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Development and Demonstration of a Cost-Effective In-Vehicle Lane Departure and Advanced Curve Speed Warning System

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A Lane-Departure Warning System (LDWS) and Advance Curve-Warning System (ACWS) are critical among several Advanced Driver-Assistance Systems (ADAS) functions, having significant potential to reduce crashes. Generally, LDWS use different image processing or optical scanning techniques to detect a lane departure. Such LDWS have some limitations such as harsh weather or irregular lane markings can influence their performance. Other LDWS use a GPS receiver with access to digital maps with lane-level resolution to improve the system's efficiency but make the overall system more complex and expensive. In this report, a lane-departure detection method is proposed, which uses a standard GPS receiver to determine the lateral shift of a vehicle by comparing a vehicle's trajectory to a reference road direction without the need of any digital maps with lane-level resolution. This method only needs road-level information from a standard digital mapping database. Furthermore, the system estimates the road curvature and provides advisory speed for a given curve simultaneously. The field test results show that the proposed system can detect a true lane departure with an accuracy of almost 100%. Although no true lane departure was left undetected, occasional false lane departures were detected about 10% of the time when the vehicle did not actually depart its lane. Furthermore, system always issues the curve warning with an advisory speed at a safe distance well ahead of time.

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DEVELOPMENT AND DEMONSTRATION OF A COST-EFFECTIVE IN-VEHICLE LANE DEPARTURE AND ADVANCED CURVE SPEED WARNING SYSTEM

FINAL REPORT

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LIST OF ABBREVIATIONS

LDWS	Lane Departure Warning System
ACWS	Advance Curve Warning System
GPS	Global Positioning System
LW	Lane Width
ADAS	Advance Driver Assistance Systems

EXECUTIVE SUMMARY

Lane Departure Warning System (LDWS) and the Advance Curve Warning System (ACWS) are two critical elements among several other Advanced Driver-Assistance Systems (ADAS) functions, which have significant potential to reduce crashes. The Majority of these crashes involve crossing of an edge line, center line, or otherwise leaving the intended lane or trajectory. Generally, LDWSs use image processing or optical scanning techniques to detect a lane departure. Most of the camera-based systems use different image processing techniques such as linear parabolic lane model or the extended edge-linking algorithm, which extract the lane markings from consecutive picture frames to calculate lateral shift of a vehicle. Some of the LDWSs can also detect the lane markings under varying lighting conditions such as nighttime. Similarly, optical scanning systems, which comprise of a linear array of infrared transmitting devices to scan the lateral area of the highway for lane marking, are inherently independent of the varying lighting conditions. Although camera and optical sensor-based systems work well in favorable weather and road conditions in day or night light, their performance deteriorates when the road conditions are not favorable, such as an absence of or irregular/broken lane markings or harsh weather conditions resulting in an inaccurate lane departure detection. Moreover, other systems use a GPS receiver with access to the lane-level resolution digital maps to improve efficiency of a camera-based system but make the overall system more complex and expensive to implement. On the other hand, majority of the ACWSs use a standard GPS receiver, a speed sensor, and access to the digital maps of lane-level resolution to detect the curve ahead. Some ACWSs are also equipped with Bluetooth Low Energy (BLE) technology along with the GPS receiver to transmit the curve information to the onboard unit. Once a curve ahead is detected and its degree of curvature is estimated, a safe distance and an advisory speed is calculated.

The authors propose a lane departure detection algorithm and an advanced curve warning algorithm using a standard GPS receiver with only road-level information, which are available commonly in any navigation device. Although the error in absolute position accuracy of a standard GPS receiver is larger than the Lane Width (LW), the error in its relative accuracy is much less (< LW). This phenomenon provides an opportunity to potentially detect a lateral lane drift of the given vehicle. Previously, the authors developed a methodology to accurately identify the relative lanes of the surrounding vehicles on freeways by utilizing the relative accuracy of a standard GPS receiver.

Using the similar concept, the authors first propose a lane departure detection algorithm to perceive an unintentional lane drift of a vehicle. The proposed algorithm compares the vehicle's trajectory to the reference road direction to determine the lateral shift of a vehicle for potential lane departure detection. The reference road direction of a given road can be obtained from a standard digital mapping database containing only road level maps without lane-level resolution, which are commonly available in any navigational system. While the vehicle is moving, the GPS receiver acquires its position coordinates periodically. At any given time, the algorithm determines the vehicle's heading and compares it with the reference road direction to estimate the instantaneous lateral distance. The instantaneous lateral distance accumulates over time and if the accumulative lateral distance crosses a certain threshold, a lane departure is detected, and an audible warning is issued.

Moreover, the authors also propose an advance curve detection algorithm that utilizes reference road direction to detect the possible curves ahead and warns the driver about the advisory speed for a given curve at a safe distance before the curve starts. The safe distance is assumed to be the distance needed to reduce a vehicle's speed from its current speed to the advisory speed of the curve by applying normal braking with a safe deceleration rate. Usually, before applying brakes, a driver needs a buffer time, called reaction time to adjust to the warning. Therefore, a driver’s reaction time will also be included in determining the safe distance. The first sub-task of ACWS is to determine the advisory speed of a given curve. In this project, two methods are explored for determining advisory speed for a given curve. In the first method, an advisory speed is determined using reference road direction for a given curve to issue the lane departure warning. However, in the second method an advisory speed value for a given curve is directly acquired from a map database. The second sub-task of ACWS is to determine the safe distance, which is calculated using the vehicle's current speed and the advisory speed for the curve, and a safe deceleration rate. If the vehicle’s current speed is higher than the advisory speed, the proposed ACWS will issue the advance curve warning at a safe distance. The warning message comprises two important pieces of information: a curve ahead warning and the advisory speed of the given curve so that the driver can adjust the vehicle’s speed accordingly.

The implementation of the proposed lane departure warning and advance curve warning algorithms was done by programing the Dedicated Short-Range Communication (DSRC) devices for performance evaluation. Extensive field tests were performed to evaluate the system’s efficiency on both straight and curved road segments. The field test results showed that the proposed system can detect and warn the driver of a true lane departure with the accuracy of almost 100% on both straight and curved road segments. Although no true lane departure was left undetected, occasional false lane departures were detected about 10% of the time when the vehicle did not truly depart its lane. A Majority of these false alarms were issued on the sharp curved sections of the road. Along with lane departure warning, the system also issued the advance curve warning about the advisory speed of a given curve if there was any curve detected. Additionally, a modification in the lane departure detection algorithm was tested, which has significant potential to reduce the frequency of false alarms on curved road sections.

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

An increasing number of modern vehicles include different Advanced Driver-Assistance Systems (ADAS) to assist in driver's safety *(1)*. Lane Departure Warning System (LDWS) and Advance Curve Warning System (ACWS) are two important ADAS features, which can prevent high-speed accidents on highways and freeways when a vehicle is about to unintentionally drift away from its lane or there is a sharp curve ahead with an advised speed. According to American Association of State Highway and Transportation Officials (AASHTO), almost 60% of the fatal accidents are caused by an unintentional lane drifting of a vehicle on major roads *(2)*. Similarly, in a Minnesota crash study, it was reported that 25 to 50 % of the severe road departure crashes in Minnesota occur on curves, even though curves account for only 10 % of the total system mileage *(3)*. Systems that predict the driver's attentive state and intent of lane change *(4-6)* and provide map-based route guidance and/or warning about unintentional lane departure *(7-8)* are all useful to reduce major road crashes. The Majority of these crashes involve crossing of an edge line, center line, or otherwise leaving the intended lane or trajectory *(9)*. According to a recent study which compared crashes with and without an LDWS, it was found that an in-vehicle LDWS was helpful in reducing crashes of all severities by 18%, with injuries by 24%, and with fatalities by 86% without considering for driver demographics (*10*).

Most available lane departure warning systems typically use a single camera and a processor to identify the imminent lane departure *(11-14)*, while other modern systems use optical scanning and Light Detection and Ranging (LIDAR) sensors *(15)*. A careful view of camera-based systems reveals that the calibration of a camera is an important element. However, there are systems available that can detect the lateral offset of a vehicle even with an uncalibrated camera *(16)*. Most of these camera-based systems use different image processing techniques such as linear parabolic lane model *(17)* or the extended edge-linking algorithm *(18)*, which extract the lane markings from consecutive picture frames to calculate lateral shift of a vehicle. Earlier camera-based systems were vulnerable to lighting conditions, hence not capable to accurately recognizing the lane markings at nighttime. However, image processing techniques have advanced over the past couple of decades overcoming the limitation of diminished lighting conditions to successfully detect lane drifting even in low lighting or nighttime *(19)*. For example, a Video-Based Lane Estimation and Tracking (VioLET) system, which uses steerable filters, is an efficient method for detecting solid-line and segmented-line markings under varying lighting and road conditions for robust and accurate lane-marking detection *(20)*. Similarly, optical scanning systems, which are comprised of a linear array of infrared transmitting devices to scan the lateral area of the highway for lane markings, are inherently independent of the varying lighting conditions *(21)*. Although camera and optical sensor-based systems work well in favorable weather and road conditions in day or night light, their performance deteriorates when the road conditions are not favorable, such as an absent or irregular/broken lane marking or harsh weather conditions resulting in inaccurate lane departure detection. Moreover, there are also some systems that integrate Global Positioning System (GPS) data with a camera-based LDWS to increase the reliability of lane departure detection in adverse road and weather conditions. However, such systems require GPS technology, inertial navigation sensor, and access to digital maps of lane-level resolution to correct the GPS position *(22)*, making such systems more complex and expensive to implement.

