Higher Order Topological Superconductors

Yuxuan Wang
University of Illinois at Urbana-Champaign

CESND, FTPI, 05/19/2018

Collaborators

Mao Lin
(UIUC)

Taylor Hughes
(UIUC)

Reference:
YW, Lin, and Hughes, arXiv:1804.01531

Family of Topological Insulators

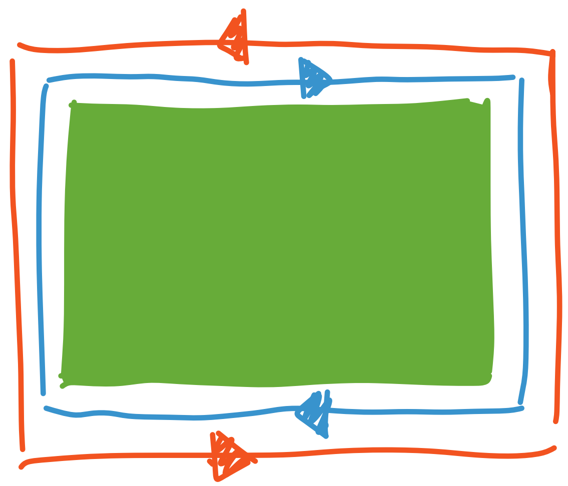
- Topological insulators in different dimensions:

1d

quantized dipole moment

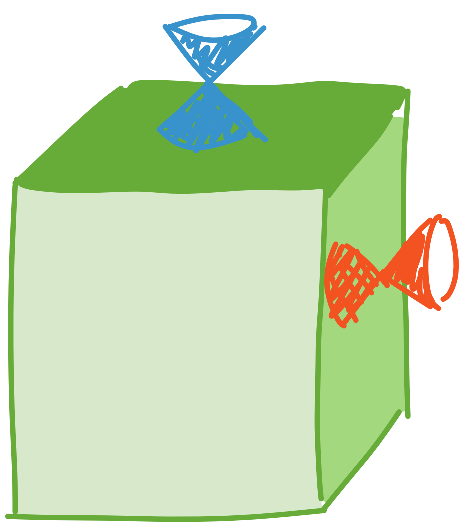
$$\vec{p} = \pm \frac{e}{2} \vec{L}$$

2d



spin Hall effect

3d



magneto-electric effect

$$\sigma_{xy}^{\mathrm{surf}} = e^2/2h$$

Family of Topological Insulators

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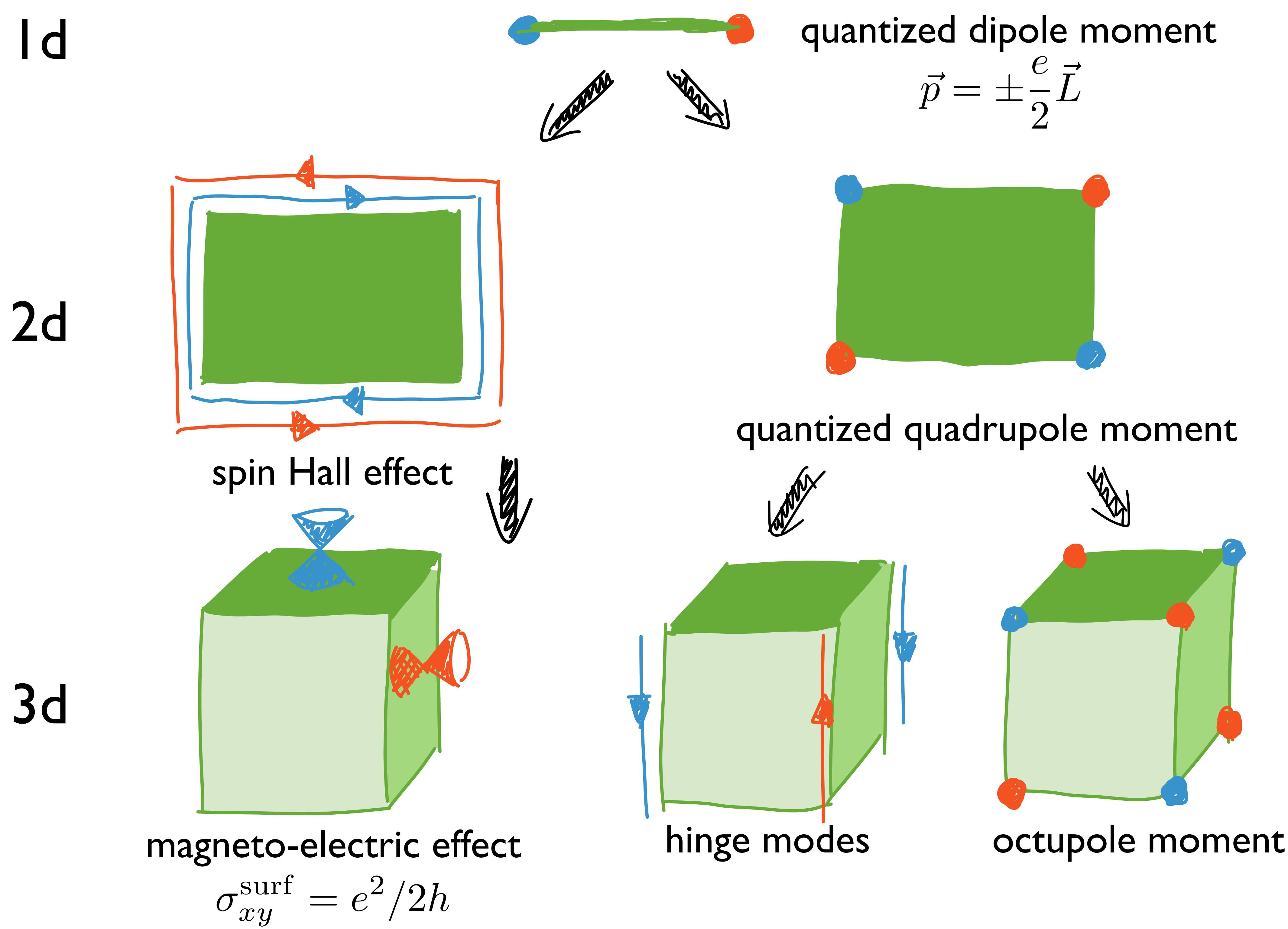

RESEARCH ARTICLE

Quantized electric multipole insulators

Wladimir A. Benalcazar[1], B. Andrei Bernevig[2], Taylor L. Hughes[1,*]

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Higher-Order Topological Insulators

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(Dated: August 15, 2017)

Selected for a Viewpoint in *Physics*
PRL 119, 246401 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Reflection-Symmetric Second-Order Topological Insulators and Superconductors

Josias Langbehn, Yang Peng,* Luka Trifunovic, Felix von Oppen, and Piet W. Brouwer
Dahlem Center for Complex Quantum Systems and Physics Department, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany

(Received 12 August 2017; published 11 December 2017)

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PRL 119, 246402 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

$(d-2)$-Dimensional Edge States of Rotation Symmetry Protected Topological States

Zhida Song,[1,2] Zhong Fang,[1,3] and Chen Fang[1,*]
[1]*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*University of Chinese Academy of Sciences, Beijing 100049, China*
[3]*Collaborative Innovation Center of Quantum Matter, Beijing 100084, China*

ved 14 August 2017; revised manuscript received 1 October 2017; published 11 December 2017)

Physics

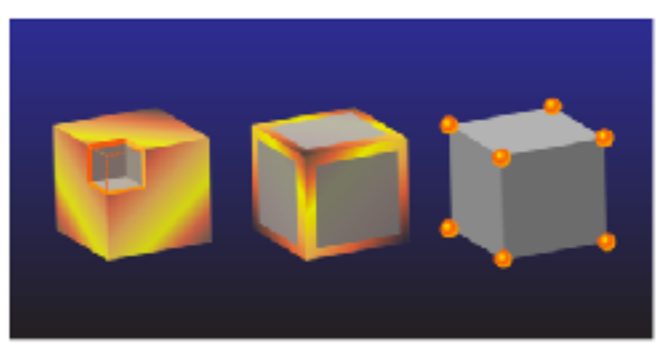
VIEWPOINT

Topological Insulators Turn a Corner

Theorists have discovered topological insulators that are insulating in their interior and on their surfaces but have conducting channels at corners or along edges.

by Siddharth A. Parameswaran* and Yuan Wan†

Identifying new phases of matter that have unusual properties is a key goal of condensed-matter physics. A famous recent example is the theoretical prediction of crystalline materials known as topological insulators (TIs), several of which have now been identified in the laboratory [1]. TIs are electronic insulators in their d-dimensional interior (bulk) but allow metallic conduction on their $(d-1)$-dimensional boundaries. This is because in their bulk these materials have an energy gap between the ground and first

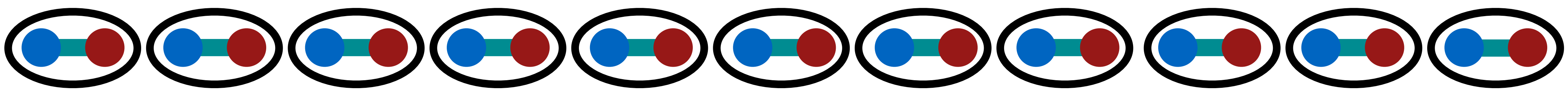


How can we realize these states?

1d Topological Insulators

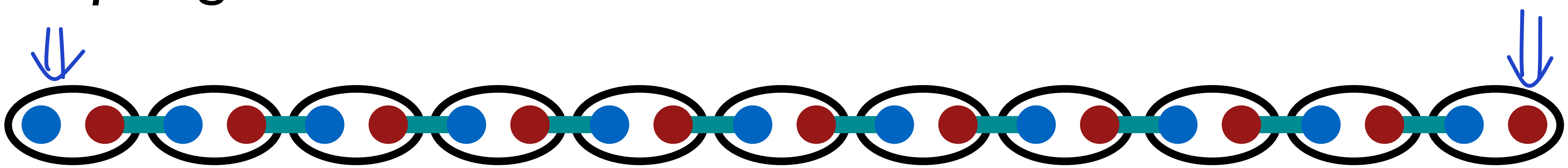
- 1d Insulators in the dimerized limit:

"trivial"



"topological"

zero modes



$$H_{\mathrm{TI}}(k) = \cos(k)\sigma^x + \sin(k)\sigma^y$$

Second Order Topological Insulators

- Consider a four-band model with mirror symmetries M_x and M_y. Banalcazar-Bernevig-Hughes Science 2017, Banalcazar-Bernevig-Hughes PRB 2017

4x4 matrices

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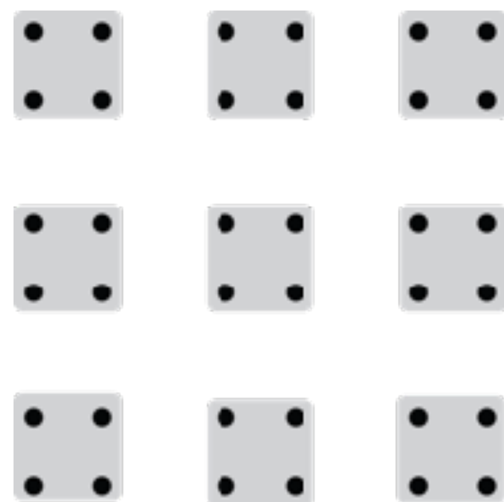

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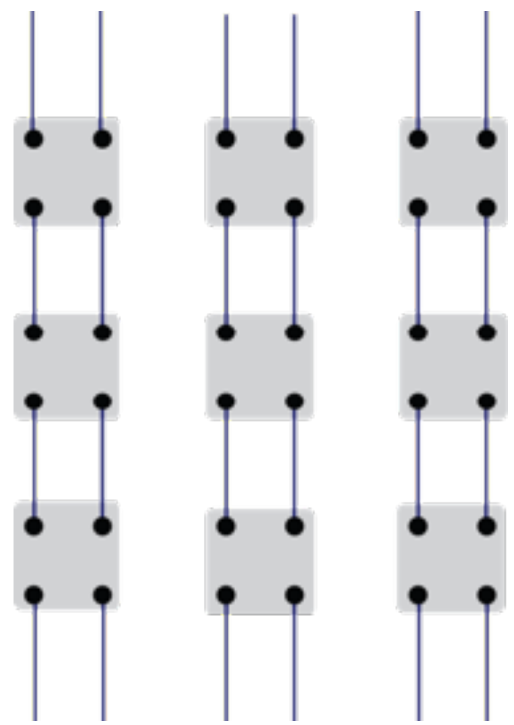

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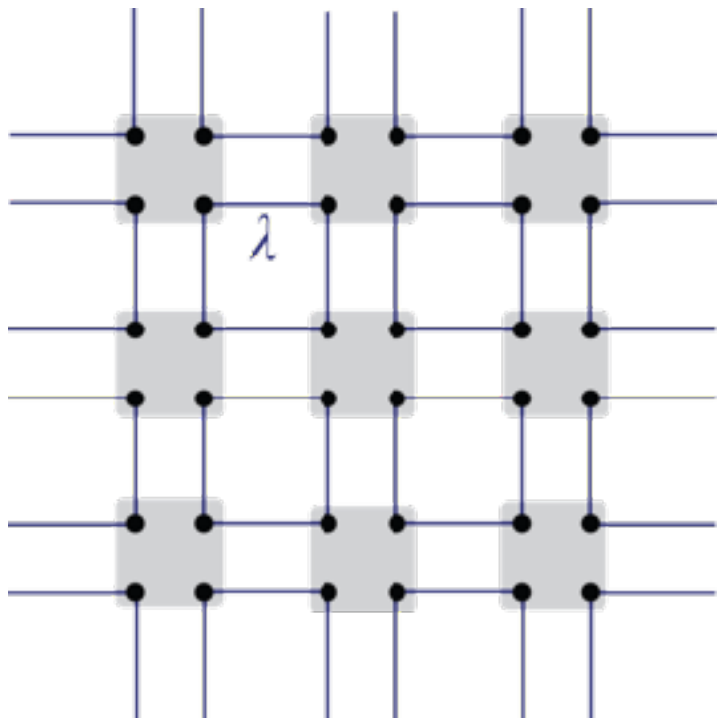

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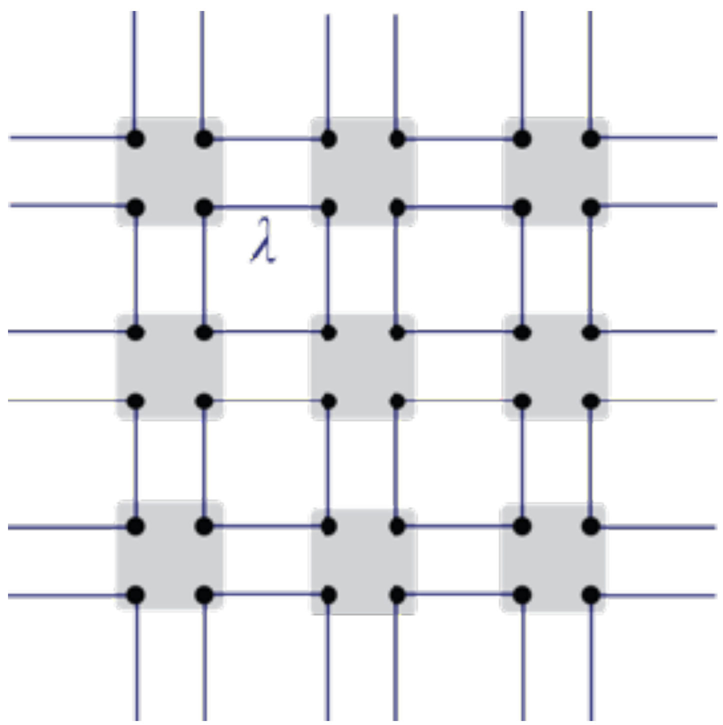

