ESTIMATING THE SLIDING VELOCITY OF THE RAINY LOBE USING INTERMEDIATE CLAST SIZE IN LODGEMENT TILLS

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Abstract

Lodgement tills are important in glacier reconstruction because their properties can be used to infer parameters such as glacier sliding velocity, ice thickness, and hydrology. These tills provide important calibration parameters for mass balance studies and numerical simulation. In Minnesota, lodgement tills of the Rainy lobe of the Late Wisconsin glacialiation exhibit significant changes in sedimentology between tills associated with the Last Glacial Maximum (LGM) and those deposited late during ice retreat. These changes include a small, but systematic increase in the intermediate size of clasts in the tills from the LGM to final ice retreat. Clast size in lodgement tills directly relates to the basal sliding velocity of ice. Small clasts drag on the bed and “lodge”, whereas large clasts simply plow into the basal sediment, but the stresses on them are too large for deposition. Therefore, we can use the size of clasts in lodgement tills to estimate sliding velocity at various times in a glacial advance.

This investigation uses field-based measurements of intermediate clast size in lodgement tills, along with the analyses of Weertman (1959, 1964), Iverson and Hooyer (2004), and Hooke (1977) to estimate sliding velocity. Clast size is determined photogrammetrically from field exposures along the path of the Rainy Lobe. Calculated sliding velocity is compared with independent calculation of the velocity based on mass balance profiles. Preconsolidation pressures of these lodgement tills and the shear strength of the subglacial sediment are also considered. These calculations provide an important calibration parameter for comparison to balance velocity calculations from mass balance studies of numerical simulation.
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of the Late Wisconsin Rainy Lobe
Introduction

Lodgement tills of the Late Wisconsin Rainy lobe are exposed across northern Minnesota, and range in age from the LGM to the final retreat of the ice from Minnesota about 10,000 BP. Over this spatial and temporal frame, the lodgement till undergoes systematic changes in texture and lithologic composition that indicate systematic changes in provenance. The changes in till texture and lithology have been used to determine the mean transport length, which is the average distance to the source of sediments that comprise the till (Larson and Mooers, 2004, 2005; Berthold, 2015). Across northern Minnesota, the tills of the Rainy lobe are ubiquitously associated with an overlying meltout facies and outwash, on top of lodgement till. The meltout facies contains a high proportion of boulders ranging in size up to 1.5 meters and occasionally even larger. Near the LGM limit of the Rainy lobe, at the Wadena drumlin field in south central Minnesota, the meltout facies is primarily composed of boulders with very little finer sediment; the boulders occur as a concentration generally one clast thick. The meltout facies increases in thickness to a maximum of 2-3 meters, with abundant large boulders, at the youngest ice margins in NE Minnesota. Within the Toimi drumlin field of NE Minnesota (Wright, 1972), the lodgment till reaches a thickness of 13 meters (Lehr and Hobbs, 1992). Continuous Rotosonic cores have allowed sampling and analysis of sedimentology of the till which can be found in the Minnesota Department of Natural Resources drill-core library.

Despite the changes in texture, lithology, and mean transport length, one characteristic of the lodgement till that is uniform throughout the region is the intermediate size of clasts in the till. This size is around 5-7 cm with the largest clasts of ~10 cm being quite rare. The homogeneity of this lodgement, its geotechnical characteristics, and the consistency of clast size allow reconstruction of the basal sliding velocity of the Rainy lobe throughout its Late Wisconsinan history. In this study, the basal sliding velocity of the Late Wisconsin Rainy lobe is reconstructed using the methodology developed by Iverson and Hooyer (2004), which is based on plowing and lodging of clasts in till below an ice sheet. The lodgement tills of the Late Wisconsin Rainy lobe do not have direct evidence of plowing clasts, such as described by Iverson and Hooyer (2004) in the till of the Lake Michigan lobe in Illinois. Rather, the geotechnical properties of the till, density, permeability, the distribution and size of clasts in the lodgement till, and the abundance of
boulders in the overlying meltout facies, are used to infer clast plowing and lodgement. We infer that clasts must plow before they lodge into the basal sediment.

The resulting magnitude of sliding velocity using the method of Iverson and Hooyer (2004) are compared with the sliding velocities using the method of Weertman (1959, 1964) and with balance velocity distributions based on mass balance reconstruction following the method of Hooke (1977), which was modified by Mooers (1990). Results indicate that calculated sliding velocities range over a factor of three across all methods and are consistent with calculated sliding velocities on modern glaciers in Greenland and West Antarctica (Bales et al., 2001).

**Background**

**Glacial History of the Rainy Lobe**

The Rainy Lobe was first identified by Elftman (1898) as being a distinct ice lobe along the southern margin of the Laurentide Ice Sheet (LIS). Elftman (1898) concluded that, during the late Wisconsinan glaciation, the Superior and Rainy ice lobes both flowed from the north to the southwest, but the Superior flowed through the Lake Superior basin, while the Rainy flowed over the Rainy Lake region of northeastern Minnesota. Both ice lobes emanated from the Labradorean Ice Center (LIC) (Figure 1a). The low-relief terrain of the Rainy Lake region lay in stark contrast to the deep Lake Superior basin traversed by the Superior Lobe; the Rainy Lobe was therefore substantially thinner and differed dynamically (Mooers, 1990a; Mooers and Lehr, 1997). These differences may have resulted in the Rainy lobe being less prone to surging, and the relatively thin profile allowed the Rainy lobe to respond more rapidly to changes in dynamics of the Laurentide Ice Sheet (Mooers and Lehr, 1997).

During the LGM, the Rainy lobe extended into west-central Minnesota to a position at the Alexandria moraine, while the Superior lobe terminated in the Twin Cities Basin in south-central Minnesota (Figure 1b). The sediments of these two lobes reflect the geologic terranes traversed by the ice. The Rainy lobe sediments contain an abundance of granitic and metamorphic lithologies characteristic of the Superior Province of the Canadian Shield and locally contain mafic intrusive rocks of the Midcontinent
Rift (Figure 2) (Mooers, 1990b; Meyer, 1996). The Superior lobe, in contrast, contains a large proportion of fine-grained mafic rocks and associated sedimentary rocks of the Midcontinent Rift (Mooers, 1990b; Meyer 1996). These lithologic assemblages, and therefore the tills of these lobes, are easily identified.

In addition to the general lithologic assemblage of the Rainy lobe sediments, there are two distinctive rock types that are readily identified and diagnostic of the area traversed. These include limestone from the Hudson Bay Basin, which outcrops in the Hudson Bay Lowlands, and erratics from the Proterozoic Omarolluk Formation, commonly termed omars, which outcrop in eastern Hudson Bay. These erratics have played an important role in working out the history of the Rainy Lobe (Mooers and Lehr, 1997) and the greater LIS (Prest, 1990; Prest et al., 2000).

Figure 1: a) The two major ice centers during the late Wisconsinan glaciation: the Labradorean and the Keewatin. Both the Rainy and the Superior originated at the Labradorean; b) the flow paths of the Rainy and Superior ice lobes, and the associated moraines and drumlin fields produced by the Rainy during the LGM.
Figure 2: Bedrock terranes of Minnesota and surrounding areas and the flow paths of the Superior and Rainy Lobes, indicated by the arrows (diagram modified from Geology of Minnesota: A Centennial volume, P.K. Sims and G.B. Morey, eds., Minnesota Geological Survey, 1972).
Wright (1957) investigated the stone orientations in the Hewitt Till of the Wadena drumlin field and supported the idea that the Rainy was flowing to the southwest until it terminated at the Alexandria Moraine (Wright, 1972). He notes that the till has a high carbonate content, but could not explain the source of these lithologies as limestone is absent from the rocks of northeastern Minnesota and adjacent Ontario. He postulated that even though the fabric in the drumlins show flow from the NE, the carbonate must have come from the Paleozoic limestone and dolomite in south-central Manitoba, the nearest carbonate terrane. Wright (1957) suggested that the ice might have come from the northwest and then was diverted to the southwest by another ice lobe. However, later studies including Goldstein (1985, 1989), Mooers (1988), Mooers and Lehr (1997), Prest (1990), and Prest et al. (2000), have convincingly shown a northeast origin of the Hudson Bay Lowlands for the carbonate.

Goldstein (1985, 1989) analyzed the Hewitt Till of the Wadena drumlin field, which was the earliest till to be laid down by the Rainy lobe. He observed systematic variation in till sedimentology including an increase in carbonate content and magnetic susceptibility with distance towards the margin. Thus, he concluded that the ice that created these drumlins flowed from the NE, but postulated that the carbonate in the till must have come from older northwestern provenance tills in the subsurface. Mooers (1988), Mooers and Lehr (1997), Larson and Mooers (2004), Larson (2008) and other studies have since contended the carbonate explanation of Goldstein (1989). These more recent studies have concluded that the carbonate must have come from the Hudson Bay lowlands along with the Omarolluk Formation erratics. The Hewitt till is a sandy, yellowish-brown till that is highly calcareous and has abundant greywacke erratics (Mooers, 1988; Berthold, 2015). The greywacke has its origin in the Proterozoic Omarolluk Formation (omars), and was transported from east of Hudson Bay (Prest, 1990; Prest et al., 2000) by the Rainy early in its advance (Larson, 2008). Similarly to the Paleozoic limestone in the Hudson Bay Lowlands, the presence of these omar erratics have been used to further support the origin of the Rainy from the LIC.

