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Dedication

To my parents, Svetlana and Sergey, and to my husband, Cameron.

Abstract

This dissertation consists of three chapters.

The first chapter studies the origin of the German labor market "miracle". I develop a search and matching model with multi-worker firms and a two-tier unemployment insurance system to explore the role of the 2005 unemployment insurance reform (the Hartz IV reform) in reducing the cyclical volatility of German employment. Lower long-term unemployment benefits reduce firms' incentives to cut employment during downturns, and render adjustment along the intensive margin relatively more important. Calibrating my model to German pre-reform data, I find that the reform reduced the volatility of employment by 68% and was the main reason behind the mild response of the German labor market to the Great Recession. A short-time work policy, praised as the key to the German "miracle," played a minor role. I also find that the reform raised an average worker's welfare by 1.18%.

In the second chapter, written jointly with Aysa Dordzhieva, we study the inertia in sovereign credit ratings. We document that in the run-up to the European debt crisis sovereign credit ratings of Italy, Portugal and Spain displayed a higher degree of inertia. We suggest that the observed inertia in sovereign ratings was the result of the optimal behavior of credit rating agencies, and it might have helped to prevent a severe banking crisis. We build a sovereign default model with a credit rating agency (CRA) that maximizes the accuracy of its credit ratings. CRA receives private information about country's fundamentals and chooses whether to update its rating or not. We assume that a rating downgrade triggers a banking crisis in the near future irrespective of the government's default decision. We show that under certain conditions it is optimal for CRA not to downgrade even if it gets a negative signal and the probability of default goes up.

Finally, the third chapter proposes a theory of the direct pass-through of sovereign default risk to firms that can generate the co-movement of sovereign and corporate spreads. I develop a model of sovereign default and bailout in an economy with productivity shocks. A key feature of the model is that the probability that the firm is going to be bailed out is endogenous and non-monotonic in the level of output. The bailout is more likely when the output is high. It is also more likely when the output is low and the government has a strong incentive to borrow: instead of repaying

the debt, the government can default and bail out the firm. When the output is in the medium range, the probability of bailout is lower because the government is rich enough not to default but is not rich enough to be able to repay its debt and afford costly bailout. Since the firm internalizes this when it makes its investment decision, it takes more risk by buying more capital when the probability of bailout is higher and hence faces lower bond prices. This non-monotonicity in the firm's capital decision implies that the government and the firm's bond prices move together as long as the level of output is not too high.

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Chapter 1

Unemployment Insurance Reform: the Origin of the German Labor Market Miracle

1.1 Introduction

Germany's jobs miracle hasn't received much attention in [the U.S.] — but it's real, it's striking. Krugman (2009)

In 2005, the German government restructured its unemployment insurance system by considerably reducing unemployment benefits for the long-term unemployed. This reform was the final step in a series of structural labor market reforms implemented in 2003-2005, known as the Hartz I-IV reforms, that aimed to reduce unemployment, increase job-search activities, and provide incentives for the unemployed to obtain jobs. Since 2005, the unemployment rate has fallen consistently, except for a slight increase during the Great Recession. The fact that the increase in the unemployment rate was the smallest among OECD countries — even though the fall in Germany's GDP was even deeper than that of the U.S. — made many observers speak of a German labor market "miracle."

There is ongoing debate regarding the origin of this German labor market miracle. Some emphasize the crucial role played by the mechanisms of labor adjustment along the intensive margin, such as a short-time work policy. As Figure 1.1 shows, the volatility of employment relative to the volatility of GDP in Germany fell after 2005 — four years before the Great Recession — compared to the U.S., where it stayed about the same. In this paper, I argue that the Hartz IV reform reduced firms' incentives to cut employment during downturns and was the main reason behind the mild response of the German labor market to the Great Recession.

Using aggregate German labor market data, I document that the volatility of employment relative to output fell by two-thirds after 2005, while the relative volatility of hours per worker increased by 10%. Prior to 2005, hours per worker were half as volatile as employment; after, they became almost twice as volatile.

To assess the quantitative implications of Germany's unemployment insurance reform on labor market volatility, I build a search and matching model with multi-worker firms and a two-tier unemployment insurance system. Firms face productivity shocks, search for new workers by posting vacancies, bargain with each of its workers over the hourly wage, and choose hours worked per worker. They can change hours per worker instantaneously, but must incur some adjustment costs in order to do so. The size of the match surplus plays a key role in generating a positive relationship between the level of unemployment benefits and employment volatility; lower long-term unemployment benefits imply a larger match surplus. Incentives to post vacancies depend on the size of percentage changes in the match surplus in response to changes in productivity. A larger surplus means that these percentage changes are smaller, as are the volatilities of vacancies and employment. Since the number of workers and the number of hours worked are substitutes in the production of goods, firms start relying more on the intensive margin of adjustment.

The introduction of multi-worker firms is motivated by the fact that in a standard one-worker one-firm setting the firm's choice of hours is independent of the search and matching frictions. As a result, changes in unemployment benefits have no effect on the volatility of hours per worker. As long as hours per worker do depend on the labor market tightness — which happens in a multi-worker firm environment with production function exhibiting decreasing returns to scale in the number of workers — the volatility of hours per worker is decreasing in the level of unemployment benefits.

I calibrate my model to German pre-reform data and find that the reform reduced the volatility of employment by 68%, while the volatility of hours per worker increased by 9%. After the reform, hours per worker became about 1.94 times as volatile as employment, which is close to what is observed in the data.

To examine the role of the short-time work policy in safeguarding jobs during the Great Recession, I introduce a government subsidy that lowers the cost of reducing

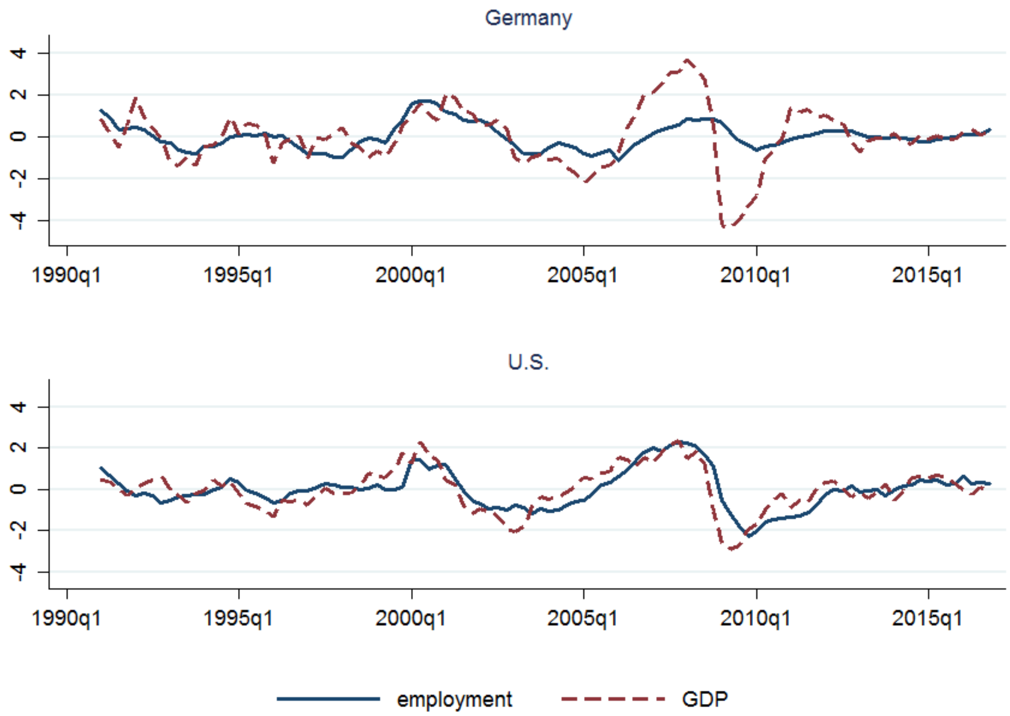

Figure 1.1: Employment and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Eurostat and OECD.

working hours during times when the economy is in recession. In Germany, these costs include social security contributions on the worker's lost hours. Between 2009 and 2011, firms that participated in the short-time work program were required to pay 50% of those contributions and 0% after the first six months. I find that the subsidy reduces employment volatility further, by an additional 0.2 percentage points. This suggests that the short-time work policy played a minor role in protecting jobs during the Great Recession.

Using the calibrated model, I construct a sequence of shocks that match the observed dynamics of output during the Great Recession, and find that in the post-reform environment with the short-time work subsidy the unemployment rate increased by 0.5 percentage points, from 5.72% to 6.22%. I perform a counterfactual exercise and find that the unemployment rate would have increased by 1.6 percentage points, from 8.4% to 10%, if long-term unemployment benefits stayed at the pre-reform level. Without the reform, the increase in unemployment would have been three times higher in absolute terms and 9% higher in relative terms.

Finally, I analyze the impact of the unemployment insurance reform on workers' welfare and find that the reform made every type of workers better off. Welfare gains range from 0.47% for long-term unemployed workers to 1.21% for currently employed workers; the welfare of an average worker went up by 1.18%.

This paper is related to several strands of literature. From a theoretical perspective, I build on the search and matching model of Mortensen and Pissarides (1994). Following Andolfatto (1996), Krause and Lubik (2010), Kudoh et al. (2016), and Cacciatore et al. (2016), I introduce multi-worker firms and allow them to adjust labor along both extensive and intensive margins. My model environment is similar to Andolfatto (1996), but with a constant capital stock that does not depreciate. I follow Fredriksson and Holmlund (2001) and introduce a two-tier unemployment insurance system, in which short-term unemployment benefits expire with some exogenous probability.

My paper is related to the literature that studies the impact of labor market institutions on business cycle fluctuations. Veracierto (2008) analyzes the impact of firing costs on cyclical fluctuations in a real business cycle model and finds that lowering firing taxes increases the volatility of employment and output. Zanetti (2011) shows that in a model with labor market frictions and nominal rigidities, increasing firing costs and lowering unemployment benefits could lead to lower volatility of employment and job flows, but higher inflation volatility. Cacciatore and Fiori (2016) introduce endogenous product creation and labor market frictions into a real business cycle model and find that joint deregulation of product and labor markets, in terms of reducing entry costs, relaxing firing restrictions and lowering unemployment benefits, reduces aggregate volatility, which leads to a sizable reduction in the welfare costs of business cycles. However, none of these studies looks at the effects of actual labor market reforms. In contrast, I quantify the effects of the Hartz IV reform on labor market volatility based on pre- and post-reform data.

I also contribute to the literature on the macroeconomic effects of the German labor market reforms of 2003-2005. Using calibrated macro models, Krause and Uhlig (2012) and Krebs and Scheffel (2013) find that the Hartz IV reform substantially reduced the long-run equilibrium unemployment rate, while Launov and Wälde (2013), using German microdata, estimate that the effect is close to zero. However, studies of the business cycle implications of Hartz reforms are rare. My paper is most closely related to Krebs and Scheffel (2017) and Gehrke et al. (2017).

Krebs and Scheffel (2017) study the effect of Hartz reforms on the output cost of recessions. They find that a reduction in unemployment benefits increases job-finding rates at all stages of the business cycle, which renders unemployment less volatile and reduces output losses during downturns. Although it is true that lower unemployment benefits lead to more vacancies being posted — which increases the level of job-finding rates — I show in this paper that the reason unemployment becomes less responsive to business cycle shocks is that the firm has fewer incentives to adjust employment over the business cycle. Thus, the volatility of vacancies, and consequently of job-finding rates and unemployment, goes down.

Gehrke et al. (2017) study Germany's labor market dynamics during the Great Recession and analyze the role of different shocks and institutions. They build a stochastic general equilibrium model with a search and matching labor market with endogenous separations and the possibility of firms' use of short-time work. They do not model unemployment insurance reform; instead, they introduce matching efficiency shocks, which they estimate from the data (together with other structural shocks), and find that positive matching efficiency shocks (likely caused by labor market reforms) were the underlying source of the unusual labor market dynamics. In this paper, I model the unemployment insurance reform explicitly. The fall in the level of unemployment and the volatility of unemployment are generated endogenously and result from firm's optimal behavior in response to the reduction in unemployment benefits.

In contrast to both Krebs and Scheffel (2017) and Gehrke et al. (2017), my model features a multi-worker firm that can adjust its labor input along both extensive and intensive margins. The data suggest that after the reform, German firms began to rely more on the intensive margin and my model can account for that.

This paper also contributes to the debate on the origin of the German labor market miracle. Short-time work subsidies are believed to have played an important role (Hijzen and Venn (2011), Brenke et al. (2011)). Cooper et al. (2017) study the employment effect of short-time work policy in a search model with heterogeneous multi-worker firms, and find that without short-time work. the miracle would disappear. They do not analyze the role played by the Hartz IV reform, arguing that it had no direct impact on firms' decisions to adjust hours. However, the data show that German firms began relying more on the intensive margin of adjustment after the reform but before the Great Recession hit and short-time work subsidies were extended.

Figure 1.2: Average net replacement rates, % of previous earnings of a single-person household earning the average income. Short-term refers to the initial phase of receiving benefits. Long-term refers to the 60th month of receiving benefits. Source: OECD.

I show that quantitatively, the impact of short-time work subsidies is small.

The remainder of the chapter is organized as follows. Section 1.2 describes the Hartz IV reform, presents evidence on changes in labor market dynamics after the reform, and provides an overview of the short-time work policy. Section 1.3 sets up the model and defines the equilibrium. Section 1.4 describes the calibration and discusses the findings. In Section 1.5, I conduct some sensitivity analysis to assess the robustness of my results. Section 1.6 concludes.

1.2 Empirical Evidence

In Section 1.2.1, I provide a brief overview of the Hartz IV reform. Section 1.2.2 describes the aggregate dynamics of the German labor market before and after the reform, and Section 1.2.3 gives an overview of the short-time work program.

1.2.1 The Hartz IV Reform

Between 2003 and 2005, the German government implemented extensive labor market reforms, known as the Hartz reforms. The first three (Hartz I-III) were aimed at improving job-search efficiency and employment flexibility. They included deregulation of the temporary work sector, improved job search assistance, and stronger incentives for the unemployed to accept a job. The Hartz IV reform, implemented on January 1, 2005, constituted a major restructuring of the unemployment insurance system that significantly reduced the size and duration of unemployment benefits.

Before the reform, the German unemployment insurance system consisted of three layers[1]. Workers who had accumulated a sufficient number of working years prior to unemployment were eligible for unemployment benefits (UB) equal to 60% of previous net earnings (67% for parents with dependent children). For workers younger than 45, the benefit was limited to 12 months; older workers were eligible for up to 32 months. After UB were exhausted, and if the worker was still unemployed, she was eligible for unemployment assistance (UA) equal to 53% of previous net earnings (57% for parents with dependent children). UA could be claimed indefinitely, subject to a means test and an annual review. Those who did not qualify for UB or UA were eligible for social assistance (SA) — a means-tested lump-sum transfer that provided the least generous support.

The reform collapsed this system into two layers. The first layer, unemployment benefits I (UB I), was essentially UB relabeled. The main change was the introduction of unemployment benefits II (UB II), which replaced UA and SA. Under the new system, workers who had exhausted their short-term benefit UB I were eligible for a means-tested lump-sum benefit that paid an amount similar to the old SA[2].

The effect of the Hartz IV reform can be seen in Figure 1.2, which shows the average net replacement rates for a single-person household (see Section 1.4.1 for details). The net replacement rate corresponds to the proportion of net income in work that is maintained after job loss. Clearly, the reform had almost no effect on short-term unemployed households, while the net replacement rate of long-term unemployed fell drastically.

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[1] Source: Engbom et al. (2015)

[2] As of 2013, UB II was equal to €345 a month plus rent allowance.

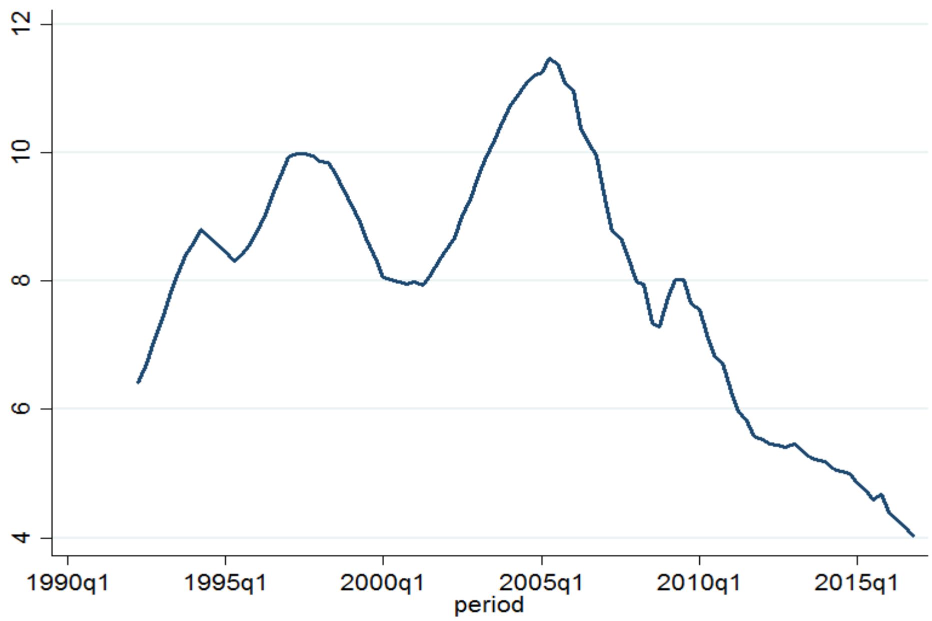

Figure 1.3: Unemployment rate, calculated as the ratio of total unemployment to active population, quarterly data, seasonally adjusted. Source: Eurostat.

1.2.2 Labor Market Dynamics

Figure 1.3 shows the unemployment rate in Germany after reunification. Prior to 2005, the unemployment rate had been on an upward trend, reaching 11% in 2005. Since 2005, the unemployment rate has fallen persistently (except for a slight increase during the Great Recession), and reached 4% by early 2017.

The Hartz IV reform has also changed the cyclical features of the German labor market. To document these changes, I use quarterly data on employment, hours per worker, total hours, and real GDP over the period 1960Q1-2013Q4 from the dataset constructed by Ohanian and Raffo (2012)[3]. They construct total hours series, H, as the product of hours worked per worker, h, and employment, N, normalized by the size of population aged 15-64 years and by the maximum number of hours per year to be shared between work and leisure (365 times 14). All variables are expressed in logs and detrended using an HP filter with smoothing parameter of 1600.

Figure 1.4 and Figure 1.5 show the cyclical fluctuations of employment and hours per worker relative to real GDP, respectively. It is clear from these graphs that the volatility of employment relative to output fell significantly after the reforms,

[3]Using quarterly data on employment, hours per worker, and total hours from Eurostat, which is available from 1991Q1 until 2016Q4, yields similar results.

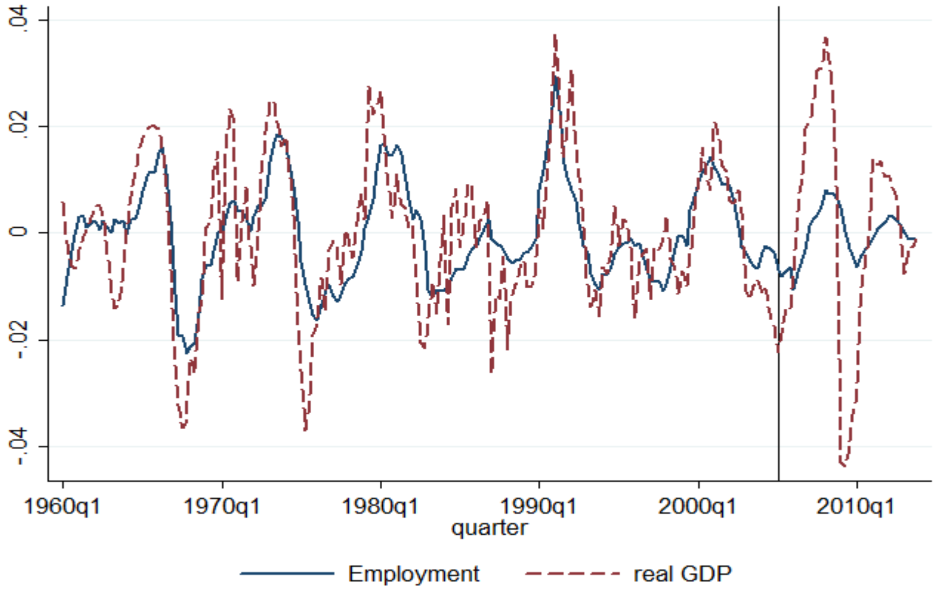

Figure 1.4: Employment and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Eurostat and OECD.

	Pre-reform	Post-reform
σ^N/σ^Y	0.671	0.224
σ^h/σ^Y	0.384	0.426
σ^h/σ^N	0.571	1.900
σ^H/σ^Y	0.723	0.516

Table 1.1: Observed and simulated relative standard deviations. Data: quarterly, seasonally adjusted, log deviation from an HP trend. Pre-reform Period: 1960Q1-2004Q4. Post-reform period: 2005Q1-2013Q4. Source: Ohanian and Raffo (2012). h – hours per worker, N – employment, H – total hours, Y – output.

while the relative volatility of hours per worker slightly increased. These findings are formalized in Table 1.1, which reports the relative volatility of employment, hours per worker, ratio between the two, and the relative volatility of total hours in the pre- and post-reform periods. The relative volatility of employment, measured as the ratio of the standard deviation of employment to the standard deviation of real GDP, fell by about two-thirds, from 0.671 to 0.224. At the same time, the relative volatility of hours per worker increased from 0.384 to 0.426. If we look at the volatility of hours per worker relative to the volatility of employment, we obtain the most striking result: Prior to the reform, hours per worker were half as volatile as employment; after, they became almost twice as volatile.

