec{\nabla}\,\delta f_p^+(\vec{x},t) = -e\vec{E}(x)\,\nabla_p f_p^{(0)}$$

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$$(\partial_t - I_-)\delta f_p^-(\vec{x},t) + \vec{v}\vec{\nabla}\delta f_p^+(\vec{x},t) = -e\vec{E}(x)\nabla_p f_p^{(0)}$$

Fast spatial diffusion along the velocity ***v*** direction, plus a slow angular diffusion that gradually randomizes the orientation of ***v***:

$$(\partial_t - D(\hat{\boldsymbol{v}}\vec{\nabla})^2 + \gamma'\partial_\theta^p)\delta f_p^-(\vec{x},t) = -e\vec{E}(x)\nabla_p f_p^{(0)}, \quad D = v^2/\gamma$$

arXiv:1905.03751

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Integrate out the even-*m* modes, derive a closed-form master equation for the odd-*m* modes, valid at the length scales $r > l_{ee}=v/\gamma$

$$(\partial_t - I_+)\delta f_p^+(\vec{x},t) + \vec{v}\vec{\nabla}\delta f_p^-(\vec{x},t) = 0,$$

$$(\partial_t - I_-)\delta f_p^-(\vec{x},t) + \vec{v}\vec{\nabla}\delta f_p^+(\vec{x},t) = -e\vec{E}(x)\nabla_p f_p^{(0)}$$

Fast spatial diffusion along the velocity ***v*** direction, plus a slow angular diffusion that gradually randomizes the orientation of ***v***:

$$(\partial_t - D(\hat{\mathbf{v}}\vec{\nabla})^2 + \gamma' \partial_\theta^p)\delta f_p^-(\vec{x},t) = -e\vec{E}(x)\nabla_p f_p^{(0)}, \quad D = v^2/\gamma$$

Analyze response to a transverse field $\vec{E}(x) = \vec{E}\cos\vec{k}\vec{x}, \quad \vec{E}\cdot\vec{k} = 0$
obtain current, momentum flux & viscosity

Conductivity and viscosity nonlocal (scale-dependent) for $l_{ee} < r < \xi$

arXiv:1905.03751

Tomographic dynamics

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$$\sigma(k) \sim \frac{\gamma}{(\gamma\gamma')^{\frac{1}{2+p}}} k^{-\frac{2p+2}{p+2}}$$

Response functions from continued fractions

Current response to transverse field $\vec{j} \perp \vec{k}, \vec{E} \perp \vec{k}$

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Ohmic momentum relaxation

Viscous momentum relaxation

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Nonlocal conductivity and viscosity;
***k* dependence = scale dependence**

The odd-parity transport regime

- Occurs between the ballistic and conventional hydro regimes, at the length scales $l_{ee} < r < \xi = \frac{v}{\sqrt{\gamma \gamma '}}$ wavenumbers $\xi^{-1} < k < 1/l_{ee}$
- Scale-dependent viscosity $\eta(k) \sim 1/k^{1/3}$
- Power-law current profile in a strip: $j(0 < x < w) \sim x^{2/3}(w - x)^{2/3}$
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- Fractional-power Poiseuille flow
- Scale-dependent conductivity: $\sigma(k) \sim 1/k^{5/3}$
- Conductance of a strip of width $l_{ee} < w < \xi$ $\quad G(w) \sim w^{8/3}$
- 2 < 8/3 < 3 (between the Gurzhi and ballistic values)