On the other hand, a majority of the ACWS use a standard GPS receiver, a speed sensor, and access to the digital maps of lane-level resolution to detect the curve ahead *(23-25)*. Some ACWS are also equipped with Bluetooth Low Energy (BLE) technology along with the GPS receiver to transmit the curve information to the onboard unit (26). Once a curve ahead is detected and its degree of curvature is estimated, a safe distance and an advisory speed is calculated. The safe distance for a given curve is defined as the distance required for a vehicle to reduce its current speed to the advisory speed of a curve. Some available systems also impose the speed control mechanism to the vehicle in order to achieve a safe speed in case the driver could not achieve it *(27)*. If a vehicle is moving on a straight section with speed higher than the advisory speed of a curve, it is beneficial to warn the driver well in advance so that the driver can adjust the speed according to the advisory speed of the curve ahead. Based on the advisory speed and the current speed of the vehicle, the proposed method will warn the driver about the advisory speed of a given curve at a safe distance before the curve starts.

In this report, lane departure detection and advance curve detection algorithms are proposed that use a standard GPS receiver with only road-level maps instead of lane-level maps. Although the error in absolute position accuracy of a standard GPS receiver is larger than the Lane Width (LW), its relative error is much less (< LW), providing an opportunity to potentially detect lateral lane drift of a vehicle (28-29). Previously, the authors developed a methodology to accurately identify the relative lanes of the surrounding vehicles on a road by utilizing the relative accuracy of a standard GPS receiver *(30)*. Using the similar concept, the authors have now developed an algorithm to detect an unintentional lane drift of a vehicle. The proposed algorithm compares the vehicle's trajectory to the reference road direction to determine the lateral shift of a vehicle for potential lane departure detection. Simultaneously, the system also calculates the degree of curvature based on curve geometry and a safe distance for a possible curve ahead warning. If a vehicle is moving on a straight section with speed higher than the advisory speed of a curve, it is beneficial to warn the driver well in advance so that the driver can adjust the speed according to the advisory speed of the curve ahead. Based on the advisory speed and the current speed of the vehicle, the proposed method will warn the driver about advisory speed of a given curve at a safe distance before the curve starts. The reference road direction of a given road, which is required by both LDWS and ACWS, can be obtained from any standard digital mapping database containing only road-level maps without lane-level resolution. Such maps are commonly available in any navigational system.

A prototype system was developed to implement both proposed algorithms (lane departure detection and advanced curve detection) and extensive field tests were performed to evaluate the system efficiency on both straight and curved road segments. The field test results showed that the proposed algorithm can detect and warn the driver of a true lane departure with an accuracy of almost 100% on both straight and curved road segments. Although no true lane departure was left undetected, occasional false lane departures were detected about 10% of the time when the vehicle did not truly depart its lane. A Majority of these false alarms were issued on the sharp curved sections of the road.

Along with lane departure warning, the system simultaneously issued advance curve warning with the information of an advisory speed for a given curve at a safe distance, if there was any curve ahead. Additionally, a modification in the lane departure detection algorithm was tested, which has significant potential to reduce the frequency of false alarms on curved road sections. Both LDWS and ACWS algorithms are explained in great detail in future chapters.

1.2 OBJECTIVES

The major objectives of the project include design and development of two algorithms; lane departure detection algorithm and advanced curve warning algorithm.

1.2.1 Lane Departure Detection Algorithm

Previously, the authors developed a methodology to accurately identify the relative lanes of the surrounding vehicles on a road by utilizing the relative accuracy of a standard GPS receiver. Using the similar concept, the authors now propose a lane departure detection algorithm to detect an unintentional lane drift of a vehicle. This proposed algorithm compares a vehicle’s trajectory to a reference road direction to determine the lateral shift of a vehicle for potential lane departure detection. The reference road direction of a given road can be obtained from a standard digital mapping database containing only road-level maps without lane-level resolution. At any given time, the algorithm determines the vehicle’s heading and compares it with the reference road direction to estimate the instantaneous lateral distance. The instantaneous lateral distance accumulates over time and if the accumulative lateral distance crosses a certain threshold, a lane departure is detected, and a warning is issued.

1.2.2 Advance Curve Detection Algorithm

Proposed advance curve detection algorithm utilizes reference road direction to detect the possible curves ahead and warn the driver about the advisory speed for a given curve at a safe distance before the curve starts. The safe distance is assumed to be the distance needed to reduce a vehicle's speed from its current speed to the advisory speed of the curve by applying normal braking with a safe deceleration rate. Usually, before applying brakes, a driver needs a buffer time called reaction time to adjust to the warning. Therefore, a driver’s reaction time will also be included in determining the safe distance. The first sub-task of ACWS is to determine the advisory speed of a given curve. Two methods are explored for determining advisory speed for a given curve. In the first method, an advisory speed is determined using reference road direction for a given curve to issue the lane departure warning. However, in the second method an advisory speed value for a given curve is directly acquired from the map database. The second sub-task of ACWS is to determine the safe distance, which is calculated using vehicle's current speed and the advisory speed for the curve, and a safe deceleration rate. If the vehicle’s current speed is higher than the advisory speed, the proposed ACWS will issue the advance curve warning at a safe distance. The warning message comprises two important pieces of information.

The first is about the curve ahead and the second is about the advisory speed so that driver can adjust its speed accordingly.

1.3 Methodology AND SYSTEM ARCHITECTURE

The proposed lane departure detection system relies on standard GPS receiver to acquire the position of a vehicle and a basic processor to execute necessary calculations to find lateral shift of the given vehicle. Generally, the absolute position accuracy of a standard GPS receiver is in the range of 3-5 m, which is not sufficient to determine any lateral lane-level drift of a vehicle's trajectory needed for lane departure detection *(28)*. However, the relative GPS accuracy is much higher and can be used for determining relative trajectory of a single vehicle *(29)*. This concept is illustrated in Figure 1 where a few adjacent GPS coordinates of a fast-moving vehicle taken by a GPS receiver are shown as red dots for a straight road in (a), and for a curved road in (b). The true positions of the vehicle are shown as green dots. Due to the GPS error, estimated location of the vehicle could be anywhere in the bigger dashed circle (Figure 1). However, the bulk of GPS error is caused by atmospheric disturbances and will remain the same for all adjacent GPS estimated positions because atmospheric disturbances will remain constant over a wide area *(32)*. Therefore, any residual GPS relative error will only be due to device-specific sources and confined to smaller dashed circles as shown in Figure 1. Additionally, in the absence of any multipath interference, most of the device specific error will also not change much in adjacent GPS readings over a short period of time. Therefore, the relative accuracy of a standard GPS receiver with no multipath interference turns out to be reasonable to determine lateral drift due to lane departure.

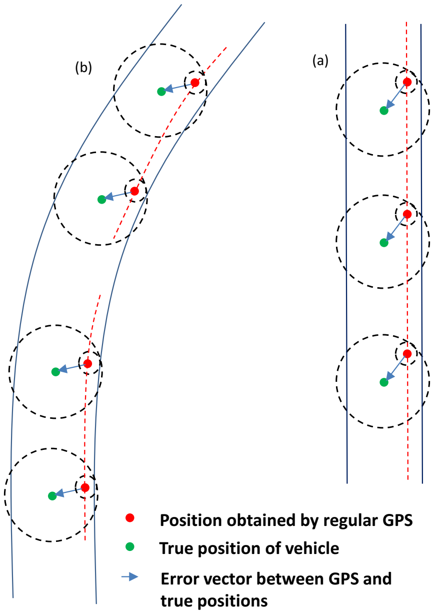


Figure 1.1 Conceptual diagram showing relative GPS accuracy versus absolute position accuracy for (a) a straight and (b) a curved road section.

The conceptual architecture of the proposed system using a standard GPS receiver is shown in Figure 2. The GPS receiver periodically obtains longitude and latitude of a vehicle's current position. The onboard processor calculates the direction of travel of the vehicle utilizing two or more consecutive positions of the vehicle acquired by the GPS. A road reference direction extracted from a digital map database with only road-level resolution is also stored in the onboard processor. By comparing the vehicle direction to the road reference direction, the processor calculates instantaneous lateral shift of the moving vehicle perpendicular to its reference road direction. Please note that the system does not rely on lane-level resolution maps for reference road direction but instead only needs road-level maps, which can be obtained from any mapping database of commonly available navigation systems. With every new acquired GPS coordinates, the system keeps accumulating the lateral distance and issues a warning to alert the driver if the accumulative lateral distance exceeds a certain threshold. There are several ways through which the driver can be alerted such as a haptic in-seat feedback, system display warning, or an audible warning.

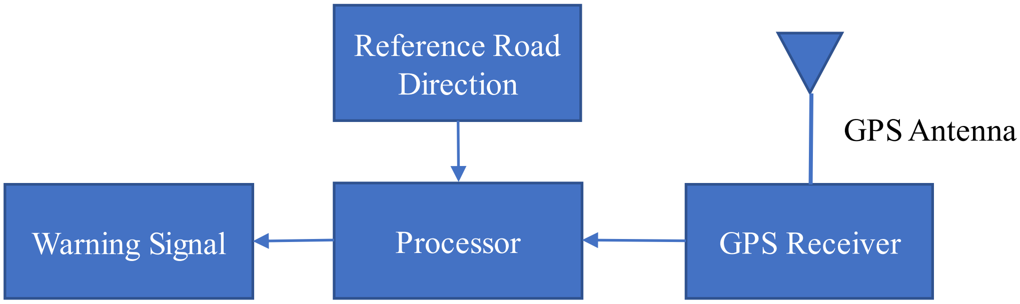


Figure 1.2 Block diagram of the proposed lane departure detection and advance curve warning detection systems.