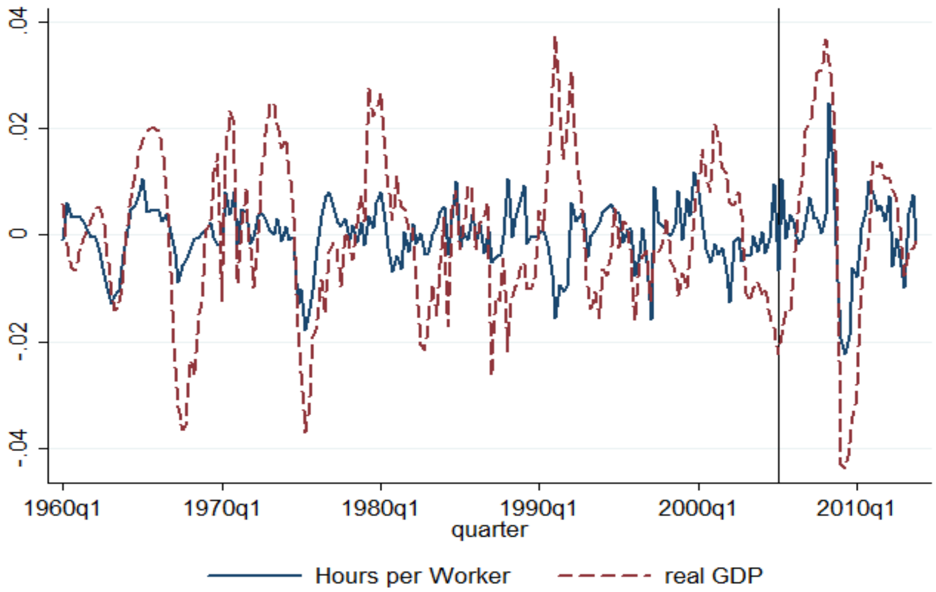

Figure 1.5: Hours per worker and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Ohanian and Raffo (2012).

To conclude, not only did the Hartz IV reform reach its main objective of raising the employment rate, but it also significantly reduced firms' incentives to adjust employment in response to shocks.

1.2.3 Short-Time Work Policy

Of all the German government's anti-crisis measures, the extension of the short-time work program (*Kurzarbeit*) received the most attention. Short-time work (STW), which has existed in Germany for almost a century, allows firms facing temporary financial difficulties to cut workers' hours and reduce wages instead of laying them off. The firm applies to the Federal Employment Agency and, of the request is approved, workers receive between 60% and 67% of their lost income from the government. The firm still has to pay social security contributions based on the worker's full-time wage. Moreover, during short-time work the firm is also responsible for the worker's share of social security contributions for the lost hours.

During the Great Recession, the maximum duration of short-time work was extended from 6 months to 24 months. Rules regarding social security contributions were loosened as well. From January 2009 to December 2011, firms only needed to pay 50% of social security contributions for the worker's lost hours during the first

6 months of short-time work (the remaining half was covered by the unemployment insurance fund). After the sixth month, the Federal Employment Agency covered 100% of those contributions. At the peak of the Great Recession in May 2009, the number of short-time workers reached 1.5 million (3.7% of total employment and 15% of employment in manufacturing sector). According to Brenke et al. (2011), the average reduction in working hours was just under 30% of the agreed working time.

1.3 Benchmark Model

In this section I present a benchmark model that is a discrete-time model of equilibrium unemployment with aggregate productivity shocks and multi-worker firms that adjust their labor input along extensive and intensive margins. The model features a two-tier unemployment insurance system with two types of unemployment benefits — short-term and long-term — and the former expire with an exogenous probability.

1.3.1 Technology and Preferences

The economy consists of a unit measure of infinitely lived workers, a unit measure of infinitely lived identical multi-worker firms, and a government. Workers and firms are risk-neutral and have a common discount factor $\beta \in (0, 1)$. There is one final good that can be used for consumption, production, and vacancy creation.

Worker's utility is given by

$$u(c_t, h_t) = c_t - \zeta \frac{h_t^{\mu+1}}{\mu + 1},$$

where c_t is consumption, h_t is hours worked, $\zeta > 0$ and μ is the inverse of Frisch elasticity.

Workers can either be employed, short-term unemployed, or long-term unemployed. Employed workers receive labor income and non-wage transfers from firms. Unemployed workers receive unemployment benefits, the level of which depends on their unemployment status. Workers own the firm (in equal shares) and receive all profits. Every worker, independent of her employment status, pays taxes that are used to finance unemployment benefits.

Firms use labor services to produce final goods according to the following technology:

$$Y_t = z_t(n_t h_t)^\alpha$$

where z_t is stochastic productivity, n_t is the number of workers, and h_t is hours worked per employee. The log of z_t follows the first-order autoregressive process

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z, \text{ where } \varepsilon_t^z \sim N(0, \sigma_z^2).$$

1.3.2 Labor Market Structure

The labor market is frictional. Unemployed workers search for jobs, and firms with vacant positions search for workers. The search is undirected, so that firms have no ability to direct their search toward a particular type of unemployed worker. Because of the search frictions, only a fraction of job-seekers find jobs, and only a fraction of vacancies are filled each period. The number of worker-firm matches in period t is determined by the following matching function:

$$m(U_t, V_t) = mU_t^\xi V_t^{1-\xi},$$

where U_t is the measure of unemployed workers searching for jobs, V_t is the total number of job vacancies, m_0 is the parameter that governs matching efficiency, and ξ is the matching elasticity. Let $\theta_t \equiv V_t/U_t$ be labor market tightness. The probability that a vacancy is matched with a worker, *vacancy filling rate*, is

$$m(U_t, V_t)/V_t = m\theta_t^{-\xi} = q_t(\theta_t).$$

Similarly, the probability that a worker is matched with a vacancy, *job finding rate*, is

$$m(U_t, V_t)/U_t = m\theta_t^{1-\xi} = \theta_t q(\theta_t).$$

Figure 1.6 illustrates labor market flows. At the end of each period, a fraction ρ^x of employed workers are hit by the exogenous separation shock and become short-term unemployed. With probability $1 - \lambda$, short-term unemployment benefits expire and the worker becomes long-term unemployed and remains in this status until she finds a job. The expected duration of short-term unemployment is $1/(1-\lambda)$. An unemployed

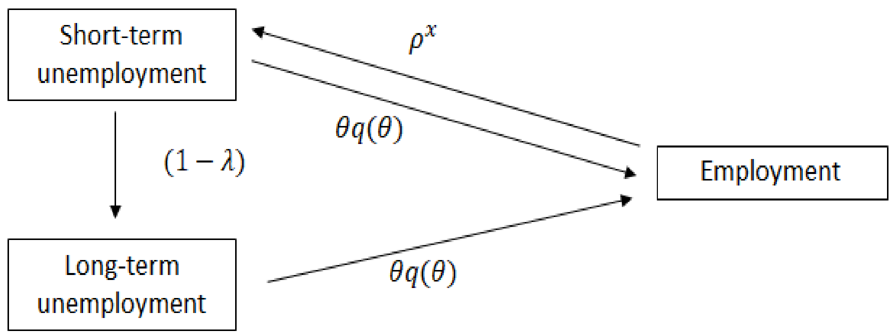

Figure 1.6: Labor market flows

worker of type i receives unemployment benefits b^i, where $i \in \{S, L\}$ and $b^S > b^L$.

Aggregate employment, N_t, evolves according to

$$N_{t+1} = (1 - \rho^x)N_t + q(\theta_t)V_t.$$

The number of searching workers is equal to the currently unemployed, $U_t = 1 - N_t$. Unemployment consists of short-term and long-term unemployment

$$U_t = U_t^S + U_t^L.$$

Let $p_t^S = U_t^S/U_t$ denote the share of short-term unemployment. Short-term unemployment evolves according to

$$U_{t+1}^S = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)U_t^S.$$

Short-term unemployment consists of workers who have just been separated from the firm and a fraction λ of previously short-term unemployed who could not find a job in this period. Long-term unemployment consists of previously long-term unemployed who could not find a job in this period and a fraction $1-\lambda$ of short-term unemployed who could not find a job and whose short-term unemployment benefits expired:

$$U_{t+1}^L = (1 - q(\theta_t)\theta_t)U_t^L + (1 - \lambda)(1 - q(\theta_t)\theta_t)U_t^S.$$

1.3.3 Timing

The aggregate state of the economy is $X_t(z_t, N_t, p_t^S)$, and the individual state of a representative firm is (n_t^S, n_t^L), where n_t^S is the number of workers whose outside option is short-term unemployment benefits; this consists of previous period workers who weren't separated and newly hired short-term unemployed. n_t^L is the number of workers whose outside option is long-term unemployment benefits — i.e., long-term unemployed hired at the end of previous period.

At the beginning of the period, z_t is realized. The firm chooses hours of work h_t and vacancies v_t. Regardless of whether the match is new or the worker had been working for the firm in the previous period, the firm and each of its workers bargain over the corresponding hourly wage, $w_t^S = w(X_t, h_t)$ or $w_t^L = w(X_t, h_t)$[4]. After that, production and consumption take place, $\rho^x(n_t^S + n_t^L)$ of current workers leave the firm, and $p_t^S q(\theta_t) v_t$ short-term unemployed and $(1 - p_t^S) q(\theta_t) v_t$ long-term unemployed are hired.

1.3.4 Workers

Each worker is characterized by a pair $\{e, b^i\}$, where $e \in \{0, 1\}$ is the employment status of a worker and $b^i \in \{b^S, b^L\}$ is the level of unemployment benefits the worker is receiving if she is currently unemployed or would receive if she weren't currently employed. A worker chooses a sequence $\{c_t\}_{t=0}^{\infty}$ to maximize her expected lifetime utility

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(c_t - \zeta \frac{h_t^{\mu+1}}{\mu + 1} \right)$$
$$\text{s.t. } c_t = e(w_t^i h_t + \Gamma_t) + (1 - e) b^i - T_t + \Pi_t,$$

where w_t^i is the hourly wage, Γ_t is non-wage transfers, T_t is lump sum taxes, and Π_t is profits.

The value of being employed for a worker who was employed in the previous period is the same as the value of being employed for a short-term unemployed worker, because both have the same outside option, b^S, and receive w_t^S when employed. Given

[4] I assume that the firm ignores the fact that the marginal product of each worker depends on the total number of workers, n_t. As Krause and Lubik (2013) show, this barely affects the dynamics of the model, but makes it much easier to solve.

that the aggregate state of the economy is X_t, the value of being employed for a short-term unemployed worker is

$$V^{WS}(X_t) = w_t^S h_t + \Gamma_t - T_t + \Pi_t - \zeta\frac{h_t^{\mu+1}}{\mu+1}+ \\ +\beta\mathbb{E}_t[(1-\rho^x)V^{WS}(X_{t+1}) + \rho^x V^{US}(X_{t+1})]. \tag{1.1}$$

The flow value of being employed equals the sum of after-tax labor income, non-wage transfers, profits, and the disutility of working h_t hours. If the match is hit by the exogenous separation shock, which happens with probability ρ^x, the worker becomes short-term unemployed in the following period. Similarly, the value of being employed for a long-term unemployed worker is

$$V^{WL}(X_t) = w_t^L h_t + \Gamma_t - T_t + \Pi_t - \zeta\frac{h_t^{\mu+1}}{\mu+1}+ \\ +\beta\mathbb{E}_t[(1-\rho^x)V^{WS}(X_{t+1}) + \rho^x V^{US}(X_{t+1})]. \tag{1.2}$$

The value of being short-term unemployed is given by

$$V^{US}(X_t) = b^S - T_t + \Pi_t + \beta\mathbb{E}_t[\lambda[\theta_t q(\theta_t)V^{WS}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{US}(X_{t+1})] \\ +(1-\lambda)[\theta_t q(\theta_t)V^{WL}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{UL}(X_{t+1})]] \tag{1.3}$$

The flow value of being short-term unemployed is equal to unemployment benefits minus taxes, $b^S - T_t$. With probability λ, the short-time unemployed worker does not change her unemployment status in the next period, and with probability $1-\lambda$, short-term unemployment benefits expire and she becomes long-term unemployed. In both cases, with probability $\theta_t q(\theta_t)$, the unemployed worker finds a job and starts working in the next period; otherwise, she remains unemployed. When the short-term unemployed worker becomes long-term unemployed, the worker stays in this status until she finds a job. The value of long-term unemployment is given by

$$V^{UL}(X_t) = b^L - T_t + \Pi_t + \beta\mathbb{E}_t[\theta_t q(\theta_t)V^{WL}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{UL}(X_{t+1})]. \tag{1.4}$$

1.3.5 Representative Firm

The profits of the representative firm with n_t^S and n_t^L workers are

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t,$$

where both wages are functions of hours worked, $w_t^S(X_t, h_t)$ and $w_t^L(X_t, h_t)$; κ is the vacancy posting cost; and $g(h_t)$ is non-wage transfers, which are given by

$$g(h_t) = \phi|h_t - h|,$$

where h is the steady-state level of hours per worker.[5] I assume that non-wage transfers are increasing in the distance between h_t and h. It is costly for the firm to require workers to work more than a "normal" number of hours, e.g., overtime bonuses. If working hours are reduced below h, the firm incurs extra costs as well; e.g., social security contributions for the lost hours in Germany's case.

Apart from paying different wages, the firm cannot treat workers differently; therefore, workers of both types work the same number of hours. Following Kudoh et al. (2016), I assume that the firm chooses hours of work per employee to focus on the composition of labor demand.

Taking as given the labor market tightness, θ_t, share of short-time unemployment, p_t^S, and the law of motion of the aggregate state, Ω, the value of a firm is

$$\begin{aligned} J(X_t, n_t^S, n_t^L) &= \max_{v_t, h_t} \{\Pi_t + \beta \mathbb{E}_t \left[J(X_{t+1}, n_{t+1}^S, n_{t+1}^L) \right] \} \\ \text{s.t. } n_{t+1}^S &= (1 - \rho^x)(n_t^S + n_t^L) + p_t^S q(\theta_t) v_t \\ n_{t+1}^L &= (1 - p_t^S) q(\theta_t) v_t \\ X_{t+1} &= \Omega(X_t) \end{aligned}$$

The next-period number of workers of type S equals the fraction $1 - \rho^x$ of the current workforce plus newly hired short-term unemployed workers. The next-period number of workers of type L equals the number of long-term unemployed workers hired this period.

[5] Non-wage transfers encompass a broad range of benefits, such as social security, health insurance, paid holidays, and overtime bonuses.

First-order conditions are given by

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p_t^S J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L) + (1-p_t^S)J_L(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{1.5}$$

$$z_t\alpha h_t^{\alpha-1}[n_t^S + n_t^L]^\alpha - \frac{\partial g(h_t)}{\partial h_t}[n_t^S + n_t^L] = w_t^S n_t^S + w_t^L n_t^L + \frac{\partial w_t^S}{\partial h_t} n_t^S h_t + \frac{\partial w_t^L}{\partial h_t} n_t^L h_t. \tag{1.6}$$

The value of having an additional short-time unemployed worker is given by

$$J_S(X_t, n_t^S, n_t^L) = \alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - w_t^S h_t - g(h_t) + \\ +(1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]. \tag{1.7}$$

The value of hiring an additional long-time unemployed worker is

$$J_L(X_t, n_t^S, n_t^L) = \alpha z_t h_t^\alpha [n_t^L + n_t^L]^{\alpha-1} - w_t^L h_t - g(h_t) + \\ +(1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]. \tag{1.8}$$

1.3.6 Bargaining

Every period, the firm and workers bargain over hourly wages, $w_t^S = w_t^S(X_t, h_t)$ or $w_t^L = w_t^L(X_t, h_t)$. Following Brügemann et al. (2018), I assume that the firm bargains with every individual worker and that each worker is treated as a marginal worker. Let $\omega \in (0,1)$ be the bargaining power of each worker. To derive the value of a marginal worker to the firm, suppose that the firm bargains with a group of type S workers of measure Δ. The threat point for the firm is $J(X_t, n_t^S - \Delta, n_t^L)$. The limit of the firm's surplus per worker as $\Delta \to 0$ gives us the value of a marginal worker:

$$\lim_{\Delta\to 0} \frac{J(X_t, n_t^S, n_t^L) - J(X_t, n_t^S - \Delta, n_t^L)}{\Delta} = J_S(X_t, n_t^S, n_t^L),$$

where $J_S(\cdot)$ is the derivative of $J(\cdot)$ with respect to n_t^S. Therefore, $w_t^S(X_t, h_t)$ is the solution to the following Nash bargaining problem:

$$\max_{w_t^X} [V^{WS}(X_t) - V^{US}(X_t)]^\omega [J_S(X_t, n_t^S, n_t^L)]^{1-\omega}.$$

Similarly, $w_t^L(X_t, h_t)$ is the solution to the following Nash bargaining problem:

$$\max_{w_t^L}[V^{WL}(X_t) - V^{UL}(S_t)]^{\omega}[J_L(X_t, n_t^S, n_t^L)]^{1-\omega}.$$

Equilibrium hourly wages are derived in Appendix A.1. The resulting expressions for equilibrium wages are quite cumbersome, but it is possible to rewrite them in a compact manner to understand the underlying logic. For example, w_t^S can be written as

$$w_t^S = \frac{\omega}{h_t}w_t^{FS} + \frac{1-\omega}{h_t}w_t^{WS},$$

where

$$\begin{aligned} w_t^{FS} &= \alpha z_t h_t^{\alpha}[n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \\ w_t^{WS} &= \zeta\frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x\frac{\omega}{1-\omega}J_S(X_{t+1}) \\ &+(1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_L(X_{t+1}) - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}] \,. \end{aligned}$$

Essentially, w_t^S is a weighted average of the marginal value of a worker to the firm per hour worked, w_t^{FS}/h_t, and the the minimum hourly wage the worker is willing to accept, w_t^{WS}/h_t. w_t^{FS} is equal to the sum of the marginal product of the worker, minus hours adjustment costs, and plus the value of keeping the worker until the next period. w_t^{WS} corresponds to the opportunity costs of not working, which includes unemployment benefits, an increase in utility from not working minus the value of entering the next period employed. The hourly wage is decreasing in the total number of employees, increasing in the level of unemployment benefits, and nonlinear in hours worked per worker.

1.3.7 Equilibrium

Given the sequence of productivity shocks $\{z_t\}_{t=0}^{\infty}$, the initial level of aggregate employment, N_0, the initial share of short-term unemployment, p_0^S, and the initial levels of employment at the representative firm, n_0^S and n_0^L, an equilibrium is a sequence of wages $\{w_t^S, w_t^L\}_{t=0}^{\infty}$, the firm's choices of hours and vacancies $\{h_t, v_t\}_{t=0}^{\infty}$, the firm's employment levels $\{n_t^S, n_t^L\}_{t=1}^{\infty}$, labor market tightness $\{\theta_t\}_{t=0}^{\infty}$, aggregate labor market *outcomes* $\{, N_{t+1}, p^S_{t+1}, U_t, V_t\}_{t=0}^{\infty}$, *taxes* $\{T_t\}_{t=0}^{\infty}$, *profits* $\{\Pi_t\}_{t=0}^{\infty}$, *non-wage transfers* $\{\Gamma_t\}_{t=0}^{\infty}$, *and the law of motion of the aggregate state* $\{\Omega_t\}_{t=0}^{\infty}$, *such that*

- w^S_t *and* w^L_t *are solutions to the corresponding Nash bargaining problems*
- h_t *and* v_t *satisfy the firm's optimality conditions*
- θ_t *satisfies*

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p^S_t(w^L_{t+1}h_{t+1} - w^S_{t+1}h_{t+1}) + J_L(X_{t+1}, n^S_{t+1}, n^L_{t+1})]$$

- *The aggregate number of vacancies satisfies*

$$V_t = \theta_t U_t$$

- *Aggregate employment is equal to the employment at the representative firm*

$$N_t = n^S_t + n^L_t$$

- *The aggregate number of vacancies is equal to the number of vacancies created by the representative firm*

$$V_t = v_t$$

- *Aggregate employment evolves according to*

$$N_{t+1} = (1 - \rho^x)N_t + q(\theta_t)V_t$$

- *The number of unemployed workers satisfies*

$$U_t = 1 - N_t$$

- n^S_t *and* n^L_t *evolve according to*

$$n^S_{t+1} = (1 - \rho^x)(n^S_t + n^L_t) + p^S_t q(\theta_t)v_t$$

$$n^L_{t+1} = (1 - p^S_t)q(\theta_t)v_t$$

- *Short-time unemployment evolves according to*

$$p^S_{t+1}U_{t+1} = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)p^S_t U_t$$

- *Non-wage transfers are given by*

$$\Gamma_t = g(h_t)$$

- *The government budget is balanced*

$$T_t = b_t^S p_t^S U_t + b_t^L (1 - p_t^S) U_t$$

- *Profits are given by*

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t$$

- Ω_t *is consistent with the law of motion of aggregate state variables*

$$(z_{t+1}, N_{t+1}, p_{t+1}^S) = \Omega_t(z_t, N_t, p_t^S) \textit{ for all } t, z_t, z_{t+1}.$$

The model is solved in Dynare. The system of equations that characterizes the equilibrium can be found in Appendix A.2

1.4 Quantitative Analysis

1.4.1 Calibration

The model is calibrated at a quarterly frequency to the German data. Table 1.2 summarizes parameters and calibration targets. The discount factor β is 0.99, which corresponds to an annual interest rate of 4.1%. Using the data from Karabarbounis and Nieman (2014), I set the labor share $\alpha = 0.64$, which corresponds to the average labor share over the period 1980-2011. Empirical estimates of Frisch elasticity are in the range of 0.5, implying $\mu = 2$ (Chetty (2012)). Following Pissarides (2009), I set the matching elasticity ξ to be equal to 0.5. I further assume that a Hosios condition holds, so that $\omega = 0.5$. Estimates for the separation rate for Germany range from 3% (Christoffel et al. (2009)) to 4% (Gartner et al. (2009)), so I select the midpoint, setting $\rho^x = 3.5\%$. The probability of staying short-term unemployed, λ, is set to 0.75 so that the expected duration of short-term unemployment in the model equals 4 quarters.