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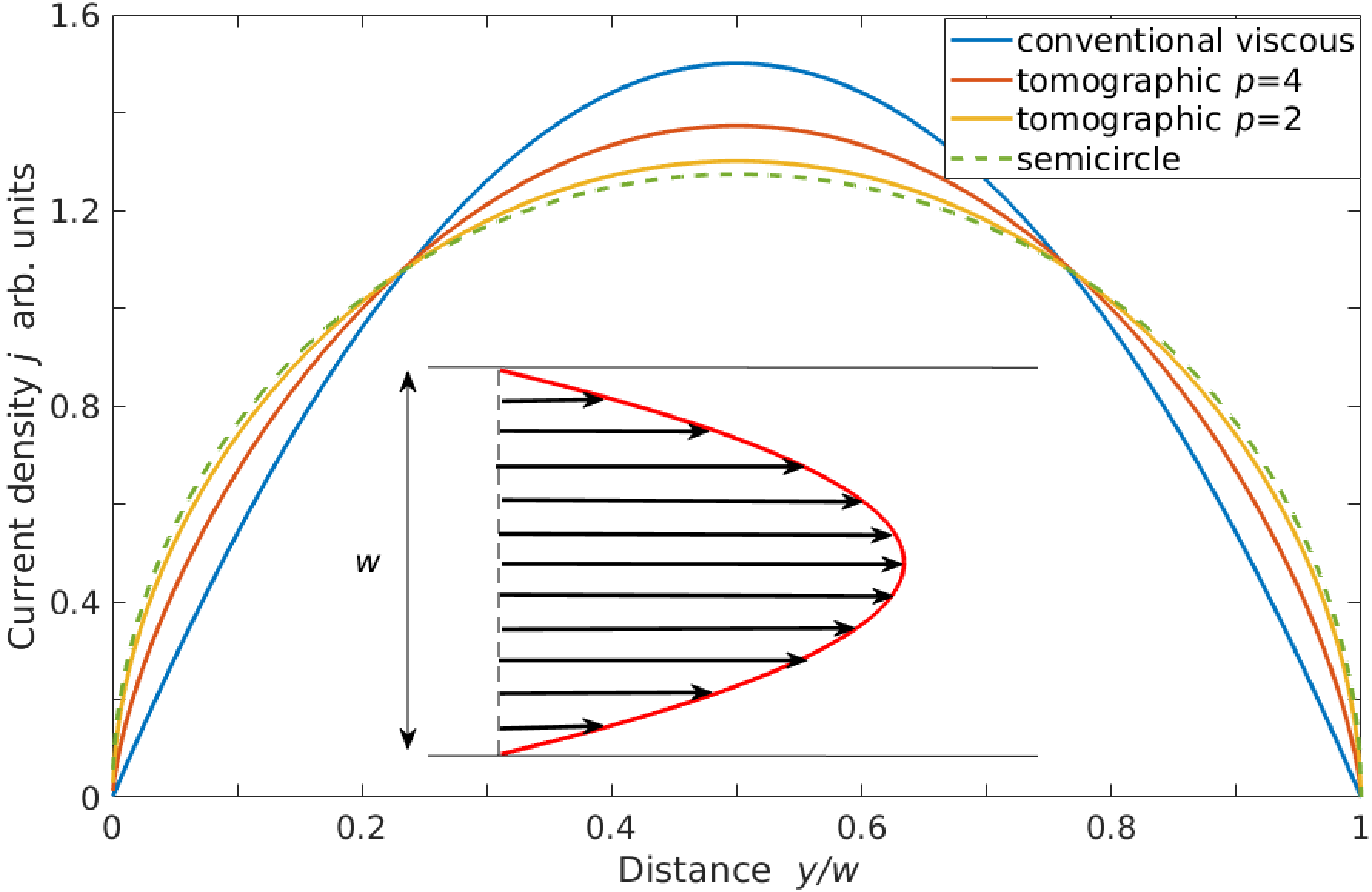
Fractional Poiseuille flows



Why all high-*m* modes matter?

Soft modes of the linearized transport operator:

$$(\partial_t + \vec{v}\vec{\nabla} - I)\delta f_p(\vec{x},t) = 0, \quad \delta f_p = \sum_m \delta f_m e^{im\theta_p}, \qquad \gamma_{m\ odd} \ll \gamma_{m\ even}$$

Plane-wave solns $\delta f \sim e^{ikx - i\omega t}$: a tight-binding model in δf_m basis

$$(\gamma_m - i\omega)\delta f_m = \frac{kv}{2}\delta f_{m-1} + \frac{kv}{2}\delta f_{m+1}$$

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γ_m=0 on every other site in the limit γ'<<γ; an infinitely long-lived dark state:

