

# A strongly correlated metal from coupled SYK models

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# Transport

$$\mathbf{j} = \underline{\sigma} \mathbf{E}$$

$$\mathbf{j}_e = -\underline{\kappa} \nabla T$$

- Arguably most important aspect of quantum materials: electrical and thermal conductivity (and crossed coefficients)
- Sensitive, versatile
- Probes extreme long wavelength, low frequency

# Theory

- *Understanding* of transport mainly through electron **quasiparticle** picture
- Boltzmann equation:

$$[\partial_t + \mathbf{v}_n(\mathbf{k}) \cdot \nabla_r - e\mathbf{E} \cdot \nabla_k] f_n = \left. \frac{\partial f_n}{\partial t} \right|_{\text{collision}}$$

Linearizing this around equilibrium gives conductivities in terms of band velocities and scattering rates

# Ultra-quantum transport

- How does transport work when quasiparticles are not adiabatically connected to electrons?
- Or when quasiparticles scatter very strongly?
- Or if there are no quasiparticles at all?



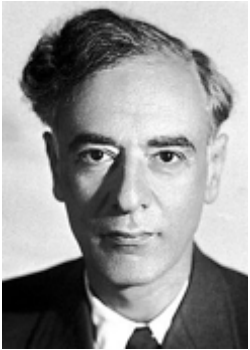
# Convergence of ideas and experiments

- Experiments on
  - non-Fermi liquid metals
  - ultra-clean semi-metals
  - thermal conductivity in quantum magnets
  - electron spin resonance of spin liquids
- Theoretical approaches
  - Gauge/gravity duality
  - SYK model and related large N theories
  - Quantum hydrodynamics
  - Field theory

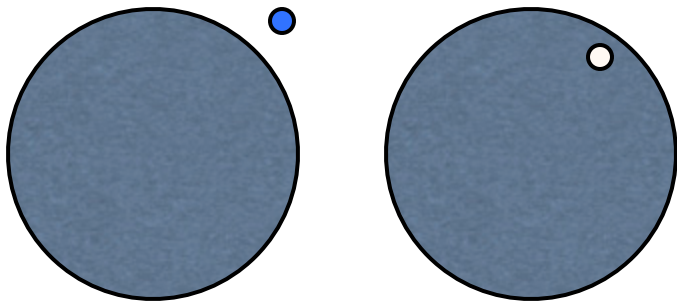
# This Talk

- Strongly correlated metals from SYK models - a route to calculable non-quasiparticle transport

# Fermi Liquid Theory



Landau provided justification for quasiparticle picture in metals when  $T \ll E_F$



Low energy excitations act like electrons and holes but with wavefunction dressing ( $Z < 1$ ), effective mass, and Landau interactions

$$E = \sum_k \epsilon_k \delta n_k + \frac{1}{2V} \sum_{k,k'} U_{k,k'} \delta n_k \delta n_{k'}$$

scattering is weak because not so many low energy qp states to scatter to

# Heavy Fermi Liquids

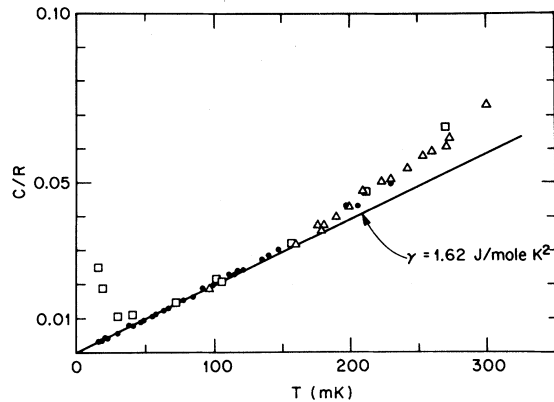


FIG. 1. Specific heat of  $\text{CeAl}_3$  at very low temperatures in zero field ( $\bullet, \Delta$ ) and in 10 kOe ( $\square$ ).

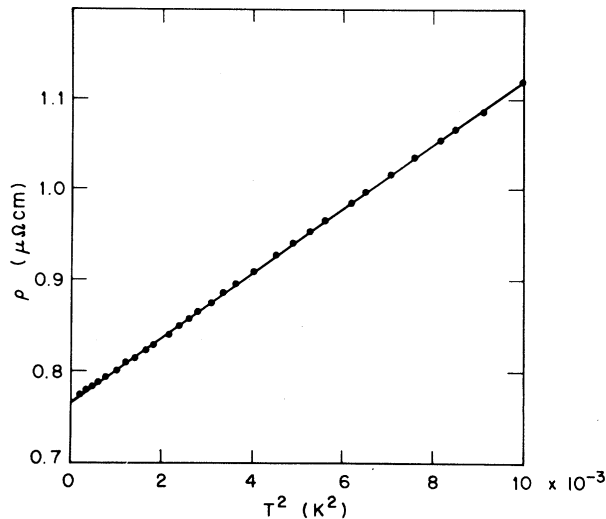


FIG. 3. Electrical resistivity of  $\text{CeAl}_3$  below 100 mK, plotted against  $T^2$ .

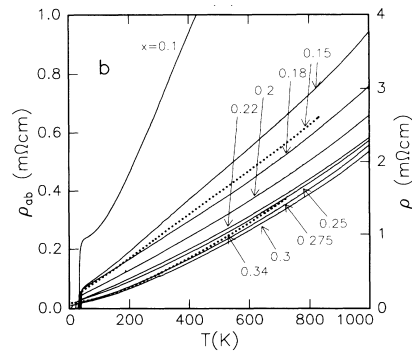
$$C \sim \gamma T$$

$$\rho(T) - \rho(0) \sim AT^2$$

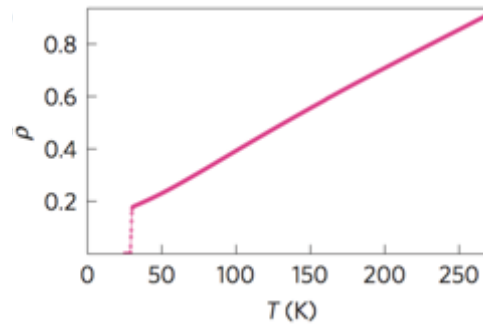
Both  $\gamma$  and  $A$  **huge**

Behave like Fermi liquid with tiny  $E_F$  and large electron mass, but only for  $T \ll E_F$

# Non-Fermi Liquids



LSCO Takagi et al, 1992



BaFe<sub>2</sub>(As<sub>1-x</sub>P<sub>x</sub>)<sub>2</sub>, Hayes et al, 2016

$$\frac{1}{\tau} \sim T ?$$

## T-linear resistivity:

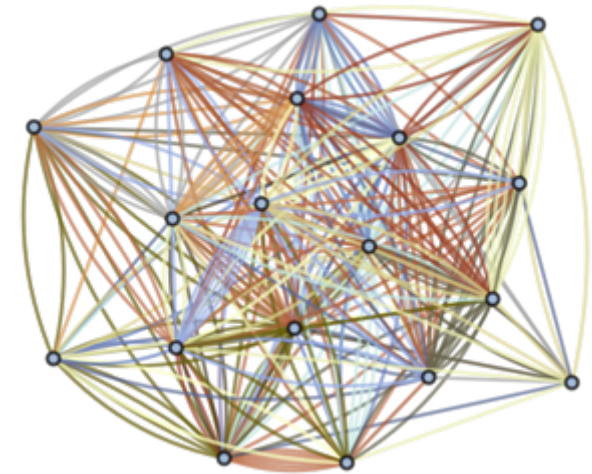
- Many materials
- Often nearby to unconventional superconductivity
- Symptom of a different type of metal?
- If no quasiparticles exist, what is the starting point?

