Non-BCS superconductivity near a quantum-critical point

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May, 2018

Where do incoherent fermions come from and how do they pair?

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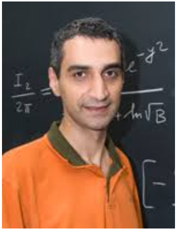

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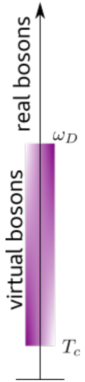
PRL **117**, 157001 (2016)

What is BCS?

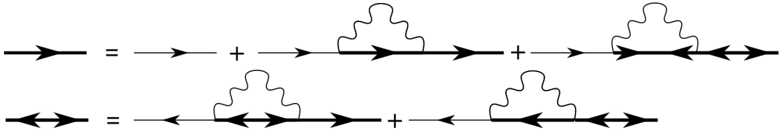
- Pairing boson is gapped – ω_D.
- Coupling is weak $\lambda \ll 1$. ($\lambda \sim g^2/\omega_D^2$)
- $T_c \sim \omega_D e^{-1/\lambda} \ll \omega_D$.
- Only virtual boson's transitions.



  - Integrate out bosons.
  - Hubbard-Stratonovich. . .
  - etc.

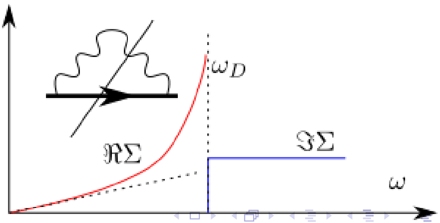
What is BCS?

We can do better. Let's increase the coupling constant.

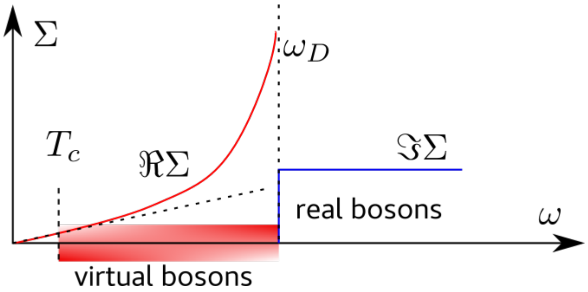


- To find T_c we need to linearize these equations.

- $\Im\Sigma(\omega < \omega_D) = 0$.



What is BCS?



- $T_c \sim \omega_D e^{-\frac{1+\lambda}{\lambda}} \ll \omega_D$.
- All fermions are coherent.
- Only virtual bosonic transitions.

Still BCS.

What is BCS?

Lessons:

- Σ suppresses T_c.
- The linearized equation

$$\Delta_m = \frac{\lambda}{2} T \sum_n \frac{\Delta_n}{|\omega_n|} \frac{\omega_D^2}{(\omega_n - \Omega_m)^2 + \omega_D^2}$$

as $T_c \ll \omega_D$ can be written

$$1 = \lambda \int_{T_c}^{\omega_D} \frac{d\omega}{|\omega|}.$$

All electronic Matsubara frequencies up to ω_D participate.

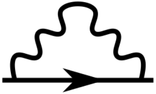
Towards larger λ.

What to keep from BCS?

- Migdal theorem – neglect vertex corrections – large N.
- Momentum factorization.

What will change

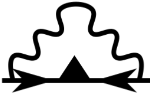
- Boson propagator.
- Electron self energy.

$\Sigma =$ 

$\Pi =$ 

 $= \frac{1}{G_0^{-1} - \Sigma}$

 $= \frac{1}{D_0^{-1} - \Pi}$

 $=$ 

Limit $\lambda \to \infty$.

We want to consider the limit $\lambda \to \infty$.

- In order to keep T_c finite we need to take $\omega_D \to 0$.

Different for different models.

- For phonon $T_c \sim \omega_D \sqrt{\lambda}$. (P. B. Allen and R. C. Dynes, PRB **12**, 905 (1975).)
- For Spin-Fermion model $T_c \sim \omega_D \lambda^2$.
- etc.

γ-Model.

