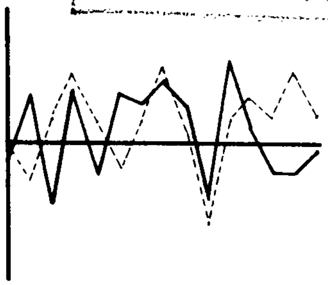
STATISTICAL ANALYSIS of CYCLICAL VARIATIONS in the NATIONAL TURKEY MARKET



M.A. Soliman

Technical Bulletin 276 - 1971
Agricultural Experiment Station
University of Minnesota

". . . when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind."

—Lord Kelvin

CONTENTS

This study was funded in part by Regional Research Project NCM-39, "Improving the Efficiency of Turkey Marketing in the North-Central Region."

M. A. Soliman is a research associate, Department of Agricultural and Applied Economics.

The author is grateful for the helpful suggestions of W. L. Fishel, Y. Hayami, and J. P. Houck, who reviewed the manuscript.

Statistical Analysis of Cyclical Variations in the National Turkey Market

M. A. Soliman

Turkey prices and production exhibited fairly recurrent cycles during the last 30 years. The fluctuations in turkey prices have caused corresponding fluctuations in producer income. The instability and uncertainty in turkey prices make it difficult, if not impossible, for producers to allocate their resources efficiently. In the past, these uncertainties have forced some marginal producers out of the turkey business.

The major objectives of this study were:

- to determine the existence of cycles in the prices and production of turkeys and apply statistical tests that enable us to reject or accept their existence,
- to determine the equilibrium path in the production and price of turkeys, and
- to investigate the nature of response of turkey producers to prices through time.

The analytical method consisted of testing whether or not a simple cobweb existed in the turkey industry with 1-year lags in prices. Then the Fourier coefficients for cycles of 3 and 4 years were estimated. The periodogram analysis was used to test the statistical significance of the cycles derived by the Fourier analysis. A correlogram was used to provide a criterion for determining the nature of the oscillations.

A second order difference equation for price and production of turkeys was estimated to determine their equilibrium path. Finally, gain to turkey producers was estimated where production control measures succeeded in dampening the cobweb pattern.

Data used in the analyses were production figures and prices received by farmers for turkeys deflated by the price received by farmers index (1957-59 = 100) for the period 1934-66.

The Turkey Industry

Understanding the principal functions of the industry is basic to understanding the forces that affect turkey production. The major counterparts in producing and delivering ready-to-cook turkey to consumers (figure 1) are:

- Primary breeders: conduct extensive genetic research and breeding to develop new strains with superior qualities.
- Hatcheries: hatch eggs from parent stock and hatching supply flocks.
- Hatching egg flocks (breeder hens): produce eggs for commercial hatching.
- Turkey producers: grow turkeys from poult phase to marketable weight.
- Processing plants: kill, pluck, wash, and dress turkey. Also freeze, package turkeys. Some are specialized in further processing.
- Cold storage: store turkeys during the heavy production season to satisfy demand in the low production months and in the high consumption period of Thanksgiving and Christmas.
- Wholesalers and retailers: carry out final steps in getting turkeys to consumers.
- Consumers: consume turkeys as whole bird or in further processed form.

The primary breeders are involved in extensive genetic research and breeding experiments. They may specialize in developing a line of turkey to comprise the male strain for the final cross breeding or the female line. Ownership and final control of these flocks are in the hands of a very few firms. Information on how these firms formulate their decisions and policies governing their operations is not available.

Note that no estimate of the number of parent breeders is included. Data are available only on the number of breeder hens present on January 1. Numbers include the multiplier and parent stocks.

Competition among these few firms is almost wholly market oriented. Each firm tries to capture a sizable portion of the market by producing a bird with superior growth rate, feed utilization, and body conformation.

The hatcheries and the hatching egg flocks are mostly on a franchise basis. They are controlled by the primary breeders in many cases. The producers, processing plant operators, cold storage operators, wholesalers, and retailers represent a separate and independent unit. As the scale of operations in the industry has grown, intra-industry arrangements have undergone major changes.

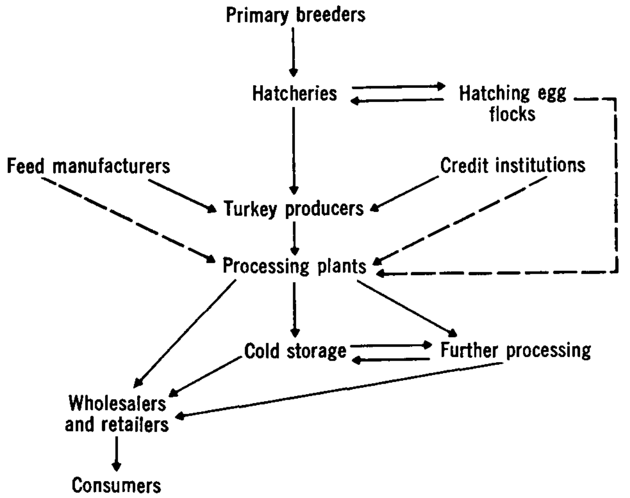


Figure 1. Principal functions of the turkey industry.

Functions have remained essentially unchanged, but the degree of interrelationship has been entirely overhauled. Today, about 80 percent of turkey produced is under some type of integration.

Integration of turkey firms proceeds horizontally, vertically, or in a circular combination. Horizontal integration occurs when two producers merge. Vertical integration occurs when processors or feed suppliers assume the turkey growing function. Circular integration occurs when a firm integrates horizontally and vertically at the same time. Most of the integration in the turkey industry is circular. Number of turkey producers declined from 87,000 to 42,000 in 1959 and 1964, respectively. Production increased from 1,440,000,000 pounds in 1959 to 1,795,000,000 pounds in 1964. Number of hatcheries declined from 551 in 1963 to 303 in 1969, while number of poults hatched increased from 102,400,000 to 115,500,000, a decline of 45 percent in number of hatcheries and an increase of 13 percent in poults hatched. Turkey slaughtering plants under federal inspection declined, with increasing volume per plant. There were 281 turkey processing plants in 1962 and only 193 in 1968; these figures include plants doing further processing. The integrated operation joins together one or more of these functions either through direct ownership, stock interest, or continuing contracts.

Biological considerations are inherent in the natural processes of growing specialized turkeys. Some of these impose more or less fixed times, as in the cases of egg incubation, the growing period of stock to their egg laying stage, or growing birds to market ready weights.

Turkey consumption is seasonal, and it is to this factor, which is roughly predictable, that the whole production cycle is adjusted. The seasonal pattern merely involves a more or less fixed time lag. That is, the setting and hatching of turkey eggs precedes the marketing of finished heavy turkeys by about 8 months.

Natural factors such as weather, death, disease, condemnations, and low hatchability seldom are of major importance so far as fluctuations of total supply are concerned.

Human decisions do affect the relationship of supply of poults and later turkey. Such decisions include relaxing the grading and sorting standards of eggs for poults, and changing the time when a laying flock will be sent to market. When prices of hatching eggs and poults are rising, hatcheries relax the size and quality requirements for eggs, accepting undersized or slightly damaged eggs. If prices are depressed, breeder hens are marketed for meat. However, liquidating the supply flocks is slow because the investment in laying flocks and related facilities is high. Practically the only relevant shortrun cost is for feed. The return to the egg producer is treated mainly as a semi-fixed cost under the typical contractual arrangements between hatcheries and egg producers.

Consumption is closely tied to production. Differences in storage and exports are of minor importance in turkey distribution, as shown in table 1. Because turkey is a perishable commodity, price must adjust so that current production continuously clears the market after allowing for storage and exports. Turkey moves into storage during the peak slaughter months and moves out when fewer turkeys are slaughtered. Differences in storage stocks have been relatively small, however, averaging only 2.8 percent of production, disregarding sign, during the period 1950-68. Less is known about the behavior of and factors affecting the cold storage section of the turkey industry. Export of turkey is small: it averaged 2.3 percent of production during 1958-68.

Previous Studies

The study of turkey price and production started mostly after World War II. Many investigations analyzing demand and supply relations have been published. Paarlberg and Watson (16)[1] explained the factors affecting turkey prices. They used the weighted average price of turkey in October, November, December, and January as the yearly price of turkey. The dependent variable used was in percentage change form:

$$\text{Price in a given year} = \frac{\text{actual price in a given year}}{\text{actual price in previous year}} \times 100.$$

They tried to investigate the existence of cycles in turkey production and prices. They concluded that a cycle of 3-5 years exists for the purchasing power of turkey and fluctuates about 15-20 percent above and below the average.[2] The production cycle was approximately the inverse of the purchasing power cycle and fluctuated only about half as much.

1 Numbers in parentheses refer to references cited.

2 Purchasing power of turkey represents using turkey to purchase other farm products, or the relationship of the price of turkeys to prices of other farm products.

