What Do We Know About the Mechanism of Superconductivity at “Intermediate Coupling?”

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Theoretical considerations are fairly general
but to be concrete:

Electron-electron interactions =

Hubbard or t-J Models on square lattice
with short-range hopping

Electron-phonon interactions =

Holstein model – linear coupling
of optical phonons to electron density

Not focusing the discussion on any particular material

These are to the theory of correlated electrons as
the Ising model is to the theory of critical phenomena

Superconductivity at weak coupling – Cooper instability

Weak electron-phonon coupling - $T_c \sim \overline{\omega}\ e^{-1/\lambda}$

$$\lambda = N(E_F)\ \alpha^2/K$$

Key role of retardation

Weak electron-electron repulsion - $T_c \sim E_F\ e^{-1/\lambda}$

$$\lambda \sim [\ N(E_F)\ U\ ]^2$$

Unconventional Superconductivity –
Key role of strongly k-dependent interactions

Chubukov and Kagan, Raghu Scalapino and SAK, …

Suppression of Superconductivity at Strong Coupling

Strong electron-phonon coupling -

Polarons, bipolarons, CDW, …

Strong electron-electron repulsion -

Ferromagnetism[1], Wigner crystal (CDW), …

1) L. Liu, H. Yao, E. Berg, and SAK, PRL **108**, 126406 (2012)

Do we REALLY find "High Temperature" Superconductivity at Intermediate Coupling?

Probably both electron-electron and electron-phonon coupling are important

I will look at them one at a time.

No small parameters:

bad news for analytic theory but
good news for numerical approaches

Ground-state of the 2D Hubbard and t-J models at intermediate couplings is unclear because of "intertwined orders"

(I realize that various people believe otherwise)

New results on Hubbard model at $T \geq 0.22\ t$

DQMC with minus sign problems −

U=6t t' = **-t/4**, 0, +t/4 $0 < \delta \leq 1/4$

16x4 and 8x 8 periodic clusters

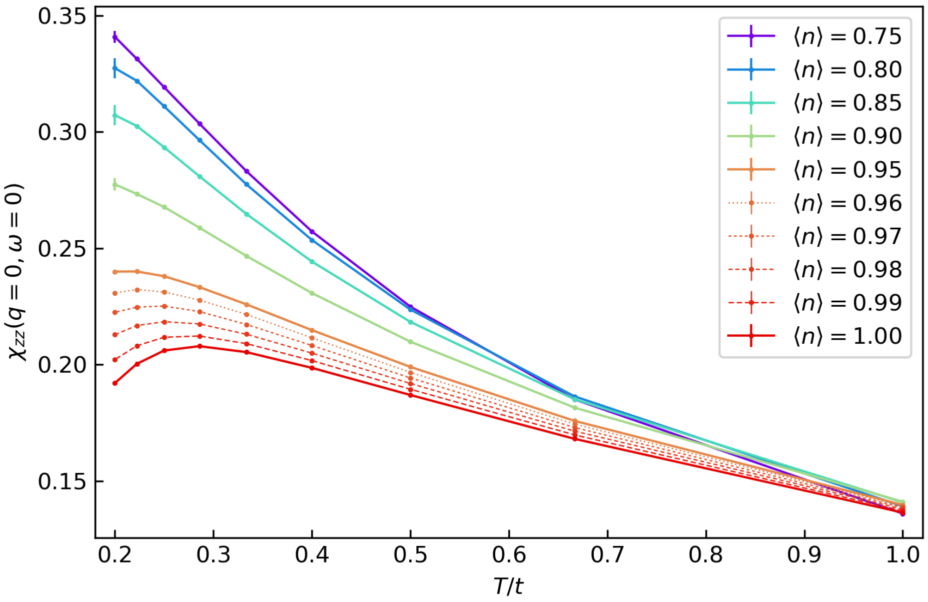
0.22 t ~ E_F / 10 (~ 650K in cuprates)

For $\delta < 5\%$ can access "pseudo-gap" regime, $T < T^*(x)$

Huang, Mendl, Jiang, Moritz, Devereaux, npj Quantum Materials (2018) and Science (2017)and unpublished

U= 6t
t'/t=-1/4

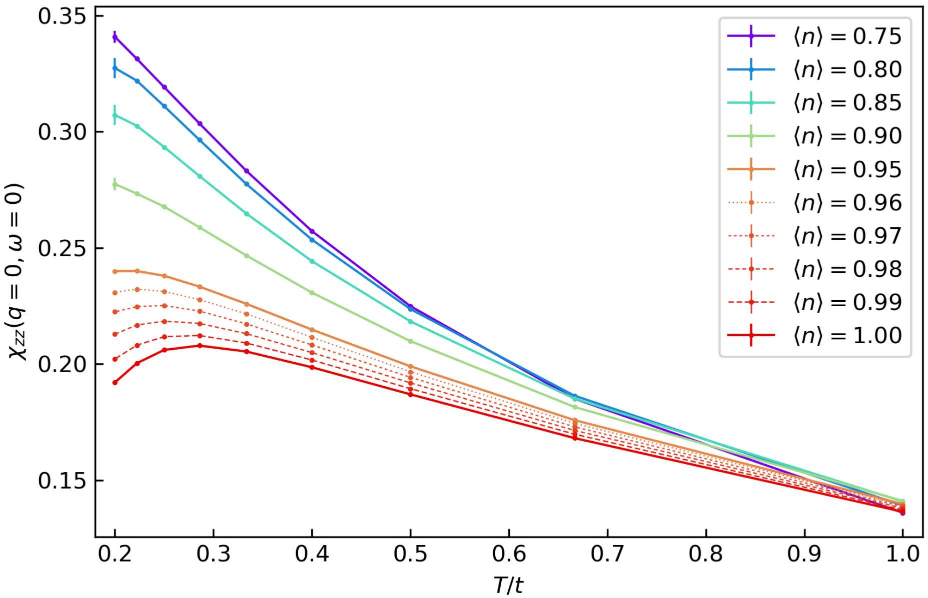
Static spin susceptibility (q=0)

