

Dynamics of Parachute Inflation

A linearised theory of parachute opening dynamics

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1. INTRODUCTION

The inflation of a parachute encompasses problems of aerodynamics, dynamics, and elasticity. The most noticeable effects of the interaction of the events are the opening force and the filling time. Attempts to analyse the opening process and to predict the opening force date back to 1927⁽¹⁾. Since then a considerable number of methods have been published. Some of them are primarily analytical, others more empirical, and several deal with isolated problems. The earliest and most analytically oriented studies are by Scheubel^(2,3) and O'Hara⁽⁴⁾.

Scheubel's leading concepts are a constant filling distance and the validity of model similarities. O'Hara introduced the effect of porosity, an assumption for the inflow velocity, and a mechanical model for the development of the parachute canopy during the inflation. His model is a truncated cone with sides which converge with the suspension lines to the confluence point while the canopy roof is a flat disc.

At the University of Minnesota a method of opening shock calculation was developed⁽⁵⁾ and published in 1961; this method included a modified O'Hara model and is based on the simultaneous equations of motion and continuity. Principal inputs to these equations are numerical values of effective porosity⁽⁶⁾ and apparent mass^(7,8). Subsequently this method was adopted in the US Air Force Parachute Handbook⁽⁹⁾.

The following analysis corrects a conceptual error which appeared in the 1961 method, and introduces a refined inflow function. This improved method was used for calculating parachute opening forces and filling times for which good field test data became available^(10,11,12). The calculated values agree well with the field test results and indicate the effects of several parameters, such as drag coefficient, volume of the inflated canopy, and inflation altitude.

2. EQUATION OF MOTION

In order to illustrate the nature and the importance of the terms contributing to the development of the parachute opening force, a diagram is taken from ref. 13 and shown as Fig. 1.

The force contributions and the total force were calculated from the terms of the equation of motion for a free flying model in the wind tunnel, utilising average values of measured time dependent functions, drag coefficients from steady state tests, and apparent mass terms from ref. 8. The measured total force is also marked in the diagram, and from the agreement between measured and calculated forces one may conclude that the equation of motion includes all important terms and in the right order.

NOTATION

C_D	drag coefficient
c	effective porosity
D	drag, diameter
F	force
m	mass
R	radius
S	area
T	dimensionless time
t	time
V	volume
v	velocity
W	weight, wind tunnel
ρ	density
σ	density ratio

Subscripts

a	apparent
f	filling
g	gravity
i	included
o	nominal, total
p	parachute, projected
s	suspended, snatch when used with velocity

Additional symbols, when used, are explained in the text.

For a free flying, horizontally deployed parachute, with $W_p \ll W_s$, $D_s \ll D_p$, and neglecting gravity terms, the equation of motion is

$$F = m_s \frac{dv}{dt} = -\frac{1}{2} \rho C_D S v^2 - \left(\frac{dm_i}{dt} + \frac{dm_a}{dt} \right) \cdot v - (m_p + m_i + m_a) \frac{dv}{dt} \quad (1)$$

In view of this equation, the objective of the following theory is the derivation of the non-steady terms with a minimum of assumptions and experimental inputs, and to provide a practically closed solution for the instantaneous opening force.

3. CANOPY GEOMETRY

The mechanical model used for the canopy geometry is related to the O'Hara shape and is shown in Fig. 2. The drag coefficients are obtained from simulated models consisting of wire frames covered with porous parachute cloth,

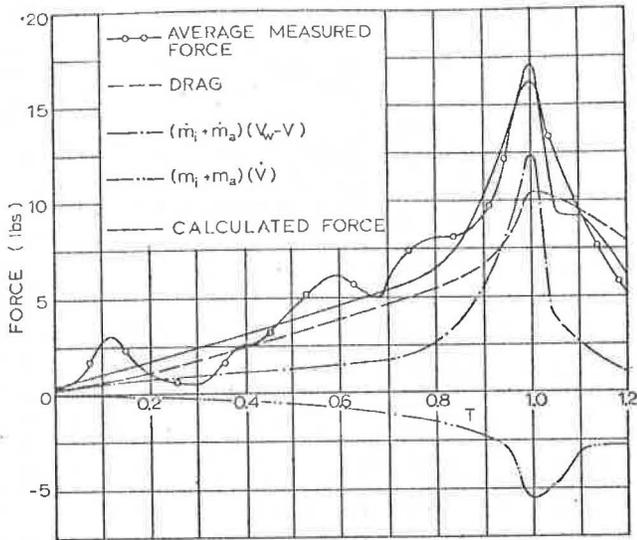


Figure 1. Measured and calculated opening forces; wind tunnel experiment with 3 ft model parachute, wind velocity 70 fps.

MIL-C-7020, Type I. More model details are described in ref. 13. The related area of the coefficients is

$$S = \pi/4 (D_o - 2L)^2.$$

Since the variation is small, a constant drag coefficient is used in the following.