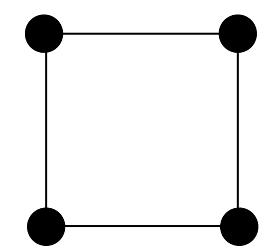

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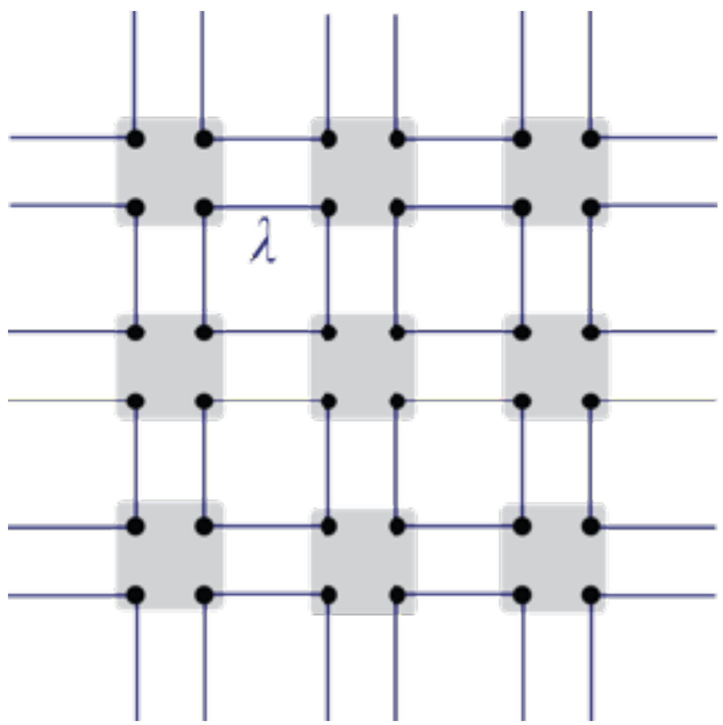

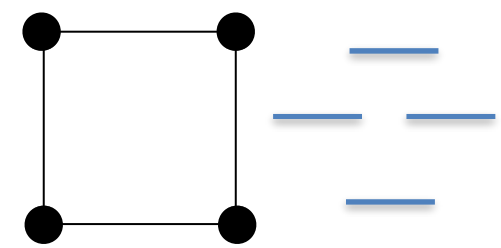

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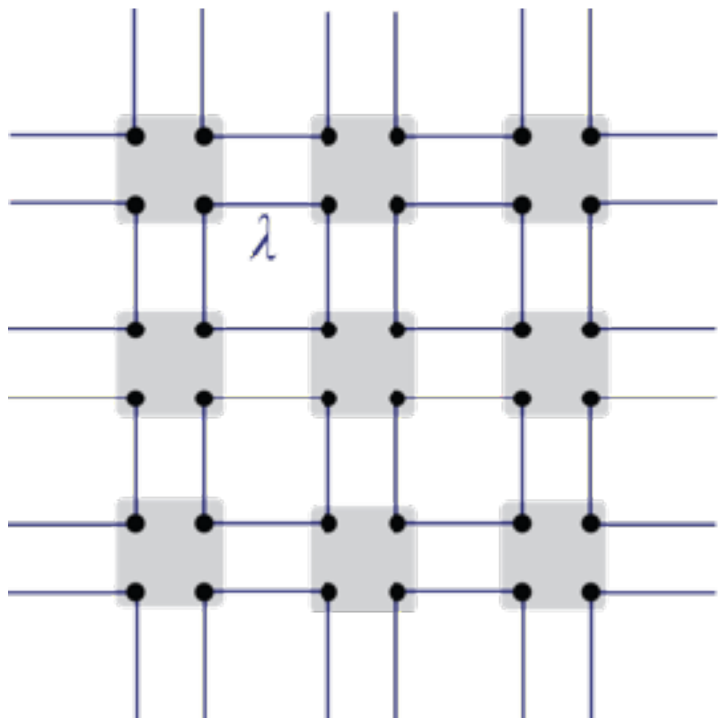

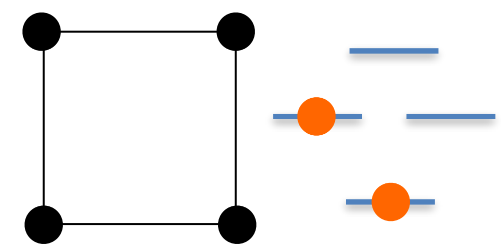

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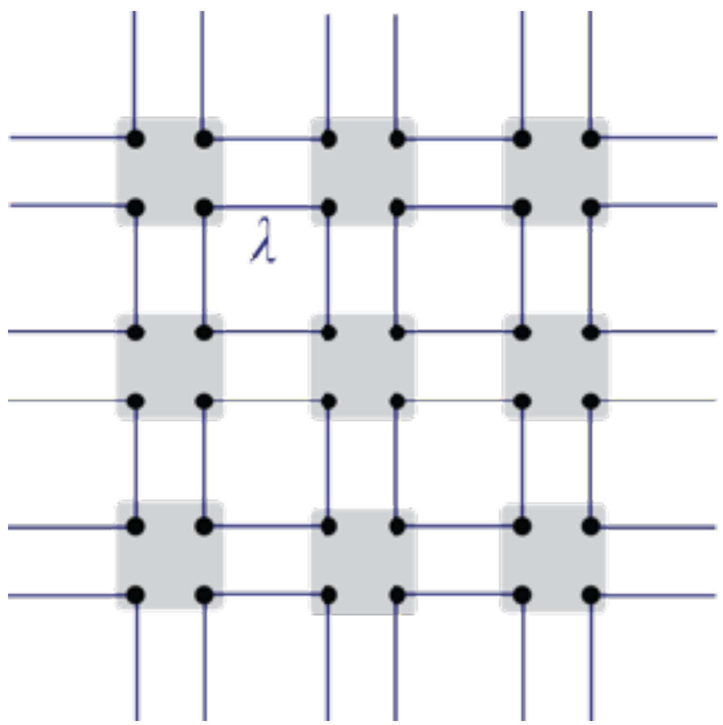

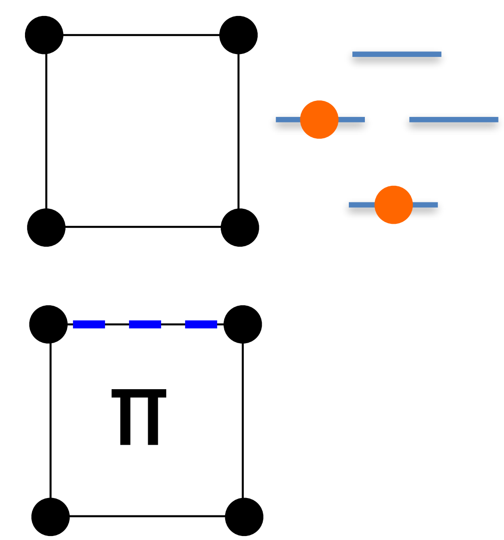

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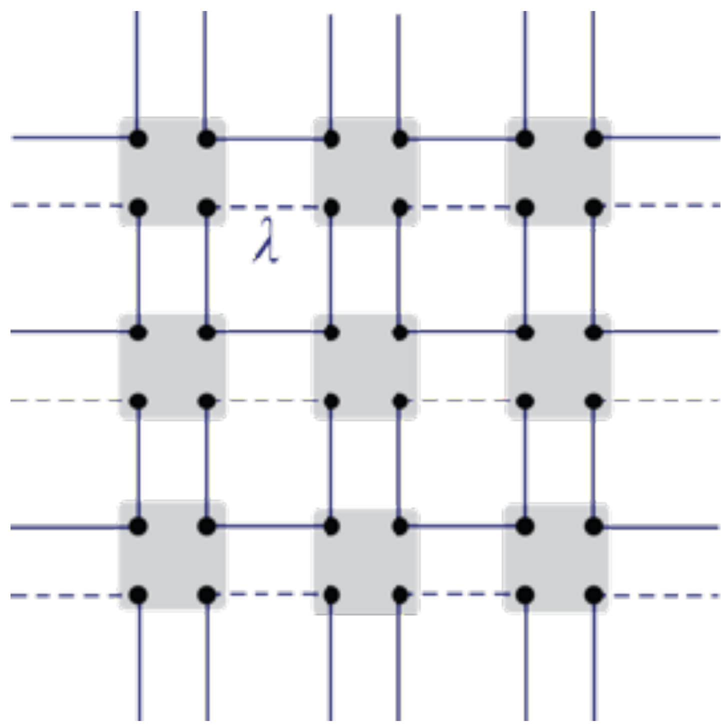

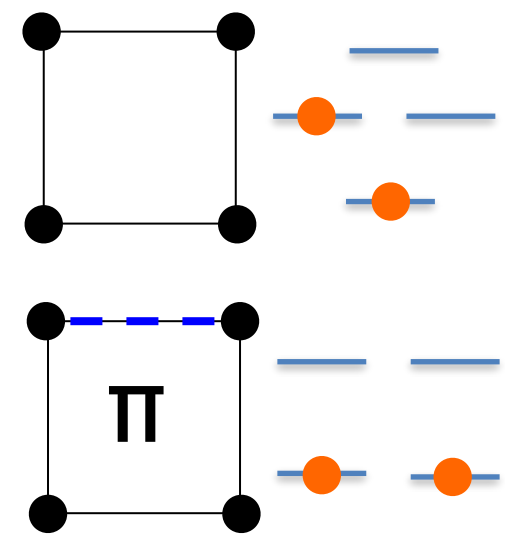

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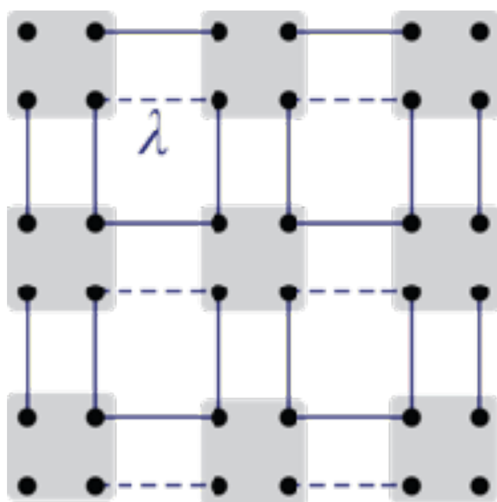

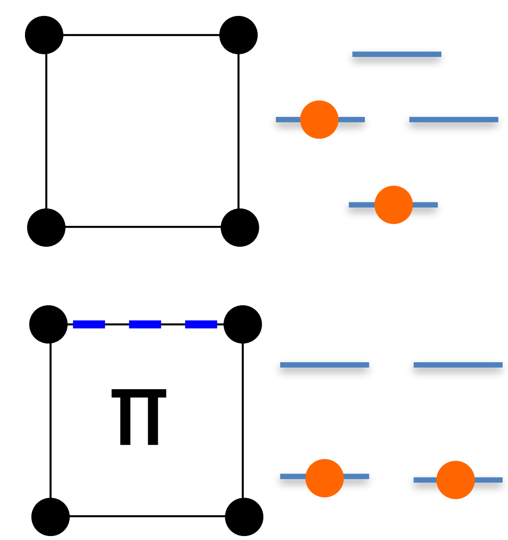

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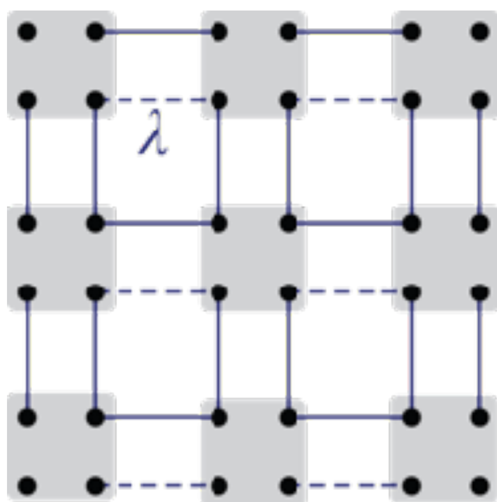

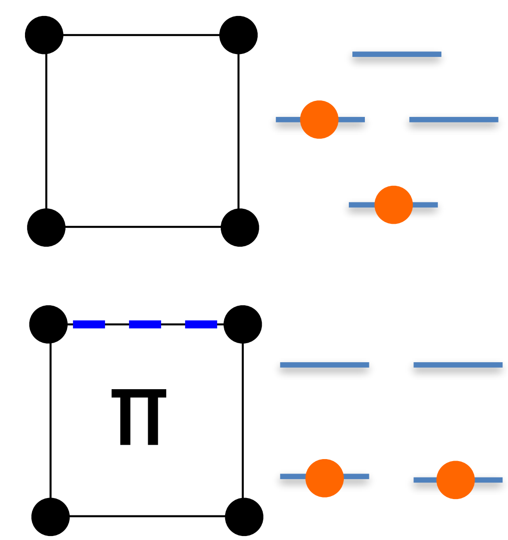

4x4 matrices

$$H_{\mathrm{TI}_2}(\mathbf{k}) = \cos k_x \Gamma_1 + \cos k_y \Gamma_2 + \sin k_x \Gamma_3 + \sin k_y \Gamma_4 \qquad \{\Gamma_i, \Gamma_j\} = 0$$

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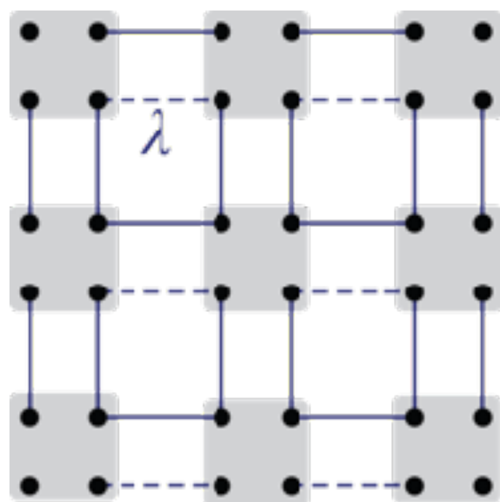

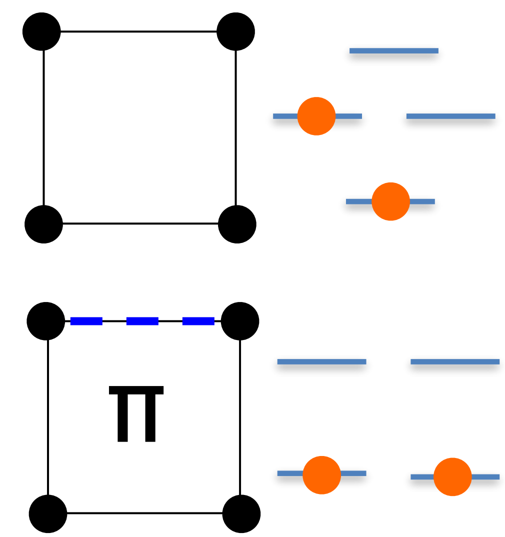

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- It has a bulk quadrupole moment. For a finite system it has four corner zero modes.