- Cannot integrate out bosons.
- Low energy particle-hole excitations.
- Landau damping for bosons.

Landau damping + momentum factorization: $\int dqD(\omega_n, q) \sim \frac{1}{|\omega_n|^\gamma}$

- color superconductivity $\gamma = 0_+$, (D. T. Son, PRD **59**, 094019 (1999); A. V. Chubukov and J. Schmalian, PRB **72**, 174520 (2005).),
- spin- and charge-mediated pairing in $D = 3 - \epsilon$ dimension $\gamma = O(\epsilon) \ll 1$, D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, PRB **82**, 045121 (2010); M. A. Metlitski, D. F. Mross, S. Sachdev, and T. Senthil, PRB **91**, 115111 (2015); S. Raghu, G. Torroba, and H. Wang, PRB **92**,205104 (2015).
- 2D pairing model with interaction peaked at $2k_F$: $\gamma = 1/4$ (N. E. Bonesteel, I. A. McDonald, and C. Nayak, PRL **77**, 3009 (1996).); S. Lederer, Y. Schattner, E. Berg, and S. A. Kivelson, PRL **114**, 097001 (2015); Z. Wang, W. Mao, and K. Bedell, PRL **87**, 257001 (2001); R. Roussev and A. J. Millis, PRB **63**, 140504 (2001); A. V. Chubukov, A. M. Finkelstein, R. Haslinger, and D. K. Morr, PRL **90**, 077002 (2003).
- etc., etc., etc.,

Instability

Equation for T_c

$$\Phi(\omega_m) = g^\gamma \pi T \sum_{n \neq m} \frac{\Phi(\omega_n)}{|\omega_n + \Sigma(\omega_n)|} \frac{1}{|\omega_m - \omega_n|^\gamma}$$

$$\Sigma(\omega_m) = g^\gamma \pi T \sum_{n \neq m} \frac{\text{sgn}(\omega_n)}{|\omega_m - \omega_n|^\gamma}$$

In order to check the competition between Σ and Φ we use large N

(S. Raghu, G. Torroba, and H. Wang, PRB **92**, 205104 (2015).)

$$\Phi(\omega_m) = \frac{g^\gamma}{N} \pi T \sum_{n \neq m} \frac{\Phi(\omega_n)}{|\omega_n + \Sigma(\omega_n)|} \frac{1}{|\omega_m - \omega_n|^\gamma}$$

$$\Sigma(\omega_m) = g^\gamma \pi T \sum_{n \neq m} \frac{\text{sgn}(\omega_n)}{|\omega_m - \omega_n|^\gamma}$$

Linear equation at $T = 0$.

At $T = 0$

$$\Sigma(\omega) = \frac{g^\gamma}{1-\gamma}|\omega|^{1-\gamma}\text{sgn}(\omega)$$

$$\Phi(\Omega) = \frac{1}{N}\frac{1-\gamma}{2}\int \frac{\Phi(\omega)}{|\omega-\Omega|^\gamma|\omega|^{1-\gamma}}\frac{d\omega}{1+(1-\gamma)|\omega/g|^\gamma}$$

Not BCS!

- No log at small ω.
- Upper cutoff is provided by bare fermionic ω.

Exact spectrum.

The linear operator defined as

$$\hat{L}\Phi = \frac{1-\gamma}{2}\int \frac{\Phi(\omega)}{|\omega-\Omega|^{\gamma}|\omega|^{1-\gamma}} \frac{d\omega}{1+(1-\gamma)|\omega/g|^{\gamma}}$$

has a continuous spectrum

$$0 < E < E_c = (1-\gamma)\frac{\Gamma^2(\gamma/2)}{\Gamma(\gamma)}\frac{\cos^2(\pi\gamma/4)}{\cos(\pi\gamma/2)}$$

- For $0 < \gamma < 1$, the largest $E = E_c > 1$, but finite.
- If $N > N_c = E_c$ – no solution.

Similar analysis for BCS.

Similar analysis of the linear operator at $T = 0$ for BCS gives

$$0 < E < E_c = O(\lambda)$$

- At small λ BCS has no solution.
- But the kernel of BCS has a log singularity, which must be cut off!