CHAPTER 2: LANE DEPARTURE WARNING SYSTEM

2.1 LANE DEPARTURE DETECTION ALGORITHM

A moving vehicle can drift away from its lane if its yaw or steering angle is not aligned with the reference road direction. Figure 2.1a depicts lane departure scenarios due to a misalignment of yaw angle (blue solid line trajectory) or steering angle (red dashed line trajectory) with the reference road direction. Both yaw and steering angles result in the vehicle drifting away from its lane causing the lateral distance of the vehicle with respect to its reference road direction to increase. While vehicle is moving, its GPS receiver periodically acquires its position coordinates. At any given time, *n*, using the current position P_n of the vehicle and its previous position P_{n-1}, the algorithm determines the vehicle's heading ϑ_v using forward azimuth equation as well as the distance between two consecutive positions D_n using haversine equation *(26)*. Once the ϑ_v is determined, it is used to calculate the deviation angle θ_n by subtracting ϑ_v from the reference road direction ϑ_{ref}. Subsequently, θ_n is used to determine the instantaneous lateral distance D_L by using Equation 2.1.

$$D_L = D_n \sin(\,\theta_n) \tag{2.1}$$

The instantaneous lateral distance is calculated upon acquiring every new set of GPS coordinates and is accumulated over time. If the accumulative lateral distance crosses a certain threshold, a lane departure is detected. This phenomenon is depicted in Figure 2.1b, where the accumulative lateral distance of a vehicle is shown versus traveled distance. When the vehicle departs from its lane from right to left, its accumulative lateral distance increases in positive direction and vice versa. A vehicle is considered to have departed its lane when the absolute value of accumulative lateral distance increases beyond a certain threshold. The intentional lane drifting (lane change scenario) can be distinguished with a presence of a lane change indicator signal. When a vehicle intentionally changes its lane, as in the scenario depicted in Figure 3b, the increase in lateral distance saturates, i.e., upon completion of lane change, any further increase in lateral distance becomes negligibly small because the vehicle starts traveling in parallel to the reference road direction. This phenomenon can be used to reset the accumulative lateral distance to zero for detecting a potential unintentional lane departure after every lane change. In case of unintentional lane departure due to drowsiness or some other negligence, the warning is issued whenever accumulative lateral distance increases beyond certain threshold and warning remains active until the vehicle's direction of travel becomes parallel to reference road direction i.e., θ_{ref}.

The proposed lane departure detection algorithm requires a reference road direction θ_{ref} and accumulative lateral distance threshold to reliably detect lane departure. Both of these aspects are further discussed below.

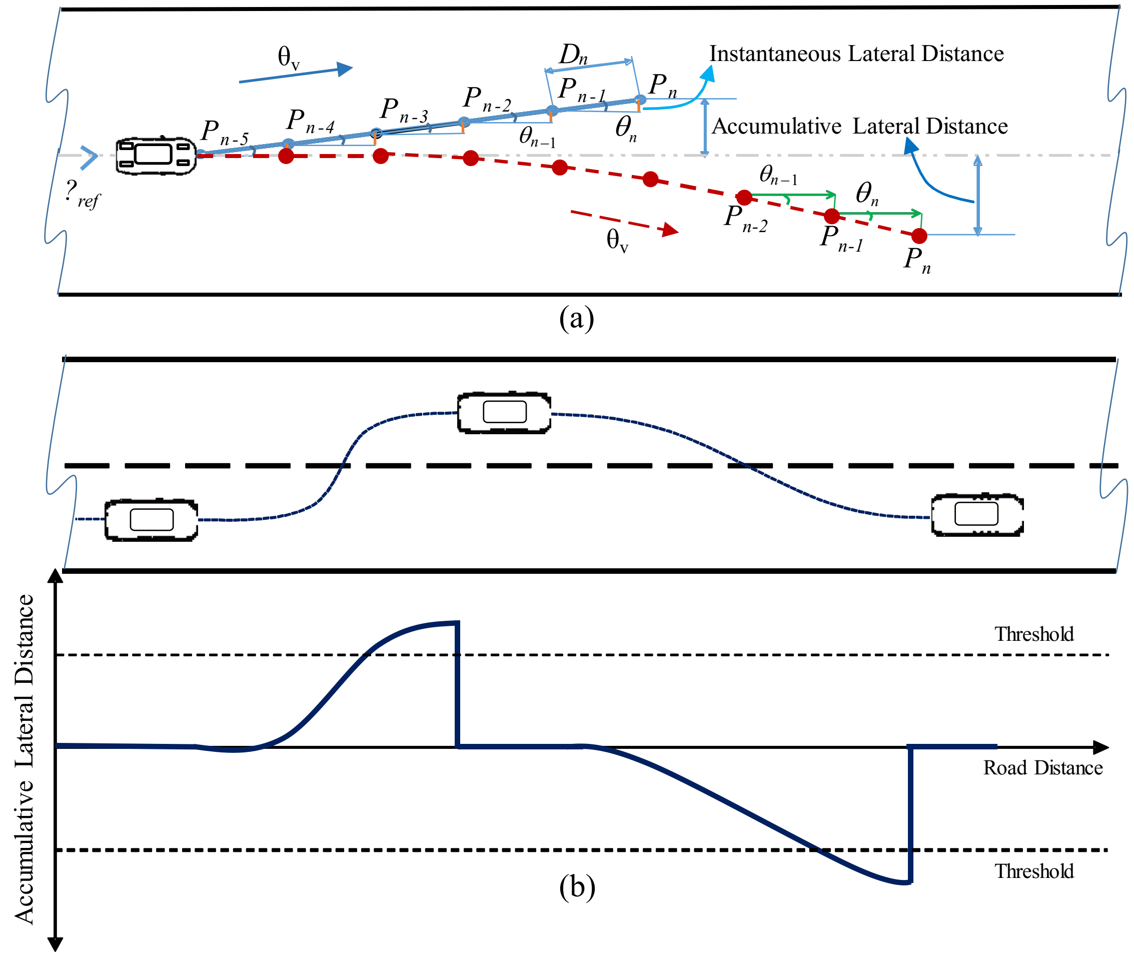


Figure 2.1 (a) Typical trajectories of a vehicle with misaligned yaw angle (blue) and steering angle (red), and (b) Schematic diagram showing lane departure detection algorithm.

2.1.1 Reference Road Direction (ϑ_{ref})

To estimate the instantaneous lateral distance, ϑ_{ref} is needed which can be extracted from Google maps or any other navigational mapping database e.g., OpenStreetMap. Typically, in any mapping database, roads are represented in several segments of individual links, each having unique characteristics such as speed, road curvature, number of lanes etc. All these links have an associated link identification (ID) that is globally unique in the mapping database for identifying and processing information about any given road segment. These mapping databases can provide road-level or lane-level information based on the quality of the survey and required services. For the proposed algorithm, only the road-level information is needed. A road-level map of a given road segment with a unique global ID has associated set of geographic latitude-longitude points placed somewhere in the middle of the road, which represent the shape of the actual physical road with some lateral error. These shape points are sparse if the road is straight and are dense whenever the road has curvature, for the obvious reason that a curved road will require more shape points to accurately represent the road curvature. The distribution of these shape points on a given road is directly proportional to how acute the road geometry is along that road segment.

A crucial element for the proposed lane departure algorithm is the accuracy of ϑ_{ref} as this becomes the basis of determining instantaneous lateral distance. Ideally, θ_{ref} should remain constant throughout any straight section of a road. However, in any practical mapping database, there will be some lateral deviation of the shape points resulting in θ_{ref} to deviate from one road link (formed by two consecutive shape points) to another as shown in Figure 2.2a. To minimize the error in θ_{ref}, a path average heading over the entire length of straight road section is taken as θ_{ref}. To find path average θ_{ref}, lateral shift between two consecutive links (combined by three shape points) is calculated. If the lateral shift between the two-consecutive links is <½LW, a path average heading of the two links is calculated and is used to find the lateral shift in the following link. Similarly, the lateral shift in the subsequent link is calculated one at a time with respect to the path average heading of the previous links. As long as the lateral shift in any subsequent link with respect to the path average heading of the previous links is <½LW, that link is included in calculation of the path average heading which is used as θ_{ref} for the entire straight section (Figure 2.2a). However, if the lateral shift in some of the links is >½LW, the corresponding shape points are considered spurious and are skipped in calculation of path average heading for θ_{ref} to minimize its lateral error as shown in Figure 2.2a where the fourth shape point from the left is not included in calculating path average θ_{ref}.

A similar technique is used for the curved section of the road to minimize the lateral error in θ_{ref} (Figure 2.2b). For a normal curved section of a road segment, the lateral shift from one link to the next link should be uniformly increasing or decreasing depending upon the direction of the curve. However, due to inherent lateral error in database shape points, the lateral shift between some of the consecutive

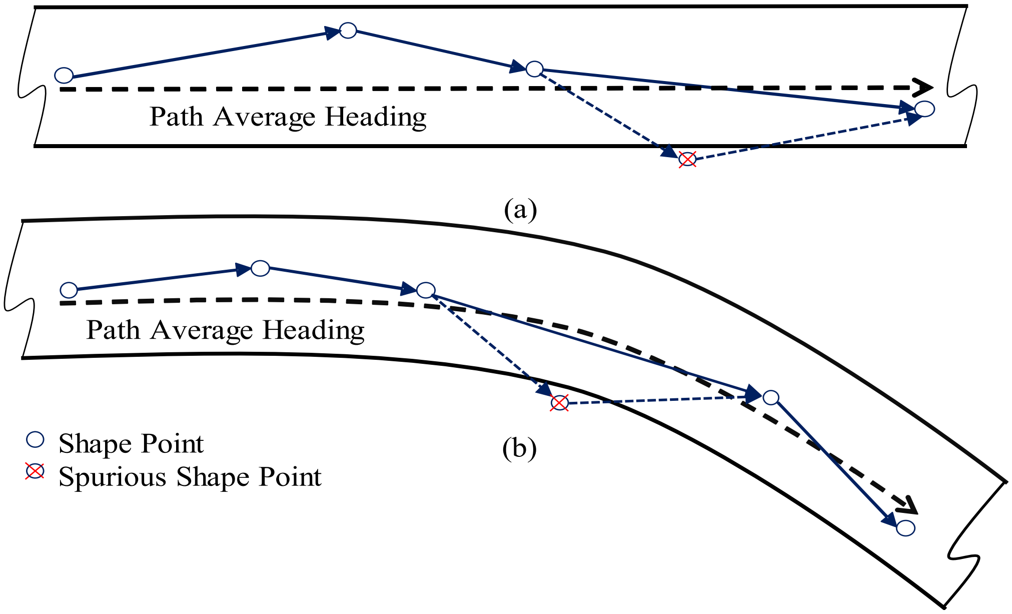


Figure 2.2 Schematic geometry showing shape points for (a) a typical straight, and (b) a typical curved road segment along with the path average heading used as a reference heading.

links could fall outside of this norm. Such shape points are considered spurious and are excluded to calculate path average θ_{ref} for the curved section of the road (Figure 4b). If the lateral shift of a subsequent link remains <½LW, that link will be included in calculation of the path average slope of θ_{ref} to minimize its lateral error as illustrated in Figure 4b.