The Rainy retreated from the Alexandria moraine about 22 ka BP (Mooers and Lehr, 1997). The extent of the retreat is uncertain but was likely no further to the east than central Minnesota. The Rainy lobe then re-advanced to the St. Croix moraine by about 15.5 ka BP (Clayton and Moran, 1982; Mooers and
Lehr, 1997). Ice flow during this St. Croix Phase had a slightly different orientation and sedimentary provenance (Schneider, 1961; Wright, 1972; Mooers, 1990; Meyer, 1996; Berthold, 2015). Till of the Rainy lobe during the St. Croix Phase is well exposed at the surface in the Brainerd drumlin field, which formed east of the St. Croix moraine (Schneider, 1961; Wright, 1972; Mooers, 1989 and 1990; Wright, 1972). Mooers (1988) observed that the till in the drumlin field is similar to the composition in the St. Croix moraine, but notes that the drumlins trend oblique to the moraine. This could be because the drumlins formed as the ice was retreating from the moraine instead of when it was advancing (Mooers, 1989). The till is brownish and less calcareous with more locally derived igneous and metamorphic lithologies.

The retreat of the Rainy lobe from the St. Croix moraine was punctuated by numerous standstills or minor re-advances (Mooers, 1988 and 1990; Mooers and Lehr, 1997), with a final retreat to the north of the international border shortly after 11 ka BP (Bjork, 1990; Mooers and Lehr, 1997). Major subglacial landforms of the Rainy lobe in NE Minnesota include the prominent Toimi drumlin field (Wright et al., 1969; Wright, 1972; Meyer, 2009). The drumlins are composed of Independence till (Wright et al., 1970). The till is grayish brown, non-calcareous, and contains an abundance of locally derived lithologies (Larson and Mooers, 2004; Berthold, 2015). The exact age of the drumlins is unknown.

Meyer (2009) studied Rogen moraines in NE Minnesota. The Rogen moraine ridges trend perpendicular to the axes of the Toimi drumlins and are transverse to ice flow. The Rogen, or ribbed moraines, form subglacially like drumlins and contain the same Independence till as the Toimi drumlins (Meyer, 2009). Meyer (2009) concluded from her GPR profile that while the Rainy was depositing at the up glacier side of the moraine, the ice was eroding at the down glacier side. Kryzer et al. (2013) suggest that the Rogen moraine is a transitional bedform that formed during the overall erosion at the glacier bed.

Larson and Mooers (2004) developed a parameter called the “erosion length scale” that can be used to determine the mean transport length of glacial sediment from its bedrock sources. The erosion length scale is the distance (or time) that it takes the sediment entrained in glacial ice to be diluted 50%. Sediments with a high proposition of far-traveled rock types have a long erosion length scale and a large
mean transport length. Thus, the more local the lithologic signature of a till, the shorter the erosion length scale and the mean transport length is.

Larson (2008), Larson et al. (2015) and Larson and Mooers (unpublished data), suggest that this till formed a continuous cover over the Canadian Shield from the Hudson Bay Basin to the LGM limit. Remnants of this once continuous till sheet can be found as isolated patches across the Canadian Shield (Figure 3). The texture, lithology, grain size, and mean transport length of the till comprising these discontinuous sediment patches is similar to the Rainy Lobe lodgement till associated with LGM in west-central Minnesota and with the basal horizon of the till in NE Minnesota associated with the Toimi drumlins. Larson (2008) postulates a continuous carbonate till sheet that spanned from Hudson Bay to the LGM. The unconsolidated sediment allowed the Rainy to advance to its maximum limit with a relatively thin ice profile (Larson, 2008). However, as the Rainy reached its maximum limit and began retreating, it was eroding its bed where the velocities were the highest in extreme northeastern Minnesota and adjacent western Ontario. This led to major lithologic compositional changes that contributed to the different till exposures along the Rainy flow path.
Figure 3: Recent terrane of south-central Canada and the upper Midwest. Patches of glaciogenic sediment on the Canadian Shield are remnants of a once continuous till sheet that spanned from Hudson Bay to south-central Minnesota along the flow path of the Rainy (the arrow) (diagram modified from Larson (2008)).
Berthold (2015) conducted an extensive study, which looked at the lithologic compositions of boulders along the flow path of the Rainy Lobe in Minnesota. The sources of the boulders were used to calculate mean transport lengths for the Wadena, Brainerd, and Toimi drumlin fields, and the Rogen Moraine based on the most abundant lithologies (Figure 4). Berthold (2015) found the dominant lithologies in the Wadena drumlin field, other than the ubiquitous granite and gneiss, to be carbonate (limestone/dolomite/Chert) and omars. The mean transport length of the carbonate was calculated to be about 1000 km and the mean transport length of the omars is about 1500 km. This supports previous studies (Mooers, 1988; Mooers and Lehr, 1997; Prest, 1990; and Prest et al., 2000) that postulate that the carbonate came from the Hudson Bay Basin and the omars from east of Hudson Bay because there is no original sediment in Minnesota that contains these erratics. However, very little carbonate and omars were found in the Brainerd drumlin field. Instead, Duluth Complex and Red Felsite were found to be the dominant rock type, which come from NE Minnesota (Berthold, 2015). The mean transport length of these rocks was calculated to be about 300 km, which suggests a more local source than for the Wadena. This supports the idea of a once continuous till sheet that has since been eroded down over time. The major lithologies in the Toimi drumlin field are also Duluth Complex and Red Felsite, but no carbonate or omars (Berthold, 2015). The mean transport length was calculated to be about 30 km because of only locally derived sediments and there was no transport of material from Hudson Bay by the time the Rainy advanced over the Toimi. The lithologic compositions of the Rogen moraine are nearly identical to the Toimi drumlin field, which suggests that they are composed of the same till, with a mean transport length of 30 km.
Lodgement Tills

Lodgement tills are primary subglacial sediment layers deposited either by pressure melting or another frictional process of a sliding glacier (Dreimanis, 1989; Benn and Evans, 2010). Clasts begin to lodge when the frictional drag between the debris and the ice bed is greater than the shear stress of the overlying ice (Boulton, 1975 and 1982; Benn and Evans, 2010). Because previous studies (Mooers, 1988; Larson, 2008) have postulated that the Rainy flowed over a soft bed of sediment with bumps, the clasts at the bed of the glacier must have lodged once the bed resistance became high enough or when they
encountered those obstacles at the bed (Benn and Evans, 2010) (Figure 5). Once a clast lodges, it will generally be rotated until it’s a-axis and a-b plane are parallel to the plane of shear and then is deposited (Benn and Evans, 2010). These tills could also be termed “hard lodgement tills” (sensu Ruszczyńska-Szenajch, 2001) since they were deposited due to friction as opposed to from melting.

Lodgement tills are typically overconsolidated because of dewatering that results from high shear stresses as the ice flows over the surface (Benn and Evans, 2010). In a study in Europe, the consolidation pressure range for lodgement tills was 140-420 kPa (Ruszczyńska-Szenajch et al., 2003) with overconsolidated tills being more skewed towards the higher end of this pressure range. The consolidation was greatest at the base of the layer and weakened upward due to increasing normal stress with depth (Ruszczyńska-Szenajch et al., 2003). Lodgement tills are dense with low permeability and porosity and thus are often unoxidized overall, since water percolation is low. Lodgement tills sometimes contain deformation structures (Ruszczyńska-Szenajch et al., 2001, 2003; Müller and Schlüchter, 2001), but have an overall fissile structure from subhorizontal joints that were created during past shearing (Benn and Evans, 2010; Ruszczyńska-Szenajch et al., 2001, 2003).

Lodgement tills are diamictons because they contain a wide variety of clast sizes from clay to course gravel that is suspended in a matrix of sand, silt, and clay. Lodgement tills often have a bimodal distribution of clast sizes with distinct modes in the sand and silt ranges (Dreimanis and Vagners, 1972). Tills with a higher sand content than silt or clay are typically more consolidated because they are less deformable (Müller and Schlüchter, 2001). The bulk density of sandy lodgement tills typically ranges from 1.9-1.95 (Müller and Schlüchter, 2001). Boulders within lodgement tills are rare because they are too big to lodge, and thus end up in the meltout till above when the glacier finally wastes. Characteristics of meltout tills will be discussed later.

In addition to clast orientation, grooves and prows are direct evidence for ploughing clasts being dragged by the ice (Iverson and Hooyer, 2004). However, if plowing structures are not visible because the bed is not exposed, the orientation of clasts still can show the dominant flow direction of the ice because lodgement till have a very strong fabric (Boulton, 1971).
Melt-out Tills

Melt-out tills can be deposited either subglacially or supraglacially depending on how the till is deposited while the glacier melts out (Boulton and Paul, 1976). The ice is debris-laden by in-situ aggregation of englacial debris and are deposited as ice stagnates (Paul and Eyles, 1990; Benn and Evans, 2010) (Figure 6). Subglacial meltout tills form as ice melts at the base of the sediment-rich debris layer (Paul and Eyles, 1990; Benn and Evans 2010). Melt-out of debris can be caused by geothermal or sensible heat sources, but these rates of melt are usually quite low when compared with other factors that contribute to thaw consolidation (Paul and Eyles, 1990; Benn and Evans 2010).