Table 1. Supply, storage, and export of turkeys, 1950-68

Year	Supply	Storage Accumulation	Storage Liquidation	Difference Quantity	Difference Percent	Export Quantity	Export Percent
	million pounds, ready-to-cook			million pounds		million pounds	
1950	615	72.67	100.20	27.53	4.5	..	
1951	703	84.43	94.56	10.13	1.4	..	
1952	795	114.18	83.80	-31.38	3.9	..	
1953	758	91.54	112.26	20.72	2.7	..	
1954	870	98.79	102.71	3.92	.5	..	
1955	818	99.30	128.31	29.01	3.5	..	
1956	957	160.55	94.39	-66.16	6.9	..	
1957	1,034	151.38	144.87	-6.51	.6	..	
1958	1,038	175.66	192.99	17.33	1.7	6	.6
1959	1,123	155.55	166.85	11.30	1.0	12	1.1
1960	1,162	215.47	197.67	-17.80	1.5	24	2.1
1961	1,506	287.78	196.99	-90.79	6.0	28	1.9
1962	1,304	218.38	266.05	47.67	3.7	37	2.8
1963	1,342	272.52	253.41	-19.11	1.4	31	2.3
1964	1,436	253.53	260.75	7.22	.5	43	3.0
1965	1,521	292.84	289.86	-2.98	.2	58	3.8
1966	1,685	326.10	241.40	-84.70	5.0	47	2.8
1967	1,883	401.40	306.60	-94.80	5.0	49	2.6
1968	1,615	319.10	362.60	43.50	2.7	40	2.5

Source: U.S. Department of Agriculture, Economic Research Service, "Poultry and Egg Situation."

Fox (8) used the diagrammatic form to present the demand and supply structure for the turkey industry. The statistical analyses were based on time series data for 1929-41.

Brandow (5) analyzed the demand for turkey relative to the demand for 24 farm products. He showed the direct price elasticities and the cross-price elasticities between each commodity and 23 other commodities. He used total quantity of liveweight turkey slaughtered under federal inspection as the dependent variable. The predetermined variables were price of turkey, price of the other 23 commodities, and trend factors.

Bawden et al. (2) estimated a supply function for turkey. They included a profit index, weighted average feed grain price received by farmers lagged 1 year, price of broilers received by farmers, and a trend variable in the estimated supply function.

Despite the highly seasonal production and consumption pattern of turkey, most statistical studies designed to measure turkey demand and the factors influencing turkey prices were based on annual time series. This approach implies that the impact of factors influencing changes in the demand for turkey from one year to the next is distributed throughout each year in roughly the same way. Recent studies emphasize the seasonal pattern in the turkey industry. Bluestone and Rojko (4) gave statistical formulas for predicting turkey prices in and out of the main marketing period and for the year.

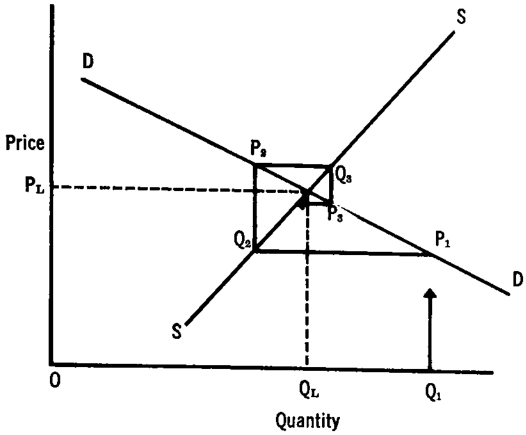


Figure 2. Nature of fluctuations in price and production, convergent fluctuation.

Swenson (22) estimated the elasticity of demand for turkey for seven different periods of the year. A recent study by the author (21) made use of quarterly data in analyzing the structure of the turkey industry. Three relations were estimated: demand, supply, and end of quarter cold storage holding. A second objective of the study was to test if the level and slope of these functions were constant among seasons of the year.

A brief survey of some of the early studies of turkey production and prices reveals knowledge of the price-making forces. The present study is more concerned with testing the existence of cycles in the turkey industry. It is a step toward realizing the prophecy of Benner (3, p. 131) in 1876, when he said: "The science of price cycles is yet in the cradle of its infancy, but waiting its time to mature full development, to unfold its principles, and declare its oracles to all mankind, and to demonstrate that causes and the laws of nature in production are not past finding out; and that man in his onward path of progress . . . will ultimately grasp the future."

Cobweb Analysis

The cobweb theorem provides an alternative explanation of cycles resulting from lags in production response.[3] Conventional economic theory assumes that, under the static conditions of pure competition, market price is established when demand and supply are equated. However, where a considerable time lag occurs between the price change for a commodity and the resulting supply response, the cobweb relationship may lead to widely fluctuating prices and quantities. Assume quantity Q_1 is produced in time period 1 and placed upon the market (figure 2). The resulting price is established at P_1. However, the low price, P_1, results in a supply of only Q_2 in time period 2. The quantities and prices show successively smaller fluctuation as they approach the equilibrium point, Q_2 and P_2, at the intersection of the demand and supply functions.[4]

[3] The cobweb theorem is discussed in Ezekiel (7) and Akerman (1).

[4] In this situation the absolute slope of the supply function is greater than that of the demand function, which constitutes convergent fluctuation. Divergent fluctuation occurs when the absolute slope of the demand function is greater than that of the supply function. Continuous fluctuation occurs when the demand and supply curves have identical slopes at any chosen price.

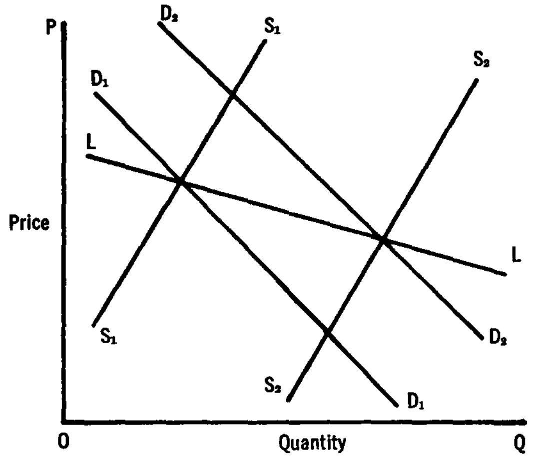


Figure 3. Shift down to the right of the supply function and shift to the right of the consumer demand function, with decreases in the equilibrium prices and increases in quantity.

Demand and supply functions shift over time. Demand shifters are price of substitutes, consumer disposable income, and changes in consumer tastes and preferences. In the case of turkey consumption, the demand schedule shifts to the right, from D_1D_1 to D_2D_2 in figure 3, as a result of changes in the demand shifters. Supply also can be shifted by change in the price of inputs, new cost-reducing methods of producing turkey, and prices of other commodities produced. Such a shift is illustrated in figure 3 from S_1S_1 to S_2S_2. The LL schedule presents the longrun equilibrium prices and quantities. It is characterized by a downward trend in prices with larger quantities.

The underlying conditions for applying the cobweb theorem are:

- Producers plan their output in the next production period on the basis of present prices.
- There are no changes in production plans once they are made.
- Price is determined by the available supply; i.e., price is set by equating demand with the constant supply.

A few empirical results indicate the nature of price expectation models used by turkey producers. They use current prices as the basis for projection and forecasting. A more realistic hypothesis is that turkey growers build their price expectations not only on current prices, but also on prices from previous years. However, the recent price probably has the greatest influence, with the weight attached to each previous price declining as the time lag increases. On the basis of the existing studies, Bawden et al. (2) and Soliman (20), the first condition for a cobweb relationship in turkey production is relatively well satisfied.

The characteristics of turkey production indicate that the second and third conditions also are reasonably fulfilled. Once the number of breeder hens is established, relatively little can be done to increase future production. However, there is greater flexibility in reducing supplies, since breeder hens can be sold for slaughter. The third condition implies no interdependence or simultaneity between the price received and the quantity supplied; i.e., quantity is assumed to be predetermined. As long as farmers do not vary the marketing weight of birds in response to shortrun price changes, the third condition is reasonably fulfilled. A rough indication of the weak response of bird weight to prices is the increase of average weight per bird despite the declining trend in turkey prices over the last 20 years.

Further evidence of a cobweb pattern in price and production in the U.S. turkey market is provided in figure 4, which shows an actual pattern of turkey prices and production with a 1-year lag. The deflated price received by turkey producers in 1933 was 41.2 cents. This price induced 300 million pounds of production in 1934. In turn, this increase in supply resulted in the lower price of 40.8 cents in 1934. Producers reacted to the decline in turkey prices and curtailed production to 298 million pounds in 1935. The reduced supply resulted in a higher price of 44.7 cents in 1935.

There is sufficient regularity in the clockwise rotation indicated in figure 4 to indicate an underlying cobweb relationship, despite the fact that the pattern spirals downward and to the right. Irregularities in this spiral are caused by other forces. For example, World War II and the period 1957-59 were transition periods with production increases despite low prices. Reasons for this expansion were availability of turkey feed at lower prices, new cost reducing developments in feeding technology, and improved breeds of turkeys. The cobweb pattern was repeated in the sixties, but at a lower price level, reflecting another similar shift in the underlying supply relationships.

Changes in turkey consumption are partly responsible for the expansion of turkey production in the 1965-67 period. Increased use of turkey in convenience food items such as rolls, pie, smoked, and cut-up portions, in addition to its traditional role as a festive meat, helped to create the irregular price and output pattern of recent years. Even with the low deflated price of 1966, farmers produced more turkeys in 1967, resulting in even lower prices in 1967.

Turkey Cycles

The existence of the cobweb phenomenon in the turkey industry suggests pronounced cycles in the production and price of turkeys. To test the existence of these cycles and measure their length, harmonic analyses were used.[5] The methods used in the analyses are developed in the next section. The statistical results of using these procedures are discussed in the following section.

5 Study of cycles received great attention by economists and statisticians in the twenties and thirties. Agricultural economists used harmonic analysis in many early cyclical analyses. It was used by Moore (14), Tintner (23), and recently by Waugh and Miller (26). More attention is now focused on spectral analysis of commodity cycles, as in Rausser and Cargill (17) and Weiss (27).