During the process of inflation the projected area of the canopy increases from a very small amount to its maximum value. The relationship between projected area and time of model and full-size parachutes has been determined⁽¹⁴⁾, but in order to keep the mathematical operations of this study relatively simple, a linear area-time function is assumed. A more complex area-time relationship leads probably to a numerical computer solution. Recently the linear assumption has also been used by McEwan⁽¹⁵⁾, who found good agreement between calculations and experiments using in part a linear relationship. A linear increase of projected area provides $S_p = \pi D_p^2/4 = Kt$, and the boundary condition of $D_p = D_{max}$ at $t = t_f$ gives $K = \pi D_{max}^2/4t_f$.

Furthermore, restricting this derivation to circular flat canopies with a nominal diameter D_o , and assuming that the fully inflated parachute is hemispherical, one finds for the maximum inflated diameter $D_{max} = 2D_o/\pi$. Combining now these expressions and introducing the dimensionless time $T = t/t_f$, one obtains for the instantaneous diameter $D_p = 2D_o T^{1/2}/\pi$.

From the geometry of the developing canopy shown in Fig. 2, one obtains for the inlet diameter d the ratio

$$\frac{d}{L_s} = \frac{D_p}{L_s + \frac{D_o}{2} - \frac{\pi D_p}{4}}$$

Introducing the projected diameter as time function, and solving for the inlet diameter d , one finds

$$d = \frac{4}{\pi} \frac{L_s D_o T^{1/2}}{2L_s + D_o - D_o T^{1/2}} \quad (2)$$

4. CONTINUITY EQUATION

The terms of projected and inlet diameters are written as functions of the dimensionless time T . For determining the filling time t_f the continuity equation will be used

which, for an inflating parachute, must show an imbalance between inflowing and outflowing masses. For incompressible flow conditions this can be expressed as

$$\frac{dV}{dt} = V_{in} \frac{d^2\pi}{4} - u \frac{D_p^2\pi}{2} \quad (3)$$

where V is the instantaneous canopy volume and u the average air velocity through the porous cloth. One notices that the mass flux through the vent is neglected. This appears to be permissible since the vents of modern parachutes are relatively small.

The flux through the cloth, expressed as an average velocity u , was already used by O'Hara and postulated as a function of Δp . A later investigation⁽⁶⁾ introduced the term of effective porosity c and established functions of the type

$$c = \frac{u}{v} = f\left(\sigma, \frac{\Delta p}{\Delta p_c}\right) \quad (4)$$

In this form v is the system velocity and $\Delta p \approx \frac{1}{2}\rho v^2$. The term Δp_c is the differential pressure which, under ambient conditions, causes sonic flow through the cloth orifices.

The inflow and outflow velocities must satisfy the following boundary conditions: at $T=0$ the inflow velocity equals the system velocity, $v_{in} = v$; and for a hemispherical canopy, one has at $T=1$ the condition

$$v_{in} \frac{D_p^2\pi}{4} = u \frac{D_p^2\pi}{2} \quad \text{or} \quad v_{in} = 2u.$$

Considering the pressure distribution over a hemisphere having the porosity of regular parachute cloth one finds an approximate average value of $\Delta p = 1.2\rho v^2/2$. Using the definition of

$$u = c \left(\frac{2\Delta p}{\rho}\right)^{1/2}$$

in combination with the expression for Δp , one finds $u = 1.1 cv$, and $v_{in} = 2.2 cv$ at $T=1$.

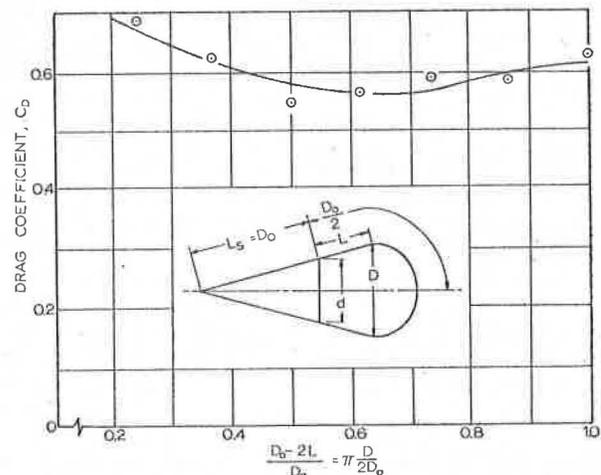


Figure 2. The mechanical model and the drag coefficient of intermediate forms.

A very simple form of the inflow function which satisfies both boundary conditions is

$$v_{in} = (1 + 2.2 cT - T) v. \quad (5)$$

The actual flow relationship is probably much more complex. However, further details are unknown at present, and eqn. 5 will be used in the following.