Comparing to the strong coupling

- No log singularity.
- No solution for $N > N_c$.

Large Σ kills superconductivity.

Not so!

Return to finite T: Σ

Self-energy ($\omega_n = \pi T(2n+1)$)

$$\Sigma(\omega_m) = g^\gamma \pi T \sum_{n\neq m} \frac{\mathrm{sgn}(\omega_n)}{|\omega_m - \omega_n|^\gamma}$$

Notice

$$\Sigma(\omega_{m=0}) = \Sigma(\omega_{m=-1}) = 0$$

and

$$\Sigma(\omega_{m\neq 0,-1}) = g^\gamma T^{1-\gamma} \bar{\Sigma}_m$$

Self-energy is not large at $m = 0, -1$.
$\bar{\Sigma}_m$ does not depend on T and g.

Return to finite T: Φ

- At large N at $T \sim T_c$ we can neglect bare ω_n in comparison with $\Sigma(\omega_n)$ in the equation for Φ.
- We separate $m = 0, -1$ terms ($\Phi_0 = \Phi_{-1}$)

$$\Phi_0 = \frac{g^\gamma}{N(2\pi T)^\gamma}\Phi_0 + \frac{1}{N}\sum_{m>0}\Phi_m \chi_{m,0}$$

$$\Phi_{m>0} = \frac{g^\gamma}{N(2\pi T)^\gamma}\Phi_0 \left[\frac{1}{m^\gamma} + \frac{1}{(m+1)^\gamma}\right] + \frac{1}{N}\sum_{n>0}\Phi_n \chi_{n,m}$$

Set of numbers $\chi_{n,m}$ does not depend on N, g, and T.

- Consider what happens at $T \sim \frac{g}{2\pi}\frac{1}{N^{1/\gamma}}$ and large N.

Large N

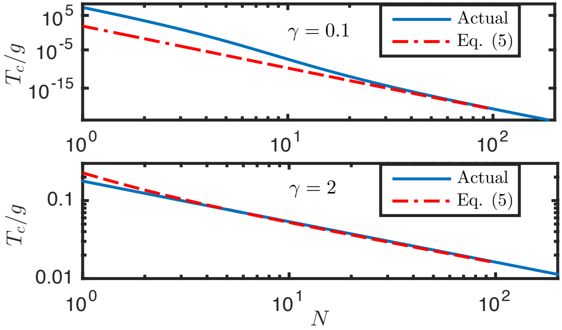
Up to the first $1/N$ correction

$$T_c \approx \frac{g}{2\pi} \frac{1}{N^{1/\gamma}} \left(1 + \frac{\delta_\gamma}{N\gamma}\right),$$

where $\delta_\gamma = O(1)$.

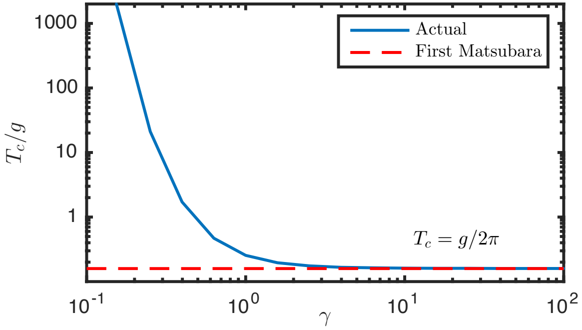
- No N_c. T_c is always finite.
- The corrections go as $1/(N\gamma)$.
- At large N or γ the transition comes from only two fermionic Matsubara frequencies and one bosonic.

Comparison to numerics.



Comparison to numerics.

$N = 1$



At small γ the result fails. We cannot neglect the bare ω term.

Conclusions

- At strong coupling and large N or γ the instability comes from the first two fermionic Matsubara frequencies.
- Very different from BCS, where all frequencies up to $\omega_D \gg T_c$ are involved.
- The upper cutoff is provided by the fermionic <u>bare</u> ω.
- No N_c.
- Unclear role of the "real" bosons.