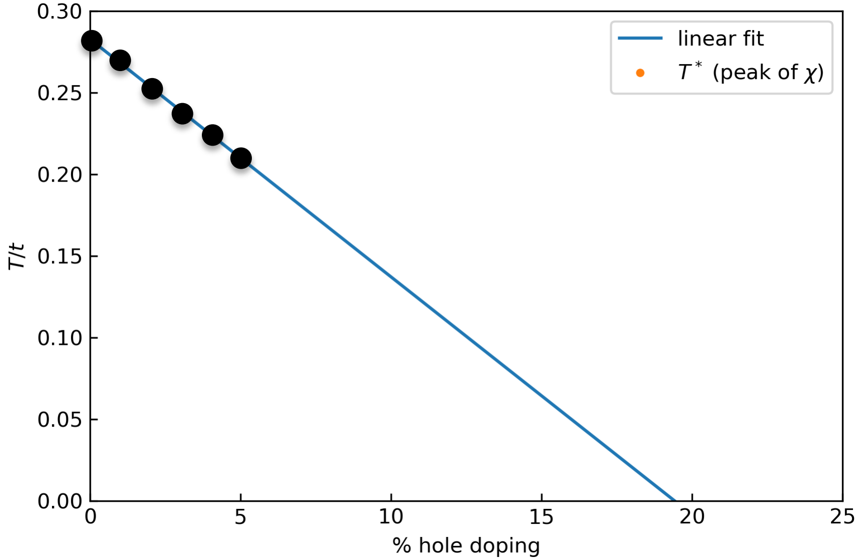


<n> =1-δ

U= 6t
t'/t=-1/4

Static spin susceptibility (q=0)





<n> =1-δ

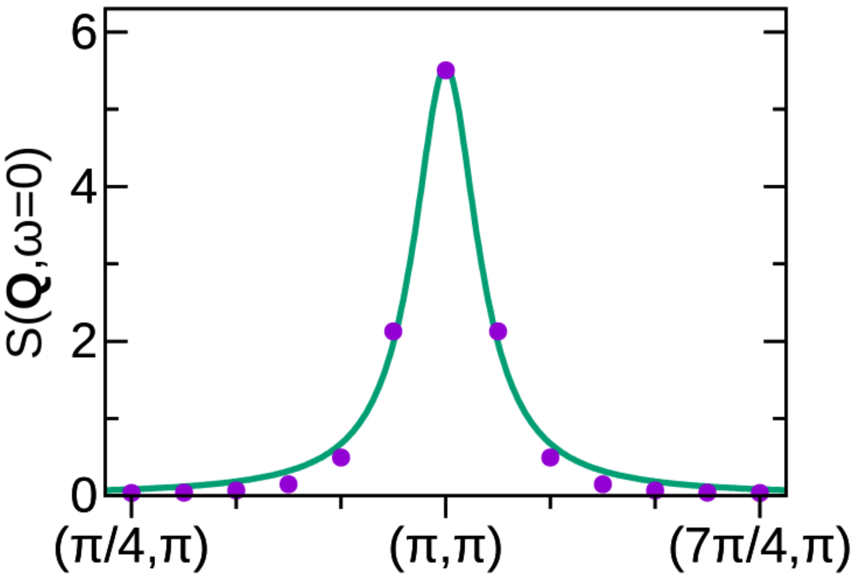
For all δ > 5%, T* (if it exists) is below lowest accessible T.