With the functions for d , D_p , and v_{in} , the mass balance eqn. (3) assumes the form of

$$\frac{dV}{dT} = t_f v \left[(1 + 2.2 cT - T) \left(\frac{4L_s D_o T^{1/2}}{\pi (2L_s + D_o - D_o T^{1/2})} \right)^2 - \frac{2.2 c D_o^2 T}{\pi} \right] \quad (6)$$

This equation can be simplified utilising the fact that, for most solid flat circular parachutes, the line length L_s is approximately equal to the nominal diameter D_o . Then, the squared term above becomes

$$\frac{4}{\pi} \frac{D_o T^{1/2}}{3 - T^{1/2}} \text{ and after curve fitting } \frac{2D_o}{\pi} T^{2/3} \quad (\text{Ref. 5}).$$

With these simplifications the mass balance equation is

$$\frac{dV}{dT} = t_f v \frac{D_o^2}{\pi} \{ [(1 + 2.2 cT - T) T^{1/3} - 2.2 c] T \}. \quad (7)$$

One notices that eqn. 7 contains the instantaneous velocity v which is also included in the equation of motion. Therefore, the next objective is the evaluation of eqn. 1.

5. EVALUATION OF THE EQUATION OF MOTION

An important term in the equation of motion is the air mass enclosed by the canopy. For the mechanical model, this mass is contained in a conical frustrum and a

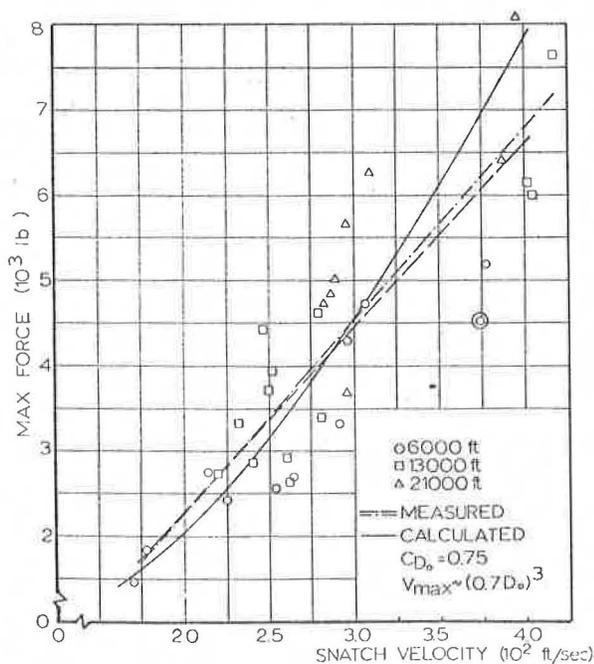


Figure 3. Data points and least mean square averages of measured and calculated opening forces, $W_s=200$ lb.

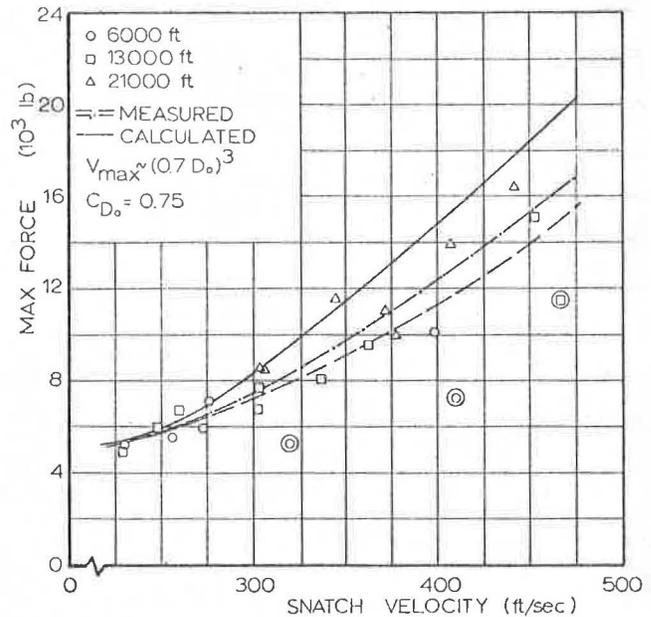


Figure 4. Data points and least mean square averages of measured and calculated opening forces, $W_s=440$ lb.

hemisphere and changes with the growth of the projected canopy diameter. The derivation of this term is shown in refs. 5 and 9, and the included mass is

$$m_i = \rho V = \frac{2\rho D_o^3}{3\pi^2} \left[1.058 - \frac{(T - 1.31)^2}{1.62} \right] \quad (8)$$

The apparent mass of fully inflated parachutes has been measured⁽⁸⁾, and the value of $m_a = 0.25 \pi R^3 \rho$ is a satisfactory approximation. However, the coefficient, 0.25, probably changes during the inflation, because in the early stages the inflating canopy is more streamlined than later. Reference 5 suggested a variation corresponding to the