In practical scenarios, a road is a combination of straight and curved sections. The above-mentioned strategy works on straight or curved sections of any given road segment. For example, a common practical scenario is a straight road section followed by a curved section or vice versa. In such cases, θ_{ref} is considered as path averaged heading for the straight road section and path averaged slope for the curved section. This is illustrated in Figure 2.3a where the heading between available shape points is plotted versus road distance in solid black line for a 3 km section of Rice Lake Rd in Duluth, MN. The

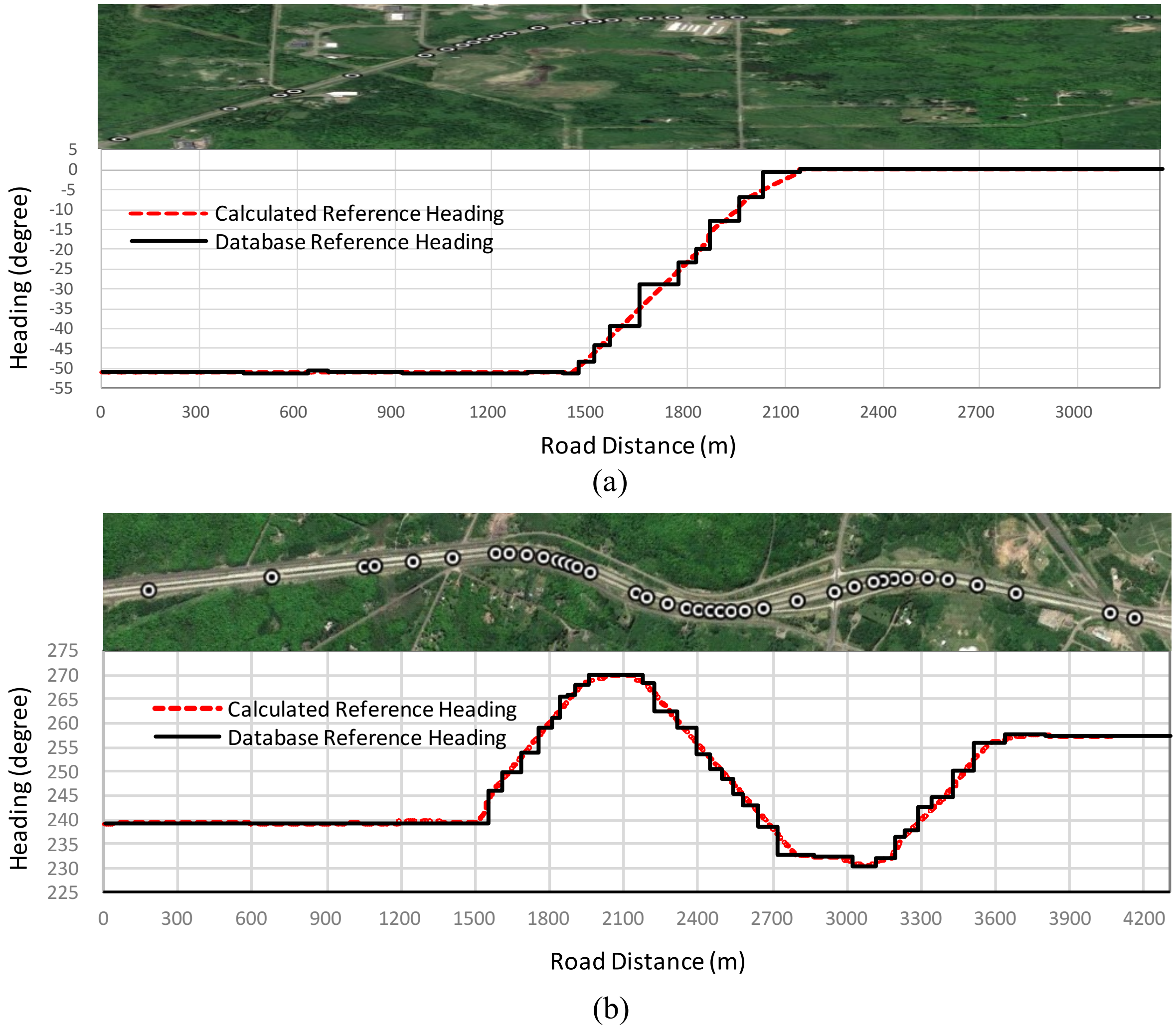


Figure 2.3 Database heading between consecutive shape points (black) and calculated reference heading (red dashed line) versus road distance for (a) a 3 km segment of Rice Lake Rd in Duluth, MN, and (b) a 4 km segment of Interstate I-35. The Google maps of the corresponding road segments with shape points are also shown for reference.

portion of Rice Lake Rd shown in Figure 2.3a has one curved section surrounded by two straight sections. The number of shape points on the straight road section is less than the number of shape points on the curved section as expected. The calculated θ_{ref} used for this road section is also shown in Figure 2.3a as red dashed line.

Similarly, a more complex curved segment on Interstate I-35 near Duluth, MN is shown in Figure 2.3b where reference heading between consecutive shape points is plotted versus road distance as solid black line for a 4 km long road segment having many curved sections as well as some straight sections. Figure 2.3b also shows calculated θ_{ref} using the strategy described above as dashed red line.

2.1.2 Threshold

Once θ_{ref} is determined, it can be used to calculate the accumulative lateral distance of a given vehicle over time to detect lane departure by comparing it to a certain threshold. The accumulative distance threshold choice is critical because a large threshold can minimize false alarms but at the cost of delayed lane departure detection. One obvious choice of the accumulative lateral distance threshold is half of the vehicle width subtracted from half of the lane width as shown in Figure 2.4a. The typical width of most common vehicles ranges from 1.6 to 2.0 m and the typical highway lane width is 3.6 m. Therefore, the threshold choice ranges between 0.8 and 1.0 m. This threshold works fine to detect a true lane departure in either direction as long as a vehicle is driven in the middle of the lane in perfect alignment with the θ_{ref} which is not always the case in reality. Instead a typical vehicle is driven in a random trajectory within its lane. Therefore, during normal driving within a lane, the lateral distance keeps

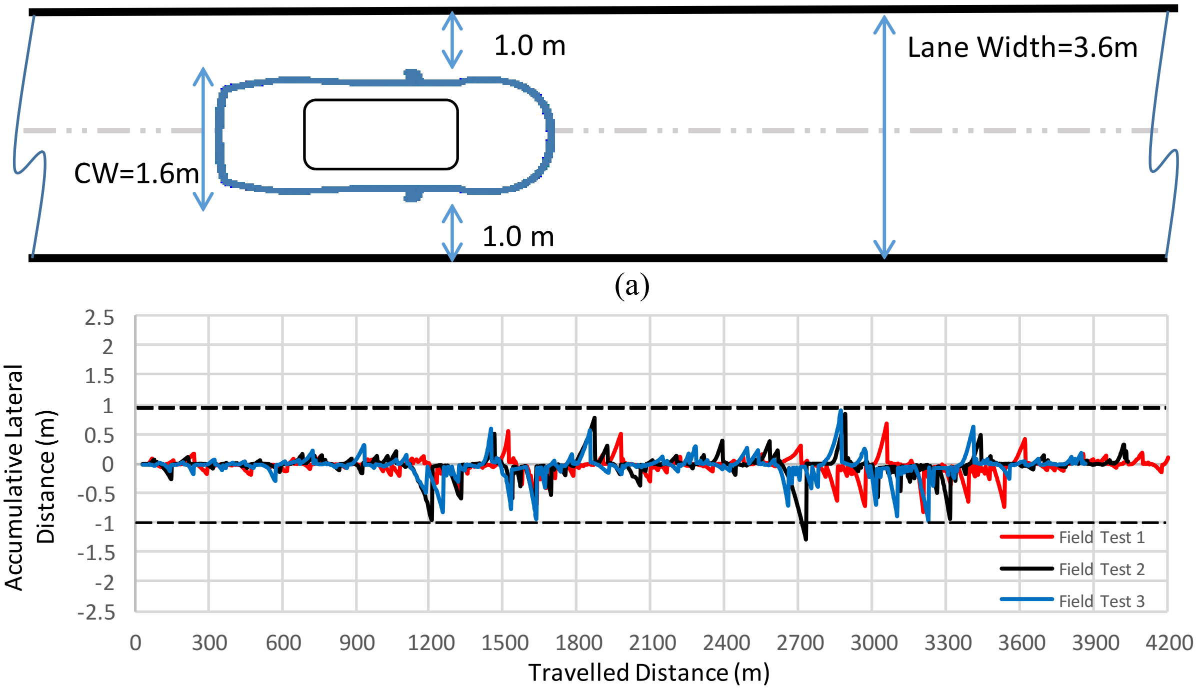


Figure 2.4 (a) Schematic geometry showing accumulative lateral distance threshold for the proposed lane departure detection algorithm, and (b) accumulative lateral distance versus traveled distance for three typical trials of normal driving.