Thaw consolidation is determined by comparing the rate of meltwater production to the rate of consolidation of the sediment (Paul and Eyles, 1990). If thaw consolidation is high, so is pore pressure (Paul and Eyles, 1990). High pore water pressures allow water to percolate, so meltout tills tend to have higher permeability, are more oxidized, and develop sandy lenses and scour fills as the layer is deposited (Benn and Evans, 2010). Debris content also has an effect on how consolidated the meltout till is (Benn and Evans, 2010). High debris content and well-drained ice will cause consolidation to be low and not much deformation will occur. However, more failure and reworking will occur if the opposite is true and the frictional strength of the sediment is low (Paul and Eyles, 1990; Benn and Evans 2010).

The degree of modification of the meltout till from the parent ice is strongly dependent on two factors: how high the pore water pressure is from thaw consolidation, and the amount of shear stress that
the till was subjected to (Benn and Evans, 2010; Paul and Eyles, 1990). If freshly deposited meltout tills have been subjected to minimal shear and low pore pressures, and thus has less deformation overall, then clasts in the till could be oriented in the direction of flow, similar to lodgement tills (Paul and Eyles, 1990). However, high shear stress and high pore pressures will result in a meltout till with extensive deformation structures and more differing characteristics from the parent ice (Paul and Eyles, 1990).

![Figure 6: Meltout till is deposited as ice stagnates and begins melting and retreating. Subglacial meltout till is usually found on top of pre-existing the lodgement till that was deposited while the ice was still advancing.](image)

**Sliding velocity**

Reconstruction of sliding velocities of past ice sheets provides valuable calibration for numerical models used in glacier and climate studies. Ice flows around obstacles on the bed by two mechanisms: regelation and enhanced creep (Weertman, 1959) (Figure 7). Regelation ($U_{\text{reg}}$) is the pressure melting and refreezing of ice as it flows over bumps. Regelation is very efficient over small scales (less than a few cm), but the ease of movement drops off exponentially as clast size increases. The pattern of sliding velocities follows a negative logarithmic trend (Figure 8). In contrast, enhanced creep ($U_{\text{crp}}$) is the flowing of ice around a clast because of internal deformation. Creep accommodates large obstacles, and the efficiency of flow increases as clast size increases in a linear trend (Weertman, 1959) (Figure 8).
Figure 7: The movement of ice by regelation vs. enhanced creep. Regelation involves the melting and refreezing of ice as it flows around the clast, but creep lets ice flow around the object by internal deformation.
In the case of lodgement tills, the obstacles on the bed are “larger” clasts moving over finer sediment. Small clasts lodge with ice sliding around them by regelation. Therefore, the largest size clast in lodgement tills, or the intermediate size, those where the drag from the sediment bed exceeded the drag force of the ice, can be used to estimate sliding velocity by the melt rate, around a clast of a given diameter (Weertman, 1957). Weertman (1964) presents a more rigorous model of incorporating uneven clast dimensions instead of cubes, but the trends of regelation and creep are still similar in both studies. Clasts that transport in basal ice impinge on the bed, inducing a drag force on the particle. These clasts then “plow” into the soft sediment, and if they drag and “lodge” into the sediment, the ice flows around them. The degree of drag is determined by their size and shape (Boulton and Paul, 1976).

Figure 8: Calculated sliding velocities based on the mechanisms of regelation ($Us_r$) and plastic deformation ($U_{sp}$), using the calculations of Weertman (1957). Regelation becomes less efficient logarithmically as clast size increases. In contrast, creep increases linearly in efficiency as clast size increases. The intersection of the two lines, where the black circle highlights, is the approximate intermediate clast size that can be more accurately used to calculate sliding velocity.
Studies have been done on the Michigan Lobe (Iverson and Hooyer, 2004) to determine approximate sliding velocities given a set of fixed parameters like ice viscosity, melting temperature depression and regelation parameter, and other variables that require an estimated value like shear stress and clast radius, in order to determine the approximate sliding velocity of the ice. Clasts above a certain size tend to stay embedded in the glacier’s sole because they are too large for the glacier to flow around it by the process of regelation. The size distribution of these clasts can be used to estimate the sliding velocity of a remnant ice sheet (Iverson and Hooyer, 2004). Execution of this method to calculate the sliding velocity of the Rainy lobe will help to verify calculated sliding velocities from exposed lodgement tills. A more extensive explanation on their methods will be discussed later.

**Methods**

**Numerical modeling**

*The nature of glacier sliding (Weertman (1957, 1964))*

Weertman (1957) considers the bed of a glacier to be irregular with bumps of many different sizes, so sliding occurs by two mechanisms that allow the ice to flow around obstacles: regelation and enhanced creep. Regelation involves the pressure melting of ice on the stoss side of an obstacle and refreezing on the lee side. The ice melts on the high pressure side of the bump because of the latent heat that is transferred back through the bump as the ice is refreezing on the low pressure end (Weertman, 1957 and 1964) (Figure 7). This sliding mechanism dominates sliding velocity with smaller obstacles since heat can easily flow through them, but the heat flow is limited by larger sizes, logarithmically (Figure 8). The rate of enhanced creep operates from stress concentrations on the bed, which allows the ice to flow around the bump and close back together on the lee side because of hydrostatic pressure (Weertman, 1957 and 1964) (Figure 7). The ice flow past the object is by plastic flow, which occurs because of the difference in stresses acting on the obstacle. This mechanism is more efficient as object size increases so sliding velocity is maximized by larger bumps (Weertman, 1957 and 1964; Nye 1952) (Figure 8). Therefore, small bumps on the glacier bed can be easily accommodated by regelation and large bumps by enhanced creep; there is an intermediate size that presents the most resistance to glacier sliding.
Sliding velocity because of regelation is given by the following equation:

\[ U_{sr} = \frac{Q}{H \rho l^2} = \frac{C \tau K_r}{H \rho r^2} \quad (1) \]

where

- \( U_{sr} \) = sliding velocity by regelation
- \( Q \) = heat flow
- \( l^2 \) = the area of the clast or bump
- \( H \) = latent heat of fusion
- \( \rho \) = density of ice
- \( C \) = change in temperature with pressure
- \( \tau \) = shear stress
- \( K_r \) = thermal conductivity of ice
- \( r^2 \) = the roughness parameter (\( L^2/l^2 \))

As indicated above, regelation shares a negative logarithmic relationship with bump size, so as \( l \) becomes larger, \( U_{sr} \) is minimized.

Sliding velocity because of enhanced creep is given by:

\[ U_{sp} = b \varepsilon l = b \left( \frac{\tau}{Br^2} \right)^n * l, \quad (2) \]

where

- \( U_{sp} \) = sliding velocity by enhanced creep or plastic deformation
- \( b \) = dimensionless constant of proportionality
\( \dot{\varepsilon} \) = the strain rate \\
\( l \) = length of the clast or bump \\
\( B \) = viscosity parameter of ice \\
\( n \) = constant from Glen’s flow law (=3) \\
\( r^2 \) is the bed roughness

In contrast to regelation, as the bump size increases, sliding velocity due to enhanced creep increases linearly.

The sliding velocity calculations for regelation and enhanced creep were determined in an excel spreadsheet, using the method of Weertman (1957), in order to compare how increasing clast size (\( l \)) affects \( U_{sr} \) and \( U_{sp} \). The intersection of curves \( U_{sr} \) and \( U_{sp} \) shows the approximate size that can accurately be used to calculate sliding velocity; smaller bumps will be accommodated by regelation and larger clasts will be accommodated by enhanced creep (Weertman, 1957). Later studies, like Weertman (1964), consider the limitations of Weertman (1957) with regards to how regelation and enhanced creep affect sliding velocity. For regelation, Weertman (1964) considers a bump with different dimensions (\( l_h \), \( l_d \), and \( l_l \)), as opposed to cubes. Although this is a more realistic assumption, the same relationship that pressure melting shares with sliding velocity remains, since the three dimensions of the clast will be averaged (Weertman, 1964). For enhanced creep, the stress on the clast is the sum of compressive and tensile components, and hydrostatic pressure is assumed to be variable depending on cavity formation on the lee side of the bump (Weertman, 1964). The resulting equation of sliding velocity, in terms of creep, considers the strain rate acting on the obstacle and the distance in the direction of motion. However, just like regelation, the sliding velocities are similar to the initial creep calculation, since the distance that creep acts across on the bump is hard to estimate (Weertman, 1964).

Iverson and Hooyer (2004)

The theory of Weertman (1957, 1964) was adapted by Iverson and Hooyer (2004) using clasts in the basal ice that drag on a soft-sediment bed. The calculated sliding velocity (\( U_r \)), is equal to the difference in the velocity of the ice (\( U \)) and the speed that the ice must be traveling in order to transport a clast of a
determined size \( (U_p) \) that is in the base of the ice and dragging on the bed. Because there is normally a frictional drag that resists clast motion, the clast will always be moving slower than the ice is. Therefore, \( U_r \) is the relative velocity that the ice and clast are moving at (Iverson and Hooyer, 2004), which is shown as such:

\[
U_r = U - U_p.
\] (3)

Using the grain size distribution of clasts in a till exposure, and combining it with geotechnical analyses and the theory of glacier sliding, the resulting sliding velocity, effective normal stress acting on the bed, and the bed shear strength can be calculated (Iverson and Hooyer, 2004; Lliboutry, 1979).