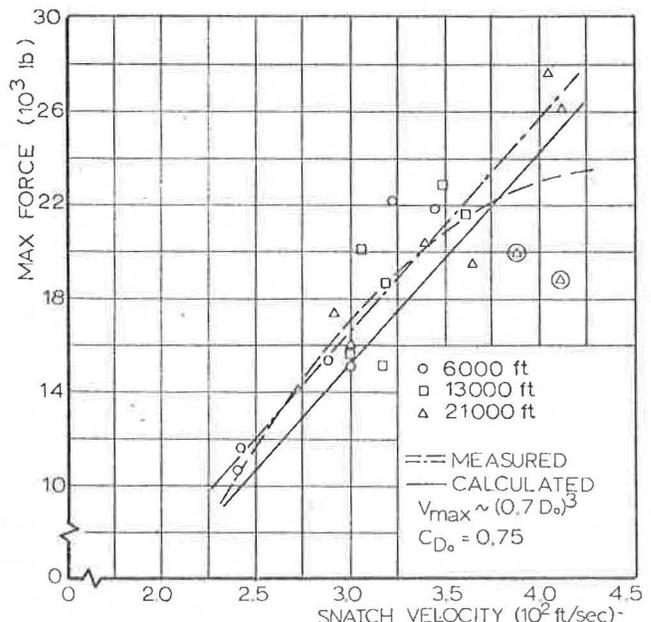


Figure 5. Data points and least mean square averages of measured and calculated opening forces, $W_s=820$ lb.

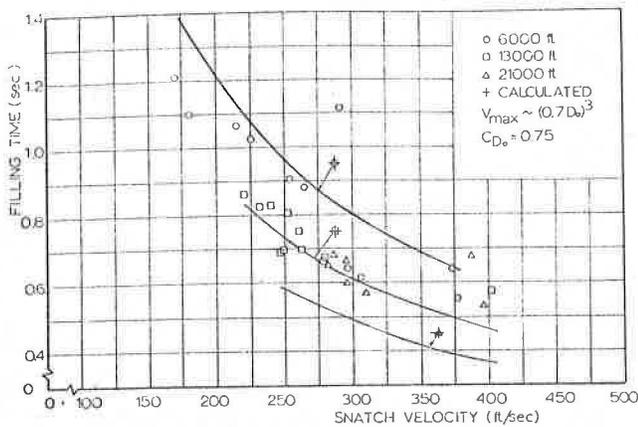


Figure 6. Measured and calculated filling times, $W_s=200$ lb.

drag area of the canopy, and with a constant drag coefficient the apparent mass may be written as $m_a = 0.25 T \pi R^3 \rho$. Introducing the radius R as function of time, one obtains

$$m_a = \frac{D_o^3}{4\pi^2} \rho T^{3/2} \quad (9)$$

The mass terms and their time derivatives can now be introduced into the equation of motion, and one finds, after some algebraic operations and simplification by curve fitting⁽⁵⁾, the equation of motion in the form of

$$2 \left[\frac{5W \cdot 10^4}{g\sigma D_o^3} + 11.25 T \right] \frac{dv}{dT} + 22.5 v = - \frac{120 (C_D S)_{\max} t_f}{D_o^3} T v^2 \quad (10)$$

This equation can be integrated and provides the system velocity

$$v = v_o \left\{ \frac{B^{v_o}}{2(11.25)^2} \left[(11.25T + A) \ln \frac{11.25T + A}{A} - 11.25T \right] + \frac{11.25T + A}{A} \right\}^{-1} \quad (11)$$

with

$$A = 5W \cdot 10^4 (g\sigma D_o^3)^{-1} \quad \text{and} \quad B = t_f 120 (C_D S)_{\max} D_o^{-5}$$

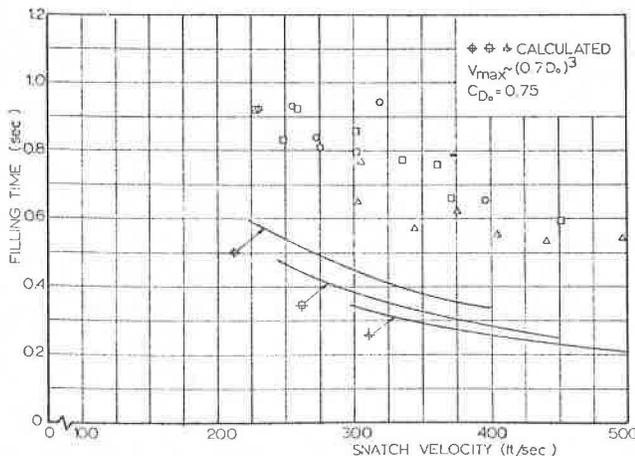


Figure 7. Measured and calculated filling times, $W_s=440$ lb.

6. DETERMINATION OF FILLING TIME AND OPENING FORCE

Combining eqns. 7 and 11 provides eqn. 12 which can easily be solved numerically for the filling time t_f , even by means of a hand calculation

$$\int_0^{v_{\max}} dV = t_f v_o \frac{D_o^2}{\pi} \int_0^1 \frac{[(1 + 2.2cT - T) T^{1/3} - 2.2c] T dT}{2(11.25)^2 \left[(11.25T + A) \ln \frac{11.25T + A}{A} - 11.25T \right] + \frac{11.25T + A}{A}} \quad (12)$$

Once the filling time t_f is known, and with dy/dt from eqn. 10, the instantaneous force can be determined from Newton's Law $F = m_s \cdot dv/dt$. Using the abbreviations A and B the force formula reads

$$F = \frac{v}{2gt_f} (vBT + 22.5) \frac{W}{A + 11.25T} \quad (13)$$

7. FILLING DISTANCE

Mueller and other authors^(1, 2, 15, 16, 17, 18) based their methods on the assumption of a constant filling distance. The analysis above indicates that under certain circumstances a constant distance of the form $L = v_o t_f$ may exist.