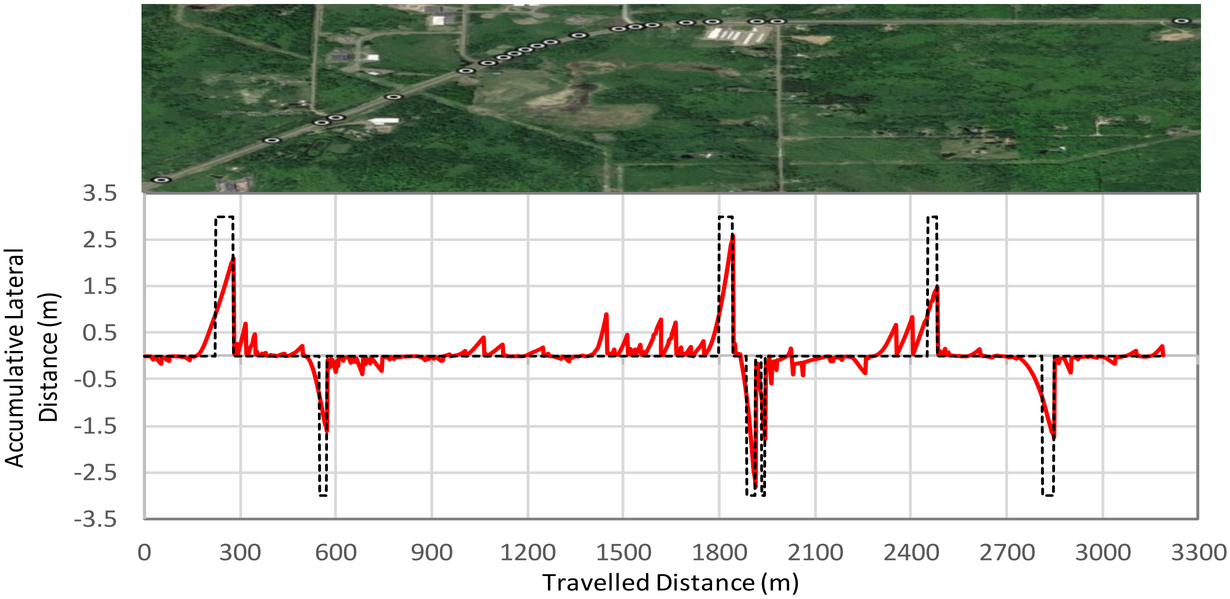
accumulating in positive or negative direction in a zig-zag fashion. To evaluate a normal driving behavior, a vehicle was driven on Interstate I-35 near Duluth, MN in the same lane, multiple times. The accumulative lateral distance versus traveled distance is shown in Figure 2.4b for three typical scenarios. During this test, no attempt was made to depart the lane. However, as expected, the test vehicle did not necessarily travel parallel to the road. Instead it traveled in a slow zig-zag pattern resulting in lateral distance to accumulate in one direction or the other. Despite slow zig-zag pattern, the accumulative lateral distance does not cross the threshold of 1 m in all three trials except once near 2,700 m point for a short period of time which would result in a false alarm (Figure 2.4b). Therefore, the normal driving behavior illustrates that 1 m threshold is a reasonable choice to detect lane departure using the proposed algorithm. If the threshold is increased, probability of the false alarm can be minimized but actual lane departure warning will be delayed.

2.2 FIELD TESTS, RESULTS AND DISCUSSION

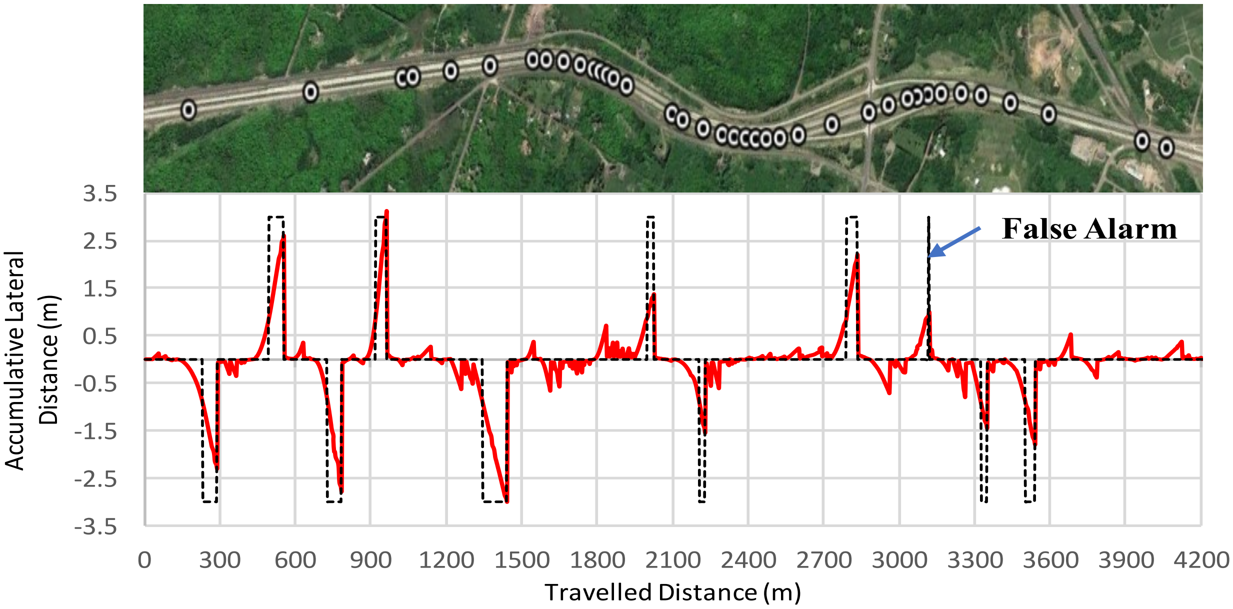
To evaluate the efficiency of the proposed algorithm, it was implemented using a dedicated short-range communication (DSRC) device as an onboard unit because it has a built-in GPS receiver and the required processing power needed to implement the proposed algorithm. Please note that the proposed algorithm can be implemented in any navigational device having a GPS receiver and necessary processing power. For the field tests, the communication aspect of DSRC device was not required so was disabled. The proposed algorithm was programmed in the DSRC device, which acquired GPS data at 10 Hz frequency to decide about potential lane departure every 100 msec. Once a lane departure was detected, a warning via an audible sound was issued. For audible warning generation, a Linux laptop was used which also helped to monitor other parameters during field tests.

The field tests were performed by driving the test vehicle multiple times on two different road segments, a 3 km long segment of Rice Lake Road in Duluth, MN, and a 4 km long segment of Interstate I-35 near Duluth, MN. Many back-and-forth lane changes were made intentionally on both road segments during the field tests. The intentional lane changes were used to evaluate the efficiency of the proposed lane departure algorithm.

First, the field tests were performed on Rice Lake Rd segment, which is single lane road with a wide shoulder which was used as a second lane for back and forth lane changes. One typical set of results involving multiple lane departures (lane changes) on Rice Lake Road is shown in Figure 2.5a where accumulative lateral distance is plotted versus traveled distance. Over the length of 3 km road segment, a total of 6 lane departures were made. Two of these six lane departures were made on the curved section of the road. The positive accumulative lateral distance represents the lane departure on the right side and vice versa. During each lane departure, an audible warning signal was generated in real time as soon as the absolute value of accumulative lateral distance increased above the threshold (1m). Upon successful completion of lane change, the vehicle's direction of travel became parallel to θ_{ref}. At this point, the accumulative lateral distance was reset to zero and the audible warning was turned off. In the proposed algorithm, a vehicle is considered to be travelling parallel to θ_{ref}, when 5 consecutive instantaneous lateral distances become negligibly small. A digital mask of the audible warning signal is also superimposed in Figure 2.5a as dashed black line showing the start and end of lane departure warning during lane changes. If the lane was changed quickly with a bigger steering angle, the audible signal was heard for a short period of time and if the lane was changed slowly with a smaller steering angle, the audible signal was heard for a longer period of time. Each of the 6 lane departures of the trial of Figure 2.5a were accurately and timely detected. However, during multiple field tests conducted, sometimes the system issued a delayed lane departure warning. This occurred during some slow lane



(a)



(b)

Figure 2.5 Accumulative lateral distance versus traveled distance of field trials on (a) 3 km section of Rice Lake Rd., showing 6 lane departures and (b) 4 km segment of Interstate I-35 showing 9 lane departures. The dashed black line represents the digital mask for the duration of audible warning signal. The Google maps of the corresponding road segments are also shown.

changes with a very small steering angle. During the period of such slow lane change, a few times, vehicle's heading became parallel to the θ_{ref} causing the accumulative distance to be reset to zero before crossing the threshold. This mainly occurred on the straight portion of the road section and was an artifact of intentional attempt of slow lane change. In real scenario of unintentional lane departure, this behavior is not likely, so a timely lane departure warning is expected to be issued using the proposed algorithm.

Similarly, the tests were repeated many times on Interstate I-35 as shown in Figure 2.5b where accumulative lateral distance is plotted versus traveled distance. Over the length of 4 km segment, a total of 9 back and forth lane departures were made. During all these 9 cases, lane departure warning was accurately generated, except once around 3,100 m distance (Figure 2.5b), where a false alarm was detected for a brief time period (about less than half a second). This false alarm occurred at the sharp curved section of the road where the vehicle remained in its own lane but deviated too much from one side of the lane causing the accumulative lateral distance to cross the threshold. This false alarm phenomenon varied with driving behavior.