I set the matching efficiency, m_0, vacancy posting costs, κ, disutility parameter,

Parameter	Description	Value	Target/Source
β	discount factor	0.99	4% annual interest rate
α	labor share	0.64	Karabarbounis and Nieman (2014)
μ	inverse of Frisch elasticity	2.00	Chetty (2012)
ξ	matching elasticity	0.50	Pissarides (2009)
ω	worker's bargaining power	0.50	($\xi = \omega$) Hosios condition
ρ	separation rate, %	3.50	Literature
λ	probability of staying ST unemployed	0.75	duration of ST unemployment
	Steady State Targets		
m	matching efficiency	0.50	job finding rate 0.35
κ	vacancy posting costs	0.10	vacancy filling rate 0.7
ζ	disutility parameter	0.69	unemployment rate 9.1%
b_0^S	pre-reform ST benefits	0.42	ST replacement rate 60%
b_0^L	pre-reform LT benefits	0.28	LT replacement rate 54%
b_1^L	post-reform LT benefits	0.05	LT replacement rate 17%
	Business Cycle Targets		
ϕ	non-wage transfers parameter	0.0072	pre-reform $\sigma^h/\sigma^N = 0.571$
	Stochastic Process Targets		
ρ_z	persistence of productivity	0.94	persistence of real GDP
σ_z	volatility of productivity, %	0.93	st.dev. of real GDP

Table 1.2: Calibration.

ζ, pre-reform short-term and long-term unemployment benefits, b_0^S and b_0^L, and post-reform long-term unemployment benefits, b_1^L, to jointly match the following steady-state targets: pre-reform short-term and long-term replacement rates, post-reform long-term replacement rate, pre-reform steady-state level of unemployment, and pre-reform job-finding and job-filling rates.

Pre- and post-reform replacement rates are taken from OECD data, which report long-term and short-term average net replacement rates for different subgroups of households over the period 2001-2014. "Short term" refers to the initial phase of unemployment, and "long term" refers to the 60th month of receiving benefits. I focus on a single-person household without children. To obtain target values for pre-reform short-term and long-term replacement rates, I compute the averages of the corresponding time series over the period 2001-2004, and get 60% and 54%. The target value for the post-reform long-term replacement rate is calculated as the average over the period 2005-2014 and is equal to 17%. In the model, the net replacement rate is defined as the ratio of unemployment benefits to labor income.

The target for the pre-reform steady-state value of the unemployment rate is set to 9.1%, which corresponds to the average employment rate over the period 1993Q1-2004Q4. Using data from Gartner et al. (2009), I set the pre-reform job-finding rate to 0.35, which corresponds to the average over the same period, 1993Q1-2004Q4. The vacancy-filling rate is set to 0.7, which is in line with estimates for European countries that have low job turnover (Amaral and Tasci (2016)).

The remaining three parameters, ϕ, ρ_z, and σ_z, are calibrated to match the ratio of volatility of hours per worker to the volatility of employment, persistence, and volatility of output in pre-reform data. The estimated persistence and volatility of the HP-filtered logarithm of real GDP are 0.79 and 1.4%, respectively.

1.4.2 Findings

Steady State

Table 1.3 shows the effect of the reform on the model's steady state. A reduction in long-term unemployment benefits reduces the outside option of long-term unemployed, so that the match surplus of hiring an additional long-term unemployed worker increases, while her hourly wage falls by half. The hourly wages of currently employed workers (and of newly hired short-time unemployed) decline slightly. As a result, the firm posts more vacancies, and hires more workers. The share of short-time unemployment increases, the aggregate unemployment goes down to 5.9%, and labor market tightness goes up from 0.5 to 1.26. The unemployment rate in the model does not fall as much as in the data. By the end of 2016, the unemployment rate in Germany reached 4%, suggesting that the model can explain about 62% of the observed fall. Since the number of workers and the number of hours worked are substitutes in the production of final goods, the firm cuts back on hours worked by each worker.

Elasticities

Following Hagedorn and Manovskii (2008), I calculate the productivity elasticities of labor market tightness, employment, hours per worker, and output assuming that the economy is at the steady state and, for simplicity, that $b^S = b^L = b$ (see Appendix A.3). The elasticity of labor market tightness, employment, hours per worker, and

Variable	Pre-reform	Post-reform
Unemployment rate	0.091	0.059
Hours per worker	1.120	1.114
Tightness	0.500	1.256
Share of ST unemployment	0.683	0.833
Vacancies	0.045	0.075
ST hourly wage	0.632	0.623
LT hourly wage	0.471	0.259
Output	1.012	1.031

Table 1.3: Steady state values of endogenous variables before and after the reform.

output are given by

$$\varepsilon_{\theta,z} = \frac{zM}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)}$$

$$\varepsilon_{n,z} = \varepsilon_{\theta,z}\frac{\rho^x(1-\xi)}{(\rho^x + \theta q(\theta))}$$

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha}\left[1-(1-\alpha)\varepsilon_{n,\theta}\varepsilon_{\theta,z}\right]$$

$$\varepsilon_{y,z} = 1 + \alpha(\varepsilon_{n,z} + \varepsilon_{h,z})$$

where $K > 0$ and $M > 0$.

Elasticities of market tightness and employment are increasing in the level of unemployment benefits, while the elasticity of hours per worker is decreasing in b. As the level of unemployment benefits goes down, the match surplus increases. Incentives to post vacancies depend on the size of percentage changes of the match surplus in response to changes in productivity. A larger surplus means that these percentage changes are smaller, as are the volatilities of vacancies, tightness, and employment. Hours per worker become more responsive to fluctuations in productivity, since the firm now has more incentives to adjust labor input along the intensive margin[6]. The impact of reducing b on the elasticity of output depends on the behavior of the sum of employment and hours per worker elasticities.

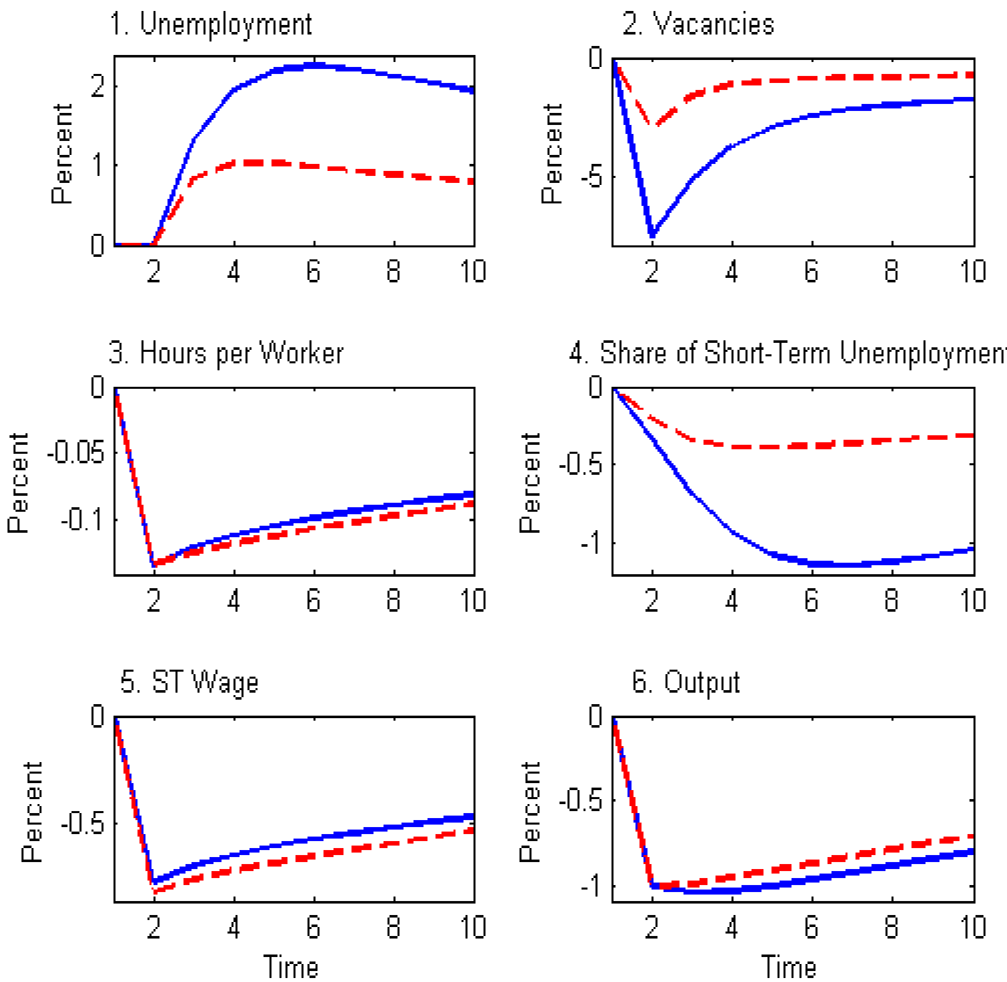

Figure 1.7: Impulse response functions to a negative one-percentage-point productivity shock. Each panel shows the percentage-point deviations from the steady state. The horizontal axes measure time, expressed in quarters. Pre-reform (solid line). Post-reform (dashed line).

	data	(1)	(2)	(3)	(4)	(5)
Pre-reform						
σ^N/σ^Y	0.671	0.201	0.199	0.189	0.280	0.293
σ^h/σ^Y	0.384	0.114	0.117	0.163	0.159	0.170
σ^h/σ^N	0.571	0.567	0.585	0.859	0.569	0.581
σ^H/σ^Y	0.723	0.306	0.310	0.334	0.432	0.459
Post-reform						
σ^N/σ^Y	0.224	0.064	0.063	0.060	0.075	0.076
σ^h/σ^Y	0.426	0.124	0.127	0.177	0.186	0.203
σ^h/σ^N	1.900	1.937	2.000	2.971	2.472	2.655
σ^H/σ^Y	0.516	0.187	0.190	0.224	0.261	0.279

Table 1.4: Observed and simulated relative standard deviations. Data: quarterly, seasonally adjusted, log deviation from an HP trend. Pre-reform Period: 1960Q1-2004Q4. Post-reform period: 2005Q1-2013Q4. Source: Ohanian and Raffo (2012). h – hours per worker, N – employment, H – total hours, Y – output. The simulated business cycle statistics are based on 5000 simulations of 10000 quarter horizon and are HP-filtered for comparison. Simulated figures are averages across simulations. (1) – benchmark model; (2) – benchmark model with $s = 0.08$; (3) – benchmark model with $s = 0.67$; (4) – benchmark model with $\gamma = 0.1$; (5) – benchmark model with $\gamma = 0.1$ and $s = 0.08$

Business Cycle Statistics

Figure 1.7 displays the pre- and post-reform impulse responses to a one-percentage-point negative productivity shock. On impact, vacancies, hours per worker, and output decline. The initial fall in hours per worker and output is almost the same in both calibrations. However, in the post-reform calibration, vacancies decline by about 3 percentage points on impact compared to 7.6 percentage points in the pre-reform calibration. Given that unemployment is unchanged in the period when the shock occurs, labor market tightness goes down. Fewer vacancies lead to an increase in unemployment one period after the shock. In the pre-reform calibration, unemployment keeps rising for 3 periods when it is 2.26 percentage points above the steady-state level, while in the post-reform calibration it starts to decline 2 periods earlier when it is just 1.0 percentage points above steady state. In the post-reform calibration, output recovers faster than in the pre-reform model, while hours per worker recover more slowly.

The third column of Table 1.4 reports the relative standard deviations of the simulated benchmark model. The relative volatility of employment falls by 68%,

[6] As it is shown in Appendix A.4, in a one-worker firm environment, the elasticity of hours per worker is a function of model parameters only.

	data	(1)	(2)	(3)	(4)	(5)
$\sigma^N_{post}/\sigma^N_{pre}$	0.328	0.319	0.317	0.294	0.270	0.259
$\sigma^h_{post}/\sigma^h_{pre}$	1.105	1.091	1.121	1.522	1.170	1.209
$\sigma^H_{post}/\sigma^H_{pre}$	0.708	0.611	0.621	0.752	0.604	0.610

Table 1.5: Observed and simulated ratios of pre- and post-reform standard deviations. h - hours per worker, N - employment, H - total hours. The simulated business cycle statistics are based on 5000 simulations of 10000 quarter horizon and are HP-filtered for comparison. Simulated figures are averages across simulations. Ratios are calculated using pre-reform standard deviations of the benchmark model. (1) – benchmark model; (2) – benchmark model with $s = 0.08$; (3) – benchmark model with $s = 0.67$; (4) – benchmark model with $\gamma = 0.1$; (5) – benchmark model with $\gamma = 0.1$ and $s = 0.08$.

from 0.201 to 0.064, while the relative volatility of hours per worker increases by 9%, from 0.114 to 0.124. The post-reform ratio of volatility of hours per worker to the volatility of employment is 1.94, which is close to the one observed in German post-reform data. In line with Shimer (2005) critique, my model does not generate enough fluctuations in vacancies to match the level of employment volatility in the data[7]. However, it does well in terms of capturing the relative changes in volatility: In the data, the relative volatility of employment fell by two-thirds, while the volatility of hours per worker went up by 10% (see Table 1.5).

Role of the Short-Time Work Policy

The effect of the short-time work policy on the labor market is twofold. Workers receive a short-time working allowance (a fraction of their lost full-time wages from the government, where full time represents some notion of "normal" — i.e., steady state — hours per worker). At the same time, it is less costly for the firm to reduce hours worked per worker below the "normal" level, because part of the social security contribution is paid by the government. In my model, I abstract from the short-time working allowance[8].

To analyze the contribution of short-time work policy to the unemployment dynamics during the Great Recession, I solve and simulate my model assuming that the firm faces lower costs of reducing hours. In particular, I assume that the government

[7] The robustness of my results to the Shimer's critique is discussed in Section 1.5.2.

[8] As Cooper et al. (2017) show, if firms were able to appropriate part of the workers' surplus that comes from the short-time working allowance, workers would be willing to accept lower wages, and it would be optimal for firms to increase employment during recessions, which is contrary to the data. In a risk-neutral environment, the model in which wages are not renegotiated to account for the short-time working allowance is essentially equivalent to the one without it.

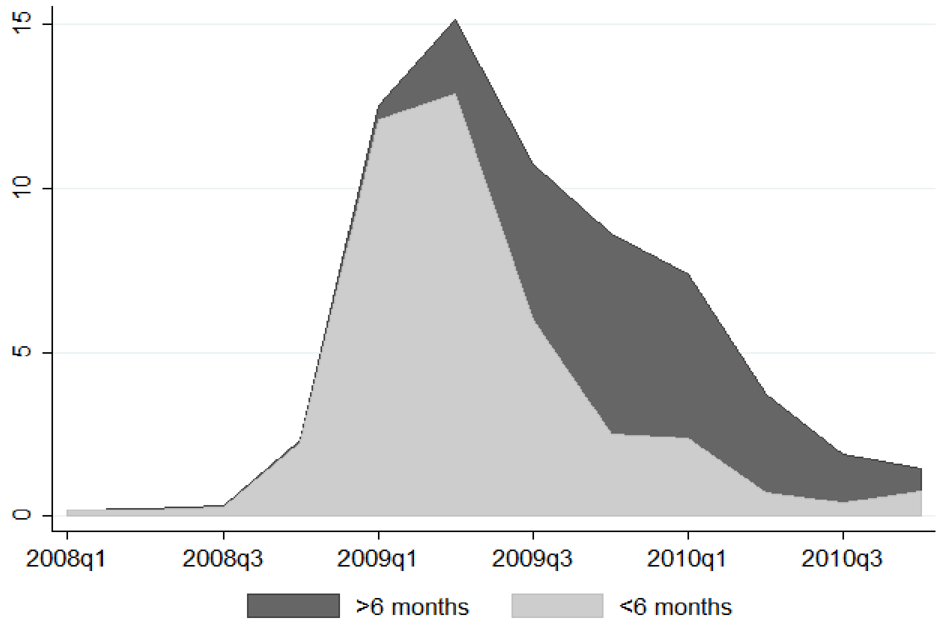

Figure 1.8: Short-time manufacturing workers by duration of short-time work (expressed as a share of total employment of manufacturing sector, %). Source: Federal Employment Agency.

introduces a subsidy s such that new non-wage transfers are

$$\tilde{g}(h_t) = \begin{cases} \phi(h_t - h),\ h_t \geq h \\ (1-s)\phi(h - h_t),\ h_t < h. \end{cases}$$

The subsidy is financed by higher taxes that are now equal to

$$T_t = b_t^S p_t^S U_t + b_t^L (1 - p_t^S) U_t + s\phi \max\{h - h_t, 0\}.$$

To pin down s, I use data on the duration of short-time work from the Federal Employment Agency (see Figure 1.8). The average subsidy is calculated as a weighted average of the share of social contributions covered by the government during the first 6 months of short time work (50%) and the share of social contribution covered by the government after the sixth month (100%). Weights are equal to the average shares of short-time workers of corresponding duration. The resulting subsidy level is 67%. To account for the fact that only a fraction of workers participated in short-time work — in 2009 the average share of manufacturing sector workers on a short-time work was 12% — I assume that only 12% of the firm's workers are eligible for this subsidy, so the resulting value of s is 0.08.

To solve the model, I use the OccBin toolkit developed by Guerrieri and Iacoviello (2015). OccBin is a library of numerical routines designed as an add-on to Dynare and is used to solve models with occasionally binding constraints. The main idea is that the model with asymmetric transfers can be represented as a model with two regimes. Under the "reference" regime, hours per worker are above the steady state level and $\tilde{\phi} = \phi$. Under the "alternative" regime, hours per worker are below the steady-state level and $\tilde{\phi} = (1 - s)\phi$. The model under each regime is log-linearized around the deterministic steady state. The algorithm employs a guess-and-verify approach: Guess the periods in which each regime applies, verify the guess, and update it if necessary.

The fourth column of Table 1.4 shows the relative standard deviations of the pre- and post-reform simulated models with the short-time work subsidy. Since changing both hours per worker and employment is costly, the subsidy makes it cheaper for the firm to reduce hours per worker, instead of posting fewer vacancies, when it is hit by a negative productivity shock. The volatility of employment goes down, while hours per worker become more volatile. The effect, however, is very small: 0.2 and 3 percentage points, respectively (see fourth column of Table 1.5). Comparing this to the 68% decline in employment volatility brought about by the reform suggests that short-time work played only a minor role in generating the German labor market miracle.

Unemployment Dynamics during the Great Recession

In this section I conduct a counterfactual exercise to analyze what would have happened with the unemployment rate during the Great Recession if there had been no unemployment insurance reform. First, I compute a sequence of productivity shocks such that the post-reform calibration with the short-time work subsidy replicates the dynamics of the cyclical component of real GDP over the period 2006Q2-2011Q1. I then feed this sequence of shocks into the pre-reform benchmark calibration and compare the dynamics of unemployment to those of the post-reform model with the short-time work subsidy[9].

[9]Due to the fact that the effect of the short-time work subsidy is small, the dynamics of the unemployment rate in the post-reform benchmark model without the subsidy is almost the same as those of the post-reform benchmark model with the short-time work subsidy

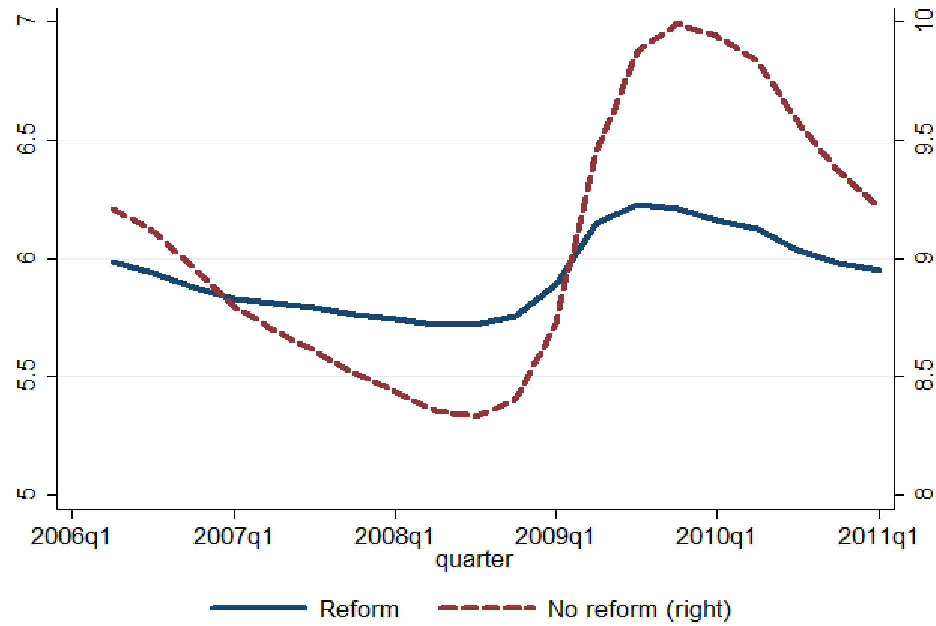

Figure 1.9: Counterfactual for the unemployment rate: reform vs no reform.