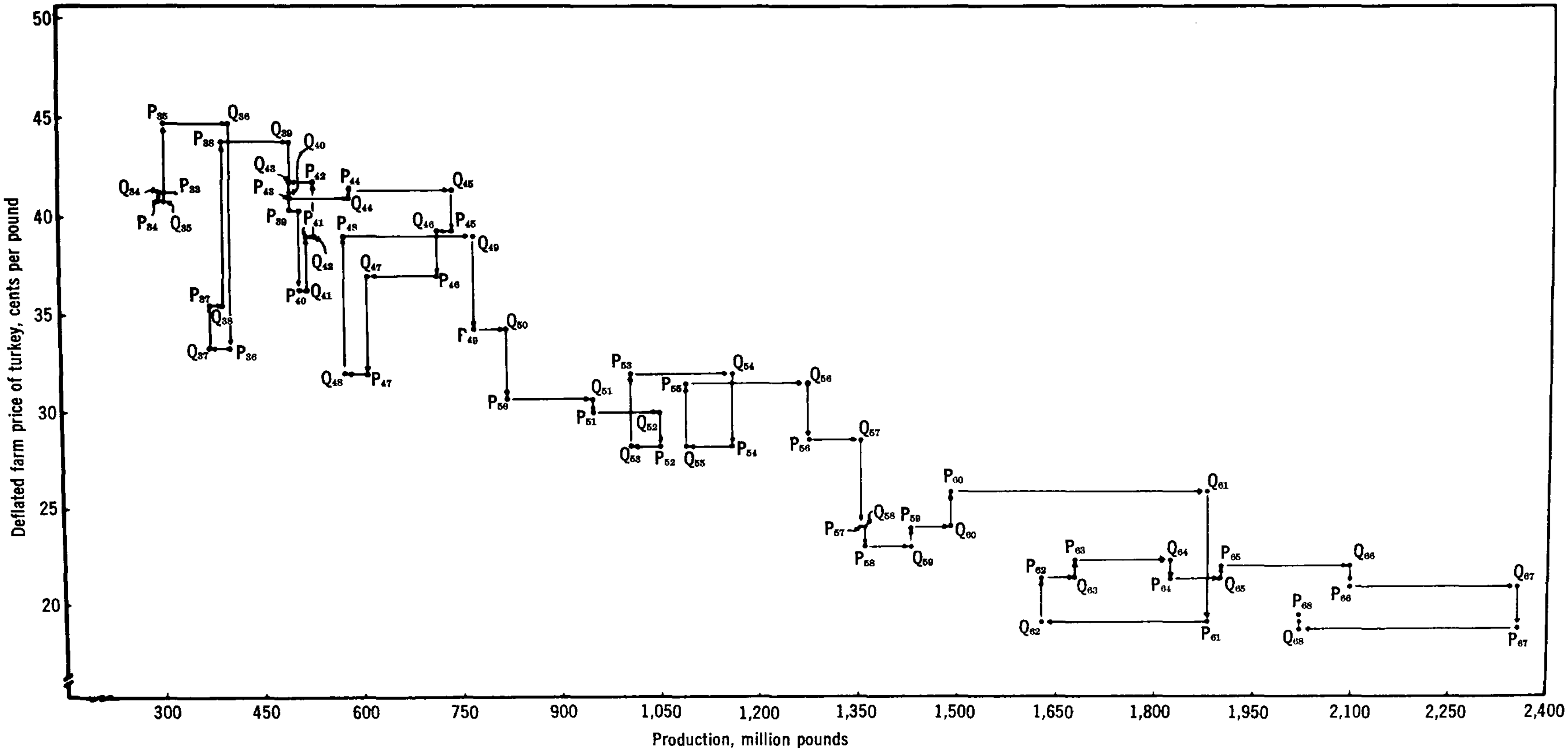


Figure 4. Relation of turkey production and farm prices, 1933-68.

Methodological Development

The following definitions commonly are used to describe movements in time series data. Of course, these movements can occur simultaneously.

1. The general trend or long term movement (T_t) corresponds to change that is maintained over many years.
2. The cycle (C_t) is a movement of quasi periodic oscillations around the trend.
3. Seasonal movement (S_t) corresponds to regular weekly , monthly, and/or yearly variations.
4. Accidental, random, unsystematic, or "irregular" (e_t) fluctuations are unpredictable movements connected with events of all kinds.

The analysis of a time series attempts to determine and isolate each of its components. If the time series components are additive, they can be expressed by:

$$Q_t = T_t + S_t + C_t + e_t$$

where:

Q_t denotes the time series.
T_t denotes the general trend.
C_t denotes the cycle.
S_t denotes the seasonal movement.
e_t denotes the random element.

Since yearly data are used, seasonal movements (S_t) cannot be identified. The observed series then can be written as:

$$Q_t = T_t + C_t + e_t$$

Harmonic analysis can be used to discover the constituent periodicities that enter into the construction of prices and production of turkeys arranged in a time sequence. For harmonic analysis, a stationary time series with no trend is required.[6] Two procedures are available to determine and eliminate the influence of a trend: the moving average procedure and the polynomial procedure.

The use of moving averages affects other components of the series by emphasizing the shorter oscillations at the expense of the longer ones. Also, the moving average of a random series tends to generate an oscillatory series if the weights result in a positive correlation between successive members of the generated series.[7] In addition, a moving average of extent k sacrifices k-1 terms of the series. This means that, by using the moving average, the derived series is k-1 terms shorter than the original series.

The polynomial method assumes that the trend can be expressed in a polynomial form as:

$$Q_t = a_o + a_1t + a_2t^2 + \ldots + a_pt^p$$

where:

Q_t denotes the time series.
t denotes time in year, month, or week, $t = 1, 2, \ldots T$.
p denotes the degree of the polynomial.

To determine the degree of the polynomial that represents the trend most adequately, the analysis of variance test can be used to test whether or not the residual variance, after fitting a polynomial of degree $p + 1$, is significantly smaller than the residual variance after fitting a polynomial of degree p.

Four main methods of studying oscillatory effects in time series are:

- counting peaks and troughs.
- using the periodogram.
- using the correlogram.
- using the cobweb theorem.

A simple method to determine the peaks and troughs of the cyclical movement is to obtain the trend for the series, subtract the annual values of the trend from the corresponding actual annual values, divide the actual deviation for each year by the corresponding annual value of the trend, and multiply by 100. Harmonic analysis was selected for accurately estimating the length of the cycle.[8] Before applying these analyses, it may be useful to discuss them.

Fourier Analysis

Assume an empirical time series, $X_1, X_2, \ldots X_N$, which has a period of length T. The stationary (free of trend) series Y_t can be represented as:

$$Y_t = \frac{1}{2} A_o + \sum_{j=1}^{T/2} \left(A_j \cos \frac{360jt}{T} + B_j \sin \frac{360jt}{T}\right)$$

where:

Y_t denotes the series.

t denotes time, $t = 1, \ldots N$.

A_o, A_j, and B_j are constants and are given by:

$$A_o = \sum_{t=1}^{N} \frac{X_t}{N}$$

$$A_j = \frac{2\left[\sum_{t=1}^{N} X_t \cos\left(\frac{360jt}{T}\right)\right]}{N}$$

$$B_j = \frac{2\left[\sum_{t=1}^{N} X_t \sin\left(\frac{360jt}{T}\right)\right]}{N}$$

In practice, it is more convenient to group the data as shown below for investigating a given period, P, where mP is equal to N or the nearest integer below N.

X_1	X_2	X_P
X_{P+1}	X_{P+2}	X_{2P}
$\vdots$	$\vdots$	$\vdots$
$X_{(m-1)P+1}$	$X_{(m-1)P+2}$	X_{mP}
Sums U_1	U_2	U_P

6 For the definition of stationarity, see Granger (10) and Kendall (12).

7 This condition is called the Slutzky-Yule effect, Kendall (12) and Tintner (23).

8 For more discussion on time series analysis, see Granger (10), Hannan (11), and Kendall (12).

If a term of period P is present in the series, the column totals (U_j) will indicate the periodic effects. But, if the remaining element is random, the effect of summing m rows will be to reduce the relative contribution of that element to the column totals. Similarly, if there are other elements with different periods, they will be out of phase in successive rows and tend to cancel out in the totals. Hence, if there are enough rows, the total (U_j) will reveal the periodic effect and will reduce any masking effects resulting from random components or oscillatory components of different periods, which would have prevented discernment of the periodic effect in the primary series. The Fourier coefficients A_P and B_P are computed from the formulas:

$$A_P = \frac{2\left[\sum_{j=1}^{P} U_j \cos\left(\frac{360j}{P}\right)\right]}{mP}$$

$$B_P = \frac{2\left[\sum_{j=1}^{P} U_j \sin\left(\frac{360j}{P}\right)\right]}{mP}$$

The square of the amplitude R_P^2 is obtained by adding the Fourier coefficients A_P^2 and B_P^2.

Periodogram Analysis

The periodogram analysis helps find the "hidden" periodicities. In the Fourier analysis, several periods were assumed and the square of the amplitude was computed for each specified period. The periodogram tests the significance of the amplitudes derived from the Fourier analysis. The process is:

1. Compute the square of the amplitude (R_P^2) as shown above. If there are no periodic fluctuations, the mean square amplitude for a random series without periodic fluctuation is: $R_m^2 = \frac{4\sigma^2}{N}$

   where σ^2 is the variance of the series X_t
2. Calculate
   $$k = \frac{R_P^2}{R_m^2}$$
3. Determine the statistical significance of the calculated k by applying one of the following tests: Schuster test, Walker test, or Fisher test.[9]

Correlogram Analysis

The correlogram is the array of the coefficients of serial correlation: $r_o (= 1), r_1, r_2, ..., r_k$

$$r_k = \frac{\frac{1}{N-k}\sum_{i=1}^{N-k}(X_i - \overline{X})\quad(X_{i+k} - \overline{X})}{\frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^2}$$

$$= \frac{\text{covariance }(X_i, X_{i+k})}{(\text{variance } X_i \text{ variance } X_{i+1})^{\frac{1}{2}}}$$

where:

i denotes time, $i = 1, 2, ..., N$.