# Sachdev-Ye-Kitaev model

A toy *exactly soluble* model  
of a non-Fermi liquid

$$H = \sum_{i < j, k < l} U_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3}$$



Like a strongly interacting quantum dot  
or atom with complicated Kanamori  
interactions between many "orbitals"



# SYK Model

Sachdev-Ye, 1993: Model has a soluble large-N limit

$$\Sigma = \text{[Diagram: a loop with a horizontal line through the center, arrows pointing clockwise]} + O(1/N)$$

In equations: very similar to DMFT:

$$G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)}$$

$$\Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau)$$

Solution:

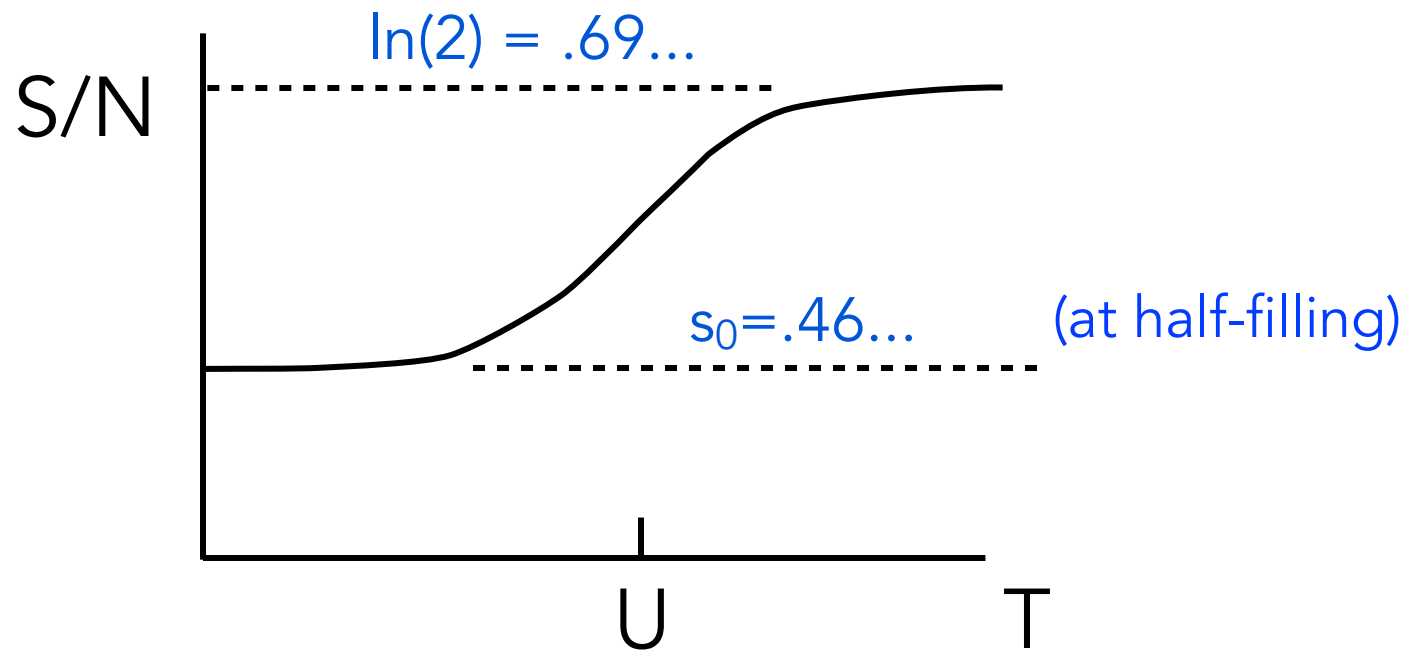
$$G(i\omega) \sim 1/\sqrt{\omega}$$

not a pole: non-Fermi liquid

# SYK Model

Why not quasiparticles?

Georges, Parcollet, Sachdev, 2001: **ground state entropy!**



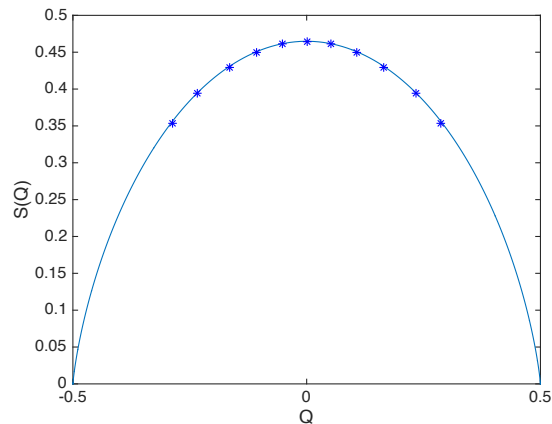
Many states available for scattering

"level spacing"  $\sim U \exp(-Ns_0)$

# Density dependence

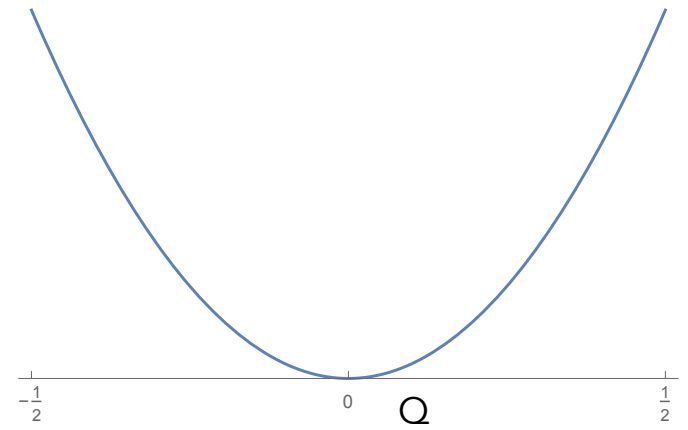
$$H \rightarrow H - \mu \mathcal{N} \quad \mathcal{N} = \sum_i c_i^\dagger c_i \quad Q = \frac{\mathcal{N}}{N} - \frac{1}{2}$$

Entropy



Davison et al, arXiv:1612.00849

Energy



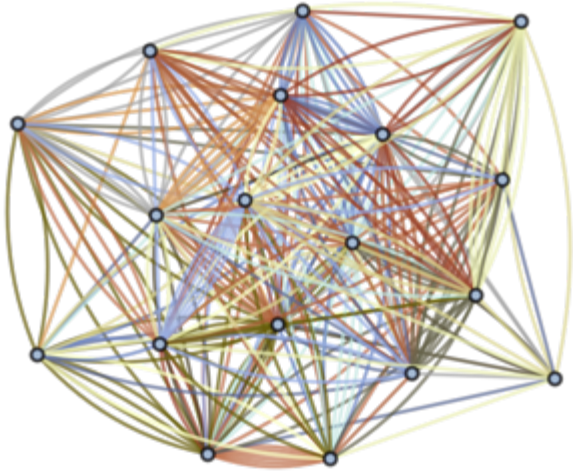
schematic

- Compressibility is constant at  $T=0$

$$K = \left. \frac{\partial Q}{\partial \mu} \right|_{\mu=0} = \frac{1.04}{U}$$

curvature =  $1/K$

# SYK Summary



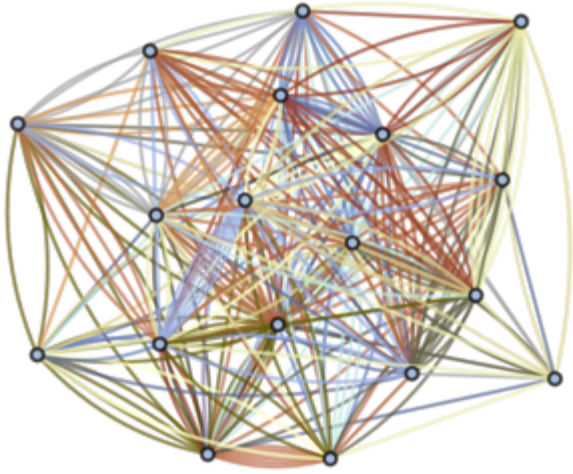
- Compressible
- Ground state entropy
- Non-Fermi liquid

$$K(T = 0) = \frac{1.04}{U}$$

$$S(T = 0)/N = .46 \dots$$

$$G(i\omega) \sim 1/\sqrt{\omega}$$

# SYK Summary



- Compressible
- Ground state entropy
- Non-Fermi liquid

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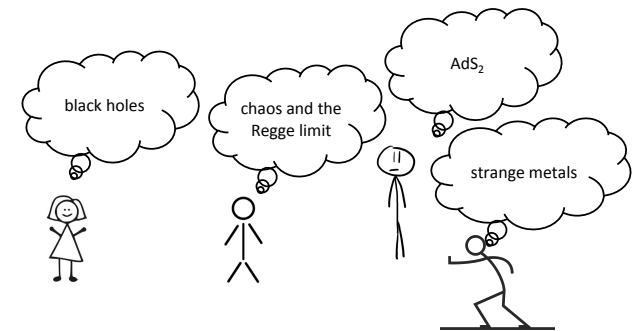
*circa*  
2015

