Essays on International Finance and Financial Crises

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Dedication

To my parents, Kyoungsuk Jung and Eunseog Lee.

Abstract

This dissertation consists of three chapters about international finance, focusing on financial crises. First and second chapters study international capital flows, motivated by the Great Recession. In the last chapter, I turn to European debt crisis and study on fiscal policy in response to sudden spikes of sovereign bond yields.

More specifically, gross capital flows, which arise from the changes in international investment positions, experienced a sudden collapse during the Great Recession in the United States and other advanced countries. The first two chapters build an open economy model of portfolio choice with two bonds and two non-tradable sectors. Equilibrium portfolios are long in domestic bonds and short in foreign bonds because the endogenous movements of real exchange rate make this portfolio a good hedge against non-tradable consumption risk. With a calibrated model, I find that the observed fluctuations in gross flows mitigated 4% of consumption drop during the Great Recession in the United States.

In the last chapter, I ask how the fiscal policy should be adjusted when the volatility of government bond yield is time varying. I first quantify the stochastic volatility of real government bond yields for Germany, Italy, Spain and Portugal. Then, I propose a small open economy model with stochastic volatility of real interest rates in order to examine the optimal fiscal policy. I find that under high volatility, consumption tax will result in the highest revenue but at the cost of biggest welfare loss compared to the benchmark low volatility. Capital and labor income taxes show similar results but to the lesser amount. Calibrating to Portuguese data, I show that 1 percentage point increase in consumption tax will yield 1.5% less total government revenue and 1.6% more welfare loss in present value terms under the high volatility than the low volatility.

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Chapter 1

Gross Capital Flows and International Diversification

One of the key stylized facts in international economics has been the explosive growth of cross-border financial transactions since the 1980s, until their collapse in 2008. With financial development and decreased capital controls in the post-Bretton Woods era, gross flows, defined as the changes in international investment positions in both assets and liabilities, increased by an order of magnitude since 1973 and reached to 34% of quarterly GDP in 2007Q1 in the United States. Subsequently, the gross flows dropped by 1.56 trillion dollars in the span of 2 years, hitting the record low in 2008Q4. The sudden collapse of gross flows during the Great Recession sparked discussions concerning the costs and benefits of integrated financial markets in developed countries, among both academics and policy makers.

The traditional approach to cross-border financial transactions has been to focus on net flows, defined as the changes in international liability positions net of asset positions.

In the United States, between 2007Q1 and 2008Q4 net flows fell by 4% of GDP, compared to a 42% drop in gross flows over the same period. This contrasts with the experience of emerging markets, which tend to have more volatile net flows and less volatile gross flows compared to developed countries (Broner et al. 2013). Motivated by these observations, I develop a simple theory of gross capital flows, accessing how much of international financial transactions are driven by economic fundamentals.

The main hypothesis in this chapter is that households have a *risk sharing* motive that leads them to hold international investment positions in order to share their non-tradable consumption risk. I assume that tradable goods are gross substitutes to non-tradable goods for households. This assumption implies that in the event of low non-tradable consumption, households want to consume more tradable goods in order to smooth their overall consumption. Then, households can share their non-tradable consumption risk by exchanging tradable goods, as long as shocks to non-tradable goods are not perfectly correlated across countries. The basic intuition is that households choose the amount of cross-border financial transactions as means to smooth out their consumption over time.

In order to examine the mechanism of risk sharing through international financial transactions, I build an open economy model of portfolio choice with two symmetric countries and two bonds. Each bond is denominated in the aggregate price index of the given country, which promises to pay one unit of aggregate consumption goods next period. There are two sectors within each country: a tradable and a non-tradable one, where the output of each sector is exogenous and follows a stochastic process. I assume that households have constant relative risk aversion (CRRA) and constant elasticity of substitution (CES) utility, consuming a basket of tradable and non-tradable goods.

The equilibrium portfolio takes a long position in domestic bonds and a short position in foreign bonds. In other words, households save (long position) in their domestic bonds, while borrowing (short position) in foreign bonds. It is because the endogenous movements of the real exchange rate make this portfolio a good hedge against non-tradable consumption risk. For example, when an adverse shock hits non-tradable consumption in one country, its non-tradable goods become more valuable relative to the tradable goods. As a result, the domestic consumption basket becomes more valuable compared to the foreign one, which implies real exchange rate appreciation. Then, households exchange their domestic bonds for tradable goods at an appreciated value and thus smooth out their total consumption over time.

Adjustments to the equilibrium portfolio take place when the output of either tradable or non-tradable sector is hit by a shock, from which the gross flows arise as a difference in gross positions. Negative shocks to output in any sector, for example, result in lower positions in both domestic and foreign bonds. The reason for these adjustments is that there is less consumption risk to insure against when output goes down. This implies that the model predicts a positive correlation of gross flows and aggregate output, as observed in the data.

To access the relevance of this model, I calibrate the model to data for the United States and the European Union (EU), which share the largest volume of cross-border financial transactions. I focus on *debt flows*, which are the sum of debt security transactions and banking flows, as opposed to *equity flows*, which are composed of foreign direct investments and portfolio equity transactions. This is due to the fact that the observed gross flows in the United States and other advanced countries are predominantly composed of debt flows. Over the sample period of 2001-2016, I estimate the exogenous output process of tradable and non-tradable sector of each country and simulate the gross flows as an endogenous outcome of the model.

I show that the simulated model captures the key business cycle statistics of the gross flows. Specifically, the model asset and liability flows are highly correlated and pro-cyclical, which is consistent with the data. In addition, gross flows are more volatile than net flows in the model, predicting the collapse during the 2008 crisis.

From the simulation results, I find that the observed fluctuations in gross flows mitigated a part of the consumption drop during the Great Recession in the US. Without international financial transactions, the model predicts a larger drop in aggregate consumption because of the households' inability to import more tradable goods when the non-tradable consumption is low. In particular, data shows that the US experienced a sharper fall in its non-tradable output compared to the EU during the recession. Both in the data and the model, lower output of the US non-tradable sector during the 2008 crisis is accompanied by a steep appreciation of the dollar against the Euro. Long positions in dollar-denominated bonds along with the appreciation of their value, which are observed in the data and predicted by the model, allow the US households to consume more tradable goods, hence smoothing their overall consumption.

The final application of my model is to analyze the effects of introducing a financial transaction tax. In 2011, the European Commission proposed the Financial Transaction Tax (FTT), which levies a 0.1% tax on security transactions in order to collect revenues from the financial sectors who got massive bailouts funded by taxpayers, but the ongoing disagreement on the effectiveness of FTT has delayed its implementation. The calibrated model suggests that imposing the tax will result in reduced benefits of international diversification. I show that through the lens of international risk sharing, the proposed tax on bond transactions will eradicate the cross-border financial transactions and prevent households from sharing non-tradable consumption risk, resulting in lower welfare.

Literature review This chapter is related to two recent strands of literature. First one is the growing literature on gross capital flows. The traditional approach to capital flows has been focused on net flows, which are defined as liability flows net of asset flows. Liability flows, also called inflows, have been the major source of capital flows especially for developing countries (Calvo et al., 1993). Sudden stops in capital flows (Mendoza 2010, Caballero and Krishnamurthy, 2001) were also mainly concerned with international borrowing, not the domestic investment abroad. However, with rapid financial globalization in the past decades, gross capital flows have grown exponentially (Obstfeld 2012) and this sparked the growing literature on gross flows.

In empirical literature of gross flows, Broner et al. (2013) document large and volatile gross flows compared to net flows, as well as the pro-cyclicality of gross flows using a wide panel of countries including both developed and developing countries. Rey (2015) also documents the fact that gross flows are pro-cyclical, and find a co-movement of gross flows with asset prices. Recently, Avdjiev et al. (2017) decompose debt inflows into different sectors and show that private debt inflows in developed countries are pro-cyclical For extreme events in gross flows such as surges and collapses, Forbes and Warnock (2012) identify episodes of large increases in gross capital flows during the economic boom and collapse during the crisis using a large panel of countries. Rothenberg and Warnock (2011) extend the sudden stop literature, which is based on net flows, through the lens of gross flows. In this chapter, I develop a theory of gross flows that is consistent with the key facts on gross flows documented in the empirical literature.

For theoretical models of gross flows, the main approach in the literature has been focused on the international investments in equity and equity flows, rather than debt securities. Pavlova and Rigobon (2007, 2008, 2010) provide a closed-form solution of international equity prices and investigate fluctuations of external wealth in relation to endogenous terms of trade fluctuations, but do not study the gross flows. Tille and Van Wincoop (2010, 2014) suggest a two-country DSGE model of equity flows, matching a broad range of characteristics of gross flows compared to net flows, yet without a quantitative analysis. Dou and Verdelhan (2015) incorporate incomplete markets and asymmetric countries in order to explain volatile equity flows quantitatively, rather than debt flows. Bruno and Shin (2015) investigate the nexus between cross-border banking and global liquidity, based on bank leverage cycle in a partial equilibrium model. More recently, Caballero and Simsek (2016) highlight the role of gross flows in creating and destroying global liquidity, rather than in international consumption diversification. Davis and Van Wincoop (2017) investigate increased correlations of asset and liability flows (outflows and inflows) in recent years, focusing on long-run trends rather than on business cycle fluctuations. With respect to these papers, I develop a quantitative general equilibrium model that focuses on the debt securities and the role of gross flows in non-tradable consumption risk sharing.

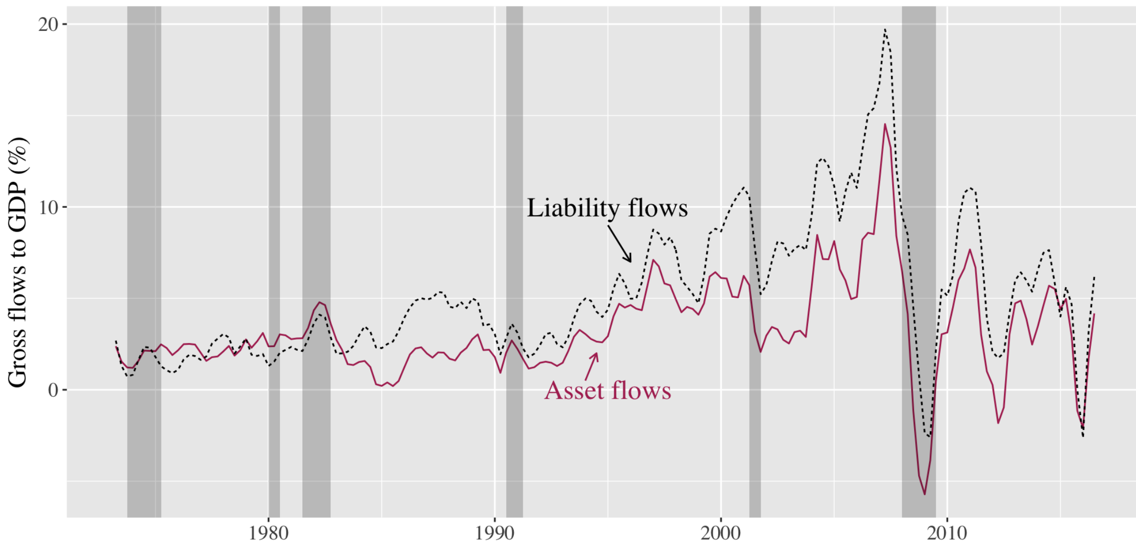
There have been extensive studies on global imbalances and international positions across countries (Mendoza et al. 2009, Gourinchas and Rey 2007, Curcuru et al. 2008, Lane and Milesi-Ferretti 2007, Caballero et al. 2008, Maggiori 2017). Heathcote and Perri (2013) propose a theory of international diversification where domestic assets provide a good hedge against labor income risk through endogenous fluctuations of the international relative price, focusing on the long-run international investment positions in equity as opposed to the fluctuations in gross flows. Coeurdacier and Gourinchas (2016) show that bonds denominated in the aggregate price index provide a hedge against real exchange rate risk, which explains the home bias in equity conditional on bond positions, but they focus on investment positions based on a 2-period model without gross flows. Compared to these papers, I build a model of international portfolio choice focusing on the gross debt flows in infinite horizon and business cycle movements.

Finally, in terms of the model solution, I globally solve a two-country model with incomplete markets using time-iteration method developed by Kubler and Schmedders (2003). This global method has been also used by Rabitsch et al. (2015), who compare the solution methods in international portfolio choice, and Coeurdacier et al. (2015) who study the welfare effects of financial integration where non-linear global solution is necessary for accurate welfare evaluation. In this chapter, global solution is used in order to analyze the portfolio decisions when the economy is subject to large shocks during the financial crisis.

The rest of Chapter 1 is organized as follows. In Section 1.1, I explain the build-up of gross flows and their fluctuations, along with international investment positions and real exchange rate movements that support the risk sharing motive in gross flows. In Section 1.2, I provide a simple environment of complete markets in order to explain the hedging motive analytically. Section 1.3 lays out the model. Section 1.4 brings the model to the data, calibrating for the US and the EU28 from 2001 to 2016. I conclude in Section 1.5.

1.1 Data

In this section, I first describe the rapid build-up of gross flows since 1970s, which lasted until the Great Recession for most of the developed countries. Then, I focus on years 2001-2016 at business cycle frequency. I document key statistics of international capital flows and investment positions and movements of the real exchange rate in the United States and the European Union, which are used to motivate the model selection and to calibrate it.



Note: Asset (solid line) and liability (dashed line) flows to GDP in the United States against rest of the world. Shaded areas are NBER recessions. Data is seasonally adjusted moving average. Data source is Bureau of Economic Analysis and GDP is at quarterly level.

Figure 1.1: Gross flows of the United States against rest of the world

1.1.1 Gross flows

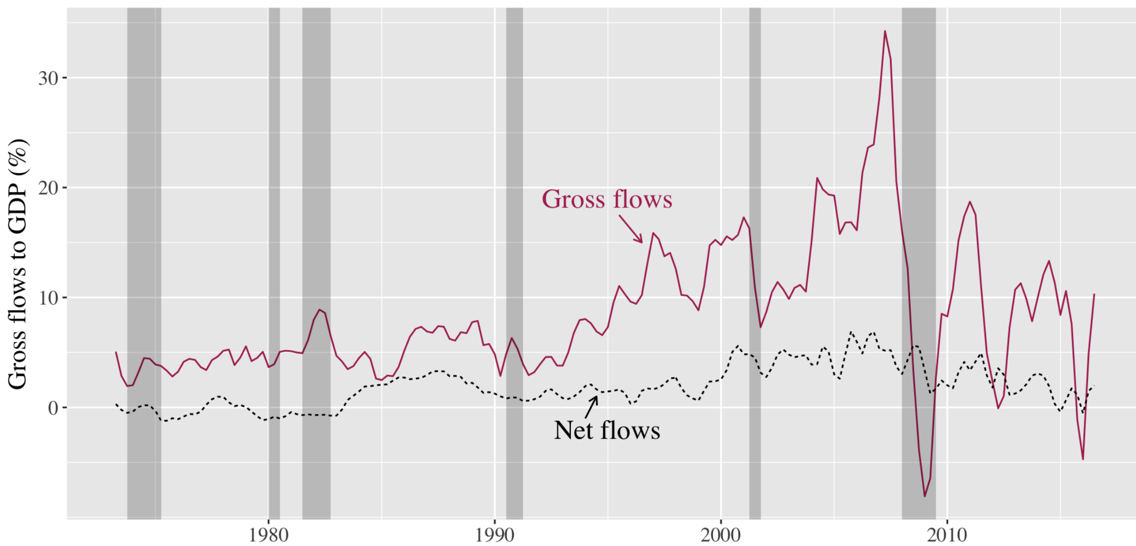
Concept of gross flows Gross capital flows are changes in international investment positions (IIP) due to transactions. IIP is a balance sheet of a country that records both *assets*, which are the financial claims on nonresidents (cross-border investments by domestic residents), and *liabilities*, which are the claims by nonresidents on residents (cross-border borrowings by domestic residents).[1] *Asset flows* are changes in asset positions due to net acquisitions of foreign financial assets by domestic residents. Analogously, *liability flows* are changes in liability positions, which are equivalent to the net incurrence of liabilities by domestic residents. Gross capital flows are defined as the sum of asset and liability flows.

[1] For more detailed concept of capital flows and international investment positions, see Balance of Payments Manual, Sixth Edition. IMF, 2010

Build-up of gross flows After the Bretton Woods system collapsed in early 1970s, international investment positions started to grow rapidly around the world. In 1970 the annual gross flows to GDP, reflecting the changes in gross positions, were merely 1.7% in the United States as Figure 1.2 shows. Over time, they grew to 13% in 1999 and 26% in 2007 based on IMF Balance of Payments. For other financial centers with smaller total output, such as the United Kingdom or Luxembourg, such increases were even more dramatic. In the UK, gross flows to GDP were 4% in 1970 but increased exponentially, reaching to 130% in 2007. The build-up of international investment positions accelerated during the economic boom, reaching their peak in 2007 for most of OECD countries (Figure in Appendix).

One of the main reasons behind the rapid growth is financial development. Lane and Milesi-Ferretti (2008) analyze that financial development and integration in the Euro area have strong correlations with increasing international financial positions using cross-country data. Obstfeld (2007) and Obstfeld and Taylor (2003) attribute "unparalleled expansion in private international asset trade" to technological advances including financial innovation, while calling out for the explanations of such phenomena in relation to international risk sharing and adjustment.

While acknowledging the trend of financial development over the past decades, the baseline approach of this chapter is to focus on the recent years since 2001 assuming that a wave of financial innovation has already taken place. I first analyze the contribution of real business cycles in gross flows by the risk sharing mechanism in Chapter 1. Then, in Chapter 2, I examine how the increase of gross flows are explained by the financial innovation, which I interpret as the loosening of credit constraints, as well as the collapse of gross flows by tightening credit constraints during the recession.



Note: Gross (solid line) and net (dashed line) flows to GDP in the United States against rest of the world. Shaded areas are NBER recessions. Data is seasonally adjusted moving average. Data source is Bureau of Economic Analysis and GDP is at quarterly level. Gross flows are the sum of asset and liability flows. Net flows are liability flows - asset flows.

Figure 1.2: Gross and net flows of the United States against rest of the world

Boom, collapse, and slow recovery Here I narrow down the focus to the recent periods of 2001Q1-2016Q2 in the United States and document the key statistics of international capital flows, which are used in model construction and calibration. Data is collected from Bureau of Economic Analysis. The sample selection is intended to capture the periods leading up to the Great Recession in the United States, who was at the center of the financial crisis, and the introduction of Euro and the European Union, which is the major financial transaction partner of the US.[2]

[2]The European Union is the largest financial transaction partner of the US, whose gross flows take 58% of total gross flows of the US at the peak of economic boom in 2007Q1 and collapsed more than 100% of the total gross flows at the height of the financial crisis in 2008Q4. More than 100% of the collapse in gross flows implies that other regions had positive gross flows whereas the EU had large negative figures for the gross flows. Asset and liability flows for the US-EU pair in 2008Q4 were -380 billion and -228 billion dollars, while the total asset and liability flows of the US against the rest of the world were -341 and -107 billion dollars. Source: Bureau of Economic Analysis, International Transactions Table 1.3.

Table 1.1: Key business cycle statistics of the US gross flows

	Total flows	Debt flows
Corr($Gross$, $GDP\ cycle$)	0.33	0.33
Corr($Asset$,$Liab$)	0.93	0.90
Std($Gross/GDP$)/Std(Net/GDP)	4.75	3.60
Std($Gross/GDP$)	0.10	0.09

Note: Sample period is from 2001Q2 to 2016Q2. All data is the US. *GDP cycle* is log HP-filtered cycles of real GDP. *Asset* and *Liab* are asset flows and debt liability flows, deflated with GDP deflator. *Gross* is gross flows and *Net* is net flows. Total flows are composed of debt flows and equity flows. Source: Bureau of Economic Analysis, author's calculations.

Asset and liability flows in the United States are plotted in Figure 1.1. Both flows are highly correlated with the correlation coefficient of 0.93. This is consistent with the empirical literature, where Broner et al. (2013) document that the correlations have been increasing over time across 103 countries, especially after 2000. Davis and Van Wincoop (2017) also document increasing correlations over the sample of 128 countries, and suggest that it is a result of increased financial and trade globalization.

Gross flows are larger in levels, and much more volatile than net flows. As Figure 1.2 shows, gross flows peak at nearly 34% of quarterly GDP in 2007Q1, and experience a sharp collapse to -8% of GDP in 2008Q4. On the other hand, net flows, which are defined as liability flows net of asset flows, move from 5% to 3% during the same period, or 7% (2006Q3) to 1% (2009Q1) from peak to trough. While the net flows also have seen a significant change during the recession, the gross flows are much more volatile than the net flows. Standard deviations are 10.18% and 2.14% for gross flows to GDP and net flows to GDP, respectively. Table 1.1 summarizes the aforementioned statistics under the *Total flows* column.

Both asset and liability flows are further decomposed into *debt* and *equity* flows (Lane and Milesi-Ferretti 2007), as Figure 1.3 summarizes in a diagram. Debt flows are composed of Portfolio investments in debt securities and Other investments, which are mostly banking flows. On the other hand, Direct investments and Portfolio investments in Equity and investment fund shares constitute equity flows. Financial derivatives, whose data is recorded in net positions only, and reserve assets, of which the amount is small in the US, are not included in the classifications.

Debt gross flows are on average 67% of the total gross flows in absolute values.[3] Among debt flows, Portfolio debt and Other investments have similar shares in absolute values, where Other investments are on average 55% of the debt gross flows. In Appendix, I plot debt and equity gross flows over the sample period in the US.

Motivated by the observation that debt flows take most of the gross flows, my model focuses only on bonds and debt flows. In Section 1.4, I compare the model simulation results with the business cycle moments introduced above focusing on the debt flows. Since the debt flows take most of the gross flows, the key characteristics described above for the total gross flows remain the same as in the debt flows. Table 1.1 summarize the business cycle statistics of both total and debt gross flows.

1.1.2 Gross positions and currency composition

In order to compare model predictions with the data, I describe international investment positions of the United States focusing on the debt instruments and their currency compositions. Debt instruments are composed of portfolio debt investments and Other investments, which are mostly bank assets and liabilities. In the following, *debt assets* and

[3] Since capital flows can take negative values, I calculate mean(abs(debt gross flows))/mean(abs(total gross flows)) = 0.67. In an alternative way, average fraction of (abs(debt asset flows)+abs(debt liability flows))/(abs(asset flows)+abs(liability flows)) = 0.94.

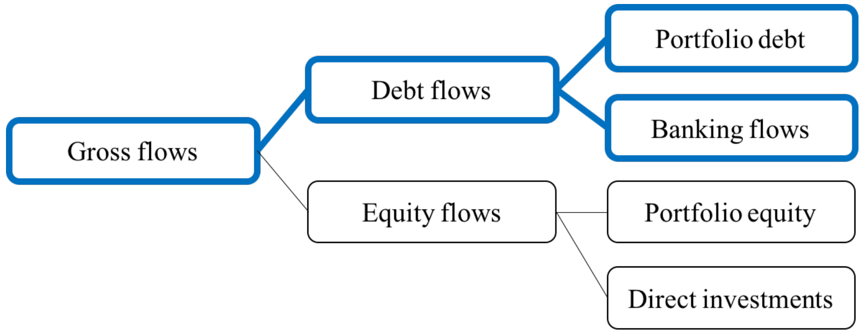


Figure 1.3: Decomposition of gross flows

debt liabilities indicate cross-border investment positions in debt instruments for asset and liability, respectively.

Bénétrix et al. (2015) estimate international currency exposures, documenting the currency compositions of external assets and liabilities for a large panel of countries from 1990 to 2012. In the United States, on average 85% of debt assets are denominated in the US dollars over the sample period, and the rest 15% is in foreign currency. Debt liabilities are on average 77% in domestic currency and 23% in foreign currency. In the data, the US shows an exceptional amount of domestic currency assets and liabilities compared to the other OECD countries. For the remaining OECD countries, except for Luxembourg, average fraction of domestic currency denomination in debt assets and debt liabilities over the same sample periods are 25% and 23%, respectively.[4]

From the above observations, I draw two main points in the view of international portfolio choice. First, on average countries have higher fraction of domestic currency in debt assets than debt liabilities. Second, the US dollar has a dominant stance in both

[4] For countries in Euro area, the average fraction of domestic currency is higher: 47% and 41% for debt assets and debt liabilities, respectively. The standard deviation of average fraction of domestic currency denomination across countries in debt assets and debt liabilities are 27% and 22%, respectively. The maximum fraction of domestic currency in debt assets is 76% (Germany) and in debt liabilities is 58% (Spain). The minimum fraction of domestic currency is 0 for both debt assets and liabilities.

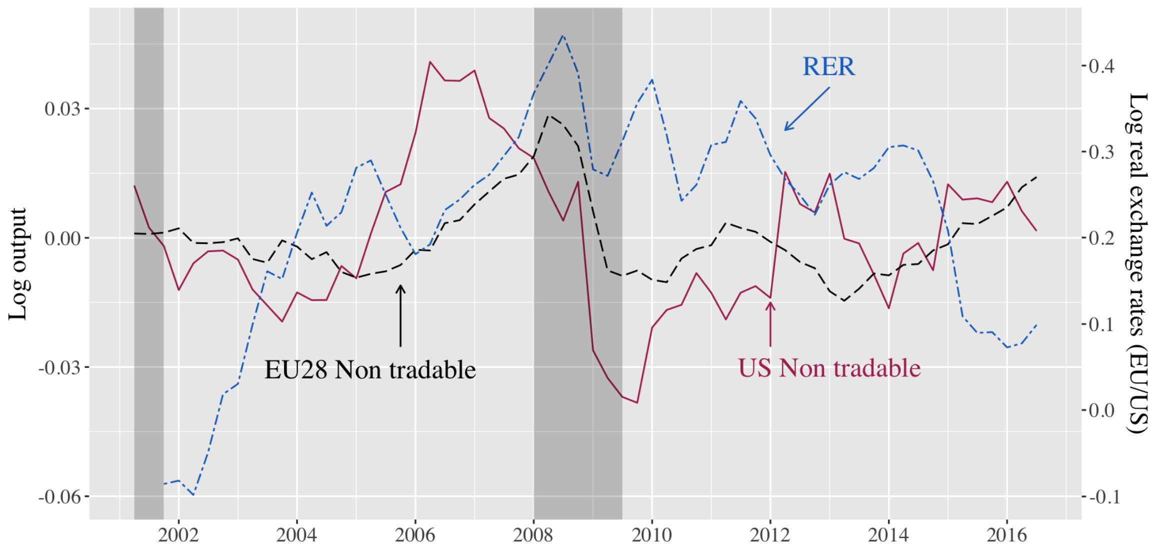
debt liabilities and debt assets, which could be a result of asymmetry between the US and other countries. In this chapter, I highlight the first point that domestic currency takes larger fraction in debt assets. In order to provide the analysis in the simplest setting, I assume symmetry between two countries throughout the chapter, which does not address the second point of observation.

Finally, a recent paper by Maggiori et al. (2017) uses a micro data of mutual funds and shows the patterns of "currency bias", where investors prefer their home currency. They find that foreign investors tend to purchase corporate bonds denominated in their home currency, across a wide range of security-level observations from different countries. Their findings give another evidence that equilibrium portfolio takes long positions in domestic bonds, while short in foreign bonds, which is the model prediction of this chapter.

1.1.3 Real exchange rates and non-tradable output

If real exchange rates (RER) are determined by the movements of the non-tradable outputs between two countries, then RER of a country would appreciate when there is a large drop in non-tradable output. It is because real exchange rates are the relative value of aggregate consumption goods between two countries, and value of non-tradable goods, which are parts of the aggregate consumption goods, would increase when the goods become scarce in the country.

Figure 1.4 shows that before and during the Great Recession, non-tradable outputs in the US and the European Union 28 member countries (EU28) support the above hypothesis on real exchange rate determinations. From 2006 to 2007, when the US non-tradable sector (red solid line) experienced positive shock on output compared to the EU28 (black dashed line), real exchange rates of the the US dollars against Euro (blue double dashed line)



Note: Log non-tradable output of the US (red solid line) and the European Union 28 (black dashed line), and log real exchange rates (blue double dashed line) of Euro in units of the US dollars. Log outputs are detrended with Hodrick-Prescott filter and log real exchange rates are calculated based on CPI. Source: Bureau of Economic Analysis, Bureau of Labor Statistics, OECD.Stat, Eurostat.

Figure 1.4: Non-tradable output of the US and the EU28, and real exchange rates of EUR/USD

depreciated. However, when the output of the US non-tradable sector dropped sharply in 2008 Q4, real exchange rates of the US dollar against Euro appreciated by 11% , which is the largest appreciation in a quarter for years 2001 to 2016. The US consumption basket remained appreciated for the next quarter, and then depreciated by 4% in 2009 Q2. These are also the periods that gross flows were at their lowest. Figure 1.2 shows that in 2008 Q4 the gross flows to GDP were *negative* 7.2%, where asset and liability flows were -5.7% and -1.5%, respectively.

1.2 Complete markets: risk sharing intuition

In this section, I analyze the risk sharing motive of bond portfolio choice under the complete markets setting. I build a simple model where households choose their domestic and foreign bond positions in order to insure themselves against adverse shocks to non-tradable consumption. I provide a closed form solution of the unique bond position, by which households achieve their first-best consumption allocations.

Physical environment There are two countries, country 1 and 2, where each has one tradable and one non-tradable sector. Output of each sector is exogenous in both countries. Non-tradable sector output in country $i \in \{1, 2\}$, denoted as y_N^i, follows a stochastic process with two possible states, which are High ($y_N(H)$) and Low ($y_N(L)$).

$$y_N^i \in \{y_N(H), y_N(L)\},\ y_N(H) > y_N(L). \tag{1.1}$$

Country $i \in \{1, 2\}$ tradable sector output, denoted as y_T^i, is given as a constant $\bar{y}_T$ for all states.

There are two symmetric states in the world economy, defined as s_1 and s_2, which belong to the set of all states $S = \{s_1, s_2\}$. In state 1 (s_1), country 1's non-tradable output is High ($y_N^1 = y_N(H)$) and country 2's non-tradable output is Low ($y_N^2 = y_N(L)$). State 2 (s_2) is the symmetric case where the non-tradable output is Low in country 1 and High in country 2.

$$s_1 = (y_N^1 = y_N(H), y_N^2 = y_N(L)),\quad s_2 = (y_N^1 = y_N(L), y_N^2 = y_N(H)) \tag{1.2}$$

In every time period, there is an equal probability of 0.5 that each state realizes, independent of time.

Consumer utility There is a continuum of identical households in each country, whose measure is 1. Each household has a Constant Relative Risk Aversion (CRRA) and Constant Elasticity of Substitution (CES) utility. Formally,

$$u(c_1(s)) = \frac{c_1(s)^{1-\gamma}}{1-\gamma} \tag{1.3}$$

where γ is the risk aversion parameter. $c_1(s)$ is the aggregate consumption basket of country 1 at state $s \in S$. There is a constant elasticity of substitution (CES) between non-tradable and tradable goods.

$$c_1(s) = (\omega_T^{\frac{1}{\sigma}}(c_T^1(s))^{\frac{\sigma-1}{\sigma}} + \omega_N^{\frac{1}{\sigma}}(c_N^1(s))^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{1.4}$$

where c_T^1 and c_N^1 are the tradable and non-tradable goods consumption, respectively. ω_T indicates weights on the tradable good consumption, while ω_N, denoting weights on non-tradable consumption, is set to $1-\omega_T$. σ is the elasticity of substitution, or *Armington elasticity*, between the two goods. Utility of the country 2 is defined analogously.

Aggregate price index for each country i in state $s \in S$, which is denoted as $P_i(s)$, is defined in a natural way given the CES utility function.

$$P_i(s) = (\omega_T + \omega_N p_N^i(s)^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{1.5}$$

where $p_N^i(s)$ is associated with the relative price of non-tradable to tradable goods in state s. Tradable goods are assumed to be the numeraire, so that their price is normalized to 1.