N denotes the number of observations.

k denotes the order of the serial correlation.

X_i denotes the value of a variable in the i^{th} time period.

The form of the correlogram provides a method of discriminating between the various types of oscillatory series. It provides a basis for distinguishing between three types of oscillatory series: moving averages, autoregressive schemes, and harmonic terms. The term correlogram denotes a graph with r_k as the ordinate and k as the abscissa.

The correlogram for a series generated by the moving average method will vanish after a certain order. The correlogram will oscillate and will not vanish, although its oscillation will dampen for a series generated by the autoregressive method. For infinite cyclical series of harmonic terms, the correlogram is a harmonic with the period equal to that of the original harmonic component; it will not vanish or be dampened.

Empirical Results

The analyses in this section were based on liveweight production and price received by farmers per pound of turkey, deflated by the price received by farmers index (1957-59 = 100). These observations are 1934-66 figures published by the U.S. Department of Agriculture. Historically, turkey production has had an increasing trend, while prices have had a decreasing trend, as shown in figure 5. Long term increase in output occurred despite a very steady decrease in prices. The explanation can be attributed to the concurrent reduction in production costs. The adoption of new breeds continues to provide impetus for expanded output. However, production and price trends have had marked oscillatory effects that were not perfectly regular.

A trend value was calculated for production and price by using a second degree polynomial function. The deviations of the actual series from the computed trend were expressed as percentages of the corresponding trend value. These percentages, considered cycle measures, are presented in figure 6. An hypothesized model results in a 3-year cycle (from trough to trough or peak to peak) in turkey production and price. A high price in one year leads to a large crop the next year; the realized large production results in a low price received by growers for that year; and the low price is followed by a small crop and higher prices.

An exact likeness of this model is not expected with the actual data. There are many exogenous variables that affect the turkey industry and distort the smooth theoretical relationships. The price of turkey depends upon demand as well as supply. Demand for turkey is affected by consumer income, population, prices, and supplies of competing meats.

On the supply side, there are many factors besides price that determine production. These factors are the

[9] For a discussion of these tests, see Davis (6, pp. 186-91).

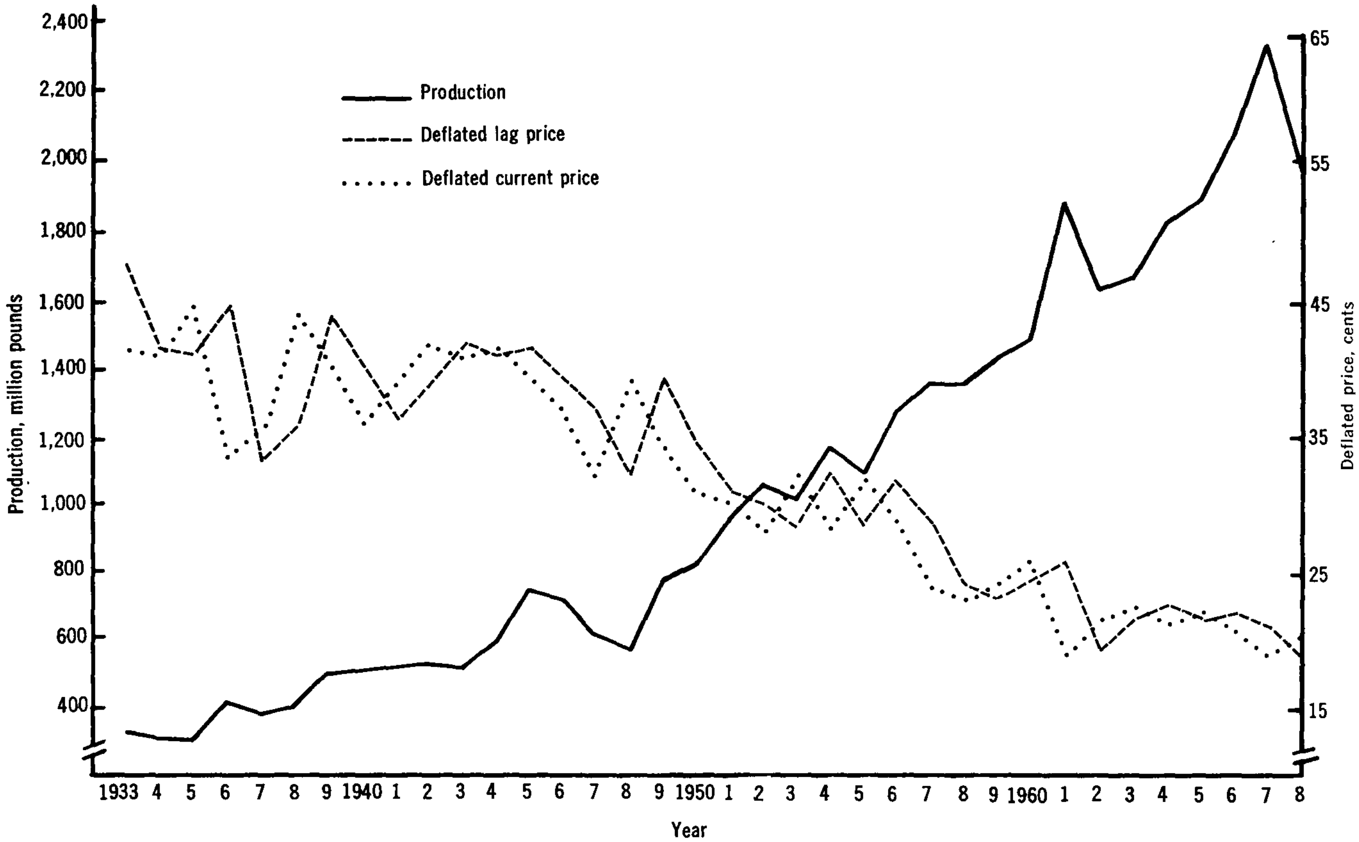
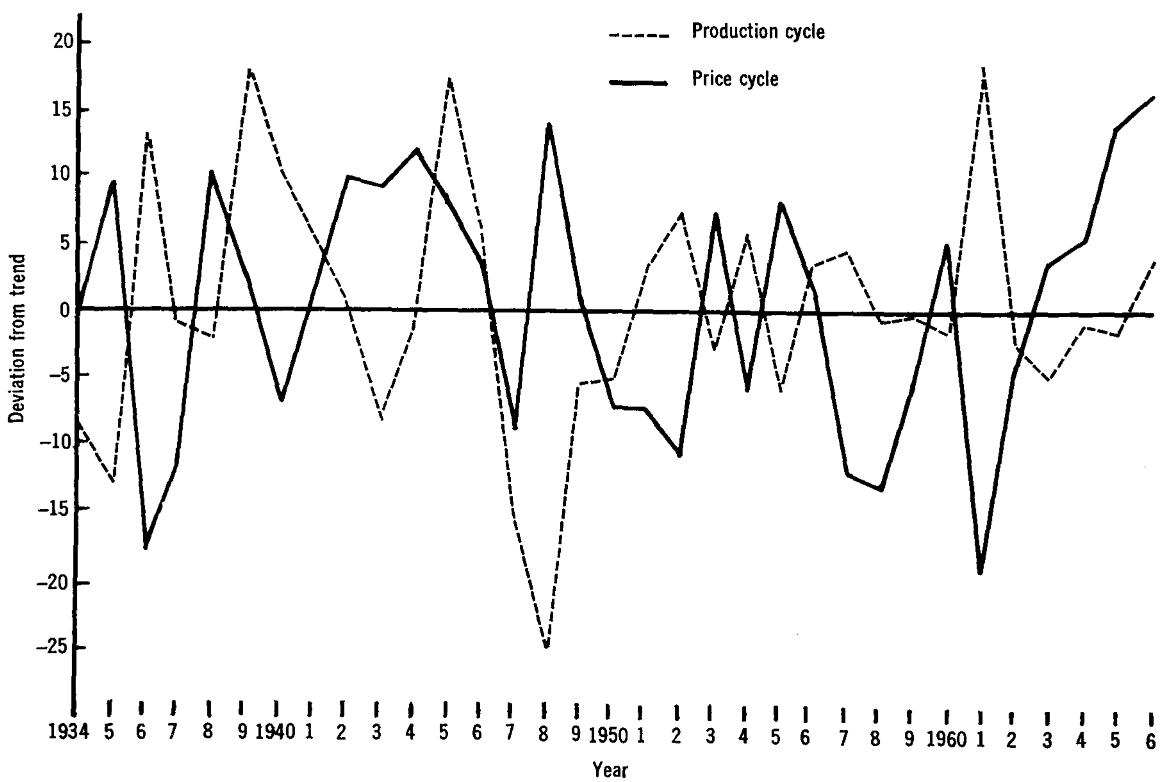


Figure 5. Production and deflated price per pound received by farmers for turkey, lagged and current prices, United States, 1933-68.