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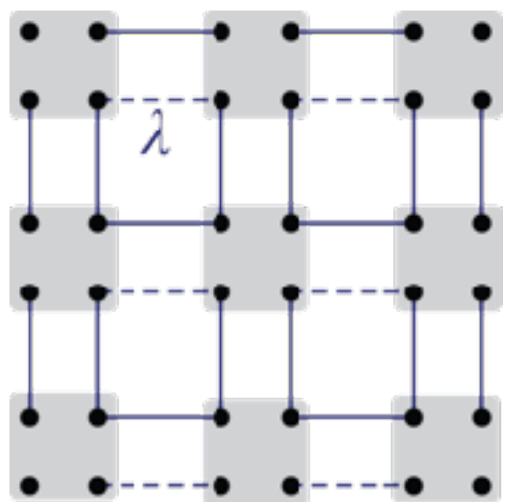

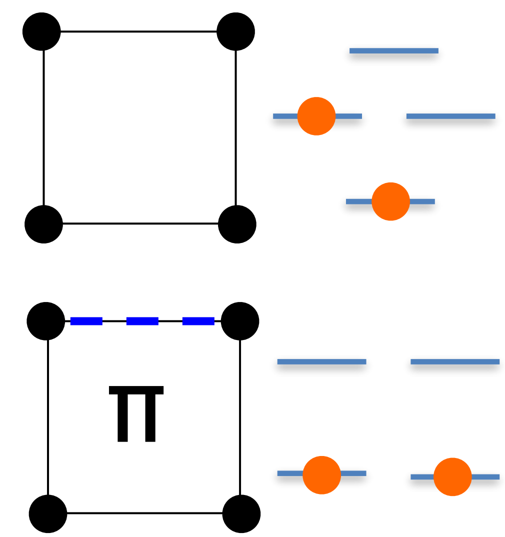

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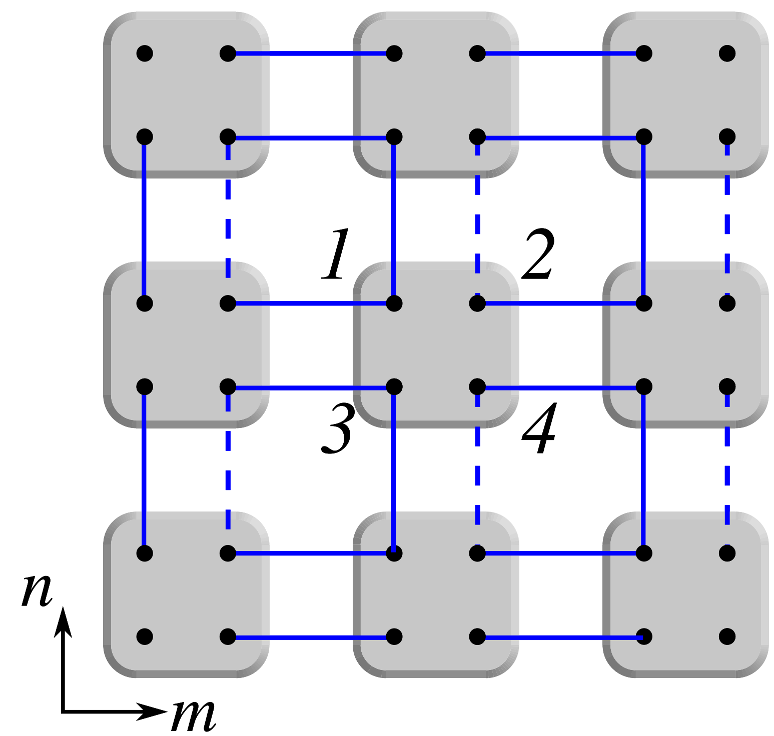
- It has a bulk quadrupole moment. For a finite system it has four corner zero modes.
- Requires an intricate hopping pattern, difficult to realize in solid state systems.

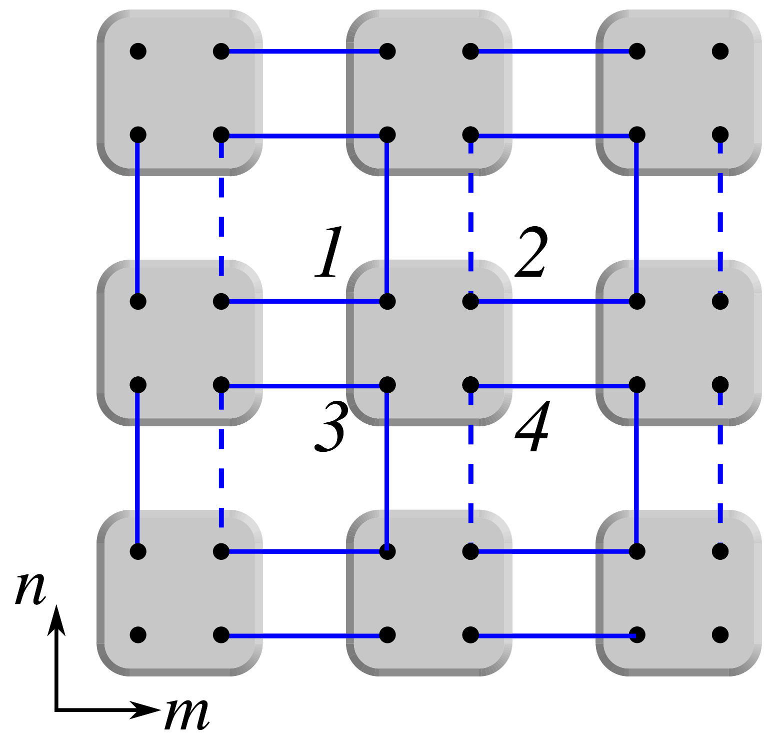
Second Order Topological Superconductors

- Generalize to topological SC ("TSC_2")?
- Replace electron operators by Majorana operators.



- Four corner Majorana modes protected by particle-hole symmetry
- Coupling between Majorana's can be *spontaneously* generated by electron pairing.
- Potential realization in condensed matter systems

Second order TSC (TSC$_2$)



$$H = -2it \sum_{(m,n)} \left[\gamma^2_{m,n}\gamma^1_{m+1,n} + \gamma^4_{m,n}\gamma^3_{m+1,n} - \gamma^2_{m,n}\gamma^4_{m,n+1} + \gamma^1_{m,n}\gamma^3_{m,n+1}\right]$$

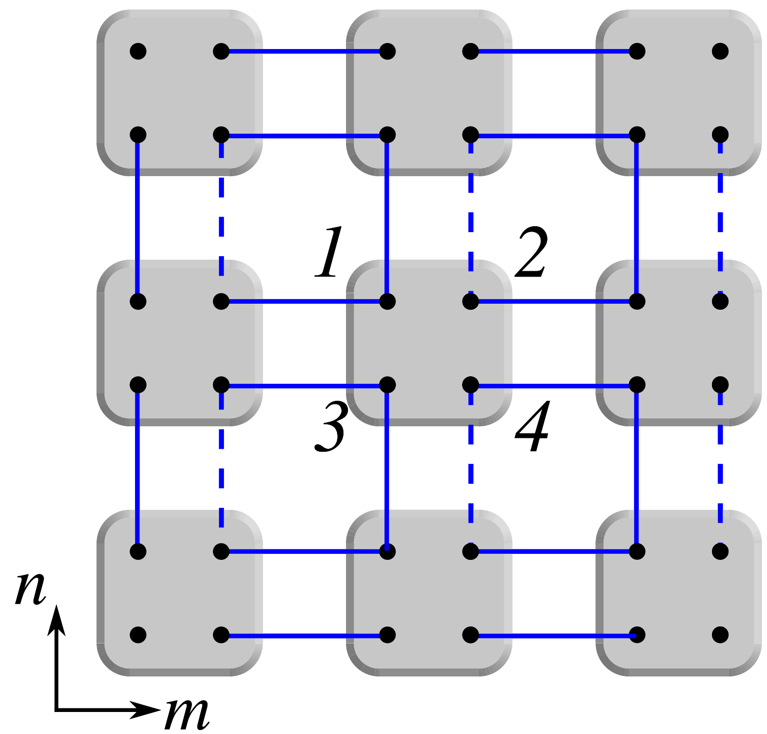
- Rewrite in terms of complex fermions, ideally with gapless band structure & suitable pairing terms.

$$\begin{aligned} c_{\uparrow,2m+1,n} &= (\gamma^1_{2m+1,n} + i\gamma^2_{2m+1,n})/\sqrt{2}, \\ c_{\downarrow,2m+1,n} &= (\gamma^3_{2m+1,n} + i\gamma^4_{2m+1,n})/\sqrt{2}, \\ c_{\uparrow,2m,n} &= (\gamma^3_{2m,n} + i\gamma^4_{2m,n})/\sqrt{2}, \\ c_{\downarrow,2m,n} &= (\gamma^1_{2m,n} + i\gamma^2_{2m,n})/\sqrt{2}. \end{aligned}$$

BdG Hamiltonian, two bands

$$\begin{aligned} \mathcal{H}(\mathbf{k}) =& t\cos k_x \sigma_x \tau_z + t\cos k_y \sigma_y \\ & + \Delta \sin k_x \sigma_x \tau_y + \Delta \sin k_y \sigma_x \tau_x \end{aligned}$$

Second order TSC (TSC_2)



$$H = -2it \sum_{(m,n)} \left[\gamma^2_{m,n}\gamma^1_{m+1,n} + \gamma^4_{m,n}\gamma^3_{m+1,n} - \gamma^2_{m,n}\gamma^4_{m,n+1} + \gamma^1_{m,n}\gamma^3_{m,n+1}\right]$$

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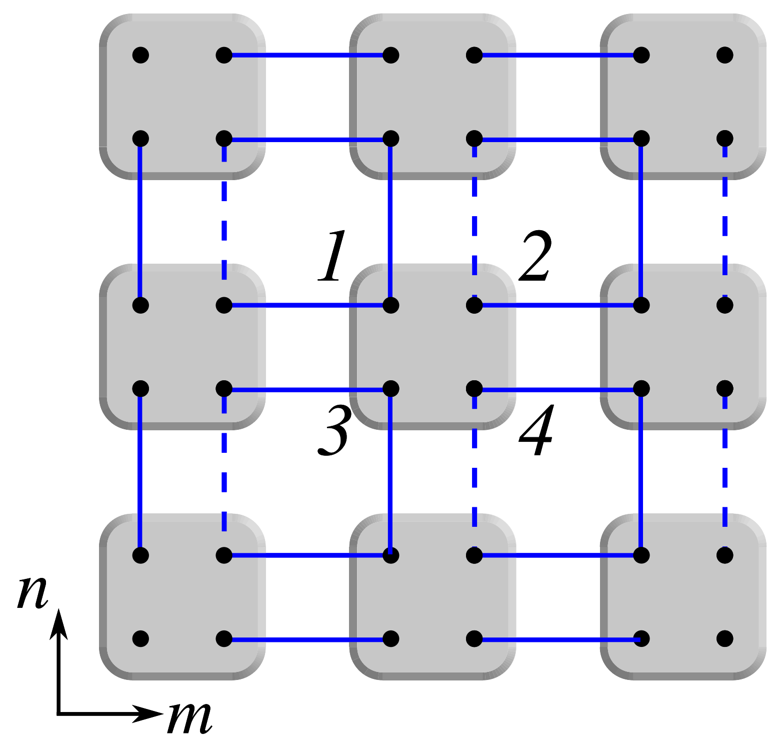
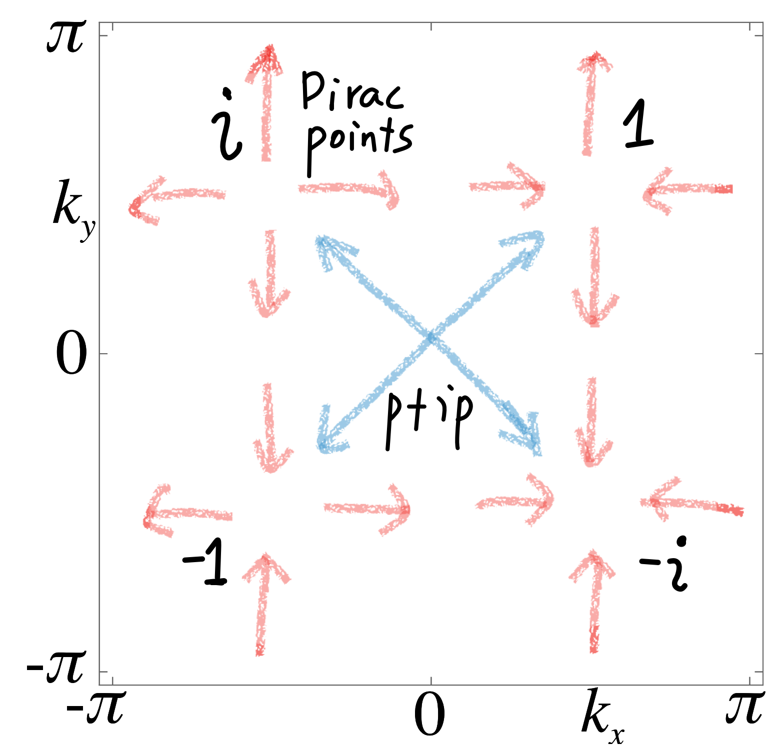
TSC$_2$ from p-wave pairing





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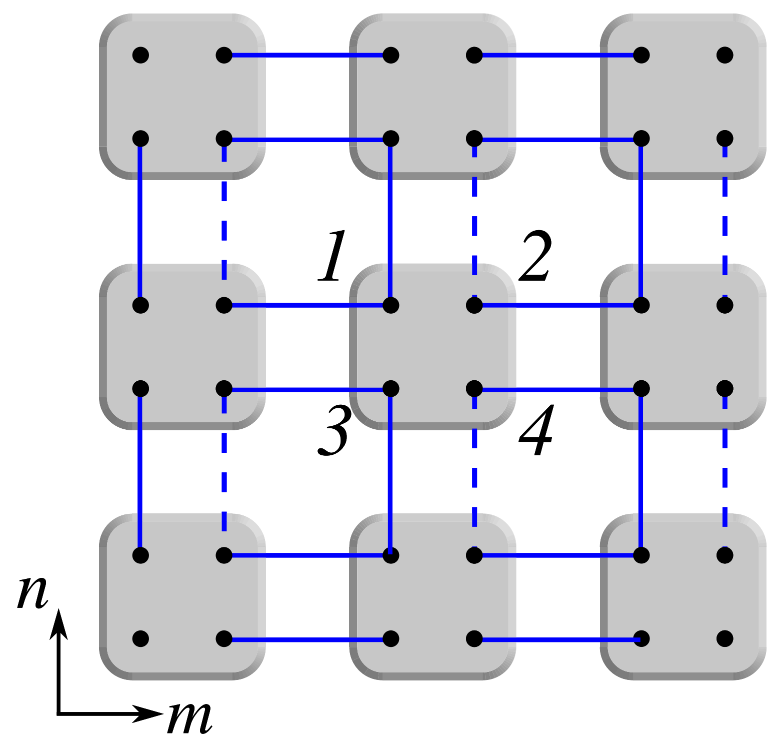
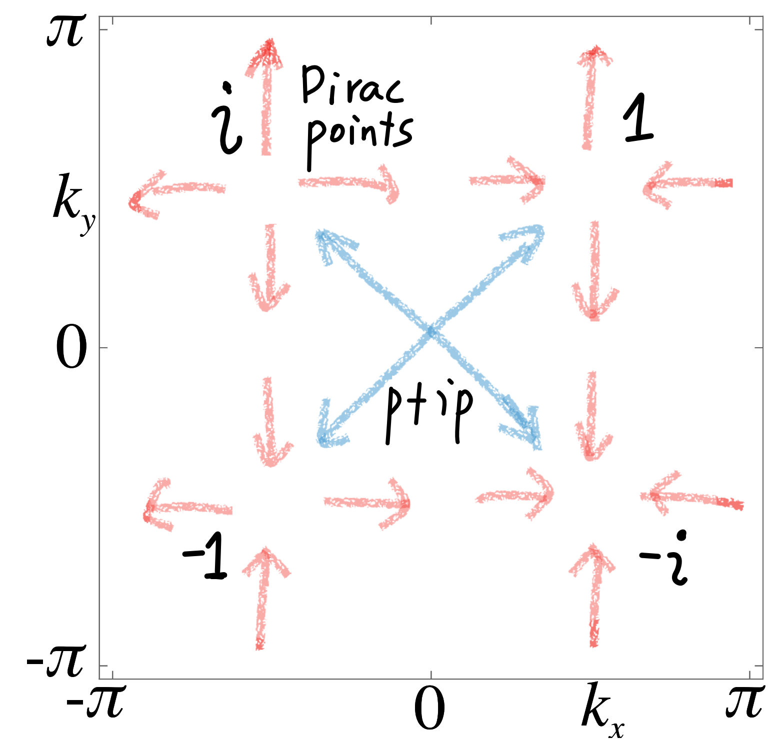
- We found "Dirac+(p+ip)" is *sufficient* for TSC$_2$

