Antiferromagnetic quantum critical metals in d=2, 3 and in-between

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Relevant References

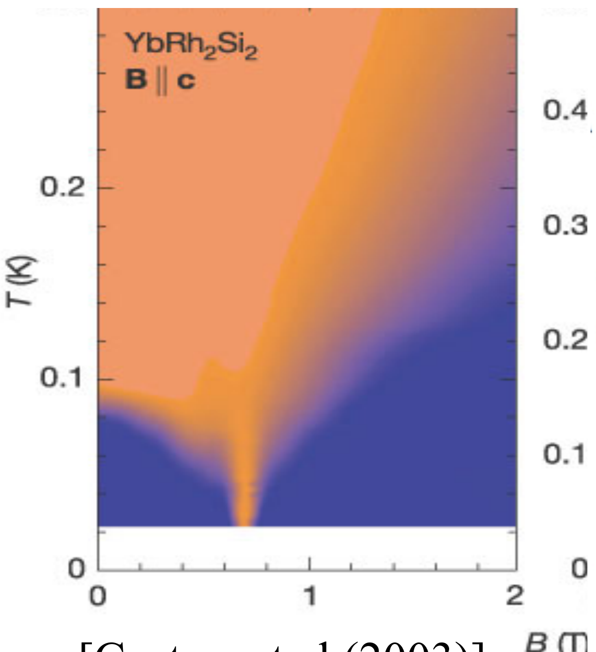
- One-loop (valid only in d=3)
  - Sur, SL, Phys. Rev. B 91, 125136 (2015)
- d=3-ε
  - Lunts, Andres, SL, Phys. Rev. B 95, 245109 (2017)
- d=2
  - Andres, Lunts, SL, Phys. Rev. X 7, 021010 (2017)
- $2<d<3$
  - Lunts, Andres, SL, **arXiv:1805.05252**

- Review paper
  - Annual Review of Condensed Matter Physics Vol. 9:227-244 (2018)

Quantum Critical Points

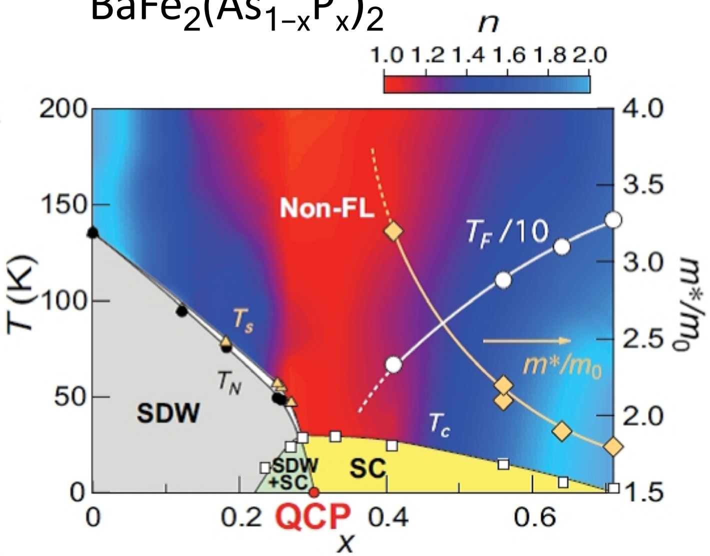
- Quantum critical points host quantum states which do not support well-defined single particle excitations
- Often, many-body eigenstates can not be written as direct product states in any known single-particle basis
  - strongly entangled both in real and momentum space

Quantum Critical Metals [heavy fermion; pnictides; cuprates]



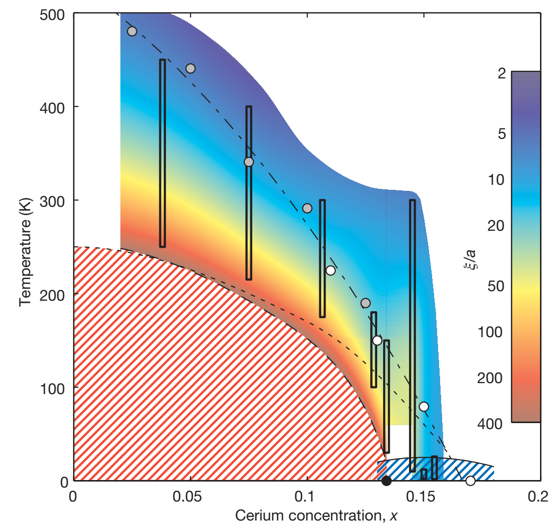
[Custers et al.(2003)]

BaFe$_2$(As$_{1-x}$P$_x$)$_2$



[Hashimoto et al. (2012)]

Nd$_{2-x}$Ce$_x$CuO$_4$



[Motoyama et al. (2012)]

- Non-Fermi liquids are characterized by anomalous exponents in spectroscopic / thermodynamic / transport properties
  - Dynamical susceptibility : $\chi \sim \omega^{-a}$
  - Specific heat : $C/T \sim T^{-b}$ or $\log(1/T)$, ..
  - Resistivity : $\rho \sim T^n$, $n < 2$