Social planner's problem Social planner maximizes the sum of two countries' flow utilities with equal weights, subject to feasibility constraint of each state.

$$U^*(s) = \max_{\{c_T^i, c_N^i\}} \; u_1(c_1(s)) + u_2(c_2(s)) \tag{1.6}$$

$$s.t. \; c_T^1(s) + c_T^2(s) = \bar{y}_T + \bar{y}_T \tag{1.7}$$

$$c_N^1(s) = y_N^1(s) \tag{1.8}$$

$$c_N^2(s) = y_N^2(s) \tag{1.9}$$

Financial market There are two one-period bonds, where each bond is denominated in the aggregate consumption basket of the given country. All bonds promise to pay one unit of aggregate consumption goods uncontingent to the states next period. In the following, I denote $a_i^j(s)$ as the amount of bond i purchased by households in country j at state s. A negative a_i^j implies borrowing, or short positions, in bond i by households j.

There is a zero net supply of each bond for all states. Market clearing conditions for the bonds are given as follows.

$$a_1^1(s) + a_1^2(s) = 0, \; a_2^1(s) + a_2^2(s) = 0 \tag{1.10}$$

Consumer's problem Individual households purchase goods and make portfolio decisions each period, under the following budget constraint at each state $s \in S$.

$$c_T^1 + p_N^1(s)c_N^1 + \sum_{i=1}^{2} P_i(s)q_i(s)a_i^{1\prime} \leq \bar{y}_T + p_N^1(s)y_N^1(s) + \sum_{i=1}^{2} P_i(s)a_i^1 \tag{1.11}$$

where a_i^1 and $a_i^{1\prime}$ are the bond purchase from the previous period and the current period, respectively, and $q_i(s)$ is the price of bond i.

Consumers maximize their expected utility at time 0 under the budget constraints.

$$\max_{c_T^1(s^t), c_N^1(s^t), a_1^1(s^t), a_2^1(s^t)} \sum_{s^t} \beta^t \left(\frac{1}{2}\right)^t u(c_1(s^t)) \tag{1.12}$$

$$s.t.\ c_T^1(s^t) + p_N^1(s^t) c_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t) q_i(s^t) a_i^1(s^t) \leq \bar{y}_T + p_N^1(s^t) y_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t) a_i^1(s^{t-1}) \tag{1.13}$$

Here, s^t is the history of states from time 0 to time t, $s^t = (s_0, s_1, \ldots, s_t)$, where s_k is the state at time k. Notice that $(1/2)^t$ is time-0 probability of history s^t realization.

Complete market solution I first solve for the social planner's allocations, and then find the bond portfolio that decentralizes the first-best allocations. First order necessary conditions of the social planner characterize the optimal tradable consumption across households at each state. These allocations critically depend on the risk aversion and elasticity of substitution between tradable and non-tradable goods. Following propositions describe the conditions under which the social planner allocates more tradable goods for the households with lower non-tradable consumption. All proofs of the propositions are in the Appendix.

Proposition 1. *If constant elasticity of substitution between non-tradable and tradable goods (σ) multiplied by constant risk aversion parameter (γ) is larger than 1 ($\sigma\gamma > 1$), then tradable goods are gross substitutes to non-tradable goods. If $\sigma\gamma < 1$, then they are gross complements.*

If tradable and non-tradable goods are gross substitutes, which is the case of $\sigma\gamma > 1$, then the demand for tradable goods increases in the event of low non-tradable output and a high non-tradable price subsequently. This condition arises from the non-separable utility function of tradable and non-tradable goods, and governs the cross-derivative of the utility (Tesar 1993). If $\sigma\gamma > 1$, then the derivative of marginal utility of tradable goods with respect to non-tradable goods ($\partial^2 u(c_N^1, c_T^1)/\partial c_N^1 \partial c_T^1$) is negative, leading to the Proposition 1. Intuitively, if the elasticity of inter-temporal substitution ($1/\gamma$) is lower than the elasticity of substitution between the two goods (σ), then in the event of low non-tradable consumption, households are willing to substitute it with tradable consumption.

Estimation results for the elasticity of substitution between tradable and non-tradable goods, or Armington elasticity, ranges from 0.2 to 3.5 (Ruhl 2008) and risk aversion parameters used in the literature spans from 1 to 100 (Lucas 2003). In this chapter, I focus on the parameter region of gross substitutes where $\sigma < 1$ but $\gamma > 1/\sigma$, so that the multiplication of the two parameters satisfy the condition $\sigma\gamma > 1$. It is in line with the estimation result by Mendoza (1995) ($\sigma = 0.74$), which is also used by Corsetti et al. (2008) who investigate the international risk sharing in a real business cycle model with net flows. Commonly used range of risk aversion parameters ($\gamma > 1.35$) together with the Armington elasticity of 0.74 satisfy the condition $\sigma\gamma > 1$.

Nevertheless, the assumption of gross substitution is at odds with Tesar (1993), who estimates the elasticity to be 0.44 and suggests a theory for the case of $\sigma\gamma < 1$. There are two reasons why this chapter opts for the region of parameters where the opposite condition, $\sigma\gamma > 1$, is satisfied. First, the estimation of Tesar (1993) includes both developing and developed countries, whereas this chapter mainly focuses on developed countries. Mendoza (1995) estimates the elasticity based on advanced economies, which is closer to the subject of this chapter. Secondly, different approaches to the financial market settings lead to the contrasting assumptions. Although Tesar (1993) also emphasizes the importance of non-tradable sector in international risk sharing, she focuses on the equity home bias, where equities are assumed to be denominated in tradable goods. On the other hand, this chapter addresses the long positions in domestic bonds, which are denominated in the aggregate consumption baskets. I find that the assumption of $\sigma\gamma > 1$ fits well in order to explain the observed portfolio choice in bonds, which I elaborate in the following propositions.

The social planner's solution equates every households' marginal utilities on tradable goods at each state. Therefore, in the case that tradable goods are gross substitutes of non-tradable goods, the social planner allocates more consumption goods to the households in the country with lower non-tradable consumption.

Proposition 2. *For any given non-tradable sector output $\{y_N^1, y_N^2\}$, the social planner's allocation of tradable consumption (c_T^{1*}, c_T^{2*}) equalizes marginal utilities of two countries with respect to tradable goods.*

$$u_T^1(c_T^{1*}, y_N^1) = u_T^2(c_T^{2*}, y_N^1) \tag{1.14}$$

Corollary 1. *If $\sigma\gamma > 1$ and $y_N^1 \leq y_N^2$, then $c_T^{1*} \geq c_T^{2*}$.*

With a complete financial market of two bonds and two states, the social planner's allocation can be decentralized in a competitive market, as long as the non-tradable output processes in two countries are not perfectly correlated. The following proposition shows the optimal bond portfolio in a closed form.

Proposition 3. *If the first best aggregate price index is positive and not the same across countries, $P_1^*(s) \neq P_2^*(s),\ s \in S$, then there is a unique bond portfolio $a^* = (a_1^{1*}, a_2^{1*})'$ that decentralizes the social planner's allocations. Specifically,*

$$a^* = \frac{c_T^{1*}(s_2) - \bar{y}_T}{\tilde{P}_1(s_2) - \tilde{P}_2(s_2)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{1.15}$$

where $\tilde{P}_i(s_j) = P_i^*(s_j)(1 - q_i^*(s_j)) = P_i^*(s_j) - 0.5\beta \sum_{j=1,2} P_i^*(s_j),\ i, j = 1, 2.$

Corollary 2. *If a^* exists, then the amount of domestic saving and foreign borrowing is* *the same.* $a_1^{1*} = -a_2^{1*}$. *Net position* $a_1^{1*} + a_2^{1*}$ *is 0.*

Corollary 3. *If* a^* *exists and* $\sigma\gamma > 1$, *then* $a_1^{1*} > 0 > a_2^{1*} = -a_1^{1*}$.

The proposition states that the domestic bond positions $a^*[1] = a_1^{1^*}$ increase with the amount of import $(c_T^{1*}(s_2) - \bar{y}_T)$ at the event of low non-tradable consumption, which is the state s_2 in country 1. Notice that if tradable goods are gross substitutes to non-tradable goods, then $c_T^{1*}(s_2) - \bar{y}_T > 0$ because country 1 households want to purchase more tradable goods in the low non-tradable state (s_2) by imports. In addition, $a^*[1] = a_1^{1^*}$ falls as the difference of aggregate prices in two bonds at state s_2 becomes larger. For example, if the consumption basket of country 1 increases sharply at the low non-tradable state, then households need to hold lower positions. Here, $P_1(s_2) > P_2(s_2)$ because country 1 has a lower non-tradable sector output than the other country in state 2 (s_2) and this makes the aggregate consumption basket of country 1 more valuable.

The optimal portfolio takes a positive position in domestic bonds and a negative position in foreign bonds. In state 2 (s_2), which is a low non-tradable state for country 1, the positive position in domestic bonds allows households in country 1 to import tradable goods due to an appreciation of the aggregate consumption basket. When the two countries are symmetric, country 2 households also save (long positions) the same amount in their domestic bonds, which is equal to a short position in foreign bonds for the country 1 households. Therefore, the optimal portfolio has zero net positions and positive gross positions.

Finally, since there is a unique portfolio that is common to all states, there are no adjustments in the portfolio across the states. Therefore, there are no gross flows in the complete market case. In the following section, I solve for the baseline model, which has an incomplete financial market and adjustments on the portfolio.

1.3 The Model

In this section, I describe the physical environment of the model and financial market structure. Following international portfolio choice models such as Baxter et al. (1998) and Tille and Van Wincoop (2010), I model an exchange economy where all goods are given as endowments, following stochastic processes. There are two countries with a continuum of identical households with measure 1 in each country. There are three goods in the world, which are one tradable and two non-tradable goods. Households share their non-tradable consumption risk by exchanging tradable goods. In the financial market, there are two internationally tradable bonds. Each bond is denominated in the aggregate price index of the given country and promises uncontingent payment next period.

In this chapter, I propose the model that highlights the risk sharing mechanism in a simple environment. I focus on the non-tradable consumption risk sharing though the equilibrium bond positions and their adjustments. I extend the model in Chapter 2.

In each period of time $t = 0, 1, \dots \infty$, an exogenous state denoted as $s_t \in S$ realizes. I denote the history of states from time 0 to time t as $s^t = (s_0, s_1, \dots s_t)$, which is also called as a node in the event tree. The root of the event tree is given as s_0. The probability of a node s^t realization is denoted as $\pi(s^t)$ in terms of time-0 probability, and the chance of node s^{t+1} realization given the history s^t is denoted as $\pi(s^{t+1}|s^t)$. Events follow a Markov process, which is specified in the following paragraph.

In the model, there are countries 1 and 2. I mostly focus on country 1, as the settings are symmetric in both countries. In the following, countries are denoted as superscripts and goods as subscripts. For example, x_i^j denotes a variable x of good i in country j.

Physical environment There are three goods in the world. Two non-tradable (NT) goods, one in each country, and a common tradable good. All outputs of goods are given as endowments. Each output has a stochastic autoregressive process of order 1 (AR1). Shocks on endowments are given by a vector of three shock variables $\boldsymbol{\varepsilon} = \{\varepsilon_T, \varepsilon_N^1, \varepsilon_N^2\}$, which are shocks on tradable, non-tradable in country 1, and non-tradable in country 2, respectively. The vector of shocks follows normal distribution of zero mean and covariance Σ independent of time, $\boldsymbol{\varepsilon} \sim \mathcal{N}(0, \Sigma)$. Therefore, realization of $\boldsymbol{\varepsilon}$ determines each time period's state s_t.

Tradable good (TR) endowment for both countries follow the same AR1 process, which is given as the following.

$$\log y_T(s^t) = \rho_T \log y_T(s^{t-1}) + \varepsilon_T(t). \tag{1.16}$$

For country i non-tradable y_N^i, process is the following.

$$\log y_N^i(s^t) = \rho_N^i \log y_N^i(s^{t-1}) + \varepsilon_N^i(t), \quad i \in \{1, 2\}. \tag{1.17}$$

In the benchmark model, it is assumed that all shocks are independent to each other, in order to show the mechanisms more clearly.

Consumer utility Each consumer is risk averse and demands a basket of non-tradable and tradable goods. Utility functions are assumed to be symmetric across countries. Flow utility has a constant relative risk aversion γ with respect to the aggregate consumption basket c_1.

$$u(c_1(s^t)) = \frac{c_1(s^t)^{1-\gamma}}{1-\gamma} \tag{1.18}$$

Aggregate consumption is a constant elasticity of substitution (CES) basket of non-tradable (c_N^1) and tradable (c_T^1) consumption, with the elasticity of substitution σ. There are weights on non-tradable (ω_N) and tradable (ω_T) goods, whose sum is one.

$$c_1(s^t) = (\omega_T^{\frac{1}{\sigma}}(c_T^1(s^t))^{\frac{\sigma-1}{\sigma}} + \omega_N^{\frac{1}{\sigma}}(c_N^1(s^t))^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{1.19}$$

I define the aggregate price index, which is naturally determined from the CES utility:

$$P_1(s^t) = (\omega_T + \omega_N p_N^1(s^t)^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{1.20}$$

Here p_N^1 is a relative price of non-tradable to tradable good, and the price of tradable goods is used as the numeraire. Foreign utility and aggregate price are defined analogously.

Consumer budget constraint Consumer budget constraint is given as the following.

$$\begin{aligned} c_T^1(s^t) + p_N^1(s^t)c_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)q_i(s^t)a_i^1(s^t) \\ \leq y_T(s^t) + p_N^1(s^t)y_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)a_i^1(s^{t-1}) \end{aligned} \tag{1.21}$$

Here, $q_i(s^t)$ is the price of bond i, which promises to pay one unit of country i aggregate consumption basket, $a_i^1(s^t)$ is the amount of bond i purchased by country 1 household in state s, and $a_i^1(s^{t-1})$ is the amount of bond i purchase in the previous period.

Consumers buy tradable (c_T^1) and non-tradable (c_N^1) goods, and make portfolio decisions (a_i^1). They are endowed by tradable (y_T) and non-tradable (y_N^1) goods, and enter the period with net financial position $\sum_{i=1}^{2} P_i(s^t)a_i^1(s^{t-1})$. Bonds are uncontingent in its own unit of payment, but consumers need to take into expected aggregate price index changes when they purchase bonds. When consumers buy 1 unit of bond i, payment tomorrow will be

$P_i(s')$, which is contingent on the state realization of tomorrow s'. Therefore, expected returns on *uncontingent* bond i in effect includes *contingent* changes in aggregate price index.

Consumer's problem Consumers maximize expected utility at time 0 under the budget constraint and borrowing constraint on each bond.

$$\max_{c_T^1(s^t), c_N^1(s^t), a_1^1(s^t), a_2^1(s^t)} \sum_{s^t} \beta^t \pi(s^t) u(c_1(s^t))$$

$$s.t. \;\; c_T^1(s^t) + p_N^1(s^t) c_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t) q_i(s^t) a_i^1(s^t) \leq y_T(s^t) + p_N^1(s^t) y_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t) a_i^1(s^{t-1})$$

$$a_i^1(s^t) \geq -\chi, \;\; \forall i = 1, 2 \tag{1.22}$$

Borrowing constraint χ is given as a large number that does not bind around the steady state in equilibrium. Later in the stochastic collateral constraint model, this constant borrowing constraint will be replaced with a fraction of collateral value.

Market clearing Goods markets clear for tradable and each non-tradable goods for each state.

$$\sum_{j=1}^{2} c_T^j(s^t) = 2 y_T(s^t) \tag{1.23}$$

$$c_N^1(s^t) = y_N^1(s^t), \;\; c_N^2(s^t) = y_N^2(s^t) \tag{1.24}$$

Bonds have zero net supply in each period.

$$\sum_{j=1}^{2} a_1^j(s^t) = 0, \quad \sum_{j=1}^{2} a_2^j(s^t) = 0 \tag{1.25}$$

Net wealth fraction and recursive formulation In order to solve the model, I transform consumer's problem in recursive form. I first define individual's *fraction of net wealth* (w), which is a country 1 consumer's net financial wealth at the beginning of period plus tradable endowment normalized by the two times of her tradable endowment. This normalization is designed in a way that in equilibrium, w is equal to the country 1's fraction of aggregate net financial wealth and tradable output normalized by the total tradable endowment in the world. Formally, net wealth fraction of individual $i \in [0, 1]$ at the node s^{t+1} is:

$$w^i(s^{t+1}) = \frac{y_T(s^{t+1}) + \sum_{i=1}^{2} P_i(s^{t+1}, W(s^{t+1}))a_i^1(s^t)}{2y_T(s^{t+1})}. \tag{1.26}$$

where $W(s^{t+1})$ is an aggregate country 1 net wealth fraction, $W(s^{t+1}) = \int_0^1 w^i(s^{t+1})di$. It is useful to define *net wealth* as well, since net wealth is the key endogenous variable that determines portfolio and consumption choice. Net wealth is denoted as $\tilde{w}$:

$$\tilde{w}^i(s^{t+1}) = \sum_{i=1}^{2} P_i(s^{t+1}, W(s^{t+1}))a_i^1(s^t) \tag{1.27}$$

Notice that portfolio decisions at the end of time period t ($a_i^1(s^t)$) determines an individual's net wealth at the beginning of period $t+1$ ($\tilde{w}^i(s^{t+1})$), depending on the realization of aggregate states and prices in time $t+1$ ($P_i(s^{t+1}, W(s^{t+1}))$). For an atomistic individual, aggregate price index P_i is taken as a function of current state and aggregate net wealth fraction. In this economy, a sufficient statistic for the endogenous states of both countries is W, which is the aggregate net wealth fraction in country 1, because of the zero net supply of bonds and identical individuals. In other words, since the sum of net positions in country 1 and 2 should be zero by the market clearing conditions and the net wealth of all individuals within a country is identical, the aggregate net wealth fraction in country 1 becomes a sufficient statistic for the endogenous states.

Each individual consumer has rational expectations on the evolution of aggregate net wealth fraction. A mapping Γ from any given aggregate net wealth fraction W in a time t node (s^t) along with an exogenous state s_{t+1} to another net wealth fraction in time $t+1$ at node $(s^{t+1} = (s^t, s_{t+1}))$ is given as

$$W(s^{t+1}) = \Gamma(W(s^t), s_{t+1}; s^t), \ \forall s_{t+1} \in S. \tag{1.28}$$

Notice that consumers form an expectation that maps today's $W(s^t)$ to tomorrow's $W(s^{t+1})$ for any pair of states $(s_t, s_{t+1}) \in S \times S$. In equilibrium, given an aggregate net wealth fraction $W(s^t)$ and a policy function $a_i^1(W(s^t), s^t)$, the following equation should be satisfied for any node s^{t+1}.

$$W(s^{t+1}) = \frac{y_T(s^{t+1}) + \sum_{i=1}^{2} P_i(s^{t+1}, W(s^{t+1}))a_i^1(W(s^t), s^t)}{2y_T(s^{t+1})} \tag{1.29}$$

Also in equilibrium, individual net wealth fraction is equal to the aggregate net wealth fraction, $w(s^{t+1}) = W(s^{t+1})$.

A formal definition of consumer's problem in a recursive form is as follows.

$$V_1(w(s); W(s), s) = \max_{c_T^1, c_N^1, a_1^{1\prime}, a_2^{1\prime}} u(c_T^1, c_N^1) + \beta \sum_{s'} \pi(s'|s) V_1(w(s'); W(s'), s') \tag{1.30}$$

$$s.t. \ \ c_T^1 + p_N^1(W(s), s)c_N^1 + \sum_{i=1}^{2} P_i(W(s), s)q_i(W(s), s)a_i^{1\prime} \tag{1.31}$$

$$\leq p_N^1(W(s), s)y_N^1(s) + w(s) \cdot 2y_T(s)$$

$$a_i^{1\prime} \geq -\chi, \ \forall i = 1, 2 \tag{1.32}$$

$$W(s') = \Gamma(W(s), s'; s), \ \forall s' \in S \tag{1.33}$$

$$w(s') = \frac{y_T(s') + \sum_{i=1}^{2} P_i(W(s'), s')a_i^{1\prime}}{2y_T(s')} \tag{1.34}$$

Here, I denote the country's net wealth fraction as W and individual's net wealth fraction as w, and suppress the history of states s^t into the state of today $s \in S$, exploiting the Markov process of shocks. Accordingly, $s' \in S$ denotes the state of next period and $a_i^{1\prime}$ is defined as the portfolio choice of today for the payments tomorrow. Consumer's problem in country 2 is defined analogously, where the country 2's aggregate net wealth fraction is $1 - W(s)$ due to the zero net supply of bonds.

Recursive competitive equilibrium Competitive recursive equilibrium is a collection of value functions $\{V_i(w(s); W(s), s)\}_{i=1,2}$, law of motion for the aggregate net wealth fraction $\Gamma(W(s), s'; s)$, consumption allocation $\{c_N^i(w(s); W(s), s), c_T^i(w(s); W(s), s)\}_{i=1,2}$, prices $\{p_N^i(W(s), s), P_i(W(s), s), q_i(W(s), s)\}_{i=1,2}$, and asset holdings $\{a_i^{j\prime}(w(s); W(s), s)\}_{i,j=1,2}$ such that 1) Given the prices and the law of motion for the aggregate net wealth fraction, consumption allocation, asset holdings, and value functions solve each consumer's problem, and 2) Markets clear.

Numerical algorithm I provide a *global solution* of portfolio choice, which implies that equilibrium is known for the time periods with large shocks far from steady state as well. It is necessary to solve the model globally, especially to address a sudden and large drop of gross capital flows as a result of large negative shocks during the 2008-2009 financial crisis. In the Appendix, I also provide a first-order dynamics of portfolio choice following Devereux and Sutherland (2011). A closed-form solution of steady state and first-order dynamics are helpful to understand which parameters affect the portfolio decision and how. For more discussions in solution method of international portfolio, Rabitsch et al. (2015) compare global and local approximation methods, which are similar algorithms used in this chapter.

In order to solve the model globally, I use the time iteration algorithm by Kubler and

Schmedders (2003), which has been applied to other international portfolio choice models such as Stepanchuk and Tsyrennikov (2015) and Dou and Verdelhan (2015). The algorithm finds equilibrium policy functions starting from an initial guess, by solving a system of first order necessary conditions and Kuhn-Tucker conditions and updating guesses over iterations.

This equilibrium is 'ε-equilibrium', meaning that the policy functions are solved up to some given critical value $\varepsilon > 0$ accuracy (Kubler and Schmedders, 2003). Specifically, denote a set of endogenous variables at iteration k as $\Omega(k) = \{w(k), c_T^i(k), a_j^i(k), q_j(k), \xi_j^i(k)\},\ i, j = 1, 2$. Here, I have used the equilibrium condition that aggregate endogenous variables are same as individual ones. Also, ξ_j^i is a parameter that solves for Kuhn-Tucker conditions (Zangwill and Garcia, 1981) in bond j in country i. Also define all endogenous variables except for net position $w(k)$ as $\tilde{\Omega}(k) = \Omega(k)/w(k)$, since $w(k)$ is an endogenous state variable. Functions that are arguments of set $\tilde{\Omega}$ have net wealth fraction and exogenous states as their input ($f : \mathbb{R}^4 \to \mathbb{R}^1,\ \forall f \in \tilde{\Omega}$), which are suppressed in the following expression.

The algorithm proceeds as follows. First, set up the initial guesses of $\tilde{\Omega}(0)$ and grids for net position w. I set equi-spaced grids for net position with 251 points[5], and set steady state prices for $q_j(0)$. I start with zero bond positions for all bonds in all countries, $a_j^i(0) = 0$. By the budget constraint and non-tradable market clearing condition, initial guess of tradable consumption in country 1 is equal to net position w multiplied by the total tradable endowment in the world ($c_T^1(0) = w \cdot 2y_T$). Finally, Garcia-Zangwill parameters ξ_j^i are set to 0. Set $\tilde{\Omega}(0) = \tilde{\Omega}^o$, where $\tilde{\Omega}^o$ is a set of *old* policy functions that is updated in every iteration. Exogenous state variables are discretized to 3 points for each shock using the method by Tauchen (1986), and critical value is set to $\varepsilon = 1.0e - 5$.

Then, start the time iteration. For any iteration $k \geq 1$, given the previous iteration's

[5] I set the range of net position grid as [0.1,0.9].

guess as *future* endogenous policy functions and prices[6], solve a system of first-order conditions and Kuhn Tucker conditions at each grid point of (net position $\times$ non-tradable endowment 1 $\times$ non-tradable endowment 2 $\times$ tradable endowment). In Appendix, I specify a set of equations in more detail. I solve the system of equations at precision of $1.0e-6$, using modified Powell's non linear solver[7] (Powell, 1970). Using the solutions in iteration k, update functions $f^o \in \tilde{\Omega}^o$ as a convex combination of $f(k) \in \tilde{\Omega}(k)$ and f^o: $f^o = \delta f(k) + (1-\delta) f^o$. I use $\delta = 1$ in the baseline case.[8].

Algorithm stops when maximum absolute difference of policy, price, and Garcia-Zangwill parameters between k^{th} iteration and *old* function across all state grid is less than critical value ε, $\max_{(w,s)} ||f(k) - f^o|| < \varepsilon, \ \forall f \in \tilde{\Omega}$.

1.4 Quantitative Analysis

I calibrate the model to the United States and the European Union in the periods 2001Q1-2016Q2. I first inspect the mechanisms of the baseline and extended models. Then, I compare the model predictions of gross capital flows to the data, using the data of sectoral output series as an input of the model simulations. Using the simulated results, I provide a welfare analysis over the sample period and implications of the European Financial Transaction Tax.

[6]Here, I need to find the mapping of today's net wealth fraction $w(s_t)$ to the tomorrow's net wealth fraction $w(s_{t+1})$ for any future state s_{t+1} by finding a root in equation 1.29. Since the solution often lies off of the grid points, I use spline methods to interpolate policy functions across endogenous state grid of $w(s_{t+1})$. I used B-spline method by Habermann and Kindermann (2007).

[7]I use HYBRD1 in Minpack. At the beginning of iteration, there are some points that cannot be solved using the zero finding routine. Then, I impute the solution by linearly interpolating the neighboring points that was accurately solved.

[8]Later in the stochastic collateral case with equity price update, I slow down to $\delta = 1\%$.

1.4.1 Data

There are two main data sets that are used in the quantitative analysis. First is international capital flows, sourced from Bureau of Economic Analysis (BEA). I focus on the US capital flows against the rest of the world, from 2001Q1 to 2016Q2. I use a quarterly data series in order to study business cycles, and the sample period starts from the first quarter of 2001 in order to analyze the periods after the introduction of the Euro and the European Union, which is the largest financial transaction partner of the US.[9] In addition, the Euro is one of the main currencies traded in the international financial market together with the US dollar. This sample period includes the observations on the most volatile capital flows in the US record, where a fast increase in capital flows from 2001 to 2007 is followed by a dramatic collapse in years 2008-2009.

The second set of data is a sectoral output series of the US and the 28 European Union member countries, which is chosen to cover the most of European Union members. Following Stockman and Tesar (1995), I divide industries into a tradable and a non-tradable sector based on the industry classifications. The non tradable sector include services, utilities, and construction. All other private industries, such as manufacturing and agriculture, comprise the tradable sector.[10] This is a rough estimate of tradable and non-tradable sectors, as exports in the service industry are increasing over time (Loungani et al., 2017).

[9]For a technical reason, there is a break in US NIPA industry classifications starting from 2001Q1.

[10]More detailed industry classifications are the followings. For the U.S., non-tradable sectors include Utilities, Construction, Retail trade, Professional and business services, Educational services, health care, and social assistance, Arts, entertainment, recreation, accommodation, and food services, Other services, except government. Tradable sectors are all other private industries, which are Agriculture, Mining, Manufacturing, Wholesale trade, Transportation and warehousing, Information. Industries are based on the 2002 North American Industry Classification System (NAICS). For the EU 28, non-tradable sectors include Other service activities, Prof., scientif., techn. activ.; admin., support service activ., Construction, Public admin.; compulsory s.s.; education; human health, Distrib. trade, repairs; transp.; accommod., food serv. activ. Tradable sectors are Industry, including energy, Information and communication, Agriculture, forestry and fishing. Classifications are based on International Standard Industrial Classification of All Economic Activities, Rev.4 (ISIC Rev.4). Details of the data description is in Appendix.

Table 1.2: Parameters

Parameter	Description	Value	Source/Data
β	Discount factor	0.98	Steady state interest rate 2%
γ	Risk aversion	4.00	
σ	Elas. of subs. btw NT and TR	0.74	Mendoza (1995)
ω_N	Weight on non-tradable	0.58	$\frac{1}{T}\sum services/total\ cons.$
ρ_T	TR persistence	0.84	Estimated from the US NIPA
σ_T	TR std.dev	0.02	
ρ_N	NT persistence	0.84	
σ_N	NT std.dev	0.01	
χ	Borrowing limit	1.80	

Note: Sample period is 2001Q1 - 2016Q2. All series are in log and real. Data source for the US is NIPA section 1 and 6. For Europe, OECD. Europe is the EU28.

However, in order to construct a time series that is internationally comparable and spans enough the time period used in the analysis, I resort to the industry based classifications. For the baseline result, I calculate the real output of each sector as the sum of each sector's nominal output in the national currency deflated by a GDP deflator.[11] In this way, I get a series for the entire sample period, 2001Q1 to 2016Q2.

Calibration Two of the key parameters in the calibration are risk aversion (γ) and elasticity of substitution between non-tradable and tradable goods (σ). When risk aversion or elasticity of substitution is high enough so that non-tradable and tradable goods are gross substitutes ($\gamma\sigma > 1$), people want to compensate themselves with tradable goods whenever non-tradable consumption is low (also see Proposition 1). I set the elasticity of substitution, or *Armington elasticity*, to 0.74, following the calibration of Mendoza (1995) as discussed in the Section 1.2. Benchmark risk aversion (γ) is 4, which is a plausible value in macro literature.

[11] I am currently working on getting a precise measure of real output, accurately deflating with each sector's prices.

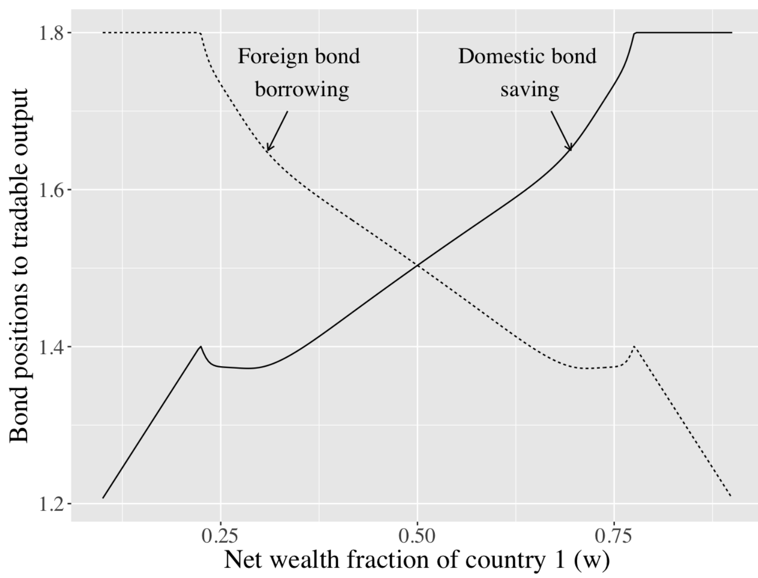
Other parameters are mostly estimated from the data. The weight on non-tradable in utility function (ω_N) is set to be the ratio of services to the total consumption averaged over the sample period for both the US and the EU28. Fraction of dividends to non-tradable consumption, which is used only in stochastic collateral model, is calculated from the fraction of rents (housing services) to total consumptions on services in the US over the sample period.

Parameters for the output series, including persistences and standard deviations of non-tradable and tradable output, are estimated based on the US series. All series are in log and detrended using the HP filter with a smoothing parameter of 1600. Non-tradable and tradable output parameters are estimated separately, assuming their independence for the brevity of analysis. Borrowing limits for the baseline model is set at a loose level, which is not binding for any of the simulation periods including the Great Recession.