Chaos

$$\langle [\mathcal{O}(0), \tilde{\mathcal{O}}^\dagger(t)]^2 \rangle \sim \frac{1}{N} e^{\lambda_L t}$$

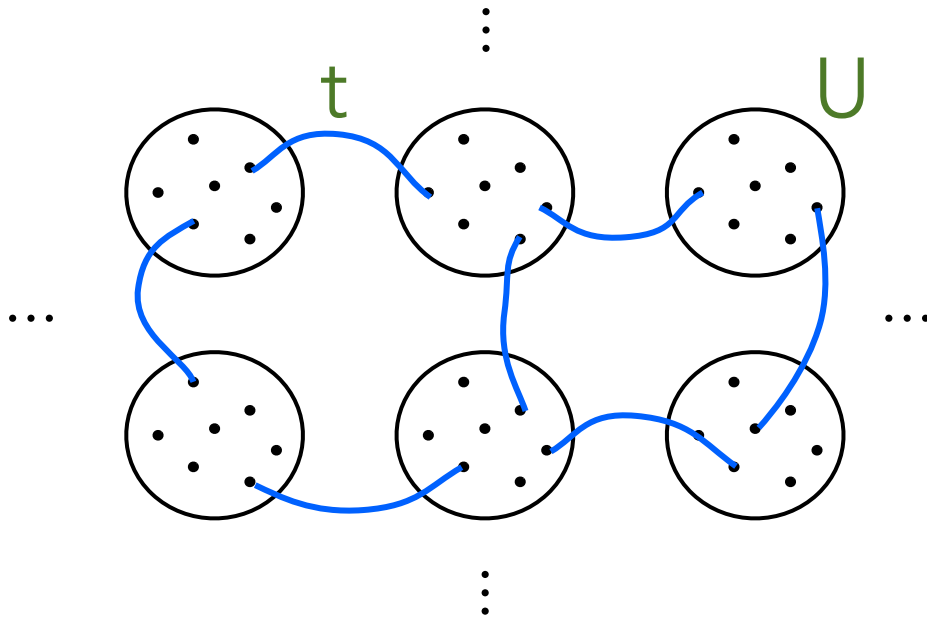
$$\lambda_L = \frac{2\pi k_B T}{\hbar}$$

Holography



slide from D. Stanford, IAS, 2017

# Building a metal

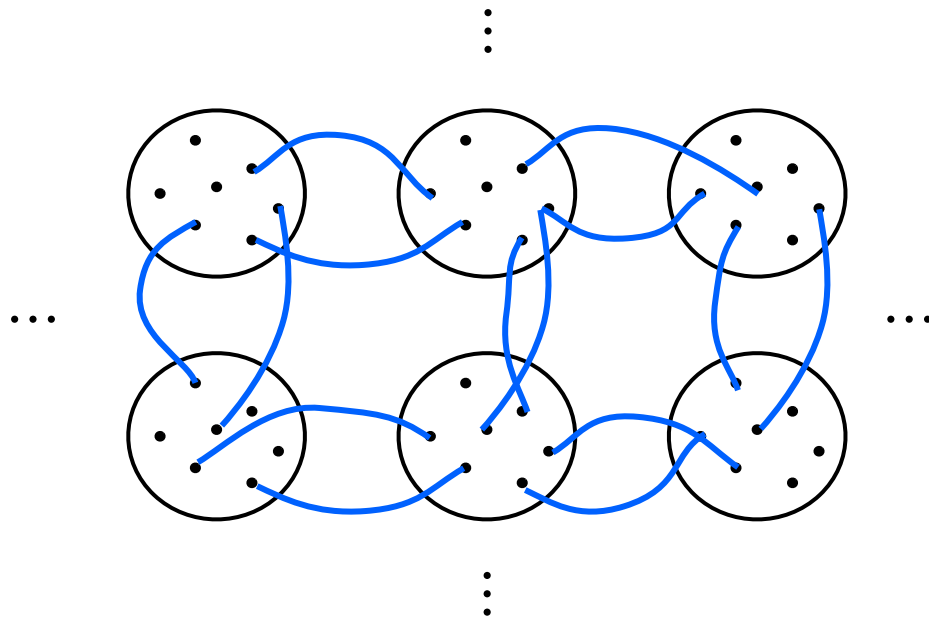


$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$

$$\overline{|t_{ij,xx'}|^2} = t_0^2 / N.$$



# Building a metal



Other work: 2-electron hopping

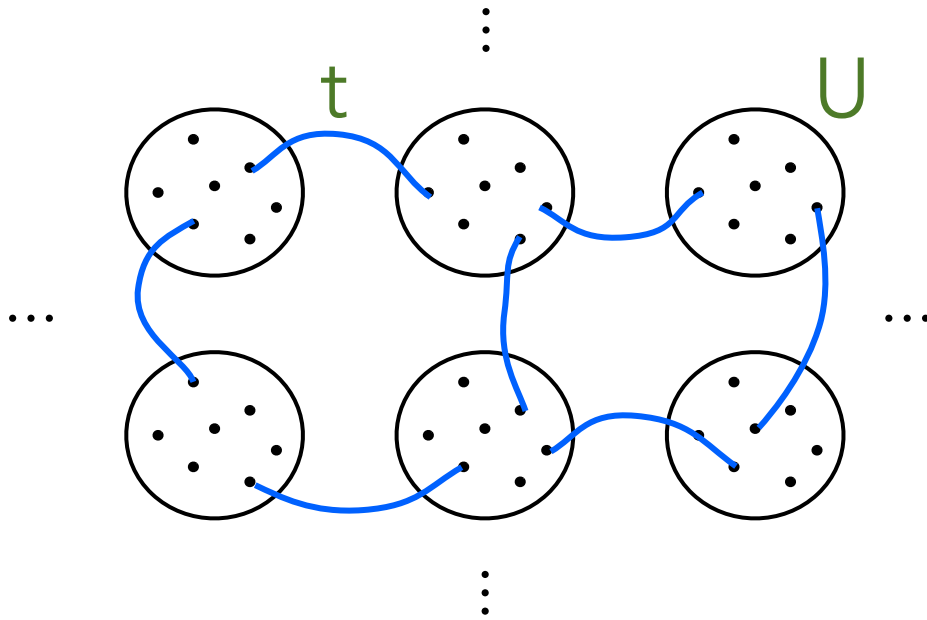
Y. Gu et al, arXiv:1609.07832

R. Davison et al, arXiv:1612.00849

$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ijkl,xx'} c_{i,x}^\dagger c_{j,x}^\dagger c_{k,x'} c_{l,x'} + \text{h.c.}$$