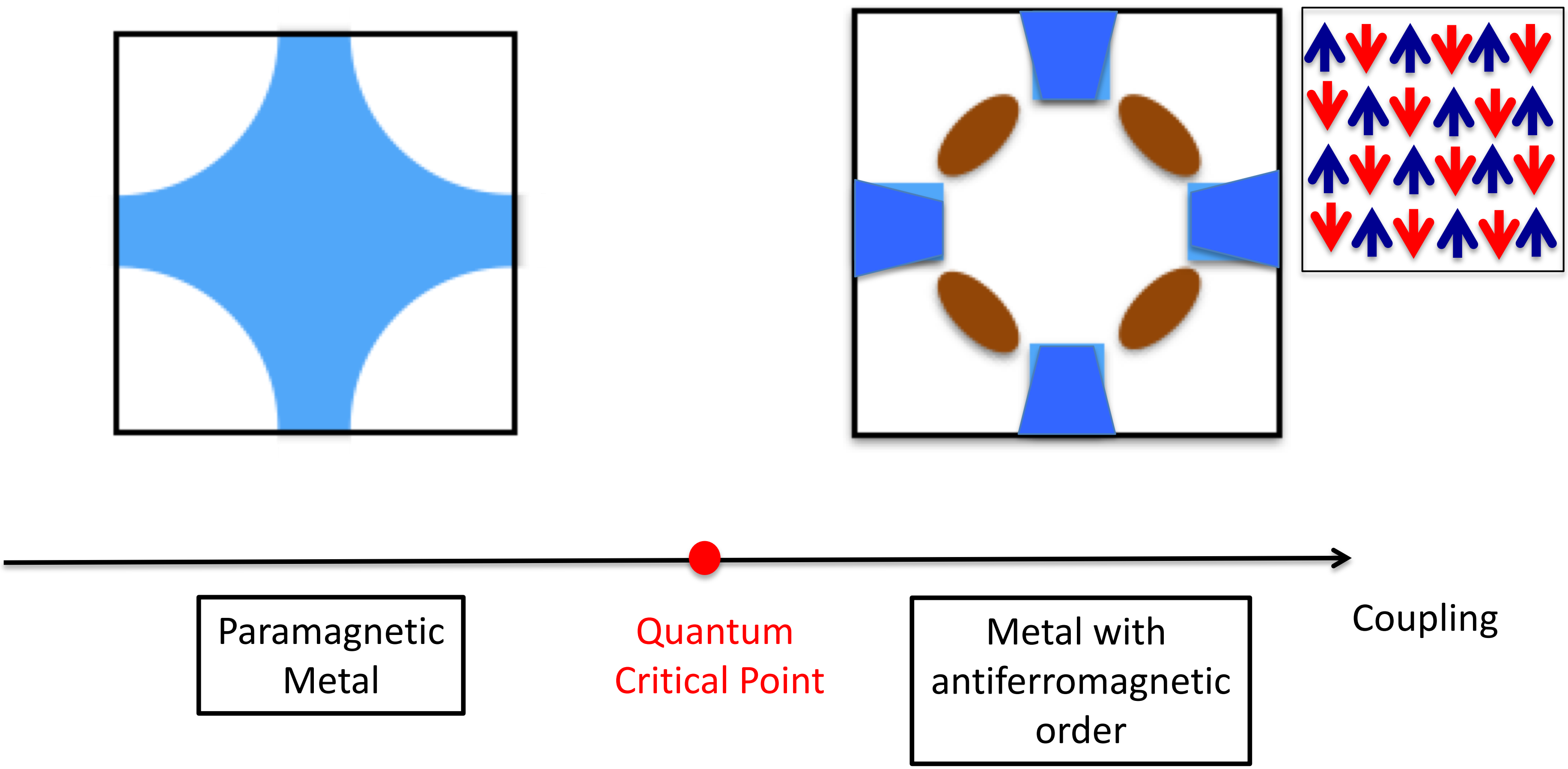
Theoretical framework

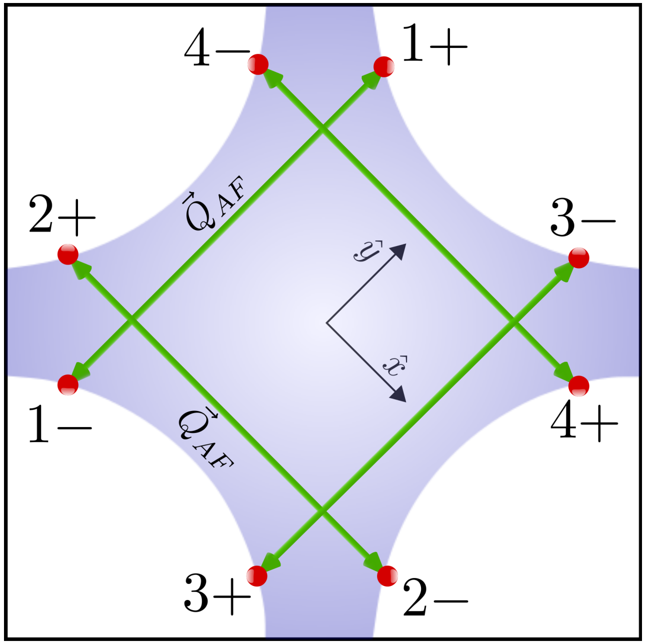
- Continuum field theories capture universal low-energy physics
- In d=3, quantum fluctuations are usually marginal, which give rise to logarithmic corrections
- In d=2, strong quantum fluctuations at long distance scales can lead to behaviors that are qualitatively different from non-interacting systems