The conditions for its existence are that, for a given parachute, the velocity v is a unique function of T , say $v = v_o f(T)$, and that the effective porosity c remains unchanged. Eqn. 7 may then be written as

$$\int_0^{v_{\max}} dV = t_f v_o \int_0^1 f(T) \frac{D_o^2}{\pi} \{ [(1 + 2.2cT - T) T^{1/3} - 2.2c] T \} dT \quad (14)$$

Since the maximum canopy volume and the integral over T are fixed values, the term $t_f v_o$ must be constant. This product, however, represents a distance and has some similarities with the filling distance which Mueller⁽¹⁾ conceptually introduced.

A certain uniqueness of $v = v_o f(T)$ has been shown in ref. 14. Therefore, one may calculate the filling time t_f

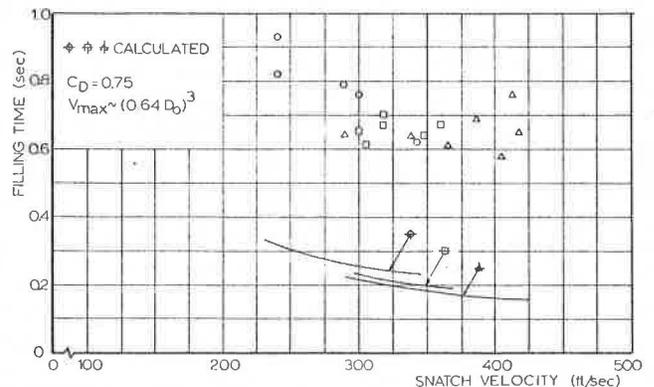


Figure 8. Measured and calculated filling times, $W_s=820$ lb.

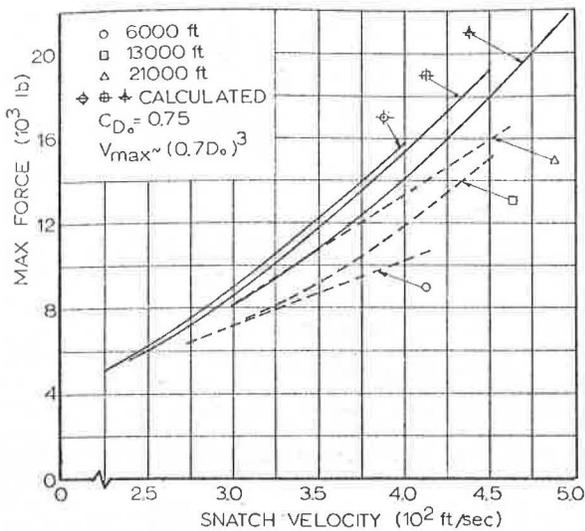


Figure 9. Measured and calculated opening forces arranged by altitude, $W_s=200$ lb.

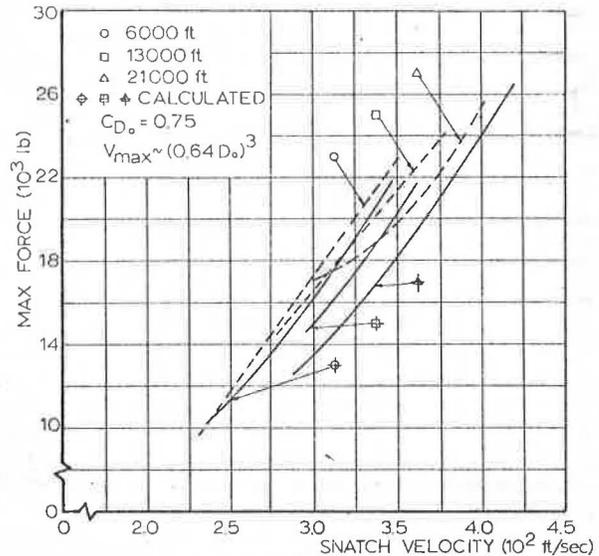


Figure 10. Measured and calculated opening forces arranged by altitude, $W_s=440$ lb.