TSC_2 from p-wave pairing





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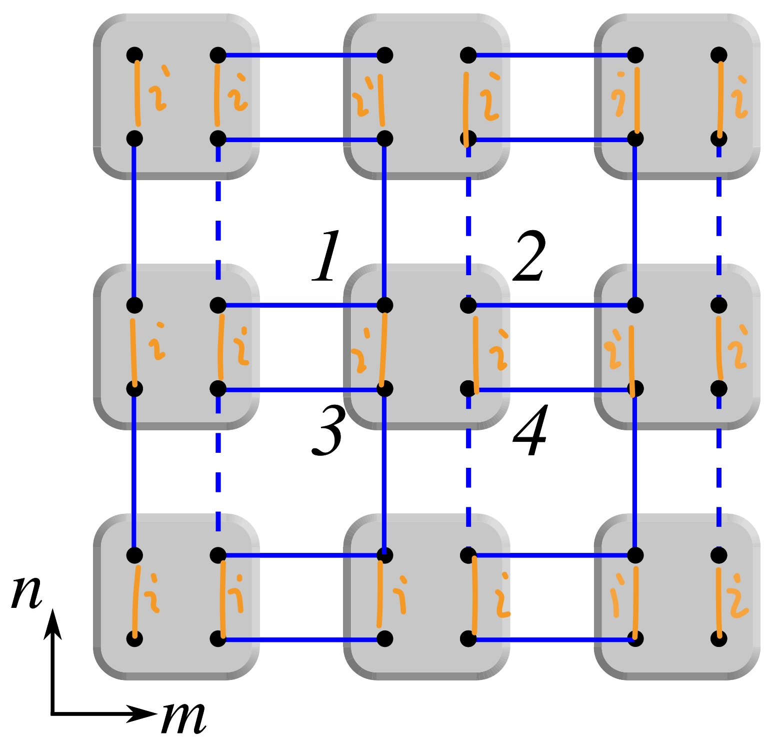
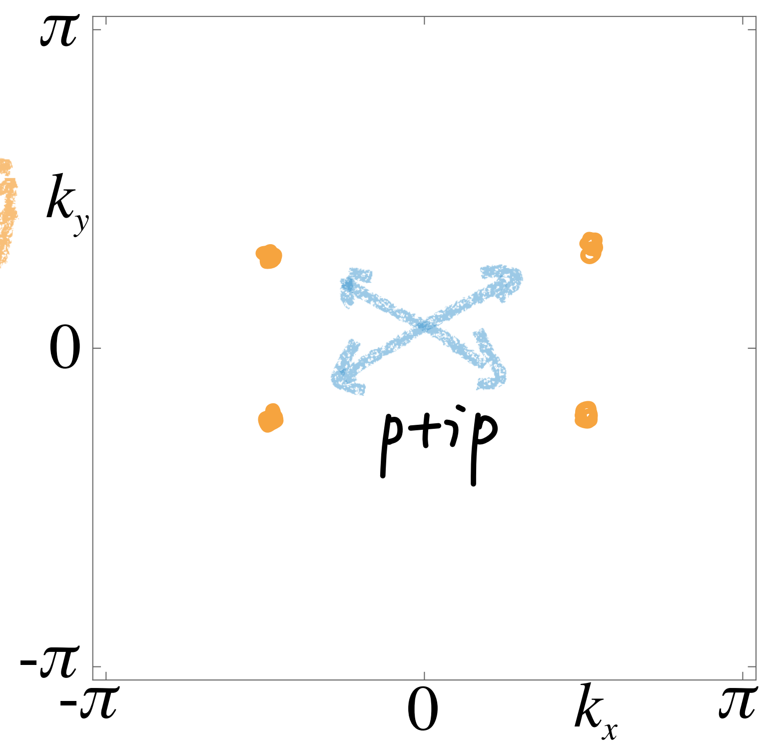
- We found "Dirac+(p+ip)" is *sufficient* for TSC_2
- p+ip pairing is usually associated with a TSC with a nonzero Chern number; here the Dirac points make our case different.

TSC$_2$ from p-wave pairing





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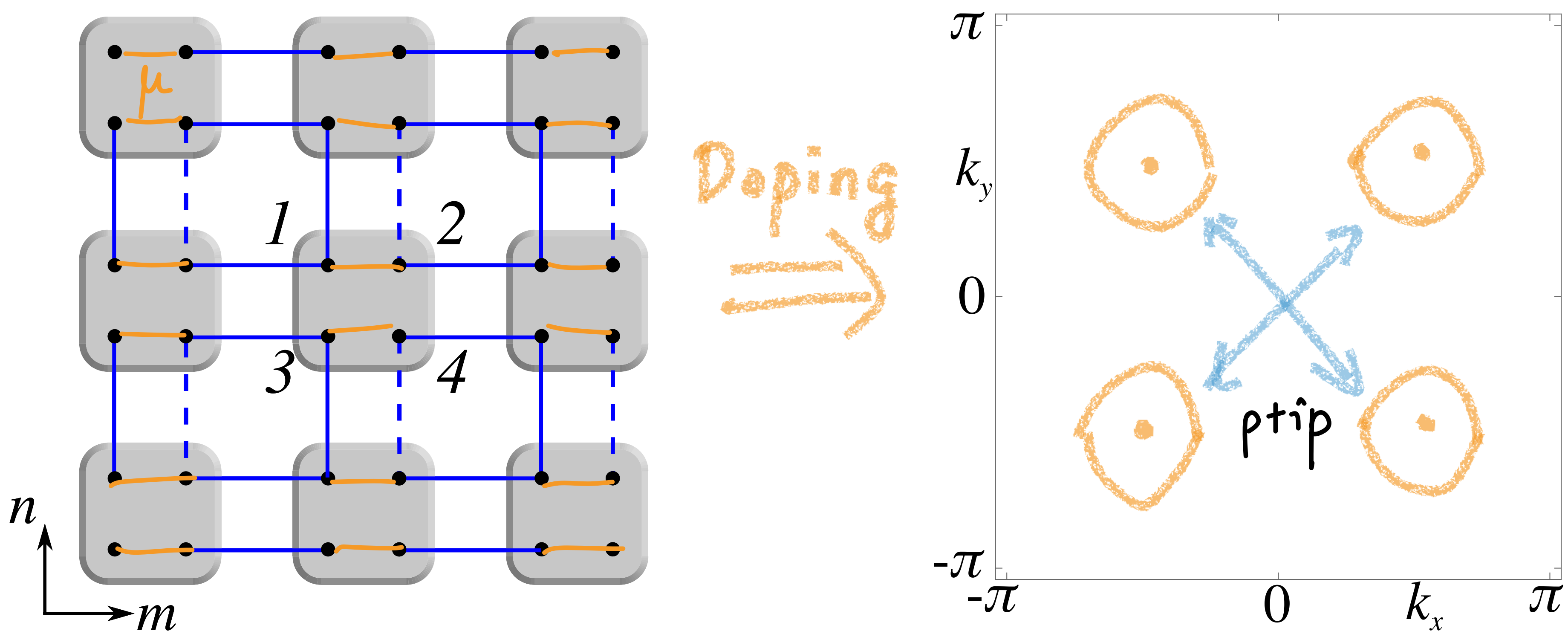
- “Dirac+(p+ip)” is *a sufficient condition* for TSC$_2$
- Intra-plaquette couplings correspond to shifting, gapping, or doping the nodes.

TSC$_2$ from p-wave pairing



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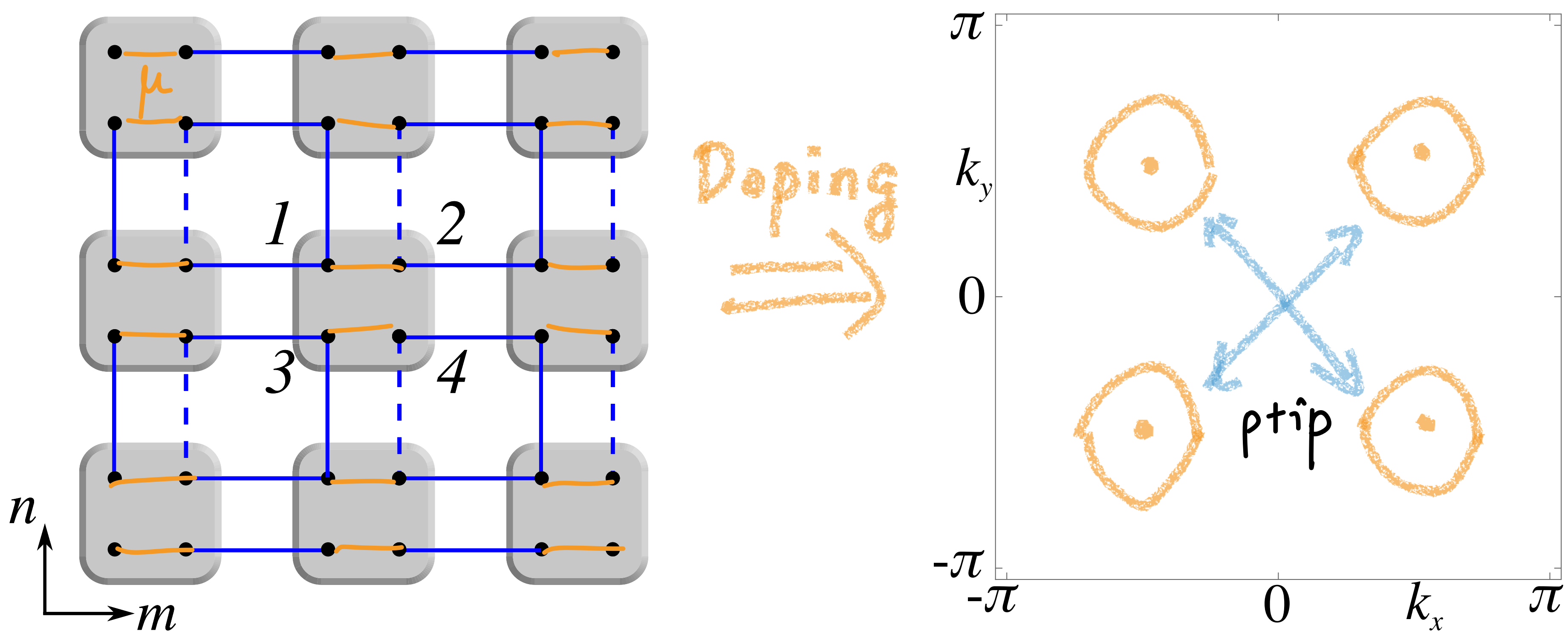

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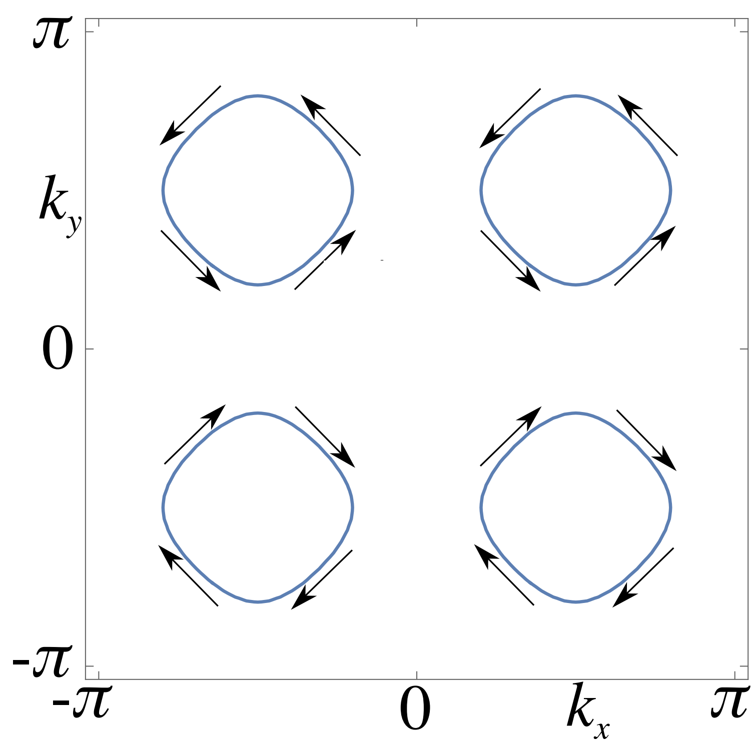
- "Dirac+(p+ip)" is *a sufficient condition* for TSC$_2$
- Intra-plaquette couplings correspond to shifting, gapping, or doping the nodes.
- They do not affect topology until critical values

p-wave pairing from electron interaction

- Let's begin with the following normal state Hamiltonian

$$\boxed{\mathcal{H}_{\rm N} = t\cos k_x \sigma_x + t\cos k_y \sigma_y - \mu}$$



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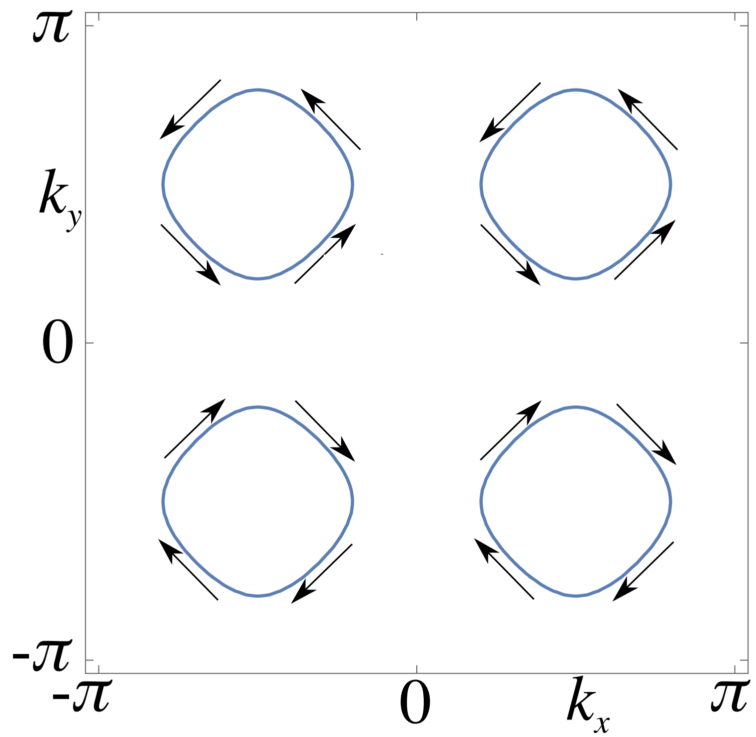
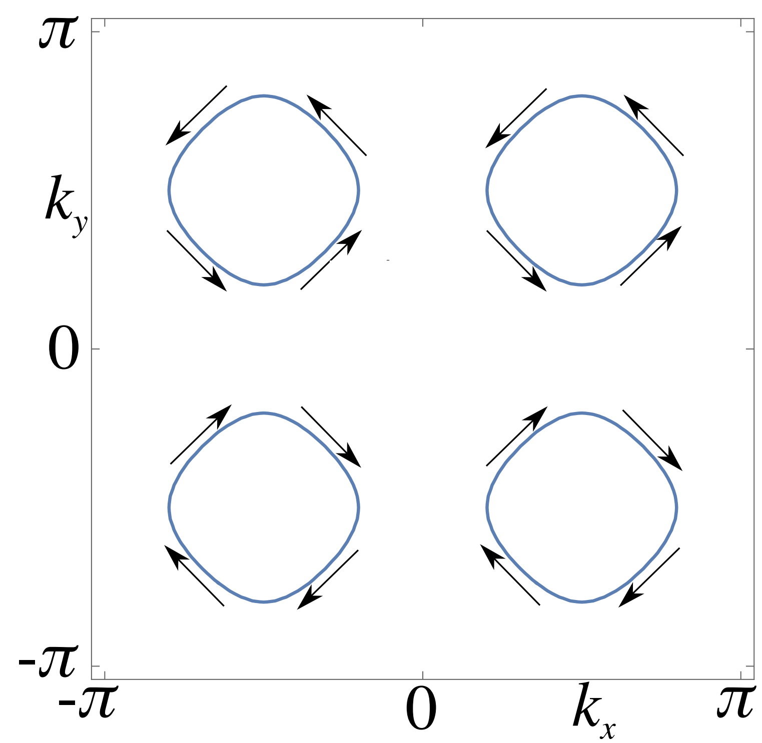


- How can p+ip pairing be *spontaneously* induced?

p-wave pairing from electron interaction

- Let's begin with the following normal state Hamiltonian

$$\boxed{\mathcal{H}_{\mathrm{N}} = t\cos k_x \sigma_x + t\cos k_y \sigma_y - \mu}$$

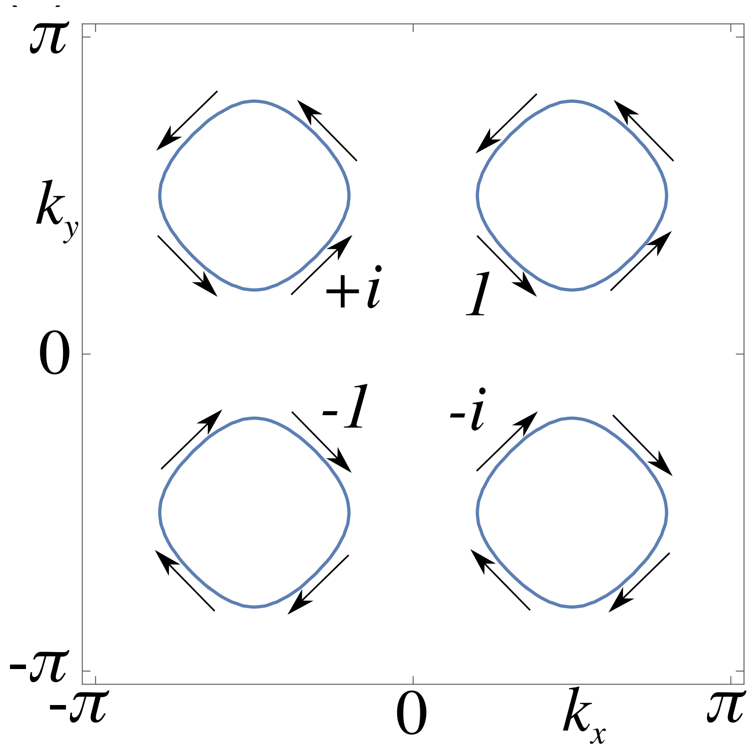


- How can p+ip pairing be *spontaneously* induced?