1.4.2 Equilibrium portfolios and risk sharing mechanism

Based on this calibration, I analyze the model solution and describe the mechanism of non-tradable consumption risk sharing. In this model, households use cross-border financial transactions in order to insure themselves from adverse shocks to non-tradable consumption. In particular, if tradable and non-tradable goods are gross substitutes, then households want to smooth their overall consumption by consuming more tradable goods when their non-tradable consumption is low.

Bond policy functions I first inspect bond policy functions at the *zero-shock state*, where there are zero shocks for all the exogenous output in tradable and two non-tradable sectors. In equilibrium, consumers save in domestic bonds and borrow in foreign bonds. At the symmetric net wealth fraction of $w = 0.5$ and zero-shock state, consumers save and

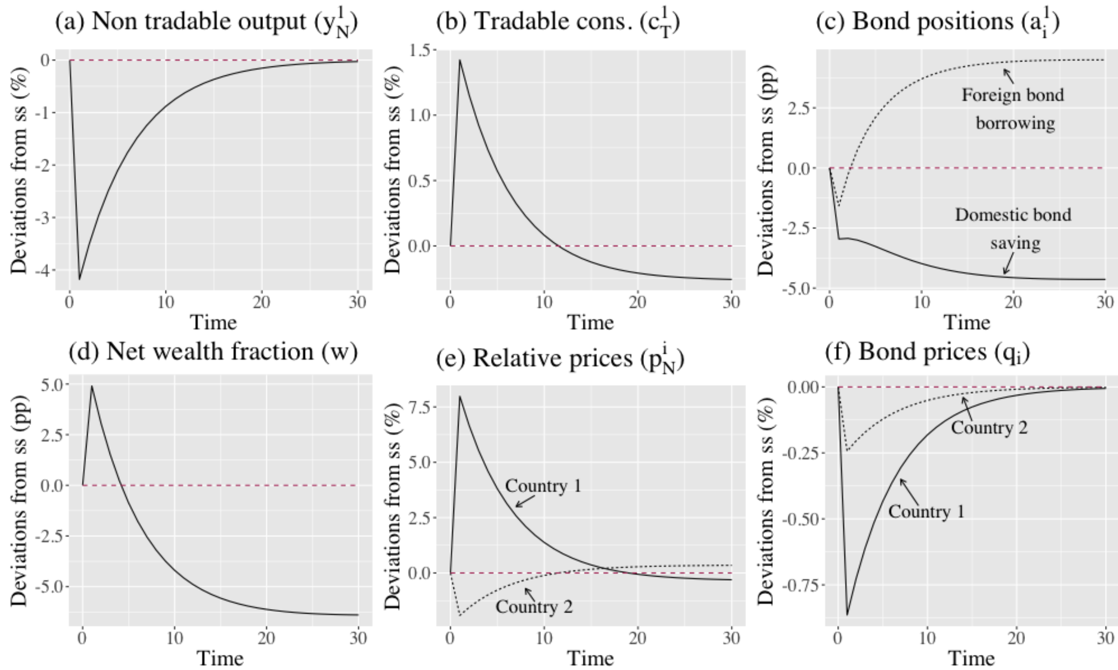


Note: Net wealth fraction of country 1 is defined as (tradable good output in country 1 + net wealth in country 1)/(world tradable good output). Solid line is policy function of domestic bond saving in country 1 (a_1^1), and dashed line is foreign bond borrowing in country 1 ($-a_2^1$).

Figure 1.5: Bond policy functions of country 1, zero-shock state

borrow same amount of bonds simultaneously, holding zero net positions. Notice that in equilibrium, if the two countries start at the symmetric wealth and the state of zero-shocks, then they remain at the same state until an exogenous shock hits. Henceforth, this is set as the initial state for the following simulations as well as impulse response functions.

Figure 1.5 depicts the policy functions for domestic and foreign bonds in country 1 at zero-shock state and net wealth fraction (w) 0.5. Country 1 households take a long position (saves) in domestic bonds for an amount of 150% of tradable goods and short (borrows) in foreign bonds for the same amount. This equilibrium portfolio implies that the country 2 households takes symmetric positions, which is long in their own domestic bonds (country 2 bonds) and short in the foreign bonds (country 1 bonds). Households in both countries take these bond positions because they provide a good hedge against adverse shocks to their

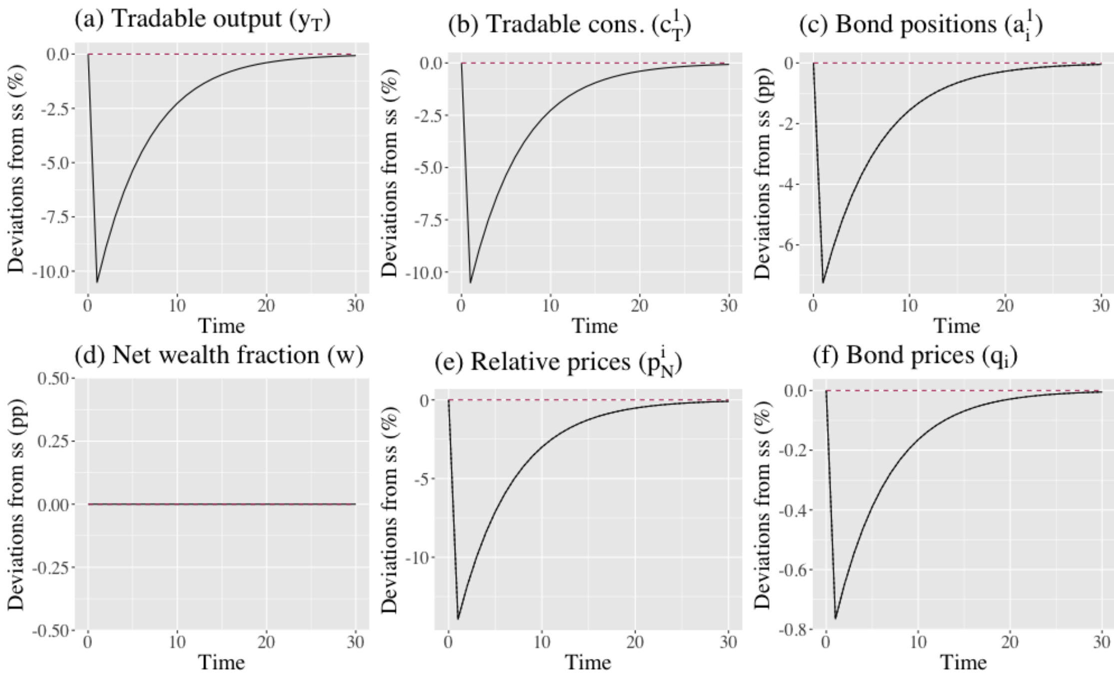


Note: Impuse response to the negative non-tradable shock in country 1. Shock is 3.6808 times of standard deviation innovation. It is the Low grid in non-tradable shock, when descritezed using Tauchen method. All graphs are percentage deviations from zero-shocks symmetric net wealth point, except for except for Fraction of net wealth and Bond positions, which are percentage point deviation from the initial point. Red dashed line is at 0 for all graphs.

Figure 1.6: Impulse response function, negative shock on country 1 NT output

non-tradable consumption. I elaborate on the mechanism with impulse response functions.

Impulse response functions It is easier to understand the risk sharing motive of portfolio choice with impulse response functions. Figure 1.6 shows responses of endogenous variables to a negative shock to the country 1 non-tradable output (y^1_N) in time 1, starting from zero-shocks symmetric wealth in time 0. All other exogenous shocks are set to be zero.



Note: Impulse response to the negative tradable shock, common to both countries. Shock is 4.9606 times of standard deviation innovation. It is the Low grid in tradable shock, when descritezed using Tauchen method. All graphs except for Fraction of net wealth and Bond positions are percentage deviations from zero-shocks symmetric net wealth point. Fraction of net wealth and Bond positions are percentage point deviation from the initial point. Red dashed line is at 0 for all graphs.

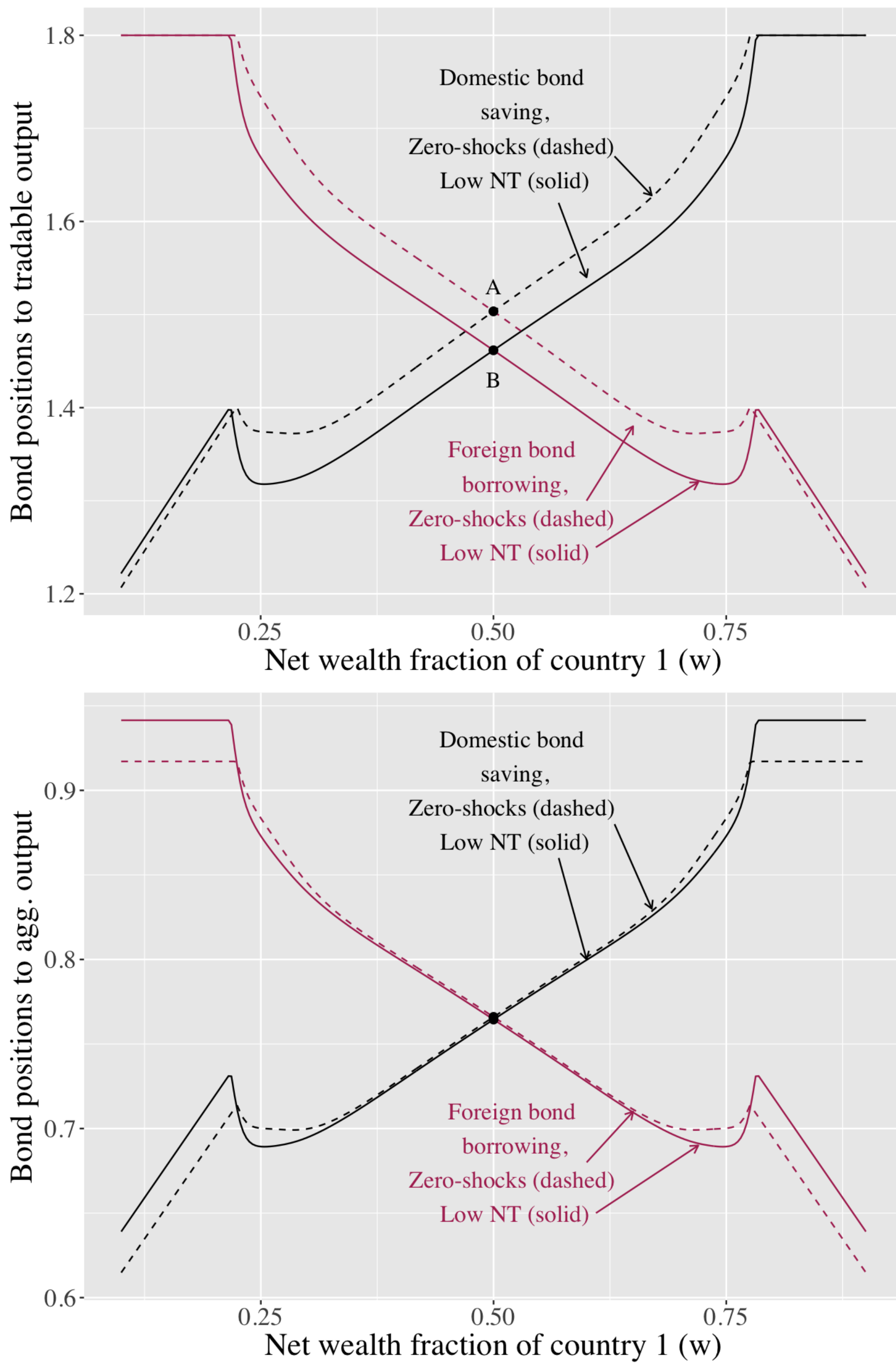
Figure 1.7: Impulse response function, negative shock on country 1 TR output

As Figure 1.6 shows, with a decrease in non-tradable output, non-tradable goods become more valuable than tradable goods in country 1 (panel (e)). Then, the value of aggregate consumption goods, which is a composite of non-tradable goods and tradable goods, also appreciates in units of tradable goods. This implies an appreciation of country 1 aggregate price index against country 2. These movements in the real exchange rates result in an increased net wealth fraction for country 1 consumers (panel (d)), because of the long positions in domestic bonds. Households import more tradable goods using their increased net wealth in order to smooth their overall consumption (panel (b)).

On the other hand, the net wealth of households in country 2 drops with a negative shock on country 1 non-tradable output because of the bond positions and the endogenous movements of real exchange rates. Since country 2 households have taken short positions (borrowed) in country 1 bonds, whose value appreciates with the negative shock, their net wealth decreases (panel (d)). Due to the reduced net wealth, tradable consumption in country 2 drops (panel (b)). Simultaneously, the relative value of non-tradable goods in country 2 decreases because they become more abundant compared to tradable goods, and this leads to a depreciation of aggregate consumption basket value in country 2 against country 1 (panel (e)). These general equilibrium effects further depreciate the aggregate prices in country 2, while transferring the net wealth to the country 1 households due to the bond positions.

Finally, bond prices drop for both countries, which is equivalent to increases in interest rates, as their demands for savings go down. I explain the movements of bond holdings (panel (f)) in more detail in the following paragraph.

In response to a negative shock to tradable sector output (y_T), which is common to both countries, the endogenous variables move symmetrically. As Figure 1.7 shows, tradable consumption decreases for both countries (panel (b)). Relative prices of non-tradable goods in both country 1 and 2 fall together because they become more abundant relative to tradable goods (panel (e)). Since the real exchange rates do not move, the net wealth fraction w remains at the initial level (panel (d)). In addition, bond prices fall simultaneously because the marginal utility of tradable goods increase (panel (e)). Households reduce their bond positions in both domestic savings and foreign borrowings (panel (c)) because there are less tradable goods that could be used to insure against the shocks to non-tradable consumption next period in expectation.



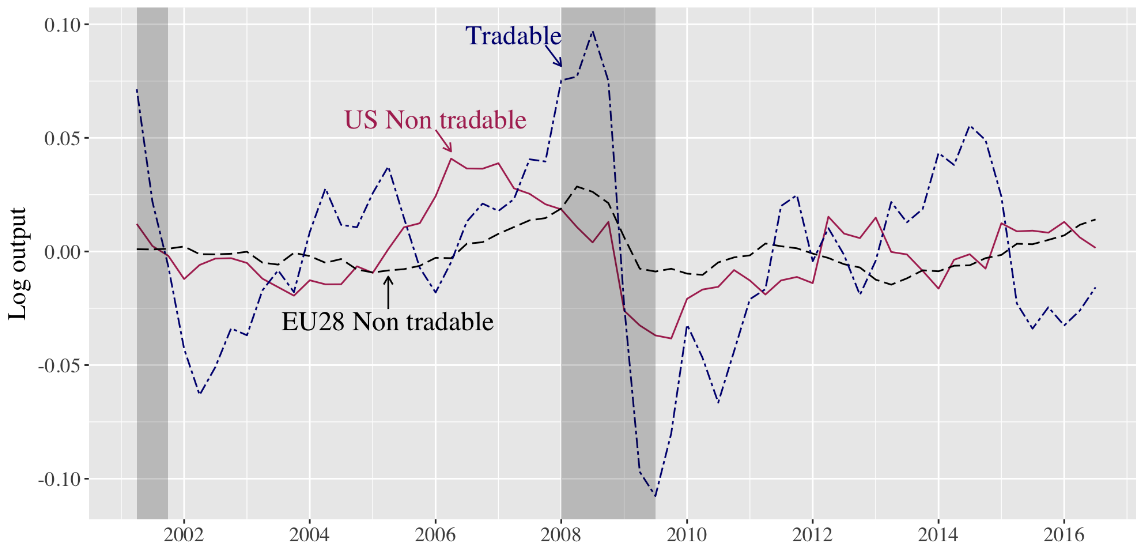
Note: Net wealth fraction of country 1 is defined as (tradable good output in country 1 + net wealth in country 1)/(world tradable good output). Solid line is policy function of low non-tradable state, where non-tradable endowment for both countries are low. Dashed line is for steady state. Black lines on upper right directions are domestic bond saving in country 1 (a_1^1), and maroon lines on bottom right directions are foreign bond borrowing in country 1 ($-a_2^1$).

Figure 1.8: Bond portfolio of country 1, zero-shocks (dashed lines) and low non-tradable goods in both countries (solid lines), in units of tradable output (top) and aggregate output (bottom).

Gross capital flows As the impulse response functions of shocks show, households decrease their holdings on both domestic and foreign bonds on the impact of negative output shocks. This results in *negative* gross capital flows, which account for the changes in both asset and liability positions. The reason for reduced bond holdings in response to a lower tradable output is rather straightforward. If the shocks are persistent, then households in both countries expect low tradable sector output next period, which they use in order to share their non-tradable consumption risk. With less tradable goods available, they hold less gross positions.

If there is an adverse shock to non-tradable sector output, then consumers adjust their bond holdings according to the changes in non-tradable consumption risk they face. The top panel of Figure 1.8 compares bond policy functions of country 1 in units of tradable goods, comparing two states. First state is the zero-shock state (dashed lines) and the other is a low non-tradable output state, by same amounts for both countries (solid lines). When there are low non-tradable goods for both countries, bond portfolio for domestic savings (solid black line) and foreign borrowings (solid maroon line) shift down compared to the zero-shocks state (dashed black and maroon lines). Starting from the zero-shock state with the symmetric net wealth, which is marked as point A, bond portfolio moves to the point B on impact, which is below the point A and at the net wealth fraction of 0.5. Since the adverse shocks are symmetric, there is no change in the distribution of net wealth across households.

In order to inspect the reasons for decreased bond holdings, I plot the same bond policy functions in units of the *aggregate* output instead of the tradable in the bottom panel of Figure 1.8. This plot shows that the bond positions in units of the aggregate output barely change across the two states. It implies that with lower non-tradable output, consumers reduce their bond positions for both domestic and foreign bonds because there is less



Note: All series are logged and detrended using Hodrick-Prescott filter of smoothing parameter 1600. Seasonally adjusted and deflated using GDP deflator of each country. Tradables are the average of the US and the EU28, where the EU28 tradable output is converted to dollars. Source: Bureau of Economic Analysis, OECD Stat, and author's calculations.

Figure 1.9: Tradable and non-tradable output series, US and EU28

non-tradable consumption risk to be shared.

1.4.3 US gross flows during the Great Recession

I test the model performance by comparing simulated gross flows to the data. In order to do so, I first extract business cycles from each non-tradable and tradable output series, by detrending them using HP-filter. Then, I feed the extracted series into the calibrated model, calculate gross flows from the model, and compare with the data. Based on the simulation results, I access the welfare implications of gross flows by comparing the tradable consumptions to the autarky state without any gross positions.

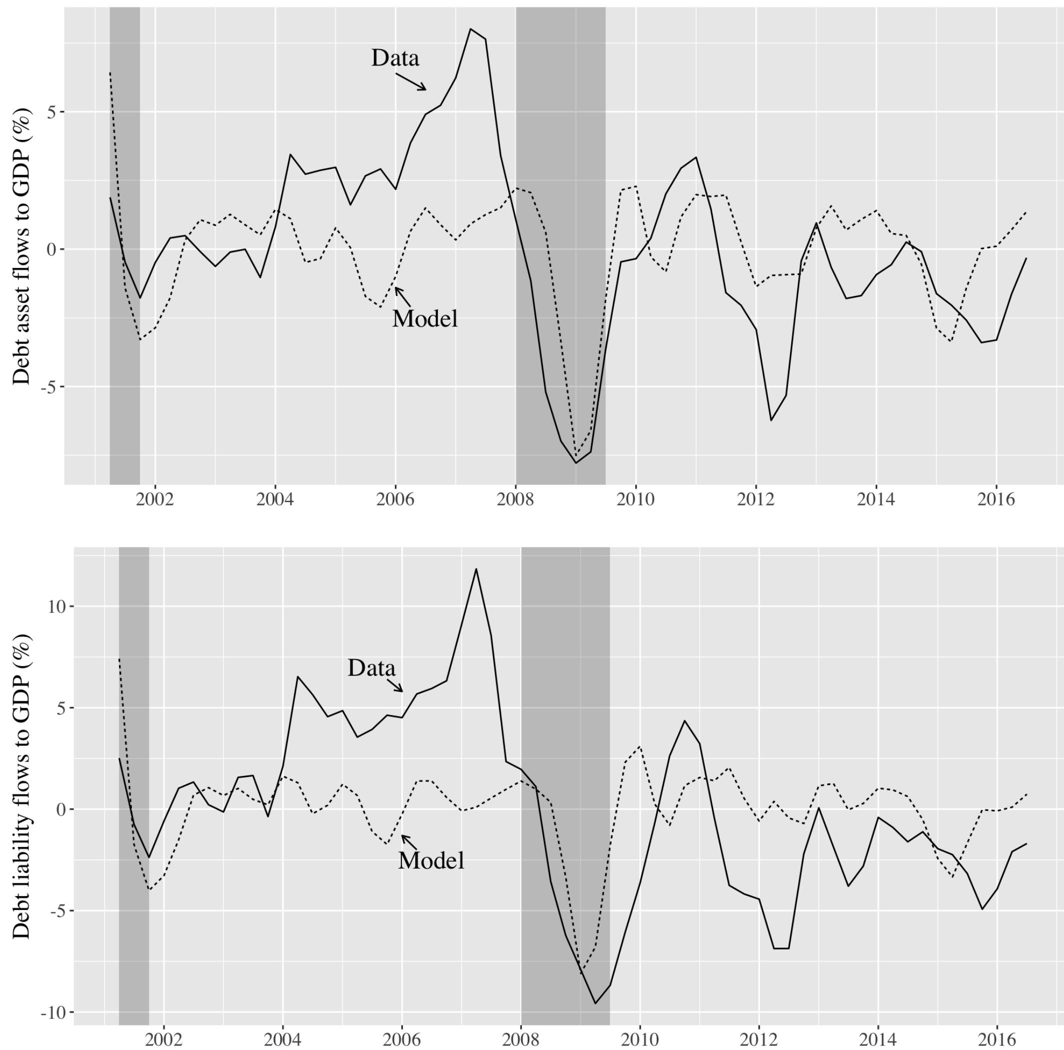
The output series that are used for simulations are in Figure 1.9. Simulations are over the sample period of 2001Q1 to 2016Q2, where 1999Q4 is set as the steady state. Figure 1.10 shows the comparison between demeaned capital flows data and model predictions.

Data is debt flows, which are composed of Portfolio debt investment and Other investment (mostly banking flows). As discussed in the Data section (Section 1.1), main focus of this chapter is on debt flows because they take most of the shares in the total gross flows. Since the symmetric model is not designed to address prolonged global imbalance, especially high level of liabilities in the US, I compare the result with demeaned data. Mean of the liability to GDP over the sample period is 4.2%, and average asset to GDP is 1.1%. Compared to the raw data, asset flows from model simulation is still close to the data, but liability flows have lower levels in the model compared to the model. Figures that plot model predictions and raw data is in Appendix.

The model explains most of the collapse in asset and liability flows to GDP for years 2008 and 2009. Especially for asset flows, model value is 7.48% at the trough of recession in 2008 Q4, which matches the data point of 7.78% closely. The model is able to account for reduced asset flows during the European debt crisis of 2012 as well as recent drops after year 2015. Liability flows perform in a similar fashion, even though the magnitude of model liability flows is smaller than the data.

Turning to the business cycle statistics, the model captures most of the key statistics during the Great Recession. In Table 1.3, I compare four key statistics of the gross flows in the data and model during the same periods. Gross flows are pro-cyclical both in the data and the model, and displays high correlation of asset and liability flows. Volatility of gross and net flows are lower in the model than in the data.

Welfare analysis Using the calibrated model, I access benefits as well as costs of the international investment positions. In the view of consumers in the United States, they



Note: Both data and model are 3-period moving average, with weights [0.25 0.5 0.25] for period [t-1,t,t+1]. Data is demeaned ratio of the US debt (Portfolio debt + Other investment) asset and liability flows over GDP, at quarterly level, seasonally adjusted. Shaded areas are NBER recessions. Source: Bureau of Economic Analysis and author's calculations.

Figure 1.10: Simulation result: baseline model and demeaned data, asset (top) and liability (bottom) flows

Table 1.3: Business cycle statistics in the benchmark model

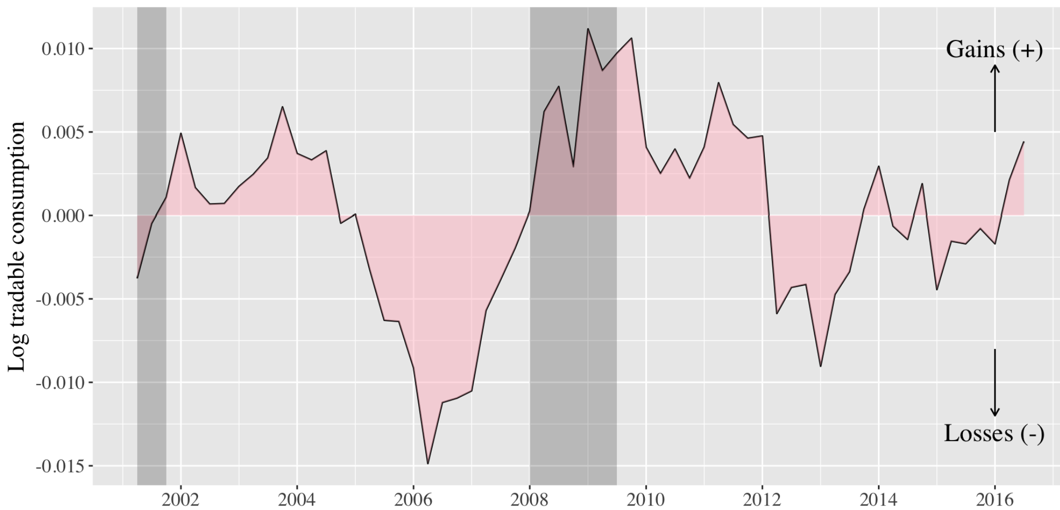
	Data	Model
Corr(*Gross*, *GDP cycle*)	0.33	0.26
Corr(*Asset*,*Liab*)	0.90	0.94
Std(*Gross*/*GDP*)/Std(*Net*/*GDP*)	3.60	5.74
Std(*Gross*/*GDP*)	0.09	0.05

Note: Sample period is from 2001Q2 to 2016Q2. All data is the US. *GDP cycle* of data is log HP-filtered cycles with smoothing parameter 1600, deflated with GDP deflator. *Asset* and *Liab* are debt asset flows and debt liability flows, deflated with GDP deflator. *Gross* is debt gross flows, sum of debt asset flows and debt liability flows. *Net* is net flows, liability flows minus asset flows. Model is a simulation result of the same sample period. Source: Bureau of Economic Analysis, author's calculations.

gained from their asset positions during the Great Recession, while losing their consumption during the economic boom in 2005-2007 and the European crisis in 2012-2013. Figure 1.11 depicts the tradable consumption gains (positive) and losses (negative) of the US against the EU28 based on the baseline model. At the height of the recession, the US tradable goods consumption increased by 1% due to the international investment positions. When the US non-tradable output was higher than Europe, notably on the eve of financial crisis and the European debt crisis, the US households exported their tradable goods to European countries.

In order to see whether there were net gains for the US consumers after netting out those consumption losses for the sample period, I calculate the present value of consumptions. As a thought experiment, I assume that the US consumers had a perfect foresight on the shocks to come. If the present value of consumption is higher than that of autarky, consumers still benefit from the international investment positions. In units of permanent consumption[12], the US consumers *gained* 0.013% of aggregate consumption. On the other hand, households in the EU28 *lost* 0.011% of their permanent consumption during the same

[12] I measure it in a conventional way. For a given consumption stream $\{c_1(s^t)\}$, I find a constant consumption level $\bar{c}_1$ such that $u(\bar{c}_1)/(1-\beta) = E_0[\beta^t u(c_1(s^t))]$. I compare the consumption certainty equivalences across different consumption streams in order to measure welfare gains or losses.



Note: Y-axis is log tradable consumption in the model minus log tradable endowment in the United States, simulated for the sample period 2001Q1 to 2016Q2.

Figure 1.11: Consumption in the baseline model compared to autarky in the US

period. In expectation, however, both consumers gain from the open financial transactions by 0.007% of permanent aggregate consumption compared to the autarky. These welfare gains seem small, but it is coming from the nature of business cycle models with low risk aversion.

1.4.4 European Financial Transaction Tax (FTT)

On 28 September 2011, the European Commission proposed a financial transaction tax that applies to financial transactions of which at least one party is located in any of 27 European Union member states. It levies 0.1% tax on securities trading, and 0.01% on derivative contracts. According to the press release by the European Commission[13],

[13] "Financial Transaction Tax: Making the financial sector pay its fair share". http://europa.eu/rapid/press-release_IP-11-1085_en.htm?locale=en

purpose of the Financial Transaction Tax (FTT) is to collect revenues from the financial sectors who got massive bailouts funded by taxpayers. They expect €57 billion of revenue every year.

Currently, implementation of the FTT is on hold and the meeting of 10 European Union finance ministers has been postponed to the end of 2017 at the earliest[14]. One of the reasons for this delay is that it is hard to reach an unanimous agreement on the benefits of the transaction tax. In the following, I argue that the proposed financial transaction tax will lower the international financial transactions close to zero, resulting in little revenues and hurting international diversifications.

In the model, I apply 0.1% tax on the transactions of foreign bonds in order to evaluate the effects of FTT. Formally, consumer's budget constraint is now as follows.

$$\begin{aligned} c_T^1(s^t) + p_N^1(s^t)c_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)q_i(s^t)a_i^1(s^t) + \tau P_2(s^t)q_2(s^t)|a_2^1(s^t)| \\ \leq y_T(s^t) + p_N^1(s^t)y_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)a_i^1(s^{t-1}) + T_1(s^t) \end{aligned} \tag{1.35}$$

where τ is the financial transaction tax, which later is set to 0.1% for quantitative analysis. $P_2(s^t)q_2(s^t)|a_2^1(s^t)|$ is the absolute value of the newly purchased foreign bonds in country 1, which becomes the tax base. Tax revenues are equally distributed among the domestic consumers in the form of lump sum transfer, $T_1(s^t)$. Government runs a balanced budget every period, which satisfies the following equation.

$$T_1(s^t) = \tau P_2(s^t)q_2(s^t)|a_2^1(s^t)|$$

Therefore, in equilibrium, taxes only distort the price of foreign bonds, not the net wealth

[14] https://www.bna.com/eu-financial-transactions-n73014461368/, Kirwin (2017)

of consumers. Country 2 imposes an analogous transaction tax on the transactions of foreign bonds $(\tau P_1(s^t)q_1(s^t)|a_1^2(s^t)|)$, making it a bilateral taxation as the FTT proposal suggests.[15]

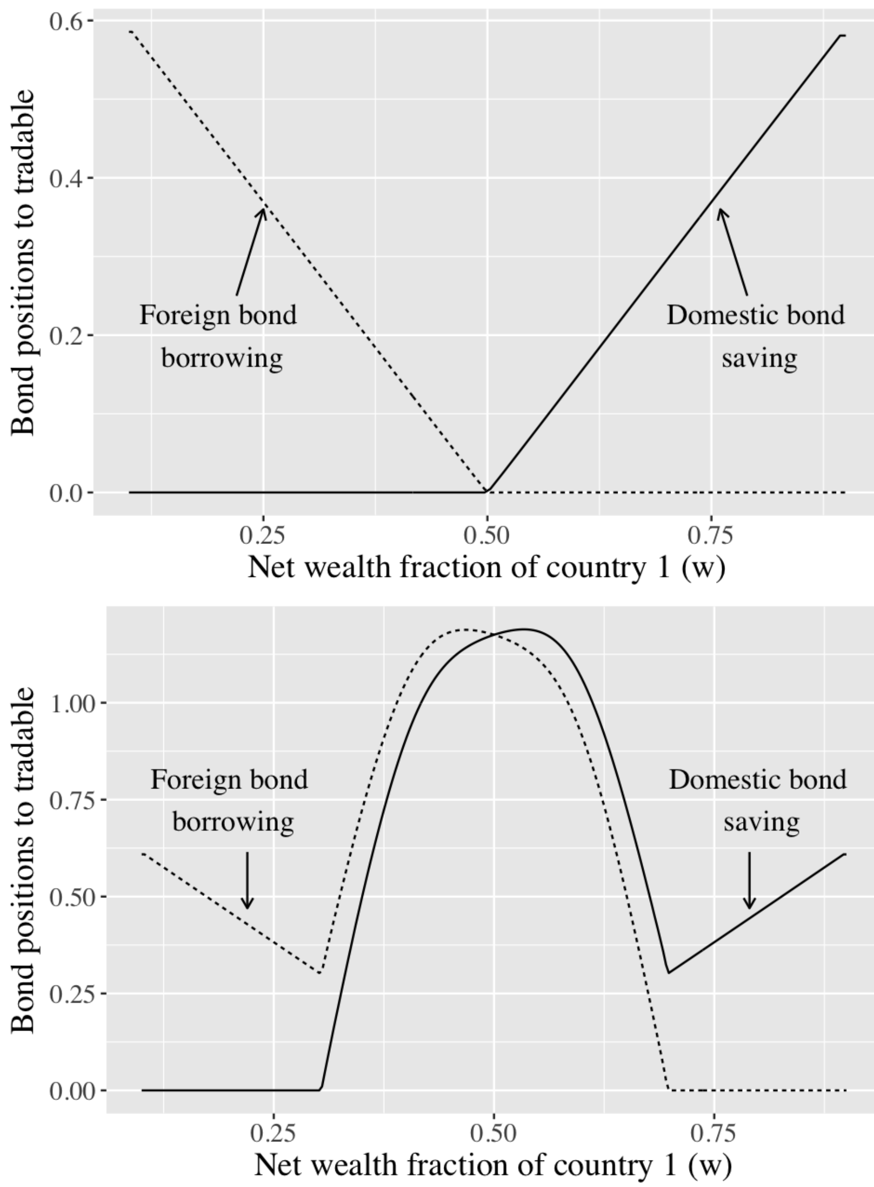
Figure 1.12 shows the bond positions of country 1 with 0.1% financial transaction tax. All the other model parameters are kept the same as in the baseline model, Table 2.1. Compared to the bond positions without the transaction tax, which is 58% of GDP for both domestic long positions and foreign short positions (Figure 1.5), bond portfolios are nearly zero at the steady state when the tax is imposed.