U= 6t
t'/t=-1/4

Short-range SDW correlations

T = 0.22 t

δ=0

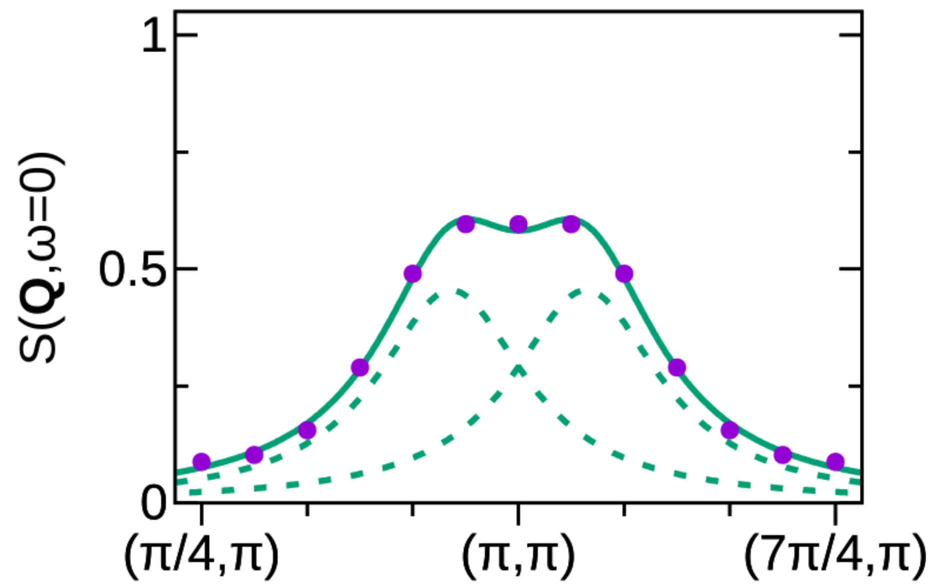


T=0.78 T*

ξ =3.4

$$\xi_T \equiv v_F/\pi T \approx 4$$

δ=1/8

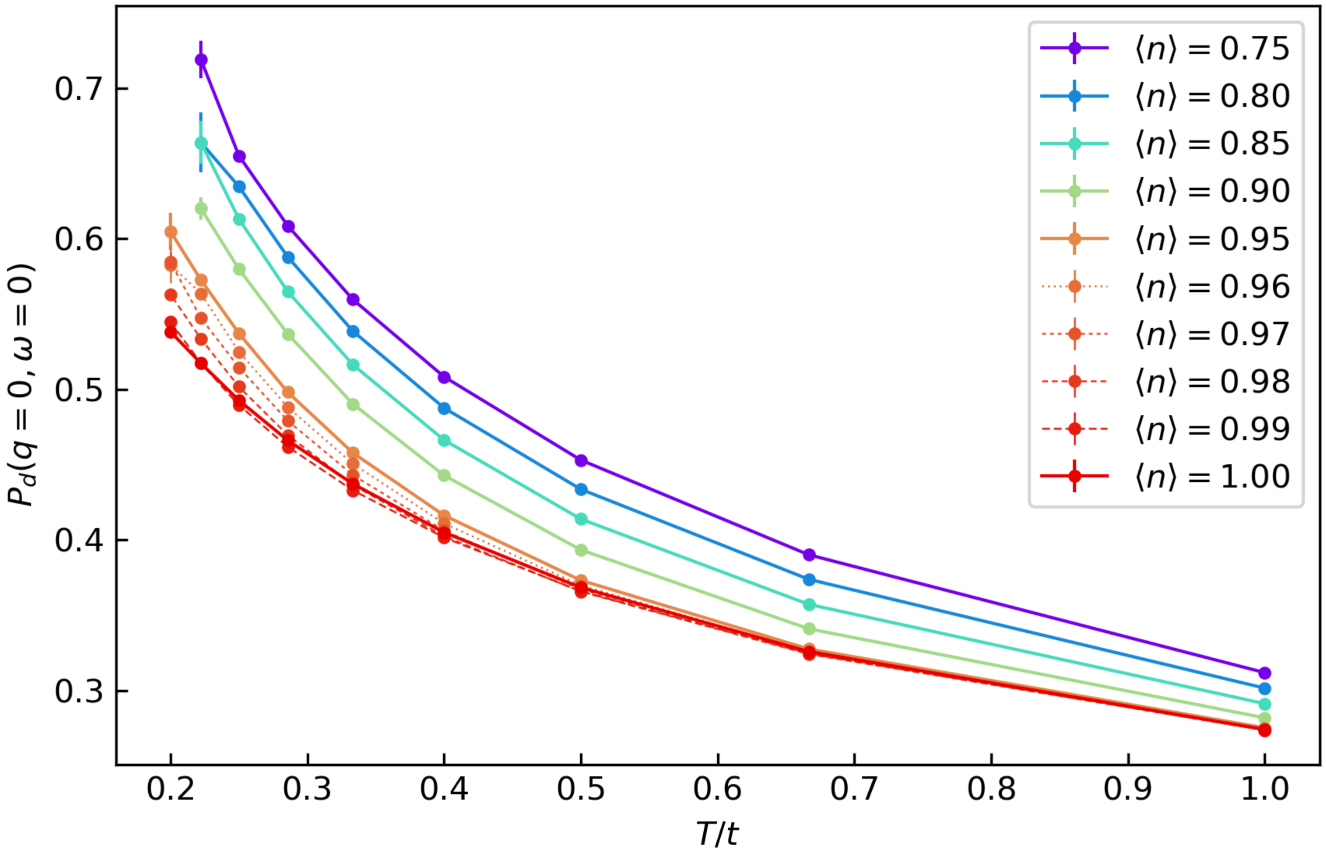


T > T*

ξ =1.5

U= 6t
t’/t=-1/4

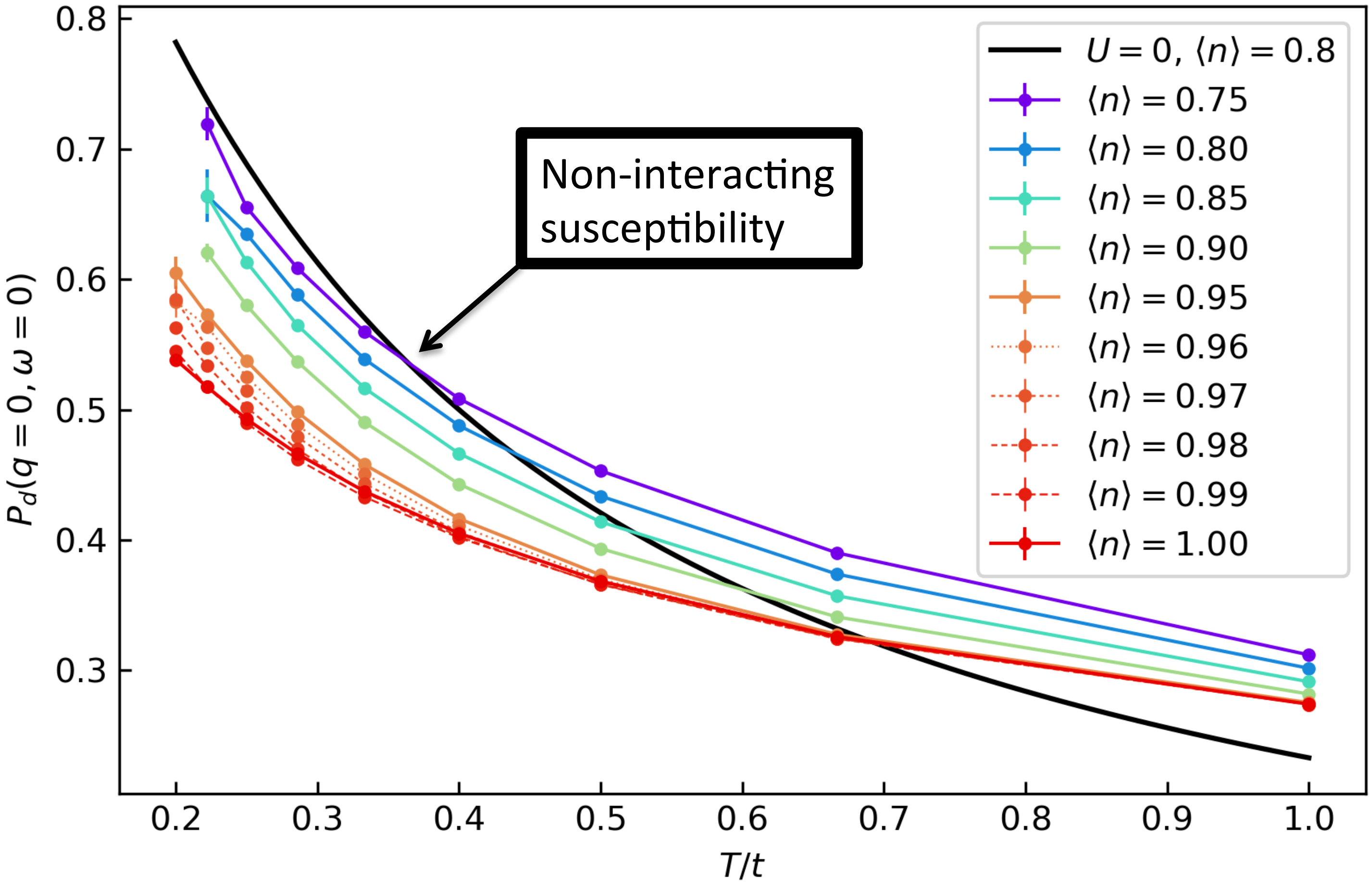
d-wave pairing susceptibility



$0 \leq \delta \leq 1/4$

U= 6t
t'/t=-1/4

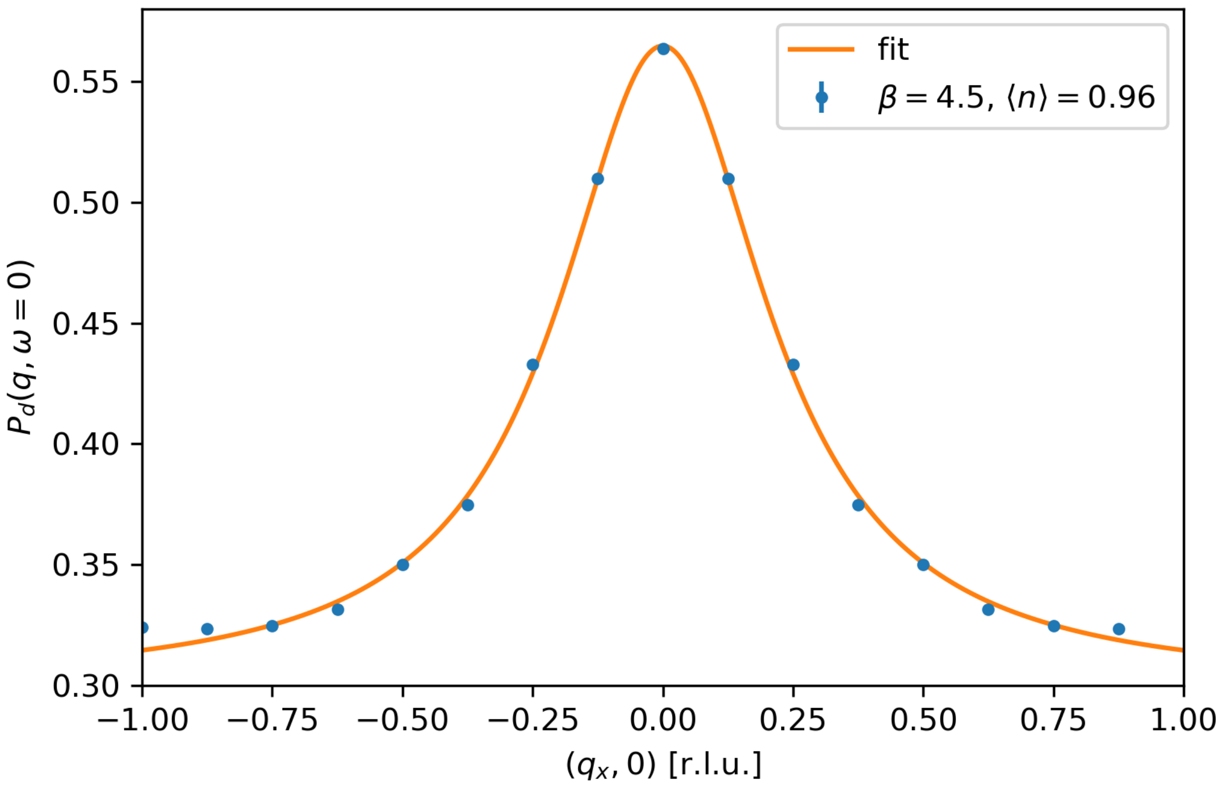
d-wave pairing susceptibility



$0 \leq x \leq ¼$

U= 6t
t'/t=-1/4

D-wave SC susceptibility: $P_d(q,\omega=0)$



Fit is to a single Lorentzian plus a constant background

ξ = 0.652

δ = 4%

T = 0.22 t = T^*

$$\xi_T \equiv v_F/\pi T \approx 4$$

<u>New results on Hubbard model at $T \geq 0.22$ t</u>

DQMC with minus sign problems –
$U=6t$, $t' =$ **$-t/4$**, 0, $+t/4$, $0 < \delta \leq 1/4$
16x4 and 8x 8 periodic clusters

0.22 t ~ E_F / 10 ~ 650K

For $x < 5\%$ can access "pseudo-gap" regime, $T < T^*(x)$

Short-range correlated (striped) SDW and d-SC order

No signatures of CDW fluctuations,