Figure 1.9 shows that in the post-reform calibration, the unemployment rate increases by 0.5 percentage points, from 5.72% to 6.22%, while without reform it would have increased by 1.6 percentage points, from 8.4% to 10%. In relative terms, without the reform the unemployment rate would have gone up by a factor of 1.19, compared to the increase by a factor of 1.09 in the reform case.

Welfare

To analyze the impact of the unemployment insurance reform on workers' welfare, I compare the welfare of workers in the pre-reform steady state to welfare along the deterministic transition path in the post-reform steady state. Pre- and post-reform steady state values of workers' value functions are derived in Appendix A.5, while the transition paths of all endogenous variables are calculated using policy functions obtained from solving the deterministic version of the model.

Suppose that the economy reaches the post-reform steady state in T periods, and consider a long-term unemployed worker. In period T, her value is $V_T^{UL} = V_{\text{post,ss}}^{UL}$, where $V_{\text{post,ss}}^{UL}$ is the post-reform steady-state value of V^{UL}. Using the deterministic version of equation (4) and $V_{\text{post,ss}}^{WL}$, I calculate V_{T-1}^{UL}. Similarly, I calculate the values of V_{T-1}^{US}, V_{T-1}^{WL}, and V_{T-1}^{WS}. I keep iterating backward until I reach period 0, when the reform was implemented and the transition started.

Comparing the welfare of each worker type, I find that the reform makes everyone better off, with welfare gains ranging from 0.5% for long-term unemployed workers to 1.2% for currently employed workers. It turns out that welfare gains stemming from lower taxes, higher profits, higher chances of finding a job, and higher employment values outweigh welfare losses from lower long-term unemployment benefits. In particular, even though the long-term unemployed suffer the most because of lower benefits, their continuation value rises a lot because hiring a long-term unemployed worker is very profitable for the firm and some of this surplus is shared with the worker in the form of higher value of employment.

To assess the impact of the reform on an average worker, I calculate the following,

$$W_{\text{no reform}} = U^S V^{US}_{\text{pre,ss}} + U^L V^{UL}_{\text{pre,ss}} + n^S V^{WS}_{\text{pre,ss}} + n^L V^{WL}_{\text{pre,ss}}$$
$$W_{\text{reform}} = U^S V^{US}_0 + U^L V^{UL}_0 + n^S V^{WS}_0 + n^L V^{WL}_0,$$

where weights are equal to the measure of each worker type in the pre-reform steady state. I find that the unemployment insurance reform improved the welfare of an average worker by 1.17%.

To calculate the impact of the reform on welfare in a stochastic setting, I generate 5,000 sequences of productivity shocks, each 10,000 quarters long, calculate the change in welfare along each, and take the average. The resulting increase in well-being ranges from 0.47% for a long-term unemployed worker to 1.21% for a currently employed worker and 1.18% for the average worker.

1.5 Robustness

1.5.1 Short-Time Work Subsidy

To assess how robust my findings are to the level of the short-time work subsidy, I solve and simulate the model assuming that all firm's workers are eligible for the subsidy, i.e., $s = 0.67$. As the fifth column of Table 1.5 shows, higher subsidy significantly increases the volatility of hours per worker, by additional 42.8 percentage points, on the top of a 9.4% increase due to the reform. It also reduces employment volatility by extra 2.3 percentage points. This suggests that even if all workers were involved in the short-time work program, its contribution to the miracle would still be small.

1.5.2 Fixed Matching Costs

Shimer (2005) argues that the standard search and matching model of the labor market fails to account for the observed high volatility of unemployment and vacancy rates. To increase the volatility, I follow Pissarides (2009) and introduce fixed matching costs that include the costs of negotiating with the successful job applicant, putting her on the firm's payroll, and training her. I assume that when a worker arrives, the firm pays a fixed fee γ before the Nash bargaining takes place. Since these costs are sunk at the time of the Nash bargaining, they do not enter Nash bargaining equations. The constant posting cost κ is now replaced by the cost $\tilde{\kappa} + \gamma q(\theta_t)$, which falls in tightness.

To see why the introduction of fixed matching costs increases the volatility of vacancies, consider first the case when $\gamma = 0$. After a positive productivity shock, the firm posts more vacancies at cost κ each. The entry of new vacancies reduces the vacancy filling rate, $q(\theta_t)$, so that the expected cost of hiring a worker, $\kappa/q(\theta_t)$, goes up. The increase in the hiring costs reduces the response of tightness to the productivity shock. When $\gamma > 0$, the hiring costs rise less than in proportion to $1/q(\theta_t)$ so that the firm's incentives to post vacancies remain high.

For any κ, there exist different combinations of $\tilde{\kappa}$ and γ such that the steady state of the model is unaffected, while the volatility of vacancies differ. Specifically, as we increase γ, the value of $\tilde{\kappa}$ falls and the volatility of vacancies goes up. Since the vacancy posting cost is assumed to be positive, the set of possible γ's is limited from above.

To assess whether my results hold in a more volatile environment, I set γ to 0.1 and recalibrate the benchmark model. First, I find that for the pre-reform steady state to remain unchanged, $\tilde{\kappa}$ should be equal to 0.03. After that, I recalibrate ϕ, ρ_z and σ_z[10]. As the sixth column of Table 1.4 shows, in the pre-reform calibration the volatility of employment goes up by about 40%, so does the volatility of hours per worker. Although the impact of the reform on employment volatility is bigger, — it falls by 73% compared to 68% in the benchmark model — the effect of the short-time work subsidy is small as before (see Table 1.5).

[10] The new values are as follows: $\phi = 0.0032$, $\rho_z = 0.954$ and $\sigma_z = 0.94\%$.

1.6 Concluding Remarks

In this paper, I study the effects of the Hartz IV reform on the cyclical volatility of the German labor market. I build a search and matching model with multi-worker firms and a two-tier unemployment insurance system. The fall in long-term unemployment benefits leads to a larger and less volatile match surplus, and reduces firms' incentives to change vacancies and employment over the business cycle. Firms start relying more on the intensive margin of adjustment. I calibrate my model to German pre-reform data and find that the effect of the reform on volatility of employment and hours per worker is consistent with the data. I show that most of the German labor market miracle can be attributed to the unemployment insurance reform of 2005, and that without it the unemployment rate would have gone up much more, both in absolute and relative terms. I also find that the short-time work program played a minor role in safeguarding jobs during the Great Recession.

From a policy perspective, my paper has two main implications. The first is that unemployment insurance reforms are more effective in reducing the cyclical volatility of employment and safeguarding jobs during downturns than temporary policies that promote labor hoarding, such as the short-time work program. However, for reforms to be welfare-improving, policymakers need to enhance efforts to ensure that gains from reforms are not appropriated solely by employers, and that newly created jobs are not inferior to existing ones in terms of duration and job security.

There are several areas for future research. Job separations in my model are exogenous, while according to recent empirical studies, in Germany changes in unemployment inflows (the job-separation rate) are more important than changes in outflows (the job-finding rate). It would be interesting to see how robust my findings are in a setting with endogenous separations. Extending my model further by introducing precautionary savings, skill heterogeneity and skill loss during unemployment would allow us to analyze in more detail the welfare implications of the Hartz IV reform. Finally, it seems that in an environment with multi-worker firms, the way in which hours per worker are chosen affects the quantitative implications of the model. In particular, the level of employment is much more sensitive to changes in unemployment benefits when firms choose hours per worker than when firms bargain over both hourly wages and hours with each individual worker. It is worth exploring this further to identify how hours per worker are actually determined, based on what we observe in the data.

Chapter 2

Inertia in Sovereign Credit Ratings: When Later is Better than Sooner

2.1 Introduction

Credit rating agencies (CRAs) have been criticized for inability to provide timely information about the creditworthiness of sovereign bonds before the outburst of the European sovereign debt crisis. Since credit ratings affect investors' perception of the underlying bond risk, delays in downgrades could mislead them into lending countries with deteriorating economies. Furthermore, after CRAs made successive downgrades in credit ratings of economically troubled countries, they were blamed for deepening the crisis by precipitating an abrupt sell-off of securities and causing investor herd behavior.

This paper documents that in the run-up to the European debt crisis sovereign credit ratings of Italy, Portugal and Spain displayed a higher degree of inertia. First, we estimate an econometric model of ratings as a function of macroeconomic variables. By comparing model-generated ratings with the actual ratings assigned by the rating agencies, we find that after 2005 ratings became stickier: downward adjustment of ratings happened when there was a sufficiently large divergence of predicted ratings from assigned ratings.

We then show that the delays in the downgrades may not be necessarily explained by methodological problems or the lack of CRAs' incentives to generate high quality ratings. On the contrary, a CRA's incentives to provide an accurate rating may be the actual reason for delays in downgrades. We find this adverse effect may actually be caused by the CRA's awareness of the impact it has on a country's economy. When the CRA discovers new information about the country's fundamentals, it updates the sovereign bond rating based on its expectation of the government's decision to default. However, the rating update also has an effect on the country's economy. In fact, "the cliff effect" – when a country's expected default probability increases as it suffers a downgrade – motivated the policy makers to pursue the reform. We show that the CRA's anticipation of a shock to the country's economy following the rating downgrade has a profound impact on its incentives to issue this downgrade in the first place. The increased uncertainty over the default decision created by the shock may make the CRA's prediction *ex post* inaccurate. Therefore, a CRA maximizing accuracy of its ratings may choose to avoid downgrades.

This paper builds on the sovereign default literature that follows Arellano (2008). The country has a benevolent government that maximizes the expected present value of utility of the representative household. The government trades one-period uncontingent bonds with risk-neutral foreign lenders and can choose to default at any time. We assume that the country receives an exogenous endowment which is composed of a persistent income shock and a transitory income shock. The CRA issues a rating to predict the country's default probability. The CRA's goal is to maximize the expected accuracy of its rating. Every period, the CRA privately observes the next period persistent component of the income shock and chooses a rating update based on this information. We model the impact of the rating downgrade by assuming that it may trigger a shock to the country's banking sector and cause a banking crisis. The banking crisis subsequently affects the country's next period output.

In our quantitative exercise, we show that the decision of the CRA to update its rating after it learns that the country's persistent income component is deteriorating depends on how bad the shock is as well as on the level of newly issued debt. If tomorrow's value of persistent income shock is very low, the CRA downgrades the bond for all levels of debt for which the probability of repayment is going down. In case tomorrow's persistent income component does not fall that much, the CRA does not downgrade the bond even if the probability of repayment falls given that the debt level is small enough.

The remainder of the chapter is organized as follows. Section 2.2 reviews the related literature. Section 2.3 describes the data and presents empirical results. Section 2.4 sets up the model and defines the equilibrium. Section 2.5 presents the results of a quantitative analysis. Section 2.6 concludes.

2.2 Related Literature

This paper is related to several strands of literature. From a theoretical perspective, we build on the sovereign default framework of Eaton and Gersovitz (1981) and Arellano (2008). Desplans (2015) is the first paper that introduces a CRA into a sovereign default model. The role of the CRA is to provide information about country's fundamentals by issuing a rating, which is a coarse signal of the country's default probability. The CRA is not allowed to withhold any information from market participants, and therefore ratings and sovereign bond yields move together. In our paper, the CRA maximizes the accuracy of its ratings and can choose how much information to disclose. We show that it might be optimal for the CRA not to downgrade the sovereign bond after receiving information that country's fundamentals are deteriorating.

Our paper contributes to the theoretical literature on strategic credit rating agencies. Mathis et al. (2009) build a model with a CRA that maximizes the expected present value of its profits. They show that when rating of risky securities is a major source of income for the CRA, it inflates ratings when its reputation is good enough. Frenkel (2015) shows that credit rating inflation may occur in concentrated markets with few issuers who repeatedly require ratings of new securities. Since issuers are better informed about the truthfulness of ratings, they can reward rating inflation with high fees. As a result, the CRA has an incentive to provide favorable ratings to create a "double reputation": investors believe that the CRA is credible, while the issuers recognize that the CRA is lenient and inflates ratings. In contrast, in our paper, rating inflation occurs even without the conflict of interest. It is generated by the CRA's incentive to issue accurate ratings and the fact that a rating downgrade leads to a banking crisis thereby increasing uncertainty regarding government's default decision.

This paper is also related to the empirical literature that studies the role of rating agencies in sovereign debt markets. Afonso et al. (2012), Cavallo et al. (2013) and

Aizenman et al. (2013) examine the effects of sovereign credit rating announcements on sovereign bond spreads in European Union countries. Afonso et al. (2012) show that rating and outlook announcements are essentially not anticipated in the previous 1 or 2 months, and that the reaction of spreads to negative rating events is much more pronounced. Cavallo et al. (2013) find that ratings explain part of the variation in macroeconomic variables, even after controlling for for the spread, which suggests that CRAs do add value. Aizenman et al. (2013) find that the association between credit rating changes and spreads follow a non-linear pattern, with ratings at the very low end and around the cut-off between speculative and low investment grade bonds being most sensitive. Ferri et al. (1999) argue that CRAs aggravated the East Asian crisis. Having failed to predict the emergence of the crisis, they became excessively conservative and downgraded East Asian countries more than the worsening in countries' economic fundamentals would justify. Mora (2006) revisits the results of Ferri et al. (1999) and finds that ratings are sticky rather than procyclical. We document that in the run-up to the European debt crisis sovereign credit ratings of Italy, Portugal and Spain displayed a higher degree of inertia.

2.3 Empirical Evidence

2.3.1 Estimation

Figure 2.1 shows the dynamics of the interest rate spreads and sovereign credit ratings of Italy, Spain and Portugal. Sovereign credit ratings are collected from Bloomberg and correspond to ratings for long-term foreign-currency denominated debt. We convert credit ratings from an alphanumeric into a numeric format according to the correspondence shown in Table 2.1. The interest rate spread is defined as the difference of the country's 5-year government bond yield and the German 5-year government bond yield. Government bond yields are taken from the Global Financial Data database. The data suggest that sovereign ratings tend to lag behind the dynamics of spreads.

The estimation approach follows Ferri et al. (1999) who model ratings as a function of a small set of macroeconomic variables. Our sample includes Italy, Portugal and Spain for a time period of 32 years: 1986 to 2017. We run a balanced panel with

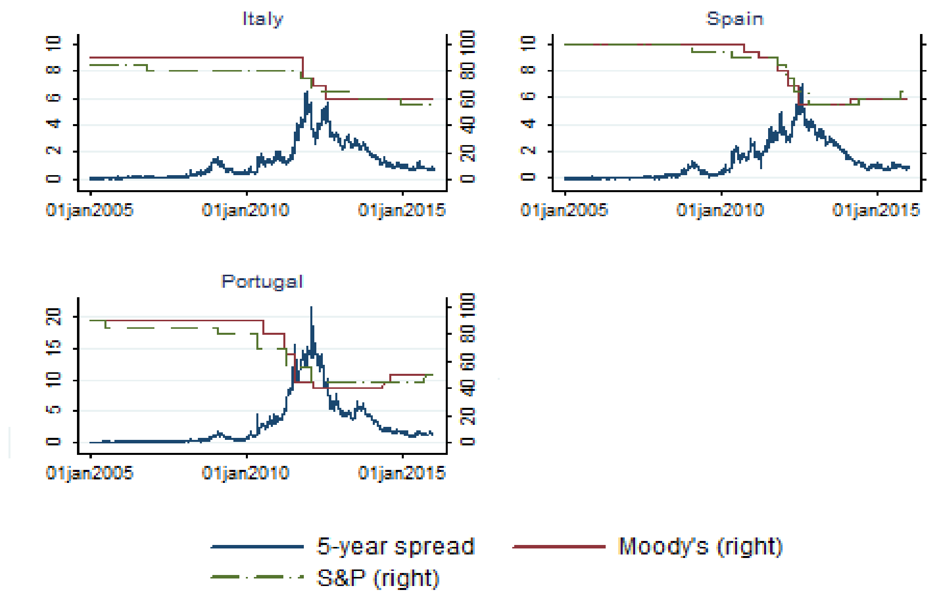

Figure 2.1: Interest rate spreads and sovereign ratings. Source: Global Financial Data and Bloomberg.

fixed effects of the form

$$\text{Rating}_{i,t} = \alpha + \beta_1 \text{Growth}_{i,t} + \beta_2 \text{Inflation}_{i,t} + \beta_3 \text{Budget Balance}_{i,t}$$
$$+\beta_4 \text{Debt}_{i,t} + \beta_5 \text{Current Account}_{i,t} + \varepsilon_{i,t},$$

where $\text{Rating}_{i,t}$ is the average rating (the average of Moody's and S&P and averaged over the year), $\text{Growth}_{i,t}$ is real GDP growth, $\text{Inflation}_{i,t}$ is CPI inflation, $\text{Budget Balance}_{i,t}$ is overall budget balance as % of GDP, $\text{Debt}_{i,t}$ is total government debt as % of GDP, and $\text{Current Account}_{i,t}$ is current account balance as % of GDP[1].

Regression results are presented in Table 2.1. All explanatory variables are significant. As expected, higher GDP growth, lower inflation rate and lower level of government debt are associated with higher ratings. Similar to Mora (2006), we find that current account deficit is associated with better rating which can be explained by the fact that better rated countries are able to borrow easily from abroad and thus run current account deficits. The negative correlation of budget balance and ratings is surprising. This may happen because the data is of annual frequency. After a rating downgrade at the beginning of the year, a country may rush to implement

[1]Source: World Bank's World Development Indicators, IMF International Financial Statistics, Eurostat and OECD.

Moody's	S&P	Numeric Values
Aaa	AAA	100
Aa1	AA+	95
Aa2	AA	90
Aa3	AA-	85
A1	A+	80
A2	A	75
A3	A-	70
Baa1	BBB+	65
Baa2	BBB	60
Baa3	BBB-	55
Ba1	BB+	50
Ba2	BB	45
Ba3	BB-	40
B1	B+	35
B2	B	30
B3	B-	25
Caa1	CCC+	20
Caa2	CCC	15
Caa3	CCC-	10
Ca	CC	5
Ca	C+	0
C	C	0
	C-	0
	D	0

Table 2.1: Converting ratings into numeric values

austerity measures resulting in lower budget deficit by the end of the year.

2.3.2 Inertia in Sovereign Ratings

We follow Mora (2006) to evaluate the extent of inertia. We regress the change in a rating during the period from July 1, year *t* to June 30, year *t+1* on the lagged error term and lagged macroeconomic variables. The error term is defined as the difference between the predicted rating (based on variables for year *t-1*) and the assigned rating on June 30, year *t*.

$$\Delta \text{Rating}_{i,t} = \alpha + \gamma \text{error}_{i,t-1} + \beta_1 \text{Growth}_{i,t-1} + \beta_2 \text{Inflation}_{i,t-1}$$
$$+\beta_3 \text{Budget Balance}_{i,t-1} + \beta_4 \text{Debt}_{i,t-1} + \beta_5 \text{Current Account}_{i,t-1} + \varepsilon_{i,t}$$

	Average Rating
GDP growth rate	0.601**
	(0.288)
Inflation rate	-0.775**
	(0.335)
Budget balance, % GDP	-0.723***
	(0.185)
Government debt, % GDP	-0.514***
	(0.0452)
Current account, % GDP	-1.103***
	(0.236)
Constant	119.6***
	(5.186)
Country effects	Yes
Observations	92
Number of countries	3
R-squared	0.874
Standard errors in parentheses	
*** p<0.01, ** p<0.05, * p<0.1	

Table 2.2: Average rating regression

If ratings are non-sticky, then γ should be equal to zero. If they are sticky, γ should be positive, i.e. there will be a rating change in the direction of the predicted rating when there is a sufficiently large divergence of predicted ratings from assigned ratings. Table 2.3 reports regression results for two time periods: pre-2005 and the whole sample. The coefficient on the lagged error term is positive but not significant in the pre-2005 period, while it is positive and statistically significant once we include post-2005 data. This indicates that after 2005 ratings started exhibiting higher degree of inertia.