The model solution shows that 0.1% FTT is large enough to completely erase the hedging positions when countries are symmetric. Without any gross positions, net wealth fraction of two countries will always remain at 0.5, same as in the autarky case. Since the positions are zero, there will be no revenues from the financial transaction taxes to be redistributed. Considering that countries lose the benefits of non-tradable consumption risk hedging, FTT results in welfare loss for both countries compared to the economy without transaction taxes.

Other studies also have suggested that the benefits of financial transaction taxes are dubious. Pomeranets (2012) empirically shows that the trading volume decreases with the transaction taxes, and the volatility may increase as well. She also casts doubt on the projected revenue collection by the European Commission, considering the effect of substitution and migration. My analysis is in line with her empirical evidence and add the dimension of international diversification, which supports the argument that benefits of FTT is unclear.

In conclusion, Financial Transaction Tax proposed by the European Commission (EC)

[15] "Commission proposes green light for enhanced cooperation on financial transactions tax". http://europa.eu/rapid/press-release_IP-12-1138_en.htm



Note: Net wealth fraction of country 1 is defined as (tradable good output in country 1 + net wealth in country 1)/(world tradable good output). Solid line is policy function of domestic bond saving in country 1 (a_1^1), and dashed line is foreign bond borrowing in country 1 ($-a_2^1$).

Figure 1.12: Bond portfolio choice of country 1 with 0.1% (top) and 0.001% (bottom) Financial Transaction Tax, steady state

may result in low revenues as the financial transaction volumes decrease as a response to the tax. Reduced amount of financial transactions implies that countries cannot benefit from the international non-tradable consumption risk sharing with hedging positions, which require large transaction volumes. Other ways of financial regulations or taxations, such as capital ratio regulations, might be more effective in achieving the intended goal of the EC without compromising the benefits of international diversification.

1.5 Conclusion

In this chapter, I document the sudden collapse of the gross capital flows during the Great Recession, focusing on the developed economies. Gross flows are the sum of changes in international liability positions (liability flows) and changes in asset positions (asset flows) due to cross-border transactions. The observed fluctuations of the gross flows are puzzling in the view of the classic models of capital flows, whose primary focus have been on the net capital flows. I propose an open economy model of portfolio choice with two bonds and non-tradable sectors that aims to answer what drives the gross flows.

The main hypothesis is *hedging motive*, where households hold international investment positions in order to insure themselves against the non-tradable consumption risks. Risks on non-tradable consumption is especially large for the households in developed economies, because share on services, which are typical non-tradable goods, account for on average 60% of their consumption. If the non-tradable sector outputs are not perfectly correlated, consumers can benefit from international diversification by holding cross-border investment positions. If tradable goods and non-tradable goods are gross substitutes, households would want to compensate themselves by consuming more tradable goods. The basic intuition in this chapter is that gross positions and flows support the allocation where consumers compensate themselves when adverse shocks hit non-tradable consumption.

When there are two bonds in the economy, denominated in each country's aggregate price index, equilibrium portfolio takes long position (save) in the domestic bonds and short (borrow) on foreign bonds. It is because real exchange rate movements make this portfolio a good hedge against the non-tradable consumption risks. When the domestic non-tradable consumption is low, then relative prices of non-tradable to tradable goods, as well as domestic aggregate price index, increase. This makes domestic currency appreciate against the foreign currency, making domestic bonds more valuable then the foreign ones. Therefore, consumers can withdraw their savings in domestic bonds and consume more tradable goods, insulating themselves from the negative non-tradable consumption shock.

Applying the model to the case of Great Recession between the US and Europe, I find that gross positions and flows mitigated a part of of consumption drop in the US during the financial crisis. I calibrate my model to the US and the EU28 from 2001Q1 to 2016Q2 and show that the model captures key business cycle moments of the gross flows, which are high correlation of asset and liability flows, pro-cyclicality of gross flows, and more volatile gross flows compared to net flows. In the long run, both countries benefit from the international financial transactions. Another application of the model is to ongoing debates on the European Financial Transaction Tax, where the European Commission proposed 0.1% tax on security transactions. I argue that in the lens of international diversification, this tax reduces welfare by reducing the benefits of consumption hedging across countries.

Throughout the analysis, I have simplified the model to symmetric countries. However, it is known that there are persistent differences across the countries in terms of financial developments or returns on financial assets, which also drive international capital flows. The observed data is an equilibrium where both developing countries, who are less financially developed, and developed countries trade assets together. In this chapter, I focus on international diversifications among developed countries, who are believed to be in a more similar environments in financial markets, and complement the existing studies on international portfolio choice and capital flows.

Chapter 2

Gross Capital Flows and Financial Frictions

In Chapter 1, I propose a model of gross capital flows with international risk sharing as the key mechanism. In this model, my main hypothesis is that households have a risk sharing motive that leads them to hold international investment positions in order to share their non-tradable consumption risk. I set up a model economy where shocks to non-tradable and tradable output drive the movements of gross capital flows over time.

While the mechanism of risk sharing based on real output fluctuations captures the key characteristics of international financial transactions, it does not fully explain the volatility of gross flows during the Great Recession. The main reason is that the rise of real output in 2007 is not sufficient to account for the surge in gross flows observed in the data.

Motivated by the fact that the movements in gross flows coincided with loosening and tightening of credit conditions, I extend the model of Chapter 1 so that short positions of households are now subject to collateral constraints. In this chapter, I show that adding credit market conditions which follow an exogenous stochastic process amplifies the simulated volatility of financial transactions across the border.

In the following, I introduce an extended model of international risk sharing with collateralizable assets such as *real estate*, which yield a part of non-tradable consumption goods every period, and collateral constraints in bond holdings. After building the model, I calibrate it to the European Union and the US as in Chapter 1 and compare the model performance to data capital flows and simulated results in Chapter 1.

In the quantitative analysis, I notice that asset values in the model do not necessarily move in the same direction as prices of non-tradable goods. When the non-tradable sector is hit by an adverse shock, the asset value (e.g. real estate price) drops while the value of non-tradable goods (e.g. rent price) relative to tradable goods goes up, which is consistent with the US data.

2.1 The Model

In this section, I extend the baseline model in Chapter 1 by adding a stochastic collateral constraint. Assumption of full commitment in bond contract by consumers is relaxed, and the total borrowing amount needs to be backed by collaterals.

I introduce non-tradable assets that yield a fraction of non-tradable consumption goods in each country. More specifically, I model *land* in the spirit of Kiyotaki and Moore (1997). Land in each country does not depreciate and drops stochastic fruits (non-tradable consumption goods), as trees in Lucas (1978). Land is not internationally traded, but used as collaterals to borrow in bonds.

Individual consumers are subject to collateral constraint, which limits total borrowings up to a fraction of land values. Collateral constraint arises because households cannot commit to repay the debt fully in the next period, which contrasts to the baseline model.

At the end of each period, after portfolio choice for the next period is made and before the repayment of bonds, borrowers decide whether to default or not. If a borrower defaults, then creditors have rights to seize the land that the borrower owns. Here, creditor cannot seize other savings that the borrower made, but can only hold the land. Assume that at the time of liquidation, land could be converted into tradable goods with some chance of success, in the spirit of Jermann and Quadrini (2012). With probability χ, the value of land is equal to the market value of land when converted into tradable goods. With probability $1-\chi$, the land does not have any value in tradable goods. In the Appendix, I describe the renegotiation procedure in more detail once the borrower defaults. Based on the expected surplus of renegotiation, collateral constraint is given such that borrowing cannot exceed more than χ fraction of land value.

Collateral constraints are subject to exogenous shocks. For simplicity, I assume that both countries' collateral constraint parameter χ follow the common process. When there is a positive shock on collateral constraint, value of land increases due to the increase in collateral value, and this further relaxes the borrowing constraint. There is a large literature that studies macroeconomic effects of credit shocks, such as Gertler et al. (2010), Guerrieri and Lorenzoni (2017), Liu et al. (2013), Khan and Thomas (2013), to name a few. In the international context, Perri and Quadrini (2011) endogenize financial frictions as a self-fulfilling pessimism to explain international synchronization during the 2008 crisis.

Fluctuations of collateral constraints amplify gross capital flow volatility. When credit constraint is relaxed, it has larger increase in gross flows than a typical positive shock on non-tradable endowment. Later in the quantitative analysis, I show that relaxed credit constraint in year 2007 before the financial crisis explains most of the boom in gross flows. When the credit constraint is tightened during the Great Recession, gross flows have sharper decline compared to the baseline model without collateral constraints.

In the following, I specify the model setup for stochastic collateral constraints, focusing on differences from the baseline model.

Physical environment The goods environment is same as in the baseline model, with two non-tradable and a common tradable endowments that follow AR1 processes. Shocks on collateral constraint are added to the baseline model. Collateral constraints of both countries, denoted as χ, follow two-state Markov process, where collateral constraint (χ) takes either High (χ_H) or Low (χ_L) values, $\chi_H > \chi_L$. Here, High implies loose collateral constraint, and Low is tight constraint. Markov switching probability is denoted as π, for example from High to Low is $Prob(\chi_L|\chi_H) = \pi(L|H)$.

Consumer utility is the same as in the baseline model. Households are risk averse to the aggregate consumption basket, which is a composite of tradable and non-tradable goods with a constant elasticity of substitution between the two. I omit a formal description of consumer utility and refer readers to the previous Baseline model section.

Consumer budget constraint Consumers purchase tradable and non-tradable goods every period, and make portfolio decisions. In addition to the two bonds, foreign and domestic, households buy shares of land (θ) that pays in non-tradable goods as dividends (d) every period. I assume that dividends are a fraction δ of total non-tradable endowments. Compared to the baseline model, consumers now have δ fraction of collateralizable non-tradable endowment, which is given as dividends from holding lands. Total amount of non-tradable consumption does not change from the baseline model in equilibrium, since each consumers hold the entire domestic land.

Consumer budget constraint at history node s^t is given by

$$
\begin{aligned}
&c_T^1(s^t) + p_N^1(s^t)c_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)q_i(s^t)a_i^1(s^t) + p_N^1(s^t)Q_1(s^t)\theta_1(s^t) \\
&\quad \le y_T(s^t) + p_N^1(s^t)\tilde{y}_N^1(s^t) + \sum_{i=1}^{2} P_i(s^t)a_i^1(s^{t-1}) + p_N^1(s^t)(Q_1(s^t) + d(s^t))\theta_1(s^{t-1}) \qquad (2.1)
\end{aligned}
$$

Here, q_i is a price of bond i in units of aggregate price index in country i. ${a_i^1}'$ is purchase of bond (borrowing) in country i by country 1 consumer, which promises one unit of payment tomorrow. Q_1 is a price of land, and ${\theta_1}'$ is a share of land purchased today. Analogously, a_i^1 is the amount of bond or borrowing from previous period and θ_1 is share of land purchased yesterday. $\tilde{y}_N^1(s) = (1-\delta)y_N^1(s)$ is the non-collateralizable fraction of non -tradable endowment $y_N^1(s)$. $d(s) = \delta y_N^1(s)$ is the dividend from land holdings, which is the collateralizable part of non-tradable endowment.

Here, atomistic consumers purchase share of lands from each other. However, notice that in equilibrium, since all consumers are identical and lands are not internationally tradable, land shares are always equal to 1 across all states. The main role of land in the model is to be used as collaterals, affecting the level of borrowings depending on the fluctuations of land prices and collateral constraint that I specify in the following.

Collateral constraint Total borrowings of an individual cannot exceed $\chi(s^t)$ fraction of the land value at any node s^t.

$$
\sum_{{a_i^1}'(s^t)<0} \underbrace{P_i(s^t)q_i(s^t)a_i^1(s^t)}_{\text{Bond i purchase by country 1}} \ge -\chi(s^t) \underbrace{Q_1(s^t)\theta_1(s^{t-1})}_{\text{Value of land in country 1}} \qquad (2.2)
$$

Notice that collateral constraint is relevant only to the bonds that a person has negative position. This is based on the assumption that when a borrower defaults on bond i, the creditor can only seize land but not the other savings of the borrower.

Market clearing Non tradable consumption must be equal to dividend payments by land in each period (d) and non-collateralizable endowment of non-tradable goods. Land shares across individuals should sum up to 1. Other market clearing conditions are same as in the Baseline model.

$$c_N^1(s^t) = d_1(s^t) + \tilde{y}_N^1(s^t) \tag{2.3}$$

$$c_N^2(s^t) = d_2(s^t) + \tilde{y}_N^2(s^t) \tag{2.4}$$

$$\sum_{j=1}^{2} c_T^j(s^t) = 2y_T(s^t) \tag{2.5}$$

Bonds have zero net supply in each period.

$$\sum_{j=1}^{2} a_1^j(s^t) = 0,\ \sum_{j=1}^{2} a_2^j(s^t) = 0 \tag{2.6}$$

$$\theta_1(s^t) = 1,\ \theta_2(s^t) = 1 \tag{2.7}$$

Consumer's problem Consumer's problem is analogous to the baseline model, except that now people are subject to collateral constraint instead of the constant borrowing limits. Also, for each individual, share of land holdings is another endogenous state variable in addition to the fraction of net wealth w. However, notice that in equilibrium, $\theta(s) \equiv 1$ in every states because all consumers are homogeneous and land is not internationally tradable. Therefore, in equilibrium, consumer's problem has in effect single endogenous variable w as in the baseline model.

In recursive form, consumer's problem is as follows.

$$V_1(w(s), \theta_1; W(s), s) = \max_{c_T^1, c_N^1, a_1^{1\prime}, a_2^{1\prime}, \theta_1'} u(c_T^1, c_N^1) + \beta \sum_{s'} \pi(s'|s) V_1(w(s'), \theta_1'; W(s'), s') \tag{2.8}$$

$$s.t.\ c_T^1 + p_N^1(W(s), s)c_N^1 + \sum_{i=1}^{2} P_i(W(s), s)q_i(W(s), s)a_i^{1\prime} + p_N^1(W(s), s)Q_1(W(s), s){\theta_1}' \tag{2.9}$$

$$\leq p_N^1(W(s), s)\tilde{y}_N^1(s) + w(s) \cdot 2y_T(s) + p_N^1(W(s), s)(Q_1(W(s), s) + d(s))\theta_1$$

$$\sum_{a_i^{1\prime}(s)<0} P_i(W(s), s)q_i(W(s), s)a_i^{1\prime}(s) \geq -\chi(s)Q_1(W(s), s)\theta_1(s_{-1}) \tag{2.10}$$

$$W(s') = \Gamma(W(s), s'; s),\ \forall s' \in S \tag{2.11}$$

$$w(s') = \frac{y_T(s') + \sum_{i=1}^{2} P_i(W(s'), s')a_i^{1\prime}}{2y_T(s')} \tag{2.12}$$

where $w(s)$ is individual in country 1 net financial wealth, normalized by total tradable endowment of the world at state s. $W(s)$ is the sum of individual net wealth in country 1.

2.2 Quantitative Analysis

I calibrate the model to the United States and the European Union in the periods 2001Q1-2016Q2. I first inspect the mechanisms of the extended model. Then, I compare the model predictions of gross capital flows to the data, using the data of sectoral output series as an input of the model simulations. Table 2.1 displays the parameters used in the calibration.

Table 2.1: Parameters

Parameter	Description	Value	Source/Data
β	Discount factor	0.98	Steady state interest rate 2%
γ	Risk aversion	4.00	
σ	Elas. of subs. btw NT and TR	0.74	Mendoza (1995)
ω_N	Weight on non-tradable	0.58	$\frac{1}{T}\sum services/total\ cons.$
ρ_T	TR persistence	0.84	Estimated from the US NIPA
σ_T	TR std.dev	0.02	
ρ_N	NT persistence	0.84	
σ_N	NT std.dev	0.01	
Collateral constraint			
δ	Fraction of dividends to NT cons.	0.28	$\frac{1}{T}\sum rents/total\ services$
χ_H	Collateral constraint, High (loose)	0.10	Baseline borrowing limit at zero shocks
χ_L	Collateral constraint, Low (tight)	0.07	Peak of gross flows in 2007Q1
$\pi(H\|H)$	χ_H persistence, High (loose)	0.75	Lasting for 4 quarters
$\pi(L\|L)$	χ_L persistence, Low (tight)	0.96	Lasting for 28 quarters

Note: Sample period is 2001Q1 - 2016Q2. All series are in log and real. Data source for the US is NIPA section 1 and 6. For Europe, OECD. Europe is the EU28.

2.2.1 Data

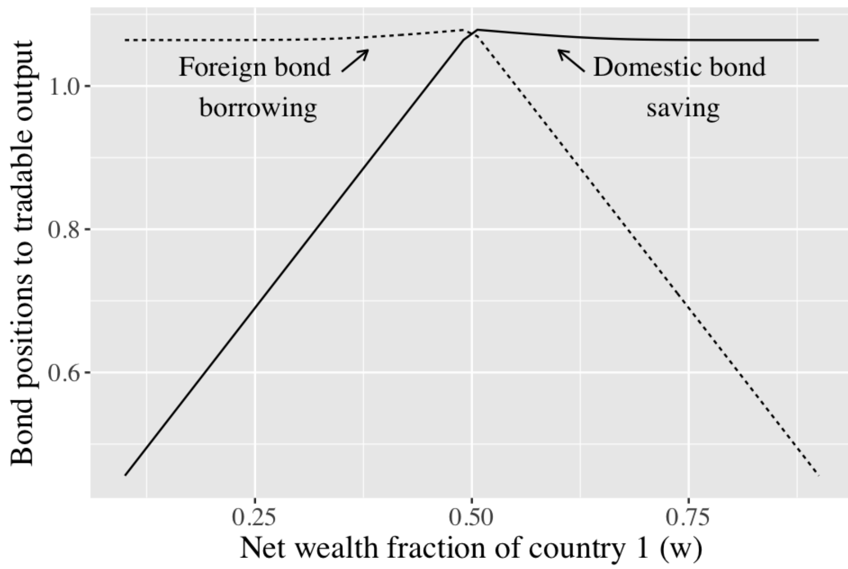
Compared to the baseline model in Chapter 1, four parameters are added to the original calibration. The High (loose) collateral constraint (χ_H) is set to the level same as in the baseline model. The Low (tight) collateral constraint (χ_L) is calibrated to match the peak of gross flows in 2007Q1 when it moves from Low to High in the simulation. Constraints are always set to be tight except for the periods 2007Q1 - 2007Q4, in which gross flows increased rapidly until their collapse in 2008. The data sets used for sectoral output and international capital flows are the same as in Chapter 1.

2.2.2 Gross flows and stochastic collateral constraints

In the extended model with collateral constraints, the risk sharing motive remains as the key driver, but households face restrictions on their ability to insure against their non-tradable consumption risk. As in the baseline model, households in each country save in their domestic bonds and borrow in foreign bonds at the same time. When the collateral constraint binds, consumers hold less gross positions than they would do without binding constraints. For example, if an adverse shock hits the non-tradable sector in country 1, then households increase their consumption on tradable goods less than they do in the baseline model.

In the following, I explain the risk sharing mechanism when the collateral constraints are binding. I contrast the amount of insurance that the households attain to that of the baseline model, using impulse response functions with respect to a negative shock to non-tradable output in country 1. Finally, I discuss the movements of land prices and its interaction with collateral constraint. When there is a negative shock on non-tradable goods, it dampens land prices, further tightening collateral constraints when the marginal utility is high, aggravating the consumption smoothing.

Bond policy functions with tight collateral constraints If collateral constraints are tight, there are zero shocks on outputs for all countries, and the households have symmetric net wealth, which is at the net wealth fraction (w) of 0.5, then every households save in their domestic bonds and borrow in foreign bonds at the same time. Figure 2.1 shows bond policy functions of country 1 households, who hold 107% of tradable output in domestic bonds and short the same amount in foreign bonds. These positions are lower than the baseline, which are 150% of tradable output in both long and short positions. In contrast to the bond policy functions without binding collateral constraint (Figure 1.5), households

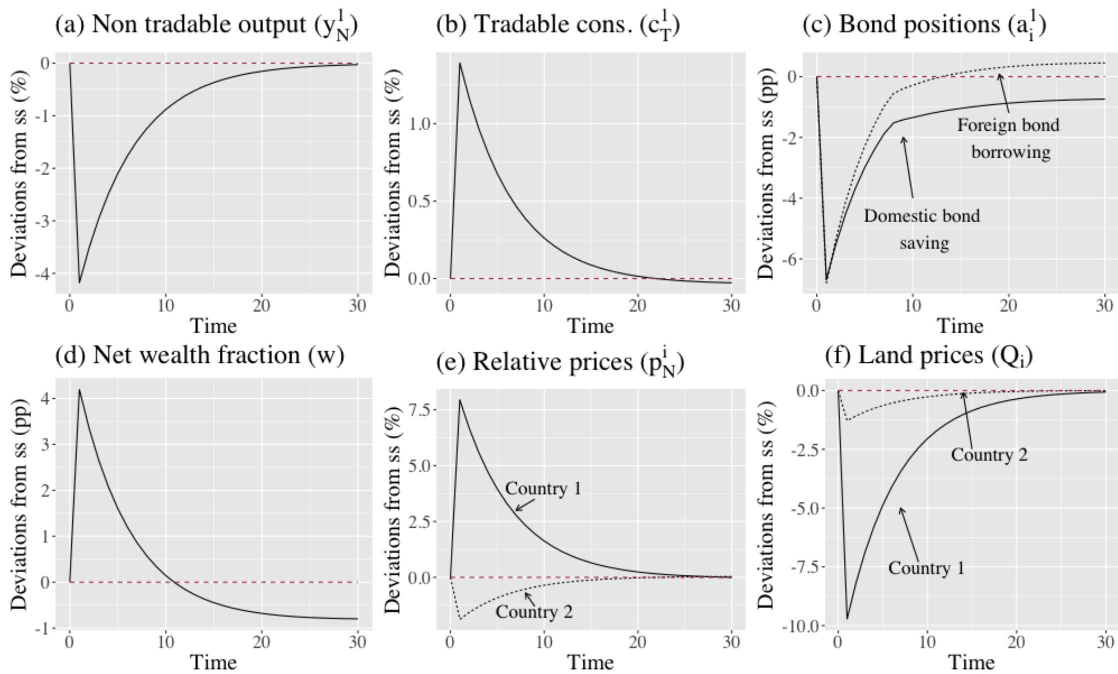


Note: Net wealth fraction of country 1 is defined as (tradable good output in country 1 + net wealth in country 1)/(world tradable good output). Solid line is policy function of domestic bond saving in country 1 (a_1^1), and dashed line is foreign bond borrowing in country 1 ($-a_2^1$).

Figure 2.1: Bond policy functions of country 1, tight collateral constraints and zero output shocks

cannot increase their foreign bond borrowings when the net wealth fraction falls below the symmetric point of 0.5 due to the binding collateral constraints.

Lower level of gross positions imply smaller amount of international diversification. Figure 2.2 plots impulse response functions with respect to a negative non-tradable endowment shock in country 1, which show changes in the degree of risk sharing compared to Figure 1.6. The logic of risk sharing holds in the same way as in the baseline model. Households hold long positions in domestic bonds, whose value appreciates when there is low non-tradable output. Net wealth of country 1 increases when the bad shock hits, and households import more tradable goods in order to smooth their aggregate consumption. However, the amount of imports are less than that of the baseline model when constraints



Note: Impuse response to the negative non-tradable shock in country 1. Shock is 3.6808 times of standard deviation innovation. It is the Low grid in non-tradable shock. All panels are percentage deviations from zero shocks and tight collateral constraints, except for Net wealth fraction and Bond positions, which are percentage point deviations. Red dashed line is at 0 for all graphs.

Figure 2.2: Impulse response function, negative shock on country 1 NT output

are in place, leading to a lower tradable consumption by 0.03%. With reduced gross positions, households cannot insure themselves as much as in the baseline setup.

Gross flows with negative non-tradable shocks When the level of output goes down for any of the countries, households in all countries cut back their savings and borrowings. The same mechanism as in the baseline model works with stochastic collateral constraints. With lower amount of risk in non-tradable consumptions or less tradable goods to be used as risk sharing, consumers purchase less insurance, which is domestic bond savings.

Two points are different from baseline model. First, amount of decrease in gross positions are larger when collateral constraints are binding as a response to an adverse non-tradable endowment shock. Second, exogenous tightening of collateral constraint also brings down gross positions. There are more fluctuations in gross flows when collateral shocks are added to the baseline model.

Impulse responses in panel (c) of Figure 2.2 show portfolio adjustment with the adverse shock on non-tradable endowment in country 1. Domestic bond savings (solid line) drops by 6.7 percentage points compared to the initial zero-shocks state, and foreign bond savings (dashed line) also decreases by 6.8 percentage points. Compared to the baseline model, where savings and borrowings fall by 3 and 1.6 percentage points respectively, bond positions shrink by more than twice as a fraction of initial point when collateral constraints are binding. In absolute quantity of bond position adjustments, not as a percentage point deviations from the initial positions, the case of collateral constraints are twice as large as the case of baseline model.

Land values and relative prices When there is a negative shock on domestic non-tradable endowment, relative prices go up but land prices drop. As panel (f) of Figure 2.2 shows, land prices drop by nearly 10 percentage in country 1 when there is negative shock on the non-tradable output. Counterpart of relative prices for land dividends are rents, and land prices are matched with the value of real estate. Speaking in terms of rents and real estate values, rents in units of tradable goods have increased while the value of real estate falls because it yields less dividends with negative shocks.

Table 2.2: Business cycle statistics in the extended model

	Data	Model	
		Baseline	Collateral
Corr($Gross$, $GDP\ cycle$)	0.33	0.26	0.05
Corr($Asset$,$Liab$)	0.90	0.94	0.99
Std($Gross/GDP$)/Std(Net/GDP)	3.60	5.74	14.43
Std($Gross/GDP$)	0.09	0.05	0.09

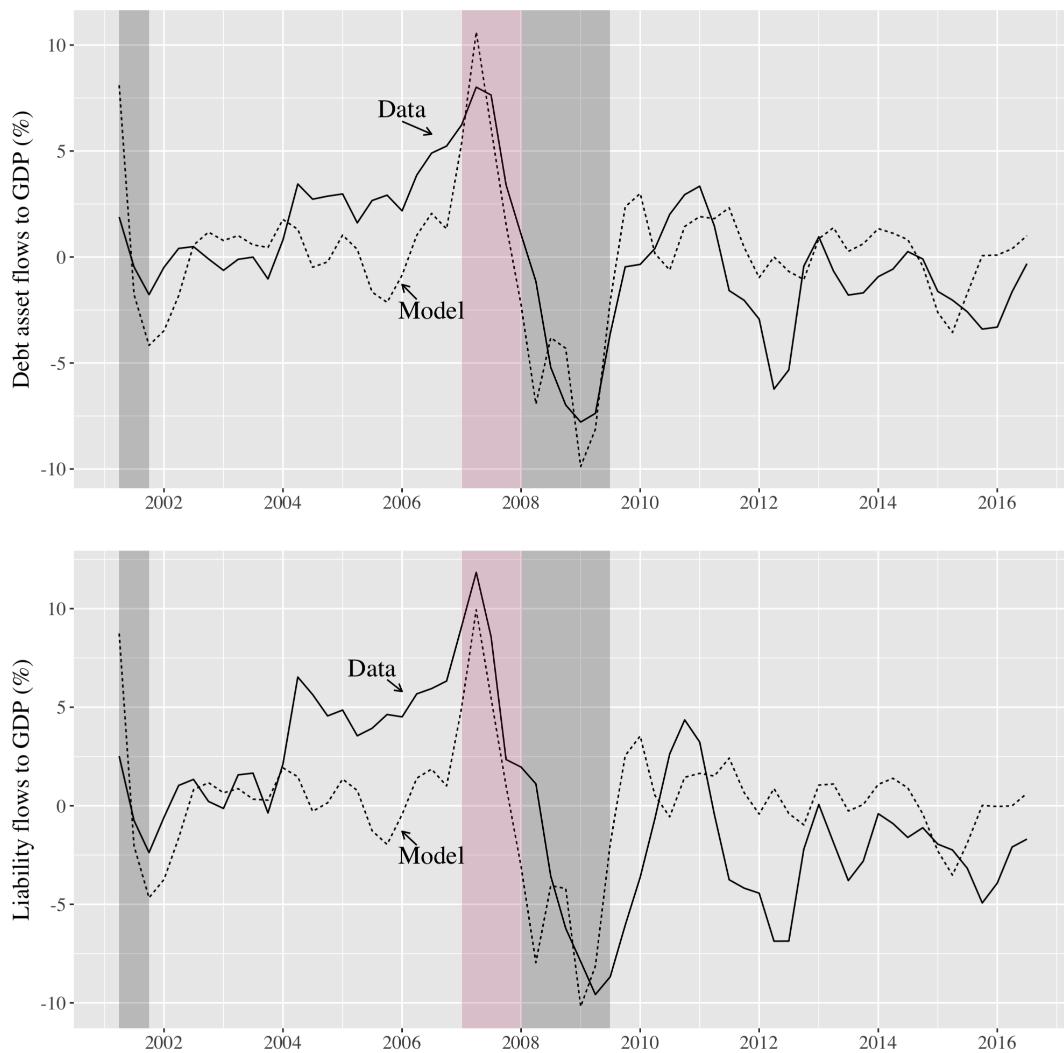
Note: Sample period is from 2001Q2 to 2016Q2. All data is the US. *GDP cycle* of data is log HP-filtered cycles with smoothing parameter 1600, deflated with GDP deflator. *Asset* and *Liab* are debt asset flows and debt liability flows, deflated with GDP deflator. *Gross* is debt gross flows, sum of debt asset flows and debt liability flows. *Net* is net flows, liability flows minus asset flows. Model is a simulation result of the same sample period. Source: Bureau of Economic Analysis, author's calculations.

2.2.3 US gross flows during the Great Recession

In the stochastic collateral constraint model, constraints are set to be tight over the all sample periods except for 2007 Q1 to 2007 Q4. Inputs for the output series are same as in the baseline model simulations in Chapter 1.