Omitting *relevant* 1-electron hopping leaves system NFL even at T=0

# Building a metal



$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$

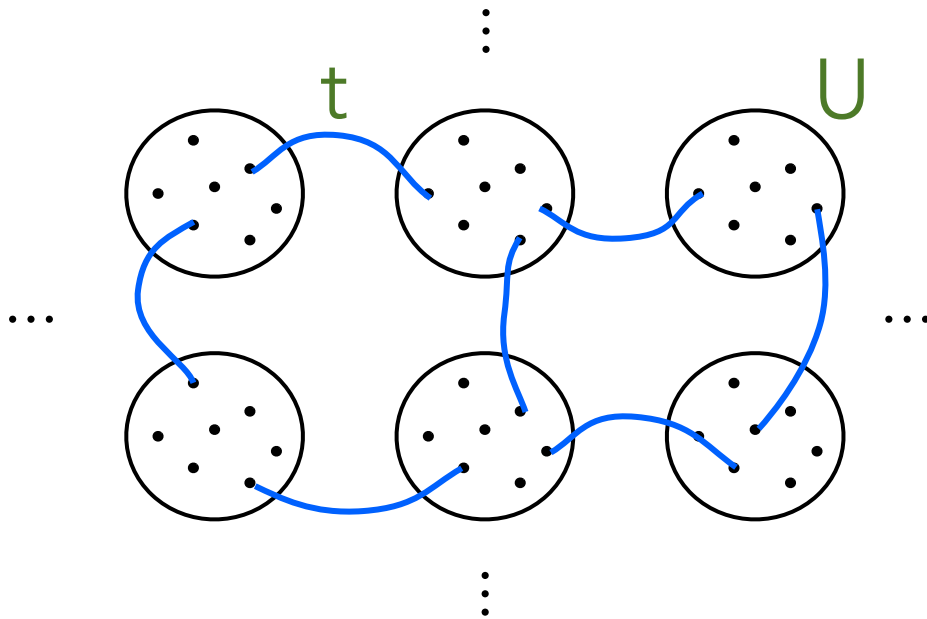


competition!

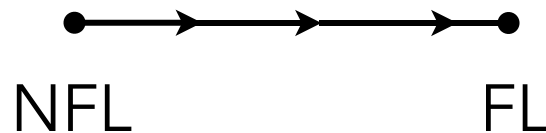
$t/U \ll 1$  interesting



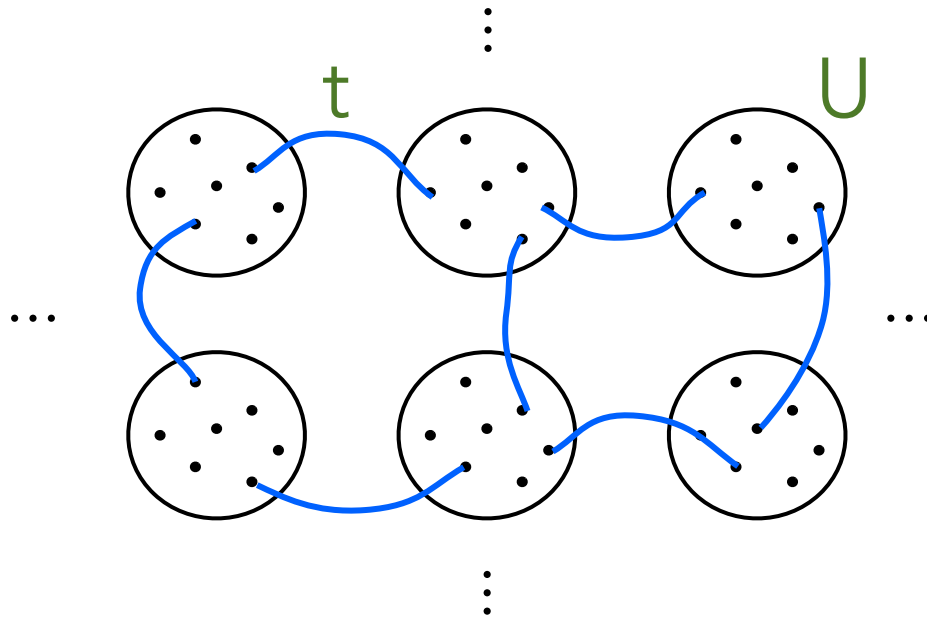
# Building a metal



$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$



# Self-consistent equations



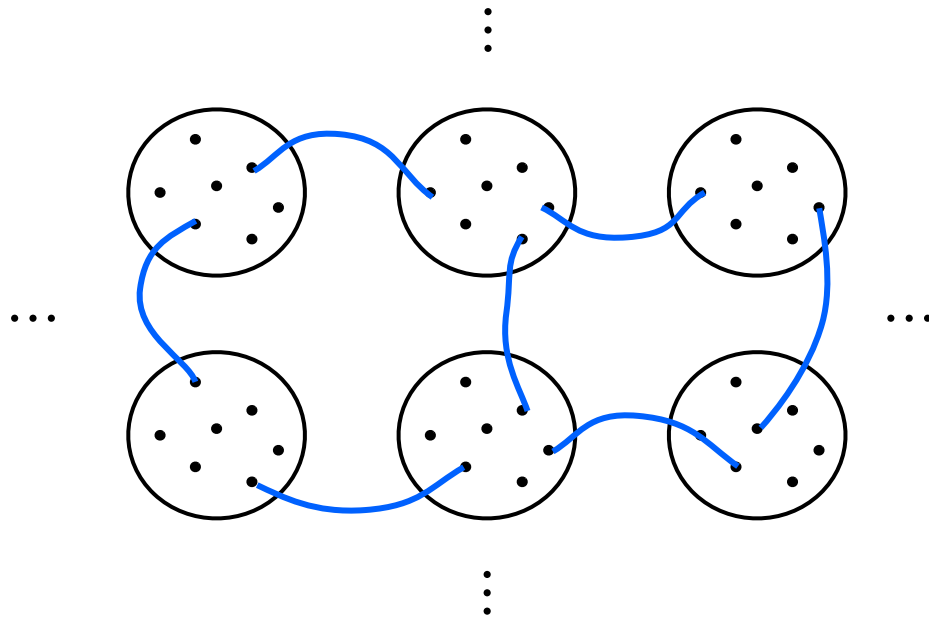
$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$
$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$



strong similarities to DMFT equations

mathematical structure appeared in early study of doped t-J model with *double* large N and infinite dimension limits: O. Parcollet+A. Georges, 1999

# Coherence scale



$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

Rescaling

$$\bar{\omega} = \frac{\omega}{\tilde{E}_c},$$

$$\bar{\tau} = \tau \tilde{E}_c,$$

$$\bar{G}(i\bar{\omega}) = \tilde{t} G(i\omega)$$

$$\bar{\Sigma}(i\bar{\omega}) = \Sigma(i\omega) / \tilde{t}$$

$$\tilde{t} = \left(\frac{z}{2}\right)^{\frac{1}{2}} t$$

$$\bar{G}(i\bar{\omega}) = \frac{\tilde{t}}{U} i\bar{\omega} - \bar{\Sigma}(i\bar{\omega})$$

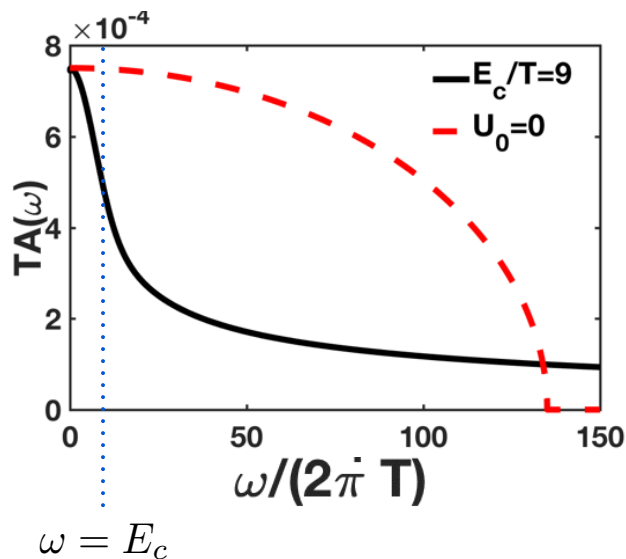
$$\bar{\Sigma}(\bar{\tau}) = -\bar{G}(\bar{\tau})^2 \bar{G}(-\bar{\tau}) + 2\bar{G}(\bar{\tau}),$$

For  $t \ll U$ , a single universal coherence scale appears

$$\tilde{E}_c = \frac{\tilde{t}^2}{U}$$

# Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.