V_s (ft/sec)	6000 ft MEASURED		V_s (ft/sec)	13000 ft MEASURED		V_s (ft/sec)	21000 ft MEASURED	
	OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)
170.0	1450	1.210	220.0	2720	0.860	282.0	4720	0.650
180.0	1830	1.100	232.0	3320	0.828	286.5	4820	0.680
214.5	2725	1.063	240.0	2850	0.805	288.0	5000	0.670
225.0	2405	1.025	247.0	4400	0.690	295.0	3690	0.660
253.0	2550	0.905	249.0	3700	0.698	295.0	5650	0.597
264.5	2685	0.885	252.0	3920	0.805	309.0	6250	0.568
291.0	3310	1.111	260.0	2920	0.750	387.0	6400	0.678
296.5	4275	0.640	262.0	2615	0.698	396.0	8050	0.530
306.0	4710	0.613	278.0	4600	0.660			
373.0	4500	0.638	280.0	3390	0.666			
377.0	5180	0.550	402.0	6150	0.570			
			404.0	5995	0.610			
			417.0	7645	0.564			
	CALCULATED			CALCULATED			CALCULATED	
175.0	1600	1.372	225.0	2580	0.815	250.0	3110	0.586
250.0	3180	0.960	250.0	3180	0.734	265.0	3500	0.554
300.0	4600	0.800	300.0	4600	0.611	300.0	4460	0.488
350.0	6120	0.685	350.0	6250	0.524	400.0	7900	0.367
400.0	7972	0.599	400.0	8165	0.459			

TABLE I
Measured and calculated opening forces and filling times of a 28 ft solid flat parachute with a 200 lb suspended weight

corresponding to an initial velocity v_0 , from the known product of filling time and velocity of the same parachute under the same altitude and approximately identical porosity conditions and proceed with the linearised theory shown above.

However, perfect uniqueness of $v = v_0 f(T)$ has not been established so far, and for the time being this mode of filling time determination should be considered an approximation.

8. MEASURED AND CALCULATED OPENING FORCES AND TIMES

The US Air Force conducted a considerable number of carefully arranged and recorded tests in which 28 ft flat circular parachutes with suspended weights of 200, 440, and 820 lb were dropped at altitudes of 6 000, 13 000, and 21 000 ft at speeds between 175 and 450 fps. The results of these tests^(10,11,12) are excellent data for checking the validity of the presented parachute opening theory.

Tables I, II, and III show measured and calculated maximum opening forces and filling times.

Figures 3 to 8 show the same data in graphical form. The drag coefficient and the maximum canopy volumes used in the calculations are indicated in the graphs, whereas the coefficients of effective porosity are listed

in Table IV. These values correspond to the snatch velocities and are taken from ref. 6. The choice of these figures is somewhat arbitrary, and this matter will be discussed later.

The measured and calculated data are summarised in curves of the least mean square averages up to and including the cubic power of the snatch velocity. One will notice that for the measured values two curves are drawn. The dashed line encompasses all field test results, whereas the dash-dot-line does not include data which appear to reflect some irregularities. In the figures the excluded data are identified by a circle about the force point.

Reviewing these figures one notices a considerable spread of the field test results, and in view of the relatively few data points and their dispersion, the least mean square average curves may be somewhat questionable.

Accepting the least mean square averages as representative information, one recognises that in most cases the field test data points are nearly as close to the least mean square curve of the calculated forces as to the one representing the field test results. It is interesting to note that the heavier loaded parachutes, Figs. 4 and 5, had several extremely low opening forces.

The measured and calculated opening times are compared in Figs. 6 to 8. One notices the same trend of all

6000 ft			13000 ft			21000 ft		
V_s (ft/sec)	MEASURED		V_s (ft/sec)	MEASURED		V_s (ft/sec)	MEASURED	
	OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)
229.0	5140	0.920	228.0	4810	0.920	303.5	18400	0.648
255.0	5490	0.930	248.0	5900	0.830	305.5	11000	0.764
272.0	5900	0.839	259.0	6620	0.920	344.0	9925	0.570
275.0	7050	0.805	302.0	6720	0.855	376.5	8420	0.620
319.0	5200	0.940	302.5	7640	0.793	406.5	16350	0.550
397.0	10100	0.650	336.5	8000	0.770	441.0	11480	0.530
408.5	7200	0.880	362.5	9420	0.755	498.5	8400	0.540
			371.5	13880	0.659			
			452.0	15050	0.590			
			466.0	11400	0.820			
CALCULATED			CALCULATED			CALCULATED		
225.0	5073	0.577	225.0	4814	0.513	300.0	7930	0.344
250.0	6263	0.519	250.0	5941	0.462	350.0	10870	0.291
275.0	7578	0.472	300.0	8563	0.384	400.0	14090	0.258
400.0	16030	0.325	350.0	11640	0.330	450.0	17930	0.227
			450.0	19230	0.257	500.0	22070	0.206

TABLE II
Measured and calculated opening forces and filling times of a 28 ft solid flat parachute with a 440 lb suspended weight

data, but the measured opening times of the tests with heavier canopy loadings are considerably longer than the calculated times. This difference can be caused by a number of reasons.

The filling time, as reported in refs. 10 to 12, is defined as the interval between the instant when the snatch force occurs and the time when the canopy reaches the same projected area as it will assume under steady state conditions.

For the calculated data the filling time is obtained from eqn. 12, which states that the canopy is inflated when the accumulated imbalance of the mass fluxes equals the maximum volume of the canopy assumed to be hemispherically shaped.

It may be argued that the canopy at the instant of $t=t_f$ is more like half an ellipsoid than a hemisphere. However, wind tunnel studies indicate that the difference of the volumes of the fully inflated canopy as an ellipsoid or as a hemisphere is negligible in view of the total canopy volume. Therefore, the difference in reported and calculated filling times cannot be caused by assuming a hemispherical canopy, but is, at least partially, a consequence of the different definitions.