- First, k and -k points in the BZ have the *same* spin — s-wave spin-singlet pairing cannot be induced! 😊

p-wave pairing from weak interaction

- Consider an attractive interaction



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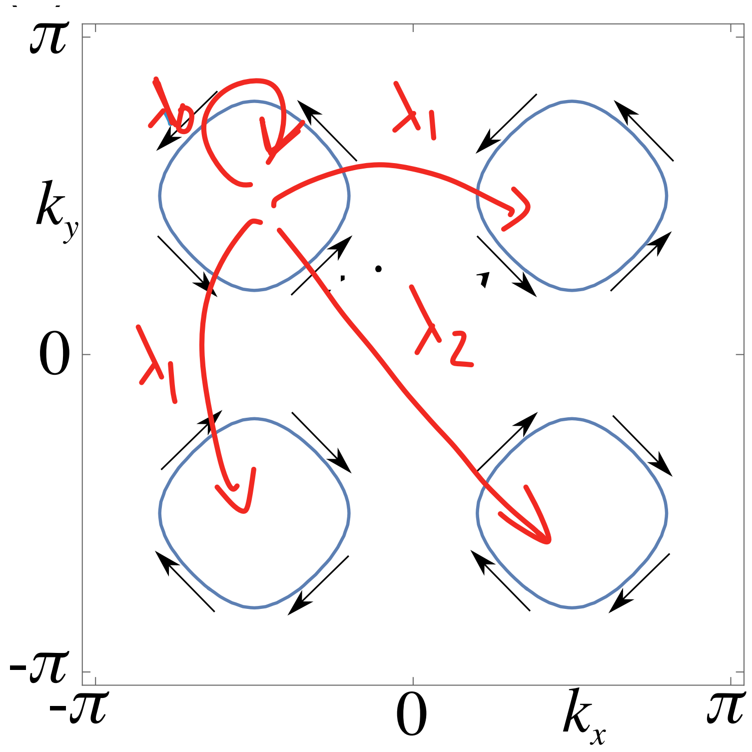

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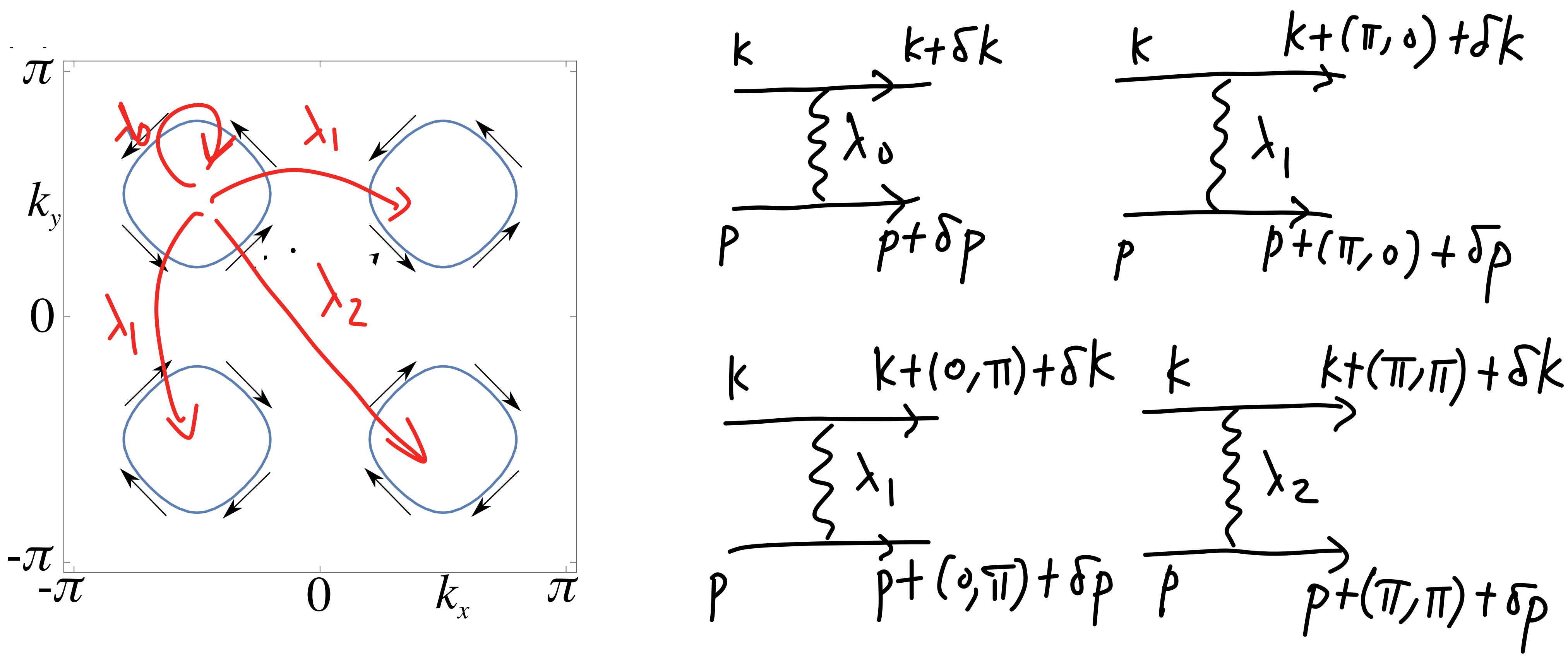

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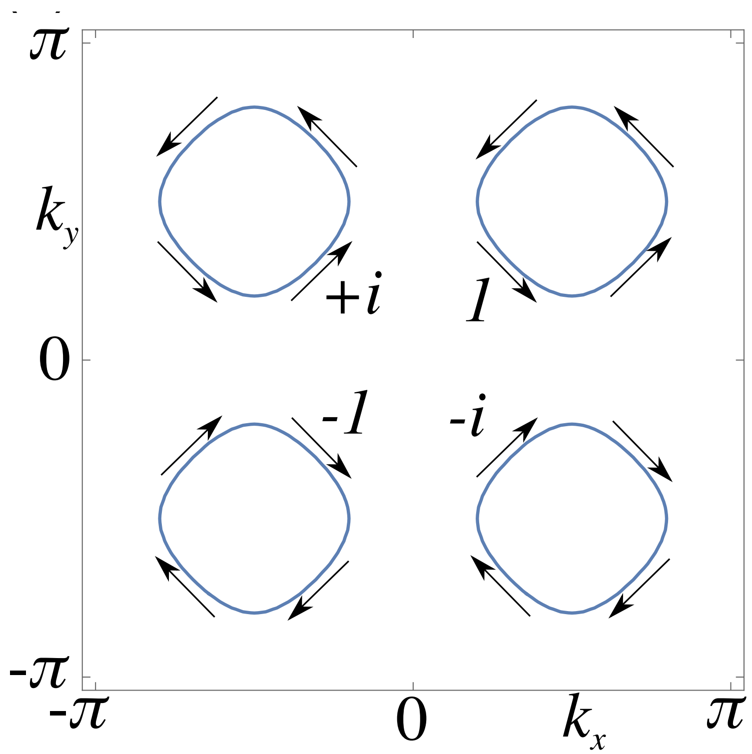

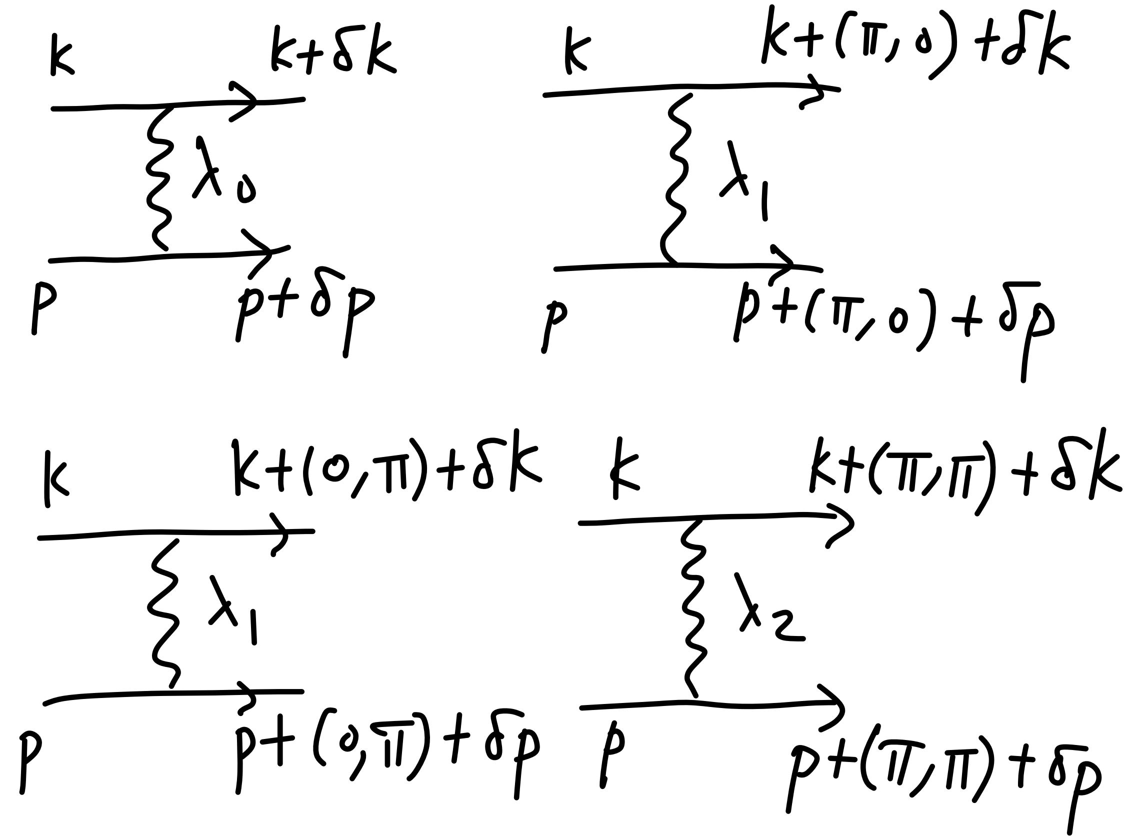

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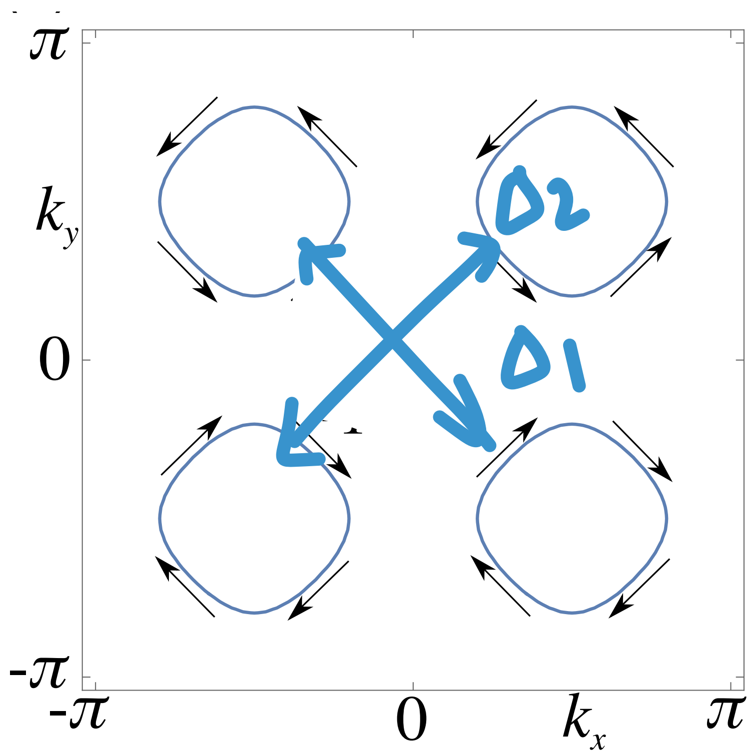

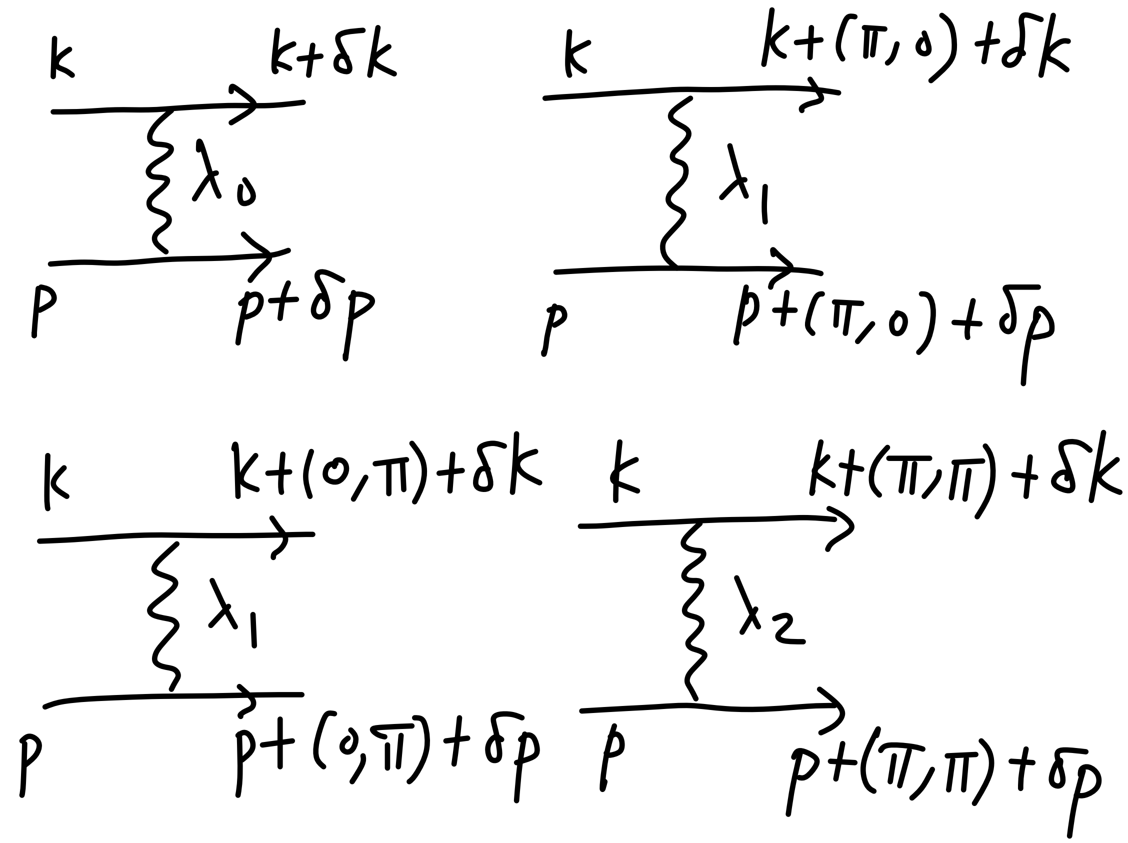

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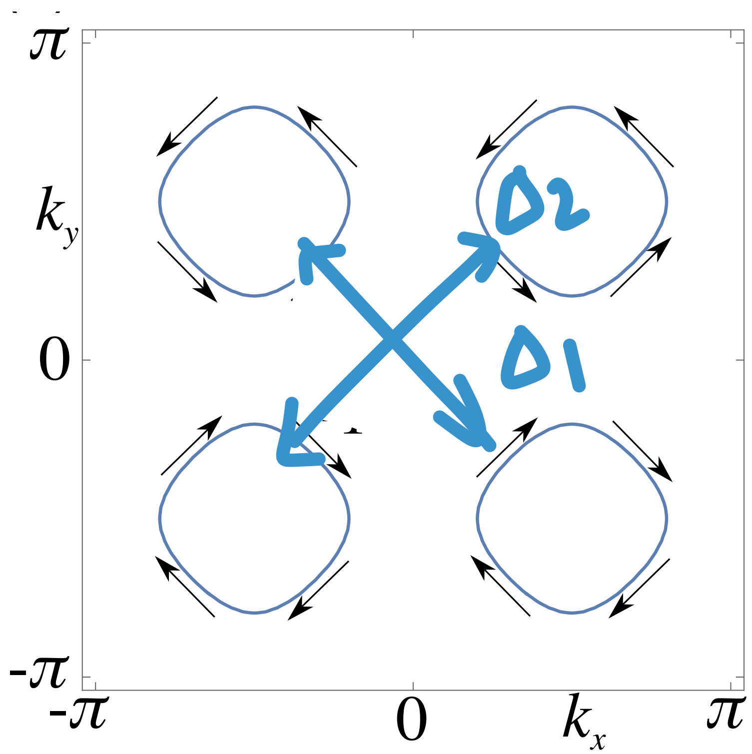