Sliding velocity can be calculated in various ways depending on which parameters are known. The radius of the largest clast that plowed \( (R_L) \), the radius of the smallest clast that plowed \( (R_S) \), and the radius of the transitional clast size \( (R_\ast) \) alone can be used to calculate sliding velocity. Equating \( U \) to \( U_r \) and if two out of the three previously-mentioned radius parameters are known \( (R_S, R_\ast, \text{ or } R_L) \), \( U \) can be solved for in terms of \( R_L \) and \( R_\ast \), as such:

\[
U = \Psi \left( \frac{R_L}{R_\ast} + \frac{1}{R_L R_\ast} \right)
\] (4)

where

\[
\Psi = \left( \frac{C_1}{B_1} \right)^{\frac{1}{2}}
\] (5)

and the ice viscosity \( (B_1) \) and regelation parameter \( (C_1) \) of normal temperate ice are used. If \( R_\ast \) is not known, then it is simply calculated using the relationship between the \( R_\ast \) and \( R_L \):
\[ R_* = \left( R_S R_L \right)^{\frac{1}{2}}. \]  

(6)

Iverson and Hooyer (2004) compiled their size distribution of clasts by only measuring the size of the clasts in their exposure, if there was a groove and/or a prow associated with it. However, if there was no clast, they would measure the prow and/or groove instead to estimate clast size. This was to ensure that there was direct evidence of ploughing clasts (Iverson and Hooyer, 2004). In this current study, it is inferred that in order for a clast to lodge into the till, it had to plow first, so every clast within the sediment layer was “plowing” previously.

In order to calculate the effective normal stress \( P_e \), no water filled cavities are assumed and \( B_1, C_1, R_* \), and the modified bearing capacity factor (N) are used as such:

\[ P_e = \left( \frac{C_1}{B_1} \right)^{\frac{1}{2}} * \left( \frac{1}{N R_*} \right) \]  

(7)

The effective normal stress value was then compared to the preconsolidation stress. If the effective normal stress is smaller than the preconsolidation stress, then the till can be classified as “overconsolidated”.

Using a similar equation, the bed shear strength \( S \) was calculated using \( B_1, C_1, N \) and \( R_* \) again, but substituting the angle of friction \( \phi \) in for 1 as such:

\[ S = \left( \frac{C_1}{B_1} \right)^{\frac{1}{2}} * \left( \frac{\tan(\phi)}{N R_*} \right) \]  

(8)

The greater the angle of friction that is used, the higher the bed shear strength (Iverson and Hooyer, 2004).

A table of applicable factors used in both Iverson and Hooyer (2004) and this study, can be referred to in Table 1. At these study locations, the surface of the till is inaccessible and thus plowing structures are not visible, so measuring the long axis of each clast is sufficient for estimating clast size. Thus, the orientation
of the clasts (long axes in the direction of flow) provide evidence for plowing and lodging. An excel sheet was set up in order to be able to input “real” $R_l$, $R_s$, and $R*$ from field data later.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice fluidity parameter</td>
<td>$B_1$</td>
<td>$1.6 \times 10^{-25}$ Pa$^{-3}$ s$^{-1}$ (normal ice) $8.7 \times 10^{-25}$ Pa$^{-3}$ s$^{-1}$ (basal ice)</td>
</tr>
<tr>
<td>Melting temperature depression</td>
<td>$C$</td>
<td>$7.4 \times 10^{-8}$ K Pa$^{-3}$</td>
</tr>
<tr>
<td>Regelation parameter</td>
<td>$C_1$</td>
<td>$2.66 \times 10^{-15}$ m$^2$ Pa$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>Sediment cohesion</td>
<td>$c$</td>
<td>0 kPa</td>
</tr>
<tr>
<td>Ice thermal conductivity</td>
<td>$K_i$</td>
<td>$2.1$ W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>Rock thermal conductivity</td>
<td>$K_r$</td>
<td>$3.0$ W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>Pressure-shadow factor</td>
<td>$k$</td>
<td>0.1–0.45</td>
</tr>
<tr>
<td>Ice volumetric latent heat of fusion</td>
<td>$L_v$</td>
<td>$3 \times 10^{-8}$ J m$^{-3}$</td>
</tr>
<tr>
<td>Modified bearing capacity factor, no cavities</td>
<td>$N$</td>
<td>2.09–6.66</td>
</tr>
<tr>
<td>Modified bearing capacity factor, with cavities</td>
<td>$N_e$</td>
<td>3.72–15.55</td>
</tr>
<tr>
<td>Bearing capacity factor</td>
<td>$N_p$</td>
<td>26–40</td>
</tr>
<tr>
<td>Radius of smallest clast that plowed</td>
<td>$R_s$</td>
<td>0.004 m</td>
</tr>
<tr>
<td>Radius of largest clast that plowed</td>
<td>$R_l$</td>
<td>&gt;0.074 m</td>
</tr>
<tr>
<td>Transition clast radius</td>
<td>$R_*$</td>
<td>0.017–0.020 m</td>
</tr>
<tr>
<td>Sediment friction angle</td>
<td>$\phi$</td>
<td>$35^\circ$</td>
</tr>
<tr>
<td>Sliding constant</td>
<td>$\Psi$</td>
<td>$3.43 \times 10^{-10}$ m$^3$ s$^{-1}$ (normal ice) $1.47 \times 10^{-10}$ m$^3$ s$^{-1}$ (basal ice)</td>
</tr>
</tbody>
</table>

Reconstructing the Rainy Lobe

The average velocity through a transverse cross-section of the glacier, or the balance velocity ($u$), along a profile of an ice sheet is determined by the mass balance ($\dot{a}$) and the ice thickness ($h$), by the expression

$$\frac{dx}{h} = u,$$  \hspace{1cm} (9)

where $x$ is the distance from the ice divide along the profile.

The mass balance controls ice discharge at any point on the ice sheet and ice thickness is determined by the driving stress (slope stress) necessary to provide the given discharge. In turn, the driving stress is modulated by the temperature distribution, which determines the viscosity. Hooke (1977) summarized this relationship between velocity and temperature in polar ice sheets by considering some of the main factors.
that control their basal and englacial temperatures: the geothermal flux, the surface temperature, and both the vertical and horizontal aspects of velocity. These models assume a steady-state profile.

The temperature profile of a stagnant ice mass is linear and controlled by the geothermal flux and surface temperature (Hooke, 1977). Sufficient basal melting at the bed could alter this linear temperature distribution as meltwater flows from one place to another at the ice bed before refreezing, since latent heat is released upon freezing (Hooke, 1977; Boulton, 1972).

For a moving glacier, vertical velocity causes the temperature profile to bow down in the accumulation zone and bow up in the ablation zone (Hooke, 1977). The vertical velocity is trends downward in the accumulation zone, which results in a steeper temperature gradient at the surface with the steepest mid glacier, and upward in the ablation zone (Figure 9). Along the glacier profile, the warmest temperatures are near the glacier bed where most of the ice deformation occurs. The frictional heat of deformation further warms the basal ice, which directly affects ice flow (Hooke, 1977).

Figure 9: Vertical velocity changes along the profile of ice sheet. It decreases downward in the accumulation area and upward in the ablation area. The black arrows show the direction of vertical velocity as you progress down-glacier from the divide.

The most widely used flow law for ice is Glen's Flow Law

\[ \varepsilon = \left( \frac{\tau}{B} \right)^n, \]  

(10)

where \( \varepsilon \) is the strain rate, \( \tau \) is the bed shear stress, \( B \) is a parameter representing the viscosity of ice, and \( n \) is a constant usually taken as 3 (Hooke, 1981).
If we consider only flow in vertical plane along a glacier flowline (x,y plane), equation 10 becomes

\[ \varepsilon_{yx} = \left( \frac{\tau_{yx}}{B} \right)^n, \]  

(11)

and

\[ \tau = \rho ghS, \]  

(12)

where \( \rho \) is the density of ice (910 kg m\(^{-3}\)), \( g \) the acceleration of gravity (9.8 m s\(^{-2}\)), \( h \) is the ice thickness with the surface being 0 and positive downward, and \( S \) is the glacier surface slope. By definition the strain rate is defined as

\[ \dot{\varepsilon} = \frac{1}{2} \left( \frac{du}{dh} + \frac{dv}{dx} \right) = \left( \frac{\tau}{B} \right)^n, \]  

(13)

where \( u \) is the horizontal (down glacier) velocity and \( v \) is the vertical velocity. If we consider a glacier of very large extent, and \( \partial v/\partial x \) is essentially 0, and

\[ \frac{du}{dh} = 2 \left( \frac{\rho gh}{B} \right)^n h^n, \]  

(14)

and

\[ \int_{u_s}^{u_h} du = 2 \left( \frac{\rho gh \tan \alpha}{B} \right)^n \int_{0}^{H} h^n dh. \]  

(15)

Integration over the glacier thickness of equation 15 yields the differential velocity between the glacier surface (\( u_s \)) and any point at depth as

\[ u_h = u_s - 2 \left( \frac{\rho gh}{B} \right)^n \frac{H^{n+1}}{n+1}. \]  

(16)

Integration of the velocity distribution given by equation 15 over the glacier thickness yields the ice discharge

\[ q = u_s H - \frac{2}{(n+1)(n+2)} \left( \frac{S_f \rho gh}{B} \right)^3 H^{n+2}. \]  

(17)
Therefore, if it is known or if the ice discharge as f(x) and ice thickness can be approximated, the surface velocity can be calculated from equation 16 and the bed velocity from equation 17. Sliding velocity is calculated by taking the difference between the surface and bed velocities.