From d=3 to d=2

- In general, it is hard to understand how weak coupling physics in d=3 evolves to non-perturbative behaviors in d=2
- Recently, the theory for the antiferromagnetic quantum critical metal has been solved both at d=3 and d=2 (exact critical exponents)
- This provides a rare opportunity to examine how physics evolves as a function of space dimension : purpose of this talk

Commensurate antiferromagnetic phase transition in metal





Minimal Theory

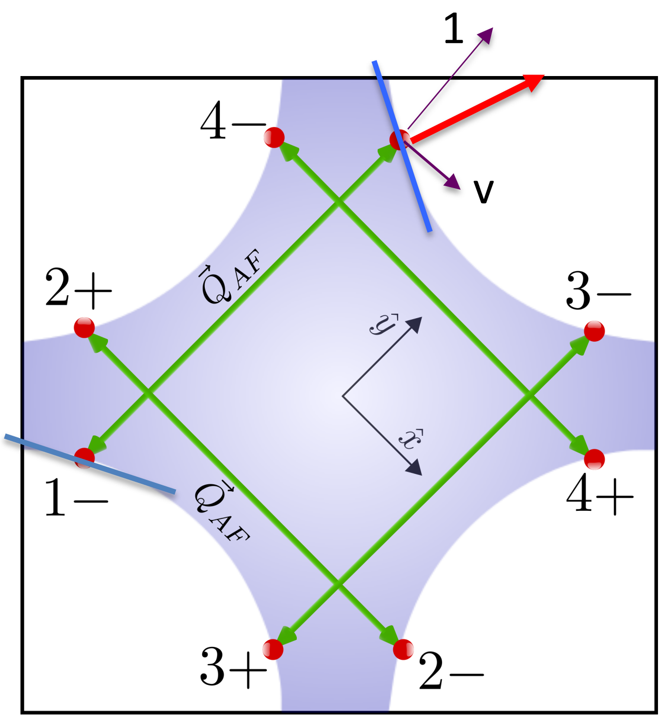
[Abanov, Chubukov, Schmalian; Metlitski, Sachdev; ..]

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = v k_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + v k_y$$

$$\begin{aligned}\mathcal{S} = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3}\, \psi_{l,\sigma}^{(m)*}(k) \left[i k_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\ & + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\ & + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]\end{aligned}$$