2.4 Model

Our model is a small open economy model of sovereign default in the spirit of Arellano (2008). The country's endowment is stochastic and consists of permanent and temporary components. A benevolent government trades one-period uncontingent bonds with risk-neutral foreign lenders. The government cannot commit to repay its debts and can choose to default at any time. A credit rating agency (CRA) produces ratings

	ΔRating	ΔRating
Lagged error term	0.0697	0.160*
	(0.113)	(0.0824)
First lags of		
GDP growth rate	0.314*	0.0913
	(0.171)	(0.238)
Inflation rate	0.187	0.912***
	(0.218)	(0.336)
Budget balance, % GDP	0.165	0.835***
	(0.160)	(0.180)
Government debt, % GDP	0.0729	0.00521
	(0.0715)	(0.0349)
Current account, % GDP	0.143	0.820***
	(0.0891)	(0.175)
Constant	-4.970	2.105
	(4.603)	(4.032)
Observations	41	77
Time period	1986-2005	1986-2017
R-squared	0.408	0.493
Number of id	3	3

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.3: Inertia regressions

that reflect the country's default probability. Upon receiving a private information about the country's endowment next period, CRA can choose to update its initial rating. A worsening of the rating leads to a banking crisis next period irrespective of the government's default decision.

A representative household is risk averse and has the following preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$ is the discount factor, c is consumption, and $u(\cdot)$ is increasing and strictly concave. The household receives a stochastic endowment of tradable good x, which is given by

$$x = y + m,$$

where y is a persistent income shock that follows a first-order Markov process with a transition function $f(y, y')$, and m is a transitory income shock drawn independently each period from a probability distribution $g(m)$.

The government is benevolent and maximizes the expected present value of utility of the representative household. It has access to international financial markets, where it can buy one period uncontingent bonds b' at a price $q_0(b', y)$. The bond price is endogenous and depends on b' and y, but not on m, because temporary shocks are serially uncorrelated and the current value of m does not contain any information about the future realization of m'.

Each period the government decides whether to repay or default on its outstanding debt. When the government decides to honor its debt, the budget constraint of the country is

$$c + q_0(b', y)b' = y + m + b$$

When the government chooses to default, the outstanding debt is erased from the country's budget constraint. The country is excluded from international financial markets and the government is not allowed to save or borrow. It re-enters international financial markets with an exogenous probability λ. While in financial autarky, the government incurs direct output costs. The budget constraint of the country when it is in financial autarky is

$$c = h(y + m),$$

where $h(y + m) \leq y + m$ and $h(\cdot)$ is increasing.

Foreign lenders have access to international financial markets where they can borrow or lend at a constant interest rate r. They price bonds in a risk-neutral manner so that they break even in expected value. Every period they choose b' to maximize expected profits π_L, taking bond prices as given

$$\pi^L = q_0 b' - \frac{\mathbb{E}[1 - d']}{1 + r} b',$$

where d' is the default decision of the government in the next period.

When the government issues new bonds b', the CRA issues the initial rating

$R_0(b', y)$, which is equal to the probability that the debt will be repaid next period. At this point, everybody has access to the same information so that the initial rating equals the probability of repayment

$$R_0(b', y) = \mathbb{E}[1 - d']$$

At the end of period, CRA learns the next period's permanent income component y' and chooses whether to update the initial rating or not. The rating downgrade triggers a banking crisis that entails direct output costs: the output in the next period is reduced by ε. CRA chooses the final rating $R_1(b', y')$ to minimize its expected error

$$\min_{R_1} \mathbb{E}[(R_1 - (1 - d'(b', y', m', \mathcal{C}'(R_1))))^2],$$

where $d'(b', y', m', \mathcal{C}'(R_1)))$ is the government's default decision that depends on the level of outstanding debt, b', the level of permanent and transitory income shocks, y' and m', and $\mathcal{C}'$ which is the indicator functions that equals 1 if there is a banking crisis and 0 otherwise.

After y' is realized at the beginning of the next period, the updated probability of repayment is equal to

$$R_1^f(b', y', \mathcal{C}'(R_1)) = \mathbb{E}[1 - d'(b', y', m', \mathcal{C}'(R_1))].$$

Note that $R_1^f(b', y', \mathcal{C}'(R_1))$ represents the true probability of repayment determined solely by fundamentals, and that it might be different from the rating R_1 issued by the CRA.

2.4.1 Timing

If the country enters the period with a good credit standing and there is no crisis, the timing is as follows

1. Country enters the period with a foreign assets position b
2. Income shock y is realized
3. Income shock m is realized
4. Government chooses whether to default or not, $d \in \{0, 1\}$

(a) If $d = 0$

- Government buys b' bonds at a price $q_0(b', y)$ and CRA reports the initial rating $R_0(b', y)$
- CRA observes y' and chooses the final rating $R_1(b', y')$. If it downgrades the bond, i.e. $R_1(b', y') < R_0(b', y)$, the country enters the next period with a banking crisis

(b) If $d = 1$

- Country enters financial autarky and its endowment is $h(y + m)$
- With probability λ it regains access to international credit markets and with probability $1 - \lambda$ it remains in financial autarky

If the country enters the period in a good credit standing and there is a crisis, the only difference from the timing above is that its endowment is $y + m - \varepsilon$ if there is no default and $h(y + m) - \varepsilon$ otherwise.

If the country enters the period in a bad credit standing and there is no crisis, it is in financial autarky and its endowment is $h(y + m)$. With probability λ it regains access to international credit markets and with probability $1-\lambda$ it remains in financial autarky.

If the country enters the period in a bad credit standing and there is a crisis, its endowment is $h(y + m) - \varepsilon$.

2.4.2 Recursive Equilibrium

Let the state of the economy be $\{b, y, m, \mathcal{C}\}$, where b is the country's initial bonds holding, y is the persistent income shock, m is the transitory income shock, $\mathcal{C} = 1$ if there is a banking crisis and 0 otherwise.

Government's Problem

Let $v(b, y, m, \mathcal{C})$ be the value of the government that has the option to default and satisfies

$$v(b, y, m, \mathcal{C}) = \max_{d \in \{0,1\}} (1 - d)v^c(b, y, m, \mathcal{C}) + dv^d(y, m, \mathcal{C})$$

where $v^c(b, y, m, \mathcal{C})$ is the value of repayment and $v^d(y, m, \mathcal{C})$ is the value of default, d is the default decision.

The value of repayment is given by

$$v^c(b,y,m,\mathcal{C}) = \max_{b'} u(y+m+b-q_0(b',y)b'-\mathcal{C}\epsilon)$$
$$+\beta\mathbb{E}_{y'|y}\left[\mathcal{I}_{\{R_1(b',y')\geq R_0(b',y)\}}\int_{m'} v(b',y',m',0)g(m')dm'\right.$$
$$\left.+\mathcal{I}_{\{R_1(b',y')<R_0(b',y)\}}\int_{m'} v(b',y',m',1)g(m')dm'\right]$$

where $\mathcal{I}$ is an indicator function.

The value of default is given by

$$v^d(y,m,\mathcal{C}) = u(h(y+m)-\mathcal{C}\epsilon)$$
$$+\beta\mathbb{E}_{y'|y}\left[\lambda\int_{m'} v(0,y',m',0)g(m')dm' + (1-\lambda)\int_{m'} v^d(y',m',0)g(m')dm'\right]$$

Lenders' Problem

Given b' and y, the initial bond price satisfies

$$q_0(b',y) = \mathbb{E}_{y'|y}\left[\int_{m'} \frac{[1-d(b',y',m',\mathcal{C}'(R_1(b',y')))]g(m')dm'}{1+r}\right]$$

where $d(b',y',m',\mathcal{C}'(R_1(b',y')))$ is default decision.

Credit Rating Agency's Problem

The initial rating is equal to the probability of repayment

$$R_0(b',y) = \mathbb{E}_{y'|y}\left[\int_{m'}[1-d(b',y',m',\mathcal{C}'(R_1(b',y')))]g(m')dm'\right]$$

After observing y', CRA decides whether to update the initial rating or not. If it does not downgrade the bond, the final rating is given by

$$R_1^{nd}(b',y') = \max\{\text{Prob}[v^c(b',y',m',0) \geq v^d(y',m',0)], R_0(b',y)\}$$

If it downgrades the bond, the final rating is

$$R_1^d(b', y') = \text{Prob}[v^c(b', y', m', 1) \geq v^d(y', m', 1)]$$

Note that if $R_1^d(b', y') \geq R_0(b', y)$, the downgrade is ruled out by definition and $R_1(b', y') = R_1^{nd}(b', y')$ and $\mathcal{C}'(R_1(b', y')) = 0$.

If $R_1^d(b', y') < R_0(b', y)$, CRA chooses not to downgrade if

$$\int_{m'} [R_1^{nd}(b', y') - (1 - d(b', y', m', 0))]^2 g(m') dm' <$$
$$\int_{m'} [R_1^d(b', y') - (1 - d(b', y', m', 1)]^2 g(m') dm'$$

so that $R_1(b', y') = R_1^{nd}(b', y')$ and $\mathcal{C}'(R_1(b', y')) = 0$. Otherwise, $R_1(b', y') = R_1^d(b', y')$ and $\mathcal{C}'(R_1(b', y')) = 1$.

Before giving the definition of an equilibrium of this economy, it is useful to define the government default and repayment sets along with downgrade, no downgrade and crisis sets. Let $\mathcal{R}(b, \mathcal{C})$ be the set of pairs (y, m) for which repayment is optimal when the bond holdings are b and the banking crisis indicator is $\mathcal{C}$

$$\mathcal{R}(b, \mathcal{C}) = [(y, m) \in Y \times M : v^c(b, y, m, \mathcal{C}) \geq v^d(y, m, \mathcal{C})]$$

Similarly, let $\mathcal{D}(b, \mathcal{C})$ be the set of (y, m) for which default is optimal

$$\mathcal{D}(b, \mathcal{C}) = [(y, m) \in Y \times M : v^c(b, y, m, \mathcal{C}) < v^d(y, m, \mathcal{C})]$$

Let $\mathcal{R}_d(b')$ be the set of y' such that CRA downgrades the bond when the new bond issue is b'

$$\mathcal{R}_d(b') = [y' \in Y : \mathcal{C}(R_1(b', y')) = 1]$$

Similarly, let $\mathcal{R}_{nd}(b')$ be the set of y' such that CRA does not downgrade the bond when the new bond issue is b'

$$\mathcal{R}_{nd}(b') = [y' \in Y : \mathcal{C}(R_1(b', y')) = 0]$$

Finally, let $\mathcal{B}(b', y)$ be the set of y' such that CRA downgrades the bond and the banking crisis happens

$$\mathcal{B}(b') = [y' \in Y : R_1(b', y') < R_0(b', y)]$$

Equilibrium: *The recursive equilibrium for this economy is a set of policy functions for consumption $c(b, y, m, \mathcal{C})$, government bond holdings $b'(b, y, m, \mathcal{C})$, repayment sets $\mathcal{R}(b, \mathcal{C})$ and default sets $\mathcal{D}(b, \mathcal{C})$, bond price $q_0(b', y)$, initial ratings $R_0(b', y)$, final ratings $R_1(b', y')$, downgrade sets $\mathcal{R}_d(b')$ and no downgrade sets $\mathcal{R}_{nd}(b')$ and crisis sets $\mathcal{B}(b', y)$ such that*

- *$c(b, y, m, \mathcal{C})$ satisfies the country's budget constraint.*
- *Taking as given the bond price function $q_0(b', y)$, the government's policy function $b'(b, y, m, \mathcal{C})$, repayment sets $\mathcal{R}(b, \mathcal{C})$ and default sets $\mathcal{D}(b, \mathcal{C})$ satisfy the government optimization problem.*
- *Initial ratings $R_0(b', y)$ reflect the initial government's default probability.*
- *CRA's final ratings $R_1(b', y')$, downgrade sets $\mathcal{R}_d(b')$, no downgrade sets $\mathcal{R}_{nd}(b')$ and crisis sets $\mathcal{B}(b', y)$ satisfy the CRA's optimization problem*
- *Bond prices $q_0(b', y)$ reflect the government's default probabilities and are consistent with lender's expected zero profits and CRA's ratings.*

2.5 Quantitative Analysis

2.5.1 Parameters and Functional Forms

We assume that the country's utility function is given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ is the coefficient of relative risk aversion. The persistent income shock y follows an AR(1) process:

$$\log(y') = \rho \log(y) + \epsilon,$$

where $\epsilon \sim N(0, \sigma_y^2)$. We set ρ and σ_y to 0.917 and 0.014, respectively. It is discretized into a 7 state Markov chain using Tauchen (1986). The transitory income shock m

Parameter	Description	Value
σ	Risk aversion	2
β	Discount factor	0.953
λ	Probability of re-entry	0.282
r	Risk-free interest rate	1.7%
$\hat{x}$	Output costs	$0.969\mathbb{E}(x)$
ρ	Autocorrelation	0.917
σ_y	Standard deviation of ϵ	0.014
ε	Banking crisis costs	0.4

Table 2.4: Parameters

is drawn independently each period from a 5-state discrete uniform distribution with support $[0, 0.4]$. Direct output costs of default are given by

$$h(x) = \begin{cases} \hat{x}, & x > \hat{x} \\ x, & x \leq \hat{x} \end{cases}$$

The risk aversion coefficient σ is set to 2, which is standard in the literature. Following Arellano (2008), the discount factor β is set to 0.953, the probability of re-entry after default λ is set to 0.282, the risk-free interest rate r is set to 1.7%, and the default cost threshold is set to 0.969. Banking crisis costs ε are set to 0.4. Table 2.4 summarizes the parameter values.

2.5.2 Numerical Exercise

This section presents the main quantitative result of our paper. The left panel of a Figure 2.2 shows the savings function $b'(b, y, m, 0)$ as a function of assets b for a high and a low y shocks. The m shock is equal to its mean value. The two y shocks are 3% above and below the trend. The government borrows more in booms than in recessions. When the indebtedness of the government is small (b is between -0.2 and 0), the government would like to borrow more. It can do so during good times, but not when the y is low because such financial contracts are unavailable and the government is at the constraint. When the government debt is very high (b is less than -0.2), the government always defaults on its debt in bad times and not allowed to borrow at all because of the exclusion from international financial markets.

The right panel of Figure 2.2 plots the initial rating R_0 as a function of b' for a high and a low y shocks. The initial rating is an increasing function of assets, so that larger levels of debt are associated with lower ratings. For the same loan size, the probability that the government will repay its debt is higher during goods times. The reason is that during recessions the government has a higher incentive to default, and since the y shock is persistent, tomorrow's y will likely be low again. Since the bond price schedule is proportional to the rating, it has the same properties: $q_0(b', y)$ is increasing in the level of assets and is countercyclical.

Figure 2.3 and Figure 2.4 show how the CRA updates its initial rating after it receives information about tomorrow's value of y'. In both cases, the today's value of y is 3% above its mean, and there is no banking crisis, i.e. $\mathcal{C} = 0$.

Figure 2.3 shows what happens when y' is in the mid range, i.e. 3% below its mean. The upper panel plots the initial rating R_0, the final rating R_1 and the true probability of repayment R_1^f as functions of assets b. The lower panel plots the expected errors of downgrading and not downgrading the sovereign debt. When the government is heavily indebted, the CRA downgrades the bond after it observes y' because the expected error in case of a downgrade is lower. The updated rating R_1 is equal to the true probability of repayment. When the government assets are positive, it will never default so there is no reason to downgrade the bond. The most interesting case is when the government's debt is small (b is between -0.1 and -0.05). If the CRA downgrades the bond, it will trigger a banking crisis and the expected accuracy of its rating will be lower than in case when the initial rating is unchanged. Therefore, the CRA does not downgrade the bond even though the true probability of repayment is below the rating thereby causing rating inflation.

The rating inflation region disappears in case when y' is very low (5.5% below the mean). As Figure 2.4 shows, for any b between 0 and 0.15 the true probability of repayment goes down so much that by downgrading the bond the CRA's rating will be very accurate.

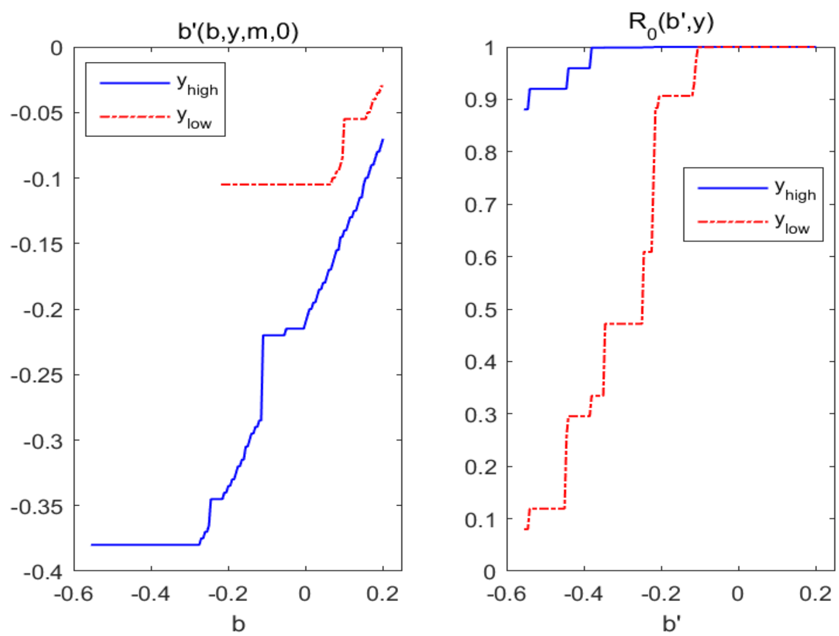

Figure 2.2: Savings and initial ratings

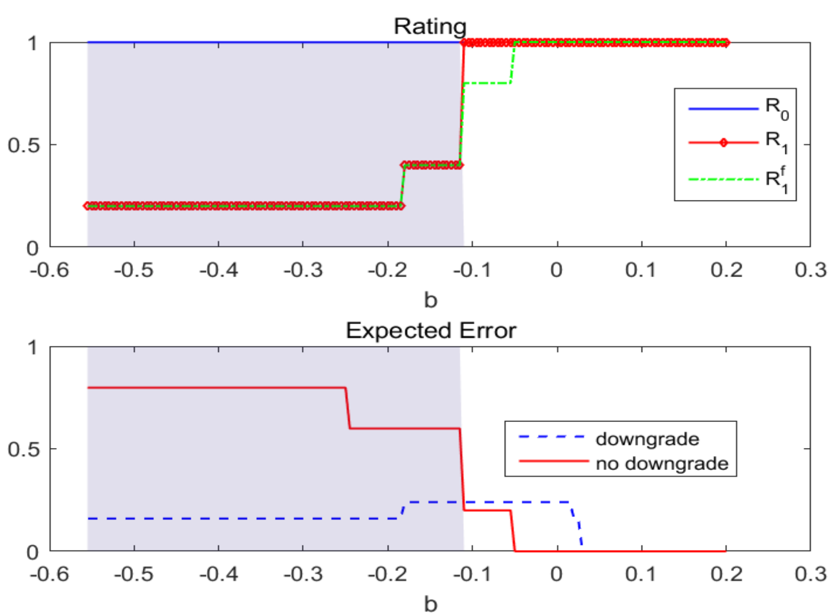

Figure 2.3: Ratings and expected errors, case $y = y_{high}, y' = y_{mid}$. Shaded area is where the bond is downgraded.

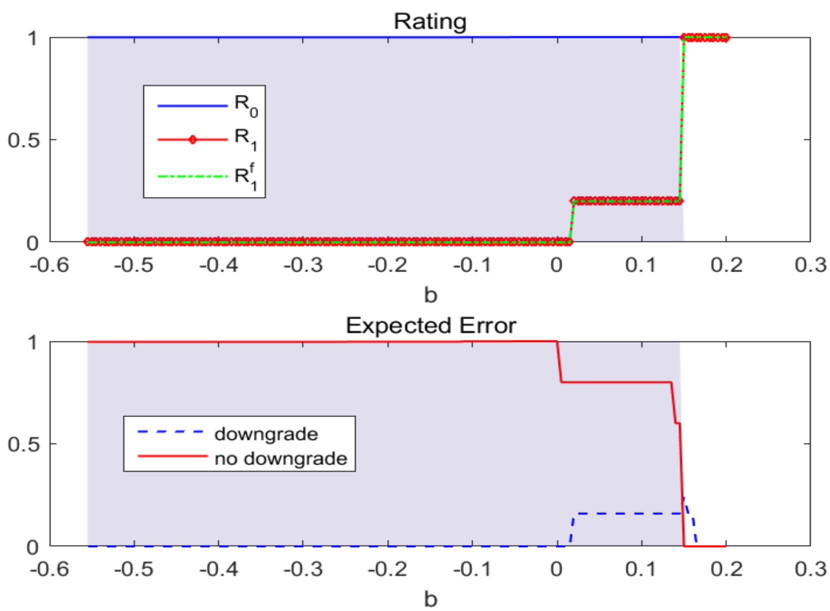

Figure 2.4: Ratings and expected errors, case $y = y_{high}, y' = y_{low}$. Shaded area is where the bond is downgraded.

2.6 Concluding Remarks

In this paper, we document that at the onset of the European debt crisis sovereign credit ratings of Italy, Portugal and Spain displayed a higher degree of inertia. We suggest that the observed delays in rating downgrades might be the result of the optimal behavior of CRAs. We build a sovereign default model with a CRA that maximizes the accuracy of its ratings and show that the CRA's anticipation of the negative effect of a rating downgrade on the economy affects the incentive to issue this downgrade in the first place. A subsequent increase in uncertainty regarding the government default decision makes the rating less accurate *ex post* so that the CRA would choose to avoid downgrades and cause rating inflation.