The relationship between mass balance and velocity is used to develop mass balance models using the method of Hooke (1977), and modified Mooers (1990), in order to estimate the sliding velocity of the Rainy Lobe along its flow path. The model assumed a constant effective shear stress (τᵢ), viscosity parameter (B), density of ice (ρ), and acceleration of gravity (g). Using an excel spreadsheet, the ice lobe was divided into 200 evenly spaced intervals, each 10,000 m long, which gives a total profile length of 2,000,000 m from the ice divide to the terminus. The equilibrium line was specified 250,000 m up glacier from the margin and the maximum accumulation rate is 100,000 m up from there (Hooke, 1977; Mooers, 1990).

The ice profile was calculated by integrating equation 12, for each interval along the ice profile, in order to solve for ice thickness (H):

\[
H = \sqrt[3]{\frac{2τᵢx}{ρg}}
\]  

where x is the distance from the margin.

An accumulation rate is specified and then integrated numerically from the ice divide to the margin. A maximum accumulation rate (acc_max) and an accumulation rate at the divide (acc_div) are assumed in order to determine the net balance (b_n) or mass balance along the entire length of the ice sheet. This integration is completed in two steps: one for the accumulation area (b_acc) and one for the ablation area (b_ab). In steady state, the volume of ice passing the ELA must be ablated between the ELA and margin, so b_acc must equal b_ab. The resulting profile and b_n can be referred to in Figure 10.

Calculation of b_acc from the ice divide to the acc_max uses the following equation:
where \( L \) is distance from the divide.

Once the maximum accumulation is reached, the rest of the accumulation zone is approximated by taking the \( \text{acc}_{\text{max}} \) and multiplying it by \( 1/\text{the number of intervals left between the acc}_{\text{max}} \) and the ELA, and then subtracting that from the previous \( b_n \).

To solve for \( b_{nab} \), we assume \( b_{nacc} = b_{nab} \) and also use the ablation gradient \( (m) \). \( m \) is calculated using

\[
m = \frac{b_{n} \cdot 2}{x^2}
\]  

(20)

\( B_{nab} \) is calculated from the ELA to the margin using this equation:

\[
B_{nab} = -m(L - L_{\text{ELA}})
\]  

(21)

The flux \( (q) \) is then calculated for every interval using equation 17 along the length of the ice sheet. The discharge increases from the divide to the ELA, and then decreases down glacier from the ELA. The flux can be converted to discharge by dividing by the density of ice to go from kg/yr to m\(^3\)/yr.

Surface velocity \( (U_s) \) is solved for by rearranging equation 17,

\[
U_s = \left(\frac{q + 0.1 \cdot \left(s_f \cdot \frac{p_0 g s^3}{x} \right) \cdot H^2}{H}\right)
\]  

(22)
and substituting in $n = 3$ (from Glen’s flow law) and setting $S_f = 1$, where $S_f$ is the shear factor, since the vertical velocity is 0.

Finally, sliding velocity can be solved for by taking the difference between the previously calculated $U_s$ and bed velocity, in order to calculate the relative sliding velocity of the ice sheet ($U_b$):

$$u_b = u_s - 2 \left( \frac{\rho g \epsilon}{H} \right)^3 \left( \frac{u_s}{4} \right)$$

(23)

As predicted, sliding velocities tend to increase from the ice divide to a spot in the ablation zone, where it starts to decrease to 0 at the margin (Figure 11).

Figure 10: The profile of the Rainy Lobe that was reconstructed using the method of Hooke (1977), as a result of the associated accumulation rates used (after Mooers, 1990).
Figure 11: The sliding velocity and ice profile of the Rainy Lobe. The sliding velocity of an ice sheet increases downglacier from the divide until a point in the ablation area, where it slows down to 0 m/yr at the margin. The profile of the ice sheet thickens upglacier from the margin. Ice thickness and sliding velocity calculations were done using the parameters from Figure 10.

Fieldwork

Site Selection

In order to estimate the sliding velocity of the Late Wisconsinan Rainy lobe, well-exposed lodgement tills are required in order to approximate the intermediate clast size within the till layer. The intermediate size is important because it is the size that is equally accommodated by the mechanisms of regelation and enhanced creep with respect to sliding velocity (Weertman, 1957 and 1964). If the intermediate clast size changes along the flow path, it can be inferred that the sliding velocity also varied.

Seven sites were chosen for analysis along the flowline of the Rainy lobe in Minnesota; two within the Wadena drumlin field (Hewitt Till, W map symbol), one within the Brainerd drumlin field (Brainerd Till, B map symbol), and four within the Toimi drumlin field (Independence Till, T map symbol), with T1 being in the proximity of the Rogen Moraine (Figure 12). The sites in the Wadena and
Brainerd drumlin fields were limited because of access restrictions (gravel pits are either privately owned or require special access) or because the lodgement till was not well-exposed. Some locations were resampled from previous studies of the Rainy Lobe (Berthold, 2015; Meyer, 2009), while others were located using 1-meter and 3-meter LiDAR or aerial photos. Surface boulder concentrations are ubiquitous across all of the locations, but no boulders are observed within any of the lodgement till (Wright, 1957; Wright, 1972; Mooers, 1988; Goldstein, 1989; Berthold, 2015) (Figure 13).
Figure 12: All of the locations that photos were taken at in NE Minnesota, indicated by pins. The letters correspond to the drumlin field that the site is located within (W = Wadena, B = Brainerd, and T = Toimi).

Figure 13: A field in Crow Wing County strewn with boulders, on top of Brainerd till (Photo by H. Mooers)
Photogrammetry/Intermediate clast size approximation

Till exposure sketches and site descriptions were recorded at each location, in order to note any differences in till thickness and lithology. Photographs of the lodgement till (and meltout till, if present and exposed) were taken by either Canon DLSR digital camera, or by DJI Phantom 4 Pro drone if the exposure was inaccessible by foot. The known height of a fixed object, like a tree or bush, was also noted at each location, in order to later estimate the scale of the photo. Fieldwork was completed during the summer of 2017.

Photos were converted to TIFF format and imported into Photoshop for enhancement before particle analyses could be determined (Figure 14a). Sharpening and contrast settings were used in order to eliminate shadows and color discrepancies in order to normalize the image. Also, the background matrix of the till exposure was darkened in order for the rock fragments within the till layer to stand out. Any slump or rock debris that was not directly incorporated into the lodgement till was also masked from the image. Each photo was saved in Photoshop and then imported into ImageJ where it was converted to 8-bit, grayscale graphics for particle analysis. The binary image was scaled so the matrix was black and the rock fragments were white (Figure 14b). The scale of the image was set, so that the clast sizes that the program measured made sense. The pixel scale was calculated by using the known size of objects at the site in cm (ie. Bushes, trees, or rock fragments) and measuring the size in pixels from Photoshop for each image to ensure that the scale was kept constant between the two cameras. Next, the “Analyze Particles” command was used in order to count and measure every detectable rock fragment. The individual clasts were determined by the program finding the edges of a particle and then measuring the size according to the scale that was set for the image. Outlines were also produced in order for the user to judge the accuracy of the scale and particle analysis (Figure 14c and 14d). If some clasts were underrepresented and broken up into smaller clasts than the actual size, the scale was changed by either a factor of 2 or 3, in order to account for image artifacts. The scale used for each site can be referred to in Table 2. The clast counts and their area sizes were imported into excel and then converted to radius in cm, by taking the square root of particle areas and dividing by 2. This method assumes particles to be squares. However, the difference between squares and circular particles is a few percent, well within the methodological error. From radius in cm, clast areas were then converted to mm in preparation for frequency distributions.
Figure 14: Shows the process of analyzing the size of clasts in site images: a) Original image at W1 b) Binary image after import into ImageJ c) Particle outlines after analysis is run and d) zoomed in version of c, to show particle outlines.

Table 2: The scale set for each site for ImageJ. The scale was approximated using a previously measured object at the site and comparing it to the pixel size measured in Photoshop.

<table>
<thead>
<tr>
<th>Site</th>
<th>Scale in pixels/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>6.1</td>
</tr>
<tr>
<td>W2</td>
<td>2.3</td>
</tr>
<tr>
<td>B1</td>
<td>3</td>
</tr>
<tr>
<td>T1</td>
<td>2</td>
</tr>
<tr>
<td>T2</td>
<td>5.6</td>
</tr>
<tr>
<td>T3</td>
<td>2.2</td>
</tr>
<tr>
<td>T4</td>
<td>2</td>
</tr>
</tbody>
</table>

The clast sizes were compiled into one cumulative spreadsheet for each site. The fragments were sorted from small to large, and then organized into “bins” using the “Data Analysis” extension in Excel, in order to show the frequency distribution of the clasts in the till layers at each site. Each bin size is 2mm, with 8mm being the smallest bin, up to 50mm, or the 48mm bin, being the upper bound. The clast size intervals and frequency columns were then converted to a log₂ scale and then graphed on a scatter plot. Because both variables have been linearized, a linear trend line was fit through the data and the x-intercept of the line was solved for. The x-intercept was then converted back to clast size in mm to be used for calculating sliding velocity, as \( R_L \), with the method of Iverson and Hooyer (2004). These sliding velocities were then
compared with the velocities using the mass balance profiles using Hooke (1977) and varied accumulation rates.