Also, one notices that the filling time differences increase with canopy loading. This can be caused by short time canopy squidding, which would be very difficult to

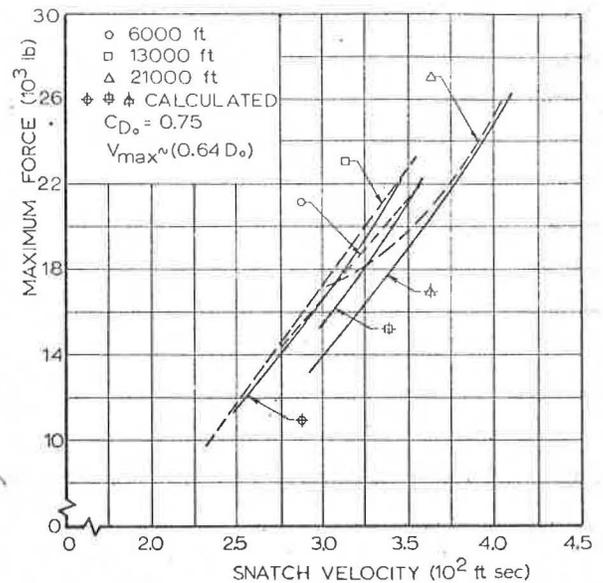


Figure 11. Measured and calculated opening forces arranged by altitude, $W_s=820$ lb.

V_s (ft/sec)	6000 ft MEASURED		V_s (ft/sec)	13000 ft MEASURED		V_s (ft/sec)	21000 ft MEASURED	
	OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)		OPENING FORCE (lb)	OPENING TIME (sec)
240.0	10500	0.93	299.0	15500	0.65	290.5	17240	0.64
241.0	11500	0.82	305.0	20000	0.61	299.0	16000	0.60
287.5	15250	0.78	316.0	15000	0.70	338.0	20200	0.64
299.5	15000	0.76	318.5	18500	0.67	364.0	19400	0.61
322.0	22000	0.62	348.0	22750	0.64	386.5	19875	0.69
343.5	21750	0.70	360.0	21500	0.67	404.5	27500	0.58
						412.0	26000	0.76
						417.8	18750	0.65

CALCULATED			CALCULATED			CALCULATED		
V_s (ft/sec)	OPENING FORCE (lb)	OPENING TIME (sec)	V_s (ft/sec)	OPENING FORCE (lb)	OPENING TIME (sec)	V_s (ft/sec)	OPENING FORCE (lb)	OPENING TIME (sec)
240.0	10410	0.308	299.0	14910	0.233	290.5	12750	0.226
241.0	10500	0.306	305.0	15360	0.227	299.0	13510	0.220
287.5	14900	0.257	316.0	16710	0.220	338.0	17040	0.192
299.5	16200	0.247	318.5	16760	0.217	364.0	19740	0.179
322.0	18730	0.230	348.0	20070	0.198	386.5	22490	0.170
343.5	21330	0.215	360.0	21460	0.191	404.5	24620	0.162
						412.0	22250	0.159
						417.8	26350	0.158

TABLE III
Measured and calculated opening forces and filling times of
a 28 ft solid flat parachute with a 800 lb suspended weight

TABLE IV
Effective porosity coefficients used in Figs. 3 through 8.

W_s (lb)	200 440	820
h (ft)	$c \cdot 10^2$	$c \cdot 10^2$
$6 \cdot 10^3$	5.05	5.4
$13 \cdot 10^3$	5.0	5.1
$21 \cdot 10^3$	4.6	4.8

record by optical means. The possibility of squidding is indicated in all three figures^(6,7,8), because of the strong dispersion of the high speed data. This is particularly pronounced in Fig. 8 which shows an increase of average filling time beginning at a velocity of 350 fps. In fact it appears that this parachute was tested close to its critical speed⁽³⁾. This interpretation agrees also with the analysis of the force recordings shown in Figs. 3 to 5. The presented theory has no provision for squidding effects and will fail, of course, in parachute deployments near the critical speed.

The question of a possible altitude effect has been asked repeatedly since Hallenbeck⁽¹⁹⁾ published the results of his high altitude drop tests. Hallenbeck's results showed a strong force increase with altitude, but the comparison was mostly based on equal indicated air speed. The data published by Berndt and DeWeese^(10,11), which are plotted versus true air speed on a log-log scale, do not indicate an altitude influence. However, when the data of refs. 10, 11, and 12 pertaining to a given altitude are summarised in least mean square averages, one obtains the information shown in Figs. 9 to 11. These curves do not include those data points which were excluded previously, and an altitude effect seems to exist.

The figures show also the calculated curves for the same altitudes. The theory predicts that the force decreases with altitude and that the altitude effect is stronger for the heavily loaded canopies. The field test data indicate a mild force increase with altitude for the lightly and moderately loaded canopies and a noticeable decrease for the canopy with high surface loading. However, the least mean square average curves are based on a relatively small number of data points, and some extremely low data points are omitted. This influences, of course, the averaged results. However, it is interesting that the field test and the theory show an unmistakable force decline for the heavily loaded canopy.