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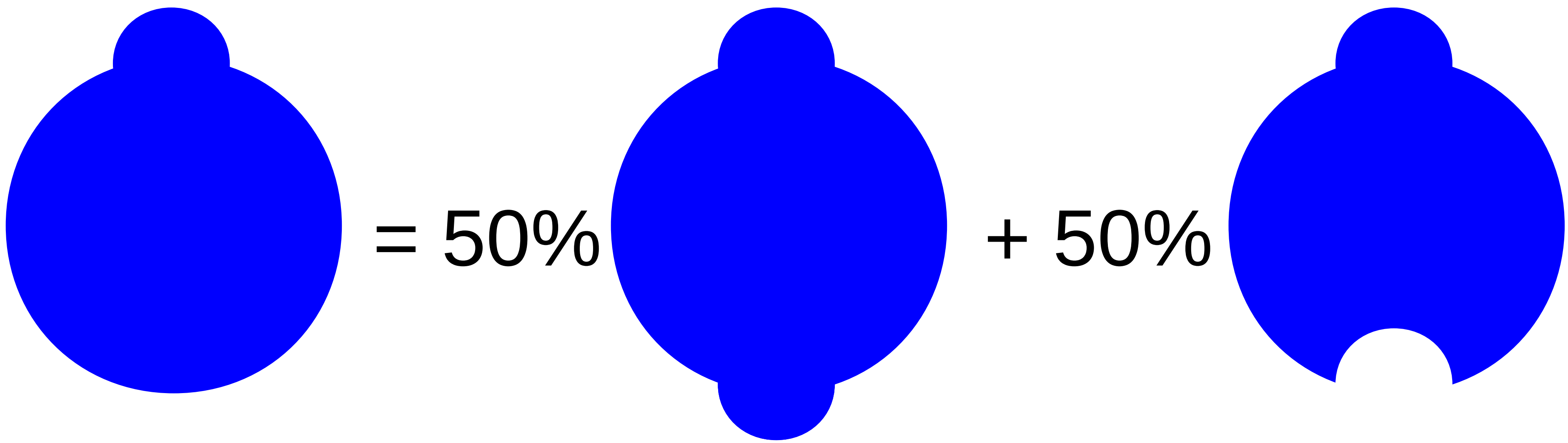
- A band of soft modes originating from the dark state: overlap w/ the current modes m=1,-1, influence conduction & viscosity
- FDT: slow relaxation implies strong current fluctuations & reduced dissipation; *k*-dependent couplings, scale dependence

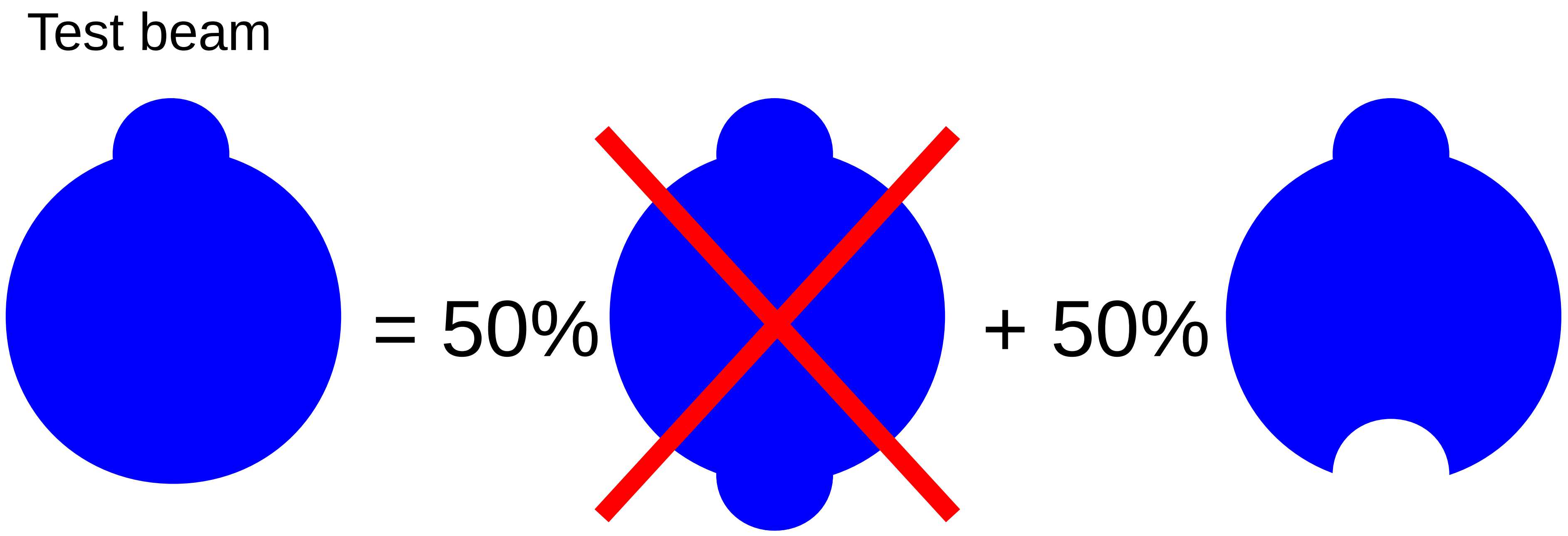
“Microscopic” manifestations:
* fermionic retroreflection and electron-hole jets
* stochastic “Andreev” scattering and magnetotransport