Geotechnical Data

Preconsolidation stress or pressure

The preconsolidation stress of a sediment layer is the maximum amount of compression that was ever applied to the sediment. For glacial till, this is directly related to effective normal stress, water flux/water pressure, and basal shear stress (Clarke, 1987; Piotrowski and Kraus, 1997; Boulton and Paul 1976). Many studies including Piotrowski and Kraus (1997), have concluded that the normal effective stress has more of an effect on sliding velocity than shear stress, which means the effective stress on the Rainy Lobe tills must have been very high. The immense pressure that the ice puts on the sediment causes expulsion of the air and water in the pores, from the sediment if it is compressed enough (Piotrowski and Kraus, 1997). If the current effective stress on a sediment is less than the maximum amount of stress that it has experienced in its history, then the sediment is termed overconsolidated (Piotrowski and Kraus 1997).

High consolidation states require that pore-water effectively drain: a lodgement till that has been fully drained will be more consolidated than a lodgement till that was consolidated while there was a high level of water at the bed (Piotrowski and Kraus, 1997).

Uniaxial or triaxial stress tests can be run on sediment samples, in order to determine the preconsolidation pressure. These tests are run in increments by gradually increasing the overburden stress until the vertical stress dips steeply (Figure 15). Any stress that is less than or equal to the preconsolidation amount will not cause the sediment layer to deform. However, once the pressure exceeds that amount, the sediment exhibits permanent deformation. Once the normal effective stress and void ratio values are plotted up, the Casagrande graphical method (Casagrande, 1936) can be used to approximate the preconsolidation pressure.
Lodgement tills have very high preconsolidation stresses, which causes them to be very dense, have lower porosities, and thus tend to be less oxidized and relatively less deformed than meltout tills.

Consolidation tests were given to this study by MNDOT, from a location south of Lake Osakis in the Wadena drumlin field. A plot in excel was created using the applied stresses, in tsf, and void ratios at each increment of the test in order to back out the preconsolidation pressure of the sediment. The porosity of the sediment was also determined using the following relationship between void ratio (e) and porosity (\( \phi \)):

\[
e = \frac{V_v}{V_s} \quad \Rightarrow \quad \frac{\phi}{1-\phi} \quad (27)
\]

\[
\phi = \frac{V_v}{V_t} \quad \Rightarrow \quad \frac{e}{1+e} \quad (28)
\]

Figure 15: Shows the approximation of the preconsolidation pressure of a sediment on a consolidation graph, using the Cassagrande graphical method (Casagrande, 1936). The arrow indicates the preconsolidation stress (modified from Piotrowski and Kraus, 1997).
where \( V_v \) is the volume of the voids, \( V_t \) is the total volume, and \( V_s \) is the volume of the solid.

The preconsolidation pressure was approximated using the Casagrande graphical method (Casagrande, 1936). This critical stress will be used to confirm our assumptions that the tills in the study area are true “lodgement tills” and are overconsolidated.

**Results**

The method of Weertman (1957) can be qualitatively used to illustrate glacier sliding. Since small bumps are accommodated by regelation and large bumps by enhanced creep, there is a transitional size that provides the greatest resistance to the glacier. This size is shown by the intersection of the \( S_r \) and \( S_p \) trends (Figure 8), since this bump is not efficiently accommodated by either regelation or enhanced creep. Sliding velocity is also sensitive to the roughness of the bed and ice viscosity; as the ice bed becomes rougher and/or the ice becomes more viscous, the sliding velocity decreases, but the critical obstacle size remains constant. Calculations were done using the method of Weertman (1957) in order to demonstrate the negative logarithmic trend of regelation and the positive linear relationship of enhanced creep as bump size increases, as well as to illustrate the concept of the critical bump size (Figure 8). In Iverson and Hooyer (2004), plowing clasts are observed, and any clast above a certain size \( R_L \) is not able to plow because the drag force is too great on the clast for the ice to drop it. Since the clasts in these exposures have lodged, they must have been plowing before being forced into the sediment by the overriding ice. Thus, even though ploughing structures are not present, since the surface of the till is not visible, clast size can still be accurately measured using the long axis of the rock fragments.

**Frequency Distributions and Sliding velocities**

Thousands of particles were analyzed at each site and organized into histograms to show the frequency distribution of clasts in the till exposures with bin sizes of 2mm (Figure 16). Any size less than 8mm is not included because it can not accurately analyzed by ImageJ, and fragments of this size are assumed to be image artifacts. Thus a “clast” in this study is defined as being a particle that is greater than or equal to 8mm. The upper bound used is 50 mm, or the 48 mm bin, because any clast larger than 50mm only had a frequency of 1 or less for each site and thus these large sizes are not considered important to this
study. T1 has unusually small clast sizes when compared to the other T-exposures. Because of very limited exposure, only one photo (from Meyer, 2009) was used for the analysis, and thus is not necessarily representative of the Independence till as a whole at that location. The general distribution for all of the locations do not vary significantly between sites and are all clustered between 8 and 10mm clast radius.

Frequency distributions converted to log$_2$ were also created for each site in order to draw a linear line of best fit through the data. The intermediate clast size was directly determined from the x-intercept of the best fit line (Figure 17). The intermediate clast size was then used to calculate sliding velocity, as $R_L$, using equation 4 in Iverson and Hooyer (2004). $R^*$ was calculated from equation 6, assuming an $R_s$ of 0.004 m or 4 mm. $R_L$ values (clast size) and their respective sliding velocities are tabulated in Table 3. Resulting clast sizes range from .0172 to .1052 m, with sliding velocities between 132-326 m/yr, and an average of 197.56 m/yr. $P_e$ and $S$ were also calculated with each $R^*$, using equation 7 and 8 respectively, for comparison to preconsolidation stresses later. $P_e$ values ranged from 356-882 kPa and $S$ values from 169-418 kPa (Table 4). Other variables used in these critical equations like $B_1$, $C_1$, and $\Psi$ were kept constant, and the values used came directly from Table 1. Since $N$ had a range, a value of 3 was used for $P_e$ and $S$ calculations. In addition, a full range of $P_e$ and $S$ values are also calculated and included in Table 5 in order to show how $P_e$ and $S$ can vary based on the scale of $N$ used in the calculation. As $N$ increases, the resulting $P_e$ and $S$ values decrease. The exposures can be observed for visual comparison in Figure 18.
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Particle size distribution for samples W1 and W2.}
\end{figure}

\begin{itemize}
\item a) Sample W1:
\item b) Sample W2:
\end{itemize}
c) B1

Particle size (mm)

Count

Cumulative percent (%)

T1

Particle size (mm)

Count

Cumulative percent (%)

38
Figure 16: Frequency distributions for each site before converting to log.: a) W1, b) W2, c) B1, d) T1, e) T2, f) T3, and g) T4. The majority of the counts are skewed to the finer end of the graph for every site. T1 has a high margin of error since there was only one photo was available for photogrammetry analysis, so less clasts were counted than for the rest. Any size less than 8mm and greater than 50mm was omitted from the graph since any size less than 8mm are image artifacts and any size greater than 50mm only had a frequency of 1 or 0, which is not helpful to this study. Cumulative percent was also calculated for each site.
c) 
B1 Frequency Distribution \[ y = -3.1915x + 18.144 \]

\[ \begin{array}{cccccc}
\text{Frequency (log}_{2}^{}) & 0 & 1 & 2 & 3 & 4 \\
\text{Clast size (log}_{2}^{}) & 0 & 1 & 2 & 3 & 4 \\
\end{array} \]

\[ \begin{array}{ccccccc}
\text{Frequency (log}_{2}^{}) & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{Clast size (log}_{2}^{}) & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array} \]

\[ \begin{array}{ccccccc}
\text{Frequency (log}_{2}^{}) & 11 & 12 & 13 & 14 & 15 & 16 \\
\text{Clast size (log}_{2}^{}) & 11 & 12 & 13 & 14 & 15 & 16 \\
\end{array} \]

d) 
T1 Frequency Distribution \[ y = -2.3622x + 9.6852 \]

\[ \begin{array}{cccccc}
\text{Frequency (log}_{2}^{}) & 0 & 1 & 2 & 3 & 4 \\
\text{Clast size (log}_{2}^{}) & 0 & 1 & 2 & 3 & 4 \\
\end{array} \]

\[ \begin{array}{cccc}
\text{Frequency (log}_{2}^{}) & 5 & 6 & 7 \\
\text{Clast size (log}_{2}^{}) & 5 & 6 & 7 \\
\end{array} \]

\[ \begin{array}{cccc}
\text{Frequency (log}_{2}^{}) & 8 & 9 & 10 \\
\text{Clast size (log}_{2}^{}) & 8 & 9 & 10 \\
\end{array} \]
e) T2 Frequency Distribution

\[ y = -1.9738x + 12.294 \]

f) T3 Frequency Distribution

\[ y = -2.9471x + 17.135 \]
Figure 17: Frequency distributions of each site after being converted to log2: a) W1, b) W2, c) B1, d) T1, e) T2, f) T3, and g) T4. The x-intercept of the trendline equation was used to calculate the intermediate clast size. The size, after being converted to mm and then m, was used to calculate sliding velocity.
Figure 18: Photographs of each till site in sequence of drumlin field: a) W1 b) W2 c) B1 d) T1 e) T2 f) T3 g) T4. A red line is drawn to separate the lodgement till and meltout till on each photo (meltout exposures were only present in the Toimi drumlin field).
Table 3: The clast sizes for each site are determined from their respective frequency distributions. The resulting sliding velocities are using Iverson and Hooyer (2004). Distance from the margin (LGM) was measured in Google Earth.