Chapter 3

The Pass-Through of Sovereign Risk to Productive Sector

3.1 Introduction

As countries in the European periphery were sliding into the sovereign debt crisis, the creditworthiness of firms in those countries started to deteriorate as well. Many empirical studies have documented that firms were facing higher costs of borrowing funds both from banks and bond markets. One of the possible reasons why banks were charging higher rates on loans is because they incurred losses on their government bond holdings and passed their increased financing costs on to firms. However, the fact that bond yields also went up indicates that markets started perceiving firms as more risky investments. Motivated by these empirical findings, I propose a theory of the direct pass-through of sovereign default risk to firms that can generate the co-movement of sovereign and firm's bond spreads. The transmission channel of sovereign risk in my model is the probability that the firm is going to be bailed out by the government. This probability is endogenous and depends on the government's incentive to default on its debt.

First, I present evidence that the behavior of spreads of non-financial corporate bonds in Spain and Italy changed dramatically when the European debt crisis began. Before 2010, the spreads of non-financial corporate bonds in each of these countries were highly correlated with the spreads of German non-financial corporate bonds. Starting from 2010, when sovereign bond spreads shot up, they followed suit and diverged from German non-financial spreads. Even though bank lending remains the biggest source of non-financial corporate funding in Europe, in the aftermath of financial crisis of 2008 European firms started to use bond markets more intensively. Gross corporate bond issuances have been growing steadily over recent years [1]. This indicates that bonds are developing into a meaningful alternative to bank loans and bond yields can be considered as one of the relevant indicators that reflect borrowing costs faced by European firms.

My model builds on the sovereign default model of Arellano (2008) and is enriched with a productive sector that is subject to productivity shocks. There are three periods. In period 1, the benevolent government decides how much to borrow. After that the firm purchases capital to be used in production in period 2 and borrows to finance this purchase. The firm has to buy capital before the next period productivity is realized which makes it a risky investment. If the realized productivity is high, the firm is solvent. It repays the debt, gives the remaining profit back to the household, invests and borrows again to produce in the third period. The only decision the government is making in period 2 when the firm is solvent is whether to repay its debt or not. In case the realized productivity is low, the firm is insolvent, has to default on its debt and shut down. However, the government may step in, bail out the firm and let it continue operating. The bailout decision of the government is endogenous and the firm takes that into account when it chooses how much to invest and borrow in period 1.

The main result of the model is that the optimal amount of capital and the firm's bond price are non-monotonic in the level of the first period output. The source of this non-monotonicity is that the government has an option to default. When the output is very low, the government has an incentive to borrow. This makes it more likely that the government will default on its debt in period 2. The default in turn increases the likelihood of a bailout because all of the second period output (less direct costs of default) can be spent not only on consumption but also on bailing out the firm. Knowing that the chances of being bailed out are higher, the firm invests and borrows more ex ante. As a result, the firm becomes riskier and faces low bond prices. For the high level of output, the government borrows much less (it might even start saving), so in period 2 there are more resources left after the debt is paid. The firm is more likely to be bailed out so it invests more, borrows more and faces low bond prices.

[1] Kaya and Meyer (2013) document a structural change in corporate financing in Europe.

When the output is within the medium range, the government is rich enough not to default on its debt but is not rich enough to be able to repay the debt and afford costly bailout at the same time. Therefore, the probability of being bailed out is lower than in case of either low or high output. It is optimal for the firm to buy less capital, so that it can finance the purchase by issuing bonds at a higher price. This non-monotonicity in the firm's capital decision implies that the government and the firm's bond prices move together as long as the level of output is not too high.

The remainder of the chapter is organized as follows. Section 3.2 reviews the related literature. Section 3.3 presents evidence on the behavior of non-financial corporate bond spreads in Spain and Italy. Section 3.4 sets up the model and defines the equilibrium. Section 3.5 presents the results of a comparative statics exercise. Section 3.6 concludes.

3.2 Related Literature

The spillover of sovereign risk to the corporate sector has been documented by many empirical studies. Cavallo and Valenzuela (2007) and Harjes (2011) document that sovereign and coporate bond yields tend to comove significantly both in emerging markets and more recently in the European periphery. Using data on syndicated loans originated by European banks, Acharya et al. (2014) show that the European sovereign debt crisis and the resulting credit crunch substantially increased the borrowing costs of firms. Bedendo and Colla (2013) provide empirical evidence for the spillover from sovereign risk to non-financial corporate credit risk using credit default swaps data for Eurozone sovereign and corporate entities. They show that an increase in sovereign risk is associated with a robust increase in the credit risk of non-financial firms. Gilchrist and Mojon (2014) study the behavior of spreads of non-financial corporate bond yields in Germany, France, Spain and Italy and show that before the European debt crisis spreads of all four countries exhibited a very strong co-movement. However, since 2010 there was a strong divergence similar to the one observed for sovereign spreads which indicates the spillover of country-specific risks into the non-financial sector.

This paper contributes to the theoretical literature on the impact of sovereign risk on corporate credit risk in developed economies, which has mainly focused on financial firms. Motivated by empirical evidence on domestic government debt holdings by banks in the European periphery countries, several recent papers analyze the links between sovereign and banking insolvency. Cooper and Nikolov (2013) and Farhi and Tirole (2014) study the interaction between the tendency of banks to hold government debt and explicit or implicit bailout guarantees provided by the government to the banking system. This setting can potentially generate co-movement in sovereign spreads and banks' borrowing costs. However, they abstract from the effect of sovereign risk on firms' borrowing costs.

Several authors have recognized the importance of the link between the sovereign default risk and the borrowing costs faced by non-financial firms. Corsetti et al. (2013) study how the 'sovereign risk channel' through which sovereign default risk raises non-financial sector funding costs affects macroeconomic dynamics and fiscal multipliers. However, in their model the connection between sovereign risk premium and non-financial sector risk premium is exogenous. Bocola (2015) endogenizes this relation in a general equilibrium model with banks that hold government bonds on their balance sheets. In his model, the news that the government may default have two effects. They tighten funding constraints of the banks thereby reducing resources available to lending. Banks also demand higher returns when lending to firms because they perceive firms as more risky investment during the default that is usually followed by severe recession. The model makes banks a key ingredient for the pass-through of sovereign risk to firms and abstracts from other sources of funding, such as corporate bonds market. However, based on the evidence presented in Gilchrist and Mojon (2014), non-financial corporate bond yield spreads in Spain and Italy went up together with sovereign bond yield spreads during the European debt crisis. Another channel through which sovereign risk can be transmitted to the corporate sector is fiscal policy. Ferra (2018) shows how a fiscal contraction (in the form of higher taxes) in the country during the sovereign debt crisis leads to a reduction in firms' profits and an increase in their default risk.

My paper is also related to recent studies that introduce production into sovereign default models. In Mendoza and Yue (2012), some imported inputs require working capital financing. When the government defaults, firms cannot borrow and have to replace those foreign intermediate goods by less efficient domestic intermediate goods. This source of inefficiency decreases output. Pei (2014) links default risk and economic activity by assuming that sovereign bonds are used as collateral by firms.

Defaulting on debt tightens firms' collateral constrains and induces them to reduce the demand for labor and cut back production. However, in these papers firms are borrowing at constant interest rates, so these models cannot explain the observed correlation between sovereign and firms' spreads, which is the focus of my research.

In this paper, I study the direct pass-through of sovereign default risk to non-financial firms. I explicitly model the behavior of firms when the sovereign default risk increases within the framework of strategic default as in Eaton and Gersovitz (1981) and Arellano (2008).

3.3 Empirical Evidence

In this section I present evidence that during the European debt crisis the sovereign risk in two European periphery countries, namely Spain and Italy, spilled over to non-financial corporations.

Figure 3.1 shows the dynamics of spreads of bond yields of non-financial corporations in Spain and Germany together with the spread of Spanish 10 year government bond yield. Data on spreads of non-financial corporations was taken from Gilchrist and Mojon (2014). Those spreads are the average spreads on the yield of non-financial corporate bonds relative to the yield on German federal government securities of matched maturities. 10 year Spanish government bond yield spread is constructed relative to 10 year German government bond yield.

Figure 3.1 demonstrates that the behavior of spreads changed at the beginning of 2010 [2]. Before 2010, bonds of Spanish non-financial corporations were considered riskier than sovereign bonds, and the correlation between the two spreads was 0.5. However, starting from 2010 two series demonstrated a strong co-movement (with correlation equal to 0.87), and financial markets started perceiving sovereign bonds as risky as non-financial corporate bonds.

Looking at the dynamics of non-financial corporate spreads of Spain and Germany, we see exactly the opposite. Before 2010 they were highly correlated (with correlation equal to 0.82) with spreads being basically the same during 2003-2009. After 2010 the correlation fell to 0.52, and two series clearly diverged throughout the whole crisis period.

[2] Corporate spreads were high in 2002 when economic growth was low, asset markets were suffering from the loss of confidence and the incidence of credit rating downgrades reached its peak.

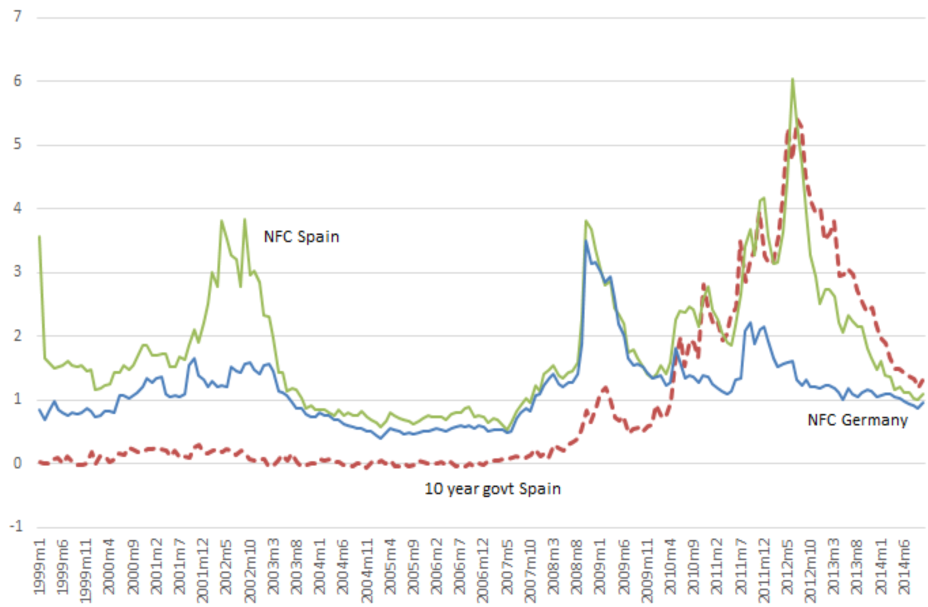

Figure 3.1: Sovereign and non-financial corporate bond yield spreads. Source: Gilchrist and Mojon (2014) and Global Financial Data.

Figure 3.2 shows the dynamics of the spreads of bond yields of Italian non-financial corporations and the spread of Italian 10 year government bond yield. Changes in the behavior of Italian spreads before and after 2010 are similar to those of Spanish spreads (correlation between sovereign and non-financial corporate spreads went up form 0.7 to 0.92).

3.4 Model

My model is a three-period model of sovereign default in the spirit of Arellano (2008) enriched with a productive sector that is subject to productivity shocks. The economy consists of a benevolent government that maximizes the utility of a representative household, a firm and risk neutral foreign creditors. There is one tradable good that can be used both for consumption and production. The only source of uncertainty in the model is the firm's productivity in period 2.

The timeline of the events is displayed on Figure 3.3. In period 1, the government moves first. Taking as given the bond price schedule, the government decides how much to borrow and how much to consume. After that, taking as given its own bond price schedule, the firm purchases the capital to be used in production in period 2

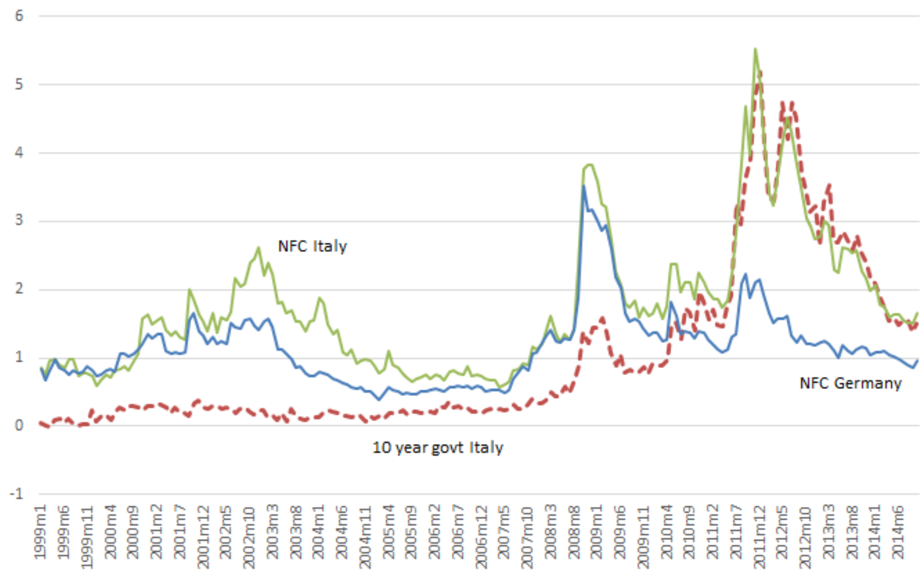

Figure 3.2: Sovereign and non-financial corporate bond yield spreads. Source: Gilchrist and Mojon (2014) and Global Financial Data.

and issues bonds to finance this purchase. In period 2, after the productivity shock is realized, production takes place and the firm is solvent if its profit is non-negative and insolvent otherwise. The government then decides whether to repay or default on its debt and whether to bail out the firm or not. These two decisions automatically determine the amount of goods that the household consumes. If the firm is bailed out, it borrows again to buy capital for period 3. Otherwise, the firm shuts down. In period 3, if the firm is operating, it produces, repays its debt and gives the profit back to the household.

3.4.1 Firm

The firm can produce consumption goods using a decreasing returns to scale technology, $zf(k)$, where z is the productivity and k is the capital stock. I assume that it takes one period to install the capital and that it depreciates fully within a period. The firm does not have any capital in period 1 but can borrow from international creditors in the form of one-period discount bonds. This means that the firm can start producing in period 2 only. The productivity in period 2, z_2, is stochastic and can take one of values in the set $Z = (z_2^1, ..., z_2^N)$ with corresponding probabilities

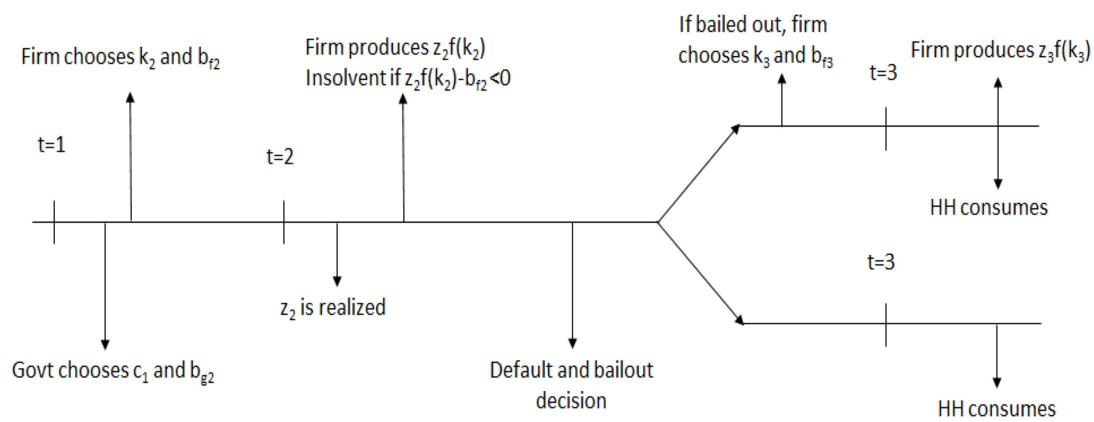

Figure 3.3: Timeline.

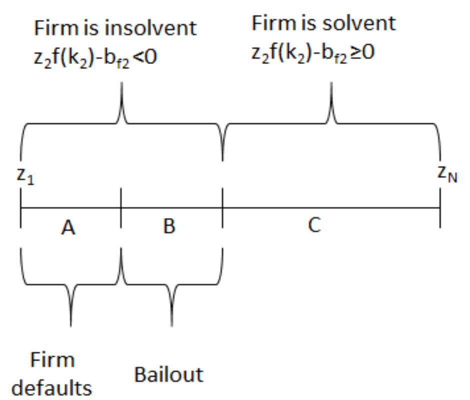

Figure 3.4: Bailout and insolvency sets.

$P = (p^1, ..., p^N)$. For simplicity, I assume that productivity in period 3, z_3, is deterministic.

The firm cannot commit to repay its debt. After the productivity shock in period 2 is realized, the firm is solvent if its profit is non-negative and insolvent otherwise. The profit of the firm goes back to the household. However, the government can decide not to receive the profit – for example in case it is negative – and let the firm default on its debt and shut down. At the same time, if the profit is negative but sufficiently close to zero, the government may choose to bail out the firm and let it produce in period 3.

Figure 3.4 shows that the set of all possible values of z_2 can be divided into 3 subsets (some of them can be empty): the firm defaults on its debt and shuts down for all z_2 in the set A, and repays its debt and continues operating for all other values of z_2. For all z_2 in the set B, the firm gets a transfer from the government to repay its debt, i.e. it is bailed out. I assume that in case of default creditors can get a fraction $\gamma \in [0,1]$ of the firm's revenue. The fraction $1-\gamma$ is lost due to liquidation costs.

The firm takes into account that its choice of k_2 and b_{f2} in period 1 affects the bailout decision of the government in period 2. Taking as given b_{g2} and the bond price schedule $q_{f2}(b_{g2}, k_2, b_{f2})$, firm chooses k_2 and b_{f2} to solve

$$\max_{k_2, b_{f2}} \mathbb{E}\left[\frac{1}{1+r}\delta(b_{g2}, z_2, k_2, b_{f2})\left(d_2 + \frac{1}{1+r}d_3\right)\right]$$
$$\text{s.t. } k_2 = q_{f2}(b_{g2}, k_2, b_{f2})b_{f2}$$
$$d_2 = z_2 f(k_2) - b_{f2}$$

where $\delta(.)$ is equal to 1 if the firm does not default on its debt and 0 otherwise. Profits of the firm in period 3, d_3, conditional that the firm is operating, are determined from the following deterministic problem:

$$d_3 = \max_{k_3, b_{f3}} z_3 f(k_3) - b_{f3}$$
$$\text{s.t. } k_3 = q_{f3} b_{f3}$$

3.4.2 Government

There is a representative household that is risk averse and has the following preferences:

$$u(c_1) + \beta\mathbb{E}[u(c_2) + \beta u(c_3)],$$

where $0 < \beta < 1$ is the discount factor, c_i is consumption in period i. The household receives a deterministic stream of tradable goods (y_1, y_2, y_3), which I am going to refer to as output. In period 1, the government trades one-period discount bonds with risk-neutral foreign creditors. It sells b_{g2} units of bonds at a price q_g, and rebates all the proceeds to the household in a lump-sum fashion. The budget constraint of the government in period 1 is

$$c_1 \leq y_1 + q_g b_{g2},$$

where b_{g2} is the amount of bonds issued by the government and q_g is the bond price.

The government internalizes that the firm's capital and borrowing decisions are affected by the choice of b_{g2}. Taking as given the bond price schedule $q_g(b_{g2})$, the government solves the following problem:

$$\max_{b_{g2},c_1} u(c_1) + \beta \mathbb{E}[V(b_{g2}, z_2, k_2(b_{g2}), b_{f2}(b_{g2}))]$$
$$\text{s.t. } c_1 \leq y_1 + q_g(b_{g2})b_{g2},$$

where $k_2(b_{g2})$ and $b_{f2}(b_{g2})$ are the optimal policy choices of the firm given b_{g2}, and $V(s)$ is the continuation utility given that the state of the economy at the beginning of period 2 is $s = (b_{g2}, z_2, k_2, b_{f2})$.

In period 2, after the productivity shock is realized, the government decides whether to repay or default on its debt and whether to bail out the firm or not (in case the firm is insolvent). If the firm is insolvent, then $V(s)$ is defined as

$$V(s) = \max\{V_{rn}(s), V_{dn}(s), V_{rb}(s), V_{db}(s)\},$$
$$\text{where } V_{rn}(s) = u(y_2 - b_{g2}) + \beta u(y_3)$$
$$V_{dn}(s) = u(\psi y_2) + \beta u(\psi y_3)$$
$$V_{rb}(s) = u(y_2 + d_2 - b_{g2}) + \beta u(y_3 + d_3)$$
$$V_{db}(s) = u(\psi(y_2 + d_2)) + \beta u(\psi(y_3 + d_3))$$

with V_{rn} denoting the value of repaying the debt and not bailing out the firm, V_{dn} – the value of defaulting and not bailing out the firm, V_{rb} – the value of repaying and bailing out the firm, V_{db} – the value of defaulting and bailing out the firm.

If the government defaults on its debt, creditors get nothing but the government incurs direct costs: its income from now on is lower by the factor of ψ. If it doesn't bail out the firm, the firm shuts down. In period 3, the household consumes its endowment, y_3, and the profits of the firm , d_3, conditional on firm non-defaulting in period 2.