Particle-hole retroreflection

Test beam

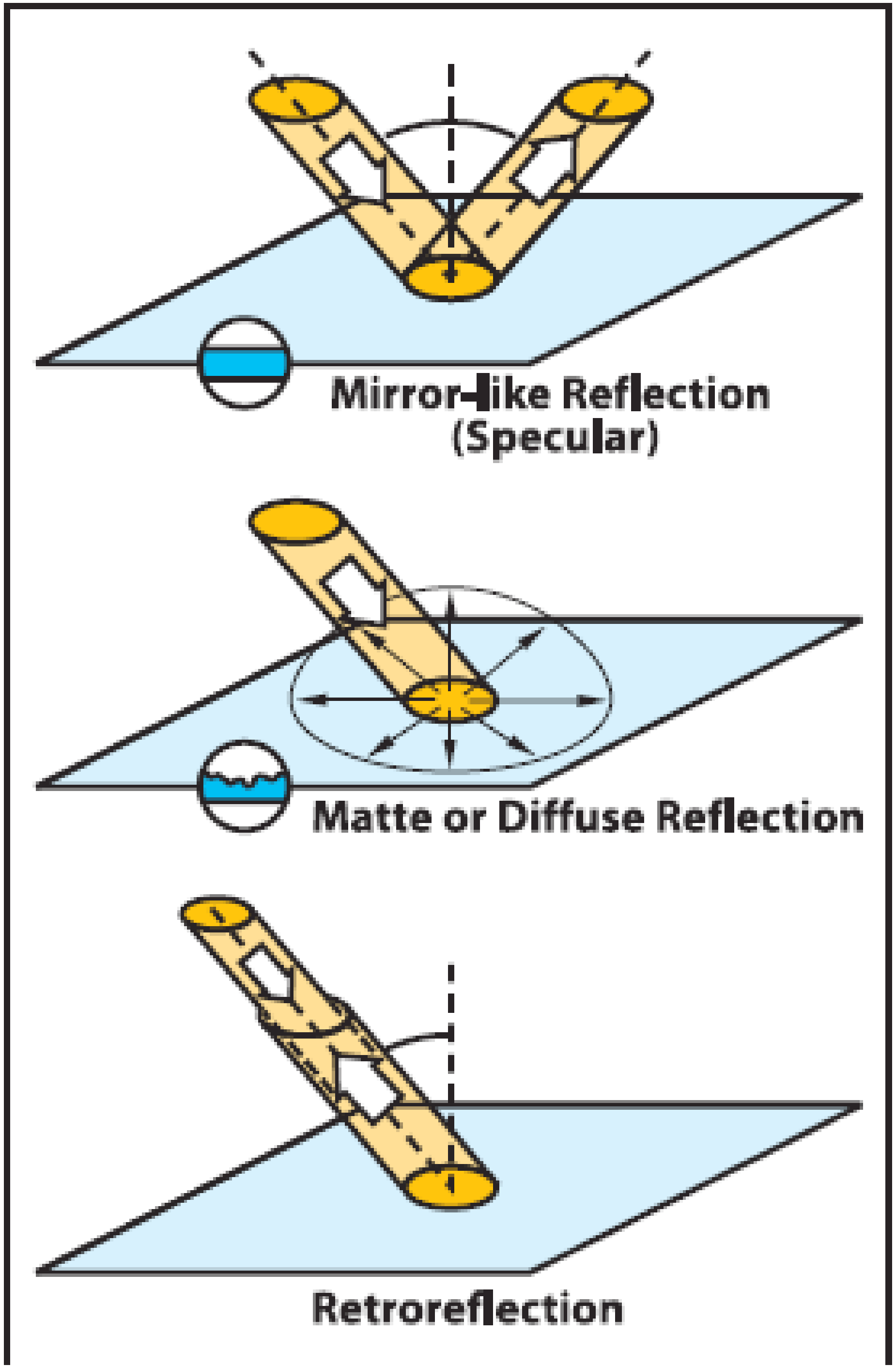


Particle-hole retroreflection

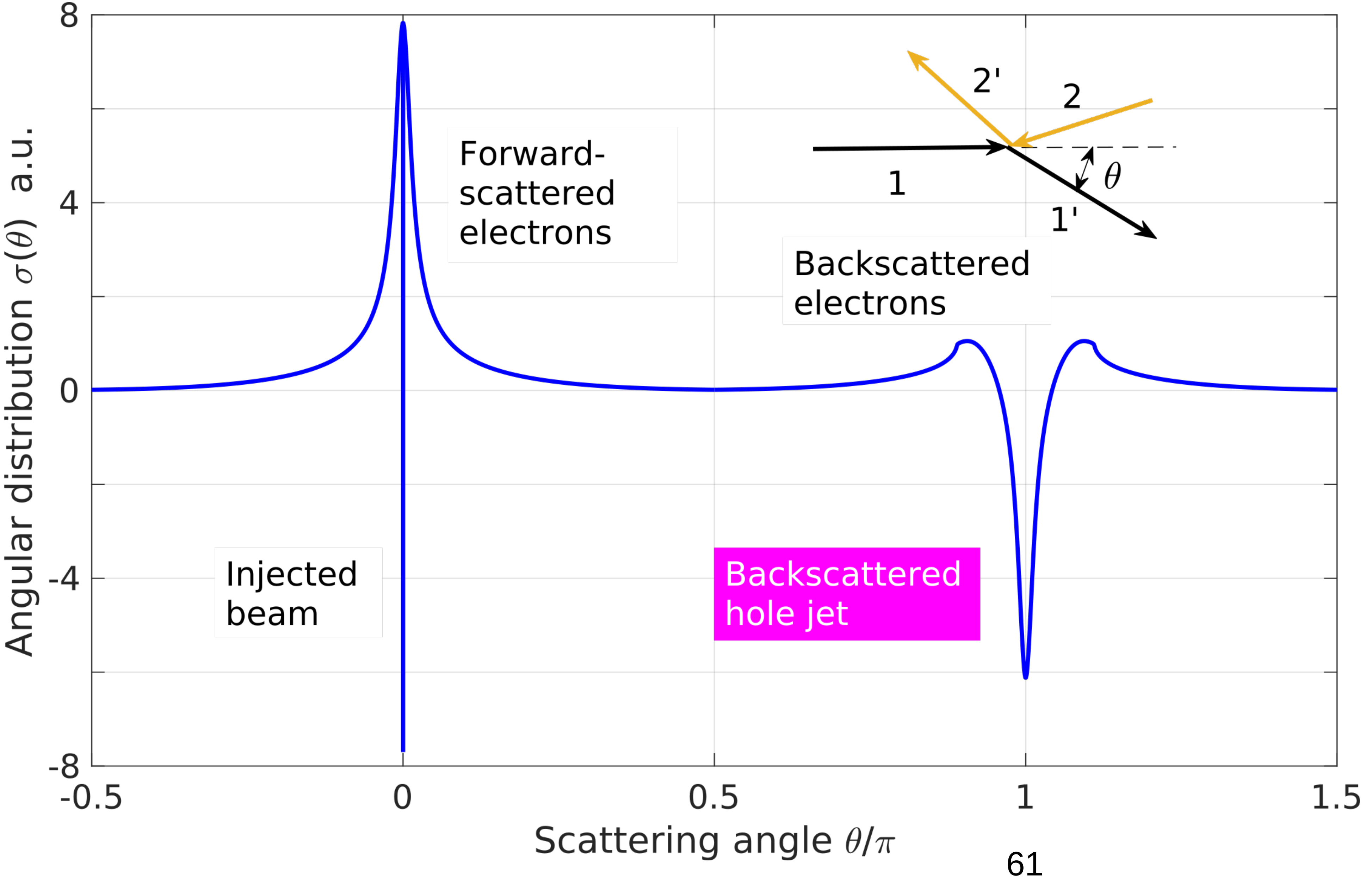


- **Fermionic jets**: multiple particle/hole switchbacks on straight paths
- Backreflected holes, stochastic Andreev scattering