Adding stochastic collateral constraint magnifies the fluctuations of gross flows. In Figure 2.3, periods of the loose collateral constraints are shaded in red. Compared to the baseline simulation (Figure 1.10), the model is capable of generating a large boom in the gross flows before the collapse as the collateral constraint is relaxed. Also, as the constraint is tightened at the beginning of the recession, gross flows turn to negative values in 2008 Q1, which implies retrenchment of international positions. The last column of Table 2.2 summarizes the business cycle statistics of the gross flows in the stochastic collateral constraint model.

The stochastic collateral constraints help the model to better match the volatility of gross flows in the model, as the last row of Table 2.2 shows. Both data and model has 9% of gross flows to GDP standard deviations over the sample period. More pronounced boom



Note: Both data and model are 3-period moving average, with weights [0.25 0.5 0.25] for period [t-1,t,t+1]. Data is demeaned ratio of the US debt (Portfolio debt + Other investment) asset and liability flows over GDP, at quarterly level, seasonally adjusted. Shaded gray areas are NBER recessions, and red areas are periods when the borrowing constraints are relaxed. Source: Bureau of Economic Analysis and author's calculations.

Figure 2.3: Simulation result: stochastic collateral constraints model and demeaned data, asset (top) and liability (bottom) flows

and collapse of gross flows increase correlation of asset and liability flows to 0.99, which is higher than the data (0.90).

Tight constraints over the periods result in muted net flows, whose standard deviation is 72% of the baseline model. It is because the lower level of gross positions restrict the ability of consumers to adjust their tradable goods consumption compared to the baseline model, resulting in lower magnitude of net flows. Combined with the increased volatility of gross flows, ratio of standard deviations of gross flows to net flows increase to 14.79 in the stochastic collateral constraints model. Finally, correlation between gross flows and GDP gets reduced over the time periods, because of sharp decrease in gross flows during the tightening of the collateral constraints and rebound in 2008 when the output was decreasing. The statistics could be improved by setting a smooth time series of collateral constraints instead of two-state Markov switching process, but at the same time it is harder to calibrate when there is a continuous series of collateral constraints. In order to provide a simple exercise, I maintain the structure of two-state collateral constraints.

Welfare analysis Using the calibrated model, I access benefits as well as costs of the international investment positions. In the model of stochastic collateral constraints, the welfare gains and losses become bigger compared to those in the baseline model (Chapter 1). When calculated based on the present value, the US consumers gained 0.02% of permanent consumption whereas the EU28 residents lost 0.02% over the sample period. It is because the collateral constraint was loosened only on the eve of the Great Recession, so that the US consumers could better insure themselves during the downturn without transferring their wealth to the EU28 for the periods of economic boom and the European debt crisis. When there is stringent collateral constraint during the European debt crisis, for example, the US residents export less tradable goods to Europe because they hold lower level of gross positions and the EU28 countries have less insurance when their non-tradable consumption is less than the United States. In Appendix, I plot the log difference of tradable consumption between stochastic collateral constraint and baseline models.

2.3 Conclusion

In this chapter, I present a model of gross capital flows with stochastic collateral constraints. It is based on the model of previous chapter (Chapter 1), in which international risk sharing works as the core mechanism that drives the movements of gross capital flows. By adding stochastic collateral constraints to the baseline model, gross flows in the extended model have higher volatility. This extension addresses a shortcoming of the model in Chapter 1, which is lower simulated volatility of gross flows during the Great Recession in the United States compared to the data. With a loosening of collateral constraint during the boom period in 2007 and a tightening at the onset of the Great Recession in 2008, simulated results of the model can better match the volatility of gross flows to that of data.

Even though the extended model is successful in bringing the simulated volatility of gross flows closer to that of data, there are several parts that call for further work. First, it is based on a simplifying assumption that there are only two states in stochastic collateral constraints: High and Low. More sophisticated specification of the states, for example autoregressive with degree 1, would help to bring the simulated gross flows even closer to the data. Related to the first point, another part that has a room for improvement is the measurement of collateral constraints. In the current calibration, periods of loosening and tightening of collateral constraints are ad hoc. More refined calibration based on data will make the quantitative analysis more convincing.

Chapter 3

Sovereign Uncertainty: Fiscal Policy Under the Debt Crisis

3.1 Introduction

Peripheral European countries implemented austerity programs while their government bond yields became more volatile. In the wake of Great Recession in 2009, public balance of Italy, Portugal, Spain and Greece deteriorated significantly. Spain and Greece recorded net borrowing of 11% and 15.3% of GDP, respectively. At the same time, government bond yields showed increasing volatility. In July 2009, Spain's 3 year government bond yield in real terms increased by 396 basis points compared to August 2008. As can be seen in Figure 3.1[1], real yields for all other governments also rose by more than 200 basis points

[1]Real government bond yield in the graph is nominal yield of 3 year maturity government bond net of Harmonized Consumer Price Index (overall index) of each country. Greek bond yield peaked at 7595 basis point in February 2012 and defaulted from March 2012 to June 2014, which is not shown in the graph due to space limit. Data is sourced from central bank of each country for nominal government bond yield, Bundes Bank for HCPI and Eurostat for government revenue to GDP.

over the same period. In the middle of economic turmoil with increasing bond yield and deepening recession, these governments undertook austerity programs over the course of 2009-2013 (Alesina et al., 2015) in an effort to reduce fiscal deficit and public debt.

How should the fiscal policy be adjusted when the volatility of government bond yield is time varying? The experience of peripheral European governments described above leads one to question the impact of fiscal policy when there is volatility shock. As recent studies show[2], volatility shocks can have significant impact on the macroeconomic variables that serve as the tax revenue base. This implies that the same change in fiscal policy can yield different results in terms of government revenue and household welfare depending on the volatility.

Classic studies based on the static Laffer curve, however, will prescribe same tax rates regardless of the volatilities. This can be misleading when the impact of volatility is sizable. In this paper, I first quantify the stochastic volatility of real government bond yields for four European countries: Germany, Italy, Spain and Portugal. From the estimation using sequential Markov Chain Monte Carlo algorithm, I show that there was significant rise in volatility in 2009-2013. Then, I propose a small open economy model that examines the dynamics of government revenue and other macroeconomic variables when fiscal policy is adjusted on the impact of real interest rate volatility shock. Calibrating to European data, I show that static model will overestimate increase in government revenue and underestimate decrease in household welfare compared to the dynamic model with stochastic volatility.

In the model, households face stochastic volatility in bond yields as well as taxes on consumption and factor incomes. The government finances its expenditure by levying taxes on the households and issuing domestic bonds. Households smooth their consumption by

[2]See Bloom (2009) and Fernández-Villaverde et al. (2011a), among others.

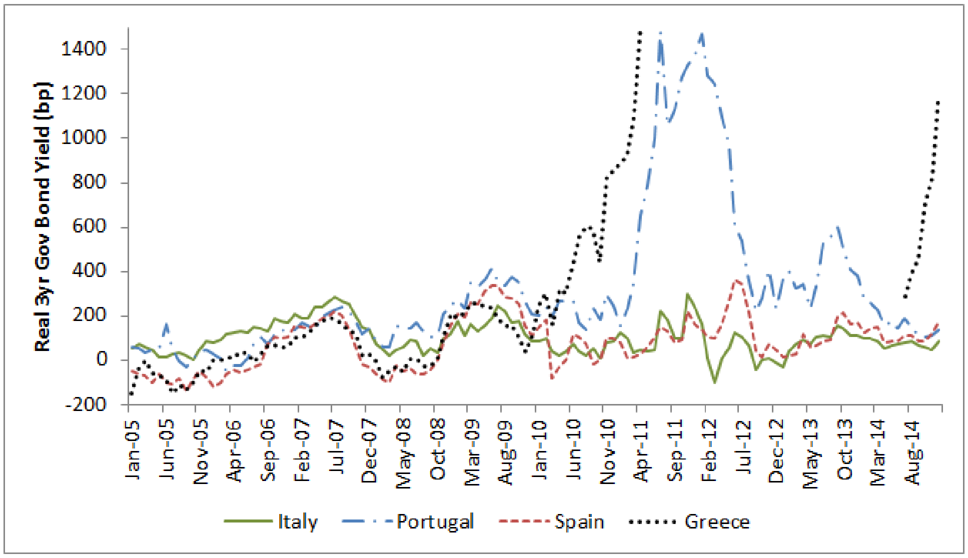


Figure 3.1: Real 3 Year Government Bond Yield

borrowing from foreigners[3].

In this environment, I first compare two economies where the real interest rate is more volatile in one economy than the other. By increasing the tax rate together with real interest rate, I analyze the difference in consumption, labor supply, investment, borrowing decisions of households, as well as government revenue and household welfare across the two economies.

For the same 1% of each tax increase, I show that household decisions vary across the different taxes and volatilities. Private agents in more volatile economy choose to run down debt level more, consume less and work more compared to those in less volatile one on impact of the shock. The intuition is that higher volatility leads to more precautionary behavior, since there is riskier bond that the agent can borrow to smooth out the negative shock. Moreover, increase in consumption tax leads to more investment since it is relatively

[3]In this paper, it is assumed for simplicity that only household will borrow from the foreigners. There is indeterminacy of private and public debt due to the presence of transfer. More details will be discussed in the section 3 of the paper.

cheaper to save in capital than to borrow when there is increase in real interest rate. For factor income tax shock, on the other hand, households in both volatile and stable economies decide to decrease the investment since factor income tax is proportional to output.

Also, I present simulated Laffer curves for both volatile and stable economies and compare them to the static economy without any real interest rate shock. In this exercise, I fix the tax rate and simulate the model under two different volatilities of real interest rate. I show that higher volatility will shift the Laffer curve down across all tax rates, but the revenue is maximized at the same tax rate. The reason why higher volatility lowers the level of revenue is that consumption and output level will be lower in the more volatile economy. Moreover, the Laffer curve will still peak at the same tax rate since difference in volatility mainly scales the household decision up and down, not changing the direction of response to the shock.

This paper mainly weaves three strands of literature: uncertainty, fiscal policy and small open economy business cycle. The model is based on the canonical small open economy model as in Neumeyer and Perri (2005) and more recently Fernández-Villaverde et al. (2011a), with imperfect financial market of risk free asset and production. The main difference from existing literature is in incorporating stochastic tax wedge to the households and focusing on the impact of volatility on government revenue. The proposed model in this paper can be served as a basic framework for small open economy business cycle accounting, expanding Chari et al. (2007) which is for the closed economy without volatility shock in real interest rate.

In both estimation and models, I follow the framework of Fernández-Villaverde et al. (2011a), which studies the impact of volatility shock in small open economy by introducing time varying volatility in the real interest rate process. However, their paper does not have government in the model, and its focus is not on the Laffer curve as in this paper.

Another study by Fernández-Villaverde et al. (2011b) examines volatility in fiscal rule and its impact in the US economy, based on time varying volatility process and particle filtering. Born and Pfeifer (2014a) investigates on uncertainty of US fiscal policy as well, but both studies are on closed economy and fiscal rules.

Finally, regarding European fiscal policy, Mendoza et al. (2014) studies effect of austerity programs in European countries hit by debt shock. Based on two-country model, the paper shows externality of fiscal policy in open economies, especially on capital taxation. However, their paper is set up in neoclassical models without shocks and mainly discuss transition to new steady state with the debt shock and austerity programs.

The rest of paper is organized as follows. Section 2 describes European debt crisis and austerity statistics in 2009-2013 and estimates stochastic volatility of real government bond yields for four European countries using particle filtering. Then, Section 3 builds the model of small open economy with government and volatility in real interest rate. Section 4 analyzes the model mechanism by impulse response functions across different taxes and presents simulated Laffer curves based on calibration. Section 5 concludes. Appendix includes more details on model equations. Data Appendix explains the source of data used in the paper and data description omitted in the main body of the paper.

3.2 Data and Estimation

In this section, I first describe public balance and government bond yield statistics for peripheral European countries: Italy, Portugal, Spain and Greece. I show that these countries undertook fiscal adjustment while suffering from increased volatility in their government bond yield. Then, I estimate the time varying volatility of real government bond yield. I set up real interest rate process composed of stochastic level and volatility, and use sequential Markov Chain Monte Carlo algorithm called particle filtering to recover the time varying volatility.

3.2.1 European Debt Crisis and Austerity

In 2009-2013, peripheral European countries were hit by a series of adverse shocks. Especially, sovereign governments suffered from mounting debt and underwent fiscal adjustment while government bond yields were increasingly volatile.

As Figure 3.2 shows, public balance of all four countries significantly deteriorated in the wake of the Great Recession in 2009. Spain and Greece recorded net borrowing of 11% and 15.3% of GDP, respectively. At the same time, government bond yields became more volatile. In July 2009, Spain's 3 year government bond yield in real terms increased by 396 basis points compared to August 2008. As can be seen in Figure 3.1, real yields for all other governments also rose by more than 200 bp over the same period.

In response, these governments implemented austerity program in 2009-2013 and targeted to reduce their fiscal deficit (Alesina et al., 2015). However, it was precisely the period that government bond volatilities were at their highest. Therefore, fiscal adjustment of sovereign government was subject to significant aggregate volatility shock. The observed time series of government revenue to GDP in Figure 3.3 can be interpreted as a realized equilibrium of fiscal adjustment under the volatility shock of real interest rate [4].

[4] It is true that GDP decreased for these countries especially in 2011-2013. However, as Figures C.1 and C.2 show in Appendix, the increase in government revenue to GDP ratio is not entirely due to drop in the GDP. Real government revenue did increase for most countries except for Greece, with Portugal increasing its revenue to the level of 2008 in year 2011.

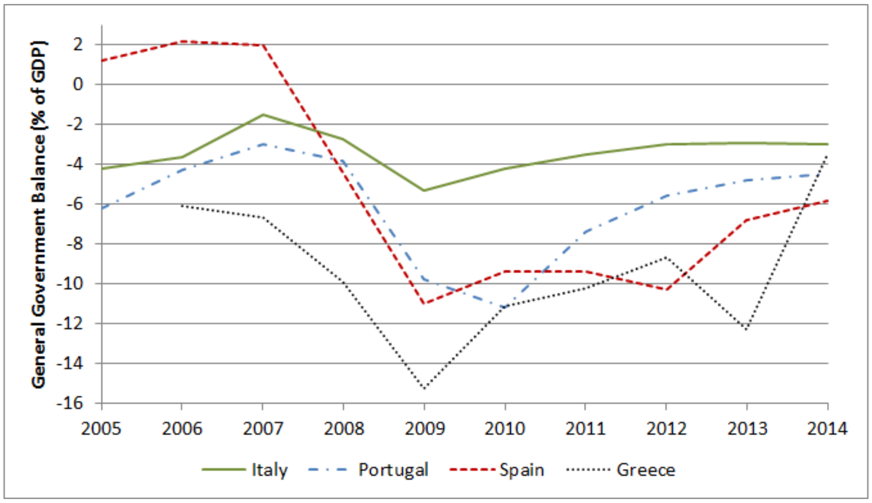


Figure 3.2: General Government Balance

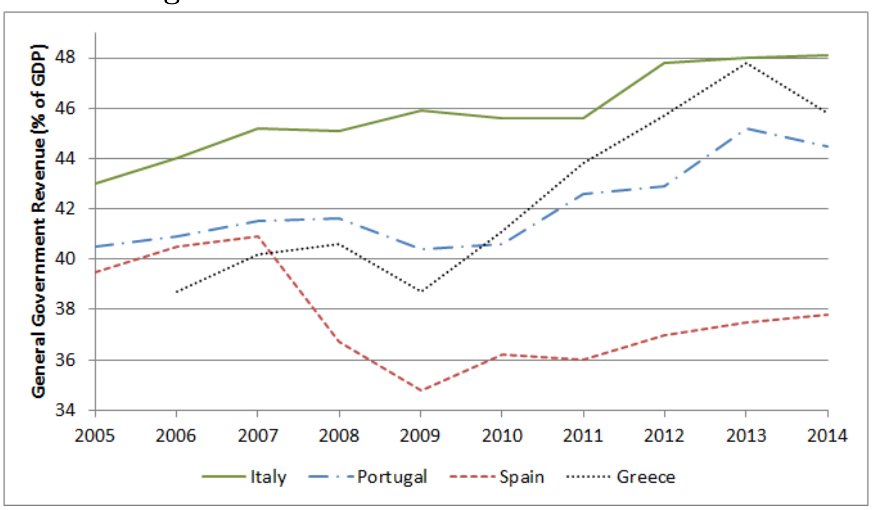


Figure 3.3: General Government Revenue to GDP

Table 3.1: Real Interest Rate Summary Statistics

	Germany	Italy	Portugal	Spain
Mean	0.8054	1.0806	2.1843	0.7223
Sd	1.5644	0.8544	3.1493	1.0965

Motivated by the peripheral European governments' experience in debt crisis and austerity program described above, this paper analyzes the impact of volatility shock on the government revenue and macroeconomic variables based on European data. As its first step, I estimate time varying volatility in real interest rate for four countries: Germany, Italy, Portugal and Spain. In the following subsection, I describe the data and estimation procedure.

3.2.2 Estimation

Real Interest Rate Data I briefly explain the data used for estimation. I construct real interest rates by subtracting Harmonized Consumer Price Index from nominal government bond yields, 3 year maturity. Then, German real interest rate is assumed to be the risk free rate, and country spread is derived by subtracting German rate from each country's real interest rate. I use monthly data from January 2000 to April 2015. Table 3.1 summarizes the mean and standard deviation of the real interest rates used in the estimation. All figures are in percent per annum. More details for the data source are in Data Appendix.

Real Interest Rate Process Real interest rate (r_t) is composed of mean real interest rate ($\bar{r}$), risk-free rate shock ($\varepsilon_{f,t}$) and country spread shock ($\varepsilon_{r,t}$). This structure follows Fernández-Villaverde et al. (2011a).

$$r_t = \bar{r} + \varepsilon_{f,t} + \varepsilon_{r,t} \tag{3.1}$$

Country spread shock $\varepsilon_{r,t}$ follows AR(1) process with level shock $u_{r,t}$ multiplied to the time varying volatility σ_t.

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + exp(\sigma_{r,t}) u_{r,t} \tag{3.2}$$

Time varying volatility process σ_t follows AR(1) process as well, with unconditional mean $\bar{\sigma}$. Standard deviation of the volatility shock $u_{\sigma,t}$ is given as η_σ.

$$\sigma_{r,t} = (1 - \rho_{\sigma_r})\bar{\sigma_r} + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_{\sigma_r} u_{\sigma_r,t} \tag{3.3}$$

Both level and volatility shocks $u_{r,t}$ and $u_{\sigma,t}$ are assume to follow $i.i.d.$ standard normal $N(0,1)$, independent to each other. Risk free rate shock follows similar process. Formally put,

$$\varepsilon_{f,t} = \rho_f \varepsilon_{f,t-1} + exp(\sigma_{f,t}) u_{f,t} \tag{3.4}$$

$$\sigma_{f,t} = (1 - \rho_{\sigma_f})\bar{\sigma_f} + \rho_{\sigma_f}\sigma_{f,t-1} + \eta_{\sigma_f} u_{\sigma_f,t} \tag{3.5}$$

where both $u_{f,t}, u_{\sigma_f,t} \sim_{i.i.d} N(0.1)$.

Estimation Result Estimation method is based on sequential Monte Carlo algorithm, or particle filtering[5]. Throughout the estimation, 2,000 particles are used for filtering and 2,000 particles are used for backward smoothing. Burn-in periods are 5,000 and Metropolis-Hastings draws are performed 20,000 times. In Markov Chain Monte Carlo algorithm, scale factors are used so that proposal density has acceptance rate of 25-40%.

[5] Refer to Fernández-Villaverde et al. (2011a), Born and Pfeifer (2014b) and Fernández-Villaverde et al. (2015) for more detailed explanation of the algorithm.

Table 3.2: Prior Distribution

	Germany	Spain	Portugal	Italy
ρ	$\mathcal{B}(0.9, 0.02)$	$\mathcal{B}(0.9, 0.02)$	$\mathcal{B}(0.9, 0.02)$	$\mathcal{B}(0.9, 0.02)$
ρ_σ	$\mathcal{B}(0.95, 0.02)$	$\mathcal{B}(0.9, 0.05)$	$\mathcal{B}(0.9, 0.05)$	$\mathcal{B}(0.9, 0.05)$
$\bar{\sigma}$	$\ln\mathcal{N}(-2, 0.3)$	$\ln\mathcal{N}(-2, 0.3)$	$\ln\mathcal{N}(-2, 0.3)$	$\ln\mathcal{N}(-2, 0.3)$
η	$\Gamma(0.2, 0.05)$	$\Gamma(0.3, 0.05)$	$\Gamma(0.3, 0.05)$	$\Gamma(0.3, 0.05)$

Note: $\mathcal{B}(mean, sd)$, $\ln\mathcal{N}(mean, sd)$, $\Gamma(mean, sd)$ indicates Beta, Log Normal and Gamma distribution, respectively. sd indicates standard deviation.

I follow Bayesian estimation with priors as in Table 3.2. Except for Germany where real interest rate is used as risk free rate, priors are set to be identical across the countries in order to keep estimation general. In Germany, persistence of stochastic volatility is set to have higher mean and lower variance than the other countries, since more stable movement is expected in risk free rate. Moreover, for the same reason, standard deviation of the volatility shock is expected to have lower mean.

Figure 3.4 shows the estimated volatility for the four countries. Estimation result shows that stochastic volatility (dash line) increased with real interest rate (solid line) for peripheral nations during the debt crisis, especially from the end of 2011 to July 2012. Portugal shows the highest increase in the volatility. In April 2008, volatility was as low as -1.168 in log scale. However, at the peak of the debt crisis in August 2011, the volatility increased up to 0.878. That is, with the same one standard deviation of level shock, real interest rate increased by 0.3% in 2008 whereas it rises by 2.4% in 2011. Time varying volatility shock can contribute to 8 times larger movement in real interest rate in Portugal during the debt crisis.

Italy also shows sharp increase in volatility during the debt crisis. In March 2009, volatility was as low as -1.163 in log scale, which means that there was 0.3% increase in real interest rate when level shock hits. However, starting from the second half of 2009,

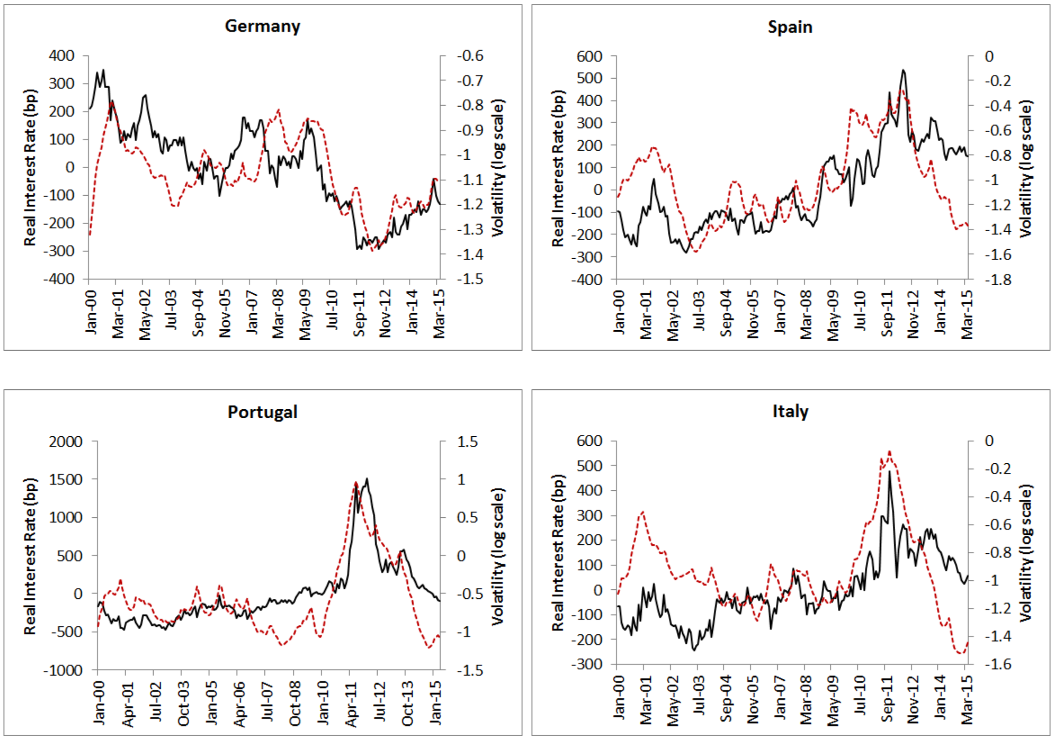


Figure 3.4: Real interest rate (solid line, left scale) and estimated volatility (dash line, right scale).

volatility started to increase continuously, hitting -0.51 in March 2011. This shows that volatility doubled two years to 0.6% in annualized interest rate terms. Volatility kept rising until November 2011, which peaked at -0.06 in log scale, which is 0.942 when exponentiated. This is more than three times bigger impact on the real interest rate compared to 2009. The volatility dropped to early 2011 level by the end of 2012, and decreased further to the lowest level in the sample period.

In case of Spain, volatility started to increase earlier than other peripheral countries. In March 2010, volatility rose to -0.415 in log scale, which is 0.66 when exponentiated. Compared to August 2009 where volatility was as low as -1.18 in log, or 0.307, the same level shock can increase the real interest shock twice as more in 2010 compare to 2009.

Table 3.3: Posterior Summary Statistics

	Germany			Spain			Portugal			Italy		
	Median	5%	95%	Median	5%	95%	Median	5%	95%	Median	5%	95%
ρ	0.9708	0.9526	0.9858	0.9419	0.9252	0.9567	0.9379	0.9200	0.9543	0.9143	0.8886	0.9369
ρ_σ	0.9496	0.9019	0.9791	0.9021	0.8292	0.9536	0.9356	0.8833	0.9677	0.9156	0.8444	0.9595
$\bar{\sigma}$	-1.2547	-1.6237	-1.0087	-1.1642	-1.4599	-0.9458	-1.0354	-1.4753	-0.6910	-1.1092	-1.4387	-0.8799
η	0.1317	0.0863	0.1924	0.1806	0.1160	0.2676	0.2208	0.1566	0.3013	0.1634	0.1082	0.2490

Note: Germany is used as risk free rate. Other countries are shown for *country spread = real interest rate - risk free rate.*

This high volatility remained until the first half of 2012, peaking at -0.28 in June 2012 and quickly declining afterwards.

On the other hand, German real interest rate level went down during the same period, showing the lowest volatility over the sample. In contrast to the peripheral countries, German volatility peaked in July 2009 at -0.85 in log scale. As the volatility in other countries goes up, Germany observed continued drop in volatility that hit its bottom in May 2012 at -1.3855. This means that impact of volatility is nearly halved compared to the summer of 2009. For one standard deviation shock, real interest rate in Germany increased by 0.43% in 2009 but only 0.25% in 2012 for annualized real interest rates. This is a sharp contrast to the other peripheral countries, whose country spread more than doubled during the same time period.

The main result of estimation is summarized in Table 3.3. I report Median and 5%, 95% quantile in posterior distribution. Parameters are in general estimated with tight bands around the median. Median of volatility level ($\bar{\sigma}$) shows that Portugal has the highest mean volatility and Germany the lowest, which is intuitive as seen in the figure above. Standard deviation of volatility shock (η) shows qualitatively similar result, preserving the ranking in the mean volatility. Autoregressive coefficients for both level shock (ρ) and volatility shock (ρ_σ) shows more persistence for Germany compared to peripheral countries.

In the following section, I build a small open economy model that can analyze the impact of fiscal policy when there is volatility shock. Based on the estimation result presented above, I calibrate in Section 4 to peripheral European economies, especially focusing on Portugal.

3.3 The Model

I build a small open economy model where sovereign government takes the real interest rate as given. Market is incomplete, so there is only risk free bond that can be traded. I follow closely to the standard setting of small open economy DSGE model. There are households, firms and government.

3.3.1 Household

I denote the state of economy at each period t as s_t, with the history of states up to period t as $s^t = (s_0, s_1, \ldots, s_t)$. Flow utility of household for any given history s^t is given as power-separable form, with consumption and leisure.

$$u(c_t(s^t), l_t(s^t)) = \frac{c_t(s^t)^{1-\nu}}{1-\nu} - \psi \frac{l_t(s^t)^{1+\eta}}{1+\eta} \tag{3.6}$$

where $c_t(s^t)$ denotes the consumption and $l_t(s^t)$ is labor supply.

There are three different marginal taxes, or wedges, that are given to the households: tax on consumption $\tau_{c,t}(s^t)$, labor income $\tau_{l,t}(s^t)$ and capital income $\tau_{k,t}(s^t)$. Taking as given the tax rates, wages $w_t(s^t)$, rental rates $R_t(s^t)$, price of bonds $q_t(s^t)$ as well as the lump sum government transfer $T_t(s^t)$, household decide on consumption goods purchase $c_t(s^t)$, investment in physical capital by $x_t(s^t)$, international bond issuance $d_{t+1}(s^t)$ and labor supply $l_t(s^t)$. Then, the household budget constraint is:

$$\begin{aligned}&(1+\tau_{c,t}(s^t))c_t(s^t)+x_t(s^t)+d_t(s^{t-1})+\Phi_d(d_{t+1}(s^t))\\&\quad=(1-\tau_{l,t}(s^t))w_t(s^t)l_t(s^t)+(1-\tau_{k,t}(s^t))R_t(s^t)k_t(s^{t-1})+q_t(s^t)d_{t+1}(s^t)+T_t(s^t)\end{aligned} \tag{3.7}$$

where the physical capital depreciates with rate δ and there is capital adjustment cost $\Phi_k(k_{t+1},k_t)$:

$$x_t(s^t)=k_{t+1}(s^t)-(1-\delta)k_t(s^{t-1})+\Phi_k(k_{t+1}(s^t),k_t(s^{t-1})),\ x_t(s^t)\geq 0 \tag{3.8}$$

$$\Phi_k(k_{t+1},k_t)=\frac{\phi_k}{2}\left(\frac{k_{t+1}-(1-\delta)k_t}{k_t}-\delta\right)^2 k_t. \tag{3.9}$$

Notice that sign convention of budget constraint indicates positive amount of $d_{t+1}(s^t)$ as debt. $d_{t+1}(s^t)$ is designed to represent the net borrowing of the economy, since there is indeterminacy of private and public bond holdings. The financial asset holdings between the government and the household will not be pinned down since the government can always borrow from the household and return it through the lump sum transfer. Therefore, for simplicity, I assume that only household can issue risk-free international bond $d_{t+1}(s^t)$.

Household also pays for the bond holding cost, $\Phi_d(d_{t+1}(s^t))$. It is a quadratic function of $(d_{t+1}(s^t)-\bar{d})$, where $\bar{d}$ is a steady state level of international borrowing.

$$\Phi_d(d_{t+1})=\frac{\phi_d}{2}(d_{t+1}-\bar{d})^2 \tag{3.10}$$

Bond holding cost has been used to prevent the small open economy from displaying random walk of real allocations. See Neumeyer and Perri (2005) for the similar application to the small open economy and Schmitt-Grohé and Uribe (2003) for other ways to close the model.

3.3.2 Firms

A representative firm produces with constant returns to scale technology, renting capital and labor from household.

$$y_t(s^t) = F(k_t(s^{t-1}), l_t(s^t)) = k_t(s^{t-1})^\alpha [e^{X_t(s^t)} l_t(s^t)]^{1-\alpha} \tag{3.11}$$

where X_t is a labor augmenting productivity shock that follows AR(1) process: $X_t = \rho_X X_{t-1} + \sigma_X u_{X,t}$ where $u_{X,t}$ follows *i.i.d.* standard normal distribution.