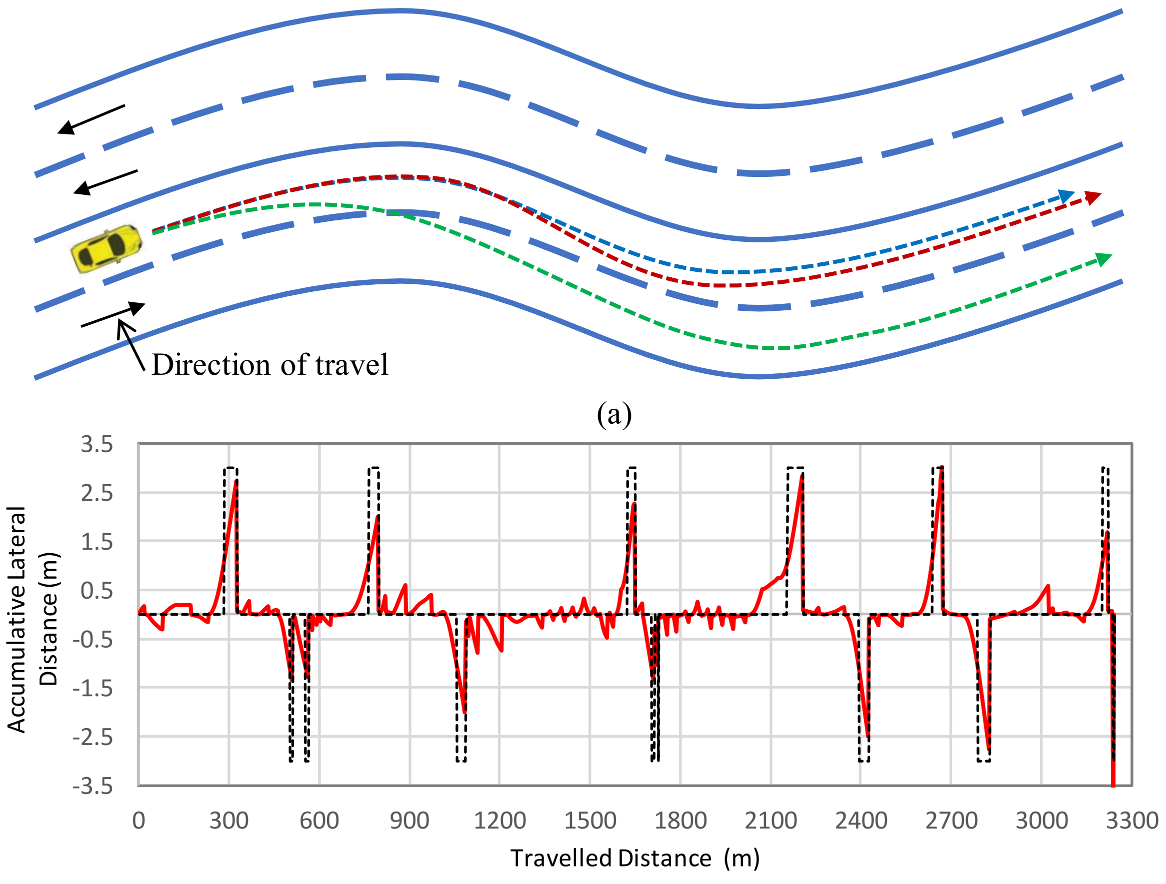


Figure 2.6 (a) Schematic of different potential trajectories of a given vehicle during multiple trips on the same road, and (b) Accumulative lateral distance versus traveled distance for a typical field trial on Rice Lake Rd, Duluth. A digital mask of audible warning signal is also superimposed as dashed black line.

Overall, the field tests were repeated at least 10 times on each of the two road segments making approximately a total of 200 lane changes and the proposed algorithm detected and issued the warning during all 200 lane changes. However, during some of the lane changes (~10% of the time), the lane departure warning was issued but delayed because of deliberate attempt of a very slow lane change. Moreover, we noticed that false alarms occurred about 10% of the time and the duration of false alarm varied between ½ second to ~3 seconds.

One of the reasons for false alarms during the curved sections of the road is due to inherent lateral error in commonly available maps with road level resolution which were used to extract θ_{ref}. This error becomes more pronounced on sharp curves. The reference road direction can be further improved by using the past trajectories of a vehicle on a given road to generate θ_{ref}. Such θ_{ref} could be much more accurate both on straight and curved sections of the road. Generally, a vehicle travels repeatedly on the same road over time. If a vehicle tends to unintentionally depart its lane on that road where it has traveled before, its own previously recorded trajectory can be used as θ_{ref} to accurately detect a future unintentional lane departure due to driver's negligence or drowsiness. Normally, a vehicle is expected to take slightly different trajectory in each new trip on the same road as shown in Figure 2.6a. A more accurate θ_{ref} can be obtained by averaging multiple past trajectories. However, it is important to exclude any intentional lane changes of a given vehicle within those trajectories before including them to obtain average θ_{ref} (Figure 2.6a). To evaluate the potential advantage of this method, a vehicle was driven normally on one of the two test sites, Rice Lake Road, to generate θ_{ref}. Using this θ_{ref}, the test vehicle was driven again on the same road and made lane change multiple times to evaluate lane departure detection. The results of one such trial is shown in Figure 2.6b, where accumulative lateral distance is plotted versus traveled distance. In this test run, a total of 10 lane change attempts were made and two of the 10 lane changes occurred on the curved section of the road. All these lane departures were detected accurately and timely. During multiple runs, the frequency of false alarms was significantly reduced. More details on this method will be discussed in a future manuscript.

CHAPTER 3: ADVANCE CURVE WARNING SYSTEM

An advanced curve speed warning system has tremendous potential to avoid vehicle accidents on sharp curves. If a vehicle is moving on a straight section with speed higher than the advisory speed of a curve ahead, it is beneficial to warn the driver well in advance so that the driver can adjust the speed according to the advisory speed of the curve ahead. Based upon the advisory speed *(V)* and the current speed of the vehicle (V_C), the proposed method will warn the driver at a safe distance (D_S) before the curve starts as shown in Figure 3.1. The safe distance is assumed to be the distance needed to reduce a vehicle's speed from its current speed to the advisory speed of the curve by applying normal braking with safe deceleration rate including standard perception and reaction time. The system would calculate this distance specific to each vehicle based on the current travel speed and required speed reduction before the start of the curve. Usually, before applying brakes, a driver needs a buffer time or a reaction time to adjust vehicle's speed to the advisory speed in the warning. Therefore, driver's reaction time will be included in determining the safe distance.

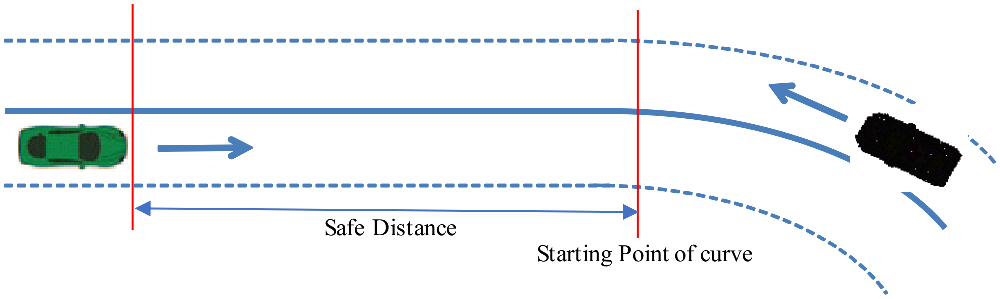


Figure 3.1 Conceptual diagram showing advance curve speed warning system

The proposed advanced curve speed warning system has following three important aspects:

Determination of the advisory speed for the specific vehicle in advance of the curve: An advisory speed needs to be determined by the vehicle well before it approaches the beginning of the curve.

Safe distance determination: Based upon the current speed of the vehicle and the advisory speed of the curve, a safe distance needs to be determined so that a warning to the driver can be issued at the point which allows the driver to adjust speed safely prior to reaching the start of the curve.

Issuing the warning: After determining the advisory speed for the curve, when a vehicle reaches within the safe distance from the start of the curve, an advance curve speed warning is issued.

In the following sections, the above three aspects are further explained in detail.

3.1 ADVISORY SPEED FOR THE CURVE

We have explored two methods for determining advisory speed for a given curve. Although, various vehicles have different capacity to handle speed on curves, for this project, we will assume just one advisory speed for all vehicles. To estimate the advisory speed, both methods obtain specific information from the same digital map database as we previously used to develop lane departure warning system as discussed in Chapter 2. In the first method, an advisory speed is determined using shape points for a given curve which we previously used to determine the reference road direction (θref) for a curved road to issue lane departure warning. However, in the second method an advisory speed value for a given curve is directly acquired from the same mapping database. These two methods are further described below.

3.1.1 Calculated Advisory Speed

As discussed in Chapter 2, LDWS uses shape points to estimate the θ_{ref} and due to constant change of θ_{ref}, proposed system can differentiate the straight and curve sections of the road segment. ACWS uses θ_{ref} to determine the degree of curvature for any given curved section of the road segment. A degree of curvature will later be used to calculate the advisory speed for that curved section. Figure 3.2 schematically describes the methodology to determine the degree of curvature needed to calculate advisory speed for an advance curve warning system. Please note that this is the same methodology which we previously used to determine the θ_{ref} for the lane departure warning system. Our lane departure detection algorithm already detects the beginning and ending points of a curve ahead. By determining the beginning and ending points of a curve, we can determine the total length of the curve *(L)* as well as the differential heading which is the difference of initial heading *(h1)* at the beginning of the curve and final heading *(h2)* at the ending of the curve. Finally, the degree of curvature *(D)* which is defined as the change of heading (in degrees) over 100 ft, is calculated using equation 3.1, where *L* is the curve length in feet.

$$D = \frac{100*|h2-h1|}{L} \tag{3.1}$$

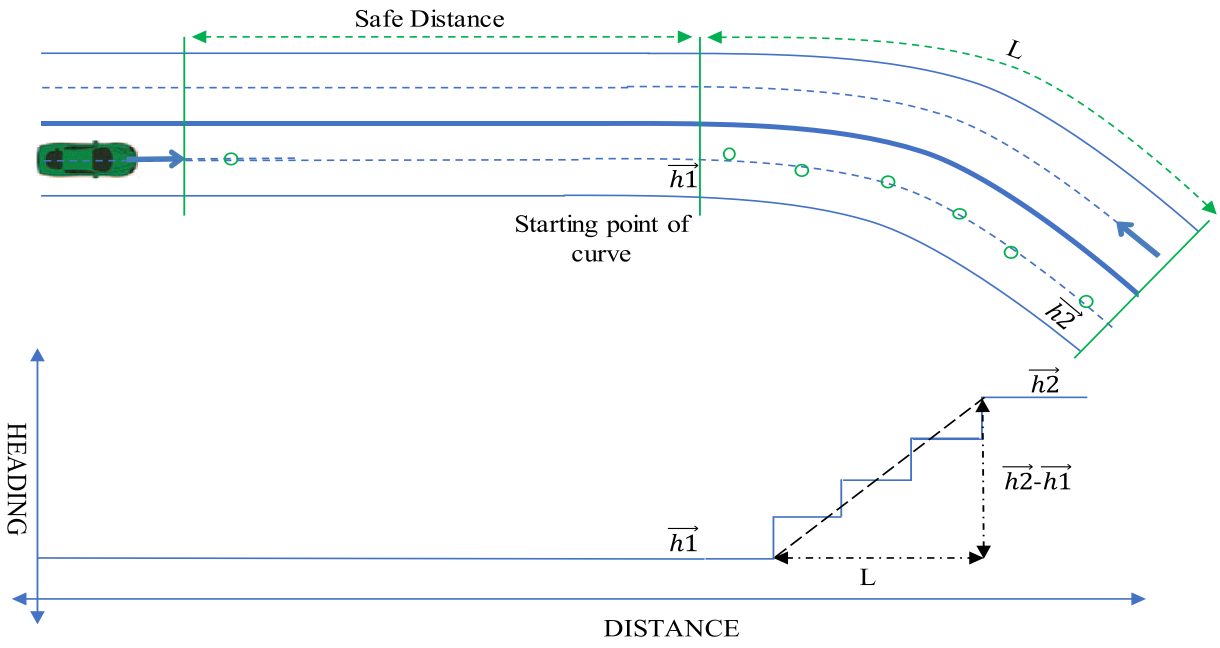


Figure 3.2 Schematic diagram showing methodology to determine beginning and ending points of a curve ahead. This will be needed to determine advisory speed of the curve.