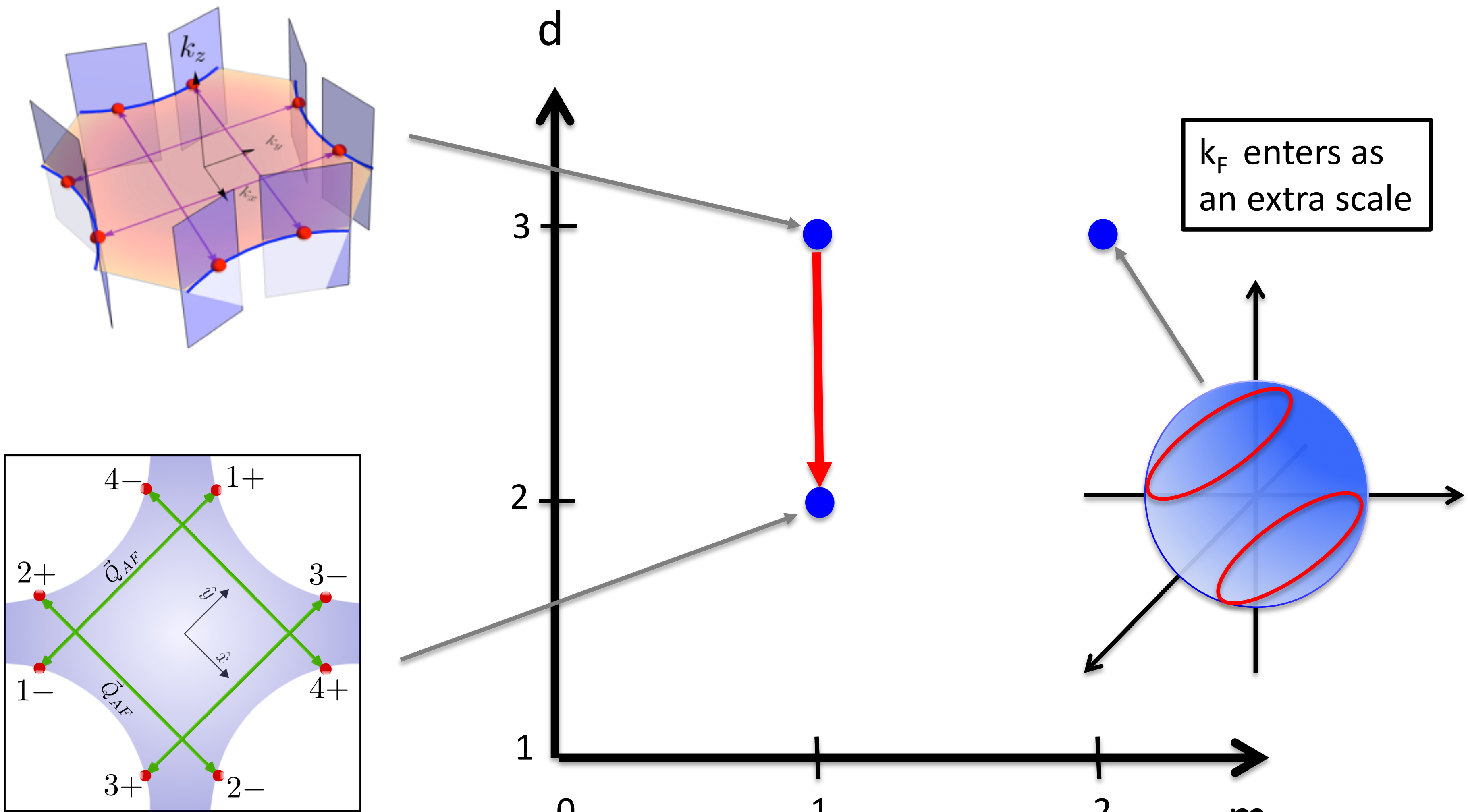
Parameters of the theory



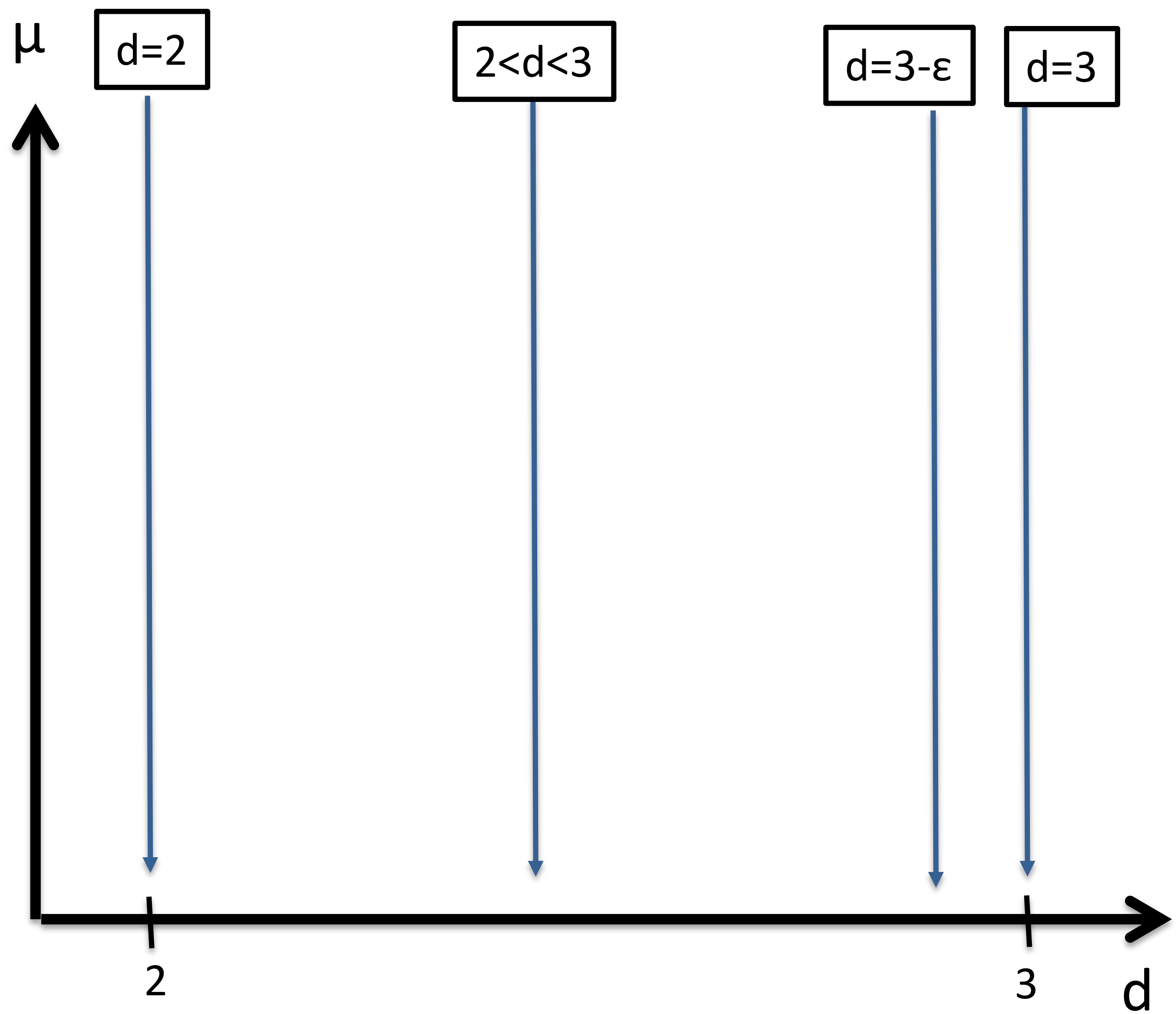
- Fermi velocity parallel to Q_{AF} set to be 1
- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : coupling between fermion and boson