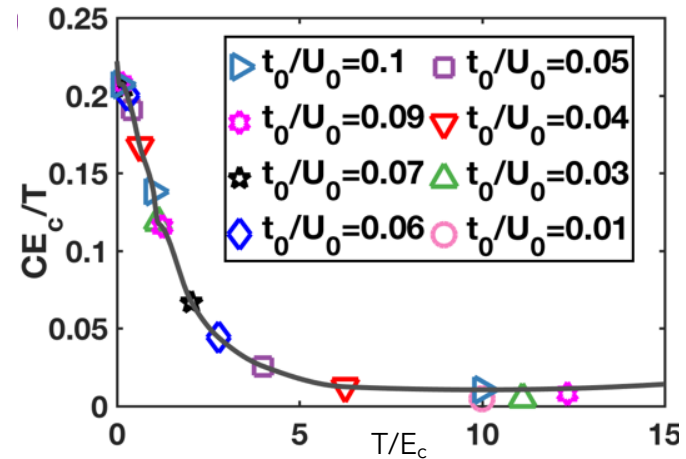
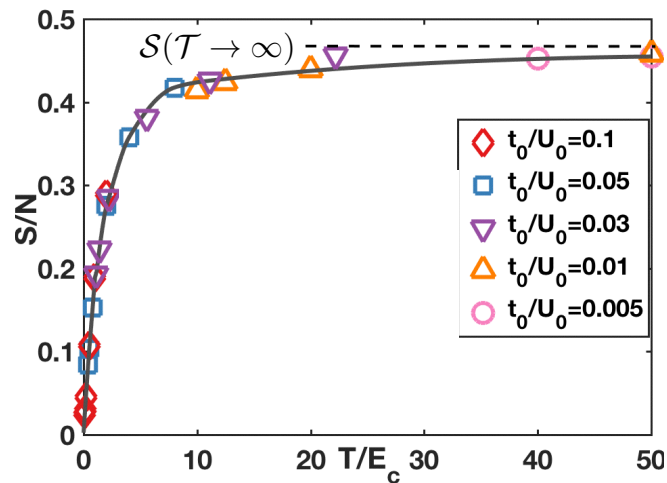
Narrow “coherence peak” appears in spectral function: heavy quasiparticles form for  $T \ll E_c$

Quasiparticle weight  $Z \sim t/U$



# Entropy

Level repulsion: entropy is released for  $T < E_c$ !



Universal scaling forms

$$S/N = \mathcal{S}(T/E_c)$$

$$C/N = T/E_c \mathcal{S}'(T/E_c)$$

$$\gamma \equiv \lim_{T \rightarrow 0} \frac{C}{T} = \frac{\mathcal{S}'(0)}{E_c}$$

Sommerfeld  
enhancement

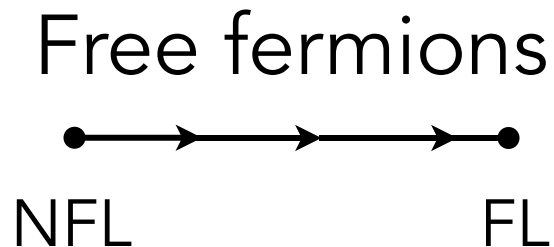
$$m^*/m \sim U/t$$

# Compressibility

For  $t \ll U$ , compressibility is almost unaffected by hopping

$$K = \left. \frac{\partial Q}{\partial \mu} \right|_{\mu=0} = \frac{1.04}{U} \ll \gamma \sim \frac{U}{t^2}$$

??How to reconcile with Sommerfeld enhancement??



??

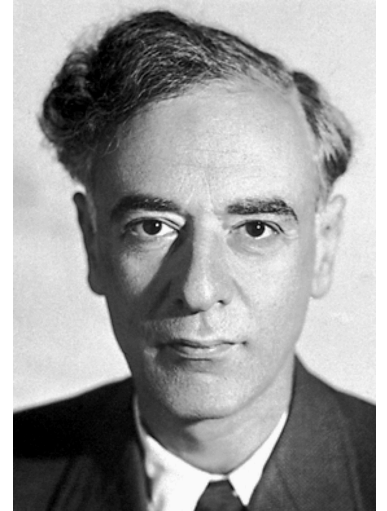
$$K = g(\epsilon_F)$$

$$\gamma = \frac{\pi^2}{3} g(\epsilon_F)$$

# Landau Fermi Liquid

- Landau Fermi Liquid Theory

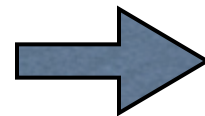
$$\delta E = \sum_a \epsilon_a \delta n_a + \frac{1}{2} \sum_{a,b} f_{ab} \delta n_a \delta n_b$$



- Compressibility is renormalized by Fermi liquid parameter  $F = g(E_F) f$

$$\gamma/K = \frac{\pi^2}{3} (1 + F)$$

$$\gamma/K \sim (U/t)^2$$

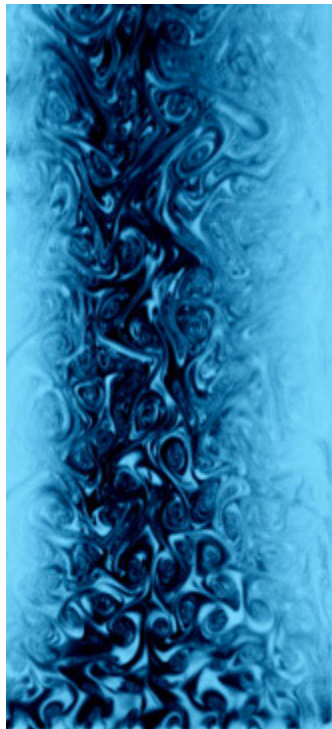


$$F \sim \left( \frac{U}{t} \right)^2 \gg 1$$

# Transport

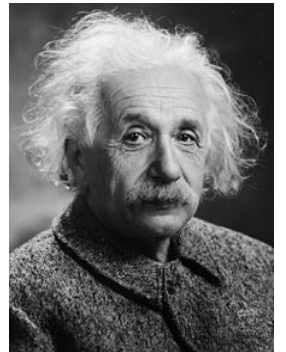
Quasiparticle picture applies only for  $T \ll E_c$

More generally, we use hydrodynamics



Einstein-Smoluchowski relation

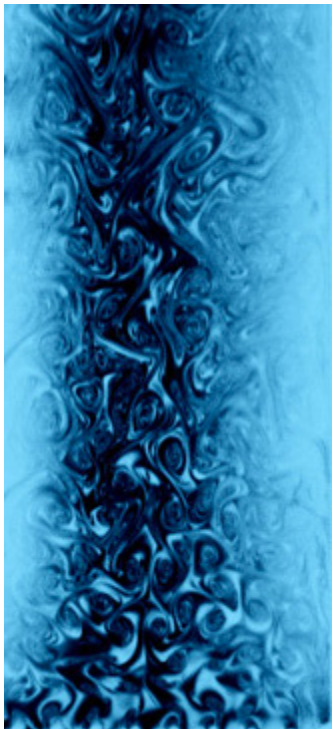
$$\sigma = e^2 \frac{\partial n}{\partial \mu} D$$



# Transport

Quasiparticle picture applies only for  $T \ll E_c$

More generally, we use hydrodynamics



$$\sigma = \lim_{\omega \rightarrow 0} \lim_{p \rightarrow 0} \frac{-i\omega}{p^2} D_{Rn}(p, \omega)$$

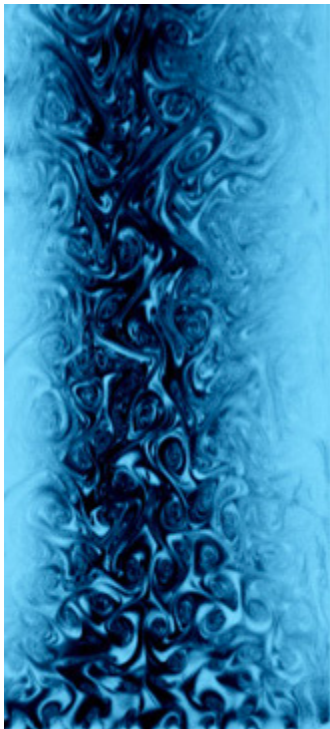
- ◆ Calculate density response using Keldysh method.
- ◆ Do analogously for thermal conductivity
- ◆ Very convenient collective field formulation - fully non-perturbative calculations possible