Figure 6. Turkey price and production cycles.



price and supply of feed and the number of breeder hens saved. The number of hens saved is influenced by weather, genetic improvements, and production practices. Cycle disruptions caused by these factors were noticed during World War II and the middle of the fifties and sixties. There have been nine complete cycles (from trough to trough) in production and eight in prices in the last 33 years. The cycles have varied from 3 to 6 years in length. The production cycle fluctuated about 17 and 25 percent above and below the average. The price cycle was about the inverse of the production cycle, fluctuating about 14 and 18 percent above and below the average.

Note from figure 6 that the amplitude of the swings seems to be decreasing. In the early years, lack of effective coordination of the vertical stages was a contributing condition underlying the high swings. Specialized and large size turkey growers are more market conscious than are growers with small operations such as were common in the thirties, forties, and early fifties.

The harmonic analysis was used to test statistically the existence and length of cycles, the results of which are shown in table 2 for production and table 3 for prices. The production cycle of 3 years is significant at the 5 percent level according to the Schuster test (18). The calculated value of k is 3.22, which is larger than the tabulated k of 3. Hence, it is unlikely that the observed ratio for a period of 3 years could have occurred by chance. The Schuster test does not verify a cycle of 3 or 4 years for turkey prices, even at the 5 percent probability level, based on the calculated k ratios.

The serial correlation coefficients of the production series are shown in table 4 and the correlogram is shown in figure 7. The correlogram is smooth and oscillatory, and no dampening can be observed. The serial correlation coefficients and the correlogram for price are given in table 5 and figure 8. The oscillations persist in a most remarkable way and are not of a dampening type. Since the correlogram for production and price for turkey is oscillatory and not vanishing or dampening, we cannot reject the existence of harmonic terms in the production and price of turkey series.

Equilibrium Path in Production and Price of Turkey

To determine the equilibrium path of quantity and price for turkey, a second order difference equation was estimated for each.[10] The time path of production and price of turkeys is determined by the magnitude and sign of the roots of the characteristic equation.

Given the initial production of 1933, Q (0) = 319, and 1934 production, Q (1) = 300, the particular solution for the second order difference equation for the quantity produced of turkey Q in time t is given by:

$$Q(t) = 638.3290\,(1.0402)^t - 71.1329\,(-.3721)^t - 390.4619$$

Since both roots are real and one is positive and exceeds unity, and the other is negative, then Q (t) will increase as t increases (appendix B, case III-3). The equilibrium path is shown in figure 9.

Table 2. Fourier coefficients, amplitude squared, mean square amplitude, and the ratio k for production of turkey for 3 and 4 year cycles

Period n	Fourier coefficients A_p	B_p	Amplitude squared R_p^2	Mean square amplitude R_m^2	Ratio $R_p^2/R_m^2 = k$
3	41.0601	24.9570	2,308.7837	717.6653	3.2171
4	29.1286	2.5166	854.8086	743.5131	1.1497

Table 3. Fourier coefficients, amplitude squared, mean square amplitude, and the ratio k for turkey farm prices for 3 and 4 year cycles

Period n	Fourier coefficients A_p	B_p	Amplitude squared R_p^2	Mean square amplitude R_m^2	Ratio $R_p^2/R_m^2 = k$
3	.1598	1.0753	1.1818	1.1699	1.0102
4	.5719	.1621	.3533	1.2128	.0291

Table 4. Correlogram of turkey production series (trend free)

k	r_k	k	r_k	k	r_k
0	1.0000	11	–.0740	22	.4800
1	–.0043	12	.1525	23	–.2231
2	.0348	13	–.3899	24	.0556
3	–.2398	14	–.1595	25	–.0233
4	–.1054	15	.1331	26	–.4531
5	.2316	16	.2905	27	.0542
6	–.1010	17	.0912	28	–.4398
7	.1864	18	–.1026	29	.0282
8	–.3202	19	–.2542	30	.9594
9	.0279	20	–.0350		
10	–.0458	21	.3550		

Table 5. Correlogram of turkey price series (trend free)

k	r_k	k	r_k	k	r_k
0	1.0000	11	.2081	22	.5425
1	.1505	12	–.2916	23	–.0126
2	–.1258	13	–.2482	24	.1702
3	.1149	14	–.0908	25	.6244
4	.1309	15	–.0984	26	–.1829
5	–.0456	16	–.2034	27	.0093
6	–.2197	17	–.0683	28	.0095
7	–.0319	18	.1093	29	–.8398
8	–.4135	19	–.5085	30	–.2712
9	–.2440	20	.1803		
10	.2214	21	.8941		

If the deflated price of turkey received by farmers in 1933 is the initial price P (0) = 41.2, and 1934 is the price in time 1, then P (1) = 40.8. The particular solution for the second order difference equation for the deflated price in time t is:

$$P(t) = 19.50704\,(.93585)^t + 1.22828\,(-.30685)^t + 22.9212$$

[10] For more discussion of different equation solutions and implications, see appendix B and Goldberg (9).

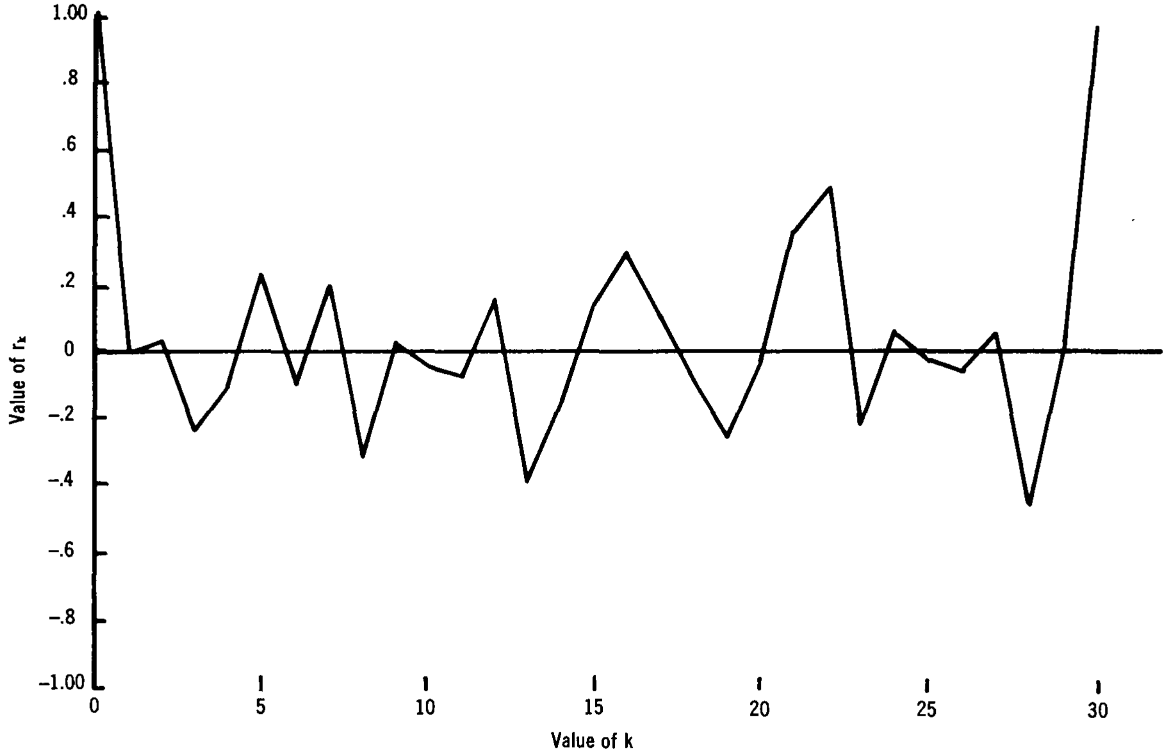
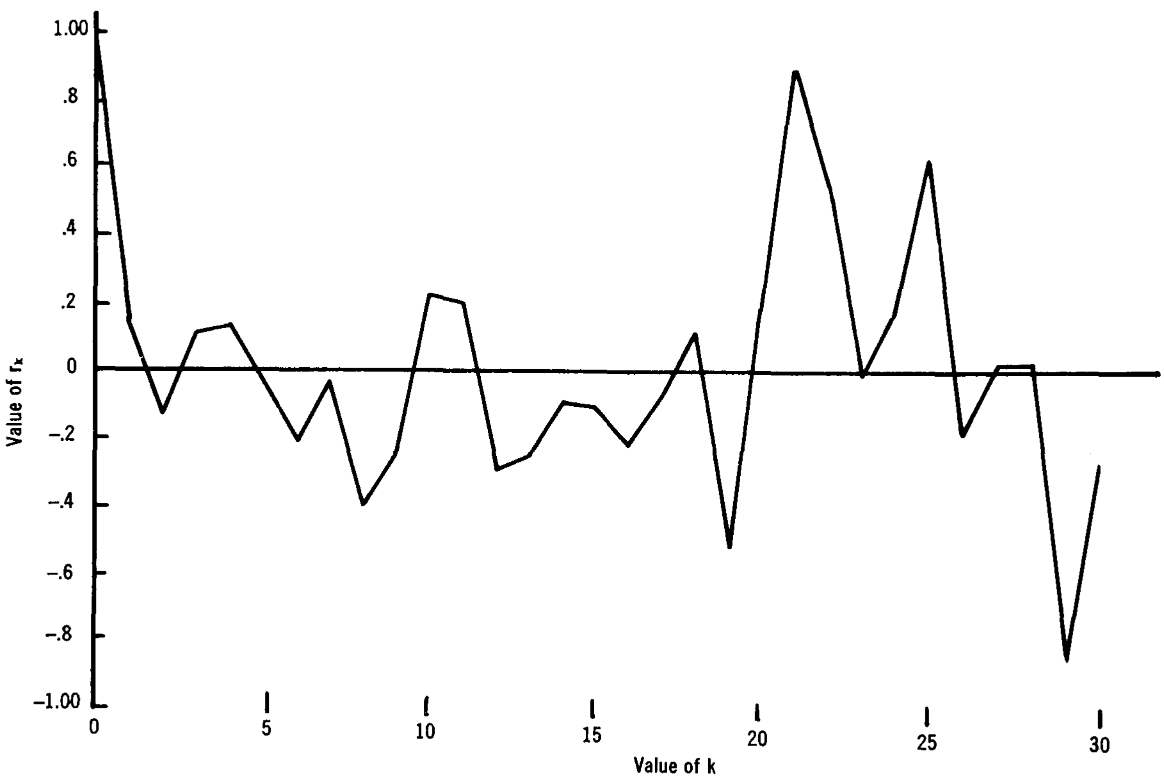


Figure 7. Correlogram of turkey production series.