and no sign of PDW tendency or orbital currents

Huang, Mendl, Jiang, Moritz, Devereaux, npj Quantum Materials (2018) and Science (2017)and unpublished

New results on t − J model on 4 x N Cylinder (at T=0)

$$U^{eff} = 4t^2/J \quad \rightarrow \quad t/J = 3 \text{ corresponds to } U = 12t$$

DMRG up to N= 160

(also with half quantum flux through cylinder)

Keeping up to 15,000 states and many details

Many different states depending on t'/t and J'/J and t/J :

Mostly CDWs with various periods ,

substantial SC with medium range SDW

H-C. Jiang, Z-Y Weng, and SAK, unpublished

J. Dodaro, H-C. Jiang, and SAK, PRB (2017).

New results on t − J model on 4 x N Cylinder

J/t = 1/3   J' = t' =0

Luther Emery liquid

$$\langle \Psi_a^\dagger(0)\Psi_b(x)\rangle \sim \Delta_a^\star \Delta_b \; x^{-K_{sc}}$$

$$\langle \rho(0)\rho(x)\rangle \sim \bar{\rho} \;+\; A_{cdw} \;\cos(Qx+\phi)\; |x|^{-K_{cdw}}$$

$$\langle \vec{S}(0)\cdot\vec{S}(x)\rangle \sim {}^{-}A_{sdw} \;\cos(2Qx+\pi x+\tilde{\phi})\; \exp[\,-|x|/\xi_s\,]$$