3.3.3 Government

Government need to finance a stream of government purchase $g_t(s^t)$ and lump sum transfer to the household $T_t(s^t)$. Linear taxation on labor, capital and consumption $\{\tau_{l,t}, \tau_{k,t}, \tau_{c,t}\}$ together with domestic bond d^g_{t+1} will raise government revenue. No arbitrage condition prescribes that government faces the same risk-free bond price as the household does, so that household is indifferent between the domestic government bond and international bond. Formally, government budget constraint is:

$$\begin{aligned} &g_t(s^t) + T_t(s^t) + d^g_t(s^{t-1}) \\ &\quad = \tau_{c,t}(s^t)c_t(s^t) + \tau_{l,t}(s^t)w_t(s^t)l_t(s^t) + \tau_{k,t}(s^t)R_t(s^t)k_t(s^t) + q_t(s^t)d^g_{t+1}(s^t). \end{aligned} \tag{3.12}$$

3.3.4 Competitive Equilibrium

Aggregate resource constraint of the economy is the following:

$$c_t(s^t) + x_t(s^t) + g_t(s^t) + d_t(s^{t-1}) + \Phi_d(d_{t+1}(s^t)) = F(k_t(s^{t-1}), l_t(s^t)) + q_t(s^t)d_{t+1}(s^t). \tag{3.13}$$

Given initial conditions $\{d_0,\ d_0^g,\ k_0\}$, a competitive equilibrium is sequence of allocations $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t),\ d_{t+1}(s^t),\ d_{t+1}^g(s^t)\}$, taxes $\{\tau_{c,t}(s^t), \tau_{l,t}(s^t), \tau_{k,t}(s^t)\}$, government expenditure and transfers $\{g_t(s^t), T_t(s^t)\}$, and prices $\{r_t(s^t), w_t(s^t),\ R(s^t)\}$ such that i) given prices, the allocations solves household's and firm's problem, ii) government budget constraint is satisfied, iii) resource constraint holds and iv) markets clear.

3.3.5 Time Varying Wedges

Tax wedges $\{\tau_{c,t}(s^t), \tau_{l,t}(s^t), \tau_{k,t}(s^t)\}$ and government spending $\{g_t(s^t)\}$ follows VAR(1) process. As in Chari et al. (2007), I assume simple processes that can capture the main wedges in the economy, which can be also interpreted as taxes. Let the vector of taxes and government spending denoted as $\tau_t(s^t) = [\tau_{c,t}(s^t), \tau_{l,t}(s^t), \tau_{k,t}(s^t), g_t(s^t)]$. Let $\rho_\tau \in \mathbb{R}^{4\times4}$ denote VAR(1) coefficient and $\sigma_\tau \in \mathbb{R}_+^{4\times4}$ be variance covariance matrix. Also, let the tax wedges have steady state mean of $\bar{\tau} = [\bar{\tau}_c, \bar{\tau}_l, \bar{\tau}_k, \bar{g}]$. Then, the formal process for the tax wedges will be:

$$\tau_t(s^t) = (I - \rho_\tau)\bar{\tau} + \rho_\tau \tau_{t-1}(s^{t-1}) + exp(\sigma_\tau)u_{\tau,t} \tag{3.14}$$

where $u_{\tau,t} \sim_{iid} N(0, I)$ is multivariate standard normal distribution.

3.3.6 Interest Rate Process with Volatility Shock

In order to inspect the role of volatility shock on the real allocations and taxation, I simplify real interest rate process into mean real interest rate $\bar{r}$ plus country spread shock $\varepsilon_{r,t}$.

$$r_t = \bar{r} + \varepsilon_{r,t} \tag{3.15}$$

Country spread shock $\varepsilon_{r,t}$ follows the same process as used in estimation section. It has AR(1) process with level shock $u_{r,t}$ multiplied to the time varying volatility σ_t.

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + exp(\sigma_t) u_{r,t} \tag{3.16}$$

Time varying volatility process σ_t follows AR(1) process as well, with unconditional mean $\bar{\sigma}$. Standard deviation of the volatility shock $u_{\sigma,t}$ is given as η_σ.

$$\sigma_t = (1 - \rho_\sigma)\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta_\sigma u_{\sigma,t} \tag{3.17}$$

Both level and volatility shocks $u_{r,t}$ and $u_{\sigma,t}$ are assume to follow *i.i.d.* standard normal $N(0,1)$, independent to each other.

3.4 Results

In this section, I solve the model by perturbation method and examine the model economy in the following ways. First, I set up two economies with fixed real interest rate volatility, high and low. By comparing the two extreme cases of volatilities, I inspect the difference in impulse response function of tax wedges and Laffer curves. This exercise shows the possible impact that volatility shock on the macroeconomic variables under time varying tax wedges as well as fixed marginal tax rates.

Next, I let the volatility vary across time stochastically and inspect its impact. I compare the effect of volatility shock on the macroeconomic variables as well as government revenues and compare it to that of level shock. I show that impulse response function of volatility and level shock and discuss with existing results in the literature.

Finally, I estimate stochastic volatility in recent European debt crisis for four countries: Germany, Italy, Spain and Portugal. Using particle filtering and backward smoothing, I present posterior estimation of the volatility parameters.

3.4.1 Model Solution

I solve the model using perturbation method at steady state. I approximate the solution up to third order as in Fernández-Villaverde et al. (2011a). The model solutions are derived from the first order necessary conditions of Lagrangian program. I drop the history of states s^t to simplify the notation.

$$u_c(c_t, l_t) = (1 + \tau_{c,t})\lambda_t \tag{3.18}$$

$$\lambda_t(q_t - \phi_d(d_{t+1} - \bar{d})) = \beta E_t \lambda_{t+1} \tag{3.19}$$

$$u_l(c_t, l_t) = \lambda_t(1 - \tau_{l,t})(1 - \alpha)y_t/l_t \tag{3.20}$$

$$\lambda_t(1 + \Phi_k^1(k_{t+1}, k_t)) = \beta E_t \left[\lambda_{t+1}\left(1 - \delta - \Phi_k^2(k_{t+2}, k_{t+1}) + (1 - \tau_{k,t+1})\alpha \frac{y_{t+1}}{k_{t+1}}\right)\right] \tag{3.21}$$

where λ_t is Lagrangian multiplier for the household budget constraint and $u_c(c_t, l_t)$, $u_l(c_t, l_t)$ indicate marginal flow utility of consumption and labor. $\Phi_k^1(\cdot, \cdot)$ and $\Phi_k^2(\cdot, \cdot)$ denote partial derivative of capital adjustment cost with respect to first and second argument, respectively.

I assume that international lenders are risk neutral. The bond price will be

$$q_t = \frac{1}{1 + r_{t+1}} \tag{3.22}$$

where r_{t+1} denotes the bond yield a small open economy faces.

The key to the model dynamics will be equation (3.19). As the real interest rate gets either higher or volatile, household will choose to run down its debt level and reduce the consumption for the next period. See Neumeyer and Perri (2005) for the level effect of real interest rate and Fernández-Villaverde et al. (2011a) for the volatility shock in the small open economy business cycle.

In this paper, I focus on the interaction of tax wedges to the real interest rate shock. When the consumption tax rate is fixed across the time periods, the Euler equation will remain the same in the economy without consumption tax wedges. However, the main difference will be in factor income tax rates that show up equations (3.20) and (3.21).

As will be shown in the following section, the distortion of labor and capital income tax will be the highest at the peak of the Laffer curve given the constant return to scale production technology. Government revenue will be lower with higher volatility in real interest rate for any combination of tax rates. The mechanism can be seen through the interaction between tax wedge and distorted marginal utility of consumption $\{\lambda_t\}$. With higher volatility of real interest rate, consumption and investment is lower compared to the less volatile economy. This will result in lower consumption tax base as well as lower factor income base when labor supply is inelastic enough. Given fixed tax rate throughout the periods, the difference in government revenue will be maximized when the low volatility economy is at its peak of the Laffer curve, since the government revenue is pro-rata to the tax base in CRS production function.

In order to solve for the steady state of the model, I assume that government expenditure, tax series and transfer to the household have steady state denoted by $\{g, \tau_c, \tau_l, \tau_k, T\}$. Given these steady state values of the government as well as the steady state debt level $\bar{d}$, allocations $\{c, l, k\}$ which are steady state consumption, labor and capital can be solved. Transfer to household is then derived as a residual from the government budget constraint.

3.4.2 Comparative Statics for Volatility Shock

In this section, in order to highlight the main feature of the result, I present a simple numerical example by comparing two economies with high and low volatilities. Since the model is solved around the steady state, I first show the Laffer curve at the steady state without any shocks. Then, I simulate the model under two different real interest rate volatilities, and compare their present value Laffer curve.

Real Interest Rate Process Throughout this section, I keep the simplest real interest rate process. I format the real interest rate as steady state level plus country spread shock, which follows AR(1) process:

$$r_t = \bar{r} + \varepsilon_t \tag{3.23}$$

$$\varepsilon_t = \rho_r \varepsilon_{t-1} + exp(\sigma_j) u_t, \;\; \sigma_j \in \{\sigma_H, \sigma_L\} \tag{3.24}$$

where $u_t \sim_{iid} N(0, 1)$. I choose two values of volatilities, σ_H for high and σ_L for low. I also include AR(1) labor augmenting productivity shock X_t where $X_t = \rho_X X_{t-1} + \sigma_X u_{X,t}$, $u_{X,t} \sim_{iid} N(0, 1)$. I assume the two shocks $\{u_t, u_{X,t}\}$ are independent.

Calibration I calibrate the model quarterly, in line with the frequency of macro aggregates data that will be used in estimation section. Calibration values are summarized in

Table 3.4. With power-separable utility function, I set the inverse of elasticity of intertemporal substitution (EIS) at 5, so that EIS is 0.2. The benchmark calibration for Frisch elasticity is 0.02. The main purpose of low Frisch elasticity is to capture one of the most stylized small open economy business cycle fact that with high and volatile real interest rate, output goes down. Since Frisch elasticity of 1-5 range will result in significant increased output with the real interest rate shock as households increase their labor supply, I keep the low Frisch elasticity in benchmark and later show the result with Frisch elasticity of 1.

For parameters in production technology, I use the constant share of capital income of 0.38 and capital depreciation rate of 7% per annum. Both figures are from Trabandt and Uhlig (2011), where static Laffer curve of US and EU-14 is updated.

Real interest rate is put as 2% per annum, which is close to Portugal's real interest rate sample mean as will be shown in Table 3.1 of estimation section. High volatility is chosen to be log(0.2), which will be 10 times bigger than the low volatility log(0.02) when exponentiated in the process.

Tax rates are set according to the average marginal effective tax rate of Portugal in 2000-2012. Labor income tax rate is 23.43%, capital income tax rate is 31.14% and consumption tax rate is 18.51%. Data is from European Commission where marginal effective tax rate relevant to the macroeconomics is calculated using the tax revenue data from each country. Marginal tax rates of other peripheral European countries used for estimation can be found in Table C.2 in Appendix[6]. Other parameters are set as in benchmark calibration, Table 3.4.

For parameters used in tax wedges and government spending processes, autoregressive

[6]For more details regarding the methodology of calculating marginal effective tax rate, see Razin and Tesar (1994) and Taxation Trends in the European Union (Eurostat et al., 2014).

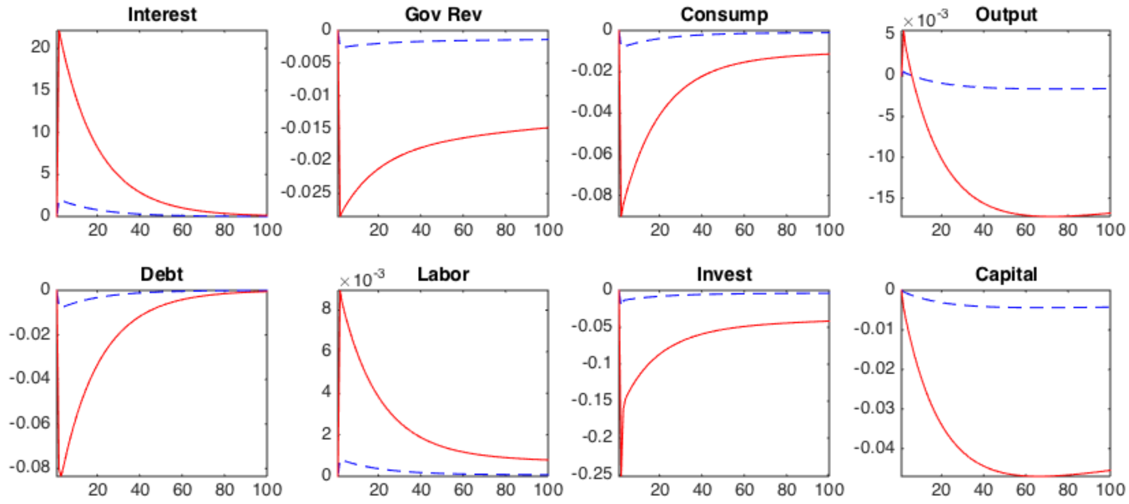
Table 3.4: Calibration

Parameter	Description	Value
ν	Inverse of elasticity of intertemporal substitution	5
η	Inverse of Frisch elasticity	50
ψ	power-separable utility function parameter	1
α	capital income share	0.38
δ	capital depreciation (per annum)	7%
ϕ_k	capital adjustment cost	5
ϕ_d	bond holding cost	1
$\bar{d}$	steady state external debt	4
$\bar{r}$	steady state real interest rate (per annum)	2%
ρ_r	autoregressive of real interest rate shock	0.95
σ_H	volatility of real interest rate shock, high	log(0.2)
σ_L	volatility of real interest rate shock, low	log(0.02)
$\bar{\tau}_l$	mean marginal effective tax rate, labor income	23.43%
$\bar{\tau}_k$	mean marginal effective tax rate, capital income	31.14%
$\bar{\tau}_c$	mean marginal effective tax rate, consumption	18.51%

coefficient is set as 0.9 and standard deviation of white noise is set as log(0.01) for all. It is assumed that there is no correlation between any of the wedges, so that each process is a autoregression of its own.

Real Interest Rate Impulse Response Function Based on the model solution and calibration, I first study the impact of real interest rate shock under different volatilities, holding every other shocks as constant at the steady state level. Figure 3.5 shows impact of real interest rate volatility on government revenue as well as tax bases and borrowing decision. Given one standard deviation shock, higher volatility economy (solid line) will have larger increase in real interest rate on impact, whose magnitude is 10 times larger than less volatile economy (dash line). All figures are scaled to the percentage deviation from the steady state.

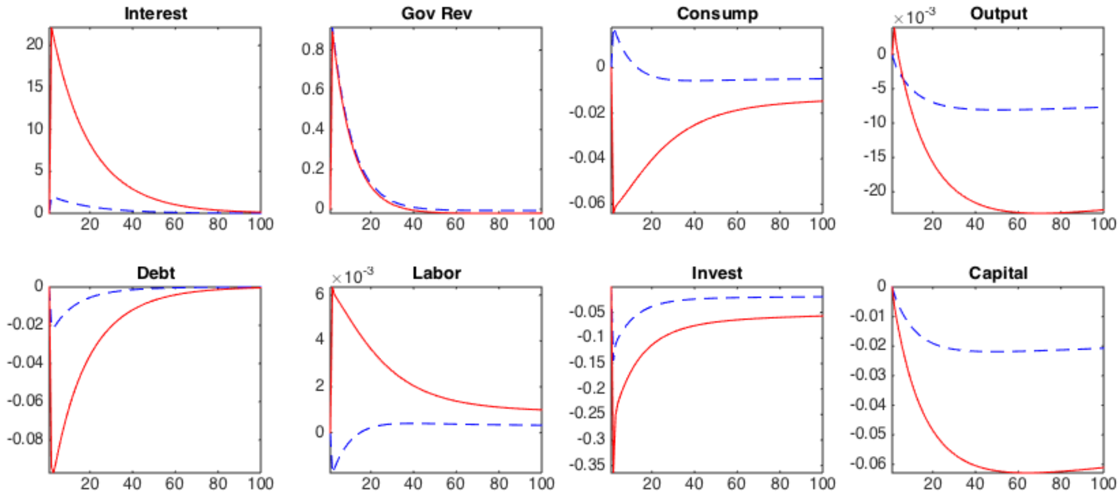
On the impact of real interest rate shock, both high and low volatility economy experience decrease in government revenue but its magnitude is bigger for more volatile economy.



Note: Solid line is for high volatility and dash line is for low volatility.

Figure 3.5: Impulse Response Function, Real Interest Rate Shock

Deteriorated consumption tax base on impact and lower output in the long run work as the main channels for reduced government revenue. As increase in real interest rate works as adverse shock in the economy, household consumption and investment decreases. Debt level decreases as external debt becomes more expensive and labor supply increases to smooth consumption throughout the periods. Given low Frisch elasticity, increase in labor supply and output on impact is transitional and marginal in its magnitude. What drives the long run transition in output is decline of capital stock as the investment is cut with higher real interest rate. Lower level of consumption together with lower capital stock and output level results in lower government revenue for both economy for more than 30 quarters, whose magnitude is larger for more volatile economy.



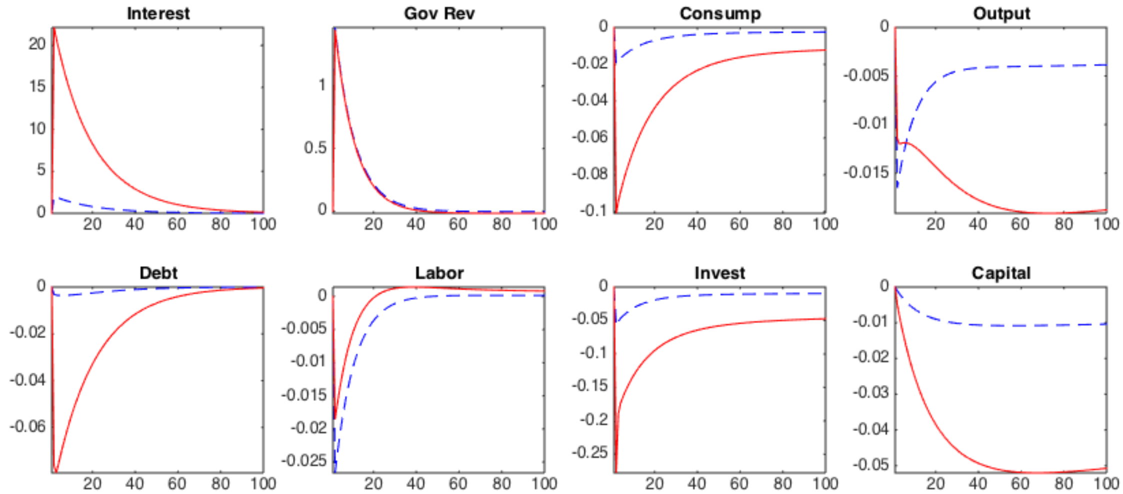
Note: Solid line is high volatility and dash line is low volatility.

Figure 3.6: IRFs, Capital Income Tax

Tax Wedge Impulse Response Function I first compare the impact of tax wedge shocks under different volatilities in real interest rate[7]. There will be one standard deviation shock for both tax wedges and real interest rates, holding everything else equal. Innovation on tax wedge will be given to only one tax wedge at a time, so that difference across the tax wedges is analyzed. The model is simulated over 100 periods, which will be 25 years based on quarterly calibration. All figures are scaled to the percentage deviation from the steady state as before.

In Figure 3.6, I show the impulse response function of the capital income tax shock. Comparing the responses of high volatility (solid line) economy to the low volatility (dash line) one, the direction of household decision is opposite in consumption and labor supply. When real interest rate is volatile, household consumes less and work more in response to the adverse shock. However, when capital tax shock is added to the stable economy,

[7]In this subsection for the Figure 3.6, Figure 3.7 and Figure 3.8, I solve the model up to the second order. For all the other subsections following them are solved up to the third orders

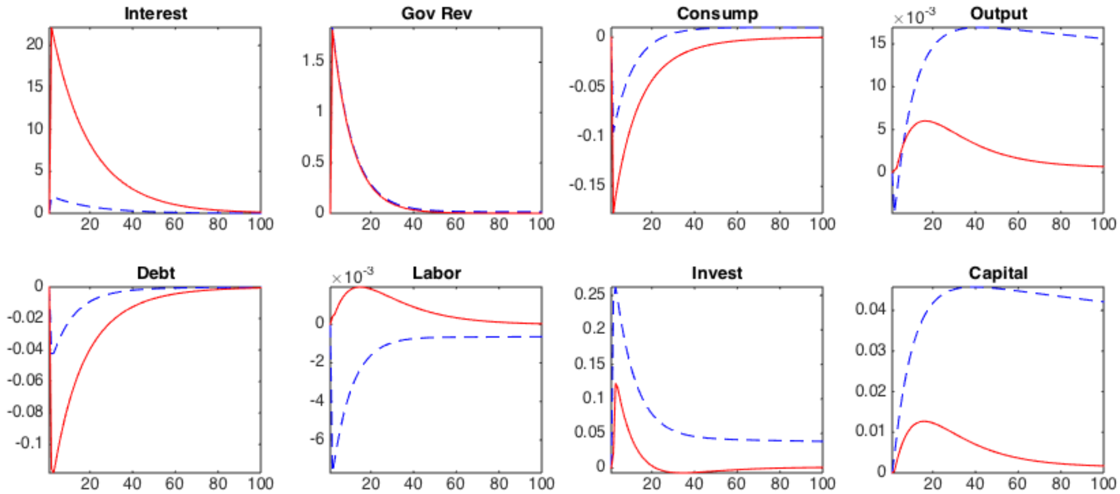


Note: Solid line is high volatility and dash line is low volatility.

Figure 3.7: IRFs, Labor Income Tax

household smooth out the negative shock over time by consuming more and working less on the impact. This is because investment and capital stock will be reduced with increased capital tax, which decreases output level over time. Moreover, households face higher cost in borrowing, so they run down their debt on impact. Therefore, in stable economy, the agents choose to increase their consumption on impact in expectation of lower long run consumption level, which is less than steady state level at the end of simulation periods.

Figure 3.7 shows response of the economy to the labor income tax shock. With increased labor tax wedge, both high and low volatility economy will reduce its labor supply. However, the difference is in its magnitude. With the same amount of increase in labor tax, stable economy will cut back the labor supply more than the volatile economy. The reason is that when there is large increase in the real interest rate, agents in the volatile economy need to reduce their borrowings and cut the consumption more than those in the stable economy. Therefore, by providing more labor supply and keeping the higher output



Note: Solid line is high volatility and dash line is low volatility.

Figure 3.8: IRFs, Consumption Tax

level on the impact than the other economy, household smooth out the negative shock.

Finally, Figure 3.8 shows the different reaction of the two economies from the consumption tax wedge shock. What contrasts from the former two shocks is that households in both economies increase their investment, whereas with capital and labor income shocks they decreased the amount of investment. It is because capital and increased future output help the household to smooth out their consumption over time, as borrowing and consumption got more expensive. Labor supply decision between the volatile and stable economy shows different direction, as the household in more volatile economy need to bump up their output level by working more on the impact of negative shock.

Overall, the impulse response functions show that the same 1 percentage point increase in tax wedge and one standard deviation real interest rate shock will bring very different reactions from the households across the tax wedges as well as the volatilities in the real interest rate.

Table 3.5: Present Value Government Revenue and Utility

	τ_k	τ_l	τ_c	r
G.Rev				
σ_L	134.6970	134.7912	134.8744	134.5550
σ_H	132.7432	132.8368	132.9192	132.6020
τ	134.9035	134.9977	135.0811	.
Utility				
σ_L	39.2724	39.2713	39.2734	39.2729
σ_H	38.6583	38.6571	38.6590	38.6588
τ	39.3374	39.3362	39.3384	.

Note: τ_x for $x \in \{k, l, c\}$ denotes the IRFs where there were both tax rate and real interest rate shock. r denotes the case where there is no tax shock. σ_L is low volatility economy and σ_H is the high volatility. τ is for only tax rate shock, as in Figure C.5 to C.5. Here, utility is presented after affine transformation of the utility function. That is, for flow utility $u(c, l)$, the figures above are calculated with $\tilde{u}(c, l) = 200 * u(c, l) + 1$.

Turning to the comparison across the impulse response functions in terms of government revenue and welfare, Table 3.5 shows the summarized results. Present value is calculated by discounting from the resulting real interest rate in the impulse response functions for high and low volatility. In the table, results for the no real interest shock and no tax shock is also included. Impulse response functions for the no interest rate shock is in Figure C.5 to C.5 in Appendix.

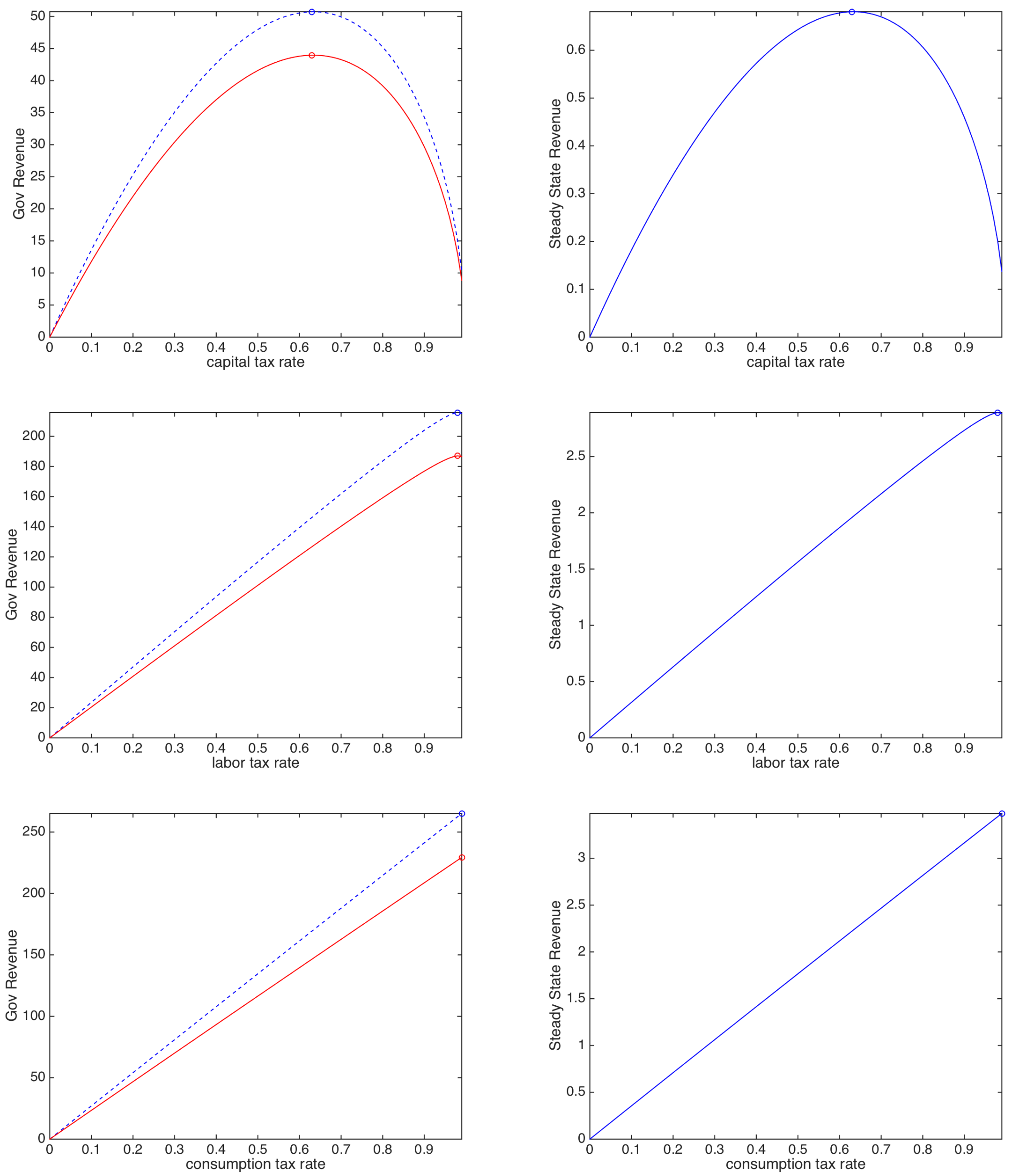
Inspecting the government revenue in Table 3.5, the present value revenue drops around 1.5% across all tax rates. Among them, consumption tax rate records the least drop, 1.4496% in ratios of high volatility to low volatility, as well as in absolute values. Labor and Capital income tax follows in the scale of revenue drop, showing 1.4486% and 1.4505% respectively.

However, high government revenue also yields biggest welfare loss. For the consumption tax increase in high volatility, welfare drops 1.5644% compared to the low volatility. In labor and capital tax, welfare decreases by 1.5640% and 1.5637%, respectively.

Therefore, the government faces the choice between high government revenue and large welfare loss when increasing the tax rates among the volatile real interest rates. Absent real interest rate shock whose volatility is time varying, government revenue collection will be overestimated whereas welfare loss will be underestimated. In the context of European peripheral government that undertook austerity programs, this study tells that household suffered more during the debt crisis for the same tax increase compared to the less volatile periods.

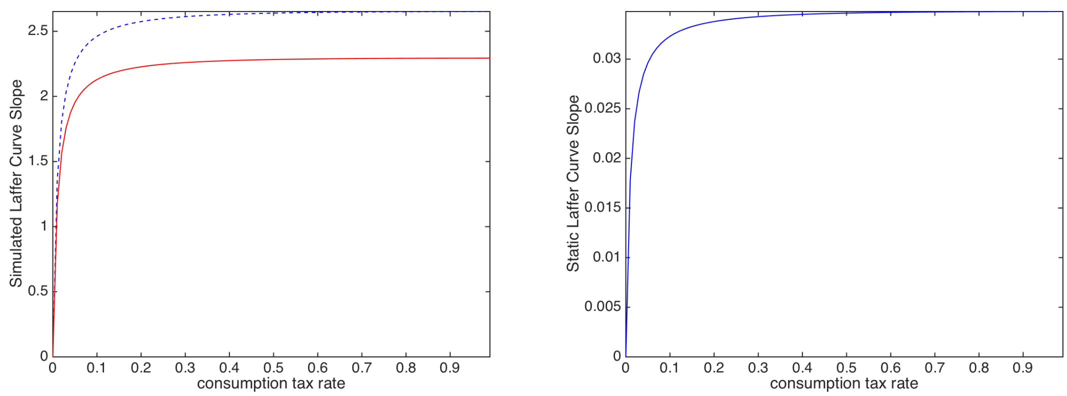
Simulated Laffer Curve In this subsection, I hold the tax wedges fixed at its steady state level and investigate the impact of volatility in government revenue by simulated Laffer curves for each taxes. Each Laffer curve is plotted holding all other taxes at 0, and varying only one tax rate each time. Present value of government revenue is derived from each simulation under different volatility, discounted by the inverse of realized gross real interest rate. Simulated Laffer curve is also compared with traditional steady state Laffer curve for each taxation. However, the simulated Laffer curve and static Laffer curve is not directly comparable as with third order approximation method, simulation value easily gets away from the steady state value. Therefore, the main focus will be the difference between high and low volatility Laffer curve and the shape-preserving characteristic of simulated Laffer curve compared to the static one.

Details of simulation method is as follows. Since the model is solved using third order approximation, the shocks are restricted to be 1 standard deviation as in Fernández-Villaverde et al. (2011a). Perturbation method is performed at the steady state rather than ergodic mean, since the comparison to static Laffer curve is more relevant when simulated around the steady state. Calibration values are as in Table 3.4 and tax grid is set from 0



Note: Laffer curves for capital income (top), labor income (center) and consumption (bottom) tax, where simulated Laffer curves are on left column and static Laffer curves are on right column. Simulated Laffer curves under high volatility is solid line and low volatility is dash line.

Figure 3.9: Laffer Curves



Note: Simulated Laffer curve slope (left) and static Laffer curve slope (right), where dash line on the left is for low volatility economy and solid line is for high volatility one.