<table>
<thead>
<tr>
<th>Site</th>
<th>Clast size from frequency distributions (mm)</th>
<th>(m)</th>
<th>Sliding velocities (m/yr)</th>
<th>Distance from the margin (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>53.24</td>
<td>0.053</td>
<td>185</td>
<td>86,375</td>
</tr>
<tr>
<td>W2</td>
<td>37.6</td>
<td>0.038</td>
<td>221</td>
<td>95,833</td>
</tr>
<tr>
<td>B1</td>
<td>55</td>
<td>0.055</td>
<td>182</td>
<td>106,819</td>
</tr>
<tr>
<td>T1</td>
<td>17.2</td>
<td>0.017</td>
<td>326</td>
<td>398,389</td>
</tr>
<tr>
<td>T2</td>
<td>75.5</td>
<td>0.075</td>
<td>155</td>
<td>284,385</td>
</tr>
<tr>
<td>T3</td>
<td>55.6</td>
<td>0.056</td>
<td>181</td>
<td>288,218</td>
</tr>
<tr>
<td>T4</td>
<td>105.2</td>
<td>0.105</td>
<td>131</td>
<td>252,426</td>
</tr>
</tbody>
</table>

Table 4: Calculated effective normal stress (Pe) and shear stress (S), using N =3, for each site using Iverson and Hooyer (2004).

<table>
<thead>
<tr>
<th>Site</th>
<th>Pe (kPa)</th>
<th>S (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>627.2</td>
<td>297.2</td>
</tr>
<tr>
<td>W2</td>
<td>527.1</td>
<td>249.7</td>
</tr>
<tr>
<td>B1</td>
<td>637.5</td>
<td>302.1</td>
</tr>
<tr>
<td>T1</td>
<td>356.5</td>
<td>168.9</td>
</tr>
<tr>
<td>T2</td>
<td>746.9</td>
<td>353.9</td>
</tr>
<tr>
<td>T3</td>
<td>640.9</td>
<td>303.7</td>
</tr>
<tr>
<td>T4</td>
<td>881.7</td>
<td>417.7</td>
</tr>
</tbody>
</table>
Table 5: Pe and S ranges for each site using the lowest and highest N values from Iverson and Hooyer (2004).

<table>
<thead>
<tr>
<th>Site</th>
<th>Pe range (kPa)</th>
<th>S range (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>282.5-900.3</td>
<td>133.9-426.6</td>
</tr>
<tr>
<td>W2</td>
<td>237.4-756.6</td>
<td>112.5-358.5</td>
</tr>
<tr>
<td>B1</td>
<td>287.2-915.1</td>
<td>136.0-433.6</td>
</tr>
<tr>
<td>T1</td>
<td>160.6-511.7</td>
<td>76.1-242.5</td>
</tr>
<tr>
<td>T2</td>
<td>336.4-1072.1</td>
<td>159.4-507.9</td>
</tr>
<tr>
<td>T3</td>
<td>288.7-920.0</td>
<td>136.8-435.9</td>
</tr>
<tr>
<td>T4</td>
<td>397.1-1265.5</td>
<td>188.2-599.6</td>
</tr>
</tbody>
</table>

Ice profiles

Mass balance calculations were conducted in order to reconstruct the Rainy Lobe ice profile from LGM to the ice divide. Equations 18-23 were used in sequence to finally solve for the sliding velocity of the Rainy Lobe along the whole length of the profile. The values for $n$, $B$, $r$, and $g$ were taken from Hooke (1977). A $t_i$ of 76,000 Pa was calculated using equation 12, using $h$ and $S$ values from the Greenland Ice Sheet. The Greenland Ice Sheet is considered an analog for the Laurentide Ice Sheet and is frequently used to model past ice sheet conditions, since the geographical location and accumulation patterns are similar between the two (Bales et. al 2001).

Various accumulation rates were considered from different studies on both the Greenland and Laurentide Ice Sheets. The average accumulation rates range from 17.5-30 g/cm$^3$, the actual accumulation rates being between 50 kg/m$^2$ and 450 kg/m$^2$, with the higher values at the accumulation max and the lower values at the ice divide (Bales et al. 2001, Hooke 1977, Sugden 1977, and Bromwich 2005) (Table 6). Ice thickness, profile length, and slope were kept constant, in addition to the other parameters ($n$, $B$, $r$, and $g$). As average accumulation increases, ice discharge also increases throughout the profile (Figure 19). Sliding velocity follows a similar trend (Figure 20). The sliding velocities from the seven sites, using the method of Iverson and Hooyer (2004), are graphed in relation to the velocity profiles determined by the method of Hooke (1977). Distance from the margin (LGM) had to also be approximated for each site, in order to plot the sliding velocities on the same graph as the ice profiles. These distances range from approximately 86,000 to 400,000 m away from the Alexandria Moraine. All of the sliding velocities, excluding T-1, are well within the ice profile velocities determined by mass-balance calculations. There is a gradual increase
in sliding velocity from the ELA to about 160,000 m away from the margin, instead of an overall decrease, which can be explained by ice thickness and discharge factors. At the ELA, the discharge decreases linearly, but the ice profile decreases exponentially, which causes a lag in the sliding velocity profile.

Figure 19: Ice thickness is constant. Even though the maximum ice discharge varies depending on the accumulation rates used, the general trend of all of the curves is similar. Dotted line is Bales et. al (2001); wide dashed line is Hooke (1977); small dashed line is Sugden (1977); and the line with no gradient is Bromwich (2005). Refer to Table 6 for the approximate accumulation rates used from each study.
Figure 20: Comparing sliding velocities using the method of Iverson and Hooyer (2004) with velocities using Hooke (1977). Stars for the seven sites are plotted in comparison to sliding velocities from reconstructed ice profiles and have different patterns based on the drumlin field that they belong to. Only 0-400,000 m away from the margin is shown to better view how the stars compare to the curves.

<table>
<thead>
<tr>
<th></th>
<th>Accumulation rate at the divide (kg/m²)</th>
<th>Accumulation rate maximum (kg/m²)</th>
<th>Average accumulation (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bales et al.</td>
<td>150</td>
<td>450</td>
<td>30</td>
</tr>
<tr>
<td>(2001)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hooke</td>
<td>350</td>
<td>150</td>
<td>27.5</td>
</tr>
<tr>
<td>(1977)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sugden</td>
<td>400</td>
<td>50</td>
<td>22.5</td>
</tr>
<tr>
<td>(1977)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bromwich</td>
<td>250</td>
<td>100</td>
<td>17.5</td>
</tr>
<tr>
<td>(2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Preconsolidation pressure

Using the uniaxial consolidation test from MNDOT and the Casagrande graphical method, the preconsolidation pressure was calculated to be approximately 141 kPa with a void ratio of 0.45 (Figure 21). The calculated porosity is 0.31, using equation 28.

![Void Ratio vs. Vertical stress (Pa)](image)

*Figure 21 shows the results of the uniaxial test from MNDOT and how overburden pressures affected the Hewitt till at this location. Calculation of the preconsolidation pressure was done using the Cassagrande method (Casagrande, 1936). Approximate preconsolidation pressure is indicated by the arrow.*

Discussion

Clast size and Sliding Velocity

Transitional clast size, which is important in determining sliding velocity, becomes larger overall up glacier from the margin. However, the estimated sliding velocities do not vary more than 90 m/yr among the sites, with the exception of T1. These calculated velocities are consistent with measured velocities on modern glacier analogs such as the Greenland ice Sheet and reconstructions of the LIS (Bales et al., 2001).
The method of Weertman (1957, 1964) calculates an intermediate clast size of around 0.006 m, using the intersection of the trends of Sr and Sp (Figure 8). This size is smaller than the observed clast sizes used to calculate sliding velocities with the method of Iverson and Hooyer (2004). The determined clast sizes from the frequency distributions of this analysis would be accommodated by enhanced creep, when compared to the U_{w} and U_{sp} trends from Figure 8, because they are larger. However, Weertman’s (1957, 1964) analysis is highly simplified to illustrate the concept, and considers only the roughness of an idealized bed, not the physical interactions of sediment and clasts embedded in glacier ice.

Iverson and Hooyer (2004) found the distribution of clasts that plowed in their exposures to be between 8 and 148 mm in diameter, with the modal size being about 23mm and the majority of clast plowing between 8 and 40mm. The clast radiuses that were used in their study were as such: R_{L} > 74 mm, R_{s} = 4mm, and R_{*} = 18-20 mm, depending on the R_{L} used. In this study, since it is difficult to determine the smallest lodging clast in the till exposure using this method of photogrammetry, R_{s} = 4mm was used for each site (Iverson and Hooyer, 2004), and R_{L} and R_{*} values were determined based on the frequency plots.

The most abundant clast size in all of these exposures is between 8 and 10 mm, which is consistent with the overall grain size distribution of the Hewitt, Brainerd, and Independence tills (sandy loam texture; Goldstein (1986); Mooers (1990); Lehr and Hobbs (1992)). Calculating sliding velocities of the Rainy lobe using the method of Iverson and Hooyer (2004), yields values that are similar to their results on the Michigan lobe of the LIS, even though their tills were finer-grained overall. Their calculated sliding velocities of 140-168 m/yr, are similar to the velocities in Table 3, which range from 132 to 326 m/yr. The similarity in values indicates that our method of assuming that lodged clasts must have been plowing previously, is reasonable.