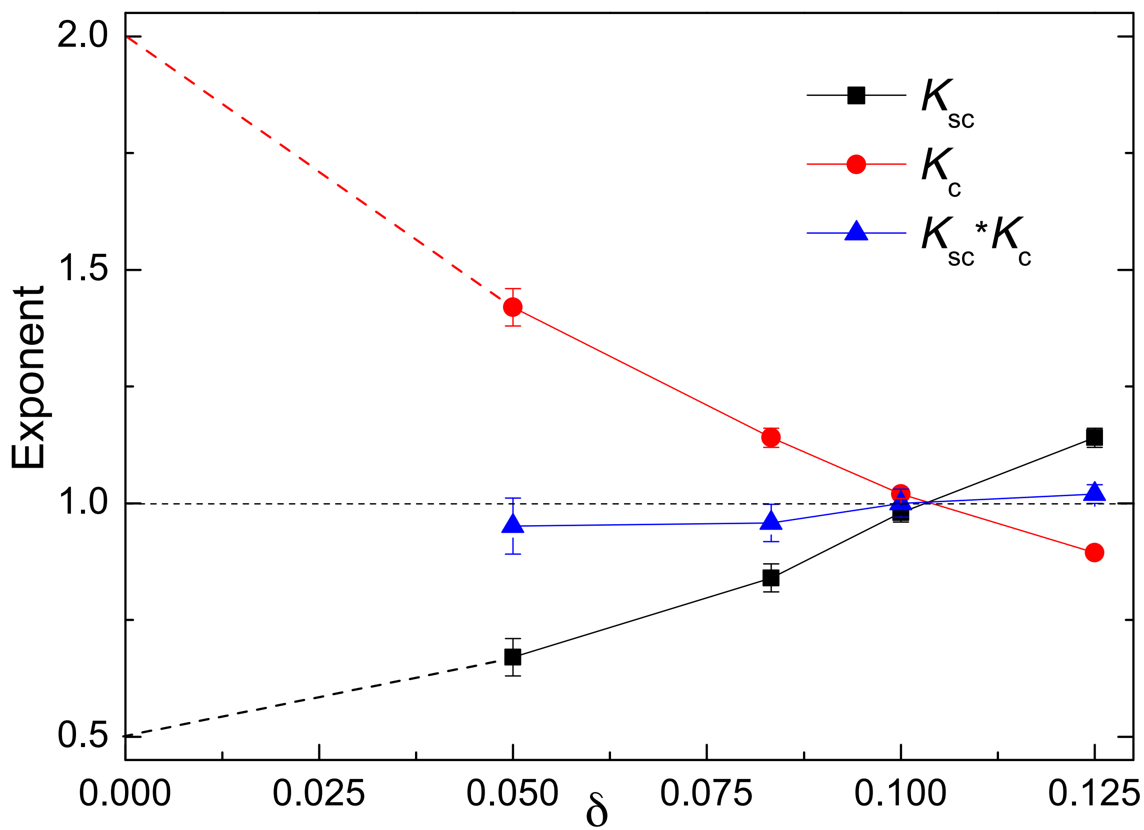
$$K_{cdw}K_{sc} = 1 \qquad S_{ent} \sim \frac{c}{6}\;\ln[L] + \ldots \;\text{ with } c = 1$$

$$\chi_{sc} \sim T^{-(2-K_{sc})} \;\text{ for }\; K_{sc} < 2$$

$$\chi_{cdw} \sim T^{-(2-K_{cdw})} \;\text{ for }\; K_{cdw} < 2$$

H-C. Jiang, Z-Y Weng, and SAK, unpublished
J. Dodaro, H-C. Jiang, and SAK, PRB (2017).

Luttinger exponents for the 4-leg t-J cylinder with J/t=1/3



First observation of power law SC in the GS of any system with $L_y > 2$

Serious competition between CDW and SC can be expected to persist for larger L_y

SC state is a nearly equal amplitude d + s (i.e. strongly nematic)

$$\chi_{sc} \sim T^{-(2-K_{sc})} \quad \text{for} \quad K_{sc} < 2$$

$$\chi_{cdw} \sim T^{-(2-K_{cdw})} \quad \text{for} \quad K_{cdw} < 2$$

2D Hubbard and t-J models at intermediate couplings

What do we (I) know now.

Exhibits tendencies to multiple orders − SC, CDW and SDW − all with comparable strength − i.e. intertwined orders

Has many of the same "players" as are seen in the cuprates

CDW (with periods between $1/2\delta \leq \lambda \leq 1/\delta$),
SDW (with period twice the CDW)
d-wave SC

The energy and T scales reasonable in relation to cuprates

Evidence (not discussed) of a "bad metal" state above T^*

No indication of any form of orbital loop current order

I am not sure what to say about PDW
− I wish the evidence in favor were stronger

Electron-phonon problem (Holstein model) at intermediate λ

(minus sign problem free DQMC)

$$H = H_{el} + H_{ph} + H_{el-ph}$$

H_{el} Tight binding model on a square lattice with <n>=0.8

H_{ph} Single dispersionless optical mode with $\omega_{\vec{k}} = \omega_0$

$$H_{el-ph} = \alpha \sum_j \hat{n}_j x_j$$

$$T \geq E_F/30$$

$$\lambda_0 = 0 \; - \; 0.6$$

$$\lambda_0 \equiv \frac{\alpha^2}{K} \; \rho(E_F) = \frac{\alpha^2}{M\omega_0^2} \; \rho(E_F)$$

$$\frac{\omega_0}{E_F} = 0.1$$

$$\frac{\omega_0}{E_F} \ll 1$$

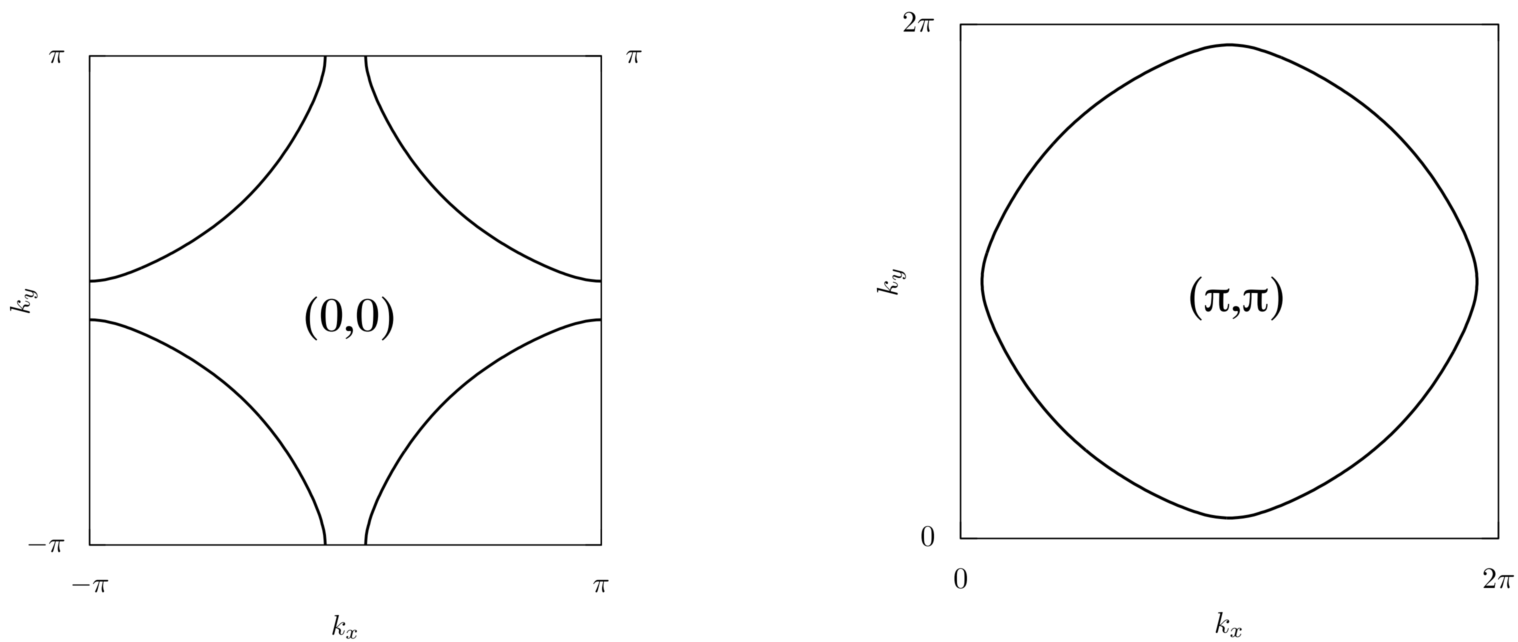
$$\lambda_0 \frac{\omega_0}{E_F} \ll 1$$