N.B. Because of randomness, momentum is not a hydrodynamic variable

# Transport

Quasiparticle picture applies only for  $T \ll E_c$

More generally, we use hydrodynamics

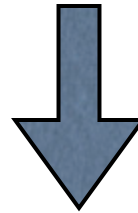


Effective action:

$$G_{x,ss'}(t, t') \rightarrow G_{x,ss'}(t - t') e^{-i(\varphi_s(x,t) - \varphi_{s'}(x,t'))}$$

$$\Sigma_{x,ss'}(t, t') \rightarrow \Sigma_{x,ss'}(t - t') e^{-i(\varphi_s(x,t) - \varphi_{s'}(x,t'))},$$

known from SYK solution



$$iS_\varphi = -2K \sum_p \int_{-\infty}^{+\infty} d\omega \varphi_{c,\omega} (i\omega^2 - D_\varphi p^2 \omega) \varphi_{q,-\omega}.$$



# Transport

Generalized resistivity

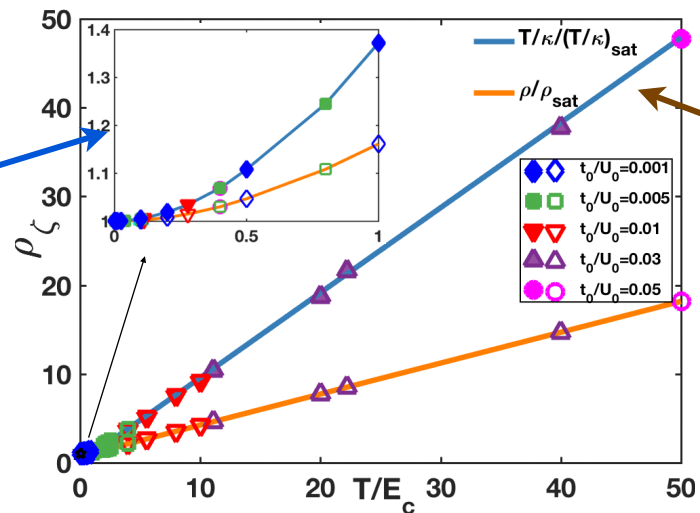
$$\rho_c = 1/\sigma$$

$$\rho_e = T/\kappa$$

scaling

$$\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta\left(\frac{T}{E_c}\right)$$

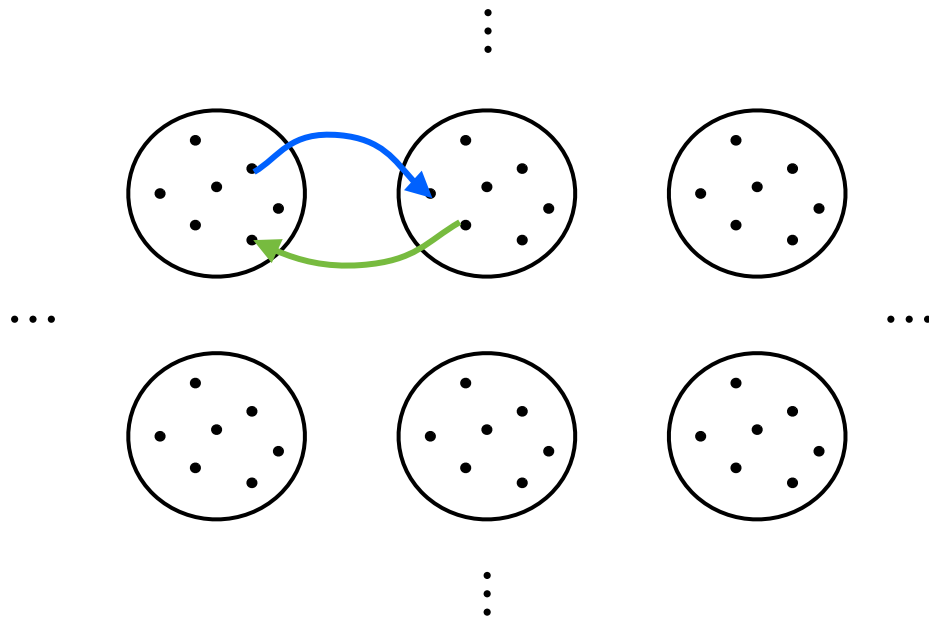
Fermi liquid  
 $R = R_0 + AT^2$   
 for  $T \ll E_c$



Linear in T for  
 $E_c \ll T \ll U$

Crossover from heavy FL to strange metal

# Incoherent tunneling



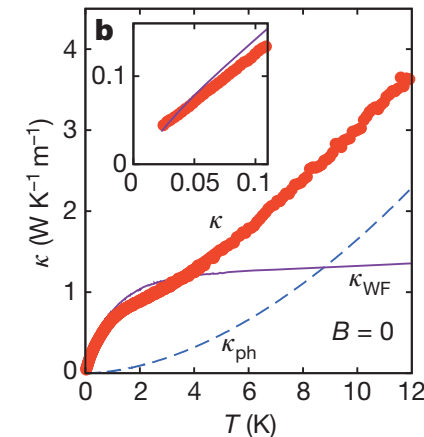
$$J_{x,x'} \sim it \left( c_x^\dagger c_{x'} - \text{h.c.} \right)$$

Kubo 
$$\sigma = \frac{\langle JJ \rangle}{\omega}$$

$$\begin{aligned} \sigma &\sim \frac{1}{\omega} \int d\tau t^2 \left\langle c_x^\dagger c_{x'} c_{x'}^\dagger c_x \right\rangle \sim \frac{t^2}{E^2} G(\tau)^2 \sim \frac{t^2}{E^2} \left( \frac{1}{\sqrt{U\tau}} \right)^2 \\ &\sim \frac{t^2}{U} \frac{1}{k_B T} \end{aligned}$$

# Wiedemann-Franz ratio

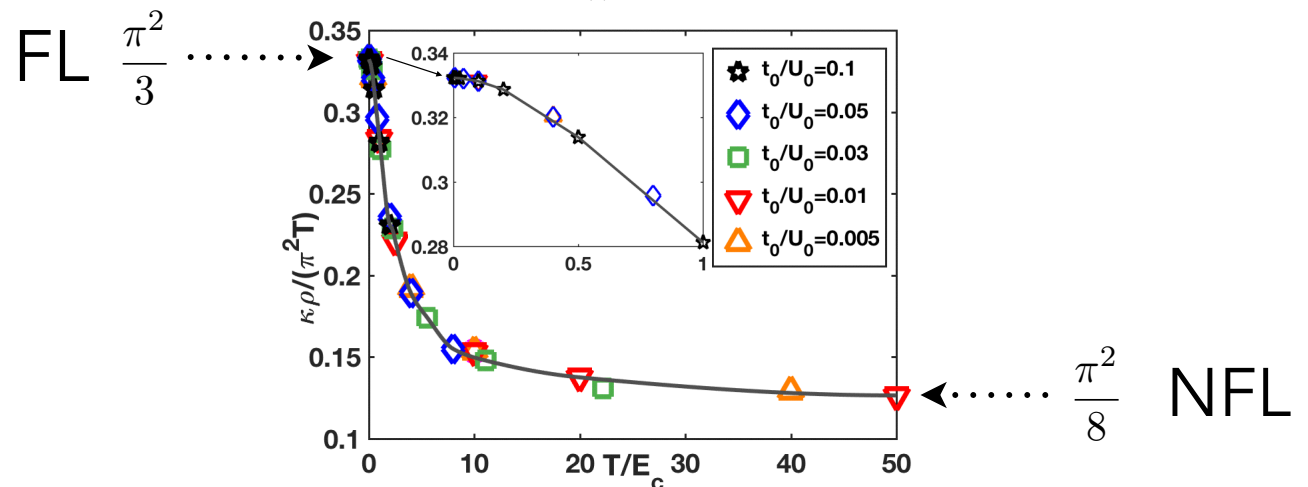
Lorenz  $L = \frac{\kappa}{\sigma T}$   
 $= \pi^2/3$  for a Fermi liquid