An advisory speed *(V)* for a given curve not only depends on degree of curvature but also rely on other factors including super-elevation *(e)* and road friction factor *(f)* and can be calculated from equation 3.2.

$$V = \sqrt{\frac{5729.578 * 15 * (e+f)}{D}} \tag{3.2}$$

Use of equation 3.2 requires determination of the super-elevation as well as friction factor in addition to the degree of curvature of the curve. Both super-elevation and friction values can be estimated empirically. According to MnDOT road design manual, the specific degree of curvature corresponds to a specific limiting friction factor value for a given road *(33)*. It contains specific friction factor values for a few discrete values of degree of curvature ranging from 2 to 21 degrees *(33)*. We have curved fitted the specified friction factor values to generate a generic formula to determine friction factor value for a given degree of curvature as shown in Figure 3.3 where friction factor values are plotted versus degree of curvature.

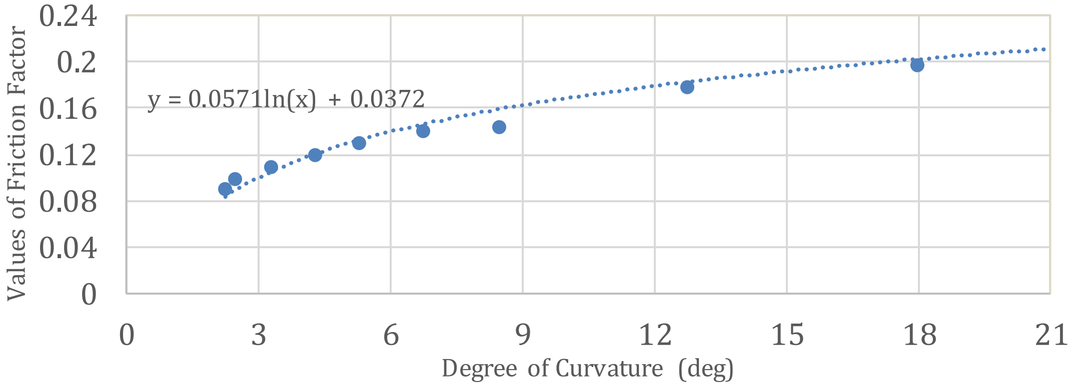


Figure 3.3 The friction factor vs. degree of curvature

Although friction factor value is fixed for a given degree of curvature, the super-elevation value for a given road can vary between 0 to 6 percent for the same degree of curvature. There is a possibility that roads with the same degree of curvature can have different super-elevation value or vice versa. Most of

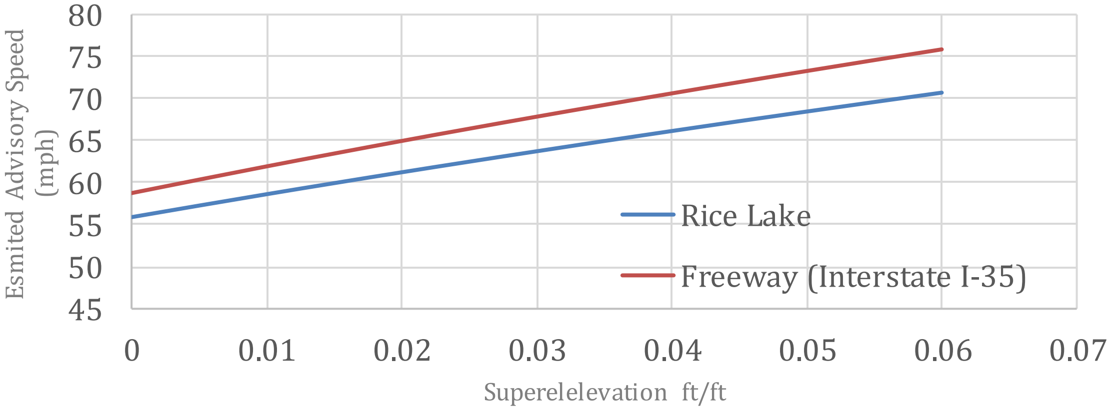


Figure 3.4 An Estimated advisory speed vs. Super-elevation values

the highways and freeways in Minnesota use 6 percent of super-elevation value (maximum recommended), but a few highways especially the old ones use a smaller super-elevation value. Figure 3.4 shows calculated advisory speed versus super-elevation values for two curved road segments (Rice Lake Road and Interstate I-35), which is also used for demonstration purposes in lane departure warning system (Chapter 2). The calculated advisory speed for the curved section of Rice Lake road ranges from 55 to 70 mph. However, the posted advisory speed for that curved section is 55 mph. This indicates that a super-elevation value of 0% is used for the Rice Lake Road. On the other hand, for the Interstate section I-35, the posted speed is 70 mph which indicates that the 4% super-elevation value is used for that segment. Without having the super-elevation information in advance, we cannot reliably calculate an advisory speed for a given road. Although the safest value of super-elevation is 0% resulting in the least advisory speed, we used a super-elevation value of 3% to calculate advisory speed as a mid-range value. An advisory speed using one fixed value of super-elevation i.e., 3% could differ from the actual posted advisory speed for that road. To mitigate this factor and to warn the driver appropriately, it is best to obtain the actual posted advisory speed from the digital map database as explained below.

3.1.2 Acquired Advisory Speed

This method directly extracts posted advisory speed from the digital maps database which we have previously used for extracting shape points to determine reference road direction. Digital maps database also has road level information including advisory speed. Previously, we used OpenStreetMap (OSM) data which is entirely open source and any user can directly obtain actual data with the help of OSM-common library by using JAVA programming language. Similarly, some commercial mapping databases, e.g., Google maps have provided several Application Programming Interfaces (APIs) to obtain the road level information. However, there is a limitation on getting the amount of data per day, free of charge, from commercial databases as opposed to open source databases.

After extracting the advisory speed directly from the map database, it is be compared with the calculated advisory speed as explained in first method (section 3.1.1). To be on the safe side, we used the lower advisory speed whether it is from the map database or from the calculation method for issuing the warning. After determining the advisory speed, next step would be finding the safe distance before issuing the warning.

3.2 SAFE DISTANCE

Safe distance is calculated using vehicle's current speed, the advisory speed of the given curve, and a safe deceleration rate. The current speed of the vehicle can be calculated from the GPS coordinates and advisory speed is determined using the methodology explained above. As far as the safe deceleration rate is concerned, according to AASHTO, approximately 90% of motorists brake with the deceleration rate of more than 3.4 m/s^2 *(34)*. This rate enables drivers to reduce their speed safely without losing control. Therefore, 3.4m/s^2 is used as a safe deceleration rate for reducing speed. Using current speed, advisory speed, and safe deceleration rate *(a)*, the safe distance is calculated by using Equation 3.3. However, Equation 3.3 does not accommodate driver's reaction time. Therefore, an adjustment is made to include the driver's reaction in calculating safe distance using Equation 3.4, where T is the reaction time for the driver. According to AASHTO, a person can take 0.9 to 2.5 seconds to react to a warning sign. To be on a safe side, we are using the longest reaction time (2.5 s) for safe distance calculations.

Safe Distance without reaction time = $\frac{{V_c}^2 - V^2}{2*a}$ (3.3)

Safe Distance with reaction time = $\frac{{V_c}^2 - V^2}{2*a} + V_c * T$ (3.4)

Figure 3.5 shows the safe distance vs. vehicle's current speed for three different values of deceleration (3.4, 6 and 8 m/s^2) for each of the two advisory speeds (55 and 70 mph). Although system uses 3.4m/s^2 as deceleration rate, the higher declaration rates (6 and 8 m/s^2) have been incorporated for reference only. It is to be noted that the higher deceleration rates show the usage of emergency brakes while reducing speed. When the vehicle is driving at the same speed as the advisory speed *($V=V_C$)*, safe distance only accounts for driver's reaction time and will have some non-zero value (Figure 3.5).