- If v=0, hot spots connected by Q_{AF} are nested

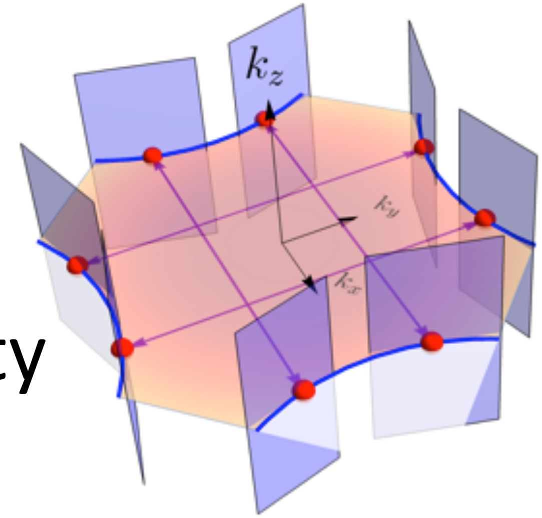
Theories in 3d



From d=3 to d=2

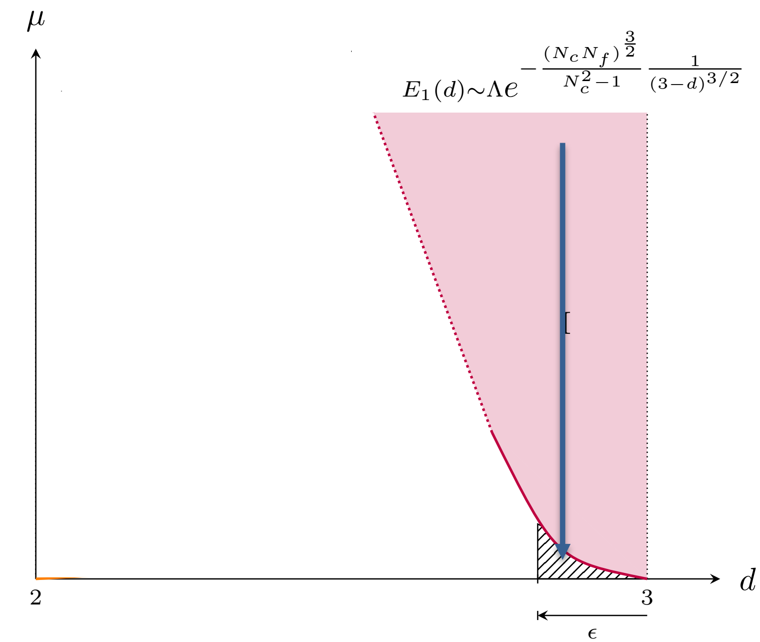


d=3



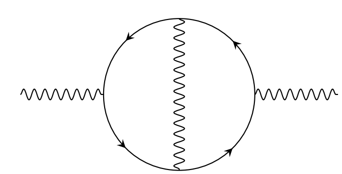
- Emergent nesting and quasi-locality
  - $v \sim c \sim \frac{1}{\log[\log(\frac{1}{\mu})]}$, $\frac{v}{c} \sim 1$
- Anomalous dimensions are controlled by $\frac{g^2}{v}$
  - $(z-1)$, $\eta \sim \frac{g^2}{v} \sim \frac{1}{\log(1/\mu)}$
- Logarithmic corrections in physical observables
  - $\chi''(\omega, Q_{AF}) \sim \frac{1}{\omega^2 \,[\log(\frac{1}{\omega})]^{8/3}}$
- One-loop is asymptotically exact