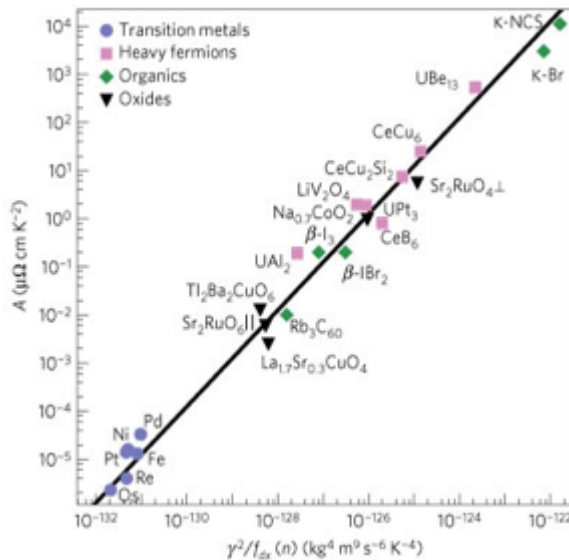
YbRh<sub>2</sub>Si<sub>2</sub>, Pfau et al (2012)

SYK lattice:

$$L = \mathcal{L}(T/E_c)$$



# Kadowaki Woods ratio



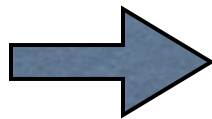
$$\rho_{\zeta}(T \ll E_c) \approx \rho_{\zeta}(0) + A_{\zeta}T^2,$$

$$KW = A/\gamma^2 \quad \text{approximately constant for many metals}$$

Scaling implies:

$$A \sim 1/(NE_c^2)$$

recall  $\gamma \sim 1/E_c$



$$KW = A/(N\gamma)^2 \sim 1/N^3$$

independent of  $t, U!$

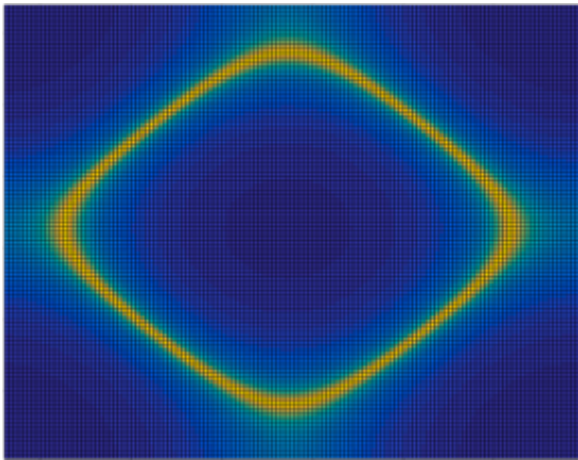


# SYK metal

- Small coherence scale  $E_c = t^2/U$
- Heavy mass  $\gamma \sim m^*/m \sim U/t$
- Small QP weight  $Z \sim t/U$
- Kadowaki-Woods  $A/\gamma^2 = \text{constant}$
- Linear in T resistivity and  $T/\kappa$
- Lorenz ratio crosses over from FL to NFL value

# SYK Fermi Surfaces?

- Extension to translationally invariant systems?



Fermi surface emerging in translationally invariant SYK model by Aavishkar Patel

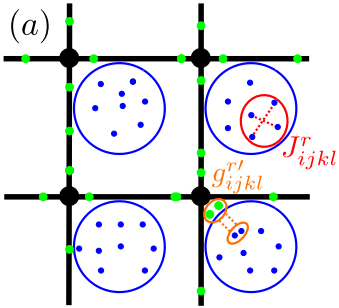
$$\begin{aligned} G(\mathbf{k}, i\omega_n)^{-1} &= i\omega_n - \epsilon(\mathbf{k}) - \Sigma(\mathbf{k}, i\omega_n), \\ \Sigma(\mathbf{x}, \tau) &= -U_0^2 G(\mathbf{x}, \tau)^2 G(-\mathbf{x}, -\tau), \end{aligned}$$

- SYK lattice, tensor models,...
- Momentum space differentiation and realistic applications?
- Relation to methods like DCA, cluster DMFT?

# SYK Fermi Surfaces?

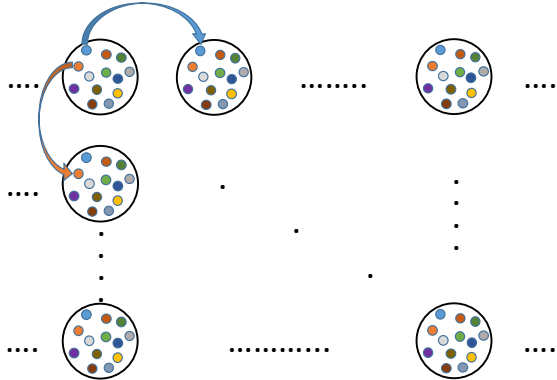
Patel, McGreevy, Arovas, Sachdev, arXiv:1712.05026

SYK dots as strong local scatterers



$$H = -t \sum_{\langle rr' \rangle; i=1}^M (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r; i=1}^M c_{ri}^\dagger c_{ri} - \mu \sum_{r; i=1}^N f_{ri}^\dagger f_{ri} \\ + \frac{1}{NM^{1/2}} \sum_{r; i,j=1}^N \sum_{k,l=1}^M g_{ijkl}^r f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r; i,j,k,l=1}^N J_{ijkl}^r f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$

Chowdhury, Werman, Berg, Senthil, arXiv:1801.06178

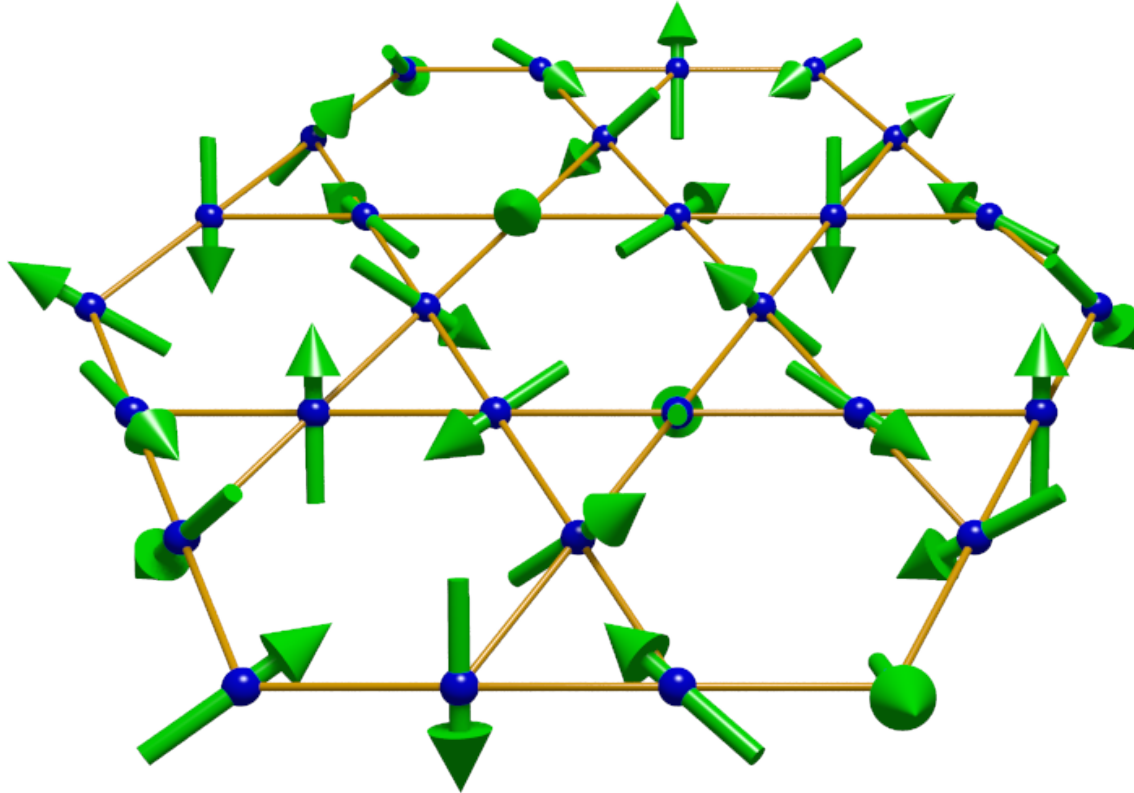


SYK models w/ full translational symmetry

$$H_c = \sum_{r,r'} \sum_{\ell} (-t_{r,r'}^c - \mu_c \delta_{rr'}) c_{r\ell}^\dagger c_{r'\ell} + \frac{1}{(2N)^{3/2}} \sum_r \sum_{ijkl} U_{ijkl}^c c_{ri}^\dagger c_{rj}^\dagger c_{rk} c_{rl},$$