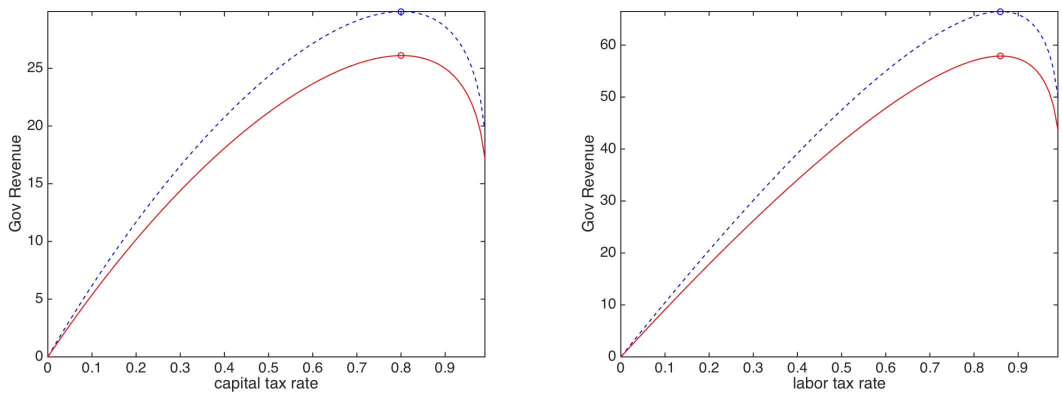
Figure 3.10: Slope of Consumption Tax Laffer Curve

to 99% with 1% steps.

In order to highlight the role of volatility in government revenue in the context of debt crisis, I show in this paper a sample Laffer curve with burn-in period 200 and simulation time period 100, which is 25 years as the model frequency is in quarters. Details of the sample path of simulation including the interest rate process is showed in Figure C.4 in Appendix.

The most curvature of Laffer curve can be observed with capital tax in Figure 3.9. Both simulated and static Laffer curve peaks at 64% of capital tax, which are marked by circles in the graph. Simulated Laffer curve on the left shows that present value government revenue is lower over all tax grids for the higher volatility real interest rate (solid line) than the lower one (dash line). Also, observe that the difference between two economies is the largest at the peak of the simulated Laffer curve.

Next, labor income tax Laffer curve shows monotonic increase of labor tax revenue due to low Frisch elasticity in calibration. Simulated and static Laffer curves all peak at



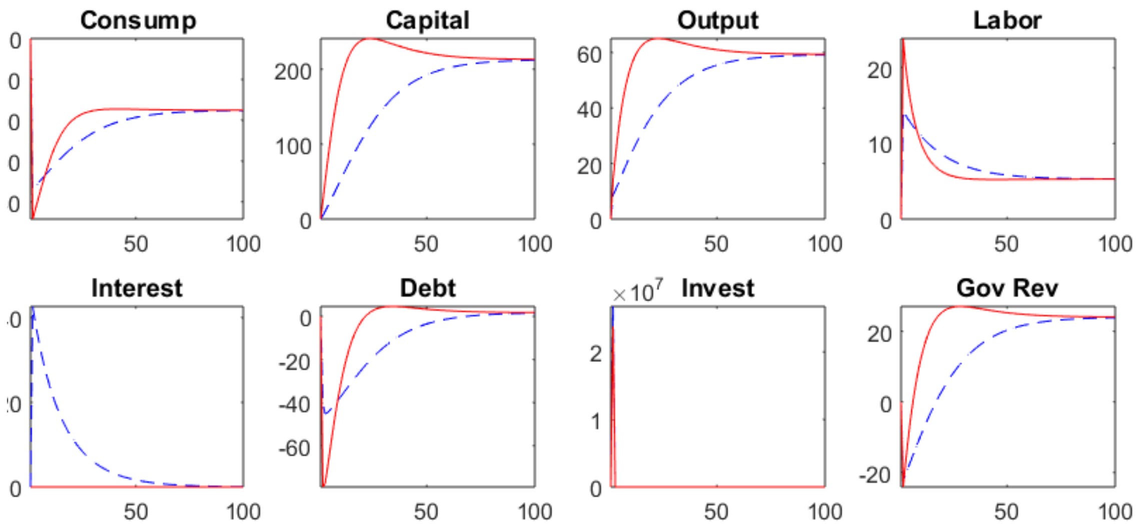
Note: Simulated Laffer curve for capital Income tax (left) and labor income tax (right). Laffer curves under high volatility is in solid line and low volatility is in dash line.

Figure 3.11: Laffer Curves, Frisch Elasticity of 1

99% labor income tax rate. Qualitative characteristics remains the same as the capital tax Laffer curve, such that the difference between the high and low volatility economy gets larger at the peak of the Laffer curve and shape of static Laffer curve is preserved in simulated one.

Finally, consumption tax Laffer curve in the bottom of Figure 3.9 shows seemingly linear line for both static and simulated ones. This is also in line with Razin and Tesar (1994) where it is shown that there is no peak of consumption Laffer curve. However, once inspected closely, Laffer curve for consumption tax is not entirely linear. As Figure 3.10 shows, the slope of each point in the Laffer curve is not constant, lower for tax rates close to 0% and converging as the tax rate is closer to 100%. Therefore, consumption Laffer curve also shows "Laffer hill" for lower tax rate, albeit subtle.

Frisch Elasticity and Laffer Curve In above result, labor income tax Laffer curve is monotone due to the low Frisch elasticity that induces almost inelastic labor supply. Therefore, I present Laffer curves with higher Frisch elasticity, which is set at 1. As can



Note: Volatility shock (solid line) and level shock (dash line)

Figure 3.12: IRFs, level and volatility shock

be seen in Figure 3.11, labor income tax rate shows much variation with higher Frisch elasticity, which peaks at 87% for both static and simulated Laffer curve. Capital income Laffer curve, on the other hand, peaks later with higher Frisch elasticity, at 81% as opposed to 64% in the benchmark case. The main characteristics of the static and simulated Laffer curves are preserved in the higher Frisch elasticity as well. Static Laffer curves for factor income taxes and consumption Laffer curves are presented in Figure C.3 in Appendix.

3.4.3 Effect of Volatility Shock on Government Revenue

In this subsection, I compare the level shock and volatility shock through the impulse response function of each shock. In particular, I closely calibrate to Portugal data using the estimated result in Section 2 and study highlight the role of volatility shock. I abstract from volatility shock in risk free rate and only put stochastic volatility in country shock.

Parameters are calibrated to median value in Table 3.3 for Portugal: autoregressive coefficient for level shock ρ is 0.9379 and for volatility shock ρ_σ is 0.9356. Mean level of volatility shock $\bar{\sigma}$ is -1.0354 and standard deviation of volatility shock is 0.2208. Mean interest is set at 2.18% per annum, which is the mean real interest rate level in Table 3.1. All other parameters are kept as in Table 3.4. Impulse response functions are in percent deviation from the steady state level.

Volatility shock impacts the household decision through the precautionary behavior, without increasing the real interest rate. Compared to the level shock, household run down the debt more, cut back the consumption and work more. Consumption in both level shock and volatility shock decreases on the impact and subsides to the new level. In all household decisions, level shock shows smoother transition compared to the volatility shock. In terms of capital accumulation, volatility shock leads the household to invest more on the impact so that capital stock goes up more than the level shock in the short run. Due to the precautionary motive, capital stock shows hump-shaped transition to the new higher level than the steady state.

As a result of household decision, government revenue decreases on the impact but increase in the long run due to higher capital stock and output level. In the short run, consumption level drops so that government revenue raised from the consumption decreases, lowering the aggregate government revenue. However, since household borrows less, invests more and provides more labor supply, output level goes up and this increases the government revenue from the factor income taxes.

Compared to the previous studies, this model has smooth capital stock transition but the investment decision fluctuates over time. This is mainly because the capital adjustment cost is put on the stock of capital rather than the investment decision. Nevertheless, the other variables such as consumption, labor supply and borrowings show consistent result as in the previous studies[8].

3.5 Conclusion

This paper studied the interaction of tax wedges and volatility shock of real interest rate in a small open economy. Based on European data, I first showed that there is significant degree of time variance in volatility of government bond yields. Then, in the model I showed that the same increase in tax wedge will give different response to the economy when combined with real interest rate shock. Also, higher volatility will lower down the peak of Laffer curve, preserving shape of the curve. After comparing static volatilities, I compared level and volatility shock and showed that their impact on real variables especially in investment decision.

In this paper, I have focused on the mechanisms and dynamics of the model, with calibrated figures outside of the model. For the future research, the quantitative model proposed in this paper can be served as a basic framework for the business cycle accounting in a small open economy. This will enable the decomposition of the wedges that impacted European countries during the debt crisis.

Finally, this paper assumes that real interest rate is subject to exogenous shocks with a stochastic process, not an equilibrium object. It will be interesting to study the interaction of government's decision on taxation and real interest rate that the economy faces. However, it is beyond the scope of this paper to incorporate the feedback effect of bond price to the fiscal policy, and I leave it to the future research.

[8]See Fernández-Villaverde et al. (2011a) Figure 5 for their result of impulse response function comparing level and volatility shocks.

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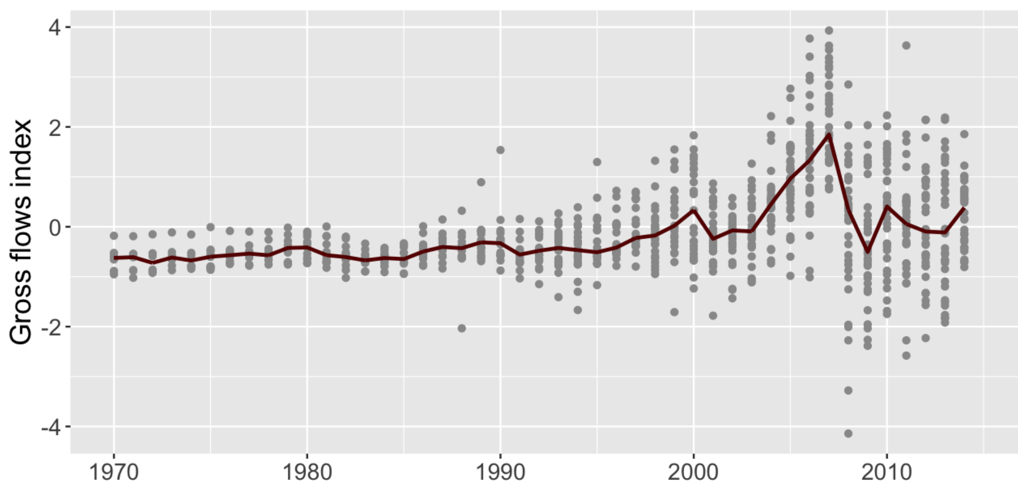
Appendix A

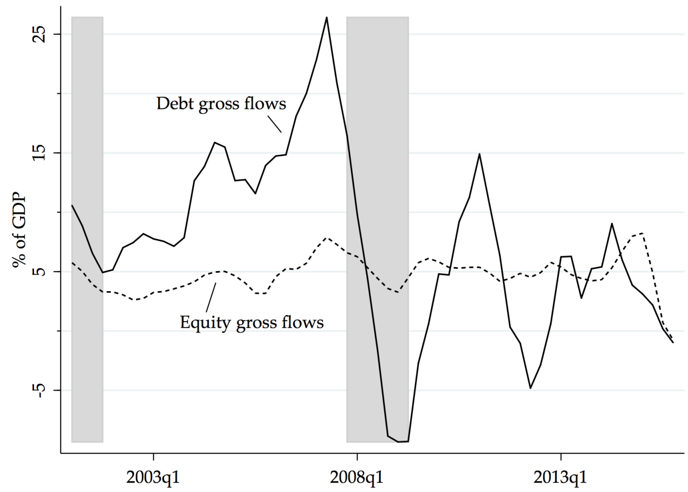
Appendix to Chapter 1

A.1 Additional Figures and Tables



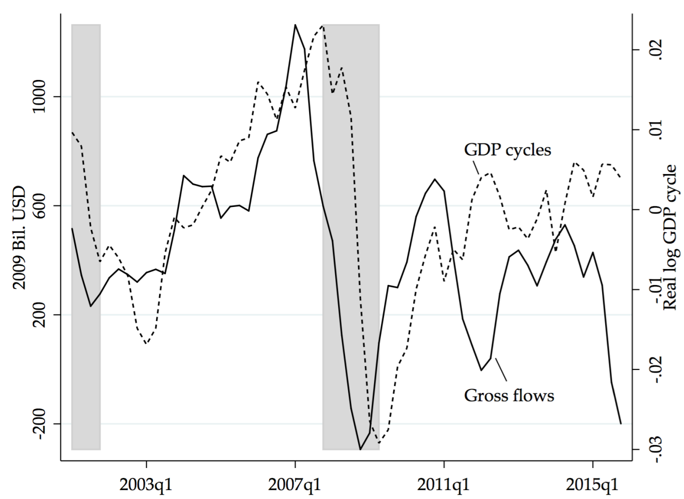
Note: Standardized annual gross flows in 2009 the US dollars from 1985 to 2014. Each dot is standardized gross flows of a country, and solid line is median of each year. Formula for standardization is: $Std.GrossFlows_{i,t} = (GrossFlows_{i,t} - \overline{GrossFlows_i})/\sqrt{Var(GrossFlows_i)}$ where i is country, t is year and $\overline{GrossFlows_i}$ is the sample average of the gross flows of a country over the sample period. Data source IMF Balance Of Payments, BMP6. Panel is unbalanced.

Figure A.1: Gross flows index of OECD countries



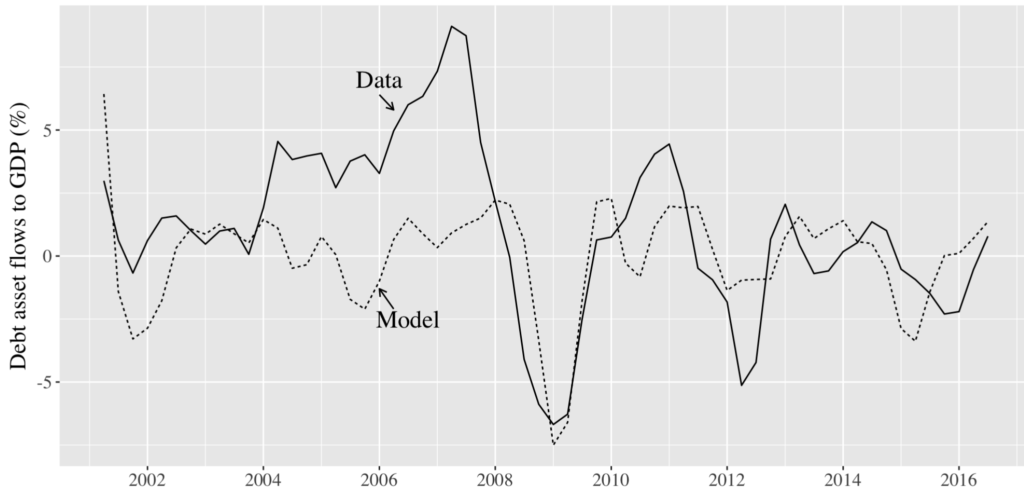
Note: Debt flows are Portfolio debt + Other investments (usually banking flows). Equity flows are Portfolio equity + Direct investments. All series are seasonally adjusted and moving average of 3 quarters. GDP is at quarterly level. Source: Bureau of Economic Analysis, FRED, author's calculations.

Figure A.2: Debt and Equity flows of the US



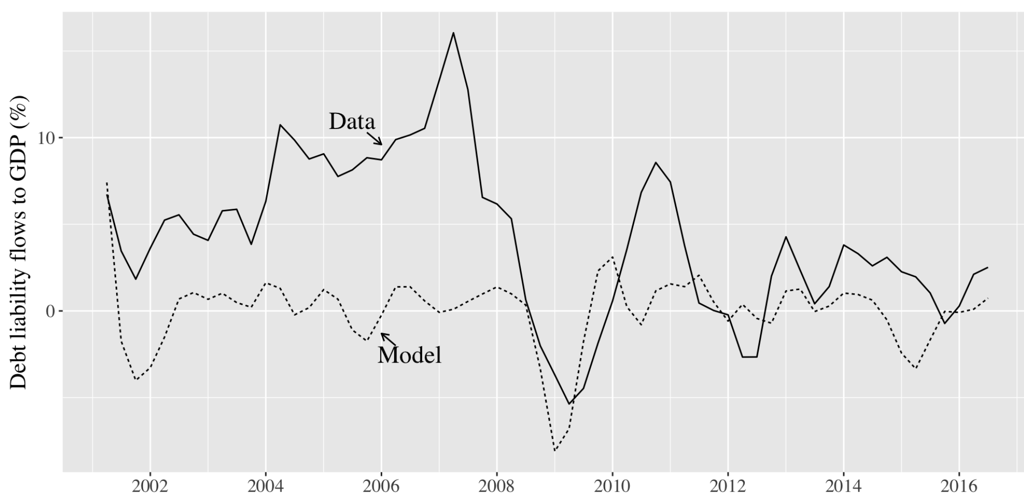
Note: Gross flows are seasonally adjusted and moving average of 3 quarters.HP-filtered GDP cycle, deflated by GDP deflator. Source: Bureau of Economic Analysis, FRED, author's calculations.

Figure A.3: GDP cycles and Gross flows of the US



Note: Both data and model are 3-period moving average, with weights [0.25 0.5 0.25] for period [t-1,t,t+1]. Data is ratio of US debt asset flows (Portfolio debt + Other investment) over GDP, at quarterly level, seasonally adjusted. Source: Bureau of Economic Analysis and author's calculations.

Figure A.4: Simulation result: baseline model and data, asset flows



Note: Both data and model are 3-period moving average, with weights [0.25 0.5 0.25] for period [t-1,t,t+1]. Data is ratio of US debt liability flows (Portfolio debt + Other investment) over GDP, at quarterly level, seasonally adjusted. Source: Bureau of Economic Analysis and author's calculations.

Figure A.5: Simulation result: baseline model and data, liability flows

A.2 Numerical Algorithm

In the following, I denote q_i^j as the bond i price evaluated by household in country j.

Country 1 First order conditions (FOCs) are:

$$u_N^1/u_T^1 = p_N^1 \tag{A.1}$$

$$(\xi_1^1)_- + \beta \sum_{s'} \pi(s,s') u_T^{1\prime} P_1' = u_T^1 P_1 q_1^1 \tag{A.2}$$

$$(\xi_2^1)_- + \beta \sum_{s'} \pi(s,s') u_T^{1\prime} P_2' = u_T^1 P_2 q_2^1 \tag{A.3}$$

$$a_1^{1\prime} + \chi = (\xi_1^1)_+ \tag{A.4}$$

$$a_2^{1\prime} + \chi = (\xi_2^1)_+ \tag{A.5}$$

where ξ_i^j is a Lagrangian multiplier of borrowing constraint on bond i in country j household. Also, $(\xi_i^j)_- = \max(0, -\xi_i^j)$, and $(\xi_i^j)_+ = \max(0, \xi_i^j)$.

Analogously, country 2 FOCs are:

$$u_N^2/u_T^2 = p_N^2 \tag{A.6}$$

$$(\xi_1^2)_- + \beta \sum_{s'} \pi(s,s') u_T^{2\prime} P_1' = u_T^2 P_1 q_1^2 \tag{A.7}$$

$$(\xi_2^2)_- + \beta \sum_{s'} \pi(s,s') u_T^{2\prime} P_2' = u_T^2 P_2 q_2^2 \tag{A.8}$$

$$a_1^{2\prime} + \chi = (\xi_1^2)_+ \tag{A.9}$$

$$a_2^{2\prime} + \chi = (\xi_2^2)_+ \tag{A.10}$$

Use goods market clearing conditions:

$$c_N^1 = y_N^1 \tag{A.11}$$

$$c_N^2 = y_N^2 \tag{A.12}$$

Use financial market clearing conditions:

$$a_1^{1\prime} + a_1^{2\prime} = 0 \tag{A.13}$$

$$a_2^{1\prime} + a_2^{2\prime} = 0 \tag{A.14}$$

Finally, use equilibrium condition that individual policy and net wealth fractions are equal to the aggregate ones, $w = W$.

Then, 7 by 7 non-linear equations are:

$$q_1^1(W(s), s) = q_1^2(W(s), s) \tag{A.15}$$

$$q_2^1(W(s), s) = q_2^2(W(s), s) \tag{A.16}$$

$$c_T^1(W(s), s) + \sum_{i=1}^{2} P_i(W(s), s) q_i(W(s), s) a_i^{1\prime}(W(s), s) \leq W(s) \cdot 2y_T(s) \tag{A.17}$$

$$a_1^{1\prime}(W(s), s) + \chi = (\xi_1^1(W(s), s))_+ \tag{A.18}$$

$$a_2^{1\prime}(W(s), s) + \chi = (\xi_2^1(W(s), s))_+ \tag{A.19}$$

$$a_1^{2\prime}(W(s), s) + \chi = (\xi_1^2(W(s), s))_+ \tag{A.20}$$

$$a_2^{2\prime}(W(s), s) + \chi = (\xi_2^2(W(s), s))_+ \tag{A.21}$$

where 7 unknowns are $\{a_1^{1\prime}, a_2^{1\prime}, c_T^1, \xi_1^1, \xi_2^1, \xi_1^2, \xi_2^2\}$.

Notice that in order to calculate bond prices, the mapping Γ of current $(W(s), s)$ to the next period's $(W(s'), s')$ and policy function of consumptions are needed. In Time iteration method, future consumption policy function is taken as given from the result of previous iteration. Also, any mapping Γ should satisfy the following equation for each future state.

$$W(s') = \frac{y_T(s') + \sum_{i=1}^{2} P_i(s', W(s'))a_i^{1'}(W(s), s)}{2y_T(s')}, \ \forall s, s' \in S \tag{A.22}$$

I used Brent algorithm to find $W(s')$ which minimizes squared error within the interval of min and max of net wealth fraction grid, and later verify the tolerance of above equation as the system reaches equilibrium.

A.3 Proofs of Complete markets: risk sharing intuition

Proposition 4. *If constant elasticity of substitution between non-tradable and tradable goods (σ) multiplied by constant risk aversion parameter (γ) is larger than 1 ($\sigma\gamma > 1$), then tradable goods are gross substitutes to non-tradable goods. If $\sigma\gamma < 1$, then they are gross complements.*

Proposition 5. *For any non-tradable endowment $\{y_N^1, y_N^2\}$, social planner's allocation of tradable goods (c_T^{1*}, c_T^{2*}) equalizes marginal utilities of two countries with respect to tradable goods.*

$$u_T^1(c_T^{1*}, y_N^1) = u_T^2(c_T^{2*}, y_N^1) \tag{A.23}$$

Corollary 4. *If $\sigma\gamma > 1$ and $y_N^1 \leq y_N^2$, then $c_T^{1*} \geq c_T^{2*}$.*

Proof. First order necessary condition of social planner equalizes marginal utility of tradable consumption between two countries.

$$u_T^1(c_1^*(s)) = u_T^2(c_2^*(s)) \tag{A.24}$$

where $u_T^i(c_i(s))$ is a marginal utility of country i at state s w.r.t. tradable goods:

$$u_T^1(c_1(s)) = c_1(s)^{-\gamma+1/\sigma}(\omega_T/c_T^1(s))^{1/\sigma} \tag{A.25}$$

$$u_T^2(c_2(s)) = c_2(s)^{-\gamma+1/\sigma}(\omega_T/c_T^2(s))^{1/\sigma} \tag{A.26}$$

Given that utility functions are identical across countries (all parameters γ, σ, ω_T are same for both countries), social planner's FOC reduces to the following equation.

$$(c_1(s)/c_2(s))^{-\sigma\gamma+1} = c_T^1(s)/c_T^2(s) \tag{A.27}$$

Other things equal, if non-tradable consumption of country 1 goes down (negative shock on non-tradable consumption), then the left hand side of above equation increases if and only if $-\sigma\gamma+1 < 0$. Therefore, in order to satisfy the above equation, tradable consumption of country 1 needs to go up with the shock. This indicates that if non-tradables and tradables are gross substitutes ($-\sigma\gamma + 1 < 0$), then social planner allocates more tradables to the country with less non-tradable endowments. If ($-\sigma\gamma+1 > 0$), then the tradable and non-tradables are gross complements and the country with lower non-tradable consumption is allocated with less tradable goods by the social planner. □

Proposition 6. *Under the condition that first best aggregate price index is positive and not the same across countries, $P_1^*(s_1)/P_2^*(s_1) \neq 1$, there is a unique bond portfolio $a^* = (a_1^{1*}, a_2^{1*})'$ that decentralizes social planner's problem in all states. Specifically,*

$$a^* = \begin{bmatrix} P_1^*(s_1)(1-q_1^*(s_1)) & P_2^*(s_1)(1-q_2^*(s_1)) \\ P_1^*(s_2)(1-q_1^*(s_2)) & P_2^*(s_2)(1-q_2^*(s_2)) \end{bmatrix}^{-1} \begin{bmatrix} c_T^{1*}(s_1) - \bar{y}_T \\ c_T^{1*}(s_2) - \bar{y}_T \end{bmatrix} \tag{A.28}$$

$$= \frac{c_T^{1*}(s_2) - \bar{y}_T}{\tilde{P}_1(s_2) - \tilde{P}_2(s_2)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{A.29}$$

where $\tilde{P}_i(s_j) = P_i^*(s_j)(1 - q_i^*(s_j)) = P_i^*(s_j) - 0.5\beta \sum_{j=1,2} P_i^*(s_j),\ i, j = 1, 2.$

Corollary 5. *If a^* exists, then the amount of domestic saving and foreign borrowing is the same. $a_1^{1*} = -a_2^{1*}$. Net position $a_1^{1*} + a_2^{1*}$ is 0.*

Corollary 6. *If a^* exists and $\sigma\gamma > 1$, then $a_1^{1*} > 0 > a_2^{1*} = -a_1^{1*}$.*

Proof. First order conditions of a consumer's problem:

$$p_N^1 = u_N(c_1)/u_T(c_1) = (\omega_N/c_N^1)^{1/\sigma}/(\omega_T/c_T^1)^{1/\sigma} \tag{A.30}$$

$$p_N^2 = u_N(c_2)/u_T(c_2) = (\omega_N/c_N^2)^{1/\sigma}/(\omega_T/c_T^2)^{1/\sigma} \tag{A.31}$$

$$q_1^1(s) \equiv \frac{\beta \sum_{s'} \pi(s, s') u_T^1(s') P_1(s')}{u_T^1(s) P_1(s)} \tag{A.32}$$

$$q_2^2(s) \equiv \frac{\beta \sum_{s'} \pi(s, s') u_T^2(s') P_2(s')}{u_T^2(s) P_2(s)} \tag{A.33}$$

Rewriting the budget constraint, for any state $s', s \in \{s_1, s_2\}$, the following should hold:

$$\begin{bmatrix} P_1(s') & P_1(s') \end{bmatrix} \begin{bmatrix} a_1^1(s) \\ a_2^1(s) \end{bmatrix} \tag{A.34}$$

$$= \left[c_T^1(s') + p_N^1(s') c_N^1(s') + \sum_{i=1}^2 P_i(s') q_i(s') a_i^1(s') - \bar{y}_T^1 - p_N^1(s') y_N^1(s') \right] \tag{A.35}$$

$$\begin{bmatrix} P_1(s') & P_2(s') \end{bmatrix} \begin{bmatrix} a_1^1(s) \\ a_2^1(s) \end{bmatrix} \tag{A.36}$$

$$= \begin{bmatrix} c_T^1(s') + p_N^1(s')c_N^1(s') - \bar{y}_T^1 - p_N^1(s')y_N^1(s') \end{bmatrix} + \begin{bmatrix} P_1(s')q_1(s') & P_2(s')q_2(s') \end{bmatrix} \begin{bmatrix} a_1^1(s') \\ a_2^1(s') \end{bmatrix} \tag{A.37}$$

For complete market, there are four states in total.

Simplifying notations and applying market clearing condition of NT:

$$A(s) \equiv \begin{bmatrix} P_1(s) & P_2(s) \end{bmatrix} \tag{A.38}$$

$$B(s) \equiv \begin{bmatrix} c_T^1(s) - \bar{y}_T^1 \end{bmatrix} \tag{A.39}$$

$$C(s) \equiv \begin{bmatrix} P_1(s)q_1(s) & P_2(s)q_2(s) \end{bmatrix} \tag{A.40}$$

$$X(s) \equiv \begin{bmatrix} a_1^1(s_i) & a_2^1(s_i) \end{bmatrix}' \tag{A.41}$$

Then, 2 (today's states) x 2 (tomorrow's states) = 4 budget constraints are:

$$A(s')X(s) = B(s') + C(s')X(s'),\ \forall s', s \in \{s_1, s_2\} \tag{A.42}$$

Probability of each state is same as half.

$$\pi(s_1) = \pi(s_2) = 0.5 \tag{A.43}$$

Conjecture that $X(s_j) = X(s_i) = X^*,\ \forall i, j$

Then, above equation is

$$A(s_j)X^* = B(s_j) + C(s_j)X^* \tag{A.44}$$

Define a matrix of coefficients

$$A = \begin{bmatrix} A(s_1) \\ A(s_2) \end{bmatrix},\ B = \begin{bmatrix} B(s_1) \\ B(s_2) \end{bmatrix},\ C = \begin{bmatrix} C(s_1) \\ C(s_2) \end{bmatrix} \tag{A.45}$$

Then

$$AX^* = B + CX^* \tag{A.46}$$

Therefore, if $(A - C)$ is invertible, then

$$X^* = (A - C)^{-1}B \tag{A.47}$$

This implies that in an environment where $(A - C)$ is invertible, there's no gross flows.

Under the social planner's allocation, two countries' marginal utilities w.r.t. tradable good are the same. That is, $u_T(c_1^*(s)) = u_T(c_2^*(s))$ for any s. Then, due to the symmetry, define $u_T^* \equiv u_T(c_1^*(s_1)) = u_T(c_2^*(s_1)) = u_T(c_1^*(s_2)) = u_T(c_2^*(s_1))$.

Moreover, due to symmetry, $P_1^*(s_1) = P_2^*(s_2)$ and $P_2^*(s_1) = P_1^*(s_2)$. Define $P^*(H) \equiv P_1^*(s_1) = P_2^*(s_2)$ and $P^*(L) \equiv P_2^*(s_1) = P_1^*(s_2)$. Analogous for bond prices.

Then, plugging in the symmetry to the bond price FOC, define $\beta\bar{P} \equiv P_1^*(s_1)q_1^*(s_1) = P_2^*(s_1)q_2^*(s_1) = P_1^*(s_2)q_1^*(s_2) = P_2^*(s_2)q_2^*(s_2) = \beta[\pi(s_1)u_T^* P^*(H) + \pi(s_2)u_T^* P^*(L)]/u_T^* = \beta \cdot 0.5 \cdot [P^*(H) + P^*(L)]$.

Define prices after subtracting discounted mean.

$$\tilde{P}(H) = P^*(H) - \beta\bar{P},\ \tilde{P}(L) = P^*(L) - \beta\bar{P} \tag{A.48}$$

Rewriting the matrix,

$$A - C = \begin{bmatrix} P_1(s_1)(1 - q_1(s_1)) & P_2(s_1)(1 - q_2(s_1)) \\ P_1(s_2)(1 - q_1(s_2)) & P_2(s_2)(1 - q_2(s_2)) \end{bmatrix} \tag{A.49}$$

$$= \begin{bmatrix} P(H)(1 - q(H)) & P(L)(1 - q(L)) \\ P(L)(1 - q(L)) & P(H)(1 - q(H)) \end{bmatrix} = \begin{bmatrix} \tilde{P}(H) & \tilde{P}(L) \\ \tilde{P}(L) & \tilde{P}(H) \end{bmatrix} \tag{A.50}$$

$$det(A - C) = \tilde{P}(H)^2 - \tilde{P}(L)^2 \tag{A.51}$$

If $det(A - C)$ is not 0, then $A - C$ is invertible.