If the firm is solvent, then $V(s)$ is defined as

$$V(s) = \max\{V_r(s), V_d(s)\},$$
$$\text{where } V_r(s) = u(y_2 + d_2 - b_{g2}) + \beta u(y_3 + d_3)$$
$$V_d(s) = u(\psi(y_2 + d_2)) + \beta u(\psi(y_3 + d_3))$$

with V_r denoting the value of repaying the debt and V_d – the value of defaulting.

3.4.3 Creditors

Foreign creditors can borrow or lend at a constant risk free rate r. They have perfect information about the productivity process of the firm and can observe the realization of productivity shock in period 2. They price bonds in a risk-neutral manner such that they break even in expected value. Creditors choose b_{g2} to maximize expected profit π_g, taking q_g as given

$$\pi_g = q_{g2} b_{g2} - \frac{1-\pi}{1+r} b_{g2},$$

where π is the probability that the government defaults. Therefore, the government bond price satisfies

$$q_{g2} = \frac{1-\pi}{1+r},$$

In a similar fashion, creditors choose b_{f2} to maximize

$$\pi_f = q_{f2} b_{f2} - \frac{\mathbb{E}[(1-\mathcal{I}_{\{d\}}) + \mathcal{I}_{\{d\}} \frac{\gamma z_2 f(k_2)}{b_{f2}}]}{1+r} b_{f2},$$

where $\mathcal{I}_{\{d\}}$ is equal to 1 if the firm defaults and 0 otherwise. If the firm defaults, creditors have a claim on the firm's revenues less liquidation costs captured by the parameter γ.

3.4.4 Equilibrium

Before giving the definition of an equilibrium of this economy, it is useful to define the government default and repayment sets along with no bailout and bailout sets. First, consider the case when the firm is insolvent. Let R be the set of z_2 for which repayment is optimal when the endogenous state of the economy is given by $s_- = (b_{g2}, k_2, b_{f2})$

$$R(s_-) = [i \in \{1, N\} : V_{rn}(s^i) \geq \max\{V_{dn}(s^i), V_{rb}(s^i), V_{db}(s^i)\} \text{ or}$$
$$V_{rb}(s^i) \geq \max\{V_{rn}(s^i), V_{dn}(s^i), V_{db}(s^i)\}],$$

where $s^i = (b_{g2}, z_2^i, k_2, b_{f2})$. Let D be the set of z_2 for which default is optimal when the endogenous state of the economy is s_-

$$D(s_-) = [i \in \{1, N\} : V_{dn}(s^i) > \max\{V_{rn}(s^i), V_{rb}(s^i), V_{db}(s^i)\} \text{ or } \\ V_{db}(s^i) > \max\{V_{rn}(s^i), V_{dn}(s^i), V_{rb}(s^i)\}]$$

Let B be the set of z_2 for which it is optimal to bail out the firm when the endogenous state of the economy is s_-

$$B(s_-) = [i \in \{1, N\} : V_{rb}(s^i) \geq \max\{V_{dn}(s^i), V_{rn}(s^i), V_{db}(s^i)\} \text{ or } \\ V_{db}(s^i) \geq \max\{V_{rn}(s^i), V_{dn}(s^i), V_{rb}(s^i)\}]$$

and let NB be the set z_2 for which it is optimal not to bail out the firm when the endogenous state of the economy is s_-

$$NB(s_-) = [i \in \{1, N\} : V_{dn}(s^i) > \max\{V_{rn}(s^i), V_{rb}(s^i), V_{db}(s^i)\} \text{ or } \\ V_{rn}(s^i) > \max\{V_{dn}(s^i), V_{db}(s^i), V_{rb}(s^i)\}]$$

If the firm is solvent, then sets $B(s_-)$ and $NB(s_-)$ are empty. Repayment and default sets are defined as

$$R(s_-) = [i \in \{1, N\} : V_r(s^i) \geq V_d(s^i)]$$

and

$$D(s_-) = [i \in \{1, N\} : V_d(s^i) > V_r(s^i)]$$

Equilibrium: *Given the endowment process (y_1, y_2, y_3) and the productivity process (Z, P, z_3), the equilibrium for this economy is a consumption bundle $\{c_1, \{c_2^i, c_3^i\}_{i=1}^N\}$; the government debt b_{g2}, repayment sets $R(s_-)$, default sets $D(s_-)$, bailout sets $B(s_-)$ and no bailout sets $NB(s_-)$; the firm's policy rules $\{f_{k2}, f_{b2}, f_{k3}, f_{b3}\}$; and price schedules for government bonds $q_g(b_{g2})$ and firm's bonds $q_f(b_{g2}, k_2, b_{f2})$ such that*

- *$\{c_1, \{c_2^i, c_3^i\}_{i=1}^N\}$ satisfies the sequence of resource constrains.*
- *For any b_{g2}, taking as given the bond price schedule $q_f(b_{g2}, k_2, b_{f2})$, $\{f_{k2}(b_{g2}), f_{b2}(b_{g2}), f_{k3}(b_{g2}), f_{b3}(b_{g2})\}$ satisfies the firm's optimization problem.*

Parameter	Value
Risk aversion	$\sigma = 3$
Capital share	$\alpha = 0.7$
Discount factor	$\beta = 0.95$
Risk free interest rate	$r = 0.05$
Output costs	$\psi = 0.92$
Liquidation costs	$\gamma = 0.5$
Productivity in period 3	$z_3 = 0.8$
Endowment in $t = 1$	$y_1 \in [0.05, 0.85]$
Endowment in $t = 2$	$y_2 = 0.6$
Endowment in $t = 3$	$y_3 = 0.5$

Table 3.1: Parameters

- *Taking as given* $\{f_{k2}, f_{b2}, f_{k3}, f_{b3}\}$ *and the bond price schedule* $q_g(b_{g2})$, b_{g2} *and repayment, default, bailout and no bailout sets satisfy the government optimization problem.*

- *Bond prices* $q_g(b_{g2})$ *and* $q_f(b_{g2}, k_2, b_{f2})$ *reflect the government's default and bailout probabilities and are consistent with creditor's expected zero profit.*

3.5 Results

3.5.1 Functional Forms and Parameters

For the functional form of the utility function I choose

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

and for the production function

$$f(k) = k^{\alpha}.$$

Productivity in period 2 can take one of 8 values

$$Z = [0.58, 0.73, 0.92, 1.16, 1.45, 1.82, 2, 29, 2.88]$$

with $p^i = 0.125$ for any $i \in \{1, 8\}$. Table 3.1 summarizes the remaining parameter values.

3.5.2 Comparative Statics

This section presents the results of a comparative statics exercise. For each value of y_1 on the grid, I solve for the equilibrium in two versions of my model: the model where the government debt is risk free and the model where the government can default on its debt.

First, consider the case when the government is committed to repay its debt. When the output in period 1 is low, the government has an incentive to borrow more (see Figure 3.5). For any productivity realization in period 2, the government debt is going to be paid in full. This means that the higher the debt is, the less resources are left for consumption and for bailout (in case the firm is insolvent). The government would less likely choose to bail out the firm (see Figure 3.11). As it can be seen from Figure 3.12, if the level of government debt is high, the firm faces (weakly) lower bond prices, i.e. it has to borrow more to buy the same amount of capital. Therefore, it is optimal for the firm to buy less capital in equilibrium (see Figure 3.6). This allows the firm to issue bonds at a higher price because the losses in case the realized productivity is low are lower and creditors can recover more (see Figure 3.7).

Suppose that the government cannot commit to repay its debt. In this case, the optimal amount of capital as well as the firm's bond price is non-monotonic in the level of period 1 output (see Figure 3.6 and Figure 3.7). For low levels of y_1, the government would like to borrow more. However, the government can choose to default if the realized productivity is low and the firm is insolvent. Since in this case creditors get nothing, the government can use all of its endowment (less default costs) not only for consumption but also for bailing out the firm. The firm faces a higher bond price schedule when the government debt is high compared to the medium level of government debt which reflects a higher probability of being bailed out (see Figure 3.9 and Figure 3.10). As a result, for low values of y_1 for which the government defaults in some states of the world – when the government bond price is less than the price of a risk free bond (see Figure 3.8) – the firm chooses a high level of capital.

Now let's see what happens when y_1 is increasing and moving out of the medium range. When y_1 is in the medium range, the government is rich enough not to default on its debt. However, it is not rich enough to be able to spend too much on bailing out the firm. As Figure 3.10 shows, the firm's bond price schedule for the medium level of government debt lies below the other two schedules. The optimal choice of

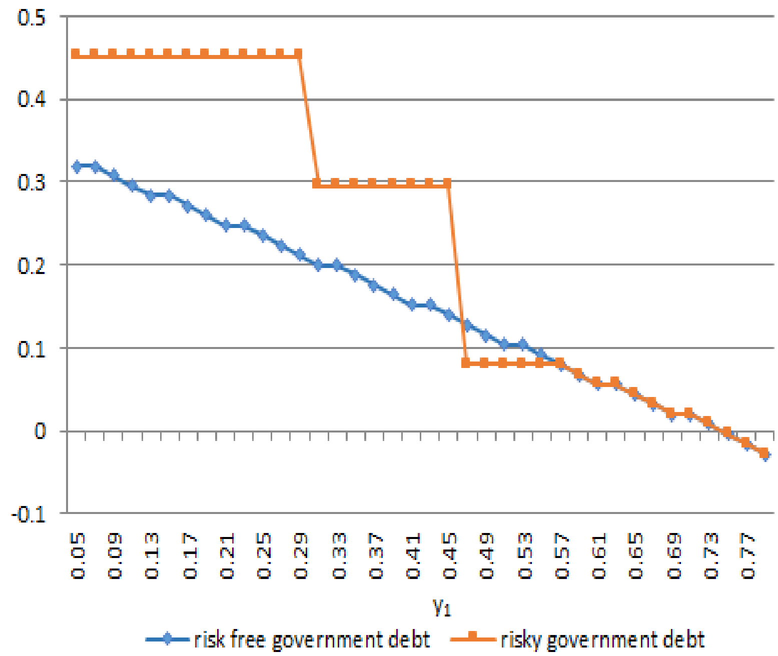

Figure 3.5: Government debt.

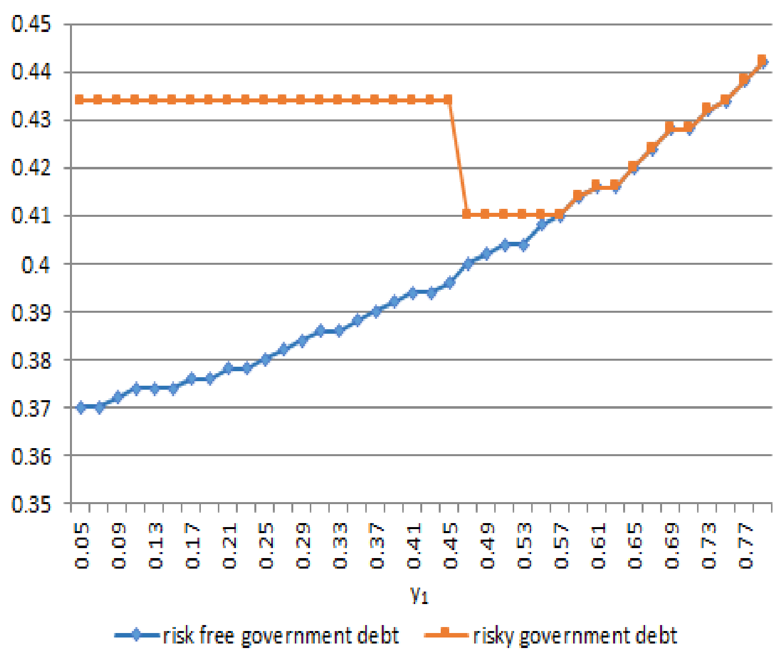

Figure 3.6: Capital

the firm in this case is to buy less capital so that it can finance the purchase by selling bonds at a higher price. When y_1 is moving out of the medium range, the same logic as for the case of risk free government debt can be applied: the government needs to borrow less, more resources are left after the debt is paid so the likelihood of bailout increases. The firms faces more favorable borrowing conditions and invests more.

As long as the level of y_1 is not too high, the equilibrium government and firm's bond prices comove. When y_1 is low, both prices are low because the government chooses to default in some states of the world and the firm is more risky as it buys more capital in anticipation of bailout. Both prices go up as y_1 increases. For high levels of y_1 the equilibria of models with and without government commitment coincide. The government borrows at a constant risk free interest rate while the firm's bond price is going down as the firm starts taking more risk.

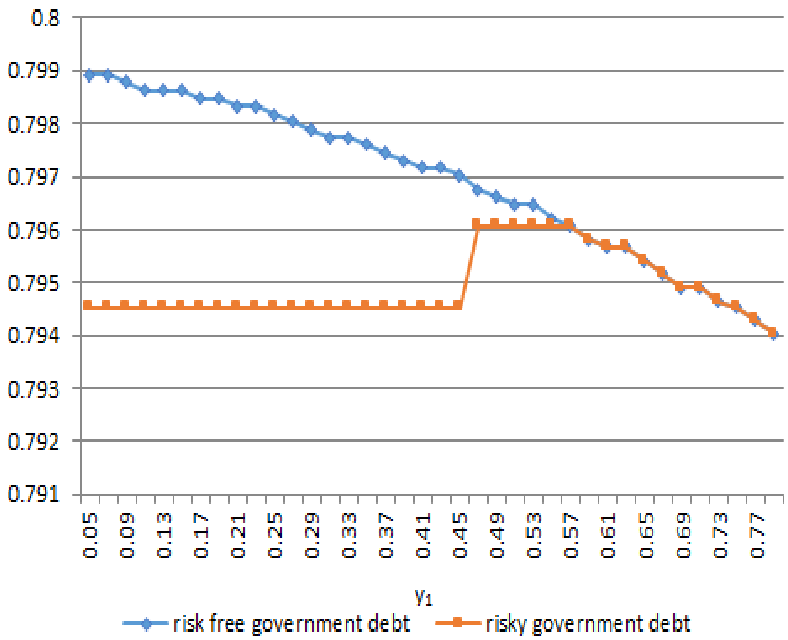

Figure 3.7: Firm's bond price

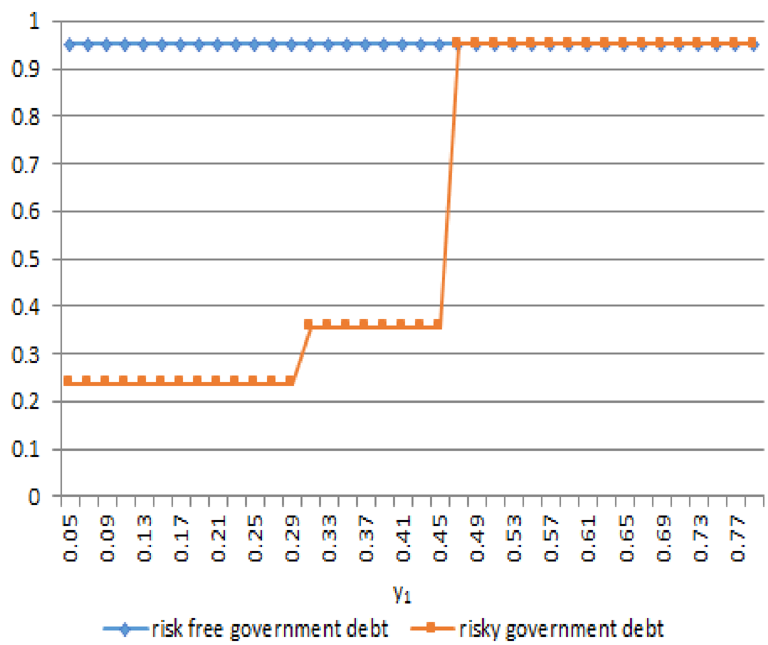

Figure 3.8: Government bond price.

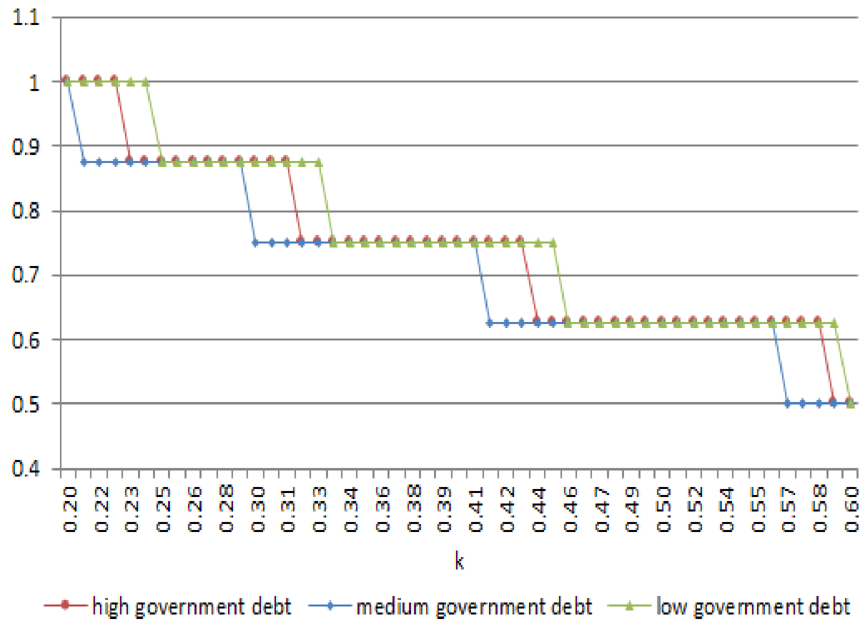

Figure 3.9: Probability of bailout (risky rovernment debt)

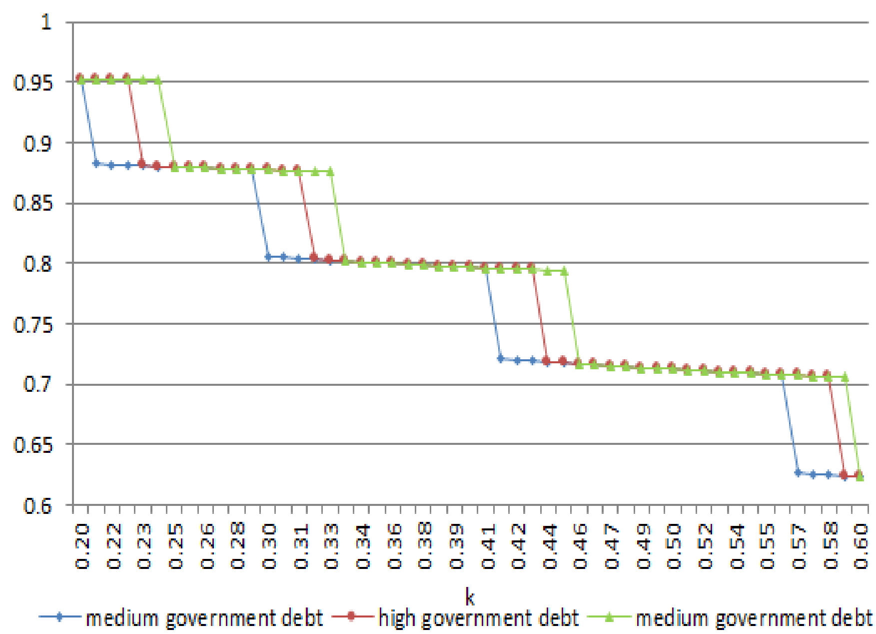

Figure 3.10: Firm's bond price schedules (risky government debt).

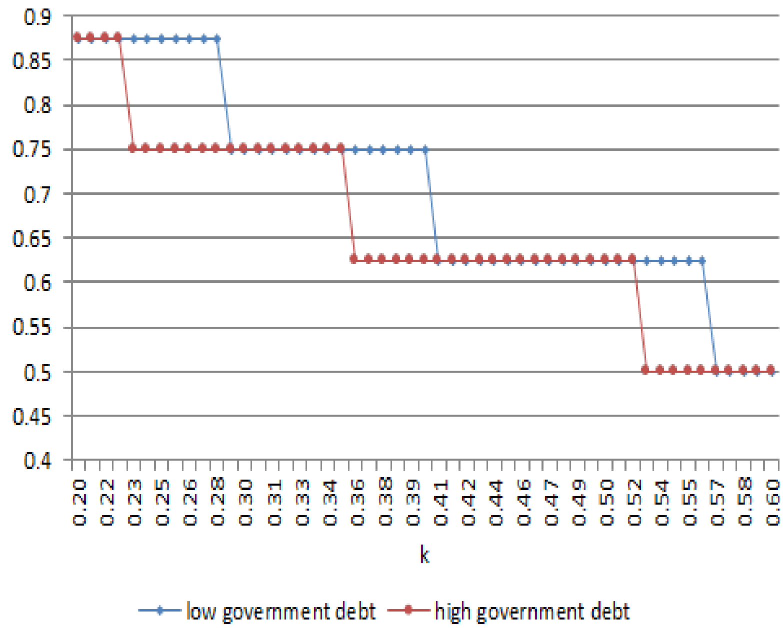

Figure 3.11: Probability of bailout (risk free government debt).