Figure 8. Correlogram of turkey price series.



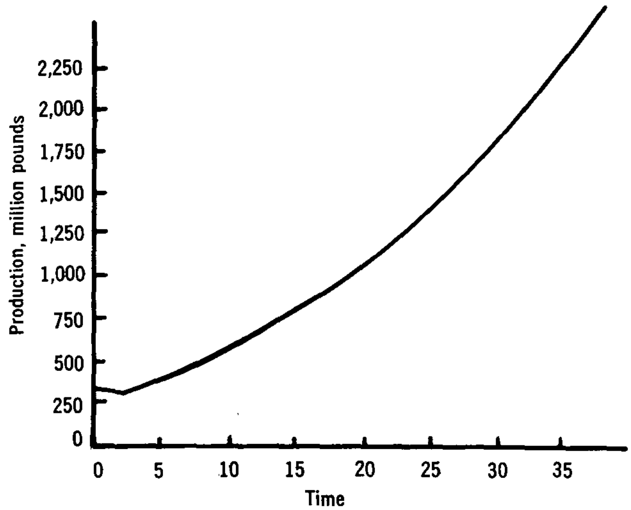


Figure 9. Equilibrium path in quantity of turkeys produced.

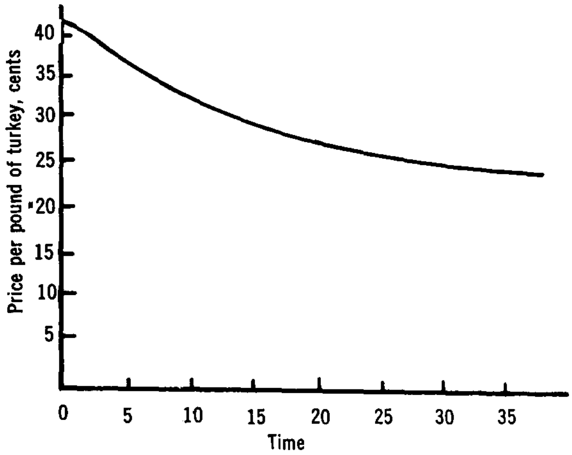


Figure 10. Equilibrium path in price of turkey.

In this case, both roots are real and the positive one has the greater absolute value. Therefore, as time increases, the price will gradually approach 22.9212 (appendix B, case III-1). This figure presents the asymptotic equilibrium deflated price of turkey at the farm level. The equilibrium path for turkey price is shown in figure 10.

Summary and Conclusions

This study explored the cyclical behavior of turkey production and prices. The objectives were to determine and measure the length of the cycles for price and production and to test the validity of a general cobweb theory underlying the turkey market. For the latter purpose, a simple cobweb pattern with a 1-year lag was developed. The study also investigated the nature of the equilibrium path in turkey production and price.

The data used were price per pound of liveweight turkey received by farmers and production for the period 1934-66. These prices were deflated by the prices received by farmers index (1957-59 = 100) in order to adjust them to changes in the general price level.

The analysis indicated the existence of nine complete cycles in production and eight in prices. They varied from 3 to 6 years in length. The amplitude of the swings in the production cycle was larger than that of prices. One explanation is the effect on prices of turkey moved in and out of storage. Storage reduces extreme fluctuations in farm prices. The amplitude of the swings generally seems to be diminishing. This can be attributed to coordination and specialization in the turkey industry, which probably have made large-scale growers more market conscious than small-scale growers.

The harmonic analyses were used to verify statistically the cycles in the turkey industry. A production cycle of 3 years was not rejected, according to the Schuster test. This test was insignificant for cycles of 3 and 4 years for turkey prices. One probable explanation is the effect of cold storage on prices. Movement of turkey into storage moderates the severe downward fluctuation of price when production increases. A cut in production increases liquidation of turkey from cold storage and prevents an extreme increase in price. The correlograms for production and price for turkey were oscillatory in a most remarkable way. Furthermore, they were not vanishing or dampening. These characteristics imply the existence of harmonic terms in both production and price.

The solution of a second order difference equation for quantity and price of turkey determines the path of both over time. Both roots for the quantity equation solution are real and the positive one exceeds unity, indicating that production will increase over time. The roots for the difference equation for price are real, and the positive one has the greater absolute value, though it is less than unity. The asymtotic equilibrium price of turkey would be 22.9212 cents.

Results from the harmonic and correlogram analyses support the existence of the cobweb relationship in the turkey industry. Extreme and irregular price changes create uncertainty, which leads to inefficiency. This causes alternate periods of overproduction and underutilization of productive resources, thereby increasing costs and, moreover, resulting in unstable and uncertain revenue to turkey producers. Instability and low prices hurt marginal producers and force some of them out of turkey production.[11] It has been estimated that producers have lost in excess of $50 million in income over the last 15 years because of this effect.[12]

[11] Price stability and welfare constitute a controversial issue. Massell (13) used the expected value of the change in producer and consumer surplus as a measure of gain. He showed a net gain to producers and consumers resulting from a stable as compared to a fluctuating price.

[12] A demand and supply function was estimated for three periods, 1951-56, 1957-60, and 1961-65. The two functions were solved to determine the equilibrium price and quantity. The estimated equilibrium quantities were: 1,100,000,000; 1,390,000,000; and 1,775,000,000 pounds for the first, second, and third periods, respectively. The estimated equilibrium prices were 31, 24.5, and 21.5 cents per pound for the first, second, and third periods, respectively. Revenue for each year was estimated and compared with actual income. By eliminating the cobweb phenomenon, some years showed gain and others losses. The consumer gets more satisfaction from fairly even consumption than he does from scarcity at one time and overabundance at another. A stable supply is therefore worth more to him than a fluctuating supply.

Turkey prices and production probably will follow the cobweb pattern in the future, based on information about prices and production for 1969. Production in 1969 was about 98 percent of 1968 production, and producers received prices 1-4 cents above 1968 prices.

The cobweb pattern, if followed, indicates an increase in output and lower turkey prices for 1970. Preliminary statistics on production and prices of turkey for 1970 support this pattern. Individual producers and producer groups should try to avoid overexpansion in their production plans. Any contemplated expansion should be in line with population growth to prevent wide fluctuations in price and, consequently, in income. Assurance of stable prices and income in the future will depend on the ability of producers to curb substantial year-to-year changes in production.

Decreasing or eliminating these price spirals would do much to reduce income uncertainty in the turkey industry.[13] Supply control measures could be used to regulate turkey production and avoid overproduction. Market quotas for turkey production could be utilized through input control by controlling the number of eggs set for hatching, controlling breeder numbers, and allocating poult quotas to producers; or through output control by allocating pounds of meat or number of meat birds to producers. The effectiveness of such programs will depend on the degree of success in controlling supply. Supply control programs could be administered by the U.S. Department of Agriculture through licenses to producers. Returns from these programs have to total more than the cost required to administer the program.

Price is not expected to become so stable as to eliminate the risks of inventory price fluctuations as well as boom and bust cycles. The reasons are: the substantial lead time required in commitments to grow turkeys; the perishability of turkey and the need to liquidate cold storage supplies to avoid quality deterioration; and the increased feeding costs if birds are kept after reaching their marketable weight.

Furthermore, factors outside the industry would upset the cyclical pattern. Such factors include the competitive supplies and prices of red meats and broilers and feed prices.