Then, asset positions are:

$$X^* = (A - C)^{-1}B \tag{A.52}$$

$$= \frac{1}{det(A - C)} \begin{bmatrix} \tilde{P}(H) & -\tilde{P}(L) \\ -\tilde{P}(L) & \tilde{P}(H) \end{bmatrix} \begin{bmatrix} c_T^*(H) - \bar{y}_T \\ c_T^*(L) - \bar{y}_T \end{bmatrix} \tag{A.53}$$

$$= \frac{1}{\tilde{P}(H)^2 - \tilde{P}(L)^2} \begin{bmatrix} \tilde{P}(H)(c_T^*(H) - \bar{y}_T) - \tilde{P}(L)(c_T^*(L) - \bar{y}_T) \\ \tilde{P}(H)(c_T^*(L) - \bar{y}_T) - \tilde{P}(L)(c_T^*(H) - \bar{y}_T) \end{bmatrix} \tag{A.54}$$

$$= \frac{c_T^*(L) - \bar{y}_T}{\tilde{P}(L) - \tilde{P}(H)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{A.55}$$

where the last equality comes from the symmetry that $c_T^*(L) - \bar{y}_T = -(c_T^*(H) - \bar{y}_T)$.

This implies that as the distance of $\tilde{P}(H)$ and $\tilde{P}(L)$ goes up, then the positions go down. On the other hand, if differences of High and Low consumption goes up, absolute value of positions increase.

Intuition is that excess returns insure the risk. Consumers need to take smaller hedging positions when higher excess returns insure risks. On the other hand, differences in optimal consumption increases, then the need for hedging also goes up other things being equal.

Define $\tilde{c}_T(H) \equiv c^*_T(H) - \bar{y}_T,\ \tilde{c}_T(L) \equiv c^*_T(L) - \bar{y}_T$

To see if the positions sum up to 0,

$$\tilde{P}(H)\tilde{c}_T(H) - \tilde{P}(L)\tilde{c}_T(L) + (\tilde{P}(H)\tilde{c}_T(L) + \tilde{P}(L)\tilde{c}_T(H)) \tag{A.56}$$

$$=\tilde{P}(H)(\tilde{c}_T(H) + \tilde{c}_T(L)) - \tilde{P}(L)(\tilde{c}_T(H) + \tilde{c}_T(L)) \tag{A.57}$$

$$=(\tilde{c}_T(H) + \tilde{c}_T(L))(\tilde{P}(H) - \tilde{P}(L)) \tag{A.58}$$

$$=0 \tag{A.59}$$

since $\tilde{c}_T(H) + \tilde{c}_T(L) = c^*_T(H) + c^*_T(L) - 2\bar{y}_T = 0$ from feasibility constraint.

□

A.4 Data: Calibration targets

- Consumption
  - Used in: ω_N
  - Calibration target is average of US and EU.

- Source (US): Bureau of Economic Analysis (BEA), National Income and Product Account (NIPA), Table 1.1.4. "Price Indexes for Gross Domestic Product", Goods (DGDSRG3), Services (DSERRG3). Quarterly data, seasonally adjusted. For quantity measures, Table 1.1.3. "Real Gross Domestic Product, Quantity Indexes". Goods (DGDSRA3), Services (DSERRA3), Services: of which on Housing and utilities (DHUTRA3), Exports (B020RA3), and Imports (B021RA3). Quarterly data, seasonally adjusted. For nominal values, Table 1.1.5. for corresponding variables are used. All series at annual rates.
- Source (EU): Eurostat, Quarterly National Account, GDP and main components (namq_10_gdp), Final consumption aggregates (namq_10_fcs), and Exports and imports by Member States of the EU/thrid countries (namq_10_exi). Seasonally and calendar adjusted. At quarterly rates. Base year 2010, used current prices and chain linked volumes.
- Sample period: (US) 1947 Q1 to 2016 Q2, (EU): 1999 Q1 - 2016 Q4
- Note on EU data construction:
  * I construct aggregate price index of goods (p_t) by weighted geometric average of consumption in each country.

$$\log p_t = \sum_{ij} \omega_{ijt} \log p_{ijt} \tag{A.60}$$

    where i is country, j is goods durability, and t is time. p_{ijt} is price index of each country/durability/time, calculated from dividing nominal consumption by chain linked volumes. Weight ω_{ijt} is defined as:

$$\omega_{ijt} = \frac{c_{ijt}}{\sum_{ij} c_{ijt}} \tag{A.61}$$

where c_{ijt} is nominal consumption of country i, durability j, time t.

* I use only the countries with complete consumption data on Eurostat. List of countries are: Estonia, Finland, France, Germany, Italy, Latvia, Luxembourg, Netherlands.

* I calculate imports from non-EU member countries in order to calculate share of foreign goods to total consumption on goods. Among the 8 countries with consumption data (listed in above bullet point), some countries were missing data (Germany and Finland, and other countries for parts of sample years). I take an average over the total sample period (average share of EU imports to total imports is 0.5452) and multiply to the total imports.

- Sectoral output
  - Used in: $\rho_T, \ \sigma_T, \ \rho_N, \ \sigma_N$
  - Source (US): Bureau of Economic Analysis, National Income and Product Account, Table 6.1D. National Income Without Capital Consumption Adjustment by Industry.
  - Source (EU): OECD.Stat, GDP at market prices - output approach.
  - Sample period: 2001Q1 - 2016Q2.
  - Tradable and non tradable sectors by industry:

* US industry classifications are based on the 2002 North American Industry Classification System (NAICS). EU 28 classifications are based on International Standard Industrial Classification of All Economic Activities, Rev.4 (ISIC Rev.4).

* US Tradables: Agriculture, Mining, Manufacturing, Wholesale trade, Transportation and warehousing, Information.

* US nontradables: Utilities, Construction, Retail trade, Professional and business services, Educational services, health care, and social assistance, Arts, entertainment, recreation, accommodation, and food services, Other services, except government.

* EU28 Tradables: Industry, including energy, Information and communication, Agriculture, forestry and fishing

* EU28 nontrdables: Other service activities, Prof., scientif., techn. activ.; admin., support service activ., Construction, Public admin.; compulsory s.s.; education; human health, Distrib. trade, repairs; transp.; accommod., food serv. activ.

- Estimation is by HP-filtering of smoothing parameter 1600 for all log values.

- Persistence and standard deviation of non tradables are from US series. For tradable series, EU28 data is converted into US dollars and then estimated average dollar tradable output of two countries. All data are deflated with US GDP deflator. (GDP deflator source: U.S. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator [GDPDEF], retrieved from FRED, Federal Reserve Bank of St. Louis, USD/EUR exchange rate source: Board of Governors of the Federal Reserve System (US), U.S. / Euro Foreign Exchange

Rate [DEXUSEU], retrieved from FRED, Federal Reserve Bank of St. Louis, simple average is taken for quarterly exchange rate).

– For sectoral output, following sections in Appendix show various ways of estimation.

A.5 Data: CPI

A.5.1 US CPI

- Source: Bureau of Labor Statistics (BLS), Consumer Price Index (CPI). All items (CUSR0000SA0, All items in U.S. city average, all urban consumers, seasonally adjusted, 1947 M01 - 2017 M07). Services (CUSR0000SAS, Services in U.S. city average, all urban consumers, seasonally adjusted, 1956 M01 - 2017 M07). Commodities (CUSR0000SAC, Commodities in U.S. city average, all urban consumers, seasonally adjusted, 1956 M01 - 2017 M07). Monthly data.
- Base year 1982-84=100

A.5.2 EA19 CPI

- Source: Eurostat, Harmonized Index of Consumer Prices (HICP, prc_hicp_midx), All items (CP00), Goods (GD), and Services (SERV). Not seasonally adjusted monthly data.
- Seasonally adjusted using R-package *seasonal*, based on X-13ARIMA-SEATS
- Base year 2015 = 100
- Sample period 2001 M01 - 2016 M12

A.5.3 Real exchange rate calculation

- Source: OECD, Consumer prices - all items (CPALTT01). Not seasonally adjusted monthly data. FRED, Dollar - Euro daily exchange rate (DEXUSEU). Sample period Jan 4 1999 - Aug 4 2017.
- Seasonally adjusted using R-package *seasonal*, based on X-13ARIMA-SEATS
- Monthly exchange rate is derived as a simple average of daily series into monthly frequency.
- Base year 2010 = 100
- Sample period 2001 M01 - 2017 M06

A.5.4 Math formula

$$\text{Relative price} = \frac{\text{price of non-tradables (services)}}{\text{price of tradables (goods)}} \tag{A.62}$$

$$\text{Ratio of relative price} = \frac{\text{Relative price of EA19}}{\text{Relative price of US}} \tag{A.63}$$

$$\text{Real exchange rate} = \text{Nominal exchange rate of USD/EUR} \cdot \frac{\text{CPI of EA19}}{\text{CPI of US}} \tag{A.64}$$

Appendix B

Appendix to Chapter 2

B.1 Numerical Algorithm

In the stochastic collateral constraint model, the basic algorithm is same as in Chapter 1 but additional first order conditions w.r.t. domestic equities and Lagrangian multipliers for the collateral constraints are added.

First order conditions of the country 1 household:

$$u_N^1/u_T^1 = p_N^1 \tag{B.1}$$

$$(\xi_1^1)_- P_1 q_1 + \beta \sum_{s'} \pi(s,s') {u_T^1}' P_1' = u_T^1 P_1 q_1 \tag{B.2}$$

$$(\xi_2^1)_- P_2 q_2 + \beta \sum_{s'} \pi(s,s') {u_T^1}' P_2' = u_T^1 P_2 q_2 \tag{B.3}$$

$$\beta \sum_{s'} \pi(s,s') [{u_T^1}' + {(\xi_1^1)_-}' \chi_1^1 + {(\xi_2^1)_-}' \chi_2^1] {p_N^1}' (Q_1' + d_1') = u_T^1 p_N^1 Q_1 \tag{B.4}$$

$$P_1 q_1 {a_1^1}' + \chi_1^1 p_N^1 Q_1 = (\xi_1^1)_+ \tag{B.5}$$

$$P_2 q_2 {a_2^1}' + \chi_2^1 p_N^1 Q_1 = (\xi_2^1)_+ \tag{B.6}$$

$$(\xi_1^1)_- = \max(0, -\xi_1^1) \tag{B.7}$$

$$(\xi_1^1)_+ = \max(0, \xi_1^1) \tag{B.8}$$

$$(\xi_2^1)_- = \max(0, -\xi_2^1) \tag{B.9}$$

$$(\xi_2^1)_+ = \max(0, \xi_2^1) \tag{B.10}$$

FOC of country 2 households are analogous. Then, the system of equations are:

$$q_1^1 = q_1^2 \tag{B.11}$$

$$q_2^1 = q_2^2 \tag{B.12}$$

$$c_T + p_N^1 c_N^1 + \sum_{i=1}^{2} P_i q_i {a_i^1}' + p_N^1 Q_1 {\theta_1}' \leq y_T + p_N^1 y_N + \sum_{i=1}^{2} P_i a_i^1 + p_N^1 (Q_1 + d_1) \theta_1 \tag{B.13}$$

$$P_1 q_1 {a_1^1}' + \chi_1^1 p_N^1 Q_1 = (\xi_1^1)_+ \tag{B.14}$$

$$P_2 q_2 {a_2^1}' + \chi_2^1 p_N^1 Q_1 = (\xi_2^1)_+ \tag{B.15}$$

$$P_1 q_1 {a_1^2}' + \chi_1^2 p_N^2 Q_2 = (\xi_1^2)_+ \tag{B.16}$$

$$P_2 q_2 {a_2^2}' + \chi_2^2 p_N^2 Q_2 = (\xi_2^2)_+ \tag{B.17}$$

B.2 Collateral constraint and default renegotiation

At the end of each period, after portfolio choice for the next period is made and before the repayment of bonds, borrowers decide whether to default or not. If a person defaults, then creditors have rights to seize the land that borrower owns. Assume that at the time of liquidation, land could be converted into tradable goods with some probability, in the spirit of Jermann and Quadrini (2012).

With probability χ, the value of land is the market value of land $p_N^i Q_i$ when converted into tradable goods, where i is the country of borrower. With probability $1-\chi$, the land does not have any value in tradables. Both parties enter renegotiation process in case of a default, and I assume that borrower has all the renegotiation power. In the following, I derive a collateral constraint where borrower is indifferent from defaulting on the bond and keeping the promise.

First, surplus of renegotiation on borrower's side when the value of land is $\kappa p_N^1 Q_i$ in tradable goods is equal to:

$$\sum_{s'} \pi(s'|s) V_i(w(s'); s', W(s')) - \sum_{j, a_j^i < 0} P_j q_j a_j^i - p_N^1 Q_i \tag{B.18}$$

where $a_j^i < 0$ is amount that borrower i owes in bond j.

In case that the land has zero value in tradable, renegotiation surplus is:

$$\sum_{s'} \pi(s'|s) V_i(w(s'); s', W(s')) - \sum_{j, a_j^i < 0} P_j q_j a_j^i \tag{B.19}$$

Then, in expectation, renegotiation value for borrower is:

$$\sum_{s'} \pi(s'|s) V_i(w(s'); s', W(s')) - \sum_{j, a_j^i < 0} P_j q_j a_j^i - \chi p_N^1 Q_i \tag{B.20}$$

For the borrower to be indifferent between defaulting and repaying the debt, value of non defaulting should be at least as big as expected renegotiation value.

$$\sum_{s'} \pi(s'|s) V_i(w(s'); s', W(s')) \geq \sum_{s'} \pi(s'|s) V_i(w(s'); s', W(s')) - \sum_{j, a_j^i < 0} P_j q_j a_j^i - \chi p_N^1 Q_i \tag{B.21}$$

Above inequality is equal to the collateral constraint after rearrangement.

Appendix C

Appendix to Chapter 3

Steady State Laffer Curve Steady state Laffer curve is characterized by the following equations.

$$\frac{\psi l^{\eta+1}}{(1-\alpha)y} = \frac{1-\tau_l}{1+\tau_c} c^{-\nu} \tag{C.1}$$

$$y = k^{\alpha} l^{1-\alpha} \tag{C.2}$$

$$1 = \beta[1 - \delta + (1-\tau_k)\alpha y/k] \tag{C.3}$$

$$c + \delta k + g = y + (1/(1+\bar{r}) - 1)\bar{d} \tag{C.4}$$

Sample Path of Simulation In order to understand how the model economy behaves under different volatilities of level shock, I compare two sample economies. One economy is given with high volatility σ_H and the other is given with low volatility, σ_L. Illustration of the sample path is given in Figure C.4.

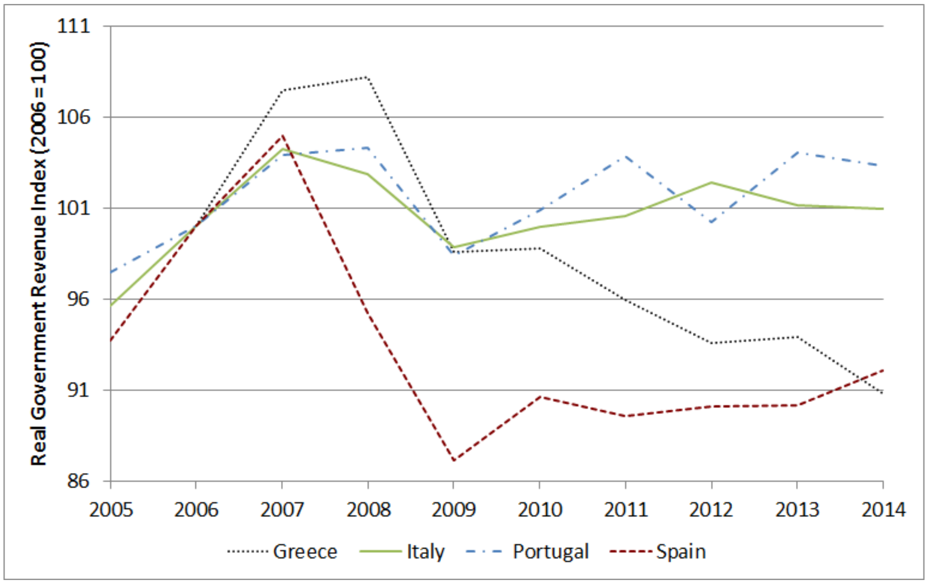


Figure C.1: Real Total General Government Revenue Index

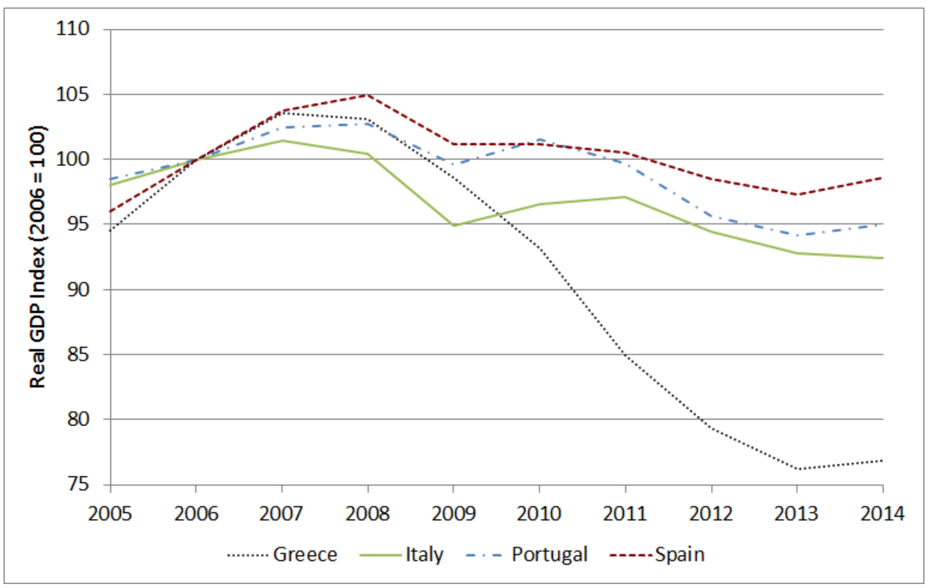


Figure C.2: Real GDP Index

Note: Real total general government revenue and real GDPs are from Eurostat. Index is calculated by the author.

Impulse Response Function I present the impulse response function when there is only tax rate shock in Figures C.5 to C.5. These figures will complement Section 4.2. Also, I present the present value of government revenue and utility for each impulse response function in Table 3.5.

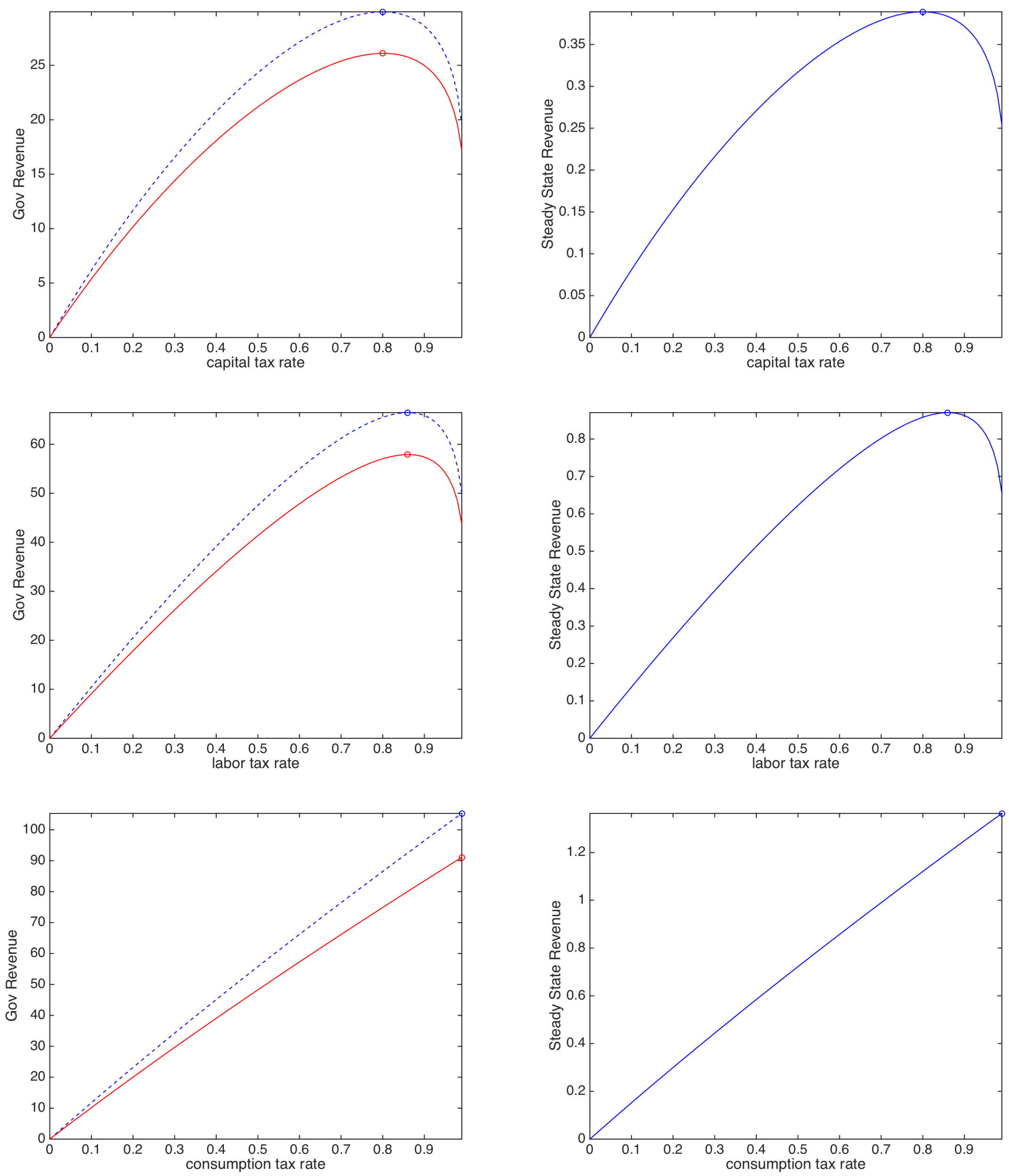
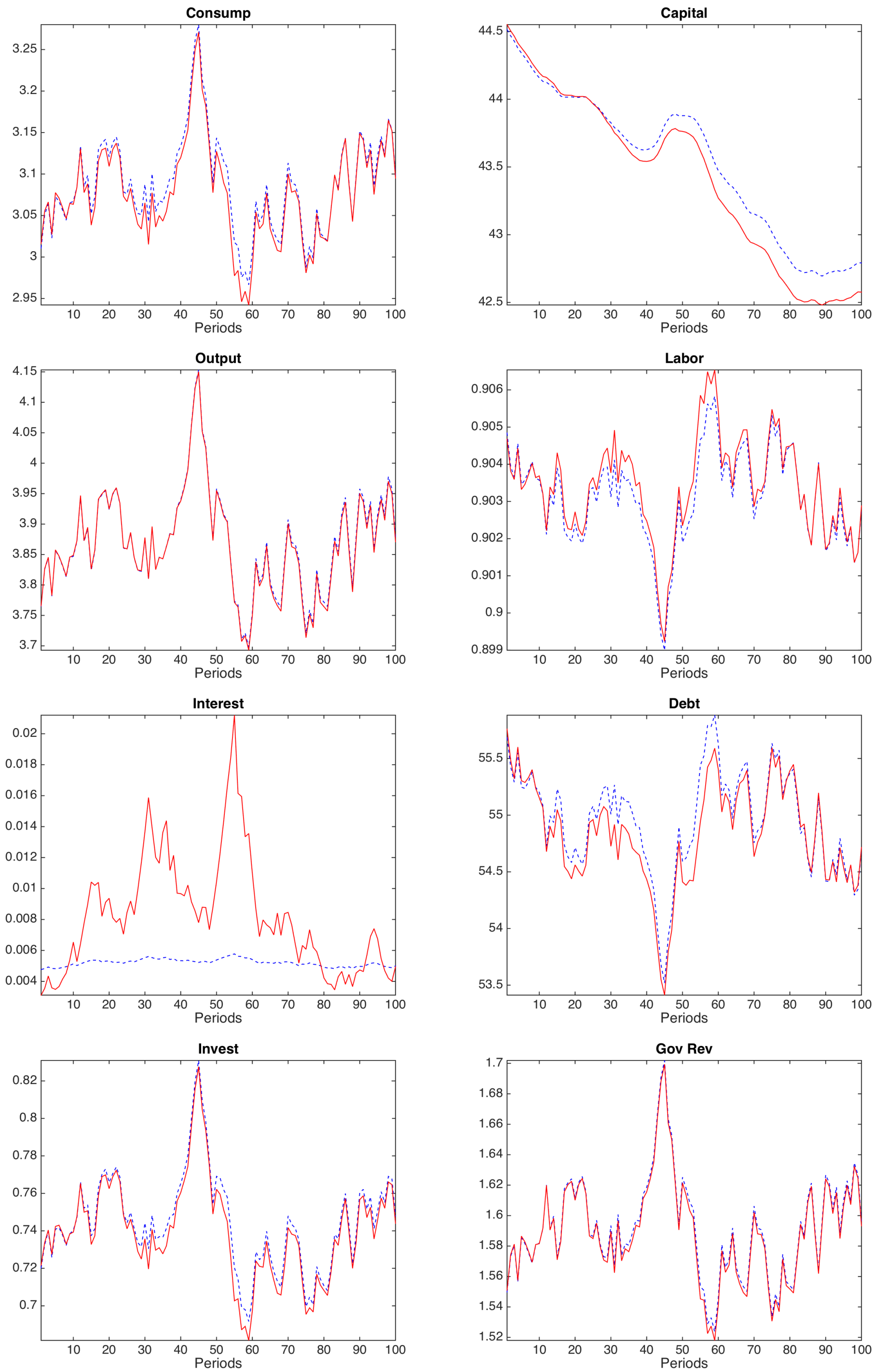


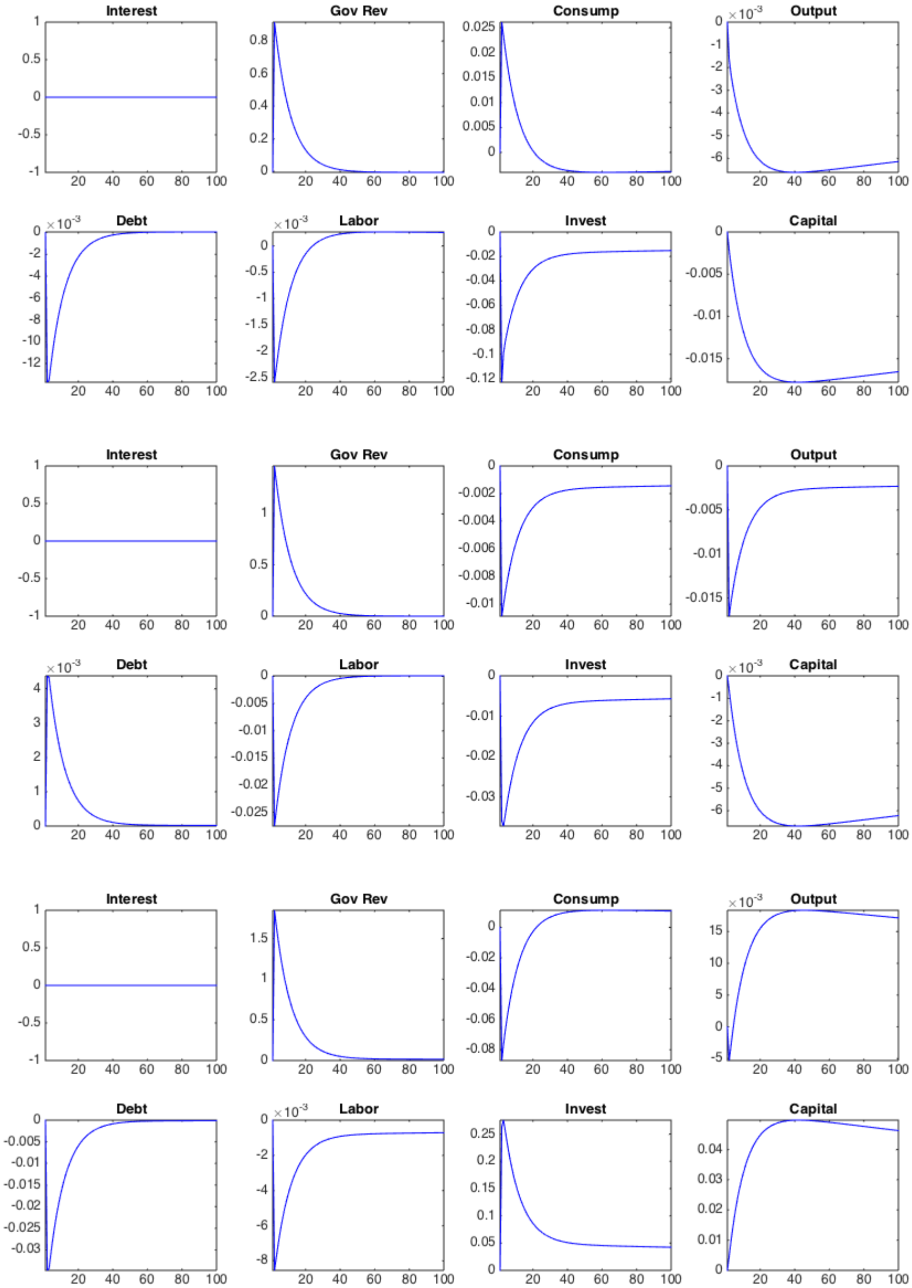
Figure C.3: Laffer Curve, Frisch Elasticity of 1

Note: Laffer curves of different taxation in simulated model with varying volatility (left column) and steady state (right column). On left column, solid line is under high volatility and dash line is under low volatility. Taxes are capital income (top), labor income (center) and consumption (bottom).



Note: Simulation sample path of high volatility (solid line) and low volatility (dash line).

Figure C.4: Real Allocation, simulation sample.



Note: Simulation sample path of high volatility (solid line) and low volatility (dash line).

Figure C.5: IRFs, Capital (top), Labor(middle) , and Consumption (bottom) Tax

C.1 Data Appendix

C.1.1 Real Interest Rate

Spain, Portugal and Italy data for country spread and Germany for risk free interest rate. Nominal interest rates are from government bond yields, 3 year maturity. Data sourced from each country's central bank. Harmonized Consumer Price Index (overall index) for each country is from Bundes Bank. Real interest rate = Yield - HCPI (percent per annum). Risk free rate is German real interest rate. Country spread = Real interest rate - Risk free rate. Monthly data from January 2000 to April 2015.

C.1.2 National Accounts

Consumption: P31_S14 Final consumption expenditure of households. Investment: P5 Gross capital formation. Net export: P6 Exports of goods and services - P7 Imports of goods and services. Output: B1GM Gross domestic product at market prices.

Quarterly real national account data from Eurostat, namq_gdp_k. From 2000Q1 to 2014Q1. Million Euro Chain linked volume 2005. Data is downloaded for not seasonally adjusted. All items are seasonally adjusted by X-13ARIMA-SEATS by US Census Bureau and then log scaled. All series are hp-filtered with $\lambda = 1600$ for the moments calculation. nx/y is a mean of (seasonally adjusted, not hp-filtered) net export to output ratio.

C.1.3 Taxation

Taxation data from European Commission "Taxation Trends in the European Union" 2014 edition. In Annex A, implicit tax rate of capital, consumption and labor. Annual data.

Table C.1: National Account Second Moment

	EL	ES	IT	PT
σ_y	0.02	0.01	0.01	0.01
σ_c/σ_y	1.43	1.20	0.69	1.08
σ_i/σ_y	5.80	3.65	3.03	3.61
nx/y	-0.10	-0.02	0.01	-0.07
σ_{nx}	0.15	0.37	1.15	0.16
$corr(nx, y)$	0.69	0.77	0.40	0.22

Table C.2: Tax Data Summary Statistics

country	τ_L	τ_C	τ_K	g/y
Mean				
Greece	33.22	15.92	.	48.17
Italy	42.11	17.62	30.89	48.58
Portugal	23.43	18.51	31.14	45.92
Spain	32.26	15.14	30.45	41.35
SD				
Greece	1.952	0.608	.	3.546
Italy	0.609	0.414	3.418	1.659
Portugal	1.062	0.863	2.490	2.889
Spain	0.894	1.228	5.470	3.685

Note: SD indicates standard deviation. Capital taxation for Greece not available in the data.

Sample period 2000 to 2012. Greece does not have implicit capital tax rate. Government Expenditure to GDP data from Eurostat. Data is annual. [gov_a_main]. Sample period is 2000 to 2013.