d=3-ε



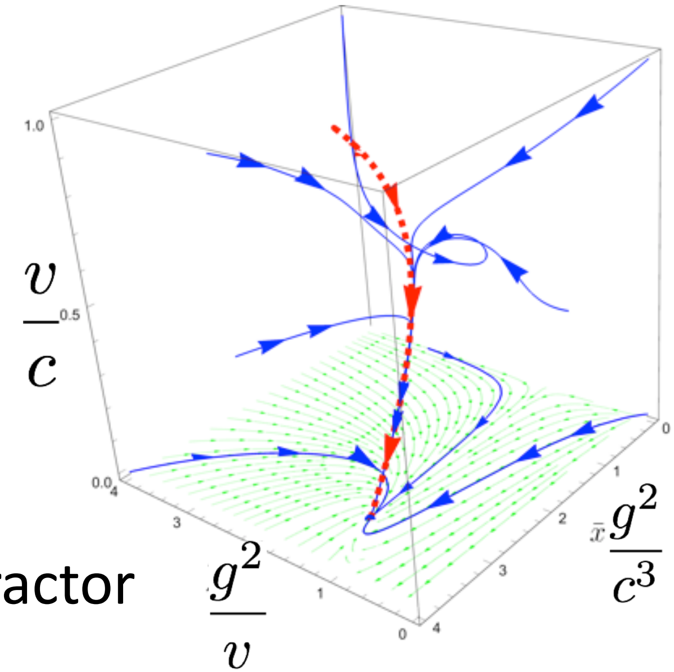
- Interaction becomes relevant
  - $\frac{g^2}{v} \sim \varepsilon, \quad v, c \ \rightarrow 0$

- Quasi-locality enhance quantum fluctuations
  - One-loop is not enough even to the leading order in ε: quantum fluctuations are not organized by number of loops
  - Higher-loop effect qualitatively changes the flow of velocities below a crossover energy scale : $\frac{v}{c} \rightarrow 0$



- $d \rightarrow 3$ and $\mu \rightarrow 0$ limits do not commute

d=3-ε

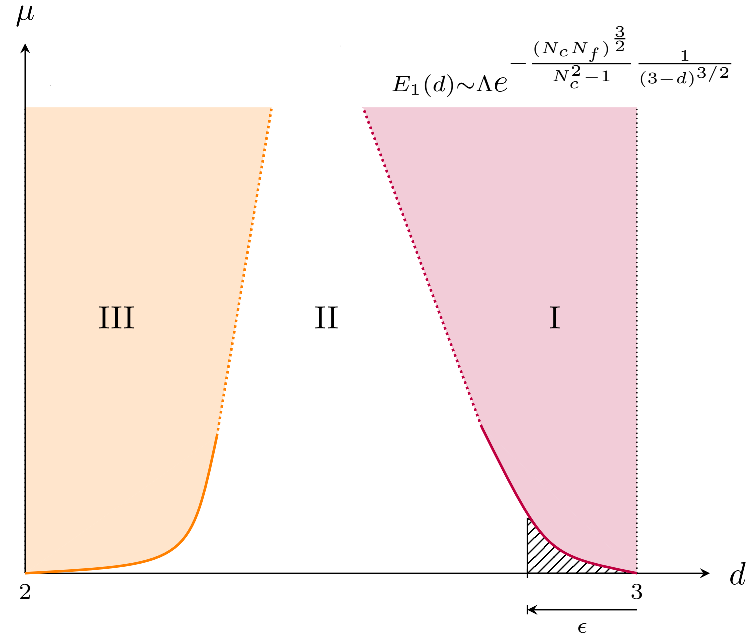


- Two stages of RG flow
  - power-law flow to 1d attractor
  - sub-logarithmic flow within the attractor
  - $\frac{v}{c} \sim \frac{1}{[log\frac{1}{\mu}]^{2/3}}$
- Anomalous dimension O(ε) + super-logarithmic corrections in physical observables
  - $\chi''(\omega, Q_{AF}) \sim \frac{1}{\omega^{2-\varepsilon}\, e^{(\log 1/\omega)^{1/3}}}$
- Electrons only weakly dressed : no anomalous dimension (only super-logarithmic corrections)

2<d<3

- c$\sim\left(\frac{v}{d-2}\right)^{1/d}$
- Even though $\varepsilon\sim 1, \frac{v}{c} \rightarrow 0$ provides a small parameter
  - Expansion in $\frac{v}{c}$ is a perturbative expansion around an interaction dominated fixed point
- Anomalous dimension for spin fluctuations : 3-d (exact)
  - $\chi''(\omega, Q_{AF}) \sim \frac{1}{\omega^{d-1}\, e^{(\log 1/\omega)^{1/d}}}$
- There is a finite basin of attraction near the stable quasi-local fixed point with v=0