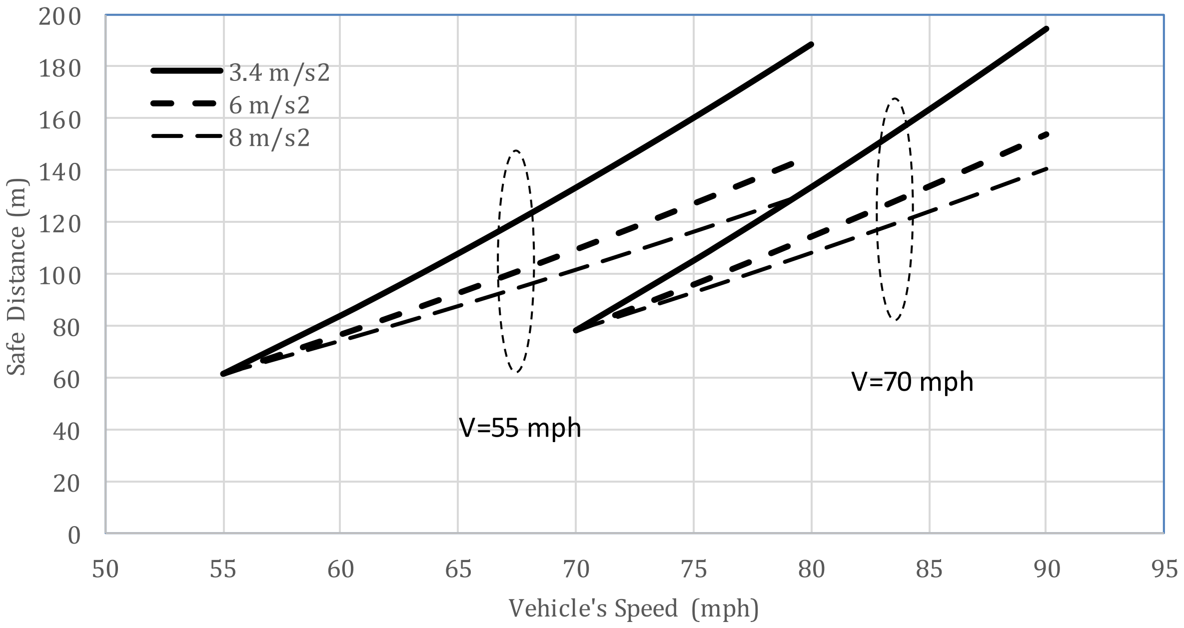


Figure 3.5 Calculated safe distance vs. vehicle's current speed.

3.3 SYSTEM WARNING GENERATION

Finally, an advance curve speed warning is issued using the same equipment as used for LDWS, i.e., a DSRC Device which has a built in GPS as well as the processing power. Based on safe distance analysis, our algorithm ensures to scan a curve ahead at least half a mile in advance to ensure that advanced curve warning can be issued in time. Half a mile criterion gives 30 seconds buffer time at the speed of 60 MPH. Once the advisory speed is determined and a safe distance is calculated, the two following cases are possible prior to issuing the warning.

- Vehicle's current speed is higher than the advisory speed.
- Vehicle's current speed is less than or equal to the advisory speed.

In both cases, we recommend the same warning, however, the safe distance will be different in both cases. Once vehicle approaches the safe distance range, the following warning will be issued.

Curve Ahead

Advisory Speed: **XX MPH**

The warning message comprises of two important pieces of information, the existence of curve and its advisory speed. By giving the warning within the safe distance, system ensures that the driver has enough time to adjust vehicle's speed comfortably. Although LDWS generates an audible warning if the vehicle departs its lane, ACWS displays a written warning on laptop screen for demonstration purpose.

CHAPTER 4: INTEGRATION AND SUMMARY

4.1 OVERVIEW

This chapter emphasizes the integration of the LDWS and ACWS and highlights the summary with conclusions and future work. Both the algorithms are integrated in such a way that if the vehicle is departing its lane and is also near to the beginning of a curve, an audible signal will be issued due to lane departure and the system will show the curve speed warning. The overall system performs calculations for both lane departure detection and advance curve detection algorithms every 100 msec. The executional flow of both LDWS and ACWS are described below.

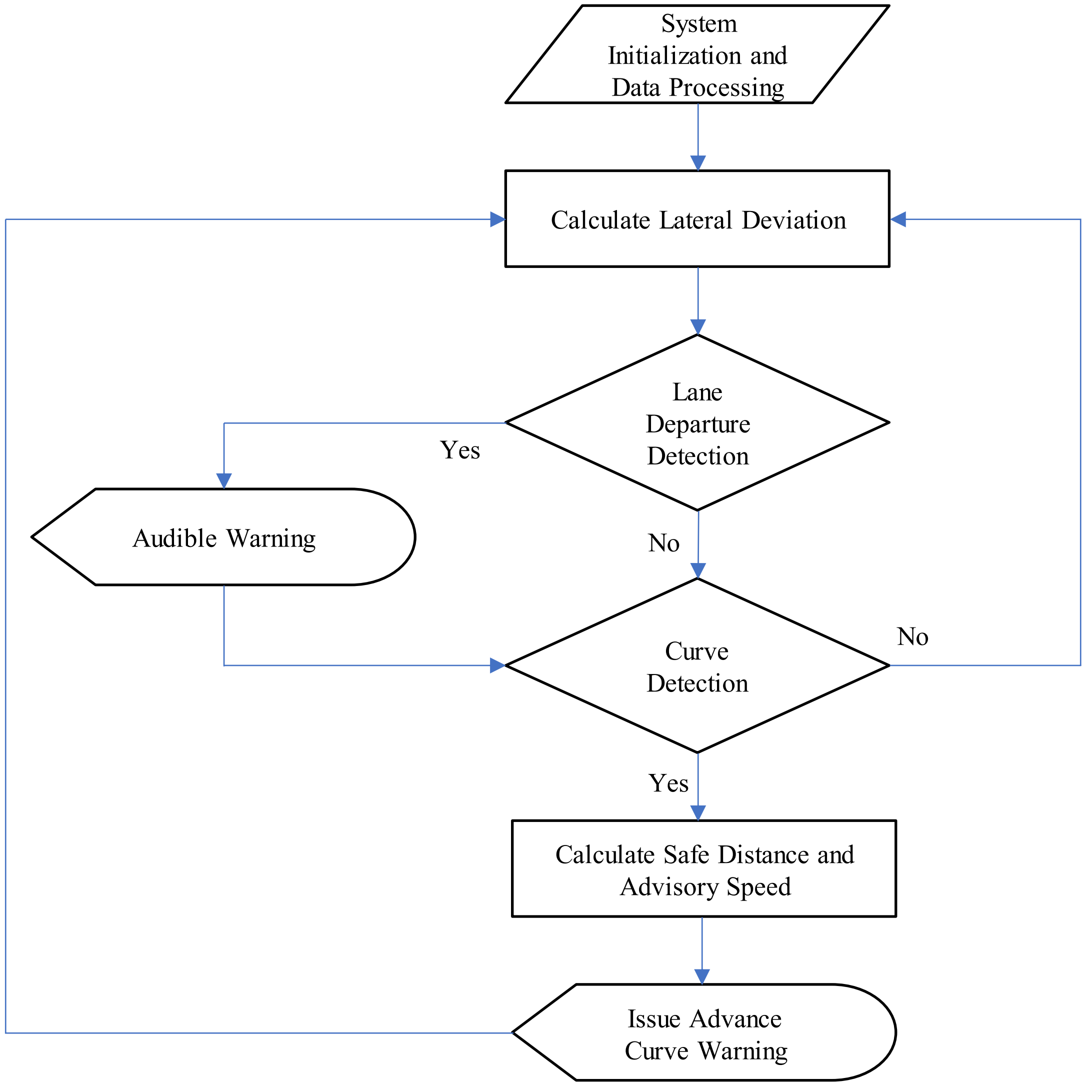


Figure 4.1 Flow diagram shows the complete functionality of LDWS and ACWS. The system updates its decisions every 100ms.

4.2 INTEGRATION

The functional flow of the implementation of the LDWS and ACWS is shown in Figure 4.1. The system periodically (every 100 ms) calculates instantaneous lateral distance and accumulates it over time. If the accumulated lateral distance crosses a certain threshold, the system will issue an audible warning to alert the driver of unintentional lane drifting. In addition to lane departure detection, the system also checks if there is any curve ahead. The developed system has the capability to differentiate between curve and straight sections of the road in real time. If there is a curve ahead, the system calculates a safe distance, based on vehicle's speed and an advisory speed for a given curve (Chapter 3). The safe distance calculation determines when to issue the advance curve warning. If the vehicle's current speed is greater than the advisory speed, the system will issue the advance curve warning at a safe distance. The ACWS displays the first message indicating the driver of the imminent curve and its advisory speed with the text "Curve Ahead and #Advisory_Speed". At the time of the warning, the vehicle will be at a safe distance away from the curve, so the driver will have enough time to reduce the vehicle's speed. Once the vehicle approaches the curve, the message changes to "On Curve" and when it leaves the curve, the message changes to "Curve Ended". This whole cycle of calculation is repeated every 100 ms and appropriate warnings are given when warranted. In this way, both the LDWS and ACWS work simultaneously and independent of each other. For the demonstration purpose, we made two videos on Rice Lake Rd and Interstate I-35. The main video screen shows the road view and the console terminal window is merged on the bottom left side of the main screen. Many lane departures were performed by the driver to test the LDWS on a straight section and a curve section. As discussed above, the ACWS displayed the position and speed information of the curve on a console.

4.3 CONCLUSION AND FUTURE WORK

In this report, two algorithms are presented, one for lane departure detection and the other for advance curve detection using a standard GPS receiver. Both algorithms were developed and tested in the field. Extensive field tests were performed to evaluate the efficiency of both algorithms on both straight and curved road segments. The field test results show that the proposed lane departure detection algorithm can detect and warn the driver of a true lane departure with an accuracy of almost 100% on both straight and curved road segments. Although no true lane departure was left undetected, occasional false lane departures were detected about 10% of the time when the vehicle did not truly depart its lane. A majority of these false alarms were issued on the sharp curved sections of the road. Along with lane departure warning, the advance curve detection algorithm simultaneously detected the possible curve ahead and issued an advance curve warning with the information of an advisory speed for a given curve at a safe distance.

The next phase of lane departure detection project will overcome the inherent error in reference road direction to improve the lane departure detection algorithm. The initial testing in the modified lane departure warning algorithm has already been tested and discussed at the end of chapter 2. This modified algorithm generates its own reference road direction via vehicle past trajectories that shows significant potential to reduce the frequency of false alarms on curved road sections. The authors will also introduce another feature to the lane departure warning algorithm that will be responsible for receiving reference road direction information from a nearby vehicle using vehicle-to-vehicle (V2V) communication.

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