$$H = -t \sum_{\langle rr' \rangle; i=1}^M (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r; i=1}^M c_{ri}^\dagger c_{ri} - \mu \sum_{r; i=1}^N f_{ri}^\dagger f_{ri} \\ + \frac{1}{NM^{1/2}} \sum_{r; i,j=1}^N \sum_{k,l=1}^M g_{ijkl}^r f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r; i,j,k,l=1}^N J_{ijkl}^r f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$

# Quantum magnets

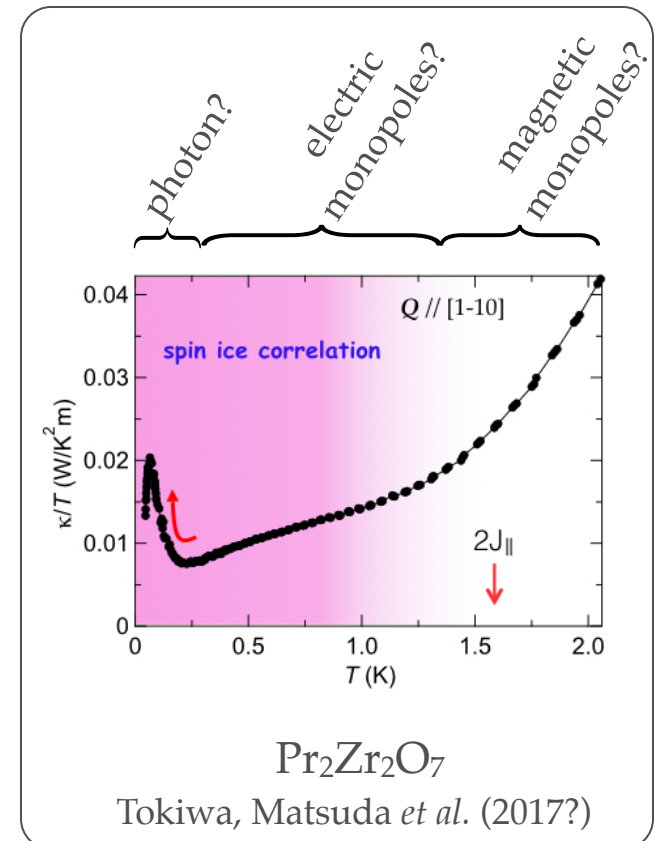
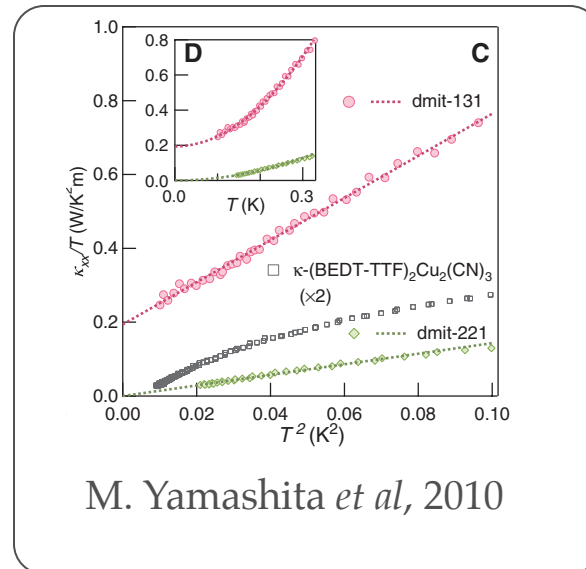
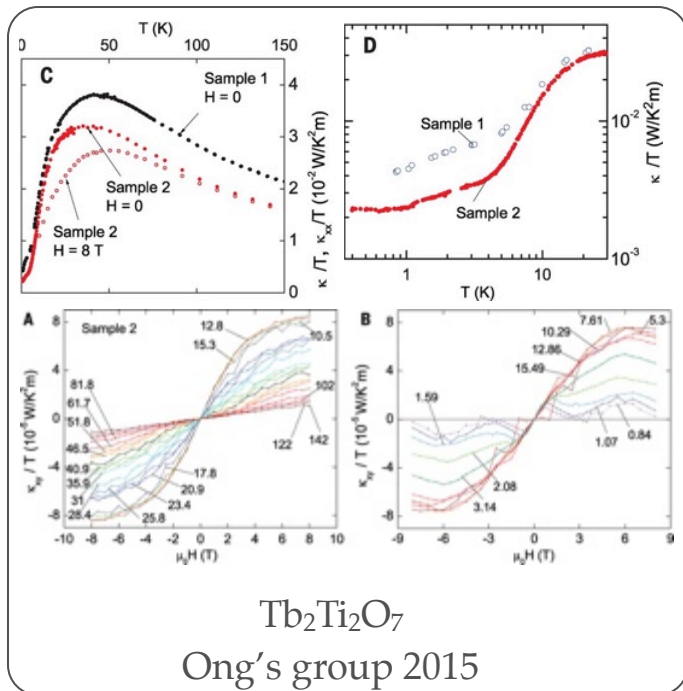


Less obvious ultra-quantum "fluids"



# Heat transport in magnets

A growing experimental effort valuable for its sensitivity, long-wavelength nature, and automatic removal of localized states



# SYK magnets?

Original SY paper actually treated spins

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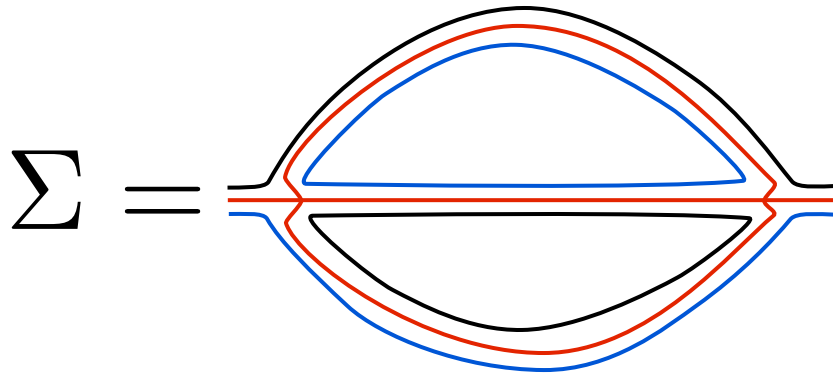
Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

Subir Sachdev and Jinwu Ye

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$

Challenge: extend to non-disordered systems

- Similar to problem of SYK Fermi surfaces
- Tensor model for large N control?

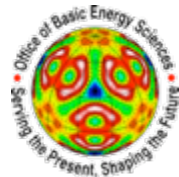


$$S_{i;abcd} = f_{i;Aab}^\dagger f_{j;Acd}$$

$$H = \frac{J}{N^{3/2}} \sum_{\langle ij \rangle} S_{i;abcd} S_{j;adcb}$$

# Conclusion

- Convergence of ideas suggests that maybe problematic metals are not “strange”, “bad”, or “incoherent” but rather they are **SYK**



GORDON AND BETTY  
**MOORE**  
FOUNDATION