Esterlis et al, PRB **97**, 140505 (2018)

Comparison of theory with numerical "experiments" on the Holstein model --- DQMC

$$H = H_{el} + H_{ph} + H_{el-ph}$$

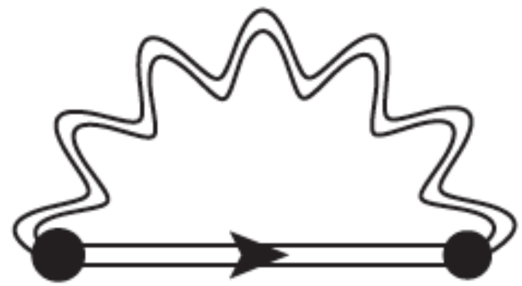
H_{el} Tight binding model on a square lattice with <n>=0.8



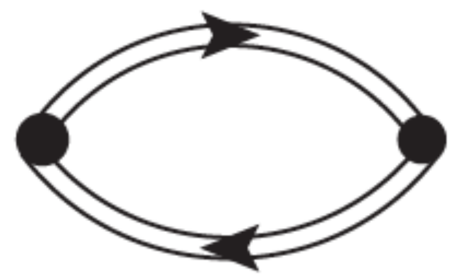
$$E_F = 1.7\ t \equiv 1.7$$

Migdal-Eliashberg Approximation

$$\Sigma(\mathbf{k},\omega_n) =$$