Sliding velocities from mass balance calculations, using the method of Hooke (1977), are also within the range of calculated values. Using the same distance from the margin as in Figure 19, the velocities range up to 292 m/yr, with the highest velocities being from Bales et. al. (2001), which uses the greatest average accumulation. Calculated sliding velocities of the Rainy lobe are within the range of sliding velocity curves as a function of distance from the Rainy lobe ice margin, except for T1, which was already mentioned as an exception. The little variance in sliding velocity suggests that the conditions of the
Rainy Lobe did not change significantly throughout its advance and retreat as recorded by the till exposures within Minnesota.

**Normal effective stress and shear strength**

Calculated Pe values of the Rainy lobe range from 356-882 kPa, with values from Iverson and Hooyer (2004) being 490-3630 kPa. In this analysis, we assume no water filled cavities to calculate Pe, while Iverson and Hooyer (2004) calculated both water and no water filled cavities resulting in a wider range. Water-filled cavities increase the normal effective stress on the bed because the weight of the ice is concentrated on a smaller area of bed material compared to being equally distributed when there is no water. The preconsolidation pressure of the MNDOT site (P1) was calculated to be 141 kPa in Figure 21, using the Casagrande graphical method (Casagrande, 1936). Even though the consolidation test yielded a smaller value than the range of values from the field sites, only one preconsolidation pressure was available for analysis and the location was close to the margin of the Rainy lobe where ice thickness, and therefore normal stress, was potentially lower. The resulting Pe values can be used to calculate the load of ice, over hydrostatic pressure, which would be required to output Pe using m(I) (Table 7):

\[
I = \frac{P_e}{\rho g}
\]

As Pe increases, so does the ice load since more ice is needed to cause more consolidation.
Table 7: Shows the approximate consolidation at each site (P1 is the preconsolidation test), and the respective ice load, under hydrostatic pressure, that would be needed in order to exert the required vertical stress on the bed.

<table>
<thead>
<tr>
<th>Site</th>
<th>$P_e$ (kPa)</th>
<th>Ice load (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>627.2</td>
<td>70.3</td>
</tr>
<tr>
<td>W2</td>
<td>527.1</td>
<td>59.1</td>
</tr>
<tr>
<td>B1</td>
<td>637.5</td>
<td>71.5</td>
</tr>
<tr>
<td>T1</td>
<td>356.5</td>
<td>40.0</td>
</tr>
<tr>
<td>T2</td>
<td>746.9</td>
<td>83.8</td>
</tr>
<tr>
<td>T3</td>
<td>641.0</td>
<td>71.9</td>
</tr>
<tr>
<td>T4</td>
<td>881.7</td>
<td>98.9</td>
</tr>
<tr>
<td>P1</td>
<td>141.0</td>
<td>21.4</td>
</tr>
</tbody>
</table>

The bed shear stress ($S$) that was calculated for the Rainy Lobe is ~76,000 Pa, using Greenland as an analog for the Laurentide Ice Sheet. $S$ values from this study are between 169 and 418 kPa (Table 3), and are similar when compared to Iverson and Hooyer (2004), which range from 340 to 2540 kPa. Since the bed shear stress of the ice is significantly less than the shear strength of the sediment by at least a factor of 3, there was probably no pervasive deformation of the soft sediment bed occurring beneath the Rainy lobe. For the applied bed shear stress to exceed the shear strength of the subglacial till, there would need to be 25% water filled cavities at the base of the glacier. This would concentrate the stress and exceed the shear strength of the subglacial till.

Lodgement tills

The lodgement tills, along the flow path of the Late Wisconsinan Rainy Lobe, are overconsolidated and have high bulk densities (Berthold, 2015). Every successive advance and retreat of the Rainy led to the deposition of a lodgement till beneath the ice and an outwash layer in the proglacial zone, from the LGM at the Alexandria Moraine to the Vermillion Moraine (Figure 22). The final advance of the Rainy lobe in Minnesota was to the Vermillion Moraine before it retreated out of Minnesota to the north into Canada (Figure 22). The approximate dates of the advances to each major moraine are taken

The Boulder Concentrations

Berthold (2015) described in detail boulder concentrations on the surface of the lodgement tills of the Rainy lobe. Boulder concentrations are a ubiquitous feature throughout the area glaciated by the Rainy lobe. The boulders range in size from 0.25 to well over 1 meter in diameter. These large clasts, however, do not occur within lodgement tills. It is suggested here that the lodgement till is devoid of larger clasts because they were too large to lodge and were carried with the ice until it stagnated and wasted away (Berthold, 2015). Therefore, locations with boulders on the surface have lodgement till beneath, and the boulders in the Wadena and Brainerd drumlin fields are essentially remnants of the meltout till. Boulders in the Toimi drumlin field are within meltout exposures since the meltout tills here are a few meters thick (Lehr and Hobbs, 1992).

Boulders in the Wadena drumlin field are carbonate and omar-rich (Berthold, 2015). These lithologies are associated with an origin in eastern Hudson Bay and from the Hudson Bay Lowlands (Larson, 2005). The Brainerd and Toimi drumlin field have little to no carbonate or omars, with nearly all Duluth Complex and Red Felsite. The absence of carbonate and omars further up glacier from the LGM indicates that the till becomes more locally derived during later advances. The Hewitt till in the Wadena drumlin field is a sandy loam and contains abundant carbonate and omars. The Brainerd and Independence tills are progressively less calcareous than the Hewitt till and more locally derived just like the boulder concentrations in the Brainerd and Toimi drumlin fields (Berthold, 2015). There is evidence in rotosonic cores to show that the Independence and Brainerd tills were being continuously deposited, even during the LGM, since the base of the Independence till is moderately calcareous, while the upper portion is slightly calcareous to non-calc当地 (Lehr and Hobbs, 1992).

Continuous till sheet

Past studies have postulated that there once was a continuous till sheet that spanned from east of Hudson Bay to the LGM in Minnesota (Larson, 2008; Berthold, 2015). Larson (2008) describes remnant till patches on the Canadian Shield to the northeast of Minnesota that are similar in carbonate content and
lithological composition to the Hewitt till. These tills are calcareous and contain erratics from eastern 
Hudson Bay (Larson, 2005). This continuous till sheet was being eroded by the Rainy Lobe at the central 
portion of the flowline, which exposed Canadian Shield rocks to be eroded during later advances. Thus, 
the Brainerd and Independence tills have more locally derived lithologies and are devoid of carbonate and 
omars. Because the intermediate clast sizes and resulting sliding velocities do not vary much from the 
Wadena drumlin field to the Toimi drumlin field, a continuous till sheet is possible.
Figure 22: Shows the progression of Rainy Lobe through each of its major advances and retreats until its final retreat out of Minnesota: a) 23 ka: the ice advanced to the Alexandria Moraine and deposited the Hewitt till (HT), b) as the ice retreated, it deposited a meltout till on top, c) 15 ka: the ice advances to the St. Croix Moraine and deposits the Brainerd till (BT), d) as the ice retreats, it deposits another meltout till layer, e) at 13.5 ka: the ice advances and deposits the Independence till (IT), and f) the ice retreats and deposits an outwash layer. The ice makes its final advance to the Vermillion Moraine after this time. The bumps at the bed left behind by the ice are homogenous with the Hewitt till, and thus we infer that the ice was cannibalizing its bed as it was advancing and retreating (Larson, 2008).

Conclusion

Lodgement tills are important for glacier reconstruction and can be used to infer parameters such as glacier sliding velocity, ice thickness, and hydrology. A combination of numerical modeling and photogrammetry can be used to calculate the sliding velocity of a remnant ice sheet using the intermediate
clast size in these lodgement tills (Weertman 1957, 1964; Hooke, 1977; Iverson and Hooyer, 2007). This critical size is important because larger clasts will be dragged along by the ice, and smaller clasts will plow and lodge into the bed. These intermediate clasts provide the most resistance to glacier sliding (Weertman 1957, 1964), and are the largest clasts that plow and lodge (Iverson and Hooyer, 2004). The intermediate size is directly related to basal sliding and thus can be used to calculate glacier sliding velocity. The flow path of the Late Wisconsin Rainy Lobe, from the LGM to the final retreat out of Minnesota, has well-exposed lodgement tills that vary in lithology and texture, but have similar clast-size frequency distributions. The Hewitt till is highly calcareous and has abundant omars, compared to the Brainerd and Independence tills, which contain more locally derived rocks from the Duluth Complex. However, even though the Brainerd and Independence tills are more locally derived than the Hewitt till, the resulting sliding velocities show that the ice sheet was moving at a similar rate throughout its flow line in Minnesota.

Lodgement tills are overconsolidated and such preserve the fabric of the matrix and the orientation of clasts within the till (Boulton, 1971). Because all of the lodgement tills analyzed in this study have high effective normal stresses, there must have been between 59 and 99 m of ice load on top of the sediment, while the Rainy Lobe was in Minnesota (Table 7). However, there was no pervasive deformation, in the absence of water filled cavities, because the shear strength of the subglacial sediment exceeded the proposed shear stress exerted on the bed. The lodgement tills of the Rainy Lobe have been previously studied to determine the mean transport length, using boulder concentrations, and to postulate a once continuous till sheet beneath the Rainy Lobe at the LGM (Larson and Mooers, 2004 and 2005; Larson, 2008; Berthold, 2015; Larson et al. 2015; Kryzer et al., 2013). However, this is the first study to use intermediate clast size, determined by photogrammetry, in order reconstruct sliding velocities along the Late Wisconsin Rainy Lobe. This study is an important part of a greater analysis to understand the behavior of the Rainy Lobe and the Laurentide Ice Sheet, during the Late Wisconsin glaciation.
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