References

1. Akerman, G. "The Cobweb Theorem: A Reconsideration." *Quart. J. of Econ.* 71: 151-60. 1957.
2. Bawden, D., H. Carter, and G. Dean. "Interregional Competition in the United States Turkey Industry." *Hilgardia* 37: 437-531.
3. Benner, S. *Benner's Prophecies of Future Ups and Downs in Prices.* 16th Edition. Cincinnati, Ohio, K. Clarke Co. 1906.
4. Bluestone, H., and A. Rojko. "Forecasting Farm Turkey Prices In and Out of the Main Marketing Period." *Agr. Econ. Res.* 18: 43-51. 1966.
5. Brandow, G. E. *Interrelation among Demands for Farm Products and Implications for Control of Market Supply.* Pa. Agr. Exp. Sta. Bull. 680. 1961.
6. Davis, H. T. "Analysis of Economic Time Series," Bloomington, Ind., Cowles Comm. Res. Econ. Monograph No. 6. 1941.
7. Ezekiel, M. "The Cobweb Theorem." *Quart. J. of Econ.* 52: 255-80. 1938.
8. Fox, K. A. *Econometric Analysis for Public Policy.* Ames, Iowa, Iowa State Univ. Press. 1958.
9. Goldberg, S. *Introduction to Difference Equations.* New York, N.Y., John Wiley & Sons, Inc. 1965.
10. Granger, C. *Spectral Analysis of Economic Time Series.* Princeton, N.J., Princeton Univ. Press. 1964.
11. Hannan, E. *Time Series Analysis.* London, England, Methuen. 1960.
12. Kendall, M., and A. Stuart. *The Advanced Theory of Statistics.* Volume 3. London, England, Charles Griffin and Company, Limited. 1966.
13. Massell, B. "Price Stabilization and Welfare." *Quart. J. of Econ.,* 83: 284-98. 1969.
14. Moore, H. L. *Generating Economic Cycles.* New York, N.Y., Macmillan, 1923. Reprinted, New York, N.Y., Augustus M. Kelley. 1967.
15. Nerlove, M. "Spectral Analysis of Seasonal Adjustment Procedures." *Econometrica* 32: 241-86. 1964.
16. Paarlberg, D., and D. J. Watson. *Factors Affecting the Price of Turkey.* Ind. Agr. Exp. Sta. Bull. 536. 1949.
17. Rausser, G. C., and T. F. Cargill. "The Existence of Broiler Cycles: An Application of Spectral Analysis." *Amer. J. Agr. Econ.* 52: 109-21. Feb. 1970.
18. Schuster, A. "The Periodogram and Its Optical Analogy." Proceedings of the Royal Society of London. Series A, Vol. 77, 1906.
19. Slutzky, E. "The Summation of Random Causes as a Source of Cyclical Processes," *Econometrica* 5. 1937.
20. Soliman, M. *Econometric Model of the Turkey Industry in the United States.* (Unpub.)
21. Soliman, M. *Quarterly Econometric Model of the Turkey Industry in the United States.* Minn. Agr. Exp. Sta. Tech. Bull. 275. 1970.
22. Swenson, C. G. "Seasonal Variation in the Demand for Turkeys." Unpub. M.S. thesis, Univ. of Minn. Library, St. Paul, Minn. 1967.
23. Tintner, G. *Econometrics.* New York, N.Y., John Wiley & Sons, Inc. 1952.
24. U.S. Department of Agriculture. Economic Research Service. *Poultry and Egg Situation.* 247-57. June 1967-June 1969.
25. Wald, A. "Long Cycles as a Result of Repeated Integration." *Amer. Math. Monthly.* 46. 1939.
26. Waugh, F. V., and M. M. Miller. "Fish Cycles: A Harmonic Analysis." *Amer. J. of Agr. Econ.* 52: 422-30. Aug. 1970.
27. Weiss, J. S., "A Spectral Analysis of World Cocoa Prices." *Amer. J. of Agr. Econ.* 52: 122-26. Feb. 1970.

[13] As shown in footnote 12, eliminating the cobweb phenomenon has no influence on the longrun trend in both prices and production.

Appendix A

Table A-1. Time series data for turkey production and farm prices

Year	Production			Price			
	Actual	Trend*	Cycle†	Actual	Deflated by PRFI‡	Trend§	Cycle†
	...............million pounds.................			........................cents........................			
1934	300	324.6023	−7.5792	15.1	40.8	41.1003	−.7307
35	298	340.6053	−12.5087	20.1	44.7	40.8084	9.5362
36	405	358.9929	12.8134	15.6	33.2	40.4887	−18.0018
37	376	379.7651	−.9914	18.1	35.5	40.1413	−11.5624
38	395	402.9219	−1.9661	17.5	43.8	39.7661	10.1440
39	494	428.4633	17.6304	15.7	40.3	39.3632	2.3799
1940	502	456.3893	9.9476	15.2	36.2	38.9326	−7.0187
41	512	486.6999	5.1983	19.9	39.0	38.4742	1.3666
42	522	519.3951	.5000	27.5	41.7	37.9881	9.7712
43	509	554.4749	−8.2014	32.7	40.9	37.4743	9.1414
44	584	591.9393	−1.3412	33.9	41.3	36.9327	11.8250
45	740	631.7883	17.1259	33.7	39.2	36.3634	7.8007
46	714	674.0219	5.9345	36.3	37.0	35.7664	3.4490
47	611	718.6401	−14.9783	36.5	32.0	35.1416	−8.9398
48	574	765.6429	−25.0311	46.8	39.3	34.4891	13.9490
49	769	815.0303	−5.6477	35.2	34.2	33.8088	1.1571
1950	817	866.8023	−5.1512	32.9	30.7	33.1008	−7.2529
51	950	920.9589	3.1488	37.5	30.0	32.3651	−7.3075
52	1,050	977.5001	7.4169	33.6	28.2	31.6016	−10.7640
53	1,008	1,036.4259	−2.7429	33.7	32.1	30.8105	7.4309
54	1,161	1,097.7363	5.7637	28.8	28.2	29.9915	−5.9733
55	1,091	1,161.4313	−6.0642	30.2	31.5	29.1449	8.0806
56	1,274	1,227.5109	3.7882	27.2	28.6	28.2705	1.1655
57	1,356	1,295.9751	4.6296	23.4	24.1	27.3683	−11.9419
58	1,356	1,366.8239	−.7919	23.9	23.0	26.4385	−13.0056
59	1,433	1,440.0573	−.4901	23.9	24.1	25.4809	−5.4193
1960	1,491	1,515.6753	−1.6280	25.4	25.7	24.4955	4.9172
61	1,878	1,593.6779	17.8406	18.9	19.1	23.4824	−18.6624
62	1,630	1,674.0651	−2.6322	21.6	21.4	22.4416	−4.6413
63	1,673	1,756.8369	−4.7720	22.3	22.3	21.3731	3.8689
64	1,824	1,841.9933	−.9768	21.0	21.4	20.2768	5.5393
65	1,901	1,929.5343	−1.4788	22.2	21.8	19.1528	13.8214
66	2,097	2,019.4599	3.8396	23.0	20.9	18.0010	16.1046

* The trend value was estimated from a second degree polynomial function

$\hat{Q}(t) = 310.9839 + 12.4261\,t + 1.1923\,t^2 + \hat{U}_t$ where t is time. t = 1, 2, ..., T, and Q(t) is the production of turkey in time t

† $\text{Cycle} = \frac{\text{Actual values} - \text{Trend} \times 100}{\text{Trend}}$

‡ Price is deflated by the price received by farmers index (1957-59 = 100).

§ The trend value was estimated from a second degree polynomial function

$\hat{P}(t) = 41.60102 - .22261\,t - .01387\,t^2 + \hat{U}_t$

P(t) is the deflated price of turkey in time t

Source: U.S. Department of Agriculture, Economic Research Service, "Poultry and Egg Situation."

Table A-2. Calculation of the Fourier coefficients (3-year) for turkey production adjusted for trend

U_1	U_2	U_3
−24.6023	−42.6053	6.0071
−3.7651	−7.9219	65.5367
45.6107	25.3001	2.6049
−45.4749	−7.9393	108.2117
39.9781	−107.6401	191.6429
−46.0303	−49.8023	29.0411
72.4999	−28.4259	63.2637
−70.4313	46.4891	60.0249
−10.8239	−7.0573	−24.6753
284.3221	−44.0651	83.8369
−17.9933	−28.5343	77.5411
223.2907	−252.2023	663.0357

$$A_p = \frac{2}{33}\left[223.2907 \cos\frac{360}{3} - 252.2023 \cos\frac{360 \times 2}{3} + 663.0357 \cos 360\right]$$

$$= \frac{2}{33}\left[223.2907\,(-.500) - 252.2023\,(-.5000) + 663.0357\right]$$

$$= 41.0601$$

$$B_p = \frac{2}{33}\left[223.2907\,(.86603) - 252.2023\,(-.86603)\right] = 24.9570$$

Table A-3. Calculation of the Fourier coefficients (3-year) for turkey prices adjusted for trend

U_1	U_2	U_3
−.3003	3.8916	−7.2887
−4.6413	4.0339	.9368
−2.7326	.5258	3.7119
3.4257	4.3673	2.8366
1.2336	−3.1416	4.8109
.3912	−2.4008	−2.3651
−3.4016	2.2895	−1.7915
2.3551	.3295	−3.2683
−3.4385	−1.3809	1.2045
−4.3824	−1.0416	.8269
1.1232	2.6472	2.8990
−10.3679	10.1199	2.5130

$$A_p = \frac{2}{33}\left[-10.3679\,(-.500) + 10.1199\,(-.500) + 2.5130\right]$$

$$A_p = \frac{2\,(2.6370)}{33} = .15982$$

$$B_p = \frac{2}{33}\left[-10.3679\,(.86603) + 10.1199\,(-.86603)\right]$$

$$= \frac{2\,(17.7430)}{33} = -1.07533$$

Appendix B — Difference Equations

A second order difference equation was estimated for production and deflated price of turkey for the period 1934-66 by the ordinary least squares estimation procedure.

Production

The estimated equation is:

(1) $\hat{Q}_t = .6681\, Q_{t-1} + .3871\, Q_{t-2} + 21.5535 + \hat{U}_t$

Q_t denotes production of turkey in time t, $t = 2, 3, \ldots T$

This equation in the equivalent standard form is:

(2) $Q_{t+2} - .6681\, Q_{t+1} - .3871\, Q_t = 21.5535$
$t = 0, 1, 2, \ldots T$

Equation 2 was solved by first finding the general solution of the corresponding homogeneous equation and then adding a solution of the complete equation. The homogeneous equation is:

(3) $\lambda^2 - .6681\,\lambda - .3871 = 0$

which is the characteristic equation. By the quadratic formula, the value of the roots of this equation is provided by:

$$\lambda = \frac{.6681 \pm \sqrt{.4464 - 4(-.3871)}}{2}$$

which provides the solutions:

$\lambda_1 = 1.0402$ $\quad\quad \lambda_2 = -.3721$

The general solution of the homogeneous equation is given by:

(4) $Q_t = k_1\,(1.0402)^t + k_2\,(-.3721)^t$.