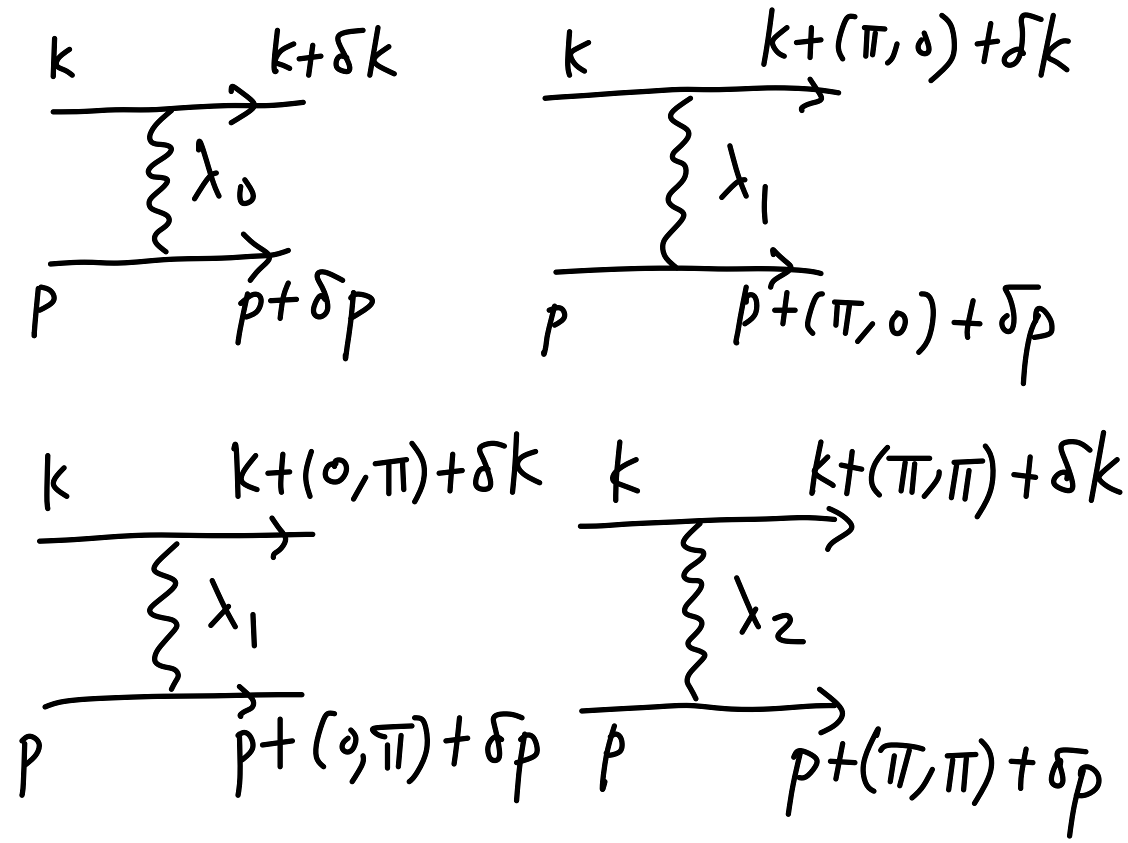

- p-wave pairing between two pockets

$$\Delta_1 = \frac{\lambda_0 - \lambda_2}{8} N(0) \ln \frac{\Lambda}{T}\ \Delta_1$$

$$\Delta_2 = \frac{\lambda_0 - \lambda_2}{8} N(0) \ln \frac{\Lambda}{T}\ \Delta_2$$

For $\lambda_0 > \lambda_2$

$$T_c = \Lambda \exp\left[\frac{8}{(\lambda_0 - \lambda_2) N(0)}\right]$$

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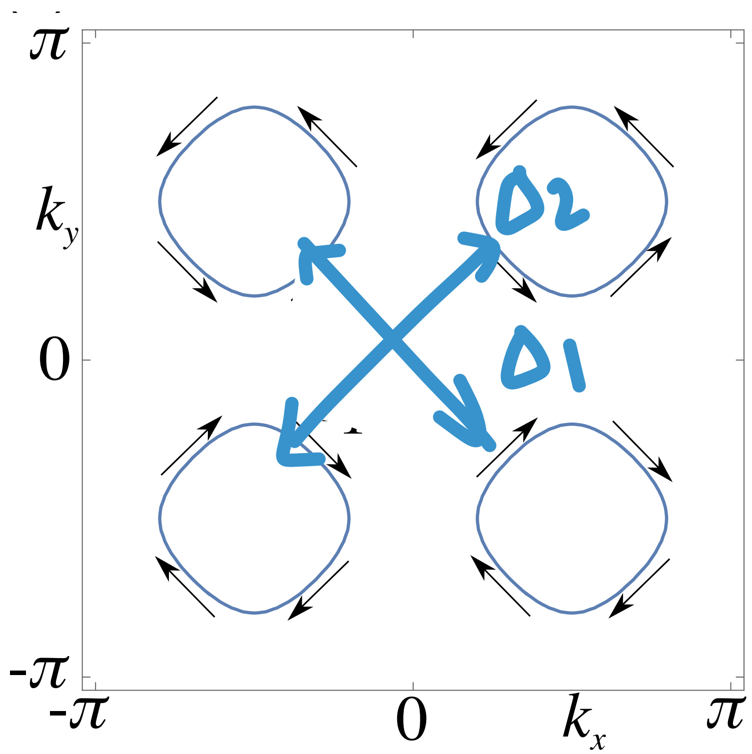

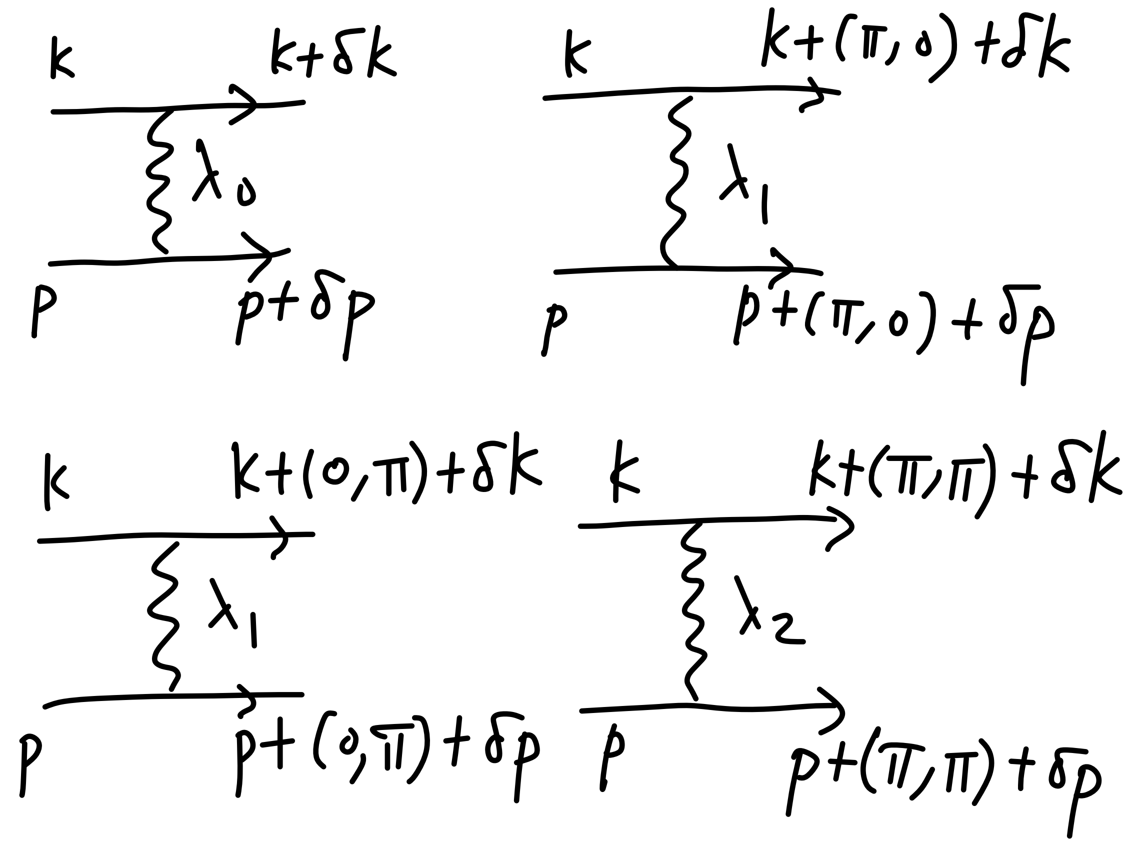

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- To further get p+ip pairing, note that $\Delta_{1,2}$ are coupled at the *quartic level* in the free energy

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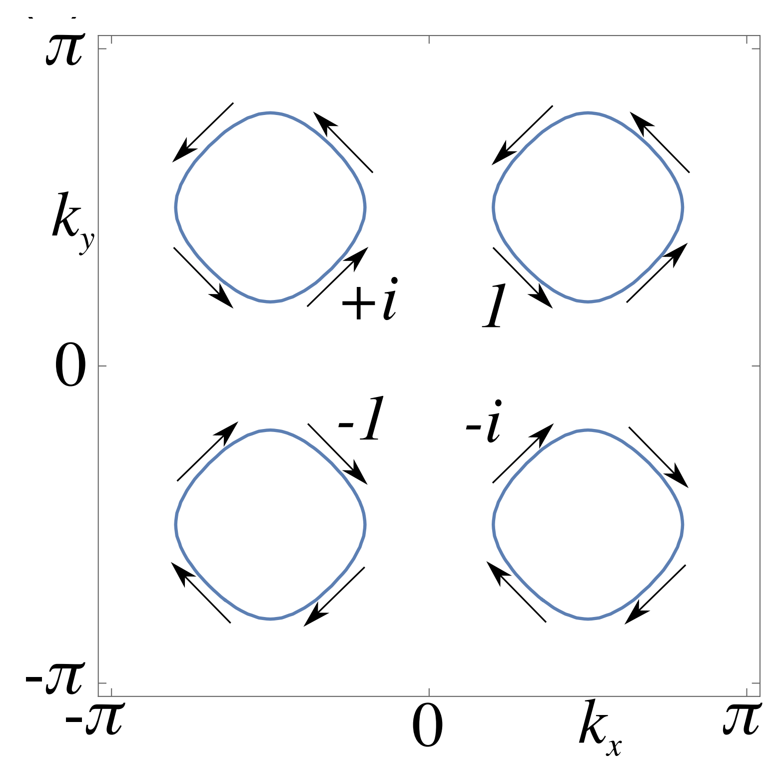
Exactly our TSC$_2$ state with p+ip pairing

- Requires a very simple form of interaction, but an exotic band structure



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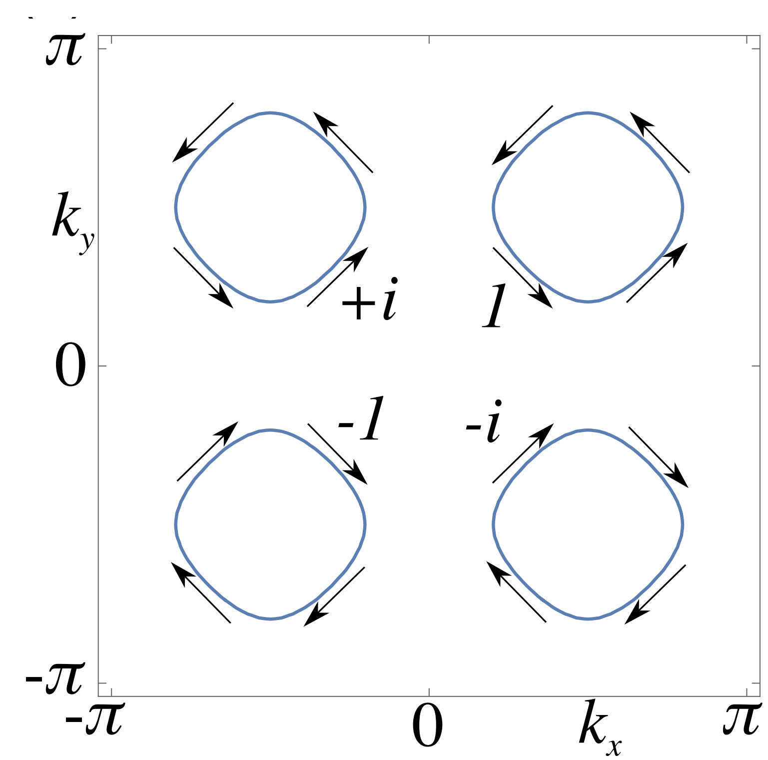
- May also be realized in cold atoms with artificial SOC

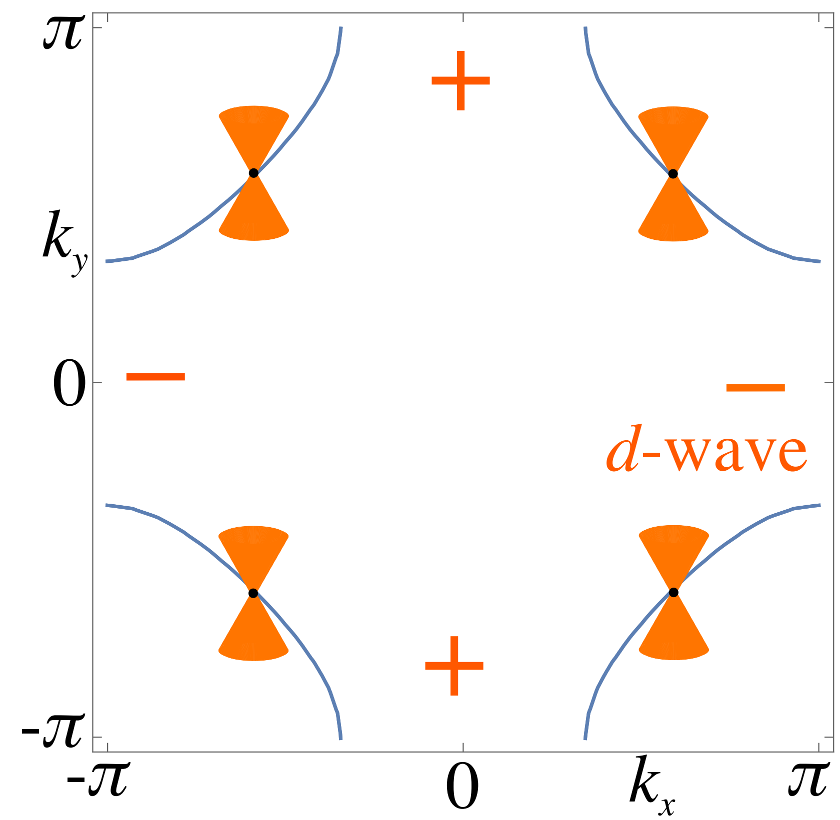
Grust-Li-Bloch-Demler PRA (2017)



Where else can one find four Dirac points?