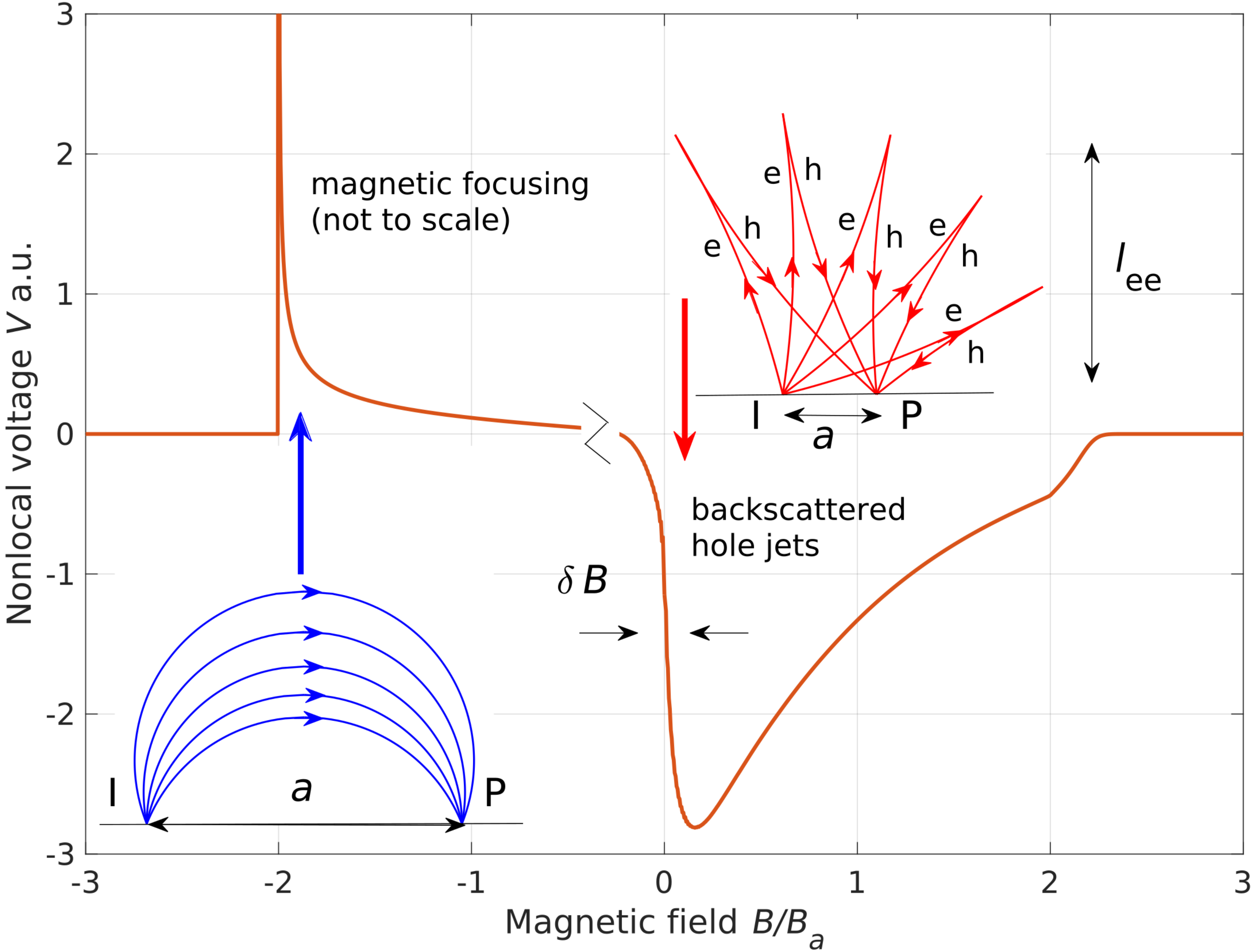
Retroreflection returns the light to its source