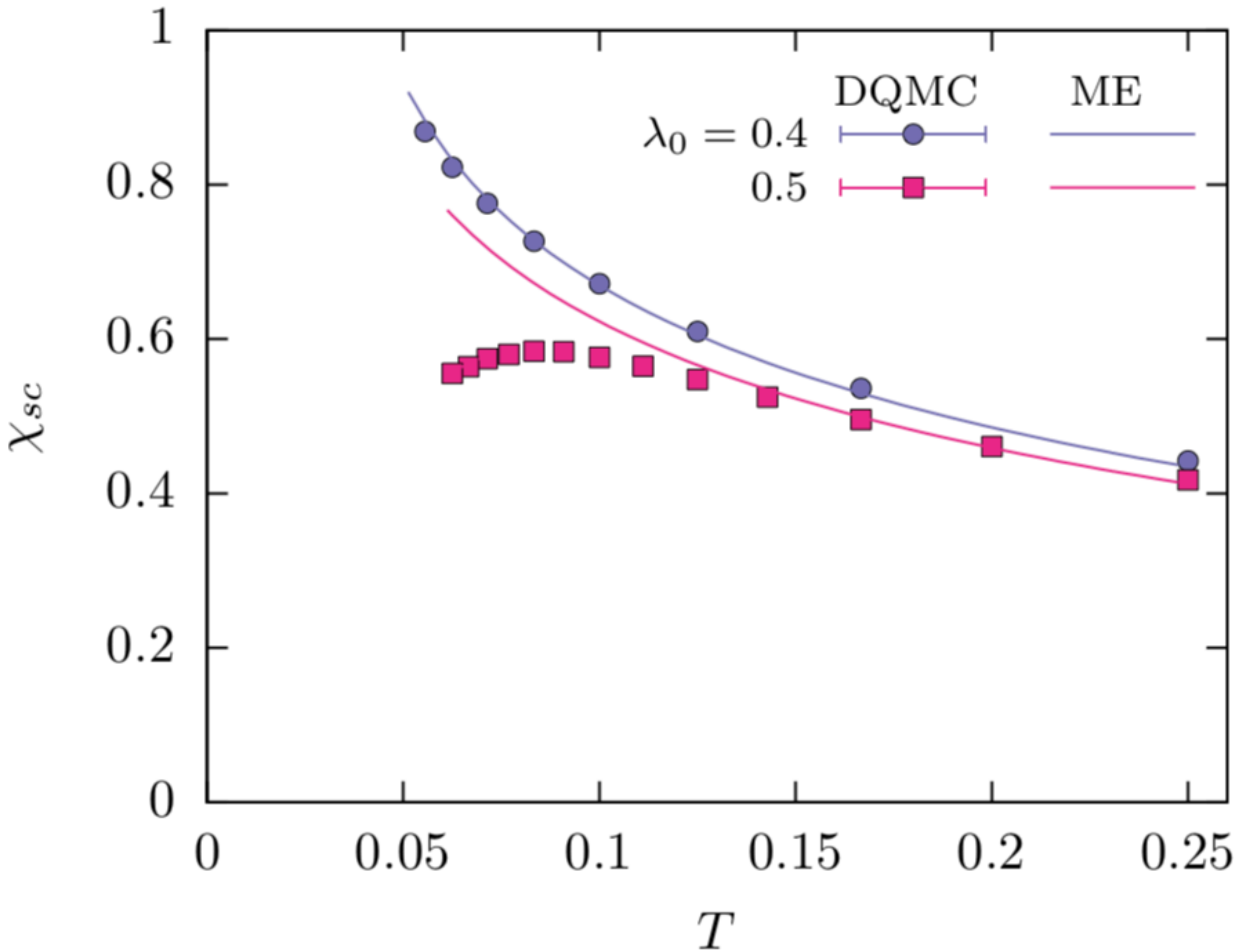
$$\Pi(\mathbf{q},\nu_n) =$$



Supposed to be valid so long as $\lambda\ (\omega_0/E_F) << 1$

$$\omega_0 = 0.1\ E_F$$

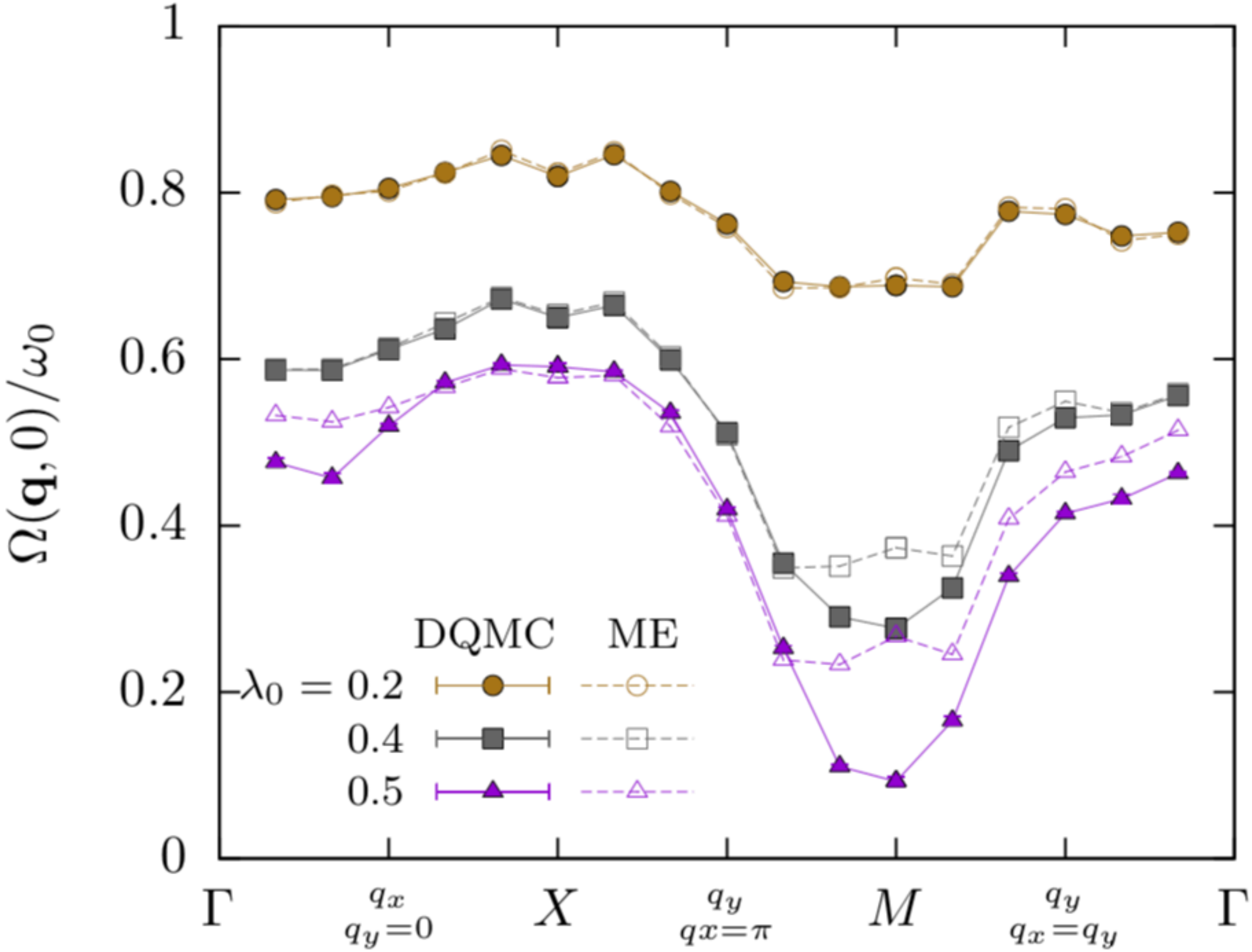
Superconducting Susceptibility



Esterlis et al, PRB **97**, 140505 (2018)

$\omega_0 = 0.1\ E_F$

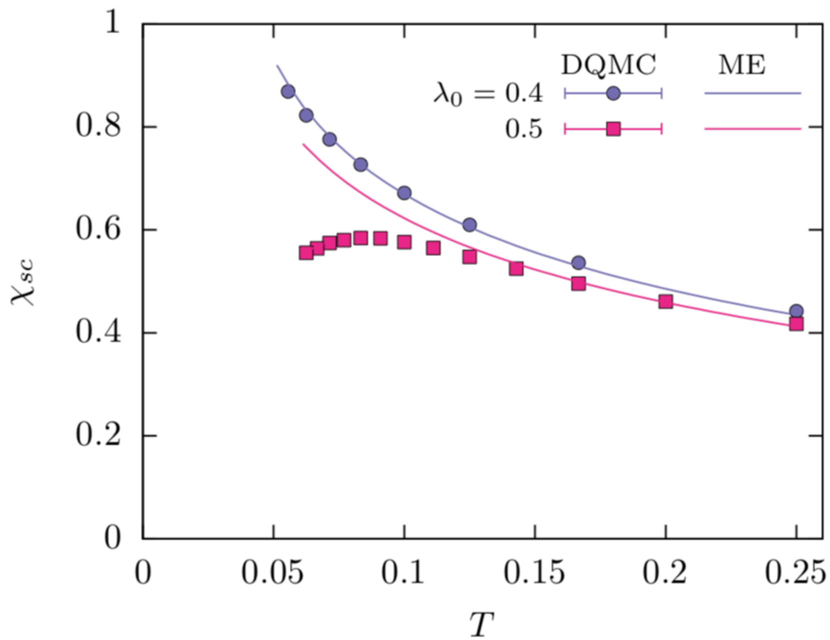
Phonon softening



Esterlis et al, PRB **97**, 140505 (2018)