TSC_2 From Combined d- and p-wave Pairing

- Dirac nodes from a single band d-wave superconductor



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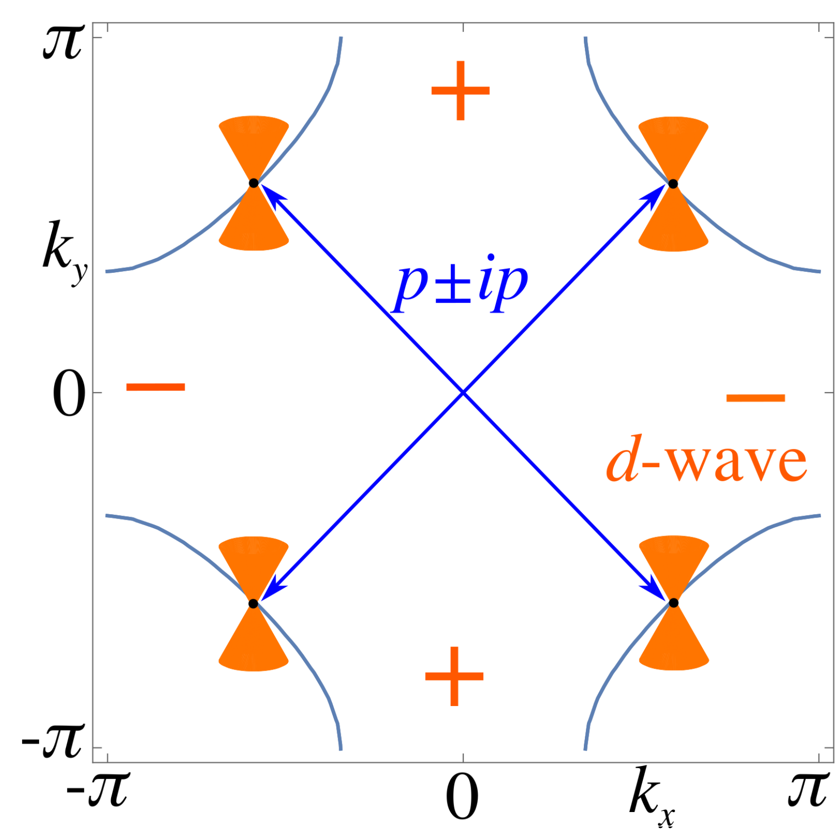

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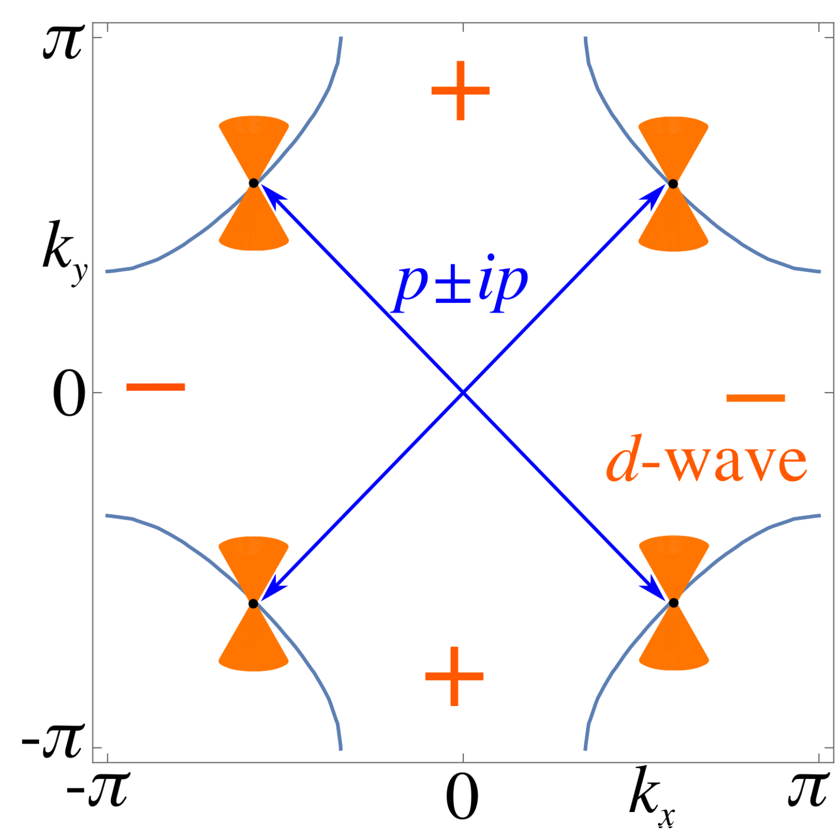

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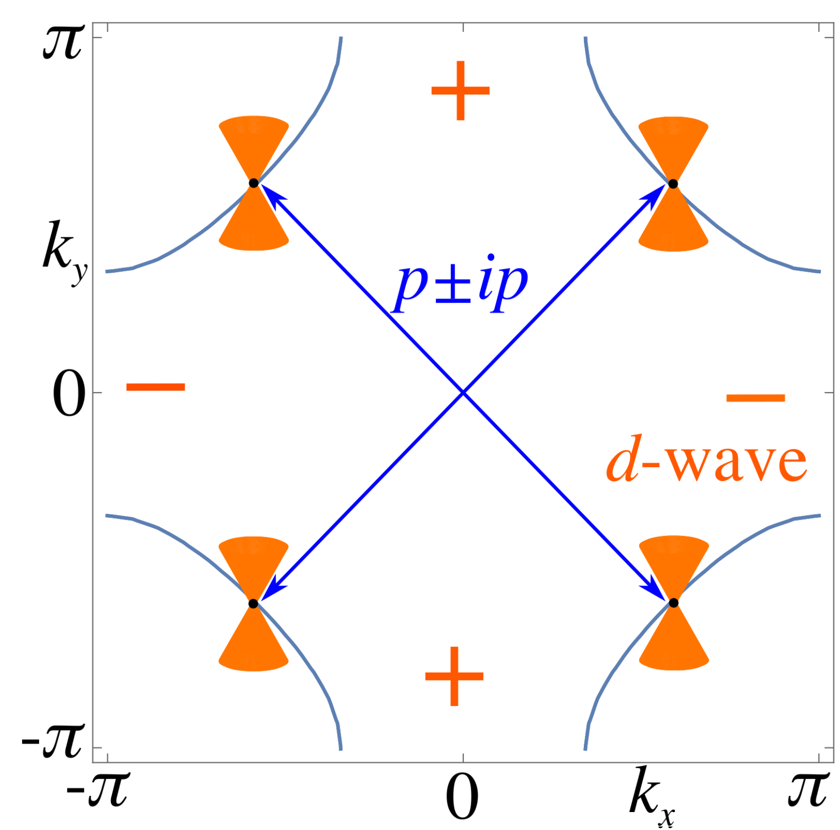

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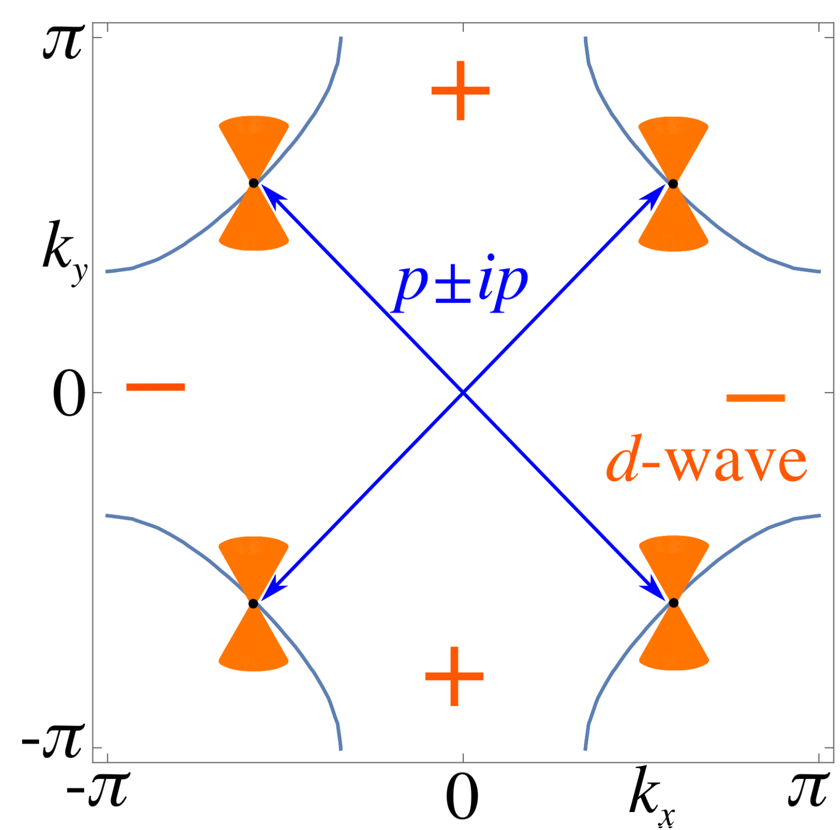
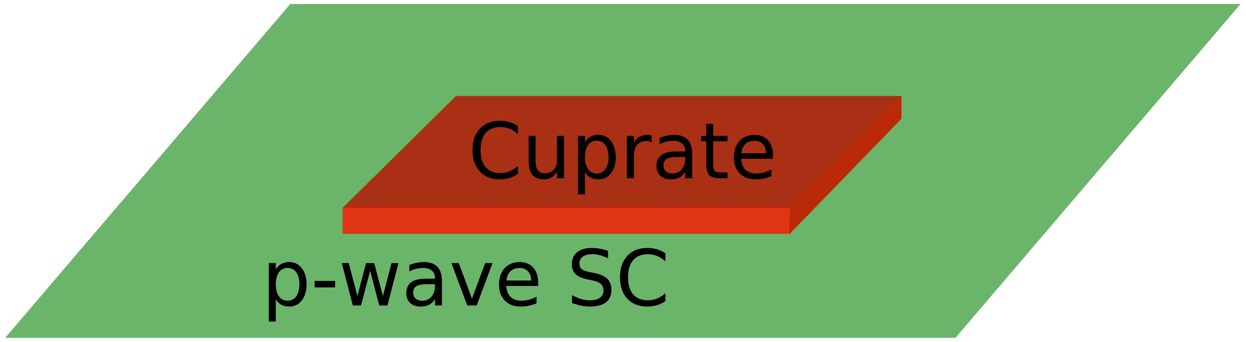


(p+id)-wave pairing on a featureless FS

p+id from a SC heterostructure

- Consider coupling between a cuprate SC on top of a p-wave SC.



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Yan-Song-Wang arXiv:1803.08545, Wang-Liu-Lu-Zhang arXiv:1804.04711

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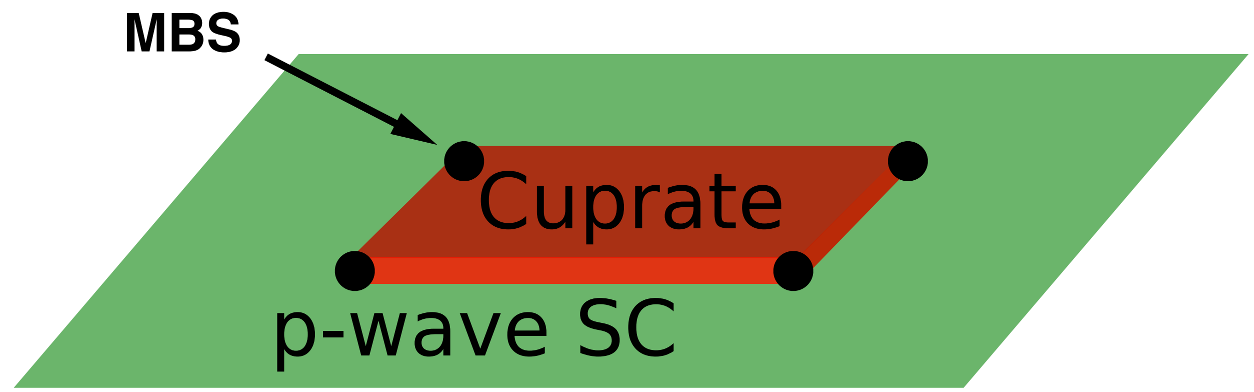

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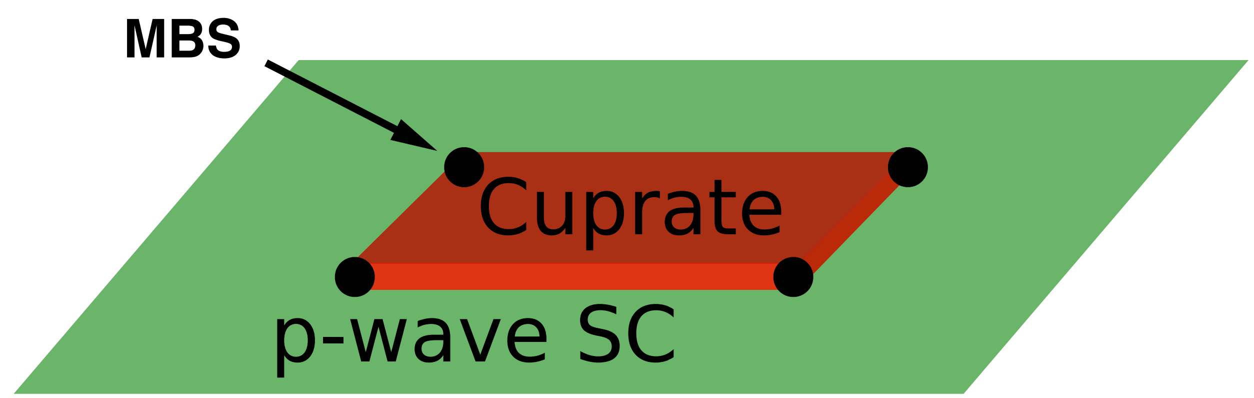

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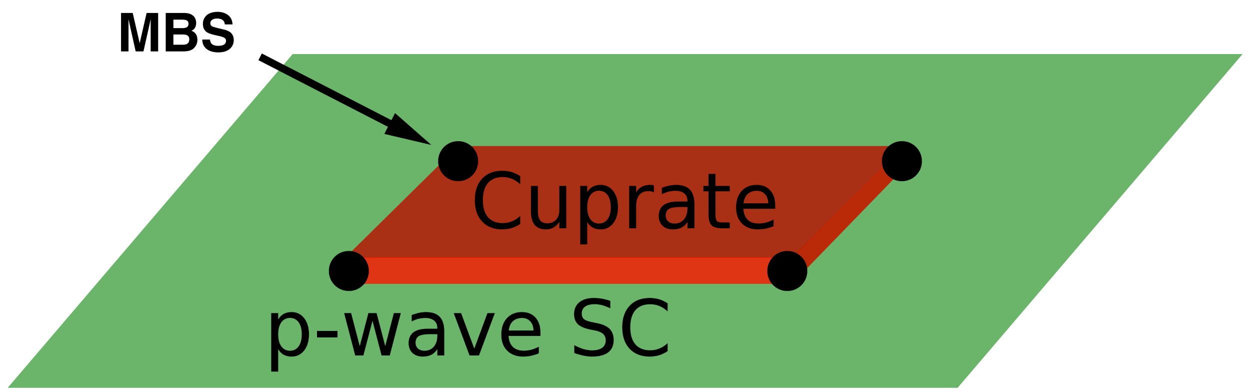