To obtain the particular solution for equation 2, we assume a trial solution of the form $y_t^* = k$, a constant. The value of k to satisfy equation 2 is provided by:

$k - .6681\,k - .3871\,k - 21.5535 = 0$

$$k = \frac{-21.5535}{.0552} = -390.4619$$

The general solution of equation 2 with $k = -390.4619$ is:

(5) $Q_t = k_1\,(1.0402)^t + k_2\,(-.3721)^t - 390.4619$

The arbitrary constants k_1 and k_2 could be found in terms of the initial values of quantities:

Production in 1933 $= Q_0 = 319$
Production in 1934 $= Q_1 = 300$

Putting $t = o$ and $t = 1$ in (5) to obtain:

(a) $k_1 - k_2 - 390.4619 = 319$
(b) $1.0402\,k_1 - .3721\,k_2 - 390.4619 = 300$

Solving equations (a) and (b) simultaneously, we have:

$k_1 = 638.3290$ $\quad\quad k_2 = -71.1329$

The particular solution of equation 2 that satisfies the initial conditions is given by:

(6) $Q_t = 638.3290\,(1.0402)^t - 71.1329\,(-.3721)^t - 390.4619$

Price

The estimated second order difference equation is:

(7) $\hat{P}_t = 1.9208 + .6290\,P_{t-1} + .2872\,P_{t-2} + \hat{U}_t$

P_t denotes the deflated price of turkey received by farmers in time t.

The characteristic equation for 7 is:

(8) $\lambda^2 - .6290\,\lambda - .2872 = 0$
$\lambda_1 = .93585$ $\quad\quad \lambda_2 = -.30685$

The general solution of equation 7 is given by:

(9) $P_t = k_1\,(.93585)^t + k_2\,(-.30685)^t$

To find the particular solution for equation 7, obtain the value of the constant k that satisfies that equation:

$k - .6290\,k - .2872\,k - 1.9208 = 0$
$k = 22.92124$

The general solution with $k = 22.92124$ is in the form:

(10) $P_t = k_1 (.93585)^t + k_2 (-.30685)^t + 22.92124$

The arbitrary constants k_1 and k_2 could be found in terms of the initial values of prices:

Price in 1933 $= P_0 = 41.2$
Price in 1934 $= P_1 = 40.8$

Putting $t = 0$ and $t = 1$ in equation (10) we have:

(c) $k_1 - k_2 + 22.92124 = 41.2$

(d) $.93585\, k_1 - .30685\, k_2 + 22.92124 = 40.8$

Solving equations (c) and (d) simultaneously, we get:

$k_1 = 19.50704$ $\qquad k_2 = 1.22828$

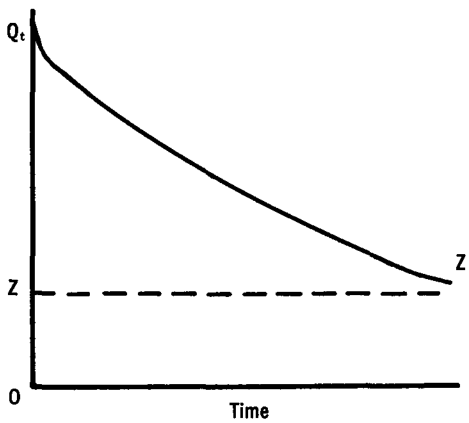
The particular solution of equation 7 that satisfies the initial conditions is given in the form:

$P_t = 19.50704 (.93585)^t + 1.22828 (-.30685)^t + 22.92124$

Interpreting difference equation models: The roots of the characteristic equation determine the time path of quantity and prices. The different cases are:

Case I. Both roots real and positive:

1. If both λ_1 and λ_2 are less than unity, then both $k_1 (\lambda_1)^t$ and $k_2 (\lambda_2)^t$ will gradually taper off to zero, so that Q_t will approach Z asymptotically, roughly as in the figure, but in discontinuous steps.

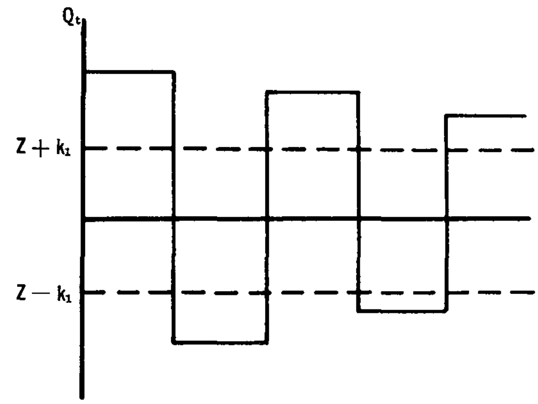


2. If one root, λ_1, is equal to one, while the other is less than one, then Q_t will gradually taper off to $k_1 + Z$, since we have:
   $Q_t = k_1 (1)^t + k_2 (\lambda_2)^t + Z = k_1 + k_2 (\lambda_2)^t + Z$
   and the second term will taper off to zero. Similarly, if $\lambda_2 = 1$ and λ_1 is less than one, Q_t will gradually approach $k_2 + Z$.
3. If either or both roots exceed unity, Q_t will explode. If λ_1 is greater than unity and k_1 and k_2 are positive, $k_1 (\lambda_1)^t$ will surpass any given finite magnitude and so will Q_t. If k_1 is negative, then $k_1 (\lambda_1)^t$ will explode downwards. If k_1 is negative and k_2 is positive, the result depends on whether λ_1 or λ_2 is greater, and what eventually happens will be little affected by the smaller root.

Case II. Both roots real, neither positive:

The value of $k_1 (\lambda_1)^t$ and $k_2 (\lambda_2)^t$ will oscillate, each being alternatively positive and negative. The results will thus be the same as in the positive roots case except that Q_t will generally oscillate.

1. If both λ_1 and λ_2 are less than unity in absolute value, Q_t will oscillate about Z and the oscillations will eventually die out.
2. If one of the two roots equals minus one, while the other is less than one in absolute value, the oscillations will tend to sink to the oscillations of, say, $k_1 (\lambda_1)^t$ if $\lambda_1 = -1$. Q_t will tend toward $k_1 (\lambda_1)^t + Z = k_1 (-1)^t + Z$, and its course over time will be as shown in the figure.



The amplitude of the fluctuations thus tends towards $2\, k_1$; that is, at the upper points Q_t tends towards $Z + k_1$, and at the lower points it tends towards $Z - k_1$.

3. Where either root is greater than one in absolute value, there will be explosive oscillations.

Case III. One root positive, the other negative:

What will happen here depends on whether the root with the greater absolute value is positive or negative; the influence of the other root eventually becomes negligible. Suppose k_1 is the larger root. Then Q_t will ultimately approximate $k_1 (\lambda_1)^t + Z$.

1. λ_1 is positive < 1, Q_t will approach Z.
2. $\lambda_1 = 1$, Q_t will approach the steady level $k_1 + Z$.
3. λ_1 positive > 1, Q_t will explode.
4. λ_1 is negative < 1 in absolute value, Q_t will tend to oscillate about Z, and the oscillations will gradually die down.
5. $\lambda_1 = -1$, Q_t will fluctuate forever between $Z + k_1$ and $Z - k_1$ after awhile.
6. λ_1 negative and greater than unity in absolute value, Q_t will oscillate explosively about Z.
7. If $\lambda_1 = -\lambda_2$, no general results in terms of λ_1 and λ_2 alone can be predicted, for what will happen depends on k_1 and k_2, which are determined by the initial conditions.

The roots obtained for the production equation are both real and the positive one exceeds unity, so Q_t will explode (case III-3). The roots of the homogeneous price equation are both real, the positive is the larger and is less than unity, so P_t will approach $Z = 22.92124$ asymptotically (case III-1).

Table B-1. Calculated equilibrium production and price of turkey

Year	Time (t)	Production*	Price†
		million pounds	cents
1933	0	319.0	41.2
34	1	300.0	40.8
35	2	290.4	40.1
36	3	331.7	38.9
37	4	355.5	37.9
38	5	387.4	36.9
39	6	418.0	36.0
1940	7	450.7	35.2
41	8	484.5	34.4
42	9	519.7	33.7
43	10	556.2	33.0
44	11	594.3	32.3
45	12	633.9	31.7
46	13	675.1	31.2
47	14	717.9	30.6
48	15	762.5	30.1
49	16	808.8	29.7
1950	17	857.0	29.2
51	18	907.2	28.8
52	19	959.3	28.5
53	20	1,013.6	28.1
54	21	1,070.0	27.8
55	22	1,128.7	27.5
56	23	1,189.8	27.2
57	24	1,253.3	26.9
58	25	1,319.4	26.6
59	26	1,388.2	26.4
1960	27	1,459.7	26.2
61	28	1,534.0	26.0
62	29	1,611.4	25.8
63	30	1,691.9	25.6
64	31	1,775.6	25.4
65	32	1,862.7	25.3
66	33	1,953.2	25.1
67	34	2,047.4	25.0
68	35	2,145.5	24.8
69	36	2,247.4	24.7
1970	37	2,353.4	24.6
71	38	2,463.7	24.5

* Calculated from the particular solution:
$Q_t = 638.3290\ (1.0402)^t - 71.1329\ (-.3721)^t - 390.4619$

† Calculated from the particular solution:
$P_t = 19.50704\ (.93585)^t + 1.22828\ (-.30685)^t + 22.92124$