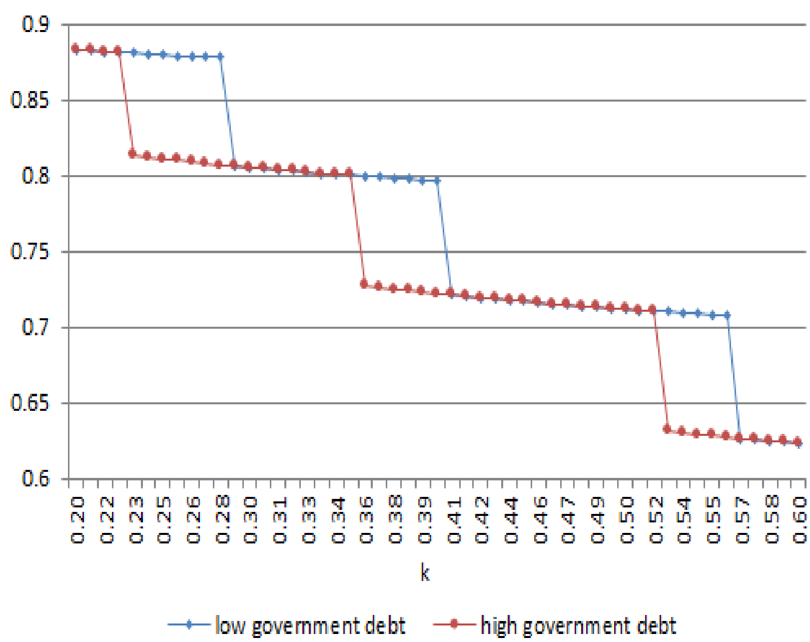

Figure 3.12: Firm's bond price schedules (risk free government debt).

3.6 Concluding Remarks

This paper develops a model of sovereign default and bailout to study the transmission of sovereign default risk to productive sector. The key feature that generates the spillover of risk is the probability of bailout which is endogenous and non-monotonic in the level of output. The probability of bailout is high when the output is high and also when it is low. In the former case, the government does not accumulate a lot of debt so it can afford to bail out the firm. In the latter case, it has a strong incentive to borrow, and hence to default. Defaulting allows the government to be able to spare some resources to bail out the firm. For the output in the medium range, the probability of bailout is lower because the government is neither rich enough nor poor enough to default on its debt. The higher is the probability of bailout, the more risk the firm takes which implies that the government and the firm's bond prices move together as long as the level of output is not too high.

One of the implications of my model is that the firm invests more when the probability of sovereign default rises which contradicts the empirical evidence that investment decreases sharply around the time of default. One way to fix this is to make the distribution of productivity shocks depend on the government's decision to default. By making productivity more volatile in case the government defaults, it might be optimal for the firm to cut investment prior to default even if the probability of bailout is higher. I plan to address this issue in future work.

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Appendix

Appendix to Chapter 1

A.1 Bargaining

The first-order conditions of the bargaining game are

$$\omega J_{St} = (1-\omega)(V_t^{WS} - V_t^{US}) \quad (1)$$

$$\omega J_{Lt} = (1-\omega)(V_t^{WL} - V_t^{UL}) \quad (2)$$

From (1) and (2), we get

$$V_t^{WS} - V_t^{WL} = w_t^S h_t - w_t^L h_t \quad (3)$$

From (7) and (8) we get

$$J_{St} - J_{Lt} = -w_t^S h_t + w_t^L h_t \quad (4)$$

Subtract (10) from (9) to get

$$\omega(J_{St} - J_{Lt}) = (1-\omega)(V_t^{WS} - V_t^{US} - V_t^{WL} + V_t^{UL}) \quad (5)$$

To derive the equilibrium wage for long-term unemployed, subtract (4) from (2) and rearrange

$$V_t^{WL} - V_t^{UL} = w_t^L h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta\mathbb{E}_t[(1-\rho^x)V_{t+1}^{WS} + \rho^x V_{t+1}^{US} - \theta_t q(\theta_t)V_{t+1}^{WL}$$
$$-(1-\theta_t q(\theta_t))V_{t+1}^{UL}]$$
$$V_t^{WL} - V_t^{UL} = w_t^L h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta\mathbb{E}_t[V_{t+1}^{WS} - \rho^x(V_{t+1}^{WS} - V_{t+1}^{US})$$
$$-V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{WL} - (1-\theta_t q(\theta_t))V_{t+1}^{UL}] \Rightarrow$$
$$V_t^{WL} - V_t^{UL} = w_t^L h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta\mathbb{E}_t[V_{t+1}^{WS} - V_{t+1}^{WL} - \rho^x(V_{t+1}^{WS} - V_{t+1}^{US}) \quad (6)$$
$$+(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL})]$$

Using (9)-(11), rewrite (14) as

$$V_t^{WL} - V_t^{UL} = w_t^L h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta\mathbb{E}_t\Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} \quad (7)$$
$$-\rho^x\frac{\omega}{1-\omega}J_{St+1} + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}\Big]$$

Plug (15) and (8) into (10) and solve for w_t^L

$$w_t^L h_t = \omega(\alpha z_t h_t^\alpha[n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}])$$
$$+(1-\omega)\Big(\zeta\frac{h_t^{\mu+1}}{\mu+1} + b^L - \Gamma_t - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x\frac{\omega}{1-\omega}J_{St+1}$$
$$+(1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}]\Big)$$

To derive the equilibrium wage for current workers or short-term unemployed, subtract (3) from (1) and rearrange

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t[(1-\rho^x)V_{t+1}^{WS} + \rho^x V_{t+1}^{US}$$

$$-\lambda[\theta_t q(\theta_t) V_{t+1}^{WS} + (1-\theta_t q(\theta_t))V_{t+1}^{US}] - (1-\lambda)[\theta_t q(\theta_t) V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}]] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Bigg[V_{t+1}^{WS} - \rho^x (V_{t+1}^{WS} - V_{t+1}^{US})$$

$$-V_{t+1}^{WL} - [-(1-\theta_t q(\theta_t))V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}]$$

$$-\lambda \left(\theta_t q(\theta_t)(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) + (1-\theta_t q(\theta_t)) \frac{w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}}{1-\omega} \right) \Bigg] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Bigg[V_{t+1}^{WS} - \rho^x (V^{WS}(S_{t+1}) - V_{t+1}^{US}) - V_{t+1}^{WL}$$

$$+(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL})$$

$$-\lambda \left(\theta_t q(\theta_t)(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) + (1-\theta_t q(\theta_t)) \frac{w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}}{1-\omega} \right) \Bigg] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Bigg[V_{t+1}^{WS} - V_{t+1}^{WL} - \rho^x (V_{t+1}^{WS} - V_{t+1}^{US}) +$$

$$(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL}) - \lambda (w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) \frac{1-\omega\theta_t q(\theta_t)}{1-\omega} \Bigg] \tag{8}$$

Using (9)-(11), rewrite (16) as

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Bigg[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} \tag{9}$$

$$-\rho^x \frac{\omega}{1-\omega} J_{St+1} + (1-\theta_t q(\theta_t)) \frac{\omega}{1-\omega} J_{Lt+1} - \lambda (w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) \frac{1-\omega\theta_t q(\theta_t)}{1-\omega} \Bigg] \tag{10}$$

Plug (17) and (7) into (9) and solve for w_t^S

$$(1-\omega)\left(w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t\Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}\right.$$

$$\left.-\rho^x\frac{\omega}{1-\omega}J_{St+1} + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1} - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\Big]\right) =$$

$$= \omega(\alpha z_t h_t^\alpha[n_t^S + n_t^L]^{\alpha-1} - w_t^S h_t - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}]) \Rightarrow$$

$$w_t^S h_t = \omega\Big(\alpha z_t h_t^\alpha[n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}]\Big) +$$

$$(1-\omega)\left(\zeta\frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta\mathbb{E}_t\Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}\right.$$

$$\left.-\rho^x\frac{\omega}{1-\omega}J_{St+1} + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1} - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\Big]\right)$$

Using expressions for w_t^S and w_t^L, the first order condition for hours per worker can be rewritten as

$$z_t\alpha h_t^{\alpha-1}[n_t^S + n_t^L]^{\alpha-1} - \frac{\partial g(h_t)}{\partial h_t} = \omega\alpha^2 z_t h_t^{\alpha-1}[n_t^S + n_t^L]^{\alpha-1} - \omega\frac{\partial g(h_t)}{\partial h_t} + (1-\omega)\zeta h_t^\mu$$

A.2 Equilibrium System of Equations

Equilibrium of the model is characterized by the following system of equations:

$$w_t^S h_t = \omega(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]) + \tag{11}$$

$$(1-\omega)\left(\zeta \frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta\mathbb{E}_t\Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x \frac{\omega}{1-\omega} J_S(X_{t+1})\right.$$

$$\left. + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega} J_L(X_{t+1}) - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\Big]\right)$$

$$w_t^L h_t = \omega(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]) \tag{12}$$

$$+(1-\omega)\left(\zeta \frac{h_t^{\mu+1}}{\mu+1} + b^L - \Gamma_t - \beta\mathbb{E}_t\Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x \frac{\omega}{1-\omega} J_S(X_{t+1})\right.$$

$$\left. + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega} J_L(X_{t+1})\Big]\right)$$

$$z_t \alpha h_t^{\alpha-1}[n_t^S + n_t^L]^\alpha - \frac{\partial g(h_t)}{\partial h_t}[n_t^S + n_t^L] = w_t^S n_t^S + w_t^L n_t^L + \frac{\partial w_t^S}{\partial h_t} n_t^S h_t + \frac{\partial w_t^L}{\partial h_t} n_t^L h_t \tag{13}$$

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p_t^S(w_{t+1}^L h_{t+1} - w_{t+1}^S h_{t+1}) + J_L(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{14}$$

$$\text{where } J_S(X_t, n_t^S, n_t^L) - J_L(X_t, n_t^S, n_t^L) = -w_t^S h_t + w_t^L h_t$$

$$J_L(X_t, n_t^S, n_t^L) = \alpha z_t h_t^\alpha [n_t^L + n_t^L]^{\alpha-1} - w_t^L h_t - g(h_t) \tag{15}$$

$$+(1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{16}$$

$$U_{t+1}^S = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)p_t^S U_t$$

$$p_t^S = U_t^S / U_t \tag{17}$$

$$\theta_t = V_t / U_t \tag{18}$$

$$U_t = \bar{N} - N_t \tag{19}$$

$$N_{t+1} = (1-\rho^x)N_t + q(\theta_t)V_t \tag{20}$$

$$n_{t+1}^L = (1 - p_t^S)q(\theta_t)v_t \tag{21}$$

$$n_{t+1}^S = N_{t+1} - n_{t+1}^L \tag{22}$$

$$T_t = b_t^S p_t^S U_t + b_t^L(1 - p_t^S)U_t \tag{23}$$

$$\Gamma_t = g(h_t) \tag{24}$$

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t \tag{25}$$

$|h_t - h|$ is approximated using a smooth function $f(h_t - h)$ given by

$$f(h_t - h) = \frac{2}{k}\log(1 + \exp^{k(h_t - h)}) - (h_t - h) - \frac{2}{k}\log(2),$$

where k is a parameter that controls the smoothness and is set to 1000.

A.3 Elasticities

Assume the economy is at the steady state and, for simplicity, that $b^S = b^L = b$. Let $n = n^S + n^L$. From the first order condition for hours per worker we obtain the equilibrium value of hours per worker

$$h = \left(\frac{\alpha z(1-\alpha\omega)n^{\alpha-1}}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}} \tag{26}$$

Plugging it into job creation condition yields

$$\kappa = q(\theta)\beta\mathbb{E}_t[J_L(X_{t+1}, n^S_{t+1}, n^L_{t+1})]$$
$$J_L(S_t, n^S_t, n^L_t) = \alpha z_t h^\alpha_t n^{\alpha-1}_t - w^L_t h_t + (1-\rho^x)\frac{\kappa}{q(\theta_t)}$$

Expression for w^L_t becomes

$$w^L h = \omega(\alpha z h^\alpha n^{\alpha-1} + \beta(1-\rho^x)\mathbb{E}_t[J_L])$$
$$+(1-\omega)\left(\zeta\frac{h^{\mu+1}}{\mu+1} + b - \beta\mathbb{E}_t\left[(1-\theta_t q(\theta_t)-\rho^x)\frac{\omega}{1-\omega}J_L\right]\right)$$

Job creation condition becomes

$$\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right] = (1-\omega)\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)$$

where

$$K = \alpha^{\frac{1+\mu}{\mu+1-\alpha}}\zeta^{\frac{-\alpha}{\mu+1-\alpha}}\left(\frac{1-\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{\mu+1-\alpha}}\left[1-\frac{1-\alpha\omega}{(\mu+1)(1-\omega)}\right]$$

Using implicit differentiation, we obtain

$$\frac{\partial\theta}{\partial z} = \frac{(1-\omega)\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\frac{\frac{\partial q(\theta)}{\partial\theta}}{q(\theta)^2} - (1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}} =$$

$$= \frac{\theta M}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)},$$

where

$$M = \frac{\kappa\left[\frac{1-\beta(1-\rho^x)}{\beta} + \omega\theta q(\theta)\right]\left[K\frac{1+\mu}{\mu+1-\alpha} z^{\frac{1+\mu}{\mu+1-\alpha}-1} n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left[q(\theta)\theta\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\xi - q(\theta)\theta(1-\omega)K z^{\frac{1+\mu}{\mu+1-\alpha}} \frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha} n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

Thus, productivity elasticity of labor market tightness is given by

$$\varepsilon_{\theta,z} = \frac{zM}{\left(K z^{\frac{1+\mu}{\mu+1-\alpha}} n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)} \tag{27}$$

Use the law of motion of the aggregate employment to calculate the productivity elasticity of employment

$$\varepsilon_{n,z} = \frac{\partial n}{\partial z}\frac{z}{n} = \frac{\rho m_0(1-\xi)\theta^{-\xi}\frac{\partial\theta}{\partial z}}{(\rho^x + \theta q(\theta))^2}\frac{z}{n} = \frac{\rho^x m_0(1-\xi)\theta^{-\xi}\frac{\partial\theta}{\partial z}}{(\rho^x + \theta q(\theta))}\frac{z}{\theta q(\theta)} \Rightarrow$$

$$\varepsilon_{n,z} = \varepsilon_{\theta,z}\frac{\rho^x(1-\xi)}{(\rho^x + \theta q(\theta))} \tag{28}$$

Finally, the productivity elasticity of hours per worker is given by

$$h = \left(\frac{\alpha z(1-\alpha\omega)n^{\alpha-1}}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}}$$

$$\varepsilon_{h,z} = \left(\frac{\alpha(1-\alpha\omega)}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}} \frac{z^{\frac{\alpha-\mu}{\mu+1-\alpha}} n^{\frac{2(\alpha-1)-\mu}{\mu+1-\alpha}}}{\mu+1-\alpha}\left[n + (\alpha-1)z\frac{\partial n}{\partial\theta}\frac{\partial\theta}{\partial z}\right]\frac{z}{h}$$

$$= \left(\frac{\alpha(1-\alpha\omega)}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}} \frac{z^{\frac{1}{\mu+1-\alpha}} n^{\frac{2(\alpha-1)-\mu}{\mu+1-\alpha}}}{\mu+1-\alpha}\left[n + (\alpha-1)z\frac{\partial n}{\partial\theta}\frac{\partial\theta}{\partial z}\right]\left(\frac{\alpha z(1-\alpha\omega)n^{\alpha-1}}{\zeta(1-\omega)}\right)^{\frac{-1}{\mu+1-\alpha}}$$

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha}\left[1 - (1-\alpha)\varepsilon_{n,\theta}\varepsilon_{\theta,z}\right] \tag{29}$$

The productivity elasticity of output is given by

$$\varepsilon_{y,z} = \frac{\partial[z(hn)^\alpha]}{\partial z}\frac{z}{y} = \left[(hn)^\alpha + z\alpha h^\alpha n^{\alpha-1}\frac{\partial n}{\partial z} + z\alpha h^{\alpha-1}n^\alpha\frac{\partial h}{\partial z}\right]\frac{z}{y} =$$
$$= 1 + \alpha\frac{\partial n}{\partial z}\frac{z}{n} + \alpha\frac{\partial h}{\partial z}\frac{z}{h} = 1 + \alpha(\varepsilon_{n,z} + \varepsilon_{h,z}) \qquad (30)$$

A.4 Model with One-Worker Firms

The economy consists of a large number of infinitely lived firms. A firm-worker pair produces $y_t = z_t h_t^\alpha$ of a single homogeneous good. The values of being employed and unemployed are the same as in the benchmark model.

The value of a short-time unemployed worker is given by

$$J_S(X_t) = \max_{h_t^S} z_t (h_t^S)^\alpha - w_t^S h_t^S - g(h_t^S) + \beta \mathbb{E}_t[(1-\rho^x) J_S(X_{t+1}) + \rho^x J_V(X_{t+1})],$$

where h_t^S denotes hours worked and $J_V(\cdot)$ is the value of unfilled vacancy. The first order condition for h_t^S is

$$z_t \alpha (h_t^S)^{\alpha-1} - \frac{\partial g(h_t^S)}{\partial h_t^S} = w_t^S + \frac{\partial w_t^S}{\partial h_t^S} h_t^S$$

The value of a long-time unemployed worker is

$$J_L(X_t) = \max_{h_t^L} z_t (h_t^L)^\alpha - w_t^L h_t^L - g(h_t^L) + \beta \mathbb{E}_t[(1-\rho^x) J_S(X_{t+1}) + \rho^x J_V(X_{t+1})].$$

The first order condition for h_t^L is

$$z_t \alpha (h_t^L)^{\alpha-1} - \frac{\partial g(h_t^L)}{\partial h_t^L} = w_t^L + \frac{\partial w_t^L}{\partial h_t^L} h_t^L$$

The value of unfilled vacancy is given by

$$J_V(X_t) = -\kappa + \beta \mathbb{E}_t[q(\theta_t)(p_t^S J_S(X_{t+1}) + (1-p_t^S) J_L(X_{t+1})) + (1-q(\theta_t)) J_V(X_{t+1})]$$

Free entry of firms implies that firms post vacancies until the value of doing so is equals to zero, $J_V(X_t) = 0$. Therefore, in equilibrium the following condition holds

$$\kappa = \beta q(\theta_t) \mathbb{E}_t[p_t^S J_S(X_{t+1}) + (1-p_t^S) J_L(X_{t+1})]$$

Each firm-worker pair bargains over hourly wages, $w_t^S(X_t, h_t^S)$ or $w_t^L(X_t, h_t^L)$, that are given by

$$w_t^S(X_t, h_t^S)h_t^S = \omega(z_t(h_t^S)^\alpha - g(h_t^S) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1})])+$$
$$(1-\omega)\left(\zeta\frac{(h_t^S)^{\mu+1}}{\mu+1} + b^S - \Gamma_t^S - \beta\mathbb{E}_t\left[w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L\right.\right.$$
$$\left.\left.-\rho^x\frac{\omega}{1-\omega}J_{St+1}) + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}) - \lambda(w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L)\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\right]\right)$$

$$w_t^L(X_t, h_t^S)h_t^L = \omega(z_t(h_t^L)^\alpha - g(h_t^L) + \beta\mathbb{E}_t[(1-\rho^x)J_{St+1})]])$$
$$+(1-\omega)\left(\zeta\frac{(h_t^L)^{\mu+1}}{\mu+1} + b^L - \Gamma_t^L - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L - \rho^x\frac{\omega}{1-\omega}J_{St+1}+\right.$$
$$\left.(1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}]\right)$$

Using expressions for wages, first order conditions for hours per worker can be rewritten as

$$z_t\alpha(h_t^S)^{\alpha-1} - \frac{\partial g(h_t^S)}{\partial h_t^S} = \omega\alpha z_t(h_t^S)^{\alpha-1} - \omega\frac{\partial g(h_t^S)}{\partial h_t^S} + (1-\omega)\zeta(h_t^S)^\mu$$
$$z_t\alpha(h_t^L)^{\alpha-1} - \frac{\partial g(h_t^L)}{\partial h_t^L} = \omega\alpha z_t(h_t^L)^{\alpha-1} - \omega\frac{\partial g(h_t^L)}{\partial h_t^L} + (1-\omega)\zeta(h_t^L)^\mu.$$

From the two equations above it follows that $h_t^S = h_t^L$.

Assuming that the economy is at the steady state, the equilibrium value of hours per worker is given by

$$h = \left(\frac{z\alpha}{\zeta}\right)^{\frac{1}{\mu+1-\alpha}}.$$

The productivity elasticity of hours per worker is given by

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha}.$$

A.5 Welfare

Suppose the economy is in the steady state. From the first order condition of the bargaining game, we have that

$$V^{WL} = V^{UL} + \frac{\omega}{1-\omega} J_L \tag{31}$$

From the job creation condition, we obtain the expression for J_L

$$J_L = \frac{\kappa}{q(\theta)\beta} + p^S(w^S h - w^L h) \tag{32}$$

Plug (39) and (40) into equation (4) to obtain

$$V^{UL} = b^L - T + \Pi + \beta[\theta q(\theta)V^{WL} + (1-\theta q(\theta))V^{UL}] \Rightarrow$$
$$V^{UL} = \frac{1}{1-\beta}\left[b^L - T + \Pi + \beta\theta q(\theta)\frac{\omega}{1-\omega} J_L\right] \Rightarrow$$
$$V^{UL} = \frac{1}{1-\beta}\left[b^L - T + \Pi + \beta\theta q(\theta)\frac{\omega}{1-\omega}\left(\frac{\kappa}{q(\theta)\beta} + p^S(w^S h - w^L h)\right)\right] \tag{33}$$

Knowing V^{UL}, V^{WL} can be obtained from (39), V^{WS} from (11). To solve for V^{US}, use equations (10) and (12).