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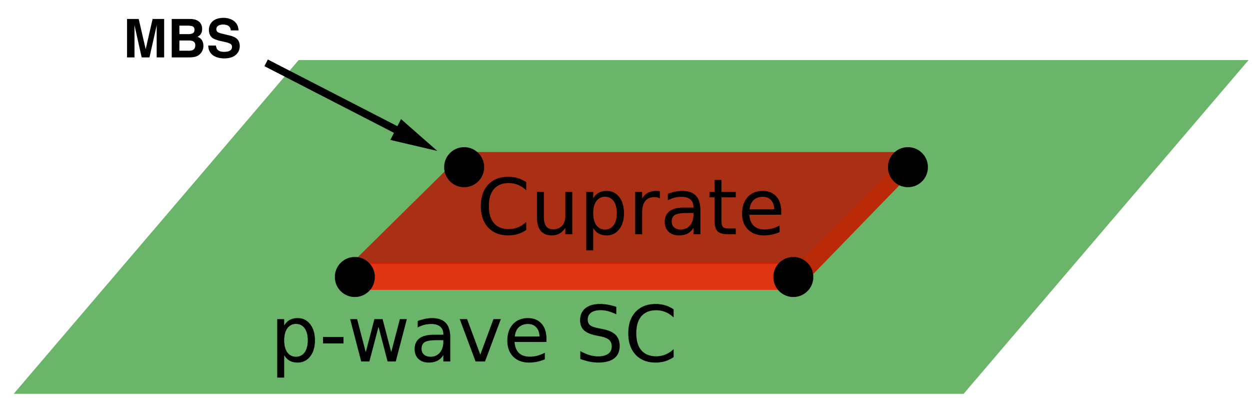

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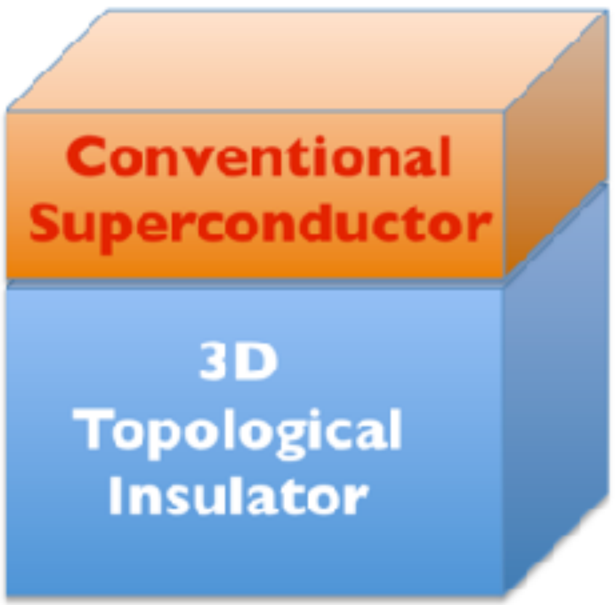
- Candidate material for p-wave SC: $Cu_xBi_2Se_3$, YPtBi

Fu-Berg 2011, Paglione Group 2016…

“Mimicking” a Topological Superconductor

- Fu-Kane: s-wave SC proximitized to a surface Dirac cone of a topological insulator



Fu & Kane (2008)

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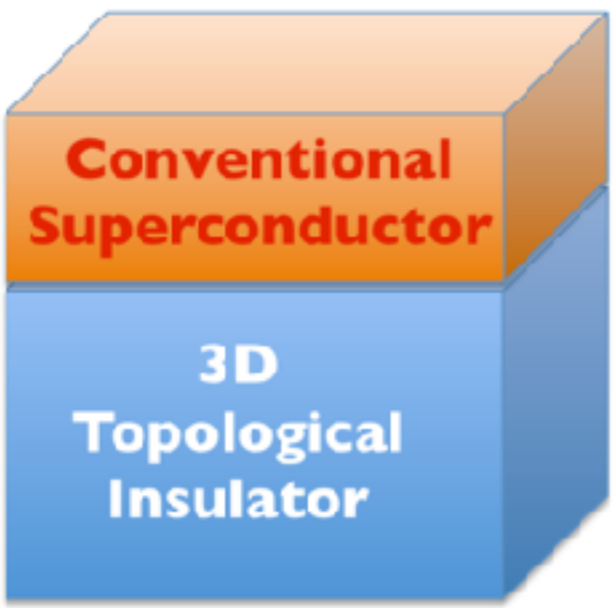

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- $FeSe_{0.45}Te_{0.55}$ is a "connate Fu-Kane SC" that has *both* s-wave SC *and* surface Dirac cone
  - Relatively high Tc at 14.5 K, even higher in monolayer
  - Gives us more room to get creative

P. Zhang et al., Science (2018)
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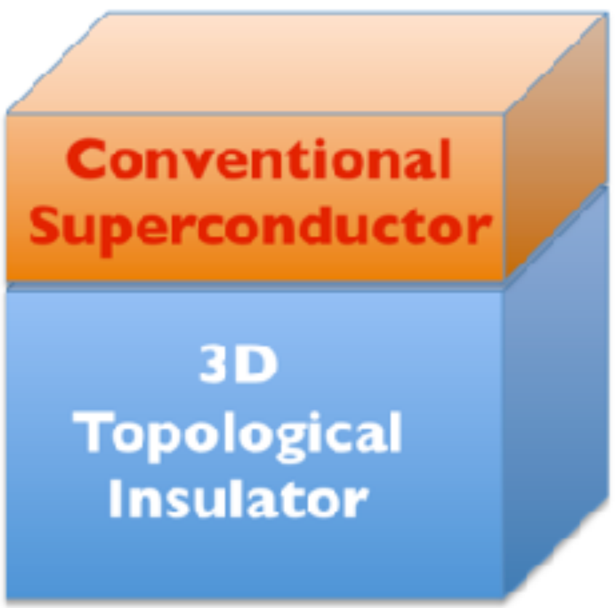

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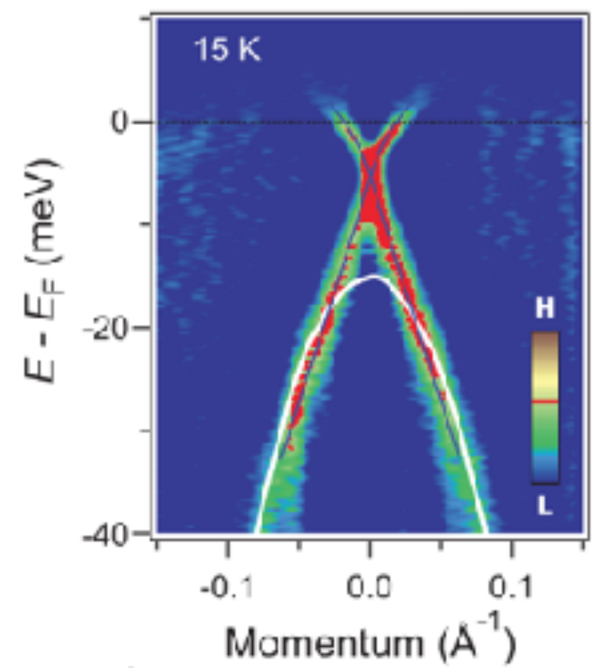

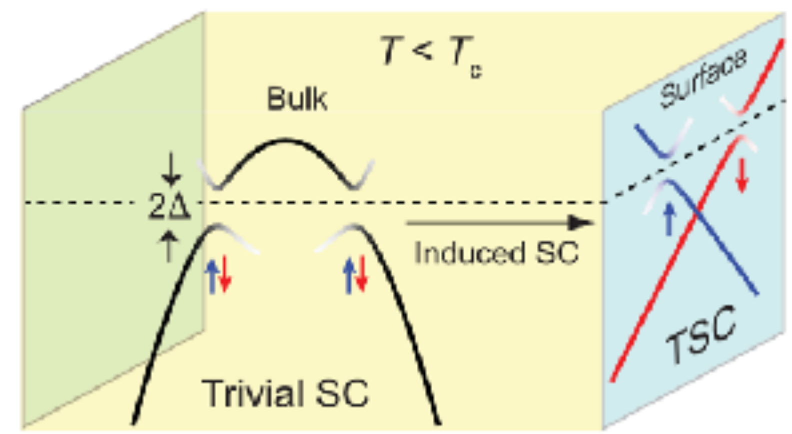

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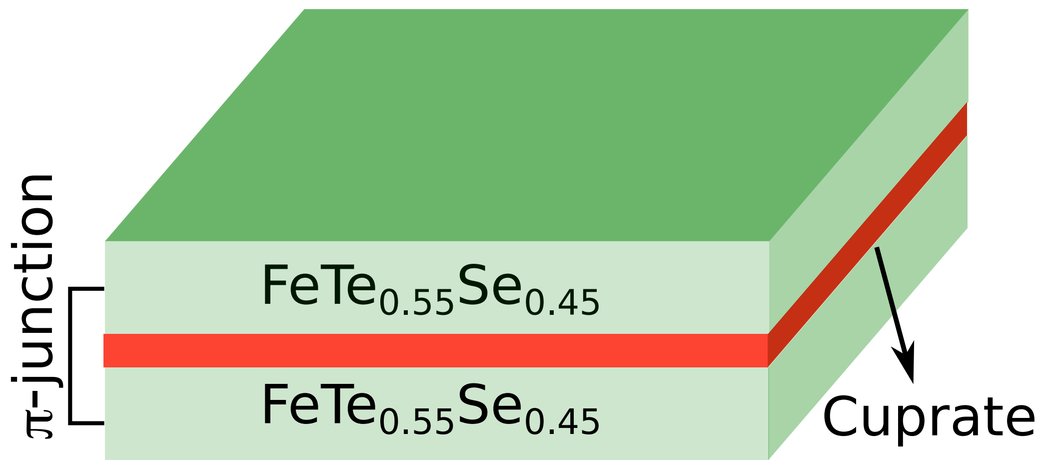

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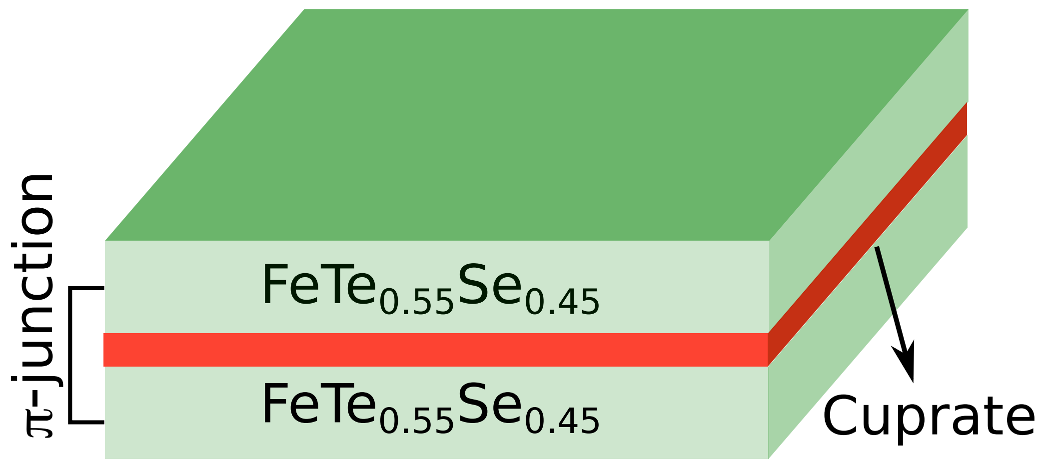

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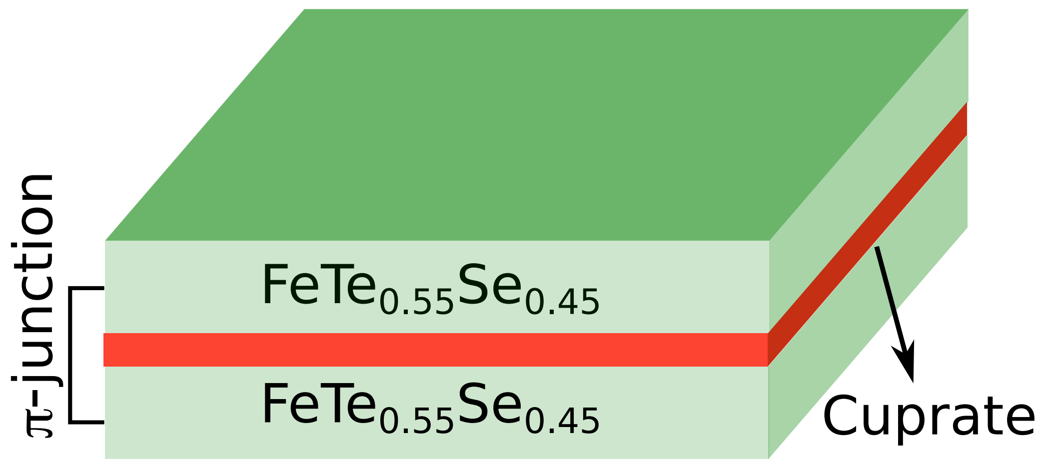

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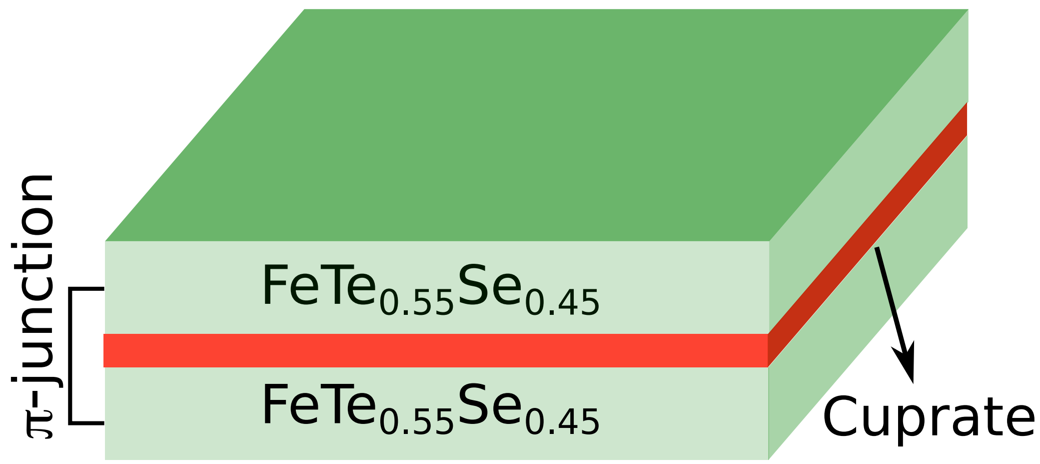

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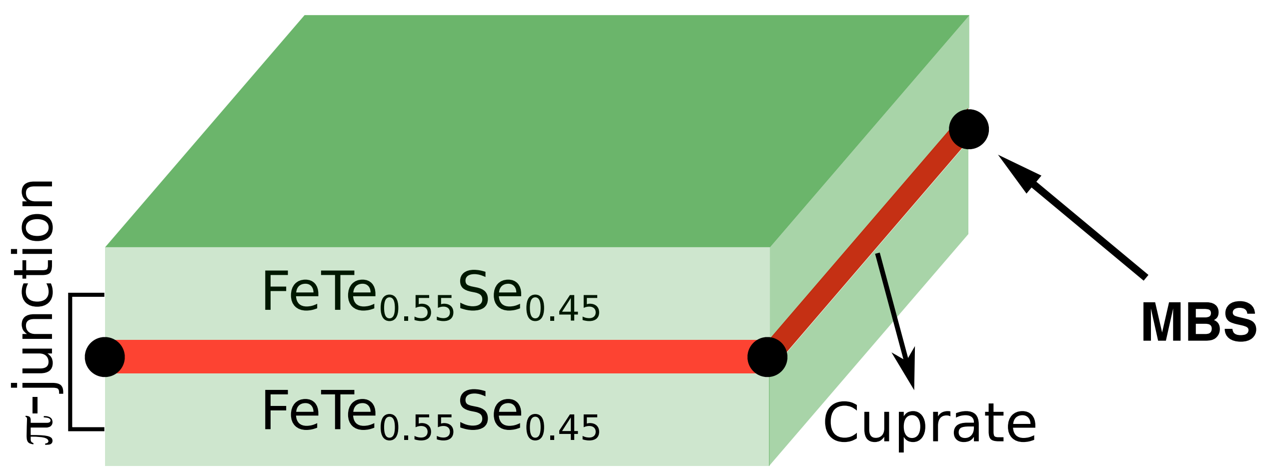

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Summary

- Extended the concept of TSC to TSC_2
- Two classes of TSC_2 phases, both spontaneously induced by weak interactions
- Potential high-Tc TSC_2 from superconducting heterostructure
- Open questions: Response theory? TSC_3 in 3d? Corner/hinge Parafermions? 2nd order SPT's?