d=2

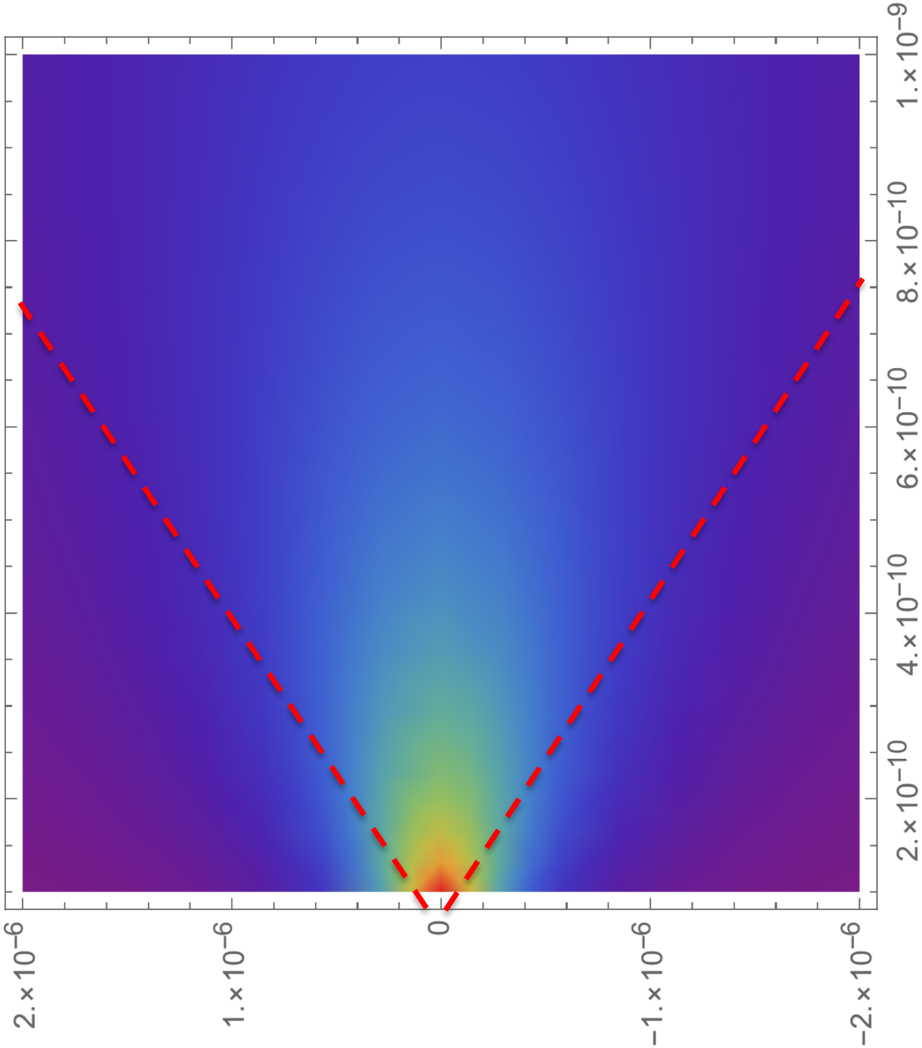


- Spin fluctuations become highly incoherent. As a result, spin fluctuations at large momenta have significant spectral weight at low energies

- This gives rise to an additional logarithmic corrections
  - $c \sim [v \log\left(\frac{1}{v}\right)]^{\frac{1}{2}}$
  - $\chi''(\omega, Q_{AF}) \sim \dfrac{1}{\omega \ \ e^{(\log 1/\omega)^{\frac{1}{2}} / \log\left[\log\left(\frac{1}{\omega}\right)\right]}}$

- $d \to 2$ and $\mu \to 0$ limits do not commute
  - The logarithmic correction is cut off at a crossover energy scale which vanishes at d=2

Dynamical Spin Susceptibility $\chi''(\omega, \vec{Q}_{AF} + \vec{q})$



ω

$$\omega \sim \sqrt{v}\,\Delta q$$

$|q_x| + |q_y|$

- Incoherent peak centered at Q_{AF} at all ω
- The width in momentum space scales linearly in energy

Spectral function at the hot spots

$$A(\omega) \sim \frac{1}{\omega e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}}$$