Two things seem to be certain: if there is an altitude effect, it is a weak one; and that more field test data are needed in order to establish more reliable averages.

9. SENSITIVITY OF THE THEORY TO NUMERICAL INPUTS

All conclusions based on the calculated results can, of course, be erroneous if the theory is very sensitive to the numerical inputs. The principal terms in the continuity equation and equation of motion are the maximum canopy value and the coefficients of drag and the effective porosity. Therefore, force and filling time calculations were carried out with different canopy volumina and coefficients.

In ref. 13 it was shown that the decrease of the maximum projected diameters from $0.70 D_0$ to $0.68 D_0$ caused the force increase of approximately 10% at a snatch velocity of 400 fps. The differences were less at lower velocities. The increase of the drag coefficient from $C_{D_0} = 0.7$ to 0.8 caused a force increase of approximately 3% at a velocity of 400 fps. Again, the differences were

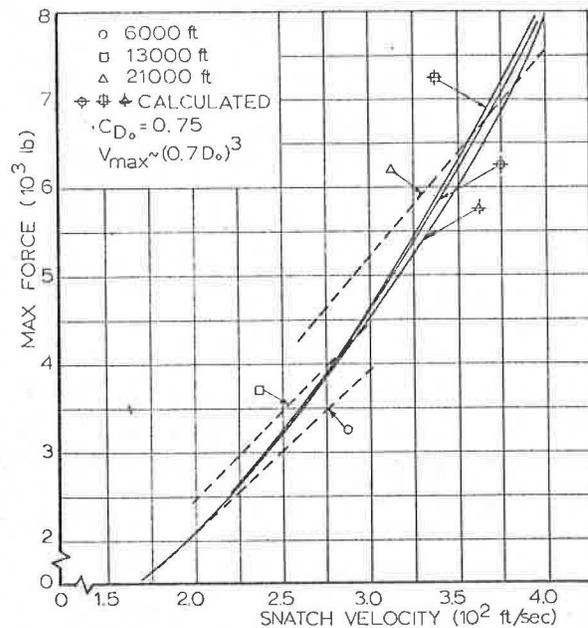


Figure 12. Measured and calculated opening forces, averaged effective porosity coefficients.

less at lower speeds. For the sensitivity check concerning the canopy volume variation, the drag coefficient was held to $C_{D_0} = 0.75$, whereas for the drag coefficient check the maximum diameter was assumed to be $D_{pmax} = 0.7 D_0$. More details of these studies are shown in ref. 13.

It appears that the uncertainties of the two parameters are not very important. It is interesting to note, however, that the theory predicts higher forces for smaller projected diameters and higher drag coefficients.

The influence of the assumed volume and the drag coefficient upon the filling time was also investigated. It was found (ref. 13) that the filling time decreases with the canopy volume and increases with the drag coefficient. Conceptually, one would expect shorter filling times for smaller volumina, whereas the lengthening of the filling time with increasing drag coefficient is surprising. However, this can be understood in view of eqn. 12 which shows in the denominator the value of B , which is a strong function of the drag coefficient.

Reference 13 shows that the filling time is more affected by volume and drag coefficient variation than the maximum forces. In view of practical applications this may be considered to be a favourable situation, since it is mostly more important to know the maximum force more accurately than the length of filling time. However, a very serious matter may arise when parachute squidding leads to infinitely long filling times.

Finally, one should investigate the influence of the effective porosity. In all calculations shown previously an effective porosity coefficient was used corresponding to the snatch velocity. For comparison all cases involving the 820 lb suspended weight were calculated with c -values, which are averages of the effective porosity pertaining to the initial and equilibrium velocities, $\bar{c} = 1/2 (c_{v_s} + c_{v_e})$. These values are listed in Table V. Figure 12 shows opening forces calculated with these porosity coefficients. Comparing Figs. 11 and 12 one notices that the forces which are calculated with the new porosity values are increased

by 5% or less, whereas the effective porosity was reduced by 17% to 25%. Also, the newly calculated forces agree better with the field test result than those shown in Fig. 11. In summary, it appears that the coefficient of effective porosity is at least as influential as the drag coefficient.

TABLE V
Averaged coefficients of effective porosity

h ft	600		1300		2100	
	c_{v_s}	c	c_{v_s}	c	c_{v_s}	c
250	0.054	0.0453	0.051	—	0.048	—
300	0.054	0.0448	0.051	0.0409	0.048	0.036
325	0.054	—	0.051	0.0404	0.048	—
350	0.054	0.0443	0.051	0.0399	0.048	0.0355
400	0.054	—	0.051	—	0.048	0.0348

Obviously the most accurate procedure would be to introduce the effective porosity as a function of velocity. However, it is questionable whether the small improvement of the force data would justify the additional effort, and it appears that the effective porosity coefficients of Table IV as well as Table V give satisfactory results.

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