Fermionic hole jets



Probing fermionic jets by magnetotransport



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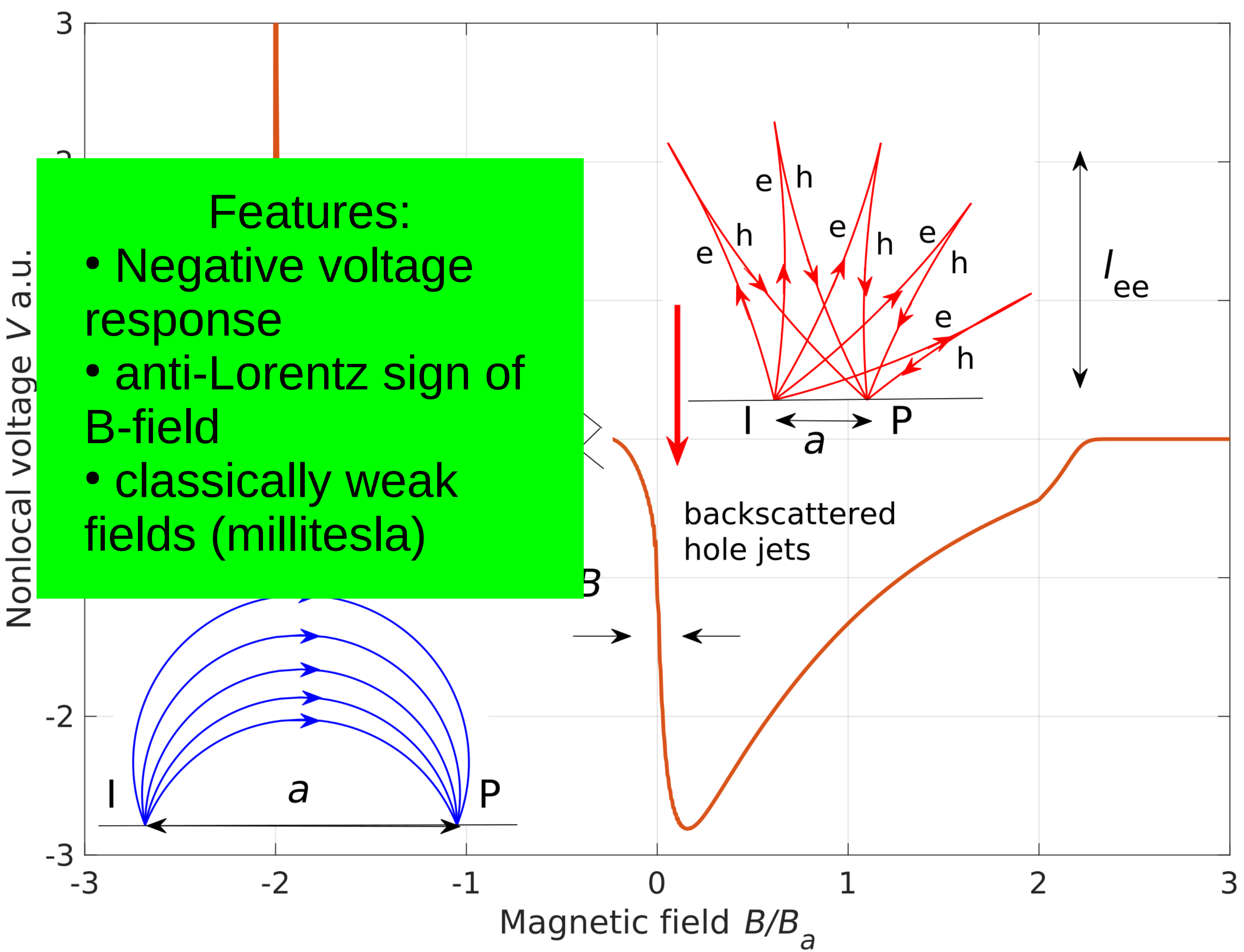
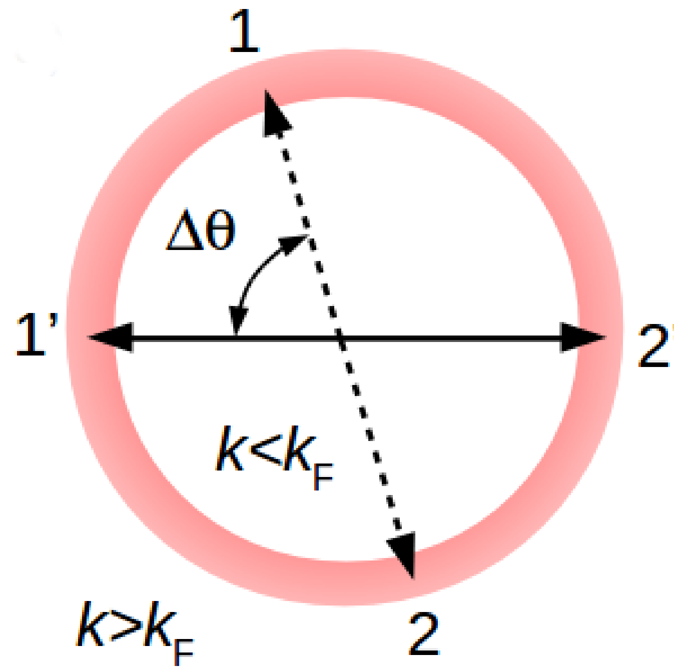
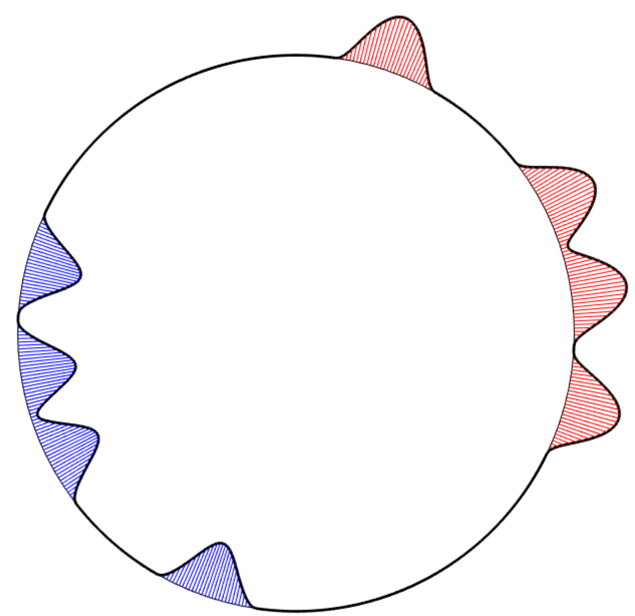