$\omega_0 = 0.1\ E_F$

Maximum T_c at $\lambda_0 = \lambda^* \approx 0.4$



Crossover from ME regime where $\lambda < \lambda^* << E_F/\omega_0$

to strong coupling regime where $\lambda > \lambda^*$ (even while $\lambda << E_F/\omega_0$)

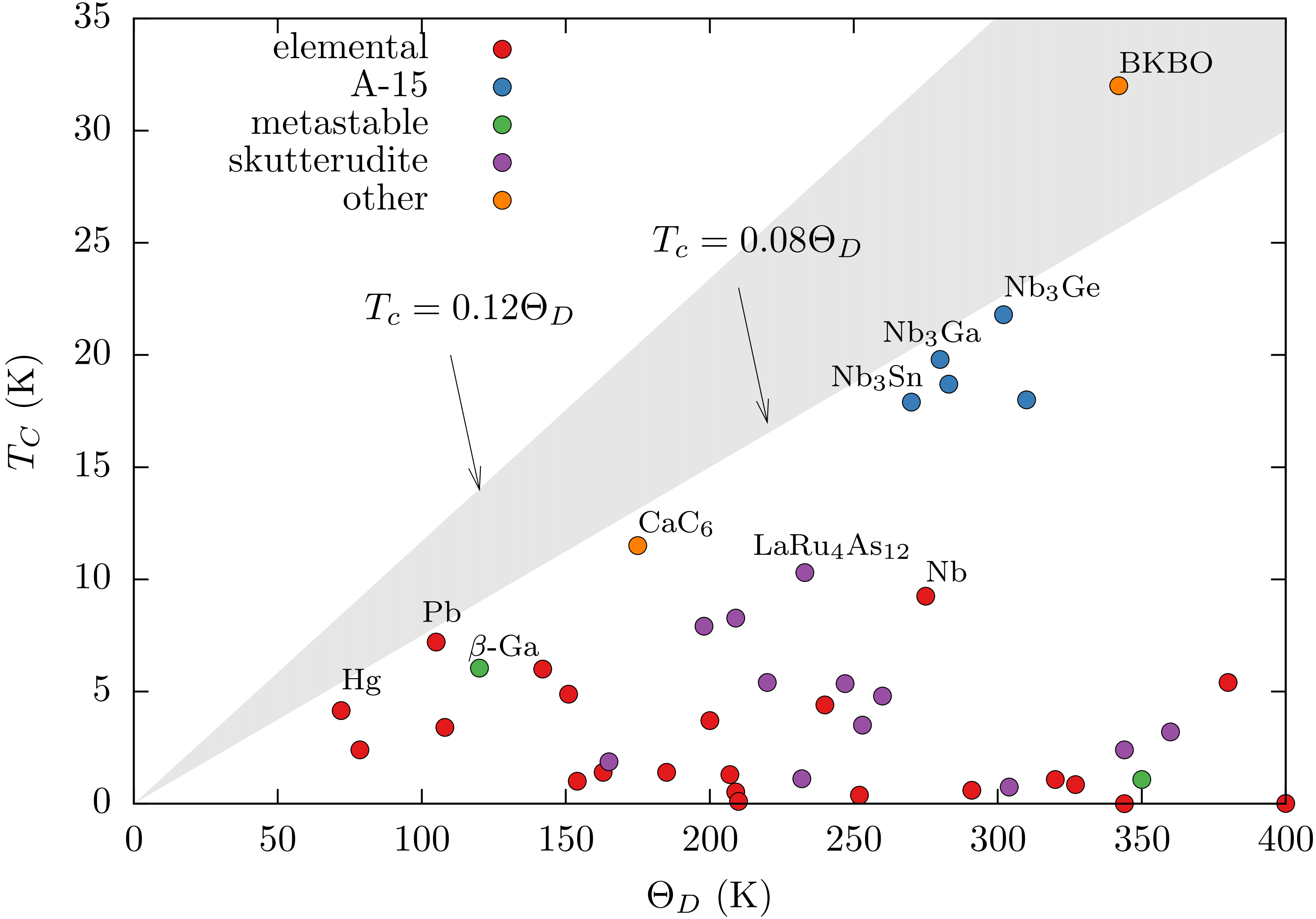
$T_c(\lambda^*) \approx 0.08\ \omega_0 \approx 0.12\ \omega_{max}$

Conjectured upper bound on "conventional" T_c

$T_c \leq A\ \omega_{max}$ $\qquad$ $A = 0.08 - 0.12$

I. Esterlis, S. A. Kivelson and D. J. Scalapino, in preparation:

Empirical evidence of the relevance of such a bound



Electron-phonon problem (Holstein model) at intermediate λ

There exists an optimal $\lambda = \lambda^*$, unrelated to the onset of a competing lattice instability.

$$k_B T_c = \hbar\omega_{max}\ A(\lambda)\ \leq\ \hbar\omega_{max}\ A(\lambda^\star)\ \approx\ \hbar\omega_{max}\ 0.08$$

For large $\lambda \sim \lambda^*$ there is a "competing" commensurate CDW totally unrelated to any Fermi surface features (strong coupling effect)

Strong deviations from FL theory at all (so far) accessible T's (Does this mean "bad metal" regime?)

Certainly no form of PDW, SDW, or orbital loop current order

To Pontificate a bit

Ηιγη $Τ_χ$ ισ α χροσσοϖερ πηενομενον ατ ιντερμεδιατε χουπλινγ

“Στρονγλψ χορρελατεδ ελεχτρον σψστεμ” προβαβλυ μεανσ
– μυστ βε υνδερστοοδ ασ αν ιντερμεδιατε χουπλινγ προβλεμ

Τηε οχχυρρενχε οφ “ιντερτωινεδ ορδερσ” ισ λικελυ αν υναϖοιδαβλε χορολλαρυ

<u>To Pontificate a bit</u>

High T_c is a crossover phenomenon at intermediate coupling

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Τηε οχχυρρενχε οφ “ιντερτωινεδ ορδερσ” ισ λικελψ αν
υναϖοιδαβλε χορολλαρψ

To Pontificate a bit

High T_c is a crossover phenomenon at intermediate coupling

"Strongly correlated electron system" probably means
- must be understood as an intermediate coupling problem

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To Pontificate a bit

High T_c is a crossover phenomenon at intermediate coupling

"Strongly correlated electron system" probably means
- must be understood as an intermediate coupling problem

The occurrence of "intertwined orders" is likely an unavoidable corollary

I still don't know whether the Hubbard model at U ~ W is a high temperature SC!

Thanks.