- No quasiparticle with weak deviation from Fermi liquid

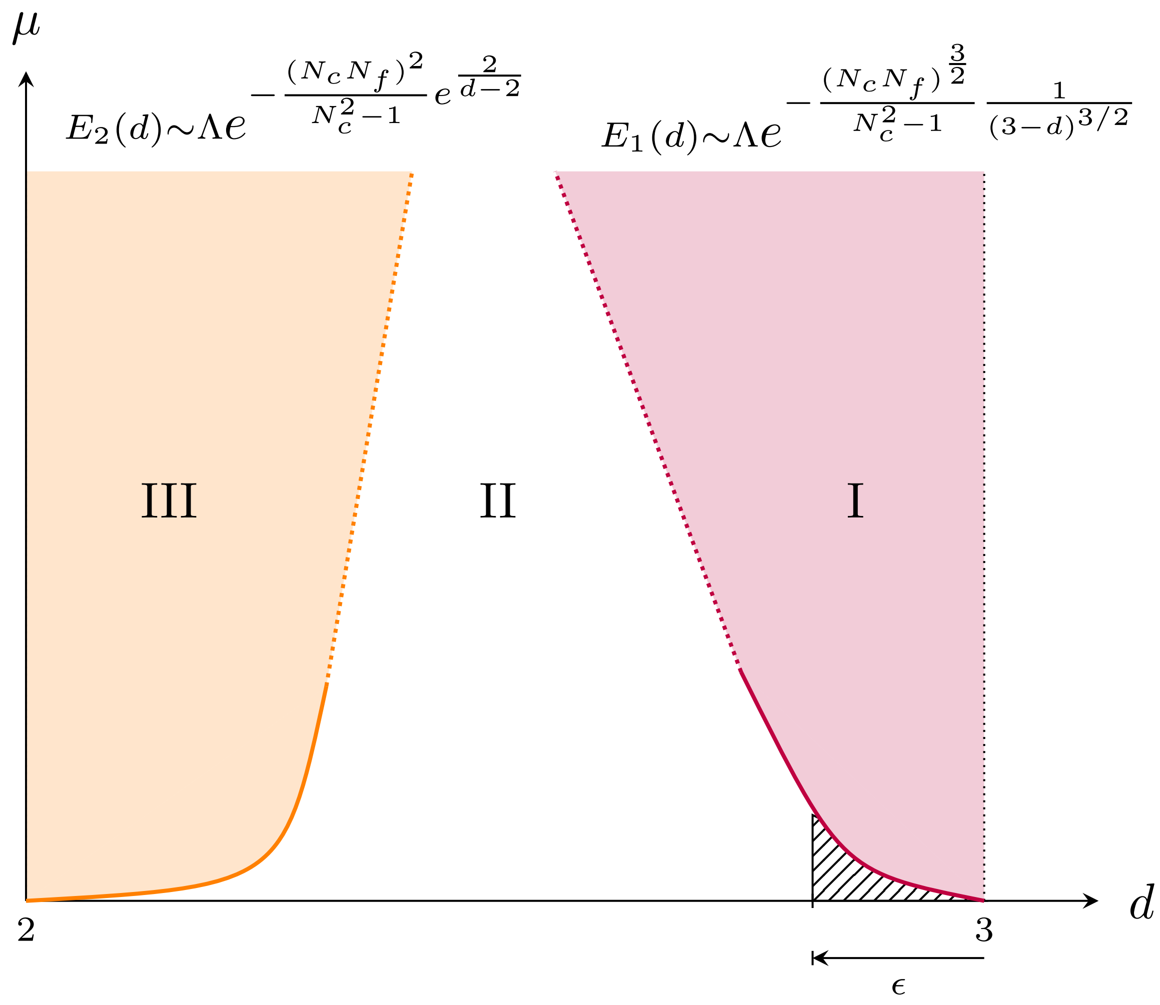
Divergent correlation length

$$\xi \sim (\lambda - \lambda_c)^{-1} e^{\frac{2}{\sqrt{3}}\left(\log \frac{1}{|\lambda - \lambda_c|}\right)^{1/2}}$$

Specific heat

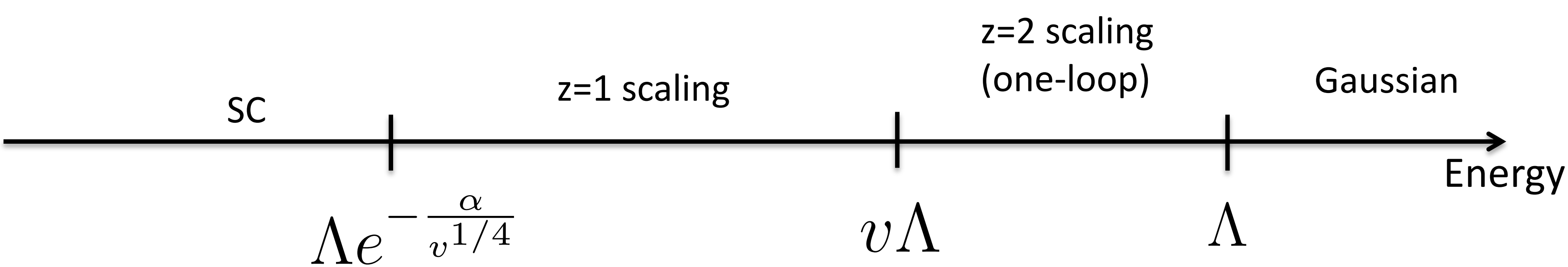
$$c \sim T e^{2\sqrt{3}\frac{\left(\log \frac{1}{T}\right)^{1/2}}{\log \log \frac{1}{T}}}$$

Two crossover scales separate the physics in d=3 and d=2



Superconductivity

- In d>2, there is no perturbative superconductivity
  - The critical strength for an SC instability goes to zero as $d \to 2$
- At d=2, there is a SC instability
  - Whether SC instability occurs within the universal scaling regime is a non-universal question
  - If bare attractive interaction is weak and v is small, there can be a large energy window between the onset of the universal scaling and T_c



Summary

- Antiferromagnetic critical metals in $2 \leq d \leq 3$
  - Exact critical exponents are smooth functions of d
  - Full low-energy scaling functions do not smoothly vary in d
  - low-energy limit and integer dimension limits do not commute