Features:
- Negative voltage response
- anti-Lorentz sign of B-field
- classically weak fields (millitesla)

New transport regime

- Head-on collisions
- Slow odd-parity modes
- Half-hydrodynamic, half-ballistic behavior
- Multiple slow modes, scale-dependent σ and η
- Phase space: loops with an odd-parity modulation





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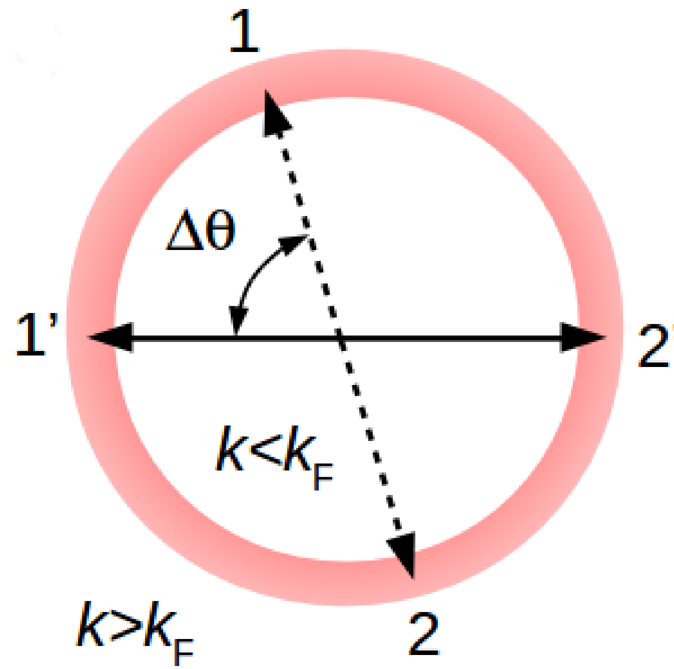
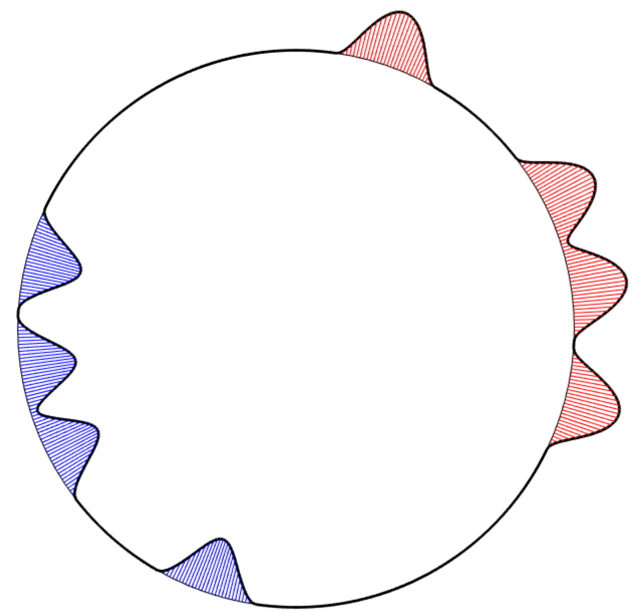
- New nonlinear effects: Odd-parity turbulence?
- Coupled jets, hydraulic logic, current-current transistors





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